

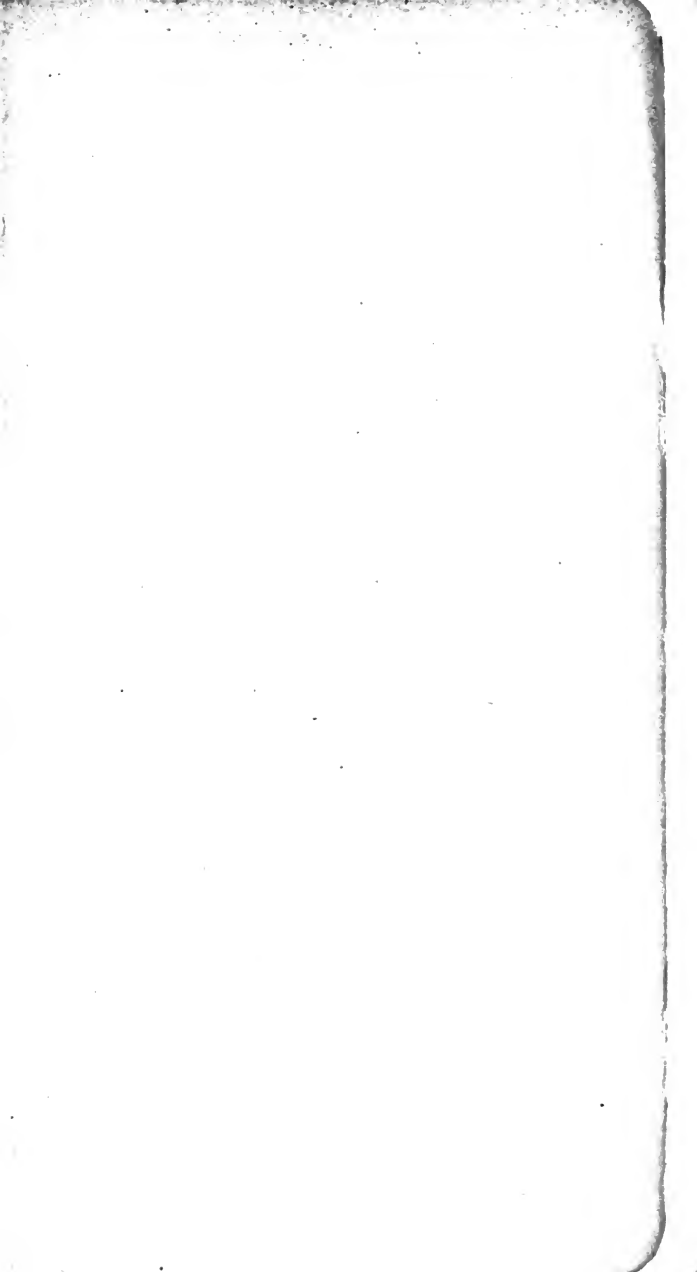


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KENT

8TH
EDITION





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A REFERENCE-BOOK OF RULES, TABLES, DATA,
AND FORMULÆ, FOR THE USE OF
ENGINEERS, MECHANICS,
AND STUDENTS.

BY

WILLIAM KENT, M.E., Sc.D.,

Consulting Engineer.

Member Amer. Soc'y Mechl. Engrs. and Amer. Inst. Mining Engrs.

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PREFACE TO THE FIRST EDITION, 1895.

MORE than twenty years ago the author began to follow the advice given by Nystrom: "Every engineer should make his own pocket-book, as he proceeds in study and practice, to suit his particular business." The manuscript pocket-book thus begun, however, soon gave place to more modern means for disposing of the accumulation of engineering facts and figures, viz., the index rerum, the scrap-book, the collection of indexed envelopes, portfolios and boxes, the card catalogue, etc. Four years ago, at the request of the publishers, the labor was begun of selecting from this accumulated mass such matter as pertained to mechanical engineering, and of condensing, digesting, and arranging it in form for publication. In addition to this, a careful examination was made of the transactions of engineering societies, and of the most important recent works on mechanical engineering, in order to fill gaps that might be left in the original collection, and insure that no important facts had been overlooked.

Some ideas have been kept in mind during the preparation of the Pocket-book that will, it is believed, cause it to differ from other works of its class. In the first place it was considered that the field of mechanical engineering was so great, and the literature of the subject so vast, that as little space as possible should be given to subjects which especially belong to civil engineering. While the mechanical engineer must continually deal with problems which belong properly to civil engineering, this latter branch is so well covered by Trautwine's "Civil Engineer's Pocket-book" that any attempt to treat it exhaustively would not only fill no "long-felt want," but would occupy space which should be given to mechanical engineering.

Another idea prominently kept in view by the author has been that he would not assume the position of an "authority" in giving rules and formulæ for designing, but only that of compiler, giving not only the name of the originator of the rule, where it was known, but also the volume and page from which it was taken, so that its derivation may be traced when desired. When different formulæ for the same problem have been found they have been given in contrast, and in many cases examples have been calculated by each to show the difference between them. In some cases these differences are quite remarkable, as will be seen under Safety-valves and Crank-pins. Occasionally the study of these differences has led to the author's devising a new formula, in which case the derivation of the formula is given.

Much attention has been paid to the abstracting of data of experiments from recent periodical literature, and numerous references to other data are given. In this respect the present work will be found to differ from other Pocket-books.

The author desires to express his obligation to the many persons who have assisted him in the preparation of the work, to manufacturers who have furnished their catalogues and given permission for the use of their tables, and to many engineers who have contributed original data and tables. The names of these persons are mentioned in their proper places in the text, and in all cases it has been endeavored to give credit to whom credit is due. The thanks of the author are also due to the following gentlemen who have given assistance in revising manuscript or proofs of the sections named: Prof. De Volson Wood, mechanics and turbines; Mr. Frank Richards, compressed air; Mr. Alfred R. Wolff, windmills; Mr. Alex. C. Humphreys, illuminating gas; Mr. Albert E. Mitchell, locomotives; Prof. James E. Denton, refrigerating-machinery; Messrs. Joseph Wetzler and Thomas W. Varley, electrical engineering; and Mr. Walter S. Dix, for valuable contributions on several subjects, and suggestions as to their treatment.

WILLIAM KENT.

PREFACE TO THE EIGHTH EDITION.

SEPTEMBER, 1910.

DURING the first ten years following the issue of the first edition of this book, in 1895, the attempt was made to keep it up to date by the method of cutting out pages and paragraphs, inserting new ones in their places, by inserting new pages lettered a, b, c, etc., and by putting some new matter in an appendix. In this way the book passed to its 7th edition in October, 1904. After 50,000 copies had been printed it was found that the electro-typed plates were beginning to wear out, so that extensive resetting of type would soon be necessary. The advances in engineering practice also had been so great that it was evident that many chapters required to be entirely rewritten. It was therefore determined to make a thorough revision of the book, and to reset the type throughout. This has now been accomplished after four years of hard labor. The size of the book has increased over 300 pages, in spite of all efforts to save space by condensation and elision of much of the old matter and by resetting many of the tables and formulæ in shorter form. A new style of type for the tables has been designed for the book, which is believed to be much more easily read than the old.

The thanks of the author are due to many manufacturers who have furnished new tables of materials and machines, and to many engineers who have made valuable contributions and helpful suggestions. He is especially indebted to his son, Robert Thurston Kent, M.E., who has done the work of revising manufacturers' tables of materials and has done practically all of the revising of the subjects of Compressed Air, Fans and Blowers, Hoisting and Conveying, and Machine Shop.

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 Col. Coll. Qly. Columbia College Quarterly.
 El. Rev. Electrical Review.
 El. World. Electrical World and Engineer.
 Engg. Engineering (London).
 Eng. News. Engineering News.
 Eng. Rec. Engineering Record.
 Engr. The Engineer (London).
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 Indust. Eng. Industrial Engineering.
 Jour. A. C. I. W. Journal of American Charcoal Iron Workers' Association.
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MATHEMATICS.

Greek Letters.

| | | | | | | | | | | | |
|----------|------------|---------|-----------|-----------|--------|----------|------------|---------|------------|------------|---------|
| A | α | Alpha | H | η | Eta | N | ν | Nu | T | τ | Tau |
| B | β | Beta | Θ | θ | Theta | Ξ | ξ | Xi | Υ | υ | Upsilon |
| Γ | γ | Gamma | I | ι | Iota | O | \omicron | Omicron | Φ | ϕ | Phi |
| Δ | δ | Delta | K | κ | Kappa | Π | π | Pi | χ | χ | Chi |
| E | ϵ | Epsilon | Λ | λ | Lambda | P | ρ | Rho | Ψ | ψ | Psi |
| Z | ζ | Zeta | M | μ | Mu | Σ | σ | Sigma | Ω | ω | Omega |

Arithmetical and Algebraical Signs and Abbreviations.

+ plus (addition).
 + positive.
 - minus (subtraction).
 - negative.
 \pm plus or minus.
 \mp minus or plus.
 = equals.
 \times multiplied by.
 ab or $a.b = a \times b$.
 \div divided by.
 $/$ divided by.
 $\frac{a}{b} = a/b = a \div b$. $15-16 = \frac{15}{16}$.
 $0.2 = \frac{2}{10}$; $0.002 = \frac{2}{1000}$.
 $\sqrt{\quad}$ square root.
 $\sqrt[3]{\quad}$ cube root.
 $\sqrt[4]{\quad}$ 4th root.
 $:$ is to, $::$ so is, $:$ to (proportion).
 $2:4::3:6$, 2 is to 4 as 3 is to 6.
 $:$ ratio; divided by.
 $2:4$, ratio of 2 to 4 = $2/4$.
 \therefore therefore.
 $>$ greater than.
 $<$ less than.
 \square square.
 \bigcirc round.
 $^{\circ}$ degrees, arc or thermometer.
 $'$ minutes or feet.
 $''$ seconds or inches.
 $'''$ accents to distinguish letters, as a' , a'' , a''' .
 a_1, a_2, a_3, a_b, a_e , read a sub 1, a sub b , etc.
 $() [] \{ \}$ — parenthesis, brackets, braces, vinculum; denoting that the numbers enclosed are to be taken together; as,
 $(a+b)c = 4+3 \times 5 = 35$.
 a^2, a^3 , a squared, a cubed.
 a^n , a raised to the n th power.
 $a^{\frac{1}{2}} = \sqrt{a^2}$, $a^{\frac{1}{3}} = \sqrt[3]{a^3}$.
 $a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$.
 $10^9 = 10$ to the 9th power = 1,000,000,000.
 $\sin a$ = the sine of a .
 $\sin^{-1} a$ = the arc whose sine is a .
 $\sin a^{-1} = \frac{1}{\sin a}$.
 \log = logarithm.
 \log_e or hyp log = hyperbolic logarithm.
 $\%$ per cent.
 \triangle angle.

\angle right angle.
 \perp perpendicular to.
 \sin , sine.
 \cos , cosine.
 \tan , tangent.
 \sec , secant.
 versin , versed sine.
 \cot , cotangent.
 cosec , cosecant.
 covers , co-versed sine.
 In Algebra, the first letters of the alphabet, a, b, c, d , etc., are generally used to denote known quantities, and the last letters, w, x, y, z , etc., unknown quantities.
Abbreviations and Symbols commonly used.
 d , differential (in calculus).
 \int , integral (in calculus).
 \int_a^b , integral between limits a and b .
 Δ , delta, difference.
 Σ , sigma, sign of summation.
 π , pi, ratio of circumference of circle to diameter = 3.14159.
 g , acceleration due to gravity = 32.16 ft. per second per second.
Abbreviations frequently used in this Book.
 $L.$, l., length in feet and inches.
 $B.$, b., breadth in feet and inches.
 $D.$, d., depth or diameter.
 $H.$, h., height, feet and inches.
 $T.$, t., thickness or temperature.
 $V.$, v., velocity.
 $F.$, force, or factor of safety.
 $f.$, coefficient of friction.
 $E.$, coefficient of elasticity.
 $R.$, r., radius.
 $W.$, w., weight.
 $P.$, p., pressure or load.
 $H.P.$, horse-power.
 $I.H.P.$, indicated horse-power.
 $B.H.P.$, brake horse-power.
 $h.$, p., high pressure.
 $i.$ p., intermediate pressure.
 $l.$ p., low pressure.
 $A.W.G.$, American Wire Gauge (Brown & Sharpe).
 $B.W.G.$, Birmingham Wire Gauge.
 $r.$ p. m., or revs. per min., revolutions per minute.
 $Q.$ = quantity, or volume.

ARITHMETIC.

The user of this book is supposed to have had a training in arithmetic as well as in elementary algebra. Only those rules are given here which are apt to be easily forgotten.

GREATEST COMMON MEASURE, OR GREATEST COMMON DIVISOR OF TWO NUMBERS.

Rule. — Divide the greater number by the less; then divide the divisor by the remainder, and so on, dividing always the last divisor by the last remainder, until there is no remainder, and the last divisor is the greatest common measure required.

LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

Rule. — Divide the given numbers by any number that will divide the greatest number of them without a remainder, and set the quotients with the undivided numbers in a line beneath.

Divide the second line as before, and so on, until there are no two numbers that can be divided; then the continued product of the divisors, last quotients, and undivided numbers will give the multiple required.

FRACTIONS.

To reduce a common fraction to its lowest terms. — Divide both terms by their greatest common divisor: $\frac{30}{52} = \frac{3}{4}$.

To change an improper fraction to a mixed number. — Divide the numerator by the denominator; the quotient is the whole number, and the remainder placed over the denominator is the fraction: $\frac{30}{4} = 7\frac{3}{4}$.

To change a mixed number to an improper fraction. — Multiply the whole number by the denominator of the fraction; to the product add the numerator; place the sum over the denominator: $17\frac{7}{8} = \frac{155}{8}$.

To express a whole number in the form of a fraction with a given denominator. — Multiply the whole number by the given denominator, and place the product over that denominator: $13 = \frac{30}{3}$.

To reduce a compound to a simple fraction, also to multiply fractions. — Multiply the numerators together for a new numerator and the denominators together for a new denominator:

$$\frac{2}{3} \text{ of } \frac{4}{3} = \frac{8}{9}, \text{ also } \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}.$$

To reduce a complex to a simple fraction. — The numerator and denominator must each first be given the form of a simple fraction; then multiply the numerator of the upper fraction by the denominator of the lower for the new numerator, and the denominator of the upper by the numerator of the lower for the new denominator:

$$\frac{7/8}{13/4} = \frac{7/8}{7/4} = \frac{28}{56} = \frac{1}{2}.$$

To divide fractions. — Reduce both to the form of simple fractions, invert the divisor, and proceed as in multiplication:

$$\frac{3}{4} \div 1\frac{1}{4} = \frac{3}{4} \div \frac{5}{4} = \frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}.$$

Cancellation of fractions. — In compound or multiplied fractions, divide any numerator and any denominator by any number which will divide them both without remainder, striking out the numbers thus divided and setting down the quotients in their stead.

To reduce fractions to a common denominator. — Reduce each fraction to the form of a simple fraction; then multiply each numerator

by all the denominators except its own for the new numerator, and all the denominators together for the common denominator:

$$\frac{1}{2}, \frac{1}{3}, \frac{3}{7} = \frac{21}{42}, \frac{14}{42}, \frac{18}{42}$$

To add fractions. — Reduce them to a common denominator, then add the numerators and place their sum over the common denominator:

$$\frac{1}{2} + \frac{1}{3} + \frac{3}{7} = \frac{21 + 14 + 18}{42} = \frac{53}{42} = 1 \frac{11}{42}.$$

To subtract fractions. — Reduce them to a common denominator, subtract the numerators and place the difference over the common denominator:

$$\frac{1}{2} - \frac{3}{7} = \frac{7 - 6}{14} = \frac{1}{14}.$$

DECIMALS.

To add decimals. — Set down the figures so that the decimal points are one above the other, then proceed as in simple addition: $18.75 + 0.012 = 18.762$.

To subtract decimals. — Set down the figures so that the decimal points are one above the other, then proceed as in simple subtraction: $18.75 - 0.012 = 18.738$.

To multiply decimals. — Multiply as in multiplication of whole numbers, then point off as many decimal places as there are in multiplier and multiplicand taken together: $1.5 \times 0.02 = .030 = 0.03$.

To divide decimals. — Divide as in whole numbers, and point off in the quotient as many decimal places as those in the dividend exceed those in the divisor. Ciphers must be added to the dividend to make its decimal places at least equal those in the divisor, and as many more as it is desired to have in the quotient: $1.5 \div 0.25 = 6$. $0.1 \div 0.3 = 0.10000 \div 0.3 = 0.3333 +$.

Decimal Equivalents of Fractions of One Inch.

| | | | | | | | |
|-------|---------|-------|---------|-------|---------|-------|---------|
| 1-64 | .015625 | 17-64 | .265625 | 33-64 | .515625 | 49-64 | .765625 |
| 1-32 | .03125 | 9-32 | .28125 | 17-32 | .53125 | 25-32 | .78125 |
| 3-64 | .046875 | 19-64 | .296875 | 35-64 | .546875 | 51-64 | .796875 |
| 1-16 | .0625 | 5-16 | .3125 | 9-16 | .5625 | 13-16 | .8125 |
| 5-64 | .078125 | 21-64 | .328125 | 37-64 | .578125 | 53-64 | .828125 |
| 3-32 | .09375 | 11-32 | .34375 | 19-32 | .59375 | 27-32 | .84375 |
| 7-64 | .109375 | 23-64 | .359375 | 39-64 | .609375 | 55-64 | .859375 |
| 1-8 | .125 | 3-8 | .375 | 5-8 | .625 | 7-8 | .875 |
| 9-64 | .140625 | 25-64 | .390625 | 41-64 | .640625 | 57-64 | .890625 |
| 5-32 | .15625 | 13-32 | .40625 | 21-32 | .65625 | 29-32 | .90625 |
| 11-64 | .171875 | 27-64 | .421875 | 43-64 | .671875 | 59-64 | .921875 |
| 3-16 | .1875 | 7-16 | .4375 | 11-16 | .6875 | 15-16 | .9375 |
| 13-64 | .203125 | 29-64 | .453125 | 45-64 | .703125 | 61-64 | .953125 |
| 7-32 | .21875 | 15-32 | .46875 | 23-32 | .71875 | 31-32 | .96875 |
| 15-64 | .234375 | 31-64 | .484375 | 47-64 | .734375 | 63-64 | .984375 |
| 1-4 | .25 | 1-2 | .50 | 3-4 | .75 | 1 | 1. |

To convert a common fraction into a decimal. — Divide the numerator by the denominator, adding to the numerator as many ciphers prefixed by a decimal point as are necessary to give the number of decimal places desired in the result: $\frac{1}{3} = 1.0000 \div 3 = 0.3333 +$.

To convert a decimal into a common fraction. — Set down the decimal as a numerator, and place as the denominator 1 with as many ciphers annexed as there are decimal places in the numerator; erase the

Product of Fractions Expressed in Decimals.

| 0 | 1 | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{3}{16}$ | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{5}{8}$ | $\frac{11}{16}$ | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{7}{8}$ | $\frac{15}{16}$ | 1 |
|-----------------|-------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|---------------|-----------------|---------------|-----------------|---------------|-----------------|-------|
| $\frac{1}{16}$ | .0625 | .0039 | | | | | | | | | | | | | | | |
| $\frac{1}{8}$ | .1250 | .0078 | .0156 | | | | | | | | | | | | | | |
| $\frac{3}{16}$ | .1875 | .0117 | .0234 | .0352 | | | | | | | | | | | | | |
| $\frac{1}{4}$ | .2500 | .0156 | .0313 | .0469 | .0625 | | | | | | | | | | | | |
| $\frac{5}{16}$ | .3125 | .0195 | .0391 | .0586 | .0781 | .0977 | | | | | | | | | | | |
| $\frac{3}{8}$ | .3750 | .0234 | .0469 | .0703 | .0937 | .1172 | .1406 | | | | | | | | | | |
| $\frac{7}{16}$ | .4375 | .0273 | .0547 | .0820 | .1093 | .1367 | .1641 | .1914 | | | | | | | | | |
| $\frac{1}{2}$ | .5000 | .0313 | .0625 | .0938 | .1250 | .1562 | .1875 | .2188 | .2500 | | | | | | | | |
| $\frac{9}{16}$ | .5625 | .0352 | .0703 | .1055 | .1406 | .1758 | .2109 | .2461 | .2813 | .3164 | | | | | | | |
| $\frac{5}{8}$ | .6250 | .0391 | .0781 | .1172 | .1562 | .1953 | .2344 | .2734 | .3125 | .3516 | .3906 | | | | | | |
| $\frac{11}{16}$ | .6875 | .0430 | .0859 | .1289 | .1719 | .2148 | .2578 | .3008 | .3438 | .3867 | .4297 | .4727 | | | | | |
| $\frac{3}{4}$ | .7500 | .0469 | .0938 | .1406 | .1875 | .2344 | .2813 | .3281 | .3750 | .4219 | .4688 | .5156 | .5625 | | | | |
| $\frac{13}{16}$ | .8125 | .0508 | .1016 | .1523 | .2031 | .2539 | .3047 | .3555 | .4063 | .4570 | .5078 | .5586 | .6094 | .6601 | | | |
| $\frac{7}{8}$ | .8750 | .0547 | .1094 | .1641 | .2187 | .2734 | .3281 | .3828 | .4375 | .4922 | .5469 | .6016 | .6563 | .7109 | .7656 | | |
| $\frac{15}{16}$ | .9375 | .0586 | .1172 | .1758 | .2344 | .2930 | .3516 | .4102 | .4688 | .5273 | .5859 | .6445 | .7031 | .7617 | .8203 | .8789 | |
| 1 | 1.000 | .0625 | .1250 | .1875 | .2500 | .3125 | .3750 | .4375 | .5000 | .5625 | .6250 | .6875 | .7500 | .8125 | .8750 | .9375 | 1.000 |

decimal point in the numerator, and reduce the fraction thus formed to its lowest terms:

$$0.25 = \frac{25}{100} = \frac{1}{4}; 0.3333 = \frac{3333}{10000} = \frac{1}{3}, \text{ nearly.}$$

To reduce a recurring decimal to a common fraction. — Subtract the decimal figures that do not recur from the whole decimal including one set of recurring figures; set down the remainder as the numerator of the fraction, and as many nines as there are recurring figures, followed by as many ciphers as there are non-recurring figures, in the denominator. Thus:

$$\begin{array}{r} \text{Subtract} \quad 0.79054054, \text{ the recurring figures being } 054. \\ \quad \quad \quad \underline{79} \\ \quad \quad \quad 78975 \\ \quad \quad \quad \underline{99900} \end{array} = (\text{reduced to its lowest terms}) \quad \frac{117}{148}$$

COMPOUND OR DENOMINATE NUMBERS.

Reduction descending. — To reduce a compound number to a lower denomination. Multiply the number by as many units of the lower denomination as makes one of the higher.

3 yards to inches: $3 \times 36 = 108$ inches.

0.04 square feet to square inches: $.04 \times 144 = 5.76$ sq. in.

If the given number is in more than one denomination proceed in steps from the highest denomination to the next lower, and so on to the lowest, adding in the units of each denomination as the operation proceeds.

3 yds. 1 ft. 7 in. to inches: $3 \times 3 = 9, + 1 = 10, 10 \times 12 = 120, + 7 = 127$ in.

Reduction ascending. — To express a number of a lower denomination in terms of a higher, divide the number by the number of units of the lower denomination contained in one of the next higher; the quotient is in the higher denomination, and the remainder, if any, in the lower.

127 inches to higher denomination.

$127 \div 12 = 10$ feet + 7 inches; $10 \text{ feet} \div 3 = 3$ yards + 1 foot.

Ans. 3 yds. 1 ft. 7 in.

To express the result in decimals of the higher denomination, divide the given number by the number of units of the given denomination contained in one of the required denomination, carrying the result to as many places of decimals as may be desired.

127 inches to yards: $127 \div 36 = 3.5277 +$ yards.

Decimals of a Foot Equivalent to Inches and Fractions of an Inch.

| Inches | 0 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ |
|--------|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 0 | 0 | .01042 | .02083 | .03125 | .04167 | .05208 | .06250 | .07292 |
| 1 | .0833 | .0938 | .1042 | .1146 | .1250 | .1354 | .1458 | .1563 |
| 2 | .1667 | .1771 | .1875 | .1979 | .2083 | .2188 | .2292 | .2396 |
| 3 | .2500 | .2604 | .2708 | .2813 | .2917 | .3021 | .3125 | .3229 |
| 4 | .3333 | .3438 | .3542 | .3646 | .3750 | .3854 | .3958 | .4063 |
| 5 | .4167 | .4271 | .4375 | .4479 | .4583 | .4688 | .4792 | .4896 |
| 6 | .5000 | .5104 | .5208 | .5313 | .5417 | .5521 | .5625 | .5729 |
| 7 | .5833 | .5938 | .6042 | .6146 | .6250 | .6354 | .6458 | .6563 |
| 8 | .6667 | .6771 | .6875 | .6979 | .7083 | .7188 | .7292 | .7396 |
| 9 | .7500 | .7604 | .7708 | .7813 | .7917 | .8021 | .8125 | .8229 |
| 10 | .8333 | .8438 | .8542 | .8646 | .8750 | .8854 | .8958 | .9063 |
| 11 | .9167 | .9271 | .9375 | .9479 | .9583 | .9688 | .9792 | .9896 |

RATIO AND PROPORTION.

Ratio is the relation of one number to another, as obtained by dividing the first number by the second. Synonymous with quotient.

Ratio of 2 to 4, or $2 : 4 = 2/4 = 1/2$.

Ratio of 4 to 2, or $4 : 2 = 2$.

Proportion is the equality of two ratios. Ratio of 2 to 4 equals ratio of 3 to 6, $2/4 = 3/6$; expressed thus, $2 : 4 :: 3 : 6$; read, 2 is to 4 as 3 is to 6. The first and fourth terms are called the extremes or outer terms, the second and third the means or inner terms.

The product of the means equals the product of the extremes:

$$2 : 4 :: 3 : 6; \quad 2 \times 6 = 12; \quad 3 \times 4 = 12.$$

Hence, given the first three terms to find the fourth, multiply the second and third terms together and divide by the first.

$$2 : 4 :: 3 : \text{what number?} \quad \text{Ans. } \frac{4 \times 3}{2} = 6.$$

Algebraic expression of proportion. — $a : b :: c : d; \quad \frac{a}{b} = \frac{c}{d}; \quad ad = bc;$

from which $a = \frac{bc}{d}; \quad d = \frac{bc}{a}; \quad b = \frac{ad}{c}; \quad c = \frac{ad}{b}.$

From the above equations may also be derived the following:

$$\begin{array}{lll} b : a :: d : c & a + b : a :: c + d : c & a + b : a - b :: c + d : c - d \\ a : c :: b : d & a + b : b :: c + d : d & a^n : b^n :: c^n : d^n \\ a : b = c : d & a - b : b :: c - d : d & \sqrt[n]{a} : \sqrt[n]{b} :: \sqrt[n]{c} : \sqrt[n]{d} \\ & a - b : a :: c - d : c & \end{array}$$

Mean proportional between two given numbers, 1st and 2d, is such a number that the ratio which the first bears to it equals the ratio which it bears to the second. Thus, $2 : 4 :: 4 : 8$; 4 is a mean proportional between 2 and 8. To find the mean proportional between two numbers, extract the square root of their product.

$$\text{Mean proportional of 2 and 8} = \sqrt{2 \times 8} = 4.$$

Single Rule of Three; or, finding the fourth term of a proportion when three terms are given. — Rule, as above, when the terms are stated in their proper order, multiply the second by the third and divide by the first. The difficulty is to state the terms in their proper order. The term which is of the same kind as the required or fourth term is made the third; the first and second must be like each other in kind and denomination. To determine which is to be made second and which first requires a little reasoning. If an inspection of the problem shows that the answer should be greater than the third term, then the greater of the other two given terms should be made the second term — otherwise the first. Thus, 3 men remove 54 cubic feet of rock in a day; how many men will remove in the same time 10 cubic yards? The answer is to be men — make men third term; the answer is to be more than three men, therefore make the greater quantity, 10 cubic yards, the second term; but as it is not the same denomination as the other term it must be reduced, = 270 cubic feet. The proportion is then stated:

$$54 : 270 :: 3 : x \text{ (the required number); } x = \frac{3 \times 270}{54} = 15 \text{ men.}$$

The problem is more complicated if we increase the number of given terms. Thus, in the above question, substitute for the words "in the same time" the words "in 3 days." First solve it as above, as if the work were to be done in the same time; then make another proportion, stating it thus: If 15 men do it in the same time, it will take fewer men to do it in 3 days; make 1 day the second term and 3 days the first term. $3 : 1 :: 15 \text{ men} : 5 \text{ men.}$

Compound Proportion, or Double Rule of Three. — By this rule are solved questions like the one just given, in which two or more statings are required by the single rule of three. In it, as in the single rule, there is one third term, which is of the same kind and denomination as the fourth or required term, but there may be two or more first and second terms. Set down the third term, take each pair of terms of the same kind separately, and arrange them as first and second by the same reasoning as is adopted in the single rule of three, making the greater of the pair the second if this pair considered alone should require the answer to be greater.

Set down all the first terms one under the other, and likewise all the second terms. Multiply all the first terms together and all the second terms together. Multiply the product of all the second terms by the third term, and divide this product by the product of all the first terms. Example: If 3 men remove 4 cubic yards in one day, working 12 hours a day, how many men working 10 hours a day will remove 20 cubic yards in 3 days?

| | | | |
|-------|------|----|--|
| Yards | 4 : | 20 | |
| Days | 3 : | 1 | |
| Hours | 10 : | 12 | |

Products 120 : 240 :: 3 : 6 men. Ans.

To abbreviate by cancellation, any one of the first terms may cancel either the third or any of the second terms; thus, 3 in first cancels 3 in third, making it 1, 10 cancels into 20 making the latter 2, which into 4 makes it 2, which into 12 makes it 6, and the figures remaining are only 1 : 6 :: 1 : 6.

INVOLUTION, OR POWERS OF NUMBERS.

Involution is the continued multiplication of a number by itself a given number of times. The number is called the root, or first power, and the products are called powers. The second power is called the square and the third power the cube. The operation may be indicated without being performed by writing a small figure called the *index* or *exponent* to the right of and a little above the root; thus, 3^3 = cube of 3, = 27.

To multiply two or more powers of the same number, add their exponents; thus, $2^2 \times 2^3 = 2^5$, or $4 \times 8 = 32 = 2^5$.

To divide two powers of the same number, subtract their exponents; thus, $2^3 \div 2^2 = 2^1 = 2$; $2^2 \div 2^4 = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$. The exponent may thus be negative. $2^3 \div 2^3 = 2^0 = 1$, whence the zero power of any number = 1. The first power of a number is the number itself. The exponent may be fractional, as $2^{\frac{1}{2}}$, $2^{\frac{1}{3}}$, which means that the root is to be raised to a power whose exponent is the numerator of the fraction, and the root whose sign is the denominator is to be extracted (see Evolution). The exponent may be a decimal, as $2^{0.5}$, $2^{0.6}$; read, two to the five-tenths power, two to the one and five-tenths power. These powers are solved by means of Logarithms (which see).

First Nine Powers of the First Nine Numbers.

| 1st Power. | 2d Power. | 3d Power. | 4th Power. | 5th Power. | 6th Power. | 7th Power. | 8th Power. | 9th Power. |
|------------|-----------|-----------|------------|------------|------------|------------|------------|------------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 |
| 3 | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 |
| 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 | 65536 | 262144 |
| 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 | 390625 | 1953125 |
| 6 | 36 | 216 | 1296 | 7776 | 46656 | 279936 | 1679616 | 10077696 |
| 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 | 5764801 | 40353607 |
| 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 |
| 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 |

The First Forty Powers of 2.

| Power. | Value. | Power. | Value. | Power. | Value. | Power. | Value. | Power. | Value. |
|--------|--------|--------|--------|--------|----------|--------|-------------|--------|---------------|
| 0 | 1 | 9 | 512 | 18 | 262144 | 27 | 134217728 | 36 | 68719476736 |
| 1 | 2 | 10 | 1024 | 19 | 524288 | 28 | 268435456 | 37 | 137438953472 |
| 2 | 4 | 11 | 2048 | 20 | 1048576 | 29 | 536870912 | 38 | 274877906944 |
| 3 | 8 | 12 | 4096 | 21 | 2097152 | 30 | 1073741824 | 39 | 549755813888 |
| 4 | 16 | 13 | 8192 | 22 | 4194304 | 31 | 2147483648 | 40 | 1099511627776 |
| 5 | 32 | 14 | 16384 | 23 | 8388608 | 32 | 4294967296 | | |
| 6 | 64 | 15 | 32768 | 24 | 16777216 | 33 | 8589934592 | | |
| 7 | 128 | 16 | 65536 | 25 | 33554432 | 34 | 17179869184 | | |
| 8 | 256 | 17 | 131072 | 26 | 67108864 | 35 | 34359738368 | | |

EVOLUTION.

Evolution is the finding of the root (or extracting the root) of any number the power of which is given.

The sign $\sqrt{\quad}$ indicates that the square root is to be extracted: $\sqrt[3]{\quad}$ $\sqrt[4]{\quad}$ $\sqrt[n]{\quad}$, the cube root, 4th root, nth root.

A fractional exponent with 1 for the numerator of the fraction is also used to indicate that the operation of extracting the root is to be performed; thus, $2^{\frac{1}{2}}$, $2^{\frac{1}{3}}$ = $\sqrt{2}$, $\sqrt[3]{2}$.

When the power of a number is indicated, the involution not being performed, the extraction of any root of that power may also be indicated by dividing the index of the power by the index of the root, indicating the division by a fraction. Thus, extract the square root of the 6th power of 2:

$$\sqrt{2^6} = 2^{\frac{6}{2}} = 2^3 = 2^3 = 8.$$

The 6th power of 2, as in the table above, is 64; $\sqrt{64} = 8$.

Difficult problems in evolution are performed by logarithms, but the square root and the cube root may be extracted directly according to the rules given below. The 4th root is the square root of the square root. The 6th root is the cube root of the square root, or the square root of the cube root; the 9th root is the cube root of the cube root; etc.

To Extract the Square Root. — Point off the given number into periods of two places each, beginning with units. If there are decimals, point these off likewise, beginning at the decimal point, and supplying as many ciphers as may be needed. Find the greatest number whose square is less than the first left-hand period, and place it as the first figure in the quotient. Subtract its square from the left-hand period, and to the remainder annex the two figures of the second period for a dividend. Double the first figure of the quotient for a partial divisor; find how many times the latter is contained in the dividend exclusive of the right-hand figure, and set the figure representing that number of times as the second figure in the quotient, and annex it to the right of the partial divisor, forming the complete divisor. Multiply this divisor by the second figure in the quotient and subtract the product from the dividend. To the remainder bring down the next period and proceed as before, in each case doubling the figures in the root already found to obtain the trial divisor. Should the product of the second figure in the root by the completed divisor be greater than the dividend, erase the second figure both from the quotient and from the divisor, and substitute the next smaller figure, or one small enough to make the product of the second figure by the divisor less than or equal to the dividend.

SQUARE ROOT.

$$\begin{array}{r}
 3.1415926536 \overline{) 1.77245 +} \\
 \underline{1} \\
 27 \overline{) 214} \\
 \underline{189} \\
 347 \overline{) 2515} \\
 \underline{2429} \\
 3542 \overline{) 8692} \\
 \underline{7084} \\
 35444 \overline{) 160865} \\
 \underline{141776} \\
 354485 \overline{) 1908936} \\
 \underline{1772425}
 \end{array}$$

CUBE ROOT.

$$\begin{array}{r}
 1,881,365,963,625 \overline{) 12345} \\
 \underline{1} \\
 300 \times 1^2 \\
 30 \times 1 \times 2 = 60 \\
 2^2 = 4 \\
 364 \overline{) 728} \\
 \underline{728} \\
 300 \times 12^2 \\
 30 \times 12 \times 3 = 1080 \\
 3^2 = 9 \\
 44289 \overline{) 132867} \\
 \underline{132867} \\
 300 \times 123^2 \\
 30 \times 123 \times 4 = 14760 \\
 4^2 = 16 \\
 4553476 \overline{) 20498963} \\
 \underline{20498963} \\
 300 \times 1234^2 \\
 30 \times 1234 \times 5 = 185100 \\
 5^2 = 25 \\
 457011925 \overline{) 2285059625} \\
 \underline{2285059625}
 \end{array}$$

To extract the square root of a fraction, extract the root of a numerator and denominator separately. $\sqrt{\frac{1}{9}} = \frac{2}{3}$, or first convert the fraction into a decimal, $\sqrt{\frac{1}{9}} = \sqrt{.4444} = .6666 +$.

To Extract the Cube Root. — Point off the number into periods of 3 figures each, beginning at the right hand, or unit's place. Point off decimals in periods of 3 figures from the decimal point. Find the greatest cube that does not exceed the left-hand period; write its root as the first figure in the required root. Subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.

Square the first figure of the root; multiply by 300, and divide the product into the dividend for a trial divisor; write the quotient after the first figure of the root as a trial second figure.

Complete the divisor by adding to 300 times the square of the first figure, 30 times the product of the first by the second figure, and the square of the second figure. Multiply this divisor by the second figure; subtract the product from the remainder. (Should the product be greater than the remainder, the last figure of the root and the complete divisor are too large; substitute for the last figure the next smaller number, and correct the trial divisor accordingly.)

To the remainder bring down the next period, and proceed as before to find the third figure of the root — that is, square the two figures of the root already found; multiply by 300 for a trial divisor, etc.

If at any time the trial divisor is greater than the dividend, bring down another period of 3 figures, and place 0 in the root and proceed.

The cube root of a number will contain as many figures as there are periods of 3 in the number.

To Extract a Higher Root than the Cube. — The fourth root is the square root of the square root; the sixth root is the cube root of the square root or the square root of the cube root. Other roots are most conveniently found by the use of logarithms.

ALLIGATION.

shows the value of a mixture of different ingredients when the quantity and value of each are known.

Let the ingredients be a, b, c, d , etc., and their respective values per unit w, x, y, z , etc.

A = the sum of the quantities = $a + b + c + d$, etc.

P = mean value or price per unit of A .

$AP = aw + bx + cy + dz$, etc.

$$P = \frac{aw + bx + cy + dz}{A}.$$

PERMUTATION

shows in how many positions any number of things may be arranged in a row; thus, the letters a, b, c may be arranged in six positions, viz. $abc, acb, cab, cba, bac, bca$.

Rule. — Multiply together all the numbers used in counting the things; thus, permutations of 1, 2, and 3 = $1 \times 2 \times 3 = 6$. In how many positions can 9 things in a row be placed?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880.$$

COMBINATION

shows how many arrangements of a few things may be made out of a greater number. Rule: Set down that figure which indicates the greater number, and after it a series of figures diminishing by 1, until as many are set down as the number of the few things to be taken in each combination. Then beginning under the last one, set down said number of few things; then going backward set down a series diminishing by 1 until arriving under the first of the upper numbers. Multiply together all the upper numbers to form one product, and all the lower numbers to form another; divide the upper product by the lower one.

How many combinations of 9 things can be made, taking 3 in each combination?

$$\frac{9 \times 8 \times 7}{1 \times 2 \times 3} = \frac{504}{6} = 84.$$

ARITHMETICAL PROGRESSION,

in a series of numbers, is a progressive increase or decrease in each successive number by the addition or subtraction of the same amount at each step, as 1, 2, 3, 4, 5, etc., or 15, 12, 9, 6, etc. The numbers are called terms, and the equal increase or decrease the difference. Examples in arithmetical progression may be solved by the following formulæ:

Let a = first term, l = last term, d = common difference, n = number of terms, s = sum of the terms:

$$l = a + (n - 1)d,$$

$$= \frac{2s}{n} - a,$$

$$s = \frac{1}{2}n[2a + (n - 1)d],$$

$$= (l + a)\frac{n}{2},$$

$$a = l - (n - 1)d,$$

$$= \frac{1}{2}d \pm \sqrt{\left(l + \frac{1}{2}d\right)^2 - 2ds},$$

$$d = \frac{l - a}{n - 1},$$

$$= \frac{l^2 - a^2}{2s - l - a};$$

$$r = \frac{l - a}{d} + 1,$$

$$= \frac{2s}{l + a},$$

$$= -\frac{1}{2}d \pm \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2},$$

$$= \frac{s}{n} + \frac{(n - 1)d}{2}.$$

$$= \frac{l + a}{2} + \frac{l^2 - a^2}{2d},$$

$$= \frac{1}{2}n[2l - (n - 1)d].$$

$$= \frac{s}{n} - \frac{(n - 1)d}{2}.$$

$$= \frac{2s}{n} - l.$$

$$= \frac{2(s - an)}{n(n - 1)},$$

$$= \frac{2(nl - s)}{n(n - 1)}.$$

$$= d - 2a \pm \sqrt{(2a - d)^2 + 8ds},$$

$$= \frac{2l + d \pm \sqrt{(2l + d)^2 - 8ds}}{2d}.$$

GEOMETRICAL PROGRESSION.

in a series of numbers, is a progressive increase or decrease in each successive number by the same multiplier or divisor at each step, as 1, 2, 4, 8, 16, etc., or 243, 81, 27, 9, etc. The common multiplier is called the ratio.

Let a = first term, l = last term, r = ratio or constant multiplier, n = number of terms, m = any term, as 1st, 2d, etc., s = sum of the terms:

$$\begin{aligned}
 l &= ar^{n-1}, & &= \frac{a + (r-1)s}{r}, & &= \frac{(r-1)sr^{n-1}}{r^n - 1}, \\
 \log l &= \log a + (n-1) \log r, & & & & l(s-l)^{n-1} - a(s-a)^{n-1} = 0. \\
 m &= ar^{m-1} & & \log m = \log a + (m-1) \log r. \\
 s &= \frac{a(r^n - 1)}{r - 1}, & &= \frac{rl - a}{r - 1}, & &= \frac{n^{-1}\sqrt[n]{l^n} - n^{-1}\sqrt[n]{a^n}}{n^{-1}\sqrt[n]{l} - n^{-1}\sqrt[n]{a}}, & &= \frac{lr^n - l}{r^n - r^{n-1}}. \\
 a &= \frac{l}{r^{n-1}}, & &= \frac{(r-1)s}{r^{n-1}}, & & \log a = \log l - (n-1) \log r. \\
 r &= \sqrt[n-1]{\frac{l}{a}}, & &= \frac{s-a}{s-l}, & & \log r = \frac{\log l - \log a}{n-1}. \\
 r^n - \frac{s}{a}r + \frac{s-a}{a} &= 0. & & & & r^n - \frac{s}{s-l}r^{n-1} + \frac{l}{s-l} = 0. \\
 n &= \frac{\log l - \log a}{\log r} + 1, & &= \frac{\log [a + (r-1)s] - \log a}{\log r}, & & \\
 &= \frac{\log l - \log a}{\log (s-a) - \log (s-l)} + 1, & &= \frac{\log l - \log [lr - (r-1)s]}{\log r} + 1.
 \end{aligned}$$

Population of the United States.

(A problem in geometrical progression.)

| Year. | Population. | Increase in 10 Years, per cent. | Annual Increase, per cent. |
|-------|-----------------|---------------------------------|----------------------------|
| 1860 | 31,443,321 | | |
| 1870 | 39,818,449* | 26.63 | 2.39 |
| 1880 | 50,155,783 | 25.96 | 2.33 |
| 1890 | 62,622,250 | 24.86 | 2.25 |
| 1900 | 76,295,220 | 21.834 | 1.994 |
| 1905 | Est. 83,577,000 | — | Est. 1.840 |
| 1910 | " 91,554,000 | Est. 20.0 | " 1.840 |

Estimated Population in Each Year from 1870 to 1909.

(Based on the above rates of increase, in even thousands.)

| | | | | | | | |
|----------|--------|----------|--------|----------|--------|----------|--------|
| 1870.... | 39,818 | 1880.... | 50,156 | 1890.... | 62,622 | 1900.... | 76,295 |
| 1871.... | 40,748 | 1881.... | 51,281 | 1891.... | 63,871 | 1901.... | 77,699 |
| 1872.... | 41,699 | 1882.... | 52,433 | 1892.... | 65,145 | 1902.... | 79,129 |
| 1873.... | 42,673 | 1883.... | 53,610 | 1893.... | 66,444 | 1903.... | 80,585 |
| 1874.... | 43,670 | 1884.... | 54,813 | 1894.... | 67,770 | 1904.... | 82,067 |
| 1875.... | 44,690 | 1885.... | 56,043 | 1895.... | 69,122 | 1905.... | 83,577 |
| 1876.... | 45,733 | 1886.... | 57,301 | 1896.... | 70,500 | 1906.... | 85,115 |
| 1877.... | 46,800 | 1887.... | 58,588 | 1897.... | 71,906 | 1907.... | 86,681 |
| 1878.... | 47,893 | 1888.... | 59,903 | 1898.... | 73,341 | 1908.... | 88,276 |
| 1879.... | 49,011 | 1889.... | 61,247 | 1899.... | 74,803 | 1909.... | 89,900 |

* Corrected by addition of 1,260,078, estimated error of the census of 1870, Census Bulletin No. 16, Dec. 12, 1890.

The preceding table has been calculated by logarithms as follows:

| | | | |
|--|-----------------------------|------------------------------------|--|
| $\log r = \log l - \log a \div (n - 1),$ | | $\log m = \log a + (m - 1) \log r$ | |
| Pop. 1900. . . 76,295,220 | $\log = 7.8824988$ | $= \log l$ | |
| " 1890. . . 62,622,250 | $\log = 7.7967285$ | $= \log a$ | |
| | diff. = .0857703 | | |
| $n = 11, n - 1 = 10;$ | diff. $\div 10 = .00857703$ | $= \log r,$ | |
| add log for 1890 | 7.7967285 | $= \log a$ | |
| | log for 1891 = 7.80530553 | No. = 63,871 . . . | |
| | add again .00857703 | | |
| | log for 1892 7.81388256 | No. = 65,145 . . . | |

Compound interest is a form of geometrical progression; the ratio being 1 plus the percentage.

PERCENTAGE: PROFIT AND LOSS: PER CENT OF EFFICIENCY.

Per cent means "by the hundred." A profit of 10 per cent means a gain of \$10 on every \$100 expended. If a thing is bought for \$1 and sold for \$2 the profit is 100 per cent; but if it is bought for \$2 and sold for \$1 the loss is not 100 per cent, but only 50 per cent.

Rule for percentage: Per cent gain or loss is the gain or loss divided by the original cost, and the quotient multiplied by 100.

Efficiency is defined in engineering as the quotient "output divided by input," that is, the energy utilized divided by the energy expended. The difference between the input and the output is the loss or waste of energy. Expressed as a fraction, efficiency is nearly always less than unity. Expressed as a per cent, it is this fraction multiplied by 100. Thus we may say that a motor has an efficiency of 0.9 or of 90 per cent.

The efficiency of a boiler is the ratio of the heat units absorbed by the boiler in heating water and making steam to the heating value of the coal burned. The saving in fuel due to increasing the efficiency of a boiler from 60 to 75% is not 25%, but only 20%. The rule is: Divide the gain in efficiency (15) by the greater figure (75). The amount of fuel used is inversely proportional to the efficiency; that is, 60 lbs. of fuel with 75% efficiency will do as much work as 75 lbs. with 60% efficiency. The saving of fuel is 15 lbs. which is 20% of 75 lbs.

INTEREST AND DISCOUNT.

Interest is money paid for the use of money for a given time; the factors are:

- p , the sum loaned, or the principal;
- t , the time in years;
- r , the rate of interest;
- i , the amount of interest for the given rate and time;
- $a = p + i$ = the amount of the principal with interest at the end of the time.

Formulae:

$$i = \text{interest} = \text{principal} \times \text{time} \times \text{rate per cent} = i = \frac{ptr}{100};$$

$$a = \text{amount} = \text{principal} + \text{interest} = p + \frac{ptr}{100};$$

$$r = \text{rate} = \frac{100i}{pt};$$

$$p = \text{principal} = \frac{100i}{tr} = a - \frac{ptr}{100};$$

$$t = \text{time} = \frac{100i}{pr}.$$

If the rate is expressed decimally as a per cent, — thus, 6 per cent = .06, — the formulæ become

$$i = prt; a = p(1 + rt); r = \frac{i}{pt}; t = \frac{i}{pr}; p = \frac{i}{tr} = \frac{a}{1 + rt}.$$

Rules for finding Interest. — Multiply the principal by the rate per annum divided by 100, and by the time in years and fractions of a year.

If the time is given in days, interest = $\frac{\text{principal} \times \text{rate} \times \text{no. of days}}{365 \times 100}$.

In banks interest is sometimes calculated on the basis of 360 days to a year, or 12 months of 30 days each.

Short rules for interest at 6 per cent, when 360 days are taken as 1 year: Multiply the principal by number of days and divide by 6000.

Multiply the principal by number of months and divide by 200.

The interest of 1 dollar for one month is $\frac{1}{2}$ cent.

Interest of 100 Dollars for Different Times and Rates.

| Time | 2% | 3% | 4% | 5% | 6% | 8% | 10% |
|------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 1 year | \$2.00 | \$3.00 | \$4.00 | \$5.00 | \$6.00 | \$8.00 | \$10.00 |
| 1 month | .16 $\frac{2}{3}$ | .25 | .33 $\frac{1}{3}$ | .41 $\frac{2}{3}$ | .50 | .66 $\frac{2}{3}$ | .83 $\frac{1}{3}$ |
| 1 day = $\frac{1}{365}$ year | .0055 $\frac{1}{2}$ | .0083 $\frac{1}{4}$ | .0111 $\frac{1}{4}$ | .0138 $\frac{3}{4}$ | .0166 $\frac{2}{3}$ | .0222 $\frac{2}{3}$ | .0277 $\frac{1}{2}$ |
| 1 day = $\frac{1}{360}$ year | .005479 | .008219 | .010959 | .013699 | .016438 | .0219178 | .0273973 |

Discount is interest deducted for payment of money before it is due.

True discount is the difference between the amount of a debt payable at a future date without interest and its present worth. The present worth is that sum which put at interest at the legal rate will amount to the debt when it is due.

To find the present worth of an amount due at a future date, divide the amount by the amount of \$1 placed at interest for the given time. The discount equals the amount minus the present worth.

What discount should be allowed on \$103 paid six months before it is due, interest being 6 per cent per annum?

$$\frac{103}{1 + 1 \times .06 \times \frac{1}{2}} = \$100 \text{ present worth, discount} = 3.00.$$

Bank discount is the amount deducted by a bank as interest on money loaned on promissory notes. It is interest calculated not on the actual sum loaned, but on the gross amount of the note, from which the discount is deducted in advance. It is also calculated on the basis of 360 days in the year, and for 3 (in some banks 4) days more than the time specified in the note. These are called days of grace, and the note is not payable till the last of these days. In some States days of grace have been abolished.

What discount will be deducted by a bank in discounting a note for \$103 payable 6 months hence? Six months = 182 days, add 3 days grace = 185 days, $\frac{103 \times 185}{6000} = \3.176 .

Compound Interest. — In compound interest the interest is added to the principal at the end of each year, (or shorter period if agreed upon).

Let p = the principal, r = the rate expressed decimally, n = no. of years, and a the amount:

$$a = \text{amount} = p(1 + r)^n; r = \text{rate} = \sqrt[n]{\frac{a}{p}} - 1,$$

$$p = \text{principal} = \frac{a}{(1 + r)^n}; \text{no. of years} = n = \frac{\log a - \log p}{\log (1 + r)}.$$

Compound Interest Table.

(Value of one dollar at compound interest, compounded yearly, at 3, 4, 5, and 6 per cent, from 1 to 50 years.)

| Years. | Per cent | | | | Years. | Per cent | | | |
|--------|----------|--------|--------|--------|--------|----------|--------|---------|---------|
| | 3 | 4 | 5 | 6 | | 3 | 4 | 5 | 6 |
| 1 | 1.03 | 1.04 | 1.05 | 1.06 | 16 | 1.6047 | 1.8730 | 2.1829 | 2.5403 |
| 2 | 1.0609 | 1.0816 | 1.1025 | 1.1236 | 17 | 1.6528 | 1.9479 | 2.2920 | 2.6928 |
| 3 | 1.0927 | 1.1249 | 1.1576 | 1.1910 | 18 | 1.7024 | 2.0258 | 2.4066 | 2.8543 |
| 4 | 1.1255 | 1.1699 | 1.2155 | 1.2625 | 19 | 1.7535 | 2.1068 | 2.5269 | 3.0256 |
| 5 | 1.1593 | 1.2166 | 1.2763 | 1.3382 | 20 | 1.8061 | 2.1911 | 2.6533 | 3.2071 |
| 6 | 1.1941 | 1.2653 | 1.3401 | 1.4185 | 21 | 1.8603 | 2.2787 | 2.7859 | 3.3995 |
| 7 | 1.2299 | 1.3159 | 1.4071 | 1.5036 | 22 | 1.9161 | 2.3699 | 2.9252 | 3.6035 |
| 8 | 1.2668 | 1.3686 | 1.4774 | 1.5938 | 23 | 1.9736 | 2.4647 | 3.0715 | 3.8197 |
| 9 | 1.3048 | 1.4233 | 1.5513 | 1.6895 | 24 | 2.0328 | 2.5633 | 3.2251 | 4.0487 |
| 10 | 1.3439 | 1.4802 | 1.6289 | 1.7908 | 25 | 2.0937 | 2.6658 | 3.3863 | 4.2919 |
| 11 | 1.3842 | 1.5394 | 1.7103 | 1.8983 | 30 | 2.4272 | 3.2433 | 4.3219 | 5.7435 |
| 12 | 1.4258 | 1.6010 | 1.7958 | 2.0122 | 35 | 2.8138 | 3.9460 | 5.5159 | 7.6862 |
| 13 | 1.4685 | 1.6651 | 1.8856 | 2.1329 | 40 | 3.2620 | 4.8009 | 7.0398 | 10.2858 |
| 14 | 1.5126 | 1.7317 | 1.9799 | 2.2609 | 45 | 3.7815 | 5.8410 | 8.9847 | 13.7648 |
| 15 | 1.5580 | 1.8009 | 2.0789 | 2.3965 | 50 | 4.3838 | 7.1064 | 11.4670 | 18.4204 |

At compound interest at 3 per cent money will double itself in $23\frac{1}{2}$ years, at 4 per cent in $17\frac{2}{3}$ years, at 5 per cent in 14.2 years, and at 6 per cent in 11.9 years.

EQUATION OF PAYMENTS.

By equation of payments we find the equivalent or average time in which one payment should be made to cancel a number of obligations due at different dates; also the number of days upon which to calculate interest or discount upon a gross sum which is composed of several smaller sums payable at different dates.

Rule. — Multiply each item by the time of its maturity in days from a fixed date, taken as a standard, and divide the sum of the products by the sum of the items: the result is the average time in days from the standard date.

A owes B \$100 due in 30 days, \$200 due in 60 days, and \$300 due in 90 days. In how many days may the whole be paid in one sum of \$600?

$$100 \times 30 + 200 \times 60 + 300 \times 90 = 42,000; \quad 42,000 \div 600 = 70 \text{ days, } \text{ans.}$$

A owes B \$100, \$200, and \$300, which amounts are overdue respectively 30, 60, and 90 days. If he now pays the whole amount, \$600, how many days' interest should he pay on that sum? *Ans.* 70 days.

PARTIAL PAYMENTS.

To compute interest on notes and bonds when partial payments have been made.

United States Rule. — Find the amount of the principal to the time of the first payment, and, subtracting the payment from it, find the amount of the remainder as a new principal to the time of the next payment.

If the payment is less than the interest, find the amount of the principal to the time when the sum of the payments equals or exceeds the interest due, and subtract the sum of the payments from this amount.

Proceed in this manner till the time of settlement.

Note. — The principles upon which the preceding rule is founded are:

1st. That payments must be applied first to discharge accrued interest, and then the remainder, if any, toward the discharge of the principal.

2d. That only unpaid principal can draw interest.

Mercantile Method. — When partial payments are made on short notes or interest accounts, business men commonly employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time it was made to the time of settlement. Subtract the amount of payments from the amount of the debt: the remainder will be the balance due.

ANNUITIES.

An **Annuity** is a fixed sum of money paid yearly, or at other equal times agreed upon. The values of annuities are calculated by the principles of compound interest.

1. Let i denote interest on \$1 for a year, then at the end of a year the amount will be $1 + i$. At the end of n years it will be $(1 + i)^n$.

2. The sum which in n years will amount to 1 is $\frac{1}{(1 + i)^n}$ or $(1 + i)^{-n}$, or the present value of 1 due in n years.

3. The amount of an annuity of 1 in any number of years n is $\frac{(1 + i)^n - 1}{i}$.

4. The present value of an annuity of 1 for any number of years n is $\frac{1 - (1 + i)^{-n}}{i}$.

5. The annuity which 1 will purchase for any number of years n is $\frac{i}{1 - (1 + i)^{-n}}$.

6. The annuity which would amount to 1 in n years is $\frac{i}{(1 + i)^n - 1}$.

Amounts, Present Values, etc., at 5% Interest.

| Years | (1) $(1 + i)^n$ | (2) $(1 + i)^{-n}$ | (3) $\frac{(1 + i)^n - 1}{i}$ | (4) $\frac{1 - (1 + i)^{-n}}{i}$ | (5) $\frac{i}{1 - (1 + i)^{-n}}$ | (6) $\frac{i}{(1 + i)^n - 1}$ |
|--------|--------------------|-----------------------|----------------------------------|-------------------------------------|-------------------------------------|----------------------------------|
| 1 . . | 1.05 | .952381 | 1.00 | .952381 | 1.05 | 1.00 |
| 2 . . | 1.1025 | .907029 | 2.05 | 1.859410 | .537805 | .487805 |
| 3 . . | 1.157625 | .863838 | 3.1525 | 2.723248 | .367209 | .317209 |
| 4 . . | 1.215506 | .822702 | 4.310125 | 3.545951 | .282012 | .232012 |
| 5 . . | 1.276282 | .783526 | 5.525631 | 4.329477 | .230975 | .180975 |
| 6 . . | 1.340096 | .746215 | 6.801913 | 5.075692 | .197017 | .147018 |
| 7 . . | 1.407100 | .710681 | 8.142008 | 5.786373 | .172820 | .122820 |
| 8 . . | 1.477455 | .676839 | 9.549109 | 6.463213 | .154722 | .104722 |
| 9 . . | 1.551328 | .644609 | 11.026564 | 7.107822 | .140690 | .090690 |
| 10 . . | 1.628895 | .613913 | 12.577893 | 7.721735 | .129505 | .079505 |

Table I. — Annuity Required to Redeem \$1000 in from 1 to 50 Years.

| Years to run | Rate of Interest, per cent. | | | | | | | | | | | | |
|-----------------|-----------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 2 | 2 1/4 | 2 1/2 | 2 3/4 | 3 | 3 1/4 | 3 1/2 | 3 3/4 | 4 | 4 1/2 | 5 | 5 1/2 | 6 |
| 2 | 495.05 | 494.43 | 493.83 | 493.22 | 492.61 | 492.00 | 491.40 | 490.80 | 490.20 | 489.00 | 487.80 | 486.62 | 485.43 |
| 3 | 326.72 | 325.94 | 325.14 | 324.35 | 323.56 | 322.75 | 321.94 | 321.13 | 320.36 | 318.77 | 317.21 | 315.63 | 314.10 |
| 4 | 242.63 | 241.74 | 240.84 | 239.93 | 239.02 | 238.14 | 237.26 | 236.38 | 235.50 | 233.74 | 232.01 | 230.29 | 228.60 |
| 5 | 192.16 | 191.18 | 190.24 | 189.30 | 188.35 | 187.42 | 186.49 | 185.56 | 184.63 | 182.79 | 180.98 | 179.13 | 177.39 |
| 6 | 158.53 | 157.53 | 156.56 | 155.58 | 154.61 | 153.64 | 152.67 | 151.73 | 150.79 | 148.88 | 147.02 | 145.18 | 143.36 |
| 7 | 134.52 | 133.51 | 132.49 | 131.50 | 130.51 | 129.54 | 128.57 | 127.59 | 126.61 | 124.67 | 122.82 | 120.96 | 119.13 |
| 8 | 116.51 | 115.48 | 114.47 | 113.46 | 112.46 | 111.47 | 110.48 | 109.50 | 108.53 | 106.60 | 104.72 | 102.86 | 101.03 |
| 9 | 102.52 | 101.48 | 100.46 | 99.45 | 98.44 | 97.44 | 96.44 | 95.46 | 94.49 | 92.57 | 90.69 | 88.83 | 87.02 |
| 10 | 91.33 | 90.29 | 89.25 | 88.24 | 87.24 | 86.24 | 85.24 | 84.26 | 83.29 | 81.38 | 79.50 | 77.67 | 75.87 |
| 11 | 82.18 | 81.14 | 80.11 | 79.09 | 78.07 | 77.08 | 76.09 | 75.12 | 74.15 | 72.25 | 70.39 | 68.57 | 66.79 |
| 12 | 74.56 | 73.52 | 72.49 | 71.47 | 70.46 | 69.47 | 68.48 | 67.51 | 66.55 | 64.67 | 62.83 | 61.03 | 59.28 |
| 13 | 68.12 | 67.08 | 66.05 | 65.04 | 64.03 | 63.05 | 62.06 | 61.10 | 60.14 | 58.27 | 56.45 | 54.68 | 52.96 |
| 14 | 62.60 | 61.56 | 60.54 | 59.53 | 58.53 | 57.55 | 56.57 | 55.62 | 54.67 | 52.82 | 51.02 | 49.28 | 47.58 |
| 15 | 57.83 | 56.79 | 55.77 | 54.77 | 53.77 | 52.79 | 51.82 | 50.88 | 49.94 | 48.11 | 46.34 | 44.62 | 42.96 |
| 16 | 53.65 | 52.62 | 51.60 | 50.60 | 49.61 | 48.64 | 47.68 | 46.75 | 45.82 | 44.01 | 42.27 | 40.58 | 38.95 |
| 17 | 49.97 | 48.94 | 47.93 | 46.94 | 45.95 | 44.99 | 44.04 | 43.12 | 42.20 | 40.42 | 38.70 | 37.04 | 35.44 |
| 18 | 46.70 | 45.67 | 44.67 | 43.69 | 42.71 | 41.76 | 40.82 | 39.90 | 38.99 | 37.24 | 35.54 | 33.92 | 32.36 |
| 19 | 43.78 | 42.76 | 41.76 | 40.78 | 39.81 | 38.87 | 37.94 | 37.04 | 36.14 | 34.40 | 32.75 | 31.15 | 29.62 |
| 20 | 41.15 | 40.14 | 39.14 | 38.18 | 37.22 | 36.29 | 35.36 | 34.47 | 33.58 | 31.87 | 30.24 | 28.68 | 27.18 |
| 25 | 31.22 | 30.24 | 29.27 | 28.35 | 27.43 | 26.55 | 25.67 | 24.84 | 24.01 | 22.44 | 20.95 | 19.55 | 18.23 |
| 30 | 24.65 | 23.70 | 22.78 | 21.90 | 21.02 | 20.19 | 19.37 | 18.60 | 17.83 | 16.39 | 15.05 | 13.80 | 12.65 |
| 35 | 20.00 | 19.09 | 18.20 | 17.37 | 16.54 | 15.77 | 15.00 | 14.29 | 13.58 | 12.27 | 11.07 | 9.97 | 8.97 |
| 40 | 16.55 | 15.68 | 14.84 | 14.05 | 13.26 | 12.54 | 11.83 | 11.17 | 10.52 | 9.34 | 8.28 | 7.32 | 6.46 |
| 45 | 13.91 | 13.07 | 12.27 | 11.52 | 10.78 | 10.12 | 9.45 | 8.85 | 8.26 | 7.20 | 6.26 | 5.43 | 4.70 |
| 50 | 11.82 | 11.02 | 10.26 | 9.56 | 8.87 | 8.25 | 7.63 | 7.09 | 6.55 | 5.60 | 4.78 | 4.06 | 3.44 |

TABLES FOR CALCULATING SINKING-FUNDS AND PRESENT VALUES.

Engineers and others connected with municipal work and industrial enterprises often find it necessary to calculate payments to sinking-funds which will provide a sum of money sufficient to pay off a bond issue or other debt at the end of a given period, or to determine the present value of certain annual charges. The accompanying tables were computed by Mr. John W. Hill, of Cincinnati, *Eng'g News*, Jan. 25, 1894.

Table I (opposite page) shows the annual sum at various rates of interest required to net \$1000 in from 2 to 50 years, and Table II shows the present value at various rates of interest of an annual charge of \$1000 for from 5 to 50 years, at five-year intervals, and for 100 years.

Table II. — Capitalization of Annuity of \$1000 for from 5 to 100 Years.

| Years. | Rate of Interest, per cent. | | | | | | | |
|--------|-----------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | 2½ | 3 | 3½ | 4 | 4½ | 5 | 5½ | 6 |
| 5 | 4,645.88 | 4,579.60 | 4,514.92 | 4,451.68 | 4,389.91 | 4,329.45 | 4,268.09 | 4,212.40 |
| 10 | 8,752.17 | 8,530.13 | 8,316.45 | 8,110.74 | 7,912.67 | 7,721.73 | 7,537.54 | 7,360.19 |
| 15 | 12,381.41 | 11,937.80 | 11,517.23 | 11,118.06 | 10,739.42 | 10,379.53 | 10,037.48 | 9,712.30 |
| 20 | 15,589.215 | 14,877.27 | 14,212.12 | 13,590.21 | 13,007.88 | 12,462.13 | 11,950.26 | 11,469.96 |
| 25 | 18,424.67 | 17,413.01 | 16,481.28 | 15,621.93 | 14,828.12 | 14,093.86 | 13,413.82 | 12,783.38 |
| 30 | 20,930.59 | 19,600.21 | 18,391.85 | 17,291.86 | 16,288.77 | 15,372.36 | 14,533.63 | 13,764.85 |
| 35 | 23,145.31 | 21,487.04 | 20,000.43 | 18,664.37 | 17,460.89 | 16,374.36 | 15,390.48 | 14,488.65 |
| 40 | 25,103.53 | 23,114.36 | 21,354.83 | 19,792.65 | 18,401.49 | 17,159.01 | 16,044.92 | 15,046.31 |
| 45 | 26,833.15 | 24,518.49 | 22,495.23 | 20,719.89 | 19,156.24 | 17,773.99 | 16,547.65 | 15,455.85 |
| 50 | 28,362.48 | 25,729.58 | 23,455.21 | 21,482.08 | 19,761.93 | 18,255.86 | 16,931.97 | 15,761.87 |
| 100 | 36,614.21 | 31,598.81 | 27,655.36 | 24,504.96 | 21,949.21 | 19,847.90 | 18,095.83 | 16,612.64 |

WEIGHTS AND MEASURES.

Long Measure. — Measures of Length.

12 inches = 1 foot.
3 feet = 1 yard.
1760 yards, or 5280 feet = 1 mile.

Additional measures of length in occasional use: 1000 mils = 1 inch; 4 inches = 1 hand; 9 inches = 1 span; 2½ feet = 1 military pace; 2 yards = 1 fathom; 5½ yards, or 16½ feet = 1 rod (formerly also called pole or perch).

Old Land Measure. — 7.92 inches = 1 link; 100 links, or 66 feet, or 4 rods = 1 chain; 10 chains, or 220 yards = 1 furlong; 8 furlongs, or 80 chains = 1 mile; 10 square chains = 1 acre.

Nautical Measure.

6080.26 feet, or 1.15156 statute miles } = 1 nautical mile, or knot.*
3 nautical miles } = 1 league.
60 nautical miles, or 69.168 statute miles } = 1 degree (at the equator).
360 degrees = circumference of the earth at the equator.

* The British Admiralty takes the round figure of 6080 ft. which is the length of the "measured mile" used in trials of vessels. The value varies from 6080.26 to 6088.44 ft. according to different measures of the earth's diameter. There is a difference of opinion among writers as to the use of the word "knot" to mean length or a distance — some holding that it should be used only to denote a rate of speed. The length between knots on the log line is 1/120 of a nautical mile, or 50.7 ft., when a half-minute glass is used; so that a speed of 10 knots is equal to 10 nautical miles per hour.

Square Measure. — Measures of Surface.

| | | |
|--|---|------------------|
| 144 square inches, or 183.35 circular inches | } | = 1 square foot. |
| 9 square feet | | |
| 30 1/4 square yards, or 272 1/4 square feet | } | = 1 square rod. |
| 10 sq. chains, or 160 sq. rods, or 4840 sq. yards, or 43560 sq. feet | | |
| 640 acres | | = 1 square mile. |

An acre equals a square whose side is 208.71 feet.

Circular Inch; Circular Mil. — A circular inch is the area of a circle 1 inch in diameter = 0.7854 square inch.

1 square inch = 1.2732 circular inches.

A circular mil is the area of a circle 1 mil, or 0.001 inch in diameter. 1000² or 1,000,000 circular mils = 1 circular inch.

1 square inch = 1,273,239 circular mils.

The mil and circular mil are used in electrical calculations involving the diameter and area of wires.

Solid or Cubic Measure. — Measures of Volume.

1728 cubic inches = 1 cubic foot.

27 cubic feet = 1 cubic yard.

1 cord of wood = a pile, $4 \times 4 \times 8$ feet = 128 cubic feet.

1 perch of masonry = $16\frac{1}{2} \times 1\frac{1}{2} \times 1$ foot = $24\frac{3}{4}$ cubic feet.

Liquid Measure.

4 gills = 1 pint.

2 pints = 1 quart.

4 quarts = 1 gallon { U. S. 231 cubic inches.
Eng. 277.274 cubic inches.

Old Liquid Measures. — $31\frac{1}{2}$ gallons = 1 barrel; 42 gallons = 1 tierce; 2 barrels, or 63 gallons = 1 hogshead; 84 gallons, or 2 tierces = 1 puncheon; 2 hogsheads, or 126 gallons = 1 pipe or butt; 2 pipes, or 3 puncheons = 1 tun.

A gallon of water at 62° F. weighs 8.3356 lbs.

The U. S. gallon contains 231 cubic inches; 7.4805 gallons = 1 cubic foot. A cylinder 7 in. diam. and 6 in. high contains 1 gallon, very nearly, or 230.9 cubic inches. The British Imperial gallon contains 277.274 cubic inches = 1.20032 U. S. gallon, or 10 lbs. of water at 62° F.

The gallon is a very troublesome unit for engineers. Much labor might be saved if it were abandoned and the cubic foot used instead. The capacity of a tank or reservoir should be stated in cubic feet, and the delivery of a pump in cubic feet per second or in millions of cubic feet in 24 hours. One cubic foot per second = 86,400 cu. ft. in 24 hours. One million cu. ft. per 24 hours = 11.5741 cu. ft. per sec.

The Miner's Inch. — (Western U. S. for measuring flow of a stream of water.) An act of the California legislature, May 23, 1901, makes the standard miner's inch 1.5 cu. ft. per minute, measured through any aperture or orifice.

The term Miner's Inch is more or less indefinite, for the reason that California water companies do not all use the same head above the centre of the aperture, and the inch varies from 1.36 to 1.73 cu. ft. per min., but the most common measurement is through an aperture 2 ins. high and whatever length is required, and through a plank $1\frac{1}{4}$ ins. thick. The lower edge of the aperture should be 2 ins. above the bottom of the measuring-box, and the plank 5 ins. high above the aperture, thus making a 6-in. head above the centre of the stream. Each square inch of this opening represents a miner's inch, which is equal to a flow of $1\frac{1}{2}$ cu. ft. per min.

Apothecaries' Fluid Measure.

60 minims = 1 fluid drachm.

8 drachms = 1 fluid ounce.

In the U. S. a fluid ounce is the 128th part of a U. S. gallon, or 1.805 cu. ins. It contains 456.3 grains of water at 39° F. In Great Britain the fluid ounce is 1.732 cu. ins. and contains 1 ounce avoirdupois, or 437.5 grains of water at 62° F.

Dry Measure, U. S.

2 pints = 1 quart. 8 quarts = 1 peck. 4 pecks = 1 bushel.

The standard U. S. bushel is the Winchester bushel, which is in cylinder form, 18½ inches diameter and 8 inches deep, and contains 2150.42 cubic inches.

A struck bushel contains 2150.42 cubic inches = 1.2445 cu. ft.; 1 cubic foot = 0.80356 struck bushel. A heaped bushel is a cylinder 18½ inches diameter and 8 inches deep, with a heaped cone not less than 6 inches high. It is equal to 1¼ struck bushels.

The British Imperial bushel is based on the Imperial gallon, and contains 8 such gallons, or 2218.192 cubic inches = 1.2837 cubic feet. The English quarter = 8 Imperial bushels.

Capacity of a cylinder in U. S. gallons = square of diameter, in inches × height in inches × .0034. (Accurate within 1 part in 100,000.)

Capacity of a cylinder in U. S. bushels = square of diameter in inches × height in inches × 0.0003652.

Shipping Measure.

Register Ton. — For register tonnage or for measurement of the entire internal capacity of a vessel:

100 cubic feet = 1 register ton.

This number is arbitrarily assumed to facilitate computation.

Shipping Ton. — For the measurement of cargo:

| | |
|-----------------|---------------------------|
| 40 cubic feet = | { 1 U. S. shipping ton. |
| | { 31.16 Imp. bushels. |
| | { 32.143 U. S. " " |
| 42 cubic feet = | { 1 British shipping ton. |
| | { 32.719 Imp. bushels. |
| | { 33.75 U. S. " " |

Carpenter's Rule. — Weight a vessel will carry = length of keel × breadth at main beam × depth of hold in feet ÷ 95 (the cubic feet allowed for a ton). The result will be the tonnage. For a double-decker instead of the depth of the hold take half the breadth of the beam.

Measures of Weight. — Avoirdupois, or Commercial Weight.

| | |
|--|--|
| 16 drachms, or 437.5 grains | = 1 ounce, oz. |
| 16 ounces, or 7000 grains | = 1 pound, lb. |
| 28 pounds | = 1 quarter, qr. |
| 4 quarters | = 1 hundredweight, cwt. = 112 lbs. |
| 20 hundred weight | = 1 ton of 2240 lbs., gross or long ton. |
| 2000 pounds | = 1 net, or short ton. |
| 2204.6 pounds | = 1 metric ton. |
| 1 stone = 14 pounds; 1 quintal = 100 pounds. | |

The drachm, quarter, hundredweight, stone, and quintal are now seldom used in the United States.

Troy Weight.

| | |
|-----------------|-------------------------------|
| 24 grains | = 1 pennyweight, dwt. |
| 20 pennyweights | = 1 ounce, oz. = 480 grains. |
| 12 ounces | = 1 pound, lb. = 5760 grains. |

Troy weight is used for weighing gold and silver. The grain is the same in Avoirdupois, Troy, and Apothecaries' weights. A carat, used in weighing diamonds = 3.168 grains = 0.205 gramme.

Apothecaries' Weight.

| | | |
|------------|-----------------------------|----------------|
| 20 grains | = 1 scruple, \mathfrak{D} | |
| 3 scruples | = 1 drachm, \mathfrak{z} | = 60 grains. |
| 8 drachms | = 1 ounce, $\mathfrak{℥}$ | = 480 grains. |
| 12 ounces | = 1 pound, lb. | = 5760 grains. |

To determine whether a balance has unequal arms. — After weighing an article and obtaining equilibrium, transpose the article and the weights. If the balance is true, it will remain in equilibrium; if untrue, the pan suspended from the longer arm will descend.

To weigh correctly on an incorrect balance. — First, by substitution. Put the article to be weighed in one pan of the balance and counterpoise it by any convenient heavy articles placed on the other pan. Remove the article to be weighed and substitute for it standard weights until equipoise is again established. The amount of these weights is the weight of the article.

Second, by transposition. Determine the apparent weight of the article as usual, then its apparent weight after transposing the article and the weights. If the difference is small, add half the difference to the smaller of the apparent weights to obtain the true weight. If the difference is 2 per cent the error of this method is 1 part in 10,000. For larger differences, or to obtain a perfectly accurate result, multiply the two apparent weights together and extract the square root of the product.

Circular Measure.

| | |
|---------------|------------------|
| 60 seconds, " | = 1 minute, '. |
| 60 minutes, ' | = 1 degree, °. |
| 90 degrees | = 1 quadrant. |
| 360 " | = circumference. |

Arc of angle of 57.3° , or $360^\circ \div 6.2832 = 1$ radian = the arc whose length is equal to the radius.

Time.

| | |
|------------|-------------|
| 60 seconds | = 1 minute. |
| 60 minutes | = 1 hour. |
| 24 hours | = 1 day. |
| 7 days | = 1 week. |

365 days, 5 hours, 48 minutes, 48 seconds = 1 year.

By the Gregorian Calendar every year whose number is divisible by 4 is a leap year, and contains 366 days, the other years containing 365 days, except that the centesimal years are leap years only when the number of the year is divisible by 400.

The comparative values of mean solar and sidereal time are shown by the following relations according to Bessel:

| | |
|---------------------------|--|
| 365.24222 mean solar days | = 366.24222 sidereal days, whence |
| 1 mean solar day | = 1.00273791 sidereal days; |
| 1 sidereal day | = 0.99726957 mean solar day; |
| 24 hours mean solar time | = $24^h 3^m 56^s.555$ sidereal time; |
| 24 hours sidereal time | = $23^h 56^m 4^s.091$ mean solar time, |

whence 1 mean solar day is $3^m 55^s.91$ longer than a sidereal day, reckoned in mean solar time.

BOARD AND TIMBER MEASURE.**Board Measure.**

In board measure boards are assumed to be one inch in thickness. To obtain the number of feet board measure (B. M.) of a board or stick of square timber, multiply together the length in feet, the breadth in feet, and the thickness in inches.

To compute the measure or surface in square feet. — When all dimensions are in feet, multiply the length by the breadth, and the product will give the surface required.

When either of the dimensions are in inches, multiply as above and divide the product by 12.

When all dimensions are in inches, multiply as before and divide product by 144.

Timber Measure.

To compute the volume of round timber. — When all dimensions are in feet, multiply the length by one quarter of the product of the mean girth and diameter, and the product will give the measurement in cubic feet. When length is given in feet, and girth and diameter in inches, divide the product by 144; when all the dimensions are in inches, divide by 1728.

To compute the volume of square timber. — When all dimensions are in feet, multiply together the length, breadth, and depth; the product will be the volume in cubic feet. When one dimension is given in inches, divide by 12; when two dimensions are in inches, divide by 144; when all three dimensions are in inches, divide by 1728.

Contents in Feet of Joists, Scantling, and Timber.

Length in Feet.

| Size. | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
|-------|----|----|----|----|----|----|----|----|----|----|
|-------|----|----|----|----|----|----|----|----|----|----|

Feet Board Measure.

| | | | | | | | | | | |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2 × 4 | 8 | 9 | 11 | 12 | 13 | 15 | 16 | 17 | 19 | 20 |
| 2 × 6 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 2 × 8 | 16 | 19 | 21 | 24 | 27 | 29 | 32 | 35 | 37 | 40 |
| 2 × 10 | 20 | 23 | 27 | 30 | 33 | 37 | 40 | 43 | 47 | 50 |
| 2 × 12 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |
| 2 × 14 | 28 | 33 | 37 | 42 | 47 | 51 | 56 | 61 | 65 | 70 |
| 3 × 8 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |
| 3 × 10 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
| 3 × 12 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 |
| 3 × 14 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 |
| 4 × 4 | 16 | 19 | 21 | 24 | 27 | 29 | 32 | 35 | 37 | 40 |
| 4 × 6 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 60 |
| 4 × 8 | 32 | 37 | 43 | 48 | 53 | 59 | 64 | 69 | 75 | 80 |
| 4 × 10 | 40 | 47 | 53 | 60 | 67 | 73 | 80 | 87 | 93 | 100 |
| 4 × 12 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 |
| 4 × 14 | 56 | 65 | 75 | 84 | 93 | 103 | 112 | 121 | 131 | 140 |
| 6 × 6 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 |
| 6 × 8 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 |
| 6 × 10 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 |
| 6 × 12 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 | 168 | 180 |
| 6 × 14 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | 196 | 210 |
| 8 × 8 | 64 | 75 | 85 | 96 | 107 | 117 | 128 | 139 | 149 | 160 |
| 8 × 10 | 80 | 93 | 107 | 120 | 133 | 147 | 160 | 173 | 187 | 200 |
| 8 × 12 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 |
| 8 × 14 | 112 | 131 | 149 | 168 | 187 | 205 | 224 | 243 | 261 | 280 |
| 10 × 10 | 100 | 117 | 133 | 150 | 167 | 183 | 200 | 217 | 233 | 250 |
| 10 × 12 | 120 | 140 | 160 | 180 | 200 | 220 | 240 | 260 | 280 | 300 |
| 10 × 14 | 140 | 163 | 187 | 210 | 233 | 257 | 280 | 303 | 327 | 350 |
| 12 × 12 | 144 | 168 | 192 | 216 | 240 | 264 | 288 | 312 | 336 | 360 |
| 12 × 14 | 168 | 196 | 224 | 252 | 280 | 308 | 336 | 364 | 392 | 420 |
| 14 × 14 | 196 | 229 | 261 | 294 | 327 | 359 | 392 | 425 | 457 | 490 |

FRENCH OR METRIC MEASURES.

The metric unit of length is the metre = 39.37 inches.

The metric unit of weight is the gram = 15.432 grains.

The following prefixes are used for subdivisions and multiples: Milli = $\frac{1}{1000}$, Centi = $\frac{1}{100}$, Deci = $\frac{1}{10}$, Deca = 10, Hecto = 100, Kilo = 1000, Myria = 10,000.

FRENCH AND BRITISH (AND AMERICAN) EQUIVALENT MEASURES.**Measures of Length.**

| FRENCH. | BRITISH and U. S. |
|------------------|--|
| 1 metre | = 39.37 inches, or 3.28083 feet, or 1.09361 yards. |
| 0.3048 metre | = 1 foot. |
| 1 centimetre | = 0.3937 inch. |
| 2.54 centimetres | = 1 inch. |
| 1 millimetre | = 0.03937 inch, or $\frac{1}{25}$ inch, nearly. |
| 25.4 millimetres | = 1 inch. |
| 1 kilometre | = 1093.61 yards, or 0.62137 mile. |

Of Surface.

| FRENCH. | BRITISH and U. S. |
|--------------------------|--|
| 1 square metre | = { 10.764 square feet, 1.196 square yards. |
| 0.836 square metre | = 1 square yard. |
| 0.0929 square metre | = 1 square foot. |
| 1 square centimetre | = 0.155 square inch. |
| 6.452 square centimetres | = 1 square inch. |
| 1 square millimetre | = 0.00155 sq. in. = 1973.5 circ. mils. |
| 645.2 square millimetres | = 1 square inch. |
| 1 centiare = 1 sq. metre | = 10.764 square feet. |
| 1 are = 1 sq. decametre | = 1076.41 " " |
| 1 hectare = 100 ares | = 107641 " " = 2.4711 acres. |
| 1 sq. kilometre | = 0.386109 sq. miles = 247.11 " " |
| 1 sq. myriametre | = 38.6109 " " |

Of Volume.

| FRENCH. | BRITISH and U. S. |
|------------------------------------|--|
| 1 cubic metre | = { 35.314 cubic feet, 1.308 cubic yards. |
| 0.7645 cubic metre | = 1 cubic yard. |
| 0.02832 cubic metre | = 1 cubic foot. |
| 1 cubic decimetre | = { 61.023 cubic inches, 0.0353 cubic foot. |
| 28.32 cubic decimetres | = 1 cubic foot. |
| 1 cubic centimetre | = 0.061 cubic inch. |
| 16.387 cubic centimetres | = 1 cubic inch. |
| 1 cubic centimetre = 1 millilitre | = 0.061 cubic inch. |
| 1 centilitre | = 0.610 " " |
| 1 decilitre | = 6.102 " " |
| 1 litre = 1 cubic decimetre | = 61.023 " " = 1.05671 quarts, U.S. |
| 1 hectolitre or decistere | = 3.5314 cubic feet = 2.8375 bushels, " " |
| 1 stère, kilolitre, or cubic metre | = 1.308 cubic yards = 28.37 bushels, " " |

Of Capacity.

| FRENCH. | BRITISH and U. S. |
|---------------------------------|---|
| 1 litre (= 1 cubic decimetre) = | { 61.023 cubic inches, 0.03531 cubic foot, 0.2642 gallon (American), 2.202 pounds of water at 62° F. |
| 28.317 litres | = 1 cubic foot. |
| 4.543 litres | = 1 gallon (British). |
| 3.785 litres | = 1 gallon (American). |

Of Weight.

| FRENCH. | BRITISH and U. S. |
|-----------------------|--------------------------------|
| 1 gramme | = 15.432 grains. |
| 0.0648 gramme | = 1 grain. |
| 28.35 gramme | = 1 ounce avoirdupois. |
| 1 kilogramme | = 2.2046 pounds. |
| 0.4536 kilogramme | = 1 pound. |
| 1 tonne or metric ton | = { 0.9842 ton of 2240 pounds, |
| 1000 kilogrammes | = { 19.68 cwts., |
| 1.016 metric tons | = { 2204.6 pounds. |
| 1016 kilogrammes | = { 1 ton of 2240 pounds. |

Mr. O. H. Titmann, in Bulletin No. 9 of the U. S. Coast and Geodetic Survey, discusses the work of various authorities who have compared the yard and the metre, and by referring all the observations to a common standard has succeeded in reconciling the discrepancies within very narrow limits. The following are his results for the number of inches in a metre according to the comparisons of the authorities named: 1817. Hassler, 39.36994 in. 1818. Kater, 39.36990 in. 1835. Baily, 39.36973 in. 1866. Clarke, 39.36970 in. 1885. Comstock, 39.36984 in. The mean of these is 39.36982 in.

The value of the metre is now defined in the U. S. laws as 39.37 inches.

French and British Equivalents of Compound Units.

| FRENCH. | BRITISH. |
|---|-------------------------------|
| 1 gramme per square millimetre | = 1.422 lbs. per sq. in. |
| 1 kilogramme per square " " | = 1422.32 " " " " |
| 1 " " " " centimetre | = 14.223 " " " " |
| 1.0335 kg. per sq. cm. = 1 atmosphere | = 14.7 " " " " |
| 0.070308 kilogramme per square centimetre | = 1 lb. per square inch. |
| 1 kilogrammetre | = 7.2330 foot-pounds. |
| 1 gramme per litre = 0.062428 lb. per cu. ft. | = 58.349 grains per U. S gal. |
| of water at 62° F. | |
| 1 grain per U. S. gallon = 1 part in 58,349 | = 1.7138 parts per 100,000 |
| = 0.017138 grammes per litre. | |

METRIC CONVERSION TABLES.

The following tables, with the subjoined memoranda, were published in 1890 by the United States Coast and Geodetic Survey, office of standard weights and measures, T. C. Mendenhall, Superintendent.

Tables for Converting U. S. Weights and Measures — Customary to Metric.

LINEAR.

| | Inches to Milli- metres. | Feet to Metres. | Yards to Metres. | Miles to Kilo- metres. |
|-----|-----------------------------|-----------------|------------------|---------------------------|
| 1 = | 25.4001 | 0.304801 | 0.914402 | 1.60935 |
| 2 = | 50.8001 | 0.609601 | 1.828804 | 3.21869 |
| 3 = | 76.2002 | 0.914402 | 2.743205 | 4.82804 |
| 4 = | 101.6002 | 1.219202 | 3.657607 | 6.43739 |
| 5 = | 127.0003 | 1.524003 | 4.572009 | 8.04674 |
| 6 = | 152.4003 | 1.828804 | 5.486411 | 9.65608 |
| 7 = | 177.8004 | 2.133604 | 6.400813 | 11.26543 |
| 8 = | 203.2004 | 2.438405 | 7.315215 | 12.87478 |
| 9 = | 228.6005 | 2.743205 | 8.229616 | 14.48412 |

SQUARE.

| | Square Inches to Square Centi- metres. | Square Feet to Square Deci- metres. | Square Yards to Square Metres. | Acres to Hectares. |
|-----|--|---|-----------------------------------|-----------------------|
| 1 = | 6.452 | 9.290 | 0.836 | 0.4047 |
| 2 = | 12.903 | 18.581 | 1.672 | 0.8094 |
| 3 = | 19.355 | 27.871 | 2.508 | 1.2141 |
| 4 = | 25.807 | 37.161 | 3.344 | 1.6187 |
| 5 = | 32.258 | 46.452 | 4.181 | 2.0234 |
| 6 = | 38.710 | 55.742 | 5.017 | 2.4281 |
| 7 = | 45.161 | 65.032 | 5.853 | 2.8328 |
| 8 = | 51.613 | 74.323 | 6.689 | 3.2375 |
| 9 = | 58.065 | 83.613 | 7.525 | 3.6422 |

CUBIC.

| | Cubic Inches to Cubic Centi- metres. | Cubic Feet to Cubic Metres. | Cubic Yards to Cubic Metres. | Bushels to Hectolitres. |
|-----|--|--------------------------------|---------------------------------|----------------------------|
| 1 = | 16.387 | 0.02832 | 0.765 | 0.35242 |
| 2 = | 32.774 | 0.05663 | 1.529 | 0.70485 |
| 3 = | 49.161 | 0.08495 | 2.294 | 1.05727 |
| 4 = | 65.549 | 0.11327 | 3.058 | 1.40969 |
| 5 = | 81.936 | 0.14158 | 3.823 | 1.76211 |
| 6 = | 98.323 | 0.16990 | 4.587 | 2.11454 |
| 7 = | 114.710 | 0.19822 | 5.352 | 2.46696 |
| 8 = | 131.097 | 0.22654 | 6.116 | 2.81938 |
| 9 = | 147.484 | 0.25485 | 6.881 | 3.17181 |

CAPACITY.

| | Fluid Drachms to Millilitres or Cubic Centi- metres. | Fluid Ounces to Millilitres. | Quarts to Litres. | Gallons to Litres. |
|-----|---|---------------------------------|-------------------|-----------------------|
| 1 = | 3.70 | 29.57 | 0.94636 | 3.78544 |
| 2 = | 7.39 | 59.15 | 1.89272 | 7.57088 |
| 3 = | 11.09 | 88.72 | 2.83908 | 11.35632 |
| 4 = | 14.79 | 118.30 | 3.78544 | 15.14176 |
| 5 = | 18.48 | 147.87 | 4.73180 | 18.92720 |
| 6 = | 22.18 | 177.44 | 5.67816 | 22.71264 |
| 7 = | 25.88 | 207.02 | 6.62452 | 26.49808 |
| 8 = | 29.57 | 236.59 | 7.57088 | 30.28352 |
| 9 = | 33.28 | 266.16 | 8.51724 | 34.06896 |

WEIGHT.

| | Grains to Milli- grammes. | Avoirdupois Ounces to Grammes. | Avoirdupois Pounds to Kilo- grammes. | Troy Ounces to Grammes. |
|-----|------------------------------|--------------------------------------|--|----------------------------|
| 1 = | 64.7989 | 28.3495 | 0.45359 | 31.10348 |
| 2 = | 129.5978 | 56.6991 | 0.90719 | 62.20696 |
| 3 = | 194.3968 | 85.0486 | 1.36078 | 93.31044 |
| 4 = | 259.1957 | 113.3981 | 1.81437 | 124.41392 |
| 5 = | 323.9946 | 141.7476 | 2.26796 | 155.51740 |
| 6 = | 388.7935 | 170.0972 | 2.72156 | 186.62089 |
| 7 = | 453.5924 | 198.4467 | 3.17515 | 217.72437 |
| 8 = | 518.3914 | 226.7962 | 3.62874 | 248.82785 |
| 9 = | 583.1903 | 255.1457 | 4.08233 | 279.93133 |

1 chain = 20.1169 metres.
 1 square mile = 259 hectares.
 1 fathom = 1.829 metres.
 1 nautical mile = 1853.27 metres.
 1 foot = 0.304801 metre.
 1 avoir. pound = 453.5924277 gram.
 15432.35639 grains = 1 kilogramme.

Tables for Converting U. S. Weights and Measures —
Metric to Customary.

LINEAR.

| | Metres to Inches. | Metres to Feet. | Metres to Yards. | Kilometres to Miles. |
|-----|----------------------|--------------------|---------------------|-------------------------|
| 1 = | 39.3700 | 3.28083 | 1.093611 | 0.62137 |
| 2 = | 78.7400 | 6.56167 | 2.187222 | 1.24274 |
| 3 = | 118.1100 | 9.84250 | 3.280833 | 1.86411 |
| 4 = | 157.4800 | 13.12333 | 4.374444 | 2.48548 |
| 5 = | 196.8500 | 16.40417 | 5.468056 | 3.10685 |
| 6 = | 236.2200 | 19.68500 | 6.561667 | 3.72822 |
| 7 = | 275.5900 | 22.96583 | 7.655278 | 4.34959 |
| 8 = | 314.9600 | 26.24667 | 8.748889 | 4.97096 |
| 9 = | 354.3300 | 29.52750 | 9.842500 | 5.59233 |

SQUARE.

| | Square Centi- metres to Square Inches. | Square Metres to Square Feet. | Square Metres to Square Yards. | Hectares to Acres. |
|-----|--|----------------------------------|-----------------------------------|-----------------------|
| 1 = | 0.1550 | 10.764 | 1.196 | 2.471 |
| 2 = | 0.3100 | 21.528 | 2.392 | 4.942 |
| 3 = | 0.4650 | 32.292 | 3.588 | 7.413 |
| 4 = | 0.6200 | 43.055 | 4.784 | 9.884 |
| 5 = | 0.7750 | 53.819 | 5.980 | 12.355 |
| 6 = | 0.9300 | 64.583 | 7.176 | 14.826 |
| 7 = | 1.0850 | 75.347 | 8.372 | 17.297 |
| 8 = | 1.2400 | 86.111 | 9.568 | 19.768 |
| 9 = | 1.3950 | 96.874 | 10.764 | 22.239 |

CUBIC.

| | Cubic Centi- metres to Cubic Inches. | Cubic Deci- metres to Cubic Inches. | Cubic Metres to Cubic Feet. | Cubic Metres to Cubic Yards. |
|-----|--|---|--------------------------------|---------------------------------|
| 1 = | 0.0610 | 61.023 | 35.314 | 1.308 |
| 2 = | 0.1220 | 122.047 | 70.629 | 2.616 |
| 3 = | 0.1831 | 183.070 | 105.943 | 3.924 |
| 4 = | 0.2441 | 244.093 | 141.258 | 5.232 |
| 5 = | 0.3051 | 305.117 | 176.572 | 6.540 |
| 6 = | 0.3661 | 366.140 | 211.887 | 7.848 |
| 7 = | 0.4272 | 427.163 | 247.201 | 9.156 |
| 8 = | 0.4882 | 488.187 | 282.516 | 10.464 |
| 9 = | 0.5492 | 549.210 | 317.830 | 11.771 |

CAPACITY.

| | Millilitres or Cubic Centi- metres to Fluid Drachms. | Centilitres to Fluid Ounces. | Litres to Quarts. | Dekalitres to Gallons. | Hektolitres to Bushels. |
|-----|---|------------------------------------|----------------------|------------------------------|-------------------------------|
| 1 = | 0.27 | 0.338 | 1.0567 | 2.6417 | 2.8375 |
| 2 = | 0.54 | 0.676 | 2.1134 | 5.2834 | 5.6750 |
| 3 = | 0.81 | 1.014 | 3.1700 | 7.9251 | 8.5125 |
| 4 = | 1.08 | 1.352 | 4.2267 | 10.5668 | 11.3500 |
| 5 = | 1.35 | 1.691 | 5.2834 | 13.2085 | 14.1875 |
| 6 = | 1.62 | 2.029 | 6.3401 | 15.8502 | 17.0250 |
| 7 = | 1.89 | 2.368 | 7.3968 | 18.4919 | 19.8625 |
| 8 = | 2.16 | 2.706 | 8.4534 | 21.1336 | 22.7000 |
| 9 = | 2.43 | 3.043 | 9.5101 | 23.7753 | 25.5375 |

WEIGHT.

| | Milligrammes to Grains. | Kilogrammes to Grains. | Hectogrammes (100 grammes) to Ounces Av. | Kilogrammes to Pounds Avoirdupois. |
|-----|----------------------------|---------------------------|--|--|
| 1 = | 0.01543 | 15432.36 | 3.5274 | 2.20462 |
| 2 = | 0.03086 | 30864.71 | 7.0548 | 4.40924 |
| 3 = | 0.04630 | 46297.07 | 10.5822 | 6.61386 |
| 4 = | 0.06173 | 61729.43 | 14.1096 | 8.81849 |
| 5 = | 0.07716 | 77161.78 | 17.6370 | 11.02311 |
| 6 = | 0.09259 | 92594.14 | 21.1644 | 13.22773 |
| 7 = | 0.10803 | 108026.49 | 24.6918 | 15.43235 |
| 8 = | 0.12346 | 123458.85 | 28.2192 | 17.63697 |
| 9 = | 0.13889 | 138891.21 | 31.7466 | 19.84159 |

WEIGHT — (Continued).

| | Quintals to Pounds Av. | Milliers or Tonnes to Pounds Av. | Grammes to Ounces. Troy. |
|-----|---------------------------|-------------------------------------|-----------------------------|
| 1 = | 220.46 | 2204.6 | 0.03215 |
| 2 = | 440.92 | 4409.2 | 0.06430 |
| 3 = | 661.38 | 6613.8 | 0.09645 |
| 4 = | 881.84 | 8818.4 | 0.12860 |
| 5 = | 1102.30 | 11023.0 | 0.16075 |
| 6 = | 1322.76 | 13227.6 | 0.19290 |
| 7 = | 1543.22 | 15432.2 | 0.22505 |
| 8 = | 1763.68 | 17636.8 | 0.25721 |
| 9 = | 1982.14 | 19841.4 | 0.28936 |

The British Avoirdupois pound was derived from the British standard Troy pound of 1758 by direct comparison, and it contains 7000 grains Troy.

The grain Troy is therefore the same as the grain Avoirdupois, and the pound Avoirdupois in use in the United States is equal to the British pound Avoirdupois.

By the concurrent action of the principal governments of the world an International Bureau of Weights and Measures has been established near Paris.

The International Standard Metre is derived from the Mètre des Archives, and its length is defined by the distance between two lines at 0° Centigrade, on a platinum-iridium bar deposited at the International Bureau.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight *in vacuo* is the same as that of the Kilogramme des Archives.

Copies of these international standards are deposited in the office of standard weights and measures of the U. S. Coast and Geodetic Survey.

The litre is equal to a cubic decimetre of water, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum; the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre.

The metric system was legalized in the United States in 1866. Many attempts were made during the 40 years following to have the U. S. Congress pass laws to make the metric system the legal standard, but they have all failed. Similar attempts in Great Britain have also failed. For arguments for and against the metric system see the report of a committee of the American Society of Mechanical Engineers, 1903, Vol. 24.

COMPOUND UNITS.

Measures of Pressure and Weight.

| | | |
|-------------------------------------|---|--|
| 1 lb. per square inch. | = | $\left\{ \begin{array}{l} 144 \text{ lbs. per square foot.} \\ 2.0355 \text{ ins. of mercury at } 32^{\circ} \text{ F.} \\ 2.0416 \text{ " " " } 62^{\circ} \text{ F.} \\ 2.309 \text{ ft. of water at } 62^{\circ} \text{ F.} \\ 27.71 \text{ ins. " " } 62^{\circ} \text{ F.} \end{array} \right.$ |
| 1 ounce per sq. in. | = | $\left\{ \begin{array}{l} 0.1276 \text{ in. of mercury at } 62^{\circ} \text{ F.} \\ 1.732 \text{ ins. of water at } 62^{\circ} \text{ F.} \end{array} \right.$ |
| 1 atmosphere (14.7 lbs. per sq.in.) | = | $\left\{ \begin{array}{l} 2116.3 \text{ lbs. per square foot.} \\ 33.947 \text{ ft. of water at } 62^{\circ} \text{ F.} \\ 30 \text{ ins. of mercury at } 62^{\circ} \text{ F.} \\ 29.922 \text{ ins. of mercury at } 32^{\circ} \text{ F.} \\ 760 \text{ millimetres of mercury at } 32^{\circ} \text{ F.} \end{array} \right.$ |

COMPOUND UNITS — (Continued).

| | | |
|-----------------------------|-----|---|
| 1 inch of water at 62° F. | = { | 0.03609 lb. or .5774 oz. per sq.in. 5.196 lbs. per square foot. 0.0736 in. of mercury at 62° F. |
| 1 inch of water at 32° F. | = { | 5.2021 lbs. per square foot. 0.036125 lb. " " inch. |
| 1 foot of water at 62° F. | = { | 0.433 lb. per square inch. 62.355 lbs. " " foot. |
| 1 inch of mercury at 62° F. | = { | 0.491 lb. or 7.86 oz. per sq. in. 1.132 ft. of water at 62° F. 13.58 ins. " " " 62° F. |

Weight of One Cubic Foot of Pure Water.

| | |
|--|-------------|
| At 32° F. (freezing-point)..... | 62.418 lbs. |
| " 39.1° F. (maximum density)..... | 62.425 " |
| " 62° F. (standard temperature)..... | 62.355 " |
| " 212° F. (boiling-point, under 1 atmosphere)..... | 59.76 " |

American gallon = 231 cubic ins. of water at 62° F. = 8.3356 lbs.
 British " = 277.274 " " " = 10 lbs.

Weight and Volume of Air.

1 cubic ft. of air at 32° F. and atmospheric pressure weighs 0.080728 lb.

1 ft. in height of air at 32° F. = { 0.0005606 lb. per sq. in.
 0.008970 ounces per sq. in.
 0.015534 inches of water at 62° F.

For air at any other temperature T° Fahr. multiply by $460 \div (460 + T)$.

1 lb. pressure per sq. ft. = 12.387 ft. of air at 32° F.

1 " " sq. in. = 1784. " " " " "

1 ounce " " sq. in. = 111.48 " " " " "

1 inch of water at 62° F. = 64.37 " " " " "

For air at any other temperature multiply by $(460 + T) \div 460$.

1 atmosphere = 14.696 lb. per sq. in. = 760 mm. or 29.921 in. of mercury.

Measures of Work, Power, and Duty.

Work. — The sustained exertion of pressure through space.

Unit of work. — One foot-pound, i.e., a pressure of one pound exerted through a space of one foot.

Horse-power. — The rate of work. Unit of horse-power = 33,000 ft.-lbs. per minute, or 550 ft.-lbs. per second = 1,980,000 ft.-lbs. per hour.

Heat unit = heat required to raise 1 lb. of water 1° F. (from 39° to 40°).

Horse-power expressed in heat units = $\frac{33000}{778} = 42.416$ heat units per minute = 0.707 heat unit per second = 2545 heat units per hour.

1 lb. of fuel per H. P. per hour = { 1,980,000 ft.-lbs. per lb. of fuel.
 2,545 heat units " " " "

1,000,000 ft.-lbs. per lb. of fuel = 1.98 lbs. of fuel per H. P. per hour.

Velocity. — Feet per second = $\frac{5280}{3600} = \frac{22}{15} \times$ miles per hour.

Gross tons per mile = $\frac{1760}{2240} = \frac{11}{14}$ lbs. per yard (single rail.)

WIRE AND SHEET-METAL GAUGES COMPARED.

| Number of Gauge. | Birmingham (or Stubs' Iron) Wire Gauge. | American or Brown and Sharpe Gauge. | Roebbling's and Washburn & Moen's Gauge. | Stubs' Steel Wire Gauge. (See also p. 30.) | British Imperial Standard Wire Gauge. (Legal Standard in Great Britain since March 1, 1884.) | | U. S. Standard Gauge for Sheet and Plate Iron and Steel. 1893. | Number of Gauge. |
|------------------|---|-------------------------------------|--|--|--|---------|--|------------------|
| | inch. | inch. | inch. | inch. | inch. | millim. | inch. | |
| 0000000 | | | .49 | | .500 | 12.7 | .5 | 7/0 |
| 000000 | | | .46 | | .464 | 11.78 | .469 | 8/0 |
| 00000 | | | .43 | | .432 | 10.97 | .438 | 5/0 |
| 0000 | .454 | .46 | .393 | | .4 | 10.16 | .406 | 4/0 |
| 000 | .425 | .40964 | .362 | | .372 | 9.45 | .375 | 3/0 |
| 00 | .38 | .3648 | .331 | | .348 | 8.84 | .344 | 2/0 |
| 0 | .34 | .32486 | .307 | | .324 | 8.23 | .313 | 0 |
| 1 | .3 | .2893 | .283 | .227 | .3 | 7.62 | .281 | 1 |
| 2 | .284 | .25763 | .263 | .219 | .276 | 7.01 | .266 | 2 |
| 3 | .259 | .22942 | .244 | .212 | .252 | 6.4 | .25 | 3 |
| 4 | .238 | .20431 | .225 | .207 | .232 | 5.89 | .234 | 4 |
| 5 | .22 | .18194 | .207 | .204 | .212 | 5.38 | .219 | 5 |
| 6 | .203 | .16202 | .192 | .201 | .192 | 4.88 | .203 | 6 |
| 7 | .18 | .14428 | .177 | .199 | .176 | 4.47 | .188 | 7 |
| 8 | .165 | .12849 | .162 | .197 | .16 | 4.06 | .172 | 8 |
| 9 | .148 | .11443 | .148 | .194 | .144 | 3.66 | .156 | 9 |
| 10 | .134 | .10189 | .135 | .191 | .128 | 3.25 | .141 | 10 |
| 11 | .12 | .09074 | .12 | .188 | .116 | 2.95 | .125 | 11 |
| 12 | .109 | .08081 | .105 | .185 | .104 | 2.64 | .109 | 12 |
| 13 | .095 | .07196 | .092 | .182 | .092 | 2.34 | .094 | 13 |
| 14 | .033 | .06408 | .08 | .180 | .08 | 2.03 | .078 | 14 |
| 15 | .072 | .05707 | .072 | .178 | .072 | 1.83 | .07 | 15 |
| 16 | .055 | .05032 | .063 | .175 | .064 | 1.63 | .0625 | 16 |
| 17 | .058 | .04526 | .054 | .172 | .056 | 1.42 | .0563 | 17 |
| 18 | .049 | .0403 | .047 | .168 | .048 | 1.22 | .05 | 18 |
| 19 | .042 | .03539 | .041 | .164 | .04 | 1.02 | .0438 | 19 |
| 20 | .035 | .03196 | .035 | .161 | .036 | .91 | .0375 | 20 |
| 21 | .032 | .02846 | .032 | .157 | .032 | .81 | .0344 | 21 |
| 22 | .028 | .02535 | .028 | .155 | .028 | .71 | .0313 | 22 |
| 23 | .025 | .02257 | .025 | .153 | .024 | .61 | .0281 | 23 |
| 24 | .022 | .0201 | .023 | .151 | .022 | .56 | .025 | 24 |
| 25 | .02 | .0179 | .02 | .148 | .02 | .51 | .0219 | 25 |
| 26 | .018 | .01594 | .018 | .146 | .018 | .46 | .0188 | 26 |
| 27 | .016 | .01419 | .017 | .143 | .0164 | .42 | .0172 | 27 |
| 28 | .014 | .01264 | .016 | .139 | .0148 | .38 | .0156 | 28 |
| 29 | .013 | .01126 | .015 | .134 | .0136 | .35 | .0141 | 29 |
| 30 | .012 | .01002 | .014 | .127 | .0124 | .31 | .0125 | 30 |
| 31 | .01 | .00893 | .013 | .120 | .0116 | .29 | .0109 | 31 |
| 32 | .009 | .00795 | .013 | .115 | .0108 | .27 | .0101 | 32 |
| 33 | .008 | .00708 | .011 | .112 | .01 | .25 | .0094 | 33 |
| 34 | .007 | .0063 | .01 | .110 | .0092 | .23 | .0086 | 34 |
| 35 | .005 | .00561 | .00 | .108 | .0084 | .21 | .0078 | 35 |
| 36 | .004 | .005 | .009 | .106 | .0076 | .19 | .007 | 36 |
| 37 | | .00445 | .0085 | .103 | .0068 | .17 | .0066 | 37 |
| 38 | | .00396 | .008 | .101 | .006 | .15 | .0063 | 38 |
| 39 | | .00353 | .0075 | .099 | .0052 | .13 | | 39 |
| 40 | | .00314 | .007 | .097 | .0048 | .12 | | 40 |
| 41 | | | | .095 | .0044 | .11 | | 41 |
| 42 | | | | .092 | .004 | .10 | | 42 |
| 43 | | | | .088 | .0036 | .09 | | 43 |
| 44 | | | | .085 | .0032 | .08 | | 44 |
| 45 | | | | .081 | .0028 | .07 | | 45 |
| 46 | | | | .079 | .0024 | .06 | | 46 |
| 47 | | | | .077 | .002 | .05 | | 47 |
| 48 | | | | .075 | .0016 | .04 | | 48 |
| 49 | | | | .092 | .0012 | .03 | | 49 |
| 50 | | | | .069 | .001 | .025 | | 50 |

EDISON, OR CIRCULAR MIL GAUGE, FOR ELECTRICAL WIRES.

| Gauge Number. | Circular Mils. | Diameter in Mils. | Gauge Number. | Circular Mils. | Diameter in Mils. | Gauge Number. | Circular Mils. | Diameter in Mils. |
|---------------|----------------|-------------------|---------------|----------------|-------------------|---------------|----------------|-------------------|
| 3 | 3,000 | 54.78 | 70 | 70,000 | 264.58 | 190 | 190,000 | 435.89 |
| 5 | 5,000 | 70.72 | 75 | 75,000 | 273.87 | 200 | 200,000 | 447.22 |
| 8 | 8,000 | 89.45 | 80 | 80,000 | 282.85 | 220 | 220,000 | 469.05 |
| 12 | 12,000 | 109.55 | 85 | 85,000 | 291.55 | 240 | 240,000 | 489.90 |
| 15 | 15,000 | 122.48 | 90 | 90,000 | 300.00 | 260 | 260,000 | 509.91 |
| 20 | 20,000 | 141.43 | 95 | 95,000 | 308.23 | 280 | 280,000 | 529.16 |
| 25 | 25,000 | 158.12 | 100 | 100,000 | 316.23 | 300 | 300,000 | 547.73 |
| 30 | 30,000 | 173.21 | 110 | 110,000 | 331.67 | 320 | 320,000 | 565.69 |
| 35 | 35,000 | 187.09 | 120 | 120,000 | 346.42 | 340 | 340,000 | 583.10 |
| 40 | 40,000 | 200.00 | 130 | 130,000 | 360.56 | 360 | 360,000 | 600.00 |
| 45 | 45,000 | 212.14 | 140 | 140,000 | 374.17 | | | |
| 50 | 50,000 | 223.61 | 150 | 150,000 | 387.30 | | | |
| 55 | 55,000 | 234.53 | 160 | 160,000 | 400.00 | | | |
| 60 | 60,000 | 244.95 | 170 | 170,000 | 412.32 | | | |
| 65 | 65,000 | 254.96 | 180 | 180,000 | 424.27 | | | |

TWIST DRILL AND STEEL WIRE GAUGE.

(Morse Twist Drill and Machine Co.)

| No. | Size. | No. | Size. | No. | Size. | No. | Size. | No. | Size. | No. | Size. |
|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|
| | inch. | | inch. | | inch. | | inch. | | inch. | | inch. |
| 1 | .2230 | 11 | .1910 | 21 | .1590 | 31 | .1200 | 41 | .0960 | 51 | .0670 |
| 2 | .2210 | 12 | .1890 | 22 | .1570 | 32 | .1160 | 42 | .0935 | 52 | .0635 |
| 3 | .2130 | 13 | .1850 | 23 | .1540 | 33 | .1130 | 43 | .0890 | 53 | .0595 |
| 4 | .2090 | 14 | .1820 | 24 | .1520 | 34 | .1110 | 44 | .0860 | 54 | .0550 |
| 5 | .2055 | 15 | .1800 | 25 | .1495 | 35 | .1100 | 45 | .0820 | 55 | .0520 |
| 6 | .2040 | 16 | .1770 | 26 | .1470 | 36 | .1065 | 46 | .0810 | 56 | .0465 |
| 7 | .2010 | 17 | .1730 | 27 | .1440 | 37 | .1040 | 47 | .0785 | 57 | .0430 |
| 8 | .1990 | 18 | .1695 | 28 | .1405 | 38 | .1015 | 48 | .0760 | 58 | .0420 |
| 9 | .1960 | 19 | .1660 | 29 | .1360 | 39 | .0995 | 49 | .0730 | 59 | .0410 |
| 10 | .1935 | 20 | .1610 | 30 | .1285 | 40 | .0980 | 50 | .0700 | 60 | .0400 |

STUBS' STEEL WIRE GAUGE.

(For Nos. 1 to 50 see table on page 29.)

| No. | Size. | No. | Size. | No. | Size. | No. | Size. | No. | Size. | No. | Size. |
|-----|-------|-----|-------|-----|---------------------|-----|-------|-----|-------|-----|-------|
| | inch. | | inch. | | inch. | | inch. | | inch. | | inch. |
| Z | .413 | P | .323 | F | .257 | 51 | .066 | 61 | .038 | 71 | .026 |
| Y | .404 | O | .316 | E | .250 | 52 | .063 | 62 | .037 | 72 | .024 |
| X | .397 | N | .302 | D | .246 | 53 | .058 | 63 | .036 | 73 | .023 |
| W | .386 | M | .295 | C | .242 | 54 | .055 | 64 | .035 | 74 | .022 |
| V | .377 | L | .290 | B | .238 | 55 | .050 | 65 | .033 | 75 | .020 |
| U | .368 | K | .281 | A | .234 | 56 | .045 | 66 | .032 | 76 | .018 |
| T | .358 | J | .277 | I | { See page 29 | 57 | .042 | 67 | .031 | 77 | .016 |
| S | .348 | I | .272 | to | | 58 | .041 | 68 | .030 | 78 | .015 |
| R | .339 | H | .266 | 50 | | 59 | .040 | 69 | .029 | 79 | .014 |
| Q | .332 | G | .261 | | | 60 | .039 | 70 | .027 | 80 | .013 |

The Stubs' Steel Wire Gauge is used in measuring drawn steel wire or drill rods of Stubs' make, and is also used by many makers of American drill rods.

THE EDISON OR CIRCULAR MIL WIRE GAUGE.

(For table of copper wires by this gauge, giving weights, electrical resistances, etc., see Copper Wire.)

Mr. C. J. Field (*Stevens Indicator*, July, 1887) thus describes the origin of the Edison gauge:

The Edison company experienced inconvenience and loss by not having a wide enough range nor sufficient number of sizes in the existing gauges. This was felt more particularly in the central-station work in making electrical determinations for the street system. They were compelled to make use of two of the existing gauges at least, thereby introducing a complication that was liable to lead to mistakes by the contractors and linemen.

In the incandescent system an even distribution throughout the entire system and a uniform pressure at the point of delivery are obtained by calculating for a given maximum percentage of loss from the potential as delivered from the dynamo. In carrying this out, on account of lack of regular sizes, it was often necessary to use larger sizes than the occasion demanded, and even to assume new sizes for large underground conductors. The engineering department of the Edison company, knowing the requirements, have designed a gauge that has the widest range obtainable and a large number of sizes which increase in a regular and uniform manner. The basis of the graduation is the sectional area, and the number of the wire corresponds. A wire of 100,000 circular mils area is No. 100; a wire of one half the size will be No. 50; twice the size No. 200.

In the older gauges, as the number increased the size decreased. With this gauge, however, the number increases with the wire, and the number multiplied by 1000 will give the circular mils.

The weight per mil-foot, 0.00000302705 pounds, agrees with a specific gravity of 8.889, which is the latest figure given for copper. The ampere capacity which is given was deduced from experiments made in the company's laboratory, and is based on a rise of temperature of 50° F. in the wire.

In 1893 Mr. Field writes, concerning gauges in use by electrical engineers:

The B. and S. gauge seems to be in general use for the smaller sizes, up to 100,000 c.m., and in some cases a little larger. From between one and two hundred thousand circular mils upwards, the Edison gauge or its equivalent is practically in use, and there is a general tendency to designate all sizes above this in circular mils, specifying a wire as 200,000, 400,000, 500,000, or 1,000,000 c.m.

In the electrical business there is a large use of copper wire and rod and other materials of these large sizes, and in ordering them, speaking of them, specifying, and in every other use, the general method is to simply specify the circular milage. I think it is going to be the only system in the future for the designation of wires, and the attaining of it means practically the adoption of the Edison gauge or the method and basis of this gauge as the correct one for wire sizes.

THE U. S. STANDARD GAUGE FOR SHEET AND PLATE IRON AND STEEL, 1893.

There is in this country no uniform or standard gauge, and the same numbers in different gauges represent different thicknesses of sheets or plates. This has given rise to much misunderstanding and friction between employers and workmen and mistakes and fraud between dealers and consumers.

An Act of Congress in 1893 established the Standard Gauge for sheet iron and steel which is given on the next page. It is based on the fact that a cubic foot of iron weighs 480 pounds.

A sheet of iron 1 foot square and 1 inch thick weighs 40 pounds, or 640 ounces, and 1 ounce in weight should be $\frac{1}{640}$ inch thick. The scale has been arranged so that each descriptive number represents a certain number of ounces in weight and an equal number of 640ths of an inch in thickness.

The law enacts that on and after July 1, 1893, the new gauge shall be used in determining duties and taxes levied on sheet and plate iron and

U. S. STANDARD GAUGE FOR SHEET AND PLATE IRON AND STEEL, 1893.

| Number of Gauge. | Approximate Thickness in Fractions of an Inch. | Approximate Thickness in Decimal Parts of an Inch. | Approximate Thickness in Millimeters. | Weight per Square Foot in Ounces Avirdupois. | Weight per Square Foot in Pounds Avirdupois. | Weight per Square Foot in Kilograms. | Weight per Square Meter in Kilograms. | Weight per Sq. Meter in Pounds Avirdupois. |
|------------------|--|--|---------------------------------------|--|--|--------------------------------------|---------------------------------------|--|
| 0000000 | 1-2 | 0.5 | 12.7 | 320 | 20. | 9.072 | 97.65 | 215.28 |
| 000000 | 15-32 | 0.46875 | 11.90625 | 300 | 18.75 | 8.505 | 91.55 | 201.82 |
| 00000 | 7-16 | 0.4375 | 11.1125 | 280 | 17.50 | 7.938 | 85.44 | 188.37 |
| 0000 | 13-32 | 0.40625 | 10.31875 | 260 | 16.25 | 7.371 | 79.33 | 174.91 |
| 000 | 3-8 | 0.375 | 9.525 | 240 | 15. | 6.804 | 73.24 | 161.46 |
| 00 | 11-32 | 0.34375 | 8.73125 | 220 | 13.75 | 6.237 | 67.13 | 148.00 |
| 0 | 5-16 | 0.3125 | 7.9375 | 200 | 12.50 | 5.67 | 61.03 | 134.55 |
| 1 | 9-32 | 0.28125 | 7.14375 | 180 | 11.25 | 5.103 | 54.93 | 121.09 |
| 2 | 17-64 | 0.265625 | 6.746875 | 170 | 10.625 | 4.819 | 51.88 | 114.37 |
| 3 | 1-4 | 0.25 | 6.35 | 160 | 10. | 4.536 | 48.82 | 107.64 |
| 4 | 15-64 | 0.234375 | 5.953125 | 150 | 9.375 | 4.252 | 45.77 | 100.91 |
| 5 | 7-32 | 0.21875 | 5.55625 | 140 | 8.75 | 3.969 | 42.72 | 94.18 |
| 6 | 13-64 | 0.203125 | 5.159375 | 130 | 8.125 | 3.685 | 39.67 | 87.45 |
| 7 | 3-16 | 0.1875 | 4.7625 | 120 | 7.5 | 3.402 | 36.62 | 80.72 |
| 8 | 11-64 | 0.171875 | 4.365625 | 110 | 6.875 | 3.118 | 33.57 | 74.00 |
| 9 | 5-32 | 0.15625 | 3.96875 | 100 | 6.25 | 2.835 | 30.52 | 67.27 |
| 10 | 9-64 | 0.140625 | 3.571875 | 90 | 5.625 | 2.552 | 27.46 | 60.55 |
| 11 | 1-8 | 0.125 | 3.175 | 80 | 5. | 2.268 | 24.41 | 53.82 |
| 12 | 7-64 | 0.109375 | 2.778125 | 70 | 4.375 | 1.984 | 21.36 | 47.09 |
| 13 | 3-32 | 0.09375 | 2.38125 | 60 | 3.75 | 1.701 | 18.31 | 40.36 |
| 14 | 5-64 | 0.078125 | 1.984375 | 50 | 3.125 | 1.417 | 15.26 | 33.64 |
| 15 | 9-128 | 0.0703125 | 1.7859375 | 45 | 2.8125 | 1.276 | 13.73 | 30.27 |
| 16 | 1-16 | 0.0625 | 1.5875 | 40 | 2.5 | 1.134 | 12.21 | 26.91 |
| 17 | 9-160 | 0.05625 | 1.42875 | 36 | 2.25 | 1.021 | 10.99 | 24.22 |
| 18 | 1-20 | 0.05 | 1.27 | 32 | 2. | 0.9072 | 9.765 | 21.53 |
| 19 | 7-160 | 0.04375 | 1.11125 | 28 | 1.75 | 0.7938 | 8.544 | 18.84 |
| 20 | 3-80 | 0.0375 | 0.9525 | 24 | 1.50 | 0.6804 | 7.324 | 16.15 |
| 21 | 11-320 | 0.034375 | 0.873125 | 22 | 1.375 | 0.6237 | 6.713 | 14.80 |
| 22 | 1-32 | 0.03125 | 0.793750 | 20 | 1.25 | 0.567 | 6.103 | 13.46 |
| 23 | 9-320 | 0.028125 | 0.714375 | 18 | 1.125 | 0.5103 | 5.49 | 12.11 |
| 24 | 1-40 | 0.025 | 0.635 | 16 | 1. | 0.4536 | 4.882 | 10.76 |
| 25 | 7-320 | 0.021875 | 0.555625 | 14 | 0.875 | 0.3969 | 4.272 | 9.42 |
| 26 | 3-160 | 0.01875 | 0.47625 | 12 | 0.75 | 0.3402 | 3.662 | 8.07 |
| 27 | 11-640 | 0.0171875 | 0.4365625 | 11 | 0.6875 | 0.3119 | 3.357 | 7.40 |
| 28 | 1-64 | 0.015625 | 0.396875 | 10 | 0.625 | 0.2835 | 3.052 | 6.73 |
| 29 | 9-640 | 0.0140625 | 0.3571875 | 9 | 0.5625 | 0.2551 | 2.746 | 6.05 |
| 30 | 1-80 | 0.0125 | 0.3175 | 8 | 0.5 | 0.2268 | 2.441 | 5.38 |
| 31 | 7-640 | 0.0109375 | 0.2778125 | 7 | 0.4375 | 0.1984 | 2.136 | 4.71 |
| 32 | 13-1280 | 0.01015625 | 0.25796875 | 6 1/2 | 0.40625 | 0.1843 | 1.983 | 4.37 |
| 33 | 3-320 | 0.009375 | 0.238125 | 6 | 0.375 | 0.1701 | 1.831 | 4.04 |
| 34 | 11-1280 | 0.00859375 | 0.21828125 | 5 1/2 | 0.34375 | 0.1559 | 1.678 | 3.70 |
| 35 | 5-640 | 0.0078125 | 0.1984375 | 5 | 0.3125 | 0.1417 | 1.526 | 3.36 |
| 36 | 9-1280 | 0.00703125 | 0.17859375 | 4 1/2 | 0.28125 | 0.1276 | 1.373 | 3.03 |
| 37 | 17-2560 | 0.00664062 | 0.16867187 | 4 1/4 | 0.26562 | 0.1205 | 1.297 | 2.87 |
| 38 | 1-160 | 0.00625 | 0.15875 | 4 | 0.25 | 0.1134 | 1.221 | 2.69 |

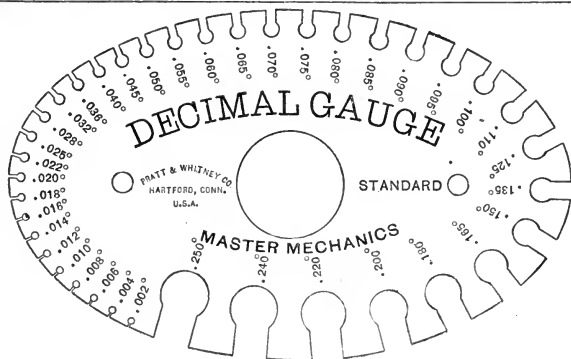
steel; and that in its application a variation of $2\frac{1}{2}$ per cent either way may be allowed.

The Decimal Gauge.—The legalization of the standard sheet-metal gauge of 1893 and its adoption by some manufacturers of sheet iron have only added to the existing confusion of gauges. A joint committee of the American Society of Mechanical Engineers and the American Railway Master Mechanics' Association in 1895 agreed to recommend the use of the decimal gauge, that is, a gauge whose number for each thickness is the number of thousandths of an inch in that thickness, and also to recommend "the abandonment and disuse of the various other gauges now in use, as tending to confusion and error." A notched gauge of oval form, shown in the cut below, has come into use as a standard form of the decimal gauge.

In 1904 The Westinghouse Electric & Mfg. Co. abandoned the use of gauge numbers in referring to wire, sheet metal, etc.

Weight of Sheet Iron and Steel. Thickness by Decimal Gauge.

| Decimal Gauge. | Approx. Fractions of an Inch. | Approx. Millimeters. | Weight per Square Foot in Pounds. | | Decimal Gauge. | Approx. Fractions of an Inch. | Approx. Millimeters. | Weight per Square Foot in Pounds. | |
|----------------|-------------------------------|----------------------|-----------------------------------|-------------------------------|----------------|-------------------------------|----------------------|-----------------------------------|-------------------------------|
| | | | Iron, 480 Lbs. per Cu. Ft. | Steel, 489.6 Lbs. per Cu. Ft. | | | | Iron, 480 Lbs. per Cu. Ft. | Steel, 489.6 Lbs. per Cu. Ft. |
| 0.002 | 1/500 | 0.05 | 0.08 | 0.082 | 0.060 | 1/16 — | 1.52 | 2.40 | 2.448 |
| 0.004 | 1/250 | 0.10 | 0.16 | 0.163 | 0.065 | 13/200 | 1.65 | 2.60 | 2.652 |
| 0.006 | 3/500 | 0.15 | 0.24 | 0.245 | 0.070 | 7/100 | 1.78 | 2.80 | 2.856 |
| 0.008 | 1/125 | 0.20 | 0.32 | 0.326 | 0.075 | 3/40 | 1.90 | 3.00 | 3.060 |
| 0.010 | 1/100 | 0.25 | 0.40 | 0.408 | 0.080 | 2/25 | 2.03 | 3.20 | 3.264 |
| 0.012 | 3/250 | 0.30 | 0.48 | 0.490 | 0.085 | 17/200 | 2.16 | 3.40 | 3.468 |
| 0.014 | 7/500 | 0.36 | 0.56 | 0.571 | 0.090 | 9/100 | 2.28 | 3.60 | 3.672 |
| 0.016 | 1/64 + | 0.41 | 0.64 | 0.653 | 0.095 | 19/200 | 2.41 | 3.80 | 3.876 |
| 0.018 | 9/500 | 0.46 | 0.72 | 0.734 | 0.100 | 1/10 | 2.54 | 4.00 | 4.080 |
| 0.020 | 1/50 | 0.51 | 0.80 | 0.816 | 0.110 | 11/100 | 2.79 | 4.40 | 4.488 |
| 0.022 | 11/500 | 0.56 | 0.88 | 0.898 | 0.125 | 1/8 | 3.18 | 5.00 | 5.100 |
| 0.025 | 1/40 | 0.64 | 1.00 | 1.020 | 0.135 | 27/200 | 3.43 | 5.40 | 5.508 |
| 0.028 | 7/250 | 0.71 | 1.12 | 1.142 | 0.150 | 3/20 | 3.81 | 6.00 | 6.120 |
| 0.032 | 1/32 + | 0.81 | 1.28 | 1.306 | 0.165 | 33/200 | 4.19 | 6.60 | 6.732 |
| 0.036 | 9/250 | 0.91 | 1.44 | 1.469 | 0.180 | 9/50 | 4.57 | 7.20 | 7.344 |
| 0.040 | 1/25 | 1.02 | 1.60 | 1.632 | 0.200 | 1/5 | 5.08 | 8.00 | 8.160 |
| 0.045 | 9/200 | 1.14 | 1.80 | 1.836 | 0.220 | 11/50 | 5.59 | 8.80 | 8.976 |
| 0.050 | 1/20 | 1.27 | 2.00 | 2.040 | 0.240 | 6/25 | 6.10 | 9.60 | 9.792 |
| 0.055 | 11/200 | 1.40 | 2.20 | 2.244 | 0.250 | 1/4 | 6.35 | 10.00 | 10.200 |



ALGEBRA.

Addition. — Add a , b , and $-c$. Ans. $a + b - c$.

Add $2a$ and $-3a$. Ans. $-a$. Add $2ab$, $-3ab$, $-c$, $-3c$. Ans. $-ab - 4c$. Add a^2 and $2a$. Ans. $a^2 + 2a$.

Subtraction. — Subtract a from b . Ans. $b - a$. Subtract $-a$ from $-b$. Ans. $-b + a$.

Subtract $b + c$ from a . Ans. $a - b - c$. Subtract $3a^2b - 9c$ from $4a^2b + c$. Ans. $a^2b + 10c$. RULE: Change the signs of the subtrahend and proceed as in addition.

Multiplication. — Multiply a by b . Ans. ab . Multiply ab by $a + b$. Ans. $a^2b + ab^2$.

Multiply $a + b$ by $a + b$. Ans. $(a + b)(a + b) = a^2 + 2ab + b^2$.

Multiply $-a$ by $-b$. Ans. ab . Multiply $-a$ by b . Ans. $-ab$. Like signs give plus, unlike signs minus.

Powers of numbers. — The product of two or more powers of any number is the number with an exponent equal to the sum of the powers: $a^2 \times a^3 = a^5$; $a^2b^2 \times ab = a^3b^3$; $-7ab \times 2ac = -14a^2bc$.

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and add the partial products: $(6a - 3b) \times 3c = 18ac - 9bc$.

To multiply two polynomials, multiply each term of one factor by each term of the other and add the partial products: $(5a - 6b) \times (3a - 4b) = 15a^2 - 38ab + 24b^2$.

The square of the sum of two numbers = sum of their squares + twice their product.

The square of the difference of two numbers = the sum of their squares - twice their product.

The product of the sum and difference of two numbers = the difference of their squares:

$$(a + b)^2 = a^2 + 2ab + b^2; (a - b)^2 = a^2 - 2ab + b^2;$$

$$(a + b) \times (a - b) = a^2 - b^2.$$

The square of half the sums of two quantities is equal to their product plus the square of half their difference: $\left(\frac{a+b}{2}\right)^2 = ab + \left(\frac{a-b}{2}\right)^2$.

The square of the sum of two quantities is equal to four times their products, plus the square of their difference: $(a + b)^2 = 4ab + (a - b)^2$.

The sum of the squares of two quantities equals twice their product, plus the square of their difference: $a^2 + b^2 = 2ab + (a - b)^2$.

The square of a trinomial = square of each term + twice the product of each term by each of the terms that follow it: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$; $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$.

The square of (any number + $\frac{1}{2}$) = square of the number + the number + $\frac{1}{4}$; = the number \times (the number + 1) + $\frac{1}{4}$; $(a + \frac{1}{2})^2 = a^2 + a + \frac{1}{4}$; $= a(a + 1) + \frac{1}{4}$. $(4\frac{1}{2})^2 = 4^2 + 4 + \frac{1}{4} = 4 \times 5 + \frac{1}{4} = 20\frac{1}{4}$.

The product of any number + $\frac{1}{2}$ by any other number + $\frac{1}{2}$ = product of the numbers + half their sum + $\frac{1}{4}$. $(a + \frac{1}{2}) \times (b + \frac{1}{2}) = ab + \frac{1}{2}(a + b) + \frac{1}{4}$. $4\frac{1}{2} \times 6\frac{1}{2} = 4 \times 6 + \frac{1}{2}(4 + 6) + \frac{1}{4} = 24 + 5 + \frac{1}{4} = 29\frac{1}{4}$.

Square, cube, 4th power, etc., of a binomial $a + b$.

$$(a + b)^2 = a^2 + 2ab + b^2; (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$

In each case the number of terms is one greater than the exponent of the power to which the binomial is raised.

2. In the first term the exponent of a is the same as the exponent of the power to which the binomial is raised, and it decreases by 1 in each succeeding term.

3. b appears in the second term with the exponent 1, and its exponent increases by 1 in each succeeding term.

4. The coefficient of the first term is 1.

5. The coefficient of the second term is the exponent of the power to which the binomial is raised.

6. The coefficient of each succeeding term is found from the next preceding term by multiplying its coefficient by the exponent of a , and dividing the product by a number greater by 1 than the exponent of b . (See Binomial Theorem, below.)

Parentheses. — When a parenthesis is preceded by a plus sign it may be removed without changing the value of the expression: $a + b + (a + b) = 2a + 2b$. When a parenthesis is preceded by a minus sign it may be removed if we change the signs of all the terms within the parenthesis: $1 - (a - b - c) = 1 - a + b + c$. When a parenthesis is within a parenthesis remove the inner one first: $a - [b - \{c - (d - e)\}] = a - [b - \{c - d + e\}] = a - [b - c + d - e] = a - b + c - d + e$.

A multiplication sign, \times , has the effect of a parenthesis, in that the operation indicated by it must be performed before the operations of addition or subtraction. $a + b \times a + b = a + ab + b$; while $(a + b) \times (a + b) = a^2 + 2ab + b^2$, and $(a + b) \times a + b = a^2 + ab + b$.

The absence of any sign between two parentheses, or between a quantity and a parenthesis, indicates that the parenthesis is to be multiplied by the quantity or parenthesis: $a(a + b + c) = a^2 + ab + ac$.

Division. — The quotient is positive when the dividend and divisor have like signs, and negative when they have unlike signs: $abc \div b = ac$; $abc \div -b = -ac$.

To divide a monomial by a monomial, write the dividend over the divisor with a line between them. If the expressions have common factors, remove the common factors:

$$a^2bx \div aby = \frac{a^2bx}{aby} = \frac{ax}{y}; \frac{a^4}{a^3} = a; \frac{a^3}{a^5} = \frac{1}{a^2} = a^{-2}.$$

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial: $(8ab - 12ac) \div 4a = 2b - 3c$.

To divide a polynomial by a polynomial, arrange both dividend and divisor in the order of the ascending or descending powers of some common letter, and keep this arrangement throughout the operation.

Divide the first term of the dividend by the first term of the divisor, and write the result as the first term of the quotient.

Multiply all the terms of the divisor by the first term of the quotient and subtract the product from the dividend. If there be a remainder, consider it as a new dividend and proceed as before: $(a^2 - b^2) \div (a + b)$.

$$\begin{array}{r} a^2 - b^2 \mid a + b. \\ a^2 + ab \mid a - b. \\ \hline - ab - b^2. \\ - ab - b^2. \\ \hline \end{array}$$

The difference of two equal odd powers of any two numbers is divisible by their difference and also by their sum:

$$(a^3 - b^3) \div (a - b) = a^2 + ab + b^2; (a^3 - b^3) \div (a + b) = a^2 - ab + b^2.$$

The difference of two equal even powers of two numbers is divisible by their difference and also by their sum: $(a^2 - b^2) \div (a - b) = a + b$.

The sum of two equal even powers of two numbers is not divisible by either the difference or the sum of the numbers; but when the exponent of each of the two equal powers is composed of an odd and an even factor, the sum of the given power is divisible by the sum of the powers expressed by the even factor. Thus $x^6 + y^6$ is not divisible by $x + y$ or by $x - y$, but is divisible by $x^2 + y^2$.

Simple equations. — An equation is a statement of equality between two expressions; as, $a + b = c + d$.

A simple equation, or equation of the first degree, is one which contains only the first power of the unknown quantity. If equal changes be made (by addition, subtraction, multiplication, or division) in both sides of an equation, the results will be equal.

Any term may be changed from one side of an equation to another, provided its sign be changed: $a + b = c + d$; $a = c + d - b$. To solve

an equation having one unknown quantity, transpose all the terms involving the unknown quantity to one side of the equation, and all the other terms to the other side; combine like terms, and divide both sides by the coefficient of the unknown quantity.

Solve $8x - 29 = 26 - 3x$. $8x + 3x = 29 + 26$; $11x = 55$; $x = 5$, ans.

Simple algebraic problems containing one unknown quantity are solved by making $x =$ the unknown quantity, and stating the conditions of the problem in the form of an algebraic equation, and then solving the equation. What two numbers are those whose sum is 48 and difference 14? Let $x =$ the smaller number, $x + 14$ the greater. $x + x + 14 = 48$. $2x = 34$, $x = 17$; $x + 14 = 31$, ans.

Find a number whose treble exceeds 50 as much as its double falls short of 40. Let $x =$ the number. $3x - 50 = 40 - 2x$; $5x = 90$; $x = 18$, ans. Proving, $54 - 50 = 40 - 36$.

Equations containing two unknown quantities. — If one equation contains two unknown quantities, x and y , an indefinite number of pairs of values of x and y may be found that will satisfy the equation, but if a second equation be given only one pair of values can be found that will satisfy both equations. Simultaneous equations, or those that may be satisfied by the same values of the unknown quantities, are solved by combining the equations so as to obtain a single equation containing only one unknown quantity. This process is called elimination.

Elimination by addition or subtraction. — Multiply the equation by such numbers as will make the coefficients of one of the unknown quantities equal in the resulting equation. Add or subtract the resulting equations according as they have unlike or like signs.

$$\text{Solve } \begin{cases} 2x + 3y = 7. \\ 4x - 5y = 3. \end{cases} \quad \begin{array}{l} \text{Multiply by 2: } 4x + 6y = 14 \\ \text{Subtract: } \underline{4x - 5y = 3} \end{array} \quad 11y - 11; y = 1.$$

Substituting value of y in first equation, $2x + 3 = 7$; $x = 2$.

Elimination by substitution. — From one of the equations obtain the value of one of the unknown quantities in terms of the other. Substitute for this unknown quantity its value in the other equation and reduce the resulting equations.

$$\text{Solve } \begin{cases} 2x + 3y = 8. & (1). \\ 3x + 7y = 7. & (2). \end{cases} \quad \text{From (1) we find } x = \frac{8 - 3y}{2}.$$

$$\text{Substitute this value in (2): } 3\left(\frac{8 - 3y}{2}\right) + 7y = 7; = 24 - 9y + 14y = 14,$$

whence $y = -2$. Substitute this value in (1): $2x - 6 = 8$; $x = 7$.

Elimination by comparison. — From each equation obtain the value of one of the unknown quantities in terms of the other. Form an equation from these equal values, and reduce this equation.

$$\text{Solve } 2x - 9y = 11. \quad (1) \text{ and } 3x - 4y = 7. \quad (2). \quad \text{From (1) we find } x = \frac{11 + 9y}{2}. \quad \text{From (2) we find } x = \frac{7 + 4y}{3}.$$

$$\text{Equating these values of } x, \frac{11 + 9y}{2} = \frac{7 + 4y}{3}; 19y = -19; y = -1.$$

Substitute this value of y in (1): $2x + 9 = 11$; $x = 1$.

If three simultaneous equations are given containing three unknown quantities, one of the unknown quantities must be eliminated between two pairs of the equations; then a second between the two resulting equations.

Quadratic equations. — A quadratic equation contains the square of the unknown quantity, but no higher power. A pure quadratic contains the square only; an affected quadratic both the square and the first power.

To solve a pure quadratic, collect the unknown quantities on one side, and the known quantities on the other; divide by the coefficient of the unknown quantity and extract the square root of each side of the resulting equation.

$$\text{Solve } 3x^2 - 15 = 0. \quad 3x^2 = 15; x^2 = 5; x = \sqrt{5}.$$

A root like $\sqrt{5}$, which is indicated, but which can be found only approximately, is called a *surd*.

Solve $3x^2 + 15 = 0$. $3x = -15$; $x^2 = -5$; $x = \sqrt{-5}$.

The square root of -5 cannot be found even approximately, for the square of any number positive or negative is positive; therefore a root which is indicated, but cannot be found even approximately, is called *imaginary*.

To solve an *affected quadratic*, 1. Convert the equation into the form $a^2x^2 \pm 2abx = c$, multiplying or dividing the equation if necessary, so as to make the coefficient of x^2 a square number.

2. Complete the square of the first member of the equation, so as to convert it to the form of $a^2x^2 \pm 2abx + b^2$, which is the square of the binomial $ax \pm b$, as follows: add to each side of the equation the square of the quotient obtained by dividing the second term by twice the square root of the first term.

3. Extract the square root of each side of the resulting equation.

Solve $3x^2 - 4x = 32$. To make the coefficient of x^2 a square number, multiply by 3: $9x^2 - 12x = 96$; $12x \div (2 \times 3x) = 2$; $2^2 = 4$.

Complete the square: $9x^2 - 12x + 4 = 100$. Extract the root: $3x - 2 = \pm 10$, whence $x = 4$ or $-2\frac{2}{3}$. The square root of 100 is either $+10$ or -10 , since the square of -10 as well as $+10^2 = 100$.

Every affected quadratic may be reduced to the form $ax^2 + bx + c = 0$.

The solution of this equation is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Problems involving quadratic equations have apparently two solutions, as a quadratic has two roots. Sometimes both will be true solutions, but generally one only will be a solution and the other be inconsistent with the conditions of the problem.

The sum of the squares of two consecutive positive numbers is 481. Find the numbers.

Let $x =$ one number, $x + 1$ the other. $x^2 + (x + 1)^2 = 481$. $2x^2 + 2x + 1 = 481$.

$x^2 + x = 240$. Completing the square, $x^2 + x + 0.25 = 240.25$. Extracting the root we obtain $x + 0.5 = \pm 15.5$; $x = 15$ or -16 . The negative root -16 is inconsistent with the conditions of the problem.

Quadratic equations containing two unknown quantities require different methods for their solution, according to the form of the equations. For these methods reference must be made to works on algebra.

Theory of exponents. — $\sqrt[n]{a}$ when n is a positive integer is one of n equal factors of a . $\sqrt[n]{a^m}$ means a is to be raised to the m th power and the n th root extracted.

$\left(\sqrt[n]{a}\right)^m$ means that the n th root of a is to be taken and the result raised to the m th power.

$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$. When the exponent is a fraction, the numerator indicates a power, and the denominator a root. $a^{6/2} = \sqrt{a^6} = a^3$; $a^{3/2} = \sqrt{a^3} = a^{1.5}$.

To extract the root of a quantity raised to an indicated power, divide the exponent by the index of the required root; as,

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}; \quad \sqrt{a^6} = a^{6/2} = a^3.$$

Subtracting 1 from the exponent of a is equivalent to dividing by a :

$$a^{2-1} = a^1 = a; \quad a^{1-1} = a^0 = \frac{a}{a} = 1; \quad a^{0-1} = a^{-1} = \frac{1}{a}; \quad a^{-1-1} = a^{-2} = \frac{1}{a^2}.$$

A number with a negative exponent denotes the reciprocal of the number with the corresponding positive exponent.

A factor under the radical sign whose root can be taken may, by having the root taken, be removed from under the radical sign:

$$\sqrt{a^2b} = \sqrt{a^2} \times \sqrt{b} = a\sqrt{b},$$

A factor outside the radical sign may be raised to the corresponding power and placed under it:

$$\sqrt{\frac{a}{b}} = \sqrt{\frac{ab}{b^2}} = \sqrt{ab \times \frac{1}{b^2}} = \frac{1}{b} \sqrt{ab}; \quad \sqrt{\frac{a}{b^2}} = \frac{1}{b} \sqrt{a}.$$

Binomial Theorem. — To obtain any power, as the n th, of an expression of the form $x + a$

$$(a + x)^n = a^n + na^{n-1}x + \frac{n(n-1)a^{n-2}}{1.2}x^2 + \frac{n(n-1)(n-2)a^{n-3}}{1.2.3}x^3 + \text{etc.}$$

The following laws hold for any term in the expansion of $(a + x)^n$.

The exponent of x is less by one than the number of terms.

The exponent of a is n minus the exponent of x .

The last factor of the numerator is greater by one than the exponent of a .

The last factor of the denominator is the same as the exponent of x .

In the r th term the exponent of x will be $r - 1$.

The exponent of a will be $n - (r - 1)$, or $n - r + 1$.

The last factor of the numerator will be $n - r + 2$.

The last factor of the denominator will be $r - 1$.

Hence the r th term = $\frac{n(n-1)(n-2) \dots (n-r+2)}{1.2.3 \dots (r-1)} a^{n-r+1} x^{r-1}$.

GEOMETRICAL PROBLEMS.

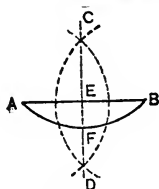


FIG. 1.

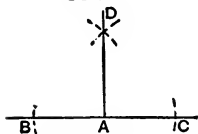


FIG. 2.

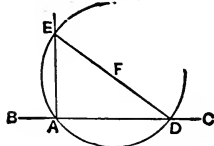


FIG. 3.

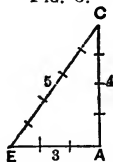


FIG. 4.

1. To bisect a straight line, or an arc of a circle (Fig. 1). — From the ends A, B , as centres, describe arcs intersecting at C and D , and draw a line through C and D which will bisect the line at E or the arc at F .

2. To draw a perpendicular to a straight line, or a radial line to a circular arc. — Same as in Problem 1. CD is perpendicular to the line AB , and also radial to the arc.

3. To draw a perpendicular to a straight line from a given point in that line (Fig. 2). — With any radius, from the given point A in the line BC , cut the line at B and C . With a longer radius describe arcs from B and C , cutting each other at D , and draw the perpendicular DA .

4. From the end A of a given line AD to erect a perpendicular AE (Fig. 3). — From any centre F , above AD , describe a circle passing through the given point A , and cutting the given line at D . Draw DF and produce it to cut the circle at E , and draw the perpendicular AE .

Second Method (Fig. 4). — From the given point A set off a distance AE equal to three parts, by any scale; and on the centres A and E , with radii of four and five parts respectively, describe arcs intersecting at C . Draw the perpendicular AC .

NOTE. — This method is most useful on very large scales, where straight edges are inapplicable. Any multiples of the numbers 3, 4, 5 may be taken with the same effect, as 6, 8, 10, or 9, 12, 15.

5. To draw a perpendicular to a straight line from any point without it (Fig. 5). — From the point *A*, with a sufficient radius cut the given line at *F* and *G*, and from these points describe arcs cutting at *E*. Draw the perpendicular *AE*.

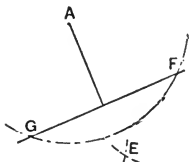


FIG. 5.

6. To draw a straight line parallel to a given line, at a given distance apart (Fig. 6). — From the centres *A*, *B*, in the given line, with the given distance as radius, describe arcs *C*, *D*, and draw the parallel lines *CD* touching the arcs.

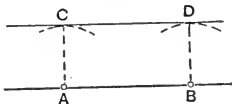


FIG. 6.

7. To divide a straight line into a number of equal parts (Fig. 7). — To divide the line *AB* into, say, five parts, draw the line *AC* at an angle from *A*; set off five equal parts; draw *B5* and draw parallels to it from the other points of division in *AC*. These parallels divide *AB* as required.

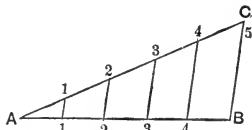


FIG. 7.

NOTE. — By a similar process a line may be divided into a number of unequal parts; setting off divisions on *AC*, proportional by a scale to the required divisions, and drawing parallels cutting *AB*. The triangles *A11*, *A22*, *A33*, etc., are *similar triangles*.

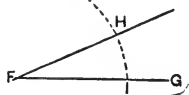
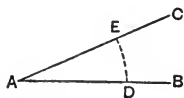


FIG. 8.

8. Upon a straight line to draw an angle equal to a given angle (Fig. 8). — Let *A* be the given angle and *FG* the line. From the point *A* with any radius describe the arc *DE*. From *F* with the same radius describe *IH*. Set off the arc *IH* equal to *DE*, and draw *FH*. The angle *F* is equal to *A*, as required.

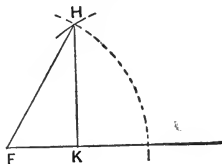


FIG. 9.

9. To draw angles of 60° and 30° (Fig. 9). — From *F*, with any radius *FI*, describe an arc *IH*; and from *I*, with the same radius, cut the arc at *H* and draw *FH* to form the required angle *IFH*. Draw the perpendicular *HK* to the base line to form the angle of 30° *FHK*.

10. To draw an angle of 45° (Fig. 10). — Set off the distance *FI*; draw the perpendicular *IH* equal to *IF*, and join *HF* to form the angle at *F*. The angle at *H* is also 45° .

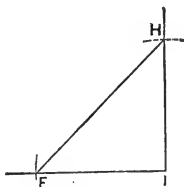


FIG. 10.

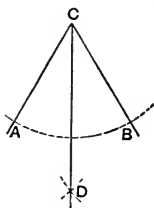


FIG. 11.

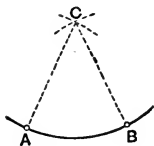


FIG. 12.

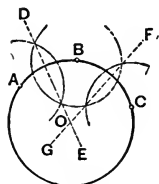


FIG. 13.

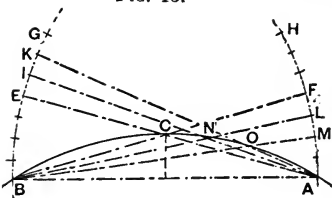


FIG. 14.

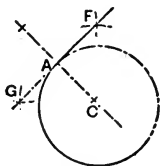


FIG. 15.

11. To bisect an angle (Fig. 11). — Let ACB be the angle; with C as a centre draw an arc cutting the sides at A, B . From A and B as centres, describe arcs cutting each other at D . Draw CD , dividing the angle into two equal parts.

12. Through two given points to describe an arc of a circle with a given radius (Fig. 12). — From the points A and B as centres, with the given radius, describe arcs cutting at C ; and from C with the same radius describe an arc AB .

13. To find the centre of a circle or of an arc of a circle (Fig. 13). — Select three points, A, B, C , in the circumference, well apart; with the same radius describe arcs from these three points, cutting each other, and draw the two lines, DE, FG , through their intersections. The point O , where they cut, is the centre of the circle or arc.

To describe a circle passing through three given points. — Let A, B, C be the given points, and proceed as in last problem to find the centre O , from which the circle may be described.

14. To describe an arc of a circle passing through three given points when the centre is not available (Fig. 14). — From the extreme points A, B , as centres, describe arcs AH, BG . Through the third point C draw AE, BF , cutting the arcs. Divide AF and BE into any number of equal parts, and set off a series of equal parts of the same length on the upper portions of the arcs beyond the points EF . Draw straight lines, BL, BM , etc., to the divisions in AF , and AI, AK , etc., to the divisions in EG . The successive intersections N, O , etc., of these lines are points in the circle required between the given points A and C , which may be drawn in; similarly the remaining part of the curve BC may be described, (See also Problem 54.)

15. To draw a tangent to a circle from a given point in the circumference (Fig. 15). — Through the given point A , draw the radial line AC , and a perpendicular to it, FG , which is the tangent required.

16. To draw tangents to a circle from a point without it (Fig. 16). — From A , with the radius AC , describe an arc BCD , and from C , with a radius equal to the diameter of the circle, cut the arc at B, D . Join BC, CD , cutting the circle at E, F , and draw AE, AF , the tangents.

NOTE. — When a tangent is already drawn, the exact point of contact may be found by drawing a perpendicular to it from the centre.

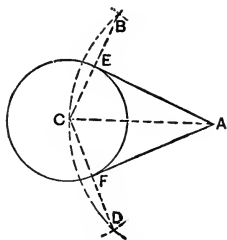


FIG. 16.

17. Between two inclined lines to draw a series of circles touching these lines and touching each other (Fig. 17). — Bisect the inclination of the given lines AB, CD , by the line NO . From a point P in this line draw the perpendicular PB to the line AB , and on P describe the circle BD , touching the lines and cutting the centre line at E . From E draw EF perpendicular to the centre line, cutting AB at F , and from F describe an arc EG , cutting AB at G . Draw GH parallel to BP , giving H , the centre of the next circle, to be described with the radius HE , and so on for the next circle IN .

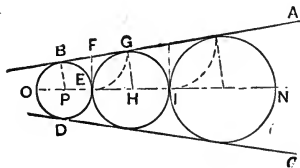


FIG. 17.

Inversely, the largest circle may be described first, and the smaller ones in succession. This problem is of frequent use in scroll-work.

18. Between two inclined lines to draw a circular segment tangent to the lines and passing through a point F on the line FC which bisects the angle of the lines (Fig. 18). — Through F draw DA at right angles to FC ; bisect the angles A and D , as in Problem 11, by lines cutting at C , and from C with radius CF draw the arc HFG required.

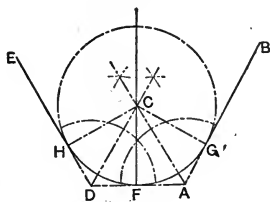


FIG. 18.

19. To draw a circular arc that will be tangent to two given lines AB and CD inclined to one another, one tangential point E being given (Fig. 19). — Draw the centre line GF . From E draw EF at right angles to AB ; then F is the centre of the circle required.

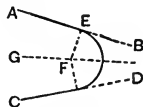


FIG. 19.

20. To describe a circular arc joining two circles, and touching one of them at a given point (Fig. 20). — To join the circles AB, FG , by an arc touching one of them at F , draw the radius EF , and produce it both ways. Set off FH equal to the radius AC of the other circle; join CH and bisect it with the perpendicular LI , cutting EF at I . On the centre I , with radius IF , describe the arc FA as required.

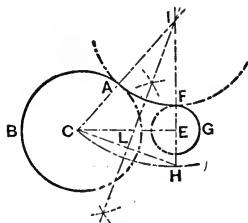


FIG. 20.

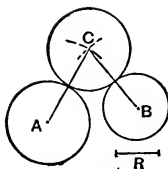


FIG. 21.

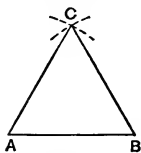


FIG. 22.

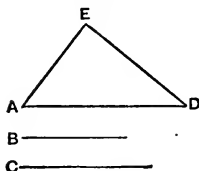


FIG. 23.

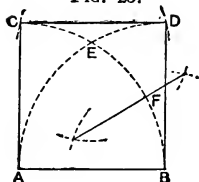


FIG. 24.

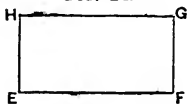


FIG. 25.

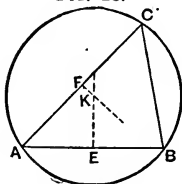


FIG. 26.

21. To draw a circle with a given radius R that will be tangent to two given circles A and B (Fig. 21). — From centre of circle A with radius equal R plus radius of A , and from centre of B with radius equal to R + radius of B , draw two arcs cutting each other in C , which will be the centre of the circle required.

22. To construct an equilateral triangle, the sides being given (Fig. 22). — On the ends of one side, A , B , with AB as radius, describe arcs cutting at C , and draw AC , CB .

23. To construct a triangle of unequal sides (Fig. 23). — On either end of the base AD , with the side B as radius, describe an arc; and with the side C as radius, on the other end of the base as a centre, cut the arc at E . Join AE , DE .

24. To construct a square on a given straight line AB (Fig. 24). — With AB as radius and A and B as centres, draw arcs AD and BC , intersecting at E . Bisect EB at F . With E as centre and EF as radius, cut the arcs AD and BC in D and C . Join AC , CD , and DB to form the square.

25. To construct a rectangle with given base EF and height EH (Fig. 25). — On the base EF draw the perpendiculars EH , FG equal to the height; and join GH .

26. To describe a circle about a triangle (Fig. 26). — Bisect two sides AB , AC of the triangle at E , F , and from these points draw perpendiculars cutting at K . On the centre K , with the radius KA , draw the circle ABC .

27. To inscribe a circle in a triangle (Fig. 27). — Bisect two of the angles A , C , of the triangle by

lines cutting at D ; from D draw a perpendicular $D E$ to any side, and with $D E$ as radius describe a circle.

When the triangle is equilateral, draw a perpendicular from one of the angles to the opposite side, and from the side set off one third of the perpendicular.

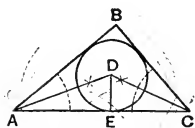


FIG. 27.

28. To describe a circle about a square, and to inscribe a square in a circle (Fig. 28). — To describe the circle, draw the diagonals $A B$, $C D$ of the square, cutting at E . On the centre E , with the radius $A E$, describe the circle.

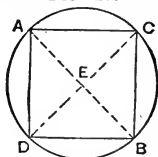


FIG. 28.

To inscribe the square. — Draw the two diameters, $A B$, $C D$, at right angles, and join the points A , B , C , D , to form the square.

NOTE. — In the same way a circle may be described about a rectangle.

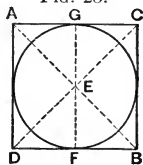


FIG. 29.

29. To inscribe a circle in a square (Fig. 29). — To inscribe the circle, draw the diagonals $A B$, $C D$ of the square, cutting at E ; draw the perpendicular $E F$ to one side, and with the radius $E F$ describe the circle.

30. To describe a square about a circle (Fig. 30). — Draw two diameters $A B$, $C D$ at right angles. With the radius of the circle and A , B , C and D as centres, draw the four half circles which cross one another in the corners of the square.

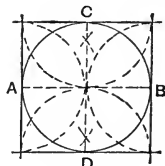


FIG. 30.

31. To inscribe a pentagon in a circle (Fig. 31). — Draw diameters $A C$, $B D$ at right angles, cutting at o . Bisect $A o$ at E , and from E , with radius $E B$, cut $A C$ at F ; from B , with radius $B F$, cut the circumference at G , H , and with the same radius step round the circle to I and K ; join the points so found to form the pentagon.

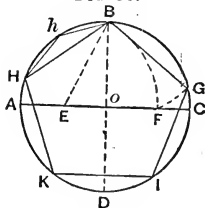


FIG. 31.

32. To construct a pentagon on a given line $A B$ (Fig. 32). — From B erect a perpendicular $B C$ half the length of $A B$; join $A C$ and prolong it to D , making $C D = B C$. Then $B D$ is the radius of the circle circumscribing the pentagon. From A and B as centres, with $B D$ as radius, draw arcs cutting each other in O , which is the centre of the circle.

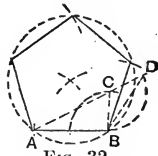


FIG. 32.

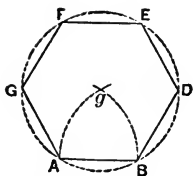


FIG. 33.

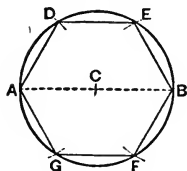


FIG. 34.

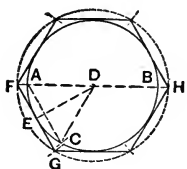


FIG. 35.

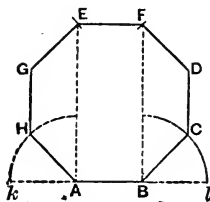


FIG. 36.

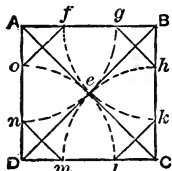


FIG. 37.

33. To construct a hexagon upon a given straight line (Fig. 33). — From A and B , the ends of the given line, with radius AB , describe arcs cutting at g ; from g , with the radius gA , describe a circle; with the same radius set off the arcs AG , GF , and BD , DE . Join the points so found to form the hexagon. The side of a hexagon = radius of its circumscribed circle.

34. To inscribe a hexagon in a circle (Fig. 34). — Draw a diameter ACB . From A and B as centres, with the radius of the circle AC , cut the circumference, at D , E , F , G , and draw AD , DE , etc., to form the hexagon. The radius of the circle is equal to the side of the hexagon; therefore the points D , E , etc., may also be found by stepping the radius six times round the circle. The angle between the diameter and the sides of a hexagon and also the exterior angle between a side and an adjacent side prolonged is 60 degrees; therefore a hexagon may conveniently be drawn by the use of a 60-degree triangle.

35. To describe a hexagon about a circle (Fig. 35). — Draw a diameter ADB , and with the radius AD , on the centre A , cut the circumference at C ; join AC , and bisect it with the radius DE ; through E draw FG , parallel to AC , cutting the diameter at F , and with the radius DF describe the circumscribing circle FH . Within this circle describe a hexagon by the preceding problem. A more convenient method is by use of a 60-degree triangle. Four of the sides make angles of 60 degrees with the diameter, and the other two are parallel to the diameter.

36. To describe an octagon on a given straight line (Fig. 36). — Produce the given line AB both ways, and draw perpendiculars AE , BF ; bisect the external angles A and B by the lines AH , BC , which make equal to AB . Draw CD and HG parallel to AE , and equal to AB ; from the centres G , D , with the radius AB , cut the perpendiculars at E , F , and draw EF to complete the octagon.

37. To convert a square into an octagon (Fig. 37). — Draw the diagonals of the square cutting at e ; from the corners A , B , C , D , with Ae as radius, describe arcs cutting the sides at gn , fk , hm , and ol , and join the points so found to form the octagon. Adjacent sides of an octagon make an angle of 135 degrees.

38. To inscribe an octagon in a circle (Fig. 38). — Draw two diameters, $A C, B D$ at right angles; bisect the arcs $A B, B C$, etc., at $e f$, etc., and join $A e, e B$, etc., to form the octagon.

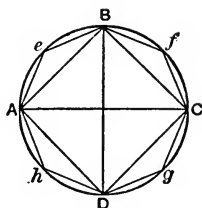


FIG. 38.

39. To describe an octagon about a circle (Fig. 39). — Describe a square about the given circle $A B$; draw perpendiculars $h k$, etc., to the diagonals, touching the circle to form the octagon.

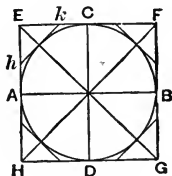


FIG. 39.

40. To describe a polygon of any number of sides upon a given straight line (Fig. 40). — Produce the given line $A B$, and on A , with the radius $A B$, describe a semicircle; divide the semi-circumference into as many equal parts as there are to be sides in the polygon — say, in this example, five sides. Draw lines from A through the divisional points D, b , and c , omitting one point a ; and on the centres B, D , with the radius $A B$, cut $A b$ at E and $A c$ at F . Draw $D E, E F, F B$ to complete the polygon.

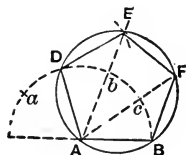


FIG. 40.

41. To inscribe a circle within a polygon (Figs. 41, 42). — When the polygon has an even number of sides (Fig. 41), bisect two opposite sides at A and B ; draw $A B$, and bisect it at C by a diagonal $D E$, and with the radius $C A$ describe the circle.

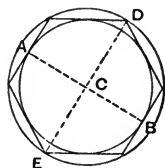


FIG. 41.

When the number of sides is odd (Fig. 42), bisect two of the sides at A and B , and draw lines $A E, B D$ to the opposite angles, intersecting at C ; from C , with the radius $C A$, describe the circle.

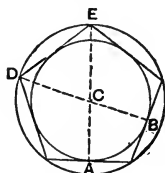


FIG. 42.

42. To describe a circle without a polygon (Figs. 41, 42). — Find the centre C as before, and with the radius $C D$ describe the circle.

43. To inscribe a polygon of any number of sides within a circle (Fig. 43). — Draw the diameter $A B$ and through the centre E draw the

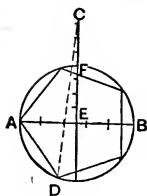


FIG. 43.

perpendicular EC , cutting the circle at F . Divide EF into four equal parts, and set off three parts equal to those from F to C . Divide the diameter AB into as many equal parts as the polygon is to have sides; and from C draw CD , through the second point of division, cutting the circle at D . Then AD is equal to one side of the polygon, and by stepping round the circumference with the length AD the polygon may be completed.

Table of Polygonal Angles.

| Number of Sides. | Angle at Centre. | Number of Sides. | Angle at Centre. | Number of Sides. | Angle at Centre. |
|------------------|------------------|------------------|------------------|------------------|------------------|
| No. | Degrees. | No. | Degrees. | No. | Degrees. |
| 3 | 120 | 9 | 40 | 15 | 24 |
| 4 | 90 | 10 | 36 | 16 | $22\frac{1}{2}$ |
| 5 | 72 | 11 | $32\frac{8}{11}$ | 17 | $21\frac{3}{17}$ |
| 6 | 60 | 12 | 30 | 18 | 20 |
| 7 | $51\frac{3}{7}$ | 13 | $27\frac{9}{13}$ | 19 | 19 |
| 8 | 45 | 14 | $25\frac{5}{7}$ | 20 | 18 |

In this table the angle at the centre is found by dividing 360 degrees, the number of degrees in a circle, by the number of sides in the polygon; and by setting off round the centre of the circle a succession of angles by means of the protractor, equal to the angle in the table due to a given number of sides, the radii so drawn will divide the circumference into the same number of parts.

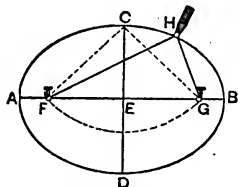


FIG. 44.

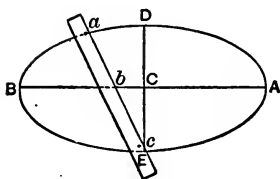


FIG. 45.

44. To describe an ellipse when the length and breadth are given (Fig. 44). — AB , transverse axis; CD , conjugate axis; F, G , foci. The sum of the distances from C to F and G , also the sum of the distances from F and G to any other point in the curve, is equal to the transverse axis. From the centre C , with AE as radius, cut the axis AB at F and G , the foci; fix a couple of pins into the axis at F and G , and loop on a thread or cord upon them equal in length to the axis AB , so as when stretched to reach to the extremity C of the conjugate axis, as shown in dot-lining. Place a pencil inside the cord as at H , and guiding the pencil in this way, keeping the cord equally in tension, carry the pencil round the pins F, G , and so describe the ellipse.

NOTE. — This method is employed in setting off elliptical garden-plots, walks, etc.

2d Method (Fig. 45). — Along the straight edge of a slip of stiff paper mark off a distance ac equal to AC , half the transverse axis; and from the same point a distance ab equal to CE , half the conjugate axis.

Place the slip so as to bring the point b on the line AB of the transverse axis, and the point c on the line DE ; and set off on the drawing the position of the point a . Shifting the slip so that the point b travels on the transverse axis, and the point c on the conjugate axis, any number of points in the curve may be found, through which the curve may be traced.

3d Method (Fig. 46). — The action of the preceding method may be embodied so as to afford the means of describing a large curve continuously by means of a bar mk , with steel points m, l, k , riveted into brass slides adjusted to the length of the semi-axis and fixed with set-screws. A rectangular cross EG , with guiding-slots is placed, coinciding with the two axes of the ellipse AC and BH . By sliding the points k, l in the slots; and carrying round the point m , the curve may be continuously described. A pen or pencil may be fixed at m .

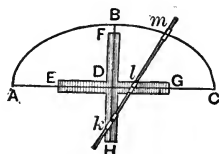


FIG. 46.

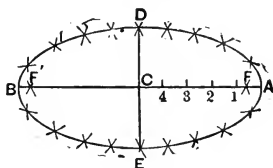


FIG. 47.

4th Method (Fig. 47). — Bisect the transverse axis at C , and through C draw the perpendicular DE , making CD and CE each equal to half the conjugate axis. From D or E , with the radius AC , cut the transverse axis at F, F' , for the foci. Divide AC into a number of parts at the points 1, 2, 3, etc. With the radius AI on F and F' as centres, describe arcs, and with the radius BI on the same centres cut these arcs as shown. Repeat the operation for the other divisions of the transverse axis. The series of intersections thus made are points in the curve, through which the curve may be traced.

5th Method (Fig. 48). — On the two axes AB, DE as diameters, on centre C , describe circles; from a number of points a, b , etc., in the circumference AFB , draw radii cutting the inner circle at a', b' , etc. From a, b , etc., draw perpendiculars to AB ; and from a', b' , etc., draw parallels to AB , cutting the respective perpendiculars at n, o , etc. The intersections are points in the curve, through which the curve may be traced.

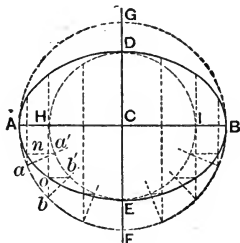


FIG. 48.

6th Method (Fig. 49). — When the transverse and conjugate diameters are given, AB, CD , draw the tangent EF parallel to AB . Produce CD , and on the centre G with the radius of half AB , describe a semicircle HDK ; from the centre G draw any number of straight lines to the points E, r , etc., in the line EF , cutting the circumference at l, m, n , etc.; from the centre O of the ellipse draw straight lines to the points E, r , etc.; and from the points l, m, n , etc., draw parallels to GC , cutting the lines OE, Or , etc., at L, M, N , etc.

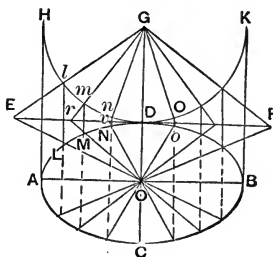


FIG. 49.

A simple method of determining the radii of curvature is illustrated in Fig. 53. Draw the straight lines af and ac , forming any angle at a . With a as a centre, and with radii ab and ac , respectively, equal to the semi-minor and semi-major axes, draw the arcs be and cd . Join ed , and through b and c respectively draw bg and cf parallel to ed , intersecting ac at g , and af at f ; af is the radius of curvature at the vertex of the minor axis; and ag the radius of curvature at the vertex of the major axis.

Lay off dh (Fig. 53) equal to one eighth of bd . Join eh , and draw ck and bl parallel to eh . Take ak for the longest radius ($= R$), al for the shortest radius ($= r$), and the arithmetical mean, or one half the sum of the semi-axes, for the third radius ($= p$), and employ these radii for the eight-centred oval as follows:

Let ab and cd (Fig. 54) be the major and minor axes. Lay off ae equal to r , and af equal to p ; also lay off cg equal to R , and ch equal to p . With g as a centre and gh as a radius, draw the arc hk ; with the centre e and radius ef draw the arc fk , intersecting hk at k . Draw the line gk and produce it, making gl equal to R . Draw ke and produce it, making km equal to p . With the centre g and radius gc ($= R$) draw the arc cl ; with the centre k and radius kl ($= p$) draw the arc lm , and with the centre e and radius em ($= r$) draw the arc ma .

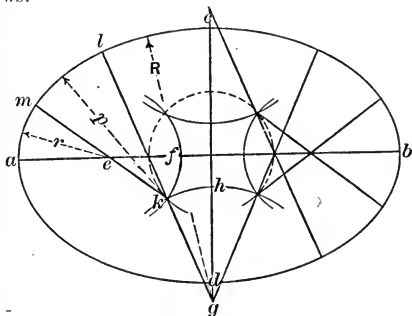


FIG. 54.

The remainder of the work is symmetrical with respect to the axes.

46. The Parabola. — A parabola (DAC , Fig. 55) is a curve such that every point in the curve is equally distant from the directrix KL and the focus F . The focus lies in the axis AB drawn from the vertex or head of the curve A , so as to divide the figure into two equal parts. The vertex A is equidistant from the directrix and the focus, or $Ae = AF$. Any line parallel to the axis is a diameter. A straight line, as EG or DC , drawn across the figure at right angles to the axis is a double ordinate, and either half of it is an ordinate. The ordinate to the axis EFG , drawn through the focus, is called the parameter of the axis. A segment of the axis, reckoned from the vertex, is an abscissa of the axis, and it is an abscissa of the ordinate drawn from the base of the abscissa. Thus, AB is an abscissa of the ordinate BC .

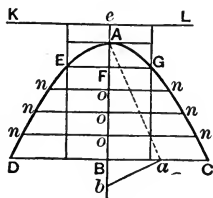


FIG. 55.

Abscissæ of a parabola are as the squares of their ordinates.

To describe a parabola when an abscissa and its ordinate are given (Fig. 55). — Bisect the given ordinate BC at a , draw Aa , and then ab perpendicular to it, meeting the axis at b . Set off Ae , AF , each equal to Bb ; and draw KL perpendicular to the axis. Then KL is the directrix and F is the focus. Through F and any number of points, o, o , etc., in the axis, draw double ordinates, no, n , etc., and from the centre F , with the radii Fe, oe , etc., cut the respective ordinates at E, G, n, n , etc. The curve may be traced through these points as shown.

2d Method: By means of a square and a cord (Fig. 56). — Place a

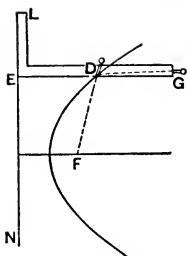


FIG. 56.

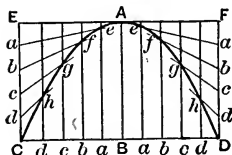


FIG. 57.

47. The Hyperbola (Fig. 58). — A hyperbola is a plane curve, such that the difference of the distances from any point of it to two fixed points is equal to a given distance. The fixed points are called the foci.

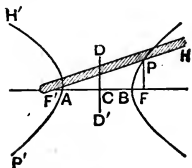


FIG. 58.

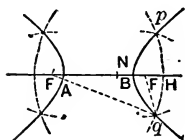


FIG. 59.

Then with F as a centre and NH as a radius describe an arc intersecting the arc before described at p and q . These will be points of the hyperbola, for $F'q - Fq$ is equal to the transverse axis AB .

If, with F as a centre and $F'H$ as a radius, an arc be described, and a second arc be described with F' as a centre and NH as a radius, two points in the other branch of the curve will be determined. Hence, by changing the centres, each pair of radii will determine two points in each branch.

The Equilateral Hyperbola. — The transverse axis of a hyperbola is

straight-edge to the directrix EN , and apply to it a square LEG . Fasten to the end G one end of a thread or cord equal in length to the edge EG , and attach the other end to the focus F ; slide the square along the straight-edge, holding the cord taut against the edge of the square by a pencil D , by which the curve is described.

3d Method: When the height and the base are given (Fig. 57). — Let AB be the given axis, and CD a double ordinate or base; to describe a parabola of which the vertex is at A . Through A draw EF parallel to CD , and through C and D draw CE and DF parallel to the axis. Divide BC and BD into any number of equal parts, say five, at a, b , etc., and divide CE and DF into the same number of parts. Through the points a, b, c, d in the base CD on each side of the axis draw perpendiculars, and through a, b, c, d in CE and DF draw lines to the vertex A , cutting the perpendiculars at e, f, g, h . These are points in the parabola, and the curve $CAAD$ may be traced as shown, passing through them.

To construct a hyperbola. — Let F' and F be the foci, and $F'F$ the distance between them. Take a ruler longer than the distance $F'F$, and fasten one of its extremities at the focus F' . At the other extremity, H , attach a thread of such a length that the length of the ruler shall exceed the length of the thread by a given distance AB . Attach the other extremity of the thread at the focus F .

Press a pencil, P , against the ruler, and keep the thread constantly tense, while the ruler is turned around F' as a centre. The point of the pencil will describe one branch of the curve.

2d Method: By points (Fig. 59). — From the focus F' lay off a distance $F'N$ equal to the transverse axis, or distance between the two branches of the curve, and take any other distance, as $F'H$, greater than $F'N$.

With F' as a centre and $F'H$ as a radius describe the arc of a circle.

the distance, on a line joining the foci, between the two branches of the curve. The conjugate axis is a line perpendicular to the transverse axis, drawn from its centre, and of such a length that the diagonal of the rectangle of the transverse and conjugate axes is equal to the distance between the foci. The diagonals of this rectangle, indefinitely prolonged, are the *asymptotes* of the hyperbola, lines which the curve continually approaches, but touches only at an infinite distance. If these asymptotes are perpendicular to each other, the hyperbola is called a *rectangular* or *equilateral hyperbola*. It is a property of this hyperbola that if the asymptotes are taken as axes of a rectangular system of coördinates (see Analytical Geometry), the product of the abscissa and ordinate of any point in the curve is equal to the product of the abscissa and ordinate of any other point; or, if p is the ordinate of any point and v its abscissa, and p_1 , and v_1 are the ordinate and abscissa of any other point, $pv = p_1v_1$; or $pv = a$ constant.

48. The Cycloid (Fig. 60). — If a circle A d be rolled along a straight line A 6, any point of the circumference as A will describe a curve, which is called a cycloid. The circle is called the generating circle, and A the generating point.

To draw a cycloid. —

Divide the circumference of the generating circle into an even number of equal parts, as A 1, 12, etc., and set off these distances on the base. Through the points 1, 2, 3, etc., on the circle draw horizontal lines, and on them set off distances $1a = A1$, $2b = A2$, $3c = A3$, etc. The points A , a , b , c , etc., will be points in the cycloid, through which draw the curve.

49. The Epicycloid (Fig. 61) is generated by a point D in one circle D C rolling upon the circumference of another circle A C B , instead of on a flat surface or line; the former being the generating circle, and the latter the fundamental circle. The generating circle is shown in four positions, in which the generating point is successively marked D , D' , D'' , D''' . A D''' B is the epicycloid.

50. The Hypocycloid (Fig. 62) is generated by a point in the generating circle rolling on the inside of the fundamental circle.

When the generating circle = radius of the other circle, the hypocycloid becomes a straight line.

51. The Tractrix or Schiele's anti-friction curve (Fig. 63). — R is the radius of the shaft, C , 1, 2, etc., the axis. From O set off on R a small distance, oa ; with radius R and centre a cut the axis at 1, join a 1, and set off a like small distance ab ; from b with radius R cut axis at 2, join b 2, and so on, thus finding points o , a , b , c , d , etc., through which the curve is to be drawn.

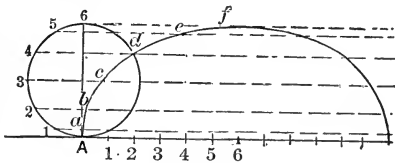


FIG. 60.

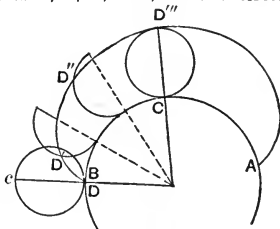


FIG. 61.

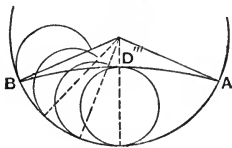


FIG. 62.

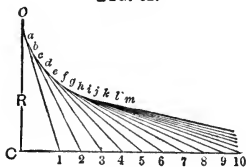


FIG. 63.

52. The Spiral. — The spiral is a curve described by a point which moves along a straight line according to any given law, the line at the same time having a uniform angular motion. The line is called the radius vector.

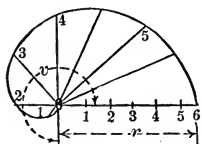


FIG. 64.

the distances 1, 2, 3, 4, etc., corresponding to the scale upon which the curve is drawn, as shown in Fig. 64.

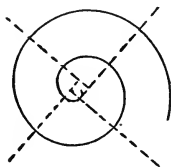


FIG. 65.

If the radius vector increases directly as the measuring angle, the spires, or parts described in each revolution, thus gradually increasing their distance from each other, the curve is known as the spiral of Archimedes (Fig. 64).

This curve is commonly used for cams. To describe it draw the radius vector in several different directions around the centre, with equal angles between them; set off

In the common spiral (Fig. 64) the pitch is uniform; that is, the spires are equidistant. Such a spiral is made by rolling up a belt of uniform thickness.

To construct a spiral with four centres (Fig. 65). — Given the pitch of the spiral, construct a square about the centre, with the sum of the four sides equal to the pitch. Prolong the sides in one direction as shown; the corners are the centres for each arc of the external angles, forming a quadrant of a spire.

53. To find the diameter of a circle into which a certain number of rings will fit on its inside (Fig. 66). — For instance, what is the diameter of a circle into which twelve $\frac{1}{2}$ -inch rings will fit, as per sketch? Assume that we have found the diameter of the required circle, and have drawn

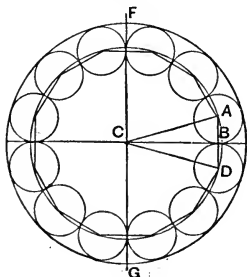


FIG. 66.

the rings inside of it. Join the centres of the rings by straight lines, as shown: we then obtain a regular polygon with 12 sides, each side being equal to the diameter of a given ring. We have now to find the diameter of a circle circumscribed about this polygon, and add the diameter of one ring to it; the sum will be the diameter of the circle into which the rings will fit. Through the centres A and D of two adjacent rings draw the radii CA and CD; since the polygon has twelve sides the angle $A C D = 30^\circ$ and $A C B = 15^\circ$. One half of the side AD is equal to AB. We now give the following proportion: The sine of the angle A C B is to AB as 1 is to the required radius. From this we

get the following rule: Divide AB by the sine of the angle A C B; the quotient will be the radius of the circumscribed circle; add to the corresponding diameter the diameter of one ring; the sum will be the required diameter FG.

54. To describe an arc of a circle which is too large to be drawn by a beam compass, by means of points in the arc, radius being given. — Suppose the radius is 20 feet and it is desired to obtain five points in an arc whose half chord is 4 feet. Draw a line equal to the half chord, full size, or on a smaller scale if more convenient, and erect a perpendicular at one end, thus making rectangular axes of coördinates. Erect perpendiculars at points 1, 2, 3, and 4 feet from the first perpendicular. Find values of y in the formula of the circle, $x^2 + y^2 = R^2$, by substituting for

x the values 0, 1, 2, 3, and 4, etc., and for R^2 the square of the radius, or 400. The values will be $y = \sqrt{R^2 - x^2} = \sqrt{400}, \sqrt{399}, \sqrt{396}, \sqrt{391}, \sqrt{384}; = 20, 19.975, 19.90, 19.774, 19.596$.

Subtract the smallest, or 19.596, leaving 0.404, 0.379, 0.304, 0.178, 0 feet. Lay off these distances on the five perpendiculars, as ordinates from the half chord, and the positions of five points on the arc will be found. Through these the curve may be drawn. (See also Problem 14.)

55. The Catenary is the curve assumed by a perfectly flexible cord when its ends are fastened at two points, the weight of a unit length being constant.

The equation of the catenary is $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$, in which e is the base of the Napierian system of logarithms.

To plot the catenary. — Let o (Fig. 67) be the origin of coördinates. Assigning to a any value as 3, the equation becomes

$$y = \frac{3}{2} \left(e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right).$$

To find the lowest point of the curve.

$$\text{Put } x = 0; \therefore y = \frac{3}{2} (e^0 + e^{-0}) = \frac{3}{2} (1 + 1) = 3.$$

$$\text{Then put } x = 1; \therefore y = \frac{3}{2} \left(e^{\frac{1}{3}} + e^{-\frac{1}{3}} \right) = \frac{3}{2} (1.396 + 0.717) = 3.17.$$

$$\text{Put } x = 2; \therefore y = \frac{3}{2} \left(e^{\frac{2}{3}} + e^{-\frac{2}{3}} \right) = \frac{3}{2} (1.948 + 0.513) = 3.69.$$

Put $x = 3, 4, 5$, etc., etc., and find the corresponding values of y . For each value of y we obtain two symmetrical points, as for example p and p' .

In this way, by making a successively equal to 2, 3, 4, 5, 6, 7, and 8, the curves of Fig. 67 were plotted.

In each case the distance from the origin to the lowest point of the curve is equal to a ; for putting $x = 0$, the general equation reduces to $y = a$.

For values of $a = 6, 7$, and 8 the catenary closely approaches the parabola. For derivation of the equation of the catenary see Bowser's Analytic Mechanics.

56. The Involute is a name given to the curve which is formed by the end of a string which is unwound from a cylinder and kept taut; consequently the string as it is unwound will always lie in the direction of a tangent to the cylinder. To describe the involute of any given circle, Fig. 68, take any point A on its circumference, draw a diameter AB , and from B draw Bb perpendicular to AB . Make Bb equal in length to half the circumference of the circle. Divide Bb and the semi-circumference into the same number of equal parts, say six. From each point of division 1, 2,

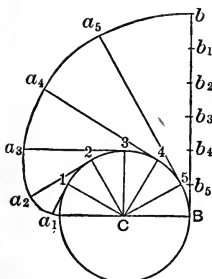


FIG. 68.

3, etc., on the circumference draw lines to the centre C of the circle. Then draw $1a_1$ perpendicular to $C1$; $2a_2$ perpendicular to $C2$; and so on. Make $1a_1$ equal to b_1b_1 ; $2a_2$ equal to b_2b_2 ; $3a_3$ equal to b_3b_3 ; and so on. Join the points A, a_1, a_2, a_3 , etc., by a curve; this curve will be the required involute.

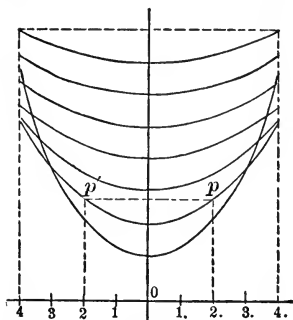


FIG. 67.

57. Method of plotting angles without using a protractor. — The radius of a circle whose circumference is 360 is 57.3 (more accurately 57.296). Striking a semicircle with a radius 57.3 by any scale, spacers set to 10 by the same scale will divide the arc into 18 spaces of 10° each, and intermediates can be measured indirectly at the rate of 1 by scale for each 1° , or interpolated by eye according to the degree of accuracy required. The following table shows the chords to the above-mentioned radius, for every 10 degrees from 0° up to 110° . By means of one of these a 10° point is fixed upon the paper next less than the required angle, and the remainder is laid off at the rate of 1 by scale for each degree.

| Angle. | Chord. | Angle. | Chord. | Angle. | Chord. |
|------------------|--------|------------------|--------|-------------------|--------|
| 1° | 0.999 | 40° | 39.192 | 80° | 73.658 |
| 10° | 9.988 | 50° | 48.429 | 90° | 81.029 |
| 20° | 19.899 | 60° | 57.296 | 100° | 87.782 |
| 30° | 29.658 | 70° | 65.727 | 110° | 93.869 |

GEOMETRICAL PROPOSITIONS.

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If a triangle is equilateral, it is equiangular, and *vice versa*.

If a straight line from the vertex of an isosceles triangle bisects the base, it bisects the vertical angle and is perpendicular to the base.

If one side of a triangle is produced, the exterior angle is equal to the sum of the two interior and opposite angles.

If two triangles are mutually equiangular, they are similar and their corresponding sides are proportional.

If the sides of a polygon are produced in the same order, the sum of the exterior angles equals four right angles. (Not true if the polygon has re-entering angles.)

In a quadrilateral, the sum of the interior angles equals four right angles.

In a parallelogram, the opposite sides are equal; the opposite angles are equal; it is bisected by its diagonal, and its diagonals bisect each other.

If three points are not in the same straight line, a circle may be passed through them.

If two arcs are intercepted on the same circle, they are proportional to the corresponding angles at the centre.

If two arcs are similar, they are proportional to their radii.

The areas of two circles are proportional to the squares of their radii.

If a radius is perpendicular to a chord, it bisects the chord and it bisects the arc subtended by the chord.

A straight line tangent to a circle meets it in only one point, and it is perpendicular to the radius drawn to that point.

If from a point without a circle tangents are drawn to touch the circle, there are but two; they are equal, and they make equal angles with the chord joining the tangent points.

If two lines are parallel chords or a tangent and parallel chord, they intercept equal arcs of a circle.

If an angle at the circumference of a circle, between two chords, is subtended by the same arc as an angle at the centre, between two radii, the angle at the circumference is equal to half the angle at the centre.

If a triangle is inscribed in a semicircle, it is right-angled.

If two chords intersect each other in a circle, the rectangle of the segments of the one equals the rectangle of the segments of the other.

And if one chord is a diameter and the other perpendicular to it, the rectangle of the segments of the diameter is equal to the square on half the other chord, and the half chord is a mean proportional between the segments of the diameter.

If an angle is formed by a tangent and chord, it is measured by one half of the arc intercepted by the chord; that is, it is equal to half the angle at the centre subtended by the chord.

Degree of a Railway Curve. — This last proposition is useful in staking out railway curves. A curve is designated as one of so many degrees, and the degree is the angle at the centre subtended by a chord of 100 ft. To lay out a curve of n degrees the transit is set at its beginning or "point of curve," pointed in the direction of the tangent, and turned through $\frac{1}{2}n$ degrees; a point 100 ft. distant in the line of sight will be a point in the curve. The transit is then swung $\frac{1}{2}n$ degrees further and a 100 ft. chord is measured from the point already found to a point in the new line of sight, which is a second point or "station" in the curve.

The radius of a 1° curve is 5729.65 ft., and the radius of a curve of any degree is 5729.65 ft. divided by the number of degrees.

MENSURATION.

PLANE SURFACES.

Quadrilateral. — A four-sided figure.

Parallelogram. — A quadrilateral with opposite sides parallel.

Varieties. — Square: four sides equal, all angles right angles. Rectangle: opposite sides equal, all angles right angles. Rhombus: four sides equal, opposite angles equal, angles not right angles. Rhomboid: opposite sides equal, opposite angles equal, angles not right angles.

Trapezium. — A quadrilateral with unequal sides.

Trapezoid. — A quadrilateral with only one pair of opposite sides parallel.

Diagonal of a square = $\sqrt{2 \times \text{side}^2} = 1.4142 \times \text{side}.$

Diag. of a rectangle = $\sqrt{\text{sum of squares of two adjacent sides}.$

Area of any parallelogram = base \times altitude.

Area of rhombus or rhomboid = product of two adjacent sides \times sine of angle included between them.

Area of a trapezoid = product of half the sum of the two parallel sides by the perpendicular distance between them.

To find the area of any quadrilateral figure. — Divide the quadrilateral into two triangles; the sum of the areas of the triangles is the area.

Or, multiply half the product of the two diagonals by the sine of the angle at their intersection.

To find the area of a quadrilateral which may be inscribed in a circle. — From half the sum of the four sides subtract each side severally; multiply the four remainders together; the square root of the product is the area.

Triangle. — A three-sided plane figure.

Varieties. — Right-angled, having one right angle; obtuse-angled, having one obtuse angle; isosceles, having two equal angles and two equal sides; equilateral, having three equal sides and equal angles.

The sum of the three angles of every triangle = $180^\circ.$

The sum of the two acute angles of a right-angled triangle = $90^\circ.$

Hypotenuse of a right-angled triangle, the side opposite the right angle, = $\sqrt{\text{sum of the squares of the other two sides.}}$ If a and b are the two sides and c the hypotenuse, $c^2 = a^2 + b^2$; $a = \sqrt{c^2 - b^2} = \sqrt{(c + b)(c - b)}.$

If the two sides are equal, side = hyp $\div 1.4142$; or hyp $\times .7071.$

To find the area of a triangle:

RULE 1. Multiply the base by half the altitude.

RULE 2. Multiply half the product of two sides by the sine of the included angle.

RULE 3. From half the sum of the three sides subtract each side severally; multiply together the half sum and the three remainders, and extract the square root of the product.

The area of an equilateral triangle is equal to one fourth the square of one of its sides multiplied by the square root of 3, = $\frac{a^2 \sqrt{3}}{4}$, a being the side; or $a^2 \times 0.433013.$

Area of a triangle given, to find base: Base = twice area \div perpendicular height.

Area of a triangle given, to find height: Height = twice area \div base.

Two sides and base given, to find perpendicular height (in a triangle in which both of the angles at the base are acute).

RULE. — As the base is to the sum of the sides, so is the difference of the sides to the difference of the divisions of the base made by drawing the perpendicular. Half this difference being added to or subtracted from half the base will give the two divisions thereof. As each side and its opposite division of the base constitutes a right-angled triangle, the perpendicular is ascertained by the rule: Perpendicular = $\sqrt{\text{hyp}^2 - \text{base}^2}$.

Areas of similar figures are to each other as the squares of their respective linear dimensions. If the area of an equilateral triangle of side = 1 is 0.433013 and its height 0.86603, what is the area of a similar triangle whose height = 1? $0.86603^2 : 1^2 :: 0.433013 : 0.57735$, Ans.

Polygon. — A plane figure having three or more sides. Regular or irregular, according as the sides or angles are equal or unequal. Polygons are named from the number of their sides and angles.

To find the area of an irregular polygon. — Draw diagonals dividing the polygon into triangles, and find the sum of the areas of these triangles.

To find the area of a regular polygon:

RULE. — Multiply the length of a side by the perpendicular distance to the centre; multiply the product by the number of sides, and divide it by 2. Or, multiply half the perimeter by the perpendicular let fall from the centre on one of the sides.

The perpendicular from the centre is equal to half of one of the sides of the polygon multiplied by the cotangent of the angle subtended by the half side.

The angle at the centre = 360° divided by the number of sides.

Table of Regular Polygons.

| No. of Sides. | Name of Polygon. | Area, Side = 1. | Area, Short diam.* = 1. | Radius of Circumscribed Circle. | | Radius of Inscribed Circle, Side = 1. | Length of Side, Radius of Circumsc. Circle = 1. | Angle at Centre. | Angle between Adjacent Sides. |
|---------------|------------------|-----------------|-------------------------|---------------------------------|-----------|---------------------------------------|---|------------------|-------------------------------|
| | | | | Perpen. from Centre = 1. | Side = 1. | | | | |
| 3 | Triangle | 0.4330 | 0.5773 | 2.000 | 0.5773 | 0.2887 | 1.732 | 120° | 60° |
| 4 | Square | 1.0000 | 1.0000 | 1.414 | 0.7071 | 0.5000 | 1.4142 | 90 | 90 |
| 5 | Pentagon | 1.7205 | 0.7265 | 1.236 | 0.8506 | 0.6882 | 1.1756 | 72 | 108 |
| 6 | Hexagon | 2.5981 | 0.8660 | 1.155 | 1.0000 | 0.866 | 1.0000 | 60 | 120 |
| 7 | Heptagon | 3.6339 | 0.7572 | 1.11 | 1.1524 | 1.0383 | 0.8677 | 51 26' | 128 4-7 |
| 8 | Octagon | 4.8284 | 0.8284 | 1.082 | 1.3066 | 1.2071 | 0.7653 | 45 | 135 |
| 9 | Nonagon | 6.1818 | 0.7688 | 1.064 | 1.4619 | 1.3737 | 0.684 | 40 | 140 |
| 10 | Decagon | 7.6942 | 0.8123 | 1.051 | 1.613 | 1.5388 | 0.618 | 36 | 144 |
| 11 | Undecagon | 9.3656 | 0.7744 | 1.042 | 1.7747 | 1.7028 | 0.5634 | 32 43' | 147 3-11 |
| 12 | Dodecagon | 11.1962 | 0.8038 | 1.035 | 1.9319 | 1.866 | 0.5176 | 30 | 150 |

* Short diameter, even number of sides, = diam. of inscribed circle; short diam., odd number of sides, = rad. of inscribed circle + rad. of circumscribed circle.

To find the area of a regular polygon, when the length of a side only is given:

RULE.—Multiply the square of the side by the figure for “area, side = 1,” opposite to the name of the polygon in the table.

Length of a side of a regular polygon inscribed in a circle = diam. $\times \sin (180^\circ \div \text{no. of sides})$.

| No. of sides | $\sin (180^\circ/n)$ | No. $\sin (180^\circ/n)$ | No. $\sin (180^\circ/n)$ | | |
|--------------|----------------------|--------------------------|--------------------------|----|---------|
| 3 | 0.86603 | 9 | 0.34202 | 15 | 0.20791 |
| 4 | .70711 | 10 | .30902 | 16 | .19509 |
| 5 | .58778 | 11 | .28173 | 17 | .18375 |
| 6 | .50000 | 12 | .25882 | 18 | .17365 |
| 7 | .43388 | 13 | .23931 | 19 | .16458 |
| 8 | .38268 | 14 | .22252 | 20 | .15643 |

To find the area of an irregular figure (Fig. 69).—Draw ordinates across its breadth at equal distances apart, the first and the last ordinate each being one half space from the ends of the figure. Find the average breadth by adding together the lengths of these lines included between the boundaries of the figure, and divide by the number of the lines added; multiply this mean breadth by the length. The greater the number of lines the nearer the approximation.

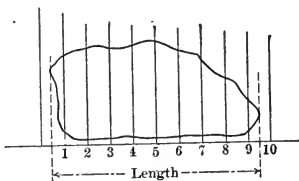


FIG. 69.

In a figure of very irregular outline, as an indicator-diagram from a high-speed steam-engine, mean lines may be substituted for the actual lines of the figure, being so traced as to intersect the undulations, so that the total area of the spaces cut off may be compensated by that of the extra spaces inclosed.

2d Method: THE TRAPEZOIDAL RULE.—Divide the figure into any sufficient number of equal parts; add half the sum of the two end ordinates to the sum of all the other ordinates; divide by the number of spaces (that is, one less than the number of ordinates) to obtain the mean ordinate, and multiply this by the length to obtain the area.

3d Method: SIMPSON'S RULE.—Divide the length of the figure into any even number of equal parts, at the common distance D apart, and draw ordinates through the points of division to touch the boundary lines. Add together the first and last ordinates and call the sum A ; add together the even ordinates and call the sum B ; add together the odd ordinates, except the first and last, and call the sum C . Then,

$$\text{area of the figure} = \frac{A + 4B + 2C}{3} \times D.$$

4th Method: DURAND'S RULE.—Add together $\frac{4}{10}$ the sum of the first and last ordinates, $\frac{11}{10}$ the sum of the second and the next to the last (or the penultimates), and the sum of all the intermediate ordinates. Multiply the sum thus gained by the common distance between the ordinates to obtain the area, or divide this sum by the number of spaces to obtain the mean ordinate.

Prof. Durand describes the method of obtaining his rule in *Engineering News*, Jan. 18, 1894. He claims that it is more accurate than Simpson's rule, and practically as simple as the trapezoidal rule. He thus describes its application for approximate integration of differential equations. Any definite integral may be represented graphically by an area. Thus, let

$$Q = \int u \, dx$$

be an integral in which u is some function of x , either known or admitting of computation or measurement. Any curve plotted with x as abscissa and u as ordinate will then represent the variation of u with x , and the

area between such curve and the axis X will represent the integral in question, no matter how simple or complex may be the real nature of the function u .

Substituting in the rule as above given the word "volume" for "area" and the word "section" for "ordinate," it becomes applicable to the determination of volumes from equidistant sections as well as of areas from equidistant ordinates.

Having approximately obtained an area by the trapezoidal rule, the area by Durand's rule may be found by adding algebraically to the sum of the ordinates used in the trapezoidal rule (that is, half the sum of the end ordinates + sum of the other ordinates) $\frac{1}{10}$ of (sum of penultimates - sum of first and last) and multiplying by the common distance between the ordinates.

5th Method. — Draw the figure on cross-section paper. Count the number of squares that are entirely included within the boundary; then estimate the fractional parts of squares that are cut by the boundary, add together these fractions, and add the sum to the number of whole squares. The result is the area in units of the dimensions of the squares. The finer the ruling of the cross-section paper the more accurate the result.

6th Method. — Use a planimeter.

7th Method. — With a chemical balance, sensitive to one milligram, draw the figure on paper of uniform thickness and cut it out carefully; weigh the piece cut out, and compare its weight with the weight per square inch of the paper as tested by weighing a piece of rectangular shape.

THE CIRCLE.

Circumference = diameter \times 3.1416, nearly; more accurately, 3.14159265359.

Approximations, $\frac{22}{7} = 3.143$; $\frac{355}{113} = 3.1415929$.

The ratio of circum. to diam. is represented by the symbol π (called *Pi*).

Area = 0.7854 \times square of the diameter.

| Multiples of π . | Multiples of $\frac{\pi}{4}$. |
|-------------------------|--------------------------------|
| $1\pi = 3.14159265359$ | $1/4\pi = 0.7853982$ |
| $2\pi = 6.28318530718$ | " $\times 2 = 1.5707963$ |
| $3\pi = 9.42477796077$ | " $\times 3 = 2.3561945$ |
| $4\pi = 12.56637061436$ | " $\times 4 = 3.1415927$ |
| $5\pi = 15.70796326795$ | " $\times 5 = 3.9269908$ |
| $6\pi = 18.84955592154$ | " $\times 6 = 4.7123890$ |
| $7\pi = 21.99114857513$ | " $\times 7 = 5.4977871$ |
| $8\pi = 25.13274122872$ | " $\times 8 = 6.2831853$ |
| $9\pi = 28.27433388231$ | " $\times 9 = 7.0685835$ |

Ratio of diam. to circumference = reciprocal of $\pi = 0.3183099$.

| | | |
|------------------------------------|--------------------|-----------------------------------|
| Reciprocal of $1/4\pi = 1.27324$. | $7/\pi = 2.22817$ | $\pi/12 = 0.261799$ |
| Multiples of $1/\pi$. | $8/\pi = 2.54648$ | $\pi/360 = 0.0087266$ |
| $1/\pi = 0.31831$ | $9/\pi = 2.86479$ | $360/\pi = 114.5915$ |
| $2/\pi = 0.63662$ | $10/\pi = 3.18310$ | $\pi^2 = 9.86960$ |
| $3/\pi = 0.95493$ | $12/\pi = 3.81972$ | $1/\pi^2 = 0.101321$ |
| $4/\pi = 1.27324$ | $\pi/2 = 1.570796$ | $\sqrt{\pi} = 1.772453$ |
| $5/\pi = 1.59155$ | $\pi/3 = 1.047197$ | $\sqrt{\frac{1}{\pi}} = 0.564189$ |
| $6/\pi = 1.90986$ | $\pi/6 = 0.523599$ | Log $\pi = 0.49714987$ |
| | | Log $\pi/4 = 1.895090$ |

Diam. in ins. = 13.5405 $\sqrt{\text{area in sq. ft.}}$

Area in sq. ft. = (diam. in inches)² \times .0054542.

D = diameter, R = radius, C = circumference, A = area.

$$C = \pi D; = 2\pi R; = \frac{4A}{D}; = 2\sqrt{\pi A}; = 3.545\sqrt{A};$$

$$A = D^2 \times .7854; = \frac{CR}{2}; = 4R^2 \times .7854; = \pi R^2; = \frac{1}{4}\pi D^2; = \frac{C^2}{4\pi}; = .07958C^2; = \frac{CD}{4}.$$

$$D = \frac{C}{\pi}; = 0.31831C; = 2\sqrt{\frac{A}{\pi}}; = 1.12838\sqrt{A};$$

$$R = \frac{C}{2\pi}; = 0.159155C; = \sqrt{\frac{A}{\pi}}; = 0.564189\sqrt{A}.$$

Areas of circles are to each other as the squares of their diameters.

To find the length of an arc of a circle:

RULE 1. As 360 is to the number of degrees in the arc, so is the circumference of the circle to the length of the arc.

RULE 2. Multiply the diameter of the circle by the number of degrees in the arc, and this product by 0.0087266.

Relations of Arc, Chord, Chord of Half the Arc, etc.

Let R = radius, D = diameter, L = length of arc,

C = chord of the arc, c = chord of half the arc,

V = rise, or height of the arc,

$$\begin{aligned}\text{Length of the arc} = L &= \frac{8c - C}{3} \text{ (very nearly), } = \frac{2c \times 10V}{60D - 27V} + 2c, \text{ nearly,} \\ &= \frac{\sqrt{C^2 + 4V^2} \times 10V^2}{15C^2 + 33V^2} + 2c, \text{ nearly.}\end{aligned}$$

$$\begin{aligned}\text{Chord of the arc } C &= 2\sqrt{c^2 - V^2}; = \sqrt{D^2 - (D - 2V)^2}; = 8c - 3L \\ &= 2\sqrt{R^2 - (R - V)^2}; = 2\sqrt{(D - V) \times V}.\end{aligned}$$

$$\text{Chord of half the arc, } c = \frac{1}{2}\sqrt{C^2 + 4V^2}; = \sqrt{D \times V}; = (3L + C) \div 8.$$

$$\text{Diameter of the circle, } D = \frac{c^2}{V}; = \frac{1/4 C^2 + V^2}{V};$$

$$\text{Rise of the arc, } V = \frac{c^2}{D}; = \frac{1}{2}(D - \sqrt{D^2 - C^2}),$$

$$\begin{aligned}\text{(or if } V \text{ is greater than radius } &\frac{1}{2}(D + \sqrt{D^2 - C^2}); \\ &= \sqrt{c^2 - 1/4 C^2}.\end{aligned}$$

Half the chord of the arc is a mean proportional between the rise and the diameter minus the rise: $\frac{1}{2}C = \sqrt{V \times (D - V)}$.

Length of the Chord subtending an angle at the centre = twice the sine of half the angle. (See Table of Sines.)

Ordinates to Circular Arcs. — C = chord, V = height of the arc, or middle ordinate, x = abscissa, or distance measured on the chord from its central point, y = ordinate, or distance from the arc to the chord at the point x , $V = R - \sqrt{R^2 - 1/4 C^2}$; $y = \sqrt{R^2 - x^2} - (R - V)$.

Length of a Circular Arc. — Huyghens's Approximation.

Length of the arc, $L = (8c - C) \div 3$. Professor Williamson shows that when the arc subtends an angle of 30° , the radius being 100,000 feet (nearly 19 miles), the error by this formula is about two inches, or $1/600,000$ part of the radius. When the length of the arc is equal to the radius, i.e., when it subtends an angle of $57^\circ.3$, the error is less than $1/7680$ part of the radius. Therefore, if the radius is 100,000 feet, the error is less than $100000/7680 = 13$ feet. The error increases rapidly with the increase of the angle subtended. For an arc of 120° the error is 1 part in 400; for an arc of 180° the error is 1.18%.

In the measurement of an arc which is described with a short radius the error is so small that it may be neglected. Describing an arc with a radius of 12 inches subtending an angle of 30° , the error is $1/50000$ of an inch.

To measure an arc when it subtends a large angle, bisect it and measure each half as before—in this case making B = length of the chord of half the arc, and b = length of the chord of one fourth the arc; then $L = (16b - 2B) \div 3$.

Formulas for a Circular Curve.

J. C. Locke, *Eng. News*, March 16, 1908.

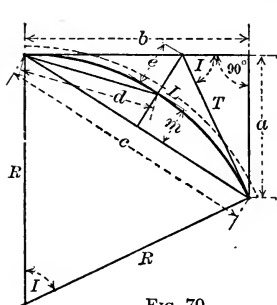


FIG. 70.

$$c = \sqrt{2Ra}, = \sqrt{a^2 + b^2},$$

$$= \sqrt{2R(R - \sqrt{(R + b)(R - b)})}$$

$$= 2\sqrt{m(2R - m)}, = 2R \sin \frac{1}{2}I,$$

$$= 2T \cos \frac{1}{2}I.$$

$$e = R \operatorname{exsec} \frac{1}{2}I, = R \tan \frac{1}{2}I \tan \frac{1}{4}I,$$

$$= T \tan \frac{1}{4}I.$$

$$b = \sqrt{a(2R - a)}, = \sqrt{\left(c + \frac{c^2}{2R}\right)\left(c - \frac{c^2}{2R}\right)},$$

$$= R \sin I, = a \cot \frac{1}{2}I.$$

$$R = \frac{a^2 + b^2}{2a}, = \frac{c^2}{2a}, = \frac{d^2}{2m}, = \frac{c^2 + 4m}{8m}.$$

$$d = \sqrt{2Rm}, = \sqrt{R(2R - \sqrt{(2R + c)(2R - c)})}, = 2R \sin \frac{1}{4}I.$$

$$m = \frac{d^2}{2R}, = R \mp \sqrt{\left(R + \frac{c}{2}\right)\left(R - \frac{c}{2}\right)}, = R \operatorname{vers} \frac{1}{2}I,$$

$$= R \sin \frac{1}{2}I \tan \frac{1}{4}I, = \frac{1}{2}c \tan \frac{1}{4}I.$$

$$a = \frac{c^2}{2R}, = R - \sqrt{(R + b)(R - b)}, = 2R (\sin \frac{1}{2}I)^2, = R \operatorname{vers} I,$$

$$= R \sin I \tan \frac{1}{2}I, = b \tan \frac{1}{2}I, = T \sin I.$$

$$T = R \tan \frac{1}{2}I. \quad I = \frac{L}{R} \times 57.295780^\circ. \quad R = \frac{L}{I} \times 57.295780^\circ.$$

$$L = IR \times 0.01745329, = \frac{8d - c}{3}.$$

$$\text{Area of Segment} = \frac{LR}{2} - \frac{R^2 \sin I}{2}, = \frac{LR}{2} - \frac{Rb}{2}.$$

Relation of the Circle to its Equal, Inscribed, and Circumscribed Squares.

| | | | |
|-------------------------|---|--------------------|------------------------------------|
| Diameter of circle | × | 0.88623 | } = side of equal square. |
| Circumference of circle | × | 0.28209 | |
| Circumference of circle | × | 1.1284 | } = perimeter of equal square. |
| Diameter of circle | × | 0.7071 | |
| Circumference of circle | × | 0.22508 | } = side of inscribed square. |
| Area of circle | × | 0.90031 ÷ diameter | |
| Area of circle | × | 1.2732 | } = area of circumscribed square. |
| Area of circle | × | 0.63662 | |
| Side of square | × | 1.4142 | } = diam. of circumscribed circle. |
| " | × | 4.4428 | |
| " | × | 1.1284 | } = diam. of equal circle. |
| " | × | 3.5449 | |
| Perimeter of square | × | 0.88623 | } = " " " |
| Square inches | × | 1.2732 | |

Sectors and Segments. — *To find the area of a sector of a circle.*

RULE 1. Multiply the arc of the sector by half its radius.

RULE 2. As 360 is to the number of degrees in the arc, so is the area of the circle to the area of the sector.

RULE 3. Multiply the number of degrees in the arc by the square of the radius and by 0.008727.

To find the area of a segment of a circle: Find the area of the sector which has the same arc, and also the area of the triangle formed by the chord of the segment and the radii of the sector.

Then take the sum of these areas, if the segment is greater than a semi-circle, but take their difference if it is less. (See Table of Segments.)

Another Method: Area of segment = $\frac{1}{2} R^2 (\text{arc} - \sin A)$, in which A is the central angle, R the radius; and arc the length of arc to radius 1.

To find the area of a segment of a circle when its chord and height only are given. First find radius, as follows:

$$\text{radius} = \frac{1}{2} \left[\frac{\text{square of half the chord}}{\text{height}} + \text{height} \right].$$

2. Find the angle subtended by the arc, as follows: half chord ÷ radius = sine of half the angle. Take the corresponding angle from a table of sines, and double it to get the angle of the arc.

3. Find area of the sector of which the segment is a part:

$$\text{area of sector} = \text{area of circle} \times \text{degrees of arc} \div 360.$$

4. Subtract area of triangle under the segment:

$$\text{Area of triangle} = \text{half chord} \times (\text{radius} - \text{height of segment}).$$

The remainder is the area of the segment.

When the chord, arc, and diameter are given, to find the area. From the length of the arc subtract the length of the chord. Multiply the remainder by the radius or one-half diameter; to the product add the chord multiplied by the height, and divide the sum by 2.

Given diameter, d , and height of segment, h .

$$\begin{aligned} \text{When } h \text{ is from } 0 \text{ to } \frac{1}{4}d, \text{ area} &= h\sqrt{1.766dh - h^2}; \\ \text{" " " " } \frac{1}{4}d \text{ to } \frac{1}{2}d, \text{ area} &= h\sqrt{0.017d^2 + 1.7dh - h^2} \end{aligned}$$

(approx.). Greatest error 0.23%, when $h = \frac{1}{4}d$.

To find the chord: From the diameter subtract the height; multiply the remainder by four times the height and extract the square root.

When the chords of the arc and of half the arc and the rise are given: To the chord of the arc add four thirds of the chord of half the arc; multiply the sum by the rise and the product by 0.40426 (approximate).

Circular Ring. — *To find the area of a ring included between the circumferences of two concentric circles:* Take the difference between the areas of the two circles; or, subtract the square of the less radius from the square of the greater, and multiply their difference by 3.14159.

$$\begin{aligned} \text{The area of the greater circle is equal to } &\pi R^2; \\ \text{and the area of the smaller,} &\pi r^2. \end{aligned}$$

Their difference, or the area of the ring, is $\pi(R^2 - r^2)$.

The Ellipse. — Area of an ellipse = product of its semi-axes $\times 3.14159$
= product of its axes $\times 0.785398$.

The Ellipse. — Circumference (approximate) = $3.1416 \sqrt{\frac{D^2 + d^2}{2}}$, D and d being the two axes.

Trautwine gives the following as more accurate: When the longer axis D is not more than five times the length of the shorter axis, d ,

$$\text{Circumference} = 3.1416 \sqrt{\frac{D^2 + d^2}{2} - \frac{(D - d)^2}{8.8}}$$

When D is more than $5d$, the divisor 8.8 is to be replaced by the following:

| | | | | | | | | | | | | | |
|-------------|---|-----|-----|------|-----|-----|-----|------|------|-----|------|------|----|
| For $D/d =$ | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 | 30 | 40 | 50 |
| Divisor = | 9 | 9.2 | 9.3 | 9.35 | 9.4 | 9.5 | 9.6 | 9.63 | 9.75 | 9.8 | 9.92 | 9.98 | 10 |

An accurate formula is $C = \pi(a + b) \left(1 + \frac{A^2}{4} + \frac{A^4}{64} + \frac{A^6}{256} + \frac{25A^8}{16384} + \dots \right)$,

in which $A = \frac{a-b}{a+b}$. — *Ingenieurs Taschenbuch*, 1896. (a and b , semi-axes.)

Carl G. Barth (*Machinery*, Sept., 1900) gives as a very close approximation to this formula

$$C = \pi(a + b) \frac{64 - 3A^4}{64 - 16A^2}.$$

Area of a segment of an ellipse the base of which is parallel to one of the axes of the ellipse. Divide the height of the segment by the axis of which it is part, and find the area of a circular segment, in a table of circular segments, of which the height is equal to the quotient; multiply the area thus found by the product of the two axes of the ellipse.

Cycloid. — A curve generated by the rolling of a circle on a plane.

Length of a cycloidal curve = $4 \times$ diameter of the generating circle.

Length of the base = circumference of the generating circle.

Area of a cycloid = $3 \times$ area of generating circle.

Helix (Screw). — A line generated by the progressive rotation of a point around an axis and equidistant from its center.

Length of a helix. — To the square of the circumference described by the generating point add the square of the distance advanced in one revolution, and take the square root of their sum multiplied by the number of revolutions of the generating point. Or,

$$\sqrt{(c^2 + h^2)n} = \text{length, } n \text{ being number of revolutions.}$$

Spirals. — Lines generated by the progressive rotation of a point around a fixed axis, with a constantly increasing distance from the axis.

A *plane spiral* is made when the point rotates in one plane.

A *conical spiral* is made when the point rotates around an axis at a progressing distance from its center, and advancing in the direction of the axis, as around a cone.

Length of a plane spiral line. — When the distance between the coils is uniform.

RULE. — Add together the greater and less diameters; divide their sum by 2; multiply the quotient by 3.1416, and again by the number of revolutions. Or, take the mean of the length of the greater and less circumferences and multiply it by the number of revolutions. Or,

$$\text{length} = \pi n \frac{d + d'}{2}, \text{ } d \text{ and } d' \text{ being the inner and outer diameters.}$$

Length of a conical spiral line. — Add together the greater and less diameters; divide their sum by 2 and multiply the quotient by 3.1416. To the square of the product of this circumference and the number of revolutions of the spiral add the square of the height of its axis and take the square root of the sum.

$$\text{Or, length} = \sqrt{\left(\pi n \frac{d + d'}{2} \right)^2 + h^2}.$$

SOLID BODIES.

Surfaces and Volumes of Similar Solids. — The surfaces of two similar solids are to each other as the squares of their linear dimensions; the volumes are as the cubes of their linear dimensions. If L = the side

of a cube or other solid, and l the side of a similar body of different size, S, s , the surfaces and V, v , the volumes respectively, $S : s :: L^2 : l^2$; $V : v :: L^3 : l^3$.

The Prism. — *To find the surface of a right prism:* Multiply the perimeter of the base by the altitude for the convex surface. To this add the areas of the two ends when the entire surface is required.

Volume of a prism = area of its base \times its altitude.

The pyramid. — Convex surface of a regular pyramid = perimeter of its base \times half the slant height. To this add area of the base if the whole surface is required.

Volume of a pyramid = area of base \times one third of the altitude.

To find the surface of a frustum of a regular pyramid: Multiply half the slant height by the sum of the perimeters of the two bases for the convex surface. To this add the areas of the two bases when the entire surface is required.

To find the volume of a frustum of a pyramid: Add together the areas of the two bases and a mean proportional between them, and multiply the sum by one third of the altitude. (Mean proportional between two numbers = square root of their product.)

Wedge. — A wedge is a solid bounded by five planes, viz.: a rectangular base, two trapezoids, or two rectangles, meeting in an edge, and two triangular ends. The altitude is the perpendicular drawn from any point in the edge to the plane of the base.

To find the volume of a wedge: Add the length of the edge to twice the length of the base, and multiply the sum by one sixth of the product of the height of the wedge and the breadth of the base.

Rectangular prismoid. — A rectangular prismoid is a solid bounded by six planes, of which the two bases are rectangles, having their corresponding sides parallel, and the four upright sides of the solid are trapezoids.

To find the volume of a rectangular prismoid: Add together the areas of the two bases and four times the area of a parallel section equally distant from the bases, and multiply the sum by one sixth of the altitude.

Cylinder. — Convex surface of a cylinder = perimeter of base \times altitude. To this add the areas of the two ends when the entire surface is required.

Volume of a cylinder = area of base \times altitude.

Cone. — Convex surface of a cone = circumference of base \times half the slant height. To this add the area of the base when the entire surface is required.

Volume of a cone = area of base \times one third of the altitude.

To find the surface of a frustum of a cone: Multiply half the side by the sum of the circumferences of the two bases for the convex surface; to this add the areas of the two bases when the entire surface is required.

To find the volume of a frustum of a cone: Add together the areas of the two bases and a mean proportional between them, and multiply the sum by one third of the altitude. Or, Vol. = $0.2618a(b^2 + c^2 + bc)$; a = altitude; b and c , diams. of the two bases.

Sphere. — *To find the surface of a sphere:* Multiply the diameter by the circumference of a great circle; or, multiply the square of the diameter by 3.14159.

Surface of sphere = $4 \times$ area of its great circle.

“ “ “ = convex surface of its circumscribing cylinder.

Surfaces of spheres are to each other as the squares of their diameters.

To find the volume of a sphere: Multiply the surface by one third of the radius; or, multiply the cube of the diameter by $\pi/6$; that is, by 0.5236.

Value of $\pi/6$ to 10 decimal places = 0.5235987756.

The volume of a sphere = $2/3$ the volume of its circumscribing cylinder.

Volumes of spheres are to each other as the cubes of their diameters.

Spherical triangle. — *To find the area of a spherical triangle:* Compute the surface of the quadrantal triangle, or one eighth of the surface of the sphere. From the sum of the three angles subtract two right angles; divide the remainder by 90, and multiply the quotient by the area of the quadrantal triangle.

Spherical polygon. — *To find the area of a spherical polygon:* Compute the surface of the quadrantal triangle. From the sum of all the angles subtract the product of two right angles by the number of sides less two; divide the remainder by 90 and multiply the quotient by the area of the quadrantal triangle.

The prismoid. — The prismoid is a solid having parallel end areas, and may be composed of any combination of prisms, cylinders, wedges, pyramids, or cones or frustums of the same, whose bases and apices lie in the end areas.

Inasmuch as cylinders and cones are but special forms of prisms and pyramids, and warped surface solids may be divided into elementary forms of them, and since frustums may also be subdivided into the elementary forms, it is sufficient to say that all prismoids may be decomposed into prisms, wedges, and pyramids. If a formula can be found which is equally applicable to all of these forms, then it will apply to any combination of them. Such a formula is called

The Prismoidal Formula.

Let A = area of the base of a prism, wedge, or pyramid;

A_1, A_2, A_m = the two end and the middle areas of a prismoid, or of any of its elementary solids; h = altitude of the prismoid or elementary solid; V = its volume;

$$V = \frac{h}{6} (A_1 + 4A_m + A_2).$$

For a prism, A_1, A_m and A_2 are equal, $= A$; $V = \frac{h}{6} \times 6A = hA$.

For a wedge with parallel ends, $A_2 = 0, A_m = \frac{1}{2} A_1$; $V = \frac{h}{6} (A_1 + 2A_1) = \frac{hA}{2}$.

For a cone or pyramid, $A_2 = 0, A_m = \frac{1}{4} A_1$; $V = \frac{h}{6} (A_1 + A_1) = \frac{hA}{3}$.

The prismoidal formula is a rigid formula for all prismoids. The only approximation involved in its use is in the assumption that the given solid may be generated by a right line moving over the boundaries of the end areas.

The area of the middle section is never the mean of the two end areas if the prismoid contains any pyramids or cones among its elementary forms. When the three sections are similar in form the *dimensions* of the middle area are always the means of the corresponding end dimensions. This fact often enables the dimensions, and hence the area of the middle section, to be computed from the end areas.

Polyedrons. — A polyedron is a solid bounded by plane polygons. A regular polyedron is one whose sides are all equal regular polygons.

To find the surface of a regular polyedron. — Multiply the area of one of the faces by the number of faces; or, multiply the square of one of the edges by the surface of a similar solid whose edge is unity.

A TABLE OF THE REGULAR POLYEDRONS WHOSE EDGES ARE UNITY.

| Names. | No. of Faces. | Surface. | Volume. |
|------------------|---------------|------------|-----------|
| Tetraedron..... | 4 | 1.7320508 | 0.1178513 |
| Hexaedron..... | 6 | 6.0000000 | 1.0000000 |
| Octaedron..... | 8 | 3.4641016 | 0.4714045 |
| Dodecaedron..... | 12 | 20.6457288 | 7.6631189 |
| Icosaedron..... | 20 | 8.6602540 | 2.1816950 |

To find the volume of a regular polyedron. — Multiply the surface by one third of the perpendicular let fall from the centre on one of the faces; or, multiply the cube of one of the edges by the solidity of a similar polyedron whose edge is unity.

Solid of revolution. — The volume of any solid of revolution is equal to the product of the area of its generating surface by the length of the path of the centre of gravity of that surface.

The convex surface of any solid of revolution is equal to the product of the perimeter of its generating surface by the length of path of its centre of gravity.

Cylindrical ring. — Let d = outer diameter; d' = inner diameter; $\frac{1}{2}(d - d')$ = thickness = t ; $\frac{1}{4}\pi t^2$ = sectional area; $\frac{1}{2}(d + d')$ = mean diameter = M ; πt = circumference of section; πM = mean circumference of ring; surface = $\pi t \times \pi M$; = $\frac{1}{4}\pi^2 (d^2 - d'^2)$; = $9.86965 t M$; = $2.46741 (d^2 - d'^2)$; volume = $\frac{1}{4}\pi t^2 M \pi$; = $2.467241 t^2 M$.

Spherical zone. — Surface of a spherical zone or segment of a sphere = its altitude \times the circumference of a great circle of the sphere. A great circle is one whose plane passes through the centre of the sphere.

Volume of a zone of a sphere. — To the sum of the squares of the radii of the ends add one third of the square of the height; multiply the sum by the height and by 1.5708.

Spherical segment. — Volume of a spherical segment with one base. — Multiply half the height of the segment by the area of the base, and the cube of the height by 0.5236 and add the two products. Or, from three times the diameter of the sphere subtract twice the height of the segment; multiply the difference by the square of the height and by 0.5236. Or, to three times the square of the radius of the base of the segment add the square of its height, and multiply the sum by the height and by 0.5236.

Spheroid or ellipsoid. — When the revolution of the generating surface of the spheroid is about the transverse diameter the spheroid is *prolate*, and when about the conjugate it is *oblate*.

Convex surface of a segment of a spheroid. — Square the diameters of the spheroid, and take the square root of half their sum; then, as the diameter from which the segment is cut is to this root so is the height of the segment to the proportionate height of the segment to the mean diameter. Multiply the product of the other diameter and 3.1416 by the proportionate height.

Convex surface of a frustum or zone of a spheroid. — Proceed as by previous rule for the surface of a segment, and obtain the proportionate height of the frustum. Multiply the product of the diameter parallel to the base of the frustum and 3.1416 by the proportionate height of the frustum.

Volume of a spheroid is equal to the product of the square of the revolving axis by the fixed axis and by 0.5236. The volume of a spheroid is two thirds of that of the circumscribing cylinder.

Volume of a segment of a spheroid. — 1. When the base is parallel to the revolving axis, multiply the difference between three times the fixed axis and twice the height of the segment, by the square of the height and by 0.5236. Multiply the product by the square of the revolving axis, and divide by the square of the fixed axis.

2. When the base is perpendicular to the revolving axis, multiply the difference between three times the revolving axis and twice the height of the segment by the square of the height and by 0.5236. Multiply the product by the length of the fixed axis, and divide by the length of the revolving axis.

Volume of the middle frustum of a spheroid. — 1. When the ends are circular, or parallel to the revolving axis: To twice the square of the middle diameter add the square of the diameter of one end; multiply the sum by the length of the frustum and by 0.2618.

2. When the ends are elliptical, or perpendicular to the revolving axis: To twice the product of the transverse and conjugate diameters of the middle section add the product of the transverse and conjugate diameters of one end; multiply the sum by the length of the frustum and by 0.2618.

Spindles. — Figures generated by the revolution of a plane area, bounded by a curve other than a circle, when the curve is revolved about a chord perpendicular to its axis, or about its double ordinate. They are designated by the name of the arc or curve from which they are generated, as Circular, Elliptic, Parabolic, etc., etc.

Convex surface of a circular spindle, zone, or segment of it. — Rule: Multiply the length by the radius of the revolving arc; multiply this arc by the central distance, or distance between the centre of the spindle and centre of the revolving arc; subtract this product from the former, double the remainder, and multiply it by 3.1416.

Volume of a circular spindle. — Multiply the central distance by half the area of the revolving segment; subtract the product from one third of the cube of half the length, and multiply the remainder by 12.5664.

Volume of frustum or zone of a circular spindle. — From the square of half the length of the whole spindle take one third of the square of half the length of the frustum, and multiply the remainder by the said half length of the frustum; multiply the central distance by the revolving area which generates the frustum; subtract this product from the former, and multiply the remainder by 6.2832.

Volume of a segment of a circular spindle. — Subtract the length of the segment from the half length of the spindle; double the remainder and ascertain the volume of a middle frustum of this length; subtract the result from the volume of the whole spindle and halve the remainder.

Volume of a cycloidal spindle = five eighths of the volume of the circumscribing cylinder. — Multiply the product of the square of twice the diameter of the generating circle and 3.927 by its circumference, and divide this product by 8.

Parabolic conoid. — *Volume of a parabolic conoid* (generated by the revolution of a parabola on its axis). — Multiply the area of the base by half the height.

Or multiply the square of the diameter of the base by the height and by 0.3927.

Volume of a frustum of a parabolic conoid. — Multiply half the sum of the areas of the two ends by the height.

Volume of a parabolic spindle (generated by the revolution of a parabola on its base). — Multiply the square of the middle diameter by the length and by 0.4189. The volume of a parabolic spindle is to that of a cylinder of the same height and diameter as 8 to 15.

Volume of the middle frustum of a parabolic spindle. — Add together 8 times the square of the maximum diameter, 3 times the square of the end diameter, and 4 times the product of the diameters. Multiply the sum by the length of the frustum and by 0.05236. This rule is applicable for calculating the content of casks of parabolic form.

Casks. — *To find the volume of a cask of any form.* — Add together 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the diameters. Multiply the sum by the length, and divide by 31,773 for the content in Imperial gallons, or by 26,470 for U. S. gallons.

This rule was framed by Dr. Hutton, on the supposition that the middle third of the length of the cask was a frustum of a parabolic spindle, and each outer third was a frustum of a cone.

To find the ullage of a cask, the quantity of liquor in it when it is not full.
1. For a *lying cask*: Divide the number of wet or dry inches by the bung diameter in inches. If the quotient is less than 0.5, deduct from it one fourth part of what it wants of 0.5. If it exceeds 0.5, add to it one fourth part of the excess above 0.5. Multiply the remainder or the sum by the whole content of the cask. The product is the quantity of liquor in the cask, in gallons, when the dividend is *wet* inches; or the empty space, if *dry* inches.

2. For a *standing cask*: Divide the number of wet or dry inches by the length of the cask. If the quotient exceeds 0.5, add to it one tenth of its excess above 0.5; if less than 0.5, subtract from it one tenth of what it wants of 0.5. Multiply the sum or the remainder by the whole content of the cask. The product is the quantity of liquor in the cask, when the dividend is wet inches; or the empty space, if dry inches.

Volume of cask (approximate) U. S. gallons = square of mean diam. \times length in inches \times 0.0034. Mean diameter = half the sum of the bung and head diameters.

Volume of an irregular solid. — Suppose it divided into parts, resembling prisms or other bodies measurable by preceding rules. Find the content of each part; the sum of the contents is the cubic contents of the solid.

The content of a small part is found nearly by multiplying half the sum of the areas of each end by the perpendicular distance between them.

The contents of small irregular solids may sometimes be found by immersing them under water in a prismatic or cylindrical vessel, and observing the amount by which the level of the water descends when the solid is withdrawn. The sectional area of the vessel being multiplied by the descent of the level gives the cubic contents.

Or, weigh the solid in air and in water; the difference is the weight of water it displaces. Divide the weight in pounds by 62.4 to obtain volume in cubic feet, or multiply it by 27.7 to obtain the volume in cubic inches.

When the solid is very large and a great degree of accuracy is not requisite, measure its length, breadth, and depth in several different places, and take the mean of the measurement for each dimension, and multiply the three means together.

When the surface of the solid is very extensive it is better to divide it into triangles, to find the area of each triangle, and to multiply it by the mean depth of the triangle for the contents of each triangular portion; the contents of the triangular sections are to be added together.

The mean depth of a triangular section is obtained by measuring the depth at each angle, adding together the three measurements, and taking one third of the sum.

PLANE TRIGONOMETRY.

Trigonometrical Functions.

Every triangle has six parts — three angles and three sides. When any three of these parts are given, provided one of them is a side, the other parts may be determined. By the *solution* of a triangle is meant the determination of the unknown parts of a triangle when certain parts are given.

The *complement* of an angle or arc is what remains after subtracting the angle or arc from 90° .

In general, if we represent any arc by A , its complement is $90^\circ - A$. Hence the complement of an arc that exceeds 90° is negative.

The *supplement* of an angle or arc is what remains after subtracting the angle or arc from 180° . If A is an arc its supplement is $180^\circ - A$. The supplement of an arc that exceeds 180° is negative.

The *sum of the three angles of a triangle is equal to 180°* . Either angle is the supplement of the other two. In a right-angled triangle, the right angle being equal to 90° , each of the acute angles is the complement of the other.

In all right-angled triangles having the same acute angle, the sides have to each other the same ratio. These ratios have received special names, as follows:

If A is one of the acute angles, a the opposite side, b the adjacent side, and c the hypotenuse.

The *sine* of the angle A is the quotient of the opposite side divided by the hypotenuse. $\text{Sin } A = \frac{a}{c}$.

The *tangent* of the angle A is the quotient of the opposite side divided by the adjacent side. $\text{Tan } A = \frac{a}{b}$.

The *secant* of the angle A is the quotient of the hypotenuse divided by the adjacent side. $\text{Sec } A = \frac{c}{b}$.

The *cosine* (cos), *cotangent* (cot), and *cosecant* (cosec) of an angle are respectively the sine, tangent, and secant of the complement of that angle. The terms sine, cosine, etc., are called trigonometrical functions.

In a circle whose radius is unity, the *sine* of an arc, or of the angle at the centre measured by that arc, is the perpendicular let fall from one extremity of the arc upon the diameter passing through the other extremity.

The *tangent* of an arc is the line which touches the circle at one extremity

of the arc, and is limited by the diameter (produced) passing through the other extremity.

The secant of an arc is that part of the produced diameter which is intercepted between the centre and the tangent.

The versed sine of an arc is that part of the diameter intercepted between the extremity of the arc and the foot of the sine.

In a circle whose radius is not unity, the trigonometric functions of an arc will be equal to the lines here defined, divided by the radius of the circle.

If $\angle ICA$ (Fig. 71) is an angle in the first quadrant, and CF = radius,

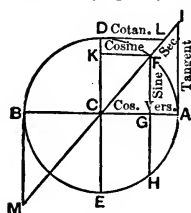


FIG. 71.

$$\text{The sine of the angle} = \frac{FG}{\text{Rad}}. \quad \text{Cos} = \frac{CG}{\text{Rad}} = \frac{KF}{\text{Rad}}.$$

$$\text{Tan} = \frac{IA}{\text{Rad}}. \quad \text{Secant} = \frac{CI}{\text{Rad}}. \quad \text{Cot} = \frac{DL}{\text{Rad}}.$$

$$\text{Cosec} = \frac{CL}{\text{Rad}}. \quad \text{Versin} = \frac{GA}{\text{Rad}}.$$

If radius is 1, then Rad in the denominator is omitted, and sine = FG , etc.

The sine of an arc = half the chord of twice the arc.

The sine of the supplement of the arc is the same as that of the arc itself. Sine of arc BDF = FG = sine arc $F A$.

The tangent of the supplement is equal to the tangent of the arc, but with a contrary sign. $\text{Tan } BDF = -BM$.

The secant of the supplement is equal to the secant of the arc, but with a contrary sign. $\text{Sec } BDF = -CM$.

Signs of the functions in the four quadrants. — If we divide a circle into four quadrants by a vertical and a horizontal diameter, the upper right-hand quadrant is called the first, the upper left the second, the lower left the third, and the lower right the fourth. The signs of the functions in the four quadrants are as follows:

| | First quad. | Second quad. | Third quad. | Fourth quad. |
|------------------------|-------------|--------------|-------------|--------------|
| Sine and cosecant, | + | + | — | — |
| Cosine and secant, | + | — | — | + |
| Tangent and cotangent, | + | — | + | — |

The values of the functions are as follows for the angles specified:

| Angle | 0 | 30 | 45 | 60 | 90 | 120 | 135 | 150 | 180 | 270 | 360 |
|-------------|----------|------------------------|-------------------------------|----------------------|----------|-----------------------|-------------------------------|------------------------|----------|----------|----------|
| Sine | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | -1 | 0 |
| Cosine | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{3}}{2}$ | -1 | 0 | 1 |
| Tangent | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ | $-\sqrt{3}$ | -1 | $-\frac{1}{\sqrt{3}}$ | 0 | ∞ | 0 |
| Cotangent | ∞ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 | $-\frac{1}{\sqrt{3}}$ | -1 | $-\sqrt{3}$ | ∞ | 0 | ∞ |
| Secant | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ∞ | -2 | $-\sqrt{2}$ | $-\frac{2}{\sqrt{3}}$ | -1 | ∞ | 1 |
| Cosecant | ∞ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ∞ | -1 | ∞ |
| Versed sine | 0 | $\frac{2-\sqrt{3}}{2}$ | $\frac{\sqrt{2}-1}{\sqrt{2}}$ | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | $\frac{\sqrt{2}+1}{\sqrt{2}}$ | $\frac{2+\sqrt{3}}{2}$ | 2 | 1 | 0 |

TRIGONOMETRICAL FORMULAE.

The following relations are deduced from the properties of similar triangles (Radius = 1):

$$\cos A : \sin A :: 1 : \tan A, \text{ whence } \tan A = \frac{\sin A}{\cos A};$$

$$\sin A : \cos A :: 1 : \cot A, \quad \text{“} \quad \cot A = \frac{\cos A}{\sin A};$$

$$\cos A : 1 :: 1 : \sec A, \quad \text{“} \quad \sec A = \frac{1}{\cos A};$$

$$\sin A : 1 :: 1 : \operatorname{cosec} A, \quad \text{“} \quad \operatorname{cosec} A = \frac{1}{\sin A};$$

$$\tan A : 1 :: 1 : \cot A \quad \text{“} \quad \tan A = \frac{1}{\cot A}.$$

The sum of the square of the sine of an arc and the square of its cosine equals unity. $\sin^2 A + \cos^2 A = 1$.

Also, $1 + \tan^2 A = \sec^2 A$; $1 + \cot^2 A = \operatorname{cosec}^2 A$.

Functions of the sum and difference of two angles:

Let the two angles be denoted by A and B , their sum $A + B = C$, and their difference $A - B$ by D .

$$\sin (A + B) = \sin A \cos B + \cos A \sin B; \dots \dots \dots (1)$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B; \dots \dots \dots (2)$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B; \dots \dots \dots (3)$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B. \dots \dots \dots (4)$$

From these four formulæ by addition and subtraction we obtain

$$\sin (A + B) + \sin (A - B) = 2 \sin A \cos B; \dots \dots \dots (5)$$

$$\sin (A + B) - \sin (A - B) = 2 \cos A \sin B; \dots \dots \dots (6)$$

$$\cos (A + B) + \cos (A - B) = 2 \cos A \cos B; \dots \dots \dots (7)$$

$$\cos (A - B) - \cos (A + B) = 2 \sin A \sin B. \dots \dots \dots (8)$$

If we put $A + B = C$, and $A - B = D$, then $A = \frac{1}{2}(C + D)$ and $B = \frac{1}{2}(C - D)$, and we have

$$\sin C + \sin D = 2 \sin \frac{1}{2}(C + D) \cos \frac{1}{2}(C - D); \dots \dots (9)$$

$$\sin C - \sin D = 2 \cos \frac{1}{2}(C + D) \sin \frac{1}{2}(C - D); \dots \dots (10)$$

$$\cos C + \cos D = 2 \cos \frac{1}{2}(C + D) \cos \frac{1}{2}(C - D); \dots \dots (11)$$

$$\cos D - \cos C = 2 \sin \frac{1}{2}(C + D) \sin \frac{1}{2}(C - D). \dots \dots (12)$$

Equation (9) may be enunciated thus: The sum of the sines of any two angles is equal to twice the sine of half the sum of the angles multiplied by the cosine of half their difference. These formulæ enable us to transform a sum or difference into a product.

The sum of the sines of two angles is to their difference as the tangent of half the sum of those angles is to the tangent of half their difference.

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)}{2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}. \quad (13)$$

The sum of the cosines of two angles is to their difference as the cotangent of half the sum of those angles is to the tangent of half their difference.

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \frac{2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)}{2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)} = \frac{\cot \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}. \quad (14)$$

The sine of the sum of two angles is to the sine of their difference as the sum of the tangents of those angles is to the difference of the tangents.

$$\frac{\sin (A + B)}{\sin (A - B)} = \frac{\tan A + \tan B}{\tan A - \tan B}; \dots \dots \dots (15)$$

$$\frac{\sin(A+B)}{\cos A \cos B} = \tan A + \tan B;$$

$$\frac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B;$$

$$\frac{\cos(A+B)}{\cos A \cos B} = 1 - \tan A \tan B;$$

$$\frac{\cos(A-B)}{\cos A \cos B} = 1 + \tan A \tan B;$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B};$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B};$$

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A};$$

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}.$$

Functions of twice an angle:

$$\sin 2A = 2 \sin A \cos A;$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A};$$

$$\cos 2A = \cos^2 A - \sin^2 A;$$

$$\cot 2A = \frac{\cot^2 A - 1}{2 \cot A}.$$

Functions of half an angle:

$$\sin \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}};$$

$$\tan \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}};$$

$$\cos \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}};$$

$$\cot \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{1 - \cos A}}.$$

For tables of Trigonometric Functions, see Mathematical Tables.

Solution of Plane Right-angled Triangles.

Let A and B be the two acute angles and C the right angle, and a , b , and c the sides opposite these angles, respectively, then we have

$$1. \sin A = \cos B = \frac{a}{c}; \quad 3. \tan A = \cot B = \frac{a}{b};$$

$$2. \cos A = \sin B = \frac{b}{c}; \quad 4. \cot A = \tan B = \frac{b}{a}.$$

1. In any plane right-angled triangle the sine of either of the acute angles is equal to the quotient of the opposite leg divided by the hypotenuse.

2. The cosine of either of the acute angles is equal to the quotient of the adjacent leg divided by the hypotenuse.

3. The tangent of either of the acute angles is equal to the quotient of the opposite leg divided by the adjacent leg.

4. The cotangent of either of the acute angles is equal to the quotient of the adjacent leg divided by the opposite leg.

5. The square of the hypotenuse equals the sum of the squares of the other two sides.

Solution of Oblique-angled Triangles.

The following propositions are proved in works on plane trigonometry. In any plane triangle —

Theorem 1. The sines of the angles are proportional to the opposite sides.

Theorem 2. The sum of any two sides is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.

Theorem 3. If from any angle of a triangle a perpendicular be drawn to the opposite side or base, the whole base will be to the sum of the other two sides as the difference of those two sides is to the difference of the segments of the base.

CASE I. Given two angles and a side, to find the third angle and the other two sides. 1. The third angle = $180^\circ - \text{sum of the two angles}$.

2. The sides may be found by the following proportion:

The sine of the angle opposite the given side is to the sine of the angle opposite the required side as the given side is to the required side.

CASE II. Given two sides and an angle opposite one of them, to find the third side and the remaining angles.

The side opposite the given angle is to the side opposite the required angle as the sine of the given angle is to the sine of the required angle.

The third angle is found by subtracting the sum of the other two from 180° , and the third side is found as in Case I.

CASE III. Given two sides and the included angle, to find the third side and the remaining angles.

The sum of the required angles is found by subtracting the given angle from 180° . The difference of the required angles is then found by Theorem II. Half the difference added to half the sum gives the greater angle, and half the difference subtracted from half the sum gives the less angle. The third side is then found by Theorem I.

Another method:

Given the sides c, b , and the included angle A , to find the remaining side a and the remaining angles B and C .

From either of the unknown angles, as B , draw a perpendicular Be to the opposite side.

Then

$$Ae = c \cos A, \quad Be = c \sin A, \quad eC = b - Ae \quad Be \div eC = \tan C.$$

Or, in other words, solve Be, Ae and BeC as right-angled triangles.

CASE IV. Given the three sides, to find the angles.

Let fall a perpendicular upon the longest side from the opposite angle, dividing the given triangle into two right-angled triangles. The two segments of the base may be found by Theorem III. There will then be given the hypotenuse and one side of a right-angled triangle to find the angles.

For areas of triangles, see Mensuration.

ANALYTICAL GEOMETRY.

Analytical geometry is that branch of Mathematics which has for its object the determination of the forms and magnitudes of geometrical magnitudes by means of analysis.

Ordinates and abscissas. — In analytical geometry two intersecting lines YY', XX' are used as *coördinate axes*, XX' being the axis of abscissas or axis of X , and YY' the axis of ordinates or axis of Y . A , the intersection, is called the origin of coördinates. The distance of any point P from the axis of Y measured parallel to the axis of X is called the *abscissa* of the point, as AD or CP , Fig. 72. Its distance from the axis of X , measured parallel to the axis of Y , is called the *ordinate*, as AC or PD . The abscissa and ordinate taken together are called the *coördinates* of the point P . The angle of intersection is usually taken as a right angle, in which case the axes of X and Y are called *rectangular coördinates*.

The abscissa of a point is designated by the letter x and the ordinate by y .

The *equations* of a point are the equations which express the distances of the point from the axis. Thus $x = a, y = b$ are the equations of the point P .

Equations referred to rectangular coördinates. — The equation of a line expresses the relation which exists between the coördinates of every point of the line.

Equation of a straight line, $y = ax \pm b$, in which a is the tangent of the angle the line makes with the axis of X , and b the distance above A in which the line cuts the axis of Y .

Every equation of the first degree between two variables is the equation

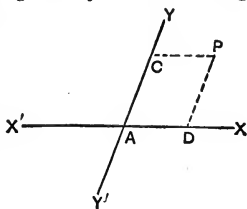


FIG. 72.

of a straight line, as $Ay + Bx + C = 0$, which can be reduced to the form $y = ax \pm b$.

Equation of the distance between two points:

$$D = \sqrt{(x'' - x')^2 + (y'' - y')^2},$$

in which $x'y'$, $x''y''$ are the coördinates of the two points.

Equation of a line passing through a given point:

$$y - y' = a(x - x'),$$

in which $x'y'$ are the coördinates of the given point, a , the tangent of the angle the line makes with the axis of x , being undetermined, since any number of lines may be drawn through a given point.

Equation of a line passing through two given points:

$$y - y' = \frac{y'' - y'}{x'' - x'}(x - x').$$

Equation of a line parallel to a given line and through a given point:

$$y - y' = a(x - x').$$

Equation of an angle V included between two given lines:

$$\text{tang } V = \frac{a' - a}{1 + a'a},$$

in which a and a' are the tangents of the angles the lines make with the axis of abscissas.

If the lines are at right angles to each other $\text{tang } V = \infty$, and

$$1 + a'a = 0.$$

Equation of an intersection of two lines, whose equations are

$$\begin{aligned} y &= ax + b, & \text{and } y &= a'x + b', \\ x &= -\frac{b - b'}{a - a'}, & \text{and } y &= \frac{ab' - a'b}{a - a'}. \end{aligned}$$

Equation of a perpendicular from a given point to a given line:

$$y - y' = -\frac{1}{a}(x - x').$$

Equation of the length of the perpendicular P :

$$P = \frac{y' - ax' - b}{\sqrt{1 + a^2}}.$$

The circle. — Equation of a circle, the origin of coördinates being at the centre, and radius = R :

$$x^2 + y^2 = R^2.$$

If the origin is at the left extremity of the diameter, on the axis of X :

$$y^2 = 2Rx - x^2.$$

If the origin is at any point, and the coördinates of the centre are $x'y'$

$$(x - x')^2 + (y - y')^2 = R^2.$$

Equation of a tangent to a circle, the coördinates of the point of tangency being $x''y''$ and the origin at the centre,

$$yy'' + xx'' = R^2.$$

The ellipse. — Equation of an ellipse, referred to rectangular coördinates with axis at the centre:

$$A^2y^2 + B^2x^2 = A^2B^2,$$

in which A is half the transverse axis and B half the conjugate axis.

Equation of the ellipse when the origin is at the vertex of the transverse axis:

$$y^2 = \frac{B^2}{A^2}(2Ax - x^2).$$

The eccentricity of an ellipse is the distance from the centre to either focus, divided by the semi-transverse axis, or

$$e = \frac{\sqrt{A^2 - B^2}}{A}.$$

The parameter of an ellipse is the double ordinate passing through the focus. It is a third proportional to the transverse axis and its conjugate, or

$$2A : 2B :: 2B : \text{parameter}; \text{ or parameter} = \frac{2B^2}{A}.$$

Any ordinate of a circle circumscribing an ellipse is to the corresponding ordinate of the ellipse as the semi-transverse axis to the semi-conjugate. Any ordinate of a circle inscribed in an ellipse is to the corresponding ordinate of the ellipse as the semi-conjugate axis to the semi-transverse.

Equation of the tangent to an ellipse, origin of axes at the centre:

$$A^2yy'' + B^2xx'' = A^2B^2,$$

$y''x''$ being the coördinates of the point of tangency.

Equation of the normal, passing through the point of tangency, and perpendicular to the tangent:

$$y - y'' = \frac{A^2y''}{B^2x''}(x - x'').$$

The normal bisects the angle of the two lines drawn from the point of tangency to the foci.

The lines drawn from the foci make equal angles with the tangent.

The parabola. — Equation of the parabola referred to rectangular coördinates, the origin being at the vertex of its axis, $y^2 = 2px$, in which $2p$ is the parameter or double ordinate through the focus.

The parameter is a third proportional to any abscissa and its corresponding ordinate, or

$$x : y :: y : 2p.$$

Equation of the tangent:

$$yy'' = p(x + x''),$$

$y''x''$ being coördinates of the point of tangency.

Equation of the normal:

$$y - y'' = -\frac{y''}{p}(x - x'').$$

The sub-normal, or projection of the normal on the axis, is constant, and equal to half the parameter.

The tangent at any point makes equal angles with the axis and with the line drawn from the point of tangency to the focus.

The hyperbola. — Equation of the hyperbola referred to rectangular coördinates, origin at the centre:

$$A^2y^2 - B^2x^2 = -A^2B^2,$$

in which A is the semi-transverse axis and B the semi-conjugate axis.

Equation when the origin is at the right vertex of the transverse axis:

$$y^2 = \frac{B^2}{A^2}(2Ax + x^2).$$

Conjugate and equilateral hyperbolas. — If on the conjugate axis,

as a transverse, and a focal distance equal to $\sqrt{A^2 + B^2}$, we construct the two branches of a hyperbola, the two hyperbolas thus constructed are called conjugate hyperbolas. If the transverse and conjugate axes are equal, the hyperbolas are called equilateral, in which case $y^2 - x^2 = -A^2$ when A is the transverse axis, and $x^2 - y^2 = -B^2$ when B is the transverse axis.

The parameter of the transverse axis is a third proportional to the transverse axis and its conjugate.

$$2A : 2B :: 2B : \text{parameter.}$$

The tangent to a hyperbola bisects the angle of the two lines drawn from the point of tangency to the foci.

The asymptotes of a hyperbola are the diagonals of the rectangle described on the axes, indefinitely produced in both directions.

The asymptotes continually approach the hyperbola, and become tangent to it at an infinite distance from the centre.

Equilateral hyperbola. — In an equilateral hyperbola the asymptotes make equal angles with the transverse axis, and are at right angles to each other. With the asymptotes as axes, and P = ordinate, V = abscissa, PV = a constant. This equation is that of the expansion of a perfect gas, in which P = absolute pressure, V = volume.

Curve of Expansion of Gases. — PV^n = a constant, or $P_1V_1^n = P_2V_2^n$, in which V_1 and V_2 are the volumes at the pressures P_1 and P_2 . When these are given, the exponent n may be found from the formula

$$n = \frac{\log P_1 - \log P_2}{\log V_2 - \log V_1}.$$

Conic sections. — Every equation of the second degree between two variables will represent either a circle, an ellipse, a parabola or a hyperbola. These curves are those which are obtained by intersecting the surface of a cone by planes, and for this reason they are called conic sections.

Logarithmic curve. — A logarithmic curve is one in which one of the coordinates of any point is the logarithm of the other.

The coordinate axis to which the lines denoting the logarithms are parallel is called the *axis of logarithms*, and the other the axis of numbers. If y is the axis of logarithms and x the axis of numbers, the equation of the curve is $y = \log x$.

If the base of a system of logarithms is a , we have $a^y = x$, in which y is the logarithm of x .

Each system of logarithms will give a different logarithmic curve. If $y = 0$, $x = 1$. Hence every logarithmic curve will intersect the axis of numbers at a distance from the origin equal to 1.

DIFFERENTIAL CALCULUS.

The differential of a variable quantity is the difference between any two of its consecutive values; hence it is indefinitely small. It is expressed by writing d before the quantity, as dx , which is read differential of x .

The term $\frac{dy}{dx}$ is called the differential coefficient of y regarded as a function of x . It is also called the first derived function or the derivative.

The differential of a function is equal to its differential coefficient multiplied by the differential of the independent variable; thus, $\frac{dy}{dx} dx = dy$.

The *limit* of a variable quantity is that value to which it continually approaches, so as at last to differ from it by less than any assignable quantity.

The differential coefficient is the limit of the ratio of the increment of the independent variable to the increment of the function.

The differential of a constant quantity is equal to 0.

The differential of a product of a constant by a variable is equal to the constant multiplied by the differential of the variable.

$$\text{If } u = Av, \quad du = A dv.$$

In any curve whose equation is $y = f(x)$, the differential coefficient $\frac{dy}{dx} = \tan a$; hence, the rate of increase of the function, or the ascension of the curve at any point, is equal to the tangent of the angle which the tangent line makes with the axis of abscissas.

All the operations of the Differential Calculus comprise but two objects:
1. To find the rate of change in a function when it passes from one state of value to another, consecutive with it.

2. To find the actual change in the function: The rate of change is the differential coefficient, and the actual change the differential.

Differentials of algebraic functions. — The differential of the sum or difference of any number of functions, dependent on the same variable, is equal to the sum or difference of their differentials taken separately:

$$\text{If } u = y + z - w, \quad du = dy + dz - dw.$$

The differential of a product of two functions dependent on the same variable is equal to the sum of the products of each by the differential of the other:

$$d(uv) = v du + u dv. \quad \frac{d(uv)}{uv} = \frac{du}{u} + \frac{dv}{v}.$$

The differential of the product of any number of functions is equal to the sum of the products which arise by multiplying the differential of each function by the product of all the others:

$$d(uts) = ts du + us dt + ut ds.$$

The differential of a fraction equals the denominator into the differential of the numerator minus the numerator into the differential of the denominator, divided by the square of the denominator:

$$dt = d\left(\frac{u}{v}\right) = \frac{v du - u dv}{v^2}.$$

If the denominator is constant, $dv = 0$, and $dt = \frac{v du}{v^2} = \frac{du}{v}.$

If the numerator is constant, $du = 0$, and $dt = -\frac{u dv}{v^2}.$

The differential of the square root of a quantity is equal to the differential of the quantity divided by twice the square root of the quantity:

$$\text{If } v = u^{1/2}, \quad \text{or } v = \sqrt{u}, \quad dv = \frac{du}{2\sqrt{u}}; = \frac{1}{2} u^{-1/2} du.$$

The differential of any power of a function is equal to the exponent multiplied by the function raised to a powerless one, multiplied by the differential of the function, $d(u^n) = nu^{n-1} du.$

Formulas for differentiating algebraic functions.

$$1. \quad d(a) = 0.$$

$$2. \quad d(ax) = a dx.$$

$$3. \quad d(x + y) = dx + dy.$$

$$4. \quad d(x - y) = dx - dy.$$

$$5. \quad d(xy) = x dy + y dx.$$

$$6. \quad d\left(\frac{x}{y}\right) = \frac{y dx - x dy}{y^2}.$$

$$7. \quad d(x^m) = mx^{m-1} dx.$$

$$8. \quad d(\sqrt{x}) = \frac{dx}{2\sqrt{x}}.$$

$$9. \quad d\left(x^{-\frac{r}{s}}\right) = -\frac{r}{s} x^{-\frac{r}{s}-1} dx.$$

To find the differential of the form $u = (a + bx^n)^m$:

Multiply the exponent of the parenthesis into the exponent of the variable within the parenthesis, into the coefficient of the variable, into the

binomial raised to a power less 1, into the variable within the parenthesis raised to a power less 1, into the differential of the variable.

$$du = d(a + bx^n)^m = mnb(a + bx^n)^{m-1} x^{n-1} dx.$$

To find the rate of change for a given value of the variable:

Find the differential coefficient, and substitute the value of the variable in the second member of the equation.

EXAMPLE. — If x is the side of a cube and u its volume, $u = x^3$, $\frac{du}{dx} = 3x^2$.

Hence the rate of change in the volume is three times the square of the edge. If the edge is denoted by 1, the rate of change is 3.

Application. The coefficient of expansion by heat of the volume of a body is three times the linear coefficient of expansion. Thus if the side of a cube expands 0.001 inch, its volume expands 0.003 cubic inch. $1.001^3 = 1.003003001$.

A **partial differential coefficient** is the differential coefficient of a function of two or more variables under the supposition that only one of them has changed its value.

A partial differential is the differential of a function of two or more variables under the supposition that only one of them has changed its value.

The total differential of a function of any number of variables is equal to the sum of the partial differentials.

If $u = f(xy)$, the partial differentials are $\frac{du}{dx} dx$, $\frac{du}{dy} dy$.

$$\text{If } u = x^2 + y^3 - z, du = \frac{du}{dx} dx + \frac{du}{dy} dy + \frac{du}{dz} dz; = 2x dx + 3y^2 dy - dz.$$

Integrals. — An integral is a functional expression derived from a differential. Integration is the operation of finding the primitive function from the differential function. It is indicated by the sign \int , which is read "the integral of." Thus $\int 2x dx = x^2$; read, the integral of $2x dx$ equals x^2 .

To integrate an expression of the form $mx^{m-1}dx$ or $x^m dx$, add 1 to the exponent of the variable, and divide by the new exponent and by the differential of the variable: $\int 3x^2 dx = x^3$. (Applicable in all cases except when $m = -1$. For $\int x^{-1} dx$ see formula 2, page 81.)

The integral of the product of a constant by the differential of a variable is equal to the constant multiplied by the integral of the differential:

$$\int ax^m dx = a \int x^m dx = a \frac{1}{m+1} x^{m+1}.$$

The integral of the algebraic sum of any number of differentials is equal to the algebraic sum of their integrals:

$$du = 2ax^2 dx - by dy - z^2 dz; \int du = \frac{2}{3} ax^3 - \frac{b}{2} y^2 - \frac{z^3}{3}.$$

Since the differential of a constant is 0, a constant connected with a variable by the sign + or - disappears in the differentiation; thus $d(a + x^m) = dx^m = mx^{m-1} dx$. Hence in integrating a differential expression we must annex to the integral obtained a constant represented by C to compensate for the term which may have been lost in differentiation. Thus if we have $dy = a dx$; $\int dy = a \int dx$. Integrating,

$$y = ax \pm C.$$

The constant C , which is added to the first integral, must have such a value as to render the functional equation true for every possible value that may be attributed to the variable. Hence, after having found the first integral equation and added the constant C , if we then make the variable equal to zero, the value which the function assumes will be the true value of C .

An indefinite integral is the first integral obtained before the value of the constant C is determined.

A particular integral is the integral after the value of C has been found.

A definite integral is the integral corresponding to a given value of the variable.

Integration between limits. — Having found the indefinite integral and the particular integral, the next step is to find the definite integral, and then the definite integral between given limits of the variable.

The integral of a function, taken between two limits, indicated by given values of x , is equal to the difference of the definite integrals corresponding to those limits. The expression

$$\int_{x'}^{x''} dy = a \int dx$$

is read: Integral of the differential of y , taken between the limits x' and x'' : the least limit, or the limit corresponding to the subtractive integral, being placed below.

Integrate $du = 9x^2 dx$ between the limits $x = 1$ and $x = 3$, u being equal to 81 when $x = 0$. $\int du = \int 9x^2 dx = 3x^3 + C$; $C = 81$ when $x = 0$, then

$$\int_{x=1}^{x=3} du = 3(3)^3 + 81, \text{ minus } 3(1)^3 + 81 = 78.$$

Integration of particular forms.

To integrate a differential of the form $du = (a + bx^n)^m x^{n-1} dx$.

1. If there is a constant factor, place it without the sign of the integral, and omit the power of the variable without the parenthesis and the differential;

2. Augment the exponent of the parenthesis by 1, and then divide this quantity, with the exponent so increased, by the exponent of the parenthesis, into the exponent of the variable within the parenthesis, into the coefficient of the variable. Whence

$$\int du = \frac{(a + bx^n)^{m+1}}{(m+1)nb} + C.$$

The differential of an arc is the hypotenuse of a right-angle triangle of which the base is dx and the perpendicular dy .

If z is an arc, $dz = \sqrt{dx^2 + dy^2}$ $z = \int \sqrt{dx^2 + dy^2}$.

Quadrature of a plane figure.

The differential of the area of a plane surface is equal to the ordinate into the differential of the abscissa.

$$ds = y dx.$$

To apply the principle enunciated in the last equation, in finding the area of any particular plane surface:

Find the value of y in terms of x , from the equation of the bounding line; substitute this value in the differential equation, and then integrate between the required limits of x .

Area of the parabola. — Find the area of any portion of the common parabola whose equation is

$$y^2 = 2px; \text{ whence } y = \sqrt{2px}.$$

Substituting this value of y in the differential equation $ds = y dx$ gives

$$\int ds = \int \sqrt{2px} dx = \sqrt{2p} \int x^{1/2} dx = \frac{2\sqrt{2p}}{3} x^{3/2} + C;$$

$$\text{or, } s = \frac{2\sqrt{2px}}{3} \times x = \frac{2}{3} xy + C.$$

If we estimate the area from the principal vertex, $x = 0$, $y = 0$, and $C = 0$; and denoting the particular integral by s' , $s' = \frac{2}{3} xy$.

That is, the area of any portion of the parabola, estimated from the vertex, is equal to $\frac{2}{3}$ of the rectangle of the abscissa and ordinate of the extreme point. The curve is therefore quadrable.

Quadrature of surfaces of revolution. — The differential of a surface of revolution is equal to the circumference of a circle perpendicular to the axis into the differential of the arc of the meridian curve.

$$ds = 2\pi y \sqrt{dx^2 + dy^2};$$

in which y is the radius of a circle of the bounding surface in a plane perpendicular to the axis of revolution, and x is the abscissa, or distance of the plane from the origin of coördinate axes.

Therefore, to find the volume of any surface of revolution:

Find the value of y and dy from the equation of the meridian curve in terms of x and dx , then substitute these values in the differential equation, and integrate between the proper limits of x .

By application of this rule we may find:

The curved surface of a cylinder equals the product of the circumference of the base into the altitude.

The convex surface of a cone equals the product of the circumference of the base into half the slant height.

The surface of a sphere is equal to the area of four great circles, or equal to the curved surface of the circumscribing cylinder.

Cubature of volumes of revolution. — A volume of revolution is a volume generated by the revolution of a plane figure about a fixed line called the axis.

If we denote the volume by V , $dV = \pi y^2 dx$.

The area of a circle described by any ordinate y is πy^2 ; hence the differential of a volume of revolution is equal to the area of a circle perpendicular to the axis into the differential of the axis.

The differential of a volume generated by the revolution of a plane figure about the axis of Y is $\pi x^2 dy$.

To find the value of V for any given volume of revolution:

Find the value of y^2 in terms of x from the equation of the meridian curve, substitute this value in the differential equation, and then integrate between the required limits of x .

By application of this rule we may find:

The volume of a cylinder is equal to the area of the base multiplied by the altitude.

The volume of a cone is equal to the area of the base into one third the altitude.

The volume of a prolate spheroid and of an oblate spheroid (formed by the revolution of an ellipse around its transverse and its conjugate axis respectively) are each equal to two thirds of the circumscribing cylinder.

If the axes are equal, the spheroid becomes a sphere and its volume = $\frac{2}{3} \pi R^2 \times D = \frac{1}{6} \pi D^3$; R being radius and D diameter.

The volume of a paraboloid is equal to half the cylinder having the same base and altitude.

The volume of a pyramid equals the area of the base multiplied by one third the altitude.

Second, third, etc., differentials. — The differential coefficient being a function of the independent variable, it may be differentiated, and we thus obtain the second differential coefficient:

$d\left(\frac{du}{dx}\right) = \frac{d^2u}{dx^2}$. Dividing by dx , we have for the second differential coefficient $\frac{d^2u}{dx^2}$, which is read: second differential of u divided by the square of the differential of x (or dx squared).

The third differential coefficient $\frac{d^3u}{dx^3}$ is read: third differential of u divided by dx cubed.

The differentials of the different orders are obtained by multiplying the differential coefficient by the corresponding powers of dx ; thus $\frac{d^3u}{dx^3} dx^3 =$ third differential of u .

Sign of the first differential coefficient. — If we have a curve whose equation is $y = fx$, referred to rectangular coördinates, the curve will recede from the axis of X when $\frac{dy}{dx}$ is positive, and approach the axis when it is negative, when the curve lies within the first angle of the coördinate axes. For all angles and every relation of y and x the curve will recede from the axis of X when the ordinate and first differential coefficient have the same sign, and approach it when they have different signs. If the tangent of the curve becomes parallel to the axis of X at any point $\frac{dy}{dx} = 0$. If the tangent becomes perpendicular to the axis of X at any point $\frac{dy}{dx} = \infty$.

Sign of the second differential coefficient. — The second differential coefficient has the same sign as the ordinate when the curve is convex toward the axis of abscissa and a contrary sign when it is concave.

Maclaurin's Theorem. — For developing into a series any function of a single variable as $u = A + Bx + Cx^2 + Dx^3 + Ex^4$, etc., in which A, B, C , etc., are independent of x :

$$u = (u)_{x=0} + \left(\frac{du}{dx}\right)_{x=0} x + \frac{1}{1 \cdot 2} \left(\frac{d^2u}{dx^2}\right)_{x=0} x^2 + \frac{1}{1 \cdot 2 \cdot 3} \left(\frac{d^3u}{dx^3}\right)_{x=0} x^3 + \text{etc.}$$

In applying the formula, omit the expressions $x = 0$, although the coefficients are always found under this hypothesis.

EXAMPLES:

$$(a+x)^m = a^m + ma^{m-1}x + \frac{m(m-1)}{2} a^{m-2}x^2 + \frac{m(m-1)(m-2)}{3} a^{m-3}x^3 + \text{etc.}$$

$$\frac{1}{a+x} = \frac{1}{a} - \frac{x}{a^2} + \frac{x^2}{a^3} - \frac{x^3}{a^4} + \dots - \frac{x^n}{a^{n+1}}, \text{etc.}$$

Taylor's Theorem. — For developing into a series any function of the sum or difference of two independent variables, as $u' = f(x \pm y)$:

$$u' = u + \frac{du}{dx} y + \frac{d^2u}{dx^2} \frac{y^2}{1 \cdot 2} + \frac{d^3u}{dx^3} \frac{y^3}{1 \cdot 2 \cdot 3} + \text{etc.},$$

In which u is what u' becomes when $y = 0$, $\frac{du}{dx}$ is what $\frac{du'}{dx}$ becomes when $y = 0$, etc.

Maxima and minima. — To find the maximum or minimum value of a function of a single variable:

1. Find the first differential coefficient of the function, place it equal to 0, and determine the roots of the equation.

2. Find the second differential coefficient, and substitute each real root,

in succession, for the variable in the second member of the equation. Each root which gives a negative result will correspond to a maximum value of the function, and each which gives a positive result will correspond to a minimum value.

EXAMPLE. — To find the value of x which will render the function y a maximum or minimum in the equation of the circle, $y^2 + x^2 = R^2$;

$$\frac{dy}{dx} = -\frac{x}{y}; \text{ making } -\frac{x}{y} = 0 \text{ gives } x = 0.$$

The second differential coefficient is: $\frac{d^2y}{dx^2} = -\frac{x^2 + y^2}{y^3}$.

When $x = 0$, $y = R$; hence $\frac{d^2y}{dx^2} = -\frac{1}{R}$, which being negative, y is a maximum for R positive.

In applying the rule to practical examples we first find an expression for the function which is to be made a maximum or minimum.

2. If in such expression a constant quantity is found as a factor, it may be omitted in the operation; for the product will be a maximum or a minimum when the variable factor is a maximum or a minimum.

3. Any value of the independent variable which renders a function a maximum or a minimum will render any power or root of that function a maximum or minimum; hence we may square both members of an equation to free it of radicals before differentiating.

By these rules we may find:

The maximum rectangle which can be inscribed in a triangle is one whose altitude is half the altitude of the triangle.

The altitude of the maximum cylinder which can be inscribed in a cone is one third the altitude of the cone.

The surface of a cylindrical vessel of a given volume, open at the top, is a minimum when the altitude equals half the diameter.

The altitude of a cylinder inscribed in a sphere when its convex surface is a maximum is $r\sqrt{2}$. r = radius.

The altitude of a cylinder inscribed in a sphere when the volume is a maximum is $2r \div \sqrt{3}$.

Maxima and Minima without the Calculus. — In the equation $y = a + bx + cx^2$, in which a , b , and c are constants, either positive or negative, if c be positive y is a minimum when $x = -b \div 2c$; if c be negative y is a maximum when $x = -b \div 2c$. In the equation $y = a + bx + c/x$, y is a minimum when $bx = c/x$.

APPLICATION. — The cost of electrical transmission is made up (1) of fixed charges, such as superintendence, repairs, cost of poles, etc., which may be represented by a ; (2) of interest on cost of the wire, which varies with the sectional area, and may be represented by bx ; and (3) of cost of the energy wasted in transmission, which varies inversely with the area of the wire, or c/x . The total cost, $y = a + bx + c/x$, is a minimum when item 2 = item 3, or $bx = c/x$.

Differential of an exponential function.

$$\text{If } u = a^x \quad \dots \dots \dots (1)$$

$$\text{then } du = da^x = a^x k dx \quad \dots \dots \dots (2)$$

in which k is a constant dependent on a .

The relation between a and k is $a^{\frac{1}{k}} = e$; whence $a = e^k \dots \dots \dots (3)$
in which $e = 2.7182818 \dots$ the base of the Naperian system of logarithms.

Logarithms. — The logarithms in the Naperian system are denoted by i , Nap. log or hyperbolic log, hyp. log, or \log_e ; and in the common system always by log.

$$k = \text{Nap. log } a, \log a = k \log_e \dots \dots \dots (4)$$

The common logarithm of e , $= \log 2.7182818 \dots = 0.4342945 \dots$, is called the modulus of the common system, and is denoted by M . Hence, if we have the Napierian logarithm of a number we can find the common logarithm of the same number by multiplying by the modulus. Reciprocally, Nap. log $=$ com. log $\times 2.3025851$.
if in equation (4) we make $a = 10$, we have

$$1 = k \log e, \text{ or } \frac{1}{k} = \log e = M.$$

That is, the modulus of the common system is equal to 1, divided by the Napierian logarithm of the common base.

From equation (2) we have

$$\frac{du}{u} = \frac{da^x}{a^x} = k dx.$$

If we make $a = 10$, the base of the common system, $x = \log u$, and

$$d(\log u) = dx = \frac{du}{u} \times \frac{1}{k} = \frac{du}{u} \times M.$$

That is, the differential of a common logarithm of a quantity is equal to the differential of the quantity divided by the quantity, into the modulus.

If we make $a = e$, the base of the Napierian system, x becomes the Napierian logarithm of u , and k becomes 1 (see equation (3)); hence $M = 1$, and

$$d(\text{Nap. log } u) = dx = \frac{du}{a^x}; = \frac{du}{u}.$$

That is, the differential of a Napierian logarithm of a quantity is equal to the differential of the quantity divided by the quantity; and in the Napierian system the modulus is 1.

Since k is the Napierian logarithm of a , $du = a^x l a dx$. That is, the differential of a function of the form a^x is equal to the function, into the Napierian logarithm of the base a , into the differential of the exponent.

If we have a differential in a fractional form, in which the numerator is the differential of the denominator, the integral is the Napierian logarithm of the denominator. Integrals of fractional differentials of other forms are given below:

Differential forms which have known integrals; exponential functions. ($l = \text{Nap. log.}$)

$$1. \quad \int a^x l a dx = a^x + C;$$

$$2. \quad \int \frac{dx}{x} = \int dx x^{-1} = lx + C;$$

$$3. \quad \int (xy^{x-1} dy + y^x ly \times dx) = y^x + C;$$

$$4. \quad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = l(x + \sqrt{x^2 \pm a^2}) + C;$$

$$5. \quad \int \frac{dx}{\sqrt{x^2 \pm 2ax}} = l(x \pm a + \sqrt{x^2 \pm 2ax}) + C;$$

$$6. \quad \int \frac{2a dx}{a^2 - x^2} = l \left(\frac{a+x}{a-x} \right) + C;$$

$$7. \quad \int \frac{2a \, dx}{x^2 - a^2} = l \left(\frac{x - a}{x + a} \right) + C;$$

$$8. \quad \int \frac{2a \, dx}{x \sqrt{a^2 + x^2}} = l \left(\frac{\sqrt{a^2 + x^2} - a}{\sqrt{a^2 + x^2} + a} \right) + C;$$

$$9. \quad \int \frac{2a \, dx}{x \sqrt{a^2 - x^2}} = l \left(\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \right) + C;$$

$$10. \quad \int \frac{x^{-2} dx}{\sqrt{x + x^{-2}}} = -l \left(\frac{1 + \sqrt{1 + a^2 x^2}}{x} \right) + C.$$

Circular functions. — Let z denote an arc in the first quadrant, y its sine, x its cosine, v its versed sine, and t its tangent; and the following notation be employed to designate an arc by any one of its functions, viz.,

$\sin^{-1} y$ denotes an arc of which y is the sine,
 $\cos^{-1} x$ " " " " " x is the cosine,
 $\tan^{-1} t$ " " " " " t is the tangent,

(read "arc whose sine is y ," etc.), — we have the following differential forms which have known integrals (r = radius):

$$\int \cos z \, dz = \sin z + C;$$

$$\int -\sin z \, dz = \cos z + C;$$

$$\int \frac{dy}{\sqrt{1 - y^2}} = \sin^{-1} y + C;$$

$$\int \frac{-dx}{\sqrt{1 - x^2}} = \cos^{-1} x + C;$$

$$\int \frac{dv}{\sqrt{2v - v^2}} = \text{versin}^{-1} v + C;$$

$$\int \frac{dt}{1 + t^2} = \tan^{-1} t + C;$$

$$\int \frac{r \, dy}{\sqrt{r^2 - y^2}} = \sin^{-1} y + C;$$

$$\int \frac{-r \, dx}{\sqrt{r^2 - x^2}} = \cos^{-1} x + C;$$

$$\int \sin z \, dz = \text{versin } z + C;$$

$$\int \frac{dz}{\cos^2 z} = \tan z + C;$$

$$\int \frac{r \, dv}{\sqrt{2rv + v^2}} = \text{versin}^{-1} v + C;$$

$$\int \frac{r^2 \, dt}{r^2 + t^2} = \tan^{-1} t + C;$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C;$$

$$\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1} \frac{u}{a} + C;$$

$$\int \frac{du}{\sqrt{2au - u^2}} = \text{versin}^{-1} \frac{u}{a} + C;$$

$$\int \frac{a \, du}{a^2 + u^2} = \tan^{-1} \frac{u}{a} + C.$$

The cycloid. — If a circle be rolled along a straight line, any point of the circumference, as P , will describe a curve which is called a cycloid. The circle is called the generating circle, and P the generating point.

The transcendental equation of the cycloid is

$$x = \text{versin}^{-1} \frac{y}{r} - \sqrt{2ry - y^2},$$

and the differential equation is $dx = \frac{y \, dx}{\sqrt{2ry - y^2}}.$

The area of the cycloid is equal to three times the area of the generating circle.

The surface described by the arc of a cycloid when revolved about its base is equal to 64 thirds of the generating circle.

The volume of the solid generated by revolving a cycloid about its base is equal to five eighths of the circumscribing cylinder.

Integral calculus. — In the integral calculus we have to return from the differential to the function from which it was derived. A number of differential expressions are given above, each of which has a known integral corresponding to it, which, being differentiated, will produce the given differential.

In all classes of functions any differential expression may be integrated when it is reduced to one of the known forms; and the operations of the integral calculus consist mainly in making such transformations of given differential expressions as shall reduce them to equivalent ones whose integrals are known.

For methods of making these transformations reference must be made to the text-books on differential and integral calculus.

THE SLIDE RULE.

The slide rule is based on the principles that the addition of logarithms multiplies the numbers which they represent, and subtracting logarithms divides the numbers. By its use the operations of multiplication, division, the finding of powers and the extraction of roots, may be performed rapidly and with an approximation to accuracy which is sufficient for many purposes. With a good 10-inch Mannheim rule the results obtained are usually accurate to $\frac{1}{4}$ of 1 per cent. Much greater accuracy is obtained with cylindrical rules like the Thacher.

The rule (see Fig. 73) consists of a fixed and a sliding part both of which are ruled with logarithmic scales; that is, with consecutive divisions spaced not equally, as in an ordinary scale, but in proportion to the logarithms of a series of numbers from 1 to 10. By moving the slide to the right or left the logarithms are added or subtracted, and multiplication or division of the numbers thereby effected. The scales on the fixed part of the rule are known as the *A* and *D* scales, and those on the slide as the *B* and *C* scales. *A* and *B* are the upper and *C* and *D* are the lower scales. The *A* and *B* scales are each divided into two, left hand and right hand, each being a reproduction, one half the size, of the *C* and *D* scales. A "runner," consisting of a transparent strip of celluloid with a vertical line on it, is used to facilitate some of the operations. The numbering on each scale begins with the figure 1, which is called

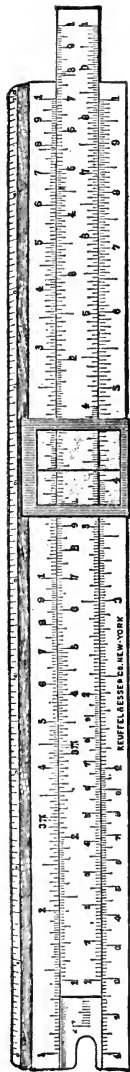


FIG. 73.

the "index" of the scale. In using the scale the figures 1, 2, 3, etc., are to be taken either as representing these numbers, or as 10, 20, 30, etc., 100, 200, 300, etc., 0.1, 0.2, 0.3, etc., that is, the numbers multiplied or divided by 10, 100, etc., as may be most convenient for the solution of a given problem.

The following examples will give an idea of the method of using the slide rule.

Proportion. — Set the first term of a proportion on the *C* scale opposite the second term on the *D* scale, then opposite the third term on the *C* scale read the fourth term on the *D* scale.

EXAMPLE. — Find the fourth term in the proportion $12 : 21 :: 30 : x$. Move the slide to the right until 12 on *C* coincides with 21 on *D*, then opposite 30 on *C* read x on *D* = 52.5. The *A* and *B* scales may be used instead of *C* and *D*.

Multiplication. — Set the index or figure 1 of the *C* scale to one of the factors on *D*.

EXAMPLE. — 25×3 . Move the slide to the right until the left index of *C* coincides with 25 on the *D* scale. Under 3 on the *C* scale will be found the product on the *D* scale, = 75.

Division. — Place the divisor on *C* opposite the dividend on *D*, and the quotient will be found on *D* under the index of *C*.

EXAMPLE. — $750 \div 25$. Move the slide to the left until 25 on *C* coincides with 750 on *D*. Under the left index of *C* is found the quotient on *D*, = 30.

Combined Multiplication and Division. — Arrange the factors to be multiplied and divided in the form of a fraction with one more factor in the numerator than in the denominator, supplying the factor 1 if necessary. Then perform alternate division and multiplication, using the runner to indicate the several partial results.

EXAMPLE. — $\frac{4 \times 5 \times 8}{3 \times 6} = 8.9$ nearly. Set 3 on *C* over 4 on *D*, set runner to 5 on *C*, then set 6 on *C* under the runner, and read under 8 on *C* the result 8.9 — on *D*.

Involution and Evolution. — The numbers on scales *A* and *B* are the squares of their coinciding numbers on the scales *C* and *D*, and also the numbers on scales *C* and *D* are the square roots of their coinciding numbers on scales *A* and *B*.

EXAMPLE. — $4^2 = 16$. Set the runner over 4 on scale *D* and read 16 on *A*.

$\sqrt{16} = 4$. Set the runner over 16 on *A* and read 4 on *D*.

In extracting square roots, if the number of digits is odd, take the number on the left-hand scale of *A*; if the number of digits is even, take the number on the right-hand scale of *A*.

To cube a number, perform the operations of squaring and multiplication.

EXAMPLE. — $2^3 = 8$. Set the index of *C* over 2 on *D*, and above 2 on *B* read the result 8 on *A*.

Extraction of the Cube Root. — Set the runner over the number on *A*, then move the slide until there is found under the runner on *B*, the same number which is found under the index of *C* on *D*; this number is the cube root desired.

EXAMPLE. — $\sqrt[3]{8} = 2$. Set the runner over 8 on *A*, move the slide along until the same number appears under the runner on *B* and under the index of *C* on *D*; this will be the number 2.

Trigonometrical Computations. — On the under side of the slide (which is reversible) are placed three scales, a scale of natural sines marked *S*, a scale of natural tangents marked *T*, and between these a scale of equal parts. To use these scales, reverse the slide, bringing its under side to the top. Coinciding with an angle on *S* its sine will be found on *A*, and coinciding with an angle on *T* will be found the tangent on *D*. Sines and tangents can be multiplied or divided like numbers.

LOGARITHMIC RULED PAPER.

W. F. Durand (*Eng. News*, Sept. 28, 1893.)

As plotted on ordinary cross-section paper the lines which express relations between two variables are usually curved, and must be plotted point by point from a table previously computed. It is only where the exponents involved in the relationship are unity that the line becomes straight and may be drawn immediately on the determination of two of its points. It is the peculiar property of logarithmic section paper that for all relationships which involve multiplication, division, raising to powers, or extraction of roots, the lines representing them are straight. Any such relationship may be represented by an equation of the form:

$y = Bx^n$. Taking logarithms we have: $\log y = \log B + n \log x$.

Logarithmic section paper is a short and ready means of plotting such logarithmic equations. The scales on each side are logarithmic instead of uniform, as in ordinary cross-section paper. The numbers and divisions marked are placed at such points that their distances from the origin are proportional to the logarithms of such numbers instead of to the numbers themselves. If we take any point, as 3, for example, on such a scale, the real distance we are dealing with is $\log 3$ to some particular base, and not 3 itself. The number at the origin of such a scale is always 1 and not 0, because 1 is the number whose logarithm is 0. This 1 may, however, represent a unit of any order, so that quantities of any size whatever may be dealt with.

If we have a series of values of x and of Bx^n , and plot on logarithmic section paper x horizontally and Bx^n vertically, the actual distances involved will be $\log x$ and $\log (Bx^n)$, or $\log B + n \log x$. But these distances will give a straight line as the locus. Hence all relationships expressible in this form are represented on logarithmic section paper by straight lines. It follows that the entire locus may be determined from any two points; that is, from any two values of Bx^n ; or, again, by any one point and the angle of inclination; that is, by one value of Bx^n and the value of n , remembering that n is the tangent of the angle of inclination to the horizontal.

A single square plotted on each edge with a logarithmic scale from 1 to 10 may be made to serve for any number whatever from 0 to ∞ . Thus to express graphically the locus of the equation: $y = x^{3/2}$. Let Fig. 74 denote a square cross-sectioned with logarithmic scales, as described. Suppose that there were joined to it and to each other on the right and above, an indefinite series of such squares similarly divided. Then, considering, in passing from one square to an adjacent one to the right or above, that the unit becomes of next higher order, such a series of squares would, with the proper variation of the unit, represent all values of either x or y between 0 and ∞ .

Suppose the original square divided on the horizontal edge into 3 parts, and on the vertical edge into 2 parts, the points of division being at A, B, D, F, G, I. Then lines joining these points, as shown, will be at an inclination to the horizontal whose tangent is $3/2$. Now, beginning at O, OF will give the value of $x^{3/2}$ for values of x from 1 to that denoted by HF, or OB, or about 4.6. For greater values of x the line would run into the adjacent square above, but the location of this line, if continued, would be exactly similar to that of BD in the square before us. Therefore the line BD will give values of $x^{3/2}$ for x between B and C, or 4.6 and 10, the corresponding values of y being of the order of tens, and ranging from 10 to 31.3. For larger values of x the unit of x is of the higher order, and we run into an adjacent square to the right without change of unit for y . In this square we should traverse a line similar to IG. Therefore, by a proper choice of units we may make use of IG for the determination of values of $x^{3/2}$ where x lies between 10 and the value at G, or about 21.5. We should then run into an adjacent square above, requiring the unit on y to be of the next higher order, and traverse a line similar to AE, which

of the m th power, while opposite to any point on Y will be m lines corresponding to the different beginnings of the m th root of the n th power. Where the complete number of lines would be quite large, it is usually unnecessary to draw them all, and the number may be limited to those necessary to cover the needed range in the values of x .

If, instead of the equation $y = x^n$, we have a constant term as a multiplier, giving an equation in the more general form $y = Bx^n$, or $Bx^{m/n}$, there will be the same number of lines and at the same inclination, but all shifted vertically through a distance equal to $\log B$. If, therefore, we start on the axis of Y at the point B , we may draw in the same series of lines and in a similar manner. In this way PQ represents the locus giving the values of the areas of circles in terms of their diameters, being the locus of the equation $A = \frac{1}{4} \pi d^2$ or $y = \frac{1}{4} \pi x^2$.

If in any case we have x in the denominator such that the equation is in the form $y = B/x^n$, this is equal to $y = Bx^{-n}$, and the same general rules hold. The lines in such case slant downward to the right instead of upward. Logarithmic ruled paper, with directions for the use, may be obtained from Keuffel & Esser Co., 127 Fulton St., New York.

MATHEMATICAL TABLES.

Formula for Interpolation.

$$a_n = a_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{1.2}d_2 + \frac{(n-1)(n-2)(n-3)}{1.2.3}d_3 + \dots$$

a_1 = the first term of the series; n , number of the required term; a_n , the required term; d_1, d_2, d_3 , first terms of successive orders of differences between a_1, a_2, a_3, a_4 , successive terms.

EXAMPLE. — Required the log of 40.7, logs of 40, 41, 42, 43 being given as below.

Terms a_1, a_2, a_3, a_4 : 1.6021 1.6128 1.6232 1.6335

| | | | |
|------------------|--------|----------|----------|
| 1st differences: | 0.0107 | 0.0104 | 0.0103 |
| 2d | " | - 0.0003 | - 0.0001 |
| 3d | " | | + 0.0002 |

For log, 40, $n = 1$; log 41, $n = 2$; for log 40.7, $n = 1.7$; $n - 1 = 0.7$; $n - 2 = -0.3$; $n - 3 = -1.3$.

$$\begin{aligned} a_n &= 1.6021 + 0.7(0.0107) + \frac{(0.7)(-0.3)(-0.0003)}{2} + \frac{(0.7)(-0.3)(-1.3)(0.0002)}{6} \\ &= 1.6021 + 0.00749 + 0.000031 + 0.000009 = 1.6096 +. \end{aligned}$$

RECIPROCAL OF NUMBERS.

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|-----|-------------|-----|-------------|-----|-------------|-----|-------------|-----|-------------|
| 1 | .00000000 | 64 | .01562500 | 127 | .00787402 | 190 | .00526316 | 253 | .00395257 |
| 2 | .50000000 | 5 | .01538461 | 8 | .00781250 | 1 | .00523566 | 4 | .00393701 |
| 3 | .33333333 | 6 | .01515151 | 9 | .00775194 | 2 | .00520833 | 5 | .00392157 |
| 4 | .25000000 | 7 | .01492537 | 130 | .00769231 | 3 | .00518135 | 6 | .00390625 |
| 5 | .20000000 | 8 | .01470588 | 1 | .00763359 | 4 | .00515464 | 7 | .00389105 |
| 6 | .16666667 | 9 | .01449275 | 2 | .00757576 | 5 | .00512820 | 8 | .00387597 |
| 7 | .14285714 | 70 | .01428571 | 3 | .00751880 | 6 | .00510204 | 9 | .00386100 |
| 8 | .12500000 | 1 | .01408451 | 4 | .00746269 | 7 | .00507614 | 260 | .00384615 |
| 9 | .11111111 | 2 | .01388889 | 5 | .00740741 | 8 | .00505051 | 1 | .00383142 |
| 10 | .10000000 | 3 | .01369863 | 6 | .00735294 | 9 | .00502513 | 2 | .00381679 |
| 11 | .09090909 | 4 | .01351351 | 7 | .00729927 | 200 | .00500000 | 3 | .00380228 |
| 12 | .08333333 | 5 | .01333333 | 8 | .00724638 | 1 | .00497512 | 4 | .00378786 |
| 13 | .07692308 | 6 | .01315789 | 9 | .00719424 | 2 | .00495049 | 5 | .00377358 |
| 14 | .07142857 | 7 | .01298701 | 140 | .00714286 | 3 | .00492611 | 6 | .00375940 |
| 15 | .06666667 | 8 | .01282051 | 1 | .00709220 | 4 | .00490196 | 7 | .00374532 |
| 16 | .06250000 | 9 | .01265823 | 2 | .00704225 | 5 | .00487805 | 8 | .00373134 |
| 17 | .05882353 | 80 | .01250000 | 3 | .00699301 | 6 | .00485437 | 9 | .00371747 |
| 18 | .05555556 | 1 | .01234568 | 4 | .00694444 | 7 | .00483092 | 270 | .00370370 |
| 19 | .05263158 | 2 | .01219512 | 5 | .00689655 | 8 | .00480769 | 1 | .00369004 |
| 20 | .05000000 | 3 | .01204819 | 6 | .00684931 | 9 | .00478469 | 2 | .00367647 |
| 1 | .04761905 | 4 | .01190476 | 7 | .00680272 | 210 | .00476190 | 3 | .00366300 |
| 2 | .04545455 | 5 | .01176471 | 8 | .00675676 | 11 | .00473934 | 4 | .00364963 |
| 3 | .04347826 | 6 | .01162791 | 9 | .00671141 | 12 | .00471698 | 5 | .00363636 |
| 4 | .04166667 | 7 | .01149425 | 150 | .00666667 | 13 | .00469484 | 6 | .00362319 |
| 5 | .04000000 | 8 | .01136364 | 1 | .00662252 | 14 | .00467290 | 7 | .00361011 |
| 6 | .03846154 | 9 | .01123595 | 2 | .00657895 | 15 | .00465116 | 8 | .00359712 |
| 7 | .03703704 | 90 | .01111111 | 3 | .00653595 | 16 | .00462963 | 9 | .00358423 |
| 8 | .03571429 | 1 | .01093901 | 4 | .00649351 | 17 | .00460829 | 280 | .00357143 |
| 9 | .03448276 | 2 | .01086956 | 5 | .00645161 | 18 | .00458716 | 1 | .00355872 |
| 30 | .03333333 | 3 | .01075269 | 6 | .00641026 | 19 | .00456621 | 2 | .00354610 |
| 1 | .03225806 | 4 | .01063830 | 7 | .00636943 | 220 | .00454545 | 3 | .00353357 |
| 2 | .03125000 | 5 | .01052632 | 8 | .00632911 | 1 | .00452489 | 4 | .00352113 |
| 3 | .03030303 | 6 | .01041667 | 9 | .00628931 | 2 | .00450450 | 5 | .00350877 |
| 4 | .02941176 | 7 | .01030928 | 160 | .00625000 | 3 | .00448430 | 6 | .00349650 |
| 5 | .02857143 | 8 | .01020498 | 1 | .00621118 | 4 | .00446429 | 7 | .00348432 |
| 6 | .02777778 | 9 | .01010101 | 2 | .00617284 | 5 | .00444444 | 8 | .00347222 |
| 7 | .02702703 | 100 | .01000000 | 3 | .00613497 | 6 | .00442478 | 9 | .00346021 |
| 8 | .02631579 | 1 | .00990099 | 4 | .00609756 | 7 | .00440529 | 290 | .00344828 |
| 9 | .02564103 | 2 | .00980392 | 5 | .00606061 | 8 | .00438596 | 1 | .00343643 |
| 40 | .02500000 | 3 | .00970874 | 6 | .00602410 | 9 | .00436681 | 2 | .00342466 |
| 1 | .02439024 | 4 | .00961538 | 7 | .00593802 | 230 | .00434783 | 3 | .00341297 |
| 2 | .02380952 | 5 | .00952331 | 8 | .00595238 | 1 | .00432900 | 4 | .00340136 |
| 3 | .02325581 | 6 | .00943396 | 9 | .00591716 | 2 | .00431034 | 5 | .00338983 |
| 4 | .02272727 | 7 | .00934579 | 170 | .00588235 | 3 | .00429184 | 6 | .00337838 |
| 5 | .02222222 | 8 | .00925926 | 1 | .00584795 | 4 | .00427350 | 7 | .00336700 |
| 6 | .02173913 | 9 | .00917431 | 2 | .00581395 | 5 | .00425532 | 8 | .00335570 |
| 7 | .02127660 | 110 | .00909091 | 3 | .00578035 | 6 | .00423729 | 9 | .00334448 |
| 8 | .02083333 | 11 | .00900091 | 4 | .00574713 | 7 | .00421941 | 300 | .00333333 |
| 9 | .02040816 | 12 | .00892857 | 5 | .00571429 | 8 | .00420168 | 1 | .00332226 |
| 50 | .02000000 | 13 | .00884956 | 6 | .00568182 | 9 | .00418410 | 2 | .00331126 |
| 1 | .01960784 | 14 | .00877193 | 7 | .00564972 | 240 | .00416667 | 3 | .00330033 |
| 2 | .01923077 | 15 | .00869565 | 8 | .00561798 | 1 | .00414938 | 4 | .00328947 |
| 3 | .01886792 | 16 | .00862069 | 9 | .00558659 | 2 | .00413223 | 5 | .00327869 |
| 4 | .01851852 | 17 | .00854701 | 180 | .00555556 | 3 | .00411523 | 6 | .00326797 |
| 5 | .01818182 | 18 | .00847458 | 1 | .00552486 | 4 | .00409836 | 7 | .00325733 |
| 6 | .01785714 | 19 | .00840336 | 2 | .00549451 | 5 | .00408163 | 8 | .00324675 |
| 7 | .01754386 | 120 | .00833333 | 3 | .00546448 | 6 | .00406504 | 9 | .00323625 |
| 8 | .01724138 | 1 | .00826446 | 4 | .00543478 | 7 | .00404858 | 310 | .00322581 |
| 9 | .01694915 | 2 | .00819672 | 5 | .00540540 | 8 | .00403226 | 11 | .00321543 |
| 60 | .01666667 | 3 | .00813008 | 6 | .00537634 | 9 | .00401606 | 12 | .00320513 |
| 1 | .01639344 | 4 | .00806452 | 7 | .00534759 | 250 | .00400000 | 13 | .00319489 |
| 2 | .01612903 | 5 | .00800000 | 8 | .00531914 | 1 | .00398406 | 14 | .00318471 |
| 3 | .01587302 | 6 | .00793651 | 9 | .00529100 | 2 | .00396825 | 15 | .00317460 |

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|-----|-------------|-----|-------------|-----|-------------|-----|-------------|-----|-------------|
| 316 | .00316456 | 381 | .00262467 | 446 | .00224215 | 511 | .00195695 | 576 | .00173611 |
| 17 | .00315457 | 2 | .00261780 | 7 | .00223714 | 12 | .00195312 | 7 | .00173310 |
| 18 | .00314465 | 3 | .00261097 | 8 | .00223214 | 13 | .00194932 | 8 | .00173010 |
| 19 | .00313480 | 4 | .00260417 | 9 | .00222717 | 14 | .00194552 | 9 | .00172712 |
| 320 | .00312500 | 5 | .00259740 | 450 | .00222222 | 15 | .00194175 | 580 | .00172414 |
| 1 | .00311526 | 6 | .00259067 | 1 | .00221729 | 16 | .00193798 | 1 | .00172117 |
| 2 | .00310559 | 7 | .00258398 | 2 | .00221239 | 17 | .00193424 | 2 | .00171821 |
| 3 | .00309597 | 8 | .00257732 | 3 | .00220751 | 18 | .00193050 | 3 | .00171527 |
| 4 | .00308642 | 9 | .00257069 | 4 | .00220264 | 19 | .00192678 | 4 | .00171233 |
| 5 | .00307692 | 390 | .00256410 | 5 | .00219780 | 520 | .00192308 | 5 | .00170940 |
| 6 | .00306748 | 1 | .00255754 | 6 | .00219298 | 1 | .00191939 | 6 | .00170648 |
| 7 | .00305810 | 2 | .00255102 | 7 | .00218818 | 2 | .00191571 | 7 | .00170358 |
| 8 | .00304873 | 3 | .00254453 | 8 | .00218341 | 3 | .00191205 | 8 | .00170068 |
| 9 | .00303951 | 4 | .00253807 | 9 | .00217865 | 4 | .00190840 | 9 | .00169779 |
| 330 | .00303030 | 5 | .00253165 | 460 | .00217391 | 5 | .00190476 | 590 | .00169491 |
| 1 | .00302115 | 6 | .00252525 | 1 | .00216920 | 6 | .00190114 | 1 | .00169205 |
| 2 | .00301205 | 7 | .00251889 | 2 | .00216450 | 7 | .00189753 | 2 | .00168919 |
| 3 | .00300300 | 8 | .00251256 | 3 | .00215983 | 8 | .00189394 | 3 | .00168634 |
| 4 | .00299401 | 9 | .00250627 | 4 | .00215517 | 9 | .00189036 | 4 | .00168350 |
| 5 | .00298507 | 400 | .00250000 | 5 | .00215054 | 530 | .00188679 | 5 | .00168067 |
| 6 | .00297619 | 1 | .00249377 | 6 | .00214592 | 1 | .00188324 | 6 | .00167785 |
| 7 | .00296736 | 2 | .00248756 | 7 | .00214133 | 2 | .00187970 | 7 | .00167504 |
| 8 | .00295858 | 3 | .00248139 | 8 | .00213675 | 3 | .00187617 | 8 | .00167224 |
| 9 | .00294985 | 4 | .00247525 | 9 | .00213220 | 4 | .00187266 | 9 | .00166945 |
| 340 | .00294118 | 5 | .00246914 | 470 | .00212766 | 5 | .00186916 | 600 | .00166667 |
| 1 | .00293255 | 6 | .00246305 | 1 | .00212314 | 6 | .00186567 | 1 | .00166389 |
| 2 | .00292398 | 7 | .00245700 | 2 | .00211864 | 7 | .00186220 | 2 | .00166113 |
| 3 | .00291545 | 8 | .00245098 | 3 | .00211416 | 8 | .00185874 | 3 | .00165837 |
| 4 | .00290698 | 9 | .00244499 | 4 | .00210970 | 9 | .00185528 | 4 | .00165563 |
| 5 | .00289855 | 410 | .00243902 | 5 | .00210526 | 540 | .00185185 | 5 | .00165289 |
| 6 | .00289017 | 11 | .00243309 | 6 | .00210034 | 1 | .00184843 | 6 | .00165016 |
| 7 | .00288184 | 12 | .00242718 | 7 | .00209644 | 2 | .00184502 | 7 | .00164745 |
| 8 | .00287356 | 13 | .00242131 | 8 | .00209205 | 3 | .00184162 | 8 | .00164474 |
| 9 | .00286533 | 14 | .00241546 | 9 | .00208768 | 4 | .00183823 | 9 | .00164204 |
| 350 | .00285714 | 15 | .00240964 | 480 | .00208333 | 5 | .00183486 | 610 | .00163934 |
| 1 | .00284900 | 16 | .00240385 | 1 | .00207900 | 6 | .00183150 | 11 | .00163666 |
| 2 | .00284091 | 17 | .00239808 | 2 | .00207469 | 7 | .00182815 | 12 | .00163399 |
| 3 | .00283286 | 18 | .00239234 | 3 | .00207039 | 8 | .00182482 | 13 | .00163132 |
| 4 | .00282486 | 19 | .00238663 | 4 | .00206612 | 9 | .00182149 | 14 | .00162866 |
| 5 | .00281690 | 420 | .00238095 | 5 | .00206186 | 550 | .00181818 | 15 | .00162602 |
| 6 | .00280899 | 1 | .00237530 | 6 | .00205761 | 1 | .00181483 | 16 | .00162338 |
| 7 | .00280112 | 2 | .00236967 | 7 | .00205339 | 2 | .00181159 | 17 | .00162075 |
| 8 | .00279330 | 3 | .00236407 | 8 | .00204918 | 3 | .00180832 | 18 | .00161812 |
| 9 | .00278551 | 4 | .00235849 | 9 | .00204499 | 4 | .00180505 | 19 | .00161551 |
| 360 | .00277778 | 5 | .00235294 | 490 | .00204082 | 5 | .00180180 | 620 | .00161290 |
| 1 | .00277008 | 6 | .00234742 | 1 | .00203666 | 6 | .00179856 | 1 | .00161031 |
| 2 | .00276243 | 7 | .00234192 | 2 | .00203252 | 7 | .00179533 | 2 | .00160772 |
| 3 | .00275482 | 8 | .00233645 | 3 | .00202840 | 8 | .00179211 | 3 | .00160514 |
| 4 | .00274725 | 9 | .00233100 | 4 | .00202429 | 9 | .00178891 | 4 | .00160256 |
| 5 | .00273973 | 430 | .00232558 | 5 | .00202020 | 560 | .00178571 | 5 | .00160000 |
| 6 | .00273224 | 1 | .00232019 | 6 | .00201613 | 1 | .00178253 | 6 | .00159744 |
| 7 | .00272480 | 2 | .00231481 | 7 | .00201207 | 2 | .00177936 | 7 | .00159490 |
| 8 | .00271739 | 3 | .00230947 | 8 | .00200803 | 3 | .00177620 | 8 | .00159236 |
| 9 | .00271003 | 4 | .00230415 | 9 | .00200401 | 4 | .00177305 | 9 | .00158982 |
| 370 | .00270270 | 5 | .00229885 | 500 | .00200000 | 5 | .00176991 | 630 | .00158730 |
| 1 | .00269542 | 6 | .00229358 | 1 | .00199601 | 6 | .00176678 | 1 | .00158479 |
| 2 | .00268817 | 7 | .00228833 | 2 | .00199203 | 7 | .00176367 | 2 | .00158228 |
| 3 | .00268096 | 8 | .00228310 | 3 | .00198807 | 8 | .00176056 | 3 | .00157978 |
| 4 | .00267380 | 9 | .00227790 | 4 | .00198413 | 9 | .00175747 | 4 | .00157729 |
| 5 | .00266667 | 440 | .00227273 | 5 | .00198020 | 570 | .00175439 | 5 | .00157480 |
| 6 | .00265957 | 1 | .00226757 | 6 | .00197628 | 1 | .00175131 | 6 | .00157233 |
| 7 | .00265252 | 2 | .00226244 | 7 | .00197239 | 2 | .00174825 | 7 | .00156986 |
| 8 | .00264550 | 3 | .00225734 | 8 | .00196850 | 3 | .00174520 | 8 | .00156740 |
| 9 | .00263852 | 4 | .00225225 | 9 | .00196464 | 4 | .00174216 | 9 | .00156494 |
| 380 | .00263158 | 5 | .00224719 | 510 | .00196078 | 5 | .00173913 | 640 | .00156250 |

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|-----|-------------|-----|-------------|-----|-------------|-----|-------------|-----|-------------|
| 641 | .00156006 | 706 | .00141643 | 771 | .00129702 | 836 | .00119617 | 901 | .00110988 |
| 2 | .00155763 | 7 | .00141443 | 2 | .00129534 | 7 | .00119474 | 2 | .00110865 |
| 3 | .00155521 | 8 | .00141243 | 3 | .00129366 | 8 | .00119332 | 3 | .00110742 |
| 4 | .00155279 | 9 | .00141044 | 4 | .00129199 | 9 | .00119189 | 4 | .00110619 |
| 5 | .00155039 | 710 | .00140845 | 5 | .00129032 | 840 | .00119048 | 5 | .00110497 |
| 6 | .00154799 | 11 | .00140647 | 6 | .00128866 | 1 | .00118906 | 6 | .00110375 |
| 7 | .00154559 | 12 | .00140449 | 7 | .00128700 | 2 | .00118765 | 7 | .00110254 |
| 8 | .00154321 | 13 | .00140252 | 8 | .00128535 | 3 | .00118624 | 8 | .00110132 |
| 9 | .00154083 | 14 | .00140056 | 9 | .00128370 | 4 | .00118483 | 9 | .00110011 |
| 650 | .00153846 | 15 | .00139860 | 780 | .00128205 | 5 | .00118343 | 910 | .00109890 |
| 1 | .00153610 | 16 | .00139665 | 1 | .00128041 | 6 | .00118203 | 11 | .00109769 |
| 2 | .00153374 | 17 | .00139470 | 2 | .00127877 | 7 | .00118064 | 12 | .00109649 |
| 3 | .00153140 | 18 | .00139276 | 3 | .00127714 | 8 | .00117924 | 13 | .00109529 |
| 4 | .00152905 | 19 | .00139082 | 4 | .00127551 | 9 | .00117786 | 14 | .00109409 |
| 5 | .00152672 | 720 | .00138889 | 5 | .00127388 | 850 | .00117647 | 15 | .00109290 |
| 6 | .00152439 | 1 | .00138696 | 6 | .00127226 | 1 | .00117509 | 16 | .00109170 |
| 7 | .00152207 | 2 | .00138504 | 7 | .00127065 | 2 | .00117371 | 17 | .00109051 |
| 8 | .00151975 | 3 | .00138313 | 8 | .00126904 | 3 | .00117233 | 18 | .00108932 |
| 9 | .00151745 | 4 | .00138121 | 9 | .00126743 | 4 | .00117096 | 19 | .00108814 |
| 660 | .00151515 | 5 | .00137931 | 790 | .00126582 | 5 | .00116959 | 920 | .00108696 |
| 1 | .00151286 | 6 | .00137741 | 1 | .00126422 | 6 | .00116822 | 1 | .00108578 |
| 2 | .00151057 | 7 | .00137552 | 2 | .00126263 | 7 | .00116686 | 2 | .00108460 |
| 3 | .00150830 | 8 | .00137363 | 3 | .00126103 | 8 | .00116550 | 3 | .00108342 |
| 4 | .00150602 | 9 | .00137174 | 4 | .00125945 | 9 | .00116414 | 4 | .00108225 |
| 5 | .00150376 | 730 | .00136986 | 5 | .00125786 | 860 | .00116279 | 5 | .00108108 |
| 6 | .00150150 | 1 | .00136799 | 6 | .00125628 | 1 | .00116144 | 6 | .00107991 |
| 7 | .00149925 | 2 | .00136612 | 7 | .00125470 | 2 | .00116009 | 7 | .00107875 |
| 8 | .00149701 | 3 | .00136426 | 8 | .00125313 | 3 | .00115875 | 8 | .00107759 |
| 9 | .00149477 | 4 | .00136240 | 9 | .00125156 | 4 | .00115741 | 9 | .00107643 |
| 670 | .00149254 | 5 | .00136054 | 800 | .00125000 | 5 | .00115607 | 930 | .00107527 |
| 1 | .00149031 | 6 | .00135870 | 1 | .00124844 | 6 | .00115473 | 1 | .00107411 |
| 2 | .00148809 | 7 | .00135685 | 2 | .00124688 | 7 | .00115340 | 2 | .00107296 |
| 3 | .00148588 | 8 | .00135501 | 3 | .00124533 | 8 | .00115207 | 3 | .00107181 |
| 4 | .00148368 | 9 | .00135318 | 4 | .00124378 | 9 | .00115075 | 4 | .00107066 |
| 5 | .00148148 | 740 | .00135135 | 5 | .00124224 | 870 | .00114942 | 5 | .00106952 |
| 6 | .00147929 | 1 | .00134953 | 6 | .00124069 | 1 | .00114811 | 6 | .00106838 |
| 7 | .00147710 | 2 | .00134771 | 7 | .00123916 | 2 | .00114679 | 7 | .00106724 |
| 8 | .00147493 | 3 | .00134589 | 8 | .00123762 | 3 | .00114547 | 8 | .00106610 |
| 9 | .00147275 | 4 | .00134409 | 9 | .00123609 | 4 | .00114416 | 9 | .00106496 |
| 680 | .00147059 | 5 | .00134228 | 810 | .00123457 | 5 | .00114286 | 940 | .00106383 |
| 1 | .00146843 | 6 | .00134048 | 11 | .00123305 | 6 | .00114155 | 1 | .00106270 |
| 2 | .00146628 | 7 | .00133869 | 12 | .00123153 | 7 | .00114025 | 2 | .00106157 |
| 3 | .00146413 | 8 | .00133690 | 13 | .00123001 | 8 | .00113895 | 3 | .00106044 |
| 4 | .00146199 | 9 | .00133511 | 14 | .00122850 | 9 | .00113766 | 4 | .00105932 |
| 5 | .00145985 | 750 | .00133333 | 15 | .00122699 | 880 | .00113636 | 5 | .00105820 |
| 6 | .00145773 | 1 | .00133156 | 16 | .00122549 | 1 | .00113507 | 6 | .00105708 |
| 7 | .00145560 | 2 | .00132979 | 17 | .00122399 | 2 | .00113379 | 7 | .00105597 |
| 8 | .00145349 | 3 | .00132802 | 18 | .00122249 | 3 | .00113250 | 8 | .00105485 |
| 9 | .00145137 | 4 | .00132626 | 19 | .00122100 | 4 | .00113122 | 9 | .00105374 |
| 690 | .00144927 | 5 | .00132450 | 820 | .00121951 | 5 | .00112994 | 950 | .00105263 |
| 1 | .00144718 | 6 | .00132275 | 1 | .00121803 | 6 | .00112867 | 1 | .00105152 |
| 2 | .00144509 | 7 | .00132100 | 2 | .00121654 | 7 | .00112740 | 2 | .00105042 |
| 3 | .00144300 | 8 | .00131926 | 3 | .00121507 | 8 | .00112613 | 3 | .00104932 |
| 4 | .00144092 | 9 | .00131752 | 4 | .00121359 | 9 | .00112486 | 4 | .00104822 |
| 5 | .00143885 | 760 | .00131579 | 5 | .00121212 | 890 | .00112360 | 5 | .00104712 |
| 6 | .00143678 | 1 | .00131406 | 6 | .00121065 | 1 | .00112233 | 6 | .00104602 |
| 7 | .00143472 | 2 | .00131234 | 7 | .00120919 | 2 | .00112103 | 7 | .00104493 |
| 8 | .00143266 | 3 | .00131062 | 8 | .00120773 | 3 | .00111982 | 8 | .00104384 |
| 9 | .00143061 | 4 | .00130890 | 9 | .00120627 | 4 | .00111857 | 9 | .00104275 |
| 700 | .00142857 | 5 | .00130719 | 830 | .00120482 | 5 | .00111732 | 960 | .00104167 |
| 1 | .00142653 | 6 | .00130543 | 1 | .00120337 | 6 | .00111607 | 1 | .00104058 |
| 2 | .00142450 | 7 | .00130378 | 2 | .00120192 | 7 | .00111483 | 2 | .00103950 |
| 3 | .00142247 | 8 | .00130203 | 3 | .00120048 | 8 | .00111359 | 3 | .00103842 |
| 4 | .00142045 | 9 | .00130039 | 4 | .00119904 | 9 | .00111235 | 4 | .00103734 |
| 5 | .00141844 | 770 | .001298370 | 5 | .00119760 | 900 | .00111111 | 5 | .00103627 |

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|------|-------------|------|-------------|------|-------------|------|-------------|------|-------------|
| 966 | .00103520 | 1031 | .000969932 | 1096 | .000912409 | 1161 | .000861326 | 1226 | .000815661 |
| 7 | .00103413 | 2 | .000968992 | 7 | .000911577 | 2 | .000860585 | 7 | .000814996 |
| 8 | .00103306 | 3 | .000968054 | 8 | .000910747 | 3 | .000859845 | 8 | .000814332 |
| 9 | .00103199 | 4 | .000967118 | 9 | .000909918 | 4 | .000859106 | 9 | .000813670 |
| 970 | .00103093 | 5 | .000966184 | 1100 | .000909091 | 5 | .000858369 | 1230 | .000813008 |
| 1 | .00102987 | 6 | .000965251 | 1 | .000908265 | 6 | .000857633 | 1 | .000812348 |
| 2 | .00102881 | 7 | .000964320 | 2 | .000907441 | 7 | .000856898 | 2 | .000811688 |
| 3 | .00102775 | 8 | .000963391 | 3 | .000906615 | 8 | .000856164 | 3 | .000811030 |
| 4 | .00102669 | 9 | .000962464 | 4 | .000905797 | 9 | .000855432 | 4 | .000810373 |
| 5 | .00102564 | 1040 | .000961538 | 5 | .000904977 | 1170 | .000854701 | 5 | .000809717 |
| 6 | .00102459 | 1 | .000960615 | 6 | .000904159 | 1 | .000853971 | 6 | .000809061 |
| 7 | .00102354 | 2 | .000959693 | 7 | .000903342 | 2 | .000853242 | 7 | .000808407 |
| 8 | .00102250 | 3 | .000958774 | 8 | .000902527 | 3 | .000852515 | 8 | .000807754 |
| 9 | .00102145 | 4 | .000957854 | 9 | .000901713 | 4 | .000851789 | 9 | .000807102 |
| 980 | .00102041 | 5 | .000956938 | 1110 | .000900901 | 5 | .000851064 | 1240 | .000806452 |
| 1 | .00101937 | 6 | .000956023 | 11 | .000900090 | 6 | .000850340 | 1 | .000805802 |
| 2 | .00101833 | 7 | .000955110 | 12 | .000899281 | 7 | .000849618 | 2 | .000805153 |
| 3 | .00101729 | 8 | .000954198 | 13 | .000898473 | 8 | .000848896 | 3 | .000804505 |
| 4 | .00101626 | 9 | .000953289 | 14 | .000897666 | 9 | .000848170 | 4 | .000803858 |
| 5 | .00101523 | 1050 | .000952381 | 15 | .000896861 | 1180 | .000847457 | 5 | .000803213 |
| 6 | .00101420 | 1 | .000951475 | 16 | .000896057 | 1 | .000846740 | 6 | .000802568 |
| 7 | .00101317 | 2 | .000950570 | 17 | .000895255 | 2 | .000846024 | 7 | .000801925 |
| 8 | .00101215 | 3 | .000949668 | 18 | .000894454 | 3 | .000845308 | 8 | .000801282 |
| 9 | .00101112 | 4 | .000948767 | 19 | .000893655 | 4 | .000844595 | 9 | .000800640 |
| 990 | .00101010 | 5 | .000947867 | 1120 | .000892857 | 5 | .000843832 | 1250 | .000800000 |
| 1 | .00100908 | 6 | .000946970 | 1 | .000892061 | 6 | .000843170 | 1 | .000799360 |
| 2 | .00100806 | 7 | .000946074 | 2 | .000891266 | 7 | .000842460 | 2 | .000798722 |
| 3 | .00100705 | 8 | .000945180 | 3 | .000890472 | 8 | .000841753 | 3 | .000798085 |
| 4 | .00100604 | 9 | .000944287 | 4 | .000889680 | 9 | .000841043 | 4 | .000797448 |
| 5 | .00100502 | 1060 | .000943396 | 5 | .000888889 | 1190 | .000840330 | 5 | .000796813 |
| 6 | .00100402 | 1 | .000942507 | 6 | .000888099 | 1 | .000839631 | 6 | .000796178 |
| 7 | .00100301 | 2 | .000941620 | 7 | .000887311 | 2 | .000838920 | 7 | .000795545 |
| 8 | .00100200 | 3 | .000940734 | 8 | .000886525 | 3 | .000838222 | 8 | .000794913 |
| 9 | .00100100 | 4 | .000939850 | 9 | .000885740 | 4 | .000837521 | 9 | .000794281 |
| 1000 | .00100000 | 5 | .000938967 | 1130 | .000884956 | 5 | .000836820 | 1260 | .000793651 |
| 1 | .000999001 | 6 | .000938036 | 1 | .000884173 | 6 | .000836120 | 1 | .000793021 |
| 2 | .000998004 | 7 | .000937201 | 2 | .000883392 | 7 | .000835422 | 2 | .000792393 |
| 3 | .000997009 | 8 | .000936330 | 3 | .000882612 | 8 | .000834724 | 3 | .000791766 |
| 4 | .000996016 | 9 | .000935454 | 4 | .000881834 | 9 | .000834028 | 4 | .000791139 |
| 5 | .000995025 | 1070 | .000934579 | 5 | .000881057 | 1200 | .000833333 | 5 | .000790514 |
| 6 | .000994036 | 1 | .000933707 | 6 | .000880282 | 1 | .000832639 | 6 | .000789889 |
| 7 | .000993049 | 2 | .000932836 | 7 | .000879508 | 2 | .000831947 | 7 | .000789265 |
| 8 | .000992063 | 3 | .000931966 | 8 | .000878735 | 3 | .000831255 | 8 | .000788643 |
| 9 | .000991080 | 4 | .000931099 | 9 | .000877963 | 4 | .000830565 | 9 | .000788022 |
| 1010 | .000990099 | 5 | .000930233 | 1140 | .000877193 | 5 | .000829875 | 1270 | .000787402 |
| 11 | .000989120 | 6 | .000929368 | 1 | .000876424 | 6 | .000829187 | 1 | .000786782 |
| 12 | .000988142 | 7 | .000928505 | 2 | .000875657 | 7 | .000828500 | 2 | .000786163 |
| 13 | .000987167 | 8 | .000927644 | 3 | .000874891 | 8 | .000827815 | 3 | .000785546 |
| 14 | .000986193 | 9 | .000926784 | 4 | .000874126 | 9 | .000827130 | 4 | .000784929 |
| 15 | .000985222 | 1030 | .000925926 | 5 | .000873362 | 1210 | .000826446 | 5 | .000784314 |
| 16 | .000984252 | 1 | .000925069 | 6 | .000872600 | 11 | .000825764 | 6 | .000783699 |
| 17 | .000983284 | 2 | .000924214 | 7 | .000871840 | 12 | .000825082 | 7 | .000783085 |
| 18 | .000982318 | 3 | .000923361 | 8 | .000871080 | 13 | .000824402 | 8 | .000782473 |
| 19 | .000981354 | 4 | .000922509 | 9 | .000870322 | 14 | .000823723 | 9 | .000781861 |
| 1020 | .000980392 | 5 | .000921659 | 1150 | .000869565 | 15 | .000823045 | 1230 | .000781250 |
| 1 | .000979432 | 6 | .000920810 | 1 | .000868810 | 16 | .000822368 | 1 | .000780640 |
| 2 | .000978474 | 7 | .000919963 | 2 | .000868056 | 17 | .000821693 | 2 | .000780031 |
| 3 | .000977517 | 8 | .000919118 | 3 | .000867303 | 18 | .000821018 | 3 | .000779423 |
| 4 | .000976562 | 9 | .000918274 | 4 | .000866551 | 19 | .000820344 | 4 | .000778816 |
| 5 | .000975610 | 1090 | .000917431 | 5 | .000865801 | 1220 | .000819672 | 5 | .000778210 |
| 6 | .000974659 | 1 | .000916590 | 6 | .000865052 | 1 | .000819001 | 6 | .000777605 |
| 7 | .000973710 | 2 | .000915757 | 7 | .000864304 | 2 | .000818331 | 7 | .000777001 |
| 8 | .000972763 | 3 | .000914913 | 8 | .000863558 | 3 | .000817661 | 8 | .000776397 |
| 9 | .000971817 | 4 | .000914077 | 9 | .000862813 | 4 | .000816993 | 9 | .000775795 |
| 1030 | .000970874 | 5 | .000913242 | 1160 | .000862069 | 5 | .000816326 | 1290 | .000775194 |

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|------|-------------|------|-------------|------|-------------|------|-------------|------|-------------|
| 1291 | .000774593 | 1356 | .000737463 | 1421 | .000703730 | 1486 | .000672948 | 1551 | .000644745 |
| 2 | .000773994 | 7 | .000736920 | 2 | .000703235 | 7 | .000672495 | 2 | .000644330 |
| 3 | .000773395 | 8 | .000736377 | 3 | .000702741 | 8 | .000672043 | 3 | .000643915 |
| 4 | .000772797 | 9 | .000735835 | 4 | .000702247 | 9 | .000671592 | 4 | .000643501 |
| 5 | .000772201 | 1360 | .000735294 | 5 | .000701754 | 1490 | .000671141 | 5 | .000643087 |
| 6 | .000771605 | 1 | .000734754 | 6 | .000701262 | 1 | .000670691 | 6 | .000642673 |
| 7 | .000771010 | 2 | .000734214 | 7 | .000700771 | 2 | .000670241 | 7 | .000642261 |
| 8 | .000770416 | 3 | .000733676 | 8 | .000700280 | 3 | .000669792 | 8 | .000641848 |
| 9 | .000769823 | 4 | .000733138 | 9 | .000699790 | 4 | .000669344 | 9 | .000641437 |
| 1300 | .000769231 | 5 | .000732601 | 1430 | .000699301 | 5 | .000668896 | 1560 | .000641026 |
| 1 | .000768639 | 6 | .000732064 | 1 | .000698812 | 6 | .000668449 | 1 | .000640615 |
| 2 | .000768049 | 7 | .000731529 | 2 | .000698324 | 7 | .000668003 | 2 | .000640205 |
| 3 | .000767459 | 8 | .000730994 | 3 | .000697837 | 8 | .000667557 | 3 | .000639795 |
| 4 | .000766871 | 9 | .000730460 | 4 | .000697350 | 9 | .000667111 | 4 | .000639386 |
| 5 | .000766283 | 1370 | .000729927 | 5 | .000696864 | 1500 | .000666667 | 5 | .000638978 |
| 6 | .000765697 | 1 | .000729395 | 6 | .000696379 | 1 | .000666223 | 6 | .000638570 |
| 7 | .000765111 | 2 | .000728863 | 7 | .000695894 | 2 | .000665779 | 7 | .000638162 |
| 8 | .000764526 | 3 | .000728332 | 8 | .000695410 | 3 | .000665336 | 8 | .000637755 |
| 9 | .000763942 | 4 | .000727802 | 9 | .000694927 | 4 | .000664894 | 9 | .000637349 |
| 1310 | .000763359 | 5 | .000727273 | 1440 | .000694444 | 5 | .000664452 | 1570 | .000636943 |
| 11 | .000762776 | 6 | .000726744 | 1 | .000693962 | 6 | .000664011 | 1 | .000636537 |
| 12 | .000762195 | 7 | .000726216 | 2 | .000693481 | 7 | .000663570 | 2 | .000636132 |
| 13 | .000761615 | 8 | .000725689 | 3 | .000693001 | 8 | .000663130 | 3 | .000635728 |
| 14 | .000761035 | 9 | .000725163 | 4 | .000692521 | 9 | .000662691 | 4 | .000635324 |
| 15 | .000760450 | 1320 | .000724638 | 5 | .000692041 | 1510 | .000662252 | 5 | .000634921 |
| 16 | .000759875 | 1 | .000724113 | 6 | .000691563 | 11 | .000661813 | 6 | .000634518 |
| 17 | .000759301 | 2 | .000723589 | 7 | .000691085 | 12 | .000661376 | 7 | .000634115 |
| 18 | .000758725 | 3 | .000723066 | 8 | .000690608 | 13 | .000660939 | 8 | .000633714 |
| 19 | .000758150 | 4 | .000722543 | 9 | .000690131 | 14 | .000660502 | 9 | .000633312 |
| 1320 | .000757576 | 5 | .000722022 | 1450 | .000689655 | 15 | .000660066 | 1580 | .000632911 |
| 1 | .000757002 | 6 | .000721501 | 1 | .000689180 | 16 | .000659631 | 1 | .000632511 |
| 2 | .000756430 | 7 | .000720980 | 2 | .000688705 | 17 | .000659196 | 2 | .000632111 |
| 3 | .000755858 | 8 | .000720461 | 3 | .000688231 | 18 | .000658761 | 3 | .000631712 |
| 4 | .000755287 | 9 | .000719942 | 4 | .000687758 | 19 | .000658328 | 4 | .000631313 |
| 5 | .000754717 | 1390 | .000719424 | 5 | .000687285 | 1520 | .000657895 | 5 | .000630915 |
| 6 | .000754148 | 1 | .000718907 | 6 | .000686813 | 1 | .000657462 | 6 | .000630517 |
| 7 | .000753579 | 2 | .000718391 | 7 | .000686341 | 2 | .000657030 | 7 | .000630120 |
| 8 | .000753012 | 3 | .000717875 | 8 | .000685871 | 3 | .000656598 | 8 | .000629723 |
| 9 | .000752445 | 4 | .000717360 | 9 | .000685401 | 4 | .000656168 | 9 | .000629327 |
| 1330 | .000751880 | 5 | .000716846 | 1460 | .000684932 | 5 | .000655738 | 1590 | .000628931 |
| 1 | .000751315 | 6 | .000716332 | 1 | .000684463 | 6 | .000655308 | 1 | .000628536 |
| 2 | .000750750 | 7 | .000715820 | 2 | .000683994 | 7 | .000654879 | 2 | .000628141 |
| 3 | .000750187 | 8 | .000715308 | 3 | .000683527 | 8 | .000654450 | 3 | .000627746 |
| 4 | .000749625 | 9 | .000714796 | 4 | .000683060 | 9 | .000654022 | 4 | .000627353 |
| 5 | .000749064 | 1400 | .000714286 | 5 | .000682594 | 1530 | .000653595 | 5 | .000626959 |
| 6 | .000748503 | 1 | .000713776 | 6 | .000682128 | 1 | .000653168 | 6 | .000626566 |
| 7 | .000747943 | 2 | .000713267 | 7 | .000681663 | 2 | .000652742 | 7 | .000626174 |
| 8 | .000747384 | 3 | .000712758 | 8 | .000681199 | 3 | .000652316 | 8 | .000625782 |
| 9 | .000746826 | 4 | .000712251 | 9 | .000680735 | 4 | .000651890 | 9 | .000625391 |
| 1340 | .000746269 | 5 | .000711744 | 1470 | .000680272 | 5 | .000651466 | 1600 | .000625000 |
| 1 | .000745712 | 6 | .000711238 | 1 | .000679810 | 6 | .000651042 | 1 | .000624619 |
| 2 | .000745156 | 7 | .000710732 | 2 | .000679348 | 7 | .000650618 | 2 | .000624241 |
| 3 | .000744602 | 8 | .000710227 | 3 | .000678887 | 8 | .000650195 | 3 | .000623865 |
| 4 | .000744048 | 9 | .000709723 | 4 | .000678426 | 9 | .000649773 | 4 | .000623489 |
| 5 | .000743494 | 1410 | .000709220 | 5 | .000677966 | 1540 | .000649351 | 1610 | .000623118 |
| 6 | .000742942 | 11 | .000708717 | 6 | .000677507 | 1 | .000648929 | 12 | .000622747 |
| 7 | .000742390 | 12 | .000708215 | 7 | .000677048 | 2 | .000648508 | 13 | .000622378 |
| 8 | .000741840 | 13 | .000707714 | 8 | .000676590 | 3 | .000648088 | 14 | .000622012 |
| 9 | .000741290 | 14 | .000707214 | 9 | .000676132 | 4 | .000647668 | 15 | .000621652 |
| 1350 | .000740741 | 15 | .000706714 | 1480 | .000675676 | 5 | .000647249 | 1620 | .000621284 |
| 1 | .000740192 | 16 | .000706215 | 1 | .000675219 | 6 | .000646830 | 2 | .000620923 |
| 2 | .000739645 | 17 | .000705716 | 2 | .000674764 | 7 | .000646412 | 3 | .000620563 |
| 3 | .000739098 | 18 | .000705219 | 3 | .000674309 | 8 | .000645995 | 4 | .000620206 |
| 4 | .000738552 | 19 | .000704722 | 4 | .000673854 | 9 | .000645578 | 5 | .000619854 |
| 5 | .000738007 | 1420 | .000704225 | 5 | .000673401 | 1550 | .000645161 | 1630 | .000619497 |

| No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. | No. | Reciprocal. |
|------|-------------|------|-------------|------|-------------|------|-------------|------|-------------|
| 1632 | .000612745 | 1706 | .000586166 | 1780 | .000561798 | 1854 | .000539374 | 1928 | .000518672 |
| 4 | .000611995 | 8 | .000585480 | 2 | .000561167 | 6 | .000538793 | 1930 | .000518135 |
| 6 | .000611247 | 1710 | .000584795 | 4 | .000560538 | 8 | .000538213 | 2 | .000517599 |
| 8 | .000610500 | 12 | .000584112 | 6 | .000559910 | 1860 | .000537634 | 4 | .000517063 |
| 1640 | .000609756 | 14 | .000583430 | 8 | .000559284 | 2 | .000537057 | 6 | .000516528 |
| 2 | .000609013 | 16 | .000582750 | 1790 | .000558659 | 4 | .000536480 | 8 | .000515996 |
| 4 | .000608272 | 18 | .000582072 | 2 | .000558035 | 6 | .000535905 | 1940 | .000515464 |
| 6 | .000607533 | 1720 | .000581395 | 4 | .000557413 | 8 | .000535332 | 2 | .000514933 |
| 8 | .000606796 | 2 | .000580720 | 6 | .000556793 | 1870 | .000534759 | 4 | .000514403 |
| 1550 | .000506061 | 4 | .000580046 | 8 | .000556174 | 2 | .000534188 | 6 | .000513874 |
| 2 | .000505327 | 6 | .000579374 | 1800 | .000555556 | 4 | .000533618 | 8 | .000513347 |
| 4 | .000504595 | 8 | .000578704 | 2 | .000554939 | 6 | .000533049 | 1950 | .000512820 |
| 6 | .000503865 | 1730 | .000578035 | 4 | .000554324 | 8 | .000532481 | 2 | .000512295 |
| 8 | .000503126 | 2 | .000577367 | 6 | .000553710 | 1880 | .000531915 | 4 | .000511770 |
| 1560 | .000502410 | 4 | .000576701 | 8 | .000553097 | 2 | .000531350 | 6 | .000511247 |
| 2 | .000501685 | 6 | .000576037 | 1810 | .000552486 | 4 | .000530785 | 8 | .000510725 |
| 4 | .000500962 | 8 | .000575374 | 12 | .000551876 | 6 | .000530222 | 1960 | .000510204 |
| 6 | .000500240 | 1740 | .000574713 | 14 | .000551268 | 8 | .000529661 | 2 | .000509684 |
| 8 | .000509520 | 2 | .000574053 | 16 | .000550661 | 1890 | .000529100 | 4 | .000509165 |
| 1570 | .000508802 | 4 | .000573394 | 18 | .000550055 | 2 | .000528541 | 6 | .000508647 |
| 2 | .000508086 | 6 | .000572737 | 1820 | .000549451 | 4 | .000527983 | 8 | .000508130 |
| 4 | .000507371 | 8 | .000572082 | 2 | .000548848 | 6 | .000527426 | 1970 | .000507614 |
| 6 | .000506658 | 1750 | .000571429 | 4 | .000548246 | 8 | .000526870 | 2 | .000507099 |
| 8 | .000505947 | 2 | .000570776 | 6 | .000547645 | 1900 | .000526316 | 4 | .000506585 |
| 1580 | .000505238 | 4 | .000570125 | 8 | .000547046 | 2 | .000525762 | 6 | .000506073 |
| 2 | .000504530 | 6 | .000569476 | 1830 | .000546448 | 4 | .000525210 | 8 | .000505561 |
| 4 | .000503824 | 8 | .000568828 | 2 | .000545851 | 6 | .000524659 | 1980 | .000505051 |
| 6 | .000503120 | 1760 | .000568132 | 4 | .000545256 | 8 | .000524109 | 2 | .000504541 |
| 8 | .000502417 | 2 | .000567537 | 6 | .000544662 | 1910 | .000523560 | 4 | .000504032 |
| 1690 | .000501716 | 4 | .000566893 | 8 | .000544069 | 12 | .000523012 | 6 | .000503524 |
| 2 | .000501017 | 6 | .000566251 | 1840 | .000543478 | 14 | .000522466 | 8 | .000503018 |
| 4 | .000500319 | 8 | .000565611 | 2 | .000542888 | 16 | .000521920 | 1990 | .000502513 |
| 6 | .000509522 | 1770 | .000564972 | 4 | .000542299 | 18 | .000521376 | 2 | .000502008 |
| 8 | .000508828 | 2 | .000564334 | 6 | .000541711 | 1920 | .000520833 | 4 | .000501504 |
| 1700 | .000508235 | 4 | .000563693 | 8 | .000541125 | 2 | .000520291 | 6 | .000501002 |
| 2 | .000507544 | 6 | .000563063 | 1850 | .000540540 | 4 | .000519750 | 8 | .000500501 |
| 4 | .000506854 | 8 | .000562430 | 2 | .000539957 | 6 | .000519211 | 2000 | .000500000 |

Use of reciprocals. — Reciprocals may be conveniently used to facilitate computations in long division. Instead of dividing as usual, multiply the dividend by the reciprocal of the divisor. The method is especially useful when many different dividends are required to be divided by the same divisor. In this case find the reciprocal of the divisor, and make a small table of its multiples up to 9 times, and use this as a multiplication-table instead of actually performing the multiplication in each case.

EXAMPLE. — 9871 and several other numbers are to be divided by 1638. The reciprocal of 1638 is .000610500.

Multiples of the reciprocal:

1. .0006105
2. .0012210
3. .0018315
4. .0024420
5. .0030525
6. .0036630
7. .0042735
8. .0048840
9. .0054945
10. .0061050

The table of multiples is made by continuous addition of 6105. The tenth line is written to check the accuracy of the addition, but it is not afterwards used.

Operation:

| | | |
|-----------------|--------|----------|
| Dividend | 9871 | |
| Take from table | 1..... | .0006105 |
| | 7..... | 0.042735 |
| | 8..... | 00.48840 |
| | 9..... | 005.4945 |

Quotient..... 6.0262455

Correct quotient by direct division..... 6.0262515

The result will generally be correct to as many figures as there are significant figures in the reciprocal, less one, and the error of the next figure will in general not exceed one. In the above example the reciprocal has six significant figures, 610500, and the result is correct to five places of figures.

**SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS OF
NUMBERS FROM 0.1 TO 1600.**

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|--------|-----------|------------|-----|---------|---------|-----------|------------|
| 0.1 | .01 | .001 | .3162 | .4642 | 3.1 | 9.61 | 29.791 | 1.761 | 1.458 |
| .15 | .0225 | .0034 | .3873 | .5313 | .2 | 10.24 | 32.768 | 1.789 | 1.474 |
| .2 | .04 | .008 | .4472 | .5848 | .3 | 10.89 | 35.937 | 1.817 | 1.489 |
| .25 | .0625 | .0156 | .500 | .6300 | .4 | 11.56 | 39.304 | 1.844 | 1.504 |
| .3 | .09 | .027 | .5477 | .6694 | .5 | 12.25 | 42.875 | 1.871 | 1.518 |
| .35 | .1225 | .0429 | .5916 | .7047 | .6 | 12.96 | 46.656 | 1.897 | 1.533 |
| .4 | .16 | .064 | .6325 | .7368 | .7 | 13.69 | 50.653 | 1.924 | 1.547 |
| .45 | .2025 | .0911 | .6708 | .7663 | .8 | 14.44 | 54.872 | 1.949 | 1.560 |
| .5 | .25 | .125 | .7071 | .7937 | .9 | 15.21 | 59.319 | 1.975 | 1.574 |
| .55 | .3025 | .1664 | .7416 | .8193 | 4. | 16. | 64. | 2. | 1.5874 |
| .6 | .36 | .216 | .7746 | .8434 | .1 | 16.81 | 68.921 | 2.025 | 1.601 |
| .65 | .4225 | .2746 | .8062 | .8662 | .2 | 17.64 | 74.088 | 2.049 | 1.613 |
| .7 | .49 | .343 | .8367 | .8879 | .3 | 18.49 | 79.507 | 2.074 | 1.626 |
| .75 | .5625 | .4219 | .8660 | .9086 | .4 | 19.36 | 85.184 | 2.098 | 1.639 |
| .8 | .64 | .512 | .8944 | .9283 | .5 | 20.25 | 91.125 | 2.121 | 1.651 |
| .85 | .7225 | .6141 | .9219 | .9473 | .6 | 21.16 | 97.336 | 2.145 | 1.663 |
| .9 | .81 | .729 | .9487 | .9655 | .7 | 22.09 | 103.823 | 2.168 | 1.675 |
| .95 | .9025 | .8574 | .9747 | .9830 | .8 | 23.04 | 110.592 | 2.191 | 1.687 |
| 1. | 1. | 1. | 1. | 1. | .9 | 24.01 | 117.649 | 2.214 | 1.698 |
| 1.05 | 1.1025 | 1.158 | 1.025 | 1.016 | 5. | 25. | 125. | 2.2361 | 1.7100 |
| 1.1 | 1.21 | 1.331 | 1.049 | 1.032 | .1 | 26.01 | 132.651 | 2.258 | 1.721 |
| 1.15 | 1.3225 | 1.521 | 1.072 | 1.048 | .2 | 27.04 | 140.608 | 2.280 | 1.732 |
| 1.2 | 1.44 | 1.728 | 1.095 | 1.063 | .3 | 28.09 | 148.877 | 2.302 | 1.744 |
| 1.25 | 1.5625 | 1.953 | 1.118 | 1.077 | .4 | 29.16 | 157.464 | 2.324 | 1.754 |
| 1.3 | 1.69 | 2.197 | 1.140 | 1.091 | .5 | 30.25 | 166.375 | 2.345 | 1.765 |
| 1.35 | 1.8225 | 2.460 | 1.162 | 1.105 | .6 | 31.36 | 175.616 | 2.366 | 1.776 |
| 1.4 | 1.96 | 2.744 | 1.183 | 1.119 | .7 | 32.49 | 185.193 | 2.387 | 1.786 |
| 1.45 | 2.1025 | 3.049 | 1.204 | 1.132 | .8 | 33.64 | 195.112 | 2.408 | 1.797 |
| 1.5 | 2.25 | 3.375 | 1.2247 | 1.1447 | .9 | 34.81 | 205.379 | 2.429 | 1.807 |
| 1.55 | 2.4025 | 3.724 | 1.245 | 1.157 | 3. | 36. | 216. | 2.4495 | 1.8171 |
| 1.6 | 2.56 | 4.096 | 1.265 | 1.170 | .1 | 37.21 | 226.981 | 2.470 | 1.827 |
| 1.65 | 2.7225 | 4.492 | 1.285 | 1.182 | .2 | 38.44 | 238.328 | 2.490 | 1.837 |
| 1.7 | 2.89 | 4.913 | 1.304 | 1.193 | .3 | 39.69 | 250.047 | 2.510 | 1.847 |
| 1.75 | 3.0625 | 5.359 | 1.323 | 1.205 | .4 | 40.96 | 262.144 | 2.530 | 1.857 |
| 1.8 | 3.24 | 5.832 | 1.342 | 1.216 | .5 | 42.25 | 274.625 | 2.550 | 1.866 |
| 1.85 | 3.4225 | 6.332 | 1.360 | 1.228 | .6 | 43.56 | 287.496 | 2.569 | 1.876 |
| 1.9 | 3.61 | 6.859 | 1.378 | 1.239 | .7 | 44.89 | 300.763 | 2.588 | 1.885 |
| 1.95 | 3.8025 | 7.415 | 1.396 | 1.249 | .8 | 46.24 | 314.432 | 2.608 | 1.895 |
| 2. | 4. | 8. | 1.4142 | 1.2599 | .9 | 47.61 | 328.509 | 2.627 | 1.904 |
| .1 | 4.41 | 9.261 | 1.449 | 1.281 | 7. | 49. | 343. | 2.6458 | 1.9129 |
| .2 | 4.84 | 10.648 | 1.483 | 1.301 | .1 | 50.41 | 357.911 | 2.665 | 1.922 |
| .3 | 5.29 | 12.167 | 1.517 | 1.320 | .2 | 51.84 | 373.248 | 2.683 | 1.931 |
| .4 | 5.76 | 13.824 | 1.549 | 1.339 | .3 | 53.29 | 389.017 | 2.702 | 1.940 |
| .5 | 6.25 | 15.625 | 1.581 | 1.357 | .4 | 54.76 | 405.224 | 2.720 | 1.949 |
| .6 | 6.76 | 17.576 | 1.612 | 1.375 | .5 | 56.25 | 421.875 | 2.739 | 1.957 |
| .7 | 7.29 | 19.683 | 1.643 | 1.392 | .6 | 57.76 | 438.976 | 2.757 | 1.966 |
| .8 | 7.84 | 21.952 | 1.673 | 1.409 | .7 | 59.29 | 456.533 | 2.775 | 1.975 |
| .9 | 8.41 | 24.389 | 1.703 | 1.426 | .8 | 60.84 | 474.552 | 2.793 | 1.983 |
| 3. | 9. | 27. | 1.7321 | 1.4422 | .9 | 62.41 | 493.039 | 2.811 | 1.992 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|---------|-----------|------------|-----|--------|--------|-----------|------------|
| 8. | 64. | 512. | 2.8284 | 2. | 45 | 2025 | 91125 | 6.7082 | 3.5569 |
| .1 | 65.61 | 531.441 | 2.846 | 2.008 | 46 | 2116 | 97336 | 6.7823 | 3.5830 |
| .2 | 67.24 | 551.368 | 2.864 | 2.017 | 47 | 2209 | 103823 | 6.8557 | 3.6088 |
| .3 | 68.89 | 571.787 | 2.881 | 2.025 | 48 | 2304 | 110592 | 6.9282 | 3.6342 |
| .4 | 70.56 | 592.704 | 2.898 | 2.033 | 49 | 2401 | 117649 | 7. | 3.6593 |
| .5 | 72.25 | 614.125 | 2.915 | 2.041 | 50 | 2500 | 125000 | 7.0711 | 3.6840 |
| .6 | 73.96 | 636.056 | 2.933 | 2.049 | 51 | 2601 | 132651 | 7.1414 | 3.7084 |
| .7 | 75.69 | 658.503 | 2.950 | 2.057 | 52 | 2704 | 140608 | 7.2111 | 3.7325 |
| .8 | 77.44 | 681.472 | 2.966 | 2.065 | 53 | 2809 | 148877 | 7.2801 | 3.7563 |
| .9 | 79.21 | 704.969 | 2.983 | 2.072 | 54 | 2916 | 157464 | 7.3485 | 3.7798 |
| 9. | 81. | 729. | 3. | 2.0801 | 55 | 3025 | 166375 | 7.4162 | 3.8030 |
| .1 | 82.81 | 753.571 | 3.017 | 2.088 | 56 | 3136 | 175616 | 7.4833 | 3.8259 |
| .2 | 84.64 | 778.688 | 3.033 | 2.095 | 57 | 3249 | 185193 | 7.5498 | 3.8485 |
| .3 | 86.49 | 804.357 | 3.050 | 2.103 | 58 | 3364 | 195112 | 7.6158 | 3.8709 |
| .4 | 88.36 | 830.584 | 3.066 | 2.110 | 59 | 3481 | 205379 | 7.6811 | 3.8930 |
| .5 | 90.25 | 857.375 | 3.082 | 2.118 | 60 | 3600 | 216000 | 7.7460 | 3.9149 |
| .6 | 92.16 | 884.736 | 3.098 | 2.125 | 61 | 3721 | 226981 | 7.8102 | 3.9365 |
| .7 | 94.09 | 912.673 | 3.114 | 2.133 | 62 | 3844 | 238328 | 7.8740 | 3.9579 |
| .8 | 96.04 | 941.192 | 3.130 | 2.140 | 63 | 3969 | 250047 | 7.9373 | 3.9791 |
| .9 | 98.01 | 970.299 | 3.146 | 2.147 | 64 | 4096 | 262144 | 8. | 4. |
| 10 | 100 | 1000 | 3.1623 | 2.1544 | 65 | 4225 | 274625 | 8.0623 | 4.0207 |
| 11 | 121 | 1331 | 3.3166 | 2.2240 | 66 | 4356 | 287496 | 8.1240 | 4.0412 |
| 12 | 144 | 1728 | 3.4641 | 2.2894 | 67 | 4489 | 300763 | 8.1854 | 4.0615 |
| 13 | 169 | 2197 | 3.6056 | 2.3513 | 68 | 4624 | 314432 | 8.2462 | 4.0817 |
| 14 | 196 | 2744 | 3.7417 | 2.4101 | 69 | 4761 | 328509 | 8.3066 | 4.1016 |
| 15 | 225 | 3375 | 3.8730 | 2.4662 | 70 | 4900 | 343000 | 8.3666 | 4.1213 |
| 16 | 256 | 4096 | 4. | 2.5198 | 71 | 5041 | 357911 | 8.4261 | 4.1408 |
| 17 | 289 | 4913 | 4.1231 | 2.5713 | 72 | 5184 | 373248 | 8.4853 | 4.1602 |
| 18 | 324 | 5832 | 4.2426 | 2.6207 | 73 | 5329 | 389017 | 8.5440 | 4.1793 |
| 19 | 361 | 6859 | 4.3589 | 2.6684 | 74 | 5476 | 405224 | 8.6023 | 4.1983 |
| 20 | 400 | 8000 | 4.4721 | 2.7144 | 75 | 5625 | 421875 | 8.6603 | 4.2172 |
| 21 | 441 | 9261 | 4.5826 | 2.7589 | 76 | 5776 | 438976 | 8.7178 | 4.2358 |
| 22 | 484 | 10648 | 4.6904 | 2.8020 | 77 | 5929 | 456533 | 8.7750 | 4.2543 |
| 23 | 529 | 12167 | 4.7958 | 2.8439 | 78 | 6084 | 474552 | 8.8318 | 4.2727 |
| 24 | 576 | 13824 | 4.8990 | 2.8845 | 79 | 6241 | 493039 | 8.8882 | 4.2908 |
| 25 | 625 | 15625 | 5. | 2.9240 | 80 | 6400 | 512000 | 8.9443 | 4.3089 |
| 26 | 676 | 17576 | 5.0990 | 2.9625 | 81 | 6561 | 531441 | 9. | 4.3267 |
| 27 | 729 | 19683 | 5.1962 | 3. | 82 | 6724 | 551368 | 9.0554 | 4.3445 |
| 28 | 784 | 21952 | 5.2915 | 3.0366 | 83 | 6889 | 571787 | 9.1104 | 4.3621 |
| 29 | 841 | 24389 | 5.3852 | 3.0723 | 84 | 7056 | 592704 | 9.1652 | 4.3795 |
| 30 | 900 | 27000 | 5.4772 | 3.1072 | 85 | 7225 | 614125 | 9.2195 | 4.3968 |
| 31 | 961 | 29791 | 5.5678 | 3.1414 | 86 | 7396 | 636056 | 9.2736 | 4.4140 |
| 32 | 1024 | 32768 | 5.6569 | 3.1748 | 87 | 7569 | 658503 | 9.3276 | 4.4310 |
| 33 | 1089 | 35937 | 5.7446 | 3.2075 | 88 | 7744 | 681472 | 9.3808 | 4.4480 |
| 34 | 1156 | 39304 | 5.8310 | 3.2396 | 89 | 7921 | 704969 | 9.4340 | 4.4647 |
| 35 | 1225 | 42875 | 5.9161 | 3.2711 | 90 | 8100 | 729000 | 9.4868 | 4.4814 |
| 36 | 1296 | 46656 | 6. | 3.3019 | 91 | 8281 | 753571 | 9.5394 | 4.4979 |
| 37 | 1369 | 50653 | 6.0828 | 3.3322 | 92 | 8464 | 778688 | 9.5917 | 4.5144 |
| 38 | 1444 | 54872 | 6.1644 | 3.3620 | 93 | 8649 | 804357 | 9.6437 | 4.5307 |
| 39 | 1521 | 59319 | 6.2450 | 3.3912 | 94 | 8836 | 830584 | 9.6954 | 4.5468 |
| 40 | 1600 | 64000 | 6.3246 | 3.4200 | 95 | 9025 | 857375 | 9.7468 | 4.5629 |
| 41 | 1681 | 68921 | 6.4031 | 3.4482 | 96 | 9216 | 884736 | 9.7980 | 4.5789 |
| 42 | 1764 | 74088 | 6.4807 | 3.4760 | 97 | 9409 | 912673 | 9.8489 | 4.5947 |
| 43 | 1849 | 79507 | 6.5574 | 3.5034 | 98 | 9604 | 941192 | 9.8995 | 4.6104 |
| 44 | 1936 | 85184 | 6.6332 | 3.5303 | 99 | 9801 | 970299 | 9.9499 | 4.6261 |

| No. | Sq. | Cube | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|-----|-------|---------|-----------|------------|-----|---------|---------|-----------|------------|
| 100 | 10000 | 1000000 | 10. | 4.6416 | 155 | 24025 | 3723875 | 12.4499 | 5.3717 |
| 101 | 10201 | 1030301 | 10.0499 | 4.6570 | 156 | 24336 | 3796416 | 12.4900 | 5.3832 |
| 102 | 10404 | 1061208 | 10.0995 | 4.6723 | 157 | 24649 | 3869893 | 12.5300 | 5.3947 |
| 103 | 10609 | 1092727 | 10.1489 | 4.6875 | 158 | 24964 | 3944312 | 12.5698 | 5.4061 |
| 104 | 10816 | 1124864 | 10.1980 | 4.7027 | 159 | 25281 | 4019679 | 12.6095 | 5.4175 |
| 105 | 11025 | 1157625 | 10.2470 | 4.7177 | 160 | 25600 | 4096000 | 12.6491 | 5.4288 |
| 106 | 11236 | 1191016 | 10.2956 | 4.7326 | 161 | 25921 | 4173281 | 12.6886 | 5.4401 |
| 107 | 11449 | 1225043 | 10.3441 | 4.7475 | 162 | 26244 | 4251528 | 12.7279 | 5.4514 |
| 108 | 11664 | 1259712 | 10.3923 | 4.7622 | 163 | 26569 | 4330747 | 12.7671 | 5.4626 |
| 109 | 11881 | 1295029 | 10.4403 | 4.7769 | 164 | 26896 | 4410944 | 12.8062 | 5.4737 |
| 110 | 12100 | 1331000 | 10.4881 | 4.7914 | 165 | 27225 | 4492125 | 12.8452 | 5.4848 |
| 111 | 12321 | 1367631 | 10.5357 | 4.8059 | 166 | 27556 | 4574296 | 12.8841 | 5.4959 |
| 112 | 12544 | 1404928 | 10.5830 | 4.8203 | 167 | 27889 | 4657463 | 12.9228 | 5.5069 |
| 113 | 12769 | 1442897 | 10.6301 | 4.8346 | 168 | 28224 | 4741632 | 12.9615 | 5.5178 |
| 114 | 12996 | 1481544 | 10.6771 | 4.8488 | 169 | 28561 | 4826809 | 13.0000 | 5.5288 |
| 115 | 13225 | 1520875 | 10.7238 | 4.8629 | 170 | 28900 | 4913000 | 13.0384 | 5.5397 |
| 116 | 13456 | 1560896 | 10.7703 | 4.8770 | 171 | 29241 | 5000211 | 13.0767 | 5.5505 |
| 117 | 13689 | 1601613 | 10.8167 | 4.8910 | 172 | 29584 | 5088448 | 13.1149 | 5.5613 |
| 118 | 13924 | 1643032 | 10.8628 | 4.9049 | 173 | 29929 | 5177717 | 13.1529 | 5.5721 |
| 119 | 14161 | 1685159 | 10.9087 | 4.9187 | 174 | 30276 | 5268024 | 13.1909 | 5.5828 |
| 120 | 14400 | 1728000 | 10.9545 | 4.9324 | 175 | 30625 | 5359375 | 13.2288 | 5.5934 |
| 121 | 14641 | 1771561 | 11.0000 | 4.9461 | 176 | 30976 | 5451776 | 13.2665 | 5.6041 |
| 122 | 14884 | 1815848 | 11.0454 | 4.9597 | 177 | 31329 | 5545233 | 13.3041 | 5.6147 |
| 123 | 15129 | 1860867 | 11.0905 | 4.9732 | 178 | 31684 | 5639752 | 13.3417 | 5.6252 |
| 124 | 15376 | 1906624 | 11.1355 | 4.9866 | 179 | 32041 | 5735339 | 13.3791 | 5.6357 |
| 125 | 15625 | 1953125 | 11.1803 | 5.0000 | 180 | 32400 | 5832000 | 13.4164 | 5.6462 |
| 126 | 15876 | 2000376 | 11.2250 | 5.0133 | 181 | 32761 | 5929741 | 13.4536 | 5.6567 |
| 127 | 16129 | 2048383 | 11.2694 | 5.0265 | 182 | 33124 | 6028568 | 13.4907 | 5.6671 |
| 128 | 16384 | 2097152 | 11.3137 | 5.0397 | 183 | 33489 | 6128487 | 13.5277 | 5.6774 |
| 129 | 16641 | 2146689 | 11.3578 | 5.0528 | 184 | 33856 | 6229504 | 13.5647 | 5.6877 |
| 130 | 16900 | 2197000 | 11.4018 | 5.0658 | 185 | 34225 | 6331625 | 13.6015 | 5.6980 |
| 131 | 17161 | 2248091 | 11.4455 | 5.0788 | 186 | 34596 | 6434856 | 13.6382 | 5.7083 |
| 132 | 17424 | 2299963 | 11.4891 | 5.0916 | 187 | 34969 | 6539203 | 13.6748 | 5.7185 |
| 133 | 17689 | 2352637 | 11.5326 | 5.1045 | 188 | 35344 | 6644672 | 13.7113 | 5.7287 |
| 134 | 17956 | 2406104 | 11.5758 | 5.1172 | 189 | 35721 | 6751269 | 13.7477 | 5.7388 |
| 135 | 18225 | 2460375 | 11.6190 | 5.1299 | 190 | 36100 | 6859000 | 13.7840 | 5.7489 |
| 136 | 18496 | 2515456 | 11.6619 | 5.1426 | 191 | 36481 | 6967871 | 13.8203 | 5.7590 |
| 137 | 18769 | 2571353 | 11.7047 | 5.1551 | 192 | 36864 | 7077888 | 13.8564 | 5.7690 |
| 138 | 19044 | 2628072 | 11.7473 | 5.1676 | 193 | 37249 | 7189057 | 13.8924 | 5.7790 |
| 139 | 19321 | 2685619 | 11.7898 | 5.1801 | 194 | 37636 | 7301384 | 13.9284 | 5.7890 |
| 140 | 19600 | 2744000 | 11.8322 | 5.1925 | 195 | 38025 | 7414875 | 13.9642 | 5.7989 |
| 141 | 19881 | 2803221 | 11.8743 | 5.2048 | 196 | 38416 | 7529536 | 14.0000 | 5.8088 |
| 142 | 20164 | 2863283 | 11.9164 | 5.2171 | 197 | 38809 | 7645373 | 14.0357 | 5.8186 |
| 143 | 20449 | 2924207 | 11.9583 | 5.2293 | 198 | 39204 | 7762392 | 14.0712 | 5.8285 |
| 144 | 20736 | 2985984 | 12.0000 | 5.2415 | 199 | 39601 | 7880599 | 14.1067 | 5.8383 |
| 145 | 21025 | 3048625 | 12.0416 | 5.2536 | 200 | 40000 | 8000000 | 14.1421 | 5.8480 |
| 146 | 21316 | 3112136 | 12.0830 | 5.2656 | 201 | 40401 | 8120601 | 14.1774 | 5.8578 |
| 147 | 21609 | 3176523 | 12.1244 | 5.2776 | 202 | 40804 | 8242408 | 14.2127 | 5.8675 |
| 148 | 21904 | 3241792 | 12.1655 | 5.2896 | 203 | 41209 | 8365427 | 14.2478 | 5.8771 |
| 149 | 22201 | 3307949 | 12.2066 | 5.3015 | 204 | 41616 | 8489664 | 14.2829 | 5.8868 |
| 150 | 22500 | 3375000 | 12.2474 | 5.3133 | 205 | 42025 | 8615125 | 14.3178 | 5.8964 |
| 151 | 22801 | 3442951 | 12.2882 | 5.3251 | 206 | 42436 | 8741816 | 14.3527 | 5.9059 |
| 152 | 23104 | 3511808 | 12.3288 | 5.3368 | 207 | 42849 | 8869743 | 14.3875 | 5.9155 |
| 153 | 23409 | 3581577 | 12.3693 | 5.3485 | 208 | 43264 | 8998912 | 14.4222 | 5.9250 |
| 154 | 23716 | 3652264 | 12.4097 | 5.3601 | 209 | 43681 | 9129329 | 14.4568 | 5.9345 |

| No. | Sq. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|-----|-------|----------|--------------|---------------|-----|---------|----------|--------------|---------------|
| 210 | 44100 | 9261000 | 14.4914 | 5.9439 | 265 | 70225 | 18609625 | 16.2788 | 6.4232 |
| 211 | 44521 | 9393931 | 14.5258 | 5.9533 | 266 | 70756 | 18821096 | 16.3095 | 6.4312 |
| 212 | 44944 | 9562128 | 14.5602 | 5.9627 | 267 | 71289 | 19034163 | 16.3401 | 6.4393 |
| 213 | 45369 | 9663597 | 14.5945 | 5.9721 | 268 | 71824 | 19248832 | 16.3707 | 6.4473 |
| 214 | 45796 | 9800344 | 14.6287 | 5.9814 | 269 | 72361 | 19465109 | 16.4012 | 6.4553 |
| 215 | 46225 | 9938375 | 14.6629 | 5.9907 | 270 | 72900 | 19683000 | 16.4317 | 6.4633 |
| 216 | 46656 | 10077696 | 14.6969 | 6.0000 | 271 | 73441 | 19902511 | 16.4621 | 6.4713 |
| 217 | 47089 | 10218313 | 14.7309 | 6.0092 | 272 | 73984 | 20123648 | 16.4924 | 6.4792 |
| 218 | 47524 | 10360232 | 14.7648 | 6.0185 | 273 | 74529 | 20346417 | 16.5227 | 6.4872 |
| 219 | 47961 | 10503459 | 14.7986 | 6.0277 | 274 | 75076 | 20570824 | 16.5529 | 6.4951 |
| 220 | 48400 | 10648000 | 14.8324 | 6.0368 | 275 | 75625 | 20796875 | 16.5831 | 6.5030 |
| 221 | 48841 | 10793861 | 14.8661 | 6.0459 | 276 | 76176 | 21024576 | 16.6132 | 6.5108 |
| 222 | 49284 | 10941048 | 14.8997 | 6.0550 | 277 | 76729 | 21253933 | 16.6433 | 6.5187 |
| 223 | 49729 | 11089567 | 14.9332 | 6.0641 | 278 | 77284 | 21484952 | 16.6733 | 6.5265 |
| 224 | 50176 | 11239424 | 14.9666 | 6.0732 | 279 | 77841 | 21717639 | 16.7033 | 6.5343 |
| 225 | 50625 | 11390625 | 15.0000 | 6.0822 | 280 | 78400 | 21952000 | 16.7332 | 6.5421 |
| 226 | 51076 | 11543176 | 15.0333 | 6.0912 | 281 | 78961 | 22188041 | 16.7631 | 6.5499 |
| 227 | 51529 | 11697083 | 15.0665 | 6.1002 | 282 | 79524 | 22425768 | 16.7929 | 6.5577 |
| 228 | 51984 | 11852352 | 15.0997 | 6.1091 | 283 | 80089 | 22665187 | 16.8226 | 6.5654 |
| 229 | 52441 | 12008989 | 15.1327 | 6.1180 | 284 | 80656 | 22906304 | 16.8523 | 6.5731 |
| 230 | 52900 | 12167000 | 15.1658 | 6.1269 | 285 | 81225 | 23149125 | 16.8819 | 6.5808 |
| 231 | 53361 | 12326391 | 15.1987 | 6.1358 | 286 | 81796 | 23393656 | 16.9115 | 6.5885 |
| 232 | 53824 | 12487168 | 15.2315 | 6.1446 | 287 | 82369 | 23639903 | 16.9411 | 6.5962 |
| 233 | 54289 | 12649337 | 15.2643 | 6.1534 | 288 | 82944 | 23887872 | 16.9706 | 6.6039 |
| 234 | 54756 | 12812904 | 15.2971 | 6.1622 | 289 | 83521 | 24137569 | 17.0000 | 6.6115 |
| 235 | 55225 | 12977875 | 15.3297 | 6.1710 | 290 | 84100 | 24389000 | 17.0294 | 6.6191 |
| 236 | 55696 | 13144256 | 15.3623 | 6.1797 | 291 | 84681 | 24642171 | 17.0587 | 6.6267 |
| 237 | 56169 | 13312053 | 15.3948 | 6.1885 | 292 | 85264 | 24897088 | 17.0880 | 6.6343 |
| 238 | 56644 | 13481272 | 15.4272 | 6.1972 | 293 | 85849 | 25153757 | 17.1172 | 6.6419 |
| 239 | 57121 | 13651919 | 15.4596 | 6.2058 | 294 | 86436 | 25412184 | 17.1464 | 6.6494 |
| 240 | 57600 | 13824000 | 15.4919 | 6.2145 | 295 | 87025 | 25672375 | 17.1756 | 6.6569 |
| 241 | 58081 | 13997521 | 15.5242 | 6.2231 | 296 | 87616 | 25934336 | 17.2047 | 6.6644 |
| 242 | 58564 | 14172488 | 15.5563 | 6.2317 | 297 | 88209 | 26198073 | 17.2337 | 6.6719 |
| 243 | 59049 | 14348907 | 15.5885 | 6.2403 | 298 | 88804 | 26463592 | 17.2627 | 6.6794 |
| 244 | 59536 | 14526784 | 15.6205 | 6.2488 | 299 | 89401 | 26730899 | 17.2916 | 6.6869 |
| 245 | 60025 | 14706125 | 15.6525 | 6.2573 | 300 | 90000 | 27000000 | 17.3205 | 6.6943 |
| 246 | 60516 | 14886936 | 15.6844 | 6.2658 | 301 | 90601 | 27270901 | 17.3494 | 6.7018 |
| 247 | 61009 | 15069223 | 15.7162 | 6.2743 | 302 | 91204 | 27543608 | 17.3781 | 6.7092 |
| 248 | 61504 | 15252992 | 15.7480 | 6.2828 | 303 | 91809 | 27818127 | 17.4069 | 6.7166 |
| 249 | 62001 | 15438249 | 15.7797 | 6.2912 | 304 | 92416 | 28094464 | 17.4356 | 6.7240 |
| 250 | 62500 | 15625000 | 15.8114 | 6.2996 | 305 | 93025 | 28372625 | 17.4642 | 6.7313 |
| 251 | 63001 | 15813251 | 15.8430 | 6.3080 | 306 | 93636 | 28652616 | 17.4929 | 6.7387 |
| 252 | 63504 | 16003008 | 15.8745 | 6.3164 | 307 | 94249 | 28934443 | 17.5214 | 6.7460 |
| 253 | 64009 | 16194277 | 15.9060 | 6.3247 | 308 | 94864 | 29218112 | 17.5499 | 6.7533 |
| 254 | 64516 | 16387064 | 15.9374 | 6.3330 | 309 | 95481 | 29503629 | 17.5784 | 6.7606 |
| 255 | 65025 | 16581375 | 15.9687 | 6.3413 | 310 | 96100 | 29791000 | 17.6068 | 6.7679 |
| 256 | 65536 | 16777216 | 16.0000 | 6.3496 | 311 | 96721 | 30080231 | 17.6352 | 6.7752 |
| 257 | 66049 | 16974593 | 16.0312 | 6.3579 | 312 | 97344 | 30371328 | 17.6635 | 6.7824 |
| 258 | 66564 | 17173512 | 16.0624 | 6.3661 | 313 | 97969 | 30664297 | 17.6918 | 6.7897 |
| 259 | 67081 | 17373979 | 16.0935 | 6.3743 | 314 | 98596 | 30959144 | 17.7200 | 6.7969 |
| 260 | 67600 | 17576000 | 16.1245 | 6.3825 | 315 | 99225 | 31255875 | 17.7482 | 6.8041 |
| 261 | 68121 | 17779581 | 16.1555 | 6.3907 | 316 | 99856 | 31554496 | 17.7764 | 6.8113 |
| 262 | 68644 | 17984728 | 16.1864 | 6.3988 | 317 | 100489 | 31855013 | 17.8045 | 6.8185 |
| 263 | 69169 | 18191447 | 16.2173 | 6.4070 | 318 | 101124 | 32157432 | 17.8326 | 6.8256 |
| 264 | 69696 | 18399744 | 16.2481 | 6.4151 | 319 | 101761 | 32461759 | 17.8606 | 6.8328 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|----------|-----------|------------|-----|--------|----------|-----------|------------|
| 320 | 102400 | 32768000 | 17.8885 | 6.8399 | 375 | 140625 | 52734375 | 19.3649 | 7.2112 |
| 321 | 103041 | 33076161 | 17.9165 | 6.8470 | 376 | 141376 | 53157376 | 19.3907 | 7.2177 |
| 322 | 103684 | 33386248 | 17.9444 | 6.8541 | 377 | 142129 | 53582633 | 19.4165 | 7.2240 |
| 323 | 104329 | 33698267 | 17.9722 | 6.8612 | 378 | 142884 | 54010152 | 19.4422 | 7.2304 |
| 324 | 104976 | 34012224 | 18.0000 | 6.8683 | 379 | 143641 | 54439939 | 19.4679 | 7.2368 |
| 325 | 105625 | 34328125 | 18.0278 | 6.8753 | 380 | 144400 | 54872000 | 19.4936 | 7.2432 |
| 326 | 106276 | 34645976 | 18.0555 | 6.8824 | 381 | 145161 | 55306341 | 19.5192 | 7.2495 |
| 327 | 106929 | 34965783 | 18.0831 | 6.8894 | 382 | 145924 | 55742968 | 19.5448 | 7.2558 |
| 328 | 107584 | 35287552 | 18.1108 | 6.8964 | 383 | 146689 | 56181887 | 19.5704 | 7.2622 |
| 329 | 108241 | 35611289 | 18.1384 | 6.9034 | 384 | 147456 | 56623104 | 19.5959 | 7.2685 |
| 330 | 108900 | 35937000 | 18.1659 | 6.9104 | 385 | 148225 | 57066625 | 19.6214 | 7.2748 |
| 331 | 109561 | 36264691 | 18.1934 | 6.9174 | 386 | 148996 | 57512456 | 19.6469 | 7.2811 |
| 332 | 110224 | 36594368 | 18.2209 | 6.9244 | 387 | 149769 | 57960603 | 19.6723 | 7.2874 |
| 333 | 110889 | 36926037 | 18.2483 | 6.9313 | 388 | 150544 | 58411072 | 19.6977 | 7.2936 |
| 334 | 111556 | 37259704 | 18.2757 | 6.9382 | 389 | 151321 | 58863869 | 19.7231 | 7.2999 |
| 335 | 112225 | 37595375 | 18.3030 | 6.9451 | 390 | 152100 | 59318000 | 19.7484 | 7.3061 |
| 336 | 112896 | 37933056 | 18.3303 | 6.9521 | 391 | 152881 | 59776471 | 19.7737 | 7.3124 |
| 337 | 113569 | 38272753 | 18.3576 | 6.9589 | 392 | 153664 | 60236288 | 19.7990 | 7.3186 |
| 338 | 114244 | 38614472 | 18.3848 | 6.9658 | 393 | 154449 | 60698457 | 19.8242 | 7.3248 |
| 339 | 114921 | 38958219 | 18.4120 | 6.9727 | 394 | 155236 | 61162984 | 19.8494 | 7.3310 |
| 340 | 115600 | 39304000 | 18.4391 | 6.9795 | 395 | 156025 | 61629875 | 19.8746 | 7.3372 |
| 341 | 116281 | 39651821 | 18.4662 | 6.9864 | 396 | 156816 | 62099136 | 19.8997 | 7.3434 |
| 342 | 116964 | 40001688 | 18.4932 | 6.9932 | 397 | 157609 | 62570773 | 19.9249 | 7.3496 |
| 343 | 117649 | 40353607 | 18.5203 | 7.0000 | 398 | 158404 | 63044792 | 19.9499 | 7.3558 |
| 344 | 118336 | 40707584 | 18.5472 | 7.0068 | 399 | 159201 | 63521159 | 19.9750 | 7.3619 |
| 345 | 119025 | 41063625 | 18.5742 | 7.0136 | 400 | 160000 | 64000000 | 20.0000 | 7.3681 |
| 346 | 119716 | 41421736 | 18.6011 | 7.0203 | 401 | 160801 | 64481201 | 20.0250 | 7.3742 |
| 347 | 120409 | 41781923 | 18.6279 | 7.0271 | 402 | 161604 | 64964808 | 20.0499 | 7.3803 |
| 348 | 121104 | 42144192 | 18.6548 | 7.0338 | 403 | 162409 | 65450827 | 20.0749 | 7.3864 |
| 349 | 121801 | 42508549 | 18.6815 | 7.0406 | 404 | 163216 | 65939264 | 20.0998 | 7.3925 |
| 350 | 122500 | 42875000 | 18.7083 | 7.0473 | 405 | 164025 | 66430125 | 20.1246 | 7.3986 |
| 351 | 123201 | 43243551 | 18.7350 | 7.0540 | 406 | 164836 | 66923416 | 20.1494 | 7.4047 |
| 352 | 123904 | 43614208 | 18.7617 | 7.0607 | 407 | 165649 | 67419143 | 20.1742 | 7.4108 |
| 353 | 124609 | 43986977 | 18.7883 | 7.0674 | 408 | 166464 | 67917312 | 20.1990 | 7.4169 |
| 354 | 125316 | 44361864 | 18.8149 | 7.0740 | 409 | 167281 | 68417929 | 20.2237 | 7.4229 |
| 355 | 126025 | 44738875 | 18.8414 | 7.0807 | 410 | 168100 | 68921000 | 20.2485 | 7.4290 |
| 356 | 126736 | 45118016 | 18.8680 | 7.0873 | 411 | 168921 | 69426531 | 20.2731 | 7.4350 |
| 357 | 127449 | 45499293 | 18.8944 | 7.0940 | 412 | 169744 | 69934528 | 20.2978 | 7.4410 |
| 358 | 128164 | 45882712 | 18.9209 | 7.1006 | 413 | 170569 | 70444997 | 20.3224 | 7.4470 |
| 359 | 128881 | 46268279 | 18.9473 | 7.1072 | 414 | 171396 | 70957944 | 20.3470 | 7.4530 |
| 360 | 129600 | 46656000 | 18.9737 | 7.1138 | 415 | 172225 | 71473375 | 20.3715 | 7.4590 |
| 361 | 130321 | 47045881 | 19.0000 | 7.1204 | 416 | 173056 | 71991296 | 20.3961 | 7.4650 |
| 362 | 131044 | 47437928 | 19.0263 | 7.1269 | 417 | 173889 | 72511713 | 20.4206 | 7.4710 |
| 363 | 131769 | 47832147 | 19.0526 | 7.1335 | 418 | 174724 | 73034632 | 20.4450 | 7.4770 |
| 364 | 132496 | 48228544 | 19.0788 | 7.1400 | 419 | 175561 | 73560059 | 20.4695 | 7.4829 |
| 365 | 133225 | 48627125 | 19.1050 | 7.1466 | 420 | 176400 | 74088000 | 20.4939 | 7.4889 |
| 366 | 133956 | 49027896 | 19.1311 | 7.1531 | 421 | 177241 | 74618461 | 20.5183 | 7.4948 |
| 367 | 134689 | 49430863 | 19.1572 | 7.1596 | 422 | 178084 | 75151448 | 20.5426 | 7.5007 |
| 368 | 135424 | 49836032 | 19.1833 | 7.1661 | 423 | 178929 | 75686967 | 20.5670 | 7.5067 |
| 369 | 136161 | 50243409 | 19.2094 | 7.1726 | 424 | 179776 | 76225024 | 20.5913 | 7.5126 |
| 370 | 136900 | 50653000 | 19.2354 | 7.1791 | 425 | 180625 | 76765625 | 20.6155 | 7.5185 |
| 371 | 137641 | 51064811 | 19.2614 | 7.1855 | 426 | 181476 | 77308776 | 20.6398 | 7.5244 |
| 372 | 138384 | 51478848 | 19.2873 | 7.1920 | 427 | 182329 | 77854483 | 20.6640 | 7.5302 |
| 373 | 139129 | 51895117 | 19.3132 | 7.1984 | 428 | 183184 | 78402752 | 20.6882 | 7.5361 |
| 374 | 139876 | 52313624 | 19.3391 | 7.2048 | 429 | 184041 | 78953589 | 20.7123 | 7.5420 |

| No. | Square | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|--------|-----------|-----------|------------|-----|--------|-----------|-----------|------------|
| 430 | 184900 | 79507000 | 20.7364 | 7.5478 | 485 | 235225 | 114084125 | 22.0227 | 7.8568 |
| 431 | 185761 | 80062991 | 20.7605 | 7.5537 | 486 | 236196 | 114791256 | 22.0454 | 7.8622 |
| 432 | 186624 | 80621568 | 20.7846 | 7.5595 | 487 | 237169 | 115501303 | 22.0681 | 7.8676 |
| 433 | 187489 | 81182737 | 20.8087 | 7.5654 | 488 | 238144 | 116214272 | 22.0907 | 7.8730 |
| 434 | 188356 | 81746504 | 20.8327 | 7.5712 | 489 | 239121 | 116930169 | 22.1133 | 7.8784 |
| 435 | 189225 | 82312875 | 20.8567 | 7.5770 | 490 | 240100 | 117649000 | 22.1359 | 7.8837 |
| 436 | 190096 | 82881856 | 20.8806 | 7.5828 | 491 | 241081 | 118370771 | 22.1585 | 7.8891 |
| 437 | 190969 | 83453453 | 20.9045 | 7.5886 | 492 | 242064 | 119095438 | 22.1811 | 7.8944 |
| 438 | 191844 | 84027672 | 20.9284 | 7.5944 | 493 | 243049 | 119823157 | 22.2036 | 7.8998 |
| 439 | 192721 | 84604519 | 20.9523 | 7.6001 | 494 | 244036 | 120553784 | 22.2261 | 7.9051 |
| 440 | 193600 | 85184000 | 20.9762 | 7.6059 | 495 | 245025 | 121287375 | 22.2486 | 7.9105 |
| 441 | 194481 | 85766121 | 21.0000 | 7.6117 | 496 | 246016 | 122023936 | 22.2711 | 7.9158 |
| 442 | 195364 | 86350888 | 21.0238 | 7.6174 | 497 | 247009 | 122763473 | 22.2935 | 7.9211 |
| 443 | 196249 | 86938307 | 21.0476 | 7.6232 | 498 | 248004 | 123505992 | 22.3159 | 7.9264 |
| 444 | 197136 | 87528384 | 21.0713 | 7.6289 | 499 | 249001 | 124251499 | 22.3383 | 7.9317 |
| 445 | 198025 | 88121125 | 21.0950 | 7.6346 | 500 | 250000 | 125000000 | 22.3607 | 7.9370 |
| 446 | 198916 | 88716536 | 21.1187 | 7.6403 | 501 | 251001 | 125751501 | 22.3830 | 7.9423 |
| 447 | 199809 | 89314623 | 21.1424 | 7.6460 | 502 | 252004 | 126506008 | 22.4054 | 7.9476 |
| 448 | 200704 | 89915392 | 21.1660 | 7.6517 | 503 | 253009 | 127263527 | 22.4277 | 7.9528 |
| 449 | 201601 | 90518849 | 21.1896 | 7.6574 | 504 | 254016 | 128024064 | 22.4499 | 7.9581 |
| 450 | 202500 | 91125000 | 21.2132 | 7.6631 | 505 | 255025 | 128787625 | 22.4722 | 7.9634 |
| 451 | 203401 | 91733851 | 21.2368 | 7.6688 | 506 | 256036 | 129554216 | 22.4944 | 7.9686 |
| 452 | 204304 | 92345408 | 21.2603 | 7.6744 | 507 | 257049 | 130323843 | 22.5167 | 7.9739 |
| 453 | 205209 | 92959677 | 21.2838 | 7.6800 | 508 | 258064 | 131096512 | 22.5389 | 7.9791 |
| 454 | 206116 | 93576664 | 21.3073 | 7.6857 | 509 | 259081 | 131872229 | 22.5610 | 7.9843 |
| 455 | 207025 | 94196375 | 21.3307 | 7.6914 | 510 | 260100 | 132651000 | 22.5832 | 7.9896 |
| 456 | 207936 | 94818816 | 21.3542 | 7.6970 | 511 | 261121 | 133432831 | 22.6053 | 7.9948 |
| 457 | 208849 | 95443993 | 21.3776 | 7.7026 | 512 | 262144 | 134217728 | 22.6274 | 8.0000 |
| 458 | 209764 | 96071912 | 21.4009 | 7.7082 | 513 | 263169 | 135005697 | 22.6495 | 8.0052 |
| 459 | 210681 | 96702579 | 21.4243 | 7.7138 | 514 | 264196 | 135796744 | 22.6716 | 8.0104 |
| 460 | 211600 | 97336000 | 21.4476 | 7.7194 | 515 | 265225 | 136590875 | 22.6936 | 8.0156 |
| 461 | 212521 | 97972181 | 21.4709 | 7.7250 | 516 | 266256 | 137388096 | 22.7156 | 8.0208 |
| 462 | 213444 | 98611128 | 21.4942 | 7.7306 | 517 | 267289 | 138188413 | 22.7376 | 8.0260 |
| 463 | 214369 | 99252847 | 21.5174 | 7.7362 | 518 | 268324 | 138991832 | 22.7596 | 8.0311 |
| 464 | 215296 | 99897344 | 21.5407 | 7.7418 | 519 | 269361 | 139798359 | 22.7816 | 8.0363 |
| 465 | 216225 | 100544625 | 21.5639 | 7.7473 | 520 | 270400 | 140608000 | 22.8035 | 8.0415 |
| 466 | 217156 | 101194696 | 21.5870 | 7.7529 | 521 | 271441 | 141420761 | 22.8254 | 8.0466 |
| 467 | 218089 | 101847563 | 21.6102 | 7.7584 | 522 | 272484 | 142236648 | 22.8473 | 8.0517 |
| 468 | 219024 | 102503232 | 21.6333 | 7.7639 | 523 | 273529 | 143055667 | 22.8692 | 8.0569 |
| 469 | 219961 | 103161709 | 21.6564 | 7.7695 | 524 | 274576 | 143877824 | 22.8910 | 8.0620 |
| 470 | 220900 | 103823000 | 21.6795 | 7.7750 | 525 | 275625 | 144703125 | 22.9129 | 8.0671 |
| 471 | 221841 | 104487111 | 21.7025 | 7.7805 | 526 | 276676 | 145531576 | 22.9347 | 8.0723 |
| 472 | 222784 | 105154048 | 21.7256 | 7.7860 | 527 | 277729 | 146363183 | 22.9565 | 8.0774 |
| 473 | 223729 | 105823817 | 21.7486 | 7.7915 | 528 | 278784 | 147197952 | 22.9783 | 8.0825 |
| 474 | 224676 | 106496424 | 21.7715 | 7.7970 | 529 | 279841 | 148035889 | 23.0000 | 8.0876 |
| 475 | 225625 | 107171875 | 21.7945 | 7.8025 | 530 | 280900 | 148877000 | 23.0217 | 8.0927 |
| 476 | 226576 | 107850176 | 21.8174 | 7.8079 | 531 | 281961 | 149721291 | 23.0434 | 8.0978 |
| 477 | 227529 | 108531333 | 21.8403 | 7.8134 | 532 | 283024 | 150568768 | 23.0651 | 8.1028 |
| 478 | 228484 | 109215352 | 21.8632 | 7.8188 | 533 | 284089 | 151419437 | 23.0868 | 8.1079 |
| 479 | 229441 | 109902239 | 21.8861 | 7.8243 | 534 | 285156 | 152273304 | 23.1084 | 8.1130 |
| 480 | 230400 | 110592000 | 21.9089 | 7.8297 | 535 | 286225 | 153130375 | 23.1301 | 8.1180 |
| 481 | 231361 | 111284641 | 21.9317 | 7.8352 | 536 | 287296 | 153990656 | 23.1517 | 8.1231 |
| 482 | 232324 | 111980168 | 21.9545 | 7.8406 | 537 | 288369 | 154854153 | 23.1733 | 8.1281 |
| 483 | 233289 | 112678587 | 21.9773 | 7.8460 | 538 | 289444 | 155720872 | 23.1948 | 8.1332 |
| 484 | 234256 | 113379904 | 22.0000 | 7.8514 | 539 | 290521 | 156590819 | 23.2164 | 8.1382 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|-----------|-----------|------------|-----|--------|-----------|-----------|------------|
| 540 | 291600 | 157464000 | 23.2379 | 8.1433 | 595 | 354025 | 210644875 | 24.3926 | 8.4108 |
| 541 | 292631 | 158340421 | 23.2594 | 8.1483 | 596 | 355216 | 211708736 | 24.4131 | 8.4155 |
| 542 | 293764 | 159220088 | 23.2809 | 8.1533 | 597 | 356409 | 212776173 | 24.4336 | 8.4202 |
| 543 | 294849 | 160103007 | 23.3024 | 8.1583 | 598 | 357604 | 213847192 | 24.4540 | 8.4249 |
| 544 | 295936 | 160989184 | 23.3238 | 8.1633 | 599 | 358801 | 214921799 | 24.4745 | 8.4296 |
| 545 | 297025 | 161878625 | 23.3452 | 8.1683 | 600 | 360000 | 216000000 | 24.4949 | 8.4343 |
| 546 | 298116 | 162771336 | 23.3666 | 8.1733 | 601 | 361201 | 217081801 | 24.5153 | 8.4390 |
| 547 | 299209 | 163667323 | 23.3880 | 8.1783 | 602 | 362404 | 218167208 | 24.5357 | 8.4437 |
| 548 | 300304 | 164566592 | 23.4094 | 8.1833 | 603 | 363609 | 219256227 | 24.5561 | 8.4484 |
| 549 | 301401 | 165469149 | 23.4307 | 8.1882 | 604 | 364816 | 220348864 | 24.5764 | 8.4530 |
| 550 | 302500 | 166375000 | 23.4521 | 8.1932 | 605 | 366025 | 221445125 | 24.5967 | 8.4577 |
| 551 | 303601 | 167284151 | 23.4734 | 8.1982 | 606 | 367236 | 222545016 | 24.6171 | 8.4623 |
| 552 | 304704 | 168196608 | 23.4947 | 8.2031 | 607 | 368449 | 223648543 | 24.6374 | 8.4670 |
| 553 | 305809 | 169112377 | 23.5160 | 8.2081 | 608 | 369664 | 224755712 | 24.6577 | 8.4716 |
| 554 | 306916 | 170031464 | 23.5372 | 8.2130 | 609 | 370881 | 225866529 | 24.6779 | 8.4763 |
| 555 | 308025 | 170953875 | 23.5584 | 8.2180 | 610 | 372100 | 226981000 | 24.6982 | 8.4809 |
| 556 | 309136 | 171879616 | 23.5797 | 8.2229 | 611 | 373321 | 228099131 | 24.7184 | 8.4856 |
| 557 | 310249 | 172808693 | 23.6008 | 8.2278 | 612 | 374544 | 229220928 | 24.7386 | 8.4902 |
| 558 | 311364 | 173741112 | 23.6220 | 8.2327 | 613 | 375769 | 230346397 | 24.7588 | 8.4948 |
| 559 | 312481 | 174676879 | 23.6432 | 8.2377 | 614 | 376996 | 231475544 | 24.7790 | 8.4994 |
| 560 | 313600 | 175616000 | 23.6643 | 8.2426 | 615 | 378225 | 232608375 | 24.7992 | 8.5040 |
| 561 | 314721 | 176558481 | 23.6854 | 8.2475 | 616 | 379456 | 233744896 | 24.8193 | 8.5086 |
| 562 | 315844 | 177504328 | 23.7065 | 8.2524 | 617 | 380689 | 234885113 | 24.8395 | 8.5132 |
| 563 | 316969 | 178453547 | 23.7276 | 8.2573 | 618 | 381924 | 236029032 | 24.8596 | 8.5178 |
| 564 | 318096 | 179406144 | 23.7487 | 8.2621 | 619 | 383161 | 237176659 | 24.8797 | 8.5224 |
| 565 | 319225 | 180362125 | 23.7697 | 8.2670 | 620 | 384400 | 238328000 | 24.8998 | 8.5270 |
| 566 | 320356 | 181321496 | 23.7908 | 8.2719 | 621 | 385641 | 239483061 | 24.9199 | 8.5316 |
| 567 | 321489 | 182284263 | 23.8118 | 8.2768 | 622 | 386884 | 240641848 | 24.9399 | 8.5362 |
| 568 | 322624 | 183250432 | 23.8328 | 8.2816 | 623 | 388129 | 241804367 | 24.9600 | 8.5408 |
| 569 | 323761 | 184220009 | 23.8537 | 8.2865 | 624 | 389376 | 242970624 | 24.9800 | 8.5453 |
| 570 | 324900 | 185193000 | 23.8747 | 8.2913 | 625 | 390625 | 244140625 | 25.0000 | 8.5499 |
| 571 | 326041 | 186169411 | 23.8956 | 8.2962 | 626 | 391876 | 245314376 | 25.0200 | 8.5544 |
| 572 | 327184 | 187149248 | 23.9165 | 8.3010 | 627 | 393129 | 246491883 | 25.0400 | 8.5590 |
| 573 | 328329 | 188132517 | 23.9374 | 8.3059 | 628 | 394384 | 247673152 | 25.0599 | 8.5635 |
| 574 | 329476 | 189119224 | 23.9583 | 8.3107 | 629 | 395641 | 248858189 | 25.0799 | 8.5681 |
| 575 | 330625 | 190109375 | 23.9792 | 8.3155 | 630 | 396900 | 250047000 | 25.0998 | 8.5726 |
| 576 | 331776 | 191102976 | 24.0000 | 8.3203 | 631 | 398161 | 251239591 | 25.1197 | 8.5772 |
| 577 | 332929 | 192100033 | 24.0208 | 8.3251 | 632 | 399424 | 252435968 | 25.1396 | 8.5817 |
| 578 | 334084 | 193100552 | 24.0416 | 8.3300 | 633 | 400689 | 253636137 | 25.1595 | 8.5862 |
| 579 | 335241 | 194104539 | 24.0624 | 8.3348 | 634 | 401956 | 254840104 | 25.1794 | 8.5907 |
| 580 | 336400 | 195112000 | 24.0832 | 8.3396 | 635 | 403225 | 256047875 | 25.1992 | 8.5952 |
| 581 | 337561 | 196122941 | 24.1039 | 8.3443 | 636 | 404496 | 257259456 | 25.2190 | 8.5997 |
| 582 | 338724 | 197137368 | 24.1247 | 8.3491 | 637 | 405769 | 258474853 | 25.2389 | 8.6043 |
| 583 | 339889 | 198155287 | 24.1454 | 8.3539 | 638 | 407044 | 259694072 | 25.2587 | 8.6088 |
| 584 | 341056 | 199176704 | 24.1661 | 8.3587 | 639 | 408321 | 260917119 | 25.2784 | 8.6132 |
| 585 | 342225 | 200201625 | 24.1868 | 8.3634 | 640 | 409600 | 262144000 | 25.2982 | 8.6177 |
| 586 | 343396 | 201230056 | 24.2074 | 8.3682 | 641 | 410881 | 263374721 | 25.3180 | 8.6222 |
| 587 | 344569 | 202262003 | 24.2281 | 8.3730 | 642 | 412164 | 264609288 | 25.3377 | 8.6267 |
| 588 | 345744 | 203297472 | 24.2487 | 8.3777 | 643 | 413449 | 265847707 | 25.3574 | 8.6312 |
| 589 | 346921 | 204336469 | 24.2693 | 8.3825 | 644 | 414736 | 267089984 | 25.3772 | 8.6357 |
| 590 | 348100 | 205379000 | 24.2899 | 8.3872 | 645 | 416025 | 268336125 | 25.3969 | 8.6401 |
| 591 | 349281 | 206425071 | 24.3105 | 8.3919 | 646 | 417316 | 269586136 | 25.4165 | 8.6446 |
| 592 | 350464 | 207474688 | 24.3311 | 8.3967 | 647 | 418609 | 270840773 | 25.4362 | 8.6490 |
| 593 | 351649 | 208527857 | 24.3516 | 8.4014 | 648 | 419904 | 272097792 | 25.4558 | 8.6535 |
| 594 | 352836 | 209584584 | 24.3721 | 8.4061 | 649 | 421201 | 273359449 | 25.4755 | 8.6579 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|-----------|-----------|------------|-----|--------|-----------|-----------|------------|
| 650 | 422500 | 274625000 | 25.4951 | 8.6624 | 705 | 497025 | 350402625 | 26.5518 | 8.9001 |
| 651 | 423801 | 275894451 | 25.5147 | 8.6668 | 706 | 498436 | 351895816 | 26.5707 | 8.9043 |
| 652 | 425104 | 277167808 | 25.5343 | 8.6713 | 707 | 499849 | 353393243 | 26.5895 | 8.9085 |
| 653 | 426409 | 278445077 | 25.5539 | 8.6757 | 708 | 501264 | 354894912 | 26.6083 | 8.9127 |
| 654 | 427716 | 279726264 | 25.5734 | 8.6801 | 709 | 502681 | 356400829 | 26.6271 | 8.9169 |
| 655 | 429025 | 281011375 | 25.5930 | 8.6845 | 710 | 504100 | 357911000 | 26.6458 | 8.9211 |
| 656 | 430336 | 282300416 | 25.6125 | 8.6890 | 711 | 505521 | 359425431 | 26.6646 | 8.9253 |
| 657 | 431649 | 283593393 | 25.6320 | 8.6934 | 712 | 506944 | 360944128 | 26.6833 | 8.9295 |
| 658 | 432964 | 284890312 | 25.6515 | 8.6978 | 713 | 508369 | 362467097 | 26.7021 | 8.9337 |
| 659 | 434281 | 286191179 | 25.6710 | 8.7022 | 714 | 509796 | 363994344 | 26.7208 | 8.9378 |
| 660 | 435600 | 287496000 | 25.6905 | 8.7066 | 715 | 511225 | 365525875 | 26.7395 | 8.9420 |
| 661 | 436921 | 288804781 | 25.7099 | 8.7110 | 716 | 512656 | 367061696 | 26.7582 | 8.9462 |
| 662 | 438244 | 290117528 | 25.7294 | 8.7154 | 717 | 514089 | 368601813 | 26.7769 | 8.9503 |
| 663 | 439569 | 291434247 | 25.7488 | 8.7198 | 718 | 515524 | 370146232 | 26.7955 | 8.9545 |
| 664 | 440896 | 292754944 | 25.7682 | 8.7241 | 719 | 516961 | 371694959 | 26.8142 | 8.9587 |
| 665 | 442225 | 294079625 | 25.7876 | 8.7285 | 720 | 518400 | 373248000 | 26.8328 | 8.9628 |
| 666 | 443556 | 295408296 | 25.8070 | 8.7329 | 721 | 519841 | 374805361 | 26.8514 | 8.9670 |
| 667 | 444889 | 296740963 | 25.8263 | 8.7373 | 722 | 521284 | 376367048 | 26.8701 | 8.9711 |
| 668 | 446224 | 298077632 | 25.8457 | 8.7416 | 723 | 522729 | 377933067 | 26.8887 | 8.9752 |
| 669 | 447561 | 299418309 | 25.8650 | 8.7460 | 724 | 524176 | 379503424 | 26.9072 | 8.9794 |
| 670 | 448900 | 300763000 | 25.8844 | 8.7503 | 725 | 525625 | 381078125 | 26.9258 | 8.9835 |
| 671 | 450241 | 302111711 | 25.9037 | 8.7547 | 726 | 527076 | 382657176 | 26.9444 | 8.9876 |
| 672 | 451584 | 303464448 | 25.9230 | 8.7590 | 727 | 528529 | 384240583 | 26.9629 | 8.9918 |
| 673 | 452929 | 304821217 | 25.9422 | 8.7634 | 728 | 529984 | 385828352 | 26.9815 | 8.9959 |
| 674 | 454276 | 306182024 | 25.9615 | 8.7677 | 729 | 531441 | 387420489 | 27.0000 | 9.0000 |
| 675 | 455625 | 307546875 | 25.9808 | 8.7721 | 730 | 532900 | 389017000 | 27.0185 | 9.0041 |
| 676 | 456976 | 308915776 | 26.0000 | 8.7764 | 731 | 534361 | 390617891 | 27.0370 | 9.0082 |
| 677 | 458329 | 310288733 | 26.0192 | 8.7807 | 732 | 535824 | 392223168 | 27.0555 | 9.0123 |
| 678 | 459684 | 311665752 | 26.0384 | 8.7850 | 733 | 537289 | 393832837 | 27.0740 | 9.0164 |
| 679 | 461041 | 313046839 | 26.0576 | 8.7893 | 734 | 538756 | 395446904 | 27.0924 | 9.0205 |
| 680 | 462400 | 314432000 | 26.0768 | 8.7937 | 735 | 540225 | 397065375 | 27.1109 | 9.0246 |
| 681 | 463761 | 315821241 | 26.0960 | 8.7980 | 736 | 541696 | 398688256 | 27.1293 | 9.0287 |
| 682 | 465124 | 317214568 | 26.1151 | 8.8023 | 737 | 543169 | 400315553 | 27.1477 | 9.0328 |
| 683 | 466489 | 318611987 | 26.1343 | 8.8066 | 738 | 544644 | 401947272 | 27.1662 | 9.0369 |
| 684 | 467856 | 320013504 | 26.1534 | 8.8109 | 739 | 546121 | 403583419 | 27.1846 | 9.0410 |
| 685 | 469225 | 321419125 | 26.1725 | 8.8152 | 740 | 547600 | 405224000 | 27.2029 | 9.0450 |
| 686 | 470596 | 322828856 | 26.1916 | 8.8194 | 741 | 549081 | 406869021 | 27.2213 | 9.0491 |
| 687 | 471969 | 324242703 | 26.2107 | 8.8237 | 742 | 550564 | 408518488 | 27.2397 | 9.0532 |
| 688 | 473344 | 325660672 | 26.2298 | 8.8280 | 743 | 552049 | 410172407 | 27.2580 | 9.0572 |
| 689 | 474721 | 327082769 | 26.2488 | 8.8323 | 744 | 553536 | 411830784 | 27.2764 | 9.0613 |
| 690 | 476100 | 328509000 | 26.2679 | 8.8366 | 745 | 555025 | 413493625 | 27.2947 | 9.0654 |
| 691 | 477481 | 329939371 | 26.2869 | 8.8408 | 746 | 556516 | 415160936 | 27.3130 | 9.0694 |
| 692 | 478864 | 331373888 | 26.3059 | 8.8451 | 747 | 558009 | 416832723 | 27.3313 | 9.0735 |
| 693 | 480249 | 332812557 | 26.3249 | 8.8493 | 748 | 559504 | 418508992 | 27.3496 | 9.0775 |
| 694 | 481636 | 334255384 | 26.3439 | 8.8536 | 749 | 561001 | 420189749 | 27.3679 | 9.0816 |
| 695 | 483025 | 335702375 | 26.3629 | 8.8578 | 750 | 562500 | 421875000 | 27.3861 | 9.0856 |
| 696 | 484416 | 337153536 | 26.3818 | 8.8621 | 751 | 564001 | 423564751 | 27.4044 | 9.0896 |
| 697 | 485809 | 338608873 | 26.4008 | 8.8663 | 752 | 565504 | 425259008 | 27.4226 | 9.0937 |
| 698 | 487204 | 340068392 | 26.4197 | 8.8706 | 753 | 567009 | 426957777 | 27.4408 | 9.0977 |
| 699 | 488601 | 341532099 | 26.4386 | 8.8748 | 754 | 568516 | 428661064 | 27.4591 | 9.1017 |
| 700 | 490000 | 343000000 | 26.4575 | 8.8790 | 755 | 570025 | 430368875 | 27.4773 | 9.1057 |
| 701 | 491401 | 344472101 | 26.4764 | 8.8833 | 756 | 571536 | 432081216 | 27.4955 | 9.1098 |
| 702 | 492804 | 345948408 | 26.4953 | 8.8875 | 757 | 573049 | 433798093 | 27.5136 | 9.1138 |
| 703 | 494209 | 347428927 | 26.5141 | 8.8917 | 758 | 574564 | 435519512 | 27.5318 | 9.1178 |
| 704 | 495616 | 348913664 | 26.5330 | 8.8959 | 759 | 576081 | 437245479 | 27.5500 | 9.1218 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|-----------|-----------|------------|-----|--------|-----------|-----------|------------|
| 760 | 577600 | 438976000 | 27.5681 | 9.1258 | 815 | 664225 | 541343375 | 28.5482 | 9.3408 |
| 761 | 579121 | 440711081 | 27.5862 | 9.1298 | 816 | 665856 | 543338496 | 28.5657 | 9.3447 |
| 762 | 580644 | 442450728 | 27.6043 | 9.1338 | 817 | 667489 | 545338513 | 28.5832 | 9.3485 |
| 763 | 582169 | 444194947 | 27.6225 | 9.1378 | 818 | 669124 | 547343432 | 28.6007 | 9.3523 |
| 764 | 583696 | 445943744 | 27.6405 | 9.1418 | 819 | 670761 | 549353259 | 28.6182 | 9.3561 |
| 765 | 585225 | 447697125 | 27.6586 | 9.1458 | 820 | 672400 | 551368000 | 28.6356 | 9.3599 |
| 766 | 586756 | 449455096 | 27.6767 | 9.1498 | 821 | 674041 | 553387661 | 28.6531 | 9.3637 |
| 767 | 588289 | 451217663 | 27.6948 | 9.1537 | 822 | 675684 | 555412248 | 28.6705 | 9.3675 |
| 768 | 589824 | 452984832 | 27.7128 | 9.1577 | 823 | 677329 | 557441767 | 28.6880 | 9.3713 |
| 769 | 591361 | 454756609 | 27.7308 | 9.1617 | 824 | 678976 | 559476224 | 28.7054 | 9.3751 |
| 770 | 592900 | 456533000 | 27.7489 | 9.1657 | 825 | 680625 | 561515625 | 28.7228 | 9.3789 |
| 771 | 594441 | 458314011 | 27.7669 | 9.1696 | 826 | 682276 | 563559976 | 28.7402 | 9.3827 |
| 772 | 595984 | 460099648 | 27.7849 | 9.1736 | 827 | 683929 | 565609283 | 28.7576 | 9.3865 |
| 773 | 597529 | 461889917 | 27.8029 | 9.1775 | 828 | 685584 | 567663552 | 28.7750 | 9.3902 |
| 774 | 599076 | 463684824 | 27.8209 | 9.1815 | 829 | 687241 | 569722789 | 28.7924 | 9.3940 |
| 775 | 600625 | 465484375 | 27.8388 | 9.1855 | 830 | 688900 | 571787000 | 28.8097 | 9.3978 |
| 776 | 602176 | 467288576 | 27.8568 | 9.1894 | 831 | 690561 | 573856191 | 28.8271 | 9.4016 |
| 777 | 603729 | 469097433 | 27.8747 | 9.1933 | 832 | 692224 | 575930368 | 28.8444 | 9.4053 |
| 778 | 605284 | 470910952 | 27.8927 | 9.1973 | 833 | 693889 | 578009537 | 28.8617 | 9.4091 |
| 779 | 606841 | 472729139 | 27.9106 | 9.2012 | 834 | 695556 | 580093704 | 28.8791 | 9.4129 |
| 780 | 608400 | 474552000 | 27.9285 | 9.2052 | 835 | 697225 | 582182875 | 28.8964 | 9.4166 |
| 781 | 609961 | 476379541 | 27.9464 | 9.2091 | 836 | 698896 | 584277056 | 28.9137 | 9.4204 |
| 782 | 611524 | 478211768 | 27.9643 | 9.2130 | 837 | 700569 | 586376253 | 28.9310 | 9.4241 |
| 783 | 613039 | 480048687 | 27.9821 | 9.2170 | 838 | 702244 | 588480472 | 28.9482 | 9.4279 |
| 784 | 614656 | 481890304 | 28.0000 | 9.2209 | 839 | 703921 | 590589719 | 28.9655 | 9.4316 |
| 785 | 616225 | 483736625 | 28.0179 | 9.2248 | 840 | 705600 | 592704000 | 28.9828 | 9.4354 |
| 786 | 617796 | 485587656 | 28.0357 | 9.2287 | 841 | 707281 | 594823321 | 29.0000 | 9.4391 |
| 787 | 619369 | 487443403 | 28.0535 | 9.2326 | 842 | 708964 | 596947688 | 29.0172 | 9.4429 |
| 788 | 620944 | 489303572 | 28.0713 | 9.2365 | 843 | 710649 | 599077107 | 29.0345 | 9.4466 |
| 789 | 622521 | 491169069 | 28.0891 | 9.2404 | 844 | 712336 | 601211584 | 29.0517 | 9.4503 |
| 790 | 624100 | 493039000 | 28.1069 | 9.2443 | 845 | 714025 | 603351125 | 29.0689 | 9.4541 |
| 791 | 625681 | 494913671 | 28.1247 | 9.2482 | 846 | 715716 | 605495736 | 29.0861 | 9.4578 |
| 792 | 627264 | 496793088 | 28.1425 | 9.2521 | 847 | 717409 | 607645423 | 29.1033 | 9.4615 |
| 793 | 628849 | 498677257 | 28.1603 | 9.2560 | 848 | 719104 | 609800192 | 29.1204 | 9.4652 |
| 794 | 630436 | 500566184 | 28.1780 | 9.2599 | 849 | 720801 | 611960049 | 29.1376 | 9.4690 |
| 795 | 632025 | 502459875 | 28.1957 | 9.2638 | 850 | 722500 | 614125000 | 29.1548 | 9.4727 |
| 796 | 633616 | 504358336 | 28.2135 | 9.2677 | 851 | 724201 | 616295051 | 29.1719 | 9.4764 |
| 797 | 635209 | 506261573 | 28.2312 | 9.2716 | 852 | 725904 | 618470208 | 29.1890 | 9.4801 |
| 798 | 636804 | 508169592 | 28.2489 | 9.2754 | 853 | 727609 | 620650477 | 29.2062 | 9.4838 |
| 799 | 638401 | 510082399 | 28.2666 | 9.2793 | 854 | 729316 | 622835864 | 29.2233 | 9.4875 |
| 800 | 640000 | 512000000 | 28.2843 | 9.2832 | 855 | 731025 | 625026375 | 29.2404 | 9.4912 |
| 801 | 641601 | 513922401 | 28.3019 | 9.2870 | 856 | 732736 | 627222016 | 29.2575 | 9.4949 |
| 802 | 643204 | 515849608 | 28.3196 | 9.2909 | 857 | 734449 | 629422793 | 29.2746 | 9.4986 |
| 803 | 644809 | 517781627 | 28.3373 | 9.2948 | 858 | 736164 | 631628712 | 29.2916 | 9.5023 |
| 804 | 646416 | 519718464 | 28.3549 | 9.2986 | 859 | 737881 | 633839779 | 29.3087 | 9.5060 |
| 805 | 648025 | 521660125 | 28.3725 | 9.3025 | 860 | 739600 | 636056000 | 29.3258 | 9.5097 |
| 806 | 649636 | 523606616 | 28.3901 | 9.3063 | 861 | 741321 | 638277381 | 29.3428 | 9.5134 |
| 807 | 651249 | 525557943 | 28.4077 | 9.3102 | 862 | 743044 | 640503928 | 29.3598 | 9.5171 |
| 808 | 652864 | 527514112 | 28.4253 | 9.3140 | 863 | 744769 | 642735647 | 29.3769 | 9.5207 |
| 809 | 654481 | 529475129 | 28.4429 | 9.3179 | 864 | 746496 | 644972544 | 29.3939 | 9.5244 |
| 810 | 656100 | 531441000 | 28.4605 | 9.3217 | 865 | 748225 | 647214625 | 29.4109 | 9.5281 |
| 811 | 657721 | 533411731 | 28.4781 | 9.3255 | 866 | 749956 | 649461896 | 29.4279 | 9.5317 |
| 812 | 659344 | 535387328 | 28.4956 | 9.3294 | 867 | 751689 | 651714363 | 29.4449 | 9.5354 |
| 813 | 660969 | 537367797 | 28.5132 | 9.3332 | 868 | 753424 | 653972032 | 29.4618 | 9.5391 |
| 814 | 662596 | 539353144 | 28.5307 | 9.3370 | 869 | 755161 | 656234909 | 29.4788 | 9.5427 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square | Cube. | Sq. Root. | Cube Root. |
|-----|---------|-----------|-----------|------------|-----|--------|-----------|-----------|------------|
| 870 | 756900 | 658503000 | 29.4958 | 9.5464 | 925 | 855625 | 791453125 | 30.4138 | 9.7435 |
| 871 | 758641 | 660776311 | 29.5127 | 9.5501 | 926 | 857476 | 794022776 | 30.4302 | 9.7470 |
| 872 | 760384 | 663054848 | 29.5296 | 9.5537 | 927 | 859329 | 796597983 | 30.4467 | 9.7505 |
| 873 | 762129 | 665338617 | 29.5466 | 9.5574 | 928 | 861184 | 799178752 | 30.4631 | 9.7540 |
| 874 | 763876 | 667627624 | 29.5635 | 9.5610 | 929 | 863041 | 801765089 | 30.4795 | 9.7575 |
| 875 | 765625 | 669921875 | 29.5804 | 9.5647 | 930 | 864900 | 804357000 | 30.4959 | 9.7610 |
| 876 | 767376 | 672221376 | 29.5973 | 9.5683 | 931 | 866761 | 806954491 | 30.5123 | 9.7645 |
| 877 | 769129 | 674526133 | 29.6142 | 9.5719 | 932 | 868624 | 809557568 | 30.5287 | 9.7680 |
| 878 | 770884 | 676836152 | 29.6311 | 9.5756 | 933 | 870489 | 812166237 | 30.5450 | 9.7715 |
| 879 | 772641 | 679151439 | 29.6479 | 9.5792 | 934 | 872356 | 814780504 | 30.5614 | 9.7750 |
| 880 | 774400 | 681472000 | 29.6648 | 9.5828 | 935 | 874225 | 817400375 | 30.5778 | 9.7785 |
| 881 | 776161 | 683797841 | 29.6816 | 9.5865 | 936 | 876096 | 820025856 | 30.5941 | 9.7819 |
| 882 | 777924 | 686128968 | 29.6985 | 9.5901 | 937 | 877969 | 822656953 | 30.6105 | 9.7854 |
| 883 | 779689 | 688465387 | 29.7153 | 9.5937 | 938 | 879844 | 825293672 | 30.6268 | 9.7889 |
| 884 | 781456 | 690807104 | 29.7321 | 9.5973 | 939 | 881721 | 827936019 | 30.6431 | 9.7924 |
| 885 | 783225 | 693154125 | 29.7489 | 9.6010 | 940 | 883600 | 830584000 | 30.6594 | 9.7959 |
| 886 | 784996 | 695506456 | 29.7658 | 9.6046 | 941 | 885481 | 833237621 | 30.6757 | 9.7993 |
| 887 | 786769 | 697864103 | 29.7825 | 9.6082 | 942 | 887364 | 835896888 | 30.6920 | 9.8028 |
| 888 | 788544 | 700227072 | 29.7993 | 9.6118 | 943 | 889249 | 838561807 | 30.7083 | 9.8063 |
| 889 | 790321 | 702595369 | 29.8161 | 9.6154 | 944 | 891136 | 841232384 | 30.7246 | 9.8097 |
| 890 | 792100 | 704969000 | 29.8329 | 9.6190 | 945 | 893025 | 843908625 | 30.7409 | 9.8132 |
| 891 | 793881 | 707347971 | 29.8496 | 9.6226 | 946 | 894916 | 846590536 | 30.7571 | 9.8167 |
| 892 | 795664 | 709732288 | 29.8664 | 9.6262 | 947 | 896809 | 849278123 | 30.7734 | 9.8201 |
| 893 | 797449 | 712121957 | 29.8831 | 9.6298 | 948 | 898704 | 851971392 | 30.7896 | 9.8236 |
| 894 | 799236 | 714516984 | 29.8998 | 9.6334 | 949 | 900601 | 854670349 | 30.8058 | 9.8270 |
| 895 | 801025 | 716917375 | 29.9166 | 9.6370 | 950 | 902500 | 857375000 | 30.8221 | 9.8305 |
| 896 | 802816 | 719323136 | 29.9333 | 9.6406 | 951 | 904401 | 860085351 | 30.8383 | 9.8339 |
| 897 | 804609 | 721734273 | 29.9500 | 9.6442 | 952 | 906304 | 862801408 | 30.8545 | 9.8374 |
| 898 | 806404 | 724150792 | 29.9666 | 9.6477 | 953 | 908209 | 865523177 | 30.8707 | 9.8408 |
| 899 | 808201 | 726572699 | 29.9833 | 9.6513 | 954 | 910116 | 868250664 | 30.8869 | 9.8443 |
| 900 | 810000 | 729000000 | 30.0000 | 9.6549 | 955 | 912025 | 870983875 | 30.9031 | 9.8477 |
| 901 | 811801 | 731432701 | 30.0167 | 9.6585 | 956 | 913936 | 873722816 | 30.9192 | 9.8511 |
| 902 | 813604 | 733870808 | 30.0333 | 9.6620 | 957 | 915849 | 876467493 | 30.9354 | 9.8546 |
| 903 | 815409 | 736314327 | 30.0500 | 9.6656 | 958 | 917764 | 879217912 | 30.9516 | 9.8580 |
| 904 | 817216 | 738763264 | 30.0666 | 9.6692 | 959 | 919681 | 881974079 | 30.9677 | 9.8614 |
| 905 | 819025 | 741217625 | 30.0832 | 9.6727 | 960 | 921600 | 884736000 | 30.9839 | 9.8648 |
| 906 | 820836 | 743677416 | 30.0998 | 9.6763 | 961 | 923521 | 887503681 | 31.0000 | 9.8683 |
| 907 | 822649 | 746142643 | 30.1164 | 9.6799 | 962 | 925444 | 890277128 | 31.0161 | 9.8717 |
| 908 | 824464 | 748613312 | 30.1330 | 9.6834 | 963 | 927369 | 893056347 | 31.0322 | 9.8751 |
| 909 | 826281 | 751089429 | 30.1496 | 9.6870 | 964 | 929296 | 895841344 | 31.0483 | 9.8785 |
| 910 | 828100 | 753571000 | 30.1662 | 9.6905 | 965 | 931225 | 898632125 | 31.0644 | 9.8819 |
| 911 | 829921 | 756058031 | 30.1828 | 9.6941 | 966 | 933156 | 901428696 | 31.0805 | 9.8854 |
| 912 | 831744 | 758550528 | 30.1993 | 9.6976 | 967 | 935089 | 904231063 | 31.0966 | 9.8888 |
| 913 | 833569 | 761048497 | 30.2159 | 9.7012 | 968 | 937024 | 907039232 | 31.1127 | 9.8922 |
| 914 | 835396 | 763551944 | 30.2324 | 9.7047 | 969 | 938961 | 909853209 | 31.1288 | 9.8956 |
| 915 | 837225 | 766060875 | 30.2490 | 9.7082 | 970 | 940900 | 912673000 | 31.1448 | 9.8990 |
| 916 | 839056 | 768575296 | 30.2655 | 9.7118 | 971 | 942841 | 915498611 | 31.1609 | 9.9024 |
| 917 | 840889 | 771095213 | 30.2820 | 9.7153 | 972 | 944784 | 918330048 | 31.1769 | 9.9058 |
| 918 | 842724 | 773620632 | 30.2985 | 9.7188 | 973 | 946729 | 921167317 | 31.1929 | 9.9092 |
| 919 | 844561 | 776151559 | 30.3150 | 9.7224 | 974 | 948676 | 924010424 | 31.2090 | 9.9126 |
| 920 | 846400 | 778688000 | 30.3315 | 9.7259 | 975 | 950625 | 926859375 | 31.2250 | 9.9160 |
| 921 | 848241 | 781229961 | 30.3480 | 9.7294 | 976 | 952576 | 929714176 | 31.2410 | 9.9194 |
| 922 | 850084 | 783777448 | 30.3645 | 9.7329 | 977 | 954529 | 932574833 | 31.2570 | 9.9227 |
| 923 | 851929 | 786330467 | 30.3809 | 9.7364 | 978 | 956484 | 935441352 | 31.2730 | 9.9261 |
| 924 | 853776 | 788889024 | 30.3974 | 9.7400 | 979 | 958441 | 938313739 | 31.2890 | 9.9295 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 930 | 960400 | 941192000 | 31.3050 | 9.9329 | 1035 | 1071225 | 1108717875 | 32.1714 | 10.1153 |
| 931 | 962361 | 944076141 | 31.3209 | 9.9363 | 1036 | 1073296 | 1111934656 | 32.1870 | 10.1186 |
| 932 | 964324 | 946966168 | 31.3369 | 9.9396 | 1037 | 1075369 | 1115157653 | 32.2025 | 10.1218 |
| 933 | 966289 | 949862087 | 31.3528 | 9.9430 | 1038 | 1077441 | 1118386872 | 32.2180 | 10.1251 |
| 934 | 968256 | 952763904 | 31.3688 | 9.9464 | 1039 | 1079522 | 1121622319 | 32.2335 | 10.1283 |
| 935 | 970225 | 955671625 | 31.3847 | 9.9497 | 1040 | 1081600 | 1124864000 | 32.2490 | 10.1316 |
| 936 | 972196 | 958585256 | 31.4006 | 9.9531 | 1041 | 1083681 | 1128111921 | 32.2645 | 10.1348 |
| 937 | 974169 | 961504803 | 31.4166 | 9.9565 | 1042 | 1085764 | 1131366088 | 32.2800 | 10.1381 |
| 938 | 976144 | 964430272 | 31.4325 | 9.9598 | 1043 | 1087849 | 1134626507 | 32.2955 | 10.1413 |
| 939 | 978121 | 967361569 | 31.4484 | 9.9632 | 1044 | 1089936 | 1137893184 | 32.3110 | 10.1446 |
| 990 | 980100 | 970299000 | 31.4643 | 9.9666 | 1045 | 1092025 | 1141166125 | 32.3265 | 10.1478 |
| 991 | 982081 | 973242271 | 31.4802 | 9.9699 | 1046 | 1094116 | 1144445336 | 32.3419 | 10.1510 |
| 992 | 984064 | 976191488 | 31.4960 | 9.9733 | 1047 | 1096209 | 1147730823 | 32.3574 | 10.1543 |
| 993 | 986049 | 979146657 | 31.5119 | 9.9766 | 1048 | 1098304 | 1151022592 | 32.3728 | 10.1575 |
| 994 | 988036 | 982107784 | 31.5278 | 9.9800 | 1049 | 1100401 | 1154320649 | 32.3883 | 10.1607 |
| 995 | 990025 | 985074875 | 31.5436 | 9.9833 | 1050 | 1102500 | 1157625000 | 32.4037 | 10.1640 |
| 996 | 992016 | 988047936 | 31.5595 | 9.9866 | 1051 | 1104601 | 1160935651 | 32.4191 | 10.1672 |
| 997 | 994009 | 991026973 | 31.5753 | 9.9900 | 1052 | 1106704 | 1164252608 | 32.4345 | 10.1704 |
| 998 | 996004 | 994011992 | 31.5911 | 9.9933 | 1053 | 1108809 | 1167575877 | 32.4500 | 10.1736 |
| 999 | 998001 | 997002999 | 31.6070 | 9.9967 | 1054 | 1110916 | 1170905464 | 32.4654 | 10.1769 |
| 1000 | 1000000 | 1000000000 | 31.6228 | 10.0000 | 1055 | 1113025 | 1174241375 | 32.4808 | 10.1801 |
| 1001 | 1002001 | 1003003001 | 31.6386 | 10.0033 | 1056 | 1115136 | 1177583616 | 32.4962 | 10.1833 |
| 1002 | 1004004 | 1006012008 | 31.6544 | 10.0067 | 1057 | 1117249 | 1180932193 | 32.5115 | 10.1865 |
| 1003 | 1006009 | 1009027027 | 31.6702 | 10.0100 | 1058 | 1119364 | 1184287112 | 32.5269 | 10.1897 |
| 1004 | 1008016 | 1012048064 | 31.6860 | 10.0133 | 1059 | 1121481 | 1187648379 | 32.5423 | 10.1929 |
| 1005 | 1010025 | 1015075125 | 31.7017 | 10.0166 | 1060 | 1123600 | 1191016000 | 32.5576 | 10.1961 |
| 1006 | 1012036 | 1018108216 | 31.7175 | 10.0200 | 1061 | 1125721 | 1194389981 | 32.5730 | 10.1993 |
| 1007 | 1014049 | 1021147343 | 31.7333 | 10.0233 | 1062 | 1127844 | 1197770328 | 32.5883 | 10.2025 |
| 1008 | 1016064 | 1024192512 | 31.7490 | 10.0266 | 1063 | 1129969 | 1201157047 | 32.6036 | 10.2057 |
| 1009 | 1018081 | 1027243729 | 31.7648 | 10.0299 | 1064 | 1132096 | 1204550144 | 32.6190 | 10.2089 |
| 1010 | 1020100 | 1030301000 | 31.7805 | 10.0332 | 1065 | 1134225 | 1207949625 | 32.6343 | 10.2121 |
| 1011 | 1022121 | 1033364331 | 31.7962 | 10.0365 | 1066 | 1136356 | 1211355496 | 32.6497 | 10.2153 |
| 1012 | 1024144 | 1036433728 | 31.8119 | 10.0398 | 1067 | 1138489 | 1214767763 | 32.6650 | 10.2185 |
| 1013 | 1026169 | 1039509197 | 31.8277 | 10.0431 | 1068 | 1140624 | 1218186432 | 32.6803 | 10.2217 |
| 1014 | 1028196 | 1042590744 | 31.8434 | 10.0465 | 1069 | 1142761 | 1221611509 | 32.6956 | 10.2249 |
| 1015 | 1030225 | 1045678375 | 31.8591 | 10.0498 | 1070 | 1144900 | 1225043000 | 32.7109 | 10.2281 |
| 1016 | 1032256 | 1048772096 | 31.8748 | 10.0531 | 1071 | 1147041 | 1228480911 | 32.7261 | 10.2313 |
| 1017 | 1034289 | 1051871913 | 31.8904 | 10.0563 | 1072 | 1149184 | 1231925248 | 32.7414 | 10.2345 |
| 1018 | 1036324 | 1054977832 | 31.9061 | 10.0596 | 1073 | 1151329 | 1235376017 | 32.7567 | 10.2376 |
| 1019 | 1038361 | 1058089859 | 31.9218 | 10.0629 | 1074 | 1153476 | 1238833224 | 32.7719 | 10.2408 |
| 1020 | 1040400 | 1061208000 | 31.9374 | 10.0662 | 1075 | 1155625 | 1242296875 | 32.7872 | 10.2440 |
| 1021 | 1042441 | 1064332261 | 31.9531 | 10.0695 | 1076 | 1157776 | 1245766976 | 32.8024 | 10.2472 |
| 1022 | 1044484 | 1067462648 | 31.9687 | 10.0728 | 1077 | 1159929 | 1249243533 | 32.8177 | 10.2503 |
| 1023 | 1046529 | 1070599167 | 31.9844 | 10.0761 | 1078 | 1162084 | 1252726552 | 32.8329 | 10.2535 |
| 1024 | 1048576 | 1073741824 | 32.0000 | 10.0794 | 1079 | 1164241 | 1256216039 | 32.8481 | 10.2567 |
| 1025 | 1050625 | 1076890625 | 32.0156 | 10.0826 | 1080 | 1166400 | 1259712000 | 32.8634 | 10.2599 |
| 1026 | 1052676 | 1080045576 | 32.0312 | 10.0859 | 1081 | 1168561 | 1263214441 | 32.8786 | 10.2630 |
| 1027 | 1054729 | 1083206683 | 32.0468 | 10.0892 | 1082 | 1170724 | 1266723368 | 32.8938 | 10.2662 |
| 1028 | 1056784 | 1086373952 | 32.0624 | 10.0925 | 1083 | 1172889 | 1270238787 | 32.9090 | 10.2693 |
| 1029 | 1058841 | 1089547389 | 32.0780 | 10.0957 | 1084 | 1175056 | 1273760704 | 32.9242 | 10.2725 |
| 1030 | 1060900 | 1092727000 | 32.0936 | 10.0990 | 1085 | 1177225 | 1277289125 | 32.9393 | 10.2757 |
| 1031 | 1062961 | 1095912791 | 32.1092 | 10.1023 | 1086 | 1179396 | 1280824056 | 32.9545 | 10.2788 |
| 1032 | 1065024 | 1099104768 | 32.1248 | 10.1055 | 1087 | 1181569 | 1284365503 | 32.9697 | 10.2820 |
| 1033 | 1067089 | 1102302937 | 32.1403 | 10.1088 | 1088 | 1183744 | 1287913472 | 32.9848 | 10.2851 |
| 1034 | 1069156 | 1105507304 | 32.1559 | 10.1121 | 1089 | 1185921 | 1291467969 | 33.0000 | 10.2883 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 1090 | 1188100 | 1295029000 | 33.0151 | 10.2914 | 1145 | 1311025 | 1501123625 | 33.8378 | 10.4617 |
| 1091 | 1190281 | 1298596571 | 33.0303 | 10.2946 | 1146 | 1313316 | 1505060136 | 33.8526 | 10.4647 |
| 1092 | 1192464 | 1302170688 | 33.0454 | 10.2977 | 1147 | 1315609 | 1509003523 | 33.8674 | 10.4678 |
| 1093 | 1194649 | 1305751357 | 33.0606 | 10.3009 | 1148 | 1317904 | 1512953792 | 33.8821 | 10.4708 |
| 1094 | 1196836 | 1309338584 | 33.0757 | 10.3040 | 1149 | 1320201 | 1516910949 | 33.8969 | 10.4739 |
| 1095 | 1199025 | 1312932375 | 33.0908 | 10.3071 | 1150 | 1322500 | 1520875000 | 33.9116 | 10.4769 |
| 1096 | 1201216 | 1316532736 | 33.1059 | 10.3103 | 1151 | 1324801 | 1524834591 | 33.9264 | 10.4799 |
| 1097 | 1203409 | 1320139673 | 33.1210 | 10.3134 | 1152 | 1327104 | 1528823808 | 33.9411 | 10.4830 |
| 1098 | 1205604 | 1323753192 | 33.1361 | 10.3165 | 1153 | 1329409 | 1532808577 | 33.9559 | 10.4860 |
| 1099 | 1207801 | 1327373299 | 33.1512 | 10.3197 | 1154 | 1331716 | 1536800264 | 33.9706 | 10.4890 |
| 1100 | 1210000 | 1331000000 | 33.1662 | 10.3223 | 1155 | 1334025 | 1540798875 | 33.9853 | 10.4921 |
| 1101 | 1212201 | 1334633301 | 33.1813 | 10.3259 | 1156 | 1336336 | 1544804416 | 34.0000 | 10.4951 |
| 1102 | 1214404 | 1338273208 | 33.1964 | 10.3290 | 1157 | 1338649 | 1548816893 | 34.0147 | 10.4981 |
| 1103 | 1216609 | 1341919727 | 33.2114 | 10.3322 | 1158 | 1340964 | 1552836312 | 34.0294 | 10.5011 |
| 1104 | 1218816 | 1345572864 | 33.2264 | 10.3353 | 1159 | 1343281 | 1556862679 | 34.0441 | 10.5042 |
| 1105 | 1221025 | 1349232625 | 33.2415 | 10.3384 | 1160 | 1345600 | 1560896000 | 34.0588 | 10.5072 |
| 1106 | 1223236 | 1352899016 | 33.2566 | 10.3415 | 1161 | 1347921 | 1564936281 | 34.0735 | 10.5102 |
| 1107 | 1225449 | 1356572043 | 33.2716 | 10.3447 | 1162 | 1350244 | 1568983523 | 34.0881 | 10.5132 |
| 1108 | 1227664 | 1360251712 | 33.2866 | 10.3478 | 1163 | 1352569 | 1573037747 | 34.1028 | 10.5162 |
| 1109 | 1229881 | 1363938029 | 33.3017 | 10.3509 | 1164 | 1354896 | 1577098944 | 34.1174 | 10.5192 |
| 1110 | 1232100 | 1367631000 | 33.3167 | 10.3540 | 1165 | 1357225 | 1581167125 | 34.1321 | 10.5223 |
| 1111 | 1234321 | 1371330631 | 33.3317 | 10.3571 | 1166 | 1359556 | 1585242296 | 34.1467 | 10.5253 |
| 1112 | 1236544 | 1375036928 | 33.3467 | 10.3602 | 1167 | 1361889 | 1589324463 | 34.1614 | 10.5283 |
| 1113 | 1238769 | 1378749897 | 33.3617 | 10.3633 | 1168 | 1364224 | 1593413632 | 34.1760 | 10.5313 |
| 1114 | 1240996 | 1382469544 | 33.3766 | 10.3664 | 1169 | 1366561 | 1597509809 | 34.1906 | 10.5343 |
| 1115 | 1243225 | 1386195875 | 33.3916 | 10.3695 | 1170 | 1368900 | 1601613000 | 34.2053 | 10.5373 |
| 1116 | 1245456 | 1389928896 | 33.4066 | 10.3726 | 1171 | 1371241 | 1605723211 | 34.2199 | 10.5403 |
| 1117 | 1247689 | 1393668613 | 33.4215 | 10.3757 | 1172 | 1373584 | 1609840448 | 34.2345 | 10.5433 |
| 1118 | 1249924 | 1397415032 | 33.4365 | 10.3788 | 1173 | 1375929 | 1613964717 | 34.2491 | 10.5463 |
| 1119 | 1252161 | 1401168159 | 33.4515 | 10.3819 | 1174 | 1378276 | 1618096024 | 34.2637 | 10.5493 |
| 1120 | 1254400 | 1404928000 | 33.4664 | 10.3850 | 1175 | 1380625 | 1622234375 | 34.2783 | 10.5523 |
| 1121 | 1256641 | 1408694561 | 33.4813 | 10.3881 | 1176 | 1382976 | 1626379776 | 34.2929 | 10.5553 |
| 1122 | 1258884 | 1412467848 | 33.4963 | 10.3912 | 1177 | 1385329 | 1630532233 | 34.3074 | 10.5583 |
| 1123 | 1261129 | 1416247867 | 33.5112 | 10.3943 | 1178 | 1387684 | 1634691752 | 34.3220 | 10.5612 |
| 1124 | 1263376 | 1420034624 | 33.5261 | 10.3973 | 1179 | 1390041 | 1638858339 | 34.3366 | 10.5642 |
| 1125 | 1265625 | 1423828125 | 33.5410 | 10.4004 | 1180 | 1392400 | 1643032000 | 34.3511 | 10.5672 |
| 1126 | 1267876 | 1427628376 | 33.5559 | 10.4035 | 1181 | 1394761 | 1647212741 | 34.3657 | 10.5702 |
| 1127 | 1270129 | 1431435383 | 33.5708 | 10.4066 | 1182 | 1397124 | 1651400568 | 34.3802 | 10.5732 |
| 1128 | 1272384 | 1435249152 | 33.5857 | 10.4097 | 1183 | 1399489 | 1655595437 | 34.3948 | 10.5762 |
| 1129 | 1274641 | 1439069689 | 33.6006 | 10.4127 | 1184 | 1401856 | 1659797504 | 34.4093 | 10.5791 |
| 1130 | 1276900 | 1442897000 | 33.6155 | 10.4158 | 1185 | 1404225 | 1664006625 | 34.4238 | 10.5821 |
| 1131 | 1279161 | 1446731091 | 33.6303 | 10.4189 | 1186 | 1406596 | 1668222856 | 34.4384 | 10.5851 |
| 1132 | 1281424 | 1450571968 | 33.6452 | 10.4219 | 1187 | 1408969 | 1672446203 | 34.4529 | 10.5881 |
| 1133 | 1283689 | 1454419637 | 33.6601 | 10.4250 | 1188 | 1411344 | 1676676672 | 34.4674 | 10.5910 |
| 1134 | 1285956 | 1458274104 | 33.6749 | 10.4281 | 1189 | 1413721 | 1680914269 | 34.4819 | 10.5940 |
| 1135 | 1288225 | 1462135375 | 33.6898 | 10.4311 | 1190 | 1416100 | 1685159000 | 34.4964 | 10.5970 |
| 1136 | 1290496 | 1466003456 | 33.7046 | 10.4342 | 1191 | 1418481 | 1689410871 | 34.5109 | 10.6000 |
| 1137 | 1292769 | 1469878353 | 33.7194 | 10.4373 | 1192 | 1420864 | 1693669388 | 34.5254 | 10.6029 |
| 1138 | 1295044 | 1473760072 | 33.7342 | 10.4404 | 1193 | 1423249 | 1697936057 | 34.5398 | 10.6059 |
| 1139 | 1297321 | 1477648619 | 33.7491 | 10.4434 | 1194 | 1425636 | 1702209384 | 34.5543 | 10.6088 |
| 1140 | 1299600 | 1481544000 | 33.7639 | 10.4464 | 1195 | 1428025 | 1706489875 | 34.5688 | 10.6118 |
| 1141 | 1301881 | 1485446221 | 33.7787 | 10.4495 | 1196 | 1430416 | 1710777536 | 34.5832 | 10.6148 |
| 1142 | 1304164 | 1489355288 | 33.7935 | 10.4525 | 1197 | 1432809 | 1715072373 | 34.5977 | 10.6177 |
| 1143 | 1306449 | 1493271207 | 33.8083 | 10.4556 | 1198 | 1435204 | 1719374392 | 34.6121 | 10.6207 |
| 1144 | 1308736 | 1497193934 | 33.8231 | 10.4586 | 1199 | 1437601 | 1723633599 | 34.6266 | 10.6235 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 1200 | 1440000 | 1728000000 | 34.6410 | 10.6266 | 1255 | 1575025 | 1976656375 | 35.4260 | 10.7865 |
| 1201 | 1442401 | 1732323601 | 34.6554 | 10.6295 | 1256 | 1577536 | 1981385216 | 35.4401 | 10.7894 |
| 1202 | 1444304 | 1736654408 | 34.6699 | 10.6325 | 1257 | 1580049 | 1986121593 | 35.4542 | 10.7922 |
| 1203 | 1447209 | 1740992427 | 34.6843 | 10.6354 | 1258 | 1582564 | 1990865512 | 35.4683 | 10.7951 |
| 1204 | 1449616 | 1745337664 | 34.6987 | 10.6384 | 1259 | 1585081 | 1995616979 | 35.4824 | 10.7980 |
| 1205 | 1452025 | 1749690125 | 34.7131 | 10.6413 | 1260 | 1587600 | 2000376000 | 35.4965 | 10.8008 |
| 1206 | 1454436 | 1754049816 | 34.7275 | 10.6443 | 1261 | 1590121 | 2005142581 | 35.5106 | 10.8037 |
| 1207 | 1455849 | 1758416743 | 34.7419 | 10.5472 | 1262 | 1592644 | 2009916728 | 35.5246 | 10.8065 |
| 1208 | 1459254 | 1762790912 | 34.7563 | 10.6501 | 1263 | 1595169 | 2014698447 | 35.5387 | 10.8094 |
| 1209 | 1451531 | 1767172329 | 34.7707 | 10.6530 | 1264 | 1597696 | 2019487744 | 35.5528 | 10.8122 |
| 1210 | 1454100 | 1771561000 | 34.7851 | 10.6560 | 1265 | 1600225 | 2024284625 | 35.5668 | 10.8151 |
| 1211 | 1465521 | 1775956931 | 34.7994 | 10.6590 | 1266 | 1602756 | 2029089096 | 35.5809 | 10.8179 |
| 1212 | 1463944 | 1780360128 | 34.8138 | 10.6619 | 1267 | 1605289 | 2033901163 | 35.5949 | 10.8208 |
| 1213 | 1471369 | 1784770597 | 34.8281 | 10.6648 | 1268 | 1607824 | 2038720832 | 35.6090 | 10.8236 |
| 1214 | 1473796 | 1789183344 | 34.8425 | 10.6678 | 1269 | 1610361 | 2043548109 | 35.6230 | 10.8265 |
| 1215 | 1476225 | 1793613375 | 34.8569 | 10.6707 | 1270 | 1612900 | 2048383000 | 35.6371 | 10.8293 |
| 1216 | 1478656 | 1793045696 | 34.8712 | 10.6736 | 1271 | 1615441 | 2053225511 | 35.6511 | 10.8322 |
| 1217 | 1481089 | 1802435313 | 34.8855 | 10.6765 | 1272 | 1617984 | 2058075648 | 35.6651 | 10.8350 |
| 1218 | 1483524 | 1806932232 | 34.8999 | 10.6795 | 1273 | 1620529 | 2062933417 | 35.6791 | 10.8378 |
| 1219 | 1485961 | 1811336459 | 34.9142 | 10.6824 | 1274 | 1623076 | 2067798824 | 35.6931 | 10.8407 |
| 1220 | 1488400 | 1815848000 | 34.9285 | 10.6853 | 1275 | 1625625 | 2072671875 | 35.7071 | 10.8435 |
| 1221 | 1490841 | 1820316361 | 34.9423 | 10.6882 | 1276 | 1628176 | 2077552576 | 35.7211 | 10.8463 |
| 1222 | 1493284 | 1824793048 | 34.9571 | 10.6911 | 1277 | 1630729 | 2082440933 | 35.7351 | 10.8492 |
| 1223 | 1495729 | 1829276567 | 34.9714 | 10.6940 | 1278 | 1633284 | 2087336952 | 35.7491 | 10.8520 |
| 1224 | 1498176 | 1833767424 | 34.9857 | 10.6970 | 1279 | 1635841 | 2092240639 | 35.7631 | 10.8548 |
| 1225 | 1500625 | 1838265625 | 35.0000 | 10.6999 | 1280 | 1638400 | 2097152000 | 35.7771 | 10.8577 |
| 1226 | 1503076 | 1842771176 | 35.0143 | 10.7028 | 1281 | 1640961 | 2102071041 | 35.7911 | 10.8605 |
| 1227 | 1505529 | 1847234033 | 35.0286 | 10.7057 | 1282 | 1643524 | 2106997768 | 35.8050 | 10.8633 |
| 1228 | 1507984 | 1851804352 | 35.0428 | 10.7086 | 1283 | 1646089 | 2111932187 | 35.8190 | 10.8661 |
| 1229 | 1510441 | 1856331939 | 35.0571 | 10.7115 | 1284 | 1648656 | 2116874304 | 35.8329 | 10.8690 |
| 1230 | 1512900 | 1860867000 | 35.0714 | 10.7144 | 1285 | 1651225 | 2121824125 | 35.8469 | 10.8718 |
| 1231 | 1515361 | 1865409391 | 35.0856 | 10.7173 | 1286 | 1653796 | 2126781656 | 35.8608 | 10.8746 |
| 1232 | 1517824 | 1869959168 | 35.0999 | 10.7202 | 1287 | 1656369 | 2131746903 | 35.8748 | 10.8774 |
| 1233 | 1520289 | 1874516337 | 35.1141 | 10.7231 | 1288 | 1658944 | 2136719872 | 35.8887 | 10.8802 |
| 1234 | 1522756 | 1879080904 | 35.1283 | 10.7260 | 1289 | 1661521 | 2141700569 | 35.9026 | 10.8831 |
| 1235 | 1525225 | 1883652375 | 35.1426 | 10.7289 | 1290 | 1664100 | 2146689000 | 35.9166 | 10.8859 |
| 1236 | 1527696 | 1888232256 | 35.1568 | 10.7318 | 1291 | 1666681 | 2151685171 | 35.9305 | 10.8887 |
| 1237 | 1530169 | 1892819053 | 35.1710 | 10.7347 | 1292 | 1669264 | 2156689088 | 35.9444 | 10.8915 |
| 1238 | 1532644 | 1897413272 | 35.1852 | 10.7376 | 1293 | 1671849 | 2161700757 | 35.9583 | 10.8943 |
| 1239 | 1535121 | 1902014919 | 35.1994 | 10.7405 | 1294 | 1674436 | 2166720184 | 35.9722 | 10.8971 |
| 1240 | 1537600 | 1906624000 | 35.2136 | 10.7434 | 1295 | 1677025 | 2171747375 | 35.9861 | 10.8999 |
| 1241 | 1540081 | 1911240521 | 35.2278 | 10.7463 | 1296 | 1679616 | 2176782336 | 36.0000 | 10.9027 |
| 1242 | 1542564 | 1915864488 | 35.2420 | 10.7491 | 1297 | 1682209 | 2181825073 | 36.0139 | 10.9055 |
| 1243 | 1545049 | 1920495907 | 35.2562 | 10.7520 | 1298 | 1684804 | 2186875592 | 36.0278 | 10.9083 |
| 1244 | 1547536 | 1925134784 | 35.2704 | 10.7549 | 1299 | 1687401 | 2191933899 | 36.0416 | 10.9111 |
| 1245 | 1550025 | 1929781125 | 35.2846 | 10.7578 | 1300 | 1690000 | 2197000000 | 36.0555 | 10.9139 |
| 1246 | 1552516 | 1934434936 | 35.2987 | 10.7607 | 1301 | 1692601 | 2202073901 | 36.0694 | 10.9167 |
| 1247 | 1555009 | 1939096223 | 35.3129 | 10.7635 | 1302 | 1695204 | 2207155603 | 36.0832 | 10.9195 |
| 1248 | 1557504 | 1943764992 | 35.3270 | 10.7664 | 1303 | 1697809 | 2212245127 | 36.0971 | 10.9223 |
| 1249 | 1560001 | 1948441249 | 35.3412 | 10.7693 | 1304 | 1700416 | 2217342464 | 36.1109 | 10.9251 |
| 1250 | 1562500 | 1953125000 | 35.3553 | 10.7722 | 1305 | 1703025 | 2222447625 | 36.1248 | 10.9279 |
| 1251 | 1565001 | 1957816251 | 35.3695 | 10.7750 | 1306 | 1705636 | 2227560616 | 36.1386 | 10.9307 |
| 1252 | 1567504 | 1962515008 | 35.3836 | 10.7779 | 1307 | 1708249 | 2232681443 | 36.1525 | 10.9335 |
| 1253 | 1570009 | 1967221277 | 35.3977 | 10.7808 | 1308 | 1710864 | 2237810112 | 36.1663 | 10.9363 |
| 1254 | 1572516 | 1971935064 | 35.4119 | 10.7837 | 1309 | 1713481 | 2242946629 | 36.1801 | 10.9391 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 1310 | 1716100 | 2248091000 | 36.1939 | 10.9418 | 1365 | 1863225 | 2543302125 | 36.9459 | 11.0929 |
| 1311 | 1718721 | 2253243231 | 36.2077 | 10.9446 | 1366 | 1865956 | 2548895896 | 36.9594 | 11.0956 |
| 1312 | 1721344 | 2258403328 | 36.2215 | 10.9474 | 1367 | 1868689 | 2554497863 | 36.9730 | 11.0983 |
| 1313 | 1723969 | 2263571297 | 36.2353 | 10.9502 | 1368 | 1871424 | 2560108032 | 36.9865 | 11.1010 |
| 1314 | 1726596 | 2268747144 | 36.2491 | 10.9530 | 1369 | 1874161 | 2565726409 | 37.0000 | 11.1037 |
| 1315 | 1729225 | 2273930875 | 36.2629 | 10.9557 | 1370 | 1876900 | 2571353000 | 37.0135 | 11.1064 |
| 1316 | 1731856 | 2279122496 | 36.2767 | 10.9585 | 1371 | 1879641 | 2576987811 | 37.0270 | 11.1091 |
| 1317 | 1734489 | 2284322013 | 36.2905 | 10.9613 | 1372 | 1882384 | 2582630848 | 37.0405 | 11.1118 |
| 1318 | 1737124 | 2289529432 | 36.3043 | 10.9640 | 1373 | 1885129 | 2588282117 | 37.0540 | 11.1145 |
| 1319 | 1739761 | 2294744759 | 36.3180 | 10.9668 | 1374 | 1887876 | 2593941624 | 37.0675 | 11.1172 |
| 1320 | 1742400 | 2299968000 | 36.3318 | 10.9696 | 1375 | 1890625 | 2599609375 | 37.0810 | 11.1199 |
| 1321 | 1745041 | 2305199161 | 36.3456 | 10.9724 | 1376 | 1893376 | 2605285376 | 37.0945 | 11.1226 |
| 1322 | 1747684 | 2310438248 | 36.3593 | 10.9752 | 1377 | 1896129 | 2610969633 | 37.1080 | 11.1253 |
| 1323 | 1750329 | 2315685267 | 36.3731 | 10.9779 | 1378 | 1898884 | 2616662152 | 37.1214 | 11.1280 |
| 1324 | 1752976 | 2320940224 | 36.3868 | 10.9807 | 1379 | 1901641 | 2622362939 | 37.1349 | 11.1307 |
| 1325 | 1755625 | 2326203125 | 36.4005 | 10.9834 | 1380 | 1904400 | 2628072000 | 37.1484 | 11.1334 |
| 1326 | 1758276 | 2331473976 | 36.4143 | 10.9862 | 1381 | 1907161 | 2633789341 | 37.1618 | 11.1361 |
| 1327 | 1760929 | 2336752783 | 36.4280 | 10.9890 | 1382 | 1909924 | 2639514968 | 37.1753 | 11.1387 |
| 1328 | 1763584 | 2342039552 | 36.4417 | 10.9917 | 1383 | 1912689 | 2645248887 | 37.1887 | 11.1414 |
| 1329 | 1766241 | 2347334289 | 36.4555 | 10.9945 | 1384 | 1915456 | 2650991104 | 37.2021 | 11.1441 |
| 1330 | 1768900 | 2352637000 | 36.4692 | 10.9972 | 1385 | 1918225 | 2656741625 | 37.2156 | 11.1468 |
| 1331 | 1771561 | 2357947691 | 36.4829 | 11.0000 | 1386 | 1920996 | 2662500456 | 37.2290 | 11.1495 |
| 1332 | 1774224 | 2363266368 | 36.4966 | 11.0028 | 1387 | 1923769 | 2668267603 | 37.2424 | 11.1522 |
| 1333 | 1776889 | 2368593037 | 36.5103 | 11.0055 | 1388 | 1926544 | 2674043072 | 37.2559 | 11.1548 |
| 1334 | 1779556 | 2373927704 | 36.5240 | 11.0083 | 1389 | 1929321 | 2679826869 | 37.2693 | 11.1575 |
| 1335 | 1782225 | 2379270375 | 36.5377 | 11.0110 | 1390 | 1932100 | 2685619000 | 37.2827 | 11.1602 |
| 1336 | 1784896 | 2384621056 | 36.5513 | 11.0138 | 1391 | 1934881 | 2691419471 | 37.2961 | 11.1629 |
| 1337 | 1787569 | 2389979753 | 36.5650 | 11.0165 | 1392 | 1937664 | 2697228288 | 37.3095 | 11.1655 |
| 1338 | 1790244 | 2395346472 | 36.5787 | 11.0193 | 1393 | 1940449 | 2703045457 | 37.3229 | 11.1682 |
| 1339 | 1792921 | 2400721219 | 36.5923 | 11.0220 | 1394 | 1943236 | 2708870984 | 37.3363 | 11.1709 |
| 1340 | 1795600 | 2406104000 | 36.6060 | 11.0247 | 1395 | 1946025 | 2714704875 | 37.3497 | 11.1736 |
| 1341 | 1798281 | 2411494821 | 36.6197 | 11.0275 | 1396 | 1948816 | 2720547136 | 37.3631 | 11.1762 |
| 1342 | 1800964 | 2416893688 | 36.6333 | 11.0302 | 1397 | 1951609 | 2726397773 | 37.3765 | 11.1789 |
| 1343 | 1803649 | 2422300607 | 36.6469 | 11.0330 | 1398 | 1954404 | 2732256792 | 37.3898 | 11.1816 |
| 1344 | 1806336 | 2427715584 | 36.6606 | 11.0357 | 1399 | 1957201 | 2738124199 | 37.4032 | 11.1842 |
| 1345 | 1809025 | 2433138625 | 36.6742 | 11.0384 | 1400 | 1960000 | 2744000000 | 37.4166 | 11.1869 |
| 1346 | 1811716 | 2438569736 | 36.6879 | 11.0412 | 1401 | 1962801 | 2749884201 | 37.4299 | 11.1896 |
| 1347 | 1814409 | 2444008923 | 36.7015 | 11.0439 | 1402 | 1965604 | 2755776808 | 37.4433 | 11.1922 |
| 1348 | 1817104 | 2449456192 | 36.7151 | 11.0466 | 1403 | 1968409 | 2761677827 | 37.4566 | 11.1949 |
| 1349 | 1819801 | 2454911549 | 36.7287 | 11.0494 | 1404 | 1971216 | 2767587264 | 37.4700 | 11.1975 |
| 1350 | 1822500 | 2460375000 | 36.7423 | 11.0521 | 1405 | 1974025 | 2773505125 | 37.4833 | 11.2002 |
| 1351 | 1825201 | 2465846551 | 36.7560 | 11.0548 | 1406 | 1976836 | 2779431416 | 37.4967 | 11.2028 |
| 1352 | 1827904 | 2471326208 | 36.7696 | 11.0575 | 1407 | 1979649 | 2785366143 | 37.5100 | 11.2055 |
| 1353 | 1830609 | 2476813977 | 36.7831 | 11.0603 | 1408 | 1982464 | 2791309312 | 37.5233 | 11.2082 |
| 1354 | 1833316 | 2482309864 | 36.7967 | 11.0630 | 1409 | 1985281 | 2797260929 | 37.5366 | 11.2108 |
| 1355 | 1836025 | 2487813875 | 36.8103 | 11.0657 | 1410 | 1988100 | 2803221000 | 37.5500 | 11.2135 |
| 1356 | 1838736 | 2493326016 | 36.8239 | 11.0684 | 1411 | 1990921 | 2809189531 | 37.5633 | 11.2161 |
| 1357 | 1841449 | 2498846293 | 36.8375 | 11.0712 | 1412 | 1993744 | 2815166528 | 37.5766 | 11.2188 |
| 1358 | 1844164 | 2504374712 | 36.8511 | 11.0739 | 1413 | 1996569 | 2821151997 | 37.5899 | 11.2214 |
| 1359 | 1846881 | 2509911279 | 36.8646 | 11.0766 | 1414 | 1999396 | 2827145944 | 37.6032 | 11.2240 |
| 1360 | 1849600 | 2515456000 | 36.8782 | 11.0793 | 1415 | 2002225 | 2833148375 | 37.6165 | 11.2267 |
| 1361 | 1852321 | 2521008881 | 36.8917 | 11.0820 | 1416 | 2005056 | 2839159296 | 37.6298 | 11.2293 |
| 1362 | 1855044 | 2526569928 | 36.9053 | 11.0847 | 1417 | 2007889 | 2845178713 | 37.6431 | 11.2320 |
| 1363 | 1857769 | 2532139147 | 36.9188 | 11.0875 | 1418 | 2010724 | 2851206632 | 37.6563 | 11.2346 |
| 1364 | 1860496 | 2537716544 | 36.9324 | 11.0902 | 1419 | 2013561 | 2857243059 | 37.6696 | 11.2373 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 1420 | 2016400 | 2863288000 | 37.6829 | 11.2399 | 1475 | 2175625 | 3209046875 | 38.4057 | 11.3832 |
| 1421 | 2019241 | 2869341461 | 37.6962 | 11.2425 | 1476 | 2178576 | 3215578176 | 38.4187 | 11.3858 |
| 1422 | 2022084 | 2875403448 | 37.7094 | 11.2452 | 1477 | 2181529 | 3222118333 | 38.4318 | 11.3883 |
| 1423 | 2024929 | 2881473967 | 37.7227 | 11.2478 | 1478 | 2184484 | 3228667352 | 38.4448 | 11.3909 |
| 1424 | 2027776 | 2887553024 | 37.7359 | 11.2505 | 1479 | 2187441 | 3235225239 | 38.4578 | 11.3935 |
| 1425 | 2030625 | 2893640625 | 37.7492 | 11.2531 | 1480 | 2190400 | 3241792000 | 38.4708 | 11.3960 |
| 1426 | 2033476 | 2899736776 | 37.7624 | 11.2557 | 1481 | 2193361 | 3248367641 | 38.4838 | 11.3986 |
| 1427 | 2036329 | 2905841483 | 37.7757 | 11.2583 | 1482 | 2196324 | 3254952168 | 38.4968 | 11.4012 |
| 1428 | 2039184 | 2911954752 | 37.7889 | 11.2610 | 1483 | 2199289 | 3261545587 | 38.5097 | 11.4037 |
| 1429 | 2042041 | 2918076589 | 37.8021 | 11.2636 | 1484 | 2202256 | 3268147904 | 38.5227 | 11.4063 |
| 1430 | 2044900 | 2924207000 | 37.8153 | 11.2662 | 1485 | 2205225 | 3274759125 | 38.5357 | 11.4089 |
| 1431 | 2047761 | 2930345991 | 37.8286 | 11.2689 | 1486 | 2208196 | 3281379256 | 38.5487 | 11.4114 |
| 1432 | 2050624 | 2936493568 | 37.8418 | 11.2715 | 1487 | 2211169 | 3288008303 | 38.5616 | 11.4140 |
| 1433 | 2053489 | 2942649737 | 37.8550 | 11.2741 | 1488 | 2214144 | 3294646272 | 38.5746 | 11.4165 |
| 1434 | 2056356 | 2948814504 | 37.8682 | 11.2767 | 1489 | 2217121 | 3301293169 | 38.5876 | 11.4191 |
| 1435 | 2059225 | 2954987875 | 37.8814 | 11.2793 | 1490 | 2220100 | 3307949000 | 38.6005 | 11.4216 |
| 1436 | 2062096 | 2961169856 | 37.8946 | 11.2820 | 1491 | 2223081 | 3314613771 | 38.6135 | 11.4242 |
| 1437 | 2064969 | 2967360453 | 37.9078 | 11.2846 | 1492 | 2226064 | 3321287488 | 38.6264 | 11.4268 |
| 1438 | 2067844 | 2973559672 | 37.9210 | 11.2872 | 1493 | 2229049 | 3327970157 | 38.6394 | 11.4293 |
| 1439 | 2070721 | 2979767519 | 37.9342 | 11.2898 | 1494 | 2232036 | 3334661784 | 38.6523 | 11.4319 |
| 1440 | 2073600 | 2985984000 | 37.9473 | 11.2924 | 1495 | 2235025 | 3341362375 | 38.6652 | 11.4344 |
| 1441 | 2076481 | 2992209121 | 37.9605 | 11.2950 | 1496 | 2238016 | 3348071936 | 38.6782 | 11.4370 |
| 1442 | 2079364 | 2998442838 | 37.9737 | 11.2977 | 1497 | 2241009 | 3354790473 | 38.6911 | 11.4395 |
| 1443 | 2082249 | 3004685307 | 37.9868 | 11.3003 | 1498 | 2244004 | 3361517992 | 38.7040 | 11.4421 |
| 1444 | 2085136 | 3010936384 | 38.0000 | 11.3029 | 1499 | 2247001 | 3368254499 | 38.7169 | 11.4446 |
| 1445 | 2088025 | 3017196125 | 38.0132 | 11.3055 | 1500 | 2250000 | 3375000000 | 38.7298 | 11.4471 |
| 1446 | 2090916 | 3023464536 | 38.0263 | 11.3081 | 1501 | 2253001 | 3381754501 | 38.7427 | 11.4497 |
| 1447 | 2093809 | 3029741623 | 38.0395 | 11.3107 | 1502 | 2256004 | 3388518008 | 38.7556 | 11.4522 |
| 1448 | 2096704 | 3036027392 | 38.0526 | 11.3133 | 1503 | 2259009 | 3395290527 | 38.7685 | 11.4548 |
| 1449 | 2099601 | 3042321849 | 38.0657 | 11.3159 | 1504 | 2262016 | 3402072064 | 38.7814 | 11.4573 |
| 1450 | 2102500 | 3048625000 | 38.0789 | 11.3185 | 1505 | 2265025 | 3408862625 | 38.7943 | 11.4598 |
| 1451 | 2105401 | 3054936851 | 38.0920 | 11.3211 | 1506 | 2268036 | 3415662216 | 38.8072 | 11.4624 |
| 1452 | 2108304 | 3061257408 | 38.1051 | 11.3237 | 1507 | 2271049 | 3422470843 | 38.8201 | 11.4649 |
| 1453 | 2111209 | 3067586677 | 38.1182 | 11.3263 | 1508 | 2274064 | 3429288512 | 38.8330 | 11.4675 |
| 1454 | 2114116 | 3073924664 | 38.1314 | 11.3289 | 1509 | 2277081 | 3436115229 | 38.8458 | 11.4700 |
| 1455 | 2117025 | 3080271375 | 38.1445 | 11.3315 | 1510 | 2280100 | 3442951000 | 38.8587 | 11.4725 |
| 1456 | 2119936 | 3086626816 | 38.1576 | 11.3341 | 1511 | 2283121 | 3449795831 | 38.8716 | 11.4751 |
| 1457 | 2122849 | 3092990993 | 38.1707 | 11.3367 | 1512 | 2286144 | 3456649728 | 38.8844 | 11.4776 |
| 1458 | 2125764 | 3099363912 | 38.1838 | 11.3393 | 1513 | 2289169 | 3463512697 | 38.8973 | 11.4801 |
| 1459 | 2128681 | 3105745579 | 38.1969 | 11.3419 | 1514 | 2292196 | 3470384744 | 38.9102 | 11.4826 |
| 1460 | 2131600 | 3112136000 | 38.2099 | 11.3445 | 1515 | 2295225 | 3477265875 | 38.9230 | 11.4852 |
| 1461 | 2134521 | 3118535181 | 38.2230 | 11.3471 | 1516 | 2298256 | 3484156096 | 38.9358 | 11.4877 |
| 1462 | 2137444 | 3124943128 | 38.2361 | 11.3496 | 1517 | 2301289 | 3491055413 | 38.9487 | 11.4902 |
| 1463 | 2140369 | 3131359847 | 38.2492 | 11.3522 | 1518 | 2304324 | 3497963832 | 38.9615 | 11.4927 |
| 1464 | 2143296 | 3137785344 | 38.2623 | 11.3548 | 1519 | 2307361 | 3504881359 | 38.9744 | 11.4953 |
| 1465 | 2146225 | 3144219625 | 38.2753 | 11.3574 | 1520 | 2310400 | 3511808000 | 38.9872 | 11.4978 |
| 1466 | 2149156 | 3150662696 | 38.2884 | 11.3600 | 1521 | 2313441 | 3518743761 | 39.0000 | 11.5003 |
| 1467 | 2152089 | 3157114563 | 38.3014 | 11.3626 | 1522 | 2316484 | 3525688468 | 39.0128 | 11.5028 |
| 1468 | 2155024 | 3163575232 | 38.3145 | 11.3652 | 1523 | 2319529 | 3532642667 | 39.0256 | 11.5054 |
| 1469 | 2157961 | 3170044709 | 38.3275 | 11.3677 | 1524 | 2322576 | 3539605824 | 39.0384 | 11.5079 |
| 1470 | 2160900 | 3176523000 | 38.3406 | 11.3703 | 1525 | 2325625 | 3546578125 | 39.0512 | 11.5104 |
| 1471 | 2163841 | 3183010111 | 38.3536 | 11.3729 | 1526 | 2328676 | 3553559576 | 39.0640 | 11.5129 |
| 1472 | 2166784 | 3189506048 | 38.3667 | 11.3755 | 1527 | 2331729 | 3560550183 | 39.0768 | 11.5154 |
| 1473 | 2169729 | 3196010817 | 38.3797 | 11.3780 | 1528 | 2334784 | 3567549952 | 39.0896 | 11.5179 |
| 1474 | 2172676 | 3202524424 | 38.3927 | 11.3806 | 1529 | 2337841 | 3574558889 | 39.1024 | 11.5204 |

| No. | Square. | Cube. | Sq. Root. | Cube Root. | No. | Square. | Cube. | Sq. Root. | Cube Root. |
|------|---------|------------|-----------|------------|------|---------|------------|-----------|------------|
| 1530 | 2340900 | 3581577000 | 39.1152 | 11.5230 | 1565 | 2449225 | 3833037125 | 39.5601 | 11.6102 |
| 1531 | 2343961 | 3588604291 | 39.1230 | 11.5255 | 1566 | 2452356 | 3840389496 | 39.5727 | 11.6126 |
| 1532 | 2347024 | 3595640768 | 39.1408 | 11.5280 | 1567 | 2455489 | 3847751263 | 39.5854 | 11.6151 |
| 1533 | 2350089 | 3602686437 | 39.1535 | 11.5305 | 1568 | 2458624 | 3855123432 | 39.5980 | 11.6176 |
| 1534 | 2353156 | 3609741304 | 39.1663 | 11.5330 | 1569 | 2461761 | 3862503009 | 39.6106 | 11.6200 |
| 1535 | 2356225 | 3616805375 | 39.1791 | 11.5355 | 1570 | 2464900 | 3869893000 | 39.6232 | 11.6225 |
| 1536 | 2359296 | 3623878656 | 39.1918 | 11.5380 | 1571 | 2468041 | 3877292411 | 39.6358 | 11.6250 |
| 1537 | 2362369 | 3630961153 | 39.2046 | 11.5405 | 1572 | 2471184 | 3884701248 | 39.6485 | 11.6274 |
| 1538 | 2365444 | 3638052872 | 39.2173 | 11.5430 | 1573 | 2474329 | 3892119517 | 39.6611 | 11.6299 |
| 1539 | 2368521 | 3645153819 | 39.2301 | 11.5455 | 1574 | 2477476 | 3899547224 | 39.6737 | 11.6324 |
| 1540 | 2371600 | 3652264000 | 39.2428 | 11.5480 | 1575 | 2480625 | 3906984375 | 39.6863 | 11.6348 |
| 1541 | 2374681 | 3659383421 | 39.2556 | 11.5505 | 1576 | 2483776 | 3914430976 | 39.6989 | 11.6373 |
| 1542 | 2377764 | 3666512038 | 39.2683 | 11.5530 | 1577 | 2486929 | 3921887033 | 39.7115 | 11.6398 |
| 1543 | 2380849 | 3673650007 | 39.2810 | 11.5555 | 1578 | 2490084 | 3929352552 | 39.7240 | 11.6422 |
| 1544 | 2383936 | 3680797184 | 39.2938 | 11.5580 | 1579 | 2493241 | 3936827539 | 39.7366 | 11.6447 |
| 1545 | 2387025 | 3687953625 | 39.3065 | 11.5605 | 1580 | 2496400 | 3944312000 | 39.7492 | 11.6471 |
| 1546 | 2390116 | 3695119336 | 39.3192 | 11.5630 | 1581 | 2499561 | 3951805941 | 39.7618 | 11.6496 |
| 1547 | 2393209 | 3702294323 | 39.3319 | 11.5655 | 1582 | 2502724 | 3959309368 | 39.7744 | 11.6520 |
| 1548 | 2396304 | 3709478592 | 39.3446 | 11.5680 | 1583 | 2505889 | 3966822287 | 39.7869 | 11.6545 |
| 1549 | 2399401 | 3716672149 | 39.3573 | 11.5705 | 1584 | 2509056 | 3974344704 | 39.7995 | 11.6570 |
| 1550 | 2402500 | 3723875000 | 39.3700 | 11.5729 | 1585 | 2512225 | 3981876625 | 39.8121 | 11.6594 |
| 1551 | 2405601 | 3731087151 | 39.3827 | 11.5754 | 1586 | 2515396 | 3989418056 | 39.8246 | 11.6619 |
| 1552 | 2408704 | 3738308608 | 39.3954 | 11.5779 | 1587 | 2518569 | 3996969003 | 39.8372 | 11.6643 |
| 1553 | 2411809 | 3745539377 | 39.4081 | 11.5804 | 1588 | 2521744 | 4004529472 | 39.8497 | 11.6668 |
| 1554 | 2414916 | 3752779464 | 39.4208 | 11.5829 | 1589 | 2524921 | 4012099469 | 39.8623 | 11.6692 |
| 1555 | 2418025 | 3760028875 | 39.4335 | 11.5854 | 1590 | 2528100 | 4019679000 | 39.8748 | 11.6717 |
| 1556 | 2421136 | 3767287616 | 39.4462 | 11.5879 | 1591 | 2531281 | 4027268071 | 39.8873 | 11.6741 |
| 1557 | 2424249 | 3774555693 | 39.4589 | 11.5903 | 1592 | 2534464 | 4034865688 | 39.8999 | 11.6765 |
| 1558 | 2427364 | 3781833112 | 39.4715 | 11.5928 | 1593 | 2537649 | 4042474857 | 39.9124 | 11.6790 |
| 1559 | 2430481 | 3789119879 | 39.4842 | 11.5953 | 1594 | 2540836 | 4050092584 | 39.9249 | 11.6814 |
| 1560 | 2433600 | 3796416000 | 39.4968 | 11.5978 | 1595 | 2544025 | 4057719375 | 39.9375 | 11.6839 |
| 1561 | 2436721 | 3803721481 | 39.5095 | 11.6003 | 1596 | 2547216 | 4065356736 | 39.9500 | 11.6863 |
| 1562 | 2439844 | 3811036328 | 39.5221 | 11.6027 | 1597 | 2550404 | 4073003173 | 39.9625 | 11.6888 |
| 1563 | 2442969 | 3818360547 | 39.5348 | 11.6052 | 1598 | 2553604 | 4080659192 | 39.9750 | 11.6912 |
| 1564 | 2446096 | 3825694144 | 39.5474 | 11.6077 | 1599 | 2556801 | 4088324799 | 39.9875 | 11.6936 |
| | | | | | 1600 | 2560000 | 4096000000 | 40.0000 | 11.6961 |

SQUARES AND CUBES OF DECIMALS.

| No. | Square | Cube. | No. | Square | Cube. | No. | Square. | Cube. |
|-----|--------|-------|-----|--------|----------|------|-----------|--------------|
| .1 | .01 | .001 | .01 | .0001 | .000 001 | .001 | .00 00 01 | .000 000 001 |
| .2 | .04 | .008 | .02 | .0004 | .000 008 | .002 | .00 00 04 | .000 000 008 |
| .3 | .09 | .027 | .03 | .0009 | .000 027 | .003 | .00 00 09 | .000 000 027 |
| .4 | .16 | .064 | .04 | .0016 | .000 064 | .004 | .00 00 16 | .000 000 064 |
| .5 | .25 | .125 | .05 | .0025 | .000 125 | .005 | .00 00 25 | .000 000 125 |
| .6 | .36 | .216 | .06 | .0036 | .000 216 | .006 | .00 00 36 | .000 000 216 |
| .7 | .49 | .343 | .07 | .0049 | .000 343 | .007 | .00 00 49 | .000 000 343 |
| .8 | .64 | .512 | .08 | .0064 | .000 512 | .008 | .00 00 64 | .000 000 512 |
| .9 | .81 | .729 | .09 | .0081 | .000 729 | .009 | .00 00 81 | .000 000 729 |
| 1.0 | 1.00 | 1.000 | .10 | .0100 | .001 000 | .010 | .00 01 00 | .000 001 000 |
| 1.2 | 1.44 | 1.728 | .12 | .0144 | .001 728 | .012 | .00 01 44 | .000 001 728 |

Note that the square has twice as many decimal places, and the cube three times as many decimal places, as the root.

FIFTH ROOTS AND FIFTH POWERS.

(Abridged from TRAUTWINE.)

| No. or Root. | Power. | No. or Root. | Power. | No. or Root. | Power. | No. or Root. | Power. | No. or Root. | Power. |
|--------------|---------|--------------|---------|--------------|---------|--------------|----------|--------------|------------|
| .10 | .000010 | 3.7 | 693.440 | 9.8 | 90392 | 21.8 | 4923597 | 40 | 102400000 |
| .15 | .000075 | 3.8 | 792.352 | 9.9 | 95099 | 22.0 | 5153632 | 41 | 115856201 |
| .20 | .000320 | 3.9 | 902.242 | 10.0 | 100000 | 22.2 | 5392186 | 42 | 130691232 |
| .25 | .000977 | 4.0 | 1024.00 | 10.2 | 110408 | 22.4 | 5639493 | 43 | 147008443 |
| .30 | .002430 | 4.1 | 1158.56 | 10.4 | 121665 | 22.6 | 5895793 | 44 | 164916224 |
| .35 | .005252 | 4.2 | 1306.91 | 10.6 | 133823 | 22.8 | 6161327 | 45 | 184528125 |
| .40 | .010240 | 4.3 | 1470.08 | 10.8 | 146933 | 23.0 | 6436343 | 46 | 205962976 |
| .45 | .018453 | 4.4 | 1649.16 | 11.0 | 161051 | 23.2 | 6721093 | 47 | 229345007 |
| .50 | .031250 | 4.5 | 1845.28 | 11.2 | 176234 | 23.4 | 7015834 | 48 | 254803963 |
| .55 | .050328 | 4.6 | 2059.63 | 11.4 | 192541 | 23.6 | 7320825 | 49 | 282475249 |
| .60 | .077760 | 4.7 | 2293.45 | 11.6 | 210034 | 23.8 | 7636332 | 50 | 312500000 |
| .65 | .116029 | 4.8 | 2548.04 | 11.8 | 228776 | 24.0 | 7962624 | 51 | 345025251 |
| .70 | .168070 | 4.9 | 2824.75 | 12.0 | 248832 | 24.2 | 8299976 | 52 | 380204032 |
| .75 | .237305 | 5.0 | 3125.00 | 12.2 | 270271 | 24.4 | 8648666 | 53 | 418195493 |
| .80 | .327680 | 5.1 | 3450.25 | 12.4 | 293163 | 24.6 | 9008978 | 54 | 459165024 |
| .85 | .443705 | 5.2 | 3802.04 | 12.6 | 317580 | 24.8 | 9381200 | 55 | 503284375 |
| .90 | .590490 | 5.3 | 4181.95 | 12.8 | 343597 | 25.0 | 9765625 | 56 | 550731776 |
| .95 | .773781 | 5.4 | 4591.65 | 13.0 | 371293 | 25.2 | 10162550 | 57 | 601692057 |
| 1.00 | 1.00000 | 5.5 | 5032.84 | 13.2 | 400746 | 25.4 | 10572278 | 58 | 656356768 |
| 1.05 | 1.27628 | 5.6 | 5507.32 | 13.4 | 432040 | 25.6 | 10995116 | 59 | 714924299 |
| 1.10 | 1.61051 | 5.7 | 6016.92 | 13.6 | 465259 | 25.8 | 11431377 | 60 | 777600000 |
| 1.15 | 2.01135 | 5.8 | 6563.57 | 13.8 | 500490 | 26.0 | 11881376 | 61 | 844596301 |
| 1.20 | 2.48832 | 5.9 | 7149.24 | 14.0 | 537824 | 26.2 | 12345437 | 62 | 916132332 |
| 1.25 | 3.05176 | 6.0 | 7776.00 | 14.2 | 577353 | 26.4 | 12823886 | 63 | 992436543 |
| 1.30 | 3.71293 | 6.1 | 8445.96 | 14.4 | 619174 | 26.6 | 13317055 | 64 | 1073741824 |
| 1.35 | 4.43403 | 6.2 | 9161.33 | 14.6 | 663383 | 26.8 | 13825281 | 65 | 1160290625 |
| 1.40 | 5.37824 | 6.3 | 9924.37 | 14.8 | 710082 | 27.0 | 14348907 | 66 | 1252332576 |
| 1.45 | 6.40973 | 6.4 | 10737 | 15.0 | 759375 | 27.2 | 14888280 | 67 | 1350125107 |
| 1.50 | 7.59375 | 6.5 | 11603 | 15.2 | 811368 | 27.4 | 15443752 | 68 | 1453933563 |
| 1.55 | 8.94661 | 6.6 | 12523 | 15.4 | 866171 | 27.6 | 16015681 | 69 | 1564031349 |
| 1.60 | 10.4358 | 6.7 | 13501 | 15.6 | 923896 | 27.8 | 16604430 | 70 | 1680700000 |
| 1.65 | 12.2298 | 6.8 | 14539 | 15.8 | 984658 | 28.0 | 17210368 | 71 | 1804229351 |
| 1.70 | 14.1986 | 6.9 | 15640 | 16.0 | 1048576 | 28.2 | 17833863 | 72 | 1934917632 |
| 1.75 | 16.4131 | 7.0 | 16807 | 16.2 | 1115771 | 28.4 | 18475309 | 73 | 2073071593 |
| 1.80 | 18.8957 | 7.1 | 18042 | 16.4 | 1186367 | 28.6 | 19135075 | 74 | 2219006624 |
| 1.85 | 21.6700 | 7.2 | 19349 | 16.6 | 1260493 | 28.8 | 19813557 | 75 | 2373046875 |
| 1.90 | 24.7610 | 7.3 | 20731 | 16.8 | 1338278 | 29.0 | 20511149 | 76 | 2535525376 |
| 1.95 | 28.1951 | 7.4 | 22190 | 17.0 | 1419357 | 29.2 | 21228253 | 77 | 2706784157 |
| 2.00 | 32.0000 | 7.5 | 23730 | 17.2 | 1503366 | 29.4 | 21965275 | 78 | 2887174368 |
| 2.05 | 36.2051 | 7.6 | 25355 | 17.4 | 1594947 | 29.6 | 22722628 | 79 | 3077056399 |
| 2.10 | 40.8410 | 7.7 | 27068 | 17.6 | 1688742 | 29.8 | 23500728 | 80 | 3276800000 |
| 2.15 | 45.9401 | 7.8 | 28872 | 17.8 | 1786899 | 30.0 | 24300000 | 81 | 3486784401 |
| 2.20 | 51.5363 | 7.9 | 30771 | 18.0 | 1889568 | 30.5 | 26393634 | 82 | 3707398432 |
| 2.25 | 57.6650 | 8.0 | 32768 | 18.2 | 1996903 | 31.0 | 28629151 | 83 | 3939040643 |
| 2.30 | 64.3634 | 8.1 | 34868 | 18.4 | 2109061 | 31.5 | 31013642 | 84 | 4182119424 |
| 2.35 | 71.6703 | 8.2 | 37074 | 18.6 | 2226203 | 32.0 | 33554432 | 85 | 4437053125 |
| 2.40 | 79.6262 | 8.3 | 39390 | 18.8 | 2348493 | 32.5 | 36259082 | 86 | 4704270176 |
| 2.45 | 88.2735 | 8.4 | 41821 | 19.0 | 2476099 | 33.0 | 39135393 | 87 | 4984209207 |
| 2.50 | 97.6562 | 8.5 | 44371 | 19.2 | 2609193 | 33.5 | 42191410 | 88 | 5277319168 |
| 2.55 | 107.820 | 8.6 | 47043 | 19.4 | 2747949 | 34.0 | 45435424 | 89 | 5584059449 |
| 2.60 | 118.814 | 8.7 | 49842 | 19.6 | 2892547 | 34.5 | 48875980 | 90 | 5904900000 |
| 2.70 | 143.489 | 8.8 | 52773 | 19.8 | 3043168 | 35.0 | 52521875 | 91 | 6240321451 |
| 2.80 | 172.104 | 8.9 | 55841 | 20.0 | 3200000 | 35.5 | 56382167 | 92 | 6590815232 |
| 2.90 | 205.111 | 9.0 | 59049 | 20.2 | 3362323 | 36.0 | 60466176 | 93 | 6956883693 |
| 3.00 | 243.000 | 9.1 | 62403 | 20.4 | 3533059 | 36.5 | 64783487 | 94 | 7339040224 |
| 3.10 | 286.292 | 9.2 | 65908 | 20.6 | 3709677 | 37.0 | 69343957 | 95 | 7737809375 |
| 3.20 | 335.544 | 9.3 | 69569 | 20.8 | 3893289 | 37.5 | 74157715 | 96 | 8153726976 |
| 3.30 | 391.354 | 9.4 | 73390 | 21.0 | 4084101 | 38.0 | 79235168 | 97 | 8587340257 |
| 3.40 | 454.354 | 9.5 | 77378 | 21.2 | 4282322 | 38.5 | 84587005 | 98 | 9039207968 |
| 3.50 | 525.219 | 9.6 | 81537 | 21.4 | 4483166 | 39.0 | 90224199 | 99 | 9509900499 |
| 3.60 | 604.662 | 9.7 | 85873 | 21.6 | 4701850 | 39.5 | 96158012 | | |

CIRCUMFERENCES AND AREAS OF CIRCLES.

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|--------|-------|---------|--------|-------|---------|--------|
| 1/64 | .04909 | .00019 | 23/8 | 7.4613 | 4.4301 | 61/8 | 19.242 | 29.465 |
| 1/32 | .09818 | .00077 | 7/16 | 7.6576 | 4.6664 | 1/4 | 19.635 | 30.680 |
| 3/64 | .14726 | .00173 | 1/2 | 7.8540 | 4.9087 | 3/8 | 20.028 | 31.919 |
| 1/16 | .19635 | .00307 | 9/16 | 8.0503 | 5.1572 | 1/2 | 20.420 | 33.183 |
| 3/32 | .29452 | .00690 | 5/8 | 8.2467 | 5.4119 | 5/8 | 20.813 | 34.472 |
| 1/8 | .39270 | .01227 | 11/16 | 8.4430 | 5.6727 | 3/4 | 21.206 | 35.785 |
| 5/32 | .49087 | .01917 | 3/4 | 8.6394 | 5.9396 | 7/8 | 21.598 | 37.122 |
| 3/16 | .58905 | .02761 | 13/16 | 8.8357 | 6.2126 | 7. | 21.991 | 38.485 |
| 7/32 | .68722 | .03758 | 7/8 | 9.0321 | 6.4918 | 1/8 | 22.384 | 39.871 |
| | | | 15/16 | 9.2284 | 6.7771 | 1/4 | 22.776 | 41.282 |
| 1/4 | .78540 | .04909 | | | | 3/8 | 23.169 | 42.718 |
| 9/32 | .88357 | .06213 | 3. | 9.4248 | 7.0686 | 1/2 | 23.562 | 44.179 |
| 5/16 | .98175 | .07670 | 1/16 | 9.6211 | 7.3662 | 5/8 | 23.955 | 45.664 |
| 11/32 | 1.0799 | .09281 | 1/8 | 9.8175 | 7.6699 | 3/4 | 24.347 | 47.173 |
| 3/8 | 1.1781 | .11045 | 3/16 | 10.014 | 7.9798 | 7/8 | 24.740 | 48.707 |
| 13/32 | 1.2763 | .12962 | 1/4 | 10.210 | 8.2958 | 8. | 25.133 | 50.265 |
| 7/16 | 1.3744 | .15033 | 5/16 | 10.407 | 8.6179 | 1/8 | 25.525 | 51.849 |
| 15/32 | 1.4726 | .17257 | 3/8 | 10.603 | 8.9462 | 1/4 | 25.918 | 53.456 |
| | | | 7/16 | 10.799 | 9.2806 | 3/8 | 26.311 | 55.088 |
| 1/2 | 1.5708 | .19635 | 1/2 | 10.996 | 9.6211 | 1/2 | 26.704 | 56.745 |
| 17/32 | 1.6690 | .22166 | 9/16 | 11.192 | 9.9678 | 5/8 | 27.096 | 58.426 |
| 9/16 | 1.7671 | .24850 | 5/8 | 11.388 | 10.321 | 3/4 | 27.489 | 60.132 |
| 19/32 | 1.8653 | .27688 | 11/16 | 11.585 | 10.680 | 7/8 | 27.882 | 61.862 |
| 5/8 | 1.9635 | .30680 | 3/4 | 11.781 | 11.045 | 9. | 28.274 | 63.617 |
| 21/32 | 2.0617 | .33824 | 13/16 | 11.977 | 11.416 | 1/8 | 28.667 | 65.397 |
| 11/16 | 2.1598 | .37122 | 7/8 | 12.174 | 11.793 | 1/4 | 29.060 | 67.201 |
| 23/32 | 2.2580 | .40574 | 15/16 | 12.370 | 12.177 | 3/8 | 29.452 | 69.029 |
| | | | 4. | 12.566 | 12.566 | 1/2 | 29.845 | 70.882 |
| 3/4 | 2.3562 | .44179 | 1/16 | 12.763 | 12.962 | 5/8 | 30.238 | 72.760 |
| 25/32 | 2.4544 | .47937 | 1/8 | 12.959 | 13.364 | 3/4 | 30.631 | 74.662 |
| 13/16 | 2.5525 | .51849 | 3/16 | 13.155 | 13.772 | 7/8 | 31.023 | 76.589 |
| 27/32 | 2.6507 | .55914 | 1/4 | 13.352 | 14.186 | 10. | 31.416 | 78.540 |
| 7/8 | 2.7489 | .60132 | 5/16 | 13.548 | 14.607 | 1/8 | 31.809 | 80.516 |
| 29/32 | 2.8471 | .64504 | 3/8 | 13.744 | 15.033 | 1/4 | 32.201 | 82.516 |
| 15/16 | 2.9452 | .69029 | 7/16 | 13.941 | 15.466 | 3/8 | 32.594 | 84.541 |
| 31/32 | 3.0434 | .73708 | 1/2 | 14.137 | 15.904 | 1/2 | 32.987 | 86.590 |
| | | | 9/16 | 14.334 | 16.349 | 5/8 | 33.379 | 88.664 |
| 1. | 3.1416 | .7854 | 5/8 | 14.530 | 16.800 | 3/4 | 33.772 | 90.763 |
| 1/16 | 3.3379 | .8866 | 11/16 | 14.726 | 17.257 | 7/8 | 34.165 | 92.886 |
| 1/8 | 3.5343 | .9940 | 3/4 | 14.923 | 17.721 | 11. | 34.558 | 95.033 |
| 3/16 | 3.7306 | 1.1075 | 13/16 | 15.119 | 18.190 | 1/8 | 34.950 | 97.205 |
| 1/4 | 3.9270 | 1.2272 | 7/8 | 15.315 | 18.665 | 1/4 | 35.343 | 99.402 |
| 5/16 | 4.1233 | 1.3530 | 15/16 | 15.512 | 19.147 | 3/8 | 35.736 | 101.62 |
| 3/8 | 4.3197 | 1.4849 | 5. | 15.708 | 19.635 | 1/2 | 36.128 | 103.87 |
| 7/16 | 4.5160 | 1.6230 | 1/16 | 15.904 | 20.129 | 5/8 | 36.521 | 106.14 |
| 1/2 | 4.7124 | 1.7671 | 1/8 | 16.101 | 20.629 | 3/4 | 36.914 | 108.43 |
| 9/16 | 4.9087 | 1.9175 | 3/16 | 16.297 | 21.135 | 7/8 | 37.306 | 110.75 |
| 5/8 | 5.1051 | 2.0739 | 1/4 | 16.493 | 21.648 | 12. | 37.699 | 113.10 |
| 11/16 | 5.3014 | 2.2365 | 5/16 | 16.690 | 22.166 | 1/8 | 38.092 | 115.47 |
| 3/4 | 5.4978 | 2.4053 | 3/8 | 16.886 | 22.691 | 1/4 | 38.485 | 117.86 |
| 13/16 | 5.6941 | 2.5802 | 7/16 | 17.082 | 23.221 | 3/8 | 38.877 | 120.28 |
| 7/8 | 5.8905 | 2.7612 | 1/2 | 17.279 | 23.758 | 1/2 | 39.270 | 122.72 |
| 15/16 | 6.0868 | 2.9483 | 9/16 | 17.475 | 24.301 | 5/8 | 39.663 | 125.19 |
| | | | 5/8 | 17.671 | 24.850 | 3/4 | 40.055 | 127.68 |
| 2. | 6.2832 | 3.1416 | 11/16 | 17.868 | 25.406 | 7/8 | 40.448 | 130.19 |
| 1/16 | 6.4795 | 3.3410 | 3/4 | 18.064 | 25.967 | 13. | 40.841 | 132.73 |
| 1/8 | 6.6759 | 3.5466 | 13/16 | 18.261 | 26.535 | 1/8 | 41.233 | 135.30 |
| 3/16 | 6.8722 | 3.7583 | 7/8 | 18.457 | 27.109 | 1/4 | 41.626 | 137.89 |
| 1/4 | 7.0686 | 3.9761 | 15/16 | 18.653 | 27.688 | 3/8 | 42.019 | 140.50 |
| 5/16 | 7.2649 | 4.2000 | 6. | 18.850 | 28.274 | 1/2 | 42.412 | 143.14 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|--------------------------------|---------|--------|--------------------------------|---------|--------|--------------------------------|---------|--------|
| 13 ⁵ / ₈ | 42.804 | 145.80 | 21 ⁷ / ₈ | 68.722 | 375.83 | 30 ¹ / ₈ | 94.640 | 712.76 |
| 3 ⁴ / ₈ | 43.197 | 148.49 | 22. | 69.115 | 380.13 | 1 ⁴ / ₈ | 95.033 | 718.69 |
| 7 ⁸ / ₈ | 43.590 | 151.20 | 1 ⁸ / ₈ | 69.508 | 384.46 | 3 ⁸ / ₈ | 95.426 | 724.64 |
| 14. | 43.982 | 153.94 | 1 ⁴ / ₈ | 69.900 | 388.82 | 1 ² / ₈ | 95.819 | 730.62 |
| 1 ⁸ / ₈ | 44.375 | 156.70 | 3 ⁸ / ₈ | 70.293 | 393.20 | 5 ⁸ / ₈ | 96.211 | 736.62 |
| 1 ⁴ / ₈ | 44.768 | 159.48 | 1 ² / ₈ | 70.686 | 397.61 | 3 ⁴ / ₈ | 96.604 | 742.64 |
| 3 ⁸ / ₈ | 45.160 | 162.30 | 5 ⁸ / ₈ | 71.079 | 402.04 | 7 ⁸ / ₈ | 96.997 | 748.69 |
| 1 ² / ₈ | 45.553 | 165.13 | 3 ⁴ / ₈ | 71.471 | 406.49 | 31. | 97.389 | 754.77 |
| 5 ⁸ / ₈ | 45.946 | 167.99 | 7 ⁸ / ₈ | 71.864 | 410.97 | 1 ⁸ / ₈ | 97.782 | 760.87 |
| 3 ⁴ / ₈ | 46.338 | 170.87 | 23. | 72.257 | 415.48 | 1 ⁴ / ₈ | 98.175 | 766.99 |
| 7 ⁸ / ₈ | 46.731 | 173.78 | 1 ⁸ / ₈ | 72.649 | 420.00 | 3 ⁸ / ₈ | 98.567 | 773.14 |
| 15. | 47.124 | 176.71 | 1 ⁴ / ₈ | 73.042 | 424.56 | 1 ² / ₈ | 98.960 | 779.31 |
| 1 ⁸ / ₈ | 47.517 | 179.67 | 3 ⁸ / ₈ | 73.435 | 429.13 | 5 ⁸ / ₈ | 99.353 | 785.51 |
| 1 ⁴ / ₈ | 47.909 | 182.65 | 1 ² / ₈ | 73.827 | 433.74 | 3 ⁴ / ₈ | 99.746 | 791.73 |
| 3 ⁸ / ₈ | 48.302 | 185.66 | 5 ⁸ / ₈ | 74.220 | 438.36 | 7 ⁸ / ₈ | 100.138 | 797.98 |
| 1 ² / ₈ | 48.695 | 188.69 | 3 ⁴ / ₈ | 74.613 | 443.01 | 32. | 100.531 | 804.25 |
| 5 ⁸ / ₈ | 49.087 | 191.75 | 7 ⁸ / ₈ | 75.006 | 447.69 | 1 ⁸ / ₈ | 100.924 | 810.54 |
| 3 ⁴ / ₈ | 49.480 | 194.83 | 24. | 75.398 | 452.39 | 1 ⁴ / ₈ | 101.316 | 816.86 |
| 7 ⁸ / ₈ | 49.873 | 197.93 | 1 ⁸ / ₈ | 75.791 | 457.11 | 3 ⁸ / ₈ | 101.709 | 823.21 |
| 16. | 50.265 | 201.06 | 1 ⁴ / ₈ | 76.184 | 461.86 | 1 ² / ₈ | 102.102 | 829.58 |
| 1 ⁸ / ₈ | 50.658 | 204.22 | 3 ⁸ / ₈ | 76.576 | 466.64 | 5 ⁸ / ₈ | 102.494 | 835.97 |
| 1 ⁴ / ₈ | 51.051 | 207.39 | 1 ² / ₈ | 76.969 | 471.44 | 3 ⁴ / ₈ | 102.887 | 842.39 |
| 3 ⁸ / ₈ | 51.444 | 210.60 | 5 ⁸ / ₈ | 77.362 | 476.26 | 7 ⁸ / ₈ | 103.280 | 848.83 |
| 1 ² / ₈ | 51.836 | 213.82 | 3 ⁴ / ₈ | 77.754 | 481.11 | 33. | 103.673 | 855.30 |
| 5 ⁸ / ₈ | 52.229 | 217.08 | 7 ⁸ / ₈ | 78.147 | 485.98 | 1 ⁸ / ₈ | 104.065 | 861.79 |
| 3 ⁴ / ₈ | 52.622 | 220.35 | 25. | 78.540 | 490.87 | 1 ⁴ / ₈ | 104.458 | 868.31 |
| 7 ⁸ / ₈ | 53.014 | 223.65 | 1 ⁸ / ₈ | 78.933 | 495.79 | 3 ⁸ / ₈ | 104.851 | 874.85 |
| 17. | 53.407 | 226.98 | 1 ⁴ / ₈ | 79.325 | 500.74 | 1 ² / ₈ | 105.243 | 881.41 |
| 1 ⁸ / ₈ | 53.800 | 230.33 | 3 ⁸ / ₈ | 79.718 | 505.71 | 5 ⁸ / ₈ | 105.636 | 888.00 |
| 1 ⁴ / ₈ | 54.192 | 233.71 | 1 ² / ₈ | 80.111 | 510.71 | 3 ⁴ / ₈ | 106.029 | 894.62 |
| 3 ⁸ / ₈ | 54.585 | 237.10 | 5 ⁸ / ₈ | 80.503 | 515.72 | 7 ⁸ / ₈ | 106.421 | 901.26 |
| 1 ² / ₈ | 54.978 | 240.53 | 3 ⁴ / ₈ | 80.896 | 520.77 | 34. | 106.814 | 907.92 |
| 5 ⁸ / ₈ | 55.371 | 243.98 | 7 ⁸ / ₈ | 81.289 | 525.84 | 1 ⁸ / ₈ | 107.207 | 914.61 |
| 3 ⁴ / ₈ | 55.763 | 247.45 | 26. | 81.681 | 530.93 | 1 ⁴ / ₈ | 107.600 | 921.32 |
| 7 ⁸ / ₈ | 56.156 | 250.95 | 1 ⁸ / ₈ | 82.074 | 536.05 | 3 ⁸ / ₈ | 107.992 | 928.06 |
| 18. | 56.549 | 254.47 | 1 ⁴ / ₈ | 82.467 | 541.19 | 1 ² / ₈ | 108.385 | 934.82 |
| 1 ⁸ / ₈ | 56.941 | 258.02 | 3 ⁸ / ₈ | 82.860 | 546.35 | 5 ⁸ / ₈ | 108.778 | 941.61 |
| 1 ⁴ / ₈ | 57.334 | 261.59 | 1 ² / ₈ | 83.252 | 551.55 | 3 ⁴ / ₈ | 109.170 | 948.42 |
| 3 ⁸ / ₈ | 57.727 | 265.18 | 5 ⁸ / ₈ | 83.645 | 556.76 | 7 ⁸ / ₈ | 109.563 | 955.25 |
| 1 ² / ₈ | 58.119 | 268.80 | 3 ⁴ / ₈ | 84.038 | 562.00 | 35. | 109.956 | 962.11 |
| 5 ⁸ / ₈ | 58.512 | 272.45 | 7 ⁸ / ₈ | 84.430 | 567.27 | 1 ⁸ / ₈ | 110.348 | 969.00 |
| 3 ⁴ / ₈ | 58.905 | 276.12 | 27. | 84.823 | 572.56 | 1 ⁴ / ₈ | 110.741 | 975.91 |
| 7 ⁸ / ₈ | 59.298 | 279.81 | 1 ⁸ / ₈ | 85.216 | 577.87 | 3 ⁸ / ₈ | 111.134 | 982.84 |
| 19. | 59.690 | 283.53 | 1 ⁴ / ₈ | 85.608 | 583.21 | 1 ² / ₈ | 111.527 | 989.80 |
| 1 ⁸ / ₈ | 60.083 | 287.27 | 3 ⁸ / ₈ | 86.001 | 588.57 | 5 ⁸ / ₈ | 111.919 | 996.78 |
| 1 ⁴ / ₈ | 60.476 | 291.04 | 1 ² / ₈ | 86.394 | 593.96 | 3 ⁴ / ₈ | 112.312 | 1003.8 |
| 3 ⁸ / ₈ | 60.868 | 294.83 | 5 ⁸ / ₈ | 86.786 | 599.37 | 7 ⁸ / ₈ | 112.705 | 1010.8 |
| 1 ² / ₈ | 61.261 | 298.65 | 3 ⁴ / ₈ | 87.179 | 604.81 | 36. | 113.097 | 1017.9 |
| 5 ⁸ / ₈ | 61.654 | 302.49 | 7 ⁸ / ₈ | 87.572 | 610.27 | 1 ⁸ / ₈ | 113.490 | 1025.0 |
| 3 ⁴ / ₈ | 62.046 | 306.35 | 28. | 87.965 | 615.75 | 1 ⁴ / ₈ | 113.883 | 1032.1 |
| 7 ⁸ / ₈ | 62.439 | 310.24 | 1 ⁸ / ₈ | 88.357 | 621.26 | 3 ⁸ / ₈ | 114.275 | 1039.2 |
| 20. | 62.832 | 314.16 | 1 ⁴ / ₈ | 88.750 | 626.80 | 1 ² / ₈ | 114.668 | 1046.3 |
| 1 ⁸ / ₈ | 63.225 | 318.10 | 3 ⁸ / ₈ | 89.143 | 632.36 | 5 ⁸ / ₈ | 115.061 | 1053.5 |
| 1 ⁴ / ₈ | 63.617 | 322.06 | 1 ² / ₈ | 89.535 | 637.94 | 3 ⁴ / ₈ | 115.454 | 1060.7 |
| 3 ⁸ / ₈ | 64.010 | 326.05 | 5 ⁸ / ₈ | 89.928 | 643.55 | 7 ⁸ / ₈ | 115.846 | 1068.0 |
| 1 ² / ₈ | 64.403 | 330.06 | 3 ⁴ / ₈ | 90.321 | 649.18 | 37. | 116.239 | 1075.2 |
| 5 ⁸ / ₈ | 64.795 | 334.10 | 7 ⁸ / ₈ | 90.713 | 654.84 | 1 ⁸ / ₈ | 116.632 | 1082.5 |
| 3 ⁴ / ₈ | 65.188 | 338.16 | 29. | 91.106 | 660.52 | 1 ⁴ / ₈ | 117.024 | 1089.8 |
| 7 ⁸ / ₈ | 65.581 | 342.25 | 1 ⁸ / ₈ | 91.499 | 666.23 | 3 ⁸ / ₈ | 117.417 | 1097.1 |
| 21. | 65.973 | 346.36 | 1 ⁴ / ₈ | 91.892 | 671.96 | 1 ² / ₈ | 117.810 | 1104.5 |
| 1 ⁸ / ₈ | 66.366 | 350.50 | 3 ⁸ / ₈ | 92.284 | 677.71 | 5 ⁸ / ₈ | 118.202 | 1111.8 |
| 1 ⁴ / ₈ | 66.759 | 354.66 | 1 ² / ₈ | 92.677 | 683.49 | 3 ⁴ / ₈ | 118.596 | 1119.2 |
| 3 ⁸ / ₈ | 67.152 | 358.84 | 5 ⁸ / ₈ | 93.070 | 689.30 | 7 ⁸ / ₈ | 118.988 | 1126.7 |
| 1 ² / ₈ | 67.544 | 363.05 | 3 ⁴ / ₈ | 93.462 | 695.13 | 38. | 119.381 | 1134.1 |
| 5 ⁸ / ₈ | 67.937 | 367.28 | 7 ⁸ / ₈ | 93.855 | 700.98 | 1 ⁸ / ₈ | 119.773 | 1141.6 |
| 3 ⁴ / ₈ | 68.330 | 371.54 | 30. | 94.248 | 706.86 | 1 ⁴ / ₈ | 120.166 | 1149.1 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|--------|-------|---------|--------|-------|---------|--------|
| 383/8 | 120.559 | 1156.6 | 465/8 | 146.477 | 1707.4 | 547/8 | 172.395 | 2365.0 |
| 1/2 | 120.951 | 1164.2 | 3/4 | 146.869 | 1716.5 | 55. | 172.788 | 2375.8 |
| 5/8 | 121.344 | 1171.7 | 7/8 | 147.262 | 1725.7 | 1/8 | 173.180 | 2386.6 |
| 3/4 | 121.737 | 1179.3 | 47. | 147.655 | 1734.9 | 1/4 | 173.573 | 2397.5 |
| 7/8 | 122.129 | 1186.9 | 1/8 | 148.048 | 1744.2 | 3/8 | 173.966 | 2408.3 |
| 39. | 122.522 | 1194.6 | 1/4 | 148.440 | 1753.5 | 1/2 | 174.358 | 2419.2 |
| 1/8 | 122.915 | 1202.3 | 3/8 | 148.833 | 1762.7 | 5/8 | 174.751 | 2430.1 |
| 1/4 | 123.308 | 1210.0 | 1/2 | 149.226 | 1772.1 | 3/4 | 175.144 | 2441.1 |
| 3/8 | 123.700 | 1217.7 | 5/8 | 149.618 | 1781.4 | 7/8 | 175.536 | 2452.0 |
| 1/2 | 124.093 | 1225.4 | 3/4 | 150.011 | 1790.8 | 56. | 175.929 | 2463.0 |
| 5/8 | 124.486 | 1233.2 | 7/8 | 150.404 | 1800.1 | 1/8 | 176.322 | 2474.0 |
| 3/4 | 124.878 | 1241.0 | 48. | 150.796 | 1809.6 | 1/4 | 176.715 | 2485.0 |
| 7/8 | 125.271 | 1248.8 | 1/8 | 151.189 | 1819.0 | 3/8 | 177.107 | 2496.1 |
| 40. | 125.664 | 1256.6 | 1/4 | 151.582 | 1828.5 | 1/2 | 177.500 | 2507.2 |
| 1/8 | 126.056 | 1264.5 | 3/8 | 151.975 | 1837.9 | 5/8 | 177.893 | 2518.3 |
| 1/4 | 126.449 | 1272.4 | 1/2 | 152.367 | 1847.5 | 3/4 | 178.285 | 2529.4 |
| 3/8 | 126.842 | 1280.3 | 5/8 | 152.760 | 1857.0 | 7/8 | 178.678 | 2540.6 |
| 1/2 | 127.235 | 1288.2 | 3/4 | 153.153 | 1866.5 | 57. | 179.071 | 2551.3 |
| 5/8 | 127.627 | 1296.2 | 7/8 | 153.545 | 1876.1 | 1/8 | 179.463 | 2563.0 |
| 3/4 | 128.020 | 1304.2 | 49. | 153.938 | 1885.7 | 1/4 | 179.856 | 2574.2 |
| 7/8 | 128.413 | 1312.2 | 1/8 | 154.331 | 1895.4 | 3/8 | 180.249 | 2585.4 |
| 41. | 128.805 | 1320.3 | 1/4 | 154.723 | 1905.0 | 1/2 | 180.642 | 2596.7 |
| 1/8 | 129.198 | 1328.3 | 3/8 | 155.116 | 1914.7 | 5/8 | 181.034 | 2608.0 |
| 1/4 | 129.591 | 1336.4 | 1/2 | 155.509 | 1924.4 | 3/4 | 181.427 | 2619.4 |
| 3/8 | 129.983 | 1344.5 | 5/8 | 155.902 | 1934.2 | 7/8 | 181.820 | 2630.7 |
| 1/2 | 130.376 | 1352.7 | 3/4 | 156.294 | 1943.9 | 58. | 182.212 | 2642.1 |
| 5/8 | 130.769 | 1360.8 | 7/8 | 156.687 | 1953.7 | 1/8 | 182.605 | 2653.5 |
| 3/4 | 131.161 | 1369.0 | 50. | 157.080 | 1963.5 | 1/4 | 182.998 | 2664.9 |
| 7/8 | 131.554 | 1377.2 | 1/8 | 157.472 | 1973.3 | 3/8 | 183.390 | 2676.4 |
| 42. | 131.947 | 1385.4 | 1/4 | 157.865 | 1983.2 | 1/2 | 183.783 | 2687.8 |
| 1/8 | 132.340 | 1393.7 | 3/8 | 158.258 | 1993.1 | 5/8 | 184.176 | 2699.3 |
| 1/4 | 132.732 | 1402.0 | 1/2 | 158.650 | 2003.0 | 3/4 | 184.569 | 2710.9 |
| 3/8 | 133.125 | 1410.3 | 5/8 | 159.043 | 2012.9 | 7/8 | 184.961 | 2722.4 |
| 1/2 | 133.518 | 1418.6 | 3/4 | 159.436 | 2022.8 | 59. | 185.354 | 2734.0 |
| 5/8 | 133.910 | 1427.0 | 7/8 | 159.829 | 2032.8 | 1/8 | 185.747 | 2745.6 |
| 3/4 | 134.303 | 1435.4 | 51. | 160.221 | 2042.8 | 1/4 | 186.139 | 2757.2 |
| 7/8 | 134.696 | 1443.8 | 1/8 | 160.614 | 2052.8 | 3/8 | 186.532 | 2768.8 |
| 43. | 135.088 | 1452.2 | 1/4 | 161.007 | 2062.9 | 1/2 | 186.925 | 2780.5 |
| 1/8 | 135.481 | 1460.7 | 3/8 | 161.399 | 2073.0 | 5/8 | 187.317 | 2792.2 |
| 1/4 | 135.874 | 1469.1 | 1/2 | 161.792 | 2083.1 | 3/4 | 187.710 | 2803.9 |
| 3/8 | 136.267 | 1477.6 | 5/8 | 162.185 | 2093.2 | 7/8 | 188.103 | 2815.7 |
| 1/2 | 136.659 | 1486.2 | 3/4 | 162.577 | 2103.3 | 60. | 188.496 | 2827.4 |
| 5/8 | 137.052 | 1494.7 | 7/8 | 162.970 | 2113.5 | 1/8 | 188.888 | 2839.2 |
| 3/4 | 137.445 | 1503.3 | 52. | 163.363 | 2123.7 | 1/4 | 189.281 | 2851.0 |
| 7/8 | 137.837 | 1511.9 | 1/8 | 163.756 | 2133.9 | 3/8 | 189.674 | 2862.9 |
| 44. | 138.230 | 1520.5 | 1/4 | 164.148 | 2144.2 | 1/2 | 190.066 | 2874.8 |
| 1/8 | 138.623 | 1529.2 | 3/8 | 164.541 | 2154.5 | 5/8 | 190.459 | 2886.6 |
| 1/4 | 139.015 | 1537.9 | 1/2 | 164.934 | 2164.8 | 3/4 | 190.852 | 2898.6 |
| 3/8 | 139.408 | 1546.6 | 5/8 | 165.326 | 2175.1 | 7/8 | 191.244 | 2910.5 |
| 1/2 | 139.801 | 1555.3 | 3/4 | 165.719 | 2185.4 | 61. | 191.637 | 2922.5 |
| 5/8 | 140.194 | 1564.0 | 7/8 | 166.112 | 2195.8 | 1/8 | 192.030 | 2934.5 |
| 3/4 | 140.586 | 1572.8 | 53. | 166.504 | 2206.2 | 1/4 | 192.423 | 2946.5 |
| 7/8 | 140.979 | 1581.6 | 1/8 | 166.897 | 2216.6 | 3/8 | 192.815 | 2958.5 |
| 45. | 141.372 | 1590.4 | 1/4 | 167.290 | 2227.0 | 1/2 | 193.208 | 2970.6 |
| 1/8 | 141.764 | 1599.3 | 3/8 | 167.683 | 2237.5 | 5/8 | 193.601 | 2982.7 |
| 1/4 | 142.157 | 1608.2 | 1/2 | 168.075 | 2248.0 | 3/4 | 193.993 | 2994.8 |
| 3/8 | 142.550 | 1617.0 | 5/8 | 168.468 | 2258.5 | 7/8 | 194.386 | 3006.9 |
| 1/2 | 142.942 | 1626.0 | 3/4 | 168.861 | 2269.1 | 62. | 194.779 | 3019.1 |
| 5/8 | 143.335 | 1634.9 | 7/8 | 169.253 | 2279.6 | 1/8 | 195.171 | 3031.3 |
| 3/4 | 143.728 | 1643.9 | 54. | 169.646 | 2290.2 | 1/4 | 195.564 | 3043.5 |
| 7/8 | 144.121 | 1652.9 | 1/8 | 170.039 | 2300.8 | 3/8 | 195.957 | 3055.7 |
| 46. | 144.513 | 1661.9 | 1/4 | 170.431 | 2311.5 | 1/2 | 196.350 | 3068.0 |
| 1/8 | 144.906 | 1670.9 | 3/8 | 170.824 | 2322.1 | 5/8 | 196.742 | 3080.3 |
| 1/4 | 145.299 | 1680.0 | 1/2 | 171.217 | 2332.8 | 3/4 | 197.135 | 3092.6 |
| 3/8 | 145.691 | 1689.1 | 5/8 | 171.609 | 2343.5 | 7/8 | 197.528 | 3104.9 |
| 1/2 | 146.084 | 1698.2 | 3/4 | 172.002 | 2354.3 | 63. | 197.920 | 3117.2 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|------------------|---------|--------|------------------|---------|--------|------------------|---------|--------|
| 63 $\frac{1}{8}$ | 198.313 | 3129.6 | 71 $\frac{3}{8}$ | 224.231 | 4001.1 | 79 $\frac{5}{8}$ | 250.149 | 4979.5 |
| $\frac{1}{4}$ | 198.706 | 3142.0 | $\frac{1}{2}$ | 224.624 | 4015.2 | $\frac{3}{4}$ | 250.542 | 4995.2 |
| $\frac{3}{8}$ | 199.098 | 3154.5 | $\frac{5}{8}$ | 225.017 | 4029.2 | $\frac{7}{8}$ | 250.935 | 5010.9 |
| $\frac{1}{2}$ | 199.491 | 3166.9 | $\frac{3}{4}$ | 225.409 | 4043.3 | 80. | 251.327 | 5026.5 |
| $\frac{5}{8}$ | 199.884 | 3179.4 | $\frac{7}{8}$ | 225.802 | 4057.4 | $\frac{1}{8}$ | 251.720 | 5042.3 |
| $\frac{3}{4}$ | 200.277 | 3191.9 | 72. | 226.195 | 4071.5 | $\frac{1}{4}$ | 252.113 | 5058.0 |
| $\frac{7}{8}$ | 200.669 | 3204.4 | $\frac{1}{8}$ | 226.587 | 4085.7 | $\frac{3}{8}$ | 252.506 | 5073.8 |
| 64. | 201.062 | 3217.0 | $\frac{1}{4}$ | 226.980 | 4099.8 | $\frac{1}{2}$ | 252.898 | 5089.6 |
| $\frac{1}{8}$ | 201.455 | 3229.6 | $\frac{3}{8}$ | 227.373 | 4114.0 | $\frac{5}{8}$ | 253.291 | 5105.4 |
| $\frac{1}{4}$ | 201.847 | 3242.2 | $\frac{1}{2}$ | 227.765 | 4128.2 | $\frac{3}{4}$ | 253.684 | 5121.2 |
| $\frac{3}{8}$ | 202.240 | 3254.8 | $\frac{5}{8}$ | 228.158 | 4142.5 | $\frac{7}{8}$ | 254.076 | 5137.1 |
| $\frac{1}{2}$ | 202.633 | 3267.5 | $\frac{3}{4}$ | 228.551 | 4156.8 | 81. | 254.469 | 5153.0 |
| $\frac{5}{8}$ | 203.025 | 3280.1 | $\frac{7}{8}$ | 228.944 | 4171.1 | $\frac{1}{8}$ | 254.862 | 5168.9 |
| $\frac{3}{4}$ | 203.418 | 3292.8 | 73. | 229.336 | 4185.4 | $\frac{1}{4}$ | 255.254 | 5184.9 |
| $\frac{7}{8}$ | 203.811 | 3305.6 | $\frac{1}{8}$ | 229.729 | 4199.7 | $\frac{3}{8}$ | 255.647 | 5200.8 |
| 65. | 204.204 | 3318.3 | $\frac{1}{4}$ | 230.122 | 4214.1 | $\frac{1}{2}$ | 256.040 | 5216.8 |
| $\frac{1}{8}$ | 204.596 | 3331.1 | $\frac{3}{8}$ | 230.514 | 4228.5 | $\frac{5}{8}$ | 256.433 | 5232.8 |
| $\frac{1}{4}$ | 204.989 | 3343.9 | $\frac{1}{2}$ | 230.907 | 4242.9 | $\frac{3}{4}$ | 256.825 | 5248.9 |
| $\frac{3}{8}$ | 205.382 | 3356.7 | $\frac{5}{8}$ | 231.300 | 4257.4 | $\frac{7}{8}$ | 257.218 | 5264.9 |
| $\frac{1}{2}$ | 205.774 | 3369.6 | $\frac{3}{4}$ | 231.692 | 4271.8 | 82. | 257.611 | 5281.0 |
| $\frac{5}{8}$ | 206.167 | 3382.4 | $\frac{7}{8}$ | 232.085 | 4286.3 | $\frac{1}{8}$ | 258.003 | 5297.1 |
| $\frac{3}{4}$ | 206.560 | 3395.3 | 74. | 232.478 | 4300.8 | $\frac{1}{4}$ | 258.396 | 5313.3 |
| $\frac{7}{8}$ | 206.952 | 3408.2 | $\frac{1}{8}$ | 232.871 | 4315.4 | $\frac{3}{8}$ | 258.789 | 5329.4 |
| 66. | 207.345 | 3421.2 | $\frac{1}{4}$ | 233.263 | 4329.9 | $\frac{1}{2}$ | 259.181 | 5345.6 |
| $\frac{1}{8}$ | 207.738 | 3434.2 | $\frac{3}{8}$ | 233.656 | 4344.5 | $\frac{5}{8}$ | 259.574 | 5361.8 |
| $\frac{1}{4}$ | 208.131 | 3447.2 | $\frac{1}{2}$ | 234.049 | 4359.2 | $\frac{3}{4}$ | 259.967 | 5378.1 |
| $\frac{3}{8}$ | 208.523 | 3460.2 | $\frac{5}{8}$ | 234.441 | 4373.8 | $\frac{7}{8}$ | 260.359 | 5394.3 |
| $\frac{1}{2}$ | 208.916 | 3473.2 | $\frac{3}{4}$ | 234.834 | 4388.5 | 83. | 260.752 | 5410.6 |
| $\frac{5}{8}$ | 209.309 | 3486.3 | $\frac{7}{8}$ | 235.227 | 4403.1 | $\frac{1}{8}$ | 261.145 | 5426.9 |
| $\frac{3}{4}$ | 209.701 | 3499.4 | 75. | 235.619 | 4417.9 | $\frac{1}{4}$ | 261.538 | 5443.3 |
| $\frac{7}{8}$ | 210.094 | 3512.5 | $\frac{1}{8}$ | 236.012 | 4432.6 | $\frac{3}{8}$ | 261.930 | 5459.6 |
| 67. | 210.487 | 3525.7 | $\frac{1}{4}$ | 236.405 | 4447.4 | $\frac{1}{2}$ | 262.323 | 5476.0 |
| $\frac{1}{8}$ | 210.879 | 3538.8 | $\frac{3}{8}$ | 236.798 | 4462.2 | $\frac{5}{8}$ | 262.716 | 5492.4 |
| $\frac{1}{4}$ | 211.272 | 3552.0 | $\frac{1}{2}$ | 237.190 | 4477.0 | $\frac{3}{4}$ | 263.108 | 5508.8 |
| $\frac{3}{8}$ | 211.665 | 3565.2 | $\frac{5}{8}$ | 237.583 | 4491.8 | $\frac{7}{8}$ | 263.501 | 5525.3 |
| $\frac{1}{2}$ | 212.058 | 3578.5 | $\frac{3}{4}$ | 237.976 | 4506.7 | 84. | 263.894 | 5541.8 |
| $\frac{5}{8}$ | 212.450 | 3591.7 | $\frac{7}{8}$ | 238.368 | 4521.5 | $\frac{1}{8}$ | 264.286 | 5558.3 |
| $\frac{3}{4}$ | 212.843 | 3605.0 | 76. | 238.761 | 4536.5 | $\frac{1}{4}$ | 264.679 | 5574.8 |
| $\frac{7}{8}$ | 213.236 | 3618.3 | $\frac{1}{8}$ | 239.154 | 4551.4 | $\frac{3}{8}$ | 265.072 | 5591.4 |
| 68. | 213.628 | 3631.7 | $\frac{1}{4}$ | 239.546 | 4566.4 | $\frac{1}{2}$ | 265.465 | 5607.9 |
| $\frac{1}{8}$ | 214.021 | 3645.0 | $\frac{3}{8}$ | 239.939 | 4581.3 | $\frac{5}{8}$ | 265.857 | 5624.5 |
| $\frac{1}{4}$ | 214.414 | 3658.4 | $\frac{1}{2}$ | 240.332 | 4596.3 | $\frac{3}{4}$ | 266.250 | 5641.2 |
| $\frac{3}{8}$ | 214.806 | 3671.8 | $\frac{5}{8}$ | 240.725 | 4611.4 | $\frac{7}{8}$ | 266.643 | 5657.8 |
| $\frac{1}{2}$ | 215.199 | 3685.3 | $\frac{3}{4}$ | 241.117 | 4626.4 | 85. | 267.035 | 5674.5 |
| $\frac{5}{8}$ | 215.592 | 3698.7 | $\frac{7}{8}$ | 241.510 | 4641.5 | $\frac{1}{8}$ | 267.428 | 5691.2 |
| $\frac{3}{4}$ | 215.984 | 3712.2 | 77. | 241.903 | 4656.6 | $\frac{1}{4}$ | 267.821 | 5707.9 |
| $\frac{7}{8}$ | 216.377 | 3725.7 | $\frac{1}{8}$ | 242.295 | 4671.8 | $\frac{3}{8}$ | 268.213 | 5724.7 |
| 69. | 216.770 | 3739.3 | $\frac{1}{4}$ | 242.688 | 4686.9 | $\frac{1}{2}$ | 268.606 | 5741.5 |
| $\frac{1}{8}$ | 217.163 | 3752.8 | $\frac{3}{8}$ | 243.081 | 4702.1 | $\frac{5}{8}$ | 268.999 | 5758.3 |
| $\frac{1}{4}$ | 217.555 | 3766.4 | $\frac{1}{2}$ | 243.473 | 4717.3 | $\frac{3}{4}$ | 269.392 | 5775.1 |
| $\frac{3}{8}$ | 217.948 | 3780.0 | $\frac{5}{8}$ | 243.866 | 4732.5 | $\frac{7}{8}$ | 269.784 | 5791.9 |
| $\frac{1}{2}$ | 218.341 | 3793.7 | $\frac{3}{4}$ | 244.259 | 4747.8 | 86. | 270.177 | 5808.8 |
| $\frac{5}{8}$ | 218.733 | 3807.3 | $\frac{7}{8}$ | 244.652 | 4763.1 | $\frac{1}{8}$ | 270.570 | 5825.7 |
| $\frac{3}{4}$ | 219.126 | 3821.0 | 78. | 245.044 | 4778.4 | $\frac{1}{4}$ | 270.962 | 5842.6 |
| $\frac{7}{8}$ | 219.519 | 3834.7 | $\frac{1}{8}$ | 245.437 | 4793.7 | $\frac{3}{8}$ | 271.355 | 5859.6 |
| 70. | 219.911 | 3848.5 | $\frac{1}{4}$ | 245.830 | 4809.0 | $\frac{1}{2}$ | 271.748 | 5876.5 |
| $\frac{1}{8}$ | 220.304 | 3862.2 | $\frac{3}{8}$ | 246.222 | 4824.4 | $\frac{5}{8}$ | 272.140 | 5893.5 |
| $\frac{1}{4}$ | 220.697 | 3876.0 | $\frac{1}{2}$ | 246.615 | 4839.8 | $\frac{3}{4}$ | 272.533 | 5910.6 |
| $\frac{3}{8}$ | 221.090 | 3889.8 | $\frac{5}{8}$ | 247.008 | 4855.2 | $\frac{7}{8}$ | 272.926 | 5927.6 |
| $\frac{1}{2}$ | 221.482 | 3903.6 | $\frac{3}{4}$ | 247.400 | 4870.7 | 87. | 273.319 | 5944.7 |
| $\frac{5}{8}$ | 221.875 | 3917.5 | $\frac{7}{8}$ | 247.793 | 4886.2 | $\frac{1}{8}$ | 273.711 | 5961.8 |
| $\frac{3}{4}$ | 222.268 | 3931.4 | 79. | 248.186 | 4901.7 | $\frac{1}{4}$ | 274.104 | 5978.9 |
| $\frac{7}{8}$ | 222.660 | 3945.3 | $\frac{1}{8}$ | 248.579 | 4917.2 | $\frac{3}{8}$ | 274.497 | 5996.0 |
| 71. | 223.053 | 3959.2 | $\frac{1}{4}$ | 248.971 | 4932.7 | $\frac{1}{2}$ | 274.889 | 6013.2 |
| $\frac{1}{8}$ | 223.446 | 3973.1 | $\frac{3}{8}$ | 249.364 | 4948.3 | $\frac{5}{8}$ | 275.282 | 6030.4 |
| $\frac{1}{4}$ | 223.838 | 3987.1 | $\frac{1}{2}$ | 249.757 | 4963.9 | $\frac{3}{4}$ | 275.675 | 6047.6 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|--------|---------|--------|--------|---------|----------|-------|---------|----------|
| 87 7/8 | 276.067 | 6064.9 | 95 7/8 | 301.200 | 7219.4 | 130 | 408.41 | 13273.23 |
| 88. | 276.460 | 6082.1 | 96. | 301.593 | 7238.2 | 131 | 411.55 | 13478.22 |
| 1/8 | 276.853 | 6099.4 | 1/8 | 301.986 | 7257.1 | 132 | 414.69 | 13684.78 |
| 1/4 | 277.246 | 6116.7 | 1/4 | 302.378 | 7276.0 | 133 | 417.83 | 13892.91 |
| 3/8 | 277.638 | 6134.1 | 3/8 | 302.771 | 7294.9 | 134 | 420.97 | 14102.61 |
| 1/2 | 278.031 | 6151.4 | 1/2 | 303.164 | 7313.8 | 135 | 424.12 | 14313.88 |
| 5/8 | 278.424 | 6168.8 | 5/8 | 303.556 | 7332.8 | 136 | 427.26 | 14526.72 |
| 3/4 | 278.816 | 6186.2 | 3/4 | 303.949 | 7351.8 | 137 | 430.40 | 14741.14 |
| 7/8 | 279.209 | 6203.7 | 7/8 | 304.342 | 7370.8 | 138 | 433.54 | 14957.12 |
| 89. | 279.602 | 6221.1 | 97. | 304.734 | 7389.8 | 139 | 436.68 | 15174.68 |
| 1/8 | 279.994 | 6238.6 | 1/8 | 305.127 | 7408.9 | 140 | 439.82 | 15393.80 |
| 1/4 | 280.387 | 6256.1 | 1/4 | 305.520 | 7428.0 | 141 | 442.96 | 15614.50 |
| 3/8 | 280.780 | 6273.7 | 3/8 | 305.913 | 7447.1 | 142 | 446.11 | 15836.77 |
| 1/2 | 281.173 | 6291.2 | 1/2 | 306.305 | 7466.2 | 143 | 449.25 | 16060.61 |
| 5/8 | 281.565 | 6308.8 | 5/8 | 306.698 | 7485.3 | 144 | 452.39 | 16286.02 |
| 3/4 | 281.958 | 6326.4 | 3/4 | 307.091 | 7504.5 | 145 | 455.53 | 16513.00 |
| 7/8 | 282.351 | 6344.1 | 7/8 | 307.483 | 7523.7 | 146 | 458.67 | 16741.55 |
| 90. | 282.743 | 6361.7 | 98. | 307.876 | 7543.0 | 147 | 461.81 | 16971.67 |
| 1/8 | 283.136 | 6379.4 | 1/8 | 308.269 | 7562.2 | 148 | 464.96 | 17203.36 |
| 1/4 | 283.529 | 6397.1 | 1/4 | 308.661 | 7581.5 | 149 | 468.10 | 17436.62 |
| 3/8 | 283.921 | 6414.9 | 3/8 | 309.054 | 7600.8 | 150 | 471.24 | 17671.46 |
| 1/2 | 284.314 | 6432.6 | 1/2 | 309.447 | 7620.1 | 151 | 474.38 | 17907.86 |
| 5/8 | 284.707 | 6450.4 | 5/8 | 309.840 | 7639.5 | 152 | 477.52 | 18145.84 |
| 3/4 | 285.100 | 6468.2 | 3/4 | 310.232 | 7658.9 | 153 | 480.66 | 18385.39 |
| 7/8 | 285.492 | 6486.0 | 7/8 | 310.625 | 7678.3 | 154 | 483.81 | 18626.50 |
| 91. | 285.885 | 6503.9 | 99. | 311.018 | 7697.7 | 155 | 486.95 | 18869.19 |
| 1/8 | 286.278 | 6521.8 | 1/8 | 311.410 | 7717.1 | 156 | 490.09 | 19113.45 |
| 1/4 | 286.670 | 6539.7 | 1/4 | 311.803 | 7736.6 | 157 | 493.23 | 19359.28 |
| 3/8 | 287.063 | 6557.6 | 3/8 | 312.196 | 7756.1 | 158 | 496.37 | 19606.68 |
| 1/2 | 287.456 | 6575.5 | 1/2 | 312.588 | 7775.6 | 159 | 499.51 | 19855.65 |
| 5/8 | 287.848 | 6593.5 | 5/8 | 312.981 | 7795.2 | 160 | 502.65 | 20106.19 |
| 3/4 | 288.241 | 6611.5 | 3/4 | 313.374 | 7814.8 | 161 | 505.80 | 20358.31 |
| 7/8 | 288.634 | 6629.6 | 7/8 | 313.767 | 7834.4 | 162 | 508.94 | 20611.99 |
| 92. | 289.027 | 6647.6 | 100 | 314.159 | 7854.0 | 163 | 512.08 | 20867.24 |
| 1/8 | 289.419 | 6665.7 | 101 | 317.30 | 8011.85 | 164 | 515.22 | 21124.07 |
| 1/4 | 289.812 | 6683.8 | 102 | 320.44 | 8171.28 | 165 | 518.36 | 21382.46 |
| 3/8 | 290.205 | 6701.9 | 103 | 323.58 | 8332.29 | 166 | 521.50 | 21642.43 |
| 1/2 | 290.597 | 6720.1 | 104 | 326.73 | 8494.87 | 167 | 524.65 | 21903.97 |
| 5/8 | 290.990 | 6738.2 | 105 | 329.87 | 8659.01 | 168 | 527.79 | 22167.08 |
| 3/4 | 291.383 | 6756.4 | 106 | 333.01 | 8824.73 | 169 | 530.93 | 22431.76 |
| 7/8 | 291.775 | 6774.7 | 107 | 336.15 | 8992.02 | 170 | 534.07 | 22698.01 |
| 93. | 292.168 | 6792.9 | 108 | 339.29 | 9160.88 | 171 | 537.21 | 22965.83 |
| 1/8 | 292.561 | 6811.2 | 109 | 342.43 | 9331.32 | 172 | 540.35 | 23235.22 |
| 1/4 | 292.954 | 6829.5 | 110 | 345.58 | 9503.32 | 173 | 543.50 | 23506.18 |
| 3/8 | 293.346 | 6847.8 | 111 | 348.72 | 9676.89 | 174 | 546.64 | 23778.71 |
| 1/2 | 293.739 | 6866.1 | 112 | 351.86 | 9852.03 | 175 | 549.78 | 24052.82 |
| 5/8 | 294.132 | 6884.5 | 113 | 355.00 | 10028.75 | 176 | 552.92 | 24328.49 |
| 3/4 | 294.524 | 6902.9 | 114 | 358.14 | 10207.03 | 177 | 556.06 | 24605.74 |
| 7/8 | 294.917 | 6921.3 | 115 | 361.28 | 10386.89 | 178 | 559.20 | 24884.56 |
| 94. | 295.310 | 6939.8 | 116 | 364.42 | 10568.32 | 179 | 562.35 | 25164.94 |
| 1/8 | 295.702 | 6958.2 | 117 | 367.57 | 10751.32 | 180 | 565.49 | 25446.90 |
| 1/4 | 296.095 | 6976.7 | 118 | 370.71 | 10935.88 | 181 | 568.63 | 25730.43 |
| 3/8 | 296.488 | 6995.3 | 119 | 373.85 | 11122.02 | 182 | 571.77 | 26015.53 |
| 1/2 | 296.881 | 7013.8 | 120 | 376.99 | 11309.73 | 183 | 574.91 | 26302.20 |
| 5/8 | 297.273 | 7032.4 | 121 | 380.13 | 11499.01 | 184 | 578.05 | 26590.44 |
| 3/4 | 297.666 | 7051.0 | 122 | 383.27 | 11689.87 | 185 | 581.19 | 26880.25 |
| 7/8 | 298.059 | 7069.6 | 123 | 386.42 | 11882.29 | 186 | 584.34 | 27171.63 |
| 95. | 298.451 | 7088.2 | 124 | 389.56 | 12076.28 | 187 | 587.48 | 27464.59 |
| 1/8 | 298.844 | 7106.9 | 125 | 392.70 | 12271.85 | 188 | 590.62 | 27759.11 |
| 1/4 | 299.237 | 7125.6 | 126 | 395.84 | 12468.98 | 189 | 593.76 | 28055.21 |
| 3/8 | 299.629 | 7144.3 | 127 | 398.98 | 12667.69 | 190 | 596.90 | 28352.87 |
| 1/2 | 300.022 | 7163.0 | 128 | 402.12 | 12867.96 | 191 | 600.04 | 28652.11 |
| 5/8 | 300.415 | 7181.8 | 129 | 405.27 | 13069.81 | 192 | 603.19 | 28952.92 |
| 3/4 | 300.807 | 7200.6 | | | | | | |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|----------|-------|---------|----------|-------|---------|-----------|
| 193 | 606.33 | 29255.30 | 260 | 816.81 | 53092.92 | 327 | 1027.30 | 83981.84 |
| 194 | 609.47 | 29559.25 | 261 | 819.96 | 53502.11 | 328 | 1030.44 | 84496.28 |
| 195 | 612.61 | 29864.77 | 262 | 823.10 | 53912.87 | 329 | 1033.58 | 85012.28 |
| 196 | 615.75 | 30171.86 | 263 | 826.24 | 54325.21 | 330 | 1036.73 | 85529.86 |
| 197 | 618.89 | 30480.52 | 264 | 829.38 | 54739.11 | 331 | 1039.87 | 86049.01 |
| 198 | 622.04 | 30790.75 | 265 | 832.52 | 55154.59 | 332 | 1043.01 | 86569.73 |
| 199 | 625.18 | 31102.55 | 266 | 835.66 | 55571.63 | 333 | 1046.15 | 87092.02 |
| 200 | 628.32 | 31415.93 | 267 | 838.81 | 55990.25 | 334 | 1049.29 | 87615.88 |
| 201 | 631.46 | 31730.87 | 268 | 841.95 | 56410.44 | 335 | 1052.43 | 88141.31 |
| 202 | 634.60 | 32047.39 | 269 | 845.09 | 56832.20 | 336 | 1055.58 | 88668.31 |
| 203 | 637.74 | 32365.47 | 270 | 848.23 | 57255.53 | 337 | 1058.72 | 89196.88 |
| 204 | 640.88 | 32685.13 | 271 | 851.37 | 57680.43 | 338 | 1061.86 | 89727.03 |
| 205 | 644.03 | 33006.36 | 272 | 854.51 | 58106.90 | 339 | 1065.00 | 90258.74 |
| 206 | 647.17 | 33329.16 | 273 | 857.65 | 58534.94 | 340 | 1068.14 | 90792.03 |
| 207 | 650.31 | 33653.53 | 274 | 860.80 | 58964.55 | 341 | 1071.28 | 91326.88 |
| 208 | 653.45 | 33979.47 | 275 | 863.94 | 59395.74 | 342 | 1074.42 | 91863.31 |
| 209 | 656.59 | 34306.98 | 276 | 867.08 | 59828.49 | 343 | 1077.57 | 92401.31 |
| 210 | 659.73 | 34636.06 | 277 | 870.22 | 60262.82 | 344 | 1080.71 | 92940.88 |
| 211 | 662.88 | 34966.71 | 278 | 873.36 | 60698.71 | 345 | 1083.85 | 93482.02 |
| 212 | 666.02 | 35298.94 | 279 | 876.50 | 61136.18 | 346 | 1086.99 | 94024.73 |
| 213 | 669.16 | 35632.73 | 280 | 879.65 | 61575.22 | 347 | 1090.13 | 94569.01 |
| 214 | 672.30 | 35968.09 | 281 | 882.79 | 62015.82 | 348 | 1093.27 | 95114.86 |
| 215 | 675.44 | 36305.03 | 282 | 885.93 | 62458.00 | 349 | 1096.42 | 95662.28 |
| 216 | 678.58 | 36643.54 | 283 | 889.07 | 62901.75 | 350 | 1099.56 | 96211.28 |
| 217 | 681.73 | 36983.61 | 284 | 892.21 | 63347.07 | 351 | 1102.70 | 96761.84 |
| 218 | 684.87 | 37325.26 | 285 | 895.35 | 63793.97 | 352 | 1105.84 | 97313.97 |
| 219 | 688.01 | 37668.48 | 286 | 898.50 | 64242.43 | 353 | 1108.98 | 97867.68 |
| 220 | 691.15 | 38013.27 | 287 | 901.64 | 64692.46 | 354 | 1112.12 | 98422.96 |
| 221 | 694.29 | 38359.63 | 288 | 904.78 | 65144.07 | 355 | 1115.27 | 98979.80 |
| 222 | 697.43 | 38707.56 | 289 | 907.92 | 65597.24 | 356 | 1118.41 | 99538.22 |
| 223 | 700.58 | 39057.07 | 290 | 911.06 | 66051.99 | 357 | 1121.55 | 100098.21 |
| 224 | 703.72 | 39408.14 | 291 | 914.20 | 66508.30 | 358 | 1124.69 | 100659.77 |
| 225 | 706.86 | 39760.78 | 292 | 917.35 | 66966.19 | 359 | 1127.83 | 101222.91 |
| 226 | 710.00 | 40115.00 | 293 | 920.49 | 67425.65 | 360 | 1130.97 | 101787.60 |
| 227 | 713.14 | 40470.78 | 294 | 923.63 | 67886.68 | 361 | 1134.11 | 102353.87 |
| 228 | 716.28 | 40828.14 | 295 | 926.77 | 68349.28 | 362 | 1137.26 | 102921.72 |
| 229 | 719.42 | 41187.07 | 296 | 929.91 | 68813.45 | 363 | 1140.40 | 103491.13 |
| 230 | 722.57 | 41547.56 | 297 | 933.05 | 69279.19 | 364 | 1143.54 | 104062.12 |
| 231 | 725.71 | 41909.63 | 298 | 936.19 | 69746.50 | 365 | 1146.68 | 104634.67 |
| 232 | 728.85 | 42273.27 | 299 | 939.34 | 70215.38 | 366 | 1149.82 | 105208.80 |
| 233 | 731.99 | 42638.48 | 300 | 942.48 | 70685.83 | 367 | 1152.96 | 105784.49 |
| 234 | 735.13 | 43005.26 | 301 | 945.62 | 71157.86 | 368 | 1156.11 | 106361.76 |
| 235 | 738.27 | 43373.61 | 302 | 948.76 | 71631.45 | 369 | 1159.25 | 106940.60 |
| 236 | 741.42 | 43743.54 | 303 | 951.90 | 72106.62 | 370 | 1162.39 | 107521.01 |
| 237 | 744.56 | 44115.03 | 304 | 955.04 | 72583.36 | 371 | 1165.53 | 108102.97 |
| 238 | 747.70 | 44488.09 | 305 | 958.19 | 73061.66 | 372 | 1168.67 | 108686.51 |
| 239 | 750.84 | 44862.73 | 306 | 961.33 | 73541.54 | 373 | 1171.81 | 109271.66 |
| 240 | 753.98 | 45238.93 | 307 | 964.47 | 74022.99 | 374 | 1174.96 | 109858.35 |
| 241 | 757.12 | 45616.71 | 308 | 967.61 | 74506.01 | 375 | 1178.10 | 110446.62 |
| 242 | 760.27 | 45996.06 | 309 | 970.75 | 74990.60 | 376 | 1181.24 | 111036.45 |
| 243 | 763.41 | 46376.98 | 310 | 973.89 | 75476.76 | 377 | 1184.38 | 111627.86 |
| 244 | 766.55 | 46759.47 | 311 | 977.04 | 75964.50 | 378 | 1187.52 | 112220.83 |
| 245 | 769.69 | 47143.52 | 312 | 980.18 | 76453.80 | 379 | 1190.66 | 112815.38 |
| 246 | 772.83 | 47529.16 | 313 | 983.32 | 76944.67 | 380 | 1193.81 | 113411.49 |
| 247 | 775.97 | 47916.36 | 314 | 986.46 | 77437.12 | 381 | 1196.95 | 114009.13 |
| 248 | 779.11 | 48305.13 | 315 | 989.60 | 77931.13 | 382 | 1200.09 | 114608.44 |
| 249 | 782.26 | 48695.47 | 316 | 992.74 | 78426.72 | 383 | 1203.23 | 115209.27 |
| 250 | 785.40 | 49087.39 | 317 | 995.88 | 78923.88 | 384 | 1206.37 | 115811.67 |
| 251 | 788.54 | 49480.87 | 318 | 999.03 | 79422.60 | 385 | 1209.51 | 116415.64 |
| 252 | 791.68 | 49875.92 | 319 | 1002.17 | 79922.90 | 386 | 1212.65 | 117021.18 |
| 253 | 794.82 | 50272.55 | 320 | 1005.31 | 80424.77 | 387 | 1215.80 | 117628.30 |
| 254 | 797.96 | 50670.75 | 321 | 1008.45 | 80928.21 | 388 | 1218.94 | 118236.98 |
| 255 | 801.11 | 51070.52 | 322 | 1011.59 | 81433.22 | 389 | 1222.08 | 118847.24 |
| 256 | 804.25 | 51471.85 | 323 | 1014.73 | 81939.80 | 390 | 1225.22 | 119459.06 |
| 257 | 807.39 | 51874.76 | 324 | 1017.87 | 82447.96 | 391 | 1228.36 | 120072.46 |
| 258 | 810.53 | 52279.24 | 325 | 1021.02 | 82957.68 | 392 | 1231.50 | 120687.46 |
| 259 | 813.67 | 52685.29 | 326 | 1024.16 | 83468.98 | 393 | 1234.65 | 121303.96 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|-----------|-------|---------|-----------|-------|---------|-----------|
| 394 | 1237.79 | 121922.07 | 461 | 1448.27 | 166913.60 | 528 | 1658.76 | 218956.44 |
| 395 | 1240.93 | 122541.75 | 462 | 1451.42 | 167638.53 | 529 | 1661.90 | 219786.61 |
| 396 | 1244.07 | 123163.00 | 463 | 1454.56 | 168365.02 | 530 | 1665.04 | 220618.34 |
| 397 | 1247.21 | 123785.82 | 464 | 1457.70 | 169093.08 | 531 | 1668.19 | 221451.65 |
| 398 | 1250.35 | 124410.21 | 465 | 1460.84 | 169822.72 | 532 | 1671.33 | 222286.53 |
| 399 | 1253.50 | 125036.17 | 466 | 1463.98 | 170553.92 | 533 | 1674.47 | 223122.98 |
| 400 | 1256.64 | 125663.71 | 467 | 1467.12 | 171286.70 | 534 | 1677.61 | 223961.00 |
| 401 | 1259.78 | 126292.81 | 468 | 1470.27 | 172021.05 | 535 | 1680.75 | 224800.59 |
| 402 | 1262.92 | 126923.48 | 469 | 1473.41 | 172756.97 | 536 | 1683.89 | 225641.75 |
| 403 | 1266.06 | 127555.73 | 470 | 1476.55 | 173494.45 | 537 | 1687.04 | 226484.48 |
| 404 | 1269.20 | 128189.55 | 471 | 1479.69 | 174233.51 | 538 | 1690.18 | 227328.79 |
| 405 | 1272.35 | 128824.93 | 472 | 1482.83 | 174974.14 | 539 | 1693.32 | 228174.66 |
| 406 | 1275.49 | 129461.89 | 473 | 1485.97 | 175716.35 | 540 | 1696.46 | 229022.10 |
| 407 | 1278.63 | 130100.42 | 474 | 1489.11 | 176460.12 | 541 | 1699.60 | 229871.12 |
| 408 | 1281.77 | 130740.52 | 475 | 1492.26 | 177205.46 | 542 | 1702.74 | 230721.71 |
| 409 | 1284.91 | 131382.19 | 476 | 1495.40 | 177952.37 | 543 | 1705.88 | 231573.86 |
| 410 | 1288.05 | 132025.43 | 477 | 1498.54 | 178700.86 | 544 | 1709.03 | 232427.59 |
| 411 | 1291.19 | 132670.24 | 478 | 1501.68 | 179450.91 | 545 | 1712.17 | 233282.89 |
| 412 | 1294.34 | 133316.63 | 479 | 1504.82 | 180202.54 | 546 | 1715.31 | 234139.76 |
| 413 | 1297.48 | 133964.58 | 480 | 1507.96 | 180955.74 | 547 | 1718.45 | 234998.20 |
| 414 | 1300.62 | 134614.10 | 481 | 1511.11 | 181710.50 | 548 | 1721.59 | 235858.21 |
| 415 | 1303.76 | 135265.20 | 482 | 1514.25 | 182466.84 | 549 | 1724.73 | 236719.79 |
| 416 | 1306.90 | 135917.86 | 483 | 1517.39 | 183224.75 | 550 | 1727.88 | 237582.94 |
| 417 | 1310.04 | 136572.10 | 484 | 1520.53 | 183984.23 | 551 | 1731.02 | 238447.67 |
| 418 | 1313.19 | 137227.91 | 485 | 1523.67 | 184745.28 | 552 | 1734.16 | 239313.96 |
| 419 | 1316.33 | 137885.29 | 486 | 1526.81 | 185507.90 | 553 | 1737.30 | 240181.83 |
| 420 | 1319.47 | 138544.24 | 487 | 1529.96 | 186272.10 | 554 | 1740.44 | 241051.26 |
| 421 | 1322.61 | 139204.76 | 488 | 1533.10 | 187037.86 | 555 | 1743.58 | 241922.27 |
| 422 | 1325.75 | 139866.85 | 489 | 1536.24 | 187805.19 | 556 | 1746.73 | 242794.85 |
| 423 | 1328.89 | 140530.51 | 490 | 1539.38 | 188574.10 | 557 | 1749.87 | 243668.99 |
| 424 | 1332.04 | 141195.74 | 491 | 1542.52 | 189344.57 | 558 | 1753.01 | 244544.71 |
| 425 | 1335.18 | 141862.54 | 492 | 1545.66 | 190116.62 | 559 | 1756.15 | 245422.00 |
| 426 | 1338.32 | 142530.92 | 493 | 1548.81 | 190890.24 | 560 | 1759.29 | 246300.86 |
| 427 | 1341.46 | 143200.86 | 494 | 1551.95 | 191665.43 | 561 | 1762.43 | 247181.30 |
| 428 | 1344.60 | 143872.38 | 495 | 1555.09 | 192442.18 | 562 | 1765.58 | 248063.30 |
| 429 | 1347.74 | 144545.46 | 496 | 1558.23 | 193220.51 | 563 | 1768.72 | 248946.87 |
| 430 | 1350.88 | 145220.12 | 497 | 1561.37 | 194000.41 | 564 | 1771.86 | 249832.01 |
| 431 | 1354.03 | 145896.35 | 498 | 1564.51 | 194781.89 | 565 | 1775.00 | 250718.73 |
| 432 | 1357.17 | 146574.15 | 499 | 1567.65 | 195564.93 | 566 | 1778.14 | 251607.01 |
| 433 | 1360.31 | 147253.52 | 500 | 1570.80 | 196349.54 | 567 | 1781.28 | 252496.87 |
| 434 | 1363.45 | 147934.46 | 501 | 1573.94 | 197135.72 | 568 | 1784.42 | 253388.30 |
| 435 | 1366.59 | 148616.97 | 502 | 1577.08 | 197923.48 | 569 | 1787.57 | 254281.29 |
| 436 | 1369.73 | 149301.05 | 503 | 1580.22 | 198712.80 | 570 | 1790.71 | 255175.86 |
| 437 | 1372.88 | 149986.70 | 504 | 1583.36 | 199503.70 | 571 | 1793.85 | 256072.00 |
| 438 | 1376.02 | 150673.93 | 505 | 1586.50 | 200296.17 | 572 | 1796.99 | 256969.71 |
| 439 | 1379.16 | 151362.72 | 506 | 1589.65 | 201090.20 | 573 | 1800.13 | 257868.99 |
| 440 | 1382.30 | 152053.08 | 507 | 1592.79 | 201885.81 | 574 | 1803.27 | 258769.85 |
| 441 | 1385.44 | 152745.02 | 508 | 1595.93 | 202682.99 | 575 | 1806.42 | 259672.27 |
| 442 | 1388.58 | 153438.53 | 509 | 1599.07 | 203481.74 | 576 | 1809.56 | 260576.26 |
| 443 | 1391.73 | 154133.60 | 510 | 1602.21 | 204282.06 | 577 | 1812.70 | 261481.83 |
| 444 | 1394.87 | 154830.25 | 511 | 1605.35 | 205083.95 | 578 | 1815.84 | 262388.96 |
| 445 | 1398.01 | 155528.47 | 512 | 1608.50 | 205887.42 | 579 | 1818.98 | 263297.67 |
| 446 | 1401.15 | 156228.26 | 513 | 1611.64 | 206692.45 | 580 | 1822.12 | 264207.94 |
| 447 | 1404.29 | 156929.62 | 514 | 1614.78 | 207499.05 | 581 | 1825.27 | 265119.79 |
| 448 | 1407.43 | 157632.55 | 515 | 1617.92 | 208307.23 | 582 | 1828.41 | 266033.21 |
| 449 | 1410.58 | 158337.06 | 516 | 1621.06 | 209116.97 | 583 | 1831.55 | 266948.20 |
| 450 | 1413.72 | 159043.13 | 517 | 1624.20 | 209928.29 | 584 | 1834.69 | 267864.76 |
| 451 | 1416.86 | 159750.77 | 518 | 1627.34 | 210741.18 | 585 | 1837.83 | 268782.89 |
| 452 | 1420.00 | 160459.99 | 519 | 1630.49 | 211555.63 | 586 | 1840.97 | 269702.59 |
| 453 | 1423.14 | 161170.77 | 520 | 1633.63 | 212371.66 | 587 | 1844.11 | 270623.86 |
| 454 | 1426.28 | 161883.13 | 521 | 1636.77 | 213189.26 | 588 | 1847.26 | 271546.70 |
| 455 | 1429.42 | 162597.05 | 522 | 1639.91 | 214008.43 | 589 | 1850.40 | 272471.12 |
| 456 | 1432.57 | 163312.55 | 523 | 1643.05 | 214829.17 | 590 | 1853.54 | 273397.10 |
| 457 | 1435.71 | 164029.62 | 524 | 1646.19 | 215651.49 | 591 | 1856.68 | 274324.66 |
| 458 | 1438.85 | 164748.26 | 525 | 1649.34 | 216475.37 | 592 | 1859.82 | 275253.78 |
| 459 | 1441.99 | 165468.47 | 526 | 1652.48 | 217300.82 | 593 | 1862.96 | 276184.48 |
| 460 | 1445.13 | 166190.25 | 527 | 1655.62 | 218127.85 | 594 | 1866.11 | 277116.75 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|-----------|-------|---------|-----------|-------|---------|-----------|
| 595 | 1869.25 | 278050.58 | 663 | 2082.88 | 345236.69 | 731 | 2296.50 | 419686.15 |
| 596 | 1872.39 | 278985.99 | 664 | 2086.02 | 346278.91 | 732 | 2299.65 | 420835.19 |
| 597 | 1875.53 | 279922.97 | 665 | 2089.16 | 347322.70 | 733 | 2302.79 | 421985.79 |
| 598 | 1878.67 | 280861.52 | 666 | 2092.30 | 348368.07 | 734 | 2305.93 | 423137.97 |
| 599 | 1881.81 | 281801.65 | 667 | 2095.44 | 349415.00 | 735 | 2309.07 | 424291.72 |
| 600 | 1884.96 | 282743.34 | 668 | 2098.58 | 350463.51 | 736 | 2312.21 | 425447.04 |
| 601 | 1888.10 | 283686.60 | 669 | 2101.73 | 351513.59 | 737 | 2315.35 | 426603.94 |
| 602 | 1891.24 | 284631.44 | 670 | 2104.87 | 352565.24 | 738 | 2318.50 | 427762.40 |
| 603 | 1894.38 | 285577.84 | 671 | 2108.01 | 353618.45 | 739 | 2321.64 | 428922.43 |
| 604 | 1897.52 | 286525.82 | 672 | 2111.15 | 354673.24 | 740 | 2324.78 | 430084.03 |
| 605 | 1900.66 | 287475.36 | 673 | 2114.29 | 355729.60 | 741 | 2327.92 | 431247.21 |
| 606 | 1903.81 | 288426.43 | 674 | 2117.43 | 356787.54 | 742 | 2331.06 | 432411.95 |
| 607 | 1906.95 | 289379.17 | 675 | 2120.58 | 357847.04 | 743 | 2334.20 | 433578.27 |
| 608 | 1910.09 | 290333.43 | 676 | 2123.72 | 358908.11 | 744 | 2337.34 | 434746.16 |
| 609 | 1913.23 | 291289.26 | 677 | 2126.86 | 359970.75 | 745 | 2340.49 | 435915.62 |
| 610 | 1916.37 | 292246.66 | 678 | 2130.00 | 361034.97 | 746 | 2343.63 | 437086.64 |
| 611 | 1919.51 | 293205.63 | 679 | 2133.14 | 362100.75 | 747 | 2346.77 | 438259.24 |
| 612 | 1922.65 | 294166.17 | 680 | 2136.28 | 363168.11 | 748 | 2349.91 | 439433.41 |
| 613 | 1925.80 | 295128.23 | 681 | 2139.42 | 364237.04 | 749 | 2353.05 | 440609.16 |
| 614 | 1928.94 | 296091.97 | 682 | 2142.57 | 365307.54 | 750 | 2356.19 | 441786.47 |
| 615 | 1932.08 | 297057.22 | 683 | 2145.71 | 366379.60 | 751 | 2359.34 | 442965.35 |
| 616 | 1935.22 | 298024.05 | 684 | 2148.85 | 367453.24 | 752 | 2362.48 | 444145.80 |
| 617 | 1938.36 | 298992.44 | 685 | 2151.99 | 368528.45 | 753 | 2365.62 | 445327.83 |
| 618 | 1941.50 | 299962.41 | 686 | 2155.13 | 369605.23 | 754 | 2368.76 | 446511.42 |
| 619 | 1944.65 | 300933.95 | 687 | 2158.27 | 370683.59 | 755 | 2371.90 | 447696.59 |
| 620 | 1947.79 | 301907.05 | 688 | 2161.42 | 371763.51 | 756 | 2375.04 | 448883.32 |
| 621 | 1950.93 | 302881.73 | 689 | 2164.56 | 372845.00 | 757 | 2378.19 | 450071.63 |
| 622 | 1954.07 | 303857.93 | 690 | 2167.70 | 373928.07 | 758 | 2381.33 | 451261.51 |
| 623 | 1957.21 | 304835.80 | 691 | 2170.84 | 375012.70 | 759 | 2384.47 | 452452.96 |
| 624 | 1960.35 | 305815.20 | 692 | 2173.98 | 376098.91 | 760 | 2387.61 | 453645.98 |
| 625 | 1963.50 | 306796.16 | 693 | 2177.12 | 377186.68 | 761 | 2390.75 | 454840.57 |
| 626 | 1966.64 | 307778.69 | 694 | 2180.27 | 378276.03 | 762 | 2393.89 | 456036.73 |
| 627 | 1969.78 | 308762.75 | 695 | 2183.41 | 379366.95 | 763 | 2397.04 | 457234.46 |
| 628 | 1972.92 | 309748.47 | 696 | 2186.55 | 380459.44 | 764 | 2400.18 | 458433.77 |
| 629 | 1976.06 | 310735.71 | 697 | 2189.69 | 381553.50 | 765 | 2403.32 | 459634.64 |
| 630 | 1979.20 | 311724.53 | 698 | 2192.83 | 382649.13 | 766 | 2406.46 | 460837.08 |
| 631 | 1982.35 | 312714.92 | 699 | 2195.97 | 383746.33 | 767 | 2409.60 | 462041.10 |
| 632 | 1985.49 | 313706.88 | 700 | 2199.11 | 384845.10 | 768 | 2412.74 | 463246.69 |
| 633 | 1988.63 | 314700.40 | 701 | 2202.26 | 385945.44 | 769 | 2415.88 | 464453.84 |
| 634 | 1991.77 | 315695.50 | 702 | 2205.40 | 387047.36 | 770 | 2419.03 | 465662.57 |
| 635 | 1994.91 | 316692.17 | 703 | 2208.54 | 388150.84 | 771 | 2422.17 | 466872.87 |
| 636 | 1998.05 | 317690.42 | 704 | 2211.68 | 389255.90 | 772 | 2425.31 | 468084.74 |
| 637 | 2001.19 | 318690.23 | 705 | 2214.82 | 390362.52 | 773 | 2428.45 | 469298.18 |
| 638 | 2004.34 | 319691.61 | 706 | 2217.96 | 391470.72 | 774 | 2431.59 | 470513.19 |
| 639 | 2007.48 | 320694.56 | 707 | 2221.11 | 392580.49 | 775 | 2434.73 | 471729.77 |
| 640 | 2010.62 | 321699.09 | 708 | 2224.25 | 393691.82 | 776 | 2437.88 | 472947.92 |
| 641 | 2013.76 | 322705.18 | 709 | 2227.39 | 394804.73 | 777 | 2441.02 | 474167.65 |
| 642 | 2016.90 | 323712.85 | 710 | 2230.53 | 395919.21 | 778 | 2444.16 | 475388.94 |
| 643 | 2020.04 | 324722.09 | 711 | 2233.67 | 397035.26 | 779 | 2447.30 | 476611.81 |
| 644 | 2023.19 | 325732.89 | 712 | 2236.81 | 398152.89 | 780 | 2450.44 | 477836.24 |
| 645 | 2026.33 | 326745.27 | 713 | 2239.96 | 399272.08 | 781 | 2453.58 | 479062.25 |
| 646 | 2029.47 | 327759.22 | 714 | 2243.10 | 400392.84 | 782 | 2456.73 | 480289.83 |
| 647 | 2032.61 | 328774.74 | 715 | 2246.24 | 401515.18 | 783 | 2459.87 | 481518.97 |
| 648 | 2035.75 | 329791.83 | 716 | 2249.38 | 402639.08 | 784 | 2463.01 | 482749.69 |
| 649 | 2038.89 | 330810.49 | 717 | 2252.52 | 403764.56 | 785 | 2466.15 | 483981.98 |
| 650 | 2042.04 | 331830.72 | 718 | 2255.66 | 404891.60 | 786 | 2469.29 | 485215.84 |
| 651 | 2045.18 | 332852.53 | 719 | 2258.81 | 406020.22 | 787 | 2472.43 | 486451.28 |
| 652 | 2048.32 | 333875.90 | 720 | 2261.95 | 407150.41 | 788 | 2475.58 | 487688.28 |
| 653 | 2051.46 | 334900.85 | 721 | 2265.09 | 408282.17 | 789 | 2478.72 | 488926.85 |
| 654 | 2054.60 | 335927.36 | 722 | 2268.23 | 409415.50 | 790 | 2481.86 | 490166.99 |
| 655 | 2057.74 | 336955.45 | 723 | 2271.37 | 410550.40 | 791 | 2485.00 | 491408.71 |
| 656 | 2060.88 | 337985.10 | 724 | 2274.51 | 411686.87 | 792 | 2488.14 | 492651.99 |
| 657 | 2064.03 | 339016.33 | 725 | 2277.65 | 412824.91 | 793 | 2491.28 | 493896.85 |
| 658 | 2067.17 | 340049.13 | 726 | 2280.80 | 413964.52 | 794 | 2494.42 | 495143.28 |
| 659 | 2070.31 | 341083.50 | 727 | 2283.94 | 415105.71 | 795 | 2497.57 | 496391.27 |
| 660 | 2073.45 | 342119.44 | 728 | 2287.08 | 416248.46 | 796 | 2500.71 | 497640.84 |
| 661 | 2076.59 | 343156.95 | 729 | 2290.22 | 417392.79 | 797 | 2503.85 | 498891.98 |
| 662 | 2079.73 | 344196.03 | 730 | 2293.36 | 418538.68 | 798 | 2506.99 | 500144.69 |

| Diam. | Circum. | Area. | Diam. | Circum. | Area. | Diam. | Circum. | Area. |
|-------|---------|-----------|-------|---------|-----------|-------|---------|-----------|
| 799 | 2510.13 | 501398.97 | 867 | 2723.76 | 590375.16 | 935 | 2937.39 | 686614.71 |
| 800 | 2513.27 | 502654.82 | 868 | 2726.90 | 591737.83 | 936 | 2940.53 | 688084.19 |
| 801 | 2516.42 | 503912.25 | 869 | 2730.04 | 593102.06 | 937 | 2943.67 | 689555.24 |
| 802 | 2519.56 | 505171.24 | 870 | 2733.19 | 594467.87 | 938 | 2946.81 | 691027.86 |
| 803 | 2522.70 | 506431.80 | 871 | 2736.33 | 595835.25 | 939 | 2949.96 | 692502.05 |
| 804 | 2525.84 | 507693.94 | 872 | 2739.47 | 597204.20 | 940 | 2953.10 | 693977.82 |
| 805 | 2528.98 | 508957.64 | 873 | 2742.61 | 598574.72 | 941 | 2956.24 | 695455.15 |
| 806 | 2532.12 | 510222.92 | 874 | 2745.75 | 599946.81 | 942 | 2959.38 | 696934.06 |
| 807 | 2535.27 | 511489.77 | 875 | 2748.89 | 601320.47 | 943 | 2962.52 | 698414.53 |
| 808 | 2538.41 | 512758.19 | 876 | 2752.04 | 602695.70 | 944 | 2965.66 | 699896.58 |
| 809 | 2541.55 | 514028.18 | 877 | 2755.18 | 604072.50 | 945 | 2968.81 | 701380.19 |
| 810 | 2544.69 | 515299.74 | 878 | 2758.32 | 605450.88 | 946 | 2971.95 | 702865.38 |
| 811 | 2547.83 | 516572.87 | 879 | 2761.46 | 606830.82 | 947 | 2975.09 | 704352.14 |
| 812 | 2550.97 | 517847.57 | 880 | 2764.60 | 608212.34 | 948 | 2978.23 | 705840.47 |
| 813 | 2554.11 | 519123.84 | 881 | 2767.74 | 609595.42 | 949 | 2981.37 | 707330.37 |
| 814 | 2557.26 | 520401.68 | 882 | 2770.88 | 610980.08 | 950 | 2984.51 | 708821.84 |
| 815 | 2560.40 | 521681.10 | 883 | 2774.03 | 612366.31 | 951 | 2987.65 | 710314.88 |
| 816 | 2563.54 | 522962.08 | 884 | 2777.17 | 613754.11 | 952 | 2990.80 | 711809.50 |
| 817 | 2566.68 | 524244.63 | 885 | 2780.31 | 615143.48 | 953 | 2993.94 | 713305.68 |
| 818 | 2569.82 | 525528.76 | 886 | 2783.45 | 616534.42 | 954 | 2997.08 | 714803.43 |
| 819 | 2572.96 | 526814.46 | 887 | 2786.59 | 617926.93 | 955 | 3000.22 | 716302.76 |
| 820 | 2576.11 | 528101.73 | 888 | 2789.73 | 619321.01 | 956 | 3003.36 | 717803.66 |
| 821 | 2579.25 | 529390.56 | 889 | 2792.88 | 620716.66 | 957 | 3006.50 | 719306.12 |
| 822 | 2582.39 | 530680.97 | 890 | 2796.02 | 622113.89 | 958 | 3009.65 | 720810.16 |
| 823 | 2585.53 | 531972.95 | 891 | 2799.16 | 623512.68 | 959 | 3012.79 | 722315.77 |
| 824 | 2588.67 | 533266.50 | 892 | 2802.30 | 624913.04 | 960 | 3015.93 | 723822.95 |
| 825 | 2591.81 | 534561.62 | 893 | 2805.44 | 626314.98 | 961 | 3019.07 | 725331.70 |
| 826 | 2594.96 | 535858.32 | 894 | 2808.58 | 627718.49 | 962 | 3022.21 | 726842.02 |
| 827 | 2598.10 | 537156.58 | 895 | 2811.73 | 629123.56 | 963 | 3025.35 | 728353.91 |
| 828 | 2601.24 | 538456.41 | 896 | 2814.87 | 630530.21 | 964 | 3028.50 | 729867.37 |
| 829 | 2604.38 | 539757.82 | 897 | 2818.01 | 631938.43 | 965 | 3031.64 | 731382.40 |
| 830 | 2607.52 | 541060.79 | 898 | 2821.15 | 633348.22 | 966 | 3034.78 | 732899.01 |
| 831 | 2610.66 | 542365.34 | 899 | 2824.29 | 634759.58 | 967 | 3037.92 | 734417.18 |
| 832 | 2613.81 | 543671.46 | 900 | 2827.43 | 636172.51 | 968 | 3041.06 | 735936.93 |
| 833 | 2616.95 | 544979.15 | 901 | 2830.58 | 637587.01 | 969 | 3044.20 | 737458.24 |
| 834 | 2620.09 | 546288.40 | 902 | 2833.72 | 639003.09 | 970 | 3047.34 | 738981.13 |
| 835 | 2623.23 | 547599.23 | 903 | 2836.86 | 640420.73 | 971 | 3050.49 | 740505.59 |
| 836 | 2626.37 | 548911.63 | 904 | 2840.00 | 641839.95 | 972 | 3053.63 | 742031.62 |
| 837 | 2629.51 | 550225.61 | 905 | 2843.14 | 643260.73 | 973 | 3056.77 | 743559.22 |
| 838 | 2632.65 | 551541.15 | 906 | 2846.28 | 644683.09 | 974 | 3059.91 | 745088.39 |
| 839 | 2635.80 | 552858.26 | 907 | 2849.42 | 646107.01 | 975 | 3063.05 | 746619.13 |
| 840 | 2638.94 | 554176.94 | 908 | 2852.57 | 647532.51 | 976 | 3066.19 | 748151.44 |
| 841 | 2642.08 | 555497.20 | 909 | 2855.71 | 648959.58 | 977 | 3069.34 | 749685.32 |
| 842 | 2645.22 | 556819.02 | 910 | 2858.85 | 650388.22 | 978 | 3072.48 | 751220.78 |
| 843 | 2648.36 | 558142.42 | 911 | 2861.99 | 651818.43 | 979 | 3075.62 | 752757.80 |
| 844 | 2651.50 | 559467.39 | 912 | 2865.13 | 653250.21 | 980 | 3078.76 | 754296.40 |
| 845 | 2654.65 | 560793.92 | 913 | 2868.27 | 654683.56 | 981 | 3081.90 | 755836.56 |
| 846 | 2657.79 | 562122.03 | 914 | 2871.42 | 656118.48 | 982 | 3085.04 | 757378.30 |
| 847 | 2660.93 | 563451.71 | 915 | 2874.56 | 657554.98 | 983 | 3088.19 | 758921.61 |
| 848 | 2664.07 | 564782.96 | 916 | 2877.70 | 658993.04 | 984 | 3091.33 | 760466.48 |
| 849 | 2667.21 | 566115.78 | 917 | 2880.84 | 660432.68 | 985 | 3094.47 | 762012.93 |
| 850 | 2670.35 | 567450.17 | 918 | 2883.98 | 661873.88 | 986 | 3097.61 | 763560.95 |
| 851 | 2673.50 | 568786.14 | 919 | 2887.12 | 663320.66 | 987 | 3100.75 | 765110.54 |
| 852 | 2676.64 | 570123.67 | 920 | 2890.27 | 664761.01 | 988 | 3103.89 | 766661.70 |
| 853 | 2679.78 | 571462.77 | 921 | 2893.41 | 666206.92 | 989 | 3107.04 | 768214.44 |
| 854 | 2682.92 | 572803.45 | 922 | 2896.55 | 667654.41 | 990 | 3110.18 | 769768.74 |
| 855 | 2686.06 | 574145.69 | 923 | 2899.69 | 669103.47 | 991 | 3113.32 | 771324.61 |
| 856 | 2689.20 | 575489.51 | 924 | 2902.83 | 670554.10 | 992 | 3116.46 | 772882.06 |
| 857 | 2692.34 | 576834.90 | 925 | 2905.97 | 672006.30 | 993 | 3119.60 | 774441.07 |
| 858 | 2695.49 | 578181.85 | 926 | 2909.11 | 673460.08 | 994 | 3122.74 | 776001.66 |
| 859 | 2698.63 | 579530.38 | 927 | 2912.26 | 674915.42 | 995 | 3125.88 | 777563.82 |
| 860 | 2701.77 | 580880.48 | 928 | 2915.40 | 676372.33 | 996 | 3129.03 | 779127.54 |
| 861 | 2704.91 | 582232.15 | 929 | 2918.54 | 677830.82 | 997 | 3132.17 | 780692.84 |
| 862 | 2708.05 | 583585.39 | 930 | 2921.68 | 679290.87 | 998 | 3135.31 | 782259.71 |
| 863 | 2711.19 | 584940.20 | 931 | 2924.82 | 680752.50 | 999 | 3138.45 | 783828.15 |
| 864 | 2714.34 | 586296.59 | 932 | 2927.96 | 682215.69 | 1000 | 3141.59 | 785398.16 |
| 865 | 2717.48 | 587654.54 | 933 | 2931.11 | 683680.46 | | | |
| 866 | 2720.62 | 589014.07 | 934 | 2934.25 | 685146.80 | | | |

CIRCUMFERENCE OF CIRCLES FROM 1 INCH TO 32 FT. 11 IN. DIAMETER.

| Diam. Feet. | 0 In. | | 1 Inch. | | 2 In. | | 3 In. | | 4 In. | | 5 In. | | 6 In. | | 7 In. | | 8 In. | | 9 In. | | 10 In. | | 11 In. | | Diam. Feet. |
|----------------|-------|--------|---------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|--------|--------|--------|--------|----------------|
| | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | Ft. | In. | |
| 0 | ... | 3 1/4 | 3 | 3 1/7 | 3 | 6 1/4 | 3 | 9 3/8 | 1 | 05/8 | 1 | 3 3/4 | 1 | 67/8 | 1 | 1 10 | 2 | 1 1/8 | 2 | 4 1/4 | 2 | 7 3/8 | 2 | 10 1/2 | 0 |
| 1 | 6 | 3 3/8 | 3 | 4 7/8 | 3 | 8 | 3 | 11 1/8 | 4 | 2 1/4 | 4 | 5 3/8 | 4 | 8 1/2 | 4 | 11 5/8 | 5 | 2 7/8 | 5 | 6 | 5 | 9 1/8 | 6 | 12 1/2 | 1 |
| 2 | 9 | 5 1/8 | 6 | 6 1/2 | 6 | 9 5/8 | 7 | 03/4 | 7 | 4 | 7 | 7 1/8 | 7 | 10 1/4 | 8 | 1 3/8 | 8 | 4 1/2 | 8 | 7 5/8 | 8 | 10 3/4 | 9 | 2 | 2 |
| 3 | 12 | 6 3/4 | 9 | 8 1/4 | 9 | 11 3/8 | 10 | 2 1/2 | 10 | 5 5/8 | 10 | 8 3/4 | 10 | 11 7/8 | 11 | 3 1/8 | 11 | 6 1/4 | 11 | 9 3/8 | 12 | 01/2 | 12 | 3 5/8 | 3 |
| 4 | 15 | 8 1/2 | 13 | 9 7/8 | 13 | 1 1/8 | 13 | 4 1/4 | 13 | 7 3/8 | 13 | 10 1/2 | 14 | 1 5/8 | 14 | 4 3/4 | 14 | 7 7/8 | 14 | 11 | 15 | 2 1/4 | 15 | 5 3/8 | 4 |
| 5 | 18 | 10 1/8 | 15 | 11 5/8 | 16 | 2 3/4 | 16 | 5 7/8 | 16 | 9 | 17 | 01/4 | 17 | 3 3/8 | 17 | 6 1/2 | 17 | 9 5/8 | 18 | 03/4 | 18 | 3 7/8 | 18 | 7 | 5 |
| 6 | 21 | 11 7/8 | 19 | 1 3/8 | 19 | 4 1/2 | 19 | 7 5/8 | 19 | 10 3/4 | 20 | 1 7/8 | 20 | 5 | 20 | 8 1/8 | 20 | 11 3/8 | 21 | 2 1/2 | 21 | 5 5/8 | 21 | 8 3/4 | 6 |
| 7 | 25 | 1 5/8 | 22 | 3 | 22 | 6 1/8 | 22 | 9 1/4 | 23 | 03/8 | 23 | 3 5/8 | 23 | 6 3/4 | 23 | 9 7/8 | 24 | 1 | 24 | 4 1/8 | 24 | 7 1/4 | 24 | 10 3/8 | 7 |
| 8 | 28 | 3 1/4 | 25 | 4 3/4 | 25 | 7 7/8 | 25 | 11 | 26 | 2 1/8 | 26 | 5 1/4 | 26 | 8 3/8 | 26 | 11 5/8 | 27 | 2 3/4 | 27 | 5 7/8 | 27 | 9 | 28 | 01/8 | 8 |
| 9 | 31 | 5 | 28 | 6 3/8 | 28 | 9 5/8 | 29 | 03/4 | 29 | 3 7/8 | 29 | 7 | 29 | 10 1/8 | 30 | 1 1/4 | 30 | 4 3/8 | 30 | 7 1/2 | 30 | 10 3/4 | 31 | 1 7/8 | 9 |
| 10 | 34 | 6 5/8 | 31 | 8 1/8 | 31 | 11 1/4 | 32 | 2 3/8 | 32 | 5 1/2 | 32 | 8 3/4 | 33 | 1 7/8 | 33 | 3 | 33 | 6 1/8 | 33 | 9 1/4 | 34 | 03/8 | 34 | 3 1/2 | 10 |
| 11 | 37 | 8 3/8 | 34 | 9 7/8 | 35 | 1 | 35 | 4 1/8 | 35 | 7 1/4 | 35 | 10 3/8 | 36 | 1 1/2 | 36 | 4 5/8 | 36 | 7 3/4 | 36 | 11 | 37 | 2 1/8 | 37 | 5 1/4 | 11 |
| 12 | 40 | 10 1/8 | 37 | 11 1/2 | 38 | 2 5/8 | 38 | 5 3/4 | 38 | 9 | 39 | 01/8 | 39 | 3 1/4 | 39 | 6 3/8 | 40 | 9 1/2 | 40 | 05/8 | 40 | 3 3/4 | 40 | 6 7/8 | 12 |
| 13 | 43 | 11 3/4 | 41 | 1 1/4 | 41 | 4 3/8 | 42 | 1 | 42 | 10 3/8 | 42 | 1 3/4 | 42 | 4 7/8 | 42 | 8 1/8 | 43 | 10 1/4 | 43 | 2 3/8 | 43 | 5 1/2 | 43 | 8 5/8 | 13 |
| 14 | 47 | 1 1/2 | 44 | 2 7/8 | 44 | 6 | 44 | 9 1/4 | 45 | 03/8 | 45 | 3 1/2 | 45 | 6 5/8 | 46 | 9 3/4 | 46 | 1 7/8 | 46 | 4 | 46 | 7 1/8 | 46 | 10 3/8 | 14 |
| 15 | 50 | 3 1/8 | 47 | 4 5/8 | 47 | 7 3/4 | 47 | 10 7/8 | 48 | 2 | 48 | 5 1/8 | 48 | 8 3/8 | 48 | 11 1/2 | 49 | 2 5/8 | 49 | 5 3/4 | 49 | 8 7/8 | 50 | 0 | 15 |
| 16 | 53 | 4 7/8 | 50 | 6 3/8 | 50 | 9 1/2 | 51 | 05/8 | 51 | 3 3/4 | 51 | 6 7/8 | 51 | 10 | 52 | 1 1/8 | 52 | 4 1/4 | 52 | 7 1/2 | 52 | 10 5/8 | 53 | 1 3/4 | 16 |
| 17 | 56 | 6 3/8 | 53 | 8 | 53 | 11 1/8 | 54 | 2 1/4 | 54 | 5 1/2 | 54 | 8 5/8 | 55 | 1 3/8 | 55 | 2 7/8 | 55 | 6 | 55 | 9 1/8 | 56 | 01/4 | 56 | 3 1/2 | 17 |
| 18 | 59 | 8 1/4 | 56 | 9 3/4 | 57 | 07/8 | 57 | 4 | 57 | 7 1/8 | 57 | 10 1/4 | 58 | 1 3/8 | 58 | 4 1/2 | 58 | 7 3/4 | 58 | 07/8 | 59 | 2 | 59 | 5 1/8 | 18 |
| 19 | 62 | 10 | 59 | 11 3/8 | 60 | 2 1/2 | 60 | 5 3/4 | 60 | 8 7/8 | 61 | 0 | 61 | 3 1/8 | 61 | 6 1/4 | 61 | 9 3/8 | 62 | 10 1/2 | 62 | 3 5/8 | 62 | 6 7/8 | 19 |
| 20 | 65 | 11 5/8 | 63 | 1 1/8 | 63 | 4 1/4 | 63 | 7 3/8 | 63 | 10 1/2 | 64 | 15/8 | 64 | 4 7/8 | 64 | 8 | 64 | 11/8 | 65 | 2 1/4 | 65 | 5 3/8 | 65 | 8 1/2 | 20 |
| 21 | 69 | 1 3/8 | 66 | 2 7/8 | 66 | 6 | 66 | 9 1/8 | 66 | 01/4 | 67 | 3 3/8 | 67 | 6 7/8 | 67 | 9 5/8 | 68 | 10 3/4 | 68 | 03/4 | 68 | 7 1/8 | 68 | 10 1/4 | 21 |
| 22 | 72 | 3 | 69 | 4 1/2 | 69 | 7 5/8 | 69 | 10 3/4 | 70 | 1 7/8 | 70 | 5 1/8 | 70 | 8 1/4 | 70 | 11 3/8 | 71 | 2 1/2 | 71 | 5 5/8 | 71 | 8 3/4 | 71 | 11 7/8 | 22 |
| 23 | 75 | 4 3/4 | 72 | 6 1/4 | 72 | 9 3/8 | 73 | 01/2 | 73 | 3 5/8 | 73 | 6 3/4 | 73 | 9 7/8 | 74 | 1 | 74 | 4 1/4 | 74 | 7 3/8 | 74 | 01/2 | 75 | 1 5/8 | 23 |
| 24 | 78 | 6 1/2 | 75 | 7 7/8 | 75 | 11 | 76 | 2 1/8 | 76 | 5 3/8 | 76 | 8 1/8 | 76 | 11 5/8 | 77 | 2 3/4 | 77 | 5 7/8 | 77 | 9 | 78 | 10 1/8 | 78 | 3 3/8 | 24 |
| 25 | 81 | 8 1/8 | 78 | 9 5/8 | 79 | 03/4 | 79 | 3 7/8 | 79 | 7 | 79 | 10 1/8 | 80 | 1 1/4 | 80 | 4 3/8 | 80 | 7 5/8 | 80 | 10 3/4 | 81 | 1 7/8 | 81 | 5 | 25 |
| 26 | 84 | 9 7/8 | 81 | 11 1/4 | 82 | 2 1/2 | 82 | 5 5/8 | 82 | 8 3/4 | 82 | 11 7/8 | 83 | 3 | 83 | 6 1/8 | 83 | 9 1/4 | 84 | 03/8 | 84 | 3 5/8 | 84 | 6 3/4 | 26 |
| 27 | 87 | 11 5/8 | 85 | 1 | 85 | 4 1/8 | 85 | 7 1/4 | 85 | 10 3/8 | 86 | 1 5/8 | 86 | 4 3/4 | 86 | 7 7/8 | 86 | 11 | 87 | 2 1/8 | 87 | 5 1/4 | 87 | 8 3/8 | 27 |
| 28 | 91 | 1 1/4 | 88 | 2 3/4 | 88 | 5 7/8 | 88 | 9 | 89 | 01/8 | 89 | 3 1/4 | 89 | 6 3/8 | 89 | 9 3/8 | 90 | 03/4 | 90 | 3 7/8 | 90 | 7 | 90 | 10 1/8 | 28 |
| 29 | 94 | 3 | 91 | 4 3/8 | 91 | 7 1/2 | 91 | 10 3/4 | 92 | 1 7/8 | 92 | 5 | 92 | 8 1/8 | 92 | 11 1/4 | 93 | 2 3/8 | 93 | 5 1/2 | 93 | 8 5/8 | 93 | 1 1/2 | 29 |
| 30 | 97 | 4 5/8 | 94 | 6 1/8 | 94 | 9 1/4 | 95 | 03/8 | 95 | 3 1/2 | 95 | 6 5/8 | 96 | 1 | 96 | 4 1/8 | 96 | 4 3/4 | 97 | 7 3/8 | 97 | 10 3/8 | 97 | 1 1/2 | 30 |
| 31 | 100 | 6 3/8 | 97 | 7 3/4 | 97 | 11 | 98 | 2 1/8 | 98 | 5 1/4 | 98 | 8 3/8 | 98 | 11 1/2 | 99 | 2 5/8 | 99 | 5 3/4 | 99 | 8 7/8 | 100 | 01/8 | 100 | 3 1/4 | 31 |
| 32 | | | 100 | 9 1/2 | 101 | 05/8 | 101 | 3 3/4 | 101 | 6 7/8 | 101 | 10 3/8 | 102 | 1 1/4 | 102 | 4 3/8 | 102 | 7 1/2 | 102 | 10 5/8 | 103 | 1 3/4 | 103 | 4 7/8 | 32 |

AREAS OF THE SEGMENTS OF A CIRCLE.

(Diameter=1; Rise or Height in parts of Diameter being given.)

RULE FOR USE OF THE TABLE.—Divide the rise or height of the segment by the diameter. Multiply the area in the table corresponding to the quotient thus found by the square of the diameter.

If the segment exceeds a semicircle its area is area of circle — area of segment whose rise is (diam. of circle — rise of given segment).

Given chord and rise, to find diameter. $\text{Diam.} = (\text{square of half chord} + \text{rise}) + \text{rise}$. The half chord is a mean proportional between the two parts into which the chord divides the diameter which is perpendicular to it.

| Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. |
|--------------------|--------|--------------------|--------|--------------------|--------|--------------------|--------|--------------------|--------|
| .001 | .00004 | .054 | .01646 | .107 | .04514 | .16 | .03111 | .213 | .12235 |
| .002 | .00012 | .055 | .01691 | .108 | .04576 | .161 | .03185 | .214 | .12317 |
| .003 | .00022 | .056 | .01737 | .109 | .04638 | .162 | .03258 | .215 | .12399 |
| .004 | .00034 | .057 | .01783 | .11 | .04701 | .163 | .03332 | .216 | .12481 |
| .005 | .00047 | .058 | .01830 | .111 | .04763 | .164 | .03406 | .217 | .12563 |
| .006 | .00062 | .059 | .01877 | .112 | .04826 | .165 | .03480 | .218 | .12646 |
| .007 | .00078 | .06 | .01924 | .113 | .04889 | .166 | .03554 | .219 | .12729 |
| .008 | .00095 | .061 | .01972 | .114 | .04953 | .167 | .03629 | .22 | .12811 |
| .009 | .00113 | .062 | .02020 | .115 | .05016 | .168 | .03704 | .221 | .12894 |
| .01 | .00133 | .063 | .02068 | .116 | .05080 | .169 | .03779 | .222 | .12977 |
| .011 | .00153 | .064 | .02117 | .117 | .05145 | .17 | .03854 | .223 | .13060 |
| .012 | .00175 | .065 | .02166 | .118 | .05209 | .171 | .03929 | .224 | .13144 |
| .013 | .00197 | .066 | .02215 | .119 | .05274 | .172 | .03904 | .225 | .13227 |
| .014 | .0022 | .067 | .02265 | .12 | .05338 | .173 | .03980 | .226 | .13311 |
| .015 | .00244 | .068 | .02315 | .121 | .05404 | .174 | .03955 | .227 | .13395 |
| .016 | .00268 | .069 | .02366 | .122 | .05469 | .175 | .03931 | .228 | .13478 |
| .017 | .00294 | .07 | .02417 | .123 | .05535 | .176 | .03907 | .229 | .13562 |
| .018 | .0032 | .071 | .02468 | .124 | .05600 | .177 | .03984 | .23 | .13646 |
| .019 | .00347 | .072 | .02520 | .125 | .05666 | .178 | .03960 | .231 | .13731 |
| .02 | .00375 | .073 | .02571 | .126 | .05733 | .179 | .03937 | .232 | .13815 |
| .021 | .00403 | .074 | .02624 | .127 | .05799 | .18 | .03913 | .233 | .13900 |
| .022 | .00432 | .075 | .02676 | .128 | .05866 | .181 | .03960 | .234 | .13984 |
| .023 | .00462 | .076 | .02729 | .129 | .05933 | .182 | .03977 | .235 | .14069 |
| .024 | .00492 | .077 | .02782 | .13 | .06000 | .183 | .03985 | .236 | .14154 |
| .025 | .00523 | .078 | .02836 | .131 | .06067 | .184 | .03992 | .237 | .14239 |
| .026 | .00555 | .079 | .02889 | .132 | .06135 | .185 | .04000 | .238 | .14324 |
| .027 | .00587 | .08 | .02943 | .133 | .06203 | .186 | .04007 | .239 | .14409 |
| .028 | .00619 | .081 | .02998 | .134 | .06271 | .187 | .04015 | .24 | .14494 |
| .029 | .00653 | .082 | .03053 | .135 | .06339 | .188 | .04033 | .241 | .14580 |
| .03 | .00687 | .083 | .03108 | .136 | .06407 | .189 | .04032 | .242 | .14666 |
| .031 | .00721 | .084 | .03163 | .137 | .06476 | .19 | .04030 | .243 | .14751 |
| .032 | .00756 | .085 | .03219 | .138 | .06545 | .191 | .04069 | .244 | .14837 |
| .033 | .00791 | .086 | .03275 | .139 | .06614 | .192 | .04047 | .245 | .14923 |
| .034 | .00827 | .087 | .03331 | .14 | .06683 | .193 | .04066 | .246 | .15009 |
| .035 | .00864 | .088 | .03387 | .141 | .06753 | .194 | .04075 | .247 | .15095 |
| .036 | .00901 | .089 | .03444 | .142 | .06822 | .195 | .04078 | .248 | .15182 |
| .037 | .00938 | .09 | .03501 | .143 | .06892 | .196 | .04086 | .249 | .15263 |
| .038 | .00976 | .091 | .03559 | .144 | .06963 | .197 | .04093 | .25 | .15355 |
| .039 | .01015 | .092 | .03616 | .145 | .07033 | .198 | .04102 | .251 | .15441 |
| .04 | .01054 | .093 | .03674 | .146 | .07103 | .199 | .04110 | .252 | .15528 |
| .041 | .01093 | .094 | .03732 | .147 | .07174 | .2 | .04118 | .253 | .15615 |
| .042 | .01133 | .095 | .03791 | .148 | .07245 | .201 | .04126 | .254 | .15702 |
| .043 | .01173 | .096 | .03850 | .149 | .07316 | .202 | .04134 | .255 | .15789 |
| .044 | .01214 | .097 | .03909 | .15 | .07387 | .203 | .04143 | .256 | .15876 |
| .045 | .01255 | .098 | .03968 | .151 | .07459 | .204 | .04150 | .257 | .15964 |
| .046 | .01297 | .099 | .04028 | .152 | .07531 | .205 | .04158 | .258 | .16051 |
| .047 | .01339 | .1 | .04087 | .153 | .07603 | .206 | .04166 | .259 | .16139 |
| .048 | .01382 | .101 | .04148 | .154 | .07675 | .207 | .04174 | .26 | .16226 |
| .049 | .01425 | .102 | .04208 | .155 | .07747 | .208 | .04182 | .261 | .16314 |
| .05 | .01468 | .103 | .04269 | .156 | .07819 | .209 | .04190 | .262 | .16402 |
| .051 | .01512 | .104 | .04330 | .157 | .07892 | .21 | .04199 | .263 | .16490 |
| .052 | .01556 | .105 | .04391 | .158 | .07965 | .211 | .04207 | .264 | .16578 |
| .053 | .01601 | .106 | .04452 | .159 | .08038 | .212 | .04215 | .265 | .16666 |

| Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. | Rise ÷ Diam. | Area. |
|--------------------|--------|--------------------|--------|--------------------|--------|--------------------|--------|--------------------|--------|
| .266 | .16755 | .313 | .21015 | .36 | .25455 | .407 | .30024 | .454 | .34676 |
| .267 | .16843 | .314 | .21108 | .361 | .25551 | .408 | .30122 | .455 | .34776 |
| .268 | .16932 | .315 | .21201 | .362 | .25647 | .409 | .30220 | .456 | .34876 |
| .269 | .17020 | .316 | .21294 | .363 | .25743 | .41 | .30319 | .457 | .34975 |
| .27 | .17109 | .317 | .21387 | .364 | .25839 | .411 | .30417 | .458 | .35075 |
| .271 | .17198 | .318 | .21480 | .365 | .25936 | .412 | .30516 | .459 | .35175 |
| .272 | .17287 | .319 | .21573 | .366 | .26032 | .413 | .30614 | .46 | .35274 |
| .273 | .17376 | .32 | .21667 | .367 | .26128 | .414 | .30712 | .461 | .35374 |
| .274 | .17465 | .321 | .21760 | .368 | .26225 | .415 | .30811 | .462 | .35474 |
| .275 | .17554 | .322 | .21853 | .369 | .26321 | .416 | .30910 | .463 | .35573 |
| .276 | .17644 | .323 | .21947 | .37 | .26418 | .417 | .31008 | .464 | .35673 |
| .277 | .17733 | .324 | .22040 | .371 | .26514 | .418 | .31107 | .465 | .35773 |
| .278 | .17823 | .325 | .22134 | .372 | .26611 | .419 | .31205 | .466 | .35873 |
| .279 | .17912 | .326 | .22228 | .373 | .26708 | .42 | .31304 | .467 | .35972 |
| .28 | .18002 | .327 | .22322 | .374 | .26805 | .421 | .31403 | .468 | .36072 |
| .281 | .18092 | .328 | .22415 | .375 | .26901 | .422 | .31502 | .469 | .36172 |
| .282 | .18182 | .329 | .22509 | .376 | .26998 | .423 | .31600 | .47 | .36272 |
| .283 | .18272 | .33 | .22603 | .377 | .27095 | .424 | .31699 | .471 | .36372 |
| .284 | .18362 | .331 | .22697 | .378 | .27192 | .425 | .31798 | .472 | .36471 |
| .285 | .18452 | .332 | .22792 | .379 | .27289 | .426 | .31897 | .473 | .36571 |
| .286 | .18542 | .333 | .22886 | .38 | .27386 | .427 | .31996 | .474 | .36671 |
| .287 | .18633 | .334 | .22980 | .381 | .27483 | .428 | .32095 | .475 | .36771 |
| .288 | .18723 | .335 | .23074 | .382 | .27580 | .429 | .32194 | .476 | .36871 |
| .289 | .18814 | .336 | .23169 | .383 | .27678 | .43 | .32293 | .477 | .36971 |
| .29 | .18905 | .337 | .23263 | .384 | .27775 | .431 | .32392 | .478 | .37071 |
| .291 | .18996 | .338 | .23358 | .385 | .27872 | .432 | .32491 | .479 | .37171 |
| .292 | .19086 | .339 | .23453 | .386 | .27969 | .433 | .32590 | .48 | .37270 |
| .293 | .19177 | .34 | .23547 | .387 | .28067 | .434 | .32689 | .481 | .37370 |
| .294 | .19268 | .341 | .23642 | .388 | .28164 | .435 | .32788 | .482 | .37470 |
| .295 | .19360 | .342 | .23737 | .389 | .28262 | .436 | .32887 | .483 | .37570 |
| .296 | .19451 | .343 | .23832 | .39 | .28359 | .437 | .32987 | .484 | .37670 |
| .297 | .19542 | .344 | .23927 | .391 | .28457 | .438 | .33086 | .485 | .37770 |
| .298 | .19634 | .345 | .24022 | .392 | .28554 | .439 | .33185 | .486 | .37870 |
| .299 | .19725 | .346 | .24117 | .393 | .28652 | .44 | .33284 | .487 | .37970 |
| .3 | .19817 | .347 | .24212 | .394 | .28750 | .441 | .33384 | .488 | .38070 |
| .301 | .19908 | .348 | .24307 | .395 | .28848 | .442 | .33483 | .489 | .38170 |
| .302 | .20000 | .349 | .24403 | .396 | .28945 | .443 | .33582 | .49 | .38270 |
| .303 | .20092 | .35 | .24498 | .397 | .29043 | .444 | .33682 | .491 | .38370 |
| .304 | .20184 | .351 | .24593 | .398 | .29141 | .445 | .33781 | .492 | .38470 |
| .305 | .20276 | .352 | .24689 | .399 | .29239 | .446 | .33880 | .493 | .38570 |
| .306 | .20368 | .353 | .24784 | .4 | .29337 | .447 | .33980 | .494 | .38670 |
| .307 | .20460 | .354 | .24880 | .401 | .29435 | .448 | .34079 | .495 | .38770 |
| .308 | .20553 | .355 | .24976 | .402 | .29533 | .449 | .34179 | .496 | .38870 |
| .309 | .20645 | .356 | .25071 | .403 | .29631 | .45 | .34278 | .497 | .38970 |
| .31 | .20738 | .357 | .25167 | .404 | .29729 | .451 | .34378 | .498 | .39070 |
| .311 | .20830 | .358 | .25263 | .405 | .29827 | .452 | .34477 | .499 | .39170 |
| .312 | .20923 | .359 | .25359 | .406 | .29926 | .453 | .34577 | .5 | .39270 |

For rules for finding the area of a segment see Mensuration, page 61.

LENGTHS OF CIRCULAR ARCS.

(Degrees being given. Radius of Circle = 1.)

FORMULA. — Length of arc = $\frac{3.1415927}{180} \times \text{radius} \times \text{number of degrees.}$

RULE. — Multiply the factor in the table (see next page) for any given number of degrees by the radius.

EXAMPLE. — Given a curve of a radius of 55 feet and an angle of 78° 20'.

Factor from table for 78°..... 1.3613568

Factor from table for 20'..... .0058178

Factor..... 1.3671746

1.3671746 × 55 = 75.19 feet.

FACTORS FOR LENGTHS OF CIRCULAR ARCS.

| Degrees. | | | | | | Minutes. | |
|----------|-----------|-----|-----------|-----|-----------|----------|----------|
| 1 | .0174533 | 61 | 1.0646508 | 121 | 2.1118484 | 1 | .0002909 |
| 2 | .0349066 | 62 | 1.0821041 | 122 | 2.1293017 | 2 | .0005818 |
| 3 | .0523599 | 63 | 1.0995574 | 123 | 2.1467550 | 3 | .0008727 |
| 4 | .0698132 | 64 | 1.1170107 | 124 | 2.1642083 | 4 | .0011636 |
| 5 | .0872665 | 65 | 1.1344640 | 125 | 2.1816616 | 5 | .0014544 |
| 6 | .1047198 | 66 | 1.1519173 | 126 | 2.1991149 | 6 | .0017453 |
| 7 | .1221730 | 67 | 1.1693706 | 127 | 2.2165682 | 7 | .0020362 |
| 8 | .1396263 | 68 | 1.1868239 | 128 | 2.2340214 | 8 | .0023271 |
| 9 | .1570796 | 69 | 1.2042772 | 129 | 2.2514747 | 9 | .0026180 |
| 10 | .1745329 | 70 | 1.2217305 | 130 | 2.2689280 | 10 | .0029089 |
| 11 | .1919862 | 71 | 1.2391838 | 131 | 2.2863813 | 11 | .0031998 |
| 12 | .2094395 | 72 | 1.2566371 | 132 | 2.3038346 | 12 | .0034907 |
| 13 | .2268928 | 73 | 1.2740904 | 133 | 2.3212879 | 13 | .0037815 |
| 14 | .2443461 | 74 | 1.2915436 | 134 | 2.3387412 | 14 | .0040724 |
| 15 | .2617994 | 75 | 1.3089969 | 135 | 2.3561945 | 15 | .0043633 |
| 16 | .2792527 | 76 | 1.3264502 | 136 | 2.3736478 | 16 | .0046542 |
| 17 | .2967060 | 77 | 1.3439035 | 137 | 2.3911011 | 17 | .0049451 |
| 18 | .3141593 | 78 | 1.3613568 | 138 | 2.4085544 | 18 | .0052360 |
| 19 | .3316126 | 79 | 1.3788101 | 139 | 2.4260077 | 19 | .0055269 |
| 20 | .3490659 | 80 | 1.3962634 | 140 | 2.4434610 | 20 | .0058178 |
| 21 | .3665191 | 81 | 1.4137167 | 141 | 2.4609142 | 21 | .0061087 |
| 22 | .3839724 | 82 | 1.4311700 | 142 | 2.4783675 | 22 | .0063995 |
| 23 | .4014257 | 83 | 1.4486233 | 143 | 2.4958208 | 23 | .0066904 |
| 24 | .4188790 | 84 | 1.4660766 | 144 | 2.5132741 | 24 | .0069813 |
| 25 | .4363323 | 85 | 1.4835299 | 145 | 2.5307274 | 25 | .0072722 |
| 26 | .4537856 | 86 | 1.5009832 | 146 | 2.5481807 | 26 | .0075631 |
| 27 | .4712389 | 87 | 1.5184364 | 147 | 2.5656340 | 27 | .0078540 |
| 28 | .4886922 | 88 | 1.5358897 | 148 | 2.5830873 | 28 | .0081449 |
| 29 | .5061455 | 89 | 1.5533430 | 149 | 2.6005406 | 29 | .0084358 |
| 30 | .5235988 | 90 | 1.5707963 | 150 | 2.6179939 | 30 | .0087266 |
| 31 | .5410521 | 91 | 1.5882496 | 151 | 2.6354472 | 31 | .0090175 |
| 32 | .5585054 | 92 | 1.6057029 | 152 | 2.6529005 | 32 | .0093084 |
| 33 | .5759587 | 93 | 1.6231562 | 153 | 2.6703538 | 33 | .0095993 |
| 34 | .5934119 | 94 | 1.6406095 | 154 | 2.6878070 | 34 | .0098902 |
| 35 | .6108652 | 95 | 1.6580628 | 155 | 2.7052603 | 35 | .0101811 |
| 36 | .6283185 | 96 | 1.6755161 | 156 | 2.7227136 | 36 | .0104720 |
| 37 | .6457718 | 97 | 1.6929694 | 157 | 2.7401669 | 37 | .0107629 |
| 38 | .6632251 | 98 | 1.7104227 | 158 | 2.7576202 | 38 | .0110538 |
| 39 | .6806784 | 99 | 1.7278760 | 159 | 2.7750735 | 39 | .0113446 |
| 40 | .6981317 | 100 | 1.7453293 | 160 | 2.7925268 | 40 | .0116355 |
| 41 | .7155850 | 101 | 1.7627825 | 161 | 2.8099801 | 41 | .0119264 |
| 42 | .7330383 | 102 | 1.7802358 | 162 | 2.8274334 | 42 | .0122173 |
| 43 | .7504916 | 103 | 1.7976891 | 163 | 2.8448867 | 43 | .0125082 |
| 44 | .7679449 | 104 | 1.8151424 | 164 | 2.8623400 | 44 | .0127991 |
| 45 | .7853982 | 105 | 1.8325957 | 165 | 2.8797933 | 45 | .0130900 |
| 46 | .8028515 | 106 | 1.8500490 | 166 | 2.8972466 | 46 | .0133809 |
| 47 | .8203047 | 107 | 1.8675023 | 167 | 2.9146999 | 47 | .0136717 |
| 48 | .8377580 | 108 | 1.8849556 | 168 | 2.9321531 | 48 | .0139626 |
| 49 | .8552113 | 109 | 1.9024089 | 169 | 2.9496064 | 49 | .0142535 |
| 50 | .8726646 | 110 | 1.9198622 | 170 | 2.9670597 | 50 | .0145444 |
| 51 | .8901179 | 111 | 1.9373155 | 171 | 2.9845130 | 51 | .0148353 |
| 52 | .9075712 | 112 | 1.9547688 | 172 | 3.0019663 | 52 | .0151262 |
| 53 | .9250245 | 113 | 1.9722221 | 173 | 3.0194196 | 53 | .0154171 |
| 54 | .9424778 | 114 | 1.9896753 | 174 | 3.0368729 | 54 | .0157080 |
| 55 | .9599311 | 115 | 2.0071286 | 175 | 3.0543262 | 55 | .0159989 |
| 56 | .9773844 | 116 | 2.0245819 | 176 | 3.0717795 | 56 | .0162897 |
| 57 | .9948377 | 117 | 2.0420352 | 177 | 3.0892328 | 57 | .0165806 |
| 58 | 1.0122910 | 118 | 2.0594885 | 178 | 3.1066861 | 58 | .0168715 |
| 59 | 1.0297443 | 119 | 2.0769418 | 179 | 3.1241394 | 59 | .0171624 |
| 60 | 1.0471976 | 120 | 2.0943951 | 180 | 3.1415927 | 60 | .0174533 |

LENGTHS OF CIRCULAR ARCS.

(Diameter = 1. Given the Chord and Height of the Arc.)

RULE FOR USE OF THE TABLE. — Divide the height by the chord. Find in the column of heights the number equal to this quotient. Take out the corresponding number from the column of lengths. Multiply this last number by the length of the given chord; the product will be length of the arc.

If the arc is greater than a semicircle, first find the diameter from the formula, Diam. = (square of half chord ÷ rise) + rise; the formula is true whether the arc exceeds a semicircle or not. Then find the circumference. From the diameter subtract the given height of arc, the remainder will be height of the smaller arc of the circle; find its length according to the rule, and subtract it from the circumference.

| Hgts. | Lgths. | Hgts. | Lgths. | Hgts. | Lgths. | Hgts. | Lgths. | Hgts. | Lgths. |
|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|
| .001 | 1.00002 | .15 | 1.05896 | .238 | 1.14480 | .326 | 1.26288 | .414 | 1.40788 |
| .005 | 1.00007 | .152 | 1.06051 | .24 | 1.14714 | .328 | 1.26588 | .416 | 1.41145 |
| .01 | 1.00027 | .154 | 1.06209 | .242 | 1.14951 | .33 | 1.26892 | .418 | 1.41503 |
| .015 | 1.00061 | .156 | 1.06368 | .244 | 1.15189 | .332 | 1.27196 | .42 | 1.41861 |
| .02 | 1.00107 | .158 | 1.06530 | .246 | 1.15428 | .334 | 1.27502 | .422 | 1.42221 |
| .025 | 1.00167 | .16 | 1.06693 | .248 | 1.15670 | .336 | 1.27810 | .424 | 1.42583 |
| .03 | 1.00240 | .162 | 1.06858 | .25 | 1.15912 | .338 | 1.28118 | .426 | 1.42945 |
| .035 | 1.00327 | .164 | 1.07025 | .252 | 1.16156 | .34 | 1.28428 | .428 | 1.43309 |
| .04 | 1.00426 | .166 | 1.07194 | .254 | 1.16402 | .342 | 1.28739 | .43 | 1.43673 |
| .045 | 1.00539 | .168 | 1.07365 | .256 | 1.16650 | .344 | 1.29052 | .432 | 1.44039 |
| .05 | 1.00565 | .17 | 1.07537 | .258 | 1.16899 | .346 | 1.29366 | .434 | 1.44405 |
| .055 | 1.00805 | .172 | 1.07711 | .26 | 1.17150 | .348 | 1.29681 | .436 | 1.44773 |
| .06 | 1.00957 | .174 | 1.07883 | .262 | 1.17403 | .35 | 1.29997 | .438 | 1.45142 |
| .055 | 1.01123 | .176 | 1.08066 | .264 | 1.17657 | .352 | 1.30315 | .44 | 1.45512 |
| .07 | 1.01302 | .178 | 1.08246 | .266 | 1.17912 | .354 | 1.30634 | .442 | 1.45883 |
| .075 | 1.01493 | .18 | 1.08423 | .268 | 1.18169 | .356 | 1.30954 | .444 | 1.46255 |
| .08 | 1.01698 | .182 | 1.08611 | .27 | 1.18429 | .358 | 1.31276 | .446 | 1.46628 |
| .085 | 1.01916 | .184 | 1.08797 | .272 | 1.18689 | .36 | 1.31599 | .448 | 1.47002 |
| .09 | 1.02146 | .186 | 1.08934 | .274 | 1.18951 | .362 | 1.31923 | .45 | 1.47377 |
| .095 | 1.02389 | .188 | 1.09174 | .276 | 1.19214 | .364 | 1.32249 | .452 | 1.47753 |
| .10 | 1.02646 | .19 | 1.09365 | .278 | 1.19479 | .366 | 1.32577 | .454 | 1.48131 |
| .102 | 1.02752 | .192 | 1.09557 | .28 | 1.19746 | .368 | 1.32905 | .456 | 1.48509 |
| .104 | 1.02860 | .194 | 1.09752 | .282 | 1.20014 | .37 | 1.33234 | .458 | 1.48889 |
| .106 | 1.02970 | .196 | 1.09949 | .284 | 1.20284 | .372 | 1.33564 | .46 | 1.49269 |
| .108 | 1.03082 | .198 | 1.10147 | .286 | 1.20555 | .374 | 1.33896 | .462 | 1.49651 |
| .11 | 1.03196 | .20 | 1.10347 | .288 | 1.20827 | .376 | 1.34229 | .464 | 1.50033 |
| .112 | 1.03312 | .202 | 1.10548 | .29 | 1.21102 | .378 | 1.34563 | .466 | 1.50416 |
| .114 | 1.03430 | .204 | 1.10752 | .292 | 1.21377 | .38 | 1.34899 | .468 | 1.50800 |
| .116 | 1.03551 | .206 | 1.10958 | .294 | 1.21654 | .382 | 1.35237 | .47 | 1.51185 |
| .118 | 1.03672 | .208 | 1.11165 | .296 | 1.21933 | .384 | 1.35575 | .472 | 1.51571 |
| .12 | 1.03797 | .21 | 1.11374 | .298 | 1.22213 | .386 | 1.35914 | .474 | 1.51958 |
| .122 | 1.03923 | .212 | 1.11584 | .30 | 1.22495 | .388 | 1.36254 | .476 | 1.52346 |
| .124 | 1.04051 | .214 | 1.11796 | .302 | 1.22778 | .39 | 1.36596 | .478 | 1.52736 |
| .126 | 1.04181 | .216 | 1.12011 | .304 | 1.23063 | .392 | 1.36939 | .48 | 1.53126 |
| .128 | 1.04313 | .218 | 1.12225 | .306 | 1.23349 | .394 | 1.37283 | .482 | 1.53518 |
| .13 | 1.04447 | .22 | 1.12444 | .308 | 1.23636 | .396 | 1.37628 | .484 | 1.53910 |
| .132 | 1.04584 | .222 | 1.12664 | .31 | 1.23926 | .398 | 1.37974 | .486 | 1.54302 |
| .134 | 1.04722 | .224 | 1.12885 | .312 | 1.24216 | .40 | 1.38322 | .488 | 1.54696 |
| .136 | 1.04862 | .226 | 1.13108 | .314 | 1.24507 | .402 | 1.38671 | .49 | 1.55091 |
| .138 | 1.05003 | .228 | 1.13331 | .316 | 1.24801 | .404 | 1.39021 | .492 | 1.55487 |
| .14 | 1.05147 | .23 | 1.13557 | .318 | 1.25095 | .406 | 1.39372 | .494 | 1.55854 |
| .142 | 1.05293 | .232 | 1.13785 | .32 | 1.25291 | .408 | 1.39724 | .496 | 1.56282 |
| .144 | 1.05441 | .234 | 1.14015 | .322 | 1.25689 | .41 | 1.40077 | .498 | 1.56681 |
| .146 | 1.05591 | .236 | 1.14247 | .324 | 1.25988 | .412 | 1.40432 | .50 | 1.57080 |
| .148 | 1.05743 | | | | | | | | |

SPHERES.

(Some errors of 1 in the last figure only. From TRAUTWINE.)

| Diam. | Sur- face. | Vol- ume. | Diam. | Sur- face. | Vol- ume. | Diam. | Sur- face. | Vol- ume. |
|---------|---------------|--------------|---------|---------------|--------------|-------|---------------|--------------|
| 1/32 | .00307 | .00002 | 3 1/4 | 33.183 | 17.974 | 9 7/8 | 306.36 | 504.21 |
| 1/16 | .01227 | .00013 | 5 1/16 | 34.472 | 19.031 | 10. | 314.16 | 523.60 |
| 3/32 | .02761 | .00043 | 3/8 | 35.784 | 20.129 | 1 1/8 | 322.06 | 543.48 |
| 1/8 | .04909 | .00102 | 7/16 | 37.122 | 21.268 | 1/4 | 330.06 | 563.86 |
| 5/32 | .07670 | .00200 | 1/2 | 38.484 | 22.449 | 3/8 | 338.16 | 584.74 |
| 3/16 | .11045 | .00345 | 9/16 | 39.872 | 23.674 | 1/2 | 346.36 | 606.13 |
| 7/32 | .15033 | .00548 | 5/8 | 41.283 | 24.942 | 5/8 | 354.66 | 628.04 |
| 1/4 | .19635 | .00818 | 11 1/16 | 42.719 | 26.254 | 3/4 | 363.05 | 650.46 |
| 9/32 | .24851 | .01165 | 3/4 | 44.179 | 27.611 | 7/8 | 371.54 | 673.42 |
| 5/16 | .30680 | .01598 | 13 1/16 | 45.664 | 29.016 | 11. | 380.13 | 696.91 |
| 11 1/32 | .37123 | .02127 | 7/8 | 47.173 | 30.466 | 1 1/8 | 388.83 | 720.95 |
| 3/8 | .44179 | .02761 | 15 1/16 | 48.708 | 31.965 | 1/4 | 397.61 | 745.51 |
| 13 1/32 | .51848 | .03511 | 4. | 50.265 | 33.510 | 3/8 | 406.49 | 770.64 |
| 7/16 | .60132 | .04385 | 1/8 | 53.456 | 36.751 | 1/2 | 415.48 | 796.33 |
| 15 1/32 | .69028 | .05393 | 1/4 | 56.745 | 40.195 | 5/8 | 424.50 | 822.58 |
| 1/2 | .78540 | .06545 | 3/8 | 60.133 | 43.847 | 3/4 | 433.73 | 849.40 |
| 9/16 | .99403 | .09319 | 1/2 | 63.617 | 47.713 | 7/8 | 443.01 | 876.79 |
| 5/8 | 1.2272 | .12783 | 5/8 | 67.201 | 51.801 | 12. | 452.39 | 904.78 |
| 11 1/16 | 1.4849 | .17014 | 3/4 | 70.883 | 56.116 | 1 1/4 | 471.44 | 962.52 |
| 3/4 | 1.7671 | .22089 | 7/8 | 74.663 | 60.663 | 1 1/2 | 490.87 | 1022.7 |
| 13 1/16 | 2.0739 | .28084 | 5. | 78.540 | 65.450 | 3/4 | 510.71 | 1085.3 |
| 7/8 | 2.4053 | .35077 | 1/8 | 82.516 | 70.482 | 13. | 530.93 | 1150.3 |
| 15 1/16 | 2.7611 | .43143 | 1/4 | 86.591 | 75.767 | 1 1/4 | 551.55 | 1218.0 |
| 1 | 3.1416 | .52360 | 3/8 | 90.763 | 81.308 | 1 1/2 | 572.55 | 1288.3 |
| 1 1/16 | 3.5466 | .62804 | 1/2 | 95.033 | 87.113 | 3/4 | 593.95 | 1361.2 |
| 1/8 | 3.9761 | .74551 | 5/8 | 99.401 | 93.189 | 14. | 615.75 | 1436.8 |
| 3/16 | 4.4301 | .87681 | 3/4 | 103.87 | 99.541 | 1 1/4 | 637.95 | 1515.1 |
| 1/4 | 4.9088 | 1.0227 | 7/8 | 108.44 | 106.18 | 1 1/2 | 660.52 | 1596.3 |
| 5/16 | 5.4119 | 1.1839 | 6. | 113.10 | 113.10 | 3/4 | 683.49 | 1680.3 |
| 3/8 | 5.9396 | 1.3611 | 1/8 | 117.87 | 120.31 | 15. | 706.85 | 1767.2 |
| 7/16 | 6.4919 | 1.5553 | 1/4 | 122.72 | 127.83 | 1 1/4 | 730.63 | 1857.0 |
| 1/2 | 7.0686 | 1.7671 | 3/8 | 127.68 | 135.66 | 1 1/2 | 754.77 | 1949.8 |
| 9/16 | 7.6699 | 1.9974 | 1/2 | 132.73 | 143.79 | 3/4 | 779.32 | 2045.7 |
| 5/8 | 8.2957 | 2.2468 | 5/8 | 137.89 | 152.25 | 16. | 804.25 | 2144.7 |
| 11 1/16 | 8.9461 | 2.5161 | 3/4 | 143.14 | 161.03 | 1 1/4 | 829.57 | 2246.8 |
| 3/4 | 9.6211 | 2.8062 | 7/8 | 148.49 | 170.14 | 1 1/2 | 855.29 | 2352.1 |
| 13 1/16 | 10.321 | 3.1177 | 7. | 153.94 | 179.59 | 3/4 | 881.42 | 2460.6 |
| 7/8 | 11.044 | 3.4514 | 1/8 | 159.49 | 189.39 | 17. | 907.93 | 2572.4 |
| 15 1/16 | 11.793 | 3.8083 | 1/4 | 165.13 | 199.53 | 1 1/4 | 934.83 | 2687.6 |
| 2. | 12.566 | 4.1888 | 3/8 | 170.87 | 210.03 | 1 1/2 | 962.12 | 2806.2 |
| 1 1/16 | 13.364 | 4.5939 | 1/2 | 176.71 | 220.89 | 3/4 | 989.80 | 2928.2 |
| 1/8 | 14.186 | 5.0243 | 5/8 | 182.66 | 232.13 | 18. | 1017.9 | 3053.6 |
| 3/16 | 15.033 | 5.4809 | 3/4 | 188.69 | 243.73 | 1 1/4 | 1046.4 | 3182.6 |
| 1/4 | 15.904 | 5.9641 | 7/8 | 194.83 | 255.72 | 1 1/2 | 1075.2 | 3315.3 |
| 5/16 | 16.800 | 6.4751 | 8. | 201.06 | 268.08 | 3/4 | 1104.5 | 3451.5 |
| 3/8 | 17.721 | 7.0144 | 1/8 | 207.39 | 280.85 | 19. | 1134.1 | 3591.4 |
| 7/16 | 18.666 | 7.5829 | 1/4 | 213.82 | 294.01 | 1 1/4 | 1164.2 | 3735.0 |
| 1/2 | 19.635 | 8.1813 | 3/8 | 220.36 | 307.58 | 1 1/2 | 1194.6 | 3882.5 |
| 9/16 | 20.629 | 8.8103 | 1/2 | 226.98 | 321.56 | 3/4 | 1225.4 | 4033.7 |
| 5/8 | 21.648 | 9.4708 | 5/8 | 233.71 | 335.95 | 20. | 1256.7 | 4188.8 |
| 11 1/16 | 22.691 | 10.164 | 3/4 | 240.53 | 350.77 | 1 1/4 | 1288.3 | 4347.8 |
| 3/4 | 23.758 | 10.889 | 7/8 | 247.45 | 366.02 | 1 1/2 | 1320.3 | 4510.9 |
| 13 1/16 | 24.850 | 11.649 | 9. | 254.47 | 381.70 | 3/4 | 1352.7 | 4677.9 |
| 7/8 | 25.967 | 12.443 | 1/8 | 261.59 | 397.83 | 21. | 1385.5 | 4849.1 |
| 15 1/16 | 27.109 | 13.272 | 1/4 | 268.81 | 414.41 | 1 1/4 | 1418.6 | 5024.3 |
| 3. | 28.274 | 14.137 | 3/8 | 270.12 | 431.44 | 1 1/2 | 1452.2 | 5203.7 |
| 1 1/16 | 29.465 | 15.039 | 1/2 | 283.53 | 448.92 | 3/4 | 1486.2 | 5387.4 |
| 1/8 | 30.680 | 15.979 | 5/8 | 291.04 | 466.87 | 22. | 1520.5 | 5575.3 |
| 3/16 | 31.919 | 16.957 | 3/4 | 289.65 | 485.31 | 1 1/4 | 1555.3 | 5767.6 |

SPHERES — *Continued.*

| Diam. | Sur- face. | Vol- ume. | Diam. | Sur- face. | Vol- ume. | Diam. | Sur- face. | Vol- ume. |
|------------------|---------------|--------------|------------------|---------------|--------------|------------------|---------------|--------------|
| 22 $\frac{1}{2}$ | 1590.4 | 5964.1 | 40 $\frac{1}{2}$ | 5153.1 | 34763 | 70 $\frac{1}{2}$ | 15615 | 183471 |
| 3/4 | 1626.0 | 6165.2 | 41. | 5281.1 | 36087 | 71. | 15837 | 187402 |
| 23. | 1661.9 | 6370.6 | $\frac{1}{2}$ | 5410.7 | 37423 | $\frac{1}{2}$ | 16061 | 191389 |
| $\frac{1}{4}$ | 1698.2 | 6580.6 | 42. | 5541.9 | 38792 | 72. | 16286 | 195433 |
| $\frac{1}{2}$ | 1735.0 | 6795.2 | $\frac{1}{2}$ | 5674.5 | 40194 | $\frac{1}{2}$ | 16513 | 199532 |
| 3/4 | 1772.1 | 7014.3 | 43. | 5808.8 | 41630 | 73. | 16742 | 203689 |
| 24. | 1809.6 | 7238.2 | $\frac{1}{2}$ | 5944.7 | 43099 | $\frac{1}{2}$ | 16972 | 207903 |
| $\frac{1}{4}$ | 1847.5 | 7466.7 | 44. | 6082.1 | 44602 | 74. | 17204 | 212175 |
| $\frac{1}{2}$ | 1885.8 | 7700.1 | $\frac{1}{2}$ | 6221.2 | 46141 | $\frac{1}{2}$ | 17437 | 216505 |
| 3/4 | 1924.4 | 7938.3 | 45. | 6361.7 | 47713 | 75. | 17672 | 220894 |
| 25. | 1963.5 | 8181.3 | $\frac{1}{2}$ | 6503.9 | 49321 | $\frac{1}{2}$ | 17908 | 225341 |
| $\frac{1}{4}$ | 2002.9 | 8429.2 | 46. | 6647.6 | 50965 | 76. | 18146 | 229848 |
| $\frac{1}{2}$ | 2042.8 | 8682.0 | $\frac{1}{2}$ | 6792.9 | 52645 | $\frac{1}{2}$ | 18386 | 234414 |
| 3/4 | 2083.0 | 8939.9 | 47. | 6939.9 | 54362 | 77. | 18626 | 239041 |
| 26. | 2123.7 | 9202.8 | $\frac{1}{2}$ | 7088.3 | 56115 | $\frac{1}{2}$ | 18869 | 243728 |
| $\frac{1}{4}$ | 2164.7 | 9470.8 | 48. | 7238.3 | 57906 | 78. | 19114 | 248475 |
| $\frac{1}{2}$ | 2206.2 | 9744.0 | $\frac{1}{2}$ | 7389.9 | 59734 | $\frac{1}{2}$ | 19360 | 253284 |
| 3/4 | 2248.0 | 10022 | 49. | 7543.1 | 61601 | 79. | 19607 | 258155 |
| 27. | 2290.2 | 10306 | $\frac{1}{2}$ | 7697.7 | 63506 | $\frac{1}{2}$ | 19856 | 263088 |
| $\frac{1}{4}$ | 2332.8 | 10595 | 50. | 7854.0 | 65450 | 80. | 20106 | 268083 |
| $\frac{1}{2}$ | 2375.8 | 10889 | $\frac{1}{2}$ | 8011.8 | 67433 | $\frac{1}{2}$ | 20358 | 273141 |
| 3/4 | 2419.2 | 11189 | 51. | 8171.2 | 69456 | 81. | 20612 | 278263 |
| 28. | 2463.0 | 11494 | $\frac{1}{2}$ | 8332.3 | 71519 | $\frac{1}{2}$ | 20867 | 283447 |
| $\frac{1}{4}$ | 2507.2 | 11805 | 52. | 8494.8 | 73622 | 82. | 21124 | 288696 |
| $\frac{1}{2}$ | 2551.8 | 12121 | $\frac{1}{2}$ | 8658.9 | 75767 | $\frac{1}{2}$ | 21382 | 294010 |
| 3/4 | 2596.7 | 12443 | 53. | 8824.8 | 77952 | 83. | 21642 | 299388 |
| 29. | 2642.1 | 12770 | $\frac{1}{2}$ | 8992.0 | 80178 | $\frac{1}{2}$ | 21904 | 304831 |
| $\frac{1}{4}$ | 2687.8 | 13103 | 54. | 9160.8 | 82448 | 84. | 22167 | 310340 |
| $\frac{1}{2}$ | 2734.0 | 13442 | $\frac{1}{2}$ | 9331.2 | 84760 | $\frac{1}{2}$ | 22432 | 315915 |
| 3/4 | 2780.5 | 13787 | 55. | 9503.2 | 87114 | 85. | 22698 | 321556 |
| 30. | 2827.4 | 14137 | $\frac{1}{2}$ | 9676.8 | 89511 | $\frac{1}{2}$ | 22966 | 327264 |
| $\frac{1}{4}$ | 2874.8 | 14494 | 56. | 9852.0 | 91953 | 86. | 23235 | 333039 |
| $\frac{1}{2}$ | 2922.5 | 14856 | $\frac{1}{2}$ | 10029 | 94438 | $\frac{1}{2}$ | 23506 | 338882 |
| 3/4 | 2970.6 | 15224 | 57. | 10207 | 96967 | 87. | 23779 | 344792 |
| 31. | 3019.1 | 15599 | $\frac{1}{2}$ | 10387 | 99541 | $\frac{1}{2}$ | 24053 | 350771 |
| $\frac{1}{4}$ | 3068.0 | 15979 | 58. | 10568 | 102161 | 88. | 24328 | 356819 |
| $\frac{1}{2}$ | 3117.3 | 16366 | $\frac{1}{2}$ | 10751 | 104826 | $\frac{1}{2}$ | 24606 | 362935 |
| 3/4 | 3166.9 | 16758 | 59. | 10936 | 107536 | 89. | 24885 | 369122 |
| 32. | 3217.0 | 17157 | $\frac{1}{2}$ | 11122 | 110294 | $\frac{1}{2}$ | 25165 | 375378 |
| $\frac{1}{4}$ | 3267.4 | 17563 | 60. | 11310 | 113098 | 90. | 25447 | 381704 |
| $\frac{1}{2}$ | 3318.3 | 17974 | $\frac{1}{2}$ | 11499 | 115949 | $\frac{1}{2}$ | 25730 | 388102 |
| 3/4 | 3369.6 | 18392 | 61. | 11690 | 118847 | 91. | 26016 | 394570 |
| 33. | 3421.2 | 18817 | $\frac{1}{2}$ | 11882 | 121794 | $\frac{1}{2}$ | 26302 | 401109 |
| $\frac{1}{4}$ | 3473.3 | 19248 | 62. | 12076 | 124789 | 92. | 26590 | 407721 |
| $\frac{1}{2}$ | 3525.7 | 19685 | $\frac{1}{2}$ | 12272 | 127832 | $\frac{1}{2}$ | 26880 | 414405 |
| 3/4 | 3578.5 | 20129 | 63. | 12469 | 130925 | 93. | 27172 | 421161 |
| 34. | 3631.7 | 20580 | $\frac{1}{2}$ | 12668 | 134067 | $\frac{1}{2}$ | 27464 | 427991 |
| $\frac{1}{4}$ | 3685.3 | 21037 | 64. | 12868 | 137259 | 94. | 27759 | 434894 |
| $\frac{1}{2}$ | 3739.3 | 21501 | $\frac{1}{2}$ | 13070 | 140501 | $\frac{1}{2}$ | 28055 | 441871 |
| 35. | 3848.5 | 22449 | 65. | 13273 | 143794 | 95. | 28353 | 448920 |
| $\frac{1}{2}$ | 3959.2 | 23425 | $\frac{1}{2}$ | 13478 | 147138 | $\frac{1}{2}$ | 28652 | 456047 |
| 36. | 4071.5 | 24429 | 66. | 13685 | 150533 | 96. | 28953 | 463248 |
| $\frac{1}{2}$ | 4185.5 | 25461 | $\frac{1}{2}$ | 13893 | 153980 | $\frac{1}{2}$ | 29255 | 470524 |
| 37. | 4300.9 | 26522 | 67. | 14103 | 157480 | 97. | 29559 | 477874 |
| $\frac{1}{2}$ | 4417.9 | 27612 | $\frac{1}{2}$ | 14314 | 161032 | $\frac{1}{2}$ | 29865 | 485302 |
| 38. | 4536.5 | 28731 | 68. | 14527 | 164637 | 98. | 30172 | 492808 |
| $\frac{1}{2}$ | 4656.7 | 29880 | $\frac{1}{2}$ | 14741 | 168295 | $\frac{1}{2}$ | 30481 | 500388 |
| 39. | 4778.4 | 31059 | 69. | 14957 | 172007 | 99. | 30791 | 508047 |
| $\frac{1}{2}$ | 4901.7 | 32270 | $\frac{1}{2}$ | 15175 | 175774 | $\frac{1}{2}$ | 31103 | 515785 |
| 40. | 5026.5 | 33510 | 70. | 15394 | 179595 | 100. | 31416 | 523598 |

CONTENTS IN CUBIC FEET AND U. S. GALLONS OF PIPES AND CYLINDERS OF VARIOUS DIAMETERS AND ONE FOOT IN LENGTH.

1 gallon = 231 cubic inches. 1 cubic foot = 7.4805 gallons.

| Diameter in Inches. | For 1 Foot in Length. | | Diameter in Inches. | For 1 Foot in Length. | | Diameter in Inches. | For 1 Foot in Length. | |
|------------------------|---------------------------------------|----------------------------------|------------------------|---------------------------------------|----------------------------------|------------------------|---------------------------------------|----------------------------------|
| | Cu. Ft. also Area in Sq. Ft. | U.S. Gals., 231 Cu. In. | | Cu. Ft. also Area in Sq. Ft. | U.S. Gals., 231 Cu. In. | | Cu. Ft. also Area in Sq. Ft. | U.S. Gals., 231 Cu. In. |
| 1/4 | .0003 | .0025 | 63/4 | .2485 | 1.859 | 19 | 1.969 | 14.73 |
| 5/16 | .0005 | .004 | 7 | .2673 | 1.999 | 19 1/2 | 2.074 | 15.51 |
| 3/8 | .0008 | .0057 | 7 1/4 | .2867 | 2.145 | 20 | 2.182 | 16.32 |
| 7/16 | .001 | .0078 | 7 1/2 | .3068 | 2.295 | 20 1/2 | 2.292 | 17.15 |
| 1/2 | .0014 | .0102 | 7 3/4 | .3276 | 2.45 | 21 | 2.405 | 17.99 |
| 9/16 | .0017 | .0129 | 8 | .3491 | 2.611 | 21 1/2 | 2.521 | 18.86 |
| 5/8 | .0021 | .0159 | 8 1/4 | .3712 | 2.777 | 22 | 2.640 | 19.75 |
| 11/16 | .0026 | .0193 | 8 1/2 | .3941 | 2.948 | 22 1/2 | 2.761 | 20.66 |
| 3/4 | .0031 | .0230 | 8 3/4 | .4176 | 3.125 | 23 | 2.885 | 21.58 |
| 13/16 | .0036 | .0269 | 9 | .4418 | 3.305 | 23 1/2 | 3.012 | 22.53 |
| 7/8 | .0042 | .0312 | 9 1/4 | .4667 | 3.491 | 24 | 3.142 | 23.50 |
| 15/16 | .0048 | .0359 | 9 1/2 | .4922 | 3.682 | 25 | 3.409 | 25.50 |
| 1 | .0055 | .0408 | 9 3/4 | .5185 | 3.879 | 26 | 3.687 | 27.58 |
| 1 1/4 | .0085 | .0638 | 10 | .5454 | 4.08 | 27 | 3.976 | 29.74 |
| 1 1/2 | .0123 | .0918 | 10 1/4 | .5730 | 4.286 | 28 | 4.276 | 31.99 |
| 1 3/4 | .0167 | .1249 | 10 1/2 | .6013 | 4.498 | 29 | 4.587 | 34.31 |
| 2 | .0218 | .1632 | 10 3/4 | .6303 | 4.715 | 30 | 4.909 | 36.72 |
| 2 1/4 | .0276 | .2066 | 11 | .66 | 4.937 | 31 | 5.241 | 39.21 |
| 2 1/2 | .0341 | .2550 | 11 1/4 | .6903 | 5.164 | 32 | 5.585 | 41.78 |
| 2 3/4 | .0412 | .3085 | 11 1/2 | .7213 | 5.396 | 33 | 5.940 | 44.43 |
| 3 | .0491 | .3672 | 11 3/4 | .7530 | 5.633 | 34 | 6.305 | 47.16 |
| 3 1/4 | .0576 | .4309 | 12 | .7854 | 5.875 | 35 | 6.681 | 49.98 |
| 3 1/2 | .0668 | .4998 | 12 1/2 | .8522 | 6.375 | 36 | 7.069 | 52.88 |
| 3 3/4 | .0767 | .5738 | 13 | .9218 | 6.895 | 37 | 7.467 | 55.86 |
| 4 | .0873 | .6528 | 13 1/2 | .994 | 7.436 | 38 | 7.876 | 58.92 |
| 4 1/4 | .0985 | .7369 | 14 | 1.069 | 7.997 | 39 | 8.296 | 62.06 |
| 4 1/2 | .1104 | .8263 | 14 1/2 | 1.147 | 8.578 | 40 | 8.727 | 65.28 |
| 4 3/4 | .1231 | .9206 | 15 | 1.227 | 9.180 | 41 | 9.168 | 68.58 |
| 5 | .1364 | 1.020 | 15 1/2 | 1.310 | 9.801 | 42 | 9.621 | 71.97 |
| 5 1/4 | .1503 | 1.125 | 16 | 1.396 | 10.44 | 43 | 10.085 | 75.44 |
| 5 1/2 | .1650 | 1.234 | 16 1/2 | 1.485 | 11.11 | 44 | 10.559 | 78.99 |
| 5 3/4 | .1803 | 1.349 | 17 | 1.576 | 11.79 | 45 | 11.045 | 82.62 |
| 6 | .1963 | 1.469 | 17 1/2 | 1.670 | 12.49 | 46 | 11.541 | 86.33 |
| 6 1/4 | .2131 | 1.594 | 18 | 1.768 | 13.22 | 47 | 12.048 | 90.10 |
| 6 1/2 | .2304 | 1.724 | 18 1/2 | 1.867 | 13.96 | 48 | 12.566 | 94.00 |

To find the capacity of pipes greater than the largest given in the table, look in the table for a pipe of one-half the given size, and multiply its capacity by 4; or one of one-third its size, and multiply its capacity by 9, etc.

To find the *weight* of water in any of the given sizes, multiply the capacity in cubic feet by 62 1/4 or the gallons by 8 1/3, or, if a closer approximation is required, by the weight of a cubic foot of water at the actual temperature in the pipe.

Given the dimensions of a cylinder in inches, to find its capacity in U. S. gallons: Square the diameter, multiply by the length and by 0.0034. If d = diameter, l = length, gallons = $\frac{d^2 \times 0.7854 \times l}{231} = 0.0034 d^2 l$. If D and L are in feet, gallons = $5.875 D^2 L$.

CYLINDRICAL VESSELS, TANKS, CISTERNS, ETC.

Diameter in Feet and Inches, Area in Square Feet, and U. S. Gallons Capacity for One Foot in Depth.

$$1 \text{ gallon} = 231 \text{ cubic inches} = \frac{1 \text{ cubic foot}}{7.4805} = 0.13368 \text{ cubic feet.}$$

| Diam. | Area. | Gals. | Diam. | Area. | Gals. | Diam. | Area. | Gals. |
|---------|---------|---------------|---------|---------|---------------|---------|---------|---------------|
| Ft. In. | Sq. ft. | 1 foot depth. | Ft. In. | Sq. ft. | 1 foot depth. | Ft. In. | Sq. ft. | 1 foot depth. |
| 1 | .785 | 5.87 | 5 8 | 25.22 | 188.66 | 19 | 283.53 | 2120.9 |
| 1 1 | .922 | 6.89 | 5 9 | 25.97 | 194.25 | 19 3 | 291.04 | 2177.1 |
| 1 2 | 1.069 | 8.00 | 5 10 | 26.73 | 199.92 | 19 6 | 298.65 | 2234.0 |
| 1 3 | 1.227 | 9.18 | 5 11 | 27.49 | 205.67 | 19 9 | 306.35 | 2291.7 |
| 1 4 | 1.396 | 10.44 | 6 | 28.27 | 211.51 | 20 | 314.16 | 2350.1 |
| 1 5 | 1.576 | 11.79 | 6 3 | 30.68 | 229.50 | 20 3 | 322.06 | 2409.2 |
| 1 6 | 1.767 | 13.22 | 6 6 | 33.18 | 248.23 | 20 6 | 330.06 | 2469.1 |
| 1 7 | 1.969 | 14.73 | 6 9 | 35.78 | 267.69 | 20 9 | 338.16 | 2529.6 |
| 1 8 | 2.182 | 16.32 | 7 | 38.48 | 287.88 | 21 | 346.36 | 2591.0 |
| 1 9 | 2.405 | 17.99 | 7 3 | 41.28 | 308.81 | 21 3 | 354.66 | 2653.0 |
| 1 10 | 2.640 | 19.75 | 7 6 | 44.18 | 330.48 | 21 6 | 363.05 | 2715.8 |
| 1 11 | 2.885 | 21.58 | 7 9 | 47.17 | 352.88 | 21 9 | 371.54 | 2779.3 |
| 2 | 3.142 | 23.50 | 8 | 50.27 | 376.01 | 22 | 380.13 | 2843.6 |
| 2 1 | 3.409 | 25.50 | 8 3 | 53.46 | 399.88 | 22 3 | 388.82 | 2908.6 |
| 2 2 | 3.687 | 27.58 | 8 6 | 56.75 | 424.48 | 22 6 | 397.61 | 2974.3 |
| 2 3 | 3.976 | 29.74 | 8 9 | 60.13 | 449.82 | 22 9 | 406.49 | 3040.8 |
| 2 4 | 4.276 | 31.99 | 9 | 63.62 | 475.89 | 23 | 415.48 | 3108.0 |
| 2 5 | 4.587 | 34.31 | 9 3 | 67.20 | 502.70 | 23 3 | 424.56 | 3175.9 |
| 2 6 | 4.909 | 36.72 | 9 6 | 70.88 | 530.24 | 23 6 | 433.74 | 3244.6 |
| 2 7 | 5.241 | 39.21 | 9 9 | 74.66 | 558.51 | 23 9 | 443.01 | 3314.0 |
| 2 8 | 5.585 | 41.78 | 10 | 78.54 | 587.52 | 24 | 452.39 | 3384.1 |
| 2 9 | 5.940 | 44.43 | 10 3 | 82.52 | 617.26 | 24 3 | 461.86 | 3455.0 |
| 2 10 | 6.305 | 47.16 | 10 6 | 86.59 | 647.74 | 24 6 | 471.44 | 3526.6 |
| 2 11 | 6.681 | 49.98 | 10 9 | 90.76 | 678.95 | 24 9 | 481.11 | 3598.9 |
| 3 | 7.069 | 52.88 | 11 | 95.03 | 710.90 | 25 | 490.87 | 3672.0 |
| 3 1 | 7.467 | 55.86 | 11 3 | 99.40 | 743.58 | 25 3 | 500.74 | 3745.8 |
| 3 2 | 7.876 | 58.92 | 11 6 | 103.87 | 776.99 | 25 6 | 510.71 | 3820.3 |
| 3 3 | 8.296 | 62.06 | 11 9 | 108.43 | 811.14 | 25 9 | 520.77 | 3895.6 |
| 3 4 | 8.727 | 65.28 | 12 | 113.10 | 846.03 | 26 | 530.93 | 3971.6 |
| 3 5 | 9.168 | 68.58 | 12 3 | 117.86 | 881.65 | 26 3 | 541.19 | 4048.4 |
| 3 6 | 9.621 | 71.97 | 12 6 | 122.72 | 918.00 | 26 6 | 551.55 | 4125.9 |
| 3 7 | 10.085 | 75.44 | 12 9 | 127.68 | 955.09 | 26 9 | 562.00 | 4204.1 |
| 3 8 | 10.559 | 78.99 | 13 | 132.73 | 992.91 | 27 | 572.56 | 4283.0 |
| 3 9 | 11.045 | 82.62 | 13 3 | 137.89 | 1031.5 | 27 3 | 583.21 | 4362.7 |
| 3 10 | 11.541 | 86.33 | 13 6 | 143.14 | 1070.8 | 27 6 | 593.96 | 4443.1 |
| 3 11 | 12.048 | 90.13 | 13 9 | 148.49 | 1110.8 | 27 9 | 604.81 | 4524.3 |
| 4 | 12.566 | 94.00 | 14 | 153.94 | 1151.5 | 28 | 615.75 | 4606.2 |
| 4 1 | 13.095 | 97.96 | 14 3 | 159.48 | 1193.0 | 28 3 | 626.80 | 4688.8 |
| 4 2 | 13.635 | 102.00 | 14 6 | 165.13 | 1235.3 | 28 6 | 637.94 | 4772.1 |
| 4 3 | 14.186 | 106.12 | 14 9 | 170.87 | 1278.2 | 28 9 | 649.18 | 4856.2 |
| 4 4 | 14.748 | 110.32 | 15 | 176.71 | 1321.9 | 29 | 660.52 | 4941.0 |
| 4 5 | 15.321 | 114.61 | 15 3 | 182.65 | 1366.4 | 29 3 | 671.96 | 5026.6 |
| 4 6 | 15.90 | 118.97 | 15 6 | 188.69 | 1411.5 | 29 6 | 683.49 | 5112.9 |
| 4 7 | 16.50 | 123.42 | 15 9 | 194.83 | 1457.4 | 29 9 | 695.13 | 5199.9 |
| 4 8 | 17.10 | 127.95 | 16 | 201.06 | 1504.1 | 30 | 706.86 | 5287.7 |
| 4 9 | 17.72 | 132.56 | 16 3 | 207.39 | 1551.4 | 30 3 | 718.69 | 5376.2 |
| 4 10 | 18.35 | 137.25 | 16 6 | 213.82 | 1599.5 | 30 6 | 730.62 | 5465.4 |
| 4 11 | 18.99 | 142.02 | 16 9 | 220.35 | 1648.4 | 30 9 | 742.64 | 5555.4 |
| 5 | 19.63 | 146.88 | 17 | 226.98 | 1697.9 | 31 | 754.77 | 5646.1 |
| 5 1 | 20.29 | 151.82 | 17 3 | 233.71 | 1748.2 | 31 3 | 766.99 | 5737.5 |
| 5 2 | 20.97 | 156.83 | 17 6 | 240.53 | 1799.3 | 31 6 | 779.31 | 5829.7 |
| 5 3 | 21.65 | 161.93 | 17 9 | 247.45 | 1851.1 | 31 9 | 791.73 | 5922.6 |
| 5 4 | 22.34 | 167.12 | 18 | 254.47 | 1903.6 | 32 | 804.25 | 6016.2 |
| 5 5 | 23.04 | 172.38 | 18 3 | 261.59 | 1956.8 | 32 3 | 816.86 | 6110.6 |
| 5 6 | 23.76 | 177.72 | 18 6 | 268.80 | 2010.8 | 32 6 | 829.58 | 6205.7 |
| 5 7 | 24.48 | 183.15 | 18 9 | 276.12 | 2065.5 | 32 9 | 842.39 | 6301.5 |

GALLONS AND CUBIC FEET.

United States Gallons in a given Number of Cubic Feet.

1 cubic foot = 7.480519 U.S. gallons; 1 gallon = 231 cu. in. = 0.13368056 cu. ft.

| Cubic Ft. | Gallons. | Cubic Ft. | Gallons. | Cubic Ft. | Gallons. |
|-----------|----------|-----------|----------|-----------|-------------|
| 0.1 | 0.75 | 50 | 374.0 | 8,000 | 59,844.2 |
| 0.2 | 1.50 | 60 | 448.8 | 9,000 | 67,324.7 |
| 0.3 | 2.24 | 70 | 523.6 | 10,000 | 74,805.2 |
| 0.4 | 2.99 | 80 | 598.4 | 20,000 | 149,610.4 |
| 0.5 | 3.74 | 90 | 673.2 | 30,000 | 224,415.6 |
| 0.6 | 4.49 | 100 | 748.0 | 40,000 | 299,220.8 |
| 0.7 | 5.24 | 200 | 1,496.1 | 50,000 | 374,025.9 |
| 0.8 | 5.98 | 300 | 2,244.2 | 60,000 | 448,831.1 |
| 0.9 | 6.73 | 400 | 2,992.2 | 70,000 | 523,636.3 |
| 1 | 7.48 | 500 | 3,740.3 | 80,000 | 598,441.5 |
| 2 | 14.96 | 600 | 4,488.3 | 90,000 | 673,246. |
| 3 | 22.44 | 700 | 5,236.4 | 100,000 | 748,051.9 |
| 4 | 29.92 | 800 | 5,984.4 | 200,000 | 1,496,103.8 |
| 5 | 37.40 | 900 | 6,732.5 | 300,000 | 2,244,155.7 |
| 6 | 44.88 | 1,000 | 7,480.5 | 400,000 | 2,992,207.6 |
| 7 | 52.36 | 2,000 | 14,961.0 | 500,000 | 3,740,259.5 |
| 8 | 59.84 | 3,000 | 22,441.6 | 600,000 | 4,488,311.4 |
| 9 | 67.32 | 4,000 | 29,922.1 | 700,000 | 5,236,363.3 |
| 10 | 74.80 | 5,000 | 37,402.6 | 800,000 | 5,984,415.2 |
| 20 | 149.6 | 6,000 | 44,883.1 | 900,000 | 6,732,467.1 |
| 30 | 224.4 | 7,000 | 52,363.6 | 1,000,000 | 7,480,519.0 |
| 40 | 299.2 | | | | |

Cubic Feet in a given Number of Gallons.

| Gallons. | Cubic Ft. | Gallons. | Cubic Ft. | Gallons. | Cubic Ft. |
|----------|-----------|----------|-----------|------------|-------------|
| 1 | .134 | 1,000 | 133.681 | 1,000,000 | 133,680.6 |
| 2 | .267 | 2,000 | 267.361 | 2,000,000 | 267,361.1 |
| 3 | .401 | 3,000 | 401.042 | 3,000,000 | 401,041.7 |
| 4 | .535 | 4,000 | 534.722 | 4,000,000 | 534,722.2 |
| 5 | .668 | 5,000 | 668.403 | 5,000,000 | 668,402.8 |
| 6 | .802 | 6,000 | 802.083 | 6,000,000 | 802,083.3 |
| 7 | .936 | 7,000 | 935.764 | 7,000,000 | 935,763.9 |
| 8 | 1.069 | 8,000 | 1,069.444 | 8,000,000 | 1,069,444.4 |
| 9 | 1.203 | 9,000 | 1,203.125 | 9,000,000 | 1,203,125.0 |
| 10 | 1.337 | 10,000 | 1,336.806 | 10,000,000 | 1,336,805.6 |

Cubic Feet per Second, Gallons in 24 hours, etc.

| | | | | |
|---|-----------|---------|-----------|-----------|
| Cu. ft. per sec. | 1/60 | 1 | 1.5472 | 2.2801 |
| Cu. ft. per min. | 1 | 60 | 92.834 | 133.681 |
| U. S. Gals. per min. | 7.480519 | 448.31 | 694.444 | 1,000. |
| " 24 hrs. | 10,771.95 | 646,317 | 1,000,000 | 1,440,000 |
| Pounds of water (at 62° F.) per min. } | 62.355 | 3741.3 | 5788.66 | 8335.65 |

The gallon is a troublesome and unnecessary measure. If hydraulic engineers and pump manufacturers would stop using it, and use cubic feet instead, many tedious calculations would be saved.

NUMBER OF SQUARE FEET IN PLATES 3 TO 32 FEET LONG, AND 1 INCH WIDE.

For other widths, multiply by the width in inches. 1 sq. in. = 0.00694/9 sq. ft.

| Ft. and Ins. Long. | Ins. Long. | Square Feet. | Ft. and Ins. Long. | Ins. Long. | Square Feet. | Ft. and Ins. Long. | Ins. Long. | Square Feet. |
|--------------------------|---------------|-----------------|--------------------------|---------------|-----------------|--------------------------|---------------|-----------------|
| 3. 0 | 36 | .25 | 7. 10 | 94 | .6528 | 12. 8 | 152 | 1.056 |
| 1 | 37 | .2569 | 11 | 95 | .6597 | 9 | 153 | 1.063 |
| 2 | 38 | .2639 | 8. 0 | 96 | .6667 | 10 | 154 | 1.069 |
| 3 | 39 | .2708 | 1 | 97 | .6736 | 11 | 155 | 1.076 |
| 4 | 40 | .2778 | 2 | 98 | .6806 | 13. 0 | 156 | 1.083 |
| 5 | 41 | .2847 | 3 | 99 | .6875 | 1 | 157 | 1.09 |
| 6 | 42 | .2917 | 4 | 100 | .6944 | 2 | 158 | 1.097 |
| 7 | 43 | .2986 | 5 | 101 | .7014 | 3 | 159 | 1.104 |
| 8 | 44 | .3056 | 6 | 102 | .7083 | 4 | 160 | 1.114 |
| 9 | 45 | .3125 | 7 | 103 | .7153 | 5 | 161 | 1.118 |
| 10 | 46 | .3194 | 8 | 104 | .7222 | 6 | 162 | 1.125 |
| 11 | 47 | .3264 | 9 | 105 | .7292 | 7 | 163 | 1.132 |
| 4. 0 | 48 | .3333 | 10 | 106 | .7361 | 8 | 164 | 1.139 |
| 1 | 49 | .3403 | 11 | 107 | .7431 | 9 | 165 | 1.146 |
| 2 | 50 | .3472 | 9. 0 | 108 | .75 | 10 | 166 | 1.153 |
| 3 | 51 | .3542 | 1 | 109 | .7569 | 11 | 167 | 1.159 |
| 4 | 52 | .3611 | 2 | 110 | .7639 | 14. 0 | 168 | 1.167 |
| 5 | 53 | .3681 | 3 | 111 | .7708 | 1 | 169 | 1.174 |
| 6 | 54 | .375 | 4 | 112 | .7778 | 2 | 170 | 1.181 |
| 7 | 55 | .3819 | 5 | 113 | .7847 | 3 | 171 | 1.188 |
| 8 | 56 | .3889 | 6 | 114 | .7917 | 4 | 172 | 1.194 |
| 9 | 57 | .3958 | 7 | 115 | .7986 | 5 | 173 | 1.201 |
| 10 | 58 | .4028 | 8 | 116 | .8056 | 6 | 174 | 1.208 |
| 11 | 59 | .4097 | 9 | 117 | .8125 | 7 | 175 | 1.215 |
| 5. 0 | 60 | .4167 | 10 | 118 | .8194 | 8 | 176 | 1.222 |
| 1 | 61 | .4236 | 11 | 119 | .8264 | 9 | 177 | 1.229 |
| 2 | 62 | .4306 | 10. 0 | 120 | .8333 | 10 | 178 | 1.236 |
| 3 | 63 | .4375 | 1 | 121 | .8403 | 11 | 179 | 1.243 |
| 4 | 64 | .4444 | 2 | 122 | .8472 | 15. 0 | 180 | 1.25 |
| 5 | 65 | .4514 | 3 | 123 | .8542 | 1 | 181 | 1.257 |
| 6 | 66 | .4583 | 4 | 124 | .8611 | 2 | 182 | 1.264 |
| 7 | 67 | .4653 | 5 | 125 | .8681 | 3 | 183 | 1.271 |
| 8 | 68 | .4722 | 6 | 126 | .875 | 4 | 184 | 1.278 |
| 9 | 69 | .4792 | 7 | 127 | .8819 | 5 | 185 | 1.285 |
| 10 | 70 | .4861 | 8 | 128 | .8889 | 6 | 186 | 1.292 |
| 11 | 71 | .4931 | 9 | 129 | .8958 | 7 | 187 | 1.299 |
| 6. 0 | 72 | .5 | 10 | 130 | .9028 | 8 | 188 | 1.306 |
| 1 | 73 | .5069 | 11 | 131 | .9097 | 9 | 189 | 1.313 |
| 2 | 74 | .5139 | 11. 0 | 132 | .9167 | 10 | 190 | 1.319 |
| 3 | 75 | .5208 | 1 | 133 | .9236 | 11 | 191 | 1.326 |
| 4 | 76 | .5278 | 2 | 134 | .9306 | 16. 0 | 192 | 1.333 |
| 5 | 77 | .5347 | 3 | 135 | .9375 | 1 | 193 | 1.34 |
| 6 | 78 | .5417 | 4 | 136 | .9444 | 2 | 194 | 1.347 |
| 7 | 79 | .5486 | 5 | 137 | .9514 | 3 | 195 | 1.354 |
| 8 | 80 | .5556 | 6 | 138 | .9583 | 4 | 196 | 1.361 |
| 9 | 81 | .5625 | 7 | 139 | .9653 | 5 | 197 | 1.368 |
| 10 | 82 | .5694 | 8 | 140 | .9722 | 6 | 198 | 1.375 |
| 11 | 83 | .5764 | 9 | 141 | .9792 | 7 | 199 | 1.382 |
| 7. 0 | 84 | .5834 | 10 | 142 | .9861 | 8 | 200 | 1.389 |
| 1 | 85 | .5903 | 11 | 143 | .9931 | 9 | 201 | 1.396 |
| 2 | 86 | .5972 | 12. 0 | 144 | 1.000 | 10 | 202 | 1.403 |
| 3 | 87 | .6042 | 1 | 145 | 1.007 | 11 | 203 | 1.41 |
| 4 | 88 | .6111 | 2 | 146 | 1.014 | 17. 0 | 204 | 1.417 |
| 5 | 89 | .6181 | 3 | 147 | 1.021 | 1 | 205 | 1.424 |
| 6 | 90 | .625 | 4 | 148 | 1.028 | 2 | 206 | 1.431 |
| 7 | 91 | .6319 | 5 | 149 | 1.035 | 3 | 207 | 1.438 |
| 8 | 92 | .6389 | 6 | 150 | 1.042 | 4 | 208 | 1.444 |
| 9 | 93 | .6458 | 7 | 151 | 1.049 | 5 | 209 | 1.451 |

SQUARE FEET IN PLATES. — *Continued.*

| Ft. and Ins. Long. | Ins. Long. | Square Feet. | Ft. and Ins. Long. | Ins. Long. | Square Feet. | Ft. and Ins. Long. | Ins. Long. | Square Feet. |
|--------------------------|---------------|-----------------|--------------------------|---------------|-----------------|--------------------------|---------------|-----------------|
| 17. 6 | 210 | 1.458 | 22. 5 | 269 | 1.868 | 27. 4 | 328 | 2.278 |
| 7 | 211 | 1.465 | 6 | 270 | 1.875 | 5 | 329 | 2.285 |
| 8 | 212 | 1.472 | 7 | 271 | 1.882 | 6 | 330 | 2.292 |
| 9 | 213 | 1.479 | 8 | 272 | 1.889 | 7 | 331 | 2.299 |
| 10 | 214 | 1.486 | 9 | 273 | 1.896 | 8 | 332 | 2.306 |
| 11 | 215 | 1.493 | 10 | 274 | 1.903 | 9 | 333 | 2.313 |
| 18. 0 | 216 | 1.5 | 11 | 275 | 1.91 | 10 | 334 | 2.319 |
| 1 | 217 | 1.507 | 23. 0 | 276 | 1.917 | 11 | 335 | 2.326 |
| 2 | 218 | 1.514 | 1 | 277 | 1.924 | 28. 0 | 336 | 2.333 |
| 3 | 219 | 1.521 | 2 | 278 | 1.931 | 1 | 337 | 2.34 |
| 4 | 220 | 1.528 | 3 | 279 | 1.938 | 2 | 338 | 2.347 |
| 5 | 221 | 1.535 | 4 | 280 | 1.944 | 3 | 339 | 2.354 |
| 6 | 222 | 1.542 | 5 | 281 | 1.951 | 4 | 340 | 2.361 |
| 7 | 223 | 1.549 | 6 | 282 | 1.958 | 5 | 341 | 2.368 |
| 8 | 224 | 1.556 | 7 | 283 | 1.965 | 6 | 342 | 2.375 |
| 9 | 225 | 1.563 | 8 | 284 | 1.972 | 7 | 343 | 2.382 |
| 10 | 226 | 1.569 | 9 | 285 | 1.979 | 8 | 344 | 2.389 |
| 11 | 227 | 1.576 | 10 | 286 | 1.986 | 9 | 345 | 2.396 |
| 19. 0 | 228 | 1.583 | 11 | 287 | 1.993 | 10 | 346 | 2.403 |
| 1 | 229 | 1.59 | 24. 0 | 288 | 2. | 11 | 347 | 2.41 |
| 2 | 230 | 1.597 | 1 | 289 | 2.007 | 29. 0 | 348 | 2.417 |
| 3 | 231 | 1.604 | 2 | 290 | 2.014 | 1 | 349 | 2.424 |
| 4 | 232 | 1.611 | 3 | 291 | 2.021 | 2 | 350 | 2.431 |
| 5 | 233 | 1.618 | 4 | 292 | 2.028 | 3 | 351 | 2.438 |
| 6 | 234 | 1.625 | 5 | 293 | 2.035 | 4 | 352 | 2.444 |
| 7 | 235 | 1.632 | 6 | 294 | 2.042 | 5 | 353 | 2.451 |
| 8 | 236 | 1.639 | 7 | 295 | 2.049 | 6 | 354 | 2.458 |
| 9 | 237 | 1.645 | 8 | 296 | 2.056 | 7 | 355 | 2.465 |
| 10 | 238 | 1.653 | 9 | 297 | 2.063 | 8 | 356 | 2.472 |
| 11 | 239 | 1.659 | 10 | 298 | 2.069 | 9 | 357 | 2.479 |
| 20. 0 | 240 | 1.667 | 11 | 299 | 2.076 | 10 | 358 | 2.486 |
| 1 | 241 | 1.674 | 25. 0 | 300 | 2.083 | 11 | 359 | 2.493 |
| 2 | 242 | 1.681 | 1 | 301 | 2.09 | 30. 0 | 360 | 2.5 |
| 3 | 243 | 1.688 | 2 | 302 | 2.097 | 1 | 361 | 2.507 |
| 4 | 244 | 1.694 | 3 | 303 | 2.104 | 2 | 362 | 2.514 |
| 5 | 245 | 1.701 | 4 | 304 | 2.111 | 3 | 363 | 2.521 |
| 6 | 246 | 1.708 | 5 | 305 | 2.118 | 4 | 364 | 2.528 |
| 7 | 247 | 1.715 | 6 | 306 | 2.125 | 5 | 365 | 2.535 |
| 8 | 248 | 1.722 | 7 | 307 | 2.132 | 6 | 366 | 2.542 |
| 9 | 249 | 1.729 | 8 | 308 | 2.139 | 7 | 367 | 2.549 |
| 10 | 250 | 1.736 | 9 | 309 | 2.146 | 8 | 368 | 2.556 |
| 11 | 251 | 1.743 | 10 | 310 | 2.153 | 9 | 369 | 2.563 |
| 21. 0 | 252 | 1.75 | 11 | 311 | 2.16 | 10 | 370 | 2.569 |
| 1 | 253 | 1.757 | 26. 0 | 312 | 2.167 | 11 | 371 | 2.576 |
| 2 | 254 | 1.764 | 1 | 313 | 2.174 | 31. 0 | 372 | 2.583 |
| 3 | 255 | 1.771 | 2 | 314 | 2.181 | 1 | 373 | 2.59 |
| 4 | 256 | 1.778 | 3 | 315 | 2.188 | 2 | 374 | 2.597 |
| 5 | 257 | 1.785 | 4 | 316 | 2.194 | 3 | 375 | 2.604 |
| 6 | 258 | 1.792 | 5 | 317 | 2.201 | 4 | 376 | 2.611 |
| 7 | 259 | 1.799 | 6 | 318 | 2.208 | 5 | 377 | 2.618 |
| 8 | 260 | 1.806 | 7 | 319 | 2.215 | 6 | 378 | 2.625 |
| 9 | 261 | 1.813 | 8 | 320 | 2.222 | 7 | 379 | 2.632 |
| 10 | 262 | 1.819 | 9 | 321 | 2.229 | 8 | 380 | 2.639 |
| 11 | 263 | 1.826 | 10 | 322 | 2.236 | 9 | 381 | 2.646 |
| 22. 0 | 264 | 1.833 | 11 | 323 | 2.243 | 10 | 382 | 2.653 |
| 1 | 265 | 1.84 | 27. 0 | 324 | 2.25 | 11 | 383 | 2.66 |
| 2 | 266 | 1.847 | 1 | 325 | 2.257 | 32. 0 | 384 | 2.667 |
| 3 | 267 | 1.854 | 2 | 326 | 2.264 | 1 | 385 | 2.674 |
| 4 | 268 | 1.861 | 3 | 327 | 2.271 | 2 | 386 | 2.681 |

NUMBER OF BARRELS (31 1-2 GALLONS) IN CISTERNS AND TANKS.

$$1 \text{ barrel} = 31 \frac{1}{2} \text{ gallons} = \frac{31.5 \times 231}{1728} = 4.21094 \text{ cu. ft. Reciprocal} = 0.237477$$

| Depth in Feet. | Diameter in Feet. | | | | | | | | | |
|----------------------|-------------------|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 4.663 | 6.714 | 9.139 | 11.937 | 15.108 | 18.652 | 22.569 | 26.859 | 31.522 | 36.557 |
| 5 | 23.3 | 33.6 | 45.7 | 59.7 | 75.5 | 93.3 | 112.8 | 134.3 | 157.6 | 182.8 |
| 6 | 28.0 | 40.3 | 54.8 | 71.6 | 90.6 | 111.9 | 135.4 | 161.2 | 189.1 | 219.3 |
| 7 | 32.6 | 47.0 | 64.0 | 83.6 | 105.8 | 130.6 | 158.0 | 188.0 | 220.7 | 255.9 |
| 8 | 37.3 | 53.7 | 73.1 | 95.5 | 120.9 | 149.2 | 180.6 | 214.9 | 252.2 | 292.5 |
| 9 | 42.0 | 60.4 | 82.3 | 107.4 | 136.0 | 167.9 | 203.1 | 241.7 | 283.7 | 329.0 |
| 10 | 46.6 | 67.1 | 91.4 | 119.4 | 151.1 | 186.5 | 225.7 | 268.6 | 315.2 | 365.6 |
| 11 | 51.3 | 73.9 | 100.5 | 131.3 | 166.2 | 205.2 | 248.3 | 295.4 | 346.7 | 402.1 |
| 12 | 56.0 | 80.6 | 109.7 | 143.2 | 181.3 | 223.8 | 270.8 | 322.3 | 378.3 | 438.7 |
| 13 | 60.6 | 87.3 | 118.8 | 155.2 | 196.4 | 242.5 | 293.4 | 349.2 | 409.8 | 475.2 |
| 14 | 65.3 | 94.0 | 127.9 | 167.1 | 211.5 | 261.1 | 316.0 | 376.0 | 441.3 | 511.8 |
| 15 | 69.9 | 100.7 | 137.1 | 179.1 | 226.6 | 279.8 | 338.5 | 402.9 | 472.8 | 548.4 |
| 16 | 74.6 | 107.4 | 146.2 | 191.0 | 241.7 | 298.4 | 361.1 | 429.7 | 504.4 | 584.9 |
| 17 | 79.3 | 114.1 | 155.4 | 202.9 | 256.8 | 317.1 | 383.7 | 456.6 | 535.9 | 621.5 |
| 18 | 83.9 | 120.9 | 164.5 | 214.9 | 271.9 | 335.7 | 406.2 | 483.5 | 567.4 | 658.0 |
| 19 | 88.6 | 127.6 | 173.6 | 226.8 | 287.1 | 354.4 | 428.8 | 510.3 | 598.9 | 694.6 |
| 20 | 93.3 | 134.3 | 182.8 | 238.7 | 302.2 | 373.0 | 451.4 | 537.2 | 630.4 | 731.1 |

| Depth in Feet. | Diameter in Feet. | | | | | | | |
|----------------------|-------------------|--------|--------|--------|--------|--------|--------|--------|
| | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 1 | 41.966 | 47.748 | 53.903 | 60.431 | 67.332 | 74.606 | 82.253 | 90.273 |
| 5 | 209.8 | 238.7 | 269.5 | 302.2 | 336.7 | 373.0 | 411.3 | 451.4 |
| 6 | 251.8 | 286.5 | 323.4 | 362.6 | 404.0 | 447.6 | 493.5 | 541.6 |
| 7 | 293.8 | 334.2 | 377.3 | 423.0 | 471.3 | 522.2 | 575.8 | 631.9 |
| 8 | 335.7 | 382.0 | 431.2 | 483.4 | 538.7 | 596.8 | 658.0 | 722.2 |
| 9 | 377.7 | 429.7 | 485.1 | 543.9 | 606.0 | 671.5 | 740.3 | 812.5 |
| 10 | 419.7 | 477.5 | 539.0 | 604.3 | 673.3 | 746.1 | 822.5 | 902.7 |
| 11 | 461.6 | 525.2 | 592.9 | 664.7 | 740.7 | 820.7 | 904.8 | 993.0 |
| 12 | 503.6 | 573.0 | 646.8 | 725.2 | 808.0 | 895.3 | 987.0 | 1083.3 |
| 13 | 545.6 | 620.7 | 700.7 | 785.6 | 875.3 | 969.9 | 1069.3 | 1173.5 |
| 14 | 587.5 | 668.5 | 754.6 | 846.0 | 942.6 | 1044.5 | 1151.5 | 1263.8 |
| 15 | 629.5 | 716.2 | 808.5 | 906.5 | 1010.0 | 1119.1 | 1233.8 | 1354.1 |
| 16 | 671.5 | 764.0 | 862.4 | 966.9 | 1077.3 | 1193.7 | 1316.0 | 1444.4 |
| 17 | 713.4 | 811.7 | 916.4 | 1027.3 | 1144.6 | 1268.3 | 1398.3 | 1534.5 |
| 18 | 755.4 | 859.5 | 970.3 | 1087.8 | 1212.0 | 1342.9 | 1480.6 | 1624.9 |
| 19 | 797.4 | 907.2 | 1024.2 | 1148.2 | 1279.3 | 1417.5 | 1562.8 | 1715.2 |
| 20 | 839.3 | 955.0 | 1078.1 | 1208.6 | 1346.6 | 1492.1 | 1645.1 | 1805.5 |

**NUMBER OF BARRELS (31 1-2 GALLONS) IN CISTERNS
AND TANKS. — Continued.**

| Depth in Feet. | Diameter in Feet. | | | | | | | |
|----------------------|-------------------|---------|---------|---------|---------|---------|---------|---------|
| | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 1 | 98.666 | 107.432 | 116.571 | 126.083 | 135.968 | 146.226 | 157.858 | 167.863 |
| 5 | 493.3 | 537.2 | 582.9 | 630.4 | 679.8 | 731.1 | 784.3 | 839.3 |
| 6 | 592.0 | 644.6 | 699.4 | 756.5 | 815.8 | 877.4 | 941.1 | 1007.2 |
| 7 | 690.7 | 752.0 | 816.0 | 882.6 | 951.8 | 1023.6 | 1098.0 | 1175.0 |
| 8 | 789.3 | 859.5 | 932.6 | 1008.7 | 1087.7 | 1169.8 | 1254.9 | 1342.9 |
| 9 | 888.0 | 966.9 | 1049.1 | 1134.7 | 1223.7 | 1316.0 | 1411.7 | 1510.8 |
| 10 | 986.7 | 1074.3 | 1165.7 | 1260.8 | 1359.7 | 1462.2 | 1568.6 | 1678.6 |
| 11 | 1085.3 | 1181.8 | 1282.3 | 1386.9 | 1495.6 | 1608.5 | 1725.4 | 1846.5 |
| 12 | 1184.0 | 1289.2 | 1398.8 | 1513.0 | 1631.6 | 1754.7 | 1882.3 | 2014.4 |
| 13 | 1282.7 | 1396.6 | 1515.4 | 1639.1 | 1767.6 | 1900.9 | 2039.2 | 2182.2 |
| 14 | 1381.3 | 1504.0 | 1632.0 | 1765.2 | 1903.6 | 2047.2 | 2196.0 | 2350.1 |
| 15 | 1480.0 | 1611.5 | 1748.6 | 1891.2 | 2039.5 | 2193.4 | 2352.9 | 2517.9 |
| 16 | 1578.7 | 1718.9 | 1865.1 | 2017.3 | 2175.5 | 2339.6 | 2509.7 | 2685.8 |
| 17 | 1677.3 | 1826.3 | 1981.7 | 2143.4 | 2311.5 | 2485.8 | 2666.6 | 2853.7 |
| 18 | 1776.0 | 1933.8 | 2098.3 | 2269.5 | 2447.4 | 2632.0 | 2823.4 | 3021.5 |
| 19 | 1874.7 | 2041.2 | 2214.8 | 2395.6 | 2583.4 | 2778.3 | 2980.3 | 3189.4 |
| 20 | 1973.3 | 2148.6 | 2321.4 | 2521.7 | 2719.4 | 2924.5 | 3137.2 | 3357.3 |

LOGARITHMS.

Logarithms (abbreviation *log*). — The log of a number is the exponent of the power to which it is necessary to raise a fixed number to produce the given number. The fixed number is called the *base*. Thus if the base is 10, the log of 1000 is 3, for $10^3 = 1000$. There are two systems of logs in general use, the *common*, in which the base is 10, and the *Naperian*, or *hyperbolic*, in which the base is 2.718281828 The Naperian base is commonly denoted by e , as in the equation $e^y = x$, in which y is the Nap. log of x . The abbreviation \log_e is commonly used to denote the Nap log.

In any system of logs, the log of 1 is 0; the log of the base, taken in that system, is 1. In any system the base of which is greater than 1, the logs of all numbers greater than 1 are positive and the logs of all numbers less than 1 are negative.

The modulus of any system is equal to the reciprocal of the Naperian log of the base of that system. The modulus of the Naperian system is 1, that of the common system is 0.4342945.

The log of a number in any system equals the modulus of that system \times the Naperian log of the number.

The *hyperbolic* or *Naperian* log of any number equals the common log \times 2.3025851.

Every log consists of two parts, an entire part called the *characteristic*, or index, and the decimal part, or *mantissa*. The mantissa only is given in the usual tables of common logs, with the decimal point omitted. The characteristic is found by a simple rule, viz., it is one less than the number of figures to the left of the decimal point in the number whose log is to be found. Thus the characteristic of numbers from 1 to 9.99 + is 0, from 10 to 99.99 + is 1, from 100 to 999 + is 2, from 0.1 to 0.99 + is - 1, from 0.01 to 0.099 + is - 2, etc. Thus

| | | | |
|-------------|-------------|------------|-------------------------------|
| log of 2000 | is 3.30103; | log of 0.2 | is - 1.30103, or 9.30103 - 10 |
| " " 200 | " 2.30103; | " " 0.02 | " - 2.30103, " 8.30103 - 10 |
| " " 20 | " 1.30103; | " " 0.002 | " - 3.30103, " 7.30103 - 10 |
| " " 2 | " 0.30103; | " " 0.0002 | " - 4.30103, " 6.30103 - 10 |

The minus sign is frequently written above the characteristic thus: log 0.002 = $\bar{3}.30103$. The characteristic only is negative, the decimal part, or mantissa, being always positive.

When a log consists of a negative index and a positive mantissa, it is usual to write the negative sign over the index, or else to add 10 to the index, and to indicate the subtraction of 10 from the resulting logarithm.

Thus log 0.2 = $\bar{1}.30103$, and this may be written $9.30103 - 10$.

In tables of logarithmic sines, etc., the -10 is generally omitted, as being understood.

Rules for use of the table of logarithms. — **To find the log of any whole number.** — For 1 to 100 inclusive the log is given complete in the small table on page 136.

For 100 to 999 inclusive the decimal part of the log is given opposite the given number in the column headed 0 in the table (including the two figures to the left, making six figures). Prefix the characteristic, or index, 2.

For 1000 to 9999 inclusive: The last four figures of the log are found opposite the first three figures of the given number and in the vertical column headed with the fourth figure of the given number; prefix the two figures under column 0, and the index, which is 3.

For numbers over 10,000 having five or more digits: Find the decimal part of the log for the first four digits as above, multiply the difference figure in the last column by the remaining digit or digits, and divide by 10 if there be only one digit more, by 100 if there be two more, and so on; add the quotient to the log of the first four digits and prefix the index, which is 4 if there are five digits, 5 if there are six digits, and so on. The table of proportional parts may be used, as shown below.

To find the log of a decimal fraction or of a whole number and a decimal. — First find the log of the quantity as if there were no decimal point, then prefix the index according to rule; the index is one less than the number of figures to the left of the decimal point.

Required log of 3.141593.

| | | | | |
|--------------|--------------|--------------|-------------|-------------|
| | | log of 3.141 | = 0.497068. | Diff. = 138 |
| From | proportional | parts | 5 | = 690 |
| " | " | " | 09 | = 1242 |
| " | " | " | 003 | = 041 |
| | | | <hr/> | |
| log 3.141593 | | | 0.4971498 | |

To find the number corresponding to a given log. — Find in the table the log nearest to the decimal part of the given log and take the first four digits of the required number from the column N and the top or foot of the column containing the log which is the next less than the given log. To find the 5th and 6th digits subtract the log in the table from the given log, multiply the difference by 100, and divide by the figure in the Diff. column opposite the log; annex the quotient to the four digits already found, and place the decimal point according to the rule; the number of figures to the left of the decimal point is one greater than the index. The number corresponding to a log is called the **anti-logarithm**.

Find the anti-log of 0.497150
Next lowest log in table corresponds to 3141 0.497068 Diff. = 82

Tabular diff. = 138; $82 \div 138 = 0.59 +$

The index being 0, the number is therefore 3.14159 +.

To multiply two numbers by the use of logarithms. — Add together the logs of the two numbers, and find the number whose log is the sum.

To divide two numbers. — Subtract the log of the divisor from the log of the dividend, and find the number whose log is the difference.

Log of a fraction. Log of $a/b = \log a - \log b$.

To raise a number to any given power. — Multiply the log of the number by the exponent of the power, and find the number whose log is the product.

To find any root of a given number. — Divide the log of the number by the index of the root. The quotient is the log of the root.

To find the reciprocal of a number. — Subtract the decimal part of the log of the number from 0, add 1 to the index and change the sign of the index. The result is the log of the reciprocal.

Required the reciprocal of 3.141593.

Log of 3.141593, as found above 0.4971498

Subtract decimal part from 0 gives 0.5028502

Add 1 to the index, and changing sign of the index gives... 1.5028502 which is the log of 0.31831.

To find the fourth term of a proportion by logarithms. — Add the logarithms of the second and third terms, and from their sum subtract the logarithm of the first term.

When one logarithm is to be subtracted from another, it may be more convenient to convert the subtraction into an addition, which may be done by first subtracting the given logarithm from 10, adding the difference to the other logarithm, and afterwards rejecting the 10.

The difference between a given logarithm and 10 is called its *arithmetical complement*, or *cologarithm*.

To subtract one logarithm from another is the same as to add its complement and then reject 10 from the result. For $a - b = 10 - b + a - 10$.

To work a proportion, then, by logarithms, add the complement of the logarithm of the first term to the logarithms of the second and third terms. The characteristic must afterwards be diminished by 10.

Example in logarithms with a negative index. — Solve by logarithms $\left(\frac{526}{1011}\right)^{2.45}$, which means divide 526 by 1011 and raise the quotient to the 2.45 power.

log 526 = 2.720986

log 1011 = 3.004751

log of quotient = 9.716235 - 10

Multiply by 2.45

.48581175

3.8864940

19.432470

23.80477575 - (10 × 2.45) = 1.30477575 = 0.20173, Ans.

LOGARITHMS OF NUMBERS FROM 1 TO 100.

| N. | Log. | N. | Log. | N. | Log. | N. | Log. | N. | Log. |
|----|----------|----|----------|----|----------|----|----------|-----|----------|
| 1 | 0.000000 | 21 | 1.322219 | 41 | 1.612784 | 61 | 1.785330 | 81 | 1.908485 |
| 2 | 0.301030 | 22 | 1.342423 | 42 | 1.623249 | 62 | 1.792392 | 82 | 1.913814 |
| 3 | 0.477121 | 23 | 1.361728 | 43 | 1.633468 | 63 | 1.799341 | 83 | 1.919078 |
| 4 | 0.602060 | 24 | 1.380211 | 44 | 1.643453 | 64 | 1.806180 | 84 | 1.924279 |
| 5 | 0.698970 | 25 | 1.397940 | 45 | 1.653213 | 65 | 1.812913 | 85 | 1.929419 |
| 6 | 0.778151 | 26 | 1.414973 | 46 | 1.662758 | 66 | 1.819544 | 86 | 1.934498 |
| 7 | 0.845098 | 27 | 1.431364 | 47 | 1.672098 | 67 | 1.826075 | 87 | 1.939519 |
| 8 | 0.903090 | 28 | 1.447158 | 48 | 1.681241 | 68 | 1.832509 | 88 | 1.944483 |
| 9 | 0.954243 | 29 | 1.462398 | 49 | 1.690196 | 69 | 1.838849 | 89 | 1.949390 |
| 10 | 1.000000 | 30 | 1.477121 | 50 | 1.698970 | 70 | 1.845098 | 90 | 1.954243 |
| 11 | 1.041393 | 31 | 1.491362 | 51 | 1.707570 | 71 | 1.851258 | 91 | 1.959041 |
| 12 | 1.079181 | 32 | 1.505150 | 52 | 1.716003 | 72 | 1.857332 | 92 | 1.963788 |
| 13 | 1.113943 | 33 | 1.518514 | 53 | 1.724276 | 73 | 1.863323 | 93 | 1.968483 |
| 14 | 1.146128 | 34 | 1.531479 | 54 | 1.732394 | 74 | 1.869232 | 94 | 1.973128 |
| 15 | 1.176091 | 35 | 1.544068 | 55 | 1.740363 | 75 | 1.875061 | 95 | 1.977724 |
| 16 | 1.204120 | 36 | 1.556303 | 56 | 1.748188 | 76 | 1.880814 | 96 | 1.982271 |
| 17 | 1.230449 | 37 | 1.568202 | 57 | 1.755875 | 77 | 1.886491 | 97 | 1.986772 |
| 18 | 1.255273 | 38 | 1.579784 | 58 | 1.763428 | 78 | 1.892095 | 98 | 1.991226 |
| 19 | 1.278754 | 39 | 1.591065 | 59 | 1.770852 | 79 | 1.897627 | 99 | 1.995635 |
| 20 | 1.301030 | 40 | 1.602060 | 60 | 1.778151 | 80 | 1.903090 | 100 | 2.000000 |

No. 100 L. 000.]

[No. 109 L. 040.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 100 | 000000 | 0434 | 0868 | 1301 | 1734 | 2166 | 2598 | 3029 | 3461 | 3891 | 432 |
| 1 | 4321 | 4751 | 5181 | 5609 | 6038 | 6466 | 6894 | 7321 | 7748 | 8174 | 428 |
| 2 | 8600 | 9026 | 9451 | 9876 | 0300 | 0724 | 1147 | 1570 | 1993 | 2415 | 424 |
| 3 | 012837 | 3259 | 3680 | 4100 | 4521 | 4940 | 5360 | 5779 | 6197 | 6616 | 420 |
| 4 | 7033 | 7451 | 7868 | 8284 | 8700 | 9116 | 9532 | 9947 | 0361 | 0775 | 416 |
| 5 | 021189 | 1603 | 2016 | 2428 | 2841 | 3252 | 3664 | 4075 | 4486 | 4896 | 412 |
| 6 | 5306 | 5715 | 6125 | 6533 | 6942 | 7350 | 7757 | 8164 | 8571 | 8978 | 408 |
| 7 | 9384 | 9789 | 0195 | 0600 | 1004 | 1408 | 1812 | 2216 | 2619 | 3021 | 404 |
| 8 | 033424 | 3826 | 4227 | 4628 | 5029 | 5430 | 5830 | 6230 | 6629 | 7028 | 400 |
| 9 | 7426 | 7825 | 8223 | 8620 | 9017 | 9414 | 9811 | 0207 | 0602 | 0998 | 397 |
| | 04 | | | | | | | | | | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|-------|-------|-------|-------|-------|-------|-------|
| 434 | 43.4 | 86.8 | 130.2 | 173.6 | 217.0 | 260.4 | 303.8 | 347.2 | 390.6 |
| 433 | 43.3 | 86.6 | 129.9 | 173.2 | 216.5 | 259.8 | 303.1 | 346.4 | 389.7 |
| 432 | 43.2 | 86.4 | 129.6 | 172.8 | 216.0 | 259.2 | 302.4 | 345.6 | 388.8 |
| 431 | 43.1 | 86.2 | 129.3 | 172.4 | 215.5 | 258.6 | 301.7 | 344.8 | 387.9 |
| 430 | 43.0 | 86.0 | 129.0 | 172.0 | 215.0 | 258.0 | 301.0 | 344.0 | 387.0 |
| 429 | 42.9 | 85.8 | 128.7 | 171.6 | 214.5 | 257.4 | 300.3 | 343.2 | 386.1 |
| 428 | 42.8 | 85.6 | 128.4 | 171.2 | 214.0 | 256.8 | 299.6 | 342.4 | 385.2 |
| 427 | 42.7 | 85.4 | 128.1 | 170.8 | 213.5 | 256.2 | 298.9 | 341.6 | 384.3 |
| 426 | 42.6 | 85.2 | 127.8 | 170.4 | 213.0 | 255.6 | 298.2 | 340.8 | 383.4 |
| 425 | 42.5 | 85.0 | 127.5 | 170.0 | 212.5 | 255.0 | 297.5 | 340.0 | 382.5 |
| 424 | 42.4 | 84.8 | 127.2 | 169.6 | 212.0 | 254.4 | 296.8 | 339.2 | 381.6 |
| 423 | 42.3 | 84.6 | 126.9 | 169.2 | 211.5 | 253.8 | 296.1 | 338.4 | 380.7 |
| 422 | 42.2 | 84.4 | 126.6 | 168.8 | 211.0 | 253.2 | 295.4 | 337.6 | 379.8 |
| 421 | 42.1 | 84.2 | 126.3 | 168.4 | 210.5 | 252.6 | 294.7 | 336.8 | 378.9 |
| 420 | 42.0 | 84.0 | 126.0 | 168.0 | 210.0 | 252.0 | 294.0 | 336.0 | 378.0 |
| 419 | 41.9 | 83.8 | 125.7 | 167.6 | 209.5 | 251.4 | 293.3 | 335.2 | 377.1 |
| 418 | 41.8 | 83.6 | 125.4 | 167.2 | 209.0 | 250.8 | 292.6 | 334.4 | 376.2 |
| 417 | 41.7 | 83.4 | 125.1 | 166.8 | 208.5 | 250.2 | 291.9 | 333.6 | 375.3 |
| 416 | 41.6 | 83.2 | 124.8 | 166.4 | 208.0 | 249.6 | 291.2 | 332.8 | 374.4 |
| 415 | 41.5 | 83.0 | 124.5 | 166.0 | 207.5 | 249.0 | 290.5 | 332.0 | 373.5 |
| 414 | 41.4 | 82.8 | 124.2 | 165.6 | 207.0 | 248.4 | 289.8 | 331.2 | 372.6 |
| 413 | 41.3 | 82.6 | 123.9 | 165.2 | 206.5 | 247.8 | 289.1 | 330.4 | 371.7 |
| 412 | 41.2 | 82.4 | 123.6 | 164.8 | 206.0 | 247.2 | 288.4 | 329.6 | 370.8 |
| 411 | 41.1 | 82.2 | 123.3 | 164.4 | 205.5 | 246.6 | 287.7 | 328.8 | 369.9 |
| 410 | 41.0 | 82.0 | 123.0 | 164.0 | 205.0 | 246.0 | 287.0 | 328.0 | 369.0 |
| 409 | 40.9 | 81.8 | 122.7 | 163.6 | 204.5 | 245.4 | 286.3 | 327.2 | 368.1 |
| 408 | 40.8 | 81.6 | 122.4 | 163.2 | 204.0 | 244.8 | 285.6 | 326.4 | 367.2 |
| 407 | 40.7 | 81.4 | 122.1 | 162.8 | 203.5 | 244.2 | 284.9 | 325.6 | 366.3 |
| 406 | 40.6 | 81.2 | 121.8 | 162.4 | 203.0 | 243.6 | 284.2 | 324.8 | 365.4 |
| 405 | 40.5 | 81.0 | 121.5 | 162.0 | 202.5 | 243.0 | 283.5 | 324.0 | 364.5 |
| 404 | 40.4 | 80.8 | 121.2 | 161.6 | 202.0 | 242.4 | 282.8 | 323.2 | 363.6 |
| 403 | 40.3 | 80.6 | 120.9 | 161.2 | 201.5 | 241.8 | 282.1 | 322.4 | 362.7 |
| 402 | 40.2 | 80.4 | 120.6 | 160.8 | 201.0 | 241.2 | 281.4 | 321.6 | 361.8 |
| 401 | 40.1 | 80.2 | 120.3 | 160.4 | 200.5 | 240.6 | 280.7 | 320.8 | 360.9 |
| 400 | 40.0 | 80.0 | 120.0 | 160.0 | 200.0 | 240.0 | 280.0 | 320.0 | 360.0 |
| 399 | 39.9 | 79.8 | 119.7 | 159.6 | 199.5 | 239.4 | 279.3 | 319.2 | 359.1 |
| 398 | 39.8 | 79.6 | 119.4 | 159.2 | 199.0 | 238.8 | 278.6 | 318.4 | 358.2 |
| 397 | 39.7 | 79.4 | 119.1 | 158.8 | 198.5 | 238.2 | 277.9 | 317.6 | 357.3 |
| 396 | 39.6 | 79.2 | 118.8 | 158.4 | 198.0 | 237.6 | 277.2 | 316.8 | 356.4 |
| 395 | 39.5 | 79.0 | 118.5 | 158.0 | 197.5 | 237.0 | 276.5 | 316.0 | 355.5 |

No. 110 L. 041.]

[No. 119 L. 078.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 110 | 041393 | 1787 | 2182 | 2576 | 2969 | 3362 | 3755 | 4148 | 4540 | 4932 | 393 |
| 1 | 5323 | 5714 | 6105 | 6495 | 6885 | 7275 | 7664 | 8053 | 8442 | 8830 | 390 |
| 2 | 9218 | 9606 | 9993 | | | | | | | | |
| | | | | 0380 | 0766 | 1153 | 1538 | 1924 | 2309 | 2694 | 386 |
| 3 | 053078 | 3463 | 3846 | 4230 | 4613 | 4996 | 5378 | 5760 | 6142 | 6524 | 383 |
| 4 | 6905 | 7286 | 7666 | 8046 | 8426 | 8805 | 9185 | 9563 | 9942 | | |
| | | | | | | | | | | 0320 | 379 |
| 5 | 060698 | 1075 | 1452 | 1829 | 2206 | 2582 | 2958 | 3333 | 3709 | 4083 | 376 |
| 6 | 4458 | 4832 | 5206 | 5580 | 5953 | 6326 | 6699 | 7071 | 7443 | 7815 | 373 |
| 7 | 8186 | 8557 | 8928 | 9298 | 9668 | | | | | | |
| | | | | | | 0038 | 0407 | 0776 | 1145 | 1514 | 370 |
| 8 | 071882 | 2250 | 2617 | 2985 | 3352 | 3718 | 4085 | 4451 | 4816 | 5182 | 366 |
| 9 | 5547 | 5912 | 6276 | 6640 | 7004 | 7368 | 7731 | 8094 | 8457 | 8819 | 363 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|-------|-------|-------|-------|-------|-------|-------|
| 395 | 39.5 | 79.0 | 118.5 | 158.0 | 197.5 | 237.0 | 276.5 | 316.0 | 355.5 |
| 394 | 39.4 | 78.8 | 118.2 | 157.6 | 197.0 | 236.4 | 275.8 | 315.2 | 354.6 |
| 393 | 39.3 | 78.6 | 117.9 | 157.2 | 196.5 | 235.8 | 275.1 | 314.4 | 353.7 |
| 392 | 39.2 | 78.4 | 117.6 | 156.8 | 196.0 | 235.2 | 274.4 | 313.6 | 352.8 |
| 391 | 39.1 | 78.2 | 117.3 | 156.4 | 195.5 | 234.6 | 273.7 | 312.8 | 351.9 |
| 390 | 39.0 | 78.0 | 117.0 | 156.0 | 195.0 | 234.0 | 273.0 | 312.0 | 351.0 |
| 389 | 38.9 | 77.8 | 116.7 | 155.6 | 194.5 | 233.4 | 272.3 | 311.2 | 350.1 |
| 388 | 38.8 | 77.6 | 116.4 | 155.2 | 194.0 | 232.8 | 271.6 | 310.4 | 349.2 |
| 387 | 38.7 | 77.4 | 116.1 | 154.8 | 193.5 | 232.2 | 270.9 | 309.6 | 348.3 |
| 386 | 38.6 | 77.2 | 115.8 | 154.4 | 193.0 | 231.6 | 270.2 | 308.8 | 347.4 |
| 385 | 38.5 | 77.0 | 115.5 | 154.0 | 192.5 | 231.0 | 269.5 | 308.0 | 346.5 |
| | | | | | | | | | |
| 384 | 38.4 | 76.8 | 115.2 | 153.6 | 192.0 | 230.4 | 268.8 | 307.2 | 345.6 |
| 383 | 38.3 | 76.6 | 114.9 | 153.2 | 191.5 | 229.8 | 268.1 | 306.4 | 344.7 |
| 382 | 38.2 | 76.4 | 114.6 | 152.8 | 191.0 | 229.2 | 267.4 | 305.6 | 343.8 |
| 381 | 38.1 | 76.2 | 114.3 | 152.4 | 190.5 | 228.6 | 266.7 | 304.8 | 342.9 |
| 380 | 38.0 | 76.0 | 114.0 | 152.0 | 190.0 | 228.0 | 266.0 | 304.0 | 342.0 |
| 379 | 37.9 | 75.8 | 113.7 | 151.6 | 189.5 | 227.4 | 265.3 | 303.2 | 341.1 |
| 378 | 37.8 | 75.6 | 113.4 | 151.2 | 189.0 | 226.8 | 264.6 | 302.4 | 340.2 |
| 377 | 37.7 | 75.4 | 113.1 | 150.8 | 188.5 | 226.2 | 263.9 | 301.6 | 339.3 |
| 376 | 37.6 | 75.2 | 112.8 | 150.4 | 188.0 | 225.6 | 263.2 | 300.8 | 338.4 |
| 375 | 37.5 | 75.0 | 112.5 | 150.0 | 187.5 | 225.0 | 262.5 | 300.0 | 337.5 |
| | | | | | | | | | |
| 374 | 37.4 | 74.8 | 112.2 | 149.6 | 187.0 | 224.4 | 261.8 | 299.2 | 336.6 |
| 373 | 37.3 | 74.6 | 111.9 | 149.2 | 186.5 | 223.8 | 261.1 | 298.4 | 335.7 |
| 372 | 37.2 | 74.4 | 111.6 | 148.8 | 186.0 | 223.2 | 260.4 | 297.6 | 334.8 |
| 371 | 37.1 | 74.2 | 111.3 | 148.4 | 185.5 | 222.6 | 259.7 | 296.8 | 333.9 |
| 370 | 37.0 | 74.0 | 111.0 | 148.0 | 185.0 | 222.0 | 259.0 | 296.0 | 333.0 |
| 369 | 36.9 | 73.8 | 110.7 | 147.6 | 184.5 | 221.4 | 258.3 | 295.2 | 332.1 |
| 368 | 36.8 | 73.6 | 110.4 | 147.2 | 184.0 | 220.8 | 257.6 | 294.4 | 331.2 |
| 367 | 36.7 | 73.4 | 110.1 | 146.8 | 183.5 | 220.2 | 256.9 | 293.6 | 330.3 |
| 366 | 36.6 | 73.2 | 109.8 | 146.4 | 183.0 | 219.6 | 256.2 | 292.8 | 329.4 |
| 365 | 36.5 | 73.0 | 109.5 | 146.0 | 182.5 | 219.0 | 255.5 | 292.0 | 328.5 |
| | | | | | | | | | |
| 364 | 36.4 | 72.8 | 109.2 | 145.6 | 182.0 | 218.4 | 254.8 | 291.2 | 327.6 |
| 363 | 36.3 | 72.6 | 108.9 | 145.2 | 181.5 | 217.8 | 254.1 | 290.4 | 326.7 |
| 362 | 36.2 | 72.4 | 108.6 | 144.8 | 181.0 | 217.2 | 253.4 | 289.6 | 325.8 |
| 361 | 36.1 | 72.2 | 108.3 | 144.4 | 180.5 | 216.6 | 252.7 | 288.8 | 324.9 |
| 360 | 36.0 | 72.0 | 108.0 | 144.0 | 180.0 | 216.0 | 252.0 | 288.0 | 324.0 |
| 359 | 35.9 | 71.8 | 107.7 | 143.6 | 179.5 | 215.4 | 251.3 | 287.2 | 323.1 |
| 358 | 35.8 | 71.6 | 107.4 | 143.2 | 179.0 | 214.8 | 250.6 | 286.4 | 322.2 |
| 357 | 35.7 | 71.4 | 107.1 | 142.8 | 178.5 | 214.2 | 249.9 | 285.6 | 321.3 |
| 356 | 35.6 | 71.2 | 106.8 | 142.4 | 178.0 | 213.6 | 249.2 | 284.8 | 320.4 |

No. 120 L. 079.]

[No. 134 L. 130.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 120 | 079181 | 9543 | 9904 | | | | | | | | |
| | | | | 0266 | 0626 | 0987 | 1347 | 1707 | 2067 | 2426 | 360 |
| 1 | 082785 | 3144 | 3503 | 3861 | 4219 | 4576 | 4934 | 5291 | 5647 | 6004 | 357 |
| 2 | 6360 | 6716 | 7071 | 7426 | 7781 | 8136 | 8490 | 8845 | 9198 | 9552 | 355 |
| 3 | 9905 | | | | | | | | | | |
| | | 0258 | 0611 | 0963 | 1315 | 1667 | 2018 | 2370 | 2721 | 3071 | 352 |
| 4 | 093422 | 3772 | 4122 | 4471 | 4820 | 5169 | 5518 | 5866 | 6215 | 6562 | 349 |
| 5 | 6910 | 7257 | 7604 | 7951 | 8298 | 8644 | 8990 | 9335 | 9681 | | |
| | | | | | | | | | | 0026 | 346 |
| 6 | 100371 | 0715 | 1059 | 1403 | 1747 | 2091 | 2434 | 2777 | 3119 | 3462 | 343 |
| 7 | 3804 | 4146 | 4487 | 4828 | 5169 | 5510 | 5851 | 6191 | 6531 | 6871 | 341 |
| 8 | 7210 | 7549 | 7888 | 8227 | 8565 | 8903 | 9241 | 9579 | 9916 | | |
| | | | | | | | | | | 0253 | 338 |
| 9 | 110590 | 0926 | 1263 | 1599 | 1934 | 2270 | 2605 | 2940 | 3275 | 3609 | 335 |
| | | | | | | | | | | | |
| 130 | 3943 | 4277 | 4611 | 4944 | 5278 | 5611 | 5943 | 6276 | 6608 | 6940 | 333 |
| 1 | 7271 | 7603 | 7934 | 8265 | 8595 | 8926 | 9256 | 9586 | 9915 | | |
| | | | | | | | | | | 0245 | 330 |
| 2 | 120574 | 0903 | 1231 | 1560 | 1888 | 2216 | 2544 | 2871 | 3198 | 3525 | 328 |
| 3 | 3852 | 4178 | 4504 | 4830 | 5156 | 5481 | 5806 | 6131 | 6456 | 6781 | 325 |
| 4 | 7105 | 7429 | 7753 | 8076 | 8399 | 8722 | 9045 | 9368 | 9690 | | |
| | 13 | | | | | | | | | 0012 | 323 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|-------|-------|-------|-------|-------|-------|-------|
| 355 | 35.5 | 71.0 | 106.5 | 142.0 | 177.5 | 213.0 | 248.5 | 284.0 | 319.5 |
| 354 | 35.4 | 70.8 | 106.2 | 141.6 | 177.0 | 212.4 | 247.8 | 283.2 | 318.6 |
| 353 | 35.3 | 70.6 | 105.9 | 141.2 | 176.5 | 211.8 | 247.1 | 282.4 | 317.7 |
| 352 | 35.2 | 70.4 | 105.6 | 140.8 | 176.0 | 211.2 | 246.4 | 281.6 | 316.8 |
| 351 | 35.1 | 70.2 | 105.3 | 140.4 | 175.5 | 210.6 | 245.7 | 280.8 | 315.9 |
| 350 | 35.0 | 70.0 | 105.0 | 140.0 | 175.0 | 210.0 | 245.0 | 280.0 | 315.0 |
| 349 | 34.9 | 69.8 | 104.7 | 139.6 | 174.5 | 209.4 | 244.3 | 279.2 | 314.1 |
| 348 | 34.8 | 69.6 | 104.4 | 139.2 | 174.0 | 208.8 | 243.6 | 278.4 | 313.2 |
| 347 | 34.7 | 69.4 | 104.1 | 138.8 | 173.5 | 208.2 | 242.9 | 277.6 | 312.3 |
| 346 | 34.6 | 69.2 | 103.8 | 138.4 | 173.0 | 207.6 | 242.2 | 276.8 | 311.4 |
| | | | | | | | | | |
| 345 | 34.5 | 69.0 | 103.5 | 138.0 | 172.5 | 207.0 | 241.5 | 276.0 | 310.5 |
| 344 | 34.4 | 68.8 | 103.2 | 137.6 | 172.0 | 206.4 | 240.8 | 275.2 | 309.6 |
| 343 | 34.3 | 68.6 | 102.9 | 137.2 | 171.5 | 205.8 | 240.1 | 274.4 | 308.7 |
| 342 | 34.2 | 68.4 | 102.6 | 136.8 | 171.0 | 205.2 | 239.4 | 273.6 | 307.8 |
| 341 | 34.1 | 68.2 | 102.3 | 136.4 | 170.5 | 204.6 | 238.7 | 272.8 | 306.9 |
| 340 | 34.0 | 68.0 | 102.0 | 136.0 | 170.0 | 204.0 | 238.0 | 272.0 | 306.0 |
| 339 | 33.9 | 67.8 | 101.7 | 135.6 | 169.5 | 203.4 | 237.3 | 271.2 | 305.1 |
| 338 | 33.8 | 67.6 | 101.4 | 135.2 | 169.0 | 202.8 | 236.6 | 270.4 | 304.2 |
| 337 | 33.7 | 67.4 | 101.1 | 134.8 | 168.5 | 202.2 | 235.9 | 269.6 | 303.3 |
| 336 | 33.6 | 67.2 | 100.8 | 134.4 | 168.0 | 201.6 | 235.2 | 268.8 | 302.4 |
| | | | | | | | | | |
| 335 | 33.5 | 67.0 | 100.5 | 134.0 | 167.5 | 201.0 | 234.5 | 268.0 | 301.5 |
| 334 | 33.4 | 66.8 | 100.2 | 133.6 | 167.0 | 200.4 | 233.8 | 267.2 | 300.6 |
| 333 | 33.3 | 66.6 | 99.9 | 133.2 | 166.5 | 199.8 | 233.1 | 266.4 | 299.7 |
| 332 | 33.2 | 66.4 | 99.6 | 132.8 | 166.0 | 199.2 | 232.4 | 265.6 | 298.8 |
| 331 | 33.1 | 66.2 | 99.3 | 132.4 | 165.5 | 198.6 | 231.7 | 264.8 | 297.9 |
| 330 | 33.0 | 66.0 | 99.0 | 132.0 | 165.0 | 198.0 | 231.0 | 264.0 | 297.0 |
| 329 | 32.9 | 65.8 | 98.7 | 131.6 | 164.5 | 197.4 | 230.3 | 263.2 | 296.1 |
| 328 | 32.8 | 65.6 | 98.4 | 131.2 | 164.0 | 196.8 | 229.6 | 262.4 | 295.2 |
| 327 | 32.7 | 65.4 | 98.1 | 130.8 | 163.5 | 196.2 | 228.9 | 261.6 | 294.3 |
| 326 | 32.6 | 65.2 | 97.8 | 130.4 | 163.0 | 195.6 | 228.2 | 260.8 | 293.4 |
| | | | | | | | | | |
| 325 | 32.5 | 65.0 | 97.5 | 130.0 | 162.5 | 195.0 | 227.5 | 260.0 | 292.5 |
| 324 | 32.4 | 64.8 | 97.2 | 129.6 | 162.0 | 194.4 | 226.8 | 259.2 | 291.6 |
| 323 | 32.3 | 64.6 | 96.9 | 129.2 | 161.5 | 193.8 | 226.1 | 258.4 | 290.7 |
| 322 | 32.2 | 64.4 | 96.6 | 128.8 | 161.0 | 193.2 | 225.4 | 257.6 | 289.8 |

No. 135 L. 130.]

[No. 149 L. 175.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 135 | 130334 | 0655 | 0977 | 1298 | 1619 | 1939 | 2260 | 2580 | 2900 | 3219 | 321 |
| 6 | 3539 | 3858 | 4177 | 4496 | 4814 | 5133 | 5451 | 5769 | 6086 | 6403 | 318 |
| 7 | 6721 | 7037 | 7354 | 7671 | 7987 | 8303 | 8618 | 8934 | 9249 | 9564 | 316 |
| 8 | 9879 | | | | | | | | | | |
| 9 | 143015 | 0194 | 0508 | 0822 | 1136 | 1450 | 1763 | 2076 | 2389 | 2702 | 314 |
| | | 3327 | 3639 | 3951 | 4263 | 4574 | 4885 | 5196 | 5507 | 5818 | 311 |
| 140 | 6128 | 6438 | 6748 | 7058 | 7367 | 7676 | 7985 | 8294 | 8603 | 8911 | 309 |
| 1 | 9219 | 9527 | 9835 | | | | | | | | |
| | | | | 0142 | 0449 | 0756 | 1063 | 1370 | 1676 | 1982 | 307 |
| 2 | 152288 | 2594 | 2900 | 3205 | 3510 | 3815 | 4120 | 4424 | 4728 | 5032 | 305 |
| 3 | 5336 | 5640 | 5943 | 6246 | 6549 | 6852 | 7154 | 7457 | 7759 | 8061 | 302 |
| 4 | 8362 | 8664 | 8965 | 9266 | 9567 | 9868 | | | | | |
| | | | | | | | 0168 | 0469 | 0769 | 1068 | 301 |
| 5 | 161368 | 1667 | 1967 | 2266 | 2564 | 2863 | 3161 | 3460 | 3758 | 4055 | 299 |
| 6 | 4353 | 4650 | 4947 | 5244 | 5541 | 5838 | 6134 | 6430 | 6726 | 7022 | 297 |
| 7 | 7317 | 7613 | 7908 | 8203 | 8497 | 8792 | 9086 | 9380 | 9674 | 9968 | 295 |
| 8 | 170262 | 0555 | 0848 | 1141 | 1434 | 1726 | 2019 | 2311 | 2603 | 2895 | 293 |
| 9 | 3186 | 3478 | 3769 | 4060 | 4351 | 4641 | 4932 | 5222 | 5512 | 5802 | 291 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|-------|-------|-------|-------|-------|-------|
| 321 | 32.1 | 64.2 | 96.3 | 128.4 | 160.5 | 192.6 | 224.7 | 256.8 | 288.9 |
| 320 | 32.0 | 64.0 | 96.0 | 128.0 | 160.0 | 192.0 | 224.0 | 256.0 | 288.0 |
| 319 | 31.9 | 63.8 | 95.7 | 127.6 | 159.5 | 191.4 | 223.3 | 255.2 | 287.1 |
| 318 | 31.8 | 63.6 | 95.4 | 127.2 | 159.0 | 190.8 | 222.6 | 254.4 | 286.2 |
| 317 | 31.7 | 63.4 | 95.1 | 126.8 | 158.5 | 190.2 | 221.9 | 253.6 | 285.3 |
| 316 | 31.6 | 63.2 | 94.8 | 126.4 | 158.0 | 189.6 | 221.2 | 252.8 | 284.4 |
| 315 | 31.5 | 63.0 | 94.5 | 126.0 | 157.5 | 189.0 | 220.5 | 252.0 | 283.5 |
| 314 | 31.4 | 62.8 | 94.2 | 125.6 | 157.0 | 188.4 | 219.8 | 251.2 | 282.6 |
| 313 | 31.3 | 62.6 | 93.9 | 125.2 | 156.5 | 187.8 | 219.1 | 250.4 | 281.7 |
| 312 | 31.2 | 62.4 | 93.6 | 124.8 | 156.0 | 187.2 | 218.4 | 249.6 | 280.8 |
| 311 | 31.1 | 62.2 | 93.3 | 124.4 | 155.5 | 186.6 | 217.7 | 248.8 | 279.9 |
| 310 | 31.0 | 62.0 | 93.0 | 124.0 | 155.0 | 186.0 | 217.0 | 248.0 | 279.0 |
| 309 | 30.9 | 61.8 | 92.7 | 123.6 | 154.5 | 185.4 | 216.3 | 247.2 | 278.1 |
| 308 | 30.8 | 61.6 | 92.4 | 123.2 | 154.0 | 184.8 | 215.6 | 246.4 | 277.2 |
| 307 | 30.7 | 61.4 | 92.1 | 122.8 | 153.5 | 184.2 | 214.9 | 245.6 | 276.3 |
| 306 | 30.6 | 61.2 | 91.8 | 122.4 | 153.0 | 183.6 | 214.2 | 244.8 | 275.4 |
| 305 | 30.5 | 61.0 | 91.5 | 122.0 | 152.5 | 183.0 | 213.5 | 244.0 | 274.5 |
| 304 | 30.4 | 60.8 | 91.2 | 121.6 | 152.0 | 182.4 | 212.8 | 243.2 | 273.6 |
| 303 | 30.3 | 60.6 | 90.9 | 121.2 | 151.5 | 181.8 | 212.1 | 242.4 | 272.7 |
| 302 | 30.2 | 60.4 | 90.6 | 120.8 | 151.0 | 181.2 | 211.4 | 241.6 | 271.8 |
| 301 | 30.1 | 60.2 | 90.3 | 120.4 | 150.5 | 180.6 | 210.7 | 240.8 | 270.9 |
| 300 | 30.0 | 60.0 | 90.0 | 120.0 | 150.0 | 180.0 | 210.0 | 240.0 | 270.0 |
| 299 | 29.9 | 59.8 | 89.7 | 119.6 | 149.5 | 179.4 | 209.3 | 239.2 | 269.1 |
| 298 | 29.8 | 59.6 | 89.4 | 119.2 | 149.0 | 178.8 | 208.6 | 238.4 | 268.2 |
| 297 | 29.7 | 59.4 | 89.1 | 118.8 | 148.5 | 178.2 | 207.9 | 237.6 | 267.3 |
| 296 | 29.6 | 59.2 | 88.8 | 118.4 | 148.0 | 177.6 | 207.2 | 236.8 | 266.4 |
| 295 | 29.5 | 59.0 | 88.5 | 118.0 | 147.5 | 177.0 | 206.5 | 236.0 | 265.5 |
| 294 | 29.4 | 58.8 | 88.2 | 117.6 | 147.0 | 176.4 | 205.8 | 235.2 | 264.6 |
| 293 | 29.3 | 58.6 | 87.9 | 117.2 | 146.5 | 175.8 | 205.1 | 234.4 | 263.7 |
| 292 | 29.2 | 58.4 | 87.6 | 116.8 | 146.0 | 175.2 | 204.4 | 233.6 | 262.8 |
| 291 | 29.1 | 58.2 | 87.3 | 116.4 | 145.5 | 174.6 | 203.7 | 232.8 | 261.9 |
| 290 | 29.0 | 58.0 | 87.0 | 116.0 | 145.0 | 174.0 | 203.0 | 232.0 | 261.0 |
| 289 | 28.9 | 57.8 | 86.7 | 115.6 | 144.5 | 173.4 | 202.3 | 231.2 | 260.1 |
| 288 | 28.8 | 57.6 | 86.4 | 115.2 | 144.0 | 172.8 | 201.6 | 230.4 | 259.2 |
| 287 | 28.7 | 57.4 | 86.1 | 114.8 | 143.5 | 172.2 | 200.9 | 229.6 | 258.3 |
| 286 | 28.6 | 57.2 | 85.8 | 114.4 | 143.0 | 171.6 | 200.2 | 228.8 | 257.4 |

No. 150 L. 176.]

[No. 169 L. 230

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 150 | 176091 | 6381 | 6670 | 6959 | 7248 | 7536 | 7825 | 8113 | 8401 | 8689 | 289 |
| 1 | 8977 | 9264 | 9552 | 9839 | | | | | | | |
| 2 | 181844 | 2129 | 2415 | 2700 | 0126 | 0413 | 0699 | 0986 | 1272 | 1558 | 287 |
| 3 | 4691 | 4975 | 5259 | 5542 | 2985 | 3270 | 3555 | 3839 | 4123 | 4407 | 285 |
| 4 | 7521 | 7803 | 8084 | 8366 | 5825 | 6108 | 6391 | 6674 | 6956 | 7239 | 283 |
| 5 | 190332 | 0612 | 0892 | 1171 | 8647 | 8928 | 9209 | 9490 | 9771 | 0051 | 281 |
| 6 | 3125 | 3403 | 3681 | 3959 | 1451 | 1730 | 2010 | 2289 | 2567 | 2846 | 279 |
| 7 | 5900 | 6176 | 6453 | 6729 | 4237 | 4514 | 4792 | 5069 | 5346 | 5623 | 278 |
| 8 | 8657 | 8932 | 9206 | 9481 | 7005 | 7281 | 7556 | 7832 | 8107 | 8382 | 276 |
| 9 | 201397 | 1670 | 1943 | 2216 | 9755 | 0029 | 0303 | 0577 | 0850 | 1124 | 274 |
| 160 | 4120 | 4391 | 4663 | 4934 | 2488 | 2761 | 3033 | 3305 | 3577 | 3848 | 272 |
| 1 | 6826 | 7096 | 7365 | 7634 | 5204 | 5475 | 5746 | 6016 | 6286 | 6556 | 271 |
| 2 | 9515 | 9783 | | | 7904 | 8173 | 8441 | 8710 | 8979 | 9247 | 269 |
| 3 | 212188 | 2454 | 0051 | 0319 | 0586 | 0853 | 1121 | 1388 | 1654 | 1921 | 267 |
| 4 | 4844 | 5109 | 2720 | 2986 | 3252 | 3518 | 3783 | 4049 | 4314 | 4579 | 266 |
| 5 | 7484 | 7747 | 5373 | 5638 | 5902 | 6166 | 6430 | 6694 | 6957 | 7221 | 264 |
| 6 | 220108 | 0370 | 8010 | 8273 | 8536 | 8798 | 9060 | 9323 | 9585 | 9846 | 262 |
| 7 | 2716 | 2976 | 0631 | 0892 | 1153 | 1414 | 1675 | 1936 | 2196 | 2456 | 261 |
| 8 | 5309 | 5568 | 3236 | 3496 | 3755 | 4015 | 4274 | 4533 | 4792 | 5051 | 259 |
| 9 | 7887 | 8144 | 5826 | 6084 | 6342 | 6600 | 6858 | 7115 | 7372 | 7630 | 258 |
| 23 | | | 8400 | 8657 | 8913 | 9170 | 9426 | 9682 | 9938 | | |
| | | | | | | | | | | 0193 | 256 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|-------|-------|-------|-------|-------|-------|
| 285 | 28.5 | 57.0 | 85.5 | 114.0 | 142.5 | 171.0 | 199.5 | 228.0 | 256.5 |
| 284 | 28.4 | 56.8 | 85.2 | 113.6 | 142.0 | 170.4 | 198.8 | 227.2 | 255.6 |
| 283 | 28.3 | 56.6 | 84.9 | 113.2 | 141.5 | 169.8 | 198.1 | 226.4 | 254.7 |
| 282 | 28.2 | 56.4 | 84.6 | 112.8 | 141.0 | 169.2 | 197.4 | 225.6 | 253.8 |
| 281 | 28.1 | 56.2 | 84.3 | 112.4 | 140.5 | 168.6 | 196.7 | 224.8 | 252.9 |
| 280 | 28.0 | 56.0 | 84.0 | 112.0 | 140.0 | 168.0 | 196.0 | 224.0 | 252.0 |
| 279 | 27.9 | 55.8 | 83.7 | 111.6 | 139.5 | 167.4 | 195.3 | 223.2 | 251.1 |
| 278 | 27.8 | 55.6 | 83.4 | 111.2 | 139.0 | 166.8 | 194.6 | 222.4 | 250.2 |
| 277 | 27.7 | 55.4 | 83.1 | 110.8 | 138.5 | 166.2 | 193.9 | 221.6 | 249.3 |
| 276 | 27.6 | 55.2 | 82.8 | 110.4 | 138.0 | 165.6 | 193.2 | 220.8 | 248.4 |
| 275 | 27.5 | 55.0 | 82.5 | 110.0 | 137.5 | 165.0 | 192.5 | 220.0 | 247.5 |
| 274 | 27.4 | 54.8 | 82.2 | 109.6 | 137.0 | 164.4 | 191.8 | 219.2 | 246.6 |
| 273 | 27.3 | 54.6 | 81.9 | 109.2 | 136.5 | 163.8 | 191.1 | 218.4 | 245.7 |
| 272 | 27.2 | 54.4 | 81.6 | 108.8 | 136.0 | 163.2 | 190.4 | 217.6 | 244.8 |
| 271 | 27.1 | 54.2 | 81.3 | 108.4 | 135.5 | 162.6 | 189.7 | 216.8 | 243.9 |
| 270 | 27.0 | 54.0 | 81.0 | 108.0 | 135.0 | 162.0 | 189.0 | 216.0 | 243.0 |
| 269 | 26.9 | 53.8 | 80.7 | 107.6 | 134.5 | 161.4 | 188.3 | 215.2 | 242.1 |
| 268 | 26.8 | 53.6 | 80.4 | 107.2 | 134.0 | 160.8 | 187.6 | 214.4 | 241.2 |
| 267 | 26.7 | 53.4 | 80.1 | 106.8 | 133.5 | 160.2 | 186.9 | 213.6 | 240.3 |
| 266 | 26.6 | 53.2 | 79.8 | 106.4 | 133.0 | 159.6 | 186.2 | 212.8 | 239.4 |
| 265 | 26.5 | 53.0 | 79.5 | 106.0 | 132.5 | 159.0 | 185.5 | 212.0 | 238.5 |
| 264 | 26.4 | 52.8 | 79.2 | 105.6 | 132.0 | 158.4 | 184.8 | 211.2 | 237.6 |
| 263 | 26.3 | 52.6 | 78.9 | 105.2 | 131.5 | 157.8 | 184.1 | 210.4 | 236.7 |
| 262 | 26.2 | 52.4 | 78.6 | 104.8 | 131.0 | 157.2 | 183.4 | 209.6 | 235.8 |
| 261 | 26.1 | 52.2 | 78.3 | 104.4 | 130.5 | 156.6 | 182.7 | 208.8 | 234.9 |
| 260 | 26.0 | 52.0 | 78.0 | 104.0 | 130.0 | 156.0 | 182.0 | 208.0 | 234.0 |
| 259 | 25.9 | 51.8 | 77.7 | 103.6 | 129.5 | 155.4 | 181.3 | 207.2 | 233.1 |
| 258 | 25.8 | 51.6 | 77.4 | 103.2 | 129.0 | 154.8 | 180.6 | 206.4 | 232.2 |
| 257 | 25.7 | 51.4 | 77.1 | 102.8 | 128.5 | 154.2 | 179.9 | 205.6 | 231.3 |
| 256 | 25.6 | 51.2 | 76.8 | 102.4 | 128.0 | 153.6 | 179.2 | 204.8 | 230.4 |
| 255 | 25.5 | 51.0 | 76.5 | 102.0 | 127.5 | 153.0 | 178.5 | 204.0 | 229.5 |

No. 170 L. 230.]

[No. 189 L. 278.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 170 | 230449 | 0704 | 0960 | 1215 | 1470 | 1724 | 1979 | 2234 | 2488 | 2742 | 255 |
| 1 | 2996 | 3250 | 3504 | 3757 | 4011 | 4264 | 4517 | 4770 | 5023 | 5276 | 253 |
| 2 | 5528 | 5781 | 6033 | 6285 | 6537 | 6789 | 7041 | 7292 | 7544 | 7795 | 252 |
| 3 | 8046 | 8297 | 8548 | 8799 | 9049 | 9299 | 9550 | 9800 | | | |
| | | | | | | | | | 0050 | 0300 | 250 |
| 4 | 240549 | 0799 | 1048 | 1297 | 1546 | 1795 | 2044 | 2293 | 2541 | 2790 | 249 |
| 5 | 3038 | 3286 | 3534 | 3782 | 4030 | 4277 | 4525 | 4772 | 5019 | 5266 | 248 |
| 6 | 5513 | 5759 | 6006 | 6252 | 6499 | 6745 | 6991 | 7237 | 7482 | 7728 | 246 |
| 7 | 7973 | 8219 | 8464 | 8709 | 8954 | 9198 | 9443 | 9687 | 9932 | | |
| | | | | | | | | | | 0176 | 245 |
| 8 | 250420 | 0664 | 0908 | 1151 | 1395 | 1638 | 1881 | 2125 | 2368 | 2610 | 243 |
| 9 | 2853 | 3096 | 3338 | 3580 | 3822 | 4064 | 4306 | 4548 | 4790 | 5031 | 242 |
| 180 | 5273 | 5514 | 5755 | 5996 | 6237 | 6477 | 6718 | 6958 | 7198 | 7439 | 241 |
| 1 | 7679 | 7918 | 8158 | 8398 | 8637 | 8877 | 9116 | 9355 | 9594 | 9833 | 239 |
| | | | | | | | | | | | |
| 2 | 260071 | 0310 | 0548 | 0787 | 1025 | 1263 | 1501 | 1739 | 1976 | 2214 | 238 |
| 3 | 2451 | 2688 | 2925 | 3162 | 3399 | 3636 | 3873 | 4109 | 4346 | 4582 | 237 |
| 4 | 4818 | 5054 | 5290 | 5525 | 5761 | 5996 | 6232 | 6467 | 6702 | 6937 | 235 |
| 5 | 7172 | 7406 | 7641 | 7875 | 8110 | 8344 | 8578 | 8812 | 9046 | 9279 | 234 |
| 6 | 9513 | 9746 | 9980 | | | | | | | | |
| | | | | 0213 | 0446 | 0679 | 0912 | 1144 | 1377 | 1609 | 233 |
| 7 | 271842 | 2074 | 2306 | 2538 | 2770 | 3001 | 3233 | 3464 | 3696 | 3927 | 232 |
| 8 | 4158 | 4389 | 4620 | 4850 | 5081 | 5311 | 5542 | 5772 | 6002 | 6232 | 230 |
| 9 | 6462 | 6692 | 6921 | 7151 | 7380 | 7609 | 7838 | 8067 | 8296 | 8525 | 229 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|-------|-------|-------|-------|-------|-------|
| 255 | 25.5 | 51.0 | 76.5 | 102.0 | 127.5 | 153.0 | 178.5 | 204.0 | 229.5 |
| 254 | 25.4 | 50.8 | 76.2 | 101.6 | 127.0 | 152.4 | 177.8 | 203.2 | 228.6 |
| 253 | 25.3 | 50.6 | 75.9 | 101.2 | 126.5 | 151.8 | 177.1 | 202.4 | 227.7 |
| 252 | 25.2 | 50.4 | 75.6 | 100.8 | 126.0 | 151.2 | 176.4 | 201.6 | 226.8 |
| 251 | 25.1 | 50.2 | 75.3 | 100.4 | 125.5 | 150.6 | 175.7 | 200.8 | 225.9 |
| 250 | 25.0 | 50.0 | 75.0 | 100.0 | 125.0 | 150.0 | 175.0 | 200.0 | 225.0 |
| 249 | 24.9 | 49.8 | 74.7 | 99.6 | 124.5 | 149.4 | 174.3 | 199.2 | 224.1 |
| 248 | 24.8 | 49.6 | 74.4 | 99.2 | 124.0 | 148.8 | 173.6 | 198.4 | 223.2 |
| 247 | 24.7 | 49.4 | 74.1 | 98.8 | 123.5 | 148.2 | 172.9 | 197.6 | 222.3 |
| 246 | 24.6 | 49.2 | 73.8 | 98.4 | 123.0 | 147.6 | 172.2 | 196.8 | 221.4 |
| 245 | 24.5 | 49.0 | 73.5 | 98.0 | 122.5 | 147.0 | 171.5 | 196.0 | 220.5 |
| | | | | | | | | | |
| 244 | 24.4 | 48.8 | 73.2 | 97.6 | 122.0 | 146.4 | 170.8 | 195.2 | 219.6 |
| 243 | 24.3 | 48.6 | 72.9 | 97.2 | 121.5 | 145.8 | 170.1 | 194.4 | 218.7 |
| 242 | 24.2 | 48.4 | 72.6 | 96.8 | 121.0 | 145.2 | 169.4 | 193.6 | 217.8 |
| 241 | 24.1 | 48.2 | 72.3 | 96.4 | 120.5 | 144.6 | 168.7 | 192.8 | 216.9 |
| 240 | 24.0 | 48.0 | 72.0 | 96.0 | 120.0 | 144.0 | 168.0 | 192.0 | 216.0 |
| 239 | 23.9 | 47.8 | 71.7 | 95.6 | 119.5 | 143.4 | 167.3 | 191.2 | 215.1 |
| 238 | 23.8 | 47.6 | 71.4 | 95.2 | 119.0 | 142.8 | 166.6 | 190.4 | 214.2 |
| 237 | 23.7 | 47.4 | 71.1 | 94.8 | 118.5 | 142.2 | 165.9 | 189.6 | 213.3 |
| 236 | 23.6 | 47.2 | 70.8 | 94.4 | 118.0 | 141.6 | 165.2 | 188.8 | 212.4 |
| 235 | 23.5 | 47.0 | 70.5 | 94.0 | 117.5 | 141.0 | 164.5 | 188.0 | 211.5 |
| | | | | | | | | | |
| 234 | 23.4 | 46.8 | 70.2 | 93.6 | 117.0 | 140.4 | 163.8 | 187.2 | 210.6 |
| 233 | 23.3 | 46.6 | 69.9 | 93.2 | 116.5 | 139.8 | 163.1 | 186.4 | 209.7 |
| 232 | 23.2 | 46.4 | 69.6 | 92.8 | 116.0 | 139.2 | 162.4 | 185.6 | 208.8 |
| 231 | 23.1 | 46.2 | 69.3 | 92.4 | 115.5 | 138.6 | 161.7 | 184.8 | 207.9 |
| 230 | 23.0 | 46.0 | 69.0 | 92.0 | 115.0 | 138.0 | 161.0 | 184.0 | 207.0 |
| 229 | 22.9 | 45.8 | 68.7 | 91.6 | 114.5 | 137.4 | 160.3 | 183.2 | 206.1 |
| 228 | 22.8 | 45.6 | 68.4 | 91.2 | 114.0 | 136.8 | 159.6 | 182.4 | 205.2 |
| 227 | 22.7 | 45.4 | 68.1 | 90.8 | 113.5 | 136.2 | 158.9 | 181.6 | 204.3 |
| 226 | 22.6 | 45.2 | 67.8 | 90.4 | 113.0 | 135.6 | 158.2 | 180.8 | 203.4 |

No. 190 L. 278.]

[No. 214 L. 332.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 190 | 278754 | 8982 | 9211 | 9439 | 9667 | 9895 | | | | | |
| | | | | | | | 0123 | 0351 | 0578 | 0806 | 228 |
| 1 | 281033 | 1261 | 1488 | 1715 | 1942 | 2169 | 2396 | 2622 | 2849 | 3075 | 227 |
| 2 | 3301 | 3527 | 3753 | 3979 | 4205 | 4431 | 4656 | 4882 | 5107 | 5332 | 226 |
| 3 | 5557 | 5782 | 6007 | 6232 | 6456 | 6681 | 6905 | 7130 | 7354 | 7578 | 225 |
| 4 | 7802 | 8026 | 8249 | 8473 | 8696 | 8920 | 9143 | 9366 | 9589 | 9812 | 223 |
| 5 | 290035 | 0257 | 0480 | 0702 | 0925 | 1147 | 1369 | 1591 | 1813 | 2034 | 222 |
| 6 | 2256 | 2478 | 2699 | 2920 | 3141 | 3363 | 3584 | 3804 | 4025 | 4246 | 221 |
| 7 | 4466 | 4687 | 4907 | 5127 | 5347 | 5567 | 5787 | 6007 | 6226 | 6446 | 220 |
| 8 | 6665 | 6884 | 7104 | 7323 | 7542 | 7761 | 7979 | 8198 | 8416 | 8635 | 219 |
| 9 | 8853 | 9071 | 9289 | 9507 | 9725 | 9943 | | | | | |
| | | | | | | | 0161 | 0378 | 0595 | 0813 | 218 |
| 200 | 301030 | 1247 | 1464 | 1681 | 1898 | 2114 | 2331 | 2547 | 2764 | 2980 | 217 |
| 1 | 3196 | 3412 | 3628 | 3844 | 4059 | 4275 | 4491 | 4706 | 4921 | 5136 | 216 |
| 2 | 5351 | 5566 | 5781 | 5996 | 6211 | 6425 | 6639 | 6854 | 7068 | 7282 | 215 |
| 3 | 7496 | 7710 | 7924 | 8137 | 8351 | 8564 | 8778 | 8991 | 9204 | 9417 | 213 |
| 4 | 9630 | 9843 | | | | | | | | | |
| | | | 0056 | 0268 | 0481 | 0693 | 0906 | 1118 | 1330 | 1542 | 212 |
| 5 | 311754 | 1966 | 2177 | 2389 | 2600 | 2812 | 3023 | 3234 | 3445 | 3656 | 211 |
| 6 | 3867 | 4078 | 4289 | 4499 | 4710 | 4920 | 5130 | 5340 | 5551 | 5760 | 210 |
| 7 | 5970 | 6180 | 6390 | 6599 | 6809 | 7018 | 7227 | 7436 | 7646 | 7854 | 209 |
| 8 | 8063 | 8272 | 8481 | 8689 | 8898 | 9106 | 9314 | 9522 | 9730 | 9938 | 208 |
| 9 | 320146 | 0354 | 0562 | 0769 | 0977 | 1184 | 1391 | 1598 | 1805 | 2012 | 207 |
| 210 | 2219 | 2426 | 2633 | 2839 | 3046 | 3252 | 3458 | 3665 | 3871 | 4077 | 206 |
| 1 | 4282 | 4488 | 4694 | 4899 | 5105 | 5310 | 5516 | 5721 | 5926 | 6131 | 205 |
| 2 | 6336 | 6541 | 6745 | 6950 | 7155 | 7359 | 7563 | 7767 | 7972 | 8176 | 204 |
| 3 | 8380 | 8583 | 8787 | 8991 | 9194 | 9398 | 9601 | 9805 | | | |
| | | | | | | | | | 0008 | 0211 | 203 |
| 4 | 330414 | 0617 | 0819 | 1022 | 1225 | 1427 | 1630 | 1832 | 2034 | 2236 | 202 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|-------|-------|-------|-------|-------|
| 225 | 22.5 | 45.0 | 67.5 | 90.0 | 112.5 | 135.0 | 157.5 | 180.0 | 202.5 |
| 224 | 22.4 | 44.8 | 67.2 | 89.6 | 112.0 | 134.4 | 156.8 | 179.2 | 201.6 |
| 223 | 22.3 | 44.6 | 66.9 | 89.2 | 111.5 | 133.8 | 156.1 | 178.4 | 200.7 |
| 222 | 22.2 | 44.4 | 66.6 | 88.8 | 111.0 | 133.2 | 155.4 | 177.6 | 199.8 |
| 221 | 22.1 | 44.2 | 66.3 | 88.4 | 110.5 | 132.6 | 154.7 | 176.8 | 198.9 |
| 220 | 22.0 | 44.0 | 66.0 | 88.0 | 110.0 | 132.0 | 154.0 | 176.0 | 198.0 |
| 219 | 21.9 | 43.8 | 65.7 | 87.6 | 109.5 | 131.4 | 153.3 | 175.2 | 197.1 |
| 218 | 21.8 | 43.6 | 65.4 | 87.2 | 109.0 | 130.8 | 152.6 | 174.4 | 196.2 |
| 217 | 21.7 | 43.4 | 65.1 | 86.8 | 108.5 | 130.2 | 151.9 | 173.6 | 195.3 |
| 216 | 21.6 | 43.2 | 64.8 | 86.4 | 108.0 | 129.6 | 151.2 | 172.8 | 194.4 |
| 215 | 21.5 | 43.0 | 64.5 | 86.0 | 107.5 | 129.0 | 150.5 | 172.0 | 193.5 |
| 214 | 21.4 | 42.8 | 64.2 | 85.6 | 107.0 | 128.4 | 149.8 | 171.2 | 192.6 |
| 213 | 21.3 | 42.6 | 63.9 | 85.2 | 106.5 | 127.8 | 149.1 | 170.4 | 191.7 |
| 212 | 21.2 | 42.4 | 63.6 | 84.8 | 106.0 | 127.2 | 148.4 | 169.6 | 190.8 |
| 211 | 21.1 | 42.2 | 63.3 | 84.4 | 105.5 | 126.6 | 147.7 | 168.8 | 189.9 |
| 210 | 21.0 | 42.0 | 63.0 | 84.0 | 105.0 | 126.0 | 147.0 | 168.0 | 189.0 |
| 209 | 20.9 | 41.8 | 62.7 | 83.6 | 104.5 | 125.4 | 146.3 | 167.2 | 188.1 |
| 208 | 20.8 | 41.6 | 62.4 | 83.2 | 104.0 | 124.8 | 145.6 | 166.4 | 187.2 |
| 207 | 20.7 | 41.4 | 62.1 | 82.8 | 103.5 | 124.2 | 144.9 | 165.6 | 186.3 |
| 206 | 20.6 | 41.2 | 61.8 | 82.4 | 103.0 | 123.6 | 144.2 | 164.8 | 185.4 |
| 205 | 20.5 | 41.0 | 61.5 | 82.0 | 102.5 | 123.0 | 143.5 | 164.0 | 184.5 |
| 204 | 20.4 | 40.8 | 61.2 | 81.6 | 102.0 | 122.4 | 142.8 | 163.2 | 183.6 |
| 203 | 20.3 | 40.6 | 60.9 | 81.2 | 101.5 | 121.8 | 142.1 | 162.4 | 182.7 |
| 202 | 20.2 | 40.4 | 60.6 | 80.8 | 101.0 | 121.2 | 141.4 | 161.6 | 181.8 |

No. 215 L. 332.]

[No. 239 L. 380.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 215 | 332438 | 2640 | 2842 | 3044 | 3246 | 3447 | 3649 | 3850 | 4051 | 4253 | 202 |
| 6 | 4454 | 4655 | 4856 | 5057 | 5257 | 5458 | 5658 | 5859 | 6059 | 6260 | 201 |
| 7 | 6460 | 6660 | 6860 | 7060 | 7260 | 7459 | 7659 | 7858 | 8058 | 8257 | 200 |
| 8 | 8456 | 8656 | 8855 | 9054 | 9253 | 9451 | 9650 | 9849 | | | |
| 9 | 340444 | 0642 | 0841 | 1039 | 1237 | 1435 | 1632 | 1830 | 0047 | 0246 | 199 |
| | | | | | | | | | 2028 | 2225 | 198 |
| 220 | 2423 | 2620 | 2817 | 3014 | 3212 | 3409 | 3606 | 3802 | 3999 | 4196 | 197 |
| 1 | 4392 | 4589 | 4785 | 4981 | 5178 | 5374 | 5570 | 5766 | 5962 | 6157 | 196 |
| 2 | 6353 | 6549 | 6744 | 6939 | 7135 | 7330 | 7525 | 7720 | 7915 | 8110 | 195 |
| 3 | 8305 | 8500 | 8694 | 8889 | 9083 | 9278 | 9472 | 9666 | 9860 | | |
| 4 | 350248 | 0442 | 0636 | 0829 | 1023 | 1216 | 1410 | 1603 | 1796 | 0054 | 194 |
| 5 | 2183 | 2375 | 2568 | 2761 | 2954 | 3147 | 3339 | 3532 | 3724 | 3916 | 193 |
| 6 | 4108 | 4301 | 4493 | 4685 | 4876 | 5068 | 5260 | 5452 | 5643 | 5834 | 192 |
| 7 | 6026 | 6217 | 6408 | 6599 | 6790 | 6981 | 7172 | 7363 | 7554 | 7744 | 191 |
| 8 | 7935 | 8125 | 8316 | 8506 | 8696 | 8886 | 9076 | 9266 | 9456 | 9646 | 190 |
| 9 | 9835 | | | | | | | | | | |
| | | 0025 | 0215 | 0404 | 0593 | 0783 | 0972 | 1161 | 1350 | 1539 | 189 |
| 230 | 361728 | 1917 | 2105 | 2294 | 2482 | 2671 | 2859 | 3048 | 3236 | 3424 | 188 |
| 1 | 3612 | 3800 | 3988 | 4176 | 4363 | 4551 | 4739 | 4926 | 5113 | 5301 | 188 |
| 2 | 5438 | 5675 | 5862 | 6049 | 6236 | 6423 | 6610 | 6796 | 6983 | 7169 | 187 |
| 3 | 7356 | 7542 | 7729 | 7915 | 8101 | 8287 | 8473 | 8659 | 8845 | 9030 | 186 |
| 4 | 9216 | 9401 | 9587 | 9772 | 9958 | | | | | | |
| 5 | 371063 | 1253 | 1437 | 1622 | 1806 | 1991 | 2175 | 0513 | 0698 | 0883 | 185 |
| 6 | 2912 | 3096 | 3280 | 3464 | 3647 | 3831 | 4015 | 2360 | 2544 | 2728 | 184 |
| 7 | 4748 | 4932 | 5115 | 5298 | 5481 | 5664 | 5846 | 4198 | 4382 | 4565 | 184 |
| 8 | 6577 | 6759 | 6942 | 7124 | 7306 | 7488 | 7670 | 6029 | 6212 | 6394 | 183 |
| 9 | 8398 | 8580 | 8761 | 8943 | 9124 | 9306 | 9487 | 7852 | 8034 | 8216 | 182 |
| 38 | | | | | | | | 9663 | 9849 | | |
| | | | | | | | | | | 0030 | 181 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|-------|-------|-------|-------|-------|
| 202 | 20.2 | 40.4 | 60.6 | 80.8 | 101.0 | 121.2 | 141.4 | 161.6 | 181.8 |
| 201 | 20.1 | 40.2 | 60.3 | 80.4 | 100.5 | 120.6 | 140.7 | 160.8 | 180.9 |
| 200 | 20.0 | 40.0 | 60.0 | 80.0 | 100.0 | 120.0 | 140.0 | 160.0 | 180.0 |
| 199 | 19.9 | 39.8 | 59.7 | 79.6 | 99.5 | 119.4 | 139.3 | 159.2 | 179.1 |
| 198 | 19.8 | 39.6 | 59.4 | 79.2 | 99.0 | 118.8 | 138.6 | 158.4 | 178.2 |
| 197 | 19.7 | 39.4 | 59.1 | 78.8 | 98.5 | 118.2 | 137.9 | 157.6 | 177.3 |
| 196 | 19.6 | 39.2 | 58.8 | 78.4 | 98.0 | 117.6 | 137.2 | 156.8 | 176.4 |
| 195 | 19.5 | 39.0 | 58.5 | 78.0 | 97.5 | 117.0 | 136.5 | 156.0 | 175.5 |
| 194 | 19.4 | 38.8 | 58.2 | 77.6 | 97.0 | 116.4 | 135.8 | 155.2 | 174.6 |
| 193 | 19.3 | 38.6 | 57.9 | 77.2 | 96.5 | 115.8 | 135.1 | 154.4 | 173.7 |
| 192 | 19.2 | 38.4 | 57.6 | 76.8 | 96.0 | 115.2 | 134.4 | 153.6 | 172.8 |
| 191 | 19.1 | 38.2 | 57.3 | 76.4 | 95.5 | 114.6 | 133.7 | 152.8 | 171.9 |
| 190 | 19.0 | 38.0 | 57.0 | 76.0 | 95.0 | 114.0 | 133.0 | 152.0 | 171.0 |
| 189 | 18.9 | 37.8 | 56.7 | 75.6 | 94.5 | 113.4 | 132.3 | 151.2 | 170.1 |
| 188 | 18.8 | 37.6 | 56.4 | 75.2 | 94.0 | 112.8 | 131.6 | 150.4 | 169.2 |
| 187 | 18.7 | 37.4 | 56.1 | 74.8 | 93.5 | 112.2 | 130.9 | 149.6 | 168.3 |
| 186 | 18.6 | 37.2 | 55.3 | 74.4 | 93.0 | 111.6 | 130.2 | 148.8 | 167.4 |
| 185 | 18.5 | 37.0 | 55.5 | 74.0 | 92.5 | 111.0 | 129.5 | 148.0 | 166.5 |
| 184 | 18.4 | 36.8 | 55.2 | 73.6 | 92.0 | 110.4 | 128.8 | 147.2 | 165.6 |
| 183 | 18.3 | 36.6 | 54.9 | 73.2 | 91.5 | 109.8 | 128.1 | 146.4 | 164.7 |
| 182 | 18.2 | 36.4 | 54.6 | 72.8 | 91.0 | 109.2 | 127.4 | 145.6 | 163.8 |
| 181 | 18.1 | 36.2 | 54.3 | 72.4 | 90.5 | 108.6 | 126.7 | 144.8 | 162.9 |
| 180 | 18.0 | 36.0 | 54.0 | 72.0 | 90.0 | 108.0 | 126.0 | 144.0 | 162.0 |
| 179 | 17.9 | 35.8 | 53.7 | 71.6 | 89.5 | 107.4 | 125.3 | 143.2 | 161.1 |

No. 240 L. 380.]

[No. 269 L. 431.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 240 | 380211 | 0392 | 0573 | 0754 | 0934 | 1115 | 1296 | 1476 | 1656 | 1837 | 181 |
| 1 | 2017 | 2197 | 2377 | 2557 | 2737 | 2917 | 3097 | 3277 | 3456 | 3636 | 180 |
| 2 | 3815 | 3995 | 4174 | 4353 | 4533 | 4712 | 4891 | 5070 | 5249 | 5428 | 179 |
| 3 | 5606 | 5785 | 5964 | 6142 | 6321 | 6499 | 6677 | 6856 | 7034 | 7212 | 178 |
| 4 | 7390 | 7568 | 7746 | 7924 | 8101 | 8279 | 8456 | 8634 | 8811 | 8989 | 178 |
| 5 | 9166 | 9343 | 9520 | 9698 | 9875 | | | | | | |
| | | | | | | 0051 | 0228 | 0405 | 0582 | 0759 | 177 |
| 6 | 390935 | 1112 | 1288 | 1464 | 1641 | 1817 | 1993 | 2169 | 2345 | 2521 | 176 |
| 7 | 2697 | 2873 | 3048 | 3224 | 3400 | 3575 | 3751 | 3926 | 4101 | 4277 | 176 |
| 8 | 4452 | 4627 | 4802 | 4977 | 5152 | 5326 | 5501 | 5676 | 5850 | 6025 | 175 |
| 9 | 6199 | 6374 | 6548 | 6722 | 6896 | 7071 | 7245 | 7419 | 7592 | 7766 | 174 |
| 250 | 7940 | 8114 | 8287 | 8461 | 8634 | 8808 | 8981 | 9154 | 9328 | 9501 | 173 |
| 1 | 9674 | 9847 | | | | | | | | | |
| | | | 0020 | 0192 | 0365 | 0538 | 0711 | 0883 | 1056 | 1228 | 173 |
| 2 | 401401 | 1573 | 1745 | 1917 | 2089 | 2261 | 2433 | 2605 | 2777 | 2949 | 172 |
| 3 | 3121 | 3292 | 3464 | 3635 | 3807 | 3978 | 4149 | 4320 | 4492 | 4663 | 171 |
| 4 | 4834 | 5005 | 5176 | 5346 | 5517 | 5688 | 5858 | 6029 | 6199 | 6370 | 171 |
| 5 | 6540 | 6710 | 6881 | 7051 | 7221 | 7391 | 7561 | 7731 | 7901 | 8070 | 170 |
| 6 | 8240 | 8410 | 8579 | 8749 | 8918 | 9087 | 9257 | 9426 | 9595 | 9764 | 169 |
| 7 | 9933 | | | | | | | | | | |
| | | 0102 | 0271 | 0440 | 0609 | 0777 | 0946 | 1114 | 1283 | 1451 | 169 |
| 8 | 411620 | 1788 | 1956 | 2124 | 2293 | 2461 | 2629 | 2796 | 2964 | 3132 | 168 |
| 9 | 3300 | 3467 | 3635 | 3803 | 3970 | 4137 | 4305 | 4472 | 4639 | 4806 | 167 |
| 260 | 4973 | 5140 | 5307 | 5474 | 5641 | 5808 | 5974 | 6141 | 6308 | 6474 | 167 |
| 1 | 6641 | 6807 | 6973 | 7139 | 7306 | 7472 | 7638 | 7804 | 7970 | 8135 | 166 |
| 2 | 8301 | 8467 | 8633 | 8798 | 8964 | 9129 | 9295 | 9460 | 9625 | 9791 | 165 |
| 3 | 9956 | | | | | | | | | | |
| | | 0121 | 0286 | 0451 | 0616 | 0781 | 0945 | 1110 | 1275 | 1439 | 165 |
| 4 | 421604 | 1768 | 1933 | 2097 | 2261 | 2426 | 2590 | 2754 | 2918 | 3082 | 164 |
| 5 | 3246 | 3410 | 3574 | 3737 | 3901 | 4065 | 4228 | 4392 | 4555 | 4718 | 164 |
| 6 | 4882 | 5045 | 5208 | 5371 | 5534 | 5697 | 5860 | 6023 | 6186 | 6349 | 163 |
| 7 | 6511 | 6674 | 6836 | 6999 | 7161 | 7324 | 7486 | 7648 | 7811 | 7973 | 162 |
| 8 | 8135 | 8297 | 8459 | 8621 | 8783 | 8944 | 9106 | 9268 | 9429 | 9591 | 162 |
| 9 | 9752 | 9914 | | | | | | | | | |
| 43 | | | 0075 | 0236 | 0398 | 0559 | 0720 | 0881 | 1042 | 1203 | 161 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|-------|-------|-------|-------|
| 178 | 17.8 | 35.6 | 53.4 | 71.2 | 89.0 | 106.8 | 124.6 | 142.4 | 160.2 |
| 177 | 17.7 | 35.4 | 53.1 | 70.8 | 88.5 | 106.2 | 123.9 | 141.6 | 159.3 |
| 176 | 17.6 | 35.2 | 52.8 | 70.4 | 88.0 | 105.6 | 123.2 | 140.8 | 158.4 |
| 175 | 17.5 | 35.0 | 52.5 | 70.0 | 87.5 | 105.0 | 122.5 | 140.0 | 157.5 |
| 174 | 17.4 | 34.8 | 52.2 | 69.6 | 87.0 | 104.4 | 121.8 | 139.2 | 156.6 |
| 173 | 17.3 | 34.6 | 51.9 | 69.2 | 86.5 | 103.8 | 121.1 | 138.4 | 155.7 |
| 172 | 17.2 | 34.4 | 51.6 | 68.8 | 86.0 | 103.2 | 120.4 | 137.6 | 154.8 |
| 171 | 17.1 | 34.2 | 51.3 | 68.4 | 85.5 | 102.6 | 119.7 | 136.8 | 153.9 |
| 170 | 17.0 | 34.0 | 51.0 | 68.0 | 85.0 | 102.0 | 119.0 | 136.0 | 153.0 |
| 169 | 16.9 | 33.8 | 50.7 | 67.6 | 84.5 | 101.4 | 118.3 | 135.2 | 152.1 |
| 168 | 16.8 | 33.6 | 50.4 | 67.2 | 84.0 | 100.8 | 117.6 | 134.4 | 151.2 |
| 167 | 16.7 | 33.4 | 50.1 | 66.8 | 83.5 | 100.2 | 116.9 | 133.6 | 150.3 |
| 166 | 16.6 | 33.2 | 49.8 | 66.4 | 83.0 | 99.6 | 116.2 | 132.8 | 149.4 |
| 165 | 16.5 | 33.0 | 49.5 | 66.0 | 82.5 | 99.0 | 115.5 | 132.0 | 148.5 |
| 164 | 16.4 | 32.8 | 49.2 | 65.6 | 82.0 | 98.4 | 114.8 | 131.2 | 147.6 |
| 163 | 16.3 | 32.6 | 48.9 | 65.2 | 81.5 | 97.8 | 114.1 | 130.4 | 146.7 |
| 162 | 16.2 | 32.4 | 48.5 | 64.8 | 81.0 | 97.2 | 113.4 | 129.6 | 145.8 |
| 161 | 16.1 | 32.2 | 48.3 | 64.4 | 80.5 | 96.6 | 112.7 | 128.8 | 144.9 |

No. 270 L. 431.]

[No. 299 L. 476.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 270 | 431364 | 1525 | 1685 | 1846 | 2007 | 2167 | 2328 | 2488 | 2649 | 2809 | 161 |
| 1 | 2969 | 3130 | 3290 | 3450 | 3610 | 3770 | 3930 | 4090 | 4249 | 4409 | 160 |
| 2 | 4569 | 4729 | 4888 | 5048 | 5207 | 5367 | 5526 | 5685 | 5844 | 6004 | 159 |
| 3 | 6163 | 6322 | 6481 | 6640 | 6799 | 6957 | 7116 | 7275 | 7433 | 7592 | 159 |
| 4 | 7751 | 7909 | 8067 | 8226 | 8384 | 8542 | 8701 | 8859 | 9017 | 9175 | 158 |
| 5 | 9333 | 9491 | 9648 | 9806 | 9964 | 0122 | 0279 | 0437 | 0594 | 0752 | 158 |
| 6 | 440909 | 1066 | 1224 | 1381 | 1538 | 1695 | 1852 | 2009 | 2166 | 2323 | 157 |
| 7 | 2480 | 2637 | 2793 | 2950 | 3106 | 3263 | 3419 | 3576 | 3732 | 3889 | 157 |
| 8 | 4045 | 4201 | 4357 | 4513 | 4669 | 4825 | 4981 | 5137 | 5293 | 5449 | 156 |
| 9 | 5604 | 5760 | 5915 | 6071 | 6226 | 6382 | 6537 | 6692 | 6848 | 7003 | 155 |
| 280 | 7158 | 7313 | 7468 | 7623 | 7778 | 7933 | 8088 | 8242 | 8397 | 8552 | 155 |
| 1 | 8706 | 8861 | 9015 | 9170 | 9324 | 9478 | 9633 | 9787 | 9941 | 0095 | 154 |
| 2 | 450249 | 0403 | 0557 | 0711 | 0865 | 1018 | 1172 | 1326 | 1479 | 1633 | 154 |
| 3 | 1786 | 1940 | 2093 | 2247 | 2400 | 2553 | 2706 | 2859 | 3012 | 3165 | 153 |
| 4 | 3318 | 3471 | 3624 | 3777 | 3930 | 4082 | 4235 | 4387 | 4540 | 4692 | 153 |
| 5 | 4845 | 4997 | 5150 | 5302 | 5454 | 5606 | 5758 | 5910 | 6062 | 6214 | 152 |
| 6 | 6366 | 6518 | 6670 | 6821 | 6973 | 7125 | 7276 | 7428 | 7579 | 7731 | 152 |
| 7 | 7882 | 8033 | 8184 | 8336 | 8487 | 8638 | 8789 | 8940 | 9091 | 9242 | 151 |
| 8 | 9392 | 9543 | 9694 | 9845 | 9995 | 0146 | 0296 | 0447 | 0597 | 0748 | 151 |
| 9 | 460898 | 1048 | 1198 | 1348 | 1499 | 1649 | 1799 | 1948 | 2098 | 2248 | 150 |
| 290 | 2398 | 2548 | 2697 | 2847 | 2997 | 3146 | 3296 | 3445 | 3594 | 3744 | 150 |
| 1 | 3893 | 4042 | 4191 | 4340 | 4490 | 4639 | 4788 | 4936 | 5085 | 5234 | 149 |
| 2 | 5383 | 5532 | 5680 | 5829 | 5977 | 6126 | 6274 | 6423 | 6571 | 6719 | 149 |
| 3 | 6868 | 7016 | 7164 | 7312 | 7460 | 7608 | 7756 | 7904 | 8052 | 8200 | 148 |
| 4 | 8347 | 8495 | 8643 | 8790 | 8938 | 9085 | 9233 | 9380 | 9527 | 9675 | 148 |
| 5 | 9822 | 9969 | 0116 | 0263 | 0410 | 0557 | 0704 | 0851 | 0998 | 1145 | 147 |
| 6 | 471292 | 1438 | 1585 | 1732 | 1878 | 2025 | 2171 | 2318 | 2464 | 2610 | 146 |
| 7 | 2756 | 2903 | 3049 | 3195 | 3341 | 3487 | 3633 | 3779 | 3925 | 4071 | 146 |
| 8 | 4216 | 4362 | 4508 | 4653 | 4799 | 4944 | 5090 | 5235 | 5381 | 5526 | 146 |
| 9 | 5671 | 5816 | 5962 | 6107 | 6252 | 6397 | 6542 | 6687 | 6832 | 6976 | 145 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|------|-------|-------|-------|
| 161 | 16.1 | 32.2 | 48.3 | 64.4 | 80.5 | 96.6 | 112.7 | 128.8 | 144.9 |
| 160 | 16.0 | 32.0 | 48.0 | 64.0 | 80.0 | 96.0 | 112.0 | 128.0 | 144.0 |
| 159 | 15.9 | 31.8 | 47.7 | 63.6 | 79.5 | 95.4 | 111.3 | 127.2 | 143.1 |
| 158 | 15.8 | 31.6 | 47.4 | 63.2 | 79.0 | 94.8 | 110.6 | 126.4 | 142.2 |
| 157 | 15.7 | 31.4 | 47.1 | 62.8 | 78.5 | 94.2 | 109.9 | 125.6 | 141.3 |
| 156 | 15.6 | 31.2 | 46.8 | 62.4 | 78.0 | 93.6 | 109.2 | 124.8 | 140.4 |
| 155 | 15.5 | 31.0 | 46.5 | 62.0 | 77.5 | 93.0 | 108.5 | 124.0 | 139.5 |
| 154 | 15.4 | 30.8 | 46.2 | 61.6 | 77.0 | 92.4 | 107.8 | 123.2 | 138.6 |
| 153 | 15.3 | 30.6 | 45.9 | 61.2 | 76.5 | 91.8 | 107.1 | 122.4 | 137.7 |
| 152 | 15.2 | 30.4 | 45.6 | 60.8 | 76.0 | 91.2 | 106.4 | 121.6 | 136.8 |
| 151 | 15.1 | 30.2 | 45.3 | 60.4 | 75.5 | 90.6 | 105.7 | 120.8 | 135.9 |
| 150 | 15.0 | 30.0 | 45.0 | 60.0 | 75.0 | 90.0 | 105.0 | 120.0 | 135.0 |
| 149 | 14.9 | 29.8 | 44.7 | 59.6 | 74.5 | 89.4 | 104.3 | 119.2 | 134.1 |
| 148 | 14.8 | 29.6 | 44.4 | 59.2 | 74.0 | 88.8 | 103.6 | 118.4 | 133.2 |
| 147 | 14.7 | 29.4 | 44.1 | 58.8 | 73.5 | 88.2 | 102.9 | 117.6 | 132.3 |
| 146 | 14.6 | 29.2 | 43.8 | 58.4 | 73.0 | 87.6 | 102.2 | 116.8 | 131.4 |
| 145 | 14.5 | 29.0 | 43.5 | 58.0 | 72.5 | 87.0 | 101.5 | 116.0 | 130.5 |
| 144 | 14.4 | 28.8 | 43.2 | 57.6 | 72.0 | 86.4 | 100.8 | 115.2 | 129.6 |
| 143 | 14.3 | 28.6 | 42.9 | 57.2 | 71.5 | 85.8 | 100.1 | 114.4 | 128.7 |
| 142 | 14.2 | 28.4 | 42.6 | 56.8 | 71.0 | 85.2 | 99.4 | 113.6 | 127.8 |
| 141 | 14.1 | 28.2 | 42.3 | 56.4 | 70.5 | 84.6 | 98.7 | 112.8 | 126.9 |
| 140 | 14.0 | 28.0 | 42.0 | 56.0 | 70.0 | 84.0 | 98.0 | 112.0 | 126.0 |

No. 300 L. 477.]

[No. 339 L. 531.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 300 | 477121 | 7266 | 7411 | 7555 | 7700 | 7844 | 7989 | 8133 | 8278 | 8422 | 145 |
| 1 | 8566 | 8711 | 8855 | 8999 | 9143 | 9287 | 9431 | 9575 | 9719 | 9863 | 144 |
| 2 | 480007 | 0151 | 0294 | 0438 | 0582 | 0725 | 0869 | 1012 | 1156 | 1299 | 144 |
| 3 | 1443 | 1586 | 1729 | 1872 | 2016 | 2159 | 2302 | 2445 | 2588 | 2731 | 143 |
| 4 | 2874 | 3016 | 3159 | 3302 | 3445 | 3587 | 3730 | 3872 | 4015 | 4157 | 143 |
| 5 | 4300 | 4442 | 4585 | 4727 | 4869 | 5011 | 5153 | 5295 | 5437 | 5579 | 142 |
| 6 | 5721 | 5863 | 6005 | 6147 | 6289 | 6430 | 6572 | 6714 | 6855 | 6997 | 142 |
| 7 | 7138 | 7280 | 7421 | 7563 | 7704 | 7845 | 7986 | 8127 | 8269 | 8410 | 141 |
| 8 | 8551 | 8692 | 8833 | 8974 | 9114 | 9255 | 9396 | 9537 | 9677 | 9818 | 141 |
| 9 | 9958 | 0099 | 0239 | 0380 | 0520 | 0661 | 0801 | 0941 | 1081 | 1222 | 140 |
| 310 | 491362 | 1502 | 1642 | 1782 | 1922 | 2062 | 2201 | 2341 | 2481 | 2621 | 140 |
| 1 | 2760 | 2900 | 3040 | 3179 | 3319 | 3458 | 3597 | 3737 | 3876 | 4015 | 139 |
| 2 | 4155 | 4294 | 4433 | 4572 | 4711 | 4850 | 4989 | 5128 | 5267 | 5406 | 139 |
| 3 | 5544 | 5683 | 5822 | 5960 | 6099 | 6238 | 6376 | 6515 | 6653 | 6791 | 139 |
| 4 | 6930 | 7068 | 7206 | 7344 | 7483 | 7621 | 7759 | 7897 | 8035 | 8173 | 138 |
| 5 | 8311 | 8448 | 8586 | 8724 | 8862 | 8999 | 9137 | 9275 | 9412 | 9550 | 138 |
| 6 | 9687 | 9824 | 9962 | 0099 | 0236 | 0374 | 0511 | 0648 | 0785 | 0922 | 137 |
| 7 | 501059 | 1196 | 1333 | 1470 | 1607 | 1744 | 1880 | 2017 | 2154 | 2291 | 137 |
| 8 | 2427 | 2564 | 2700 | 2837 | 2973 | 3109 | 3246 | 3382 | 3518 | 3655 | 136 |
| 9 | 3791 | 3927 | 4063 | 4199 | 4335 | 4471 | 4607 | 4743 | 4878 | 5014 | 136 |
| 320 | 5150 | 5286 | 5421 | 5557 | 5693 | 5828 | 5964 | 6099 | 6234 | 6370 | 136 |
| 1 | 6505 | 6640 | 6776 | 6911 | 7046 | 7181 | 7316 | 7451 | 7586 | 7721 | 135 |
| 2 | 7856 | 7991 | 8126 | 8260 | 8395 | 8530 | 8664 | 8799 | 8934 | 9068 | 135 |
| 3 | 9203 | 9337 | 9471 | 9606 | 9740 | 9874 | 0009 | 0143 | 0277 | 0411 | 134 |
| 4 | 510545 | 0679 | 0813 | 0947 | 1081 | 1215 | 1349 | 1482 | 1616 | 1750 | 134 |
| 5 | 1883 | 2017 | 2151 | 2284 | 2418 | 2551 | 2684 | 2818 | 2951 | 3084 | 133 |
| 6 | 3218 | 3351 | 3484 | 3617 | 3750 | 3883 | 4016 | 4149 | 4282 | 4415 | 133 |
| 7 | 4548 | 4681 | 4813 | 4946 | 5079 | 5211 | 5344 | 5476 | 5609 | 5741 | 133 |
| 8 | 5874 | 6006 | 6139 | 6271 | 6403 | 6535 | 6668 | 6800 | 6932 | 7064 | 132 |
| 9 | 7196 | 7328 | 7460 | 7592 | 7724 | 7855 | 7987 | 8119 | 8251 | 8382 | 132 |
| 330 | 8514 | 8646 | 8777 | 8909 | 9040 | 9171 | 9303 | 9434 | 9566 | 9697 | 131 |
| 1 | 9828 | 9959 | 0090 | 0221 | 0353 | 0484 | 0615 | 0745 | 0876 | 1007 | 131 |
| 2 | 521138 | 1269 | 1400 | 1530 | 1661 | 1792 | 1922 | 2053 | 2183 | 2314 | 131 |
| 3 | 2444 | 2575 | 2705 | 2835 | 2966 | 3096 | 3226 | 3356 | 3486 | 3616 | 130 |
| 4 | 3746 | 3876 | 4006 | 4136 | 4266 | 4396 | 4526 | 4656 | 4785 | 4915 | 130 |
| 5 | 5045 | 5174 | 5304 | 5434 | 5563 | 5693 | 5822 | 5951 | 6081 | 6210 | 129 |
| 6 | 6339 | 6469 | 6598 | 6727 | 6856 | 6985 | 7114 | 7243 | 7372 | 7501 | 129 |
| 7 | 7630 | 7759 | 7888 | 8016 | 8145 | 8274 | 8402 | 8531 | 8660 | 8788 | 129 |
| 8 | 8917 | 9045 | 9174 | 9302 | 9430 | 9559 | 9687 | 9815 | 9943 | 0072 | 128 |
| 9 | 530200 | 0328 | 0456 | 0584 | 0712 | 0840 | 0968 | 1096 | 1223 | 1351 | 128 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|------|------|-------|-------|
| 139 | 13.9 | 27.8 | 41.7 | 55.6 | 69.5 | 83.4 | 97.3 | 111.2 | 125.1 |
| 138 | 13.8 | 27.6 | 41.4 | 55.2 | 69.0 | 82.8 | 96.6 | 110.4 | 124.2 |
| 137 | 13.7 | 27.4 | 41.1 | 54.8 | 68.5 | 82.2 | 95.9 | 109.6 | 123.3 |
| 136 | 13.6 | 27.2 | 40.8 | 54.4 | 68.0 | 81.6 | 95.2 | 108.8 | 122.4 |
| 135 | 13.5 | 27.0 | 40.5 | 54.0 | 67.5 | 81.0 | 94.5 | 108.0 | 121.5 |
| 134 | 13.4 | 26.8 | 40.2 | 53.6 | 67.0 | 80.4 | 93.8 | 107.2 | 120.6 |
| 133 | 13.3 | 26.6 | 39.9 | 53.2 | 66.5 | 79.8 | 93.1 | 106.4 | 119.7 |
| 132 | 13.2 | 26.4 | 39.6 | 52.8 | 66.0 | 79.2 | 92.4 | 105.6 | 118.8 |
| 131 | 13.1 | 26.2 | 39.3 | 52.4 | 65.5 | 78.6 | 91.7 | 104.8 | 117.9 |
| 130 | 13.0 | 26.0 | 39.0 | 52.0 | 65.0 | 78.0 | 91.0 | 104.0 | 117.0 |
| 129 | 12.9 | 25.8 | 38.7 | 51.6 | 64.5 | 77.4 | 90.3 | 103.2 | 116.1 |
| 128 | 12.8 | 25.6 | 38.4 | 51.2 | 64.0 | 76.8 | 89.6 | 102.4 | 115.2 |
| 127 | 12.7 | 25.4 | 38.1 | 50.8 | 63.5 | 76.2 | 88.9 | 101.6 | 114.3 |

No. 340 L. 531.]

[No. 379 L. 579.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 340 | 531479 | 1607 | 1734 | 1862 | 1990 | 2117 | 2245 | 2372 | 2500 | 2627 | 128 |
| 1 | 2754 | 2882 | 3009 | 3136 | 3264 | 3391 | 3518 | 3645 | 3772 | 3899 | 127 |
| 2 | 4026 | 4153 | 4280 | 4407 | 4534 | 4661 | 4787 | 4914 | 5041 | 5167 | 127 |
| 3 | 5294 | 5421 | 5547 | 5674 | 5800 | 5927 | 6053 | 6180 | 6306 | 6432 | 126 |
| 4 | 6558 | 6685 | 6811 | 6937 | 7063 | 7189 | 7315 | 7441 | 7567 | 7693 | 126 |
| 5 | 7819 | 7945 | 8071 | 8197 | 8322 | 8448 | 8574 | 8699 | 8825 | 8951 | 126 |
| 6 | 9076 | 9202 | 9327 | 9452 | 9578 | 9703 | 9829 | 9954 | | | |
| 7 | 540329 | 0455 | 0580 | 0705 | 0830 | 0955 | 1080 | 1205 | 0079 | 0204 | 125 |
| 8 | 1579 | 1704 | 1829 | 1953 | 2078 | 2203 | 2327 | 2452 | 1330 | 1454 | 125 |
| 9 | 2825 | 2950 | 3074 | 3199 | 3323 | 3447 | 3571 | 3696 | 2576 | 2701 | 125 |
| 350 | 4068 | 4192 | 4316 | 4440 | 4564 | 4688 | 4812 | 4936 | 3820 | 3944 | 124 |
| 1 | 5307 | 5431 | 5555 | 5678 | 5802 | 5925 | 6049 | 6172 | 6296 | 6419 | 124 |
| 2 | 6543 | 6666 | 6789 | 6913 | 7036 | 7159 | 7282 | 7405 | 7529 | 7652 | 123 |
| 3 | 7775 | 7898 | 8021 | 8144 | 8267 | 8389 | 8512 | 8635 | 8758 | 8881 | 123 |
| 4 | 9003 | 9126 | 9249 | 9371 | 9494 | 9616 | 9739 | 9861 | 9984 | | |
| 5 | 550228 | 0351 | 0473 | 0595 | 0717 | 0840 | 0962 | 1084 | 0106 | 0106 | 123 |
| 6 | 1450 | 1572 | 1694 | 1816 | 1938 | 2060 | 2181 | 2303 | 1206 | 1328 | 122 |
| 7 | 2668 | 2790 | 2911 | 3033 | 3155 | 3276 | 3398 | 3519 | 2425 | 2547 | 122 |
| 8 | 3883 | 4004 | 4126 | 4247 | 4368 | 4489 | 4610 | 4731 | 3640 | 3762 | 121 |
| 9 | 5094 | 5215 | 5336 | 5457 | 5578 | 5699 | 5820 | 5940 | 4852 | 4973 | 121 |
| 360 | 6303 | 6423 | 6544 | 6664 | 6785 | 6905 | 7026 | 7146 | 6061 | 6182 | 121 |
| 1 | 7507 | 7627 | 7748 | 7868 | 7988 | 8108 | 8228 | 8349 | 7267 | 7387 | 120 |
| 2 | 8709 | 8829 | 8948 | 9068 | 9188 | 9308 | 9428 | 9548 | 8469 | 8589 | 120 |
| 3 | 9907 | | | | | | | | 9667 | 9787 | 120 |
| 4 | 561101 | 0026 | 0146 | 0265 | 0385 | 0504 | 0624 | 0743 | 0863 | 0982 | 119 |
| 5 | 2293 | 1221 | 1340 | 1459 | 1578 | 1698 | 1817 | 1936 | 2055 | 2174 | 119 |
| 6 | 3481 | 2412 | 2531 | 2650 | 2769 | 2887 | 3006 | 3125 | 3244 | 3362 | 119 |
| 7 | 4666 | 3600 | 3718 | 3837 | 3955 | 4074 | 4192 | 4311 | 4429 | 4548 | 119 |
| 8 | 5848 | 4784 | 4903 | 5021 | 5139 | 5257 | 5376 | 5494 | 5612 | 5730 | 118 |
| 9 | 7026 | 5966 | 6084 | 6202 | 6320 | 6437 | 6555 | 6673 | 6791 | 6909 | 118 |
| 370 | 8202 | 7144 | 7262 | 7379 | 7497 | 7614 | 7732 | 7849 | 7967 | 8084 | 118 |
| 1 | 9374 | 8319 | 8436 | 8554 | 8671 | 8788 | 8905 | 9023 | 9140 | 9257 | 117 |
| 2 | 570543 | 9491 | 9608 | 9725 | 9842 | 9959 | | | | | |
| 3 | 1709 | 0076 | 0193 | 0309 | 0426 | 0543 | 0660 | 0776 | 0893 | 0426 | 117 |
| 4 | 2872 | 1243 | 1359 | 1476 | 1592 | 1709 | 1825 | 1942 | 2059 | 2175 | 117 |
| 5 | 4031 | 2407 | 2523 | 2639 | 2755 | 2872 | 2988 | 3104 | 3220 | 3336 | 116 |
| 6 | 5188 | 3452 | 3568 | 3684 | 3800 | 3915 | 4031 | 4147 | 4263 | 4379 | 116 |
| 7 | 6341 | 4610 | 4726 | 4841 | 4957 | 5072 | 5188 | 5303 | 5419 | 5534 | 115 |
| 8 | 7492 | 5765 | 5880 | 5996 | 6111 | 6226 | 7607 | 7722 | 7836 | 7951 | 115 |
| 9 | 8639 | 6917 | 7032 | 7147 | 7262 | 7377 | 8066 | 8181 | 8295 | 8410 | 115 |
| | | 9212 | 9326 | 9441 | 9555 | 9669 | | | | | 114 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|------|------|-------|-------|
| 128 | 12.8 | 25.6 | 38.4 | 51.2 | 64.0 | 76.8 | 89.6 | 102.4 | 115.2 |
| 127 | 12.7 | 25.4 | 38.1 | 50.8 | 63.5 | 76.2 | 88.9 | 101.6 | 114.3 |
| 126 | 12.6 | 25.2 | 37.8 | 50.4 | 63.0 | 75.6 | 88.2 | 100.8 | 113.4 |
| 125 | 12.5 | 25.0 | 37.5 | 50.0 | 62.5 | 75.0 | 87.5 | 100.0 | 112.5 |
| 124 | 12.4 | 24.8 | 37.2 | 49.6 | 62.0 | 74.4 | 86.8 | 99.2 | 111.6 |
| 123 | 12.3 | 24.6 | 36.9 | 49.2 | 61.5 | 73.8 | 86.1 | 98.4 | 110.7 |
| 122 | 12.2 | 24.4 | 36.6 | 48.8 | 61.0 | 73.2 | 85.4 | 97.6 | 109.8 |
| 121 | 12.1 | 24.2 | 36.3 | 48.4 | 60.5 | 72.6 | 84.7 | 96.8 | 108.9 |
| 120 | 12.0 | 24.0 | 36.0 | 48.0 | 60.0 | 72.0 | 84.0 | 96.0 | 108.0 |
| 119 | 11.9 | 23.8 | 35.7 | 47.6 | 59.5 | 71.4 | 83.3 | 95.2 | 107.1 |

[No. 380 L. 579.]

[No. 414 L. 617.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 380 | 579784 | 9898 | | | | | | | | | |
| | | | 0012 | 0126 | 0241 | 0355 | 0469 | 0583 | 0697 | 0811 | 114 |
| 1 | 580925 | 1039 | 1153 | 1267 | 1381 | 1495 | 1608 | 1722 | 1836 | 1950 | |
| 2 | 2063 | 2177 | 2291 | 2404 | 2518 | 2631 | 2745 | 2858 | 2972 | 3085 | |
| 3 | 3199 | 3312 | 3426 | 3539 | 3652 | 3765 | 3879 | 3992 | 4105 | 4218 | |
| 4 | 4331 | 4444 | 4557 | 4670 | 4783 | 4896 | 5009 | 5122 | 5235 | 5348 | 113 |
| 5 | 5461 | 5574 | 5686 | 5799 | 5912 | 6024 | 6137 | 6250 | 6362 | 6475 | |
| 6 | 6587 | 6700 | 6812 | 6925 | 7037 | 7149 | 7262 | 7374 | 7486 | 7599 | |
| 7 | 7711 | 7823 | 7935 | 8047 | 8160 | 8272 | 8384 | 8496 | 8608 | 8720 | 112 |
| 8 | 8832 | 8944 | 9056 | 9167 | 9279 | 9391 | 9503 | 9615 | 9726 | 9838 | |
| 9 | 9950 | | | | | | | | | | |
| | | 0061 | 0173 | 0284 | 0396 | 0507 | 0619 | 0730 | 0842 | 0953 | |
| 390 | 591065 | 1176 | 1287 | 1399 | 1510 | 1621 | 1732 | 1843 | 1955 | 2066 | |
| 1 | 2177 | 2288 | 2399 | 2510 | 2621 | 2732 | 2843 | 2954 | 3064 | 3175 | 111 |
| 2 | 3286 | 3397 | 3508 | 3618 | 3729 | 3840 | 3950 | 4061 | 4171 | 4282 | |
| 3 | 4393 | 4503 | 4614 | 4724 | 4834 | 4945 | 5055 | 5165 | 5276 | 5386 | |
| 4 | 5496 | 5606 | 5717 | 5827 | 5937 | 6047 | 6157 | 6267 | 6377 | 6487 | 110 |
| 5 | 6597 | 6707 | 6817 | 6927 | 7037 | 7146 | 7256 | 7366 | 7476 | 7586 | |
| 6 | 7695 | 7805 | 7914 | 8024 | 8134 | 8243 | 8353 | 8462 | 8572 | 8681 | |
| 7 | 8791 | 8900 | 9009 | 9119 | 9228 | 9337 | 9446 | 9556 | 9665 | 9774 | 109 |
| 8 | 9883 | 9992 | | | | | | | | | |
| 9 | 600973 | 1082 | 1191 | 1299 | 1408 | 1517 | 1625 | 1734 | 1843 | 1951 | |
| 400 | 2060 | 2169 | 2277 | 2386 | 2494 | 2603 | 2711 | 2819 | 2928 | 3036 | |
| 1 | 3144 | 3253 | 3361 | 3469 | 3577 | 3686 | 3794 | 3902 | 4010 | 4118 | 108 |
| 2 | 4226 | 4334 | 4442 | 4550 | 4658 | 4766 | 4874 | 4982 | 5089 | 5197 | |
| 3 | 5305 | 5413 | 5521 | 5628 | 5736 | 5844 | 5951 | 6059 | 6166 | 6274 | |
| 4 | 6381 | 6489 | 6596 | 6704 | 6811 | 6919 | 7026 | 7133 | 7241 | 7348 | 107 |
| 5 | 7455 | 7562 | 7669 | 7777 | 7884 | 7991 | 8098 | 8205 | 8312 | 8419 | |
| 6 | 8526 | 8633 | 8740 | 8847 | 8954 | 9061 | 9167 | 9274 | 9381 | 9488 | |
| 7 | 9594 | 9701 | 9808 | 9914 | | | | | | | |
| 8 | 610660 | 0767 | 0873 | 0979 | 1086 | 1192 | 1298 | 1405 | 1511 | 1617 | 106 |
| 9 | 1723 | 1829 | 1936 | 2042 | 2148 | 2254 | 2360 | 2466 | 2572 | 2678 | |
| 410 | 2784 | 2890 | 2996 | 3102 | 3207 | 3313 | 3419 | 3525 | 3630 | 3736 | |
| 1 | 3842 | 3947 | 4053 | 4159 | 4264 | 4370 | 4475 | 4581 | 4686 | 4792 | |
| 2 | 4897 | 5003 | 5108 | 5213 | 5319 | 5424 | 5529 | 5634 | 5740 | 5845 | |
| 3 | 5950 | 6055 | 6160 | 6265 | 6370 | 6476 | 6581 | 6686 | 6790 | 6895 | 105 |
| 4 | 7000 | 7105 | 7210 | 7315 | 7420 | 7525 | 7629 | 7734 | 7839 | 7943 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|------|------|------|-------|
| 118 | 11.8 | 23.6 | 35.4 | 47.2 | 59.0 | 70.8 | 82.6 | 94.4 | 106.2 |
| 117 | 11.7 | 23.4 | 35.1 | 46.8 | 58.5 | 70.2 | 81.9 | 93.6 | 105.3 |
| 116 | 11.6 | 23.2 | 34.8 | 46.4 | 58.0 | 69.6 | 81.2 | 92.8 | 104.4 |
| 115 | 11.5 | 23.0 | 34.5 | 46.0 | 57.5 | 69.0 | 80.5 | 92.0 | 103.5 |
| 114 | 11.4 | 22.8 | 34.2 | 45.6 | 57.0 | 68.4 | 79.8 | 91.2 | 102.6 |
| 113 | 11.3 | 22.6 | 33.9 | 45.2 | 56.5 | 67.8 | 79.1 | 90.4 | 101.7 |
| 112 | 11.2 | 22.4 | 33.6 | 44.8 | 56.0 | 67.2 | 78.4 | 89.6 | 100.8 |
| 111 | 11.1 | 22.2 | 33.3 | 44.4 | 55.5 | 66.6 | 77.7 | 88.8 | 99.9 |
| 110 | 11.0 | 22.0 | 33.0 | 44.0 | 55.0 | 66.0 | 77.0 | 88.0 | 99.0 |
| 109 | 10.9 | 21.8 | 32.7 | 43.6 | 54.5 | 65.4 | 76.3 | 87.2 | 98.1 |
| 108 | 10.8 | 21.6 | 32.4 | 43.2 | 54.0 | 64.8 | 75.6 | 86.4 | 97.2 |
| 107 | 10.7 | 21.4 | 32.1 | 42.8 | 53.5 | 64.2 | 74.9 | 85.6 | 96.3 |
| 106 | 10.6 | 21.2 | 31.8 | 42.4 | 53.0 | 63.6 | 74.2 | 84.8 | 95.4 |
| 105 | 10.5 | 21.0 | 31.5 | 42.0 | 52.5 | 63.0 | 73.5 | 84.0 | 94.5 |
| 104 | 10.4 | 20.8 | 31.2 | 41.6 | 52.0 | 62.4 | 72.8 | 83.2 | 93.6 |

No. 415 L. 618.]

[No. 459 L. 662

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 415 | 618043 | 8153 | 8257 | 8362 | 8466 | 8571 | 8676 | 8780 | 8884 | 8989 | 105 |
| 6 | 9093 | 9198 | 9302 | 9406 | 9511 | 9615 | 9719 | 9824 | 9928 | 0032 | |
| 7 | 620136 | 0240 | 0344 | 0448 | 0552 | 0656 | 0760 | 0864 | 0968 | 1072 | 104 |
| 8 | 1176 | 1280 | 1384 | 1488 | 1592 | 1695 | 1799 | 1903 | 2007 | 2110 | |
| 9 | 2214 | 2318 | 2421 | 2525 | 2628 | 2732 | 2835 | 2939 | 3042 | 3146 | |
| 420 | 3249 | 3353 | 3456 | 3559 | 3663 | 3766 | 3869 | 3973 | 4076 | 4179 | |
| 1 | 4232 | 4385 | 4488 | 4591 | 4695 | 4798 | 4901 | 5004 | 5107 | 5210 | 103 |
| 2 | 5312 | 5415 | 5518 | 5621 | 5724 | 5827 | 5929 | 6032 | 6135 | 6238 | |
| 3 | 6340 | 6443 | 6546 | 6648 | 6751 | 6853 | 6956 | 7058 | 7161 | 7263 | |
| 4 | 7366 | 7463 | 7571 | 7673 | 7775 | 7878 | 7980 | 8082 | 8185 | 8287 | |
| 5 | 8389 | 8491 | 8593 | 8695 | 8797 | 8900 | 9002 | 9104 | 9206 | 9308 | 102 |
| 6 | 9410 | 9512 | 9613 | 9715 | 9817 | 9919 | 0021 | 0123 | 0224 | 0326 | |
| 7 | 630428 | 0530 | 0631 | 0733 | 0835 | 0936 | 1038 | 1139 | 1241 | 1342 | |
| 8 | 1444 | 1545 | 1647 | 1748 | 1849 | 1951 | 2052 | 2153 | 2255 | 2356 | |
| 9 | 2457 | 2559 | 2660 | 2761 | 2862 | 2963 | 3064 | 3165 | 3266 | 3367 | |
| 430 | 3468 | 3569 | 3670 | 3771 | 3872 | 3973 | 4074 | 4175 | 4276 | 4376 | 101 |
| 1 | 4477 | 4578 | 4679 | 4779 | 4880 | 4981 | 5081 | 5182 | 5283 | 5383 | |
| 2 | 5484 | 5584 | 5685 | 5785 | 5886 | 5986 | 6087 | 6187 | 6287 | 6388 | |
| 3 | 6488 | 6588 | 6688 | 6789 | 6889 | 6989 | 7089 | 7189 | 7290 | 7390 | |
| 4 | 7490 | 7590 | 7690 | 7790 | 7890 | 7990 | 8090 | 8190 | 8290 | 8389 | 100 |
| 5 | 8489 | 8589 | 8689 | 8789 | 8888 | 8988 | 9088 | 9188 | 9287 | 9387 | |
| 6 | 9486 | 9586 | 9686 | 9785 | 9885 | 9984 | 0084 | 0183 | 0283 | 0382 | |
| 7 | 640431 | 0581 | 0680 | 0779 | 0879 | 0978 | 1077 | 1177 | 1276 | 1375 | |
| 8 | 1474 | 1573 | 1672 | 1771 | 1871 | 1970 | 2069 | 2168 | 2267 | 2366 | |
| 9 | 2465 | 2563 | 2662 | 2761 | 2860 | 2959 | 3058 | 3156 | 3255 | 3354 | 99 |
| 440 | 3453 | 3551 | 3650 | 3749 | 3847 | 3946 | 4044 | 4143 | 4242 | 4340 | |
| 1 | 4439 | 4537 | 4636 | 4734 | 4832 | 4931 | 5029 | 5127 | 5226 | 5324 | |
| 2 | 5422 | 5521 | 5619 | 5717 | 5815 | 5913 | 6011 | 6110 | 6208 | 6306 | |
| 3 | 6404 | 6502 | 6600 | 6698 | 6796 | 6894 | 6992 | 7089 | 7187 | 7285 | 98 |
| 4 | 7383 | 7481 | 7579 | 7676 | 7774 | 7872 | 7969 | 8067 | 8165 | 8262 | |
| 5 | 8360 | 8458 | 8555 | 8653 | 8750 | 8848 | 8945 | 9043 | 9140 | 9237 | |
| 6 | 9335 | 9432 | 9530 | 9627 | 9724 | 9821 | 9919 | 0016 | 0113 | 0210 | |
| 7 | 650303 | 0405 | 0502 | 0599 | 0696 | 0793 | 0890 | 0987 | 1084 | 1181 | |
| 8 | 1278 | 1375 | 1472 | 1569 | 1666 | 1762 | 1859 | 1956 | 2053 | 2150 | 97 |
| 9 | 2246 | 2343 | 2440 | 2536 | 2633 | 2730 | 2826 | 2923 | 3019 | 3116 | |
| 450 | 3213 | 3309 | 3405 | 3502 | 3598 | 3695 | 3791 | 3888 | 3984 | 4080 | |
| 1 | 4177 | 4273 | 4369 | 4465 | 4562 | 4658 | 4754 | 4850 | 4946 | 5042 | |
| 2 | 5138 | 5235 | 5331 | 5427 | 5523 | 5619 | 5715 | 5810 | 5906 | 6002 | 96 |
| 3 | 6093 | 6194 | 6290 | 6386 | 6482 | 6577 | 6673 | 6769 | 6864 | 6960 | |
| 4 | 7056 | 7152 | 7247 | 7343 | 7438 | 7534 | 7629 | 7725 | 7820 | 7916 | |
| 5 | 8011 | 8107 | 8202 | 8298 | 8393 | 8488 | 8584 | 8679 | 8774 | 8870 | |
| 6 | 8965 | 9060 | 9155 | 9250 | 9346 | 9441 | 9536 | 9631 | 9726 | 9821 | |
| 7 | 9916 | 0011 | 0106 | 0201 | 0296 | 0391 | 0486 | 0581 | 0676 | 0771 | 95 |
| 8 | 660865 | 0960 | 1055 | 1150 | 1245 | 1339 | 1434 | 1529 | 1623 | 1718 | |
| 9 | 1813 | 1907 | 2002 | 2096 | 2191 | 2286 | 2380 | 2475 | 2569 | 2663 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|------|------|------|------|------|------|------|------|------|
| 105 | 10.5 | 21.0 | 31.5 | 42.0 | 52.5 | 63.0 | 73.5 | 84.0 | 94.5 |
| 104 | 10.4 | 20.8 | 31.2 | 41.6 | 52.0 | 62.4 | 72.8 | 83.2 | 93.6 |
| 103 | 10.3 | 20.6 | 30.9 | 41.2 | 51.5 | 61.8 | 72.1 | 82.4 | 92.7 |
| 102 | 10.2 | 20.4 | 30.6 | 40.8 | 51.0 | 61.2 | 71.4 | 81.6 | 91.8 |
| 101 | 10.1 | 20.2 | 30.3 | 40.4 | 50.5 | 60.6 | 70.7 | 80.8 | 90.9 |
| 100 | 10.0 | 20.0 | 30.0 | 40.0 | 50.0 | 60.0 | 70.0 | 80.0 | 90.0 |
| 99 | 9.9 | 19.8 | 29.7 | 39.6 | 49.5 | 59.4 | 69.3 | 79.2 | 89.1 |

No. 460 L. 632.]

[No. 499 L. 698.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 460 | 662758 | 2852 | 2947 | 3041 | 3135 | 3230 | 3324 | 3418 | 3512 | 3607 | |
| 1 | 3701 | 3795 | 3889 | 3983 | 4078 | 4172 | 4266 | 4360 | 4454 | 4548 | |
| 2 | 4642 | 4736 | 4830 | 4924 | 5018 | 5112 | 5206 | 5299 | 5393 | 5487 | 94 |
| 3 | 5581 | 5675 | 5769 | 5862 | 5956 | 6050 | 6143 | 6237 | 6331 | 6424 | |
| 4 | 6518 | 6612 | 6705 | 6799 | 6892 | 6986 | 7079 | 7173 | 7266 | 7360 | |
| 5 | 7453 | 7546 | 7640 | 7733 | 7826 | 7920 | 8013 | 8106 | 8199 | 8293 | |
| 6 | 8386 | 8479 | 8572 | 8665 | 8759 | 8852 | 8945 | 9038 | 9131 | 9224 | |
| 7 | 9317 | 9410 | 9503 | 9596 | 9689 | 9782 | 9875 | 9967 | | | |
| | | | | | | | | | 0060 | 0153 | 93 |
| 8 | 670246 | 0339 | 0431 | 0524 | 0617 | 0710 | 0802 | 0895 | 0988 | 1080 | |
| 9 | 1173 | 1265 | 1358 | 1451 | 1543 | 1636 | 1728 | 1821 | 1913 | 2005 | |
| 470 | 2098 | 2190 | 2283 | 2375 | 2467 | 2560 | 2652 | 2744 | 2836 | 2929 | |
| 1 | 3021 | 3113 | 3205 | 3297 | 3390 | 3482 | 3574 | 3666 | 3758 | 3850 | |
| 2 | 3942 | 4034 | 4126 | 4218 | 4310 | 4402 | 4494 | 4586 | 4677 | 4769 | 92 |
| 3 | 4861 | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 | |
| 4 | 5778 | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 | |
| 5 | 6694 | 6785 | 6876 | 6968 | 7059 | 7151 | 7242 | 7333 | 7424 | 7516 | |
| 6 | 7607 | 7698 | 7789 | 7881 | 7972 | 8063 | 8154 | 8245 | 8336 | 8427 | |
| 7 | 8518 | 8609 | 8700 | 8791 | 8882 | 8973 | 9064 | 9155 | 9246 | 9337 | 91 |
| 8 | 9428 | 9519 | 9610 | 9700 | 9791 | 9882 | 9973 | | | | |
| | | | | | | | | 0063 | 0154 | 0245 | |
| 9 | 680336 | 0426 | 0517 | 0607 | 0698 | 0789 | 0879 | 0970 | 1060 | 1151 | |
| 480 | 1241 | 1332 | 1422 | 1513 | 1603 | 1693 | 1784 | 1874 | 1964 | 2055 | |
| 1 | 2145 | 2235 | 2326 | 2416 | 2506 | 2596 | 2686 | 2777 | 2867 | 2957 | |
| 2 | 3047 | 3137 | 3227 | 3317 | 3407 | 3497 | 3587 | 3677 | 3767 | 3857 | 90 |
| 3 | 3947 | 4037 | 4127 | 4217 | 4307 | 4396 | 4486 | 4576 | 4666 | 4756 | |
| 4 | 4845 | 4935 | 5025 | 5114 | 5204 | 5294 | 5383 | 5473 | 5563 | 5652 | |
| 5 | 5742 | 5831 | 5921 | 6010 | 6100 | 6189 | 6279 | 6368 | 6458 | 6547 | |
| 6 | 6636 | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 | |
| 7 | 7529 | 7618 | 7707 | 7796 | 7886 | 7975 | 8064 | 8153 | 8242 | 8331 | |
| 8 | 8420 | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 | 89 |
| 9 | 9309 | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | | | |
| | | | | | | | | | 0019 | 0107 | |
| 490 | 690196 | 0285 | 0373 | 0462 | 0550 | 0639 | 0728 | 0816 | 0905 | 0993 | |
| 1 | 1081 | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 | |
| 2 | 1965 | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 | |
| 3 | 2847 | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 | 88 |
| 4 | 3727 | 3815 | 3903 | 3991 | 4078 | 4166 | 4254 | 4342 | 4430 | 4517 | |
| 5 | 4605 | 4693 | 4781 | 4868 | 4956 | 5044 | 5131 | 5219 | 5307 | 5394 | |
| 6 | 5482 | 5569 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 | |
| 7 | 6356 | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 | |
| 8 | 7229 | 7317 | 7404 | 7491 | 7578 | 7665 | 7752 | 7839 | 7926 | 8014 | 87 |
| 9 | 8100 | 8188 | 8275 | 8362 | 8449 | 8535 | 8622 | 8709 | 8796 | 8883 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 98 | 9.8 | 19.6 | 29.4 | 39.2 | 49.0 | 58.8 | 68.6 | 78.4 | 88.2 |
| 97 | 9.7 | 19.4 | 29.1 | 38.8 | 48.5 | 58.2 | 67.9 | 77.6 | 87.3 |
| 96 | 9.6 | 19.2 | 28.8 | 38.4 | 48.0 | 57.6 | 67.2 | 76.8 | 86.4 |
| 95 | 9.5 | 19.0 | 28.5 | 38.0 | 47.5 | 57.0 | 66.5 | 76.0 | 85.5 |
| 94 | 9.4 | 18.8 | 28.2 | 37.6 | 47.0 | 56.4 | 65.8 | 75.2 | 84.6 |
| 93 | 9.3 | 18.6 | 27.9 | 37.2 | 46.5 | 55.8 | 65.1 | 74.4 | 83.7 |
| 92 | 9.2 | 18.4 | 27.6 | 36.8 | 46.0 | 55.2 | 64.4 | 73.6 | 82.8 |
| 91 | 9.1 | 18.2 | 27.3 | 36.4 | 45.5 | 54.6 | 63.7 | 72.8 | 81.9 |
| 90 | 9.0 | 18.0 | 27.0 | 36.0 | 45.0 | 54.0 | 63.0 | 72.0 | 81.0 |
| 89 | 8.9 | 17.8 | 26.7 | 35.6 | 44.5 | 53.4 | 62.3 | 71.2 | 80.1 |
| 88 | 8.8 | 17.6 | 26.4 | 35.2 | 44.0 | 52.8 | 61.6 | 70.4 | 79.2 |
| 87 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 |
| 86 | 8.6 | 17.2 | 25.8 | 34.4 | 43.0 | 51.6 | 60.2 | 68.8 | 77.4 |

No. 500 L. 698.]

[No. 544 L. 736.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 500 | 698970 | 9057 | 9144 | 9231 | 9317 | 9404 | 9491 | 9578 | 9664 | 9751 | |
| 1 | 938 | 9924 | | | | | | | | | |
| 2 | 700704 | 0790 | 0011 | 0098 | 0184 | 0271 | 0358 | 0444 | 0531 | 0617 | |
| 3 | 1568 | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 | |
| 4 | 2431 | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 | |
| 5 | 3291 | 3377 | 3463 | 3549 | 3635 | 3721 | 3807 | 3893 | 3979 | 4065 | 86 |
| 6 | 4151 | 4236 | 4322 | 4408 | 4494 | 4579 | 4665 | 4751 | 4837 | 4922 | |
| 7 | 5008 | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 | |
| 8 | 5864 | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 | |
| 9 | 6718 | 6803 | 6888 | 6974 | 7059 | 7144 | 7229 | 7315 | 7400 | 7485 | |
| 510 | 7570 | 7655 | 7740 | 7826 | 7911 | 7996 | 8081 | 8166 | 8251 | 8336 | |
| 1 | 8421 | 8506 | 8591 | 8676 | 8761 | 8846 | 8931 | 9015 | 9100 | 9185 | 85 |
| 2 | 9270 | 9355 | 9440 | 9524 | 9609 | 9694 | 9779 | 9863 | 9948 | | |
| 3 | 710117 | 0202 | 0287 | 0371 | 0456 | 0540 | 0625 | 0710 | 0794 | 0879 | |
| 4 | 0963 | 1048 | 1132 | 1217 | 1301 | 1385 | 1470 | 1554 | 1639 | 1723 | |
| 5 | 1807 | 1892 | 1976 | 2060 | 2144 | 2229 | 2313 | 2397 | 2481 | 2566 | |
| 6 | 2650 | 2734 | 2818 | 2902 | 2986 | 3070 | 3154 | 3238 | 3323 | 3407 | |
| 7 | 3491 | 3575 | 3659 | 3742 | 3826 | 3910 | 3994 | 4078 | 4162 | 4246 | 84 |
| 8 | 4330 | 4414 | 4497 | 4581 | 4665 | 4749 | 4833 | 4916 | 5000 | 5084 | |
| 9 | 5167 | 5251 | 5335 | 5418 | 5502 | 5586 | 5669 | 5753 | 5836 | 5920 | |
| 520 | 6003 | 6087 | 6170 | 6254 | 6337 | 6421 | 6504 | 6588 | 6671 | 6754 | |
| 1 | 6838 | 6921 | 7004 | 7088 | 7171 | 7254 | 7338 | 7421 | 7504 | 7587 | |
| 2 | 7671 | 7754 | 7837 | 7920 | 8003 | 8086 | 8169 | 8253 | 8336 | 8419 | 83 |
| 3 | 8502 | 8585 | 8668 | 8751 | 8834 | 8917 | 9000 | 9083 | 9165 | 9248 | |
| 4 | 9331 | 9414 | 9497 | 9580 | 9663 | 9745 | 9828 | 9911 | 9994 | | |
| 5 | 720159 | 0242 | 0325 | 0407 | 0490 | 0573 | 0655 | 0738 | 0821 | 0903 | |
| 6 | 0986 | 1068 | 1151 | 1233 | 1316 | 1398 | 1481 | 1563 | 1646 | 1728 | |
| 7 | 1811 | 1893 | 1975 | 2058 | 2140 | 2222 | 2305 | 2387 | 2469 | 2552 | |
| 8 | 2634 | 2716 | 2798 | 2881 | 2963 | 3045 | 3127 | 3209 | 3291 | 3374 | |
| 9 | 3456 | 3538 | 3620 | 3702 | 3784 | 3866 | 3948 | 4030 | 4112 | 4194 | 82 |
| 530 | 4276 | 4358 | 4440 | 4522 | 4604 | 4685 | 4767 | 4849 | 4931 | 5013 | |
| 1 | 5095 | 5176 | 5258 | 5340 | 5422 | 5503 | 5585 | 5667 | 5748 | 5830 | |
| 2 | 5912 | 5993 | 6075 | 6156 | 6238 | 6320 | 6401 | 6483 | 6564 | 6646 | |
| 3 | 6727 | 6809 | 6890 | 6972 | 7053 | 7134 | 7216 | 7297 | 7379 | 7460 | |
| 4 | 7541 | 7623 | 7704 | 7785 | 7866 | 7948 | 8029 | 8110 | 8191 | 8273 | |
| 5 | 8354 | 8435 | 8516 | 8597 | 8678 | 8759 | 8841 | 8922 | 9003 | 9084 | |
| 6 | 9165 | 9246 | 9327 | 9408 | 9489 | 9570 | 9651 | 9732 | 9813 | 9893 | 81 |
| 7 | 9974 | | | | | | | | | | |
| 8 | 730782 | 0055 | 0136 | 0217 | 0298 | 0378 | 0459 | 0540 | 0621 | 0702 | |
| 9 | 1589 | 0863 | 0944 | 1024 | 1105 | 1186 | 1266 | 1347 | 1428 | 1508 | |
| 540 | 2394 | 2474 | 2555 | 2635 | 2715 | 2796 | 2876 | 2956 | 3037 | 3117 | |
| 1 | 3197 | 3278 | 3358 | 3438 | 3518 | 3598 | 3679 | 3759 | 3839 | 3919 | |
| 2 | 3999 | 4079 | 4160 | 4240 | 4320 | 4400 | 4480 | 4560 | 4640 | 4720 | 80 |
| 3 | 4800 | 4880 | 4960 | 5040 | 5120 | 5200 | 5279 | 5359 | 5439 | 5519 | |
| 4 | 5599 | 5679 | 5759 | 5838 | 5918 | 5998 | 6078 | 6157 | 6237 | 6317 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 87 | 8.7 | 17.4 | 26.1 | 34.8 | 43.5 | 52.2 | 60.9 | 69.6 | 78.3 |
| 86 | 8.6 | 17.2 | 25.8 | 34.4 | 43.0 | 51.6 | 60.2 | 68.8 | 77.4 |
| 85 | 8.5 | 17.0 | 25.5 | 34.0 | 42.5 | 51.0 | 59.5 | 68.0 | 76.5 |
| 84 | 8.4 | 16.8 | 25.2 | 33.6 | 42.0 | 50.4 | 58.8 | 67.2 | 75.6 |

No. 545 L. 736.]

[No. 584 L. 767.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 545 | 736397 | 6476 | 6556 | 6635 | 6715 | 6795 | 6874 | 6954 | 7034 | 7113 | |
| 6 | 7193 | 7272 | 7352 | 7431 | 7511 | 7590 | 7670 | 7749 | 7829 | 7908 | |
| 7 | 7987 | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 | |
| 8 | 8781 | 8860 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 | |
| 9 | 9572 | 9651 | 9731 | 9810 | 9889 | 9968 | | | | | |
| | | | | | | | 0047 | 0126 | 0205 | 0284 | 79 |
| 550 | 740363 | 0442 | 0521 | 0600 | 0678 | 0757 | 0836 | 0915 | 0994 | 1073 | |
| 1 | 1152 | 1230 | 1309 | 1388 | 1467 | 1546 | 1624 | 1703 | 1782 | 1860 | |
| 2 | 1939 | 2018 | 2096 | 2175 | 2254 | 2332 | 2411 | 2489 | 2568 | 2647 | |
| 3 | 2725 | 2804 | 2882 | 2961 | 3039 | 3118 | 3196 | 3275 | 3353 | 3431 | |
| 4 | 3510 | 3588 | 3667 | 3745 | 3823 | 3902 | 3980 | 4058 | 4136 | 4215 | |
| 5 | 4293 | 4371 | 4449 | 4528 | 4606 | 4684 | 4762 | 4840 | 4919 | 4997 | |
| 6 | 5075 | 5153 | 5231 | 5309 | 5387 | 5465 | 5543 | 5621 | 5699 | 5777 | 78 |
| 7 | 5855 | 5933 | 6011 | 6089 | 6167 | 6245 | 6323 | 6401 | 6479 | 6556 | |
| 8 | 6634 | 6712 | 6790 | 6868 | 6945 | 7023 | 7101 | 7179 | 7256 | 7334 | |
| 9 | 7412 | 7489 | 7567 | 7645 | 7722 | 7800 | 7878 | 7955 | 8033 | 8110 | |
| 560 | 8188 | 8266 | 8343 | 8421 | 8498 | 8576 | 8653 | 8731 | 8808 | 8885 | |
| 1 | 8963 | 9040 | 9118 | 9195 | 9272 | 9350 | 9427 | 9504 | 9582 | 9659 | |
| 2 | 9736 | 9814 | 9891 | 9968 | | | | | | | |
| | | | | | 0045 | 0123 | 0200 | 0277 | 0354 | 0431 | |
| 3 | 750508 | 0586 | 0663 | 0740 | 0817 | 0894 | 0971 | 1048 | 1125 | 1202 | |
| 4 | 1279 | 1356 | 1433 | 1510 | 1587 | 1664 | 1741 | 1818 | 1895 | 1972 | 77 |
| 5 | 2048 | 2125 | 2202 | 2279 | 2356 | 2433 | 2509 | 2586 | 2663 | 2740 | |
| 6 | 2816 | 2893 | 2970 | 3047 | 3123 | 3200 | 3277 | 3353 | 3430 | 3506 | |
| 7 | 3583 | 3660 | 3736 | 3813 | 3889 | 3966 | 4042 | 4119 | 4195 | 4272 | |
| 8 | 4348 | 4425 | 4501 | 4578 | 4654 | 4730 | 4807 | 4883 | 4960 | 5036 | |
| 9 | 5112 | 5189 | 5265 | 5341 | 5417 | 5494 | 5570 | 5646 | 5722 | 5799 | |
| 570 | 5875 | 5951 | 6027 | 6103 | 6180 | 6256 | 6332 | 6408 | 6484 | 6560 | |
| 1 | 6636 | 6712 | 6788 | 6864 | 6940 | 7016 | 7092 | 7168 | 7244 | 7320 | 76 |
| 2 | 7396 | 7472 | 7548 | 7624 | 7700 | 7775 | 7851 | 7927 | 8003 | 8079 | |
| 3 | 8155 | 8230 | 8306 | 8382 | 8458 | 8533 | 8609 | 8685 | 8761 | 8836 | |
| 4 | 8912 | 8988 | 9063 | 9139 | 9214 | 9290 | 9366 | 9441 | 9517 | 9592 | |
| 5 | 9668 | 9743 | 9819 | 9894 | 9970 | | | | | | |
| | | | | | | 0045 | 0121 | 0196 | 0272 | 0347 | |
| 6 | 760422 | 0498 | 0573 | 0649 | 0724 | 0799 | 0875 | 0950 | 1025 | 1101 | |
| 7 | 1176 | 1251 | 1326 | 1402 | 1477 | 1552 | 1627 | 1702 | 1778 | 1853 | |
| 8 | 1928 | 2003 | 2078 | 2153 | 2228 | 2303 | 2378 | 2453 | 2529 | 2604 | 75 |
| 9 | 2679 | 2754 | 2829 | 2904 | 2978 | 3053 | 3128 | 3203 | 3278 | 3353 | |
| 580 | 3428 | 3503 | 3578 | 3653 | 3727 | 3802 | 3877 | 3952 | 4027 | 4101 | |
| 1 | 4176 | 4251 | 4326 | 4400 | 4475 | 4550 | 4624 | 4699 | 4774 | 4848 | |
| 2 | 4923 | 4998 | 5072 | 5147 | 5221 | 5296 | 5370 | 5445 | 5520 | 5594 | |
| 3 | 5669 | 5743 | 5818 | 5892 | 5966 | 6041 | 6115 | 6190 | 6264 | 6338 | |
| 4 | 6413 | 6487 | 6562 | 6636 | 6710 | 6785 | 6859 | 6933 | 7007 | 7082 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 83 | 8.3 | 16.6 | 24.9 | 33.2 | 41.5 | 49.8 | 58.1 | 66.4 | 74.7 |
| 82 | 8.2 | 16.4 | 24.6 | 32.8 | 41.0 | 49.2 | 57.4 | 65.6 | 73.8 |
| 81 | 8.1 | 16.2 | 24.3 | 32.4 | 40.5 | 48.6 | 56.7 | 64.8 | 72.9 |
| 80 | 8.0 | 16.0 | 24.0 | 32.0 | 40.0 | 48.0 | 56.0 | 64.0 | 72.0 |
| 79 | 7.9 | 15.8 | 23.7 | 31.6 | 39.5 | 47.4 | 55.3 | 63.2 | 71.1 |
| 78 | 7.8 | 15.6 | 23.4 | 31.2 | 39.0 | 46.8 | 54.6 | 62.4 | 70.2 |
| 77 | 7.7 | 15.4 | 23.1 | 30.8 | 38.5 | 46.2 | 53.9 | 61.6 | 69.3 |
| 76 | 7.6 | 15.2 | 22.8 | 30.4 | 38.0 | 45.6 | 53.2 | 60.8 | 68.4 |
| 75 | 7.5 | 15.0 | 22.5 | 30.0 | 37.5 | 45.0 | 52.5 | 60.0 | 67.5 |
| 74 | 7.4 | 14.8 | 22.2 | 29.6 | 37.0 | 44.4 | 51.8 | 59.2 | 66.6 |

No. 585 L. 767.]

[No. 629 L. 799.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 585 | 767156 | 7230 | 7304 | 7379 | 7453 | 7527 | 7601 | 7675 | 7749 | 7823 | 74 |
| 6 | 7898 | 7972 | 8046 | 8120 | 8194 | 8268 | 8342 | 8416 | 8490 | 8564 | |
| 7 | 8638 | 8712 | 8786 | 8860 | 8934 | 9008 | 9082 | 9156 | 9230 | 9303 | |
| 8 | 9377 | 9451 | 9525 | 9599 | 9673 | 9746 | 9820 | 9894 | 9968 | 0042 | |
| 9 | 770115 | 0189 | 0263 | 0336 | 0410 | 0484 | 0557 | 0631 | 0705 | 0778 | 73 |
| 590 | 0852 | 0926 | 0999 | 1073 | 1146 | 1220 | 1293 | 1367 | 1440 | 1514 | |
| 1 | 1587 | 1661 | 1734 | 1808 | 1881 | 1955 | 2028 | 2102 | 2175 | 2248 | |
| 2 | 2322 | 2395 | 2468 | 2542 | 2615 | 2688 | 2762 | 2835 | 2908 | 2981 | |
| 3 | 3055 | 3128 | 3201 | 3274 | 3348 | 3421 | 3494 | 3567 | 3640 | 3713 | |
| 4 | 3786 | 3860 | 3933 | 4006 | 4079 | 4152 | 4225 | 4298 | 4371 | 4444 | |
| 5 | 4517 | 4590 | 4663 | 4736 | 4809 | 4882 | 4955 | 5028 | 5100 | 5173 | |
| 6 | 5246 | 5319 | 5392 | 5465 | 5538 | 5610 | 5683 | 5756 | 5829 | 5902 | |
| 7 | 5974 | 6047 | 6120 | 6193 | 6265 | 6338 | 6411 | 6483 | 6556 | 6629 | |
| 8 | 6701 | 6774 | 6846 | 6919 | 6992 | 7064 | 7137 | 7209 | 7282 | 7354 | |
| 9 | 7427 | 7499 | 7572 | 7644 | 7717 | 7789 | 7862 | 7934 | 8006 | 8079 | |
| 600 | 8151 | 8224 | 8296 | 8368 | 8441 | 8513 | 8585 | 8658 | 8730 | 8802 | 72 |
| 1 | 8874 | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 9380 | 9452 | 9524 | |
| 2 | 9596 | 9669 | 9741 | 9813 | 9885 | 9957 | 0029 | 0101 | 0173 | 0245 | |
| 3 | 780317 | 0389 | 0461 | 0533 | 0605 | 0677 | 0749 | 0821 | 0893 | 0965 | |
| 4 | 1037 | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 1540 | 1612 | 1684 | |
| 5 | 1755 | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 | |
| 6 | 2473 | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 | |
| 7 | 3189 | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 | |
| 8 | 3904 | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 | |
| 9 | 4617 | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 | |
| 610 | 5330 | 5401 | 5472 | 5543 | 5615 | 5686 | 5757 | 5828 | 5899 | 5970 | 71 |
| 1 | 6041 | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 | |
| 2 | 6751 | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 | |
| 3 | 7460 | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 | |
| 4 | 8168 | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 | |
| 5 | 8875 | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9510 | |
| 6 | 9581 | 9651 | 9722 | 9792 | 9863 | 9933 | 0004 | 0074 | 0144 | 0215 | |
| 7 | 790285 | 0356 | 0426 | 0496 | 0567 | 0637 | 0707 | 0778 | 0848 | 0918 | |
| 8 | 0988 | 1059 | 1129 | 1199 | 1269 | 1340 | 1410 | 1480 | 1550 | 1620 | |
| 9 | 1691 | 1761 | 1831 | 1901 | 1971 | 2041 | 2111 | 2181 | 2252 | 2322 | |
| 620 | 2392 | 2462 | 2532 | 2602 | 2672 | 2742 | 2812 | 2882 | 2952 | 3022 | 70 |
| 1 | 3092 | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 | |
| 2 | 3790 | 3860 | 3930 | 4000 | 4070 | 4139 | 4209 | 4279 | 4349 | 4418 | |
| 3 | 4488 | 4558 | 4627 | 4697 | 4767 | 4836 | 4906 | 4976 | 5045 | 5115 | |
| 4 | 5185 | 5254 | 5324 | 5393 | 5463 | 5532 | 5602 | 5672 | 5741 | 5811 | |
| 5 | 5880 | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 | |
| 6 | 6574 | 6644 | 6713 | 6782 | 6852 | 6921 | 6990 | 7060 | 7129 | 7198 | |
| 7 | 7268 | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 | |
| 8 | 7960 | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 | |
| 9 | 8651 | 8720 | 8789 | 8858 | 8927 | 8996 | 9065 | 9134 | 9203 | 9272 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 75 | 7.5 | 15.0 | 22.5 | 30.0 | 37.5 | 45.0 | 52.5 | 60.0 | 67.5 |
| 74 | 7.4 | 14.8 | 22.2 | 29.6 | 37.0 | 44.4 | 51.8 | 59.2 | 66.6 |
| 73 | 7.3 | 14.6 | 21.9 | 29.2 | 36.5 | 43.8 | 51.1 | 58.4 | 65.7 |
| 72 | 7.2 | 14.4 | 21.6 | 28.8 | 36.0 | 43.2 | 50.4 | 57.6 | 64.8 |
| 71 | 7.1 | 14.2 | 21.3 | 28.4 | 35.5 | 42.6 | 49.7 | 56.8 | 63.9 |
| 70 | 7.0 | 14.0 | 21.0 | 28.0 | 35.0 | 42.0 | 49.0 | 56.0 | 63.0 |
| 69 | 6.9 | 13.8 | 20.7 | 27.6 | 34.5 | 41.4 | 48.3 | 55.2 | 62.1 |

No. 630 L. 799.]

[No. 674 L. 829.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 630 | 799341 | 9409 | 9478 | 9547 | 9616 | 9685 | 9754 | 9823 | 9892 | 9961 | |
| 1 | 800029 | 0098 | 0167 | 0236 | 0305 | 0373 | 0442 | 0511 | 0580 | 0648 | |
| 2 | 0717 | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 | |
| 3 | 1404 | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 | |
| 4 | 2089 | 2158 | 2226 | 2295 | 2363 | 2432 | 2500 | 2568 | 2637 | 2705 | |
| 5 | 2774 | 2842 | 2910 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 | |
| 6 | 3457 | 3525 | 3594 | 3662 | 3730 | 3798 | 3867 | 3935 | 4003 | 4071 | |
| 7 | 4139 | 4208 | 4276 | 4344 | 4412 | 4480 | 4548 | 4616 | 4685 | 4753 | |
| 8 | 4821 | 4889 | 4957 | 5025 | 5093 | 5161 | 5229 | 5297 | 5365 | 5433 | 68 |
| 9 | 5501 | 5569 | 5637 | 5705 | 5773 | 5841 | 5908 | 5976 | 6044 | 6112 | |
| 640 | 806180 | 6248 | 6316 | 6384 | 6451 | 6519 | 6587 | 6655 | 6723 | 6790 | |
| 1 | 6858 | 6926 | 6994 | 7061 | 7129 | 7197 | 7264 | 7332 | 7400 | 7467 | |
| 2 | 7535 | 7603 | 7670 | 7738 | 7806 | 7873 | 7941 | 8008 | 8076 | 8143 | |
| 3 | 8211 | 8279 | 8346 | 8414 | 8481 | 8549 | 8616 | 8684 | 8751 | 8818 | |
| 4 | 8886 | 8953 | 9021 | 9088 | 9156 | 9223 | 9290 | 9358 | 9425 | 9492 | |
| 5 | 9560 | 9627 | 9694 | 9762 | 9829 | 9896 | 9964 | | | | |
| 6 | 810233 | 0300 | 0367 | 0434 | 0501 | 0569 | 0636 | 0703 | 0770 | 0837 | |
| 7 | 0904 | 0971 | 1039 | 1106 | 1173 | 1240 | 1307 | 1374 | 1441 | 1508 | 67 |
| 8 | 1575 | 1642 | 1709 | 1776 | 1843 | 1910 | 1977 | 2044 | 2111 | 2178 | |
| 9 | 2245 | 2312 | 2379 | 2445 | 2512 | 2579 | 2646 | 2713 | 2780 | 2847 | |
| 650 | 2913 | 2980 | 3047 | 3114 | 3181 | 3247 | 3314 | 3381 | 3448 | 3514 | |
| 1 | 3581 | 3648 | 3714 | 3781 | 3848 | 3914 | 3981 | 4048 | 4114 | 4181 | |
| 2 | 4248 | 4314 | 4381 | 4447 | 4514 | 4581 | 4647 | 4714 | 4780 | 4847 | |
| 3 | 4913 | 4980 | 5046 | 5113 | 5179 | 5246 | 5312 | 5378 | 5445 | 5511 | |
| 4 | 5578 | 5644 | 5711 | 5777 | 5843 | 5910 | 5976 | 6042 | 6109 | 6175 | |
| 5 | 6241 | 6308 | 6374 | 6440 | 6506 | 6573 | 6639 | 6705 | 6771 | 6838 | |
| 6 | 6904 | 6970 | 7036 | 7102 | 7169 | 7235 | 7301 | 7367 | 7433 | 7499 | |
| 7 | 7565 | 7631 | 7698 | 7764 | 7830 | 7896 | 7962 | 8028 | 8094 | 8160 | |
| 8 | 8226 | 8292 | 8358 | 8424 | 8490 | 8556 | 8622 | 8688 | 8754 | 8820 | 66 |
| 9 | 8885 | 8951 | 9017 | 9083 | 9149 | 9215 | 9281 | 9346 | 9412 | 9478 | |
| 660 | 9544 | 9610 | 9676 | 9741 | 9807 | 9873 | 9939 | | | | |
| 1 | 820201 | 0267 | 0333 | 0399 | 0464 | 0530 | 0595 | 0661 | 0727 | 0792 | |
| 2 | 0858 | 0924 | 0989 | 1055 | 1120 | 1186 | 1251 | 1317 | 1382 | 1448 | |
| 3 | 1514 | 1579 | 1645 | 1710 | 1775 | 1841 | 1906 | 1972 | 2037 | 2103 | |
| 4 | 2168 | 2233 | 2299 | 2364 | 2430 | 2495 | 2560 | 2626 | 2691 | 2756 | |
| 5 | 2822 | 2887 | 2952 | 3018 | 3083 | 3148 | 3213 | 3279 | 3344 | 3409 | |
| 6 | 3474 | 3539 | 3605 | 3670 | 3735 | 3800 | 3865 | 3930 | 3996 | 4061 | |
| 7 | 4126 | 4191 | 4256 | 4321 | 4386 | 4451 | 4516 | 4581 | 4646 | 4711 | |
| 8 | 4776 | 4841 | 4906 | 4971 | 5036 | 5101 | 5166 | 5231 | 5296 | 5361 | 65 |
| 9 | 5426 | 5491 | 5556 | 5621 | 5686 | 5751 | 5815 | 5880 | 5945 | 6010 | |
| 670 | 6075 | 6140 | 6204 | 6269 | 6334 | 6399 | 6464 | 6528 | 6593 | 6658 | |
| 1 | 6723 | 6787 | 6852 | 6917 | 6981 | 7046 | 7111 | 7175 | 7240 | 7305 | |
| 2 | 7369 | 7434 | 7499 | 7563 | 7628 | 7692 | 7757 | 7821 | 7886 | 7951 | |
| 3 | 8015 | 8080 | 8144 | 8209 | 8273 | 8338 | 8402 | 8467 | 8531 | 8595 | |
| 4 | 8660 | 8724 | 8789 | 8853 | 8918 | 8982 | 9046 | 9111 | 9175 | 9239 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 68 | 6.8 | 13.6 | 20.4 | 27.2 | 34.0 | 40.8 | 47.6 | 54.4 | 61.2 |
| 67 | 6.7 | 13.4 | 20.1 | 26.8 | 33.5 | 40.2 | 46.9 | 53.6 | 60.3 |
| 66 | 6.6 | 13.2 | 19.8 | 26.4 | 33.0 | 39.6 | 46.2 | 52.8 | 59.4 |
| 65 | 6.5 | 13.0 | 19.5 | 26.0 | 32.5 | 39.0 | 45.5 | 52.0 | 58.5 |
| 64 | 6.4 | 12.8 | 19.2 | 25.6 | 32.0 | 38.4 | 44.8 | 51.2 | 57.6 |

No. 675 L. 829.]

[No. 719 L. 857.]

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 675 | 829304 | 9368 | 9432 | 9497 | 9561 | 9625 | 9690 | 9754 | 9818 | 9882 | |
| 6 | 9947 | | | | | | | | | | |
| | | 0011 | 0075 | 0139 | 0204 | 0268 | 0332 | 0396 | 0460 | 0525 | |
| 7 | 830589 | 0653 | 0717 | 0781 | 0845 | 0909 | 0973 | 1037 | 1102 | 1166 | |
| 8 | 1230 | 1294 | 1358 | 1422 | 1486 | 1550 | 1614 | 1678 | 1742 | 1806 | 64 |
| 9 | 1870 | 1934 | 1998 | 2062 | 2126 | 2189 | 2253 | 2317 | 2381 | 2445 | |
| 680 | 2509 | 2573 | 2637 | 2700 | 2764 | 2828 | 2892 | 2956 | 3020 | 3083 | |
| 1 | 3147 | 3211 | 3275 | 3338 | 3402 | 3466 | 3530 | 3593 | 3657 | 3721 | |
| 2 | 3784 | 3848 | 3912 | 3975 | 4039 | 4103 | 4166 | 4230 | 4294 | 4357 | |
| 3 | 4421 | 4484 | 4548 | 4611 | 4675 | 4739 | 4802 | 4866 | 4929 | 4993 | |
| 4 | 5056 | 5120 | 5183 | 5247 | 5310 | 5373 | 5437 | 5500 | 5564 | 5627 | |
| 5 | 5691 | 5754 | 5817 | 5881 | 5944 | 6007 | 6071 | 6134 | 6197 | 6261 | |
| 6 | 6324 | 6387 | 6451 | 6514 | 6577 | 6641 | 6704 | 6767 | 6830 | 6894 | |
| 7 | 6957 | 7020 | 7083 | 7146 | 7210 | 7273 | 7336 | 7399 | 7462 | 7525 | |
| 8 | 7588 | 7652 | 7715 | 7778 | 7841 | 7904 | 7967 | 8030 | 8093 | 8156 | 63 |
| 9 | 8219 | 8282 | 8345 | 8408 | 8471 | 8534 | 8597 | 8660 | 8723 | 8786 | |
| 690 | 8849 | 8912 | 8975 | 9038 | 9101 | 9164 | 9227 | 9289 | 9352 | 9415 | |
| 1 | 9478 | 9541 | 9604 | 9667 | 9729 | 9792 | 9855 | 9918 | 9981 | | |
| | | | | | | | | | | 0043 | |
| 2 | 840106 | 0169 | 0232 | 0294 | 0357 | 0420 | 0482 | 0545 | 0608 | 0671 | |
| 3 | 0733 | 0796 | 0859 | 0921 | 0984 | 1046 | 1109 | 1172 | 1234 | 1297 | |
| 4 | 1359 | 1422 | 1485 | 1547 | 1610 | 1672 | 1735 | 1797 | 1860 | 1922 | |
| 5 | 1985 | 2047 | 2110 | 2172 | 2235 | 2297 | 2360 | 2422 | 2484 | 2547 | |
| 6 | 2609 | 2672 | 2734 | 2796 | 2859 | 2921 | 2983 | 3046 | 3108 | 3170 | |
| 7 | 3233 | 3295 | 3357 | 3420 | 3482 | 3544 | 3606 | 3669 | 3731 | 3793 | |
| 8 | 3855 | 3918 | 3980 | 4042 | 4104 | 4166 | 4229 | 4291 | 4353 | 4415 | |
| 9 | 4477 | 4539 | 4601 | 4664 | 4726 | 4788 | 4850 | 4912 | 4974 | 5036 | |
| 700 | 5098 | 5160 | 5222 | 5284 | 5346 | 5408 | 5470 | 5532 | 5594 | 5656 | 62 |
| 1 | 5718 | 5780 | 5842 | 5904 | 5966 | 6028 | 6090 | 6151 | 6213 | 6275 | |
| 2 | 6337 | 6399 | 6461 | 6523 | 6585 | 6646 | 6708 | 6770 | 6832 | 6894 | |
| 3 | 6955 | 7017 | 7079 | 7141 | 7202 | 7264 | 7326 | 7388 | 7449 | 7511 | |
| 4 | 7573 | 7634 | 7696 | 7758 | 7819 | 7881 | 7943 | 8004 | 8066 | 8128 | |
| 5 | 8189 | 8251 | 8312 | 8374 | 8435 | 8497 | 8559 | 8620 | 8682 | 8743 | |
| 6 | 8805 | 8866 | 8928 | 8989 | 9051 | 9112 | 9174 | 9235 | 9297 | 9358 | |
| 7 | 9419 | 9431 | 9542 | 9604 | 9665 | 9726 | 9788 | 9849 | 9911 | 9972 | |
| 8 | 850033 | 0095 | 0156 | 0217 | 0279 | 0340 | 0401 | 0462 | 0524 | 0585 | |
| 9 | 0646 | 0707 | 0769 | 0830 | 0891 | 0952 | 1014 | 1075 | 1136 | 1197 | |
| 710 | 1258 | 1320 | 1381 | 1442 | 1503 | 1564 | 1625 | 1686 | 1747 | 1809 | |
| 1 | 1870 | 1931 | 1992 | 2053 | 2114 | 2175 | 2236 | 2297 | 2358 | 2419 | 61 |
| 2 | 2480 | 2541 | 2602 | 2663 | 2724 | 2785 | 2846 | 2907 | 2968 | 3029 | |
| 3 | 3090 | 3150 | 3211 | 3272 | 3333 | 3394 | 3455 | 3516 | 3577 | 3637 | |
| 4 | 3698 | 3759 | 3820 | 3881 | 3941 | 4002 | 4063 | 4124 | 4185 | 4245 | |
| 5 | 4306 | 4367 | 4428 | 4488 | 4549 | 4610 | 4670 | 4731 | 4792 | 4852 | |
| 6 | 4913 | 4974 | 5034 | 5095 | 5156 | 5216 | 5277 | 5337 | 5398 | 5459 | |
| 7 | 5519 | 5580 | 5640 | 5701 | 5761 | 5822 | 5882 | 5943 | 6003 | 6064 | |
| 8 | 6124 | 6185 | 6245 | 6306 | 6366 | 6427 | 6487 | 6548 | 6608 | 6668 | |
| 9 | 6729 | 6789 | 6850 | 6910 | 6970 | 7031 | 7091 | 7152 | 7212 | 7272 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 65 | 6.5 | 13.0 | 19.5 | 26.0 | 32.5 | 39.0 | 45.5 | 52.0 | 58.5 |
| 64 | 6.4 | 12.8 | 19.2 | 25.6 | 32.0 | 38.4 | 44.8 | 51.2 | 57.6 |
| 63 | 6.3 | 12.6 | 18.9 | 25.2 | 31.5 | 37.8 | 44.1 | 50.4 | 56.7 |
| 62 | 6.2 | 12.4 | 18.6 | 24.8 | 31.0 | 37.2 | 43.4 | 49.6 | 55.8 |
| 61 | 6.1 | 12.2 | 18.3 | 24.4 | 30.5 | 36.6 | 42.7 | 48.8 | 54.9 |
| 60 | 6.0 | 12.0 | 18.0 | 24.0 | 30.0 | 36.0 | 42.0 | 48.0 | 54.0 |

No. 720 L. 857.]

[No. 764 L. 883.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 720 | 857332 | 7393 | 7453 | 7513 | 7574 | 7634 | 7694 | 7755 | 7815 | 7875 | 60 |
| 1 | 7935 | 7995 | 8056 | 8116 | 8176 | 8236 | 8297 | 8357 | 8417 | 8477 | |
| 2 | 8537 | 8597 | 8657 | 8718 | 8778 | 8838 | 8898 | 8958 | 9018 | 9078 | |
| 3 | 9138 | 9198 | 9258 | 9318 | 9379 | 9439 | 9499 | 9559 | 9619 | 9679 | |
| 4 | 9739 | 9799 | 9859 | 9918 | 9978 | | | | | | |
| 5 | 860338 | 0398 | 0458 | 0518 | 0578 | 0637 | 0697 | 0757 | 0817 | 0877 | |
| 6 | 0937 | 0996 | 1056 | 1116 | 1176 | 1236 | 1295 | 1355 | 1415 | 1475 | |
| 7 | 1534 | 1594 | 1654 | 1714 | 1773 | 1833 | 1893 | 1952 | 2012 | 2072 | |
| 8 | 2131 | 2191 | 2251 | 2310 | 2370 | 2430 | 2489 | 2549 | 2608 | 2668 | |
| 9 | 2728 | 2787 | 2847 | 2906 | 2966 | 3025 | 3085 | 3144 | 3204 | 3263 | |
| 730 | 3323 | 3382 | 3442 | 3501 | 3561 | 3620 | 3680 | 3739 | 3799 | 3858 | 59 |
| 1 | 3917 | 3977 | 4036 | 4096 | 4155 | 4214 | 4274 | 4333 | 4392 | 4452 | |
| 2 | 4511 | 4570 | 4630 | 4689 | 4748 | 4808 | 4867 | 4926 | 4985 | 5045 | |
| 3 | 5104 | 5163 | 5222 | 5282 | 5341 | 5400 | 5459 | 5519 | 5578 | 5637 | |
| 4 | 5696 | 5755 | 5814 | 5874 | 5933 | 5992 | 6051 | 6110 | 6169 | 6228 | |
| 5 | 6287 | 6346 | 6405 | 6465 | 6524 | 6583 | 6642 | 6701 | 6760 | 6819 | |
| 6 | 6878 | 6937 | 6996 | 7055 | 7114 | 7173 | 7232 | 7291 | 7350 | 7409 | |
| 7 | 7467 | 7526 | 7585 | 7644 | 7703 | 7762 | 7821 | 7880 | 7939 | 7998 | |
| 8 | 8056 | 8115 | 8174 | 8233 | 8292 | 8350 | 8409 | 8468 | 8527 | 8586 | |
| 9 | 8644 | 8703 | 8762 | 8821 | 8879 | 8938 | 8997 | 9056 | 9114 | 9173 | |
| 740 | 9232 | 9290 | 9349 | 9408 | 9466 | 9525 | 9584 | 9642 | 9701 | 9760 | 58 |
| 1 | 9818 | 9877 | 9935 | 9994 | | | | | | | |
| 2 | 870404 | 0462 | 0521 | 0579 | 0638 | 0696 | 0755 | 0813 | 0872 | 0930 | |
| 3 | 0989 | 1047 | 1106 | 1164 | 1223 | 1281 | 1339 | 1398 | 1456 | 1515 | |
| 4 | 1573 | 1631 | 1690 | 1748 | 1806 | 1865 | 1923 | 1981 | 2040 | 2098 | |
| 5 | 2156 | 2215 | 2273 | 2331 | 2389 | 2448 | 2506 | 2564 | 2622 | 2681 | |
| 6 | 2739 | 2797 | 2855 | 2913 | 2972 | 3030 | 3088 | 3146 | 3204 | 3262 | |
| 7 | 3321 | 3379 | 3437 | 3495 | 3553 | 3611 | 3669 | 3727 | 3785 | 3844 | |
| 8 | 3902 | 3960 | 4018 | 4076 | 4134 | 4192 | 4250 | 4308 | 4366 | 4424 | |
| 9 | 4482 | 4540 | 4598 | 4656 | 4714 | 4772 | 4830 | 4888 | 4945 | 5003 | |
| 750 | 5061 | 5119 | 5177 | 5235 | 5293 | 5351 | 5409 | 5466 | 5524 | 5582 | 57 |
| 1 | 5640 | 5698 | 5756 | 5813 | 5871 | 5929 | 5987 | 6045 | 6102 | 6160 | |
| 2 | 6218 | 6276 | 6333 | 6391 | 6449 | 6507 | 6564 | 6622 | 6680 | 6737 | |
| 3 | 6795 | 6853 | 6910 | 6968 | 7026 | 7083 | 7141 | 7199 | 7256 | 7314 | |
| 4 | 7371 | 7429 | 7487 | 7544 | 7602 | 7659 | 7717 | 7774 | 7832 | 7889 | |
| 5 | 7947 | 8004 | 8062 | 8119 | 8177 | 8234 | 8292 | 8349 | 8407 | 8464 | |
| 6 | 8522 | 8579 | 8637 | 8694 | 8752 | 8809 | 8866 | 8924 | 8981 | 9039 | |
| 7 | 9096 | 9153 | 9211 | 9268 | 9325 | 9383 | 9440 | 9497 | 9555 | 9612 | |
| 8 | 9669 | 9726 | 9784 | 9841 | 9898 | 9956 | | | | | |
| 9 | 880242 | 0299 | 0356 | 0413 | 0471 | 0528 | 0585 | 0642 | 0699 | 0756 | |
| 760 | 0814 | 0871 | 0928 | 0985 | 1042 | 1099 | 1156 | 1213 | 1271 | 1328 | 57 |
| 1 | 1385 | 1442 | 1499 | 1556 | 1613 | 1670 | 1727 | 1784 | 1841 | 1898 | |
| 2 | 1955 | 2012 | 2069 | 2126 | 2183 | 2240 | 2297 | 2354 | 2411 | 2468 | |
| 3 | 2525 | 2581 | 2638 | 2695 | 2752 | 2809 | 2866 | 2923 | 2980 | 3037 | |
| 4 | 3093 | 3150 | 3207 | 3264 | 3321 | 3377 | 3434 | 3491 | 3548 | 3605 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 59 | 5.9 | 11.8 | 17.7 | 23.6 | 29.5 | 35.4 | 41.3 | 47.2 | 53.1 |
| 58 | 5.8 | 11.6 | 17.4 | 23.2 | 29.0 | 34.8 | 40.6 | 46.4 | 52.2 |
| 57 | 5.7 | 11.4 | 17.1 | 22.8 | 28.5 | 34.2 | 39.9 | 45.6 | 51.3 |
| 56 | 5.6 | 11.2 | 16.8 | 22.4 | 28.0 | 33.6 | 39.2 | 44.8 | 50.4 |

No. 765 L. 883.]

[No. 809 L. 908.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 765 | 883661 | 3718 | 3775 | 3832 | 3888 | 3945 | 4002 | 4059 | 4115 | 4172 | |
| 6 | 4229 | 4285 | 4342 | 4399 | 4455 | 4512 | 4569 | 4625 | 4682 | 4739 | |
| 7 | 4795 | 4852 | 4909 | 4965 | 5022 | 5078 | 5135 | 5192 | 5248 | 5305 | |
| 8 | 5361 | 5418 | 5474 | 5531 | 5587 | 5644 | 5700 | 5757 | 5813 | 5870 | |
| 9 | 5926 | 5983 | 6039 | 6096 | 6152 | 6209 | 6265 | 6321 | 6378 | 6434 | |
| 770 | 6491 | 6547 | 6604 | 6660 | 6716 | 6773 | 6829 | 6885 | 6942 | 6998 | |
| 1 | 7054 | 7111 | 7167 | 7223 | 7280 | 7336 | 7392 | 7449 | 7505 | 7561 | |
| 2 | 7617 | 7674 | 7730 | 7786 | 7842 | 7898 | 7955 | 8011 | 8067 | 8123 | |
| 3 | 8179 | 8236 | 8292 | 8348 | 8404 | 8460 | 8516 | 8573 | 8629 | 8685 | |
| 4 | 8741 | 8797 | 8853 | 8909 | 8965 | 9021 | 9077 | 9134 | 9190 | 9246 | |
| 5 | 9302 | 9358 | 9414 | 9470 | 9526 | 9582 | 9638 | 9694 | 9750 | 9806 | 56 |
| 6 | 9862 | 9918 | 9974 | 0030 | 0086 | 0141 | 0197 | 0253 | 0309 | 0365 | |
| 7 | 890421 | 0477 | 0533 | 0589 | 0645 | 0700 | 0756 | 0812 | 0868 | 0924 | |
| 8 | 0980 | 1035 | 1091 | 1147 | 1203 | 1259 | 1314 | 1370 | 1426 | 1482 | |
| 9 | 1537 | 1593 | 1649 | 1705 | 1760 | 1816 | 1872 | 1928 | 1983 | 2039 | |
| 780 | 2095 | 2150 | 2206 | 2262 | 2317 | 2373 | 2429 | 2484 | 2540 | 2595 | |
| 1 | 2651 | 2707 | 2762 | 2818 | 2873 | 2929 | 2985 | 3040 | 3096 | 3151 | |
| 2 | 3207 | 3262 | 3318 | 3373 | 3429 | 3484 | 3540 | 3595 | 3651 | 3706 | |
| 3 | 3762 | 3817 | 3873 | 3928 | 3984 | 4039 | 4094 | 4150 | 4205 | 4261 | |
| 4 | 4316 | 4371 | 4427 | 4482 | 4538 | 4593 | 4648 | 4704 | 4759 | 4814 | |
| 5 | 4870 | 4925 | 4980 | 5036 | 5091 | 5146 | 5201 | 5257 | 5312 | 5367 | |
| 6 | 5423 | 5478 | 5533 | 5588 | 5644 | 5699 | 5754 | 5809 | 5864 | 5920 | |
| 7 | 5975 | 6030 | 6085 | 6140 | 6195 | 6251 | 6306 | 6361 | 6416 | 6471 | |
| 8 | 6526 | 6581 | 6636 | 6692 | 6747 | 6802 | 6857 | 6912 | 6967 | 7022 | |
| 9 | 7077 | 7132 | 7187 | 7242 | 7297 | 7352 | 7407 | 7462 | 7517 | 7572 | 55 |
| 790 | 7627 | 7682 | 7737 | 7792 | 7847 | 7902 | 7957 | 8012 | 8067 | 8122 | |
| 1 | 8176 | 8231 | 8286 | 8341 | 8396 | 8451 | 8506 | 8561 | 8615 | 8670 | |
| 2 | 8725 | 8780 | 8835 | 8890 | 8944 | 8999 | 9054 | 9109 | 9164 | 9218 | |
| 3 | 9273 | 9328 | 9383 | 9437 | 9492 | 9547 | 9602 | 9656 | 9711 | 9766 | |
| 4 | 9821 | 9875 | 9930 | 9985 | 0039 | 0094 | 0149 | 0203 | 0258 | 0312 | |
| 5 | 900367 | 0422 | 0476 | 0531 | 0586 | 0640 | 0695 | 0749 | 0804 | 0859 | |
| 6 | 0913 | 0968 | 1022 | 1077 | 1131 | 1186 | 1240 | 1295 | 1349 | 1404 | |
| 7 | 1458 | 1513 | 1567 | 1622 | 1676 | 1731 | 1785 | 1840 | 1894 | 1948 | |
| 8 | 2003 | 2057 | 2112 | 2166 | 2221 | 2275 | 2329 | 2384 | 2438 | 2492 | |
| 9 | 2547 | 2601 | 2655 | 2710 | 2764 | 2818 | 2873 | 2927 | 2981 | 3036 | |
| 800 | 3090 | 3144 | 3199 | 3253 | 3307 | 3361 | 3416 | 3470 | 3524 | 3578 | |
| 1 | 3633 | 3687 | 3741 | 3795 | 3849 | 3904 | 3958 | 4012 | 4066 | 4120 | |
| 2 | 4174 | 4229 | 4283 | 4337 | 4391 | 4445 | 4499 | 4553 | 4607 | 4661 | |
| 3 | 4716 | 4770 | 4824 | 4878 | 4932 | 4986 | 5040 | 5094 | 5148 | 5202 | |
| 4 | 5256 | 5310 | 5364 | 5418 | 5472 | 5526 | 5580 | 5634 | 5688 | 5742 | 54 |
| 5 | 5796 | 5850 | 5904 | 5958 | 6012 | 6066 | 6119 | 6173 | 6227 | 6281 | |
| 6 | 6335 | 6389 | 6443 | 6497 | 6551 | 6604 | 6658 | 6712 | 6766 | 6820 | |
| 7 | 6874 | 6927 | 6981 | 7035 | 7089 | 7143 | 7196 | 7250 | 7304 | 7358 | |
| 8 | 7411 | 7465 | 7519 | 7573 | 7626 | 7680 | 7734 | 7787 | 7841 | 7895 | |
| 9 | 7949 | 8002 | 8056 | 8110 | 8163 | 8217 | 8270 | 8324 | 8378 | 8431 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 57 | 5.7 | 11.4 | 17.1 | 22.8 | 28.5 | 34.2 | 39.9 | 45.6 | 51.3 |
| 56 | 5.6 | 11.2 | 16.8 | 22.4 | 28.0 | 33.6 | 39.2 | 44.8 | 50.4 |
| 55 | 5.5 | 11.0 | 16.5 | 22.0 | 27.5 | 33.0 | 38.5 | 44.0 | 49.5 |
| 54 | 5.4 | 10.8 | 16.2 | 21.6 | 27.0 | 32.4 | 37.8 | 43.2 | 48.6 |

No. 810 L. 908.]

[No. 854 L. 931.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 810 | 908485 | 8539 | 8592 | 8646 | 8699 | 8753 | 8807 | 8860 | 8914 | 8967 | 53 |
| 1 | 9021 | 9074 | 9128 | 9181 | 9235 | 9289 | 9342 | 9396 | 9449 | 9503 | |
| 2 | 9556 | 9610 | 9663 | 9716 | 9770 | 9823 | 9877 | 9930 | 9984 | 0037 | |
| 3 | 910091 | 0144 | 0197 | 0251 | 0304 | 0358 | 0411 | 0464 | 0518 | 0571 | |
| 4 | 0624 | 0678 | 0731 | 0784 | 0838 | 0891 | 0944 | 0998 | 1051 | 1104 | |
| 5 | 1158 | 1211 | 1264 | 1317 | 1371 | 1424 | 1477 | 1530 | 1584 | 1637 | |
| 6 | 1690 | 1743 | 1797 | 1850 | 1903 | 1956 | 2009 | 2063 | 2116 | 2169 | |
| 7 | 2222 | 2275 | 2328 | 2381 | 2435 | 2488 | 2541 | 2594 | 2647 | 2700 | |
| 8 | 2753 | 2806 | 2859 | 2913 | 2966 | 3019 | 3072 | 3125 | 3178 | 3231 | |
| 9 | 3284 | 3337 | 3390 | 3443 | 3496 | 3549 | 3602 | 3655 | 3708 | 3761 | |
| 820 | 3814 | 3867 | 3920 | 3973 | 4026 | 4079 | 4132 | 4184 | 4237 | 4290 | 52 |
| 1 | 4343 | 4396 | 4449 | 4502 | 4555 | 4608 | 4660 | 4713 | 4766 | 4819 | |
| 2 | 4872 | 4925 | 4977 | 5030 | 5083 | 5136 | 5189 | 5241 | 5294 | 5347 | |
| 3 | 5400 | 5453 | 5505 | 5558 | 5611 | 5664 | 5716 | 5769 | 5822 | 5875 | |
| 4 | 5927 | 5980 | 6033 | 6085 | 6138 | 6191 | 6243 | 6296 | 6349 | 6401 | |
| 5 | 6454 | 6507 | 6559 | 6612 | 6664 | 6717 | 6770 | 6822 | 6875 | 6927 | |
| 6 | 6980 | 7033 | 7085 | 7138 | 7190 | 7243 | 7295 | 7348 | 7400 | 7453 | |
| 7 | 7506 | 7558 | 7611 | 7663 | 7716 | 7768 | 7820 | 7873 | 7925 | 7978 | |
| 8 | 8030 | 8083 | 8135 | 8188 | 8240 | 8293 | 8345 | 8397 | 8450 | 8502 | |
| 9 | 8555 | 8607 | 8659 | 8712 | 8764 | 8816 | 8869 | 8921 | 8973 | 9026 | |
| 830 | 9078 | 9130 | 9183 | 9235 | 9287 | 9340 | 9392 | 9444 | 9496 | 9549 | 51 |
| 1 | 9601 | 9653 | 9706 | 9758 | 9810 | 9862 | 9914 | 9967 | 0019 | 0071 | |
| 2 | 920123 | 0176 | 0228 | 0280 | 0332 | 0384 | 0436 | 0489 | 0541 | 0593 | |
| 3 | 0645 | 0697 | 0749 | 0801 | 0853 | 0906 | 0958 | 1010 | 1062 | 1114 | |
| 4 | 1166 | 1218 | 1270 | 1322 | 1374 | 1426 | 1478 | 1530 | 1582 | 1634 | |
| 5 | 1686 | 1738 | 1790 | 1842 | 1894 | 1946 | 1998 | 2050 | 2102 | 2154 | |
| 6 | 2206 | 2258 | 2310 | 2362 | 2414 | 2466 | 2518 | 2570 | 2622 | 2674 | |
| 7 | 2725 | 2777 | 2829 | 2881 | 2933 | 2985 | 3037 | 3089 | 3140 | 3192 | |
| 8 | 3244 | 3296 | 3348 | 3399 | 3451 | 3503 | 3555 | 3607 | 3658 | 3710 | |
| 9 | 3762 | 3814 | 3865 | 3917 | 3969 | 4021 | 4072 | 4124 | 4176 | 4228 | |
| 840 | 4279 | 4331 | 4383 | 4434 | 4486 | 4538 | 4589 | 4641 | 4693 | 4744 | 51 |
| 1 | 4796 | 4848 | 4899 | 4951 | 5003 | 5054 | 5106 | 5157 | 5209 | 5261 | |
| 2 | 5312 | 5364 | 5415 | 5467 | 5518 | 5570 | 5621 | 5673 | 5725 | 5776 | |
| 3 | 5828 | 5879 | 5931 | 5982 | 6034 | 6085 | 6137 | 6188 | 6240 | 6291 | |
| 4 | 6342 | 6394 | 6445 | 6497 | 6548 | 6600 | 6651 | 6702 | 6754 | 6805 | |
| 5 | 6857 | 6908 | 6959 | 7011 | 7062 | 7114 | 7165 | 7216 | 7268 | 7319 | |
| 6 | 7370 | 7422 | 7473 | 7524 | 7576 | 7627 | 7678 | 7730 | 7781 | 7832 | |
| 7 | 7883 | 7935 | 7986 | 8037 | 8088 | 8140 | 8191 | 8242 | 8293 | 8345 | |
| 8 | 8396 | 8447 | 8498 | 8549 | 8601 | 8652 | 8703 | 8754 | 8805 | 8857 | |
| 9 | 8908 | 8959 | 9010 | 9061 | 9112 | 9163 | 9215 | 9266 | 9317 | 9368 | |
| 850 | 9419 | 9470 | 9521 | 9572 | 9623 | 9674 | 9725 | 9776 | 9827 | 9879 | 51 |
| 1 | 9930 | 9981 | 0032 | 0083 | 0134 | 0185 | 0236 | 0287 | 0338 | 0389 | |
| 2 | 930440 | 0491 | 0542 | 0592 | 0643 | 0694 | 0745 | 0796 | 0847 | 0898 | |
| 3 | 0949 | 1000 | 1051 | 1102 | 1153 | 1204 | 1254 | 1305 | 1356 | 1407 | |
| 4 | 1458 | 1509 | 1560 | 1610 | 1661 | 1712 | 1763 | 1814 | 1865 | 1915 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 53 | 5.3 | 10.6 | 15.9 | 21.2 | 26.5 | 31.8 | 37.1 | 42.4 | 47.7 |
| 52 | 5.2 | 10.4 | 15.6 | 20.8 | 26.0 | 31.2 | 36.4 | 41.6 | 46.8 |
| 51 | 5.1 | 10.2 | 15.3 | 20.4 | 25.5 | 30.6 | 35.7 | 40.8 | 45.9 |
| 50 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |

No. 855 L. 931.]

[No. 899 L. 954.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 855 | 931966 | 2017 | 2068 | 2118 | 2169 | 2220 | 2271 | 2322 | 2372 | 2423 | |
| 6 | 2474 | 2524 | 2575 | 2626 | 2677 | 2727 | 2778 | 2829 | 2879 | 2930 | |
| 7 | 2981 | 3031 | 3082 | 3133 | 3183 | 3234 | 3285 | 3335 | 3386 | 3437 | |
| 8 | 3487 | 3538 | 3589 | 3639 | 3690 | 3740 | 3791 | 3841 | 3892 | 3943 | |
| 9 | 3993 | 4044 | 4094 | 4145 | 4195 | 4246 | 4296 | 4347 | 4397 | 4448 | |
| 860 | 4498 | 4549 | 4599 | 4650 | 4700 | 4751 | 4801 | 4852 | 4902 | 4953 | |
| 1 | 5003 | 5054 | 5104 | 5154 | 5205 | 5255 | 5306 | 5356 | 5406 | 5457 | |
| 2 | 5507 | 5558 | 5608 | 5658 | 5709 | 5759 | 5809 | 5860 | 5910 | 5960 | |
| 3 | 6011 | 6061 | 6111 | 6162 | 6212 | 6262 | 6313 | 6363 | 6413 | 6463 | |
| 4 | 6514 | 6564 | 6614 | 6665 | 6715 | 6765 | 6815 | 6865 | 6916 | 6966 | |
| 5 | 7016 | 7066 | 7116 | 7167 | 7217 | 7267 | 7317 | 7367 | 7418 | 7468 | |
| 6 | 7518 | 7568 | 7618 | 7668 | 7718 | 7769 | 7819 | 7869 | 7919 | 7969 | |
| 7 | 8019 | 8069 | 8119 | 8169 | 8219 | 8269 | 8320 | 8370 | 8420 | 8470 | 50 |
| 8 | 8520 | 8570 | 8620 | 8670 | 8720 | 8770 | 8820 | 8870 | 8920 | 8970 | |
| 9 | 9020 | 9070 | 9120 | 9170 | 9220 | 9270 | 9320 | 9369 | 9419 | 9469 | |
| 870 | 9519 | 9569 | 9619 | 9669 | 9719 | 9769 | 9819 | 9869 | 9918 | 9968 | |
| 1 | 940018 | 0068 | 0118 | 0168 | 0218 | 0267 | 0317 | 0367 | 0417 | 0467 | |
| 2 | 0516 | 0566 | 0616 | 0666 | 0716 | 0765 | 0815 | 0865 | 0915 | 0964 | |
| 3 | 1014 | 1064 | 1114 | 1163 | 1213 | 1263 | 1313 | 1362 | 1412 | 1462 | |
| 4 | 1511 | 1561 | 1611 | 1660 | 1710 | 1760 | 1809 | 1859 | 1909 | 1958 | |
| 5 | 2008 | 2058 | 2107 | 2157 | 2207 | 2256 | 2306 | 2355 | 2405 | 2455 | |
| 6 | 2504 | 2554 | 2603 | 2653 | 2702 | 2752 | 2801 | 2851 | 2901 | 2950 | |
| 7 | 3000 | 3049 | 3099 | 3148 | 3198 | 3247 | 3297 | 3346 | 3396 | 3445 | |
| 8 | 3495 | 3544 | 3593 | 3643 | 3692 | 3742 | 3791 | 3841 | 3890 | 3939 | |
| 9 | 3989 | 4038 | 4088 | 4137 | 4186 | 4236 | 4285 | 4335 | 4384 | 4433 | |
| 880 | 4483 | 4532 | 4581 | 4631 | 4680 | 4729 | 4779 | 4828 | 4877 | 4927 | |
| 1 | 4976 | 5025 | 5074 | 5124 | 5173 | 5222 | 5272 | 5321 | 5370 | 5419 | |
| 2 | 5469 | 5518 | 5567 | 5616 | 5665 | 5715 | 5764 | 5813 | 5862 | 5912 | |
| 3 | 5961 | 6010 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | |
| 4 | 6452 | 6501 | 6551 | 6600 | 6649 | 6698 | 6747 | 6796 | 6845 | 6894 | |
| 5 | 6943 | 6992 | 7041 | 7090 | 7139 | 7189 | 7238 | 7287 | 7336 | 7385 | |
| 6 | 7434 | 7483 | 7532 | 7581 | 7630 | 7679 | 7728 | 7777 | 7826 | 7875 | 49 |
| 7 | 7924 | 7973 | 8022 | 8070 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | |
| 8 | 8413 | 8462 | 8511 | 8560 | 8608 | 8657 | 8706 | 8755 | 8804 | 8853 | |
| 9 | 8902 | 8951 | 8999 | 9048 | 9097 | 9146 | 9195 | 9244 | 9292 | 9341 | |
| 890 | 9390 | 9439 | 9488 | 9536 | 9585 | 9634 | 9683 | 9731 | 9780 | 9829 | |
| 1 | 9878 | 9926 | 9975 | 0024 | 0073 | 0121 | 0170 | 0219 | 0267 | 0316 | |
| 2 | 950365 | 0414 | 0462 | 0511 | 0560 | 0608 | 0657 | 0706 | 0754 | 0803 | |
| 3 | 0851 | 0900 | 0949 | 0997 | 1046 | 1095 | 1143 | 1192 | 1240 | 1289 | |
| 4 | 1338 | 1386 | 1435 | 1483 | 1532 | 1580 | 1629 | 1677 | 1726 | 1775 | |
| 5 | 1823 | 1872 | 1920 | 1969 | 2017 | 2066 | 2114 | 2163 | 2211 | 2260 | |
| 6 | 2308 | 2356 | 2405 | 2453 | 2502 | 2550 | 2599 | 2647 | 2696 | 2744 | |
| 7 | 2792 | 2841 | 2889 | 2938 | 2986 | 3034 | 3083 | 3131 | 3180 | 3228 | |
| 8 | 3276 | 3325 | 3373 | 3421 | 3470 | 3518 | 3566 | 3615 | 3663 | 3711 | |
| 9 | 3760 | 3808 | 3856 | 3905 | 3953 | 4001 | 4049 | 4098 | 4146 | 4194 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|------|------|------|------|------|------|------|------|
| 51 | 5.1 | 10.2 | 15.3 | 20.4 | 25.5 | 30.6 | 35.7 | 40.8 | 45.9 |
| 50 | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 | 30.0 | 35.0 | 40.0 | 45.0 |
| 49 | 4.9 | 9.8 | 14.7 | 19.6 | 24.5 | 29.4 | 34.3 | 39.2 | 44.1 |
| 48 | 4.8 | 9.6 | 14.4 | 19.2 | 24.0 | 28.8 | 33.6 | 38.4 | 43.2 |

No. 900 L. 954.]

[No. 944 L. 975.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 900 | 954243 | 4291 | 4339 | 4387 | 4435 | 4484 | 4532 | 4580 | 4628 | 4677 | 48 |
| 1 | 4725 | 4773 | 4821 | 4869 | 4918 | 4966 | 5014 | 5062 | 5110 | 5158 | |
| 2 | 5207 | 5255 | 5303 | 5351 | 5399 | 5447 | 5495 | 5543 | 5592 | 5640 | |
| 3 | 5688 | 5736 | 5784 | 5832 | 5880 | 5928 | 5976 | 6024 | 6072 | 6120 | |
| 4 | 6168 | 6216 | 6265 | 6313 | 6361 | 6409 | 6457 | 6505 | 6553 | 6601 | |
| 5 | 6649 | 6697 | 6745 | 6793 | 6840 | 6888 | 6936 | 6984 | 7032 | 7080 | |
| 6 | 7128 | 7176 | 7224 | 7272 | 7320 | 7368 | 7416 | 7464 | 7512 | 7559 | |
| 7 | 7607 | 7655 | 7703 | 7751 | 7799 | 7847 | 7894 | 7942 | 7990 | 8038 | |
| 8 | 8086 | 8134 | 8181 | 8229 | 8277 | 8325 | 8373 | 8421 | 8468 | 8516 | |
| 9 | 8564 | 8612 | 8659 | 8707 | 8755 | 8803 | 8850 | 8898 | 8946 | 8994 | |
| 910 | 9041 | 9089 | 9137 | 9185 | 9232 | 9280 | 9328 | 9375 | 9423 | 9471 | 47 |
| 1 | 9518 | 9566 | 9614 | 9661 | 9709 | 9757 | 9804 | 9852 | 9900 | 9947 | |
| 2 | 9995 | | | | | | | | | | |
| 3 | 960471 | 0042 | 0090 | 0138 | 0185 | 0233 | 0280 | 0328 | 0376 | 0423 | |
| 4 | 0946 | 0518 | 0566 | 0613 | 0661 | 0709 | 0756 | 0804 | 0851 | 0899 | |
| 5 | 1421 | 0994 | 1041 | 1089 | 1136 | 1184 | 1231 | 1279 | 1326 | 1374 | |
| 6 | 1895 | 1469 | 1516 | 1563 | 1611 | 1658 | 1706 | 1753 | 1801 | 1848 | |
| 7 | 2369 | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 | |
| 8 | 2843 | 2417 | 2464 | 2511 | 2559 | 2606 | 2653 | 2701 | 2748 | 2795 | |
| 9 | 3316 | 2890 | 2937 | 2985 | 3032 | 3079 | 3126 | 3174 | 3221 | 3268 | |
| 920 | 3788 | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 | 46 |
| 1 | 4260 | 3835 | 3882 | 3929 | 3977 | 4024 | 4071 | 4118 | 4165 | 4212 | |
| 2 | 4731 | 4307 | 4354 | 4401 | 4448 | 4495 | 4542 | 4590 | 4637 | 4684 | |
| 3 | 5202 | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 | |
| 4 | 5672 | 5249 | 5296 | 5343 | 5390 | 5437 | 5484 | 5531 | 5578 | 5625 | |
| 5 | 6142 | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 | |
| 6 | 6611 | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 | |
| 7 | 7080 | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 | |
| 8 | 7548 | 7127 | 7173 | 7220 | 7267 | 7314 | 7361 | 7408 | 7454 | 7501 | |
| 9 | 8016 | 7595 | 7642 | 7688 | 7735 | 7782 | 7829 | 7875 | 7922 | 7969 | |
| 930 | 8483 | 8062 | 8109 | 8156 | 8203 | 8249 | 8296 | 8343 | 8390 | 8436 | 45 |
| 1 | 8950 | 8530 | 8576 | 8623 | 8670 | 8716 | 8763 | 8810 | 8856 | 8903 | |
| 2 | 9416 | 8996 | 9043 | 9090 | 9136 | 9183 | 9229 | 9276 | 9323 | 9369 | |
| 3 | 9882 | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 | |
| 4 | 970347 | 9928 | 9975 | | | | | | | | |
| 5 | 0812 | 0393 | 0440 | 0021 | 0068 | 0114 | 0161 | 0207 | 0254 | 0300 | |
| 6 | 1276 | 0858 | 0904 | 0486 | 0533 | 0579 | 0626 | 0672 | 0719 | 0765 | |
| 7 | 1740 | 1322 | 1369 | 0951 | 0997 | 1044 | 1090 | 1137 | 1183 | 1229 | |
| 8 | 2203 | 1786 | 1832 | 1415 | 1461 | 1508 | 1554 | 1601 | 1647 | 1693 | |
| 9 | 2666 | 2249 | 2295 | 1879 | 1925 | 1971 | 2018 | 2064 | 2110 | 2157 | |
| 940 | 3128 | 2712 | 2758 | 2804 | 2851 | 2897 | 2943 | 2989 | 3035 | 3082 | 44 |
| 1 | 3590 | 3174 | 3220 | 3266 | 3313 | 3359 | 3405 | 3451 | 3497 | 3543 | |
| 2 | 4051 | 3636 | 3682 | 3728 | 3774 | 3820 | 3866 | 3913 | 3959 | 4005 | |
| 3 | 4512 | 4097 | 4143 | 4189 | 4235 | 4281 | 4327 | 4374 | 4420 | 4466 | |
| 4 | 4972 | 4558 | 4604 | 4650 | 4696 | 4742 | 4788 | 4834 | 4880 | 4926 | |
| | | 5018 | 5064 | 5110 | 5156 | 5202 | 5248 | 5294 | 5340 | 5386 | 46 |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|-----|------|------|------|------|------|------|------|
| 47 | 4.7 | 9.4 | 14.1 | 18.8 | 23.5 | 28.2 | 32.9 | 37.6 | 42.3 |
| 46 | 4.6 | 9.2 | 13.8 | 18.4 | 23.0 | 27.6 | 32.2 | 36.8 | 41.4 |

No. 945 L. 975.]

[No. 980 L. 995.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 945 | 975432 | 5478 | 5524 | 5570 | 5616 | 5662 | 5707 | 5753 | 5799 | 5845 | |
| 6 | 5891 | 5937 | 5983 | 6029 | 6075 | 6121 | 6167 | 6212 | 6258 | 6304 | |
| 7 | 6350 | 6396 | 6442 | 6488 | 6533 | 6579 | 6625 | 6671 | 6717 | 6763 | |
| 8 | 6808 | 6854 | 6900 | 6946 | 6992 | 7037 | 7083 | 7129 | 7175 | 7220 | |
| 9 | 7266 | 7312 | 7358 | 7403 | 7449 | 7495 | 7541 | 7586 | 7632 | 7678 | |
| 950 | 7724 | 7769 | 7815 | 7861 | 7906 | 7952 | 7998 | 8043 | 8089 | 8135 | |
| 1 | 8181 | 8226 | 8272 | 8317 | 8363 | 8409 | 8454 | 8500 | 8546 | 8591 | |
| 2 | 8637 | 8683 | 8728 | 8774 | 8819 | 8865 | 8911 | 8956 | 9002 | 9047 | |
| 3 | 9093 | 9138 | 9184 | 9230 | 9275 | 9321 | 9366 | 9412 | 9457 | 9503 | |
| 4 | 9548 | 9594 | 9639 | 9685 | 9730 | 9776 | 9821 | 9867 | 9912 | 9958 | |
| 5 | 980003 | 0049 | 0094 | 0140 | 0185 | 0231 | 0276 | 0322 | 0367 | 0412 | |
| 6 | 0458 | 0503 | 0549 | 0594 | 0640 | 0685 | 0730 | 0776 | 0821 | 0867 | |
| 7 | 0912 | 0957 | 1003 | 1048 | 1093 | 1139 | 1184 | 1229 | 1275 | 1320 | |
| 8 | 1366 | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 | |
| 9 | 1819 | 1864 | 1909 | 1954 | 2000 | 2045 | 2090 | 2135 | 2181 | 2226 | |
| 960 | 2271 | 2316 | 2362 | 2407 | 2452 | 2497 | 2543 | 2588 | 2633 | 2678 | |
| 1 | 2723 | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 3040 | 3085 | 3130 | |
| 2 | 3175 | 3220 | 3265 | 3310 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 | |
| 3 | 3626 | 3671 | 3716 | 3762 | 3807 | 3852 | 3897 | 3942 | 3987 | 4032 | |
| 4 | 4077 | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 | |
| 5 | 4527 | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 | 45 |
| 6 | 4977 | 5022 | 5067 | 5112 | 5157 | 5202 | 5247 | 5292 | 5337 | 5382 | |
| 7 | 5426 | 5471 | 5516 | 5561 | 5606 | 5651 | 5696 | 5741 | 5786 | 5830 | |
| 8 | 5875 | 5920 | 5965 | 6010 | 6055 | 6100 | 6144 | 6189 | 6234 | 6279 | |
| 9 | 6324 | 6369 | 6413 | 6458 | 6503 | 6548 | 6593 | 6637 | 6682 | 6727 | |
| 970 | 6772 | 6817 | 6861 | 6906 | 6951 | 6996 | 7040 | 7085 | 7130 | 7175 | |
| 1 | 7219 | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 | |
| 2 | 7666 | 7711 | 7756 | 7800 | 7845 | 7890 | 7934 | 7979 | 8024 | 8068 | |
| 3 | 8113 | 8157 | 8202 | 8247 | 8291 | 8336 | 8381 | 8425 | 8470 | 8514 | |
| 4 | 8559 | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 8960 | |
| 5 | 9005 | 9049 | 9094 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 | |
| 6 | 9450 | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 9850 | |
| 7 | 9895 | 9939 | 9983 | 0028 | 0072 | 0117 | 0161 | 0206 | 0250 | 0294 | |
| 8 | 990339 | 0383 | 0428 | 0472 | 0516 | 0561 | 0605 | 0650 | 0694 | 0738 | |
| 9 | 0783 | 0827 | 0871 | 0916 | 0960 | 1004 | 1049 | 1093 | 1137 | 1182 | |
| 980 | 1226 | 1270 | 1315 | 1359 | 1403 | 1448 | 1492 | 1536 | 1580 | 1625 | |
| 1 | 1669 | 1713 | 1758 | 1802 | 1846 | 1890 | 1935 | 1979 | 2023 | 2067 | |
| 2 | 2111 | 2156 | 2200 | 2244 | 2288 | 2333 | 2377 | 2421 | 2465 | 2509 | |
| 3 | 2554 | 2598 | 2642 | 2686 | 2730 | 2774 | 2819 | 2863 | 2907 | 2951 | |
| 4 | 2995 | 3039 | 3083 | 3127 | 3172 | 3216 | 3260 | 3304 | 3348 | 3392 | |
| 5 | 3436 | 3480 | 3524 | 3568 | 3613 | 3657 | 3701 | 3745 | 3789 | 3833 | |
| 6 | 3877 | 3921 | 3965 | 4009 | 4053 | 4097 | 4141 | 4185 | 4229 | 4273 | |
| 7 | 4317 | 4361 | 4405 | 4449 | 4493 | 4537 | 4581 | 4625 | 4669 | 4713 | |
| 8 | 4757 | 4801 | 4845 | 4889 | 4933 | 4977 | 5021 | 5065 | 5108 | 5152 | 44 |
| 9 | 5196 | 5240 | 5284 | 5328 | 5372 | 5416 | 5460 | 5504 | 5547 | 5591 | |

PROPORTIONAL PARTS.

| Diff. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|-----|-----|------|------|------|------|------|------|------|
| 46 | 4.6 | 9.2 | 13.8 | 18.4 | 23.0 | 27.6 | 32.2 | 36.8 | 41.4 |
| 45 | 4.5 | 9.0 | 13.5 | 18.0 | 22.5 | 27.0 | 31.5 | 36.0 | 40.5 |
| 44 | 4.4 | 8.8 | 13.2 | 17.6 | 22.0 | 26.4 | 30.8 | 35.2 | 39.6 |
| 43 | 4.3 | 8.6 | 12.9 | 17.2 | 21.5 | 25.8 | 30.1 | 34.4 | 38.7 |

No. 990 L. 995.]

[No. 999 L. 999.

| N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Diff. |
|-----|--------|------|------|------|------|------|------|------|------|------|-------|
| 990 | 995635 | 5679 | 5723 | 5767 | 5811 | 5854 | 5898 | 5942 | 5986 | 6030 | 44 |
| 1 | 6074 | 6117 | 6161 | 6205 | 6249 | 6293 | 6337 | 6380 | 6424 | 6468 | |
| 2 | 6512 | 6555 | 6599 | 6643 | 6687 | 6731 | 6774 | 6818 | 6862 | 6906 | |
| 3 | 6949 | 6993 | 7037 | 7080 | 7124 | 7168 | 7212 | 7255 | 7299 | 7343 | |
| 4 | 7386 | 7430 | 7474 | 7517 | 7561 | 7605 | 7648 | 7692 | 7736 | 7779 | |
| 5 | 7823 | 7867 | 7910 | 7954 | 7998 | 8041 | 8085 | 8129 | 8172 | 8216 | |
| 6 | 8259 | 8303 | 8347 | 8390 | 8434 | 8477 | 8521 | 8564 | 8608 | 8652 | |
| 7 | 8695 | 8739 | 8782 | 8826 | 8869 | 8913 | 8956 | 9000 | 9043 | 9087 | |
| 8 | 9131 | 9174 | 9218 | 9261 | 9305 | 9348 | 9392 | 9435 | 9479 | 9522 | |
| 9 | 9565 | 9609 | 9652 | 9696 | 9739 | 9783 | 9826 | 9870 | 9913 | 9957 | 43 |

HYPERBOLIC LOGARITHMS.

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|------|-------|------|-------|------|-------|------|--------|------|--------|
| 1.01 | .0099 | 1.45 | .3716 | 1.89 | .6366 | 2.33 | .8458 | 2.77 | 1.0188 |
| 1.02 | .0198 | 1.46 | .3784 | 1.90 | .6419 | 2.34 | .8502 | 2.78 | 1.0225 |
| 1.03 | .0296 | 1.47 | .3853 | 1.91 | .6471 | 2.35 | .8544 | 2.79 | 1.0260 |
| 1.04 | .0392 | 1.48 | .3920 | 1.92 | .6523 | 2.36 | .8587 | 2.80 | 1.0296 |
| 1.05 | .0488 | 1.49 | .3988 | 1.93 | .6575 | 2.37 | .8629 | 2.81 | 1.0332 |
| 1.06 | .0583 | 1.50 | .4055 | 1.94 | .6627 | 2.38 | .8671 | 2.82 | 1.0367 |
| 1.07 | .0677 | 1.51 | .4121 | 1.95 | .6678 | 2.39 | .8713 | 2.83 | 1.0403 |
| 1.08 | .0770 | 1.52 | .4187 | 1.96 | .6729 | 2.40 | .8755 | 2.84 | 1.0438 |
| 1.09 | .0862 | 1.53 | .4253 | 1.97 | .6780 | 2.41 | .8796 | 2.85 | 1.0473 |
| 1.10 | .0953 | 1.54 | .4318 | 1.98 | .6831 | 2.42 | .8838 | 2.86 | 1.0508 |
| 1.11 | .1044 | 1.55 | .4383 | 1.99 | .6881 | 2.43 | .8879 | 2.87 | 1.0543 |
| 1.12 | .1133 | 1.56 | .4447 | 2.00 | .6931 | 2.44 | .8920 | 2.88 | 1.0578 |
| 1.13 | .1222 | 1.57 | .4511 | 2.01 | .6981 | 2.45 | .8961 | 2.89 | 1.0613 |
| 1.14 | .1310 | 1.58 | .4574 | 2.02 | .7031 | 2.46 | .9002 | 2.90 | 1.0647 |
| 1.15 | .1398 | 1.59 | .4637 | 2.03 | .7080 | 2.47 | .9042 | 2.91 | 1.0682 |
| 1.16 | .1484 | 1.60 | .4700 | 2.04 | .7129 | 2.48 | .9083 | 2.92 | 1.0716 |
| 1.17 | .1570 | 1.61 | .4762 | 2.05 | .7178 | 2.49 | .9123 | 2.93 | 1.0750 |
| 1.18 | .1655 | 1.62 | .4824 | 2.06 | .7227 | 2.50 | .9163 | 2.94 | 1.0784 |
| 1.19 | .1740 | 1.63 | .4886 | 2.07 | .7275 | 2.51 | .9203 | 2.95 | 1.0818 |
| 1.20 | .1823 | 1.64 | .4947 | 2.08 | .7324 | 2.52 | .9243 | 2.96 | 1.0852 |
| 1.21 | .1906 | 1.65 | .5008 | 2.09 | .7372 | 2.53 | .9282 | 2.97 | 1.0886 |
| 1.22 | .1988 | 1.66 | .5068 | 2.10 | .7419 | 2.54 | .9322 | 2.98 | 1.0919 |
| 1.23 | .2070 | 1.67 | .5128 | 2.11 | .7467 | 2.55 | .9361 | 2.99 | 1.0953 |
| 1.24 | .2151 | 1.68 | .5188 | 2.12 | .7514 | 2.56 | .9400 | 3.00 | 1.0986 |
| 1.25 | .2231 | 1.69 | .5247 | 2.13 | .7561 | 2.57 | .9439 | 3.01 | 1.1019 |
| 1.26 | .2311 | 1.70 | .5306 | 2.14 | .7608 | 2.58 | .9478 | 3.02 | 1.1056 |
| 1.27 | .2390 | 1.71 | .5365 | 2.15 | .7655 | 2.59 | .9517 | 3.03 | 1.1081 |
| 1.28 | .2469 | 1.72 | .5423 | 2.16 | .7701 | 2.60 | .9555 | 3.04 | 1.1113 |
| 1.29 | .2546 | 1.73 | .5481 | 2.17 | .7747 | 2.61 | .9594 | 3.05 | 1.1154 |
| 1.30 | .2624 | 1.74 | .5539 | 2.18 | .7793 | 2.62 | .9632 | 3.06 | 1.1187 |
| 1.31 | .2700 | 1.75 | .5596 | 2.19 | .7839 | 2.63 | .9670 | 3.07 | 1.1219 |
| 1.32 | .2776 | 1.76 | .5653 | 2.20 | .7885 | 2.64 | .9708 | 3.08 | 1.1246 |
| 1.33 | .2852 | 1.77 | .5710 | 2.21 | .7930 | 2.65 | .9746 | 3.09 | 1.1284 |
| 1.34 | .2927 | 1.78 | .5766 | 2.22 | .7975 | 2.66 | .9783 | 3.10 | 1.1312 |
| 1.35 | .3001 | 1.79 | .5822 | 2.23 | .8020 | 2.67 | .9821 | 3.11 | 1.1349 |
| 1.36 | .3075 | 1.80 | .5878 | 2.24 | .8065 | 2.68 | .9858 | 3.12 | 1.1378 |
| 1.37 | .3148 | 1.81 | .5933 | 2.25 | .8109 | 2.69 | .9895 | 3.13 | 1.1410 |
| 1.38 | .3221 | 1.82 | .5988 | 2.26 | .8154 | 2.70 | .9933 | 3.14 | 1.1442 |
| 1.39 | .3293 | 1.83 | .6043 | 2.27 | .8198 | 2.71 | .9969 | 3.15 | 1.1474 |
| 1.40 | .3365 | 1.84 | .6098 | 2.28 | .8242 | 2.72 | 1.0006 | 3.16 | 1.1506 |
| 1.41 | .3436 | 1.85 | .6152 | 2.29 | .8286 | 2.73 | 1.0043 | 3.17 | 1.1537 |
| 1.42 | .3507 | 1.86 | .6206 | 2.30 | .8329 | 2.74 | 1.0080 | 3.18 | 1.1569 |
| 1.43 | .3577 | 1.87 | .6259 | 2.31 | .8372 | 2.75 | 1.0116 | 3.19 | 1.1600 |
| 1.44 | .3646 | 1.88 | .6313 | 2.32 | .8416 | 2.76 | 1.0152 | 3.20 | 1.1632 |

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|------|--------|------|--------|------|--------|------|--------|------|--------|
| 3.21 | 1.1663 | 3.87 | 1.3533 | 4.53 | 1.5107 | 5.19 | 1.6467 | 5.85 | 1.7664 |
| 3.22 | 1.1694 | 3.88 | 1.3558 | 4.54 | 1.5129 | 5.20 | 1.6487 | 5.86 | 1.7681 |
| 3.23 | 1.1725 | 3.89 | 1.3584 | 4.55 | 1.5151 | 5.21 | 1.6506 | 5.87 | 1.7699 |
| 3.24 | 1.1756 | 3.90 | 1.3610 | 4.56 | 1.5173 | 5.22 | 1.6525 | 5.88 | 1.7716 |
| 3.25 | 1.1787 | 3.91 | 1.3635 | 4.57 | 1.5195 | 5.23 | 1.6544 | 5.89 | 1.7733 |
| 3.26 | 1.1817 | 3.92 | 1.3661 | 4.58 | 1.5217 | 5.24 | 1.6563 | 5.90 | 1.7750 |
| 3.27 | 1.1848 | 3.93 | 1.3686 | 4.59 | 1.5239 | 5.25 | 1.6582 | 5.91 | 1.7766 |
| 3.28 | 1.1878 | 3.94 | 1.3712 | 4.60 | 1.5261 | 5.26 | 1.6601 | 5.92 | 1.7783 |
| 3.29 | 1.1909 | 3.95 | 1.3737 | 4.61 | 1.5282 | 5.27 | 1.6620 | 5.93 | 1.7800 |
| 3.30 | 1.1939 | 3.96 | 1.3762 | 4.62 | 1.5304 | 5.28 | 1.6639 | 5.94 | 1.7817 |
| 3.31 | 1.1969 | 3.97 | 1.3788 | 4.63 | 1.5326 | 5.29 | 1.6658 | 5.95 | 1.7834 |
| 3.32 | 1.1999 | 3.98 | 1.3813 | 4.64 | 1.5347 | 5.30 | 1.6677 | 5.96 | 1.7851 |
| 3.33 | 1.2030 | 3.99 | 1.3838 | 4.65 | 1.5369 | 5.31 | 1.6696 | 5.97 | 1.7867 |
| 3.34 | 1.2060 | 4.00 | 1.3863 | 4.66 | 1.5390 | 5.32 | 1.6715 | 5.98 | 1.7884 |
| 3.35 | 1.2090 | 4.01 | 1.3888 | 4.67 | 1.5412 | 5.33 | 1.6734 | 5.99 | 1.7901 |
| 3.36 | 1.2119 | 4.02 | 1.3913 | 4.68 | 1.5433 | 5.34 | 1.6752 | 6.00 | 1.7918 |
| 3.37 | 1.2149 | 4.03 | 1.3938 | 4.69 | 1.5454 | 5.35 | 1.6771 | 6.01 | 1.7934 |
| 3.38 | 1.2179 | 4.04 | 1.3962 | 4.70 | 1.5476 | 5.36 | 1.6790 | 6.02 | 1.7951 |
| 3.39 | 1.2208 | 4.05 | 1.3987 | 4.71 | 1.5497 | 5.37 | 1.6808 | 6.03 | 1.7967 |
| 3.40 | 1.2238 | 4.06 | 1.4012 | 4.72 | 1.5518 | 5.38 | 1.6827 | 6.04 | 1.7984 |
| 3.41 | 1.2267 | 4.07 | 1.4036 | 4.73 | 1.5539 | 5.39 | 1.6845 | 6.05 | 1.8001 |
| 3.42 | 1.2296 | 4.08 | 1.4061 | 4.74 | 1.5560 | 5.40 | 1.6864 | 6.06 | 1.8017 |
| 3.43 | 1.2326 | 4.09 | 1.4085 | 4.75 | 1.5581 | 5.41 | 1.6882 | 6.07 | 1.8034 |
| 3.44 | 1.2355 | 4.10 | 1.4110 | 4.76 | 1.5602 | 5.42 | 1.6901 | 6.08 | 1.8050 |
| 3.45 | 1.2384 | 4.11 | 1.4134 | 4.77 | 1.5623 | 5.43 | 1.6919 | 6.09 | 1.8066 |
| 3.46 | 1.2413 | 4.12 | 1.4159 | 4.78 | 1.5644 | 5.44 | 1.6938 | 6.10 | 1.8083 |
| 3.47 | 1.2442 | 4.13 | 1.4183 | 4.79 | 1.5665 | 5.45 | 1.6956 | 6.11 | 1.8099 |
| 3.48 | 1.2470 | 4.14 | 1.4207 | 4.80 | 1.5686 | 5.46 | 1.6974 | 6.12 | 1.8116 |
| 3.49 | 1.2499 | 4.15 | 1.4231 | 4.81 | 1.5707 | 5.47 | 1.6993 | 6.13 | 1.8132 |
| 3.50 | 1.2528 | 4.16 | 1.4255 | 4.82 | 1.5728 | 5.48 | 1.7011 | 6.14 | 1.8148 |
| 3.51 | 1.2556 | 4.17 | 1.4279 | 4.83 | 1.5748 | 5.49 | 1.7029 | 6.15 | 1.8165 |
| 3.52 | 1.2585 | 4.18 | 1.4303 | 4.84 | 1.5769 | 5.50 | 1.7047 | 6.16 | 1.8181 |
| 3.53 | 1.2613 | 4.19 | 1.4327 | 4.85 | 1.5790 | 5.51 | 1.7066 | 6.17 | 1.8197 |
| 3.54 | 1.2641 | 4.20 | 1.4351 | 4.86 | 1.5810 | 5.52 | 1.7084 | 6.18 | 1.8213 |
| 3.55 | 1.2669 | 4.21 | 1.4375 | 4.87 | 1.5831 | 5.53 | 1.7102 | 6.19 | 1.8229 |
| 3.56 | 1.2698 | 4.22 | 1.4398 | 4.88 | 1.5851 | 5.54 | 1.7120 | 6.20 | 1.8245 |
| 3.57 | 1.2726 | 4.23 | 1.4422 | 4.89 | 1.5872 | 5.55 | 1.7138 | 6.21 | 1.8262 |
| 3.58 | 1.2754 | 4.24 | 1.4446 | 4.90 | 1.5892 | 5.56 | 1.7156 | 6.22 | 1.8278 |
| 3.59 | 1.2782 | 4.25 | 1.4469 | 4.91 | 1.5913 | 5.57 | 1.7174 | 6.23 | 1.8294 |
| 3.60 | 1.2809 | 4.26 | 1.4493 | 4.92 | 1.5933 | 5.58 | 1.7192 | 6.24 | 1.8310 |
| 3.61 | 1.2837 | 4.27 | 1.4516 | 4.93 | 1.5953 | 5.59 | 1.7210 | 6.25 | 1.8326 |
| 3.62 | 1.2865 | 4.28 | 1.4540 | 4.94 | 1.5974 | 5.60 | 1.7228 | 6.26 | 1.8342 |
| 3.63 | 1.2892 | 4.29 | 1.4563 | 4.95 | 1.5994 | 5.61 | 1.7246 | 6.27 | 1.8358 |
| 3.64 | 1.2920 | 4.30 | 1.4586 | 4.96 | 1.6014 | 5.62 | 1.7263 | 6.28 | 1.8374 |
| 3.65 | 1.2947 | 4.31 | 1.4609 | 4.97 | 1.6034 | 5.63 | 1.7281 | 6.29 | 1.8390 |
| 3.66 | 1.2975 | 4.32 | 1.4633 | 4.98 | 1.6054 | 5.64 | 1.7299 | 6.30 | 1.8405 |
| 3.67 | 1.3002 | 4.33 | 1.4656 | 4.99 | 1.6074 | 5.65 | 1.7317 | 6.31 | 1.8421 |
| 3.68 | 1.3029 | 4.34 | 1.4679 | 5.00 | 1.6094 | 5.66 | 1.7334 | 6.32 | 1.8437 |
| 3.69 | 1.3056 | 4.35 | 1.4702 | 5.01 | 1.6114 | 5.67 | 1.7352 | 6.33 | 1.8453 |
| 3.70 | 1.3083 | 4.36 | 1.4725 | 5.02 | 1.6134 | 5.68 | 1.7370 | 6.34 | 1.8469 |
| 3.71 | 1.3110 | 4.37 | 1.4748 | 5.03 | 1.6154 | 5.69 | 1.7387 | 6.35 | 1.8485 |
| 3.72 | 1.3137 | 4.38 | 1.4770 | 5.04 | 1.6174 | 5.70 | 1.7405 | 6.36 | 1.8500 |
| 3.73 | 1.3164 | 4.39 | 1.4793 | 5.05 | 1.6194 | 5.71 | 1.7422 | 6.37 | 1.8516 |
| 3.74 | 1.3191 | 4.40 | 1.4816 | 5.06 | 1.6214 | 5.72 | 1.7440 | 6.38 | 1.8532 |
| 3.75 | 1.3218 | 4.41 | 1.4839 | 5.07 | 1.6233 | 5.73 | 1.7457 | 6.39 | 1.8547 |
| 3.76 | 1.3244 | 4.42 | 1.4861 | 5.08 | 1.6253 | 5.74 | 1.7475 | 6.40 | 1.8563 |
| 3.77 | 1.3271 | 4.43 | 1.4884 | 5.09 | 1.6273 | 5.75 | 1.7492 | 6.41 | 1.8579 |
| 3.78 | 1.3297 | 4.44 | 1.4907 | 5.10 | 1.6292 | 5.76 | 1.7509 | 6.42 | 1.8594 |
| 3.79 | 1.3324 | 4.45 | 1.4929 | 5.11 | 1.6312 | 5.77 | 1.7527 | 6.43 | 1.8610 |
| 3.80 | 1.3350 | 4.46 | 1.4951 | 5.12 | 1.6332 | 5.78 | 1.7544 | 6.44 | 1.8625 |
| 3.81 | 1.3376 | 4.47 | 1.4974 | 5.13 | 1.6351 | 5.79 | 1.7561 | 6.45 | 1.8641 |
| 3.82 | 1.3403 | 4.48 | 1.4996 | 5.14 | 1.6371 | 5.80 | 1.7579 | 6.46 | 1.8656 |
| 3.83 | 1.3429 | 4.49 | 1.5019 | 5.15 | 1.6390 | 5.81 | 1.7596 | 6.47 | 1.8672 |
| 3.84 | 1.3455 | 4.50 | 1.5041 | 5.16 | 1.6409 | 5.82 | 1.7613 | 6.48 | 1.8687 |
| 3.85 | 1.3481 | 4.51 | 1.5063 | 5.17 | 1.6429 | 5.83 | 1.7630 | 6.49 | 1.8703 |
| 3.86 | 1.3507 | 4.52 | 1.5085 | 5.18 | 1.6448 | 5.84 | 1.7647 | 6.50 | 1.8718 |

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No | Log. |
|------|--------|------|--------|------|--------|------|--------|-------|--------|
| 6.51 | 1.8733 | 7.15 | 1.9671 | 7.79 | 2.0528 | 8.66 | 2.1587 | 9.94 | 2.2966 |
| 6.52 | 1.8749 | 7.16 | 1.9685 | 7.80 | 2.0541 | 8.68 | 2.1610 | 9.96 | 2.2986 |
| 6.53 | 1.8764 | 7.17 | 1.9699 | 7.81 | 2.0554 | 8.70 | 2.1633 | 9.98 | 2.3006 |
| 6.54 | 1.8779 | 7.18 | 1.9713 | 7.82 | 2.0567 | 8.72 | 2.1656 | 10.00 | 2.3026 |
| 6.55 | 1.8795 | 7.19 | 1.9727 | 7.83 | 2.0580 | 8.74 | 2.1679 | 10.25 | 2.3279 |
| 6.56 | 1.8810 | 7.20 | 1.9741 | 7.84 | 2.0592 | 8.76 | 2.1702 | 10.50 | 2.3513 |
| 6.57 | 1.8825 | 7.21 | 1.9754 | 7.85 | 2.0605 | 8.78 | 2.1725 | 10.75 | 2.3749 |
| 6.58 | 1.8840 | 7.22 | 1.9769 | 7.86 | 2.0618 | 8.80 | 2.1748 | 11.00 | 2.3979 |
| 6.59 | 1.8856 | 7.23 | 1.9782 | 7.87 | 2.0631 | 8.82 | 2.1770 | 11.25 | 2.4201 |
| 6.60 | 1.8871 | 7.24 | 1.9796 | 7.88 | 2.0643 | 8.84 | 2.1793 | 11.50 | 2.4430 |
| 6.61 | 1.8886 | 7.25 | 1.9810 | 7.89 | 2.0656 | 8.86 | 2.1815 | 11.75 | 2.4636 |
| 6.62 | 1.8901 | 7.26 | 1.9824 | 7.90 | 2.0669 | 8.88 | 2.1838 | 12.00 | 2.4849 |
| 6.63 | 1.8916 | 7.27 | 1.9838 | 7.91 | 2.0681 | 8.90 | 2.1861 | 12.25 | 2.5052 |
| 6.64 | 1.8931 | 7.28 | 1.9851 | 7.92 | 2.0694 | 8.92 | 2.1883 | 12.50 | 2.5262 |
| 6.65 | 1.8946 | 7.29 | 1.9865 | 7.93 | 2.0707 | 8.94 | 2.1905 | 12.75 | 2.5455 |
| 6.66 | 1.8961 | 7.30 | 1.9879 | 7.94 | 2.0719 | 8.96 | 2.1928 | 13.00 | 2.5649 |
| 6.67 | 1.8976 | 7.31 | 1.9892 | 7.95 | 2.0732 | 8.98 | 2.1950 | 13.25 | 2.5840 |
| 6.68 | 1.8991 | 7.32 | 1.9906 | 7.96 | 2.0744 | 9.00 | 2.1972 | 13.50 | 2.6027 |
| 6.69 | 1.9006 | 7.33 | 1.9920 | 7.97 | 2.0757 | 9.02 | 2.1994 | 13.75 | 2.6211 |
| 6.70 | 1.9021 | 7.34 | 1.9933 | 7.98 | 2.0769 | 9.04 | 2.2017 | 14.00 | 2.6391 |
| 6.71 | 1.9036 | 7.35 | 1.9947 | 7.99 | 2.0782 | 9.06 | 2.2039 | 14.25 | 2.6567 |
| 6.72 | 1.9051 | 7.36 | 1.9961 | 8.00 | 2.0794 | 9.08 | 2.2061 | 14.50 | 2.6740 |
| 6.73 | 1.9066 | 7.37 | 1.9974 | 8.01 | 2.0807 | 9.10 | 2.2083 | 14.75 | 2.6913 |
| 6.74 | 1.9081 | 7.38 | 1.9988 | 8.02 | 2.0819 | 9.12 | 2.2105 | 15.00 | 2.7081 |
| 6.75 | 1.9095 | 7.39 | 2.0001 | 8.03 | 2.0832 | 9.14 | 2.2127 | 15.50 | 2.7408 |
| 6.76 | 1.9110 | 7.40 | 2.0015 | 8.04 | 2.0844 | 9.16 | 2.2148 | 16.00 | 2.7726 |
| 6.77 | 1.9125 | 7.41 | 2.0028 | 8.05 | 2.0857 | 9.18 | 2.2170 | 16.50 | 2.8034 |
| 6.78 | 1.9140 | 7.42 | 2.0041 | 8.06 | 2.0869 | 9.20 | 2.2192 | 17.00 | 2.8332 |
| 6.79 | 1.9155 | 7.43 | 2.0055 | 8.07 | 2.0882 | 9.22 | 2.2214 | 17.50 | 2.8621 |
| 6.80 | 1.9169 | 7.44 | 2.0069 | 8.08 | 2.0894 | 9.24 | 2.2235 | 18.00 | 2.8904 |
| 6.81 | 1.9184 | 7.45 | 2.0082 | 8.09 | 2.0906 | 9.26 | 2.2257 | 18.50 | 2.9178 |
| 6.82 | 1.9199 | 7.46 | 2.0096 | 8.10 | 2.0919 | 9.28 | 2.2279 | 19.00 | 2.9444 |
| 6.83 | 1.9213 | 7.47 | 2.0108 | 8.11 | 2.0931 | 9.30 | 2.2300 | 19.50 | 2.9703 |
| 6.84 | 1.9228 | 7.48 | 2.0122 | 8.12 | 2.0943 | 9.32 | 2.2322 | 20.00 | 2.9957 |
| 6.85 | 1.9242 | 7.49 | 2.0136 | 8.13 | 2.0956 | 9.34 | 2.2343 | 21 | 3.0445 |
| 6.86 | 1.9257 | 7.50 | 2.0149 | 8.14 | 2.0968 | 9.36 | 2.2364 | 22 | 3.0910 |
| 6.87 | 1.9272 | 7.51 | 2.0162 | 8.15 | 2.0980 | 9.38 | 2.2386 | 23 | 3.1355 |
| 6.88 | 1.9286 | 7.52 | 2.0176 | 8.16 | 2.0992 | 9.40 | 2.2407 | 24 | 3.1781 |
| 6.89 | 1.9301 | 7.53 | 2.0189 | 8.17 | 2.1005 | 9.42 | 2.2428 | 25 | 3.2189 |
| 6.90 | 1.9315 | 7.54 | 2.0202 | 8.18 | 2.1017 | 9.44 | 2.2450 | 26 | 3.2581 |
| 6.91 | 1.9330 | 7.55 | 2.0215 | 8.19 | 2.1029 | 9.46 | 2.2471 | 27 | 3.2958 |
| 6.92 | 1.9344 | 7.56 | 2.0229 | 8.20 | 2.1041 | 9.48 | 2.2492 | 28 | 3.3322 |
| 6.93 | 1.9359 | 7.57 | 2.0242 | 8.22 | 2.1066 | 9.50 | 2.2513 | 29 | 3.3673 |
| 6.94 | 1.9373 | 7.58 | 2.0255 | 8.24 | 2.1090 | 9.52 | 2.2534 | 30 | 3.4012 |
| 6.95 | 1.9387 | 7.59 | 2.0268 | 8.26 | 2.1114 | 9.54 | 2.2555 | 31 | 3.4340 |
| 6.96 | 1.9402 | 7.60 | 2.0281 | 8.28 | 2.1138 | 9.56 | 2.2576 | 32 | 3.4657 |
| 6.97 | 1.9416 | 7.61 | 2.0295 | 8.30 | 2.1163 | 9.58 | 2.2597 | 33 | 3.4965 |
| 6.98 | 1.9430 | 7.62 | 2.0308 | 8.32 | 2.1187 | 9.60 | 2.2618 | 34 | 3.5263 |
| 6.99 | 1.9445 | 7.63 | 2.0321 | 8.34 | 2.1211 | 9.62 | 2.2638 | 35 | 3.5553 |
| 7.00 | 1.9459 | 7.64 | 2.0334 | 8.36 | 2.1235 | 9.64 | 2.2659 | 36 | 3.5835 |
| 7.01 | 1.9473 | 7.65 | 2.0347 | 8.38 | 2.1258 | 9.66 | 2.2680 | 37 | 3.6109 |
| 7.02 | 1.9488 | 7.66 | 2.0360 | 8.40 | 2.1282 | 9.68 | 2.2701 | 38 | 3.6376 |
| 7.03 | 1.9502 | 7.67 | 2.0373 | 8.42 | 2.1306 | 9.70 | 2.2721 | 39 | 3.6636 |
| 7.04 | 1.9516 | 7.68 | 2.0386 | 8.44 | 2.1330 | 9.72 | 2.2742 | 40 | 3.6889 |
| 7.05 | 1.9530 | 7.69 | 2.0399 | 8.46 | 2.1353 | 9.74 | 2.2762 | 41 | 3.7136 |
| 7.06 | 1.9544 | 7.70 | 2.0412 | 8.48 | 2.1377 | 9.76 | 2.2783 | 42 | 3.7377 |
| 7.07 | 1.9559 | 7.71 | 2.0425 | 8.50 | 2.1401 | 9.78 | 2.2803 | 43 | 3.7612 |
| 7.08 | 1.9573 | 7.72 | 2.0438 | 8.52 | 2.1424 | 9.80 | 2.2824 | 44 | 3.7842 |
| 7.09 | 1.9587 | 7.73 | 2.0451 | 8.54 | 2.1448 | 9.82 | 2.2844 | 45 | 3.8067 |
| 7.10 | 1.9601 | 7.74 | 2.0464 | 8.56 | 2.1471 | 9.84 | 2.2865 | 46 | 3.8286 |
| 7.11 | 1.9615 | 7.75 | 2.0477 | 8.58 | 2.1494 | 9.86 | 2.2885 | 47 | 3.8501 |
| 7.12 | 1.9629 | 7.76 | 2.0490 | 8.60 | 2.1518 | 9.88 | 2.2905 | 48 | 3.8712 |
| 7.13 | 1.9643 | 7.77 | 2.0503 | 8.62 | 2.1541 | 9.90 | 2.2925 | 49 | 3.8918 |
| 7.14 | 1.9657 | 7.78 | 2.0516 | 8.64 | 2.1564 | 9.92 | 2.2946 | 50 | 3.9120 |

NATURAL TRIGONOMETRICAL FUNCTIONS.

| • | M. | Sine. | Co-Vers. | Cosec. | Tang. | Cotan. | Se-cant. | Ver. Sin. | Cosine. | | |
|----|----|----------|-----------|----------|--------|----------|----------|-----------|---------|----|----|
| 0 | 0 | .00000 | 1.0000 | Infinite | .00000 | Infinite | 1.0000 | .00000 | 1.0000 | 90 | 0 |
| | 15 | .00436 | .99564 | 229.18 | .00436 | 229.18 | 1.0000 | .00001 | .99999 | | 45 |
| | 30 | .00873 | .99127 | 114.59 | .00873 | 114.59 | 1.0000 | .00004 | .99996 | | 30 |
| | 45 | .01309 | .98691 | 76.397 | .01309 | 76.390 | 1.0001 | .00009 | .99991 | | 15 |
| 1 | 0 | .01745 | .98255 | 57.299 | .01745 | 57.290 | 1.0001 | .00015 | .99985 | 89 | 0 |
| | 15 | .02181 | .97819 | 45.840 | .02182 | 45.829 | 1.0002 | .00024 | .99976 | | 45 |
| | 30 | .02618 | .97382 | 38.202 | .02618 | 38.188 | 1.0003 | .00034 | .99966 | | 30 |
| | 45 | .03054 | .96946 | 32.746 | .03055 | 32.730 | 1.0005 | .00047 | .99953 | | 15 |
| 2 | 0 | .03490 | .96510 | 28.654 | .03492 | 28.636 | 1.0006 | .00061 | .99939 | 88 | 0 |
| | 15 | .03926 | .96074 | 25.471 | .03929 | 25.452 | 1.0008 | .00077 | .99923 | | 45 |
| | 30 | .04362 | .95638 | 22.926 | .04366 | 22.904 | 1.0009 | .00095 | .99905 | | 30 |
| | 45 | .04798 | .95202 | 20.843 | .04803 | 20.819 | 1.0011 | .00115 | .99885 | | 15 |
| 3 | 0 | .05234 | .94766 | 19.107 | .05241 | 19.081 | 1.0014 | .00137 | .99863 | 87 | 0 |
| | 15 | .05669 | .94331 | 17.639 | .05678 | 17.611 | 1.0016 | .00161 | .99839 | | 45 |
| | 30 | .06105 | .93895 | 16.380 | .06116 | 16.350 | 1.0019 | .00187 | .99813 | | 30 |
| | 45 | .06540 | .93460 | 15.290 | .06554 | 15.257 | 1.0021 | .00214 | .99786 | | 15 |
| 4 | 0 | .06976 | .93024 | 14.336 | .06993 | 14.301 | 1.0024 | .00244 | .99756 | 86 | 0 |
| | 15 | .07411 | .92589 | 13.494 | .07431 | 13.457 | 1.0028 | .00275 | .99725 | | 45 |
| | 30 | .07846 | .92154 | 12.745 | .07870 | 12.706 | 1.0031 | .00308 | .99692 | | 30 |
| | 45 | .08281 | .91719 | 12.076 | .08309 | 12.035 | 1.0034 | .00343 | .99656 | | 15 |
| 5 | 0 | .08716 | .91284 | 11.474 | .08749 | 11.430 | 1.0038 | .00381 | .99619 | 85 | 0 |
| | 15 | .09150 | .90850 | 10.929 | .09189 | 10.883 | 1.0042 | .00420 | .99580 | | 45 |
| | 30 | .09585 | .90415 | 10.433 | .09629 | 10.385 | 1.0046 | .00460 | .99540 | | 30 |
| | 45 | .10019 | .89981 | 9.9812 | .10069 | 9.9310 | 1.0051 | .00503 | .99497 | | 15 |
| 6 | 0 | .10453 | .89547 | 9.5668 | .10510 | 9.5144 | 1.0055 | .00548 | .99452 | 84 | 0 |
| | 15 | .10887 | .89113 | 9.1855 | .10952 | 9.1309 | 1.0060 | .00594 | .99406 | | 45 |
| | 30 | .11320 | .88680 | 8.8337 | .11393 | 8.7769 | 1.0065 | .00643 | .99357 | | 30 |
| | 45 | .11754 | .88246 | 8.5079 | .11836 | 8.4490 | 1.0070 | .00693 | .99307 | | 15 |
| 7 | 0 | .12187 | .87813 | 8.2055 | .12278 | 8.1443 | 1.0075 | .00745 | .99255 | 83 | 0 |
| | 15 | .12620 | .87380 | 7.9240 | .12722 | 7.8606 | 1.0081 | .00800 | .99200 | | 45 |
| | 30 | .13053 | .86947 | 7.6613 | .13165 | 7.5958 | 1.0086 | .00856 | .99144 | | 30 |
| | 45 | .13485 | .86515 | 7.4156 | .13609 | 7.3479 | 1.0092 | .00913 | .99086 | | 15 |
| 8 | 0 | .13917 | .86083 | 7.1853 | .14054 | 7.1154 | 1.0098 | .00973 | .99027 | 82 | 0 |
| | 15 | .14349 | .85651 | 6.9690 | .14499 | 6.8969 | 1.0105 | .01035 | .98965 | | 45 |
| | 30 | .14781 | .85219 | 6.7655 | .14945 | 6.6912 | 1.0111 | .01098 | .98902 | | 30 |
| | 45 | .15212 | .84788 | 6.5736 | .15391 | 6.4971 | 1.0118 | .01164 | .98836 | | 15 |
| 9 | 0 | .15643 | .84357 | 6.3924 | .15838 | 6.3138 | 1.0125 | .01231 | .98769 | 81 | 0 |
| | 15 | .16074 | .83926 | 6.2211 | .16286 | 6.1402 | 1.0132 | .01300 | .98700 | | 45 |
| | 30 | .16505 | .83495 | 6.0589 | .16734 | 5.9758 | 1.0139 | .01371 | .98629 | | 30 |
| | 45 | .16935 | .83065 | 5.9049 | .17183 | 5.8197 | 1.0147 | .01444 | .98556 | | 15 |
| 10 | 0 | .17365 | .82635 | 5.7588 | .17633 | 5.6713 | 1.0154 | .01519 | .98481 | 80 | 0 |
| | 15 | .17794 | .82206 | 5.6198 | .18083 | 5.5301 | 1.0162 | .01596 | .98404 | | 45 |
| | 30 | .18224 | .81776 | 5.4874 | .18534 | 5.3955 | 1.0170 | .01675 | .98325 | | 30 |
| | 45 | .18652 | .81348 | 5.3612 | .18986 | 5.2672 | 1.0179 | .01755 | .98245 | | 15 |
| 11 | 0 | .19081 | .80919 | 5.2408 | .19438 | 5.1446 | 1.0187 | .01837 | .98163 | 79 | 0 |
| | 15 | .19509 | .80491 | 5.1258 | .19891 | 5.0273 | 1.0196 | .01921 | .98079 | | 45 |
| | 30 | .19937 | .80063 | 5.0158 | .20345 | 4.9152 | 1.0205 | .02008 | .97992 | | 30 |
| | 45 | .20364 | .79636 | 4.9106 | .20800 | 4.8077 | 1.0214 | .02095 | .97905 | | 15 |
| 12 | 0 | .20791 | .79209 | 4.8097 | .21256 | 4.7046 | 1.0223 | .02185 | .97815 | 78 | 0 |
| | 15 | .21218 | .78782 | 4.7130 | .21712 | 4.6057 | 1.0233 | .02277 | .97723 | | 45 |
| | 30 | .21644 | .78356 | 4.6202 | .22169 | 4.5107 | 1.0243 | .02370 | .97630 | | 30 |
| | 45 | .22070 | .77930 | 4.5311 | .22628 | 4.4194 | 1.0253 | .02466 | .97534 | | 15 |
| 13 | 0 | .22495 | .77505 | 4.4454 | .23087 | 4.3315 | 1.0263 | .02563 | .97437 | 77 | 0 |
| | 15 | .22920 | .77080 | 4.3630 | .23547 | 4.2468 | 1.0273 | .02662 | .97338 | | 45 |
| | 30 | .23345 | .76655 | 4.2837 | .24008 | 4.1653 | 1.0284 | .02763 | .97237 | | 30 |
| | 45 | .23769 | .76231 | 4.2072 | .24470 | 4.0867 | 1.0295 | .02866 | .97134 | | 15 |
| 14 | 0 | .24192 | .75808 | 4.1336 | .24933 | 4.0108 | 1.0306 | .02970 | .97030 | 76 | 0 |
| | 15 | .24615 | .75385 | 4.0625 | .25397 | 3.9375 | 1.0317 | .03077 | .96923 | | 45 |
| | 30 | .25038 | .74962 | 3.9939 | .25862 | 3.8667 | 1.0329 | .03185 | .96815 | | 30 |
| | 45 | .25460 | .74540 | 3.9277 | .26328 | 3.7983 | 1.0341 | .03295 | .96705 | | 15 |
| 15 | 0 | .25882 | .74118 | 3.8637 | .26795 | 3.7320 | 1.0353 | .03407 | .96593 | 75 | 0 |
| | | Co-sine. | Ver. Sin. | Secant. | Cotan | Tang. | Cosec. | Co-Vers. | Sine. | ° | M. |

From 75° to 90° read from bottom of table upwards.

| ° | M. | Sine. | Co-Vers. | Cosec. | Tang. | Cotan. | Secant. | Ver. Sin. | Cosine. | | |
|----|----|----------|-----------|----------|--------|--------|---------|-----------|---------|----|----|
| 15 | 0 | .25882 | .74118 | 3.8637 | .26795 | 3.7320 | 1.0353 | .03407 | .96593 | 75 | 0 |
| | 15 | .26303 | .73697 | 3.8018 | .27263 | 3.6680 | 1.0365 | .03521 | .96479 | | 45 |
| | 30 | .26724 | .73276 | 3.7420 | .27732 | 3.6059 | 1.0377 | .03637 | .96363 | | 30 |
| | 45 | .27144 | .72856 | 3.6840 | .28203 | 3.5457 | 1.0390 | .03754 | .96246 | 74 | 15 |
| 16 | 0 | .27564 | .72436 | 3.6280 | .28674 | 3.4874 | 1.0403 | .03874 | .96126 | | 0 |
| | 15 | .27983 | .72017 | 3.5736 | .29147 | 3.4308 | 1.0416 | .03995 | .96005 | | 45 |
| | 30 | .28402 | .71598 | 3.5209 | .29621 | 3.3759 | 1.0429 | .04118 | .95882 | | 30 |
| | 45 | .28820 | .71180 | 3.4699 | .30096 | 3.3226 | 1.0443 | .04243 | .95757 | | 15 |
| 17 | 0 | .29237 | .70763 | 3.4203 | .30573 | 3.2709 | 1.0457 | .04370 | .95630 | 73 | 0 |
| | 15 | .29654 | .70346 | 3.3722 | .31051 | 3.2205 | 1.0471 | .04498 | .95502 | | 45 |
| | 30 | .30070 | .69929 | 3.3255 | .31530 | 3.1716 | 1.0485 | .04628 | .95372 | | 30 |
| | 45 | .30486 | .69514 | 3.2801 | .32010 | 3.1240 | 1.0500 | .04760 | .95240 | | 15 |
| 18 | 0 | .30902 | .69098 | 3.2361 | .32492 | 3.0777 | 1.0515 | .04894 | .95106 | 72 | 0 |
| | 15 | .31316 | .68684 | 3.1932 | .32975 | 3.0326 | 1.0530 | .05030 | .94970 | | 45 |
| | 30 | .31730 | .68270 | 3.1515 | .33459 | 2.9887 | 1.0545 | .05168 | .94832 | | 30 |
| | 45 | .32144 | .67856 | 3.1110 | .33945 | 2.9459 | 1.0560 | .05307 | .94693 | | 15 |
| 19 | 0 | .32557 | .67443 | 3.0715 | .34433 | 2.9042 | 1.0576 | .05448 | .94552 | 71 | 0 |
| | 15 | .32969 | .67031 | 3.0331 | .34921 | 2.8636 | 1.0592 | .05591 | .94409 | | 45 |
| | 30 | .33381 | .66619 | 2.9957 | .35412 | 2.8239 | 1.0608 | .05736 | .94264 | | 30 |
| | 45 | .33792 | .66208 | 2.9593 | .35904 | 2.7852 | 1.0625 | .05882 | .94118 | | 15 |
| 20 | 0 | .34202 | .65798 | 2.9238 | .36397 | 2.7475 | 1.0642 | .06031 | .93969 | 70 | 0 |
| | 15 | .34612 | .65388 | 2.8892 | .36892 | 2.7106 | 1.0659 | .06181 | .93819 | | 45 |
| | 30 | .35021 | .64979 | 2.8554 | .37388 | 2.6746 | 1.0676 | .06333 | .93667 | | 30 |
| | 45 | .35429 | .64571 | 2.8225 | .37887 | 2.6395 | 1.0694 | .06486 | .93514 | | 15 |
| 21 | 0 | .35837 | .64163 | 2.7904 | .38386 | 2.6051 | 1.0711 | .06642 | .93358 | 69 | 0 |
| | 15 | .36244 | .63756 | 2.7591 | .38888 | 2.5715 | 1.0729 | .06799 | .93201 | | 45 |
| | 30 | .36650 | .63350 | 2.7285 | .39391 | 2.5386 | 1.0748 | .06958 | .93042 | | 30 |
| | 45 | .37056 | .62944 | 2.6986 | .39896 | 2.5065 | 1.0766 | .07119 | .92881 | | 15 |
| 22 | 0 | .37461 | .62539 | 2.6695 | .40403 | 2.4751 | 1.0785 | .07282 | .92718 | 68 | 0 |
| | 15 | .37865 | .62135 | 2.6410 | .40911 | 2.4443 | 1.0804 | .07446 | .92554 | | 45 |
| | 30 | .38268 | .61732 | 2.6131 | .41421 | 2.4142 | 1.0824 | .07612 | .92388 | | 30 |
| | 45 | .38671 | .61329 | 2.5859 | .41933 | 2.3847 | 1.0844 | .07780 | .92220 | | 15 |
| 23 | 0 | .39073 | .60927 | 2.5593 | .42447 | 2.3559 | 1.0864 | .07950 | .92050 | 67 | 0 |
| | 15 | .39474 | .60526 | 2.5333 | .42963 | 2.3276 | 1.0884 | .08121 | .91879 | | 45 |
| | 30 | .39875 | .60125 | 2.5078 | .43481 | 2.2998 | 1.0904 | .08294 | .91706 | | 30 |
| | 45 | .40275 | .59725 | 2.4829 | .44001 | 2.2727 | 1.0925 | .08469 | .91531 | | 15 |
| 24 | 0 | .40674 | .59326 | 2.4586 | .44523 | 2.2460 | 1.0946 | .08645 | .91355 | 66 | 0 |
| | 15 | .41072 | .58928 | 2.4348 | .45047 | 2.2199 | 1.0968 | .08824 | .91176 | | 45 |
| | 30 | .41469 | .58531 | 2.4114 | .45573 | 2.1943 | 1.0989 | .09004 | .90996 | | 30 |
| | 45 | .41866 | .58134 | 2.3886 | .46101 | 2.1692 | 1.1011 | .09186 | .90814 | | 15 |
| 25 | 0 | .42262 | .57738 | 2.3662 | .46631 | 2.1445 | 1.1034 | .09369 | .90631 | 65 | 0 |
| | 15 | .42657 | .57343 | 2.3443 | .47163 | 2.1203 | 1.1056 | .09554 | .90446 | | 45 |
| | 30 | .43051 | .56949 | 2.3228 | .47697 | 2.0965 | 1.1079 | .09741 | .90259 | | 30 |
| | 45 | .43445 | .56555 | 2.3018 | .48234 | 2.0732 | 1.1102 | .09930 | .90070 | | 15 |
| 26 | 0 | .43837 | .56163 | 2.2812 | .48773 | 2.0503 | 1.1126 | .10121 | .89879 | 64 | 0 |
| | 15 | .44229 | .55771 | 2.2610 | .49314 | 2.0278 | 1.1150 | .10313 | .89687 | | 45 |
| | 30 | .44620 | .55380 | 2.2412 | .49858 | 2.0057 | 1.1174 | .10507 | .89493 | | 30 |
| | 45 | .45010 | .54990 | 2.2217 | .50404 | 1.9840 | 1.1198 | .10702 | .89298 | | 15 |
| 27 | 0 | .45399 | .54601 | 2.2027 | .50952 | 1.9626 | 1.1223 | .10899 | .89101 | 63 | 0 |
| | 15 | .45787 | .54213 | 2.1840 | .51503 | 1.9416 | 1.1248 | .11098 | .88902 | | 45 |
| | 30 | .46175 | .53825 | 2.1657 | .52057 | 1.9210 | 1.1274 | .11299 | .88701 | | 30 |
| | 45 | .46561 | .53439 | 2.1477 | .52612 | 1.9007 | 1.1300 | .11501 | .88499 | | 15 |
| 28 | 0 | .46947 | .53053 | 2.1300 | .53171 | 1.8807 | 1.1326 | .11705 | .88295 | 62 | 0 |
| | 15 | .47332 | .52668 | 2.1127 | .53732 | 1.8611 | 1.1352 | .11911 | .88089 | | 45 |
| | 30 | .47716 | .52284 | 2.0957 | .54295 | 1.8418 | 1.1379 | .12118 | .87882 | | 30 |
| | 45 | .48099 | .51901 | 2.0790 | .54862 | 1.8228 | 1.1406 | .12327 | .87673 | | 15 |
| 29 | 0 | .48481 | .51519 | 2.0627 | .55431 | 1.8040 | 1.1433 | .12538 | .87462 | 61 | 0 |
| | 15 | .48862 | .51138 | 2.0466 | .56003 | 1.7856 | 1.1461 | .12750 | .87250 | | 45 |
| | 30 | .49242 | .50758 | 2.0308 | .56577 | 1.7675 | 1.1490 | .12964 | .87036 | | 30 |
| | 45 | .49622 | .50378 | 2.0152 | .57155 | 1.7496 | 1.1518 | .13180 | .86820 | | 15 |
| 30 | 0 | .50000 | .50000 | 2.0000 | .57735 | 1.7320 | 1.1547 | .13397 | .86603 | 60 | 0 |
| | | Co-sine. | Ver. Sin. | Se-cant. | Cotan. | Tang. | Cosec. | Co-Vers. | Sine. | ° | M. |

From 60° to 75° read from bottom of table upwards.

| ° | M. | Sine. | Co- Vers. | Cosec. | Tang. | Cotan. | Secant. | Ver. Sin. | Cosine | | |
|----|----|--------|--------------|--------------|--------|--------|---------|--------------|--------|----|----|
| 30 | 0 | .50000 | .50000 | 2.0000 | .57735 | 1.7320 | 1.1547 | .13397 | .86603 | 60 | 0 |
| | 15 | .50377 | .49623 | 1.9850 | .58318 | 1.7147 | 1.1576 | .13616 | .86384 | | 45 |
| | 30 | .50754 | .49246 | 1.9703 | .58904 | 1.6977 | 1.1606 | .13837 | .86163 | | 30 |
| | 45 | .51129 | .48871 | 1.9558 | .59494 | 1.6808 | 1.1636 | .14059 | .85941 | | 15 |
| 31 | 0 | .51504 | .48496 | 1.9416 | .60086 | 1.6643 | 1.1666 | .14283 | .85717 | 59 | 0 |
| | 15 | .51877 | .48123 | 1.9276 | .60681 | 1.6479 | 1.1697 | .14509 | .85491 | | 45 |
| | 30 | .52250 | .47750 | 1.9139 | .61280 | 1.6319 | 1.1728 | .14736 | .85264 | | 30 |
| | 45 | .52621 | .47379 | 1.9004 | .61882 | 1.6160 | 1.1760 | .14965 | .85035 | | 15 |
| 32 | 0 | .52992 | .47008 | 1.8871 | .62487 | 1.6003 | 1.1792 | .15195 | .84805 | 58 | 0 |
| | 15 | .53361 | .46639 | 1.8740 | .63095 | 1.5849 | 1.1824 | .15427 | .84573 | | 45 |
| | 30 | .53730 | .46270 | 1.8612 | .63707 | 1.5697 | 1.1857 | .15661 | .84339 | | 30 |
| | 45 | .54097 | .45903 | 1.8485 | .64322 | 1.5547 | 1.1890 | .15896 | .84104 | | 15 |
| 33 | 0 | .54464 | .45536 | 1.8361 | .64941 | 1.5399 | 1.1924 | .16133 | .83867 | 57 | 0 |
| | 15 | .54829 | .45171 | 1.8238 | .65563 | 1.5253 | 1.1958 | .16371 | .83629 | | 45 |
| | 30 | .55194 | .44806 | 1.8118 | .66188 | 1.5108 | 1.1992 | .16611 | .83389 | | 30 |
| | 45 | .55557 | .44443 | 1.7999 | .66818 | 1.4966 | 1.2027 | .16853 | .83147 | | 15 |
| 34 | 0 | .55919 | .44081 | 1.7883 | .67451 | 1.4826 | 1.2062 | .17096 | .82904 | 56 | 0 |
| | 15 | .56280 | .43720 | 1.7768 | .68087 | 1.4687 | 1.2098 | .17341 | .82659 | | 45 |
| | 30 | .56641 | .43359 | 1.7655 | .68728 | 1.4550 | 1.2134 | .17587 | .82413 | | 30 |
| | 45 | .57000 | .43000 | 1.7544 | .69372 | 1.4415 | 1.2171 | .17835 | .82165 | | 15 |
| 35 | 0 | .57358 | .42642 | 1.7434 | .70021 | 1.4281 | 1.2208 | .18085 | .81915 | 55 | 0 |
| | 15 | .57715 | .42285 | 1.7327 | .70673 | 1.4150 | 1.2245 | .18336 | .81664 | | 45 |
| | 30 | .58070 | .41930 | 1.7220 | .71329 | 1.4019 | 1.2283 | .18588 | .81412 | | 30 |
| | 45 | .58425 | .41575 | 1.7116 | .71990 | 1.3891 | 1.2322 | .18843 | .81157 | | 15 |
| 36 | 0 | .58779 | .41221 | 1.7013 | .72654 | 1.3764 | 1.2361 | .19098 | .80902 | 54 | 0 |
| | 15 | .59131 | .40869 | 1.6912 | .73323 | 1.3638 | 1.2400 | .19356 | .80644 | | 45 |
| | 30 | .59482 | .40518 | 1.6812 | .73996 | 1.3514 | 1.2440 | .19614 | .80386 | | 30 |
| | 45 | .59832 | .40168 | 1.6713 | .74673 | 1.3392 | 1.2480 | .19875 | .80125 | | 15 |
| 37 | 0 | .60181 | .39819 | 1.6616 | .75355 | 1.3270 | 1.2521 | .20136 | .79864 | 53 | 0 |
| | 15 | .60529 | .39471 | 1.6521 | .76042 | 1.3151 | 1.2563 | .20400 | .79600 | | 45 |
| | 30 | .60876 | .39124 | 1.6427 | .76733 | 1.3032 | 1.2605 | .20665 | .79335 | | 30 |
| | 45 | .61222 | .38778 | 1.6334 | .77428 | 1.2915 | 1.2647 | .20931 | .79069 | | 15 |
| 38 | 0 | .61566 | .38434 | 1.6243 | .78129 | 1.2799 | 1.2690 | .21199 | .78801 | 52 | 0 |
| | 15 | .61909 | .38091 | 1.6153 | .78834 | 1.2685 | 1.2734 | .21468 | .78532 | | 45 |
| | 30 | .62251 | .37749 | 1.6064 | .79543 | 1.2572 | 1.2778 | .21739 | .78261 | | 30 |
| | 45 | .62592 | .37408 | 1.5976 | .80258 | 1.2460 | 1.2822 | .22012 | .77988 | | 15 |
| 39 | 0 | .62932 | .37068 | 1.5890 | .80978 | 1.2349 | 1.2868 | .22285 | .77715 | 51 | 0 |
| | 15 | .63271 | .36729 | 1.5805 | .81703 | 1.2239 | 1.2913 | .22561 | .77439 | | 45 |
| | 30 | .63608 | .36392 | 1.5721 | .82434 | 1.2131 | 1.2960 | .22838 | .77162 | | 30 |
| | 45 | .63944 | .36056 | 1.5639 | .83169 | 1.2024 | 1.3007 | .23116 | .76884 | | 15 |
| 40 | 0 | .64279 | .35721 | 1.5557 | .83910 | 1.1918 | 1.3054 | .23396 | .76604 | 50 | 0 |
| | 15 | .64612 | .35388 | 1.5477 | .84656 | 1.1812 | 1.3102 | .23677 | .76323 | | 45 |
| | 30 | .64945 | .35055 | 1.5398 | .85408 | 1.1708 | 1.3151 | .23959 | .76041 | | 30 |
| | 45 | .65276 | .34724 | 1.5320 | .86165 | 1.1606 | 1.3200 | .24244 | .75756 | | 15 |
| 41 | 0 | .65606 | .34394 | 1.5242 | .86929 | 1.1504 | 1.3250 | .24529 | .75471 | 49 | 0 |
| | 15 | .65935 | .34065 | 1.5166 | .87698 | 1.1403 | 1.3301 | .24816 | .75184 | | 45 |
| | 30 | .66262 | .33738 | 1.5092 | .88472 | 1.1303 | 1.3352 | .25104 | .74896 | | 30 |
| | 45 | .66588 | .33412 | 1.5018 | .89253 | 1.1204 | 1.3404 | .25394 | .74606 | | 15 |
| 42 | 0 | .66913 | .33087 | 1.4945 | .90040 | 1.1106 | 1.3456 | .25686 | .74314 | 48 | 0 |
| | 15 | .67237 | .32763 | 1.4873 | .90834 | 1.1009 | 1.3509 | .25978 | .74022 | | 45 |
| | 30 | .67559 | .32441 | 1.4802 | .91633 | 1.0913 | 1.3563 | .26272 | .73728 | | 30 |
| | 45 | .67880 | .32120 | 1.4732 | .92439 | 1.0818 | 1.3618 | .26568 | .73432 | | 15 |
| 43 | 0 | .68200 | .31800 | 1.4663 | .93251 | 1.0724 | 1.3673 | .26865 | .73135 | 47 | 0 |
| | 15 | .68518 | .31482 | 1.4595 | .94071 | 1.0630 | 1.3729 | .27163 | .72837 | | 45 |
| | 30 | .68835 | .31165 | 1.4527 | .94896 | 1.0538 | 1.3786 | .27463 | .72537 | | 30 |
| | 45 | .69151 | .30849 | 1.4461 | .95729 | 1.0446 | 1.3843 | .27764 | .72236 | | 15 |
| 44 | 0 | .69466 | .30534 | 1.4396 | .96569 | 1.0355 | 1.3902 | .28066 | .71934 | 46 | 0 |
| | 15 | .69779 | .30221 | 1.4331 | .97416 | 1.0265 | 1.3961 | .28370 | .71630 | | 45 |
| | 30 | .70091 | .29909 | 1.4267 | .98270 | 1.0176 | 1.4020 | .28675 | .71325 | | 30 |
| | 45 | .70401 | .29599 | 1.4204 | .99131 | 1.0088 | 1.4081 | .28981 | .71019 | | 15 |
| 45 | 0 | .70711 | .29289 | 1.4142 | 1.0000 | 1.0000 | 1.4142 | .29289 | .70711 | 45 | 0 |
| | | Cosine | Ver. Sin. | Se- cant. | Cotan. | Tang. | Cosec. | Co- Ver. | Sine. | ° | M. |

From 45° to 60° read from bottom of table upwards.

LOGARITHMIC SINES, ETC.

| Deg. | Sine. | Cosec. | Versin. | Tangent | Cotan. | Covers. | Secant. | Cosine. | Deg. |
|------|---------|-----------|---------|----------|-----------|----------|-----------|----------|------|
| 0 | In.Neg. | Infinite. | In.Neg. | In.Neg. | Infinite. | 10.00000 | 10.00000 | 10.00000 | 90 |
| 1 | 8.24186 | 11.75814 | 6.18271 | 8.24192 | 11.75808 | 9.99235 | 10.00007 | 9.99993 | 89 |
| 2 | 8.54282 | 11.45718 | 6.78474 | 8.54308 | 11.45692 | 9.98457 | 10.00026 | 9.99974 | 88 |
| 3 | 8.71880 | 11.28120 | 7.13687 | 8.71940 | 11.28060 | 9.97665 | 10.00060 | 9.99940 | 87 |
| 4 | 8.84358 | 11.15642 | 7.38667 | 8.84464 | 11.15536 | 9.96860 | 10.00106 | 9.99894 | 86 |
| 5 | 8.94030 | 11.05970 | 7.58039 | 8.94195 | 11.05805 | 9.96040 | 10.00166 | 9.99834 | 85 |
| 6 | 9.01923 | 10.98077 | 7.73863 | 9.02162 | 10.97838 | 9.95205 | 10.00239 | 9.99761 | 84 |
| 7 | 9.08589 | 10.91411 | 7.87238 | 9.08914 | 10.91086 | 9.94356 | 10.00325 | 9.99675 | 83 |
| 8 | 9.14356 | 10.85644 | 7.98820 | 9.14780 | 10.85220 | 9.93492 | 10.00425 | 9.99575 | 82 |
| 9 | 9.19433 | 10.80567 | 8.09032 | 9.19971 | 10.80029 | 9.92612 | 10.00538 | 9.99462 | 81 |
| 10 | 9.23967 | 10.76033 | 8.18162 | 9.24632 | 10.75368 | 9.91717 | 10.00665 | 9.99335 | 80 |
| 11 | 9.28060 | 10.71940 | 8.26418 | 9.28865 | 10.71135 | 9.90805 | 10.00805 | 9.99195 | 79 |
| 12 | 9.31788 | 10.68212 | 8.33950 | 9.32747 | 10.67253 | 9.89877 | 10.00960 | 9.99040 | 78 |
| 13 | 9.35209 | 10.64791 | 8.40875 | 9.36336 | 10.63664 | 9.88933 | 10.01128 | 9.98872 | 77 |
| 14 | 9.38368 | 10.61632 | 8.47282 | 9.39677 | 10.60323 | 9.87971 | 10.01310 | 9.98690 | 76 |
| 15 | 9.41300 | 10.58700 | 8.53243 | 9.42805 | 10.57195 | 9.86992 | 10.01506 | 9.98494 | 75 |
| 16 | 9.44034 | 10.55966 | 8.58814 | 9.45750 | 10.54250 | 9.85996 | 10.01716 | 9.98284 | 74 |
| 17 | 9.46594 | 10.53406 | 8.64043 | 9.48534 | 10.51466 | 9.84981 | 10.01940 | 9.98060 | 73 |
| 18 | 9.48998 | 10.51002 | 8.68969 | 9.51178 | 10.48822 | 9.83947 | 10.02179 | 9.97821 | 72 |
| 19 | 9.51264 | 10.48736 | 8.73625 | 9.53697 | 10.46303 | 9.82894 | 10.02433 | 9.97567 | 71 |
| 20 | 9.53405 | 10.46595 | 8.78037 | 9.56107 | 10.43893 | 9.81821 | 10.02701 | 9.97299 | 70 |
| 21 | 9.55433 | 10.44567 | 8.82230 | 9.58418 | 10.41582 | 9.80729 | 10.02985 | 9.97015 | 69 |
| 22 | 9.57358 | 10.42642 | 8.86223 | 9.60641 | 10.39359 | 9.79615 | 10.03283 | 9.96717 | 68 |
| 23 | 9.59188 | 10.40812 | 8.90034 | 9.62785 | 10.37215 | 9.78481 | 10.03597 | 9.96403 | 67 |
| 24 | 9.60931 | 10.39069 | 8.93679 | 9.64858 | 10.35142 | 9.77325 | 10.03927 | 9.96073 | 66 |
| 25 | 9.62595 | 10.37405 | 8.97170 | 9.66867 | 10.33133 | 9.76146 | 10.04272 | 9.95728 | 65 |
| 26 | 9.64184 | 10.35816 | 9.00521 | 9.68818 | 10.31182 | 9.74945 | 10.04634 | 9.95366 | 64 |
| 27 | 9.65705 | 10.34295 | 9.03740 | 9.70717 | 10.29283 | 9.73720 | 10.05012 | 9.94988 | 63 |
| 28 | 9.67161 | 10.32839 | 9.06838 | 9.72567 | 10.27433 | 9.72471 | 10.05407 | 9.94593 | 62 |
| 29 | 9.68557 | 10.31443 | 9.09823 | 9.74375 | 10.25625 | 9.71197 | 10.05818 | 9.94182 | 61 |
| 30 | 9.69897 | 10.30103 | 9.12702 | 9.76144 | 10.23856 | 9.69897 | 10.06247 | 9.93753 | 60 |
| 31 | 9.71184 | 10.28816 | 9.15483 | 9.77877 | 10.22123 | 9.68571 | 10.06693 | 9.93307 | 59 |
| 32 | 9.72421 | 10.27579 | 9.18171 | 9.79579 | 10.20421 | 9.67217 | 10.07158 | 9.92842 | 58 |
| 33 | 9.73611 | 10.26389 | 9.20771 | 9.81252 | 10.18748 | 9.65836 | 10.07641 | 9.92359 | 57 |
| 34 | 9.74756 | 10.25244 | 9.23290 | 9.82899 | 10.17101 | 9.64425 | 10.08143 | 9.91857 | 56 |
| 35 | 9.75859 | 10.24141 | 9.25731 | 9.84523 | 10.15477 | 9.62984 | 10.08664 | 9.91336 | 55 |
| 36 | 9.76922 | 10.23078 | 9.28099 | 9.86126 | 10.13874 | 9.61512 | 10.09204 | 9.90796 | 54 |
| 37 | 9.77946 | 10.22054 | 9.30398 | 9.87711 | 10.12289 | 9.60008 | 10.09765 | 9.90235 | 53 |
| 38 | 9.78934 | 10.21066 | 9.32631 | 9.89281 | 10.10719 | 9.58471 | 10.10347 | 9.89653 | 52 |
| 39 | 9.79887 | 10.20113 | 9.34802 | 9.90837 | 10.09163 | 9.56900 | 10.10750 | 9.89050 | 51 |
| 40 | 9.80807 | 10.19193 | 9.36913 | 9.92381 | 10.07619 | 9.55293 | 10.111575 | 9.88425 | 50 |
| 41 | 9.81694 | 10.18306 | 9.38968 | 9.93916 | 10.06084 | 9.53648 | 10.12222 | 9.87778 | 49 |
| 42 | 9.82551 | 10.17449 | 9.40969 | 9.95444 | 10.04556 | 9.51966 | 10.12893 | 9.87107 | 48 |
| 43 | 9.83378 | 10.16622 | 9.42918 | 9.96966 | 10.03034 | 9.50243 | 10.13587 | 9.86413 | 47 |
| 44 | 9.84177 | 10.15823 | 9.44818 | 9.98484 | 10.01516 | 9.48479 | 10.14307 | 9.85693 | 46 |
| 45 | 9.84949 | 10.15052 | 9.46671 | 10.00000 | 10.00000 | 9.46671 | 10.15052 | 9.84949 | 45 |
| | Cosine. | Secant. | Covers. | Cotan. | Tangent | Versin. | Cosec. | Sine. | |

From 45° to 90° read from bottom of table upwards.

MATERIALS.

THE CHEMICAL ELEMENTS.

Common Elements (42).

| Chemical Symbol. | Name. | Atomic Weight. | Chemical Symbol. | Name. | Atomic Weight. | Chemical Symbol. | Name. | Atomic Weight. |
|---------------------|----------|-------------------|---------------------|-----------|-------------------|---------------------|------------|-------------------|
| Al | Aluminum | 27.1 | F | Fluorine | 19. | Pd | Palladium | 106.5 |
| Sb | Antimony | 120.2 | Au | Gold | 197.2 | P | Phosphorus | 31. |
| As | Arsenic | 75.0 | H | Hydrogen | 1.01 | Pt | Platinum | 194.8 |
| Ba | Barium | 137.4 | I | Iodine | 127.0 | K | Potassium | 39.1 |
| Bi | Bismuth | 208.5 | Ir | Iridium | 193.0 | Si | Silicon | 28.4 |
| B | Boron | 11.0 | Fe | Iron | 55.9 | Ag | Silver | 107.9 |
| Br | Bromine | 80.0 | Pb | Lead | 206.9 | Na | Sodium | 23. |
| Cd | Cadmium | 112.4 | Li | Lithium | 7.03 | Sr | Strontium | 87.6 |
| Ca | Calcium | 40.1 | Mg | Magnesium | 24.36 | S | Sulphur | 32.1 |
| C | Carbon | 12. | Mn | Manganese | 55. | Sn | Tin | 119. |
| Cl | Chlorine | 35.4 | Hg | Mercury | 200. | Ti | Titanium | 48.1 |
| Cr | Chromium | 52.1 | Ni | Nickel | 58.7 | W | Tungsten | 184.0 |
| Co | Cobalt | 59. | N | Nitrogen | 14.04 | Va | Vanadium | 51.2 |
| Cu | Copper | 63.6 | O | Oxygen | 16. | Zn | Zinc | 65.4 |

The atomic weights of many of the elements vary in the decimal place as given by different authorities. The above are the most recent values referred to O = 16 and H = 1.008. When H is taken as 1, O = 15.879, and the other figures are diminished proportionately. (See *Jour. Am. Chem. Soc.*, March, 1896.)

Rare Elements (27).

| | | | |
|----------------|-----------------|----------------|----------------|
| Beryllium, Be. | Indium, In. | Ruthenium, Ru. | Thallium, Tl. |
| Cæsium, Cs. | Lanthanum, La. | Samarium, Sm. | Thorium, Th. |
| Cerium, Ce. | Molybdenum, Mo. | Scandium, Sc. | Uranium, U. |
| Erbium, Er. | Niobium, Nb. | Selenium, Se. | Ytterbium, Yr. |
| Gallium, Ga. | Osmium, Os. | Tantalum, Ta. | Yttrium, Y. |
| Germanium, Ge. | Rhodium, R. | Tellurium, Te. | Zirconium, Zr. |
| Glucinum, G. | Rubidium, Rb. | Terbium, Tb. | |

Elements recently discovered (1900-1905): Argon, A, 39.9; Krypton, Kr, 81.8; Neon, Ne, 20.0; Xenon, X, 128.0; constituents of the atmosphere, which contains about 1 per cent by volume of Argon, and very small quantities of the others. Helium, He, 4.0; Radium, Ra, 225.0; Gadolinium, Gd, 156.0; Neodymium, Nd, 143.6; Præsdymium, Pr, 140.5; Thulium, Tm, 171.0.

SPECIFIC GRAVITY.

The specific gravity of a substance is its weight as compared with the weight of an equal bulk of pure water.

To find the specific gravity of a substance.

W = weight of body in air; w = weight of body submerged in water.

$$\text{Specific gravity} = \frac{W}{W - w}$$

Given sp. gr. referred to water at 62° F., to find weight per cubic foot multiply by 62.355. Given weight per cubic foot, to find sp. gr. multiply by 0.016037. Given sp. gr., to find weight per cubic inch multiply by 0.036085.

| | Specific Gravity. Range accord- ing to several Authorities. | Specific Grav- ity. Approx. Mean Value, used in Calculation of Weight. | Weight per Cubic Foot, lbs. | Weight per Cubic Inch, lbs. |
|---|---|---|---|---|
| Aluminum | 2.56 to 2.71 | 2.67 | 166.5 | 0.0963 |
| Antimony | 6.66 to 6.86 | 6.76 | 421.6 | 0.2439 |
| Bismuth | 9.74 to 9.90 | 9.82 | 612.4 | 0.3544 |
| Brass: Copper + Zinc. | | | | |
| 80 20 | | 8.60 | 536.3 | 0.3103 |
| 70 30 | 7.8 to 8.6 | 8.40 | 523.8 | 0.3031 |
| 60 40 | | 8.36 | 521.3 | 0.3017 |
| 50 50 | | 8.20 | 511.4 | 0.2959 |
| Bronze {Cop., 95 to 80} {Tin, 5 to 20} | 8.52 to 8.96 | 8.853 | 552. | 0.3195 |
| Cadmium | 8.6 to 8.7 | 8.65 | 539. | 0.3121 |
| Calcium | 1.58 | 1.58 | 98.5 | 0.0570 |
| Chromium | 5.0 | 5.0 | 311.8 | 0.1804 |
| Cobalt | 8.5 to 8.6 | 8.55 | 533.1 | 0.3085 |
| Gold, pure | 19.245 to 19.361 | 19.258 | 1200.9 | 0.6949 |
| Copper | 8.69 to 8.92 | 8.853 | 552. | 0.3195 |
| Iridium | 22.38 to 23. | 22.38 | 1396. | 0.8076 |
| Iron, Cast | 6.85 to 7.48 | 7.218 | 450. | 0.2604 |
| Iron, Wrought | 7.4 to 7.9 | 7.70 | 480. | 0.2779 |
| Lead | 11.07 to 11.44 | 11.38 | 709.7 | 0.4106 |
| Manganese | 7. to 8. | 8. | 499. | 0.2887 |
| Magnesium | 1.69 to 1.75 | 1.75 | 109. | 0.0641 |
| Mercury | 13.60 to 13.62 | 13.62 | 849.3 | 0.4915 |
| | 13.58 | 13.58 | 846.8 | 0.4900 |
| | 13.37 to 13.38 | 13.38 | 834.4 | 0.4828 |
| Nickel | 8.279 to 8.93 | 8.8 | 548.7 | 0.3175 |
| Platinum | 20.33 to 22.07 | 21.5 | 1347.0 | 0.7758 |
| Potassium | 0.865 | 0.865 | 53.9 | 0.0312 |
| Silver | 10.474 to 10.511 | 10.505 | 655.1 | 0.3791 |
| Sodium | 0.97 | 0.97 | 60.5 | 0.0350 |
| Steel | 7.69* to 7.932† | 7.854 | 489.6 | 0.2834 |
| Tin | 7.291 to 7.409 | 7.350 | 458.3 | 0.2652 |
| Titanium | 5.3 | 5.3 | 330.5 | 0.1913 |
| Tungsten | 17. to 17.6 | 17.3 | 1078.7 | 0.6243 |
| Zinc | 6.86 to 7.20 | 7.00 | 436.5 | 0.2526 |

In the first column of figures the lowest are usually those of cast metals, which are more or less porous; the highest are of metals finely rolled or drawn into wire.

Specific Gravity of Liquids at 60° F.

| | | | |
|---------------------------|-------|-------------------|---------------|
| Acid, Muriatic..... | 1.200 | Oil, Olive..... | 0.92 |
| “ Nitric..... | 1.217 | “ Palm..... | 0.97 |
| “ Sulphuric..... | 1.849 | “ Petroleum..... | 0.78 to 0.88 |
| Alcohol, pure..... | 0.794 | “ Rape..... | 0.92 |
| “ 95 per cent..... | 0.816 | “ Turpentine..... | 0.87 |
| “ 50 per cent..... | 0.934 | “ Whale..... | 0.92 |
| Ammonia, 27.9 per cent... | 0.891 | Tar..... | 1. |
| Bromine..... | 2.97 | Vinegar..... | 1.08 |
| Carbon disulphide..... | 1.26 | Water..... | 1. |
| Ether, Sulphuric..... | 0.72 | Water, Sea..... | 1.026 to 1.03 |
| Oil, Linseed..... | 0.94 | | |

Compression of the following Fluids under a Pressure of 15 lbs. per Square Inch.

| | | | |
|--------------|------------|--------------|------------|
| Water..... | 0.00004663 | Ether..... | 0.00006158 |
| Alcohol..... | 0.0000216 | Mercury..... | 0.00000265 |

The Hydrometer.

The hydrometer is an instrument for determining the density of liquids. It is usually made of glass, and consists of three parts: (1) the upper part, a graduated stem or fine tube of uniform diameter; (2) a bulb, or enlargement of the tube, containing air; and (3) a small bulb at the bottom, containing shot or mercury which causes the instrument to float in a vertical position. The graduations are figures representing either specific gravities, or the numbers of an arbitrary scale, as in Baumé's Twaddell's, Beck's, and other hydrometers.

There is a tendency to discard all hydrometers with arbitrary scales and to use only those which read in terms of the specific gravity directly.

Baumé's Hydrometer and Specific Gravities Compared.

Formule { Heavy liquids, Sp. gr. = $145 \div (145 - \text{deg. Be.})$
 { Light liquids, Sp. gr. = $140 \div (130 + \text{deg. Be.})$

| Degrees Baumé | Liquids Heavier than Water, Sp. Gr. | Liquids Lighter than Water, Sp. Gr. | Degrees Baumé | Liquids Heavier than Water, Sp. Gr. | Liquids Lighter than Water, Sp. Gr. | Degrees Baumé | Liquids Heavier than Water, Sp. Gr. | Liquids Lighter than Water, Sp. Gr. |
|---------------|-------------------------------------|-------------------------------------|---------------|-------------------------------------|-------------------------------------|---------------|-------------------------------------|-------------------------------------|
| 0.0 | 1.000 | | 19.0 | 1.151 | 0.940 | 38.0 | 1.355 | 0.833 |
| 1.0 | 1.007 | | 20.0 | 1.160 | 0.933 | 39.0 | 1.368 | 0.828 |
| 2.0 | 1.014 | | 21.0 | 1.169 | 0.927 | 40.0 | 1.381 | 0.824 |
| 3.0 | 1.021 | | 22.0 | 1.179 | 0.921 | 41.0 | 1.394 | 0.819 |
| 4.0 | 1.028 | | 23.0 | 1.189 | 0.915 | 42.0 | 1.408 | 0.814 |
| 5.0 | 1.036 | | 24.0 | 1.198 | 0.909 | 44.0 | 1.436 | 0.805 |
| 6.0 | 1.043 | | 25.0 | 1.208 | 0.903 | 46.0 | 1.465 | 0.796 |
| 7.0 | 1.051 | | 26.0 | 1.219 | 0.897 | 48.0 | 1.495 | 0.787 |
| 8.0 | 1.058 | | 27.0 | 1.229 | 0.892 | 50.0 | 1.526 | 0.778 |
| 9.0 | 1.066 | | 28.0 | 1.239 | 0.886 | 52.0 | 1.559 | 0.769 |
| 10.0 | 1.074 | 1.000 | 29.0 | 1.250 | 0.881 | 54.0 | 1.593 | 0.761 |
| 11.0 | 1.082 | 0.993 | 30.0 | 1.261 | 0.875 | 56.0 | 1.629 | 0.753 |
| 12.0 | 1.090 | 0.986 | 31.0 | 1.272 | 0.870 | 58.0 | 1.667 | 0.745 |
| 13.0 | 1.099 | 0.979 | 32.0 | 1.283 | 0.864 | 60.0 | 1.706 | 0.737 |
| 14.0 | 1.107 | 0.972 | 33.0 | 1.295 | 0.859 | 65.0 | 1.813 | 0.718 |
| 15.0 | 1.115 | 0.966 | 34.0 | 1.306 | 0.854 | 70.0 | 1.933 | 0.700 |
| 16.0 | 1.124 | 0.959 | 35.0 | 1.318 | 0.849 | 75.0 | 2.071 | 0.683 |
| 17.0 | 1.133 | 0.952 | 36.0 | 1.330 | 0.843 | | | |
| 18.0 | 1.142 | 0.946 | 37.0 | 1.343 | 0.838 | | | |

Specific Gravity and Weight of Gases at Atmospheric Pressure and 32° F.

(For other temperatures and pressures see Physical Properties of Gases.)

| | Density, Air = 1. | Density, H = 1. | Grammes per Litre. | Lbs. per Cu. Ft. | Cubic Ft. per Lb. |
|--|----------------------|--------------------|-----------------------|---------------------|----------------------|
| Air..... | 1.0000 | 14.444 | 1.2931 | 0.080728 | 12.388 |
| Oxygen, O..... | 1.1052 | 15.963 | 1.4291 | 0.08921 | 11.209 |
| Hydrogen, H..... | 0.0692 | 1.000 | 0.0895 | 0.00559 | 178.931 |
| Nitrogen, N..... | 0.9701 | 14.012 | 1.2544 | 0.07831 | 12.770 |
| Carbon monoxide, CO.. | 0.9671 | 13.968 | 1.2505 | 0.07807 | 12.810 |
| Carbon dioxide, CO ₂ .. | 1.5197 | 21.950 | 1.9650 | 0.12267 | 8.152 |
| Methane, marsh-gas, CH ₄ | 0.5530 | 7.987 | 0.7150 | 0.04464 | 22.429 |
| Ethylene, C ₂ H ₄ | 0.9674 | 13.973 | 1.2510 | 0.07809 | 12.805 |
| Acetylene, C ₂ H ₂ | 0.8982 | 12.973 | 1.1614 | 0.07251 | 13.792 |
| Ammonia, NH ₃ | 0.5889 | 8.506 | 0.7615 | 0.04754 | 21.036 |
| Water vapor, H ₂ O.... | 0.6218 | 8.981 | 0.8041 | 0.05020 | 19.922 |

Specific Gravity and Weight of Wood.

| | Specific Gravity. | Weight per Cubic Foot, Pounds. | | Specific Gravity. | Weight per Cubic Foot, Pounds. | |
|-------------|-------------------|--------------------------------|----|-------------------|--------------------------------|--|
| | Avg. | | | Avg. | | |
| Alder..... | 0.56 to 0.80 | 0.68 | 42 | Hornbeam.. | 0.76 0.76 47 | |
| Apple..... | 0.73 to 0.79 | 0.76 | 47 | Juniper.... | 0.56 0.56 35 | |
| Ash..... | 0.60 to 0.84 | 0.72 | 45 | Larch..... | 0.56 0.56 35 | |
| Bamboo.... | 0.31 to 0.40 | 0.35 | 22 | Lignum vitae | 0.65 to 1.33 1.00 62 | |
| Beech..... | 0.62 to 0.85 | 0.73 | 46 | Linden ... | 0.604 37 | |
| Birch..... | 0.56 to 0.74 | 0.65 | 41 | Locust..... | 0.728 46 | |
| Box..... | 0.91 to 1.33 | 1.12 | 70 | Mahogany.. | 0.56 to 1.06 0.81 51 | |
| Cedar..... | 0.49 to 0.75 | 0.62 | 39 | Maple..... | 0.57 to 0.79 0.68 42 | |
| Cherry..... | 0.61 to 0.72 | 0.66 | 41 | Mulberry... | 0.56 to 0.90 0.73 46 | |
| Chestnut... | 0.46 to 0.66 | 0.56 | 35 | Oak, Live.. | 0.96 to 1.26 1.11 69 | |
| Cork..... | 0.24 | 0.24 | 15 | Oak, White. | 0.69 to 0.86 0.77 48 | |
| Cypress.... | 0.41 to 0.66 | 0.53 | 33 | Oak, Red .. | 0.73 to 0.75 0.74 46 | |
| Dogwood ... | 0.76 | 0.76 | 47 | Pine, White | 0.35 to 0.55 0.45 28 | |
| Ebony..... | 1.13 to 1.33 | 1.23 | 76 | “ Yellow | 0.46 to 0.76 0.61 38 | |
| Elm..... | 0.55 to 0.78 | 0.61 | 38 | Poplar..... | 0.38 to 0.58 0.48 30 | |
| Fir..... | 0.48 to 0.70 | 0.59 | 37 | Spruce..... | 0.40 to 0.50 0.45 28 | |
| Gum..... | 0.84 to 1.00 | 0.92 | 57 | Sycamore... | 0.59 to 0.62 0.60 37 | |
| Hackmatack | 0.59 | 0.59 | 37 | Teak..... | 0.66 to 0.98 0.82 51 | |
| Hemlock.... | 0.36 to 0.41 | 0.38 | 24 | Walnut | 0.50 to 0.67 0.58 36 | |
| Hickory.... | 0.69 to 0.94 | 0.77 | 48 | Willow | 0.49 to 0.59 0.54 34 | |
| Holly..... | 0.76 | 0.76 | 47 | | | |

Weight and Specific Gravity of Stones, Brick, Cement, etc. (Pure Water = 1.00.)

| | Lb. per Cu. Ft. | Sp. Gr. |
|---------------------------------|-----------------|--------------|
| Asphaltum | 87 | 1.39 |
| Brick, Soft | 100 | 1.6 |
| " Common | 112 | 1.79 |
| " Hard | 125 | 2.0 |
| " Pressed | 135 | 2.16 |
| " Fire | 140 to 150 | 2.24 to 2.4 |
| " Sand-lime | 136 | 2.18 |
| Brickwork in mortar | 100 | 1.6 |
| " cement | 112 | 1.79 |
| Cement, American, natural | ... | 2.8 to 3.2 |
| " Portland | ... | 3.05 to 3.15 |
| " " loose | 92 | |
| " " in barrel | 115 | |
| Clay | 120 to 150 | 1.92 to 2.4 |
| Concrete | 120 to 155 | 1.92 to 2.48 |
| Earth, loose | 72 to 80 | 1.15 to 1.28 |
| " rammed | 90 to 110 | 1.44 to 1.76 |
| Emery | 250 | 4. |
| Glass | 156 to 172 | 2.5 to 2.75 |
| " flint | 180 to 196 | 2.88 to 3.14 |
| Gneiss } | 160 to 170 | 2.56 to 2.72 |
| Granite } | 100 to 120 | 1.6 to 1.92 |
| Gravel | 130 to 150 | 2.08 to 2.4 |
| Gypsum | 200 to 220 | 3.2 to 3.52 |
| Hornblende | 55 to 57 | 0.88 to 0.92 |
| Ice | 50 to 60 | 0.8 to 0.96 |
| Lime, quick, in bulk | 140 to 185 | 2.30 to 2.90 |
| Limestone | 150 | 2.4 |
| Magnesia, Carbonate | 160 to 180 | 2.56 to 2.88 |
| Marble | 140 to 160 | 2.24 to 2.56 |
| Masonry, dry rubble | 140 to 180 | 2.24 to 2.88 |
| " dressed | 175 | 2.80 |
| Mica | 90 to 100 | 1.44 to 1.6 |
| Mortar | 104 to 120 | 1.67 to 1.92 |
| Mud, soft flowing | 72 | 1.15 |
| Pitch | 93 to 113 | 1.50 to 1.81 |
| Plaster of Paris | 165 | 2.64 |
| Quartz | 90 to 110 | 1.44 to 1.76 |
| Sand | 118 to 129 | 1.89 to 2.07 |
| Sand, wet | 140 to 150 | 2.24 to 2.4 |
| Sandstone | 170 to 180 | 2.72 to 2.88 |
| Slate | 166 to 175 | 2.65 to 2.8 |
| Soapstone | 135 to 200 | 2.16 to 3.4 |
| Stone, various | 170 to 200 | 2.72 to 3.4 |
| Trap | 110 to 120 | 1.76 to 1.92 |
| Tile | | |

PROPERTIES OF THE USEFUL METALS.

Aluminum, Al. — Atomic weight 27.1. Specific gravity 2.6 to 2.7. The lightest of all the useful metals except magnesium. A soft, ductile, malleable metal, of a white color, approaching silver, but with a bluish cast. Very non-corrosive. Tenacity about one third that of wrought iron. Formerly a rare metal, but since 1890 its production and use have greatly increased on account of the discovery of cheap processes for reducing it from the ore. Melts at 1215° F. For further description see Aluminum, under Strength of Materials, page 357.

Antimony (Stibium), Sb. — At. wt. 120.2. Sp. gr. 6.7 to 6.8. A brittle metal of a bluish-white color and highly crystalline or laminated structure. Melts at 842° F. Heated in the open air it burns with a bluish-white flame. Its chief use is for the manufacture of certain alloys, as type-metal (antimony 1, lead 4), britannia (antimony 1, tin 9), and various anti-friction metals (see Alloys). Cubical expansion by heat from 32° to 212° F., 0.0070. Specific heat 0.050.

Bismuth, Bi. — At. wt. 208.5. Bismuth is of a peculiar light reddish color, highly crystalline, and so brittle that it can readily be pulverized. It melts at 510° F., and boils at about 2300° F. Sp. gr. 9.823 at 54° F., and 10.055 just above the melting-point. Specific heat about 0.0301 at ordinary temperatures. Coefficient of cubical expansion from 32° to 212°, 0.0040. Conductivity for heat about $\frac{1}{56}$ and for electricity only about $\frac{1}{89}$ of that of silver. Its tensile strength is about 6400 lbs. per square inch. Bismuth expands in cooling, and Tribe has shown that this expansion does not take place until after solidification. Bismuth is the most diamagnetic element known, a sphere of it being repelled by a magnet.

Cadmium, Cd. — At. wt. 112.4. Sp. gr. 8.6 to 8.7. A bluish-white metal, lustrous, with a fibrous fracture. Melts below 500° F. and volatilizes at about 680° F. It is used as an ingredient in some fusible alloys with lead, tin, and bismuth. Cubical expansion from 32° to 212° F., 0.0094.

Copper, Cu. — At. wt. 63.6. Sp. gr. 8.81 to 8.95. Fuses at about 1930° F. Distinguished from all other metals by its reddish color. Very ductile and malleable, and its tenacity is next to iron. Tensile strength 20,000 to 30,000 lbs. per square inch. Heat conductivity 73.6% of that of silver, and superior to that of other metals. Electric conductivity equal to that of gold and silver. Expansion by heat from 32° to 212° F., 0.0051 of its volume. Specific heat 0.093. (See Copper under Strength of Materials; also Alloys.)

Gold (Aurum), Au. — At. wt. 197.2. Sp. gr., when pure and pressed in a die, 19.34. Melts at about 1915° F. The most malleable and ductile of all metals. One ounce Troy may be beaten so as to cover 160 sq. ft. of surface. The average thickness of gold-leaf is $\frac{1}{282000}$ of an inch, or 100 sq. ft. per ounce. One grain may be drawn into a wire 500 ft. in length. The ductility is destroyed by the presence of $\frac{1}{2000}$ part of lead, bismuth, or antimony. Gold is hardened by the addition of silver or of copper. U. S. gold coin is 90 parts gold and 10 parts alloy, which is chiefly copper with a little silver. By jewelers the fineness of gold is expressed in carats, pure gold being 24 carats, three-fourths fine 18 carats, etc.

Iridium, Ir. — Iridium is one of the rarer metals. It has a white lustre, resembling that of steel; its hardness is about equal to that of the ruby; in the cold it is quite brittle, but at white heat it is somewhat malleable. It is one of the heaviest of metals, having a specific gravity of 22.38. It is extremely infusible and almost absolutely inoxidizable.

For uses of iridium, methods of manufacturing it, etc., see paper by W. L. Dudley on the "Iridium Industry," Trans. A. I. M. E., 1884.

Iron (Ferrum), Fe. — At. wt. 55.9. Sp. gr.: Cast, 6.85 to 7.48; Wrought, 7.4 to 7.9. Pure iron is extremely infusible, its melting point being above 3000° F., but its fusibility increases with the addition of carbon, cast iron fusing about 2500° F. Conductivity for heat 11.9, and for electricity 12 to 14.8, silver being 100. Expansion in bulk by heat: cast iron 0.0033, and wrought iron 0.0035, from 32° to 212° F. Specific heat: cast iron 0.1298, wrought iron 0.1138, steel 0.1165. Cast iron exposed to continued heat becomes permanently expanded $1\frac{1}{2}$ to 3 per cent of its length. Grate-bars should therefore be allowed about 4 per cent play. (For other properties see Iron and Steel under Strength of Materials.)

Lead (Plumbum), Pb. — At. wt. 206.9. Sp. gr. 11.07 to 11.44 by different authorities. Melts at about 625° F., softens and becomes pasty at about 617° F. If broken by a sudden blow when just below the melting-point it is quite brittle and the fracture appears crystalline. Lead is very malleable and ductile, but its tenacity is such that it can be drawn into wire with great difficulty. Tensile strength, 1600 to 2400 lbs. per square inch. Its elasticity is very low, and the metal

flows under very slight strain. Lead dissolves to some extent in pure water, but water containing carbonates or sulphates forms over it a film of insoluble salt which prevents further action.

Magnesium, Mg. — At. wt. 24.36. Sp. gr. 1.69 to 1.75. Silver-white, brilliant, malleable, and ductile. It is one of the lightest of metals, weighing only about two thirds as much as aluminum. In the form of filings, wire, or thin ribbons it is highly combustible, burning with a light of dazzling brilliancy, useful for signal-lights and for flash-lights for photographers. It is nearly non-corrosive, a thin film of carbonate of magnesia forming on exposure to damp air, which protects it from further corrosion. It may be alloyed with aluminum, 5 per cent Mg added to Al giving about as much increase of strength and hardness as 10 per cent of copper. Cubical expansion by heat 0.0083, from 32° to 212° F. Melts at 1200° F. Specific heat 0.25.

Manganese, Mn. — At. wt. 55. Sp. gr. 7 to 8. The pure metal is not used in the arts, but alloys of manganese and iron, called spiegeleisen when containing below 25 per cent of manganese, and ferro-manganese when containing from 25 to 90 per cent, are used in the manufacture of steel. Metallic manganese, when alloyed with iron, oxidizes rapidly in the air, and its function in steel manufacture is to remove the oxygen from the bath of steel whether it exists as oxide of iron or as occluded gas.

Mercury (Hydrargyrum), Hg. — At. wt. 199.8. A silver-white metal, liquid at temperatures above - 39° F., and boils at 680° F. Unchangeable as gold, silver, and platinum in the atmosphere at ordinary temperatures, but oxidizes to the red oxide when near its boiling-point. Sp. gr.: when liquid 13.58 to 13.59, when frozen 14.4 to 14.5. Easily tarnished by sulphur fumes, also by dust, from which it may be freed by straining through a cloth. No metal except iron or platinum should be allowed to touch mercury. The smallest portions of tin, lead, zinc, and even copper to a less extent, cause it to tarnish and lose its perfect liquidity. Coefficient of cubical expansion from 32° to 212° F. 0.0182; per deg. 0.000101.

Nickel, Ni. — At. wt. 58.7. Sp. gr. 8.27 to 8.93. A silvery-white metal with a strong lustre, not tarnishing on exposure to the air. Ductile, hard, and as tenacious as iron. It is attracted to the magnet and may be made magnetic like iron. Nickel is very difficult of fusion, melting at about 3000° F. Chiefly used in alloys with copper, as german-silver, nickel-silver, etc., and also in the manufacture of steel to increase its hardness and strength, also for nickel-plating. Cubical expansion from 32° to 212° F., 0.0038. Specific heat 0.109.

Platinum, Pt. — At. wt. 194.8. A whitish steel-gray metal, malleable, very ductile, and as unalterable by ordinary agencies as gold. When fused and refined it is as soft as copper. Sp. gr. 21.15. It is fusible only by the oxyhydrogen blowpipe or in strong electric currents. When combined with iridium it forms an alloy of great hardness, which has been used for gun-vents and for standard weights and measures. The most important uses of platinum in the arts are for vessels for chemical laboratories and manufactories, and for the connecting wires in incandescent electric lamps and for electrical contact points. Cubical expansion from 32° to 212° F., 0.0027, less than that of any other metal except the rare metals, and almost the same as glass.

Silver (Argentum), Ag. — At. wt. 107.9. Sp. gr. 10.1 to 11.1, according to condition and purity. It is the whitest of the metals, very malleable and ductile, and in hardness intermediate between gold and copper. Melts at about 1750° F. Specific heat 0.056. Cubical expansion from 32° to 212° F., 0.0058. As a conductor of electricity it is equal to copper. As a conductor of heat it is superior to all other metals.

Tin (Stannum), Sn. — At. wt. 119. Sp. gr. 7.293. White, lustrous, soft, malleable, of little strength, tenacity about 3500 lbs. per square inch. Fuses at 442° F. Not sensibly volatile when melted at ordinary heats. Heat conductivity 14.5, electric conductivity 12.4; silver being 100 in each case. Expansion of volume by heat 0.0069 from 32° to 212° F. Specific heat 0.055. Its chief uses are for coating of sheet-iron (called tin plate) and for making alloys with copper and other metals.

Zinc, Zn. — At. wt. 65.4. Sp. gr. 7.14. Melts at 780° F. Volatilizes and burns in the air when melted, with bluish-white fumes of zinc oxide. It is ductile and malleable, but to a much less extent than copper, and its tenacity, about 5000 to 6000 lbs. per square inch, is about one tenth that of wrought iron. It is practically non-corrosive in the atmosphere, a thin film of carbonate of zinc forming upon it. Cubical expansion between 32° and 212° F., 0.0088. Specific heat 0.096. Electric conductivity 29, heat conductivity 36, silver being 100. Its principal uses are for coating iron surfaces, called "galvanizing," and for making brass and other alloys.

Table Showing the Order of

| Malleability. | Ductility. | Tenacity. | Infusibility. |
|---------------|------------|-----------|---------------|
| Gold | Platinum | Iron | Platinum |
| Silver | Silver | Copper | Iron |
| Aluminum | Iron | Aluminum | Copper |
| Copper | Copper | Platinum | Gold |
| Tin | Gold | Silver | Silver |
| Lead | Aluminum | Zinc | Aluminum |
| Zinc | Zinc | Gold | Zinc |
| Platinum | Tin | Tin | Lead |
| Iron | Lead | Lead | Tin |

MEASURES AND WEIGHTS OF VARIOUS MATERIALS
(APPROXIMATE).

Brickwork. — Brickwork is estimated by the thousand, and for various thicknesses of wall runs as follows:

| | |
|--|---------------------------------|
| 8 1/4-in. wall, or 1 brick in thickness, | 14 bricks per superficial foot. |
| 12 3/4 " " " 1 1/2 " " " | 21 " " " " |
| 17 " " " 2 " " " | 28 " " " " |
| 21 1/2 " " " 2 1/2 " " " | 35 " " " " |

An ordinary brick measures about 8 1/4 × 4 × 2 inches, which is equal to 66 cubic inches, or 26.2 bricks to a cubic foot. The average weight is 4 1/2 lbs.

Fuel. — A bushel of bituminous coal weighs 76 pounds and contains 2688 cubic inches = 1.554 cubic feet. 29.47 bushels = 1 gross ton.

One acre of bituminous coal contains 1600 tons of 2240 pounds per foot of thickness of coal worked. 15 to 25 per cent must be deducted for waste in mining.

| | |
|--|--------------------|
| 41 to 45 cubic feet bituminous coal when broken down | = 1 ton, 2240 lbs. |
| 34 to 41 " " anthracite prepared for market... | = 1 ton, 2240 lbs. |
| 123 " " " of charcoal..... | = 1 ton, 2240 lbs. |
| 70.9 " " " coke..... | = 1 ton, 2240 lbs. |
| 1 cubic foot of anthracite coal..... | = 55 to 66 lbs. |
| 1 " " " bituminous coal..... | = 50 to 55 lbs. |
| 1 " " Cumberland (semi-bituminous) coal..... | = 53 lbs. |
| 1 " " Cannel coal..... | = 50.3 lbs. |
| 1 " " Charcoal (hardwood)..... | = 18.5 lbs. |
| 1 " " (pine)..... | = 18 lbs. |

A bushel of coke weighs 40 pounds (35 to 42 pounds).

A bushel of charcoal. — In 1881 the American Charcoal-Iron Workers' Association adopted for use in its official publications for the standard bushel of charcoal 2748 cubic inches, or 20 pounds. A ton of charcoal is to be taken at 2000 pounds. This figure of 20 pounds to the bushel was taken as a fair average of different bushels used throughout the country, and it has since been established by law in some States.

Ores, Earths, etc.

| | |
|---|---------------------|
| 13 cubic feet of ordinary gold or silver ore, in mine | = 1 ton = 2000 lbs. |
| 20 " " " broken quartz..... | = 1 ton = 2000 lbs. |
| 18 feet of gravel in bank..... | = 1 ton. |
| 27 cubic feet of gravel when dry..... | = 1 ton. |
| 25 " " " sand..... | = 1 ton. |
| 18 " " " earth in bank..... | = 1 ton. |
| 27 " " " earth when dry..... | = 1 ton. |
| 17 " " " clay..... | = 1 ton. |

Cement.—Portland, per bbl. net, 376 lbs., per bag, net 94 lbs.
Natural, per bbl. net, 282 lbs., per bag net..... 94 lbs.

Lime.—A struck bushel..... 72 to 75 lbs.

Grain.—A struck bushel of wheat = 60 lbs.; of corn = 56 lbs.; of oats = 30 lbs.

Salt.—A struck bushel of salt, coarse, Syracuse, N. Y. = 56 lbs.; Turk's Island = 76 to 80 lbs.

WEIGHT OF RODS, BARS, PLATES, TUBES, AND SPHERES OF DIFFERENT MATERIALS.

Notation: b = breadth, t = thickness, s = side of square, D = external diameter, d = internal diameter, all in inches.

Sectional areas: of square bars = s^2 ; of flat bars = bt ; of round rods = $0.7854 D^2$; of tubes = $0.7854 (D^2 - d^2) = 3.1416 (Dt - t^2)$.

Volume of 1 foot in length: of square bars = $12s^2$; of flat bars = $12bt$; of round bars = $9.4248D^2$; of tubes = $9.4248 (D^2 - d^2) = 37.699 (Dt - t^2)$, in cu. in.

Weight per foot length = volume \times weight per cubic inch of material. Weight of a sphere = $\text{diam.}^3 \times 0.5236 \times$ weight per cubic inch.

| Material. | Specific Gravity. | Weight per Cubic Foot, Lbs. | Weight of Plates 1 Inch Thick per Sq. Ft., Lbs. | Weight of Square Bars per Foot Length, Lbs. | Weight of Flat Bars per Foot Length, Lbs. | Weight per Cubic Inch, Lbs. | Relative Weights. Wrought Iron = 1 | Weight of Round Rod per Foot Length, Lbs. | Weight of Spheres or Balls, Lbs. |
|-----------------------------------|-------------------|-----------------------------|---|---|---|-----------------------------|------------------------------------|---|----------------------------------|
| Cast iron..... | 7.218 | 450. | 37.5 | $s^2 \times \frac{31}{8}$ | $bt \times \frac{31}{8}$ | .2604 | 15-16 | $D^2 \times 2.454$ | $D^3 \times .1363$ |
| Wrought iron..... | 7.7 | 480. | 40. | $\frac{31}{3}$ | $\frac{31}{3}$ | .2779 | 1. | 2.618 | .1455 |
| Steel..... | 7.854 | 489.6 | 40.8 | 3.4 | 3.4 | .2833 | 1.02 | 2.670 | .1484 |
| Copper & Bronze (copper and tin) | 8.855 | 552. | 46. | 3.833 | 3.833 | .3195 | 1.15 | 3.011 | .1673 |
| Brass { 65 copper 35 Zinc... } | 8.393 | 523.2 | 43.6 | 3.633 | 3.633 | .3029 | 1.09 | 2.854 | .1586 |
| Lead..... | 11.38 | 709.6 | 59.1 | 4.93 | 4.93 | .4106 | 1.48 | 3.870 | .2150 |
| Aluminum..... | 2.67 | 166.5 | 13.9 | 1.16 | 1.16 | .0963 | 0.347 | 0.908 | .0504 |
| Glass..... | 2.62 | 163.4 | 13.6 | 1.13 | 1.13 | .0945 | 0.34 | 0.891 | .0495 |
| Pine wood, dry.... | 0.481 | 30.0 | 2.5 | 0.21 | 0.21 | .0174 | 1-16 | 0.164 | .0091 |

Weight per cylindrical in., 1 in. long, = coefficient of D^2 in next to last col. + 12.

For tubes use the coefficient of D^2 in next to last column, as for rods, and multiply it into $(D^2 - d^2)$; or multiply it by 4 $(Dt - t^2)$.

For hollow spheres use the coefficient of D^3 in the last column and multiply it into $(D^3 - d^3)$.

For hexagons multiply the weight of square bars by 0.866 (short diam. of hexagon = side of square). **For octagons** multiply by 0.8284.

COMMERCIAL SIZES OF IRON AND STEEL BARS.

Flats.

| Width. | Thickness. | Width. | Thickness. | Width. | Thickness. |
|--------|---------------|--------|---------------|--------|------------|
| 3/4 | 1/8 to 5/8 | 1 7/8 | 1/2 to 1 1/2 | 4 | 1/4 to 2 |
| 7/8 | 1/8 to 3/4 | 2 | 1/8 to 1 3/4 | 4 1/2 | 1/4 to 2 |
| 1 | 1/8 to 15/16 | 2 1/4 | 1/4 to 1 3/4 | 5 | 1/4 to 2 |
| 1 1/8 | 1/8 to 1 | 2 3/8 | 1/4 to 1 1/8 | 5 1/2 | 1/4 to 2 |
| 1 1/4 | 1/8 to 1 1/8 | 2 1/2 | 3/16 to 1 3/4 | 6 | 1/4 to 2 |
| 1 3/8 | 1/8 to 1 1/8 | 2 5/8 | 1/4 to 1 1/8 | 6 1/2 | 1/4 to 2 |
| 1 1/2 | 1/8 to 1 1/4 | 2 3/4 | 1/4 to 1 1/8 | 7 | 1/4 to 2 |
| 1 5/8 | 1/4 to 1 1/4 | 3 | 1/4 to 2 | 7 1/2 | 1/4 to 2 |
| 1 3/4 | 3/16 to 1 1/2 | 3 1/2 | 1/4 to 2 | | |

Commercial Sizes of Iron and Steel Bars.

Rounds: Iron. 1/4 to 1 3/8 in., advancing by 1/16 in.; 1 3/8 in. to 5 in., advancing by 1/8 in. **Steel.** 1/4 in. to 1 1/8 in., advancing by 1/32 in.; 1 1/8 in. to 2 in., advancing by 1/16 in.; 2 in. to 4 in., advancing by 1/8 in.; 4 to 6 3/4 in., advancing by 1/4 in. Also the following intermediate sizes: 23/64, 25/64, 29/64, 31/64, 33/64, 35/64, 39/64, 47/64, 53/64, 55/64, 63/64, 1 7/64 and 1 15/32 in.

Squares: Iron. 5/16 to 1 1/4 in., advancing by 1/16 in.; 1 1/4 to 3 in., advancing by 1/8 in. **Steel.** 1/4 to 2 in., advancing by 1/16 in.; 2 1/8 in.; 2 1/4 to 4 in., advancing by 1/4 in.; 4 1/2 in.; 5 in.

Half rounds: Iron. 7/16, 1/2, 5/8, 1 1/16, 3/4, 1, 1 1/8, 1 1/4, 1 1/2, 1 3/4, and 2 in. **Steel.** 3/8, 25/64, 13/32, 7/16, 29/64, 15/32, 1/2, 33/64, 17/32, 9/16, 19/32, 5/8, 21/32, 11/16, 23/32, 3/4, 25/32, 13/16, 27/32, 7/8, 29/32, 15/16, 1, 1 1/32, 1 1/8, 1 1/4, 1 3/8, 1 1/2, 1 3/4, 2, 2 1/2, and 3 in. Weights of half rounds, one half of corresponding rounds. See table, page 180.

Ovals: Iron. 1/2 × 1/4, 5/8 × 5/16, 3/4 × 3/8, and 7/8 × 7/16 in. **Steel.** 5/8 × 5/16, 1/2 × 3/8, 17/32 × 9/32, 9/16 × 3/8, 19/32 × 9/32, 3/4 × 5/16, 3/4 × 3/8, 7/8 × 5/16, 7/8 × 7/16, 1 × 1/2, and 1 1/8 × 9/16 in.

Half Ovals: Iron. 1/2 × 1/8, 5/8 × 5/32, 3/4 × 3/16, 7/8 × 7/32, 1 1/2 × 1/2, 1 3/4 × 5/8, 1 7/8 × 5/8 in.

Round Edge Flats: Iron. 1 1/2 × 1/2, 1 3/4 × 5/8, 1 7/8 × 5/8 in. **Steel.** 1 × 3/16, 1 × 1/4, 1 × 5/16, 1 × 3/8, 1 × 7/16, 1 1/4 × 3/16, 1 1/4 × 1/4, 1 1/4 × 5/16, 1 1/4 × 3/8, 1 1/4 × 7/16 in.; 1 1/2 × 1/4 to 1 1/2 × 1 in., advancing by 1/16 in.; 1 3/4 × 1/4 to 1 3/4 × 1 in., advancing by 1/16 in.; 2 × 1/4 to 2 × 1 in., advancing by 1/16 in.; 2 1/4 × 1/4 to 2 1/4 × 1 in., advancing by 1/16 in.; 2 1/2 × 1/4 to 2 1/2 × 1 in., advancing by 1/16 in.; 2 3/4 × 1/4 to 2 3/4 × 1 in., advancing by 1/16 in.; 3 × 1/4 to 3 × 1 in., advancing by 1/16 in.

Bands: Iron. 1/2 to 1 1/8 in., advancing by 1/8 in., 7 to 16 B. W. G.; 1 1/4 to 5 in.; advancing by 1/4 in., 7 to 16 gauge up to 3 in., 4 to 14 gauge, 3 1/4 to 5 in.

WEIGHTS OF SQUARE AND ROUND BARS OF WROUGHT IRON IN POUNDS PER LINEAL FOOT.

Iron weighing 480 lb. per cubic foot. For steel add 2 per cent.

| Thickness or Diameter in Inches. | Weight of Square Bar 1 Ft. Long. | Weight of Round Bar 1 Ft. Long. | Thickness or Diameter in Inches. | Weight of Square Bar 1 Ft. Long. | Weight of Round Bar 1 Ft. Long. | Thickness or Diameter in Inches. | Weight of Square Bar 1 Ft. Long. | Weight of Round Bar 1 Ft. Long. |
|--|--|---------------------------------------|--|--|---------------------------------------|--|--|---------------------------------------|
| 0 | | | | | | | | |
| 1/16 | 0.013 | 0.010 | 11/16 | 24.03 | 18.91 | 3/8 | 96.30 | 75.64 |
| 1/8 | .052 | .041 | 3/4 | 25.21 | 19.80 | 7/16 | 98.55 | 77.40 |
| 3/16 | .117 | .092 | 13/16 | 26.37 | 20.71 | 1/2 | 100.8 | 79.19 |
| 1/4 | .208 | .164 | 7/8 | 27.55 | 21.64 | 5/16 | 103.1 | 81.00 |
| 5/16 | .326 | .256 | 15/16 | 28.76 | 22.59 | 5/8 | 105.5 | 82.83 |
| 3/8 | .469 | .368 | | 30.00 | 23.56 | 11/16 | 107.8 | 84.69 |
| 7/16 | .638 | .501 | 1/16 | 31.26 | 24.55 | 3/4 | 110.2 | 86.56 |
| 1/2 | .833 | .654 | 1/8 | 32.55 | 25.57 | 13/16 | 112.6 | 88.45 |
| 9/16 | 1.055 | .828 | 3/16 | 33.87 | 26.60 | 7/8 | 115.1 | 90.36 |
| 5/8 | 1.302 | 1.023 | 1/4 | 35.21 | 27.65 | 15/16 | 117.5 | 92.29 |
| 11/16 | 1.576 | 1.237 | 5/16 | 36.58 | 28.73 | | 120.0 | 94.25 |
| 3/4 | 1.875 | 1.473 | 3/8 | 37.97 | 29.82 | 1/8 | 125.1 | 98.22 |
| 13/16 | 2.201 | 1.728 | 7/16 | 39.39 | 30.94 | 1/4 | 130.2 | 102.3 |
| 7/8 | 2.552 | 2.004 | 1/2 | 40.83 | 32.07 | 3/8 | 135.5 | 106.4 |
| 15/16 | 2.930 | 2.301 | 9/16 | 42.30 | 33.23 | 1/2 | 140.8 | 110.6 |
| | 3.333 | 2.618 | 5/8 | 43.80 | 34.40 | 5/8 | 146.3 | 114.9 |
| 1 | | | 11/16 | 45.33 | 35.60 | 3/4 | 151.9 | 119.3 |
| 1/16 | 3.763 | 2.955 | 3/4 | 46.88 | 36.82 | 7/8 | 157.6 | 123.7 |
| 1/8 | 4.219 | 3.313 | 13/16 | 48.45 | 38.05 | | 163.3 | 128.3 |
| 3/16 | 4.701 | 3.692 | 7/8 | 50.05 | 39.31 | 1/8 | 169.2 | 132.9 |
| 1/4 | 5.208 | 4.091 | 15/16 | 51.68 | 40.59 | 1/4 | 175.2 | 137.6 |
| 5/16 | 5.742 | 4.510 | | 53.33 | 41.89 | 3/8 | 181.3 | 142.4 |
| 3/8 | 6.302 | 4.950 | 1/16 | 55.01 | 43.21 | 1/2 | 187.5 | 147.3 |
| 7/16 | 6.888 | 5.410 | 1/8 | 56.72 | 44.55 | 5/8 | 193.8 | 152.2 |
| 1/2 | 7.500 | 5.890 | 3/16 | 58.45 | 45.91 | 3/4 | 200.2 | 157.2 |
| 9/16 | 8.138 | 6.392 | 1/4 | 60.21 | 47.29 | 7/8 | 206.7 | 162.4 |
| 5/8 | 8.802 | 6.913 | 5/16 | 61.99 | 48.69 | | 213.3 | 167.6 |
| 11/16 | 9.492 | 7.455 | 3/8 | 63.80 | 50.11 | 1/4 | 226.9 | 178.2 |
| 3/4 | 10.21 | 8.018 | 7/16 | 65.64 | 51.55 | 1/2 | 240.8 | 189.2 |
| 13/16 | 10.95 | 8.601 | 1/2 | 67.50 | 53.01 | 3/4 | 255.2 | 200.4 |
| 7/8 | 11.72 | 9.204 | 9/16 | 69.39 | 54.50 | | 270.0 | 212.1 |
| 15/16 | 12.51 | 9.828 | 5/8 | 71.30 | 56.00 | 1/4 | 285.2 | 224.0 |
| | 13.33 | 10.47 | 11/16 | 73.24 | 57.52 | 1/2 | 300.8 | 236.3 |
| 2 | | | 3/4 | 75.21 | 59.07 | 3/4 | 316.9 | 248.9 |
| 1/16 | 14.18 | 11.14 | 13/16 | 77.20 | 60.63 | | 333.3 | 261.8 |
| 1/8 | 15.05 | 11.82 | 7/8 | 79.22 | 62.22 | 1/4 | 350.2 | 275.1 |
| 3/16 | 15.95 | 12.53 | 15/16 | 81.26 | 63.82 | 1/2 | 367.5 | 288.6 |
| 1/4 | 16.88 | 13.25 | | 83.33 | 65.45 | 3/4 | 385.2 | 302.5 |
| 5/16 | 17.83 | 14.00 | | 85.43 | 67.10 | | 403.3 | 316.8 |
| 3/8 | 18.80 | 14.77 | 1/16 | 87.55 | 68.76 | 1/4 | 421.9 | 331.3 |
| 7/16 | 19.80 | 15.55 | 1/8 | 89.70 | 70.45 | 1/2 | 440.8 | 346.2 |
| 1/2 | 20.83 | 16.36 | 3/16 | 91.88 | 72.16 | 3/4 | 460.2 | 361.4 |
| 9/16 | 21.89 | 17.19 | 1/4 | 94.08 | 73.89 | | 480. | 377. |
| 5/8 | 22.97 | 18.04 | 5/16 | | | 12 | | |

WEIGHT OF IRON AND STEEL SHEETS.**- Weights in Pounds per Square Foot.**

(For weights by the Decimal Gauge, see page 33.)

| Thickness by Birmingham Gauge. | | | | U. S. Standard Gauge, 1893. (See p. 32.) | | | |
|--------------------------------|-----------------------|-------|--------|--|---------------------------|---------|--------|
| No. of Gauge. | Thick-ness in Inches. | Iron. | Steel. | No. of Gauge. | Thick-ness, In. (Approx.) | Iron. | Steel. |
| 0000 | 0.454 | 18.16 | 18.52 | 0000000 | 0.5 | 20. | 20.40 |
| 000 | .425 | 17.00 | 17.34 | 000000 | 0.4688 | 18.75 | 19.125 |
| 00 | .38 | 15.20 | 15.50 | 00000 | 0.4375 | 17.50 | 17.85 |
| 0 | .34 | 13.60 | 13.87 | 0000 | 0.4063 | 16.25 | 16.575 |
| 1 | .3 | 12.00 | 12.24 | .000 | 0.375 | 15. | 15.30 |
| 2 | .284 | 11.36 | 11.59 | 00 | 0.3438 | 13.75 | 14.025 |
| 3 | .259 | 10.36 | 10.57 | 0 | 0.3125 | 12.50 | 12.75 |
| 4 | .238 | 9.52 | 9.71 | 1 | 0.2813 | 11.25 | 11.475 |
| 5 | .22 | 8.80 | 8.98 | 2 | 0.2656 | 10.625 | 10.837 |
| 6 | .203 | 8.12 | 8.28 | 3 | 0.25 | 10. | 10.20 |
| 7 | .18 | 7.20 | 7.34 | 4 | 0.2344 | 9.375 | 9.562 |
| 8 | .165 | 6.60 | 6.73 | 5 | 0.2188 | 8.75 | 8.925 |
| 9 | .148 | 5.92 | 6.04 | 6 | 0.2031 | 8.125 | 8.287 |
| 10 | .134 | 5.36 | 5.47 | 7 | 0.1875 | 7.5 | 7.65 |
| 11 | .12 | 4.80 | 4.90 | 8 | 0.1719 | 6.875 | 7.012 |
| 12 | .109 | 4.36 | 4.45 | 9 | 0.1563 | 6.25 | 6.375 |
| 13 | .095 | 3.80 | 3.88 | 10 | 0.1405 | 5.625 | 5.737 |
| 14 | .083 | 3.32 | 3.39 | 11 | 0.125 | 5. | 5.10 |
| 15 | .072 | 2.88 | 2.94 | 12 | 0.1094 | 4.375 | 4.462 |
| 16 | .065 | 2.60 | 2.65 | 13 | 0.0938 | 3.75 | 3.825 |
| 17 | .058 | 2.32 | 2.37 | 14 | 0.0781 | 3.125 | 3.187 |
| 18 | .049 | 1.96 | 2.00 | 15 | 0.0703 | 2.8125 | 2.869 |
| 19 | .042 | 1.68 | 1.71 | 16 | 0.0625 | 2.5 | 2.55 |
| 20 | .035 | 1.40 | 1.43 | 17 | 0.0563 | 2.25 | 2.295 |
| 21 | .032 | 1.28 | 1.31 | 18 | 0.05 | 2. | 2.04 |
| 22 | .028 | 1.12 | 1.14 | 19 | 0.0438 | 1.75 | 1.785 |
| 23 | .025 | 1.00 | 1.02 | 20 | 0.0375 | 1.50 | 1.53 |
| 24 | .022 | .88 | .898 | 21 | 0.0344 | 1.375 | 1.402 |
| 25 | .02 | .80 | .816 | 22 | 0.0312 | 1.25 | 1.275 |
| 26 | .018 | .72 | .734 | 23 | 0.0281 | 1.125 | 1.147 |
| 27 | .016 | .64 | .653 | 24 | 0.025 | 1. | 1.02 |
| 28 | .014 | .56 | .571 | 25 | 0.0219 | 0.875 | 0.892 |
| 29 | .013 | .52 | .530 | 26 | 0.0188 | 0.75 | 0.765 |
| 30 | .012 | .48 | .490 | 27 | 0.0172 | 0.6875 | 0.701 |
| 31 | .01 | .40 | .408 | 28 | 0.0156 | 0.625 | 0.637 |
| 32 | .009 | .36 | .367 | 29 | 0.0141 | 0.5625 | 0.574 |
| 33 | .008 | .32 | .326 | 30 | 0.0125 | 0.5 | 0.51 |
| 34 | .007 | .28 | .286 | 31 | 0.0109 | 0.4375 | 0.446 |
| 35 | .005 | .20 | .204 | 32 | 0.0102 | 0.40625 | 0.414 |
| 36 | .004 | .16 | .163 | 33 | 0.0094 | 0.375 | 0.382 |
| | | | | 34 | 0.0086 | 0.34375 | 0.351 |
| | | | | 35 | 0.0078 | 0.3125 | 0.319 |
| | | | | 36 | 0.0070 | 0.28125 | 0.287 |
| | | | | 37 | 0.0066 | 0.26562 | 0.271 |
| | | | | 38 | 0.0063 | 0.25 | 0.255 |

| | Iron. | Steel. |
|----------------------------|--------|--------|
| Specific gravity..... | 7.7 | 7.854 |
| Weight per cubic foot..... | 480. | 489.6 |
| Weight per cubic inch..... | 0.2778 | 0.2833 |

As there are many gauges in use differing from each other, and even the thicknesses of a certain specified gauge, as the Birmingham, are not assumed the same by all manufacturers, orders for sheets and wires should always state the weight per square foot, or the thickness in thousandths of an inch.

WEIGHTS OF FLAT ROLLED IRON IN POUNDS PER LINEAL FOOT. **WIDTHS from 1 in. to 12 in.**

Iron weighing 480 lbs. per cubic foot. For steel add 2 per cent.

| Thick- ness in Inches. | 1". | 1 1/4". | 1 1/2". | 1 3/4". | 2". | 2 1/4". | 2 1/2". | 2 3/4". | 3". | 3 1/4". | 3 1/2". | 3 3/4". | 4". | 4 1/4". | 4 1/2". | 4 3/4". |
|------------------------------|-------|---------|---------|---------|-------|---------|---------|---------|-------|---------|---------|---------|-------|---------|---------|---------|
| 1/16 | 0.208 | 0.260 | 0.313 | 0.365 | 0.417 | 0.469 | 0.521 | 0.573 | 0.625 | 0.677 | 0.729 | 0.781 | 0.833 | 0.885 | 0.938 | 0.990 |
| 1/8 | .417 | .521 | .625 | .729 | .833 | .938 | 1.041 | 1.15 | 1.25 | 1.35 | 1.46 | 1.56 | 1.67 | 1.77 | 1.88 | 1.98 |
| 3/16 | .625 | .781 | .938 | 1.09 | 1.25 | 1.41 | 1.56 | 1.72 | 1.88 | 2.03 | 2.19 | 2.34 | 2.50 | 2.66 | 2.81 | 2.97 |
| 1/4 | .833 | 1.04 | 1.25 | 1.46 | 1.67 | 1.88 | 2.08 | 2.29 | 2.50 | 2.71 | 2.92 | 3.13 | 3.33 | 3.54 | 3.75 | 3.96 |
| 5/16 | 1.04 | 1.30 | 1.56 | 1.82 | 2.08 | 2.34 | 2.60 | 2.86 | 3.13 | 3.39 | 3.65 | 3.91 | 4.17 | 4.43 | 4.69 | 4.95 |
| 3/8 | 1.25 | 1.56 | 1.88 | 2.19 | 2.50 | 2.81 | 3.13 | 3.44 | 3.75 | 4.06 | 4.38 | 4.69 | 5.00 | 5.31 | 5.63 | 5.94 |
| 7/16 | 1.46 | 1.82 | 2.19 | 2.55 | 2.92 | 3.28 | 3.65 | 4.01 | 4.38 | 4.74 | 5.10 | 5.47 | 5.83 | 6.20 | 6.56 | 6.93 |
| 1/2 | 1.67 | 2.08 | 2.50 | 2.92 | 3.33 | 3.75 | 4.17 | 4.58 | 5.00 | 5.42 | 5.83 | 6.25 | 6.67 | 7.08 | 7.50 | 7.92 |
| 9/16 | 1.88 | 2.34 | 2.81 | 3.28 | 3.75 | 4.22 | 4.69 | 5.16 | 5.63 | 6.09 | 6.56 | 7.03 | 7.50 | 7.97 | 8.44 | 8.91 |
| 5/8 | 2.08 | 2.60 | 3.13 | 3.65 | 4.17 | 4.69 | 5.21 | 5.73 | 6.25 | 6.77 | 7.29 | 7.81 | 8.33 | 8.85 | 9.38 | 9.90 |
| 11/16 | 2.29 | 2.86 | 3.44 | 4.01 | 4.58 | 5.16 | 5.73 | 6.30 | 6.88 | 7.45 | 8.02 | 8.59 | 9.17 | 9.74 | 10.31 | 10.89 |
| 3/4 | 2.50 | 3.13 | 3.75 | 4.38 | 5.00 | 5.63 | 6.25 | 6.88 | 7.50 | 8.13 | 8.75 | 9.38 | 10.00 | 10.63 | 11.25 | 11.88 |
| 13/16 | 2.71 | 3.39 | 4.06 | 4.74 | 5.42 | 6.09 | 6.77 | 7.45 | 8.13 | 8.80 | 9.48 | 10.16 | 10.83 | 11.51 | 12.19 | 12.86 |
| 7/8 | 2.92 | 3.65 | 4.38 | 5.10 | 5.83 | 6.56 | 7.29 | 8.02 | 8.75 | 9.48 | 10.21 | 10.94 | 11.67 | 12.40 | 13.13 | 13.85 |
| 15/16 | 3.13 | 3.91 | 4.69 | 5.47 | 6.25 | 7.03 | 7.81 | 8.59 | 9.38 | 10.16 | 10.94 | 11.72 | 12.50 | 13.28 | 14.06 | 14.84 |
| 1 | 3.33 | 4.17 | 5.00 | 5.83 | 6.67 | 7.50 | 8.33 | 9.17 | 10.00 | 10.83 | 11.67 | 12.50 | 13.33 | 14.17 | 15.00 | 15.83 |
| 1 1/16 | 3.54 | 4.43 | 5.31 | 6.20 | 7.08 | 7.97 | 8.85 | 9.74 | 10.63 | 11.51 | 12.40 | 13.28 | 14.17 | 15.05 | 15.94 | 16.82 |
| 1 1/8 | 3.75 | 4.69 | 5.63 | 6.56 | 7.50 | 8.44 | 9.38 | 10.31 | 11.25 | 12.19 | 13.13 | 14.06 | 15.00 | 15.94 | 16.88 | 17.81 |
| 1 1/4 | 3.96 | 4.95 | 5.94 | 6.93 | 7.92 | 8.91 | 9.90 | 10.89 | 11.88 | 12.86 | 13.85 | 14.84 | 15.83 | 16.82 | 17.81 | 18.80 |
| 1 1/2 | 4.17 | 5.21 | 6.25 | 7.29 | 8.33 | 9.38 | 10.42 | 11.46 | 12.50 | 13.54 | 14.58 | 15.63 | 16.67 | 17.71 | 18.75 | 19.79 |
| 1 3/8 | 4.37 | 5.47 | 6.56 | 7.66 | 8.75 | 9.84 | 10.94 | 12.03 | 13.13 | 14.22 | 15.31 | 16.41 | 17.50 | 18.59 | 19.69 | 20.78 |
| 1 3/4 | 4.58 | 5.73 | 6.88 | 8.02 | 9.17 | 10.31 | 11.46 | 12.60 | 13.75 | 14.90 | 16.04 | 17.19 | 18.33 | 19.48 | 20.63 | 21.77 |
| 1 7/8 | 4.79 | 5.99 | 7.19 | 8.39 | 9.58 | 10.78 | 11.98 | 13.18 | 14.38 | 15.57 | 16.77 | 17.97 | 19.17 | 20.36 | 21.56 | 22.76 |
| 2 | 5.00 | 6.25 | 7.50 | 8.75 | 10.00 | 11.25 | 12.50 | 13.75 | 15.00 | 16.25 | 17.50 | 18.75 | 20.00 | 21.25 | 22.50 | 23.75 |
| 2 1/16 | 5.21 | 6.51 | 7.81 | 9.11 | 10.42 | 11.72 | 13.02 | 14.32 | 15.63 | 16.93 | 18.23 | 19.53 | 20.83 | 22.14 | 23.44 | 24.74 |
| 2 1/8 | 5.42 | 6.77 | 8.13 | 9.48 | 10.83 | 12.19 | 13.54 | 14.90 | 16.25 | 17.60 | 18.96 | 20.31 | 21.67 | 23.02 | 24.38 | 25.73 |
| 2 1/4 | 5.63 | 7.03 | 8.44 | 9.84 | 11.25 | 12.66 | 14.06 | 15.47 | 16.88 | 18.28 | 19.69 | 21.09 | 22.50 | 23.91 | 25.31 | 26.72 |
| 2 1/2 | 5.83 | 7.29 | 8.75 | 10.21 | 11.67 | 13.13 | 14.58 | 16.04 | 17.50 | 18.96 | 20.42 | 21.88 | 23.33 | 24.79 | 26.25 | 27.71 |
| 2 3/8 | 6.04 | 7.55 | 9.06 | 10.57 | 12.08 | 13.59 | 15.10 | 16.61 | 18.13 | 19.64 | 21.15 | 22.66 | 24.17 | 25.68 | 27.19 | 28.70 |
| 2 3/4 | 6.25 | 7.81 | 9.38 | 10.94 | 12.50 | 14.06 | 15.63 | 17.19 | 18.75 | 20.31 | 21.88 | 23.44 | 25.00 | 26.56 | 28.13 | 29.69 |
| 2 7/8 | 6.46 | 8.07 | 9.69 | 11.30 | 12.92 | 14.53 | 16.15 | 17.76 | 19.38 | 20.99 | 22.60 | 24.22 | 25.83 | 27.45 | 29.06 | 30.68 |
| 3 | 6.67 | 8.33 | 10.00 | 11.67 | 13.33 | 15.00 | 16.67 | 18.33 | 20.00 | 21.67 | 23.33 | 25.00 | 26.67 | 28.33 | 30.00 | 31.67 |

Widths.

| Thick- ness in Inches. | Widths. | | | | | | | | | | | | | | | |
|------------------------------|---------|---------|---------|---------|-------|---------|---------|---------|-------|---------|-------|---------|-------|-------|-------|-------|
| | 5". | 5 1/4". | 5 1/2". | 5 3/4". | 6". | 6 1/4". | 6 1/2". | 6 3/4". | 7". | 7 1/2". | 8". | 8 1/2". | 9". | 10". | 11". | 12". |
| 1/16 | 1.04 | 1.09 | 1.15 | 1.20 | 1.25 | 1.30 | 1.35 | 1.41 | 1.46 | 1.56 | 1.67 | 1.77 | 1.88 | 2.08 | 2.29 | 2.50 |
| 1/8 | 2.08 | 2.19 | 2.29 | 2.40 | 2.50 | 2.60 | 2.71 | 2.81 | 2.92 | 3.13 | 3.33 | 3.54 | 3.75 | 4.17 | 4.58 | 5.00 |
| 3/16 | 3.13 | 3.28 | 3.44 | 3.59 | 3.75 | 3.91 | 4.06 | 4.22 | 4.38 | 4.69 | 5.00 | 5.31 | 5.63 | 6.25 | 6.88 | 7.50 |
| 1/4 | 4.17 | 4.38 | 4.58 | 4.79 | 5.00 | 5.21 | 5.42 | 5.63 | 5.83 | 6.25 | 6.67 | 7.08 | 7.50 | 8.33 | 9.17 | 10.00 |
| 5/16 | 5.21 | 5.47 | 5.73 | 5.99 | 6.25 | 6.51 | 6.77 | 7.03 | 7.29 | 7.81 | 8.33 | 8.85 | 9.38 | 10.42 | 11.46 | 12.50 |
| 3/8 | 6.25 | 6.56 | 6.88 | 7.19 | 7.50 | 7.81 | 8.13 | 8.44 | 8.75 | 9.38 | 10.00 | 10.63 | 11.25 | 12.50 | 13.75 | 15.00 |
| 7/16 | 7.29 | 7.66 | 8.02 | 8.39 | 8.75 | 9.11 | 9.48 | 9.84 | 10.21 | 10.94 | 11.67 | 12.40 | 13.13 | 14.58 | 16.04 | 17.50 |
| 1/2 | 8.33 | 8.75 | 9.17 | 9.58 | 10.00 | 10.42 | 10.83 | 11.25 | 11.67 | 12.50 | 13.33 | 14.17 | 15.00 | 16.67 | 18.33 | 20.00 |
| 9/16 | 9.38 | 9.84 | 10.31 | 10.78 | 11.25 | 11.72 | 12.19 | 12.66 | 13.13 | 14.06 | 15.00 | 15.94 | 16.88 | 18.75 | 20.63 | 22.50 |
| 5/8 | 10.42 | 10.94 | 11.46 | 11.98 | 12.50 | 13.02 | 13.54 | 14.06 | 14.58 | 15.63 | 16.67 | 17.71 | 18.75 | 20.83 | 22.92 | 25.00 |
| 11/16 | 11.46 | 12.03 | 12.60 | 13.18 | 13.75 | 14.32 | 14.90 | 15.47 | 16.04 | 17.19 | 18.33 | 19.48 | 20.63 | 22.92 | 25.21 | 27.50 |
| 3/4 | 12.50 | 13.13 | 13.75 | 14.38 | 15.00 | 15.63 | 16.25 | 16.88 | 17.50 | 18.75 | 20.00 | 21.25 | 22.50 | 25.00 | 27.50 | 30.00 |
| 13/16 | 13.54 | 14.22 | 14.90 | 15.57 | 16.25 | 16.93 | 17.60 | 18.28 | 18.96 | 20.31 | 21.67 | 23.02 | 24.38 | 27.08 | 29.79 | 32.50 |
| 7/8 | 14.58 | 15.31 | 16.04 | 16.77 | 17.50 | 18.23 | 18.96 | 19.69 | 20.42 | 21.88 | 23.33 | 24.79 | 26.25 | 29.17 | 32.08 | 35.00 |
| 15/16 | 15.63 | 16.41 | 17.19 | 17.97 | 18.75 | 19.53 | 20.31 | 21.09 | 21.88 | 23.44 | 25.00 | 26.56 | 28.13 | 31.25 | 34.38 | 37.50 |
| 1 | 16.67 | 17.50 | 18.33 | 19.17 | 20.00 | 20.83 | 21.67 | 22.50 | 23.33 | 25.00 | 26.67 | 28.33 | 30.00 | 33.33 | 36.67 | 40.00 |
| 1 1/8 | 18.75 | 19.69 | 20.63 | 21.56 | 22.50 | 23.44 | 24.38 | 25.31 | 26.25 | 28.13 | 30.00 | 31.88 | 33.75 | 37.50 | 41.25 | 45.00 |
| 1 1/4 | 20.83 | 21.88 | 22.92 | 23.96 | 25.00 | 26.04 | 27.08 | 28.13 | 29.17 | 31.25 | 33.33 | 35.42 | 37.50 | 41.67 | 45.83 | 50.00 |
| 1 3/8 | 22.92 | 24.06 | 25.21 | 26.35 | 27.50 | 28.65 | 29.79 | 30.94 | 32.08 | 34.38 | 36.67 | 38.96 | 41.25 | 45.83 | 50.42 | 55.00 |
| 1 1/2 | 25.00 | 26.25 | 27.50 | 28.75 | 30.00 | 31.25 | 32.50 | 33.75 | 35.00 | 37.50 | 40.00 | 42.50 | 45.00 | 50.00 | 55.00 | 60.00 |
| 1 5/8 | 27.08 | 28.44 | 29.79 | 31.15 | 32.50 | 33.85 | 35.21 | 36.56 | 37.92 | 40.63 | 43.33 | 46.04 | 48.75 | 54.17 | 59.58 | 65.00 |
| 1 3/4 | 29.17 | 30.63 | 32.08 | 33.54 | 35.00 | 36.46 | 37.92 | 39.38 | 40.83 | 43.75 | 46.67 | 49.58 | 52.50 | 58.33 | 64.17 | 70.00 |
| 1 7/8 | 31.25 | 32.81 | 34.38 | 35.94 | 37.50 | 39.06 | 40.63 | 42.19 | 43.75 | 46.88 | 50.00 | 53.13 | 56.25 | 62.50 | 68.75 | 75.00 |
| 2 | 33.33 | 35.00 | 36.67 | 38.33 | 40.00 | 41.67 | 43.33 | 45.00 | 46.67 | 50.00 | 53.33 | 56.67 | 60.00 | 66.67 | 73.33 | 80.00 |

Other sizes. — Weight of other sizes can easily be obtained from the above table by means of combinations or divisions. Thus, for example,

Weight of $12 \times 11/4$ equals weight of 12×1 plus weight of $12 \times 1/4$ 50.00
 Or, twice weight of $12 \times 5/8$, as it is twice as thick 50.00
 Weight of $6 \times 11/16$ equals midway weight between $6 \times 17/8$ and 6×2 38.75
 Weight of $24 \times 15/16$, being twice as wide as $12 \times 15/16$, weighs 75.00

WEIGHTS OF STEEL BLOOMS.

Soft steel. 1 cubic inch = 0.284 lb. 1 cubic foot = 490.75 lbs.

| Size, Inches | Lengths. | | | | | | | | | | | | |
|-----------------|----------|-------|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|
| | 1" | 6" | 12" | 18" | 24" | 30" | 36" | 42" | 48" | 54" | 60" | 66" | |
| 12 | × 6 | 20.45 | 123 | 245 | 363 | 491 | 613 | 736 | 859 | 982 | 1104 | 1227 | 1350 |
| | × 5 | 17.04 | 102 | 204 | 307 | 409 | 511 | 613 | 716 | 818 | 920 | 1022 | 1125 |
| | × 4 | 13.63 | 82 | 164 | 245 | 327 | 409 | 491 | 573 | 654 | 736 | 818 | 900 |
| 11 | × 6 | 18.75 | 113 | 225 | 338 | 450 | 563 | 675 | 788 | 900 | 1013 | 1125 | 1238 |
| | × 5 | 15.62 | 94 | 188 | 281 | 375 | 469 | 562 | 656 | 750 | 843 | 937 | 1031 |
| | × 4 | 12.50 | 75 | 150 | 225 | 300 | 375 | 450 | 525 | 600 | 675 | 750 | 825 |
| 10 | × 8 | 22.72 | 136 | 273 | 409 | 545 | 682 | 818 | 954 | 1091 | 1227 | 1363 | 1500 |
| | × 7 | 19.88 | 120 | 239 | 358 | 477 | 596 | 715 | 835 | 955 | 1074 | 1193 | 1312 |
| | × 6 | 17.04 | 102 | 204 | 307 | 409 | 511 | 613 | 716 | 818 | 920 | 1022 | 1125 |
| | × 5 | 14.20 | 85 | 170 | 256 | 341 | 426 | 511 | 596 | 682 | 767 | 852 | 937 |
| | × 4 | 11.36 | 68 | 136 | 205 | 273 | 341 | 409 | 477 | 546 | 614 | 682 | 750 |
| | × 3 | 8.52 | 51 | 102 | 153 | 204 | 255 | 306 | 358 | 409 | 460 | 511 | 562 |
| 9 | × 8 | 20.45 | 123 | 245 | 368 | 491 | 613 | 736 | 859 | 982 | 1104 | 1227 | 1350 |
| | × 7 | 17.89 | 107 | 215 | 322 | 430 | 537 | 644 | 751 | 859 | 966 | 1073 | 1181 |
| | × 6 | 15.34 | 92 | 184 | 276 | 368 | 460 | 552 | 644 | 736 | 828 | 920 | 1012 |
| | × 5 | 12.78 | 77 | 153 | 230 | 307 | 383 | 460 | 537 | 614 | 690 | 767 | 844 |
| | × 4 | 10.22 | 61 | 123 | 184 | 245 | 307 | 368 | 429 | 490 | 552 | 613 | 674 |
| | × 3 | 7.66 | 46 | 92 | 138 | 184 | 230 | 276 | 322 | 368 | 414 | 460 | 506 |
| 8 | × 8 | 18.18 | 109 | 218 | 327 | 436 | 545 | 655 | 764 | 873 | 982 | 1091 | 1200 |
| | × 7 | 15.9 | 95 | 191 | 286 | 382 | 477 | 572 | 668 | 763 | 859 | 954 | 1049 |
| | × 6 | 13.63 | 82 | 164 | 245 | 327 | 409 | 491 | 573 | 654 | 736 | 818 | 900 |
| | × 5 | 11.36 | 68 | 136 | 205 | 273 | 341 | 409 | 477 | 546 | 614 | 682 | 750 |
| | × 4 | 9.09 | 55 | 109 | 164 | 218 | 273 | 327 | 382 | 436 | 491 | 545 | 600 |
| | × 3 | 6.82 | 41 | 82 | 123 | 164 | 204 | 245 | 286 | 327 | 368 | 409 | 450 |
| 7 | × 7 | 13.92 | 83 | 167 | 251 | 334 | 418 | 501 | 585 | 668 | 752 | 835 | 919 |
| | × 6 | 11.93 | 72 | 143 | 215 | 286 | 358 | 430 | 501 | 573 | 644 | 716 | 788 |
| | × 5 | 9.94 | 60 | 119 | 179 | 238 | 298 | 358 | 417 | 477 | 536 | 596 | 656 |
| | × 4 | 7.95 | 48 | 96 | 143 | 191 | 239 | 286 | 334 | 382 | 429 | 477 | 525 |
| | × 3 | 5.96 | 36 | 72 | 107 | 143 | 179 | 214 | 250 | 286 | 322 | 358 | 393 |
| 6 1/2 × 6 1/2 | × 4 | 12. | 72 | 144 | 216 | 288 | 360 | 432 | 504 | 576 | 648 | 720 | 792 |
| | × 4 | 7.38 | 44 | 89 | 133 | 177 | 221 | 266 | 310 | 354 | 399 | 443 | 487 |
| | × 6 | 10.22 | 61 | 123 | 184 | 245 | 307 | 368 | 429 | 490 | 551 | 613 | 674 |
| | × 5 | 8.52 | 51 | 102 | 153 | 204 | 255 | 307 | 358 | 409 | 460 | 511 | 562 |
| | × 4 | 6.82 | 41 | 82 | 123 | 164 | 204 | 245 | 286 | 327 | 368 | 409 | 450 |
| 5 1/2 × 5 1/2 | × 3 | 5.11 | 31 | 61 | 92 | 123 | 153 | 184 | 214 | 245 | 276 | 307 | 337 |
| | × 4 | 8.59 | 52 | 103 | 155 | 206 | 258 | 309 | 361 | 412 | 464 | 515 | 567 |
| | × 4 | 6.25 | 37 | 75 | 112 | 150 | 188 | 225 | 262 | 300 | 337 | 375 | 412 |
| | × 5 | 7.10 | 43 | 85 | 128 | 170 | 213 | 256 | 298 | 341 | 383 | 426 | 469 |
| | × 4 | 5.68 | 34 | 68 | 102 | 136 | 170 | 205 | 239 | 273 | 307 | 341 | 375 |
| 4 1/2 × 4 1/2 | × 4 | 5.75 | 35 | 69 | 104 | 138 | 173 | 207 | 242 | 276 | 311 | 345 | 380 |
| | × 4 | 5.11 | 31 | 61 | 92 | 123 | 153 | 184 | 215 | 246 | 276 | 307 | 338 |
| | × 4 | 4.54 | 27 | 55 | 82 | 109 | 136 | 164 | 191 | 218 | 246 | 272 | 300 |
| | × 3 1/2 | 3.97 | 24 | 48 | 72 | 96 | 119 | 143 | 167 | 181 | 215 | 238 | 262 |
| | × 3 | 3.40 | 20 | 41 | 61 | 82 | 102 | 122 | 143 | 163 | 184 | 204 | 224 |
| 3 1/2 × 3 1/2 | × 4 | 3.48 | 21 | 42 | 63 | 84 | 104 | 125 | 146 | 167 | 188 | 209 | 230 |
| | × 3 | 2.98 | 18 | 36 | 54 | 72 | 89 | 107 | 125 | 143 | 161 | 179 | 197 |
| | × 3 | 2.56 | 15 | 31 | 46 | 61 | 77 | 92 | 108 | 123 | 138 | 154 | 169 |

SIZES AND WEIGHTS OF ROOFING MATERIALS.

Corrugated Iron or Steel Plates.—Weight per 100 Sq. Ft., Lb.

(American Sheet and Tin Plate Co., 1905.)

SCHEDULE OF WEIGHTS.

| Corrugations. U. S. Std. Sheet Metal Gauge. | 5/8 in. | | 1 1/4 × 3/8 in. | | 2 × 1 1/2 in. | | 2 1/2 × 1 1/2 in. | | 3 × 3/4 in. | | 5 × 7/8 in. | |
|---|----------|------------------|-----------------|------------------|---------------|------------------|-------------------|------------------|-------------|------------------|-------------|------------------|
| | Painted. | Galvan- ized. | Painted. | Galvan- ized. | Painted. | Galvan- ized. | Painted. | Galvan- ized. | Painted. | Galvan- ized. | Painted. | Galvan- ized. |
| 28 | 72 | 87 | 72 | 87 | 68 | 85 | 68 | 85 | 68 | 85 | 68 | 85 |
| 27 | 79 | 94 | 79 | 94 | 76 | 91 | 76 | 91 | 76 | 91 | 76 | 91 |
| 26 | 86 | 101 | 86 | 101 | 83 | 98 | 83 | 98 | 83 | 98 | 83 | 98 |
| 25 | 100 | 115 | 100 | 115 | 96 | 111 | 96 | 111 | 96 | 111 | 96 | 111 |
| 24 | 114 | 129 | 114 | 129 | 110 | 124 | 110 | 124 | 110 | 124 | 110 | 124 |
| 23 | | | 128 | 143 | 123 | 138 | 123 | 138 | 123 | 138 | 123 | 138 |
| 22 | | | 142 | 157 | 136 | 151 | 136 | 151 | 136 | 151 | 136 | 151 |
| 21 | | | 156 | 171 | 150 | 165 | 150 | 165 | 150 | 165 | 150 | 165 |
| 20 | | | 170 | 185 | 163 | 178 | 163 | 178 | 163 | 178 | 163 | 178 |
| 18 | | | | | 217 | 232 | 217 | 232 | 217 | 232 | 217 | 232 |
| 16 | | | | | 271 | 286 | 271 | 286 | 271 | 286 | 271 | 286 |

Covering width of plates, lapped one corrugation, 24 in. Standard lengths, 5, 6, 7, 8, 9, and 10 ft.; maximum length, 12 ft.

Ordinary corrugated sheets should have a lap of 1 1/2 or 2 corrugations side-lap for roofing in order to secure water-tight side seams; if the roof is rather steep 1 1/2 corrugations will answer. Some manufacturers make a special high-edge corrugation on sides of sheets, and thereby are enabled to secure a water-proof side-lap with one corrugation only, thus saving from 6% to 12% of material to cover a given area.

No. 28 gauge corrugated iron is generally used for applying to wooden buildings; but for applying to iron framework No. 24 gauge or heavier should be adopted.

Galvanizing sheet iron adds about 2 1/2 oz. to its weight per square foot.

Corrugated Arches.

For corrugated curved sheets for floor and ceiling construction in fire-proof buildings, No. 16, 18, or 20 gauge iron is commonly used, and sheets may be curved from 4 to 10 in. rise — the higher the rise the stronger the arch. By a series of tests it has been demonstrated that corrugated arches give the most satisfactory results with a base length not exceeding 6 ft., and 5 ft. or even less is preferable where great strength is required. These corrugated arches are made with 1 1/4 × 3/8, 2 1/2 × 1 1/2, 3 × 3/4 and 5 × 7/8 in. corrugations, and in the same width of sheet as above mentioned.

Terra-Cotta.

Porous terra-cotta roofing 3 in. thick weighs 16 lb. per square foot and 2 in. thick, 12 lb. per square foot.

Ceiling made of the same material 2 in. thick weighs 11 lb. per square foot.

Tiles.

Flat tiles 6 1/4 × 10 1/2 × 5/8 in. weigh from 1480 to 1850 lb. per square of roof (100 square feet), the lap being one-half the length of the tile.

Tiles with grooves and fillets weigh from 740 to 925 lb. per square of roof.

Pan-tiles 14 1/2 × 10 1/2 laid 10 in. to the weather weigh 850 lb. per square.

Standard Weights and Gauges of Tin Plates.

American Sheet and Tin Plate Co., Pittsburg, Pa.

| Trade term. | 56 lb. | 60 lb. | 65 lb. | 70 lb. | 75 lb. | 80 lb. | 85 lb. | 90 lb. | 95 lb. | 100 lb. |
|-----------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|
| Nearest wire gauge No. | 38 | 37 | 35 | 35 | 34 | 33 | 32 | 31 | 31 | 30 1/2 |
| Weight per sq. ft., lb. | 0.257 | 0.275 | 0.298 | 0.322 | 0.345 | 0.367 | 0.390 | 0.413 | 0.436 | 0.459 |
| Weight, box, 14×20 in., lb. | 56 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 |

| Trade term. | IC | IXL | IX | IXX | IXXX | IXXXX | IXXXXX |
|-------------------------|-------|-------|-------|-------|-------|-------|--------|
| Nearest wire gauge No. | 30 | 28 | 28 | 27 | 26 | 25 | 24 |
| Weight per sq. ft., lb. | 0.491 | 0.588 | 0.619 | 0.712 | 0.803 | 0.895 | 0.987 |
| Weight, box, 14×20, lb. | 107 | 128 | 135 | 155 | 175 | 195 | 215 |

| Trade term. | DC | DX | DXX | DXXX | DXXXX |
|---|-------|-------|--------|-------|-------|
| Weight, per sq. ft., lb. | 0.637 | 0.826 | 0.962 | 1.10 | 1.23 |
| Nearest equivalent in I plates. | IX | IXXX | IXXXXX | I-6 X | I-7 X |
| 12 1/2 × 17 in. 100 sheets, per box, lbs. | 94 | 122 | 142 | 162 | 182 |
| 17 × 25 in. 50 sheets, per box, lbs. | 94 | 122 | 142 | 162 | 182 |
| 15 × 21 in. 100 sheets, per box, lbs. | 140 | 181 | 211 | 241 | 271 |

Sizes and Net Weight per Box of 100-lb. (0.459 lb. per sq. ft.) Tin Plates.

| Size of Sheets. | Sheets per Box. | Weight per Box. | Size of Sheets. | Sheets per Box. | Weight per Box. | Size of Sheets. | Sheets per Box. | Weight per Box. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 10 × 14 | 225 | 100 | 15 × 15 | 225 | 161 | 14 × 31 | 112 | 155 |
| 14 × 20 | 112 | 100 | 16 × 16 | 225 | 183 | 11 1/4 × 22 3/4 | 112 | 91 |
| 20 × 28 | 112 | 200 | 17 × 17 | 225 | 206 | 13 1/4 × 17 3/4 | 112 | 84 |
| 10 × 20 | 225 | 143 | 18 × 18 | 112 | 116 | 13 1/4 × 19 1/4 | 112 | 91 |
| 11 × 22 | 225 | 172 | 19 × 19 | 112 | 129 | 13 1/2 × 19 1/2 | 112 | 94 |
| 11 1/2 × 23 | 225 | 189 | 20 × 20 | 112 | 143 | 13 1/2 × 19 3/4 | 112 | 95 |
| 12 × 12 | 225 | 103 | 21 × 21 | 112 | 158 | 14 × 18 3/4 | 124 | 103 |
| 12 × 24 | 112 | 103 | 22 × 22 | 112 | 172 | 14 × 19 1/4 | 120 | 103 |
| 13 × 13 | 225 | 121 | 23 × 23 | 112 | 189 | 14 × 21 | 112 | 105 |
| 13 × 26 | 112 | 121 | 24 × 24 | 112 | 204 | 14 × 22 | 112 | 110 |
| 14 × 14 | 225 | 140 | 26 × 26 | 112 | 241 | 14 × 22 1/4 | 112 | 111 |
| 14 × 28 | 112 | 140 | 16 × 20 | 112 | 114 | 15 1/2 × 23 | 112 | 127 |

For weight per box of other than 100-lb. plates, multiply by the figures in the fourth line of the two upper tables, and divide by 100. Thus for IX plates 20 × 28 in., $200 \times 135 \div 100 = 270$.

Tin Plates are made of soft sheet steel coated with tin. The words "charcoal" and "coke" plates are trade terms retained from the time

when high-grade tin plates were made from charcoal iron and lower grade from coke iron (sheet iron made with coke as fuel). The terms are now used to distinguish the percentage of tin coating, and the finish. Coke plates, with light coating, are used for cans. Charcoal plates are designated by letters A to A.A.A.A.A, the latter having the heaviest coating and the highest polish. Plates lighter than 65-lb. per base box (14 × 20 in., 112 plates) are called taggers tin.

Terne Plates, or Roofing Tin, are coated with an alloy of tin and lead. In the "U. S. Eagle, N.M." brand the alloy is 32% tin, 68% lead. The weight per 112 sheets of this brand before and after coating is as follows:

| | IC 14 × 20 | IC 20 × 28 | IX 14 × 20 | IX 20 × 28 |
|---------------|---------------|----------------|----------------|-----------------|
| Black plates | 95 to 100 lb. | 190 to 200 lb. | 125 to 130 lb. | 250 to 260 lb.. |
| After coating | 115 to 120 | 230 to 240 | 145 to 150 | 290 to 300 |

Long terne sheets are made in gauges, Nos. 20 to 30, from 20 to 40 in wide and up to 120 in. long. Continuous roofing tin, 10, 14, 20 and 28 in wide, is made from terne coated sheets 72, 84 and 96 in. long, single lock seam and soldered.

A box of 112 sheets 14 × 20 in. will cover approximately 192 sq. ft. of roof, flat seam, or 583 sheets 1000 sq. ft. For standing seam roofing a sheet 20 × 28 in. will cover 475 sq. in., or 303 sheets 1000 sq. ft. A box of 112 sheets 20 × 28 in. will cover approximately 370 sq. ft.

The common sizes of tin plates are 10 × 14 in. and multiples of that measure. The sizes most generally used are 14 × 20 and 20 × 28 in.

Specifications for Tin and Terne Plate. (Penna. R.R. Co., 1903.)

| | Material Desired. | | |
|-----------------------------------|-------------------|-----------------|-----------------|
| | Tin Plate. | No. 1 Terne. | No. 2 Terne. |
| Kind of coating..... | Pure tin | 26 tin, 74 lead | 16 tin, 84 lead |
| Amount of coating per sq. ft..... | 0.023 lb. | 0.46 lb. | 0.023 lb. |
| Weight per sq. ft. of — | | | |
| Grade IC..... | 0.496 " | 0.519 " | 0.496 " |
| Grade IX..... | 0.625 " | 0.648 " | 0.625 " |
| Grade IXX..... | 0.716 " | 0.739 " | 0.716 " |
| Grade IXXX..... | 0.808 " | 0.831 " | 0.808 " |
| Grade IXXXX..... | 0.900 " | 0.923 " | 0.900 " |
| Will be rejected if less than | | | |
| Amount of coating per sq. ft..... | 0.0183 lb. | 0.0413 lb. | 0.0183 lb. |
| Weight per sq. ft. of — | | | |
| Grade IC..... | 0.468 " | 0.490 " | 0.468 " |
| Grade IX..... | 0.590 " | 0.612 " | 0.590 " |
| Grade IXX..... | 0.676 " | 0.699 " | 0.676 " |
| Grade IXXX..... | 0.763 " | 0.787 " | 0.763 " |
| Grade IXXXX..... | 0.850 " | 0.874 " | 0.850 " |

Each sheet in a shipment of tin or terne plate must (1) be cut as nearly exact to size ordered as possible; (2) must be rectangular and flat and free from flaws; (3) must double seam successfully under reasonable treatment; (4) must show a smooth edge with no sign of fracture when bent through an angle of 180 degrees and flattened down with a wooden mallet; (5) must be so nearly like every other sheet in the shipment, both in thickness and in uniformity and amount of coating, that no difficulty will arise in the shops, due to varying thickness of sheets.

Slate.

Number and superficial area of slate required for one square of roof.
(1 square = 100 square feet.)

| Size, Inches. | Num- ber per Square. | Area in Sq. Ft. | Size, Inches. | Num- ber per Square. | Area in Sq. Ft. | Size, Inches. | Num- ber per Square. | Area in Sq. Ft. |
|------------------|----------------------------|--------------------|------------------|----------------------------|--------------------|------------------|----------------------------|--------------------|
| 6×12 | 533 | 267 | 9×16 | 246 | | 16×20 | 137 | |
| 7×12 | 457 | | 10×16 | 221 | | 12×22 | 126 | 231 |
| 8×12 | 400 | | 9×18 | 213 | 240 | 14×22 | 108 | |
| 9×12 | 355 | | 10×18 | 192 | | 12×24 | 114 | 228 |
| 7×14 | 374 | 254 | 12×18 | 160 | 240 | 14×24 | 98 | |
| 8×14 | 327 | | 10×20 | 169 | 235 | 16×24 | 86 | |
| 9×14 | 291 | | 11×20 | 154 | | 14×26 | 89 | 225 |
| 10×14 | 261 | | 12×20 | 141 | | 16×26 | 78 | |
| 8×16 | 277 | 246 | 14×20 | 121 | | | | |

As slate is usually laid, the number of square feet of roof covered by one slate can be obtained from the following formula:

$$\frac{\text{width} \times (\text{length} - 3 \text{ inches})}{288} = \text{the number of square feet of roof covered.}$$

Weight of slate of various lengths and thicknesses required for one square of roof: based on the number of slate required for one square of roof, taking the weight of a cubic foot of slate at 175 pounds.

| Length in Inches. | Weight in Pounds per Square for the Thickness. | | | | | | | |
|-------------------------|--|----------|---------|---------|---------|---------|---------|-------|
| | 1/8 In. | 3/16 In. | 1/4 In. | 3/8 In. | 1/2 In. | 5/8 In. | 3/4 In. | 1 In. |
| 12 | 483 | 724 | 967 | 1450 | 1936 | 2419 | 2902 | 3872 |
| 14 | 460 | 688 | 920 | 1379 | 1842 | 2301 | 2760 | 3683 |
| 16 | 445 | 667 | 890 | 1336 | 1784 | 2229 | 2670 | 3567 |
| 18 | 434 | 650 | 869 | 1303 | 1740 | 2174 | 2607 | 3480 |
| 20 | 425 | 637 | 851 | 1276 | 1704 | 2129 | 2553 | 3408 |
| 22 | 418 | 626 | 836 | 1254 | 1675 | 2093 | 2508 | 3350 |
| 24 | 412 | 617 | 825 | 1238 | 1653 | 2066 | 2478 | 3306 |
| 26 | 407 | 610 | 815 | 1222 | 1631 | 2039 | 2445 | 3263 |

Pine Shingles.

Number and weight of shingles required to cover one square of roof:

| | | | | | |
|---|-----|-------|-----|-------|-----|
| Inches exposed to weather..... | 4 | 4 1/2 | 5 | 5 1/2 | 6 |
| Number of shingles per square of roof..... | 900 | 800 | 720 | 655 | 600 |
| Weight of shingles on one square, pound.... | 216 | 192 | 173 | 157 | 144 |

The number of shingles per square is for common gable-roofs. For hip-roofs add five per cent to these figures.

Skylight Glass.

The weights of various sizes and thicknesses of fluted or rough plate-glass required for one square of roof.

| Dimensions in Inches. | Thickness in Inches. | Area in Square Feet. | Weight in Lbs. per Square of Roof. |
|-----------------------|----------------------|----------------------|------------------------------------|
| 12× 48 | 3/16 | 3.997 | 250 |
| 15× 60 | 1/4 | 6.246 | 350 |
| 20× 100 | 3/8 | 13.880 | 500 |
| 94× 156 | 1/2 | 101.768 | 700 |

In the above table no allowance is made for lap.

If ordinary window-glass is used, single thick glass (about 1/16 inch) will weigh about 82 lb. per square, and double thick glass (about 1/8 inch) will weigh about 164 lb. per square, *no allowance being made for lap*. A box of ordinary window-glass contains as nearly 50 square feet as the size of the panes will admit of. Panes of any size are made to order by the manufacturers, but a great variety of sizes are usually kept in stock, ranging from 6 × 8 inches to 36 × 60 inches.

APPROXIMATE WEIGHT OF MATERIALS FOR ROOFS.

American Sheet and Tin Plate Co.

| Material. | Average Weight, Lb. per Sq. Ft. |
|--|---------------------------------|
| Corrugated galvanized iron, No. 20, unboarded | 21/4 |
| Copper, 16 oz. standing seam..... | 11/4 |
| Felt and asphalt, without sheathing..... | 2 |
| Glass, 1/8 in. thick..... | 13/4 |
| Hemlock sheathing, 1 in. thick..... | 2 |
| Lead, about 1/8 in. thick..... | 6 to 8 |
| Lath and plaster ceiling (ordinary)..... | 6 to 8 |
| Mackite, 1 in. thick, with plaster..... | 10 |
| Neponset roofing, felt, 2 layers..... | 1/2 |
| Spruce sheathing, 1 in. thick..... | 21/2 |
| Slate, 3/16 in. thick, 3 in. double lap | 63/4 |
| Slate, 1/8 in. thick, 3 in. double lap..... | 41/2 |
| Shingles, 6 in. × 18 in., 1/3 to weather | 2 |
| Skylight of glass, 3/16 to 1/2 in., inc. frame..... | 4 to 10 |
| Slag roof, 4-ply | 4 |
| Terne plate, IC, without sheathing | 1/2 |
| Terne plate, IX, without sheathing..... | 5/8 |
| Tiles (plain), 10 1/2 in. × 6 1/4 in. × 5/8 - 5 1/4 in. to weather | 18 |
| Tiles (Spanish) 14 1/2 in. × 10 1/2 in. - 7 1/4 in. to weather..... | 8 1/2 |
| White pine sheathing, 1 in. thick..... | 2 1/2 |
| Yellow pine sheathing, 1 in. thick | 4 |

WEIGHT OF CAST-IRON PIPES OR COLUMNS.

In Pounds per Lineal Foot.

Cast iron = 450 lbs. per cubic foot.

| Bore. | Thick. of Metal. | Weight per Foot. | Bore. | Thick. of Metal. | Weight per Foot. | Bore. | Thick. of Metal. | Weight per Foot. |
|-------|------------------------|---------------------|--------|------------------------|---------------------|-------|------------------------|---------------------|
| Ins. | Ins. | Lbs. | Ins. | Ins. | Lbs. | Ins. | Ins. | Lbs. |
| 3 | 3/8 | 12.4 | 10 | 3/4 | 79.2 | 22 | 3/4 | 167.5 |
| | 1/2 | 17.2 | 10 1/2 | 1/2 | 54.0 | | 7/8 | 196.5 |
| | 5/8 | 22.2 | | 5/8 | 68.2 | 23 | 3/4 | 174.9 |
| 3 1/2 | 3/8 | 14.3 | | 3/4 | 82.8 | | 7/8 | 205.1 |
| | 1/2 | 19.6 | 11 | 1/2 | 56.5 | | 1 | 235.6 |
| | 5/8 | 25.3 | | 5/8 | 71.3 | 24 | 3/4 | 182.2 |
| 4 | 3/8 | 16.1 | | 3/4 | 86.5 | | 7/8 | 213.7 |
| | 1/2 | 22.1 | 11 1/2 | 1/2 | 58.9 | | 1 | 245.4 |
| | 5/8 | 28.4 | | 5/8 | 74.4 | 25 | 3/4 | 189.6 |
| 4 1/2 | 3/8 | 18.0 | | 3/4 | 90.2 | | 7/8 | 222.3 |
| | 1/2 | 24.5 | 12 | 1/2 | 61.4 | | 1 | 255.3 |
| | 5/8 | 31.5 | | 5/8 | 77.5 | 26 | 3/4 | 197.0 |
| 5 | 3/8 | 19.8 | | 3/4 | 93.9 | | 7/8 | 230.9 |
| | 1/2 | 27.0 | 12 1/2 | 1/2 | 63.8 | | 1 | 265.1 |
| | 5/8 | 34.4 | | 5/8 | 80.5 | 27 | 3/4 | 204.3 |
| 5 1/2 | 3/8 | 21.6 | | 3/4 | 97.6 | | 7/8 | 239.4 |
| | 1/2 | 29.4 | 13 | 1/2 | 66.3 | | 1 | 274.9 |
| | 5/8 | 37.6 | | 5/8 | 83.6 | 28 | 3/4 | 211.7 |
| 6 | 3/8 | 23.5 | | 3/4 | 101.2 | | 7/8 | 248.1 |
| | 1/2 | 31.9 | 14 | 1/2 | 71.2 | | 1 | 284.7 |
| | 5/8 | 40.7 | | 5/8 | 89.7 | 29 | 3/4 | 219.1 |
| 6 1/2 | 3/8 | 25.3 | | 3/4 | 108.6 | | 7/8 | 256.6 |
| | 1/2 | 34.4 | 15 | 5/8 | 95.9 | | 1 | 294.5 |
| | 5/8 | 43.7 | | 3/4 | 116.0 | 30 | 7/8 | 265.2 |
| 7 | 3/8 | 27.2 | | 7/8 | 136.4 | | 1 | 304.3 |
| | 1/2 | 36.8 | 16 | 5/8 | 102.0 | | 1 1/8 | 343.7 |
| | 5/8 | 46.8 | | 3/4 | 123.3 | 31 | 7/8 | 273.8 |
| 7 1/2 | 3/8 | 29.0 | | 7/8 | 145.0 | | 1 | 314.2 |
| | 1/2 | 39.3 | 17 | 5/8 | 108.2 | | 1 1/8 | 354.8 |
| | 5/8 | 49.9 | | 3/4 | 130.7 | 32 | 7/8 | 282.4 |
| 8 | 3/8 | 30.8 | | 7/8 | 153.6 | | 1 | 324.0 |
| | 1/2 | 41.7 | 18 | 5/8 | 114.3 | | 1 1/8 | 365.8 |
| | 5/8 | 52.9 | | 3/4 | 138.1 | 33 | 7/8 | 291.0 |
| 8 1/2 | 1/2 | 44.2 | | 7/8 | 162.1 | | 1 | 333.8 |
| | 5/8 | 56.0 | 19 | 5/8 | 120.4 | | 1 1/8 | 376.9 |
| | 3/4 | 68.1 | | 3/4 | 145.4 | 34 | 7/8 | 299.6 |
| 9 | 1/2 | 46.6 | | 7/8 | 170.7 | | 1 | 343.7 |
| | 5/8 | 59.1 | 20 | 5/8 | 126.6 | | 1 1/8 | 388.0 |
| | 3/4 | 71.8 | | 2 1/4 | 152.8 | 35 | 7/8 | 308.1 |
| 9 1/2 | 1/2 | 49.1 | | 7/8 | 179.3 | | 1 | 353.4 |
| | 5/8 | 62.1 | 21 | 5/8 | 132.7 | | 1 1/8 | 399.0 |
| | 3/4 | 75.5 | | 3/4 | 160.1 | 36 | 7/8 | 316.6 |
| 10 | 1/2 | 51.5 | | 7/8 | 187.9 | | 1 | 363.1 |
| | 5/8 | 65.2 | 22 | 5/8 | 138.8 | | 1 1/8 | 410.0 |

The weight of the two flanges may be reckoned = weight of one foot.

STANDARD THICKNESSES AND WEIGHTS OF CAST-IRON PIPE.

(U. S. Cast-Iron Pipe & Fd'y Co., 1908.)

| Nominal In- side Diam. Ins. | Class A. 100 ft. Head. 43 lb. Pressure. | | | Class B. 200 ft. Head. 86 lb. Pressure. | | |
|--------------------------------|---|---------|--------|---|---------|--------|
| | Thick- ness, Ins. | Wt. per | | Thick- ness, Ins. | Wt. per | |
| | | Ft. | L'gth. | | Ft. | L'gth. |
| 3 | 0.39 | 14.5 | 175 | 0.42 | 16.2 | 194 |
| 4 | .42 | 20.0 | 240 | .45 | 21.7 | 260 |
| 6 | .44 | 30.8 | 370 | .48 | 33.3 | 400 |
| 8 | .46 | 42.9 | 515 | .51 | 47.5 | 570 |
| 10 | .50 | 57.1 | 685 | .57 | 63.8 | 765 |
| 12 | .54 | 72.5 | 870 | .62 | 82.1 | 985 |
| 14 | .57 | 89.6 | 1075 | .66 | 102.5 | 1230 |
| 16 | .60 | 108.3 | 1300 | .70 | 125.0 | 1500 |
| 18 | .64 | 129.2 | 1550 | .75 | 150.0 | 1800 |
| 20 | .67 | 150.0 | 1800 | .80 | 175.0 | 2100 |
| 24 | .76 | 204.2 | 2450 | .89 | 233.3 | 2800 |
| 30 | .88 | 291.7 | 3500 | 1.03 | 333.3 | 4000 |
| 36 | .99 | 391.7 | 4700 | 1.15 | 454.2 | 5450 |
| 42 | 1.10 | 512.5 | 6150 | 1.28 | 591.7 | 7100 |
| 48 | 1.26 | 666.7 | 8000 | 1.42 | 750.0 | 9000 |
| 54 | 1.35 | 800.0 | 9600 | 1.55 | 933.3 | 11200 |
| 60 | 1.39 | 916.7 | 11000 | 1.67 | 1104.2 | 13250 |
| 72 | 1.62 | 1283.4 | 15400 | 1.95 | 1545.8 | 18550 |
| 84 | 1.72 | 1633.4 | 19600 | 2.22 | 2104.2 | 25250 |

| Nominal In- side Diam. Ins. | Class C. 300 ft. Head. 130 lb. Pressure. | | | Class D. 400 ft. Head. 173 lb. Pressure. | | |
|--------------------------------|--|---------|--------|--|---------|--------|
| | Thick- ness, Ins. | Wt. per | | Thick- ness, Ins. | Wt. per | |
| | | Ft. | L'gth. | | Ft. | L'gth. |
| 3 | 0.45 | 17.1 | 205 | 0.48 | 18.0 | 216 |
| 4 | .48 | 23.3 | 280 | .52 | 25.0 | 300 |
| 6 | .51 | 35.8 | 430 | .55 | 38.3 | 460 |
| 8 | .56 | 52.1 | 625 | .60 | 55.8 | 670 |
| 10 | .62 | 70.8 | 850 | .68 | 76.7 | 920 |
| 12 | .68 | 91.7 | 1100 | .75 | 100.0 | 1200 |
| 14 | .74 | 116.7 | 1400 | .82 | 129.2 | 1550 |
| 16 | .80 | 143.8 | 1725 | .89 | 158.3 | 1900 |
| 18 | .87 | 175.0 | 2100 | .96 | 191.7 | 2300 |
| 20 | .92 | 208.3 | 2500 | 1.03 | 229.2 | 2750 |
| 24 | 1.04 | 279.2 | 3350 | 1.16 | 306.7 | 3680 |
| 30 | 1.20 | 400.0 | 4800 | 1.37 | 450.0 | 5400 |
| 36 | 1.36 | 545.8 | 6550 | 1.58 | 625.0 | 7500 |
| 42 | 1.54 | 716.7 | 8600 | 1.78 | 825.0 | 9900 |
| 48 | 1.71 | 908.3 | 10900 | 1.96 | 1050.0 | 12600 |
| 54 | 1.90 | 1141.7 | 13700 | 2.23 | 1341.7 | 16100 |
| 60 | 2.00 | 1341.7 | 16100 | 2.38 | 1583.3 | 19000 |
| 72 | 2.39 | 1904.2 | 22850 | ... | ... | ... |
| 84 | ... | ... | ... | ... | ... | ... |

The above weights are per length to lay 12 feet, including standard sockets; proportionate allowance to be made for any variation.

Standard Thicknesses and Weights of Cast-Iron Pipe.

FOR FIRE-LINES AND OTHER HIGH-PRESSURE SERVICE.

(U. S. Cast-Iron Pipe & Fd'y Co., 1908.)

| Nominal Inside Diam. In. | Class E. 500 ft. Head. 217 lb. | | | | Class F. 600 ft. Head. 260 lb. | | | | Class G. 700 ft. Head. 304 lb. | | | | Class H. 800 ft. Head. 347 lb. | | | |
|--------------------------|--------------------------------------|---------|-------|--|--------------------------------------|---------|-------|--|--------------------------------------|---------|-------|--|--------------------------------------|---------|-------|--|
| | Thick- ness, Ins. | Wt. per | | | Thick- ness, Ins. | Wt. per | | | Thick- ness, Ins. | Wt. per | | | Thick- ness, Ins. | Wt. per | | |
| | | Ft. | L'gth | | | Ft. | L'gth | | | Ft. | L'gth | | | Ft. | L'gth | |
| 6 | 0.58 | 41.7 | 500 | | 0.61 | 43.3 | 520 | | 0.65 | 47.1 | 565 | | 0.69 | 49.6 | 595 | |
| 8 | .66 | 61.7 | 740 | | .71 | 65.7 | 790 | | .75 | 70.8 | 850 | | .80 | 75.0 | 900 | |
| 10 | .74 | 86.3 | 1035 | | .80 | 92.1 | 1105 | | .86 | 100.9 | 1210 | | .92 | 106.7 | 1280 | |
| 12 | .82 | 113.8 | 1365 | | .89 | 122.1 | 1465 | | .97 | 135.4 | 1625 | | 1.04 | 143.8 | 1725 | |
| 14 | .90 | 145.0 | 1740 | | .99 | 157.5 | 1890 | | 1.07 | 174.2 | 2090 | | 1.16 | 186.7 | 2240 | |
| 16 | .98 | 179.6 | 2155 | | 1.08 | 195.4 | 2345 | | 1.18 | 219.2 | 2620 | | 1.27 | 232.5 | 2790 | |
| 18 | 1.07 | 220.4 | 2645 | | 1.17 | 238.4 | 2860 | | 1.28 | 267.1 | 3205 | | 1.39 | 286.7 | 3440 | |
| 20 | 1.15 | 263.0 | 3155 | | 1.27 | 286.3 | 3435 | | 1.39 | 320.8 | 3850 | | 1.51 | 344.6 | 4135 | |
| 24 | 1.31 | 359.6 | 4315 | | 1.45 | 392.9 | 4715 | | ... | ... | ... | | ... | ... | ... | |
| 30 | 1.55 | 521.7 | 6260 | | 1.73 | 585.4 | 7025 | | ... | ... | ... | | ... | ... | ... | |
| 36 | 1.80 | 725.0 | 8700 | | 2.02 | 820.0 | 9840 | | ... | ... | ... | | ... | ... | ... | |

The above weights are per length to lay 12 feet, including standard sockets; proportionate allowance to be made for any variation.

Weight of Underground Pipes. (Adopted by the Natl. Fire Protection Association, 1905). Weights are not to be less than those specified when the normal pressures do not exceed 125 lbs. per sq. in. When the normal pressures are in excess of 125 lbs. heavier pipes should be used. The weights given include sockets.

| | | | | | | | |
|-----------------------------|----|----|----|----|----|-----|-----|
| Pipe, ins. | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| Weights per foot, lbs. | 19 | 32 | 48 | 67 | 87 | 109 | 133 |

THICKNESS OF CAST-IRON WATER-PIPES.

P. H. Baermann, in a paper read before the Engineers' Club of Philadelphia in 1882, gave twenty different formulas for determining the thickness of cast-iron pipes under pressure. The formulas are of three classes:

1. Depending upon the diameter only.
2. Those depending upon the diameter and head, and which add a constant.
3. Those depending upon the diameter and head, contain an additive or subtractive term depending upon the diameter, and add a constant.

The more modern formulas are of the third class, and are as follows:

| | | |
|---|-----------------|--------|
| $t = 0.00008hd + 0.01d + 0.36$ | Shedd, | No. 1. |
| $t = 0.00006hd + 0.0133d + 0.296$ | Warren Foundry, | No. 2. |
| $t = 0.000058hd + 0.0152d + 0.312$ | Francis, | No. 3. |
| $t = 0.000048hd + 0.013d + 0.32$ | Dupuit, | No. 4. |
| $t = 0.00004hd + 0.1\sqrt{d} + 0.15$ | Box, | No. 5. |
| $t = 0.000135hd + 0.4 - 0.0011d$ | Whitman, | No. 6. |
| $t = 0.00006(h + 230)d + 0.333 - 0.0033d$ | Fanning, | No. 7. |
| $t = 0.00015hd + 0.25 - 0.0052d$ | Meggs, | No. 8. |

Safe Pressures, etc., for Cast-iron Pipe. — (Continued.)

Size of Pipe.

| Thickness. | 22" | | 24" | | 27" | | 30" | | 33" | | 36" | | 42" | | 48" | | 60" | |
|------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|------------------|-------------|
| | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. | Pressure in Lbs. | Head in Ft. |
| 11/16..... | 40 | 92 | 30 | 69 | 19 | 64 | | | | | | | | | | | | |
| 3/4..... | 60 | 138 | 49 | 113 | 36 | 83 | 24 | 55 | | | | | | | | | | |
| 13/16..... | 80 | 184 | 68 | 157 | 52 | 120 | 39 | 90 | | | | | | | | | | |
| 7/8..... | 101 | 233 | 86 | 198 | 69 | 159 | 54 | 124 | 42 | 97 | 32 | 74 | | | | | | |
| 15/16..... | 121 | 279 | 105 | 242 | 85 | 196 | 69 | 159 | 55 | 127 | 44 | 101 | | | | | | |
| 1..... | 142 | 327 | 124 | 286 | 102 | 235 | 84 | 194 | 69 | 159 | 57 | 131 | 38 | 88 | 24 | 55 | | |
| 1 1/8..... | 182 | 419 | 161 | 371 | 135 | 311 | 114 | 263 | 96 | 221 | 82 | 189 | 59 | 136 | 43 | 99 | | |
| 1 1/4..... | 224 | 516 | 199 | 458 | 169 | 389 | 144 | 332 | 124 | 286 | 107 | 247 | 81 | 187 | 62 | 143 | 34 | 78 |
| 1 3/8..... | | | 237 | 546 | 202 | 465 | 174 | 401 | 151 | 348 | 132 | 304 | 103 | 237 | 81 | 187 | 49 | 113 |
| 1 1/2..... | | | | | 236 | 544 | 204 | 470 | 178 | 410 | 157 | 362 | 124 | 286 | 99 | 228 | 64 | 147 |
| 1 5/8..... | | | | | | | 234 | 538 | 205 | 472 | 182 | 409 | 145 | 334 | 116 | 272 | 79 | 182 |
| 1 3/4..... | | | | | | | | | 233 | 537 | 207 | 477 | 167 | 385 | 136 | 313 | 94 | 217 |
| 1 7/8..... | | | | | | | | | | | | | 188 | 433 | 155 | 357 | 109 | 251 |
| 2..... | | | | | | | | | | | | | 210 | 484 | 174 | 401 | 124 | 286 |
| 2 1/8..... | | | | | | | | | | | | | | | 193 | 445 | 139 | 320 |
| 2 1/4..... | | | | | | | | | | | | | | | 212 | 488 | 154 | 355 |
| 2 1/2..... | | | | | | | | | | | | | | | | | 184 | 424 |
| 2 3/4..... | | | | | | | | | | | | | | | | | 214 | 482 |

NOTE. — The absolute safe static pressure which may be put upon pipe is given by the formula $P = 2TS/5D$, in which formula P is the pressure per square inch; T , the thickness of the shell; S , the ultimate strength per square inch of the metal in tension; and D , the inside diameter of the pipe. In the tables S is taken as 18,000 pounds per square inch, with a working strain of one-fifth this amount or 3600 pounds per square inch. The formula for the absolute safe static pressure then is: $P = 7200/D$.

It is, however, usual to allow for "water-ram" by increasing the thickness enough to provide for 100 pounds additional static pressure, and, to insure sufficient metal for good casting and for wear and tear, a further increase equal to 0.333 ($1 - 0.01D$).

The expression for the thickness then becomes

$$T = \frac{(P + 100)D}{7200} + 0.333 \left(1 - \frac{D}{100}\right),$$

and for safe working pressure

$$P = \frac{7200}{D} \left(T - 0.333 \left(1 - \frac{D}{100}\right)\right) - 100.$$

The additional section provided as above represents an increased value under static pressure for the different sizes of pipe as follows (see table in margin). So that to test the pipes up to one-fifth of the ultimate strength of the material, the pressures in the marginal table should be added to the pressure-values given in the table above.

| Size of Pipe. | Lbs. |
|---------------|------|
| 4" | 676 |
| 6 | 476 |
| 8 | 346 |
| 10 | 316 |
| 12 | 276 |
| 14 | 248 |
| 16 | 226 |
| 18 | 209 |
| 20 | 196 |
| 22 | 185 |
| 24 | 176 |
| 27 | 165 |
| 30 | 156 |
| 33 | 149 |
| 36 | 143 |
| 42 | 133 |
| 48 | 126 |
| 60 | 116 |

CAST-IRON PIPE-FITTINGS.

Approximate Weights (The Massillon Iron & Steel Co.).

| Inches. | Crosses. | Tees. | Inches. | Crosses. | Tees. | Inches. | Crosses. | Tees. | Inches. | Crosses. | Tees. |
|---------|----------|-------|---------|----------|-------|---------|----------|-------|---------|----------|-------|
| 3×3 | 85 | 65 | 14×14 | 665 | 525 | 20×12 | 900 | 800 | 36×36 | 4160 | 3490 |
| 4×4 | 115 | 90 | 14×10 | 530 | 445 | 20×8 | 730 | 665 | 36×30 | 3475 | 3010 |
| 4×3 | 105 | 85 | 14×6 | 390 | 350 | 20×3 | 565 | 545 | 36×24 | 2920 | 2585 |
| 6×6 | 165 | 130 | 14×3 | 330 | 310 | 24×24 | 1800 | 1565 | 36×20 | 2550 | 2315 |
| 6×3 | 125 | 105 | 16×16 | 810 | 735 | 24×18 | 1480 | 1280 | 36×18 | 2370 | 2175 |
| 8×8 | 290 | 230 | 16×12 | 715 | 615 | 24×14 | 1215 | 1085 | 36×16 | 2240 | 2070 |
| 8×6 | 230 | 195 | 16×8 | 585 | 520 | 24×10 | 1035 | 945 | 36×14 | 2060 | 1930 |
| 8×4 | 205 | 175 | 16×3 | 415 | 395 | 24×6 | 840 | 800 | 36×12 | 1940 | 1835 |
| 8×3 | 185 | 165 | 18×18 | 1055 | 860 | 24×3 | 725 | 705 | 36×10 | 1810 | 1730 |
| 10×10 | 380 | 300 | 18×14 | 865 | 735 | 30×30 | 2850 | 2415 | 36×8 | 1700 | 1635 |
| 10×6 | 280 | 240 | 18×10 | 695 | 610 | 30×20 | 2020 | 1790 | 36×6 | 1555 | 1515 |
| 10×3 | 225 | 205 | 18×6 | 550 | 510 | 30×16 | 1755 | 1585 | 36×4 | 1445 | 1415 |
| 12×12 | 495 | 395 | 18×3 | 455 | 435 | 30×12 | 1475 | 1370 | 36×3 | 1380 | 1360 |
| 12×8 | 405 | 345 | 20×20 | 1335 | 1100 | 30×8 | 1255 | 1190 | | | |
| 12×3 | 275 | 255 | 20×16 | 1100 | 935 | 30×4 | 1030 | 1005 | | | |

These tables are greatly abridged from the original, many intermediate sizes being omitted.

| Inches. | Branches. | | | Inches. | Branches. | | | Inches. | Branches. | | |
|---------|-----------|-----|-----|---------|-----------|------|------|---------|-----------|------|------|
| | 30° | 45° | 60° | | 30° | 45° | 60° | | 30° | 45° | 60° |
| 3×3 | 70 | 70 | 60 | 14×3 | 360 | 295 | 305 | 24×16 | 1865 | 1520 | 1345 |
| 4×4 | 115 | 95 | 85 | 16×16 | 1185 | 910 | 815 | 24×12 | 1500 | 1235 | 1100 |
| 4×3 | 100 | 80 | 75 | 16×12 | 885 | 710 | 635 | 24×8 | 1175 | 1055 | 915 |
| 6×6 | 180 | 145 | 130 | 16×8 | 670 | 560 | 520 | 24×3 | 825 | 770 | 695 |
| 6×3 | 135 | 105 | 100 | 16×3 | 460 | 385 | 385 | 30×30 | 4445 | 3390 | 2905 |
| 8×8 | 310 | 250 | 230 | 18×18 | 1415 | 1080 | 935 | 30×20 | 3005 | 2365 | 2220 |
| 8×6 | 240 | 205 | 190 | 18×14 | 1105 | 865 | 770 | 30×16 | 2475 | 2025 | 1770 |
| 8×3 | 185 | 160 | 150 | 18×10 | 850 | 670 | 635 | 30×12 | 1990 | 1695 | 1495 |
| 10×10 | 450 | 370 | 320 | 18×6 | 630 | 510 | 500 | 30×8 | 1630 | 1400 | 1250 |
| 10×6 | 300 | 255 | 235 | 18×3 | 510 | 435 | 410 | 30×3 | 1180 | 1125 | 960 |
| 10×3 | 235 | 195 | 190 | 20×20 | 1935 | 1455 | 1400 | 36×36 | 6595 | 4565 | 4115 |
| 12×12 | 650 | 545 | 445 | 20×16 | 1550 | 1190 | 1045 | 36×24 | 4405 | 3335 | 2990 |
| 12×8 | 470 | 385 | 345 | 20×12 | 1195 | 935 | 860 | 36×18 | 3370 | 2695 | 2360 |
| 12×3 | 300 | 255 | 240 | 20×8 | 930 | 750 | 690 | 36×14 | 2805 | 2340 | 2050 |
| 14×14 | 830 | 650 | 565 | 20×3 | 635 | 550 | 520 | 36×10 | 2295 | 2040 | 1760 |
| 14×10 | 625 | 505 | 455 | 24×24 | 2795 | 2140 | 1840 | 36×6 | 1860 | 1610 | 1415 |
| 14×6 | 450 | 365 | 355 | 24×18 | 2035 | 1675 | 1450 | 36×3 | 1505 | 1360 | 1245 |

Split Tees.

| | | | | | | | | | | | |
|-----|-----|------|-----|------|-----|------|-----|------|------|------|------|
| 3×3 | 65 | 8×8 | 165 | 12×8 | 275 | 16×8 | 380 | 20×8 | 595 | 30×3 | 1090 |
| 4×4 | 85 | 8×3 | 125 | 12×3 | 235 | 16×3 | 340 | 20×3 | 555 | 36×8 | 1460 |
| 6×6 | 115 | 10×8 | 220 | 14×8 | 325 | 18×8 | 485 | 24×8 | 780 | 36×3 | 1420 |
| 6×3 | 100 | 10×3 | 180 | 14×3 | 285 | 18×3 | 445 | 30×8 | 1130 | | |

Split Sleeves.

| Inches. | Branches. | | | Inches. | Branches. | | | Inches. | Branches. | | |
|---------|-----------|-----|-----|---------|-----------|-----|-----|---------|-----------|-----|------|
| | 30° | 45° | 60° | | 30° | 45° | 60° | | 30° | 45° | 60° |
| 3 | 50 | 8 | 110 | 12 | 220 | 16 | 325 | 20 | 540 | 30 | 1075 |
| 4 | 60 | 10 | 165 | 14 | 270 | 18 | 430 | 24 | 725 | 36 | 1405 |
| 6 | 85 | | | | | | | | | | |

Taper Plugs.

| | | | | | | | | | | | |
|---|----|----|----|----|----|----|-----|----|-----|----|-----|
| 3 | 7 | 8 | 25 | 12 | 60 | 16 | 95 | 20 | 170 | 30 | 430 |
| 4 | 10 | 10 | 40 | 14 | 80 | 18 | 135 | 24 | 260 | 36 | 600 |
| 6 | 20 | | | | | | | | | | |

Reducers.

| | | | | | | | | | | | |
|-----|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|------|
| 4×3 | 50 | 10×8 | 175 | 14×12 | 295 | 16×6 | 220 | 20×14 | 430 | 30×24 | 865 |
| 6×4 | 70 | 10×3 | 115 | 14×8 | 220 | 18×16 | 435 | 20×10 | 380 | 30×18 | 825 |
| 6×3 | 60 | 12×10 | 230 | 14×4 | 165 | 18×12 | 345 | 24×20 | 645 | 36×30 | 1455 |
| 8×6 | 100 | 12×6 | 165 | 16×14 | 355 | 18×8 | 280 | 24×16 | 555 | 36×18 | 950 |
| 8×3 | 55 | 12×3 | 140 | 16×10 | 290 | 20×18 | 520 | 24×12 | 485 | | |

Increasesers.

| | | | | | | | | | | | |
|------|-----|------|-----|-------|-----|-------|-----|-------|------|-------|------|
| 3×4 | 55 | 4×14 | 210 | 8×14 | 235 | 12×14 | 300 | 16×18 | 455 | 20×36 | 1310 |
| 3×8 | 95 | 6×8 | 115 | 8×18 | 320 | 12×18 | 385 | 16×30 | 965 | 24×30 | 940 |
| 3×12 | 165 | 6×12 | 195 | 10×12 | 245 | 12×24 | 510 | 18×20 | 485 | 24×36 | 1380 |
| 4×6 | 75 | 6×16 | 275 | 10×16 | 335 | 14×16 | 375 | 18×36 | 1220 | 30×38 | 1475 |
| 4×10 | 150 | 8×10 | 190 | 10×20 | 410 | 14×24 | 600 | 20×24 | 700 | | |

| Size. | Elbows. | 45° bends. | 22½° bends. | Shoe elbows. | S pipes. | Drip boxes. | 90° Y pipes. | 60° Y pipes. | 45° Y pipes. | 30° Y pipes. | Caps. | Sleeves. |
|-------|---------|------------|-------------|--------------|----------|-------------|--------------|--------------|--------------|--------------|-------|----------|
| 3 | 50 | 40 | 35 | 75 | 50 | 355 | 50 | 55 | 65 | 70 | 20 | 35 |
| 4 | 60 | 50 | 45 | 105 | 70 | 370 | 70 | 75 | 85 | 95 | 25 | 45 |
| 6 | 95 | 70 | 60 | 145 | 115 | 395 | 95 | 105 | 120 | 125 | 40 | 60 |
| 8 | 155 | 115 | 100 | 210 | 190 | 450 | 160 | 175 | 205 | 210 | 60 | 75 |
| 10 | 215 | 160 | 130 | 360 | 295 | 485 | 200 | 235 | 270 | 290 | 85 | 100 |
| 12 | 290 | 210 | 170 | 450 | 420 | 575 | 265 | 300 | 380 | 425 | 110 | 125 |
| 14 | 355 | 260 | 210 | 595 | 500 | 690 | 320 | 360 | 480 | 520 | 145 | 150 |
| 16 | 495 | 355 | 280 | 640 | 775 | 890 | 415 | 500 | 615 | 755 | 165 | 175 |
| 18 | 575 | 405 | 320 | 880 | 910 | 1080 | 475 | 565 | 735 | 875 | 235 | 200 |
| 20 | 745 | 515 | 410 | 1160 | 1195 | 1190 | 580 | 725 | 950 | 1135 | 290 | 240 |
| 24 | 1040 | 715 | 555 | 1590 | 1680 | 1785 | 830 | 1000 | 1330 | 1600 | 435 | 345 |
| 30 | 1580 | 1060 | 800 | 2450 | 2345 | 2410 | 1145 | 1470 | 2005 | 2270 | 680 | 475 |
| 36 | 2230 | 1490 | 1120 | 3540 | 3495 | 3225 | 1600 | 2070 | 2720 | 3315 | 1015 | 630 |

STANDARD PIPE FLANGES (CAST IRON).

Adopted August, 1894, at a conference of committees of the American Society of Mechanical Engineers, and the Master Steam and Hot Water Fitters' Association, with representatives of leading manufacturers and users of pipe, — *Trans. A. S. M. E.*, xxi, 29.

The list is divided into two groups; for medium and high pressures, the first ranging up to 75 lbs. per square inch, and the second up to 200 lbs.

| Pipe Size, Inches. | Pipe Thickness, $\frac{P + 100}{0.48} d + 0.333 \left(1 - \frac{d}{100}\right)$ | Thickness, Nearest Fraction, Inches. | Stress on Pipe per Square Inch at 200 Lbs. | Radius of Fillet, Inches. | Flange Diameters, Inches. | Flange, Thicknesses at Edge, Inches. | Width Flange Face, Inches. | Bolt Circle Diameter, Inches. | Number of Bolts. | Bolt Diameter, Inches. | Bolt Length, Inches. | Stress on Each Bolt, per Square Inch, at Bottom of Thread at 200 Lbs. |
|--------------------|--|--------------------------------------|--|---------------------------|---------------------------|--------------------------------------|----------------------------|-------------------------------|------------------|------------------------|----------------------|---|
| 2 | 0.409 | 7/16 | 460 | 1/8 | 6 | 5/8 | 2 | 43/4 | 4 | 5/8 | 2 | 825 |
| 2 1/2 | .429 | 7/16 | 550 | 1/8 | 7 | 11/16 | 2 1/4 | 51/2 | 4 | 5/8 | 2 1/4 | 1050 |
| 3 | .448 | 7/16 | 690 | 1/8 | 7 1/2 | 3/4 | 2 1/4 | 6 | 4 | 5/8 | 2 1/2 | 1330 |
| 3 1/2 | .466 | 1/2 | 700 | 1/8 | 8 1/2 | 13/16 | 2 1/2 | 7 | 4 | 5/8 | 2 1/2 | 2530 |
| 4 | .486 | 1/2 | 800 | 1/8 | 9 | 15/16 | 2 1/2 | 7 1/2 | 4 | 3/4 | 2 3/4 | 2100 |
| 4 1/2 | .498 | 1/2 | 900 | 1/8 | 9 1/4 | 15/16 | 2 3/8 | 7 3/4 | 8 | 3/4 | 3 | 1430 |
| 5 | .525 | 1/2 | 1000 | 1/8 | 10 | 15/16 | 2 1/2 | 8 1/2 | 8 | 3/4 | 3 | 1630 |
| 6 | .563 | 9/16 | 1060 | 1/8 | 11 | 1 | 2 1/2 | 9 1/2 | 8 | 3/4 | 3 | 2360 |
| 7 | .60 | 5/8 | 1120 | 1/8 | 12 1/2 | 1 1/16 | 2 3/4 | 10 3/4 | 8 | 3/4 | 3 1/4 | 3200 |
| 8 | .639 | 5/8 | 1280 | 1/8 | 13 1/2 | 1 1/8 | 2 3/4 | 11 3/4 | 8 | 3/4 | 3 1/2 | 4190 |
| 9 | .678 | 11/16 | 1310 | 3/16 | 15 | 1 1/8 | 3 | 13 1/4 | 12 | 3/4 | 3 1/2 | 3610 |
| 10 | .713 | 3/4 | 1330 | 3/16 | 16 | 1 3/16 | 3 | 14 1/4 | 12 | 7/8 | 3 5/8 | 2970 |
| 12 | .79 | 13/16 | 1470 | 3/16 | 19 | 1 1/4 | 3 1/2 | 17 | 12 | 7/8 | 3 3/4 | 4280 |
| 14 | .864 | 7/8 | 1600 | 3/16 | 21 | 1 3/8 | 3 1/2 | 18 3/4 | 12 | 1 | 4 1/4 | 4280 |
| 15 | .904 | 15/16 | 1600 | 3/16 | 22 1/4 | 1 3/8 | 3 5/8 | 20 | 16 | 1 | 4 1/4 | 3660 |
| 16 | .946 | 1 | 1600 | 3/16 | 23 1/2 | 1 7/16 | 3 3/4 | 21 1/4 | 16 | 1 | 4 1/4 | 4210 |
| 18 | 1.02 | 1 1/16 | 1690 | 3/16 | 25 | 1 9/16 | 3 1/2 | 22 3/4 | 16 | 1 1/8 | 4 3/4 | 4540 |
| 20 | 1.09 | 1 1/8 | 1780 | 3/16 | 27 1/2 | 1 11/16 | 3 3/4 | 25 | 20 | 1 1/8 | 5 | 4490 |
| 22 | 1.18 | 1 3/16 | 1850 | 1/4 | 29 1/2 | 1 13/16 | 3 3/4 | 27 1/4 | 20 | 1 1/4 | 5 1/2 | 4320 |
| 24 | 1.25 | 1 1/4 | 1920 | 1/4 | 31 1/2 | 1 1/4 | 3 3/4 | 29 1/4 | 20 | 1 1/4 | 5 1/2 | 5130 |
| 26 | 1.30 | 1 5/16 | 1980 | 1/4 | 33 3/4 | 1 3/8 | 3 7/8 | 31 1/4 | 24 | 1 1/4 | 5 3/4 | 5030 |
| 28 | 1.38 | 1 3/8 | 2040 | 1/4 | 36 | 1 7/16 | 4 | 33 1/2 | 28 | 1 1/4 | 6 | 5000 |
| 30 | 1.48 | 1 1/2 | 2000 | 1/4 | 38 | 2 1/8 | 4 | 35 1/2 | 28 | 1 3/8 | 6 1/4 | 4590 |
| 36 | 1.71 | 1 3/4 | 1920 | 1/4 | 44 1/2 | 2 3/8 | 4 1/4 | 42 3/4 | 32 | 1 3/8 | 6 1/2 | 5790 |
| 42 | 1.87 | 2 | 2100 | 1/4 | 51 | 2 5/8 | 4 1/2 | 48 1/2 | 36 | 1 1/2 | 7 1/4 | 5700 |
| 48 | 2.17 | 2 1/4 | 2130 | 1/4 | 57 1/2 | 2 3/4 | 4 3/4 | 54 3/4 | 44 | 1 1/2 | 7 3/4 | 6090 |

NOTES. — Sizes up to 24 inches are designed for 200 lbs. or less.

Sizes from 24 to 48 inches are divided into two scales, one for 200 lbs. the other for less.

The sizes of bolts given are for high pressure. For medium pressures the diameters are 1/8 in. less for pipes 2 to 20 in. diameter inclusive, and 1/4 in. less for larger sizes, except 48-in. pipe, for which the size of bolt is 1 3/8 in.

When two lines of figures occur under one heading, the single columns are for both medium and high pressures. Beginning with 24 inches, the left-hand columns are for medium and the right-hand lines are for high pressures.

The sudden increase in diameters at 16 inches is due to the possible insertion of wrought-iron pipe, making with a nearly constant width of gasket a greater diameter desirable.

When wrought-iron pipe is used, if thinner flanges than those given are sufficient, it is proposed that bosses be used, to bring the nuts up to the standard lengths. This avoids the use of a reinforcement around the pipe.

Figures in the 3d, 4th, 5th, and last columns refer only to pipe for high pressure.

In drilling valve flanges a vertical line parallel to the spindles should be midway between two holes on the upper side of the flanges.

FLANGE DIMENSIONS, ETC., FOR EXTRA HEAVY PIPE FITTINGS (UP TO 250 LBS. PRESSURE).

Adopted by a Conference of Manufacturers, June 28, 1901.

| Size of Pipe. | Diam. of Flange. | Thickness of Flange. | Diameter of Bolt Circle. | Number of Bolts. | Size of Bolts. |
|---------------|------------------|----------------------|--------------------------|------------------|----------------|
| Inches. | Inches. | Inches. | Inches. | | Inches. |
| 2 | 6 1/2 | 7/8 | 5 | 4 | 5/8 |
| 2 1/2 | 7 1/2 | 1 | 5 7/8 | 4 | 3/4 |
| 3 | 8 1/4 | 1 1/8 | 6 5/8 | 8 | 5/8 |
| 3 1/2 | 9 | 1 3/16 | 7 1/4 | 8 | 5/8 |
| 4 | 10 | 1 1/4 | 7 7/8 | 8 | 3/4 |
| 4 1/2 | 10 1/2 | 1 5/16 | 8 1/2 | 8 | 3/4 |
| 5 | 11 | 1 3/8 | 9 1/4 | 8 | 3/4 |
| 6 | 12 1/2 | 1 7/16 | 10 5/8 | 12 | 3/4 |
| 7 | 14 | 1 1/2 | 11 7/8 | 12 | 7/8 |
| 8 | 15 | 1 5/8 | 13 | 12 | 7/8 |
| 9 | 16 | 1 3/4 | 14 | 12 | 7/8 |
| 10 | 17 1/2 | 1 7/8 | 15 1/4 | 16 | 7/8 |
| 12 | 20 | 2 | 17 3/4 | 16 | 7/8 |
| 14 | 22 1/2 | 2 1/8 | 20 | 20 | 7/8 |
| 15 | 23 1/2 | 2 3/16 | 21 | 20 | 1 |
| 16 | 25 | 2 1/4 | 22 1/2 | 20 | 1 |
| 18 | 27 | 2 3/8 | 24 1/2 | 24 | 1 |
| 20 | 29 1/2 | 2 1/2 | 26 3/4 | 24 | 1 1/8 |
| 22 | 31 1/2 | 2 5/8 | 28 3/4 | 28 | 1 1/8 |
| 24 | 34 | 2 3/4 | 31 1/4 | 28 | 1 1/8 |

STANDARD STRAIGHT-WAY GATE VALVES.

(Crane Co.)

Iron Body. Brass Trimmings. Wedge Gate.

Dimensions in Inches: A, nominal size; B, face to face, flanged; C, diam. of flanges; D, thickness of flanges; K, end to end, screwed; N, center to top of non-rising stem; O, diam. of wheel; S, center to top of rising stem, open; P, size of by-pass; F, end to end, hub; T, diam. of hub; X, number of turns to open.

| A | B | C | D | K | N | O | S | Y | P | X |
|-------|--------|--------|---------|---------|--------|-------|---------|--------|---|--------|
| 1 1/2 | 6 1/2 | 5 1/4 | 9/16 | 5 | 10 1/2 | 5 1/2 | | | | 6 |
| 2 | 7 | 6 | 5/8 | 5 7/16 | 11 3/4 | 5 1/2 | 14 | | | 7 |
| 2 1/2 | 7 1/2 | 7 | 11/16 | 5 7/8 | 12 3/4 | 5 1/2 | 15 3/4 | | | 8 |
| 3 | 8 | 7 1/2 | 3/4 | 6 1/8 | 14 1/4 | 6 1/2 | 18 1/2 | | | 10 1/4 |
| 3 1/2 | 8 1/2 | 8 1/2 | 13/16 | 6 1/2 | 15 1/4 | 7 1/2 | 20 3/4 | | | 10 1/8 |
| 4 | 9 | 9 | 15/16 | 6 7/8 | 16 1/4 | 9 | 23 1/2 | | | 8 3/4 |
| 4 1/2 | 9 1/2 | 9 1/4 | 15/16 | 7 1/4 | 17 5/8 | 9 | 24 3/4 | | | 9 |
| 5 | 10 | 10 | 15/16 | 7 5/16 | 19 | 10 | 28 | | | 11 |
| 6 | 10 1/2 | 11 | 1 | 7 3/4 | 20 3/4 | 10 | 31 3/4 | | | 12 5/8 |
| 7 | 11 | 12 1/2 | 1 1/16 | 8 1/4 | 23 | 12 | 37 1/4 | | | 15 1/4 |
| 8 | 11 1/2 | 13 1/2 | 1 1/8 | 8 11/16 | 26 | 14 | 41 | | | 16 |
| 9 | 12 | 15 | 1 1/8 | 9 1/4 | 28 | 14 | 44 1/4 | | | 18 3/4 |
| 10 | 13 | 16 | 1 3/16 | 9 7/8 | 30 1/4 | 16 | 49 1/2 | | | 20 1/2 |
| 12 | 14 | 19 | 1 1/4 | 11 5/8 | 35 1/4 | 18 | 57 1/4 | | | 24 1/8 |
| 14 | 15 | 21 | 1 3/8 | | 39 1/4 | 20 | 66 1/2 | 19 1/2 | 2 | 28 1/4 |
| 15 | 15 | 22 1/4 | 1 3/8 | | 41 1/8 | 20 | 69 3/4 | 21 | 2 | 31 1/2 |
| 16 | 16 | 23 1/2 | 1 7/16 | | 42 3/4 | 22 | 74 3/4 | 23 3/4 | 3 | 33 1/4 |
| 18 | 17 | 25 | 1 9/16 | | 48 3/4 | 24 | 86 | 24 3/4 | 3 | 35 1/2 |
| 20 | 18 | 27 1/2 | 1 11/16 | | 52 1/2 | 24 | 91 | 27 3/4 | 4 | 42 1/4 |
| 22 | 19 | 29 1/2 | 1 13/16 | | 55 1/2 | 27 | 100 | 29 | 4 | 46 |
| 24 | 20 | 32 | 1 7/8 | | 62 | 30 | 109 | 30 1/2 | 4 | 50 |
| 26 | 23 | 34 1/4 | 2 | | 65 7/8 | 30 | 117 1/2 | 32 | 4 | 65 |
| 28 | 26 | 36 1/2 | 2 1/16 | | 70 | 36 | 125 | 33 | 4 | 80 |
| 30 | 30 | 38 3/4 | 2 1/8 | | 75 1/2 | 36 | 133 | 34 | 4 | 92 1/2 |
| 36 | 36 | 45 3/4 | 2 3/8 | | 83 | | 158 1/2 | 39 | 6 | 108 |

EXTRA HEAVY STRAIGHT-WAY GATE VALVES.

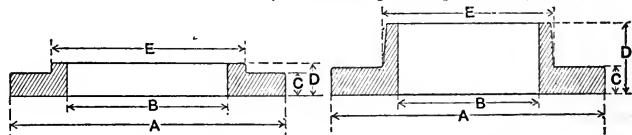
Ferrosteel. Hard Metal Seats. Wedge Gate.

| A | B | K | C | D | N | S | O | P | Y | X |
|-------|--------|--------|--------|--------|--------|--------|-------|-------|--------|----|
| 1 1/4 | 6 1/2 | 5 1/2 | 5 | 3/4 | 8 3/4 | 10 5/8 | 5 | | | 12 |
| 1 1/2 | 7 1/2 | 6 1/4 | 6 | 13/16 | 9 5/8 | 12 1/4 | 5 1/2 | | | 11 |
| 2 | 8 1/2 | 7 | 6 1/2 | 7/8 | 10 1/2 | 13 3/4 | 6 1/2 | | | 14 |
| 2 1/2 | 9 1/2 | 8 | 7 1/2 | 1 | 12 7/8 | 16 | 7 1/2 | | | 15 |
| 3 | 11 1/8 | 9 | 8 1/4 | 1 1/8 | 14 5/8 | 19 1/2 | 9 | | | 14 |
| 3 1/2 | 11 7/8 | 10 | 9 | 13/16 | 15 1/2 | 22 | 10 | | | 16 |
| 4 | 12 | 11 | 10 | 1 1/4 | 17 3/4 | 24 1/2 | 12 | | | 18 |
| 4 1/2 | 13 1/4 | 12 1/4 | 10 1/2 | 1 5/16 | 18 3/4 | 27 | 12 | | | 21 |
| 5 | 15 | 13 1/2 | 11 | 1 3/8 | 20 1/4 | 29 3/4 | 14 | | | 23 |
| 6 | 15 7/8 | 15 7/8 | 12 1/2 | 1 7/16 | 23 | 34 1/8 | 16 | 1 1/4 | 13 | 28 |
| 7 | 16 1/4 | 16 1/4 | 14 | 1 1/2 | 24 3/4 | 38 | 18 | 1 1/4 | 14 1/8 | 30 |
| 8 | 16 1/2 | 16 1/2 | 15 | 1 5/8 | 28 3/4 | 42 3/4 | 20 | 1 1/2 | 15 7/8 | 34 |
| 9 | 17 | 17 | 16 | 1 3/4 | 30 1/2 | 47 | 20 | 1 1/2 | 16 3/8 | 40 |
| 10 | 18 | 18 | 17 1/2 | 1 7/8 | 33 3/4 | 52 3/4 | 22 | 1 1/2 | 16 7/8 | 39 |
| 12 | 19 3/4 | | 20 | 2 | 37 1/4 | 60 | 24 | 2 | 19 7/8 | 46 |
| 14 | 22 1/2 | | 22 1/2 | 2 1/8 | 42 3/4 | 67 3/4 | 24 | 2 | 20 5/8 | 52 |
| 15 | 22 1/2 | | 23 1/2 | 2 3/8 | 42 3/4 | 67 3/4 | 24 | 2 | 20 5/8 | 52 |
| 16 | 24 | | 25 | 2 1/4 | | 75 1/4 | 27 | 3 | 25 1/4 | 60 |
| 18 | 26 | | 27 | 2 3/8 | | 82 1/4 | 30 | 3 | 26 1/2 | 67 |
| 20 | 28 | | 29 1/2 | 2 1/2 | | 91 1/2 | 30 | 4 | 30 1/2 | 74 |
| 22 | 29 1/2 | | 31 1/2 | 2 5/8 | | 101 | 36 | 4 | 32 1/4 | 82 |
| 24 | 31 | | 34 | 2 3/4 | | 109 | 36 | 4 | 33 | 88 |

For dimensions of Medium Valves and Extra Heavy Hydraulic Valves, See Crane Company's catalogue.

FORGED AND ROLLED STEEL FLANGES.

Dimensions in Inches. (American Spiral Pipe Works, 1908.)

**Standard Companion Flanges.****Standard Shrink Flanges.**

| Nominal Size, Inches. | Outside Diam. | Actual Bore | Thick-ness. | Depth of Hub. | Diam. of Hub. | Nominal Size, Inches. | Outside Diam. | Bore Diam. | Thick-ness. | Depth of Hub. | Diam. of Hub. |
|-----------------------|---------------|-------------|-------------|---------------|---------------|-----------------------|---------------|------------|-------------|---------------|---------------|
| | A | B | C | D | E | | A | B | C | D | E |
| 2 | 6 | 2 1/8 | 5/8 | 1 | 3 1/8 | 4 | 9 | 4 3/8 | 1 5/16 | 2 3/16 | 5 3/4 |
| 2 1/2 | 7 | 2 1/2 | 1 1/16 | 1 1/16 | 3 5/8 | 4 1/2 | 9 1/4 | 4 7/8 | 1 5/16 | 2 1/4 | 6 1/8 |
| 3 | 7 1/2 | 3 1/8 | 3/4 | 1 1/8 | 4 5/16 | 5 | 10 | 5 7/16 | 1 5/16 | 2 5/16 | 6 7/8 |
| 3 1/2 | 8 1/2 | 3 5/8 | 13/16 | 1 3/16 | 4 7/8 | 6 | 11 | 6 1/2 | 1 | 2 7/16 | 7 7/8 |
| 4 | 9 | 4 1/8 | 15/16 | 1 3/16 | 5 3/8 | 7 | 12 1/2 | 7 1/2 | 1 1/16 | 2 1/2 | 9 |
| 4 1/2 | 9 1/4 | 4 5/8 | 1 1/16 | 1 1/4 | 5 13/16 | 8 | 13 1/2 | 8 1/2 | 1 1/8 | 2 5/8 | 10 1 |
| 5 | 10 | 5 1/8 | 1 5/16 | 1 5/16 | 6 7/16 | 9 | 15 | 9 1/2 | 1 1/8 | 2 3/4 | 11 1/8 |
| 6 | 11 | 6 3/16 | 1 | 1 7/16 | 7 9/16 | 10 | 16 | 10 5/8 | 1 3/16 | 3 | 12 1/8 |
| 7 | 12 1/2 | 7 3/16 | 1 1/16 | 1 1/2 | 8 5/8 | 12 | 19 | 12 5/8 | 1 1/4 | 3 3/8 | 14 7/8 |
| 8 | 13 1/2 | 8 3/16 | 1 1/8 | 1 5/8 | 9 11/16 | 14 | 21 | 13 7/8 | 1 3/8 | 3 3/8 | 15 7/8 |
| 9 | 15 | 9 3/16 | 1 1/8 | 1 3/4 | 10 5/8 | 15 | 22 1/4 | 14 7/8 | 1 3/8 | 3 1/2 | 16 |
| 10 | 16 | 10 5/16 | 1 3/16 | 1 7/8 | 11 15/16 | 16 | 23 1/2 | 15 7/8 | 1 7/16 | 3 5/8 | 18 2/4 |
| 12 | 19 | 12 5/16 | 1 1/4 | 2 1/16 | 14 1/8 | 18 | 25 | 17 7/8 | 1 9/16 | 3 7/8 | 20 1/8 |
| 14 | 21 | 13 1/2 | 1 3/8 | 2 3/16 | 15 7/16 | 20 | 27 1/2 | 19 7/8 | 1 11/16 | 4 1/8 | 22 1/4 |

FORGED AND ROLLED STEEL FLANGES. — Continued

| Extra Heavy Companion Flanges. | | | | | | Extra Heavy High Hub Flanges. | | | | | |
|--------------------------------|---------------|------------|------------|---------------|---------------|-------------------------------|---------------|-------|------------|---------------|---------------|
| Nominal Size, Inches. | Outside Diam. | Bore Diam. | Thickness. | Depth of Hub. | Diam. of Hub. | Nominal Size, Inches. | Outside Diam. | Bore. | Thickness. | Depth of Hub. | Diam. of Hub. |
| | A | B | C | D | E | | A | B | C | D | E |
| 2 | 61/2 | 21/8 | 7/8 | 13/8 | 33/8 | 4 | 10 | 43/8 | 11/8 | 31/8 | 53/4 |
| 2 1/2 | 71/2 | 21/2 | 1 | 17/16 | 41/16 | 4 1/2 | 10 1/2 | 47/8 | 11/4 | 31/4 | 61/4 |
| 3 | 81/4 | 31/8 | 1 | 19/16 | 41 1/16 | 5 | 11 | 57/16 | 11/4 | 31/4 | 7 |
| 3 1/2 | 9 | 35/8 | 1 1/8 | 15/8 | 55/16 | 6 | 12 1/2 | 61/2 | 1 1/4 | 31/4 | 7 15/16 |
| 4 | 10 | 41/8 | 1 1/8 | 13/4 | 513/16 | 7 | 14 | 71/2 | 15/16 | 33/8 | 91/8 |
| 4 1/2 | 10 1/2 | 45/8 | 1 1/4 | 1 13/16 | 61/4 | 8 | 15 | 81/2 | 13/8 | 31/2 | 105/16 |
| 5 | 11 | 51/8 | 1 1/4 | 17/8 | 613/16 | 9 | 16 | 91/2 | 17/16 | 35/8 | 113/8 |
| 6 | 12 1/2 | 63/16 | 1 1/4 | 2 | 77/8 | 10 | 17 1/2 | 105/8 | 1 1/2 | 33/4 | 125/8 |
| 7 | 14 | 73/16 | 15/16 | 2 1/16 | 91/8 | 11 | 18 3/4 | 115/8 | 19/16 | 37/8 | 135/8 |
| 8 | 15 | 83/16 | 13/8 | 23/16 | 101/8 | 12 | 20 | 125/8 | 15/8 | 4 | 143/4 |
| 9 | 16 | 93/16 | 17/16 | 21/4 | 113/16 | 14 | 22 1/2 | 137/8 | 13/4 | 43/8 | 163/16 |
| 10 | 17 1/2 | 105/16 | 11/2 | 23/8 | 129/16 | 15 | 23 1/2 | 147/8 | 1 13/16 | 41/2 | 171/4 |
| 12 | 20 | 125/16 | 15/8 | 29/16 | 145/8 | 16 | 25 | 157/8 | 17/8 | 43/4 | 181/2 |
| 14 | 22 1/2 | 131/2 | 13/4 | 21 1/16 | 1513/16 | 18 | 27 | 177/8 | 2 | 5 | 203/4 |
| 15 | 23 1/2 | 141/2 | 133/16 | 213/16 | 173/16 | 20 | 29 1/2 | 197/8 | 2 1/4 | 5 1/2 | 221/2 |
| 16 | 25 | 151/2 | 17/8 | 31/16 | 181/4 | | | | | | |

Forged Steel Flanges for Riveted Pipe.

Riveted Pipe Manufacturers' Standard.*

| Nominal Size, Ins. | Outside Diam. | Thickness of Flange.* | No. of Bolts. | Size of Bolts. | Diam. of Bolt Circle. | Nominal Size, Ins. | Outside Diam. | Thickness of Flange.* | No. of Bolts | Size of Bolts. | Diam. of Bolt Circle. |
|--------------------|---------------|-----------------------|---------------|----------------|-----------------------|--------------------|---------------|-----------------------|--------------|----------------|-----------------------|
| 3 | 6 | 5/16 | 4 | 7/16 | 43/4 | 16 | 21 1/4 | 5/8 | 3/4 | 12 | 19 1/4 |
| 4 | 7 | 5/16 | 8 | 7/16 | 515/16 | 18 | 23 1/4 | 5/8 | 3/4 | 16 | 21 1/4 |
| 5 | 8 | 5/16 | 8 | 7/16 | 615/16 | 20 | 25 1/4 | 5/8 | 3/4 | 16 | 23 1/8 |
| 6 | 9 | 3/8 | 8 | 1/2 | 77/8 | 22 | 28 1/4 | 11/16 | 3/4 | 16 | 26 |
| 7 | 10 | 3/8 | 8 | 1/2 | 9 | 24 | 30 | 11/16 | 7/8 | 16 | 27 3/4 |
| 8 | 11 | 3/8 | 8 | 1/2 | 10 | 26 | 32 | | 13/8 | 24 | 29 3/4 |
| 9 | 13 | 3/8 | 8 | 1/2 | 11 1/4 | 28 | 34 | | 13/8 | 28 | 31 3/4 |
| 10 | 14 | 3/8 | 8 | 1/2 | 12 1/4 | 30 | 36 | | 13/8 | 28 | 33 3/4 |
| 11 | 15 | 7/16 | 12 | 1/2 | 133/8 | 32 | 38 | | | 28 | 35 3/4 |
| 12 | 16 | 7/16 | 12 | 1/2 | 141/4 | 34 | 40 | | | 28 | 37 3/4 |
| 13 | 17 | 7/16 | 12 | 1/2 | 151/4 | 36 | 42 | | 1 1/2 | 32 | 39 3/4 |
| 14 | 18 | 7/16 | 12 | 1/2 | 161/4 | 40 | 46 | | 1 1/2 | 32 | 43 3/4 |
| 15 | 19 | 9/16 | 12 | 1/2 | 177/16 | | | | | | |

* Flanges for riveted pipe are also made with the outside diameter and the drilling dimensions the same as those of the A. S. M. E. standard (page 198), and with the thickness as given in the second column of figures under "Thickness of Flange" in the above table.

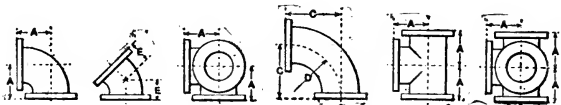
Curved Forged Steel Flanges are also made for boilers and tanks. See catalogue of American Spiral Pipe Works, Chicago.

The Rockwood Pipe Joint. — The system of flanged joints now in common use for high pressures, made by slipping a flange over the pipe, expanding the end of the pipe by rolling or peening, and then facing it in a lathe, so that when the flanges of two pipes are bolted together the bearing of the joint is on the ends of the pipes themselves and not on the flanges, was patented by George I. Rockwood, April 5, 1897, No. 580,058, and first described in *Eng. Rec.*, July 20, 1895. The joint as made by different manufacturers is known by various trade names, as Walmanco, Van Stone, etc.

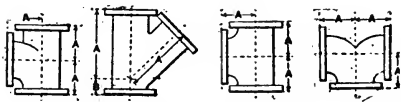
WROUGHT-IRON (OR STEEL) WELDED PIPE.

For discussion of the Briggs Standard of Wrought-iron Pipe Dimensions, see Report of the Committee of the A. S. M. E. in "Standard Pipe and Pipe Threads," 1886. Trans., Vol. VIII, p. 29. The diameter of the bottom of the thread is derived from the formula $D - (0.05D + 1.9) \times \frac{1}{n}$, in which D = outside diameter of the tubes, and n the number of threads to the inch. The diameter of the top of the thread is derived from the formula $0.8 \frac{1}{n} \times 2 + d$, or $1.6 \frac{1}{n} + d$, in which d is the diameter at the bottom of the thread at the end of the pipe.
(Continued on page 207).

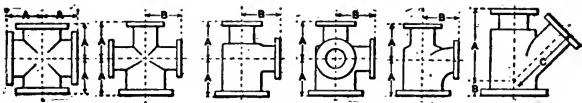
FORMS OF CAST-IRON FLANGES. (See tables on pages 203 to 206).



(1) Elbow. (2) 45° Elbow. (3) Side Outlet Elbow. (4) Long Turn Elbow. (5) Tee. (6) Side Outlet T.



(7) Single Sweep T. (8) Lateral. (9) Double Sweep T. (10) Double Branch Elbow.



(11) Cross. (12) Reducing Cross. (13) Reducing Tee. (14) Red T with Side Outlet. (15) Red Single Sweep Tee. (16) Reducing Lateral.

DIMENSIONS OF STANDARD CAST-IRON FLANGED FITTINGS.

For Steam Working Pressures up to 125 Pounds. (Crane Company, 1908.)

| Size..... | In. | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 16 |
|--|-----|-------|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|-------|--------|--------|--------|
| AA-Face to Face..... | In. | 7 1/2 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 20 | 22 | 24 | 28 | 29 | 30 |
| A-Center to Face..... | In. | 3 3/4 | 4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 | 7 1/2 | 8 | 8 1/2 | 9 | 10 | 11 | 12 | 14 | 14 1/2 | 15 |
| B-Center to Face..... | In. | 3 3/4 | 4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 | 7 1/2 | 8 | 8 1/2 | 9 | 10 | 11 | 12 | 14 | 14 1/2 | 15 |
| C-Center to Face, Long Radius Ells In. | In. | | | | | | | 9 | 9 1/2 | 10 1/4 | 11 1/2 | 12 3/4 | 14 | 15 1/4 | 16 1/2 | 19 | 21 1/2 | 22 3/4 | 24 |
| D-Radius, Long Radius Ells..... | In. | 2 | 2 1/4 | 2 1/2 | 3 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 1/2 | 8 | 9 | 10 | 11 | 12 |
| E-Center to Face of 45° Ells..... | In. | 4 1/2 | 5 | 6 | 7 | 7 1/2 | 8 1/2 | 9 | 9 1/4 | 10 | 11 | 12 1/2 | 13 1/2 | 15 | 16 | 19 | 21 | 22 1/4 | 23 1/2 |
| Diameter of Flange..... | In. | 1 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 1 1/8 | 1 1/4 | 1 1/2 | 1 5/8 | 1 3/4 | 1 7/8 | 2 | 2 1/8 | 2 1/2 | 2 3/4 | 2 7/8 | 3 |
| Thickness of Flange..... | In. | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Number of Bolts..... | In. | 7/16 | 1/2 | 5/8 | 5/8 | 5/8 | 5/8 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 | 3/4 |
| Size of Bolts..... | In. | 1 1/2 | 1 3/4 | 2 | 2 1/4 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 |
| Length of Bolts..... | In. | 3 3/8 | 3 7/8 | 4 3/4 | 5 1/2 | 6 | 7 | 7 1/2 | 7 3/4 | 8 1/2 | 9 1/2 | 10 3/4 | 11 3/4 | 13 1/4 | 14 1/4 | 17 | 18 3/4 | 20 | 21 1/4 |
| Bolt Circle..... | In. | | | | | | | | | | | | | | | | | | |

Reducing Fittings, sizes 1 1/4 to 9 inch, inclusive, the dimensions do not change from above table by any reductions in the size of run or outlet except Double Sweep Tees, in which the reduced end is longer than the regular fitting.

VARIATIONS FROM THE ABOVE GENERAL DIMENSIONS OF STANDARD FLANGED FITTINGS, SIZES 10 TO 16 INCH WITH SMALL OUTLET, SHORT BODY PATTERN.

| Size..... | In. | 10 | 12 | 14 | 15 | 16 |
|----------------------------------|-----|---------------|---------------|---------------|---------------|----------------|
| Outlets..... | In. | 6 and smaller | 8 and smaller | 9 and smaller | 9 and smaller | 10 and smaller |
| AA-Face to Face of Run..... | In. | 18 | 20 | 22 | 23 | 24 |
| A-Center to Face of Run..... | In. | 9 | 10 | 11 | 11 1/2 | 12 |
| B-Center to Face of Outlets..... | In. | 9 1/2 | 11 | 13 | 13 1/2 | 14 |

If the outlet is larger than given in this lower table, use the upper table of Long Body Pattern.

For Steam Working Pressures up to 250 Pounds.

| Size..... | In. | 11/4 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 |
|---|-----|-------|-------|-------|-------|-------|--------|-------|--------|--------|--------|--------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| A-A-Face to Face..... | In. | 8 1/2 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 20 | 21 | 23 | 26 | 29 | 30 | 32 | 34 | 37 | 40 | 44 |
| A-Center to Face..... | In. | 4 1/4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 | 7 1/2 | 8 | 8 1/2 | 9 | 10 | 10 1/2 | 11 1/2 | 13 | 14 1/2 | 15 | 16 | 17 | 18 1/2 | 20 | 22 |
| B-Center to Face..... | In. | 4 1/4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 | 7 1/2 | 8 | 8 1/2 | 9 | 10 | 10 1/2 | 11 1/2 | 13 | 14 1/2 | 15 | 16 | 17 | 18 1/2 | 20 | 22 |
| C-Center to Face of Long Radius Ells..... | In. | | | 6 1/2 | 7 | 7 3/4 | 8 1/2 | 9 | 9 1/2 | 10 1/4 | 11 1/2 | 12 3/4 | 14 | 15 1/4 | 16 1/2 | 19 | 21 1/2 | 22 3/4 | 24 | 26 1/2 | 29 | 31 1/2 | 34 |
| D-Radius of Long Radius Ells..... | In. | | | 5 1/4 | 5 5/8 | 6 1/4 | 6 7/8 | 7 3/8 | 7 3/4 | 8 1/2 | 9 5/8 | 10 7/8 | 12 | 13 | 14 1/8 | 16 1/2 | 18 7/8 | 20 | 21 1/4 | 23 5/8 | 26 | 28 3/8 | 30 3/4 |
| E-Center to Face of 45° Ells..... | In. | 2 1/2 | 2 3/4 | 3 | 3 1/2 | 3 1/2 | 4 | 4 1/2 | 4 1/2 | 5 | 5 1/2 | 6 | 6 | 6 1/2 | 7 | 8 | 8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 1/2 |
| Diameter of Flange In. | In. | 5 | 6 | 6 1/2 | 7 1/2 | 8 1/4 | 9 | 10 | 10 1/2 | 11 | 12 1/2 | 14 | 15 | 16 | 17 1/2 | 20 | 22 1/2 | 23 1/2 | 25 | 27 | 29 1/2 | 31 1/2 | 34 |
| Thickness of Flange In. | In. | 3/4 | 13/16 | 7/8 | 1 | 1 1/8 | 1 3/16 | 1 1/4 | 1 5/16 | 1 3/8 | 1 7/16 | 1 1/2 | 1 5/8 | 1 3/4 | 1 7/8 | 2 | 2 1/8 | 2 3/16 | 2 1/4 | 2 3/8 | 2 1/2 | 2 5/8 | 2 3/4 |
| Number of Bolts..... | In. | 4 | 4 | 4 | 4 | 4 | 8 | 8 | 8 | 8 | 12 | 12 | 12 | 12 | 16 | 16 | 20 | 20 | 20 | 24 | 24 | 28 | 28 |
| Size of Bolts..... | In. | 1/2 | 5/8 | 5/8 | 3/4 | 3/4 | 5/8 | 3/4 | 3/4 | 3/4 | 3/4 | 7/8 | 7/8 | 7/8 | 7/8 | 7/8 | 7/8 | 1 | 1 | 1 | 1 1/8 | 1 1/8 | 1 1/8 |
| Length of Bolts..... | In. | 2 1/4 | 2 1/2 | 2 1/2 | 3 | 3 | 3 1/4 | 3 1/2 | 3 1/2 | 3 3/4 | 3 3/4 | 4 | 4 1/4 | 4 1/2 | 4 3/4 | 5 | 5 1/4 | 5 1/2 | 5 3/4 | 6 | 6 1/2 | 6 3/4 | 7 |
| Bolt Circle..... | In. | 3 3/4 | 4 1/2 | 5 | 5 7/8 | 6 5/8 | 7 1/4 | 7 7/8 | 8 1/2 | 9 1/4 | 10 5/8 | 11 7/8 | 13 | 14 | 15 1/4 | 17 3/4 | 20 | 21 | 22 1/2 | 24 1/2 | 26 3/4 | 28 3/4 | 31 1/4 |

Reducing Fittings, sizes 1 1/4 to 9 inch, inclusive, the dimensions do not change from above by any reductions in the size of run or outlet, except Double Sweep Tees, in which the reduced end is always longer than the regular fitting.

VARIATIONS FROM THE ABOVE GENERAL DIMENSIONS OF EXTRA HEAVY REDUCING FLANGED FITTINGS, SIZES 10 TO 24 INCH WITH SMALL OUTLET, SHORT BODY PATTERN.

| Size..... | In. | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 |
|---------------------------------|-----|---------------|---------------|---------------|---------------|----------------|----------------|----------------|----------------|----------------|
| Size of Outlets..... | In. | 6 and smaller | 8 and smaller | 9 and smaller | 9 and smaller | 10 and smaller | 12 and smaller | 14 and smaller | 15 and smaller | 15 and smaller |
| A-A-Face to Face of Run..... | In. | 18 | 21 | 22 | 23 | 24 | 27 | 30 | 30 | 30 |
| A-Center to Face of Run..... | In. | 9 | 10 1/2 | 11 | 11 1/2 | 12 | 13 1/2 | 15 | 15 | 15 |
| B-Center to Face of Outlet..... | In. | 11 | 12 1/2 | 13 1/2 | 13 1/2 | 15 | 16 1/2 | 17 1/2 | 18 1/2 | 19 1/2 |

If the outlet is larger than given in this lower table, use the upper table of Long Body Pattern.

LOW PRESSURE CAST-IRON FLANGED FITTINGS
For Steam Working Pressures up to 25 Pounds.

| | | | | | | | | | | | | | | | | | | | | | |
|-----------------------------------|-----|-------|--------|--------|--------|--------|--------|--------|--------|---------|--------|--------|---------|--------|--------|---------|--------|--------|--------|--------|--------|
| Size..... | In. | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 |
| AA-Face to Face..... | In. | 24 | 28 | 29 | 30 | 33 | 36 | 40 | 44 | 46 | 48 | 50 | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 |
| A-Center to Face..... | In. | 12 | 14 | 14 1/2 | 15 | 16 1/2 | 18 | 20 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 |
| B-Center to Face..... | In. | 12 | 14 | 14 1/2 | 15 | 16 1/2 | 18 | 20 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 |
| E-Center to Face of 45° Ells..... | In. | 7 1/2 | 8 | 8 1/2 | 9 1/2 | 10 | 11 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| Diameter of Flange..... | In. | 19 | 21 | 22 1/4 | 23 1/2 | 25 | 27 1/2 | 29 1/2 | 32 | 34 1/4 | 36 1/2 | 38 3/4 | 41 1/2 | 43 1/2 | 45 3/4 | 48 1/2 | 50 1/2 | 52 3/4 | 55 1/4 | 57 1/4 | 59 1/2 |
| Thickness of Flange..... | In. | 1 | 1 1/8 | 1 3/16 | 1 1/4 | 1 5/16 | 1 3/8 | 1 1/2 | 1 5/8 | 1 11/16 | 1 1/2 | 1 3/4 | 1 13/16 | 1 7/8 | 1 7/8 | 1 15/16 | 2 | 2 1/16 | 2 1/8 | 2 1/4 | 2 3/8 |
| Number of Bolts..... | In. | 12 | 16 | 16 | 16 | 16 | 20 | 20 | 20 | 24 | 28 | 28 | 32 | 32 | 32 | 36 | 36 | 40 | 40 | 44 | 44 |
| Size of Bolts..... | In. | 7/8 | 1 | 1 | 1 1/8 | 1 1/8 | 1 1/8 | 1 1/4 | 1 1/4 | 1 1/4 | 1 1/4 | 1 3/8 | 1 3/8 | 1 3/8 | 1 3/8 | 1 1/2 | 1 1/2 | 1 1/2 | 1 1/2 | 1 1/2 | 1 1/2 |
| Length of Bolts..... | In. | 3 1/2 | 4 | 4 | 4 1/2 | 4 3/4 | 5 | 5 1/4 | 5 1/4 | 5 1/2 | 5 3/4 | 6 | 6 | 6 | 6 1/4 | 6 1/2 | 6 1/2 | 6 3/4 | 6 3/4 | 7 | 7 1/4 |
| Bolt Circle..... | In. | 17 | 18 3/4 | 20 | 21 1/4 | 22 3/4 | 25 | 27 1/4 | 29 1/2 | 31 3/4 | 34 | 36 | 38 1/2 | 40 1/2 | 42 3/4 | 45 1/4 | 47 1/4 | 49 1/2 | 51 3/4 | 53 3/4 | 56 |

VARIATIONS FROM THE ABOVE GENERAL DIMENSIONS OF REDUCING FITTINGS WITH SMALL OUTLET — SHORT BODY PATTERN.

| | | | | | | | | | | | | | | | | | | | | | |
|--|-----|----|----|--------|----|--------|----|----|----|----|----|----|----|----|----|----|----|----|--------|----|--------|
| Size..... | In. | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 |
| Size of Outlets, In., and smaller..... | In. | 8 | 9 | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 18 | 20 | 20 | 22 | 22 | 22 | 26 | 28 | 32 | 34 | 36 |
| AA-Face to Face of Run..... | In. | 20 | 22 | 23 | 24 | 26 | 28 | 28 | 30 | 32 | 32 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 51 | 54 | 56 |
| A-Center to Face of Run..... | In. | 10 | 11 | 11 1/2 | 12 | 13 | 14 | 14 | 15 | 16 | 16 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 1/2 | 27 | 28 |
| B-Center to Face of Outlet..... | In. | 11 | 13 | 13 1/2 | 14 | 15 1/2 | 17 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 27 | 28 | 29 | 30 | 31 | 32 | 33 1/2 |

If the outlet is larger than given in this lower table, use the upper table of Long Body Pattern.

EXTRA HEAVY HYDRAULIC FERROSTEEL FLANGED FITTINGS
For Water Working Pressures up to 800 Pounds.

Tested to 2000 Pounds Pressure per Square Inch.

| | | | | | | | | | | | | |
|--|-----|--------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Size..... | In. | 11 1/2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |
| Diameter of Flanges..... | In. | 6 1/2 | 7 1/2 | 8 3/4 | 11 1/2 | 13 1/2 | 15 | 16 | 17 | 18 1/2 | 21 | 23 1/2 |
| Center to Face of Run or Reducing Sizes, Tees and Crosses..... | In. | 5 1/2 | 6 | 6 1/2 | 8 1/2 | 9 1/2 | 11 | 11 1/2 | 12 1/2 | 13 1/2 | 15 | 16 1/2 |
| Center to Face of Elbows..... | In. | 7 1/4 | 8 | 8 3/4 | 10 3/4 | 12 1/2 | 13 3/4 | 15 | 16 1/2 | 17 3/4 | 20 3/4 | 23 3/4 |
| Radius of Sweep of Elbows..... | In. | 5 5/8 | 6 1/4 | 6 7/8 | 8 1/2 | 9 5/8 | 12 | 13 | 14 1/8 | 16 1/2 | 19 1/2 | 24 3/4 |
| Center to Face of 45° Elbows..... | In. | 3 1/4 | 3 1/2 | 4 | 5 1/4 | 5 1/2 | 6 | 7 | 7 1/4 | 7 3/4 | 8 1/2 | 9 1/2 |

DIMENSIONS OF STRAIGHT LATERALS.

| | | | | | | | | | | | | | | | | | | | | |
|---------------------------|--------|-------|----|--------|----|--------|--------|--------|--------|--------|--------|--------|--------|----|--------|--------|----|----|--------|--------|
| Size.....In. | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 |
| Face to Face of Run...In. | 10 1/2 | 12 | 13 | 14 1/2 | 15 | 15 1/2 | 17 | 18 | 20 1/2 | 22 | 24 | 25 1/2 | 30 | 33 | 34 1/2 | 36 1/2 | 39 | 43 | 46 | 49 1/2 |
| A-Center to Face.....In. | 8 | 9 1/2 | 10 | 11 1/2 | 12 | 12 1/2 | 13 1/2 | 14 1/2 | 16 1/2 | 17 1/2 | 19 1/2 | 20 1/2 | 24 1/2 | 27 | 28 1/2 | 30 | 32 | 35 | 37 1/2 | 40 1/2 |
| B-Center to Face.....In. | 2 1/2 | 2 1/2 | 3 | 3 | 3 | 3 | 3 1/2 | 3 1/2 | 4 | 4 1/2 | 4 1/2 | 5 | 5 1/2 | 6 | 6 | 6 1/2 | 7 | 8 | 8 1/2 | 9 |

| | | | | | | | | | | | | | | | | | | | | |
|-------------------------|--------|--------|----|--------|--------|--------|--------|--------|--------|----|----|--------|--------|--------|--------|--------|----|--------|--------|--------|
| Size.....In. | 11 1/2 | 13 | 14 | 15 1/2 | 16 1/2 | 18 | 18 1/2 | 21 1/2 | 23 | 25 | 26 | 28 1/2 | 32 1/2 | 36 1/2 | 38 | 40 1/2 | 44 | 46 1/2 | 51 | 54 1/2 |
| Face to Face of Run In. | 9 | 10 1/2 | 11 | 12 1/2 | 13 1/2 | 14 1/2 | 15 | 17 1/2 | 18 1/2 | 20 | 21 | 23 | 26 1/2 | 30 | 31 1/2 | 33 | 36 | 38 | 41 1/2 | 44 1/2 |
| A-Center to Face...In. | 2 1/2 | 2 1/2 | 3 | 3 | 3 | 3 1/2 | 3 1/2 | 4 | 4 1/2 | 5 | 5 | 5 1/2 | 6 | 6 1/2 | 6 1/2 | 7 1/2 | 8 | 8 1/2 | 9 1/2 | 10 |
| B-Center to Face...In. | | | | | | | | | | | | | | | | | | | | |

DIMENSIONS OF REDUCING LATERALS.

| | | | | | | | | | | | | | | | | |
|-----------------------------|-------------------|-------------------|---------------|---------------|-------------------|---------------|-------------------|---------------|---------------|---------------|---------------|---------------|---------------|----------------|----------------|----------------|
| Size.....In. | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 22 | 24 |
| Size of Branch.....In. | 2 1/2 and smaller | 2 1/2 and smaller | 3 and smaller | 3 and smaller | 3 1/2 and smaller | 4 and smaller | 4 1/2 and smaller | 5 and smaller | 6 and smaller | 7 and smaller | 7 and smaller | 8 and smaller | 9 and smaller | 10 and smaller | 10 and smaller | 12 and smaller |
| Face to Face of Run.....In. | 13 | 13 | 14 | 15 | 16 | 16 | 17 | 18 | 20 | 22 | 23 | 24 | 26 | 28 | 29 | 32 |
| A-Center to Face.....In. | 11 | 11 | 12 | 13 1/2 | 14 1/2 | 14 1/2 | 15 1/2 | 17 | 19 | 21 | 22 | 23 | 25 | 27 | 28 1/2 | 31 1/2 |
| B-Center to Face.....In. | 2 | 2 | 2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 3 | 3 1/2 | 3 1/2 | 3 1/2 | 3 1/2 | 3 1/2 | 3 1/2 | 3 1/2 | 3 1/2 |
| C-Center to Face.....In. | 11 | 11 | 12 | 13 1/2 | 15 | 15 1/2 | 16 1/2 | 18 | 20 1/2 | 23 | 24 | 25 1/2 | 27 1/2 | 29 1/2 | 31 1/2 | 34 1/2 |

| | | | | | | | | | | | | | | | | |
|---------------------------|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----|----|----|
| Size.....In. | 14 | 15 | 16 | 17 | 18 | 20 | 21 | 23 | 26 | 29 | 30 | 32 | 34 | 37 | 40 | 44 |
| Face to Face of Run...In. | 12 | 12 1/2 | 13 1/2 | 14 1/2 | 15 1/2 | 17 1/2 | 18 1/2 | 20 1/2 | 23 1/2 | 26 1/2 | 27 1/2 | 29 | 31 | 34 | 37 | 41 |
| A-Center to Face.....In. | 2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 2 1/2 | 3 | 3 | 3 | 3 | 3 |
| B-Center to Face.....In. | 13 | 13 1/2 | 14 1/2 | 15 1/2 | 16 1/2 | 18 1/2 | 19 1/2 | 21 1/2 | 24 1/2 | 27 1/2 | 28 1/2 | 30 1/2 | 32 1/2 | 36 | 39 | 43 |

If the Branch is larger than given in this lower table, use the upper table.

The dimensions of Reducing Flanged Fittings are always regulated by the reduction of the outlet. Fittings reducing on the run only, the long body pattern (upper table) will always be used. For general dimensions and templates for drilling, see page 203.

(Continued from page 202.) The sizes for the diameters at the bottom and top of the thread at the end of the pipe are as follows:

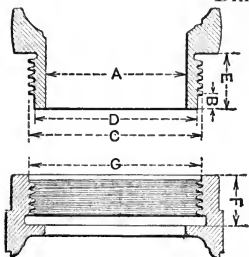
| Diam. of Pipe, Nomi- nal. | Diam. at Bot- tom of Thread. | Diam. at Top of Thread. | Diam. of Pipe, Nomi- nal. | Diam. at Bot- tom of Thread. | Diam. at Top of Thread. | Diam. of Pipe, Nomi- nal. | Diam. at Bot- tom of Thread. | Diam. at Top of Thread. |
|------------------------------------|---------------------------------------|----------------------------------|------------------------------------|---------------------------------------|----------------------------------|------------------------------------|---------------------------------------|----------------------------------|
| in. | in. | in. | in. | in. | in. | in. | in. | in. |
| 1/8 | 0.334 | 0.393 | 2 1/2 | 2.620 | 2.820 | 8 | 8.334 | 8.534 |
| 1/4 | .433 | .522 | 3 | 3.241 | 3.441 | 9 | 9.327 | 9.527 |
| 3/8 | .568 | .658 | 3 1/2 | 3.738 | 3.938 | 10 | 10.445 | 10.645 |
| 1/2 | .701 | .815 | 4 | 4.234 | 4.434 | 11 | 11.439 | 11.639 |
| 3/4 | .911 | 1.025 | 4 1/2 | 4.731 | 4.931 | 12 | 12.433 | 12.633 |
| 1 | 1.144 | 1.283 | 5 | 5.290 | 5.490 | 13 | 13.675 | 13.875 |
| 1 1/4 | 1.488 | 1.627 | 6 | 6.346 | 6.546 | 14 | 14.669 | 14.869 |
| 1 1/2 | 1.727 | 1.866 | 7 | 7.340 | 7.540 | 15 | 15.663 | 15.863 |
| 2 | 2.223 | 2.339 | | | | | | |

Having the taper, length of full-threaded portion, and the sizes at bottom and top of thread at the end of the pipe, as given in the table, taps and dies can be made to secure these points correctly, the length of the imperfect threaded portions on the pipe, and the length the tap is run into the fittings beyond the point at which the size is as given, or, in other words, beyond the end of the pipe, having no effect upon the standard. The angle of the thread is 60°, and it is slightly rounded off at top and bottom, so that, instead of its depth being 0.866 its pitch, as is the case with a full V-thread, it is $\frac{4}{5}$ the pitch, or equal to $0.8 \div n$, n being the number of threads per inch.

Taper of conical tube ends, 1 in 32 to axis of tube = $\frac{3}{4}$ inch to the foot total taper.

NATIONAL STANDARD HOSE COUPLINGS.

Dimensions in Inches.



| | | | | |
|--------|--------|--------|--------|--------|
| A..... | 2 1/2 | 3 | 3 1/2 | 4 1/2 |
| B..... | 1/4 | 1/4 | 1/4 | 1/4 |
| C..... | 3 1/16 | 3 5/8 | 4 1/4 | 5 3/4 |
| D..... | 2.8715 | 3.3763 | 4.0013 | 5.3970 |
| E..... | 1 | 1 1/8 | 1 1/8 | 1 3/8 |
| N..... | 7 1/2 | 6 | 6 | 4 |
| F..... | 7/8 | 1 | 1 | 1 1/4 |
| G..... | 3.0925 | 3.6550 | 4.28 | 5.80 |

The threads to be of the 60° V. pattern with 0.01 in. cut off the top of thread and 0.01 in. left in the bottom of the 2 1/2-in., 3-in., and 3 1/2-in. couplings, and 0.02 in. in like manner for the 4 1/2 in. couplings.

A = inside diameter of hose couplings, N = number of threads per inch.

DIMENSIONS OF STANDARD WELDED PIPE.

Referring to the table on the next page, the weights per foot are based upon steel weighing 0.2833 lb. per cu. in. and up to and including 15 ins. on an average length of 20 ft. 0 in. including the coupling, although shipping lengths of small sizes will usually average less than 20 ft. long. Above 15 ins. the weights given are for plain end pipe. All dimensions and weights are nominal. The limits of variation in weight are 5 per cent above and 5 per cent below. Taper of threads is $\frac{3}{4}$ in. in the diameter per ft. length. Weight of contained water is based on a temperature of 62° F. and 0.036035 lb. to the cubic inch.

Dimensions of Standard Welded Pipe.

Briggs' Standard to 10 in. National Tube Company Standard above 10 in. (National Tube Co., Feb., 1910.)

| Size. | Diameter. | | Thickness of Metal. | No. of Threads per inch. | Weight of Pipe per Lin. ft. | Circumference. | | Length of Pipe per sq. ft. of Surface. | | Sectional Areas. | | | | Length of pipe containing 1 cubic foot. | U. S. gallons contained in 1 lin. ft. | Water contained in 1 lin. ft. of pipe. |
|-------|-----------|--------|---------------------|--------------------------|-----------------------------|----------------|--------|--|--------|------------------|-----------|----------|---------|---|---------------------------------------|--|
| | Exter. | Inter. | | | | Exter. | Inter. | Exter. | Inter. | Exter. | Internal. | Sq. ins. | Sq. ft. | | | |
| Ins. | Ins. | Ins. | Ins. | Ins. | Lbs. | Ins. | Ins. | Ft. | Ft. | Sq. ins. | Sq. ft. | Sq. ins. | Sq. ft. | Lin. ft. | Gal. | Lbs. |
| 1/8 | 0.405 | 0.269 | 0.068 | 27 | 0.246 | 1.272 | 0.845 | 9.431 | 14.200 | 0.129 | 0.0009 | 0.057 | 0.0004 | 2533.8 | 0.0030 | 0.0246 |
| 1/4 | 0.540 | 0.364 | 0.088 | 18 | 0.426 | 1.696 | 1.144 | 7.074 | 10.494 | 0.229 | 0.0016 | 0.104 | 0.0007 | 1383.8 | 0.0054 | 0.0451 |
| 3/8 | 0.675 | 0.493 | 0.091 | 18 | 0.570 | 2.121 | 1.549 | 5.659 | 7.748 | 0.358 | 0.0025 | 0.191 | 0.0013 | 754.36 | 0.0099 | 0.0827 |
| 1/2 | 0.840 | 0.622 | 0.109 | 14 | 0.855 | 2.639 | 2.039 | 4.547 | 6.141 | 0.554 | 0.0038 | 0.304 | 0.0021 | 473.91 | 0.0158 | 0.1316 |
| 3/4 | 1.050 | 0.824 | 0.113 | 14 | 1.140 | 3.299 | 2.589 | 3.638 | 4.636 | 0.866 | 0.0060 | 0.533 | 0.0037 | 270.03 | 0.0277 | 0.2309 |
| 1 | 1.315 | 1.049 | 0.133 | 11 1/2 | 1.690 | 4.131 | 3.296 | 2.905 | 3.641 | 1.358 | 0.0094 | 0.864 | 0.0060 | 166.62 | 0.0449 | 0.3742 |
| 1 1/4 | 1.660 | 1.380 | 0.140 | 11 1/2 | 2.290 | 5.215 | 4.335 | 2.301 | 2.768 | 2.164 | 0.0150 | 1.496 | 0.0104 | 96.275 | 0.0777 | 0.6477 |
| 1 1/2 | 1.900 | 1.610 | 0.145 | 11 1/2 | 2.740 | 5.969 | 5.058 | 2.010 | 2.372 | 2.835 | 0.0197 | 2.036 | 0.0141 | 70.733 | 0.1058 | 0.8816 |
| 2 | 2.375 | 2.067 | 0.154 | 8 | 3.690 | 7.461 | 6.494 | 1.608 | 1.848 | 4.430 | 0.0308 | 3.356 | 0.0233 | 42.913 | 0.1743 | 1.4530 |
| 2 1/2 | 2.875 | 2.469 | 0.203 | 8 | 5.850 | 9.032 | 7.757 | 1.329 | 1.547 | 6.492 | 0.0451 | 4.788 | 0.0332 | 30.077 | 0.2487 | 2.0732 |
| 3 | 3.500 | 3.068 | 0.216 | 8 | 7.660 | 10.996 | 9.638 | 1.091 | 1.245 | 9.621 | 0.0668 | 7.393 | 0.0513 | 19.479 | 0.3840 | 3.2012 |
| 3 1/2 | 4.000 | 3.548 | 0.226 | 8 | 9.240 | 12.566 | 11.146 | 0.955 | 1.077 | 12.566 | 0.0873 | 9.887 | 0.0687 | 14.565 | 0.5136 | 4.2812 |
| 4 | 4.500 | 4.026 | 0.237 | 8 | 10.900 | 14.137 | 12.648 | 0.849 | 0.949 | 15.904 | 0.1104 | 12.730 | 0.0884 | 11.312 | 0.6613 | 5.5125 |
| 4 1/2 | 5.000 | 4.506 | 0.247 | 8 | 12.700 | 15.708 | 14.156 | 0.764 | 0.848 | 19.635 | 0.1364 | 15.947 | 0.1107 | 9.0301 | 0.8284 | 6.9053 |
| 5 | 5.563 | 5.047 | 0.258 | 8 | 14.900 | 17.477 | 15.856 | 0.687 | 0.757 | 24.306 | 0.1688 | 20.006 | 0.1389 | 7.1979 | 1.0393 | 8.6629 |
| 5 1/2 | 6.625 | 6.065 | 0.280 | 8 | 19.200 | 20.813 | 19.054 | 0.577 | 0.630 | 34.472 | 0.2394 | 28.890 | 0.2006 | 4.9844 | 1.5008 | 12.5101 |
| 6 | 7.625 | 7.023 | 0.301 | 8 | 23.800 | 23.955 | 22.063 | 0.501 | 0.544 | 45.664 | 0.3171 | 38.738 | 0.2690 | 3.7173 | 2.0124 | 16.7743 |
| 7 | 8.625 | 7.981 | 0.322 | 8 | 28.900 | 27.096 | 25.073 | 0.443 | 0.479 | 58.426 | 0.4057 | 50.027 | 0.3474 | 2.8784 | 2.5988 | 21.6627 |
| 8 | 9.625 | 8.941 | 0.342 | 8 | 34.300 | 30.238 | 28.089 | 0.397 | 0.427 | 72.760 | 0.5053 | 62.786 | 0.4360 | 2.2955 | 3.2616 | 27.1876 |
| 9 | 10.750 | 10.020 | 0.365 | 8 | 41.200 | 33.772 | 31.479 | 0.355 | 0.381 | 90.763 | 0.6303 | 78.854 | 0.5476 | 1.8262 | 4.0963 | 34.1456 |
| 10 | 11.750 | 11.000 | 0.375 | 8 | 46.400 | 36.914 | 34.558 | 0.325 | 0.347 | 108.434 | 0.7530 | 95.033 | 0.6600 | 1.5153 | 4.9368 | 41.1513 |
| 11 | 12.750 | 12.000 | 0.375 | 8 | 56.100 | 40.955 | 37.699 | 0.300 | 0.318 | 127.676 | 0.8666 | 113.097 | 0.7854 | 1.2732 | 5.8752 | 48.9735 |
| 12 | 14.000 | 13.250 | 0.375 | 8 | 66.700 | 44.982 | 41.626 | 0.273 | 0.288 | 153.938 | 1.0690 | 137.886 | 0.9575 | 1.0443 | 7.1629 | 59.7077 |
| 13 | 15.000 | 14.250 | 0.375 | 8 | 80.700 | 47.124 | 44.768 | 0.255 | 0.268 | 176.715 | 1.2272 | 159.485 | 1.1075 | 0.90291 | 8.2849 | 69.0603 |
| 14 | 16.000 | 15.250 | 0.375 | 8 | 94.900 | 50.265 | 47.909 | 0.239 | 0.250 | 201.062 | 1.3963 | 182.654 | 1.2684 | 0.78838 | 9.4885 | 79.0931 |
| 15 | 18.000 | 17.250 | 0.375 | 8 | 116.000 | 56.549 | 54.192 | 0.212 | 0.221 | 254.469 | 1.7671 | 233.705 | 1.6230 | 0.61616 | 12.1405 | 101.9920 |
| | 20.000 | 19.250 | 0.375 | | 138.000 | 62.832 | 60.476 | 0.191 | 0.198 | 314.159 | 2.1817 | 291.039 | 2.0211 | 0.49478 | 15.1189 | 126.0272 |
| | 22.000 | 21.250 | 0.375 | | 160.000 | 69.115 | 66.759 | 0.174 | 0.180 | 380.133 | 2.6398 | 354.656 | 2.4629 | 0.40603 | 18.4237 | 153.5736 |
| | 24.000 | 23.250 | 0.375 | | 194.600 | 75.398 | 73.042 | 0.159 | 0.164 | 452.389 | 3.1416 | 424.557 | 2.9483 | 0.33918 | 22.0549 | 183.8419 |

WROUGHT-IRON WELDED TUBES, EXTRA STRONG.

Standard Dimensions.
(National Tube Co., 1902.)

| Nominal Diameter. | Actual Outside Diameter. | Thickness, Extra Strong. | Thickness, Double Extra Strong. | Actual Inside Diameter, Extra Strong. | Actual Inside Diameter, Double Extra Strong. |
|-------------------|--------------------------|--------------------------|---------------------------------|---------------------------------------|--|
| Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| 1/8 | 0.405 | 0.100 | | 0.205 | |
| 1/4 | 0.54 | 0.123 | | 0.294 | |
| 3/8 | 0.675 | 0.127 | | 0.421 | |
| 1/2 | 0.84 | 0.149 | 0.298 | 0.542 | 0.244 |
| 3/4 | 1.05 | 0.157 | 0.314 | 0.736 | 0.422 |
| 1 | 1.315 | 0.182 | 0.364 | 0.951 | 0.587 |
| 1 1/4 | 1.66 | 0.194 | 0.388 | 1.272 | 0.884 |
| 1 1/2 | 1.9 | 0.203 | 0.406 | 1.494 | 1.088 |
| 2 | 2.375 | 0.221 | 0.442 | 1.933 | 1.491 |
| 2 1/2 | 2.875 | 0.280 | 0.560 | 2.315 | 1.755 |
| 3 | 3.5 | 0.304 | 0.608 | 2.892 | 2.284 |
| 3 1/2 | 4.0 | 0.321 | 0.642 | 3.358 | 2.716 |
| 4 | 4.5 | 0.341 | 0.682 | 3.818 | 3.136 |

STANDARD SIZES, ETC., OF LAP-WELDED CHARCOAL-IRON BOILER-TUBES.

(National Tube Co.)

| External Diameter. | Internal Diameter. | Standard Thickness. | Internal Circumference. | External Circumference. | Internal Area. | | External Area. | | Length of Tube per Sq. Ft. of Inside Surface. | Length of Tube per Sq. Ft. of Outside Surface. | Length of Tube per Sq. Ft. of Mean Surface. | Weight per Lineal Foot. |
|--------------------|--------------------|---------------------|-------------------------|-------------------------|----------------|---------|----------------|---------|---|--|---|-------------------------|
| in. | in. | in. | in. | in. | sq. in. | sq. ft. | sq. in. | sq. ft. | ft. | ft. | ft. | lb. |
| 1 | 0.810 | .095 | 2.545 | 3.142 | 0.515 | .0036 | 0.785 | .0055 | 4.479 | 3.820 | 4.149 | 0.90 |
| 1 1/4 | 1.060 | .095 | 3.330 | 3.927 | 0.882 | .0061 | 1.227 | .0085 | 3.604 | 3.056 | 3.330 | 1.15 |
| 1 1/2 | 1.310 | .095 | 4.115 | 4.712 | 1.348 | .0094 | 1.767 | .0123 | 2.916 | 2.547 | 2.732 | 1.40 |
| 1 3/4 | 1.560 | .095 | 4.901 | 5.498 | 1.911 | .0133 | 2.405 | .0167 | 2.448 | 2.183 | 2.316 | 1.65 |
| 2 | 1.810 | .095 | 5.686 | 6.283 | 2.573 | .0179 | 3.142 | .0218 | 2.110 | 1.910 | 2.010 | 1.91 |
| 2 1/4 | 2.060 | .095 | 6.472 | 7.069 | 3.333 | .0231 | 3.976 | .0276 | 1.854 | 1.698 | 1.776 | 2.16 |
| 2 1/2 | 2.282 | .109 | 7.169 | 7.854 | 4.090 | .0284 | 4.909 | .0341 | 1.674 | 1.528 | 1.601 | 2.75 |
| 2 3/4 | 2.532 | .109 | 7.955 | 8.639 | 5.035 | .0350 | 5.940 | .0412 | 1.508 | 1.389 | 1.449 | 3.04 |
| 3 | 2.782 | .109 | 8.740 | 9.425 | 6.079 | .0422 | 7.069 | .0491 | 1.373 | 1.273 | 1.322 | 3.33 |
| 3 1/4 | 3.010 | .120 | 9.456 | 10.210 | 7.116 | .0494 | 8.296 | .0576 | 1.269 | 1.175 | 1.222 | 3.96 |
| 3 1/2 | 3.260 | .120 | 10.242 | 10.996 | 8.347 | .0580 | 9.621 | .0668 | 1.172 | 1.091 | 1.132 | 4.28 |
| 3 3/4 | 3.510 | .120 | 11.027 | 11.781 | 9.676 | .0672 | 11.045 | .0767 | 1.088 | 1.019 | 1.054 | 4.60 |
| 4 | 3.732 | .134 | 11.724 | 12.566 | 10.939 | .0760 | 12.566 | .0873 | 1.024 | 0.955 | 0.990 | 5.47 |
| 4 1/2 | 4.232 | .134 | 13.295 | 14.137 | 14.066 | .0977 | 15.904 | .1104 | 0.903 | 0.849 | 0.876 | 6.17 |
| 5 | 4.704 | .148 | 14.778 | 15.708 | 17.379 | .1207 | 19.635 | .1364 | 0.812 | 0.764 | 0.788 | 7.58 |
| 6 | 5.670 | .165 | 17.813 | 18.850 | 25.250 | .1750 | 28.274 | .1963 | 0.674 | 0.637 | 0.656 | 10.16 |
| 7 | 6.670 | .165 | 20.954 | 21.991 | 34.942 | .2427 | 38.485 | .2673 | 0.573 | 0.546 | 0.560 | 11.90 |
| 8 | 7.670 | .165 | 24.096 | 25.133 | 46.204 | .3209 | 50.266 | .3491 | 0.498 | 0.477 | 0.488 | 13.65 |
| 9 | 8.640 | .180 | 27.143 | 28.274 | 58.630 | .4072 | 63.617 | .4418 | 0.442 | 0.424 | 0.433 | 16.76 |
| 10 | 9.594 | .203 | 30.141 | 31.416 | 72.292 | .5020 | 78.540 | .5454 | 0.398 | 0.382 | 0.390 | 21.00 |
| 11 | 10.560 | .220 | 33.175 | 34.558 | 87.583 | .6082 | 95.033 | .6600 | 0.362 | 0.347 | 0.355 | 25.00 |
| 12 | 11.542 | .229 | 36.260 | 37.699 | 104.629 | .7266 | 113.098 | .7854 | 0.331 | 0.318 | 0.325 | 28.50 |
| 13 | 12.524 | .238 | 39.345 | 40.841 | 123.190 | .8555 | 132.733 | .9217 | 0.305 | 0.294 | 0.300 | 32.06 |
| 14 | 13.504 | .248 | 42.424 | 43.982 | 143.224 | .9946 | 153.938 | 1.0690 | 0.283 | 0.273 | 0.278 | 36.00 |
| 15 | 14.482 | .259 | 45.497 | 47.124 | 164.721 | 1.1439 | 176.715 | 1.2272 | 0.264 | 0.255 | 0.260 | 40.60 |
| 16 | 15.458 | .271 | 48.563 | 50.266 | 187.671 | 1.3033 | 201.062 | 1.3963 | 0.247 | 0.239 | 0.243 | 45.20 |
| 17 | 16.432 | .284 | 51.623 | 53.407 | 212.066 | 1.4727 | 226.981 | 1.5763 | 0.232 | 0.225 | 0.229 | 49.90 |
| 18 | 17.416 | .292 | 54.714 | 56.549 | 238.225 | 1.6543 | 254.470 | 1.7671 | 0.219 | 0.212 | 0.216 | 54.82 |
| 19 | 18.400 | .300 | 57.805 | 59.690 | 265.905 | 1.8466 | 283.529 | 1.9690 | 0.208 | 0.201 | 0.205 | 59.48 |
| 20 | 19.360 | .320 | 60.821 | 62.832 | 294.375 | 2.0443 | 314.159 | 2.1817 | 0.197 | 0.191 | 0.194 | 66.77 |
| 21 | 20.320 | .340 | 63.837 | 65.974 | 324.294 | 2.2520 | 346.361 | 2.4053 | 0.188 | 0.182 | 0.185 | 73.40 |

In estimating the effective steam-heating or boiler surface of tubes, the surface in contact with air or gases of combustion (whether internal or external to the tubes) is to be taken.

For heating liquids by steam, superheating steam, or transferring heat from one liquid or gas to another, the mean surface of the tubes is to be taken.

Outside Area of Tubes.

To find the square feet of surface, S , in a tube of a given length, L , in feet, and diameter, d , in inches, multiply the length in feet by the diameter in inches and by 0.2618. Or, $S = \frac{3.1416 dL}{12} = 0.2618 dL$. For the diameters in the table below, multiply the length in feet by the figures given opposite the diameter.

| Inches, Diameter. | Square Feet per Foot Length. | Inches, Diameter. | Square Feet per Foot Length. | Inches, Diameter. | Square Feet per Foot Length. |
|----------------------|------------------------------------|----------------------|------------------------------------|----------------------|---------------------------------------|
| $\frac{1}{4}$ | 0.0654 | $2\frac{1}{4}$ | 0.5890 | 5 | 1.3090 |
| $\frac{1}{2}$ | .1309 | $2\frac{1}{2}$ | .6545 | 6 | 1.5708 |
| $\frac{3}{4}$ | .1963 | $2\frac{3}{4}$ | .7199 | 7 | 1.8326 |
| 1 | .2618 | 3 | .7854 | 8 | 2.0944 |
| $1\frac{1}{4}$ | .3272 | $3\frac{1}{4}$ | .8508 | 9 | 2.3562 |
| $1\frac{1}{2}$ | .3927 | $3\frac{1}{2}$ | .9163 | 10 | 2.6180 |
| $1\frac{3}{4}$ | .4581 | $3\frac{3}{4}$ | .9817 | 11 | 2.8798 |
| 2 | .5236 | 4 | 1.0472 | 12 | 3.1416 |

RIVETED IRON PIPE.

(Abendroth & Root Mfg. Co.)

Sheets punched and rolled, ready for riveting, are packed in convenient form for shipment. The following table shows the iron and rivets required for punched and formed sheets.

| Number Square Feet of Iron Required to Make 100 Lineal Feet Punched and Formed Sheets when put Together. | | | Approximate No. of Rivets 1 Inch apart Required for 100 Lineal Feet Punched and Formed Sheets. | Number Square Feet of Iron Required to Make 100 Lineal Feet Punched and Formed Sheets when put Together. | | | Approximate No. of Rivets 1 Inch apart Required for 100 Lineal Feet Punched and Formed Sheets. |
|--|----------------------------------|-----------------|--|--|----------------------------------|-----------------|--|
| Diam- eter in Inches. | Width of Lap in Inches. | Square Feet. | | Diam- eter in Inches. | Width of Lap in Inches. | Square Feet. | |
| 3 | 1 | 90 | 1600 | 14 | $1\frac{1}{2}$ | 397 | 2800 |
| 4 | 1 | 116 | 1700 | 15 | $1\frac{1}{2}$ | 423 | 2900 |
| 5 | $1\frac{1}{2}$ | 150 | 1800 | 16 | $1\frac{1}{2}$ | 452 | 3000 |
| 6 | $1\frac{1}{2}$ | 178 | 1900 | 18 | $1\frac{1}{2}$ | 506 | 3200 |
| 7 | $1\frac{1}{2}$ | 206 | 2000 | 20 | $1\frac{1}{2}$ | 562 | 3500 |
| 8 | $1\frac{1}{2}$ | 234 | 2200 | 22 | $1\frac{1}{2}$ | 617 | 3700 |
| 9 | $1\frac{1}{2}$ | 258 | 2300 | 24 | $1\frac{1}{2}$ | 670 | 3900 |
| 10 | $1\frac{1}{2}$ | 289 | 2400 | 26 | $1\frac{1}{2}$ | 725 | 4100 |
| 11 | $1\frac{1}{2}$ | 314 | 2500 | 28 | $1\frac{1}{2}$ | 779 | 4400 |
| 12 | $1\frac{1}{2}$ | 343 | 2600 | 30 | $1\frac{1}{2}$ | 836 | 4600 |
| 13 | $1\frac{1}{2}$ | 369 | 2700 | 36 | $1\frac{1}{2}$ | 998 | 5200 |

Weight and Strength of Riveted Hydraulic Pipe.

(Abner Doble Co., San Francisco, 1906.)

 S = Safe head in feet. W = Weight in pounds.

| Thickness. | | 4-in. | | 5-in. | | 6-in. | | 7-in. | | 8-in. | |
|------------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|
| Gauge. | In. | S | W | S | W | S | W | S | W | S | W |
| 18 | 0.050 | 555 | 2.8 | 444 | 3.5 | 370 | 4.1 | 317 | 4.7 | 277 | 5.3 |
| 16 | .062 | 693 | 3.7 | 555 | 4.4 | 462 | 5.2 | 396 | 5.9 | 346 | 6.7 |
| 14 | .078 | 866 | 4.4 | 693 | 5.5 | 578 | 6.4 | 495 | 7.3 | 433 | 8.2 |
| 12 | .109 | | | | | 808 | 8.8 | 693 | 10.0 | 606 | 11.5 |
| 10 | .140 | | | | | | | | | 777 | 14.5 |
| | | 9-in. | | 10-in. | | 11-in. | | 12-in. | | 14-in. | |
| 16 | 0.062 | 308 | 7.5 | 277 | 8.3 | 252 | 9.0 | 231 | 9.9 | 198 | 11.4 |
| 14 | .078 | 385 | 9.2 | 346 | 10.2 | 314 | 11.0 | 289 | 12.2 | 248 | 14.0 |
| 12 | .109 | 539 | 12.6 | 485 | 14.2 | 439 | 15.2 | 404 | 16.7 | 346 | 19.2 |
| 10 | .140 | 693 | 16.4 | 623 | 18.0 | 565 | 19.3 | 519 | 21.0 | 445 | 24.2 |
| 8 | .171 | | | 761 | 21.5 | 693 | 23.5 | 635 | 25.6 | 543 | 29.5 |
| | 3/16 | | | 832 | 23.5 | 757 | 25.5 | 693 | 27.7 | 594 | 31.9 |
| | | 15-in. | | 16-in. | | 18-in. | | 20-in. | | 22-in. | |
| 16 | 0.062 | 185 | 12.0 | 173 | 12.8 | 154 | 14.5 | 139 | 16.0 | 126 | 17.7 |
| 14 | .078 | 231 | 14.0 | 217 | 16.0 | 193 | 17.8 | 173 | 19.6 | 157 | 21.2 |
| 12 | .109 | 323 | 20.3 | 303 | 21.5 | 270 | 24.4 | 242 | 27.3 | 220 | 29.2 |
| 10 | .140 | 415 | 25.7 | 388 | 27.3 | 346 | 30.7 | 311 | 34.5 | 283 | 37.1 |
| 8 | .171 | 507 | 30.4 | 475 | 33.3 | 422 | 38.4 | 380 | 41.5 | 346 | 45.2 |
| | 3/16 | 555 | 34.0 | 520 | 36.0 | 462 | 40.5 | 416 | 45.0 | 378 | 49.0 |
| | 1/4 | 739 | 45.5 | 693 | 48.2 | 616 | 54.1 | 555 | 59.6 | 505 | 65.5 |
| | 5/16 | | | 866 | 60.6 | 770 | 67.7 | 693 | 74.6 | 631 | 81.5 |
| | 3/8 | | | | | 924 | 81.3 | 831 | 89.5 | 757 | 98.0 |
| | 7/16 | | | | | | | 970 | 105.0 | 883 | 114.5 |
| | | 24-in. | | 26-in. | | 30-in. | | 36-in. | | 42-in. | |
| 14 | 0.078 | 144 | 23.7 | 133 | 25.5 | | | | | | |
| 12 | .109 | 202 | 32.5 | 186 | 34.5 | 162 | 39.5 | 134 | 47.7 | | |
| 10 | .140 | 259 | 40.5 | 239 | 43.7 | 208 | 50.3 | 173 | 60.0 | 148 | 69.5 |
| 8 | .171 | 317 | 49.2 | 293 | 53.0 | 254 | 60.5 | 211 | 75.0 | 181 | 84.7 |
| | 3/16 | 346 | 53.0 | 320 | 57.5 | 277 | 65.5 | 231 | 79.0 | 198 | 91.5 |
| | 1/4 | 462 | 71.0 | 427 | 76.5 | 370 | 87.5 | 308 | 105.5 | 264 | 122.0 |
| | 5/16 | 578 | 88.5 | 533 | 95.5 | 462 | 109.0 | 385 | 130.0 | 330 | 151.0 |
| | 3/8 | 693 | 106.0 | 640 | 114.5 | 555 | 130.5 | 462 | 156.0 | 396 | 180.5 |
| | 7/16 | 808 | 124.5 | 747 | 134.0 | 647 | 151.5 | 539 | 182.5 | 462 | 211.0 |
| | 1/2 | 924 | 142.0 | 854 | 153.0 | 739 | 174.5 | 616 | 207.0 | 528 | 240.5 |
| | 5/8 | | | 1066 | 191.0 | 924 | 220.0 | 770 | 260.0 | 660 | 302.0 |
| | 3/4 | | | | | 1108 | 264.0 | 924 | 312.5 | 792 | 361.5 |
| | 7/8 | | | | | | | 1078 | 366.0 | 924 | 424.0 |
| | | 48-in. | | 54-in. | | 60-in. | | 66-in. | | 72-in. | |
| 8 | 0.171 | 158 | 98.0 | 141 | 110.0 | 127 | 121.0 | | | | |
| | 3/16 | 173 | 106.0 | 154 | 119.0 | 139 | 131.0 | 127 | 144.5 | 115 | 158.0 |
| | 1/4 | 231 | 142.0 | 205 | 159.0 | 185 | 175.0 | 168 | 193.0 | 154 | 211.0 |
| | 5/16 | 289 | 177.0 | 256 | 198.0 | 231 | 218.0 | 210 | 239.0 | 193 | 260.0 |
| | 3/8 | 346 | 212.0 | 308 | 237.0 | 277 | 261.0 | 252 | 286.5 | 231 | 312.0 |
| | 7/16 | 404 | 249.0 | 359 | 277.5 | 323 | 303.0 | 294 | 334.0 | 270 | 365.0 |
| | 1/2 | 462 | 284.0 | 411 | 316.5 | 370 | 349.0 | 336 | 382.0 | 308 | 414.0 |
| | 5/8 | 578 | 354.0 | 513 | 399.5 | 462 | 440.0 | 420 | 480.0 | 385 | 520.0 |
| | 3/4 | 693 | 430.0 | 616 | 479.5 | 555 | 528.0 | 504 | 577.5 | 462 | 624.0 |
| | 7/8 | 808 | 505.0 | 719 | 563.5 | 647 | 620.0 | 588 | 677.0 | 539 | 732.0 |
| | 1 | 924 | 582.0 | 822 | 647.5 | 739 | 712.0 | 672 | 777.5 | 616 | 840.0 |

Pipe made of sheet steel plate, ultimate tensile strength 55,000 lbs. per sq. in., double-riveted longitudinal joints and single-riveted circular joints. Strength of longitudinal joints, 70%. Strain by safe pressure, 1/4 of ultimate strength.

WEIGHT OF ONE SQUARE FOOT OF SHEET-IRON FOR RIVETED PIPE.

Thickness by the Birmingham Wire-Gauge.

| No. of Gauge. | Thick-ness, In. | Weight in Lbs., Black. | Weight in Lbs., Galvan-ized. | No. of Gauge. | Thick-ness, In. | Weight in Lbs., Black. | Weight in Lbs., Galvan-ized. |
|---------------|-----------------|------------------------|------------------------------|---------------|-----------------|------------------------|------------------------------|
| 26 | 0.018 | 0.80 | 0.91 | 18 | 0.049 | 1.82 | 2.16 |
| 24 | .022 | 1.00 | 1.16 | 16 | .065 | 2.50 | 2.67 |
| 22 | .028 | 1.25 | 1.40 | 14 | .083 | 3.12 | 3.34 |
| 20 | .035 | 1.56 | 1.67 | 12 | .109 | 4.37 | 4.73 |

SPIRAL RIVETED PIPE.

Approximate Bursting Strength. Pounds per Square Inch.

(American Spiral Pipe Works.)

| Inside Diam. Inches. | Thickness. — U. S. Standard Gauge. | | | | | | | | |
|----------------------|------------------------------------|---------|---------|---------|---------|---------|--------|--------|---------------|
| | No. 20. | No. 18. | No. 16. | No. 14. | No. 12. | No. 10. | No. 8. | No. 6. | No. 3 (1/4"). |
| 3 | 1500 | 2000 | | | | | | | |
| 4 | 1125 | 1500 | 1875 | | | | | | |
| 5 | 900 | 1200 | 1500 | | | | | | |
| 6 | | 1000 | 1250 | 1560 | 2170 | | | | |
| 7 | | 860 | 1070 | 1340 | 1860 | | | | |
| 8 | | 750 | 935 | 1170 | 1640 | | | | |
| 9 | | | 835 | 1045 | 1460 | | | | |
| 10 | | | 750 | 935 | 1310 | | | | |
| 11 | | | 680 | 850 | 1200 | | | | |
| 12 | | | 625 | 780 | 1080 | 1410 | | | |
| 13 | | | 575 | 720 | 1010 | 1295 | | | |
| 14 | | | 535 | 670 | 940 | 1210 | | | |
| 15 | | | | 625 | 875 | 1125 | | | |
| 16 | | | | 585 | 820 | 1050 | 1290 | 1520 | 1880 |
| 18 | | | | 520 | 730 | 940 | 1140 | 1360 | 1660 |
| 20 | | | | 470 | 660 | 840 | 1030 | 1220 | 1500 |
| 22 | | | | 425 | 595 | 765 | 940 | 1108 | 1364 |
| 24 | | | | 390 | 540 | 705 | 820 | 1015 | 1250 |
| 26 | | | | | 505 | 650 | 795 | 935 | 1154 |
| 28 | | | | | 470 | 605 | 735 | 870 | 1071 |
| 30 | | | | | 435 | 560 | 685 | 810 | 1000 |
| 32 | | | | | 410 | 525 | 645 | 760 | 940 |
| 34 | | | | | 380 | 490 | 600 | 715 | 880 |
| 36 | | | | | 365 | 470 | 570 | 680 | 830 |
| 40 | | | | | 330 | 420 | 515 | 610 | 750 |

FORGED STEEL FLANGES FOR RIVETED PIPE.

(American Spiral Pipe Works.)

| Nominal Size. Inches. | Outside Diameter. Inches. | Bore. Inches. | Bolt Cir- cle. Inches. | Number of Bolts. | Size of Bolts. | Nominal Size. Inches. | Outside Diameter. Inches. | Bore. Inches. | Bolt Cir- cle. Inches. | Number of Bolts. | Size of Bolts. |
|-----------------------------|---------------------------------|------------------|------------------------------|---------------------|-------------------|-----------------------------|---------------------------------|------------------|------------------------------|---------------------|-------------------|
| 3 | 6 | 33/16 | 4 3/4 | 4 | 7/16 | 16 | 21 1/4 | 16 1/4 | 19 1/4 | 12 | 1 1/2 |
| 4 | 7 | 43/16 | 5 15/16 | 8 | 7/16 | 18 | 23 1/4 | 18 5/16 | 21 1/4 | 16 | 5/8 |
| 5 | 8 | 53/16 | 6 13/16 | 8 | 7/16 | 20 | 25 1/4 | 20 5/16 | 23 1/8 | 16 | 5/8 |
| 6 | 9 | 63/16 | 7 7/8 | 8 | 1 1/2 | 22 | 28 1/4 | 22 3/8 | 26 | 16 | 5/8 |
| 7 | 10 | 73/16 | 9 | 8 | 1 1/2 | 24 | 30 | 24 3/8 | 27 3/4 | 16 | 5/8 |
| 8 | 11 | 83/16 | 10 | 8 | 1 1/2 | 26 | 32 | 26 3/8 | 29 3/4 | 24 | 3/4 |
| 9 | 13 | 9 1/4 | 11 1/4 | 8 | 1 1/2 | 28 | 34 | 28 3/8 | 31 3/4 | 28 | 3/4 |
| 10 | 14 | 10 1/4 | 12 1/4 | 8 | 1 1/2 | 30 | 36 | 30 3/8 | 33 3/4 | 28 | 3/4 |
| 11 | 15 | 11 1/4 | 13 3/8 | 12 | 1 1/2 | 32 | 38 | 32 3/8 | 35 3/4 | 28 | 3/4 |
| 12 | 16 | 12 1/4 | 14 1/4 | 12 | 1 1/2 | 34 | 40 | 34 3/8 | 37 3/4 | 28 | 3/4 |
| 13 | 17 | 13 1/4 | 15 1/4 | 12 | 1 1/2 | 36 | 42 | 36 3/8 | 39 3/4 | 32 | 3/4 |
| 14 | 18 | 14 1/4 | 16 1/4 | 12 | 1 1/2 | 40 | 46 | 40 3/8 | 43 3/4 | 32 | 3/4 |
| 15 | 19 | 15 1/4 | 17 7/16 | 12 | 1 1/2 | | | | | | |

BENT AND COILED PIPES.

(National Pipe Bending Co., New Haven, Conn.)

Coils and Bends of Iron and Steel Pipe.

| | | | | | | | | | | |
|--|-------|-------|-------|-------|----|-------|-------|-----|-------|-----|
| Size of pipe.....Inches | 1/4 | 3/8 | 1/2 | 3/4 | 1 | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 |
| Least outside diameter of coil.....Inches | 2 | 2 1/2 | 3 1/2 | 4 1/2 | 6 | 8 | 12 | 16 | 24 | 32 |
| <hr/> | | | | | | | | | | |
| Size of pipe.....Inches | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |
| Least outside diameter of coil.....Inches | 40 | 48 | 52 | 58 | 66 | 80 | 92 | 105 | 130 | 156 |

Lengths continuous welded up to 3-in. pipe or coupled as desired.

Coils and Bends of Drawn Brass and Copper Tubing.

| | | | | | | | | |
|--|-------|-------|-------|-------|-------|-------|-------|-------|
| Size of tube, outside diameter..Inches | 1/4 | 3/8 | 1/2 | 5/8 | 3/4 | 1 | 1 1/4 | 1 3/8 |
| Least outside diameter of coil..Inches | 1 | 1 1/2 | 2 | 2 1/2 | 3 | 4 | 6 | 7 |
| <hr/> | | | | | | | | |
| Size of tube, outside diameter..Inches | 1 1/2 | 1 5/8 | 1 3/4 | 2 | 2 1/4 | 2 3/8 | 2 1/2 | 2 3/4 |
| Least outside diameter of coil..Inches | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |

Lengths continuous brazed, soldered, or coupled as desired.

90° Bends in Iron or Steel Pipe.

(Whitlock Coil Pipe Co., Hartford, Conn.)

| | | | | | | | | | | | |
|----------------------|----|--------|--------|-------|----|----|----|----|----|----|----|
| Size pipe, I.D..... | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |
| Radius of bend | 12 | 13 | 15 | 17 | 20 | 23 | 26 | 30 | 36 | 42 | 48 |
| End..... | 3 | 3 1/2 | 3 1/2 | 4 | 4 | 4 | 5 | 5 | 5 | 6 | 6 |
| Center to face..... | 15 | 16 1/2 | 18 1/2 | 21 | 24 | 27 | 31 | 35 | 41 | 48 | 54 |

| | | | | | | | | | | | |
|----------------------|----|----|----|----|-----|-----|-----|-----|-----|--|--|
| Size pipe, O.D..... | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | | |
| Radius of bend | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 140 | 160 | | |
| End..... | 7 | 7 | 7 | 8 | 8 | 8 | 10 | 10 | 10 | | |
| Center to face..... | 67 | 77 | 87 | 98 | 108 | 118 | 130 | 150 | 170 | | |

The radii given are for the center of the pipe. "End" means the length of straight pipe, in addition to the 90° bend, at each end of the pipe. "Center to face" means the perpendicular distance from the center of one end of the bent pipe to a plane passing across the other end.

Flexibility of Pipe Bends. (*Valve World*, Feb., 1906.)—So far as can be ascertained, no thorough attempt has ever been made to determine the maximum amount of expansion which a U-loop, or quarter bend, would take up in a straight run of pipe having both ends anchored. The Crane Company have adopted five diameters of the pipe as a standard radius, which come nearer than any other to suiting average requirements, and at the same time produce a symmetrical article. Bends shorter than this can be made, but they are extremely stiff, tend to buckle in bending, and the metal in the outer wall is stretched beyond a desirable point.

In 1905 the Crane Company made a few experiments with 8-inch U and quarter bends to ascertain the amount of expansion they would take up. The U-bend was made of steel pipe 0.32 inch thick, weighing 28 lbs. per foot, with extra heavy cast-iron flanges screwed on and refaced. It was connected by elbows to two straight pipes, *N*, 67 ft., *S*, 82 ft., which were firmly anchored at their outer ends. Steam was then let into the pipes with results as follows:

| | | | | |
|----------|---|--|-------------------|--------------------------|
| 80 lbs. | Expansion, | | Total 1 7/8 in. | Flange broke. |
| 50 lbs. | Expansion, <i>N</i> , 7/8, <i>S</i> , 11/8. | | Total 2 in. | |
| 100 lbs. | Expansion, <i>N</i> , 13/16, <i>S</i> , 11/2. | | Total 2 11/16 in. | |
| 150 lbs. | Expansion, <i>N</i> , 11/8, <i>S</i> , 17/8. | | Total 3 in. | |
| 200 lbs. | Expansion, <i>N</i> , 11/2, <i>S</i> , 17/8. | | Total 3 3/8 in. | Flange broke at 208 lbs. |

Quarter bend, full weight pipe. Straight pipe 148 ft., one end. 80 lbs. Total expansion 1 3/8. Flange leaked.

Quarter bend, extra heavy pipe. Expanded 7/8 in. when a flange broke. Replaced with a new flange, which broke when the expansion was 1 1/8 in.

SEAMLESS BRASS TUBE, IRON-PIPE SIZES.

(For actual dimensions see tables of Wrought-iron Pipe.)

| Nominal Size. | Weight per Foot. | Nom. Size. | Weight per Foot. | Nom. Size. | Weight per Foot. | Nom. Size. | Weight per Foot. |
|---------------|------------------|------------|------------------|------------|------------------|------------|------------------|
| ins. | lbs. | ins. | lbs. | ins. | lbs. | ins. | lbs. |
| 1/8 | .25 | 3/4 | 1.25 | 2 | 4.0 | 4 | 12.70 |
| 1/4 | .43 | 1 | 1.70 | 2 1/2 | 5.75 | 4 1/2 | 13.90 |
| 3/8 | .62 | 1 1/4 | 2.50 | 3 | 8.30 | 5 | 15.75 |
| 1/2 | .90 | 1 1/2 | 3. | 3 1/2 | 10.90 | 6 | 18.31 |

WEIGHT PER FOOT OF SEAMLESS BRASS TUBES.

(Waterbury Brass Co., 1908.)

| A.W.G. 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | |
|----------|--------|--------|--------|--------|---------|---------|--------|---------|---------|---------|-------|--------|
| In.* | .20431 | .16202 | .12849 | .10189 | .080808 | .064084 | .05082 | .040303 | .031961 | .025347 | .0201 | .01594 |
| In.† | | | | | | | | | | | | |
| 1/8 | ... | ... | ... | ... | ... | 0.043 | 0.039 | 0.034 | 0.028 | 0.024 | 0.020 | |
| 3/16 | ... | ... | ... | ... | 0.090 | .08 | .068 | .057 | .047 | .038 | .032 | |
| 1/4 | ... | ... | ... | 0.174 | 0.16 | .14 | .12 | .097 | .080 | .065 | .053 | .043 |
| 5/16 | ... | ... | ... | .25 | .22 | .18 | .15 | .13 | .104 | .084 | .067 | .054 |
| 3/8 | ... | ... | 0.36 | .32 | .27 | .23 | .19 | .15 | .126 | .102 | .082 | .066 |
| 1/2 | ... | 0.63 | .55 | .47 | .39 | .32 | .26 | .21 | .17 | .139 | .111 | .089 |
| 5/8 | 0.99 | .87 | .74 | .61 | .51 | .42 | .34 | .27 | .22 | .174 | .140 | .112 |
| 3/4 | 1.29 | 1.10 | .92 | .76 | .62 | .51 | .41 | .33 | .26 | .211 | .169 | .135 |
| 7/8 | 1.58 | 1.33 | 1.11 | .91 | .74 | .60 | .48 | .39 | .31 | .248 | .198 | .158 |
| 1 | 1.88 | 1.57 | 1.29 | 1.06 | .86 | .69 | .56 | .45 | .36 | .283 | .227 | .181 |
| 1 1/8 | 2.17 | 1.80 | 1.48 | 1.20 | .97 | .79 | .63 | .50 | .40 | .321 | .256 | ... |
| 1 1/4 | 2.47 | 2.03 | 1.66 | 1.35 | 1.09 | .88 | .70 | .56 | .45 | .358 | .285 | ... |
| 1 3/8 | 2.76 | 2.27 | 1.85 | 1.50 | 1.21 | .97 | .78 | .62 | .50 | .395 | .314 | ... |
| 1 1/2 | 3.05 | 2.50 | 2.03 | 1.64 | 1.32 | 1.06 | .85 | .68 | .54 | .43 | .343 | ... |
| 1 3/4 | 3.64 | 2.97 | 2.40 | 1.94 | 1.56 | 1.25 | 1.00 | .79 | .63 | .50 | .401 | ... |
| 2 | 4.23 | 3.44 | 2.77 | 2.23 | 1.79 | 1.43 | 1.14 | .91 | .73 | .58 | .459 | ... |

| A.W.G. 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | |
|----------|--------|--------|--------|--------|--------|---------|---------|--------|---------|---------|---------|--------|
| In.* | .25763 | .20431 | .16202 | .12849 | .10189 | .080808 | .064084 | .05082 | .040303 | .031961 | .025347 | .01594 |
| In.† | | | | | | | | | | | | |
| 21¼ | 5.92 | 4.82 | 3.90 | 3.15 | 2.53 | 2.02 | 1.62 | 1.29 | 1.03 | 0.82 | 0.65 | ... |
| 21½ | 6.67 | 5.41 | 4.37 | 3.52 | 2.82 | 2.26 | 1.80 | 1.44 | 1.14 | .91 | .73 | ... |
| 23¼ | 7.41 | 6.00 | 4.84 | 3.89 | 3.11 | 2.49 | 1.99 | 1.58 | 1.26 | 1.00 | .80 | ... |
| 3 | 8.16 | 6.59 | 5.31 | 4.26 | 3.41 | 2.72 | 2.17 | 1.73 | 1.38 | 1.09 | .87 | ... |
| 31¼ | 8.90 | 7.18 | 5.77 | 4.63 | 3.70 | 2.96 | 2.36 | 1.88 | 1.49 | 1.19 | .94 | ... |
| 31½ | 9.64 | 7.77 | 6.24 | 5.00 | 4.00 | 3.19 | 2.54 | 2.02 | 1.61 | 1.28 | 1.02 | ... |
| 33¼ | 10.39 | 8.36 | 6.71 | 5.37 | 4.29 | 3.42 | 2.73 | 2.17 | 1.72 | 1.37 | 1.09 | ... |
| 4 | 11.13 | 8.95 | 7.18 | 5.74 | 4.58 | 3.66 | 2.91 | 2.32 | 1.84 | 1.46 | 1.16 | ... |
| 41¼ | 11.87 | 9.54 | 7.64 | 6.11 | 4.88 | 3.89 | 3.10 | 2.46 | 1.96 | 1.55 | ... | ... |
| 41½ | 12.62 | 10.13 | 8.11 | 6.48 | 5.17 | 4.12 | 3.28 | 2.61 | 2.07 | 1.64 | ... | ... |
| 43¼ | 13.36 | 10.72 | 8.58 | 6.85 | 5.47 | 4.36 | 3.47 | 2.76 | 2.19 | 1.74 | ... | ... |
| 5 | 14.10 | 11.31 | 9.05 | 7.22 | 5.76 | 4.59 | 3.65 | 2.90 | 2.31 | 1.83 | ... | ... |
| 51¼ | 14.85 | 11.90 | 9.51 | 7.59 | 6.05 | 4.82 | 3.84 | 3.05 | 2.42 | ... | ... | ... |
| 51½ | 15.59 | 12.49 | 9.98 | 7.97 | 6.35 | 5.06 | 4.02 | 3.20 | 2.54 | ... | ... | ... |
| 53¼ | 16.33 | 13.08 | 10.45 | 8.34 | 6.64 | 5.29 | 4.21 | 3.34 | 2.65 | ... | ... | ... |
| 6 | 17.08 | 13.67 | 10.92 | 8.71 | 6.94 | 5.52 | 4.39 | 3.49 | 2.77 | ... | ... | ... |
| 61¼ | 17.82 | 14.26 | 11.38 | 9.08 | 7.23 | 5.76 | 4.58 | 3.64 | ... | ... | ... | ... |
| 61½ | 18.56 | 14.84 | 11.85 | 9.45 | 7.52 | 5.99 | 4.76 | 3.78 | ... | ... | ... | ... |
| 63¼ | 19.31 | 15.43 | 12.32 | 9.82 | 7.82 | 6.22 | 4.95 | 3.93 | ... | ... | ... | ... |
| 7 | 20.05 | 16.02 | 12.79 | 10.19 | 8.11 | 6.46 | 5.13 | 4.08 | ... | ... | ... | ... |
| 71¼ | 20.79 | 16.61 | 13.25 | 10.56 | 8.41 | 6.69 | 5.32 | ... | ... | ... | ... | ... |
| 71½ | 21.54 | 17.20 | 13.72 | 10.93 | 8.70 | 6.92 | 5.50 | ... | ... | ... | ... | ... |
| 73¼ | 22.28 | 17.79 | 14.19 | 11.30 | 8.99 | 7.15 | 5.69 | ... | ... | ... | ... | ... |
| 8 | 23.03 | 18.48 | 14.66 | 11.67 | 9.29 | 7.39 | 5.87 | ... | ... | ... | ... | ... |

* Thickness of Wall.

† Outside diameter.

Seamless brass tubes are made from 1/8 in. to 1 in. outside diameter, varying by 1/16 in., and from 1 1/8 in. to 8 in. outside diameter, varying by 1/8 in., and in all gauges from No. 2 to No. 26 A. W. G. within the limits of the above table. To determine the weight per foot of a tube of a given inside diameter, add to the weights given above the weights given below, under the corresponding gauge numbers.

For copper tubing add 5% to the weights given above.

| A.W.G. | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Lb. per ft. | 1.532 | .9637 | .6061 | .3811 | .2397 | .1507 | .0948 | .0596 | .0375 | .0236 | .0148 | .0093 | .0059 |

LEAD AND TIN LINED LEAD PIPE.

(United Lead Co., New York, 1908.)

| Cali-ber. | Letter. | Weight per Foot and Rod. | Thickness in 1/100 In. | Cali-ber. | Letter. | Weight per Foot. | Thickness in 1/100 In. |
|-----------|---------|--------------------------|------------------------|-----------|---------|---------------------|------------------------|
| 3/8 in. | E | 7 lbs. per rod | 5 | 1 in. | E | 1 1/2 lbs. per foot | 10 |
| | D | 10 oz. per foot | 6 | | D | 2 " " " | 11 |
| | C | 12 " " " | 8 | | C | 2 1/2 " " " | 14 |
| | B | 1 lb. " " " | 12 | | B | 3 1/4 " " " | 17 |
| | A | 1 1/4 " " " | 16 | | A | 4 " " " | 21 |
| | AA | 1 1/2 " " " | 19 | | AA | 4 3/4 " " " | 24 |
| | AAA | 1 3/4 " " " | 27 | | AAA | 6 " " " | 30 |
| 7/16 in. | | 13 oz. " " " | | 1 1/4 in. | E | 2 " " " | 10 |
| | | 1 lb. " " " | | | D | 2 1/2 " " " | 12 |
| 1/2 in. | E | 9 lbs. per rod | 7 | | C | 3 " " " | 14 |
| | D | 3 1/4 lb. per foot | 9 | | B | 3 3/4 " " " | 16 |
| | C | 1 " " " | 11 | | A | 4 3/4 " " " | 19 |
| | B | 1 1/4 " " " | 13 | | AA | 5 3/4 " " " | 25 |
| | Spe'l | 1 1/2 " " " | 14 | | AAA | 6 3/4 " " " | 28 |
| | A | 1 3/4 " " " | 16 | 1 1/2 in. | E | 3 " " " | 12 |
| | AA | 2 " " " | 19 | | D | 3 1/2 " " " | 14 |
| | Spe'l | 2 1/2 " " " | 23 | | C | 4 1/4 " " " | 17 |
| | AAA | 3 " " " | 25 | | B | 5 " " " | 19 |
| 5/8 in. | E | 12 " per rod | 8 | | A | 6 1/2 " " " | 23 |
| | D | 1 " per foot | 9 | | AA | 7 1/2 " " " | 25 |
| | C | 1 1/2 " " " | 13 | | Spe'l | 8 " " " | 27 |
| | B | 2 " " " | 16 | | AAA | 8 1/2 " " " | 28 |
| | A | 2 1/2 " " " | 20 | 1 3/4 in. | D | 4 " " " | 13 |
| | AA | 2 3/4 " " " | 22 | | C | 5 " " " | 17 |
| | AAA | 3 1/2 " " " | 25 | | B | 6 " " " | 19 |
| 3/4 in. | E | 1 " " " | 8 | | Spe'l | 6 1/2 " " " | 21 |
| | D | 1 1/4 " " " | 10 | | A | 7 " " " | 23 |
| | C | 1 3/4 " " " | 12 | | AA | 8 1/2 " " " | 27 |
| | Spe'l | 2 " " " | 14 | | AAA | 10 " " " | 30 |
| | B | 2 1/4 " " " | 16 | 2 in. | D | 4 3/4 " " " | 15 |
| | A | 3 " " " | 20 | | C | 6 " " " | 18 |
| | AA | 3 1/2 " " " | 23 | | B | 7 " " " | 22 |
| | AAA | 4 3/4 " " " | 30 | | A | 8 " " " | 25 |
| | | | | | AA | 9 " " " | 27 |
| | | | | | AAA | 11 3/4 " " " | 30 |

WEIGHT OF LEAD PIPE WHICH SHOULD BE USED FOR A GIVEN HEAD OF WATER.

(United Lead Co., New York, 1908.)

| Head or Number of Feet Fall. | Pres-sure per sq. inch. | Caliber and Weight per Foot. | | | | | | |
|------------------------------|-------------------------|------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| | | Letter. | 3/8 inch. | 1/2 inch. | 5/8 inch. | 3/4 inch. | 1 inch. | 1 1/4 in. |
| 30 ft. | 15 lb. | D | 10 oz. | 3/4 lb. | 1 lb. | 1 1/4 lb. | 2 lb. | 2 1/2 lb. |
| 50 ft. | 25 lb. | C | 12 oz. | 1 lb. | 1 1/2 lb. | 1 3/4 lb. | 2 1/2 lb. | 3 lb. |
| 75 ft. | 38 lb. | B | 1 lb. | 1 1/4 lb. | 2 lb. | 2 1/4 lb. | 3 1/4 lb. | 3 3/4 lb. |
| 100 ft. | 50 lb. | A | 1 1/4 lb. | 1 3/4 lb. | 2 1/2 lb. | 3 lb. | 4 lb. | 4 3/4 lb. |
| 150 ft. | 75 lb. | AA | 1 1/2 lb. | 2 lb. | 2 3/4 lb. | 3 1/2 lb. | 4 3/4 lb. | 5 3/4 lb. |
| 200 ft. | 100 lb. | AAA | 1 3/4 lb. | 3 lb. | 3 1/2 lb. | 4 3/4 lb. | 6 lb. | 6 3/4 lb. |

To find the thickness of lead pipe required when the head of water is given. (Chadwick Lead Works.)

RULE. — Multiply the head in feet by size of pipe wanted, expressed decimally, and divide by 750; the quotient will be the thickness required, in one-hundredths of an inch.

EXAMPLE. — Required thickness of half-inch pipe for a head of 25 feet.

$$25 \times 0.50 \div 750 = 0.16 \text{ inch.}$$

LEAD WASTE-PIPE.

| | |
|--------------------------------------|-------------------------------------|
| 1 1/2 in., 2 and 3 pounds per foot. | 4 in., 5, 6, and 8 pounds per foot. |
| 2 " 3 and 4 pounds per foot. | 4 1/2 " 6 and 8 pounds per foot. |
| 3 " 3 1/2, 5, and 6 pounds per foot. | 5 " 8, 10, and 12 pounds per foot. |
| 3 1/2 " 4 pounds per foot. | 6 " 12 pounds per foot. |

COMMERCIAL SIZES OF LEAD AND TIN TUBING.

1/8 inch.

1/4 inch.

SHEET LEAD.

Weight per square foot, 2 1/2, 3, 3 1/2, 4, 4 1/2, 5, 6, 8, 9, 10 lb. and upwards. Other weights rolled to order.

BLOCK-TIN PIPE.

| | |
|--------------------------------------|-----------------------------------|
| 3/8 in., 4, 5, 6 and 8 oz. per foot. | 1 in., 15 and 18 oz. per foot. |
| 1/2 " 6, 7 1/2 and 10 " " " | 1 1/4 " 11 1/4 and 11 1/2 lb. " " |
| 5/8 " 8 and 10 " " " | 1 1/2 " 2 and 2 1/2 lb. " " |
| 3/4 " 10 and 12 " " " | 2 " 2 1/2 and 3 lb. " " |

TIN-LINED AND LEAD-LINED IRON PIPE.

Iron and steel pipes are frequently lined with tin or lead for use as water service pipes, ventilation pipes, and for carrying corrosive liquids. See catalogue of Lead Lined Iron Pipe Co., Wakefield, Mass.

WOODEN STAVE PIPE.

Pipes made of wooden staves, banded with steel hoops, are made by the Excelsior Wooden Pipe Co., San Francisco, in sizes from 10 inches to 10 feet in diameter, and are extensively used for long-distance piping, especially in the Western States. The hoops are made of steel rods with upset and threaded ends. When buried below the hydraulic grade line and kept full of water, these pipes are practically indestructible. For the economic design and use of stave pipe see paper by A. L. Adams, *Trans. A.S.C.E.*, vol. xli.

WEIGHT PER FT. OF COPPER RODS, LB.

(Waterbury Brass Co., 1908.)

| In. | Round. | Square. | In. | Round. | Square. | In. | Round. | Square. |
|-----|--------|---------|-------|--------|---------|-------|--------|---------|
| 1/8 | 0.047 | 0.060 | 1 1/8 | 3.831 | 4.88 | 2 1/8 | 13.668 | 17.42 |
| 1/4 | .189 | .241 | 1 1/4 | 4.723 | 6.01 | 2 1/4 | 15.325 | 19.51 |
| 3/8 | .426 | .542 | 1 3/8 | 5.723 | 7.24 | 2 3/8 | 17.075 | 21.74 |
| 1/2 | .757 | .964 | 1 1/2 | 6.811 | 8.67 | 2 1/2 | 18.916 | 24.09 |
| 5/8 | 1.182 | 1.51 | 1 5/8 | 7.993 | 10.18 | 2 5/8 | 20.856 | 26.56 |
| 3/4 | 1.703 | 2.17 | 1 3/4 | 9.27 | 11.80 | 2 3/4 | 22.891 | 29.05 |
| 7/8 | 2.318 | 2.95 | 1 7/8 | 10.642 | 13.55 | 2 7/8 | 25.019 | 31.86 |
| 1 | 3.03 | 3.86 | 2 | 12.108 | 15.42 | 3 | 27.243 | 34.69 |

To find the weight of octagon rod, multiply the weight of round rod by 1.084.

To find the weight of hexagon rod, multiply the weight of round rod by 1.12.

WEIGHT OF COPPER AND BRASS WIRE AND PLATES.
Brown & Sharpe Gauge.
(From tables of leading manufacturers.)

| No. of Gauge. | Weight of Wire per 1000 Lineal Feet. | | No. of Gauge. | Thickness or Diameter. | Weight of Plates per Square Foot. | | Weight of Wire per 1000 Lineal Feet. | | Weight of Plates per Square Foot. | |
|---------------|--------------------------------------|----------------|-------------------------|------------------------|-----------------------------------|---------------|--------------------------------------|---------------|-----------------------------------|--------------|
| | Copper. | Brass. | | | Copper. | Brass. | Copper. | Brass. | Copper. | Brass. |
| 0000 | Lbs. 640.5 | Lbs. 605.18 | 21 | Inch. 0.46000 | Lbs. 20.84 | Lbs. 19.69 | Lbs. 2.45 | Lbs. 2.317 | Lbs. 1.29 | Lbs. 1.22 |
| 000 | 508.0 | 479.91 | 22 | .40964 | 18.56 | 17.53 | 1.95 | 1.838 | 1.15 | 1.08 |
| 00 | 402.8 | 380.67 | 23 | .36480 | 16.53 | 15.61 | 1.54 | 1.457 | 1.02 | 0.966 |
| 0 | 319.5 | 301.82 | 24 | .32486 | 14.72 | 13.90 | 1.22 | 1.155 | 0.911 | .860 |
| 1 | 253.3 | 239.35 | 25 | .28930 | 13.11 | 12.38 | 0.970 | 0.916 | .811 | .766 |
| 2 | 200.9 | 189.82 | 26 | .25763 | 11.67 | 11.03 | .769 | .727 | .722 | .682 |
| 3 | 159.3 | 150.52 | 27 | .22942 | 10.39 | 9.82 | .610 | .576 | .643 | .608 |
| 4 | 126.4 | 119.38 | 28 | .20431 | 9.26 | 8.74 | .484 | .457 | .573 | .541 |
| 5 | 100.2 | 94.67 | 29 | .18194 | 8.24 | 7.79 | .384 | .362 | .510 | .482 |
| 6 | 79.46 | 75.08 | 30 | .16202 | 7.34 | 6.93 | .304 | .287 | .454 | .429 |
| 7 | 63.01 | 59.55 | 31 | .14428 | 6.54 | 6.18 | .241 | .228 | .404 | .382 |
| 8 | 49.98 | 47.22 | 32 | .12849 | 5.82 | 5.50 | .191 | .181 | .360 | .340 |
| 9 | 39.64 | 37.44 | 33 | .11443 | 5.18 | 4.90 | .152 | .143 | .321 | .303 |
| 10 | 31.43 | 29.69 | 34 | .10189 | 4.62 | 4.36 | .120 | .114 | .286 | .270 |
| 11 | 24.92 | 23.55 | 35 | .090742 | 4.11 | 3.88 | .0956 | .0902 | .254 | .240 |
| 12 | 19.77 | 18.68 | 36 | .080808 | 3.66 | 3.46 | .0757 | .0715 | .226 | .214 |
| 13 | 15.65 | 14.81 | 37 | .071961 | 3.26 | 3.08 | .0600 | .0567 | .202 | .191 |
| 14 | 12.44 | 11.75 | 38 | .064084 | 2.90 | 2.74 | .0476 | .0450 | .180 | .170 |
| 15 | 9.86 | 9.32 | 39 | .057068 | 2.59 | 2.44 | .0376 | .0357 | .160 | .151 |
| 16 | 7.82 | 7.59 | 40 | .050820 | 2.30 | 2.18 | .0299 | .0283 | .142 | .135 |
| 17 | 6.20 | 5.86 | Specific gravity..... | | 2.05 | 1.94 | 8.880 | 8.386 | 8.698 | 8.218 |
| 18 | 4.92 | 4.65 | | | 1.83 | 1.73 | 555. | 524.16 | 5.436 | 513.6 |
| 19 | 3.90 | 3.68 | Weight per cubic ft.... | | 1.63 | 1.54 | | | | |
| 20 | 3.09 | 2.92 | | | 1.45 | 1.38 | | | | |

WEIGHT OF SHEET AND BAR BRASS.

| Thickness, Side or Diam., Inches. | Sheets per sq. ft., Lbs. | Square Bars 1 ft. long, Lbs. | Round Bars 1 ft. long, Lbs. | Thickness, Side or Diam., Inches. | Sheets per sq. ft., Lbs. | Square Bars 1 ft. long, Lbs. | Round Bars 1 ft. long, Lbs. |
|--|-----------------------------------|---------------------------------------|--------------------------------------|--|-----------------------------------|---------------------------------------|--------------------------------------|
| 1/16 | 2.72 | 0.014 | 0.011 | 1 1/16 | 46.32 | 4.10 | 3.22 |
| 1/8 | 5.45 | .056 | .045 | 1 1/8 | 49.05 | 4.59 | 3.61 |
| 3/16 | 8.17 | .128 | .100 | 1 3/16 | 51.77 | 5.12 | 4.02 |
| 1/4 | 10.90 | .227 | .178 | 1 1/4 | 54.50 | 5.67 | 4.45 |
| 5/16 | 13.62 | .355 | .278 | 1 5/16 | 57.22 | 6.26 | 4.91 |
| 3/8 | 16.35 | .510 | .401 | 1 3/8 | 59.95 | 6.86 | 5.39 |
| 7/16 | 19.07 | .695 | .545 | 1 7/16 | 62.67 | 7.50 | 5.89 |
| 1/2 | 21.80 | .907 | .712 | 1 1/2 | 65.40 | 8.16 | 6.41 |
| 9/16 | 24.52 | 1.15 | .902 | 1 9/16 | 68.12 | 8.86 | 6.95 |
| 5/8 | 27.25 | 1.42 | 1.11 | 1 5/8 | 70.85 | 9.59 | 7.53 |
| 11/16 | 29.97 | 1.72 | 1.35 | 1 11/16 | 73.57 | 10.34 | 8.12 |
| 3/4 | 32.70 | 2.04 | 1.60 | 1 3/4 | 76.30 | 11.12 | 8.73 |
| 13/16 | 35.42 | 2.40 | 1.88 | 1 13/16 | 79.02 | 11.93 | 9.36 |
| 7/8 | 38.15 | 2.78 | 2.18 | 1 7/8 | 81.75 | 12.76 | 10.01 |
| 15/16 | 40.87 | 3.19 | 2.50 | 1 15/16 | 84.47 | 13.63 | 10.70 |
| 1 | 43.60 | 3.63 | 2.85 | 2 | 87.20 | 14.52 | 11.40 |

WEIGHT OF ALUMINUM SHEETS, SQUARE AND ROUND BARS.

(Specific Gravity 2.68; 1 cu. in. = 0.0973 lb.)

| Thickness or Diameter, Inches. | Sheets per Sq. Ft., Lbs. | Round Bars per Ft., Lbs. | Square Bars per Ft., Lbs. | Thickness or Diameter, Inches. | Sheets per Sq. Ft., Lbs. | Round Bars per Ft., Lbs. | Square Bars per Ft., Lbs. |
|---|-----------------------------------|-----------------------------------|------------------------------------|---|-----------------------------------|-----------------------------------|------------------------------------|
| 1/16 | 0.876 | 0.004 | 0.005 | 3/4 | 10.508 | 0.516 | 0.657 |
| 1/8 | 1.751 | .014 | .018 | 7/8 | 12.260 | .702 | .894 |
| 1/4 | 3.503 | .057 | .073 | 1 | 14.011 | .917 | 1.168 |
| 3/8 | 5.254 | .129 | .164 | 1 1/4 | 17.514 | 1.433 | 1.824 |
| 1/2 | 7.006 | .229 | .292 | 1 1/2 | 21.017 | 2.063 | 2.627 |
| 5/8 | 8.757 | .358 | .456 | 2 | 28.022 | 3.668 | 4.671 |

For further particulars regarding aluminum, see pp. 174, 357.

SCREW-THREADS, WHITWORTH (ENGLISH) STANDARD.

| Diam. | Pitch. | Diam. | Pitch. | Diam. | Pitch. | Diam. | Pitch. | Diam. | Pitch. |
|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|
| 1/4 | 20 | 5/8 | 11 | 1 | 8 | 1 3/4 | 5 | 3 | 3 1/2 |
| 5/16 | 18 | 11/16 | 11 | 1 1/8 | 7 | 1 7/8 | 4 1/2 | 3 1/4 | 3 1/4 |
| 3/8 | 16 | 3/4 | 10 | 1 1/4 | 7 | 2 | 4 1/2 | 3 1/2 | 3 1/4 |
| 7/16 | 14 | 13/16 | 10 | 1 3/8 | 6 | 2 1/4 | 4 | 3 3/4 | 3 |
| 1/2 | 12 | 7/8 | 9 | 1 1/2 | 6 | 2 1/2 | 4 | 4 | 3 |
| 9/16 | 12 | 15/16 | 9 | 1 5/8 | 5 | 2 3/4 | 3 1/2 | | |

In the Whitworth or English system the angle of the thread is 55 degrees, and the point and root of the thread are rounded to a radius of $0.1373 \times \text{pitch}$. The depth of the thread is $0.6403 \times \text{pitch}$.

SCREW-THREADS, SELLERS OR U. S. STANDARD.

| BOLTS AND THREADS. | | | | | | HEX. NUTS AND HEADS. | | | | | |
|--------------------|-------------------|--------------------------|----------------|----------------------------------|---------------------------------------|----------------------|----------------------|--------------------|-------------------|--------------------|----------------------------|
| Diam. of Bolt. | Threads per Inch. | Diam. of Root of Thread. | Width of Flat. | Area of Bolt Body in Sq. Inches. | Area of Root of Thread in Sq. Inches. | Short Diam., Rough. | Short Diam., Finish. | Long Diam., Rough. | Thickness, Rough. | Thickness, Finish. | Long Diam. Sq. Nuts Rough. |
| In. | | In. | In. | | | In. | In. | In. | In. | In. | |
| 1/4 | 20 | 0.185 | 0.0062 | 0.049 | 0.027 | 1/2 | 7/16 | 37/64 | 1/4 | 3/16 | 7/10 |
| 5/16 | 18 | .240 | .0074 | .077 | .045 | 19/32 | 17/32 | 11/16 | 5/16 | 1/4 | 10/12 |
| 3/8 | 16 | .294 | .0078 | .110 | .068 | 11/16 | 5/8 | 51/64 | 3/8 | 5/16 | 63/64 |
| 7/16 | 14 | .344 | .0089 | .150 | .093 | 25/32 | 23/32 | 9/10 | 7/16 | 3/8 | 17/64 |
| 1/2 | 13 | .400 | .0096 | .196 | .126 | 7/8 | 13/16 | 1 | 1/2 | 7/16 | 115/64 |
| 9/16 | 12 | .454 | .0104 | .249 | .162 | 31/32 | 29/32 | 1 1/8 | 9/16 | 1/2 | 123/64 |
| 5/8 | 11 | .507 | .0113 | .307 | .202 | 1 1/16 | 1 | 1 7/32 | 5/8 | 9/16 | 11/2 |
| 3/4 | 10 | .620 | .0125 | .442 | .302 | 1 1/4 | 1 3/16 | 1 7/16 | 3/4 | 1 1/16 | 1 49/64 |
| 7/8 | 9 | .731 | .0138 | .601 | .420 | 1 7/16 | 1 3/8 | 1 21/32 | 7/8 | 1 3/16 | 2 1/32 |
| 1 | 8 | .837 | .0156 | .785 | .550 | 1 5/8 | 1 9/16 | 1 7/8 | 1 | 1 5/16 | 2 19/64 |
| 1 1/8 | 7 | .940 | .0178 | .994 | .694 | 1 13/16 | 1 3/4 | 2 3/32 | 1 1/8 | 1 11/16 | 2 9/16 |
| 1 1/4 | 7 | 1.065 | .0178 | 1.227 | .893 | 2 | 1 15/16 | 2 5/16 | 1 1/4 | 1 13/16 | 2 53/64 |
| 1 3/8 | 6 | 1.160 | .0208 | 1.485 | 1.057 | 2 3/16 | 2 1/8 | 2 17/32 | 1 3/8 | 1 5/16 | 3 3/32 |
| 1 1/2 | 6 | 1.284 | .0208 | 1.767 | 1.295 | 2 3/8 | 2 5/16 | 2 3/4 | 1 1/2 | 1 7/16 | 3 23/64 |
| 1 5/8 | 5 1/2 | 1.389 | .0227 | 2.074 | 1.515 | 2 9/16 | 2 1/2 | 2 31/32 | 1 5/8 | 1 9/16 | 3 5/8 |
| 1 3/4 | 5 | 1.491 | .0250 | 2.405 | 1.746 | 2 3/4 | 2 11/16 | 3 3/16 | 1 3/4 | 1 11/16 | 3 57/64 |
| 1 7/8 | 5 | 1.616 | .0250 | 2.761 | 2.051 | 2 5/16 | 2 7/8 | 3 13/32 | 1 7/8 | 1 13/16 | 4 5/32 |
| 2 | 4 1/2 | 1.712 | .0277 | 3.142 | 2.302 | 3 1/8 | 3 1/16 | 3 5/8 | 2 | 1 15/16 | 4 27/64 |
| 2 1/4 | 4 1/2 | 1.962 | .0277 | 3.976 | 3.023 | 3 1/2 | 3 7/16 | 4 1/16 | 2 1/4 | 2 3/16 | 4 61/64 |
| 2 1/2 | 4 | 2.176 | .0312 | 4.909 | 3.719 | 3 7/8 | 3 13/16 | 4 1/2 | 2 1/2 | 2 7/16 | 5 31/64 |
| 2 3/4 | 4 | 2.426 | .0312 | 5.940 | 4.620 | 4 1/4 | 4 3/16 | 4 29/32 | 2 3/4 | 2 11/16 | 6 |
| 3 | 3 1/2 | 2.629 | .0357 | 7.069 | 5.428 | 4 5/8 | 4 9/16 | 5 3/8 | 3 | 2 15/16 | 6 17/32 |
| 3 1/4 | 3 1/2 | 2.879 | .0357 | 8.296 | 6.510 | 5 | 4 15/16 | 5 13/16 | 3 1/4 | 3 3/16 | 7 1/16 |
| 3 1/2 | 3 1/4 | 3.100 | .0384 | 9.621 | 7.548 | 5 3/8 | 5 5/16 | 6 7/64 | 3 1/2 | 3 7/16 | 7 39/64 |
| 3 3/4 | 3 | 3.317 | .0413 | 11.045 | 8.641 | 5 3/4 | 5 11/16 | 6 21/32 | 3 3/4 | 3 11/16 | 8 1/8 |
| 4 | 3 | 3.567 | .0413 | 12.566 | 9.993 | 6 1/8 | 6 1/16 | 7 3/32 | 4 | 3 15/16 | 8 41/64 |
| 4 1/4 | 2 7/8 | 3.798 | .0435 | 14.186 | 11.329 | 6 1/2 | 6 7/16 | 7 9/16 | 4 1/4 | 4 3/16 | 9 3/16 |
| 4 1/2 | 2 3/4 | 4.028 | .0454 | 15.904 | 12.743 | 6 7/8 | 6 13/16 | 7 31/32 | 4 1/2 | 4 7/16 | 9 3/4 |
| 4 3/4 | 2 5/8 | 4.256 | .0476 | 17.721 | 14.226 | 7 1/4 | 7 3/16 | 8 13/32 | 4 3/4 | 4 11/16 | 10 1/4 |
| 5 | 2 1/2 | 4.480 | .0500 | 19.635 | 15.763 | 7 5/8 | 7 9/16 | 8 27/32 | 5 | 4 15/16 | 10 49/64 |
| 5 1/4 | 2 1/2 | 4.730 | .0500 | 21.648 | 17.572 | 8 | 7 15/16 | 9 9/32 | 5 1/4 | 5 3/16 | 11 23/64 |
| 5 1/2 | 2 3/8 | 4.953 | .0526 | 23.758 | 19.267 | 8 3/8 | 8 5/16 | 9 23/32 | 5 1/2 | 5 7/16 | 11 7/8 |
| 5 3/4 | 2 3/8 | 5.203 | .0526 | 25.967 | 21.262 | 8 3/4 | 8 11/16 | 10 5/32 | 5 3/4 | 5 11/16 | 12 3/8 |
| 6 | 2 1/4 | 5.423 | .0555 | 28.274 | 23.098 | 9 1/8 | 9 1/16 | 10 19/32 | 6 | 5 15/16 | 12 15/16 |

In 1864 a committee of the Franklin Institute recommended the adoption of the system of screw-threads and bolts which was devised by Mr. William Sellers of Philadelphia. This system is now in general use in the United States, and it is commonly called the United States Standard.

The rule for proportioning the thread is as follows: Divide the pitch, or, what is the same thing, the side of the thread, into eight equal parts; take off one part from the top and fill in one part in the bottom of the thread; then the flat top and bottom will equal one-eighth of the pitch, the wearing surface will be three-quarters of the pitch, and the diameter of screw at bottom of the thread will be expressed by the formula,

$$\text{diam. of bolt} - (1.299 \div \text{no. of threads per inch}).$$

For a sharp V-thread with angle of 60 degrees the formula is,

$$\text{diam. of bolt} - (1.733 \div \text{no. of threads per inch}).$$

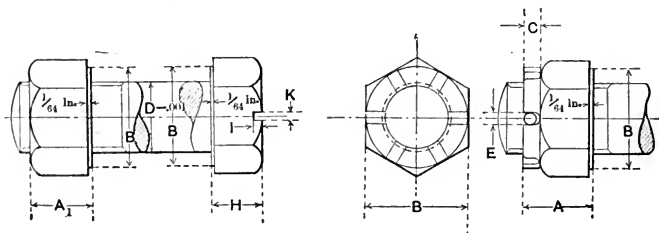
The angle of the thread in the Sellers system is 60 degrees.

Thickness of Nuts and Bolt Heads.—In the above table the thickness of nuts and heads (rough) is given as equal to the diameter of the bolt. Many manufacturers make the thickness of nuts about $\frac{7}{8}$, and of bolt heads $\frac{3}{4}$, of the diam. of the bolt.

Automobile Screws and Nuts.—The Association of Licensed Automobile M'frs (1906) adopted standard specifications for hexagon head screws, castle and plain nuts known as the A.L.A.M. standard. Material to be steel, elastic limit not less than 60,000 lbs. per sq. in., tensile strength not less than 100,000 lbs. per sq. in. U. S. Standard thread is used, the threaded portion of screws being $1\frac{1}{2}$ times the diameter. The castle nut has a boss on the upper surface with six slots for a locking pin through the bolt.

Standard Automobile Screws, Castle and Plain Nuts.

All dimensions in inches. P = pitch, or number of threads per inch. d = diam. of cotter pin. $P \div 8$ = flat top.



| D | P | B | A_1 | H | K | I | A | C | E | d |
|-----------------|-----|-----------------|-----------------|-----------------|----------------|----------------|-----------------|----------------|----------------|----------------|
| $\frac{1}{4}$ | 28 | $\frac{3}{8}$ | $\frac{7}{32}$ | $\frac{3}{16}$ | $\frac{1}{16}$ | $\frac{3}{32}$ | $\frac{9}{32}$ | $\frac{3}{32}$ | $\frac{5}{64}$ | $\frac{1}{16}$ |
| $\frac{5}{16}$ | 24 | $\frac{1}{2}$ | $\frac{17}{64}$ | $\frac{15}{64}$ | $\frac{1}{16}$ | $\frac{7}{64}$ | $\frac{21}{64}$ | $\frac{3}{32}$ | $\frac{5}{64}$ | $\frac{1}{16}$ |
| $\frac{3}{8}$ | 24 | $\frac{9}{16}$ | $\frac{21}{64}$ | $\frac{9}{32}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{13}{32}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{3}{32}$ |
| $\frac{7}{16}$ | 20 | $\frac{11}{16}$ | $\frac{3}{8}$ | $\frac{21}{64}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{29}{64}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{3}{32}$ |
| $\frac{1}{2}$ | 20 | $\frac{3}{4}$ | $\frac{7}{16}$ | $\frac{3}{8}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{9}{16}$ | $\frac{3}{16}$ | $\frac{1}{8}$ | $\frac{3}{32}$ |
| $\frac{9}{16}$ | 18 | $\frac{7}{8}$ | $\frac{31}{64}$ | $\frac{27}{64}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{39}{64}$ | $\frac{3}{16}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |
| $\frac{5}{8}$ | 18 | $\frac{15}{16}$ | $\frac{35}{64}$ | $\frac{15}{32}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{23}{32}$ | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |
| $\frac{11}{16}$ | 16 | 1 | $\frac{19}{32}$ | $\frac{33}{64}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{49}{64}$ | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |
| $\frac{3}{4}$ | 16 | $\frac{11}{8}$ | $\frac{21}{32}$ | $\frac{9}{16}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{13}{16}$ | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |
| $\frac{7}{8}$ | 14 | $\frac{11}{4}$ | $\frac{49}{64}$ | $\frac{21}{32}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | $\frac{29}{32}$ | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |
| 1 | 14 | $\frac{17}{16}$ | $\frac{7}{8}$ | $\frac{3}{4}$ | $\frac{3}{32}$ | $\frac{1}{8}$ | 1 | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{1}{8}$ |

INTERNATIONAL STANDARD THREAD (METRIC SYSTEM).

P = pitch, = 1 - no. of threads per millimeter.

Depth of thread = $0.6495 P$.

Flat top and bottom of thread = one-eighth pitch.

Diam. at bottom of thread = diam. of bolt - $1.299 P$.

| | | | | | | | | | | | | | | |
|------------|-----|-----|------|------|-----|-----|------|-----|-----|-----|-----|-----|-----|----|
| Diam., mm. | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 27 |
| Pitch, mm. | 1.0 | 1.0 | 1.25 | 1.25 | 1.5 | 1.5 | 1.75 | 2. | 2. | 2.5 | 2.5 | 2.5 | 3. | 3. |
| Diam., mm. | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 80 |
| Pitch, mm. | 3.5 | 3.5 | 4. | 4. | 4.5 | 4.5 | 5. | 5.5 | 5.5 | 6. | 6. | 6.5 | 6.5 | 7. |

BRITISH ASSOCIATION STANDARD THREAD.

The angle between the threads is $47\frac{1}{2}^\circ$. The depth of the thread is $0.6 \times$ the pitch. The tops and bottoms of the threads are rounded with a radius of $\frac{2}{11}$ of the pitch.

| | | | | | | | |
|---------------|------|------|------|------|------|------|------|
| Number | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Diameter, mm. | 6.0 | 5.3 | 4.7 | 4.1 | 3.64 | 3.2 | 2.8 |
| Pitch, mm. | 1.00 | 0.90 | 0.81 | 0.73 | 0.66 | 0.59 | 0.53 |

| | | | | | | | |
|-------------------|------|------|------|------|------|------|------|
| Number | 7 | 8 | 9 | 10 | 12 | 14 | 19 |
| Diameter, mm..... | 2.5 | 2.2 | 1.9 | 1.7 | 1.3 | 1.0 | .79 |
| Pitch, mm..... | 0.48 | 0.43 | 0.39 | 0.35 | 0.28 | 0.23 | 0.19 |

LIMIT GAUGES FOR IRON FOR SCREW-THREADS.

In adopting the Sellers, or Franklin Institute, or United States Standard, as it is variously called, a difficulty arose from the fact that it is the habit of iron manufacturers to make iron over-size, and as there are no over-size screws in the Sellers system, if iron is too large it is necessary to cut it away with the dies. So great is this difficulty, that the practice of making taps and dies over-size has become very general. If the Sellers system is adopted it is essential that iron should be obtained of the correct size, or very nearly so. Of course no high degree of precision is possible in rolling iron, and when exact sizes were demanded, the question arose how much allowable variation there should be from the true size. It was proposed to make limit-gauges for inspecting iron with two openings, one larger and the other smaller than the standard size, and then specify that the iron should enter the large end and not enter the small one. The following table of dimensions for the limit-gauges was adopted by the Master Car-Builders' Association in 1883.

| Size of Iron. In. | Large End of Gauge. | Small End of Gauge. | Differ- ence. | Size of Iron. In. | Large End of Gauge. | Small End of Gauge. | Differ- ence. |
|-------------------------|---------------------------|---------------------------|------------------|-------------------------|---------------------------|---------------------------|------------------|
| 1/4 | 0.2550 | 0.2450 | 0.010 | 5/8 | 0.6330 | 0.6170 | 0.016 |
| 5/16 | 0.3180 | 0.3070 | 0.011 | 3/4 | 0.7585 | 0.7415 | 0.017 |
| 3/8 | 0.3810 | 0.3690 | 0.012 | 7/8 | 0.8840 | 0.8660 | 0.018 |
| 7/16 | 0.4440 | 0.4310 | 0.013 | 1 | 1.0095 | 0.9905 | 0.019 |
| 1/2 | 0.5070 | 0.4930 | 0.014 | 1 1/8 | 1.1350 | 1.1150 | 0.020 |
| 9/16 | 0.5700 | 0.5550 | 0.015 | 1 1/4 | 1.2605 | 1.2395 | 0.021 |

Caliper gauges with the above dimensions, and standard reference gauges for testing them, are made by the Pratt & Whitney Co.

THE MAXIMUM VARIATION IN SIZE OF ROUGH IRON FOR U. S. STANDARD BOLTS.

Am. Mach., May 12, 1892.

By the adoption of the Sellers or U. S. Standard, thread taps and dies keep their size much longer in use when flatted in accordance with this system than when made sharp "V", though it has been found advisable in practice in most cases to make the taps of somewhat larger outside diameter than the nominal size, thus carrying the threads further towards the V-shape and giving corresponding clearance to the tops of the threads when in the nuts or tapped holes.

Makers of taps and dies often have calls for taps and dies, U. S. Standard, "for rough iron."

An examination of rough iron will show that much of it is rolled out of round to an amount exceeding the limit of variation in size allowed.

In view of this it may be desirable to know what the extreme variation in iron may be, consistent with the maintenance of U. S. Standard threads, i.e., threads which are standard when measured upon the angles, the only place where it seems advisable to have them fit closely. Mr. Chas. A. Bauer, the general manager of the Warder, Bushnell & Glessner Co., at Springfield, Ohio, in 1884 adopted a plan which may be stated as follows: All bolts, whether cut from rough or finished stock, are standard size at the bottom and at the sides or angles of the threads, the variation for fit of the nut and allowance for wear of taps being made in the machine taps. Nuts are punched with holes of such size as to give 85 per cent of a full thread, experience showing that the metal of wrought nuts will then crowd into the threads of the taps sufficiently to give practically a full thread, while if punched smaller some of the metal will be cut out by the tap at the bottom of the threads, which is of course undesirable. Machine taps are made enough larger than the nominal

to bring the tops of the threads up sharp, plus the amount allowed for fit and wear of taps. This allows the iron to be enough above the nominal diameter to bring the threads up full (sharp) at top, while if it is small the only effect is to give a flat at top of threads; neither condition affecting the actual size of the thread at the point at which it is intended to bear. Limit gauges are furnished to the mills, by which the iron is rolled, the maximum size being shown in the third column of the table. The minimum diameter is not given, the tendency in rolling being nearly always to exceed the nominal diameter.

In making the taps the threaded portion is turned to the size given in the eighth column of the table, which gives 6 to 7 thousandths of an inch allowance for fit and wear of tap. Just above the threaded portion of the tap a place is turned to the size given in the ninth column, these sizes being the same as those of the regular U. S. Standard bolt, at the bottom of the thread, plus the amount allowed for fit and wear of tap; or, in other words, $d' = \text{U. S. Standard } d + (D' - D)$. Gauges like the one in the cut, Fig. 75, are furnished for this sizing. In finishing the threads of the



FIG. 75.

tap a tool is used which has a removable cutter finished accurately to gauge by grinding, this tool being correct U. S. Standard as to angle, and flat at the point. It is fed in and the threads chased until the flat point just touches the portion of the tap which has been turned to size d' . Care having been taken with the form of the tool, with its grinding on the top face (a fixture being provided for this to insure its being ground properly), and also with the setting of the tool properly in the lathe, the result is that the threads of the tap are correctly sized without further attention.

STANDARD SIZES OF SCREW-THREADS FOR BOLTS AND TAPS.

(CHAS. A. BAUER.)

| <i>A</i> | <i>n</i> | <i>D</i> | <i>d</i> | <i>h</i> | <i>f</i> | <i>D' - D</i> | <i>D'</i> | <i>d'</i> | <i>H</i> |
|----------|----------|----------|----------|----------|----------|---------------|-----------|-----------|----------|
| | | Inches | Inches | Inches | Inches | Inches | Inches | Inches | Inches |
| 1/4 | 20 | 0.2608 | 0.1855 | 0.0379 | 0.0062 | 0.006 | 0.2668 | 0.1915 | 0.2024 |
| 5/16 | 18 | 0.3245 | 0.2403 | 0.0421 | 0.0070 | 0.006 | 0.3305 | 0.2463 | 0.2589 |
| 3/8 | 16 | 0.3885 | 0.2938 | 0.0474 | 0.0078 | 0.006 | 0.3945 | 0.2998 | 0.3139 |
| 7/16 | 14 | 0.4530 | 0.3447 | 0.0541 | 0.0089 | 0.006 | 0.4590 | 0.3507 | 0.3670 |
| 1/2 | 13 | 0.5166 | 0.4000 | 0.0582 | 0.0096 | 0.006 | 0.5226 | 0.4060 | 0.4236 |
| 9/16 | 12 | 0.5805 | 0.4543 | 0.0631 | 0.0104 | 0.007 | 0.5875 | 0.4613 | 0.4802 |
| 5/8 | 11 | 0.6447 | 0.5069 | 0.0689 | 0.0114 | 0.007 | 0.6517 | 0.5139 | 0.5346 |
| 3/4 | 10 | 0.7717 | 0.6201 | 0.0758 | 0.0125 | 0.007 | 0.7787 | 0.6271 | 0.6499 |
| 7/8 | 9 | 0.8991 | 0.7307 | 0.0842 | 0.0139 | 0.007 | 0.9061 | 0.7377 | 0.7630 |
| 1 | 8 | 1.0271 | 0.8376 | 0.0947 | 0.0156 | 0.007 | 1.0341 | 0.8446 | 0.8731 |
| 1 1/8 | 7 | 1.1559 | 0.9394 | 0.1083 | 0.0179 | 0.007 | 1.1629 | 0.9464 | 0.9789 |
| 1 1/4 | 7 | 1.2809 | 1.0644 | 0.1083 | 0.0179 | 0.007 | 1.2879 | 1.0714 | 1.1039 |

A = nominal diameter of bolt.

D = actual diameter of bolt.

d = diameter of bolt at bottom of thread.

n = number of threads per inch.

f = flat of bottom of thread.

h = depth of thread.

D' and *d'* = diameters of tap.

H = hole in nut before tapping.

$D = A + 0.2165/n$.

$d = A - 1.29904/n$.

$h = 0.7577/n = (D - d)/2$.

$f = 0.125/n$.

$H = D' - \frac{1.288}{n} = D' - 0.85(2h)$.

STANDARD SET-SCREWS AND CAP-SCREWS.
 American, Hartford, and Worcester Machine-Screw Companies.
 (Compiled by W. S. Dix, 1895.)

| | | | | | | | |
|-------------------------|--------------------|-------------------|-------------------|-------------------|---------------|----------------|---------------------|
| (See tables below) | (A) | (B) | (C) | (D) | (E) | (F) | (G) |
| Diameter of screw..... | $\frac{1}{8}$ | $\frac{3}{16}$ | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ |
| Threads per inch..... | 40 | 24 | 20 | 18 | 16 | 14 | 12 |
| Size of tap drill*..... | No. 43 | No. 30 | No. 5 | 17/64 | 21/64 | 3/8 | 27/64 |
| Diameter of screw..... | (H) $\frac{9}{16}$ | (I) $\frac{5}{8}$ | (J) $\frac{3}{4}$ | (K) $\frac{7}{8}$ | (L) 1 | (M) 1 | (N) $\frac{11}{16}$ |
| Threads per inch..... | 12 | 11 | 10 | 9 | 8 | 7 | 7 |
| Size of tap drill*..... | 31/64 | 17/32 | 21/32 | 49/64 | 7/8 | 63/64 | 11/8 |

* For cast iron. For numbers of twist-drills, see page 30.

| Set-screws. | | | Hex. Head Cap-screws. | | | Sq. Head Cap-screws. | | |
|----------------------------|--------------------------|---------------------------------|----------------------------|---------------------------|----------------------------------|----------------------------|---------------------------|----------------------------------|
| Short Diam. of Head. | Long Diam. of H'd. | Lengths (under Head). | Short Diam. of Head. | Long Diam. of Head. | Lengths (under Head). | Short Diam. of Head. | Long Diam. of Head. | Lengths (under Head). |
| (C) $\frac{1}{4}$ | 0.35 | $\frac{3}{4}$ to 3 | $\frac{7}{16}$ | 0.51 | $\frac{3}{4}$ to 3 | $\frac{3}{8}$ | 0.53 | $\frac{3}{4}$ to 3 |
| (D) $\frac{5}{16}$ | .44 | $\frac{3}{4}$ to $\frac{31}{4}$ | $\frac{1}{2}$ | .58 | $\frac{3}{4}$ to $\frac{31}{4}$ | $\frac{7}{16}$ | .62 | $\frac{3}{4}$ to $\frac{31}{4}$ |
| (E) $\frac{3}{8}$ | .53 | $\frac{3}{4}$ to $\frac{31}{2}$ | $\frac{9}{16}$ | .65 | $\frac{3}{4}$ to $\frac{31}{2}$ | $\frac{1}{2}$ | .71 | $\frac{3}{4}$ to $\frac{31}{2}$ |
| (F) $\frac{7}{16}$ | .62 | $\frac{3}{4}$ to $\frac{33}{4}$ | $\frac{5}{8}$ | .72 | $\frac{3}{4}$ to $\frac{33}{4}$ | $\frac{9}{16}$ | .80 | $\frac{3}{4}$ to $\frac{33}{4}$ |
| (G) $\frac{1}{2}$ | .71 | $\frac{3}{4}$ to 4 | $\frac{3}{4}$ | .87 | $\frac{3}{4}$ to 4 | $\frac{5}{8}$ | .89 | $\frac{3}{4}$ to 4 |
| (H) $\frac{9}{16}$ | .80 | $\frac{3}{4}$ to $\frac{41}{4}$ | $\frac{13}{16}$ | .94 | $\frac{3}{4}$ to $\frac{41}{4}$ | $\frac{11}{16}$ | .98 | $\frac{3}{4}$ to $\frac{41}{4}$ |
| (I) $\frac{5}{8}$ | .89 | $\frac{3}{4}$ to $\frac{41}{2}$ | $\frac{7}{8}$ | 1.01 | 1 to $\frac{41}{2}$ | $\frac{3}{4}$ | 1.06 | 1 to $\frac{41}{2}$ |
| (J) $\frac{3}{4}$ | 1.06 | 1 to $\frac{43}{4}$ | 1 | 1.15 | $\frac{11}{4}$ to $\frac{43}{4}$ | $\frac{7}{8}$ | 1.24 | $\frac{11}{4}$ to $\frac{43}{4}$ |
| (K) $\frac{7}{8}$ | 1.24 | $\frac{11}{4}$ to 5 | $\frac{11}{8}$ | 1.30 | $\frac{11}{2}$ to 5 | $\frac{11}{8}$ | 1.60 | $\frac{11}{2}$ to 5 |
| (L) 1 | 1.42 | $\frac{11}{2}$ to 5 | $\frac{11}{4}$ | 1.45 | $\frac{13}{4}$ to 5 | $\frac{11}{4}$ | 1.77 | $\frac{13}{4}$ to 5 |
| (M) $\frac{11}{8}$ | 1.60 | $\frac{13}{4}$ to 5 | $\frac{13}{8}$ | 1.59 | 2 to 5 | $\frac{13}{8}$ | 1.95 | 2 to 5 |
| (N) $\frac{11}{4}$ | 1.77 | 2 to 5 | $\frac{11}{2}$ | 1.73 | 2 to 5 | $\frac{11}{2}$ | 2.13 | $\frac{21}{4}$ to 5 |

| Round and Fillister Head Cap-screws. | | Flat Head Cap-screws. | | Button-head Cap- screws. | |
|---|----------------------------------|-----------------------|---------------------------------|-----------------------------|---------------------------------|
| Diam. of Head. | Lengths (under Head). | Diam. of Head. | Lengths (including Head). | Diam. of Head. | Lengths (under Head). |
| (A) $\frac{3}{16}$ | $\frac{3}{4}$ to $\frac{21}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ to $\frac{13}{4}$ | $\frac{7}{32}$ (.225) | $\frac{3}{4}$ to $\frac{13}{4}$ |
| (B) $\frac{1}{4}$ | $\frac{3}{4}$ to $\frac{23}{4}$ | $\frac{3}{8}$ | $\frac{3}{4}$ to 2 | $\frac{5}{16}$ | $\frac{3}{4}$ to 2 |
| (C) $\frac{3}{8}$ | $\frac{3}{4}$ to 3 | $\frac{15}{32}$ | $\frac{3}{4}$ to $\frac{21}{4}$ | $\frac{7}{16}$ | $\frac{3}{4}$ to $\frac{21}{4}$ |
| (D) $\frac{7}{16}$ | $\frac{3}{4}$ to $\frac{31}{4}$ | $\frac{5}{8}$ | $\frac{3}{4}$ to $\frac{23}{4}$ | $\frac{9}{16}$ | $\frac{3}{4}$ to $\frac{21}{2}$ |
| (E) $\frac{9}{16}$ | $\frac{3}{4}$ to $\frac{31}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ to 3 | $\frac{5}{8}$ | $\frac{3}{4}$ to $\frac{23}{4}$ |
| (F) $\frac{5}{8}$ | $\frac{3}{4}$ to $\frac{33}{4}$ | $\frac{13}{16}$ | 1 to 3 | $\frac{3}{4}$ | $\frac{3}{4}$ to 3 |
| (G) $\frac{3}{4}$ | $\frac{3}{4}$ to 4 | $\frac{7}{8}$ | $\frac{11}{4}$ to 3 | $\frac{13}{16}$ | 1 to 3 |
| (H) $\frac{13}{16}$ | 1 to $\frac{41}{4}$ | 1 | $\frac{11}{2}$ to 3 | $\frac{15}{16}$ | $\frac{11}{4}$ to 3 |
| (I) $\frac{7}{8}$ | $\frac{11}{4}$ to $\frac{41}{2}$ | $\frac{11}{8}$ | $\frac{13}{4}$ to 3 | 1 | $\frac{11}{2}$ to 3 |
| (J) 1 | $\frac{11}{2}$ to $\frac{43}{4}$ | $\frac{13}{8}$ | 2 to 3 | $\frac{11}{4}$ | $\frac{13}{4}$ to 3 |
| (K) $\frac{11}{8}$ | $\frac{13}{4}$ to 5 | | | | |
| (L) $\frac{11}{4}$ | 2 to 5 | | | | |

Threads are U. S. Standard. Cap-screws are threaded $\frac{3}{4}$ length up to and including 1 inch diameter \times 4 inches long, and $\frac{1}{2}$ length above. Lengths increase by $\frac{1}{4}$ inch each regular size between the limits given. Lengths of heads, except flat and button, equal diameter of screws.

The angle of the cone of the flat-head screw is 76 degrees, the sides making angles of 52 degrees with the top.

THE ACME SCREW THREAD.

The Acme Thread is an adaptation of the commonly used style of worm thread and is intended to take the place of the square thread. It is a little shallower than the worm thread, but the same depth as the square thread and much stronger than the latter. The angle of the thread is 29°.

The various parts of the Acme Thread are obtained as follows:

Width of point of tool for screw or tap thread =

$(0.3707 \div \text{No. of Threads per in.}) - 0.0052$.

Width of screw or nut thread = $0.3707 \div \text{No. of Threads per in.}$

Diam. of Tap = Diam. of Screw + 0.020.

Diam. of Tap or } = Diam. of Screw - $\frac{1}{\text{No. of Threads per in.}}$ + 0.020.
Screw at Root }

Depth of Thread = $(1 \div 2 \times \text{No. of Threads per in.}) + 0.010$.

MACHINE SCREWS.—A.S.M.E. Standard.

The American Society of Mechanical Engineers (1907) received a report on standard machine screws from its committee on that subject. The included angle of the thread is 60 degrees and a flat is made at the top and bottom of the thread of one-eighth the basic diameter. A uniform increment of 0.013 inch exists between all sizes from 0 to 10 and 0.026 inch in the remaining sizes. The pitches are a function of the diameter as expressed by the formula

$$\text{Threads per inch} = \frac{6.5}{D + 0.02}$$

The minimum tap conforms to the basic standard in all respects except diameter. The difference between the minimum tap and the maximum screw provides an allowance for error in pitch and for wear of the tap in service.

A. S. M. E. STANDARD MACHINE SCREWS.

(Corbin Screw Corporation.)

| Size. | | Outside Diameters. | | | Pitch Diameters. | | | Root Diameters. | | |
|-------|-----------------------------|--------------------|----------|-------------|------------------|----------|-------------|-----------------|----------|-------------|
| No. | Out. Dia. and Thds. per In. | Minimum. | Maximum. | Difference. | Minimum. | Maximum. | Difference. | Minimum. | Maximum. | Difference. |
| 0 | 0.060-80 | 0.0572 | 0.060 | 0.0028 | 0.0505 | 0.0519 | 0.0014 | 0.0410 | 0.0438 | 0.0028 |
| 1 | .073-72 | .070 | .073 | .003 | .0625 | .064 | .0015 | .052 | .055 | .0030 |
| 2 | .086-64 | .0828 | .086 | .0032 | .0743 | .0759 | .0016 | .0624 | .0657 | .0033 |
| 3 | .099-56 | .0955 | .099 | .0035 | .0857 | .0874 | .0017 | .0721 | .0758 | .0037 |
| 4 | .112-48 | .1082 | .112 | .0038 | .0966 | .0985 | .0019 | .0807 | .0849 | .0042 |
| 5 | .125-44 | .1210 | .125 | .0040 | .1082 | .1102 | .0020 | .0910 | .0955 | .0045 |
| 6 | .138-40 | .1338 | .138 | .0042 | .1197 | .1218 | .0021 | .1007 | .1055 | .0048 |
| 7 | .151-36 | .1466 | .151 | .0044 | .1308 | .1330 | .0022 | .1097 | .1149 | .0052 |
| 8 | .164-36 | .1596 | .164 | .0044 | .1438 | .146 | .0022 | .1227 | .1279 | .0052 |
| 9 | .177-32 | .1723 | .177 | .0047 | .1544 | .1567 | .0023 | .1307 | .1364 | .0057 |
| 10 | .190-30 | .1852 | .190 | .0048 | .166 | .1684 | .0024 | .1407 | .1467 | .0060 |
| 12 | .216-28 | .2111 | .216 | .0049 | .1904 | .1928 | .0024 | .1633 | .1696 | .0063 |
| 14 | .242-24 | .2368 | .242 | .0052 | .2123 | .2149 | .0026 | .1808 | .1879 | .0071 |
| 16 | .268-22 | .2626 | .268 | .0054 | .2358 | .2385 | .0027 | .2014 | .209 | .0076 |
| 18 | .294-20 | .2884 | .294 | .0056 | .2587 | .2615 | .0028 | .2208 | .229 | .0082 |
| 20 | .320-20 | .3144 | .320 | .0056 | .2847 | .2875 | .0028 | .2468 | .255 | .0082 |
| 22 | .346-18 | .3402 | .346 | .0058 | .3070 | .3099 | .0029 | .2649 | .2738 | .0089 |
| 24 | .372-16 | .366 | .372 | .0060 | .3284 | .3314 | .0030 | .281 | .2908 | .0098 |
| 26 | .398-16 | .392 | .398 | .0060 | .3544 | .3574 | .0030 | .307 | .3168 | .0098 |
| 28 | .424-14 | .4178 | .424 | .0062 | .3745 | .3776 | .0031 | .3204 | .3312 | .0108 |
| 30 | .450-14 | .4438 | .450 | .0062 | .4005 | .4036 | .0031 | .3464 | .3572 | .0108 |

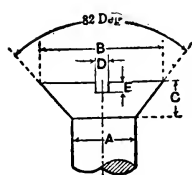
A. S. M. E. STANDARD TAPS.
(Corbin Screw Corporation.)

| Size. | | Outside Diameters. | | | Pitch Diameters. | | | Root Diameters. | | | Tap Drill Diameters. |
|-------|-------------------------------|--------------------|-----------|---------------|------------------|-----------|---------------|-----------------|-----------|---------------|----------------------|
| No. | Out. Dia. and Thds. per Inch. | Mini-mum. | Maxi-mum. | Dif-fer-ence. | Mini-mum. | Maxi-mum. | Dif-fer-ence. | Mini-mum. | Maxi-mum. | Dif-fer-ence. | |
| 0 | 0.060-80 | 0.0609 | 0.0632 | 0.0023 | 0.0528 | 0.0538 | 0.001 | 0.0447 | 0.0466 | 0.0019 | 0.0465 |
| 1 | .073-72 | .074 | .0765 | .0025 | .065 | .066 | .001 | .056 | .058 | .002 | .0595 |
| 2 | .086-64 | .0871 | .0898 | .0027 | .0770 | .0781 | .0011 | .0668 | .0689 | .0021 | .070 |
| 3 | .099-56 | .1002 | .1033 | .0031 | .0886 | .0897 | .0011 | .077 | .0793 | .0023 | .0785 |
| 4 | .112-48 | .1133 | .1168 | .0035 | .0998 | .101 | .0012 | .0852 | .0887 | .0025 | .089 |
| 5 | .125-44 | .1263 | .1301 | .0038 | .1116 | .1129 | .0013 | .0968 | .0995 | .0027 | .0995 |
| 6 | .138-40 | .1394 | .1435 | .0041 | .1232 | .1246 | .0014 | .1069 | .1097 | .0028 | .110 |
| 7 | .151-36 | .1525 | .1569 | .0044 | .1345 | .1359 | .0014 | .1164 | .1193 | .0029 | .120 |
| 8 | .164-36 | .1655 | .1699 | .0044 | .1475 | .1489 | .0014 | .1294 | .1323 | .0029 | .136 |
| 9 | .177-32 | .1786 | .1835 | .0049 | .1583 | .1598 | .0015 | .138 | .1411 | .0031 | .1405 |
| 10 | .190-30 | .1916 | .1968 | .0052 | .170 | .1716 | .0016 | .1483 | .1515 | .0032 | .152 |
| 12 | .216-28 | .2176 | .2232 | .0056 | .1944 | .1961 | .0017 | .1712 | .1745 | .0033 | .173 |
| 14 | .242-24 | .2438 | .250 | .0062 | .2167 | .2184 | .0017 | .1897 | .1932 | .0035 | .1935 |
| 16 | .268-22 | .2698 | .2765 | .0067 | .2403 | .2421 | .0018 | .2108 | .2144 | .0036 | .213 |
| 18 | .294-20 | .2959 | .3031 | .0072 | .2634 | .2652 | .0018 | .2309 | .2346 | .0037 | .234 |
| 20 | .320-20 | .3219 | .3291 | .0072 | .2894 | .2912 | .0018 | .2569 | .2606 | .0037 | .261 |
| 22 | .346-18 | .3479 | .3559 | .0080 | .3118 | .3138 | .0020 | .2757 | .2796 | .0039 | .281 |
| 24 | .372-16 | .374 | .3828 | .0088 | .3334 | .3354 | .0020 | .2928 | .2968 | .0040 | .2968 |
| 26 | .398-16 | .400 | .4088 | .0088 | .3594 | .3614 | .0020 | .3188 | .3228 | .0040 | .323 |
| 28 | .424-14 | .4261 | .4359 | .0098 | .3797 | .3818 | .0021 | .3333 | .3374 | .0041 | .339 |
| 30 | .450-14 | .4521 | .4619 | .0098 | .4057 | .4078 | .0021 | .3593 | .3634 | .0041 | .368 |

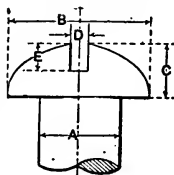
SPECIAL TAPS.

| | | | | | | | | | | | |
|----|----------|--------|--------|--------|-------|--------|--------|--------|--------|--------|-------|
| 1 | 0.073-64 | 0.0741 | 0.0768 | 0.0027 | 0.064 | 0.0651 | 0.0011 | 0.0538 | 0.0559 | 0.0021 | 0.055 |
| 2 | .086-56 | .0872 | .0903 | .0031 | .0756 | .0767 | .0011 | .064 | .0663 | .0023 | .067 |
| 3 | .099-48 | .1003 | .1038 | .0035 | .0868 | .088 | .0012 | .0732 | .0757 | .0025 | .076 |
| 4 | .112-40 | .1134 | .1175 | .0041 | .0972 | .0986 | .0014 | .0809 | .0837 | .0028 | .082 |
| | .36 | .1135 | .1179 | .0044 | .0955 | .0969 | .0014 | .0774 | .0803 | .0029 | .081 |
| 5 | .125-40 | .1264 | .1305 | .0041 | .1102 | .1116 | .0014 | .0939 | .0967 | .0028 | .098 |
| | .36 | .1255 | .1309 | .0044 | .1085 | .1099 | .0014 | .0904 | .0933 | .0029 | .0935 |
| 6 | .138-36 | .1395 | .1439 | .0044 | .1215 | .1229 | .0014 | .1034 | .1063 | .0029 | .1065 |
| | .32 | .1396 | .1445 | .0049 | .1193 | .1208 | .0015 | .099 | .1021 | .0031 | .1015 |
| 7 | .151-32 | .1526 | .1575 | .0049 | .1323 | .1338 | .0015 | .112 | .1151 | .0031 | .116 |
| | .30 | .1526 | .1578 | .0052 | .131 | .1326 | .0016 | .1093 | .1125 | .0032 | .113 |
| 8 | .164-32 | .1656 | .1705 | .0049 | .1453 | .1468 | .0015 | .125 | .1281 | .0031 | .1285 |
| | .30 | .1656 | .1708 | .0052 | .144 | .1456 | .0016 | .1223 | .1255 | .0032 | .1285 |
| 9 | .177-30 | .1786 | .1838 | .0052 | .1569 | .1585 | .0016 | .1353 | .1385 | .0032 | .1405 |
| | .24 | .1788 | .185 | .0062 | .1517 | .1534 | .0017 | .1247 | .1282 | .0035 | .1285 |
| 10 | .190-32 | .1916 | .1965 | .0049 | .1713 | .1728 | .0015 | .151 | .1541 | .0031 | .154 |
| | .24 | .1918 | .198 | .0062 | .1647 | .1664 | .0017 | .1377 | .1412 | .0035 | .1405 |
| 12 | .216-24 | .2178 | .224 | .0062 | .1907 | .1917 | .0017 | .1637 | .1672 | .0035 | .166 |
| 14 | .242-20 | .2439 | .2511 | .0072 | .2114 | .2132 | .0018 | .1789 | .1826 | .0037 | .182 |
| 16 | .268-20 | .2699 | .2771 | .0072 | .2374 | .2392 | .0018 | .2049 | .2086 | .0037 | .209 |
| 18 | .294-18 | .2959 | .3039 | .0080 | .2598 | .2618 | .0020 | .2237 | .2276 | .0039 | .228 |
| 20 | .320-18 | .3219 | .3299 | .0080 | .2858 | .2878 | .0020 | .2497 | .2536 | .0039 | .257 |
| 22 | .346-16 | .348 | .3568 | .0088 | .3074 | .3094 | .0020 | .2668 | .2708 | .0040 | .272 |
| 24 | .372-18 | .3739 | .3819 | .0080 | .3378 | .3398 | .0020 | .3017 | .3056 | .0039 | .3125 |
| 26 | .398-14 | .4001 | .4099 | .0098 | .3537 | .3558 | .0021 | .3073 | .3114 | .0041 | .316 |
| 28 | .424-16 | .426 | .4348 | .0088 | .3854 | .3874 | .0020 | .3448 | .3488 | .0040 | .348 |
| 30 | .450-16 | .452 | .4608 | .0088 | .4114 | .4134 | .0020 | .3708 | .3748 | .0040 | .377 |

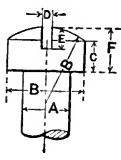
DIMENSIONS OF MACHINE SCREW HEADS, A. S. M. E. STANDARD.



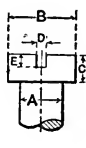
FLAT HEAD.
(1)



ROUND HEAD.
(2)



OVAL FILLISTER HEAD.
(3)



FLAT FILLISTER HEAD.
(4)

Dimensions.

A = Diam. of Body. D = Width of Slot = $0.173 A + 0.015$.

| | | | | |
|---|---------------------------|-----------------------|------------------|-----------------|
| B = Diameter of Head, and rad. of oval (3). | (1) | (2) | (3) | (4) |
| C = Height of Head or Side of Head (3). | $2A - 0.008$ | $1.85A - 0.005$ | $1.64A - 0.009$ | $1.64A - 0.009$ |
| E = Width of Slot. | $\frac{A - 0.008}{1.739}$ | $0.7A$ | $0.66A - 0.002$ | $0.66A - 0.002$ |
| F = Height of Head (3). | $\frac{1}{3}C$ | $\frac{1}{2}C + 0.01$ | $\frac{1}{2}F$ | $\frac{1}{2}C$ |
| | | | $0.134 B \} + C$ | |

| A | B (1) | B (2) | B (3,4) | C (1) | C (2) | C (3,4) | D | E (1) | E (2) | E (3) | E (4) | F (3) |
|------|-------|-------|---------|-------|-------|---------|------|-------|-------|-------|-------|-------|
| .060 | .112 | .106 | .0894 | .029 | .042 | .0376 | .025 | .010 | .031 | .025 | .019 | .0496 |
| .073 | .138 | .130 | .1107 | .037 | .051 | .0461 | .028 | .012 | .035 | .030 | .023 | .0609 |
| .086 | .164 | .154 | .132 | .045 | .060 | .0548 | .030 | .015 | .040 | .036 | .027 | .0725 |
| .099 | .190 | .178 | .153 | .052 | .069 | .0633 | .032 | .017 | .044 | .042 | .032 | .0838 |
| .112 | .216 | .202 | .1747 | .060 | .078 | .0719 | .034 | .020 | .049 | .048 | .036 | .0953 |
| .125 | .242 | .226 | .196 | .067 | .087 | .0805 | .037 | .022 | .053 | .053 | .040 | .1068 |
| .138 | .262 | .250 | .217 | .075 | .096 | .089 | .039 | .025 | .058 | .059 | .044 | .1180 |
| .151 | .294 | .274 | .2386 | .082 | .105 | .0976 | .041 | .027 | .062 | .065 | .049 | .1296 |
| .164 | .320 | .298 | .2599 | .090 | .114 | .1062 | .043 | .030 | .067 | .071 | .053 | .1410 |
| .177 | .346 | .322 | .2813 | .097 | .123 | .1148 | .046 | .032 | .071 | .076 | .057 | .1524 |
| .190 | .372 | .346 | .3026 | .105 | .133 | .1234 | .048 | .035 | .076 | .082 | .062 | .1639 |
| .216 | .424 | .394 | .3452 | .120 | .151 | .1405 | .052 | .040 | .085 | .093 | .070 | .1868 |
| .242 | .472 | .443 | .3879 | .135 | .169 | .1577 | .057 | .045 | .094 | .105 | .079 | .2097 |
| .268 | .528 | .491 | .4305 | .150 | .187 | .1748 | .061 | .050 | .103 | .116 | .087 | .2325 |
| .294 | .580 | .539 | .4731 | .164 | .205 | .192 | .066 | .055 | .112 | .128 | .096 | .2554 |
| .320 | .632 | .587 | .5158 | .179 | .224 | .2092 | .070 | .060 | .122 | .140 | .104 | .2783 |
| .346 | .682 | .635 | .5584 | .194 | .242 | .2263 | .075 | .065 | .131 | .150 | .113 | .3011 |
| .372 | .732 | .683 | .601 | .209 | .260 | .2435 | .079 | .070 | .140 | .162 | .122 | .3240 |
| .398 | .788 | .731 | .6437 | .224 | .278 | .2606 | .084 | .075 | .149 | .173 | .130 | .3469 |
| .424 | .840 | .779 | .6863 | .239 | .296 | .2778 | .088 | .080 | .158 | .185 | .139 | .3698 |
| .450 | .892 | .827 | .727 | .254 | .315 | .295 | .093 | .085 | .167 | .201 | .147 | .4024 |

WEIGHT OF 100 BOLTS WITH SQUARE HEADS. (Hoopes & Townsend.)

| Diam. inches. | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 3/4 | 7/8 | 1 | 1 1/8 | 1 1/4 | 1 3/8 | 1 1/2 | 1 3/4 | 2 |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Length. inches. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. |
| 1 1/2 | 3.9 | 6.2 | 9.7 | 14.7 | 20.4 | 26.0 | 37.0 | 58.0 | | | | | | | | |
| 2 | 4.6 | 7.2 | 11.3 | 16.5 | 22.4 | 29.0 | 39.9 | 63.2 | 97.7 | 145 | | | | | | |
| 2 1/2 | 5.4 | 8.2 | 12.9 | 18.5 | 25.0 | 32.2 | 44.1 | 69.0 | 105.6 | 153 | | | | | | |
| 3 | 6.2 | 9.3 | 14.5 | 20.5 | 27.8 | 35.4 | 48.3 | 75.2 | 113.8 | 163 | 240 | 309 | 350 | 480 | | |
| 3 1/2 | 6.9 | 10.4 | 16.1 | 22.6 | 30.6 | 38.7 | 52.5 | 81.4 | 122.0 | 174 | 253 | 325 | 370 | 500 | | |
| 4 | 7.6 | 11.5 | 17.7 | 24.7 | 33.4 | 42.0 | 56.7 | 87.6 | 130.2 | 185 | 267 | 342 | 390 | 520 | 800 | |
| 4 1/2 | 8.3 | 12.6 | 19.2 | 26.8 | 36.2 | 45.3 | 60.9 | 93.8 | 138.4 | 196 | 281 | 359 | 410 | 545 | 833 | |
| 5 | 9.0 | 13.7 | 20.7 | 28.9 | 39.0 | 48.6 | 65.1 | 100.0 | 146.6 | 207 | 295 | 376 | 430 | 570 | 866 | 1370 |
| 5 1/2 | 9.7 | 14.8 | 22.2 | 31.0 | 41.8 | 51.9 | 69.2 | 106.1 | 154.9 | 218 | 309 | 394 | 450 | 595 | 900 | 1414 |
| 6 | 10.4 | 15.9 | 23.7 | 33.1 | 44.6 | 55.2 | 73.4 | 112.2 | 163.2 | 229 | 323 | 412 | 470 | 620 | 934 | 1458 |
| 6 1/2 | 11.1 | 17.0 | 25.2 | 35.2 | 47.4 | 58.5 | 77.6 | 118.3 | 171.5 | 240 | 337 | 430 | 490 | 645 | 968 | 1502 |
| 7 | 11.8 | 18.1 | 26.7 | 37.3 | 50.2 | 61.8 | 81.8 | 124.4 | 179.8 | 251 | 351 | 448 | 510 | 670 | 1002 | 1546 |
| 7 1/2 | 12.5 | 19.2 | 28.2 | 39.4 | 53.1 | 65.1 | 86.0 | 130.5 | 187.1 | 262 | 365 | 466 | 530 | 695 | 1036 | 1590 |
| 8 | 13.2 | 20.3 | 29.7 | 41.5 | 56.0 | 68.5 | 90.0 | 136.6 | 195.4 | 273 | 379 | 484 | 550 | 725 | 1070 | 1634 |
| 9 | | | 33.1 | 45.7 | 61.5 | 75.2 | 98.0 | 148.8 | 212.0 | 295 | 407 | 518 | 590 | 775 | 1138 | 1722 |
| 10 | | | 36.5 | 49.9 | 67.0 | 81.9 | 106.3 | 161.0 | 229.0 | 317 | 435 | 552 | 630 | 825 | 1206 | 1810 |
| 11 | | | 40.0 | 54.1 | 72.5 | 88.7 | 114.6 | 173.2 | 246.0 | 339 | 463 | 586 | 670 | 875 | 1274 | 1898 |
| 12 | | | 43.5 | 58.3 | 78.0 | 95.5 | 122.9 | 184.4 | 263.0 | 361 | 491 | 620 | 710 | 925 | 1342 | 1986 |
| 13 | | | | | 83.5 | 102.3 | 131.2 | 196.6 | 280.0 | 383 | 519 | 655 | 751 | 975 | 1410 | 2074 |
| 14 | | | | | 89.0 | 109.1 | 139.5 | 208.8 | 297.0 | 405 | 547 | 690 | 793 | 1025 | 1478 | 2162 |
| 15 | | | | | 94.5 | 116.0 | 148.0 | 221.0 | 314.0 | 427 | 575 | 725 | 835 | 1075 | 1548 | 2250 |
| 16 | | | | | 100.0 | 123.0 | 156.5 | 233.2 | 331.0 | 449 | 603 | 760 | 877 | 1125 | 1616 | 2338 |
| 17 | | | | | 105.5 | 130.0 | 165.0 | 245.4 | 348.0 | 471 | 631 | 795 | 919 | 1175 | 1684 | 2426 |
| 18 | | | | | 111.0 | 137.0 | 173.5 | 257.6 | 365.0 | 493 | 659 | 830 | 961 | 1225 | 1752 | 2514 |
| 19 | | | | | 116.5 | 144.0 | 182.0 | 269.8 | 382.0 | 515 | 687 | 865 | 1003 | 1275 | 1820 | 2602 |
| 20 | | | | | 122.0 | 151.0 | 190.5 | 282.0 | 399.0 | 537 | 715 | 900 | 1045 | 1325 | 1888 | 2690 |
| 21 | | | | | | | 198.0 | 294.0 | 416.0 | 559 | 743 | 935 | 1087 | 1375 | 1956 | 2778 |
| 22 | | | | | | | 206.0 | 306.0 | 437.0 | 581 | 771 | 970 | 1129 | 1425 | 2024 | 2866 |
| 23 | | | | | | | 215.0 | 318.0 | 454.0 | 603 | 799 | 1005 | 1171 | 1475 | 2092 | 2954 |
| 24 | | | | | | | 224.0 | 330.0 | 470.0 | 625 | 827 | 1040 | 1213 | 1525 | 2160 | 3042 |
| 25 | | | | | | | | | | | 855 | 1075 | 1255 | 1575 | 2228 | 3130 |

ROUND HEAD RIVETS.

Approximate Number in One Pound. (Garland Nut & Rivet Co.)

| Diameter. Length. | 7/16 | 3/8 | 5/16 | 1/4 | 7/32 | 3/16 | 5/32 | 1/8 | Diameter. Length. | 7/16 | 3/8 | 5/16 | 1/4 | 7/32 | 3/16 | 5/32 | 1/8 |
|----------------------|--------|-----|------|-----|------|------|------|-----|----------------------|--------|-----|------|-----|------|------|------|-----|
| 3/8 | | | 68 | 103 | 145 | 184 | 194 | 204 | 1 5/8 | 10 1/2 | 15 | 22 | 34 | 46 | 65 | 76 | 87 |
| 1/2 | | 31 | 51 | 80 | 108 | 155 | 165 | 175 | 1 3/4 | 10 | 14 | 21 | 32 | 43 | 62 | 72 | 81 |
| 5/8 | | 28 | 45 | 70 | 94 | 135 | 148 | 160 | 1 7/8 | 9 1/2 | 13 | 20 | 30 | 41 | 58 | 69 | |
| 3/4 | 17 | 24 | 39 | 63 | 84 | 119 | 132 | 144 | 2 | 9 | 12 | 19 | 29 | 39 | 55 | 67 | |
| 7/8 | 15 | 22 | 35 | 56 | 75 | 106 | 121 | 135 | 2 1/4 | 8 1/4 | 11 | 17 | 27 | 35 | 49 | | |
| 1 | 14 | 20 | 32 | 50 | 68 | 96 | 111 | 126 | 2 1/2 | 7 3/4 | 10 | 16 | 24 | 32 | 45 | | |
| 1 1/8 | 13 | 19 | 30 | 46 | 62 | 88 | 102 | 116 | 2 3/4 | 7 1/4 | 9 | 14 | 22 | 29 | 42 | | |
| 1 1/4 | 12 | 18 | 28 | 43 | 57 | 81 | 94 | 108 | 3 | 6 3/4 | 8 | 13 | 20 | 27 | 39 | | |
| 1 3/8 | 11 1/2 | 17 | 26 | 40 | 53 | 74 | 87 | 100 | 3 1/2 | 6 | 7 | 11 | 18 | 23 | 34 | | |
| 1 1/2 | 11 | 16 | 24 | 37 | 50 | 69 | 81 | 93 | 4 | 5 | 6 | 10 | 16 | 20 | 30 | | |

Small rivets are made to fit holes of their rated size; the actual diameter may vary slightly from the decimals given below:

| | | | | | | | |
|-------------------|------|------|------|------|------|-------|------|
| Size..... | 3/32 | 7/64 | 1/8 | 9/64 | 5/32 | 11/64 | 3/16 |
| Approx. diam..... | .094 | .109 | .125 | .140 | .155 | .170 | .185 |
| Size..... | 7/32 | 1/4 | 9/32 | 5/16 | 3/8 | 7/16 | |
| Approx. diam..... | .215 | .245 | .275 | .305 | .365 | .425 | |

TRACK BOLTS.
With United States Standard Hexagon Nuts.

| Wt. of Rail. Lb. per Yard. | Bolts. | Nuts. | No. in Keg, 200 Lb. | Kegs per Mile. | Wt. of Rail. Lbs. per Yard. | Bolts. | Nuts. | No. in Keg, 200 Lb. | Kegs per Mile. |
|-------------------------------|---|---------|------------------------|----------------|--------------------------------|--------------------|----------|------------------------|----------------|
| 45 to 85 | $\left\{ \begin{array}{l} 3/4 \times 4 1/4 \\ 3/4 \times 4 \\ 3/4 \times 3 3/4 \\ 3/4 \times 3 1/2 \\ 3/4 \times 3 1/4 \end{array} \right.$ | $1 1/4$ | 230 | 6.3 | 30 to 40 | $5/8 \times 3 1/2$ | $1 1/16$ | 375 | 4. |
| | | $1 1/4$ | 240 | 6. | | $5/8 \times 3$ | $1 1/16$ | 410 | 3.7 |
| | | $1 1/4$ | 254 | 5.7 | | $5/8 \times 2 3/4$ | $1 1/16$ | 435 | 3.3 |
| | | $1 1/4$ | 260 | 5.5 | | $5/8 \times 2 1/2$ | $1 1/16$ | 465 | 3.1 |
| | | $1 1/4$ | 266 | 5.4 | | $1/2 \times 3$ | $7/8$ | 715 | 2. |
| | $\left\{ \begin{array}{l} 3/4 \times 3 1/4 \\ 3/4 \times 3 \\ 3/4 \times 3 \end{array} \right.$ | $1 1/4$ | 283 | 5.1 | 20 to 30 | $1/2 \times 2 1/2$ | $7/8$ | 760 | 2. |
| | | $1 1/4$ | | | | $1/2 \times 2 1/4$ | $7/8$ | 800 | 2. |
| | | $1 1/4$ | | | | $1/2 \times 2$ | $7/8$ | 820 | 2. |
| | | | | | | | | | |
| | | | | | | | | | |

WROUGHT WASHERS, MANUFACTURERS' STANDARD.
 (Upson Nut Co., Cleveland, 1906.)

| Diam. | Hole. | Thick- ness B.W.G. | Bolt. | No. in 200 Lb. | Diam. | Hole. | Thick- ness B.W.G. | Bolt. | No. in 200 Lb. |
|---------|----------|--------------------------|--------|-------------------|---------|----------|--------------------------|---------|-------------------|
| In. | In. | No. | In. | | In. | In. | No. | In. | |
| $9/16$ | $1/4$ | 18 | $3/16$ | 85200 | $2 1/2$ | $1 1/16$ | 9 | 1 | 1200 |
| $3/4$ | $5/16$ | 16 | $1/4$ | 34800 | $2 3/4$ | $1 1/4$ | 9 | $1 1/8$ | 888 |
| $7/8$ | $3/8$ | 16 | $5/16$ | 26200 | 3 | $1 3/8$ | 9 | $1 1/4$ | 900 |
| 1 | $7/16$ | 14 | $3/8$ | 14400 | $3 1/4$ | $1 1/2$ | 8 | $1 5/8$ | 600 |
| $1 1/4$ | $1/2$ | 14 | $7/16$ | 8400 | $3 1/2$ | $1 5/8$ | 8 | $1 1/2$ | 570 |
| $1 3/8$ | $9/16$ | 12 | $1/2$ | 5800 | $3 3/4$ | $1 3/4$ | 8 | $1 5/8$ | 460 |
| $1 1/2$ | $5/8$ | 12 | $9/16$ | 4600 | 4 | $1 7/8$ | 8 | $1 3/4$ | 432 |
| $1 3/4$ | $1 1/16$ | 10 | $5/8$ | 2600 | $4 1/4$ | 2 | 8 | $1 7/8$ | 366 |
| 2 | $1 3/16$ | 10 | $3/4$ | 2200 | $4 1/2$ | $2 1/8$ | 8 | 2 | 356 |
| $2 1/4$ | $1 5/16$ | 9 | $7/8$ | 1600 | | | | | |

SIZES OF CAST WASHERS.
 (Upson Nut Co., Cleveland, 1906.)

| Diam. | Hole. | Thick. | Bolt. | Weight. Lbs. | Diam. | Hole. | Thick. | Bolt. | Weight. Lbs. |
|---------|-------|----------|-------|-----------------|---------|---------|----------|---------|-----------------|
| In. | In. | In. | In. | | In. | In. | In. | In. | |
| $2 1/4$ | $5/8$ | $1 1/16$ | $1/2$ | $1/2$ | 4 | $1 1/8$ | $1 5/16$ | 1 | $1 5/8$ |
| $2 3/4$ | $3/4$ | $3/4$ | $5/8$ | $5/8$ | $4 1/2$ | $1 1/4$ | 1 | $1 1/8$ | $2 1/4$ |
| 3 | $7/8$ | $1 3/16$ | $3/4$ | $3/4$ | 5 | $1 3/8$ | $1 1/8$ | $1 1/4$ | 3 |
| $3 1/2$ | 1 | $7/8$ | $7/8$ | $1 1/4$ | 6 | $1 3/4$ | $1 1/4$ | $1 1/2$ | 5 |

CONE-HEAD BOILER RIVETS, WEIGHT PER 100.

(Hoopes & Townsend.)

| Diam., in., Scant. | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 | 1 | 1 1/8* | 1 1/4* |
|-----------------------|-------|------|-------|-------|-------|-------|-------|-------|--------|--------|
| Length. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. |
| 3/4 inch | 8.75 | 13.7 | 16.20 | | | | | | | |
| 7/8 " | 9.35 | 14.4 | 17.22 | | | | | | | |
| 1 " | 10.00 | 15.2 | 18.25 | 21.70 | 26.55 | | | | | |
| 1 1/8 " | 10.70 | 16.0 | 19.28 | 23.10 | 28.00 | | | | | |
| 1 1/4 " | 11.40 | 16.8 | 20.31 | 24.50 | 29.45 | 37.0 | 46 | 60 | | |
| 1 3/8 " | 12.10 | 17.6 | 21.34 | 25.90 | 30.90 | 38.6 | 48 | 63 | 95 | |
| 1 1/2 " | 12.80 | 18.4 | 22.37 | 27.30 | 32.35 | 40.2 | 50 | 65 | 98 | 133 |
| 1 5/8 " | 13.50 | 19.2 | 23.40 | 28.70 | 33.80 | 41.9 | 52 | 67 | 101 | 137 |
| 1 3/4 " | 14.20 | 20.0 | 24.43 | 30.10 | 35.25 | 43.5 | 54 | 69 | 104 | 141 |
| 1 7/8 " | 14.90 | 20.8 | 25.46 | 31.50 | 36.70 | 45.2 | 56 | 71 | 107 | 145 |
| 2 " | 15.60 | 21.6 | 26.49 | 32.90 | 38.15 | 47.0 | 58 | 74 | 110 | 149 |
| 2 1/8 " | 16.30 | 22.4 | 27.52 | 34.30 | 39.60 | 48.7 | 60 | 77 | 114 | 153 |
| 2 1/4 " | 17.00 | 23.2 | 28.55 | 35.70 | 41.05 | 50.3 | 62 | 80 | 118 | 157 |
| 2 3/8 " | 17.70 | 24.0 | 29.58 | 37.10 | 42.50 | 51.9 | 64 | 83 | 121 | 161 |
| 2 1/2 " | 18.40 | 24.8 | 30.61 | 38.50 | 43.95 | 53.5 | 66 | 86 | 124 | 165 |
| 2 5/8 " | 19.10 | 25.6 | 31.64 | 39.90 | 45.40 | 55.1 | 68 | 89 | 127 | 169 |
| 2 3/4 " | 19.80 | 26.4 | 32.67 | 41.30 | 46.85 | 56.8 | 70 | 92 | 130 | 173 |
| 2 7/8 " | 20.50 | 27.2 | 33.70 | 42.70 | 48.30 | 58.4 | 72 | 95 | 133 | 177 |
| 3 " | 21.20 | 28.0 | 34.73 | 44.10 | 49.75 | 60.0 | 74 | 98 | 137 | 181 |
| 3 1/4 " | 22.60 | 29.7 | 36.79 | 46.90 | 52.65 | 63.3 | 78 | 103 | 144 | 189 |
| 3 1/2 " | 24.00 | 31.5 | 38.85 | 49.70 | 55.55 | 66.5 | 82 | 108 | 151 | 197 |
| 3 3/4 " | 25.40 | 33.3 | 40.91 | 52.50 | 58.45 | 69.8 | 86 | 113 | 158 | 205 |
| 4 " | 26.80 | 35.2 | 42.97 | 55.30 | 61.35 | 73.0 | 90 | 118 | 165 | 213 |
| 4 1/4 " | 28.20 | 36.9 | 45.03 | 58.10 | 64.25 | 76.3 | 94 | 124 | 172 | 221 |
| 4 1/2 " | 29.60 | 38.6 | 47.09 | 60.90 | 67.15 | 79.5 | 98 | 130 | 179 | 229 |
| 4 3/4 " | 31.00 | 40.3 | 49.15 | 63.70 | 70.05 | 82.8 | 102 | 136 | 186 | 237 |
| 5 " | 32.40 | 42.0 | 51.27 | 66.50 | 72.95 | 86.0 | 106 | 142 | 193 | 245 |
| 5 1/4 " | 33.80 | 43.7 | 53.27 | 69.20 | 75.85 | 89.3 | 110 | 148 | 200 | 254 |
| 5 1/2 " | 35.20 | 45.4 | 55.33 | 72.00 | 78.75 | 92.5 | 114 | 154 | 206 | 263 |
| 5 3/4 " | 36.60 | 47.1 | 57.39 | 74.80 | 81.65 | 95.7 | 118 | 160 | 212 | 272 |
| 6 " | 38.00 | 48.8 | 59.45 | 77.60 | 84.55 | 99.0 | 122 | 166 | 218 | 281 |
| 6 1/2 " | 40.80 | 52.0 | 63.57 | 83.30 | 90.35 | 105.5 | 130 | 177 | 231 | 297 |
| 7 " | 43.60 | 55.2 | 67.69 | 88.90 | 95.15 | 112.0 | 138 | 188 | 245 | 314 |
| Heads..... | 5.50 | 8.40 | 11.50 | 13.20 | 18.00 | 23.0 | 29.0 | 38.0 | 55.0 | 77.5 |

* These two sizes are calculated for exact diameter.

TURNBUCKLES.

(Cleveland City Forge and Iron Co.)

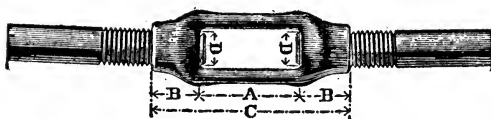
Standard sizes made with right and left threads. D = outside diameter

FIG. 76.

of screw. A = length in clear between heads = 6 ins. for all sizes,
 B = length of tapped heads = $1\frac{1}{2} D$ nearly. C = 6 ins. + $3 D$ nearly.

TINNERS' RIVETS. FLAT HEADS.

Garland Nut & Rivet Co.

| Diam., in. | Length, in. | Wt. per 1000. | Diam., in. | Length, in. | Wt. per 1000. | Diam., in. | Length, in. | Wt. per 1000. | Diam., in. | Length, in. | Wt. per 1000. |
|---------------|----------------|------------------|---------------|----------------|------------------|---------------|----------------|------------------|---------------|----------------|------------------|
| 0.070 | 1/8 | 4 oz. | .115 | 13/64 | 1 lb. | 0.160 | 5/16 | 3 lbs. | 0.225 | 7/16 | 8 |
| .080 | 9/64 | 6 | .120 | 7/32 | 1 1/4 | .163 | 21/64 | 3 1/2 | .230 | 29/64 | 9 |
| .090 | 5/32 | 8 | .125 | 15/64 | 1 1/2 | .173 | 11/32 | 4 | .233 | 15/32 | 10 |
| .094 | 11/64 | 10 | .133 | 1/4 | 1 3/4 | .185 | 3/8 | 5 | .253 | 1/2 | 12 |
| .101 | 3/16 | 12 | .140 | 17/64 | 2 | .200 | 25/64 | 6 | .275 | 33/64 | 14 |
| .109 | 3/16 | 14 | .147 | 9/32 | 2 1/2 | .215 | 13/32 | 7 | .293 | 17/32 | 16 |

MATERIAL REQUIRED FOR ONE MILE OF SINGLE TRACK RAILROAD.

(American Bureau of Inspection and Tests, 1908.)

Cross Ties.

| 33-Foot Rail. | | 30-Foot Rail. | | Spacing of Ties, Center to Center. |
|-------------------|----------------|----------------|----------------|--|
| Ties per Rail. | Ties per Mile. | Ties per Rail. | Ties per Mile. | |
| 20 | 3200 | 18 | 3168 | 1 ft. 6 in. |
| 18 | 2880 | 16 | 2816 | 1 " 9 " |
| 16 | 2560 | 14 | 2464 | 2 " 0 " |

Rails.

| Weight per Yard. Lb. | Gross Tons Per Mile. | Weight per Yard. Lb. | Gross Tons Per Mile. | Weight per Yard. Lb. | Gross Tons per Mile. |
|----------------------------|-------------------------|----------------------------|-------------------------|----------------------------|-------------------------|
| 100 | 157 1/7 | 67 | 105 2/7 | 40 | 62 6/7 |
| 90 | 141 3/7 | 65 | 102 1/7 | 35 | 55 |
| 85 | 133 4/7 | 60 | 94 2/7 | 30 | 47 1/7 |
| 80 | 125 5/7 | 56 | 88 | 25 | 39 2/7 |
| 75 | 117 6/7 | 52 | 81 5/7 | 20 | 31 3/7 |
| 72 | 113 1/7 | 50 | 78 4/7 | 16 | 25 1/7 |
| 70 | 110 | 45 | 70 5/7 | 12 | 18 6/7 |

Decimal Equivalent for 1/7 = 0.143. 2/7 = 0.286. 3/7 = 0.429. 4/7 = 0.571.
5/7 = 0.714. 6/7 = 0.857.

To find gross tons per mile of track multiply weight of rail (pounds per yard) by 11 and divide by 7. To find feet of rail per gross ton divide 6720 by weight of rail per yard.

Splices and Bolts.

| Length of Rails Used. | Number of Joints or Rails. | Number of Bolts Using Four-Hole Splices. | Number of Bolts Using Six-Hole Splices. |
|-----------------------|----------------------------|--|---|
| 33 ft. | 320 | 1280 | 1920 |
| 30 " | 352 | 1408 | 2112 |

Spikes.

| Size Measured Under Head. | Av. No. per Keg of 200 Lbs. | Kegs per Mile (4 Spikes to a Tie). | | | | | | | |
|------------------------------|--------------------------------|------------------------------------|-------|-------|------------------------|-------|------|------------------------|--|
| | | Using 33-Ft. Rails. | | | Using 30-Ft. Rails. | | | Ties 2 Ft. C. to C. | |
| | | 20 | 18 | 16 | 18 | 16 | 14 | | |
| | | Ties per Rail. | | | Ties per Rail. | | | | |
| 6 × 5/8 | 260 | 49.2 | 44.3 | 39.4 | 48.7 | 43.3 | 37.9 | 40.6 | |
| 6 × 9/16 | 350 | 36.6 | 32.9 | 29.3 | 36.2 | 32.2 | 28.2 | 30.2 | |
| 5 1/2 × 5/8 | 290 | 44.1 | 39.7 | 35.3 | 43.7 | 38.8 | 34.0 | 36.4 | |
| 5 1/2 × 9/16 | 375 | 34.1 | 30.7 | 27.3 | 33.8 | 30.0 | 26.3 | 28.2 | |
| 5 × 9/16 | 400 | 32.0 | 28.8 | 25.6 | 31.7 | 28.2 | 24.6 | 26.4 | |
| 5 × 1 1/2 | 450 | 28.5 | 25.6 | 22.8 | 28.2 | 25.0 | 21.9 | 23.5 | |
| 4 1/2 × 1 1/2 | 530 | 24.2 | 21.8 | 19.3 | 23.9 | 21.3 | 18.6 | 19.9 | |
| 4 1/2 × 7/16 | 680 | 18.8 | 17.0 | 15.1 | 18.6 | 16.6 | 14.5 | 15.5 | |
| Spikes per mile..... | | 12800 | 11520 | 10240 | 12672 | 11264 | 9856 | 10560 | |

In ordering spikes allow
for extras, waste, etc.

In ordering spikes allow for extras, waste, etc.

WROUGHT SPIKES.**Number of Nails in Keg of 150 Pounds.**

| Length, Inches. | 1/4 in. | 5/16 in. | 3/8 in. | Length, Inches. | 1/4 in. | 5/16 in. | 3/8 in. | 7/16 in. | 1/2 in. |
|-----------------|---------|----------|---------|-----------------|---------|----------|---------|----------|---------|
| 3 | 2250 | | | 7 | 1161 | 662 | 482 | 445 | 306 |
| 3 1/2 | 1890 | 1208 | | 8 | | 635 | 455 | 384 | 256 |
| 4 | 1650 | 1135 | | 9 | | 573 | 424 | 300 | 240 |
| 4 1/2 | 1464 | 1064 | | 10 | | | 391 | 270 | 222 |
| 5 | 1380 | 930 | 742 | 11 | | | | 249 | 203 |
| 6 | 1292 | 868 | 570 | 12 | | | | 236 | 180 |

For sizes and weights of wire spikes see Steel Wire Nails, page 235.

BOAT SPIKES.**Number in Keg of 200 Pounds.**

| Length. | 1/4 | 5/16 | 3/8 | 1/2 |
|-------------|-------|-------|-------|-------|
| 4 inch..... | 2375 | | | |
| 5 "..... | 2050 | 1230 | 940 | |
| 6 "..... | 1825 | 1175 | 800 | 450 |
| 7 "..... | | 990 | 650 | 375 |
| 8 "..... | | 880 | 600 | 335 |
| 9 "..... | | | 525 | 300 |
| 10 "..... | | | 475 | 275 |

LENGTH AND NUMBER OF CUT NAILS TO THE POUND.

| Size. | Length. | Common. | Clinch. | Fence. | Finishing. | Fine. | Barrel. | Casing. | Brads. | Tobacco | Cut Spikes |
|-------|---------|---------|---------|--------|------------|-------|---------|---------|--------|---------|------------|
| 3/4 | 3/4 In. | | | | | | 800 | | | | |
| 7/8 | 7/8 | | | | | | 500 | | | | |
| 2d | 1 | 800 | | | 1100 | 1000 | 376 | | | | |
| 3d | 1 1/4 | 480 | | | 720 | 760 | 224 | | | | |
| 4d | 1 1/2 | 288 | | | 523 | 368 | 180 | 398 | | | |
| 5d | 1 3/4 | 200 | | | 410 | | | | | 130 | |
| 6d | 2 | 168 | 95 | 84 | 268 | | | 224 | 126 | 96 | |
| 7d | 2 1/4 | 124 | 74 | 64 | 188 | | | | 98 | 82 | |
| 8d | 2 1/2 | 88 | 62 | 48 | 146 | | | 128 | 75 | 68 | |
| 9d | 2 3/4 | 70 | 53 | 36 | 130 | | | 110 | 65 | | |
| 10d | 3 | 58 | 46 | 30 | 102 | | | 91 | 55 | | 28 |
| 12d | 3 1/4 | 44 | 42 | 24 | 76 | | | 71 | 40 | | |
| 16d | 3 1/2 | 34 | 38 | 20 | 62 | | | 54 | 27 | | 22 |
| 20d | 4 | 23 | 33 | 16 | 54 | | | 40 | | | 14 1/2 |
| 30d | 4 1/2 | 18 | 20 | | | | | 33 | | | 12 1/2 |
| 40d | 5 | 14 | | | | | | 27 | | | 9 1/2 |
| 50d | 5 1/2 | 10 | | | | | | | | | 8 |
| 60d | 6 | 8 | | | | | | | | | 6 |

DIMENSIONS OF WOOD SCREWS.

| No. | Threads per In. | Diam. of Body. | Lengths. | No. | Threads per In. | Diam. of Body. | Lengths. |
|-----|-----------------|----------------|-----------|-----|-----------------|----------------|-----------|
| | | In. | In. | | | In. | In. |
| 2 | 56 | 0.0842 | 3/16-1/2 | 12 | 20, 24 | 0.2158 | 3/8-13/4 |
| 3 | 48 | .0973 | 3/16-5/8 | 14 | 20, 24 | .2421 | 3/8-2 |
| 4 | 32, 36, 40 | .1105 | 3/16-3/4 | 16 | 16, 18, 20 | .2684 | 3/8-2 1/4 |
| 5 | 32, 36, 40 | .1236 | 3/16-7/8 | 18 | 16, 18 | .2947 | 1/2-2 1/2 |
| 6 | 30, 32 | .1368 | 3/16-1 | 20 | 16, 18 | .3210 | 1/2-2 3/4 |
| 7 | 30, 32 | .1500 | 1/4-1 1/8 | 22 | 16, 18 | .3474 | 1/2-3 |
| 8 | 30, 32 | .1631 | 1/4-1 1/4 | 24 | 14, 16 | .3737 | 1/2-3 |
| 9 | 24, 30, 32 | .1763 | 1/4-1 3/8 | 26 | 14, 16 | .4000 | 3/4-3 |
| 10 | 24, 30, 32 | .1894 | 1/4-1 1/2 | 28 | 14, 16 | .4263 | 7/8-8 |
| | | | | 30 | 14, 16 | .4520 | 13- |

WEIGHTS AND DIMENSIONS OF LAG SCREWS.

| Length in Inches. | Diameter in Inches. | | | | |
|-------------------|---------------------|--------------|--------------|--------------|--------------|
| | 3/8 | 7/16 | 1/2 | 5/8 | 3/4 |
| | Lb. per 100. | Lb. per 100. | Lb. per 100. | Lb. per 100. | Lb. per 100. |
| 1 1/2 | 6.88 | | | | |
| 1 3/4 | 7.50 | 11.75 | 16.88 | | |
| 2 | 8.25 | 12.62 | 17.18 | | |
| 2 1/4 | 9.25 | 12.88 | 18.07 | | |
| 2 1/2 | 9.62 | 13.28 | 19.18 | | |
| 3 | 10.82 | 16.62 | 22.00 | 34.07 | |
| 3 1/2 | 11.50 | 18.18 | 24.00 | 35.88 | |
| 4 | 13.31 | 18.88 | 26.82 | 39.25 | 64.00 |
| 4 1/2 | 14.82 | 19.50 | 28.25 | 42.62 | 67.88 |
| 5 | 16.50 | 21.25 | 30.37 | 47.75 | 71.37 |
| 5 1/2 | 17.37 | 23.56 | 33.88 | 51.62 | 79.37 |
| 6 | 18.82 | 25.31 | 35.37 | 55.12 | 86.62 |
| 7 | | | 38.94 | 61.88 | 92.75 |
| 8 | | | 44.37 | 68.75 | 97.50 |
| 9 | | | | 77.00 | 108.75 |
| 10 | | | | 90.00 | 124.75 |

SIZES, LENGTH, AND NUMBER TO THE POUND OF STANDARD STEEL WIRE NAILS.
(American Steel and Wire Co., 1908.)

| Sizes. | Length, Inches. | Common Nails and Brads. | Clinch. | Fence. | Smooth and Barbed Finishing. | Fine. | Barrel. | Casing, and Smooth and Barbed Box. | Flooring Brads. | Boat Nails. | | Barbed Car Nail. | | Slat- ing. | Barbed Roofing. | Shingle. | Tobacco. | Lining. | Wire Spikes. | Length, Inches. | Sizes. |
|-----------|-----------------|----------------------------|---------|--------|---------------------------------|-------|---------|--|-----------------|-------------|--------|---------------------|--------|---------------|-----------------|----------|----------|---------|--------------|-----------------|--------|
| | | | | | | | | | | Light. | Heavy. | Light. | Heavy. | | | | | | | | |
| 3/4 | 3/4 | | | | | 2077 | 1346 | | | | | | | | 714 | | | 2077 | | 3/4 | |
| 7/8 | 7/8 | | | | | 1781 | 906 | | | | | | | | 409 | | | 1781 | | 7/8 | |
| 1 | 1 | | | | 1351 | 1558 | 775 | 1010 | | | | | | 411 | 411 | | | 155 | | 1 | |
| 1 1/8 | 1 1/8 | | | | | 1140 | 700 | | | | | | | | | | | | | 1 1/8 | |
| 3d fine. | 1 1/4 | | | | 807 | | 568 | 913 | | | | | | 225 | 251 | 429 | | | | 1 1/4 | |
| 3d common | 1 3/8 | | | | | | 400 | 635 | | | | | | | 230 | | | | | 1 3/8 | |
| 4d | 1 1/2 | | | | 584 | 760 | 357 | 473 | | 82 | 44 | 274 | 165 | 187 | 176 | 274 | 274 | | | 1 1/2 | |
| 5d | 1 3/4 | | | | 500 | | | 406 | | | | 142 | 118 | 142 | 151 | 235 | 235 | | | 1 3/4 | |
| 6d | 2 | | | | 309 | | | 236 | | 62 | 32 | 124 | 103 | 103 | 103 | 204 | 157 | | | 2 | |
| 7d | 2 1/4 | | | | 238 | | | 210 | | | | 92 | 76 | | 139 | 139 | 139 | | | 2 1/4 | |
| 8d | 2 1/2 | | | | 189 | | | 145 | | 50 | 26 | | | | 125 | 99 | 125 | | | 2 1/2 | |
| 9d | 2 3/4 | | | | 172 | | | 132 | | | | 82 | 69 | | 114 | 90 | 114 | | | 2 3/4 | |
| 10d | 3 | | | | 121 | | | 94 | | 22 | 14 | 57 | 50 | | | 83 | 69 | | | 3 | |
| 12d | 3 1/4 | | | | 113 | | | 87 | | | | 50 | 42 | | | | | | | 3 1/4 | |
| 16d | 3 1/2 | | | | 90 | | | 71 | | 20 | 13 | 43 | 35 | | | | | | | 3 1/2 | |
| 20d | 4 | | | | 62 | | | 52 | | 18 | 10 | 31 | 26 | | | | | | | 4 | |
| 30d | 4 1/2 | | | | | | | 46 | | 16 | | 28 | 24 | | | | | | | 4 1/2 | |
| 40d | 5 | | | | | | | 35 | | | | 21 | 18 | | | | | | | 5 | |
| 50d | 5 1/2 | | | | | | | | | | | 17 | 15 | | | | | | | 5 1/2 | |
| 60d | 6 | | | | | | | | | | | 15 | 13 | | | | | | | 6 | |

3 3/8 lb. of 4d Common, or 2 3/8 lb. of 3d Common, will lay 1000 shingles.
3 1/4 lb. of 3d Fine will put on 1000 laths — 4 nails to the lath.

APPROXIMATE NUMBER OF WIRE NAILS PER POUND.

(American Steel and Wire Co., 1908.)

| Wire Gauge. B.W.G. | Length, Inches. | | | | | | | | | | | | | |
|-----------------------|-----------------|-----|-----|-----|-----|-----|---|-------|-------|-------|---|-------|---|-------|
| | 3/16 | 1/4 | 3/8 | 1/2 | 5/8 | 3/4 | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/2 | 3 | 3 1/2 |
| 3/8 in. | | | | | | | | | | | | | | |
| 0 | | | | | | | | | | | | | | |
| 5/16 in. | | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | | | |
| 11 | | | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | | | |
| 13 | | | | | | | | | | | | | | |
| 14 | | | | | | | | | | | | | | |
| 15 | | | | | | | | | | | | | | |
| 16 | | | | | | | | | | | | | | |
| 17 | | | | | | | | | | | | | | |
| 18 | | | | | | | | | | | | | | |
| 19 | | | | | | | | | | | | | | |
| 20 | | | | | | | | | | | | | | |
| 21 | | | | | | | | | | | | | | |
| 22 | | | | | | | | | | | | | | |

These approximate numbers are an average only, and the figures given may be varied either way by changes in the dimensions of the heads or points. Brads and no-head nails will run more to the pound than the table shows, and large or thick-headed nails will run less.

PROPERTIES OF STEEL WIRE.

(John A. Roebling's Sons Co., 1908.)

| No., Roebling Gauge. | Diam., in. | Area, square inches. | Breaking strain, 100, 000 lb. per sq. inch. | Weight in pounds. | | Feet in 2000 lb. |
|----------------------------|---------------|----------------------------|--|-------------------|--------------|---------------------|
| | | | | Per 1000 ft. | Per mile. | |
| 000000 | 0.460 | 0.166191 | 16,619 | 558.4 | 2,948 | 3,582 |
| 00000 | 0.430 | 0.145221 | 14,522 | 487.9 | 2,576 | 4,099 |
| 0000 | 0.393 | 0.121304 | 12,130 | 407.6 | 2,152 | 4,907 |
| 000 | 0.362 | 0.102922 | 10,292 | 345.8 | 1,826 | 5,783 |
| 00 | 0.331 | 0.086049 | 8,605 | 289.1 | 1,527 | 6,917 |
| 0 | 0.307 | 0.074023 | 7,402 | 248.7 | 1,313 | 8,041 |
| 1 | 0.283 | 0.062902 | 6,290 | 211.4 | 1,116 | 9,463 |
| 2 | 0.263 | 0.054325 | 5,433 | 182.5 | 964 | 10,957 |
| 3 | 0.244 | 0.046760 | 4,676 | 157.1 | 830 | 12,730 |
| 4 | 0.225 | 0.039761 | 3,976 | 133.6 | 705 | 14,970 |
| 5 | 0.207 | 0.033654 | 3,365 | 113.1 | 597 | 17,687 |
| 6 | 0.192 | 0.028953 | 2,895 | 97.3 | 514 | 20,559 |
| 7 | 0.177 | 0.024606 | 2,461 | 82.7 | 437 | 24,191 |
| 8 | 0.162 | 0.020612 | 2,061 | 69.3 | 366 | 28,878 |
| 9 | 0.148 | 0.017203 | 1,720 | 57.8 | 305 | 34,600 |
| 10 | 0.135 | 0.014314 | 1,431 | 48.1 | 254 | 41,584 |
| 11 | 0.120 | 0.011310 | 1,131 | 38.0 | 201 | 52,631 |
| 12 | 0.105 | 0.008659 | 866 | 29.1 | 154 | 68,752 |
| 13 | 0.092 | 0.006648 | 665 | 22.3 | 118 | 89,525 |
| 14 | 0.080 | 0.005027 | 503 | 16.9 | 89.2 | 118,413 |
| 15 | 0.072 | 0.004071 | 407 | 13.7 | 72.2 | 146,198 |
| 16 | 0.063 | 0.003117 | 312 | 10.5 | 55.3 | 191,022 |
| 17 | 0.054 | 0.002290 | 229 | 7.70 | 40.6 | 259,909 |
| 18 | 0.047 | 0.001735 | 174 | 5.83 | 30.8 | 343,112 |
| 19 | 0.041 | 0.001320 | 132 | 4.44 | 23.4 | 450,856 |
| 20 | 0.035 | 0.000962 | 96 | 3.23 | 17.1 | 618,620 |
| 21 | 0.032 | 0.000804 | 80 | 2.70 | 14.3 | 740,193 |
| 22 | 0.028 | 0.000616 | 62 | 2.07 | 10.9 | 966,651 |
| 23 | 0.025 | 0.000491 | 49 | 1.65 | 8.71 | ... |
| 24 | 0.023 | 0.000415 | 42 | 1.40 | 7.37 | ... |
| 25 | 0.020 | 0.000314 | 31 | 1.06 | 5.58 | ... |
| 26 | 0.018 | 0.000254 | 25 | 0.855 | 4.51 | ... |
| 27 | 0.017 | 0.000227 | 23 | .763 | 4.03 | ... |
| 28 | 0.016 | 0.000201 | 20 | .676 | 3.57 | ... |
| 29 | 0.015 | 0.000177 | 18 | .594 | 3.14 | ... |
| 30 | 0.014 | 0.000154 | 15 | .517 | 2.73 | ... |
| 31 | 0.0135 | 0.000143 | 14 | .481 | 2.54 | ... |
| 32 | 0.013 | 0.000133 | 13 | .446 | 2.36 | ... |
| 33 | 0.011 | 0.000095 | 9.5 | .319 | 1.69 | ... |
| 34 | 0.010 | 0.000079 | 7.9 | .264 | 1.39 | ... |
| 35 | 0.0095 | 0.000071 | 7.1 | .238 | 1.26 | ... |
| 36 | 0.009 | 0.000064 | 6.4 | .214 | 1.13 | ... |

The above table was calculated on a basis of 483.84 lb. per cu. ft. for steel wire. Iron wire is a trifle lighter. The breaking strains are calculated for 100,000 lb. per sq. in. throughout, simply for convenience, so that the breaking strains of wires of any strength per sq. in. may be quickly determined by multiplying the values given in the tables by the ratio between the strength per square inch and 100,000. Thus, a No. 15 wire, with a strength per sq. in. of 150,000 lb., has a breaking strain of $407 \times \frac{150,000}{100,000} = 610.5$ lb.

GALVANIZED IRON WIRE FOR TELEGRAPH AND TELEPHONE LINES.

(Trenton Iron Co.)

WEIGHT PER MILE-OHM. — This term is to be understood as distinguishing the *resistance of material* only, and means the weight of such material required per mile to give the resistance of one ohm. To ascertain the mileage resistance of any wire, divide the "weight per mile-ohm" by the weight of the wire per mile. Thus in a grade of Extra Best Best, of which the weight per mile-ohm is 5000, the mileage resistance of No. 6 (weight per mile 525 lbs.) would be about $9\frac{1}{2}$ ohms; and No. 14 steel wire, 6500 lbs. weight per mile-ohm (95 lbs. weight per mile), would show about 69 ohms.

Sizes of Wire used in Telegraph and Telephone Lines.

No. 4. Has not been much used until recently; is now used on important lines where the multiplex systems are applied.

No. 5. Little used in the United States.

No. 6. Used for important circuits between cities.

No. 8. Medium size for circuits of 400 miles or less.

No. 9. For similar locations to No. 8, but on somewhat shorter circuits; until lately was the size most largely used in this country.

Nos. 10, 11. For shorter circuits, railway telegraphs, private lines, police and fire-alarm lines, etc.

No. 12. For telephone lines, police and fire-alarm lines, etc.

Nos. 13, 14. For telephone lines and short private lines; steel wire is used most generally in these sizes.

The coating of telegraph wire with zinc as a protection against oxidation is now generally admitted to be the most efficacious method.

The grades of line wire are generally known to the trade as "Extra Best Best" (E. B. B.), "Best Best" (B. B.), and "Steel."

"Extra Best Best" is made of the very best iron, as nearly pure as any commercial iron, soft, tough, uniform, and of very high conductivity, its weight per mile-ohm being about 5000 lbs.

The "Best Best" is of iron, showing in mechanical tests almost as good results as the E. B. B., but is not quite as soft, and somewhat lower in conductivity; weight per mile-ohm about 5700 lbs.

The "Steel" wire is well suited for telephone or short telegraph lines, and the weight per mile-ohm is about 6500 lbs.

The following are (approximately) the weights per mile of various sizes of galvanized telegraph wire, drawn by Trenton Iron Co.'s gauge:

| | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|-----|
| No. | 4, | 5, | 6, | 7, | 8, | 9, | 10, | 11, | 12, | 13, | 14. |
| Lbs. | 720, | 610, | 525, | 450, | 375, | 310, | 250, | 200, | 160, | 125, | 95. |

TESTS OF TELEGRAPH WIRE.

The following data are taken from a table given by Mr. Prescott relating to tests of E. B. B. galvanized wire furnished the Western Union Telegraph Co.

| Size of Wire | Diam., Inch. | Weight. | | Length, Feet per pound. | Resistance, Temp. 75.8° Fahr. | | Ratio of Breaking Weight to Weight per mile. |
|--------------|-----------------|---------------------|---------------------|----------------------------------|----------------------------------|-------------------|--|
| | | Grains per foot. | Pounds per mile. | | Feet per ohm | Ohms per mile. | |
| 4 | 0.238 | 1043.2 | 886.6 | 6.00 | 958 | 5.51 | |
| 5 | .220 | 891.3 | 673.0 | 7.85 | 727 | 7.26 | |
| 6 | .203 | 758.9 | 572.2 | 9.20 | 618 | 8.54 | 3.05 |
| 7 | .180 | 596.7 | 449.9 | 11.70 | 578 | 10.86 | 3.40 |
| 8 | .165 | 501.4 | 378.1 | 14.00 | 409 | 12.92 | 3.07 |
| 9 | .148 | 403.4 | 304.2 | 17.4 | 328 | 16.10 | 3.38 |
| 10 | .134 | 330.7 | 249.4 | 21.2 | 269 | 19.60 | 3.37 |
| 11 | .120 | 265.2 | 200.0 | 26.4 | 216 | 24.42 | 2.97 |
| 12 | .109 | 218.8 | 165.0 | 32.0 | 179 | 29.60 | 3.43 |
| 14 | .083 | 126.9 | 95.7 | 55.2 | 104 | 51.00 | 3.05 |

JOINTS IN TELEGRAPH WIRES. — The fewer the joints in a line the better. All joints should be carefully made and well soldered over, for a bad joint may cause as much resistance to the electric current as several miles of wire.

SPECIFICATIONS FOR GALVANIZED IRON WIRE.

Issued by the British Postal Telegraph Authorities.

| Weight per Mile. | | | Diameter. | | | Tests for Strength and Ductility. | | | | | | Resistance per Mile of Standard at 60° F. | Constant = Standard Weight × Resistance. |
|--------------------|----------|------|--------------------|----------|-------|-----------------------------------|------------------------|---------------------------|----------------------|---------------------------|----------------------|---|--|
| Required Standard. | Allowed. | | Required Standard. | Allowed. | | Breaking Weight. | No. of Twists in 6 in. | Least Breaking Weight for | Min. Twists in 6 in. | Least Breaking Weight for | Min. Twists in 6 in. | | |
| | Min. | Max. | | Min. | Max. | | | | | | | Min. | |
| lb. | lb. | lb. | mils. | mils. | mils. | lb. | | lb. | | lb. | | ohms. | |
| 800 | 767 | 833 | 242 | 237 | 247 | 2480 | 15 | 2550 | 14 | 2620 | 13 | 6.75 | 5400 |
| 600 | 571 | 629 | 209 | 204 | 214 | 1860 | 17 | 1910 | 16 | 1960 | 15 | 9.00 | 5400 |
| 450 | 424 | 477 | 181 | 176 | 186 | 1390 | 19 | 1425 | 18 | 1460 | 17 | 12.00 | 5400 |
| 400 | 377 | 424 | 171 | 166 | 176 | 1240 | 21 | 1270 | 20 | 1300 | 19 | 13.50 | 5400 |
| 200 | 190 | 213 | 121 | 118 | 125 | 620 | 30 | 638 | 28 | 655 | 26 | 27.00 | 5400 |

STRENGTH OF PIANO-WIRE.

The average strength of English piano-wire is given as follows by Webster, Horsfals & Lean:

| Size, Music-wire Gauge. | Equivalent Diameters, Inch. | Ultimate Tensile Strength, Pounds. | Size, Music-wire Gauge. | Equivalent Diameters, Inch. | Ultimate Tensile Strength, Pounds. |
|-------------------------|-----------------------------|------------------------------------|-------------------------|-----------------------------|------------------------------------|
| 12 | 0.029 | 225 | 18 | 0.041 | 395 |
| 13 | .031 | 250 | 19 | .043 | 425 |
| 14 | .033 | 285 | 20 | .045 | 500 |
| 15 | .035 | 305 | 21 | .047 | 540 |
| 16 | .037 | 340 | 22 | .052 | 650 |
| 17 | .039 | 360 | | | |

These strengths range from 300,000 to 340,000 lbs. per sq. in. The composition of this wire is as follows: Carbon, 0.570; silicon, 0.090; sulphur, 0.011; phosphorus, 0.018; manganese, 0.425.

" PLOUGH " -STEEL WIRE.

The term "plough," given in England to steel wire of high quality was derived from the fact that such wire is used for the construction of ropes used for ploughing purposes. It is to be hoped that the term will not be used in this country, as it tends to confusion of terms. Plough-steel is known here in some steel-works as the quality of plate steel used for the mold-boards of ploughs, for which a very ordinary grade is good enough.

Experiments by Dr. Percy on the English plough-steel (so-called) gave the following results: Specific gravity, 7.814; carbon, 0.828 per cent; manganese, 0.587 per cent; silicon, 0.143 per cent; sulphur, 0.009 per cent; phosphorus, nil; copper, 0.030 per cent. No traces of chromium, titanium, or tungsten were found. The breaking strains of the wire were as follows:

| | | | | |
|--------------------------|---------|---------|---------|---------|
| Diameter, inch | 0.093 | 0.132 | 0.159 | 0.191 |
| Pounds per sq. inch . . | 344,960 | 257,600 | 224,000 | 201,600 |

The elongation was only from 0.75 to 1.1 per cent.

DIMENSIONS, WEIGHT, AND RESISTANCE OF PURE COPPER WIRE. (Edison or Circular Mil Gauge.) (See page 31.)

| E. W. G. Gauge Number. | Circular Mils. | Maximum Amperes. $\sqrt{\text{C.M.}^2}$ (104) | Diameter in Mils. Mil = 0.001 in. | Weight. | | Length. | | Resistance, at 75° Fahr. | | E. W. G. Gauge Number. |
|------------------------------|-------------------|--|--|-------------------|------------------|-----------------|------------------|-----------------------------|-----------------|------------------------------|
| | | | | Lbs. per Foot. | Lbs. per Ohm. | Feet per Lb. | Feet per Ohm. | Ohms per Lb. | Ohms per Ft. | |
| 3 | 3000 | 12.5 | 54.78 | 0.009084 | 2.597 | 110.087 | 285.9 | 0.3850405 | 0.003497600 | 3 |
| 5 | 5000 | 18.3 | 70.72 | 0.015139 | 7.214 | 66.054 | 476.5 | .1386225 | .002098640 | 5 |
| 8 | 8000 | 26.0 | 89.45 | 0.024220 | 18.464 | 41.288 | 762.3 | 0.051602 | .001311780 | 8 |
| 12 | 12000 | 35.2 | 109.55 | 0.036328 | 41.538 | 27.527 | 1143.4 | 0.0240743 | .000874578 | 12 |
| 15 | 15000 | 41.6 | 122.48 | 0.045410 | 64.902 | 22.022 | 1429.2 | 0.0154178 | .000699663 | 15 |
| 20 | 20000 | 51.6 | 141.43 | 0.060548 | 115.372 | 16.516 | 1905.7 | 0.0086664 | .000524745 | 20 |
| 25 | 25000 | 61.0 | 158.12 | 0.075682 | 180.278 | 13.213 | 2382.0 | 0.0055470 | .000434984 | 25 |
| 30 | 30000 | 70.0 | 173.21 | 0.090817 | 259.722 | 11.011 | 2859.9 | 0.0038522 | .000349840 | 30 |
| 35 | 35000 | 78.6 | 187.09 | 0.105955 | 353.340 | 9.4381 | 3334.9 | 0.0028301 | .000299863 | 35 |
| 40 | 40000 | 86.8 | 200.00 | 0.121082 | 461.440 | 8.2589 | 3811.0 | 0.0021671 | .000262400 | 40 |
| 45 | 45000 | 94.9 | 212.14 | 0.136227 | 584.098 | 7.3407 | 4287.7 | 0.0017120 | .000233227 | 45 |
| 50 | 50000 | 102.7 | 223.61 | 0.151357 | 721.026 | 6.6069 | 4763.8 | 0.0013868 | .000209914 | 50 |
| 55 | 55000 | 110.3 | 234.53 | 0.166501 | 872.547 | 5.9059 | 5240.5 | 0.0011467 | .000190821 | 55 |
| 60 | 60000 | 117.7 | 244.95 | 0.181625 | 1038.258 | 5.5060 | 5716.5 | 0.00096315 | .000174931 | 60 |
| 65 | 65000 | 125.0 | 254.96 | 0.196772 | 1218.586 | 5.0820 | 6192.9 | 0.00082057 | .000161465 | 65 |
| 70 | 70000 | 132.1 | 264.58 | 0.211901 | 1413.264 | 4.7192 | 6669.4 | 0.00070758 | .000149937 | 70 |
| 75 | 75000 | 139.1 | 273.87 | 0.227043 | 1622.457 | 4.4044 | 7146.0 | 0.00061635 | .000139938 | 75 |
| 80 | 80000 | 146.0 | 282.85 | 0.242176 | 1845.952 | 4.1292 | 7622.3 | 0.00054172 | .000131193 | 80 |
| 85 | 85000 | 152.8 | 291.55 | 0.257303 | 2083.759 | 3.8865 | 8098.4 | 0.00047990 | .000123480 | 85 |
| 90 | 90000 | 159.5 | 300.00 | 0.272434 | 2336.405 | 3.6706 | 8574.7 | 0.00042807 | .000116622 | 90 |
| 95 | 95000 | 166.1 | 308.23 | 0.287587 | 2603.046 | 3.4773 | 9051.6 | 0.00038415 | .000110477 | 95 |
| 100 | 100000 | 172.6 | 316.23 | 0.302709 | 2884.082 | 3.3035 | 9527.6 | 0.00034673 | .000104960 | 100 |
| 110 | 110000 | 185.4 | 331.67 | 0.332991 | 3489.958 | 3.0031 | 10480.6 | 0.00028656 | .000095410 | 110 |
| 120 | 120000 | 198.0 | 346.42 | 0.363267 | 4153.433 | 2.7528 | 11433.6 | 0.00024070 | .000084460 | 120 |
| 130 | 130000 | 210.2 | 360.56 | 0.393527 | 4874.226 | 2.5411 | 12386.0 | 0.00020514 | .000073030 | 130 |
| 140 | 140000 | 222.2 | 374.17 | 0.423797 | 5652.899 | 2.3596 | 13338.7 | 0.00017690 | .000074970 | 140 |
| 150 | 150000 | 234.0 | 387.30 | 0.454061 | 6484.573 | 2.2023 | 14291.3 | 0.00015409 | .000069997 | 150 |
| 160 | 160000 | 245.6 | 400.00 | 0.484328 | 7383.042 | 2.0647 | 15243.9 | 0.00013544 | .000065600 | 160 |
| 170 | 170000 | 257.0 | 412.32 | 0.514622 | 8335.525 | 1.9432 | 16197.4 | 0.00011995 | .000061735 | 170 |
| 180 | 180000 | 268.3 | 424.27 | 0.544884 | 9344.686 | 1.8353 | 17149.9 | 0.00010701 | .000058309 | 180 |
| 190 | 190000 | 279.4 | 435.89 | 0.575140 | 10411.241 | 1.7387 | 18102.1 | 0.00009604 | .000055242 | 190 |
| 200 | 200000 | 290.4 | 447.22 | 0.605427 | 11536.681 | 1.6517 | 19055.4 | 0.00008667 | .000052478 | 200 |
| 220 | 220000 | 312.0 | 469.05 | 0.665975 | 13959.567 | 1.5016 | 20961.1 | 0.00007163 | .000047707 | 220 |
| 240 | 240000 | 333.0 | 489.90 | 0.726498 | 16612.114 | 1.3765 | 22866.0 | 0.00006019 | .000043733 | 240 |
| 260 | 260000 | 353.5 | 509.91 | 0.787058 | 19496.997 | 1.2706 | 24772.1 | 0.00005129 | .000040368 | 260 |
| 280 | 280000 | 373.7 | 529.16 | 0.847605 | 22612.233 | 1.1798 | 26677.8 | 0.00004422 | .000037484 | 280 |
| 300 | 300000 | 393.6 | 547.73 | 0.908140 | 25957.464 | 1.1012 | 28583.1 | 0.00003852 | .000034986 | 300 |
| 320 | 320000 | 413.1 | 565.69 | 0.968672 | 29533.696 | 1.0323 | 30488.3 | 0.00003386 | .000032799 | 320 |
| 340 | 340000 | 432.3 | 583.10 | 1.029214 | 33340.181 | 0.9716 | 32393.8 | 0.00002999 | .000030870 | 340 |
| 360 | 360000 | 451.3 | 600.00 | 1.089738 | 37376.652 | 0.9177 | 34298.7 | 0.00002675 | .000029155 | 360 |

1 Mil Foot = 9.718 B. A. Units at 0° C. (Dr. Matthiessen.)

Sizes, Weights and Strengths of Hard-Copper Telegraph and Telephone Wire.

(J. A. Roebling's Sons Co., 1908.)

| Size B. & S. Gauge. | Diam., in. | Wt., lbs. per mile. | Breaking strain, lbs. | Resistance, In- ternational ohms per mile at 75° F. | Approx. size, Roebling gauge of E. B. B. iron wire of equal resistance. | Size B. & S. Gauge. | Diam., in. | Wt., lbs. per mile. | Breaking strain, lbs. | Resistance, In- ternational ohms per mile at 75° F. | Approx. size, Roebling gauge of E. B. B. iron wire of equal resistance. |
|------------------------|------------|------------------------|--------------------------|--|---|------------------------|------------|------------------------|--------------------------|--|---|
| 9 | 0.114 | 208 | 653 | 4.39 | 2 | 13 | 0.072 | 83 | 274 | 11.01 | 6 1/2 |
| 10 | 0.102 | 166 | 540 | 5.49 | 3 | 14 | 0.064 | 65 | 220 | 13.94 | 8 |
| 11 | 0.091 | 132 | 426 | 6.90 | 4 | 15 | 0.057 | 52 | 174 | 17.57 | 9 |
| 12 | 0.081 | 105 | 334 | 8.70 | 6 | 16 | 0.051 | 42 | 139 | 21.95 | 10 |

In handling this wire the greatest care should be observed to avoid kinks, bends, scratches, or cuts. Joints should be made only with McIntire connectors. On account of its conductivity being about five times that of E. B. B. iron wire, and its breaking strength over three times its weight per mile, copper may be used of which the section is smaller and the weight less than an equivalent iron wire, allowing a greater number of wires to be strung on the poles. Besides this advantage, the reduction of section materially decreases the electrostatic capacity, while its non-magnetic character lessens the self-induction of the line, both of which features tend to increase the possible speed of signaling in telegraphing, and to give greater clearness of enunciation over telephone lines, especially those of great length.

Weight of Bare and Insulated Copper Wire, Pounds.

(John A. Roebling's Sons Co., 1908.)

| Size B. & S. Gauge. | Weight per 1000 Feet, Solid. | | | | | Weight per Mile, Solid. | | | | |
|------------------------|------------------------------|------------------|------------------|-------------------------------|------------------|-------------------------|------------------|------------------|-------------------------------|------------------|
| | Bare. | Weather-proof. | | Fire and Weather Proof. | Slow Burning. | Bare. | Weather-proof. | | Fire and Weather Proof. | Slow Burning. |
| | | Double Braid. | Triple Braid. | | | | Double Braid. | Triple Braid. | | |
| 0000 | 641 | 723 | 767 | 862 | 925 | 3384 | 3817 | 4050 | 4550 | 4890 |
| 000 | 509 | 587 | 629 | 710 | 760 | 2687 | 3098 | 3320 | 3750 | 4020 |
| 00 | 403 | 467 | 502 | 562 | 600 | 2127 | 2467 | 2650 | 2970 | 3170 |
| 0 | 320 | 377 | 407 | 462 | 495 | 1689 | 1989 | 2150 | 2440 | 2610 |
| 1 | 253 | 294 | 316 | 340 | 365 | 1335 | 1553 | 1670 | 1800 | 1930 |
| 2 | 202 | 239 | 260 | 280 | 300 | 1066 | 1264 | 1370 | 1480 | 1585 |
| 3 | 159 | 185 | 199 | 230 | 270 | 840 | 977 | 1050 | 1220 | 1425 |
| 4 | 126 | 151 | 164 | 190 | 220 | 665 | 795 | 865 | 1000 | 1160 |
| 5 | 100 | 122 | 135 | 155 | 190 | 528 | 646 | 710 | 820 | 1000 |
| 6 | 79 | 100 | 112 | 127 | 160 | 417 | 529 | 590 | 670 | 840 |
| 8 | 50 | 66 | 75 | 85 | 110 | 264 | 349 | 395 | 450 | 580 |
| 9 | 39 | 54 | 62 | .. | .. | 206 | 283 | 325 | .. | .. |
| 10 | 32 | 46 | 53 | 60 | 80 | 169 | 241 | 280 | 315 | 420 |
| 12 | 20 | 30 | 35 | 42 | 55 | 106 | 158 | 185 | 220 | 290 |
| 14 | 12.4 | 20 | 25 | 30 | 40 | 66 | 107 | 130 | 160 | 210 |
| 16 | 7.9 | 16 | 20 | 24 | 30 | 42 | 83 | 105 | 130 | 160 |
| 18 | 4.8 | 12 | 16 | 19 | 24 | 25 | 64 | 85 | 100 | 130 |
| 20 | 3.1 | 9 | 12 | .. | .. | 16 | 48 | 65 | .. | .. |

Stranded Copper Feed Wire, Weight in Pounds.

(John A. Roebling's Sons Co., 1908.)

| Size, Circular Mils. | Weight per 1000 Feet. | | | | | Weight per Mile. | | | | |
|----------------------------|-----------------------|------------------|------------------|-------------------------------|------------------|------------------|------------------|------------------|-------------------------------|------------------|
| | Bare. | Weather-proof | | Fire and Weather Proof. | Slow Burning. | Bare. | Weather-proof | | Fire and Weather Proof. | Slow Burning. |
| | | Double Braid. | Triple Braid. | | | | Double Braid. | Triple Braid. | | |
| 2,000,000 | 6100 | 6690 | 7008 | ... | 7540 | 32208 | 35323 | 37000 | ... | 39800 |
| 1,750,000 | 5338 | 5894 | 6193 | ... | 6700 | 28184 | 31119 | 32700 | ... | 35400 |
| 1,500,000 | 4575 | 5098 | 5380 | ... | 5830 | 24156 | 26915 | 28400 | ... | 30800 |
| 1,250,000 | 3813 | 4264 | 4508 | ... | 4940 | 20132 | 22516 | 23800 | ... | 20000 |
| 1,000,000 | 3050 | 3456 | 3674 | 3860 | 3980 | 16104 | 18246 | 19400 | 20400 | 26100 |
| 900,000 | 2745 | 3127 | 3332 | 3520 | 3640 | 14493 | 16513 | 17600 | 18600 | 11000 |
| 800,000 | 2440 | 2799 | 2992 | 3180 | 3280 | 12883 | 14779 | 15800 | 16800 | 19200 |
| 750,000 | 2288 | 2635 | 2822 | 3000 | 3100 | 12080 | 13913 | 14900 | 15850 | 17300 |
| 700,000 | 2135 | 2471 | 2650 | 2820 | 2920 | 11272 | 13045 | 14000 | 14900 | 16300 |
| 600,000 | 1830 | 2093 | 2235 | 2350 | 2460 | 9662 | 11052 | 11800 | 12400 | 15400 |
| 500,000 | 1525 | 1765 | 1894 | 1990 | 2080 | 8052 | 9318 | 10000 | 10500 | 13100 |
| 450,000 | 1373 | 1601 | 1724 | 1820 | 1900 | 7249 | 8452 | 9100 | 9600 | 10000 |
| 400,000 | 1220 | 1436 | 1553 | 1650 | 1700 | 6441 | 7584 | 8200 | 8700 | 9000 |
| 350,000 | 1068 | 1248 | 1345 | 1440 | 1500 | 5639 | 6589 | 7100 | 7600 | 7900 |
| 300,000 | 915 | 1083 | 1174 | 1270 | 1310 | 4831 | 5721 | 6200 | 6700 | 6900 |
| 250,000 | 762 | 907 | 985 | 1060 | 1120 | 4023 | 4788 | 5200 | 5600 | 5900 |
| B. & S. Gauge. | | | | | | | | | | |
| 0000 | 645 | 745 | 800 | 900 | 960 | 3405 | 3935 | 4220 | 4750 | 5070 |
| 000 | 513 | 604 | 653 | 735 | 785 | 2708 | 3190 | 3450 | 3880 | 4150 |
| 00 | 406 | 482 | 522 | 583 | 625 | 2143 | 2544 | 2760 | 3080 | 3300 |
| 0 | 322 | 388 | 424 | 480 | 510 | 1700 | 2051 | 2240 | 2530 | 2700 |
| 1 | 255 | 303 | 328 | 355 | 380 | 1346 | 1599 | 1735 | 1870 | 2000 |
| 2 | 203 | 246 | 270 | 290 | 335 | 1071 | 1301 | 1425 | 1540 | 1770 |
| 3 | 160 | 190 | 206 | 240 | 280 | 844 | 1004 | 1090 | 1270 | 1480 |
| 4 | 127 | 155 | 170 | 195 | 230 | 670 | 820 | 900 | 1030 | 1220 |
| 5 | 101 | 126 | 140 | 160 | 195 | 533 | 668 | 740 | 845 | 1030 |
| 6 | 80 | 103 | 115 | 132 | 165 | 422 | 544 | 610 | 695 | 870 |
| 8 | 50 | 68 | 78 | 87 | 105 | 264 | 359 | 410 | 460 | 555 |

Approximate Rules for the Resistance of Copper Wire.—The resistance of any copper wire at 20° C. or 68° F., according to Matthiessen's standard, is $R = \frac{10.35l}{d^2}$, in which R is the resistance in international ohms, l the length of the wire in feet, and d its diameter in mils. (1 mil = $\frac{1}{1000}$ inch.)

A No. 10 Wire, A.W.G., 0.1019 in. diameter (practically 0.1 in.), 1000 ft. in length, has a resistance of 1 ohm at 68° F. and weighs 31.4 lbs.

If a wire of a given length and size by the American or Brown & Sharpe gauge has a certain resistance, a wire of the same length and three numbers higher has twice the resistance, six numbers higher four times the resistance, etc.

| | | | | | | | | | |
|------------------------|----------------|---------------|---------------|---------------|----|---------------|---------------|---------------|----------------|
| Wire gauge, A.W.G. No. | 000 | 1 | 4 | 7 | 10 | 13 | 16 | 19 | 22 |
| Relative resistance | 16 | 8 | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ |
| section or weight | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 | 16 |

See wire table, A.W.G., under Electrical Engineering.

SPECIFICATIONS FOR HARD-DRAWN COPPER WIRE.

The British Post Office authorities require that hard-drawn copper wire supplied to them shall be of the lengths, sizes, weights, strengths, and conductivities as set forth in the annexed table.

| Weight per Statute Mile, lb. | | | Approximate Equivalent Diameter, mils. | | | Minimum Breaking Weight, lb. | Minimum No. of Twists in 3 Inches. | Maximum Resistance per Mile of Wire (when hard) at 60° F., ohms. | Minimum Weight of each Piece of Wire, lbs. |
|------------------------------|--------------------|--------------------|--|--------------------|--------------------|------------------------------|------------------------------------|--|--|
| Required Standard. | Minimum. | Maximum. | Standard. | Minimum. | Maximum. | | | | |
| 100 | 971 $\frac{1}{2}$ | 1021 $\frac{1}{2}$ | 79 | 78 | 80 | 330 | 30 | 9.10 | 50 |
| 150 | 1461 $\frac{1}{4}$ | 1533 $\frac{3}{4}$ | 97 | 951 $\frac{1}{2}$ | 98 | 490 | 25 | 6.05 | 50 |
| 200 | 195 | 205 | 112 | 1101 $\frac{1}{2}$ | 1131 $\frac{1}{4}$ | 650 | 20 | 4.53 | 50 |
| 400 | 390 | 410 | 158 | 1551 $\frac{1}{2}$ | 1601 $\frac{1}{4}$ | 1300 | 10 | 2.27 | 50 |

WIRES OF DIFFERENT METALS AND ALLOYS.

(J. Bucknall Smith's Treatise on Wire.)

Brass Wire is commonly composed of an alloy of 1 $\frac{3}{4}$ to 2 parts of copper to one part of zinc. The tensile strength ranges from 20 to 40 tons per square inch, increasing with the percentage of zinc in the alloy.

German or Nickel Silver, an alloy of copper, zinc, and nickel, is practically brass whitened by the addition of nickel. It has been drawn into wire as fine as 0.002 inch diameter.

Platinum wire may be drawn into the finest sizes. On account of its high price its use is practically confined to special scientific instruments and electrical appliances in which resistances to high temperature, oxygen, and acids are essential. It expands less than other metals when heated. Its coefficient of expansion being almost the same as that of glass permits its being sealed in glass without fear of cracking the latter. It is therefore used in incandescent electric lamps.

Phosphor-bronze Wire contains from 2 to 6 per cent of tin and from 1 $\frac{2}{3}$ to 1 $\frac{1}{8}$ per cent of phosphorus. The presence of phosphorus is detrimental to electric conductivity.

"Delta-metal" wire is made from an alloy of copper, iron, and zinc. Its strength ranges from 45 to 62 tons per square inch. It is used for some kinds of wire rope, also for wire gauze. It is not subject to deposits of verdigris. It has great toughness, even when its tensile strength is over 60 tons per square inch.

Aluminum Wire.—Specific gravity 0.268. Tensile strength only about 10 tons per square inch. It has been drawn as fine as 11,400 yards to the ounce, or 0.042 grain per yard.

Aluminum Bronze, 90 copper, 10 aluminum, has high strength and ductility; is inoxidizable, sonorous. Its electric conductivity is 12.6 per cent.

Silicon Bronze, patented in 1882 by L. Weiler of Paris, is made as follows: Fluosilicate of potash, pounded glass, chloride of sodium and calcium, carbonate of soda and lime, are heated in a plumbago crucible, and after the reaction takes place the contents are thrown into the molten bronze to be treated. Silicon-bronze wire has a conductivity of from 40 to 98 per cent of that of copper wire and four times more than that of iron, while its tensile strength is nearly that of steel, or 28 to 55 tons per square inch of section. The conductivity decreases as the tensile strength increases. Wire whose conductivity equals 95 per cent of that of pure copper gives a tensile strength of 28 tons per square inch, but when its conductivity is 34 per cent of pure copper, its strength is 50 tons per square inch. It is being largely used for telegraph wires. It has great resistance to oxidation.

Ordinary Drawn and Annealed Copper Wire has a strength of from 15 to 20 tons per square inch.

WIRE ROPES.

STANDARD HOISTING ROPE.

Composed of 6 Strands and a Hemp Center, 19 Wires to the Strand.

(John A. Roebling's Sons Co., 1908.)

See also pamphlets of John A. Roebling's Sons Co., Trenton Iron Co., A. Leschen & Sons Rope Co., and other makers.

SWEDISH IRON.

| Trade number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Stress, tons (2000 lb.). | Allowable Working Stress, tons (2000 lb.). | Min. Size of Drum or Sheave, ft. | Trade Number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Stress, tons (2000 lb.). | Allowable Working Stress, tons (2000 lb.). | Min. Size of Drum or Sheave, ft. |
|---------------|---------------|----------------------|------------------|---|--|----------------------------------|---------------|---------------|----------------------|------------------|---|--|----------------------------------|
| | 23/4 | 85/8 | 11.95 | 114 | 22.8 | 16 | 8 | 1 | 3 | 1.58 | 17 | 3.40 | 5 1/4 |
| | 21 1/2 | 77/8 | 9.85 | 95 | 18.9 | 15 | 9 | 7/8 | 23/4 | 1.20 | 13 | 2.60 | 4 1/2 |
| 1 | 21 1/4 | 71/8 | 8.00 | 78 | 15.60 | 13 | 10 | 3/4 | 21 1/4 | 0.89 | 9.7 | 1.94 | 4 |
| 2 | 2 | 61/4 | 6.30 | 62 | 12.40 | 12 | 10 1/4 | 5/8 | 2 | 0.62 | 6.8 | 1.36 | 3 1/2 |
| 3 | 13/4 | 51/2 | 4.85 | 48 | 9.60 | 10 | 10 1/2 | 9/16 | 13/4 | 0.50 | 5.5 | 1.10 | 23/4 |
| 4 | 15/8 | 5 | 4.15 | 42 | 8.40 | 8 1/2 | 10 3/4 | 1/2 | 11 1/2 | 0.39 | 4.4 | 0.88 | 2 1/4 |
| 5 | 11 1/2 | 43/4 | 3.55 | 36 | 7.20 | 7 1/2 | 10a | 7/16 | 11 1/4 | 0.30 | 3.4 | 0.68 | 2 |
| 5 1/2 | 13/8 | 41/4 | 3.00 | 31 | 6.20 | 7 | 10b | 3/8 | 11/8 | 0.22 | 2.5 | 0.50 | 1 1/2 |
| 6 | 11/4 | 4 | 2.45 | 25 | 5.00 | 6 1/2 | 10c | 5/16 | 1 | 0.15 | 1.7 | 0.34 | 1 |
| 7 | 11/8 | 3 1/2 | 2.00 | 21 | 4.20 | 6 | 10d | 1/4 | 3/4 | 0.10 | 1.2 | 0.24 | 3/4 |

CAST STEEL.

| | | | | | | | | | | | | | |
|-------|--------|-------|-------|-----|------|-------|--------|------|--------|------|------|------|-------|
| | 23/4 | 85/8 | 11.95 | 228 | 45.6 | 10 | 8 | 1 | 3 | 1.58 | 34 | 6.80 | 4 |
| | 21 1/2 | 77/8 | 9.85 | 190 | 37.9 | 9 1/2 | 9 | 7/8 | 23/4 | 1.20 | 26 | 5.20 | 3 1/2 |
| 1 | 21 1/4 | 71/8 | 8.00 | 156 | 31.2 | 8 1/2 | 10 | 3/4 | 21 1/4 | 0.89 | 19.4 | 3.88 | 3 |
| 2 | 2 | 61/4 | 6.30 | 124 | 24.8 | 8 | 10 1/4 | 5/8 | 2 | 0.62 | 13.6 | 2.72 | 2 1/4 |
| 3 | 13/4 | 51/2 | 4.85 | 96 | 19.2 | 7 1/4 | 10 1/2 | 9/16 | 13/4 | 0.50 | 11.0 | 2.20 | 13/4 |
| 4 | 15/8 | 5 | 4.15 | 84 | 16.8 | 6 1/4 | 10 3/4 | 1/2 | 11 1/2 | 0.39 | 8.8 | 1.76 | 1 1/2 |
| 5 | 11 1/2 | 43/4 | 3.55 | 72 | 14.4 | 5 3/4 | 10a | 7/16 | 11 1/4 | 0.30 | 6.8 | 1.36 | 1 1/4 |
| 5 1/2 | 13/8 | 41/4 | 3.00 | 62 | 12.4 | 5 1/2 | 10b | 3/8 | 11/8 | 0.22 | 5.0 | 1.00 | 1 |
| 6 | 11/4 | 4 | 2.45 | 50 | 10.0 | 5 | 10c | 5/16 | 1 | 0.15 | 3.4 | 0.68 | 2/3 |
| 7 | 11/8 | 3 1/2 | 2.00 | 42 | 8.40 | 4 1/2 | 10d | 1/4 | 3/4 | 0.10 | 2.4 | 0.48 | 1/2 |

This rope is almost universally employed for hoisting purposes on account of its flexibility. It is made of 6 strands, each of which is formed by twisting 19 wires together, and a hemp core or center. Sometimes the hemp center is replaced by a wire strand, which adds from 7 to 10 per cent to the strength of the rope; but the wear on the center is as great as on the outside strands, and its use is not generally advised. This rope is very pliable, and will wind on moderate-sized drums and pass over reasonably small sheaves without injury. Where it is possible, drums and sheaves larger than those indicated in the lists should be adopted, particularly when high speeds are employed or when the working strain is greater than one-fifth of the breaking strain, as the bending of a rope around a sheave is more destructive the heavier the strain on the rope and the smaller the sheave. The working strains for these tables have been calculated at about one-fifth the breaking strains. It is necessary, however, in some cases, — where the speed of the rope is excessive, — to take it at one-eighth or one-tenth of the breaking strain.

Before deciding upon iron or steel for ropes, it is better to have advice from the manufacturers of wire rope.

In substituting steel for iron, it is well to use the same size of rope, thereby taking full advantage of the increased wearing capacity of steel over iron. The best steel is the only one to use, as inferior grades are not as serviceable as good iron, because the constant vibrations to which ropes are subjected cause the poor steel to become brittle and unsafe.

TRANSMISSION OR HAULAGE ROPE.

Composed of 6 Strands and a Hemp Center, 7 Wires to the Strand.

SWEDISH IRON.

| Trade Number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.) | Allowable Working Strain, tons (2000 lb.) | Min. Size of Drum or Sheave, ft. | Trade Number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.) | Allowable Working Strain, tons (2000 lb.) | Min. Size of Drum or Sheave, ft. |
|---------------|---------------|----------------------|------------------|--|---|----------------------------------|---------------|---------------|----------------------|------------------|--|---|----------------------------------|
| 11 | 1 1/2 | 43/4 | 3.55 | 34 | 6.80 | 13 | 19 | 5/8 | 2 | 0.62 | 6.6 | 1.32 | 5 1/4 |
| 12 | 1 3/8 | 4 1/4 | 3.00 | 29 | 5.80 | 12 | 20 | 9/16 | 13/4 | 0.50 | 5.3 | 1.06 | 4 1/2 |
| 13 | 1 1/4 | 4 | 2.45 | 24 | 4.80 | 10 3/4 | 21 | 1/2 | 1 1/2 | 0.39 | 4.2 | 0.84 | 4 |
| 14 | 1 1/8 | 3 1/2 | 2.00 | 20 | 4.00 | 9 1/2 | 22 | 7/16 | 1 1/4 | 0.30 | 3.3 | 0.66 | 3 1/4 |
| 15 | 1 | 3 | 1.58 | 16 | 3.20 | 8 1/2 | 23 | 3/8 | 1 1/8 | 0.22 | 2.4 | 0.48 | 2 3/4 |
| 16 | 7/8 | 2 3/4 | 1.20 | 12 | 2.40 | 7 1/2 | 24 | 5/16 | 1 | 0.15 | 1.7 | 0.34 | 2 1/2 |
| 17 | 3/4 | 2 1/4 | 0.89 | 9.3 | 1.86 | 6 3/4 | 25 | 9/32 | 7/8 | 0.125 | 1.4 | 0.28 | 2 1/4 |
| 18 | 11/16 | 2 1/8 | 0.75 | 7.9 | 1.58 | 6 | | | | | | | |

CAST STEEL.

| | | | | | | | | | | | | | |
|----|-------|-------|------|------|------|-------|----|------|-------|-------|------|------|-------|
| 11 | 1 1/2 | 43/4 | 3.55 | 68 | 13.6 | 8 1/2 | 19 | 5/8 | 2 | 0.62 | 13.2 | 2.64 | 3 1/2 |
| 12 | 1 3/8 | 4 1/4 | 3.00 | 58 | 11.6 | 8 | 20 | 9/16 | 13/4 | 0.50 | 10.6 | 2.12 | 3 |
| 13 | 1 1/4 | 4 | 2.45 | 48 | 9.60 | 7 1/4 | 21 | 1/2 | 1 1/2 | 0.39 | 8.4 | 1.68 | 2 1/2 |
| 14 | 1 1/8 | 3 1/2 | 2.00 | 40 | 8.00 | 6 1/4 | 22 | 7/16 | 1 1/4 | 0.30 | 6.6 | 1.32 | 2 1/4 |
| 15 | 1 | 3 | 1.58 | 32 | 6.40 | 5 3/4 | 23 | 3/8 | 1 1/8 | 0.22 | 4.8 | 0.96 | 2 |
| 16 | 7/8 | 2 3/4 | 1.20 | 24 | 4.80 | 5 | 24 | 5/16 | 1 | 0.15 | 3.4 | 0.68 | 1 3/4 |
| 17 | 3/4 | 2 1/4 | 0.89 | 18.6 | 3.72 | 4 1/2 | 25 | 9/32 | 7/8 | 0.125 | 2.8 | 0.56 | 1 1/2 |
| 18 | 11/16 | 2 1/8 | 0.75 | 15.8 | 3.16 | 4 | | | | | | | |

This rope is much stiffer than standard hoisting rope. It is made of 6 strands, each of which is composed of 7 wires, and a hemp core or center. It may have, if it is desired, a wire center, which adds from 7 to 10 per cent to its strength, but it is then open to the objections already noted on page 226. The wires of this variety of rope are 12/3 times greater in diameter than those of the standard hoisting rope, and hence the rope is much less pliable, and will not bend around as small sheaves. It is well adapted for haulages and transmissions, because the wires are large and are not quickly worn through. It will resist the rough usage of mine haulages and the great wear due to passing over a large number of pulleys and rollers. The wires are fewer in number, however, and a greater factor of safety is desirable than for hoisting rope, because the breakage of one or two wires takes away considerable amount of the total strength. In using steel, instead of iron rope, it is necessary to have the best quality. For transmissions, the sizes from 1 1/8 in. diameter down give excellent satisfaction, when properly selected. Both the regular and Lang constructions are extensively used for haulages and inclined planes.

PLOUGH-STEEL ROPE.

Composed of 6 Strands and a Hemp Center.

19 WIRES TO THE STRAND.

| Trade Number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Allowable Working Strain, tons (2000 lb.). | Min. Size of Drum or Sheave, ft. | Trade Number. | Diameter, in. | Approx. circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Allowable Working Strain, tons (2000 lb.). | Min. Size of Drum or Sheave, ft. |
|---------------|---------------|----------------------|------------------|---|--|----------------------------------|---------------|---------------|----------------------|------------------|---|--|----------------------------------|
| | 23/4 | 85/8 | 11.95 | 305 | 61.0 | 11 | 8 | 1 | 3 | 1.58 | 44 | 8.80 | 41/4 |
| | 21/2 | 77/8 | 9.85 | 254 | 50.8 | 10 | 9 | 7/8 | 23/4 | 1.20 | 34 | 6.80 | 33/4 |
| 1 | 21/4 | 71/8 | 8.00 | 208 | 41.6 | 9 | 10 | 3/4 | 21/4 | 0.89 | 25 | 5.00 | 31/2 |
| 2 | 2 | 61/4 | 6.30 | 165 | 33.0 | 8 | 101/4 | 5/8 | 2 | 0.62 | 18 | 3.60 | 3 |
| 3 | 13/4 | 51/2 | 4.85 | 128 | 25.6 | 71/2 | 101/2 | 9/16 | 13/4 | 0.50 | 14.5 | 2.90 | 21/2 |
| 4 | 15/8 | 5 | 4.15 | 111 | 22.2 | 6 | 103/4 | 1/2 | 11/2 | 0.39 | 11.4 | 2.28 | 2 |
| 5 | 11/2 | 43/4 | 3.55 | 96 | 19.2 | 51/2 | 10a | 7/16 | 11/4 | 0.30 | 8.85 | 1.77 | 11/2 |
| 51/2 | 13/8 | 41/4 | 3.00 | 82 | 16.4 | 51/4 | 10b | 3/8 | 11/8 | 0.22 | 6.55 | 1.31 | 1 |
| 6 | 11/4 | 4 | 2.45 | 67 | 13.4 | 5 | 10c | 5/16 | 1 | 0.15 | 4.50 | 0.90 | 7/8 |
| 7 | 11/8 | 31/2 | 2.00 | 56 | 11.2 | 41/2 | 10d | 1/4 | 3/4 | 0.10 | 3.00 | 0.60 | 2/3 |

7 WIRES TO THE STRAND.

| | | | | | | | | | | | | | |
|----|-------|------|------|----|------|------|----|------|------|-------|------|------|------|
| 11 | 11/2 | 43/4 | 3.55 | 91 | 18.2 | 81/2 | 19 | 5/8 | 2 | 0.62 | 17 | 3.40 | 3 |
| 12 | 13/8 | 41/4 | 3.00 | 78 | 15.6 | 8 | 20 | 9/16 | 13/4 | 0.50 | 14 | 2.80 | 23/4 |
| 13 | 11/4 | 4 | 2.45 | 64 | 12.8 | 71/4 | 21 | 1/2 | 11/2 | 0.39 | 11 | 2.20 | 21/2 |
| 14 | 11/8 | 31/2 | 2.00 | 53 | 10.6 | 61/4 | 22 | 7/16 | 11/4 | 0.30 | 8.55 | 1.71 | 2 |
| 15 | 1 | 3 | 1.58 | 42 | 8.40 | 51/2 | 23 | 3/8 | 11/8 | 0.22 | 6.35 | 1.27 | 11/2 |
| 16 | 7/8 | 23/4 | 1.20 | 32 | 6.40 | 5 | 24 | 5/16 | 1 | 0.15 | 4.35 | 0.87 | 11/4 |
| 17 | 3/4 | 21/4 | 0.89 | 24 | 4.80 | 4 | 25 | 9/32 | 7/8 | 0.125 | 3.65 | 0.73 | 1 |
| 18 | 11/16 | 21/8 | 0.75 | 21 | 4.20 | 31/2 | | | | | | | |

Plough-steel wire is made of high grade of crucible steel, and will stand a strain of from 95 to 175 tons per sq. in. Plough-steel ropes are used instead of cast-steel or iron where it is necessary to reduce the dead weight, as, for instance, with heavy or extremely long ropes when the weight of the rope is a large item. They are also employed when the load on the rope of an existing plant has been materially increased and the sheaves and drums cannot be altered to meet the new requirements. In this case a plough-steel rope of the same size can be used with an increase in strength of 50 to 100 per cent. Plough-steel is, therefore, applicable to conditions involving great wear and rough usage. It is advisable to reduce all bends to a minimum and to use somewhat larger drums and sheaves than are suitable for the ordinary cast-steel rope, having a strength of 60 to 80 tons per sq. in. It is well to obtain advice upon the adaptability of plough-steel ropes before using them.

"LANG LAY" ROPE.

In wire rope, as ordinarily made, the component strands are laid up into rope in a direction opposite to that in which the wires are laid into strands; that is, if the wires in the strands are laid from right to left, the strands are laid into rope from left to right. In the "Lang Lay," sometimes known as "Universal Lay," the wires are laid into strands and the strands into rope in the same direction; that is, if the wire is laid in the strands from right to left, the strands are also laid into rope from right to left. Its use has been found desirable under certain conditions

and for certain purposes, mostly for haulage plants, inclined planes, and street railway cables, although it has also been used for vertical hoists in mines, etc. Its advantages are that it is somewhat more flexible than rope of the same diameter and composed of the same number of wires laid up in the ordinary manner; and (especially) that owing to the fact that the wires are laid more axially in the rope, longer surfaces of the wire are exposed to wear, and the endurance of the rope is thereby increased. (Trenton Iron Co.)

CABLE-TRACTION ROPES.

According to English practice, cable-traction ropes, of about $3\frac{1}{2}$ in. circumference, are commonly constructed with six strands of 7 or 15 wires, the lays in the strands varying from, say, 3 in. to $3\frac{1}{2}$ in., and the lays in the ropes from, say, $7\frac{1}{2}$ in. to 9 in. In the United States, however, strands of 19 wires are generally preferred, as being more flexible; but, on the other hand, the smaller external wires wear out more rapidly. The Market-street Street Railway Company, San Francisco, has used ropes $1\frac{1}{4}$ in. diam., composed of six strands of 19 steel wires, weighing $2\frac{1}{2}$ lb. per foot, the longest continuous length being 24,125 ft. The Chicago City Railroad Co. has employed cables of identical construction, the longest length being 27,700 ft. On the New York and Brooklyn Bridge cable-railway steel ropes 11,500 ft. long, containing 114 wires, have been used.

GALVANIZED IRON WIRE ROPE.

For Ships' Rigging and Derrick Guys.

Composed of 6 Strands and a Hemp Center, 7 or 12 Wires to the Strand.

| Approx. diam., in. | Circum., in. | Wt. per ft., lb. | Approx. Break- ing Strain, tons (2000 lb.). | Circum., in., of New Manila Rope of Equal Strength. | Approx. diam., in. | Circum., in. | Wt. per ft., lb. | Approx. Break- ing Strain, tons (2000 lb.). | Circum., in., of New Manila Rope of Equal Strength. |
|-----------------------|----------------|------------------|---|--|-----------------------|----------------|------------------|---|--|
| $1\frac{3}{4}$ | $5\frac{1}{2}$ | 4.85 | 44 | 11 | 1 | 3 | 1.44 | 13 | $5\frac{3}{4}$ |
| $1\frac{11}{16}$ | $5\frac{1}{4}$ | 4.40 | 40 | $10\frac{1}{2}$ | $\frac{7}{8}$ | $2\frac{3}{4}$ | 1.21 | 11 | $5\frac{1}{4}$ |
| $1\frac{5}{8}$ | 5 | 4.00 | 36 | 10 | $\frac{13}{16}$ | $2\frac{1}{2}$ | 1.00 | 9.0 | 5 |
| $1\frac{1}{2}$ | $4\frac{3}{4}$ | 3.60 | 32 | $9\frac{1}{2}$ | $\frac{3}{4}$ | $2\frac{1}{4}$ | 0.81 | 7.3 | $4\frac{3}{4}$ |
| $1\frac{7}{16}$ | $4\frac{1}{2}$ | 3.25 | 29 | 9 | $\frac{5}{8}$ | 2 | 0.64 | 5.8 | $4\frac{1}{2}$ |
| $1\frac{3}{8}$ | $4\frac{1}{4}$ | 2.90 | 26 | $8\frac{1}{2}$ | $\frac{9}{16}$ | $1\frac{3}{4}$ | 0.49 | 4.4 | $3\frac{3}{4}$ |
| $1\frac{1}{4}$ | 4 | 2.55 | 23 | 8 | $\frac{1}{2}$ | $1\frac{1}{2}$ | 0.36 | 3.2 | 3 |
| $1\frac{3}{16}$ | $3\frac{3}{4}$ | 2.25 | 20 | $7\frac{1}{2}$ | $\frac{7}{16}$ | $1\frac{1}{4}$ | 0.25 | 2.3 | $2\frac{1}{2}$ |
| $1\frac{1}{8}$ | $3\frac{1}{2}$ | 1.95 | 18 | $6\frac{1}{2}$ | $\frac{3}{8}$ | $1\frac{1}{8}$ | 0.20 | 1.8 | $2\frac{1}{4}$ |
| $1\frac{1}{16}$ | $3\frac{1}{4}$ | 1.70 | 15 | 6 | $\frac{5}{16}$ | 1 | 0.16 | 1.4 | 2 |

5 STRANDS, 7 WIRES EACH.

| | | | | | | | | | |
|----------------|---------------|-------|------|----------------|----------------|---------------|-------|------|----------------|
| $\frac{9}{32}$ | $\frac{7}{8}$ | 0.123 | 1.1 | $\frac{13}{4}$ | $\frac{7}{32}$ | $\frac{5}{8}$ | 0.063 | 0.56 | $1\frac{1}{4}$ |
| $\frac{1}{4}$ | $\frac{3}{4}$ | 0.090 | 0.81 | $1\frac{1}{2}$ | $\frac{3}{16}$ | $\frac{1}{2}$ | 0.040 | 0.36 | $1\frac{1}{8}$ |

Galvanized wire rope has almost entirely superseded manila rope for shrouds and stays aboard ship. It is cheaper in first cost, is not affected by weather, and does not stretch and contract with changes in atmospheric conditions; on the other hand, it is quite as elastic as manila rope. It is only $\frac{1}{5}$ or $\frac{1}{6}$ as large by bulk as a manila rope of equal strength, and offers only half as much surface to the wind, and weighs less. It is much less liable to accidents by cutting or chafing.

If galvanized rope of greater strength than that shown in the table is desired, galvanized open hearth, cast-steel or plough-steel wire rope can be obtained.

STEEL FLAT ROPES.

(J. A. Roebling's Sons Co.)

Steel-wire Flat Ropes are composed of a number of strands, alternately twisted to the right and left, laid alongside of each other, and sewed together with soft iron wires. These ropes are used at times in place of round ropes in the shafts of mines. They wind upon themselves on a narrow winding-drum, which takes up less room than one necessary for a round rope. The soft-iron sewing-wires wear out sooner than the steel strands, and then it becomes necessary to sew the rope with new iron wires.

| Width and Thickness, in. | Weight per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Allowable Working Strain, tons (2000 lb.). | Width and Thickness, in. | Weight per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Allowable Working Strain, tons (2000 lb.). |
|-----------------------------------|---------------------|---|--|-----------------------------------|---------------------|---|--|
| $\frac{3}{8} \times 2$ | 1.19 | 18 | 3.6 | $\frac{1}{2} \times 3$ | 2.38 | 36 | 7.2 |
| $\frac{3}{8} \times 2\frac{1}{2}$ | 1.86 | 28 | 5.6 | $\frac{1}{2} \times 3\frac{1}{2}$ | 2.97 | 45 | 9.0 |
| $\frac{3}{8} \times 3$ | 2.00 | 30 | 6.0 | $\frac{1}{2} \times 4$ | 3.30 | 50 | 10.0 |
| $\frac{3}{8} \times 3\frac{1}{2}$ | 2.50 | 38 | 7.6 | $\frac{1}{2} \times 4\frac{1}{2}$ | 4.00 | 60 | 12.0 |
| $\frac{3}{8} \times 4$ | 2.86 | 43 | 8.6 | $\frac{1}{2} \times 5$ | 4.27 | 64 | 12.8 |
| $\frac{3}{8} \times 4\frac{1}{2}$ | 3.12 | 47 | 9.4 | $\frac{1}{2} \times 5\frac{1}{2}$ | 4.82 | 72 | 14.4 |
| $\frac{3}{8} \times 5$ | 3.40 | 50 | 10.0 | $\frac{1}{2} \times 6$ | 5.10 | 77 | 15.4 |
| $\frac{3}{8} \times 5\frac{1}{2}$ | 3.90 | 55 | 11.0 | $\frac{1}{2} \times 7$ | 5.90 | 89 | 17.8 |

GALVANIZED STEEL CABLES.**For Suspension Bridges. (Roebling's.)**

Composed of 6 Strands — With Wire Center.

| Diam., in. | Wt. per foot, lb. | Approx. Breaking Strain, tons (2000 lb.). | Diam., in. | Wt. per foot, lb. | Approx. Breaking Strain, tons. | Diam., in. | Wt. per foot, lb. | Approx. Breaking Strain, tons. |
|----------------|-------------------|---|----------------|-------------------|--------------------------------|----------------|-------------------|--------------------------------|
| $\frac{23}{4}$ | 12.7 | 310 | $\frac{21}{4}$ | 8.52 | 208 | $\frac{13}{4}$ | 5.10 | 124 |
| $\frac{25}{8}$ | 11.6 | 283 | $\frac{21}{8}$ | 7.60 | 185 | $\frac{15}{8}$ | 4.34 | 106 |
| $\frac{21}{2}$ | 10.5 | 256 | 2 | 6.73 | 164 | $\frac{11}{2}$ | 3.70 | 90 |
| $\frac{23}{8}$ | 9.50 | 232 | $\frac{17}{8}$ | 5.90 | 144 | $\frac{13}{8}$ | 3.10 | 75 |

GALVANIZED CAST-STEEL YACHT RIGGING.

6 Strands and a Hemp Center. 7 or 19 Wires to the Strand.

| Approx. Diam., in. | Circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Circum. of New Manila Rope of Equal Strength | Approx. Diam., in. | Circum., in. | Wt. per ft., lb. | Approx. Breaking Strain, tons (2000 lb.). | Circum. of New Manila Rope of Equal Strength |
|--------------------|----------------|------------------|---|--|--------------------|----------------|------------------|---|--|
| $\frac{11}{4}$ | 4 | 2.55 | 53 | 13 | $\frac{5}{8}$ | 2 | 0.64 | 14.0 | 6 |
| $\frac{13}{16}$ | $3\frac{3}{4}$ | 2.25 | 47 | 12 | $\frac{9}{16}$ | $\frac{13}{4}$ | 0.49 | 10.8 | $5\frac{1}{4}$ |
| $\frac{11}{8}$ | $3\frac{1}{2}$ | 1.95 | 41 | 11 | $\frac{1}{2}$ | $\frac{11}{2}$ | 0.36 | 8.1 | $4\frac{3}{4}$ |
| $\frac{11}{16}$ | $3\frac{1}{4}$ | 1.70 | 36 | 10 | $\frac{15}{32}$ | $\frac{13}{8}$ | 0.30 | 6.8 | $4\frac{1}{2}$ |
| 1 | 3 | 1.44 | 31 | 9 | $\frac{7}{16}$ | $\frac{11}{4}$ | 0.25 | 5.7 | $4\frac{1}{4}$ |
| $\frac{7}{8}$ | $2\frac{3}{4}$ | 1.21 | 26 | $8\frac{1}{2}$ | $\frac{3}{8}$ | $\frac{11}{8}$ | 0.20 | 4.5 | $3\frac{3}{4}$ |
| $\frac{13}{16}$ | $2\frac{1}{2}$ | 1.00 | 22 | 8 | $\frac{5}{16}$ | 1 | 0.16 | 3.7 | 3 |
| $\frac{3}{4}$ | $2\frac{1}{4}$ | 0.81 | 17.6 | 7 | | | | | |

GALVANIZED STEEL-WIRE STRAND.**For Smokestack Guys, Signal Strand, etc.**

(J. A. Roebling's Sons Co., 1908.)

This strand is composed of 7 wires, twisted together into a single strand.

| Diam., in. | Wt. per 1000 ft., lb. | Approx. Breaking Strain, lb. | Diam., in. | Wt. per 1000 ft. | Approx. Breaking Strain, lb. |
|------------|--------------------------|------------------------------------|------------|---------------------|------------------------------------|
| 1/2 | 510 | 8500 | 7/32..... | 95 | 1800 |
| 7/16..... | 415 | 6500 | 3/16..... | 75 | 1400 |
| 3/8 | 295 | 5000 | 5/32..... | 55 | 900 |
| 5/16..... | 210 | 3800 | 1/8 | 32 | 500 |
| 1/4 | 125 | 2300 | 3/32..... | 20 | 400 |

Galvanized strand is made on application of wire of any strength from 60,000 lb. to 350,000 lb. per sq. in. When used to run over sheaves or pulleys the use of soft-iron stock is advisable.

FLEXIBLE STEEL-WIRE HAWSERS.

These hawsers are extensively used. They are made with six strands of twelve wires each, hemp centers being inserted in the individual strands as well as in the completed rope. The material employed is crucible cast steel, galvanized, and guaranteed to fulfill Lloyd's requirements. They are only one-third the weight of hempen hawsers, and are sufficiently pliable to work round any bits to which hempen rope of equivalent strength can be applied.

13-inch tarred Russian hemp hawser weighs about 39 lbs. per fathom.

10-inch white manila hawser weighs about 20 lbs. per fathom.

1 1/8-inch stud chain weighs about 68 lbs. per fathom.

4-inch galvanized steel hawser weighs about 12 lbs. per fathom.

Each of the above named has about the same tensile strength.

GALVANIZED STEEL HAWSERS.**For Mooring, Sea or Lake Towing.**

Composed of 6 Strands and a Hemp Center, each Strand consisting of 12 Wires and a Hemp Core or of 37 Wires.

| Approx. Diam., in. | Circum., in. | Wt. per ft., lb. | | Approx. Breaking Strain, tons (2000 lb.). | |
|-----------------------|-----------------|--------------------|--------------------|--|--------------------|
| | | 12-Wire Strand. | 37-Wire Strand. | 12-Wire Strand. | 37-Wire Strand. |
| 2 1/16 | 6 1/2 | 4.56 | 6.76 | 83 | 179 |
| 2 | 6 1/4 | 4.20 | 6.25 | 77 | 166 |
| 1 15/16 | 6 | 3.88 | 5.76 | 71 | 155 |
| 1 13/16 | 5 3/4 | 3.56 | 5.29 | 66 | 142 |
| 1 3/4 | 5 1/2 | 3.25 | 4.85 | 61 | 131 |
| 1 11/16 | 5 1/4 | 2.95 | 4.41 | 57 | 120 |
| 1 5/8 | 5 | 2.70 | 4.00 | 53 | 109 |
| 1 1/2 | 4 3/4 | 2.42 | 3.60 | 45 | 99 |
| 1 7/16 | 4 1/2 | 2.18 | 3.24 | 42 | 90 |
| 1 3/8 | 4 1/4 | 1.94 | 2.90 | 39 | 81 |
| 1 1/4 | 4 | 1.72 | 2.55 | 32 | 72 |
| 1 3/16 | 3 3/4 | 1.51 | 2.25 | 29 | 62 |
| 1 1/8 | 3 1/2 | 1.32 | 1.95 | 27 | 55 |
| 1 1/16 | 3 1/4 | 1.14 | 1.69 | 24 | 46 |
| 1 | 3 | .97 | 1.44 | 21.5 | 40 |
| 7/8 | 2 3/4 | .81 | 1.21 | 16.4 | 34 |
| 13/16 | 2 1/2 | .67 | 1.00 | 14.4 | 28 |
| 3/4 | 2 1/4 | .54 | .81 | 12.3 | 23 |

Notes on the Use of Wire Rope. (J. A. Roebling's Sons Co.)

(See also notes under various tables of wire ropes.)

Several kinds of wire rope are manufactured. The most pliable variety contains nineteen wires to the strand. The ropes with twelve wires and seven wires in the strand are stiffer, and are better adapted for standing rope, guys, and rigging. Orders should state the use of the rope, and advice will be given.

Wire rope is as pliable as new hemp rope of the same strength; the former will therefore run over the same-sized sheaves and pulleys as the latter. But the greater the diameter of the sheaves, pulleys, or drums, the longer wire rope will last. The minimum size of drum is given in the table. Experience has demonstrated that the wear increases with the speed. It is, therefore, better to increase the load than the speed.

Wire rope must not be coiled or uncoiled like hemp rope. — When mounted on a reel, the latter should be mounted on a spindle or flat turn-table to pay off the rope. When forwarded in a small coil, without reel, roll it over the ground like a wheel, and run off the rope in that way. All untwisting or kinking must be avoided.

To preserve wire rope, apply raw linseed-oil with a piece of sheepskin, wool inside; or mix the oil with equal parts of Spanish brown or lamp-black.

To preserve wire rope under water or under ground, take mineral or vegetable tar, and add one bushel of fresh-slacked lime to one barrel of tar, which will neutralize the acid. Boil it well, and saturate the rope with the hot tar. To give the mixture body, add some sawdust.

The grooves of cast-iron pulleys and sheaves should be filled with well-seasoned blocks of hard wood, set on end, to be renewed when worn out. This end-wood will save wear and increase adhesion. The smaller pulleys or rollers which support the ropes on inclined planes should be constructed on the same plan. When large sheaves run at high velocity, the grooves should be lined with leather, set on end, or with India rubber. This is done in the case of sheaves used in the transmission of power between distant points by means of rope, which frequently runs at the rate of 4000 feet per minute.

Locked Wire Rope.

Fig. 77 shows what is known as the Patent Locked Steel Wire Rope made by the Trenton Iron Co. It is claimed to wear two to three times



FIG. 77.

as long as an ordinary wire rope of equal diameter and of like material, with an increased life for sheaves and rollers. Sizes made are from $\frac{1}{2}$ to 2 inches diameter, with a minimum diameter of sheave of 4 and 12 feet respectively.

CHAINS.

Weight per Foot, Proof Test and Breaking Weight.
(Pennsylvania Railroad Specifications, 1903.)

| Nominal Diameter of Wire. Inches. | Description. | Maximum Length of 100 Links. Inches. | Weight per Foot. Lbs. | Proof Test. Lbs. | Breaking Weight. Lbs. |
|-----------------------------------|--------------------------|--------------------------------------|-----------------------|------------------|-----------------------|
| 5/32 | Twisted chain..... | 103 1/8 | 0.20 | | |
| 3/16 | "..... | 96 1/4 | 0.35 | | |
| 3/16 | Perfection twisted chain | 151 1/4 | 0.27 | | |
| 1/4 | Straight-link chain.... | 102 | 0.70 | 1,600 | 3,200 |
| 5/16 | " " "..... | 114 3/4 | 1.10 | 2,500 | 5,000 |
| 3/8 | " " "..... | 114 3/4 | 1.60 | 3,600 | 7,200 |
| 3/8 | Crane chain..... | 113 5/8 | 1.60 | 4,140 | 8,280 |
| 7/16 | Straight-link chain.... | 127 1/2 | 2.07 | 4,900 | 9,800 |
| 7/16 | Crane chain..... | 126 1/4 | 2.07 | 5,635 | 11,270 |
| 1/2 | Straight-link chain.... | 153 | 2.50 | 6,400 | 12,800 |
| 1/2 | Crane chain..... | 151 1/2 | 2.60 | 7,360 | 14,720 |
| 5/8 | Straight-link chain.... | 178 1/2 | 4.08 | 10,000 | 20,000 |
| 5/8 | Crane chain..... | 176 3/4 | 4.18 | 11,500 | 23,000 |
| 3/4 | Straight-link chain.... | 204 | 5.65 | 14,400 | 28,800 |
| 3/4 | Crane chain..... | 202 | 5.75 | 16,560 | 33,120 |
| 7/8 | " "..... | 252 1/2 | 7.70 | 22,540 | 45,080 |
| 1 | " "..... | 277 3/4 | 9.80 | 29,440 | 58,880 |
| 1 | Straight-link chain.... | 280 1/2 | 9.80 | 25,600 | 51,200 |
| 1 1/8 | Crane chain..... | 303 | 12.65 | 38,260 | 76,520 |
| 1 1/4 | " "..... | 353 1/2 | 15.50 | 46,000 | 92,000 |
| 1 1/2 | " "..... | 416 5/8 | 22.50 | 66,240 | 132,480 |
| 1 3/4 | " "..... | 479 3/4 | 30.00 | 90,160 | 180,320 |
| 2 | " "..... | 555 1/2 | 39.00 | 117,760 | 235,520 |

Elongation of all sizes, 10 per cent. All chain must stand the proof test without deformation. A piece 2 ft. long out of each 200 ft. is tested to destruction.

British Admiralty Proving Tests of Chain Cables. — Stud-links. Minimum size in inches and 16ths. Proving test in tons of 2240 lbs.

| | | | | | | | | | | | | | |
|-------------|--------|--------|-------|--------|--------|-------|-------|--------|--------|-------|--------|-------|-------|
| Min. Size: | 1 1/8 | 3/4 | 1 1/4 | 1 1/2 | 1 3/4 | 1 | 1 1/8 | 1 1/4 | 1 1/2 | 1 3/4 | 1 1/2 | 1 3/4 | 1 3/4 |
| Test, tons: | 8 1/2 | 10.1 | 11.9 | 13 1/2 | 15 3/4 | 18 | 20.3 | 22 1/4 | 25 1/2 | 28.1 | 31 | 34 | |
| Min. Size: | 1 7/8 | 1 1/2 | 1 1/8 | 1 1/4 | 1 1/2 | 1 3/4 | 1 1/2 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/4 | |
| Test, tons: | 37 3/4 | 40 1/2 | 43 9 | 47 1/2 | 51 1/4 | 55.1 | 59.1 | 63 1/4 | 67 1/2 | 72 | 81 1/2 | | |

Wrought-iron Chain Cables. — The strength of a chain link is less than twice that of a straight bar of a sectional area equal to that of one side of the link. A weld exists at one end and a bend at the other, each requiring at least one heat, which produces a decrease in the strength. The report of the committee of the U. S. Testing Board (1879), on tests of wrought-iron and chain cables, contains the following conclusions. That beyond doubt, when made of American bar iron, with cast-iron studs, the studded link is inferior in strength to the unstudded one.

"That when proper care is exercised in the selection of material, a variation of 5 to 17 per cent of the strongest may be expected in the resistance of cables. Without this care, the variation may rise to 25 per cent.

"That with proper material and construction the ultimate resistance of the chain may be expected to vary from 155 to 170 per cent of that of the bar used in making the links, and show an average of about 163 per cent.

"That the proof test of a chain cable should be about 50 per cent of the ultimate resistance of the weakest link."

The decrease of the resistance of the studded below the unstudded cable is probably due to the fact that in the former the sides of the link do not remain parallel to each other up to failure, as they do in the latter. The result is an increase of stress in the studded link over the unstudded in the proportion of unity, to the secant of half the inclination of the sides of the former to each other.

From a great number of tests of bars and unfinished cables, the committee considered that the average ultimate resistance, and proof tests of chain cables made of the bars, whose diameters are given, should be such as are shown in the accompanying table.

ULTIMATE RESISTANCE AND PROOF TESTS OF CHAIN CABLES.

| Diam. of Bar. | Average resist. = 163% of Bar. | Proof Test. | Diam. of Bar. | Average resist. = 163% of Bar. | Proof Test. |
|---------------------|-----------------------------------|-------------|---------------------|-----------------------------------|-------------|
| Inches. | Pounds. | Pounds. | Inches. | Pounds. | Pounds. |
| 1 | 71,172 | 33,840 | 1 9/16 | 162,283 | 77,159 |
| 1 1/16 | 79,544 | 37,820 | 1 5/8 | 174,475 | 82,956 |
| 1 1/8 | 88,445 | 42,053 | 1 11/16 | 187,075 | 88,947 |
| 1 3/16 | 97,731 | 46,468 | 1 3/4 | 200,074 | 95,128 |
| 1 1/4 | 107,440 | 51,084 | 1 13/16 | 213,475 | 101,499 |
| 1 5/16 | 117,577 | 55,903 | 1 7/8 | 227,271 | 108,058 |
| 1 3/8 | 128,129 | 60,920 | 1 15/16 | 241,463 | 114,806 |
| 1 7/16 | 139,103 | 66,138 | 2 | 256,040 | 121,737 |
| 1 1/2 | 150,485 | 71,550 | | | |

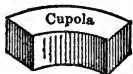
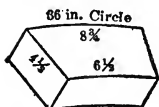
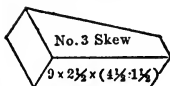
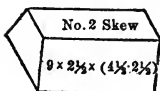
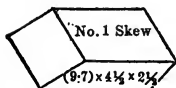
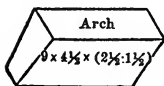
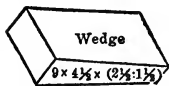
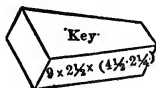
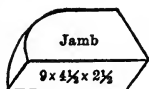
Pitch, Breaking, Proof and Working Strains of Chains.
(Bradlee & Co., Philadelphia.)

| Size of Chain, in. | Pitch, in. | Approx. Wt. per ft. lb. | Outside Width, in. | D. B. G. Special Crane. | | | Crane. | | |
|--------------------|------------|-------------------------|--------------------|-------------------------|-----------------------------------|--|-----------------|-----------------------------------|--|
| | | | | Proof Test, lb. | Average Break- ing Strain, lb. | Ordinary Safe Load, General Use, lb. | Proof Test, lb. | Average Break- ing Strain, lb. | Ordinary Safe Load, General Use, lb. |
| 1/4 | 25/32 | 3/4 | 15/16 | 1,932 | 3,864 | 1,288 | 1,680 | 3,360 | 1,12 |
| 5/16 | 27/32 | 1 | 1 1/8 | 2,898 | 5,796 | 1,932 | 2,520 | 5,040 | 1,68 |
| 3/8 | 31/32 | 1 1/2 | 1 5/8 | 4,186 | 8,372 | 2,790 | 3,640 | 7,280 | 2,420 |
| 7/16 | 1 5/32 | 2 | 1 1/2 | 5,796 | 11,592 | 3,864 | 5,040 | 10,080 | 3,360 |
| 1/2 | 1 11/32 | 2 1/2 | 1 13/16 | 7,728 | 15,456 | 5,152 | 6,720 | 13,440 | 4,487 |
| 9/16 | 1 15/32 | 3 3/10 | 2 | 9,660 | 19,320 | 6,440 | 8,400 | 16,800 | 5,600 |
| 5/8 | 1 23/32 | 4 1/10 | 2 3/16 | 11,914 | 23,828 | 7,942 | 10,360 | 20,720 | 6,900 |
| 11/16 | 1 13/16 | 5 | 2 3/8 | 14,490 | 28,980 | 9,660 | 12,600 | 25,200 | 8,400 |
| 3/4 | 1 15/16 | 6 2/10 | 2 9/16 | 17,388 | 34,776 | 11,592 | 15,120 | 30,240 | 10,087 |
| 13/16 | 2 1/16 | 6 7/10 | 2 3/4 | 20,286 | 40,572 | 13,524 | 17,640 | 35,280 | 11,760 |
| 7/8 | 2 3/16 | 8 3/8 | 2 15/16 | 22,484 | 44,968 | 14,989 | 20,440 | 40,880 | 13,620 |
| 15/16 | 2 7/16 | 9 | 3 3/16 | 25,872 | 51,744 | 17,248 | 23,520 | 47,040 | 15,680 |
| 1 | 2 1/2 | 10 1/2 | 3 3/8 | 29,568 | 59,136 | 19,712 | 26,880 | 53,760 | 17,927 |
| 1 1/16 | 2 5/8 | 12 | 3 9/16 | 33,264 | 66,538 | 22,176 | 30,240 | 60,480 | 20,160 |
| 1 1/8 | 2 3/4 | 13 5/8 | 3 13/16 | 37,576 | 75,152 | 25,050 | 34,160 | 68,320 | 22,770 |
| 1 3/16 | 3 1/16 | 13 7/10 | 4 | 41,888 | 83,776 | 27,925 | 38,080 | 76,160 | 25,380 |
| 1 1/4 | 3 1/8 | 16 | 4 3/16 | 46,200 | 92,400 | 30,800 | 42,000 | 84,000 | 28,003 |
| 1 5/16 | 3 3/8 | 16 1/2 | 4 3/8 | 50,512 | 101,024 | 33,674 | 45,920 | 91,840 | 30,617 |
| 1 3/8 | 3 9/16 | 19 1/4 | 4 9/16 | 55,748 | 111,496 | 37,165 | 50,680 | 101,360 | 33,780 |
| 1 7/16 | 3 11/16 | 19 7/10 | 4 3/4 | 60,368 | 120,736 | 40,245 | 54,880 | 109,760 | 36,583 |
| 1 1/2 | 3 7/8 | 23 | 5 1/8 | 66,528 | 133,056 | 44,352 | 60,480 | 120,960 | 40,327 |
| 1 9/16 | 4 | 25 | 5 5/16 | 70,762 | 141,524 | 47,174 | 65,520 | 131,140 | 43,187 |
| 1 3/4 | 4 3/4 | 31 | 5 7/8 | 82,320 | 164,640 | 54,880 | | | |
| 2 | 5 3/4 | 40 | 6 3/4 | 107,520 | 215,040 | 71,680 | | | |
| 2 1/4 | 6 3/4 | 52 3/4 | 7 5/8 | 136,080 | 272,160 | 90,720 | | | |
| 2 1/2 | 7 | 64 1/2 | 8 3/8 | 168,000 | 336,000 | 112,000 | | | |
| 2 3/4 | 7 1/4 | 73 | 9 1/8 | 193,088 | 386,176 | 128,725 | | | |
| 3 | 7 3/4 | 86 | 9 7/8 | 217,728 | 435,456 | 145,152 | | | |

The distance from center of one link to center of next is equal to the inside length of link, but in practice 1/32 in. is allowed for weld. This is approximate, and where exactness is required, chain should be made so.

FOR CHAIN SHEAVES. — The diameter, if possible, should be not less than thirty times the diameter of chain used.

EXAMPLE. — For 1-inch chain use 30-inch sheaves.



SIZES OF FIRE-BRICK.

| | |
|--------------------------------|---|
| 9-inch straight... | 9 × 4 1/2 × 2 1/2 inches. |
| Soap..... | 9 × 2 1/4 × 2 1/2 " |
| Checker..... | 9 × 3 × 3 " |
| No. 1 Split..... | 9 × 4 1/2 × 1 1/4 " |
| No. 2 Split..... | 9 × 4 1/2 × 2 " |
| Jamb..... | 9 × 4 1/2 × 2 1/2 " |
| No. 1 key..... | 9 × 2 1/2 thick × 4 1/2 to 4 inches wide. 112 bricks to circle 12 feet inside diam. |
| No. 2 key..... | 9 × 2 1/2 thick × 4 1/2 to 3 1/2 inches wide. 65 bricks to circle 6 ft. inside diam. |
| No. 3 key..... | 9 × 2 1/2 thick × 4 1/2 to 3 inches wide. 41 bricks to circle 3 ft. inside diam. |
| No. 4 key..... | 9 × 2 1/2 thick × 4 1/2 to 2 1/4 inches wide. 26 bricks to circle 1 1/2 ft. inside diam. |
| No. 1 wedge (or bullhead)..... | 9 × 4 1/2 wide 2 × 1/2 to 2 in. thick, tapering lengthwise. 102 bricks to circle 5 ft. inside diam. |
| No. 2 wedge..... | 9 × 4 1/2 × 2 1/2 to 1 1/2 in. thick. 63 bricks to circle 2 1/2 ft. inside diam. |
| No. 1 arch..... | 9 × 4 1/2 × 2 1/2 to 2 in. thick, tapering breadthwise. 72 bricks to circle 4 ft. inside diam. |
| No. 2 arch..... | 9 × 4 1/2 × 2 1/2 to 1 1/2. 42 bricks to circle 2 ft. inside diam. |
| No. 1 skew..... | 9 to 7 × 4 1/2 to 2 1/2. Bevel on one end. |
| No. 2 skew..... | 9 × 2 1/2 × 4 1/2 to 2 1/2. Equal bevel on both edges. |
| No. 3 skew..... | 9 × 2 1/2 × 4 1/2 to 1 1/2. Taper on one edge. |
| 24-inch circle..... | 8 1/4 to 5 1/8 × 4 1/2 × 2 1/2. Edges curved, 9 bricks line a 24-inch circle. |
| 36-inch circle..... | 8 3/4 to 6 1/2 × 4 1/2 × 2 1/2. 13 bricks line a 36-inch circle. |
| 48-inch circle..... | 8 3/4 to 7 1/4 × 4 1/2 × 2 1/2. 17 bricks line a 48-inch circle. |
| 13 1/2-inch straight..... | 13 1/2 × 2 1/2 × 6. |
| 13 1/2-inch key No. 1..... | 13 1/2 × 2 1/2 × 6 to 5 inch. |
| 1..... | 90 bricks turn a 12-ft. circle. |
| 3 1/2-inch key No. 2..... | 13 1/2 × 2 1/2 × 6 to 4 3/8 inch. 52 bricks turn a 6-ft. circle. |
| Bridge wall, No. 1..... | 13 × 6 1/2 × 6. |
| Bridge wall, No. 2..... | 13 × 6 1/2 × 3. |
| Mill tile..... | 18, 20, or 24 × 6 × 3. |
| Stock-hole tiles..... | 18, 20, or 24 × 9 × 4. |
| 18-inch block..... | 18 × 9 × 6. |
| Flat back..... | 9 × 6 × 2 1/2. |
| Flat back arch..... | 9 × 6 × 3 1/2 to 2 1/2. 22-inch radius, 56 bricks to circle. |
| Locomotive tile..... | 32 × 10 × 3. 36 × 8 × 3. |
| | 34 × 10 × 3. 40 × 10 × 3. |
| | 34 × 8 × 3. |

Tiles, slabs, and blocks, various sizes 12 to 30 in. long, 8 to 30 in. wide, 2 to 6 in. thick.

Cupola brick, 4 and 6 in. high, 4 and 6 in. radial width, to line shells 23 to 66 in. diameter.

A 9-inch straight brick weighs 7 lb. and contains 100 cubic inches. (= 120 lb. per cubic foot. Specific gravity 1.93.)

One cubic foot of wall requires 17 9-inch bricks, one cubic yard requires 460. Where keys, wedges, and other "shapes" are used, add 10 per cent in estimating the number required.

One ton of fire-clay should be sufficient to lay 3000 ordinary bricks. To secure the best results, fire-bricks should be laid in the same clay

from which they are manufactured. It should be used as a thin paste, and not as mortar. The thinner the joint the better the furnace wall. In ordering bricks, the service for which they are required should be stated.

NUMBER OF FIRE-BRICK REQUIRED FOR VARIOUS CIRCLES.

| Diam. of Circle. | Key Bricks. | | | | | Arch Bricks. | | | | Wedge Bricks. | | | |
|------------------------|-------------|--------|--------|--------|--------|--------------|--------|-------|--------|---------------|--------|-------|--------|
| | No. 4. | No. 3. | No. 2. | No. 1. | Total. | No. 2. | No. 1. | 9-in. | Total. | No. 2. | No. 1. | 9-in. | Total. |
| ft. in. | | | | | | | | | | | | | |
| 1 6 | 25 | | | | 25 | | | | | | | | |
| 2 0 | 17 | 13 | | | 30 | 42 | | | 42 | | | | |
| 2 6 | 9 | 25 | | | 34 | 31 | 18 | | 49 | 60 | | | 60 |
| 3 0 | | 38 | | | 38 | 21 | 36 | | 57 | 48 | 20 | | 68 |
| 3 6 | | 32 | 10 | | 42 | 10 | 54 | | 64 | 36 | 40 | | 76 |
| 4 0 | | 25 | 21 | | 46 | | 72 | | 72 | 24 | 59 | | 83 |
| 4 6 | | 19 | 32 | | 51 | | 72 | 8 | 80 | 12 | 79 | | 91 |
| 5 0 | | 13 | 42 | | 55 | | 72 | 15 | 87 | | 98 | | 98 |
| 5 6 | | 6 | 53 | | 59 | | 72 | 23 | 95 | | 98 | 8 | 106 |
| 6 0 | | | 63 | | 63 | | 72 | 30 | 102 | | 98 | 15 | 113 |
| 6 6 | | | 58 | 9 | 67 | | 72 | 38 | 110 | | 98 | 23 | 121 |
| 7 0 | | | 52 | 19 | 71 | | 72 | 45 | 117 | | 98 | 30 | 128 |
| 7 6 | | | 47 | 29 | 76 | | 72 | 53 | 125 | | 98 | 38 | 136 |
| 8 0 | | | 42 | 38 | 80 | | 72 | 60 | 132 | | 98 | 46 | 144 |
| 8 6 | | | 37 | 47 | 84 | | 72 | 68 | 140 | | 98 | 53 | 151 |
| 9 0 | | | 31 | 57 | 88 | | 72 | 75 | 147 | | 98 | 61 | 159 |
| 9 6 | | | 26 | 66 | 92 | | 72 | 83 | 155 | | 98 | 68 | 166 |
| 10 0 | | | 21 | 76 | 97 | | 72 | 90 | 162 | | 98 | 76 | 174 |
| 10 6 | | | 16 | 85 | 101 | | 72 | 98 | 170 | | 98 | 83 | 181 |
| 11 0 | | | 11 | 94 | 105 | | 72 | 105 | 177 | | 98 | 91 | 189 |
| 11 6 | | | 5 | 104 | 109 | | 72 | 113 | 185 | | 98 | 98 | 196 |
| 12 0 | | | | 113 | 113 | | 72 | 121 | 193 | | 98 | 106 | 204 |
| 12 6 | | | | 113 | 117 | | | | | | | | |

For larger circles than 12 feet use 113 No. 1 Key, and as many 9-inch brick as may be needed in addition.

WEIGHTS OF LOGS, LUMBER, ETC.

Weight of Green Logs to Scale 1000 Feet, Board Measure.

| | |
|---|---------------------|
| Yellow pine (Southern)..... | 8,000 to 10,000 lb. |
| Norway pine (Michigan)..... | 7,000 to 8,000 " |
| White pine (Michigan) { off of stump..... | 7,000 to 7,000 " |
| { out of water..... | 7,000 to 8,000 " |
| White pine (Pennsylvania), bark off..... | 5,000 to 6,000 " |
| Hemlock (Pennsylvania), bark off..... | 6,000 to 7,000 " |

Four acres of water are required to store 1,000,000 feet of logs.

Weight of 1000 Feet of Lumber, Board Measure.

| | | |
|----------------------------|----------------|------------------|
| Yellow or Norway pine..... | Dry, 3,000 lb. | Green, 5,000 lb. |
| White pine..... | 2,500 " | " 4,000 " |

Weight of 1 Cord of Seasoned Wood, 128 Cu. Ft. per Cord.

| | |
|----------------------------------|-----------|
| Hickory or sugar maple..... | 4,500 lb. |
| White oak..... | 3,850 " |
| Beech, red oak or black oak..... | 3,250 " |
| Poplar, chestnut or elm..... | 2,350 " |
| Pine (white or Norway)..... | 2,000 " |
| Hemlock bark, dry..... | 2,200 " |

ANALYSES OF FIRE CLAYS.

| Brand. | Titanic Acid, TiO ₂ | Silica, SiO ₂ | Alumina, Al ₂ O ₃ | Moisture, H ₂ O | Iron, Fe ₂ O ₃ | Lime, CaO | Magnesia, MgO | Potash, K ₂ O | Soda, Na ₂ O | Total Im- purities. | Loss. |
|---|-----------------------------------|--------------------------|--|-------------------------------|--------------------------------------|-----------|------------------|--------------------------|-------------------------|------------------------|----------------------|
| Mt. Savage ¹ | | 50.46 | 35.90 | 12.744 | 1.50 | 0.13 | 0.02 | Trace | | 1.65 | |
| Mt. Savage ² | 1.15 | 56.80 | 30.08 | 10.50 | 1.12 | | | 0.80 | | 1.92 | |
| Mt. Savage ³ | 1.53 | 44.40 | 33.56 | 14.575 | 1.08 | Tr. | 0.11 | 0.247 | | 1.47 | |
| Mt. Savage ⁴ | | 56.15 | 33.30 | 9.68 | 0.59 | 0.17 | 0.12 | | | 0.88 | |
| Strasburg, O..... | 0.45 | 55.87 | 41.39 | | 1.60 | 0.40 | 0.30 | 0.29 | 0.20 | 2.79 | |
| Cumberland, Md..... | 1.15 | 56.80 | 30.08 | 7.69 | 1.67 | | | 2.30 | | 3.97 | |
| Woodbridge, N. J..... | | 67.84 | 21.83 | 5.98 | 1.57 | 0.28 | 0.24 | 2.24 | | 4.33 | |
| Carter Co., Ky..... | | 68.01 | 24.09 | 3.03 | 1.01 | 3.01 | | | | 4.02 | |
| Clearfield Co., Pa..... | | 48.35 | 36.37 | 10.56 | 2.00 | 0.07 | 0.12 | 2.54 | | 4.73 | |
| Clearfield ⁵ and Cambria Cos., Pa. ⁶ | | 44.80 | 39.00 | 14.70 | 0.30 | 0.20 | 1.00 | | | | |
| Clinton Co., Pa..... | 1.46 | 63.18 | 23.70 | 6.87 | 1.20 | 0.17 | 0.47 | 2.52 | | 4.55 | SO ₂ 0.19 |
| Clarion Co., Pa..... | 1.02 | 44.61 | 38.01 | 13.63 | 1.25 | 0.08 | 0.41 | 1.74 | | 3.47 | |
| Farrandville, Pa..... | | 45.26 | 37.85 | 13.30 | 2.03 | 0.08 | 0.02 | 1.26 | | 3.59 | 0.20 |
| St. Louis Co., Mo..... | | 67.47 | 19.33 | 10.45 | 2.56 | 0.41 | 0.07 | 1.07 | | 5.14 | |
| Stourbridge, Eng..... | | 73.82 | 15.88 | 6.45 | 2.95 | Tr. | Tr. | 0.90 | | 3.85 | |

¹ Mass. Inst. of Technology 1871. ² Report on Clays of New Jersey. Prof. G. H. Cook, 1877. ³ Second Geological Survey of Penna., 1878. ⁴ Dr. Otto Wuth (2 samples), 1885. ⁵ Flint clay from Clearfield and Cambria counties, Pa., average of hundreds of analyses by Harbison-Walker Refractories Co., Pittsburg, Pa. ⁶ Same material calcined. All other analyses from catalogue of Stowe-Fuller Co., 1907.

Refractoriness of Some American Fire-Brick. — (R. F. Weber, A. I. M. E., 1904.) Prof. Heinrich Ries notes that the fusibility of New Jersey brick is influenced largely by its percentage of silica, but also in part by the texture of the clay. It was found that the fusion-point of almost any of the New Jersey fire-bricks could be reduced four or five cones by grinding the brick sufficiently fine to pass through a 100-mesh sieve.

Mr. Weber draws the conclusion from his tests of 44 bricks that it is evident that the refractoriness of a fire-brick depends on the total quantity of fluxes present, the silica percentage and the coarseness of grain; moreover, chemical analysis alone cannot be used as an index of the refractoriness except within rather wide limits. The following table shows the composition, fusion-point, and physical properties of six most refractory and of five least refractory of the 44 bricks.

| Number of Sample. | Locality. | SiO ₂ . | Al ₂ O ₃ . | Fe ₂ O ₃ . | TiO ₂ . | Alkaline Earths and Alkalies. | Sum of Fluxes. | Cone of Fusion. |
|-------------------|-------------------|--------------------|----------------------------------|----------------------------------|--------------------|-------------------------------|----------------|-----------------|
| | | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | Per cent. | No. |
| 1.... | Missouri..... | 51.59 | 38.26 | 1.84 | 1.97 | 6.34 | 10.25 | 32 to 33 |
| 2.... | Kentucky..... | 54.90 | 38.19 | 2.18 | 1.55 | 3.18 | 6.91 | 32 to 33 |
| 3.... | Pennsylvania..... | 53.05 | 41.16 | 2.65 | 1.80 | 1.34 | 5.79 | 32 to 33 |
| 4.... | Colorado..... | 93.57 | 2.53 | 0.62 | 0.27 | 3.01 | 3.90 | 32 to 33 |
| 5.... | Kentucky..... | 44.77 | 43.08 | 2.78 | 2.54 | 6.83 | 12.15 | 31 to 32 |
| 6.... | New York..... | 68.70 | 20.75 | 1.20 | 5.54 | 3.81 | 10.55 | 31 to 32 |
| 40.... | Pennsylvania..... | 61.28 | 27.13 | 2.90 | 1.37 | 7.31 | 11.58 | 26 |
| 41.... | Pennsylvania..... | 74.83 | 16.40 | 3.26 | 0.77 | 4.74 | 8.77 | 26 |
| 42.... | Alabama..... | 67.19 | 25.05 | 2.83 | 0.71 | 4.22 | 7.76 | 26 |
| 43.... | Indiana..... | 60.76 | 31.66 | 5.67 | 1.58 | 0.33 | 7.58 | 26 |
| 44.... | Kentucky..... | 60.58 | 32.49 | 2.25 | 1.69 | 2.99 | 6.93 | 26 |

¹ Fairly uniform, angular flint-clay particles, constituting body of brick. Largest pieces 5 to 6 mm. in diameter. White.

² Coarse-grained; angular pieces of flint-clay as large as 9 mm. Average 4 to 5 mm. Light buff.

³ Coarse, angular flint-clay particles, varying from 1 to 5 mm. in diameter. Average 4 to 5 mm. Buff.

⁴ Fine-grained quartz particles. Largest 2 to 3 mm. in diameter. White.

⁵ Medium grain; flint-clay particles, fairly uniform in size, 3 to 4 mm. Light buff.

⁶ Coarse grain; quartz particles, 4 to 5 mm. in diameter, forming about 50 per cent of brick. White.

⁴⁰ Fine grain; small, white flint-clay particles, not over 2 mm. in diameter and not abundant. Buff.

⁴¹ Medium grain; pieces of quartz with pinkish color and angular flint-clay particles. About 3 mm. in diameter. Buff.

⁴² Fine grain; even texture. Few coarse particles. Brown.

⁴³ Fine grain; some particles as large as 1 to 2 mm. in diameter. Buff.

⁴⁴ Angular, dark-colored, flinty-clay particles. Maximum size 5 mm. Throughout a reddish-brown matrix.

SLAG BRICKS AND SLAG BLOCKS.

Slag bricks are made by mixing granulated basic slag and slaked lime, molding the mixture in a brick press or by hand, and drying. The silica in the slag ranges from 22.5% to 35%; the alumina and iron oxide together, from 16.1% to 21%; the lime, from 40% to 51.5%. The granulated slag is dried and pulverized. Powdered slaked lime is added in sufficient quantity to bring the total calcium oxide in the mixture up to about 55%. Usually a small amount of water is added. The mixture is then molded into shape, and the bricks are then dried for six to ten days in the open air. Slag bricks weigh less than clay bricks of equal size, require less mortar in laying up, and are at least equal to them in crushing strength.

Slag blocks are made by running molten slag direct from the furnaces into molds. If properly made, they are stronger than slag bricks. They are, however, impervious to air and moisture; and on that account dwellings constructed of them are apt to be damp. Their chief uses are for foundations or for paving blocks. The properties required in a slag paving block, viz: density, resistance to abrasion, toughness, and roughness of surface, vary with the chemical composition of the slag, the rapidity of cooling, and the character of the molds used. Blocks cast in sand molds, and heavily covered with loose sand, cool slowly, and give much better results than those cast in iron molds. — E. C. Eckel, *Eng. News*, April 30, 1903.

MAGNESIA BRICKS.

"Foreign Abstracts" of the Institution of Civil Engineers, 1893, gives a paper by C. Bischof on the production of magnesia bricks. The material most in favor at present is the magnesite of Styria, which, although less pure considered as a source of magnesia than the Greek, has the property of fritting at a high temperature without melting.

At a red heat magnesium carbonate is decomposed into carbonic acid and caustic magnesia, which resembles lime in becoming hydrated and recarbonated when exposed to the air, and possesses a certain plasticity, so that it can be moulded when subjected to a heavy pressure. By long-continued or stronger heating the material becomes dead-burnt, giving a form of magnesia of high density, sp. gr. 3.8, as compared with 3.0 in the plastic form, which is unalterable in the air but devoid of plasticity. A mixture of two volumes of dead-burnt with one of plastic magnesia can be moulded into bricks which contract but little in firing. Other binding materials that have been used are: clay up to 10 or 15 per cent; gas-tar, perfectly freed from water, soda, silica, vinegar as a solution of magnesium acetate which is readily decomposed by heat, and carbolates of alkalis or lime. Among magnesium compounds a weak solution of magnesium chloride may also be used. For setting the bricks lightly burnt, caustic magnesia, with a small proportion of silica to render it less refractory, is recommended. The strength of the bricks may be increased by adding iron, either as oxide or silicate. If a porous product is required, sawdust or starch may be added to the mixture. When dead-burnt magnesia is used alone, soda is said to be the best binding material. See also papers by A. E. Hunt, *Trans. A. I. M. E.*, xvi, 720, and by T. Egleston, *Trans. A. I. M. E.*, xiv, 458.

The average composition of magnesite, crude and calcined, is given as follows by the Harbison-Walker Refractories Co., Pittsburg (1907).

| | Grecian. | | Styrian. | |
|-----------------------|----------|-----------|----------|-----------|
| | Crude. | Calcined. | Crude. | Calcined. |
| Carbonate of magnesia | 97.00% | | 92.50% | |
| Magnesia..... | | 94.00% | | 85.50% |
| Silica..... | 1.25 | 2.75 | 1.50 | 3.00 |
| Alumina..... | 0.40 | 0.70 | 0.50 | 1.00 |
| Iron Oxide..... | 0.40 | 0.80 | 3.90 | 8.00 |
| Lime..... | 0.75 | 1.50 | 1.25 | 2.50 |
| Loss..... | | 0.40 | | 0.50 |
| | 100.05 | 100.15 | 99.65 | 100.50 |

With the calcined Styrian magnesite of the above analysis it is not necessary to use a binder either for making brick or for forming the bottom of an open-hearth furnace.

ASBESTOS.

The following analyses of asbestos are given by J. T. Donald, *Eng. and M. Jour.*, June 27, 1891.

| | Italian. | Canadian. | |
|--------------------|----------|------------|------------|
| | | Broughton. | Templeton. |
| Silica..... | 40.30% | 40.57% | 40.52% |
| Magnesia..... | 43.37 | 41.50 | 42.05 |
| Ferrous oxide..... | .87 | 2.81 | 1.97 |
| Alumina..... | 2.27 | .90 | 2.10 |
| Water..... | 13.72 | 13.55 | 13.46 |
| | 100.53 | 99.33 | 100.10 |

Chemical analysis throws light upon an important point in connection with asbestos, i.e., the cause of the harshness of the fibre of some varieties. Asbestos is principally a hydrous silicate of magnesia, i.e., silicate of magnesia combined with water. When harsh fibre is analyzed it is found to contain less water than the soft fibre. In fibre of very fine quality from Black Lake analysis showed 14.38% of water, while a harsh-fibred sample gave only 11.70%. If soft fibre be heated to a temperature that will drive off a portion of the combined water, there results a substance so brittle that it may be crumbled between thumb and finger. There is evidently some connection between the consistency of the fibre and the amount of water in its composition.

STRENGTH OF MATERIALS.

Stress and Strain. — There is much confusion among writers on strength of materials as to the definition of these terms. An external force applied to a body, so as to pull it apart, is resisted by an internal force, or resistance, and the action of these forces causes a displacement of the molecules, or deformation. By some writers the external force is called a stress, and the internal force a strain; others call the external force a strain, and the internal force a stress; this confusion of terms is not of importance, as the words stress and strain are quite commonly used synonymously, but the use of the word strain to mean molecular displacement, deformation, or distortion, as is the custom of some, is a corruption of the language. See *Engineering News*, June 23, 1892. Some authors in order to avoid confusion never use the word strain in their writings. Definitions by leading authorities are given below.

Stress. — A stress is a force which acts in the interior of a body, and resists the external forces which tend to change its shape. A deformation is the amount of change of shape of a body caused by the stress. The word strain is often used as synonymous with stress, and sometimes it is also used to designate the deformation. (Merriman.)

The force by which the molecules of a body resist a strain at any point is called the stress at that point.

The summation of the displacements of the molecules of a body for a given point is called the distortion or strain at the point considered. (Burr.)

Stresses are the forces which are applied to bodies to bring into action their elastic and cohesive properties. These forces cause alterations of the forms of the bodies upon which they act. *Strain* is a name given to the kind of alteration produced by the stresses. The distinction between stress and strain is not always observed, one being used for the other. (Wood.)

The use of the word stress as synonymous with "stress per square inch," or with "strength per square inch," should be condemned as lacking in precision.

Stresses are of different kinds, viz.: *tensile*, *compressive*, *transverse*, *torsional*, and *shearing* stresses.

A *tensile stress*, or pull, is a force tending to elongate a piece. A *compressive stress*, or push, is a force tending to shorten it. A *transverse stress* tends to bend it. A *torsional stress* tends to twist it. A *shearing stress* tends to force one part of it to slide over the adjacent part.

Tensile, compressive, and shearing stresses are called simple stresses. Transverse stress is compounded of tensile and compressive stresses, and torsional of tensile and shearing stresses.

To these five varieties of stresses might be added *tearing* stress, which is either tensile or shearing, but in which the resistance of different portions of the material are brought into play in detail, or one after the other, instead of simultaneously, as in the simple stresses.

Effects of Stresses. — The following general laws for cases of simple tension or compression have been established by experiment (Merriman):

1. When a small stress is applied to a body, a small deformation is produced, and on the removal of the stress the body springs back to its original form. For small stresses, then, materials may be regarded as perfectly elastic.

2. Under small stresses the deformations are approximately proportional to the forces or stresses which produce them, and also approximately proportional to the length of the bar or body.

3. When the stress is great enough a deformation is produced which is partly permanent, that is, the body does not spring back entirely to its original form on removal of the stress. This permanent part is termed a set. In such cases the deformations are not proportional to the stress.

4. When the stress is greater still the deformation rapidly increases and the body finally ruptures.

5. A sudden stress, or shock, is more injurious than a steady stress or than a stress gradually applied.

Elastic Limit. — The elastic limit is defined as that load at which the deformations cease to be proportional to the stresses, or at which the rate of stretch (or other deformation) begins to increase. It is also defined as the load at which a permanent set first becomes visible. The last definition is not considered as good as the first, as it is found that with some materials a set occurs with any load, no matter how small, and that with others a set which might be called permanent vanishes with lapse of time, and as it is impossible to get the point of first set without removing the whole load after each increase of load, which is frequently inconvenient. The elastic limit, defined, however, as that stress at which the extensions begin to increase at a higher rate than the applied stresses, usually corresponds very nearly with the point of first measurable permanent set.

Apparent Elastic Limit. — Prof. J. B. Johnson (Materials of Construction, p. 19) defines the "apparent elastic limit" as "the point on the stress diagram [a plotted diagram in which the ordinates represent loads and the abscissas the corresponding elongations] at which the rate of deformation is 50% greater than it is at the origin," [the minimum rate]. An equivalent definition, proposed by the author, is that point at which the modulus of extension ($\text{length} \times \text{increment of load per unit of section} \div \text{increment of elongation}$) is two thirds of the maximum. For steel, with a modulus of elasticity of 30,000,000, this is equivalent to that point at which the increase of elongation in an 8-inch specimen for 1000 lbs. per sq. in. increase of load is 0.0004 in.

Yield-point. — The term yield-point has recently been introduced into the literature of the strength of materials. It is defined as that point at which the rate of stretch suddenly increases rapidly with no increase of the load. The difference between the elastic limit, strictly defined as the point at which the rate of stretch begins to increase, and the yield-point, may in some cases be considerable. This difference, however, will not be discovered in short test-pieces unless the readings of elongations are made by an exceedingly fine instrument, as a micrometer reading to 0.0001 inch. In using a coarser instrument, such as calipers reading to 1/100 of an inch, the elastic limit and the yield-point will appear to be simultaneous. Unfortunately for precision of language, the term yield-point was not introduced until long after the term elastic limit had been almost universally adopted to signify the same physical fact which is now defined by the term yield-point, that is, not the point at which the first change in rate, observable only by a microscope, occurs, but that later point (more or less indefinite as to its precise position) at which the increase is great enough to be seen by the naked eye. A most convenient method of determining the point at which a sudden increase of rate of stretch occurs in short specimens, when a testing-machine in which the pulling is done by screws is used, is to note the weight on the beam at the instant that the beam "drops." During the earlier portion of the test, as the extension is steadily increased by the uniform but slow rotation of the screws, the poise is moved steadily along the beam to keep it in equipoise; suddenly a point is reached at which the beam drops, and will not rise until the elongation has been considerably increased by the further rotation of the screws, the advancing of the poise meanwhile being suspended. This point corresponds practically to the point at which the rate of elongation suddenly increases, and to the point at which an appreciable permanent set is first found. It is also the point which has hitherto been called in practice and in text-books the elastic limit, and it will probably continue to be so called, although the use of the newer term "yield-point" for it, and the restriction of the term elastic limit to mean the earlier point at which the rate of stretch begins to increase, as determinable only by micrometric measurements, is more precise and scientific. In order to obtain the yield-point by the drop of the beam with approximate accuracy, the screws of the testing machine must be run very slowly as the yield-point is approached, so as to cause an elongation of not more than, say, 0.005 in. per minute.

In tables of strength of materials hereafter given, the term elastic limit is used in its customary meaning, the point at which the rate of stress has begun to increase as observable by ordinary instruments or by the drop of the beam. With this definition it is practically synonymous with yield-point.

Coefficient (or Modulus) of Elasticity. — This is a term expressing the relation between the amount of extension or compression of a material and the load producing that extension or compression.

It is defined as the load per unit of section divided by the extension per unit of length.

Let P be the applied load, k the sectional area of the piece, l the length of the part extended, λ the amount of the extension, and E the coefficient of elasticity. Then $P \div k =$ the load on a unit of section; $\lambda \div l =$ the elongation of a unit of length.

$$E = \frac{P}{k} \div \frac{\lambda}{l} = \frac{Pl}{k\lambda}.$$

The coefficient of elasticity is sometimes defined as the figure expressing the load which would be necessary to elongate a piece of one square inch section to double its original length, provided the piece would not break, and the ratio of extension to the force producing it remained constant. This definition follows from the formula above given, thus: If $k =$ one square inch, l and λ each = one inch, then $E = P$.

Within the elastic limit, when the deformations are proportional to the stresses, the coefficient of elasticity is constant, but beyond the elastic limit it decreases rapidly.

In cast iron there is generally no apparent limit of elasticity, the deformations increasing at a faster rate than the stresses, and a permanent set being produced by small loads. The coefficient of elasticity therefore is not constant during any portion of a test, but grows smaller as the load increases. The same is true in the case of timber. In wrought iron and steel, however, there is a well-defined elastic limit, and the coefficient of elasticity within that limit is nearly constant.

Resilience, or Work of Resistance of a Material. — Within the elastic limit, the resistance increasing uniformly from zero stress to the stress at the elastic limit, the work done by a load applied gradually is equal to one half the product of the final stress by the extension or other deformation. Beyond the elastic limit, the extensions increasing more rapidly than the loads, and the strain diagram (a plotted diagram showing the relation of extensions to stresses) approximating a parabolic form, the work is approximately equal to two thirds the product of the maximum stress by the extension.

The amount of work required to break a bar, measured usually in inch-pounds, is called its resilience; the work required to strain it to the elastic limit is called its elastic resilience. (See below.)

Under a load applied suddenly the momentary elastic distortion is equal to twice that caused by the same load applied gradually.

When a solid material is exposed to percussive stress, as when a weight falls upon a beam transversely, the work of resistance is measured by the product of the weight into the total fall.

Elastic Resilience. — In a rectangular beam tested by transverse stress, supported at the ends and loaded in the middle,

$$P = \frac{2}{3} \frac{Rbd^2}{l}; \quad (1) \qquad \Delta = \frac{1}{4} \frac{Pl^3}{Ebd^3}; \quad (2)$$

in which, if P is the load in pounds at the elastic limit, $R =$ the modulus of transverse strength, or the stress on the extreme fibre, at the elastic limit, $E =$ modulus of elasticity, $\Delta =$ deflection, l , b , and $d =$ length, breadth, and depth in inches. Substituting for P in (2) its value in (1), $\Delta = 1/6 Rl^2 \div Ed$.

The elastic resilience = half the product of the load and deflection = $1/2 P \Delta$, and the elastic resilience per cubic inch = $1/2 P \Delta \div lbd$.

Substituting the values of P and Δ , this reduces to elastic resilience per cubic inch = $\frac{1}{18} \frac{R^2}{E}$, which is independent of the dimensions; and therefore the elastic resilience per cubic inch for transverse strain may be used as a modulus expressing one valuable quality of a material.

Similarly for tension: Let P = tensile stress in pounds per square inch at the elastic limit; e = elongation per unit of length at the elastic limit; E = modulus of elasticity = $P \div e$; whence $e = P \div E$.

Then elastic resilience per cubic inch = $\frac{1}{2} Pe = \frac{1}{2} \frac{P^2}{E}$

Elevation of Ultimate Resistance and Elastic Limit. — It was first observed by Prof. R. H. Thurston, and Commander L. A. Beardslee, U. S. N., independently, in 1873, that if wrought iron be subjected to a stress beyond its elastic limit, but not beyond its ultimate resistance, and then allowed to "rest" for a definite interval of time, a considerable increase of elastic limit and ultimate resistance may be experienced. In other words, the application of stress and subsequent "rest" increases the resistance of wrought iron. This "rest" may be an entire release from stress or a simple holding the test-piece at a given intensity of stress.

Commander Beardslee prepared twelve specimens and subjected them to a stress equal to the ultimate resistance of the material, without breaking the specimens. These were then allowed to rest, entirely free from stress, from 24 to 30 hours, after which they were again stressed until broken. The gain in ultimate resistance by the rest was found to vary from 4.4 to 17 per cent.

This elevation of elastic and ultimate resistance appears to be peculiar to iron and steel; it has not been found in other metals.

Relation of the Elastic Limit to Endurance under Repeated Stresses (condensed from *Engineering*, August 7, 1891). — When engineers first began to test materials, it was soon recognized that if a specimen was loaded beyond a certain point it did not recover its original dimensions on removing the load, but took a permanent set; this point was called the elastic limit. Since below this point a bar appeared to recover completely its original form and dimensions on removing the load, it appeared obvious that it had not been injured by the load, and hence the working load might be deduced from the elastic limit by using a small factor of safety.

Experience showed, however, that in many cases a bar would not carry safely a stress anywhere near the elastic limit of the material as determined by these experiments, and the whole theory of any connection between the elastic limit of a bar and its working load became almost discredited, and engineers employed the ultimate strength only in deducing the safe working load to which their structures might be subjected. Still, as experience accumulated it was observed that a higher factor of safety was required for a live load than for a dead one.

In 1871 Wöhler published the results of a number of experiments on bars of iron and steel subjected to live loads. In these experiments the stresses were put on and removed from the specimens without impact, but it was, nevertheless, found that the breaking stress of the materials was in every case much below the statical breaking load. Thus, a bar of Krupp's axle steel having a tenacity of 49 tons per square inch broke with a stress of 28.6 tons per square inch, when the load was completely removed and replaced without impact 170,000 times. These experiments were made on a large number of different brands of iron and steel, and the results were concordant in showing that a bar would break with an alternating stress of only, say, one third the statical breaking strength of the material, if the repetitions of stress were sufficiently numerous. At the same time, however, it appeared from the general trend of the experiments that a bar would stand an indefinite number of alternations of stress, provided the stress was kept below the limit.

Prof. Bauschinger defines the elastic limit as the point at which stress ceases to be sensibly proportional to extension, the latter being measured with a mirror apparatus reading to $1/5000$ of a millimetre, or about $1/100000$ in. This limit is always below the yield-point, and may on occasion be zero. On loading a bar above the yield-point, this point rises with the stress, and the rise continues for weeks, months, and possibly for years if the bar is left at rest under its load. On the other hand, when a bar is loaded beyond its true elastic limit, but below its yield-point, this limit rises, but reaches a maximum as the yield-point is approached, and then falls rapidly, reaching even to zero. On leaving the bar at rest under a stress exceeding that of its primitive breaking-

down point the elastic limit begins to rise again, and may, if left a sufficient time, rise to a point much exceeding its previous value.

A bar has two limits of elasticity, one for tension and one for compression. Bauschinger loaded a number of bars in tension until stress ceased to be sensibly proportional to deformation. The load was then removed and the bar tested in compression until the elastic limit in this direction had been exceeded. This process raises the elastic limit in compression, as would be found on testing the bar in compression a second time. In place of this, however, it was now again tested in tension, when it was found that the artificial raising of the limit in compression had lowered that in tension below its previous value. By repeating the process of alternately testing in tension and compression, the two limits took up points at equal distances from the line of no load, both in tension and compression. These limits Bauschinger calls natural elastic limits of the bar, which for wrought iron correspond to a stress of about $8\frac{1}{2}$ tons per square inch, but this is practically the limiting load to which a bar of the same material can be strained alternately in tension and compression, without breaking when the loading is repeated sufficiently often, as determined by Wöhler's method.

As received from the rolls the elastic limit of the bar in tension is above the natural elastic limit of the bar as defined by Bauschinger, having been artificially raised by the deformations to which it has been subjected in the process of manufacture. Hence, when subjected to alternating stresses, the limit in tension is immediately lowered, while that in compression is raised until they both correspond to equal loads. Hence, in Wöhler's experiments, in which the bars broke at loads nominally below the elastic limits of the material, there is every reason for concluding that the loads were really greater than true elastic limits of the material. This is confirmed by tests on the connecting-rods of engines, which work under alternating stresses of equal intensity. Careful experiments on old rods show that the elastic limit in compression is the same as that in tension, and that both are far below the tension elastic limit of the material as received from the rolls.

The common opinion that straining a metal beyond its elastic limit injures it appears to be untrue. It is not the mere straining of a metal beyond one elastic limit that injures it, but the straining, many times repeated, beyond its two elastic limits. Sir Benjamin Baker has shown that in bending a shell plate for a boiler the metal is of necessity strained beyond its elastic limit, so that stresses of as much as 7 tons to 15 tons per square inch may obtain in it as it comes from the rolls, and unless the plate is annealed, these stresses will still exist after it has been built into the boiler. In such a case, however, when exposed to the additional stress due to the pressure inside the boiler, the overstrained portions of the plate will relieve themselves by stretching and taking a permanent set, so that probably after a year's working very little difference could be detected in the stresses in a plate built into the boiler as it came from the bending rolls, and in one which had been annealed, before riveting into place, and the first, in spite of its having been strained beyond its elastic limits, and not subsequently annealed, would be as strong as the other.

Resistance of Metals to Repeated Shocks.

More than twelve years were spent by Wöhler at the instance of the Prussian Government in experimenting upon the resistance of iron and steel to repeated stresses. The results of his experiments are expressed in what is known as Wöhler's law, which is given in the following words in Dubois's translation of Weyrauch:

"Rupture may be caused not only by a steady load which exceeds the carrying strength, but also by repeated applications of stresses, none of which are equal to the carrying strength. The differences of these stresses are measures of the disturbance of continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes."

A practical illustration of the meaning of the first portion of this law may be given thus: If 50,000 pounds once applied will just break a bar of iron or steel, a stress very much less than 50,000 pounds will break it if repeated sufficiently often,

This is fully confirmed by the experiments of Fairbairn and Spangenberg, as well as those of Wöhler; and, as is remarked by Weyrauch, it may be considered as a long-known result of common experience. It partially accounts for what Mr. Holley has called the "intrinsically ridiculous factor of safety of six."

Another "long-known result of experience" is the fact that rupture may be caused by a succession of *shocks* or *impacts*, none of which alone would be sufficient to cause it. Iron axles, the piston-rods of steam hammers, and other pieces of metal subject to continuously repeated shocks, invariably break after a certain length of service. They have a "life" which is limited.

Several years ago Fairbairn wrote: "We know that in some cases wrought iron subjected to continuous vibration assumes a crystalline structure, and that the cohesive powers are much deteriorated, but we are ignorant of the causes of this change." We are still ignorant, not only of the causes of this change, but of the conditions under which it takes place. Who knows whether wrought iron subjected to very slight continuous vibration will endure forever? or whether to insure final rupture each of the continuous small shocks must amount at least to a certain percentage of single heavy shock (both measured in foot-pounds), which would cause rupture with one application? Wöhler found in testing iron by repeated stresses (not impacts) that in one case 400,000 applications of a stress of 500 centners to the square inch caused rupture, while a similar bar remained sound after 48,000,000 applications of a stress of 300 centners to the square inch (1 centner = 110.2 lbs.).

Who knows whether or not a similar law holds true in regard to repeated shocks? Suppose that a bar of iron would break under a single impact of 1000 foot-pounds, how many times would it be likely to bear the repetition of 100 foot-pounds, or would it be safe to allow it to remain for fifty years subjected to a continual succession of blows of even 10 foot-pounds each?

Mr. William Metcalf published in the *Metallurgical Review*, Dec., 1877, the results of some tests of the life of steel of different percentages of carbon under impact. Some small steel pitmans were made, the specifications for which required that the unloaded machine should run $4\frac{1}{2}$ hours at the rate of 1200 revolutions per minute before breaking.

The steel was all of uniform quality, except as to carbon. Here are the results. The

| | | | |
|---------|--|------------|----------------------------------|
| 0.30 C. | ran | 1 h. 21 m. | Heated and bent before breaking. |
| 0.49 C. | " | 1 h. 28 m. | " |
| 0.53 C. | " | 4 h. 57 m. | Broke without heating. |
| 0.65 C. | " | 3 h. 50 m. | Broke at weld where imperfect. |
| 0.80 C. | " | 5 h. 40 m. | " |
| 0.84 C. | " | 18 h. | " |
| 0.87 C. | Broke in weld near the end. | | |
| 0.96 C. | Ran 4.55 m., and the machine broke down. | | |

Some other experiments by Mr. Metcalf confirmed his conclusion, viz. that high-carbon steel was better adapted to resist repeated shocks and vibrations than low-carbon steel.

These results, however, would scarcely be sufficient to induce any engineer to use 0.84 carbon steel in a car-axle or a bridge-rod. Further experiments are needed to confirm or overthrow them.

(See description of proposed apparatus for such an investigation in the author's paper in *Trans. A. I. M. E.*, vol. viii, p. 76, from which the above extract is taken.)

Stresses Produced by Suddenly Applied Forces and Shocks.

(Mansfield Merriman, *R. R. & Eng. Jour.*, Dec., 1889.)

Let P be the weight which is dropped from a height h upon the end of a bar, and let y be the maximum elongation which is produced. The work performed by the falling weight, then, is $W = P(h + y)$, and this must equal the internal work of the resisting molecular stresses. The stress in the bar, which is at first 0, increases up to a certain limit Q , which is greater than P ; and if the elastic limit be not exceeded the elongation increases uniformly with the stress, so that the internal work is equal to

the mean stress $\frac{1}{2}Q$ multiplied by the total elongation y , or $W = \frac{1}{2}Qy$. Whence, neglecting the work that may be dissipated in heat,

$$\frac{1}{2}Qy = Ph + Py.$$

If e be the elongation due to the static load P , within the elastic limit $y = \frac{Q}{P}e$; whence $Q = P\left(1 + \sqrt{1 + 2\frac{h}{e}}\right)$, which gives the momentary maximum stress. Substituting this value of Q , there results $y = e\left(1 + \sqrt{1 + 2\frac{h}{e}}\right)$, which is the value of the momentary maximum elongation.

A shock results when the force P , before its action on the bar, is moving with velocity, as is the case when a weight P falls from a height h . The above formulas show that this height h may be small if e is a small quantity, and yet very great stresses and deformations be produced. For instance, let $h = 4e$, then $Q = 4P$ and $y = 4e$; also let $h = 12e$, then $Q = 6P$ and $y = 6e$. Or take a wrought-iron bar 1 in. square and 5 ft. long: under a steady load of 5000 lbs. this will be compressed about 0.012 in., supposing that no lateral flexure occurs; but if a weight of 5000 lbs. drops upon its end from the small height of 0.048 in. there will be produced the stress of 20,000 lbs.

A suddenly applied force is one which acts with the uniform intensity P upon the end of the bar, but which has no velocity before acting upon it. This corresponds to the case of $h = 0$ in the above formulas, and gives $Q = 2P$ and $y = 2e$ for the maximum stress and maximum deformation. Probably the action of a rapidly moving train upon a bridge produces stresses of this character. For a further discussion of this subject, in which the inertia of the bar is considered, see Merriman's *Mechanics of Materials*, 10th ed., 1908.

Increasing the Tensile Strength of Iron Bars by Twisting them.—Ernest L. Ransome of San Francisco obtained a patent, in 1888, for an "improvement in strengthening and testing wrought metal and steel rods or bars, consisting in twisting the same in a cold state. . . . Any defect in the lamination of the metal which would otherwise be concealed is revealed by twisting, and imperfections are shown at once. The treatment may be applied to bolts, suspension-rods or bars subjected to tensile strength of any description."

Jesse J. Shuman (Am. Soc. Test. Mat., 1907) describes several series of experiments on the effect of twisting square steel bars. Following are some of the results:

| | | | | | |
|---|--------|----------------|--------|----------------|----------------|
| Soft Bessemer steel bars $\frac{1}{2}$ in. square. Tens. Strength, plain bar, 60,400. | | | | | |
| No. of turns per foot..... | 3 | $4\frac{3}{4}$ | 5 | $5\frac{3}{4}$ | $5\frac{7}{8}$ |
| Yield point, lbs. per sq. in..... | 65,600 | 72,400 | 84,800 | 84,000 | 80,800 |
| Ult. strength "..... | 83,200 | 89,600 | 92,000 | 90,000 | 88,800 |
| Elongation in 8 in., %..... | 10 | 5.75 | 6.25 | 7.5 | 3.75 |

| | | | | | |
|---|--------|----------------|----------------|---------|----------------|
| Bessemer, 0.25 carbon, $\frac{1}{2}$ in. sq. Tens. strength, plain bar, 75,000. | | | | | |
| No. of turns per foot..... | 3 | $4\frac{1}{2}$ | $4\frac{7}{8}$ | 5 | $5\frac{1}{2}$ |
| Yield point, lbs. per sq. in..... | 83,600 | 83,200 | 88,800 | 84,200 | 84,200 |
| Ult. strength "..... | 99,600 | 99,200 | 104,000 | 102,000 | 100,800 |
| Elongation in 8 in., %..... | 8 | 4.5 | 4 | 5.75 | 6 |

Bars of each grade twisted off when given more turns than stated.

Soft Bessemer, square bars, different sizes.

| | | | | | | | | | |
|-------------------------------|---------------|----------------|---------------|----------------|----------------|----------------|------|----------------|----------------|
| Size, in. sq..... | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 | $1\frac{1}{8}$ | $1\frac{1}{4}$ |
| No. of turns per ft..... | 4 | $3\frac{1}{2}$ | 3 | $2\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{1}{4}$ | 1 | $\frac{7}{8}$ | $\frac{3}{4}$ |
| Yield point, increase %*..... | 111 | 82 | 64 | 83 | 85.5 | 77 | 82 | 64 | 59 |
| Ult. strength " %*..... | 37 | 38.6 | 41 | 33.5 | 34.3 | 29.7 | 22.8 | 20.1 | 28.9 |

Mr. Schuman recommends that in twisting bars for reinforced concrete, in order not to be in danger of approaching the breaking point, the number of turns should be about half the number at which the steel is at its maximum strength, which for Bessemer of about 60,000 lbs. tensile strength means one complete twist in 8 to 10 times the size of the bar.

Steel bars strengthened by twisting are largely used in reinforced concrete.

* Average of two tests each.]

TENSILE STRENGTH.

The following data are usually obtained in testing by tension in a testing-machine a sample of a material of construction:

The load and the amount of extension at the elastic limit.

The maximum load applied before rupture.

The elongation of the piece, measured between gauge-marks placed a stated distance apart before the test; and the reduction of area at the point of fracture.

The load at the elastic limit and the maximum load are recorded in pounds per square inch of the original area. The elongation is recorded as a percentage of the stated length between the gauge-marks, and the reduction of area as a percentage of the original area. The coefficient of elasticity is calculated from the ratio the extension within the elastic limit per inch of length bears to the load per square inch producing that extension.

On account of the difficulty of making accurate measurements of the fractured area of a test-piece, and of the fact that elongation is more valuable than reduction of area as a measure of ductility and of resilience or work of resistance before rupture, modern experimenters are abandoning the custom of reporting reduction of area. The data now calculated from the results of a tensile test for commercial purposes are: 1. Tensile strength in pounds per square inch of original area. 2. Elongation per cent of a stated length between gauge-marks, usually 8 inches. 3. Elastic limit in pounds per square inch of original area.

The short or grooved test specimen gives with most metals, especially with wrought iron and steel, an apparent tensile strength much higher than the real strength. This form of test-piece is now almost entirely abandoned. Pieces 2 in. in length between marks are used for forgings.

The following results of the tests of six specimens from the same $\frac{1}{4}$ -in. steel bar illustrate the apparent elevation of elastic limit and the changes in other properties due to change in length of stems which were turned down in each specimen to 0.798 in. diameter. (Jas. E. Howard, Eng. Congress 1893, Section G.)

| Description of Stem. | Elastic Limit, Lbs. per Sq. In. | Tensile Strength, Lbs. per Sq. In. | Contraction of Area, per cent. |
|--|------------------------------------|---------------------------------------|-----------------------------------|
| 1.00 in. long. | 64,900 | 94,400 | 49.0 |
| 0.50 in. long. | 65,320 | 97,800 | 43.4 |
| 0.25 in. long. | 68,000 | 102,420 | 39.6 |
| Semicircular groove, 0.4 in. radius. | 75,000 | 116,380 | 31.6 |
| Semicircular groove, $\frac{1}{8}$ in. radius. | 86,000, about | 134,960 | 23.0 |
| V-shaped groove. | 90,000, about | 117,000 | Indeterminate. |

Test plates made by the author in 1879 of straight and grooved test-pieces of boiler-plate steel cut from the same gave the following results:

5 straight pieces, 56,605 to 59,012 lbs. T. S. Aver. 57,566 lbs.

4 grooved " 64,341 to 67,400 " " " 65,452 "

Excess of the short or grooved specimen, 21 per cent, or 12,114 lbs.

Measurement of Elongation. — In order to be able to compare records of elongation, it is necessary not only to have a uniform length of section between gauge-marks (say 8 inches), but to adopt a uniform method of measuring the elongation to compensate for the difference between the apparent elongation when the piece breaks near one of the gauge-marks, and when it breaks midway between them. The following method is recommended (*Trans. A. S. M. E.*, vol. xi, p. 622):

Mark on the specimen divisions of $\frac{1}{2}$ inch each. After fracture measure from the point of fracture the length of 8 of the marked spaces on each fractured portion (or 7 + on one side and 8 + on the other if the fracture is not at one of the marks). The sum of these measurements, less 8 inches, is the elongation of 8 inches of the original length. If the fracture is so near one end of the specimen that 7 + spaces are not left on the shorter portion, then take the measurement of as many spaces (with the fractional part next to the fracture) as are left, and for the spaces lacking add the measurement of as many corresponding spaces of the longer portion as are necessary to make the 7 + spaces.

Shapes of Specimens for Tensile Tests. — The shapes shown in Fig. 78 were recommended by the author in 1882 when he was connected with the Pittsburgh Testing Laboratory. They are now in most general use; the earlier forms, with 5 inches or less in length between shoulders, being almost entirely abandoned.

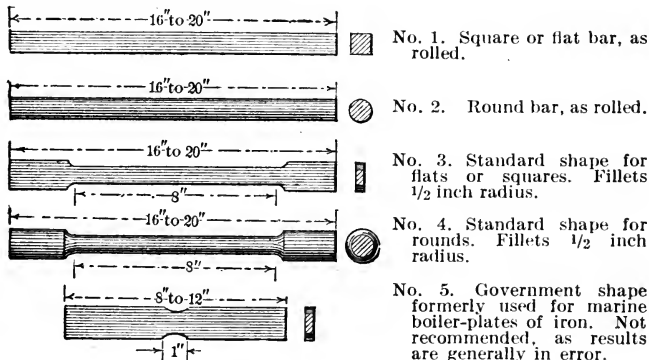


FIG. 78.

Precautions Required in making Tensile Tests. — The testing-machine itself should be tested, to determine whether its weighing apparatus is accurate, and whether it is so made and adjusted that in the test of a properly made specimen the line of strain of the testing-machine is absolutely in line with the axis of the specimen.

The specimen should be so shaped that it will not give an incorrect record of strength.

It should be of uniform minimum section for not less than eight inches of its length. Eight inches is the standard length for bars. For forgings and castings and in special cases shorter lengths are used; these show greater percentages of elongation, and the length between gauge marks should therefore always be stated in the record.

Regard must be had to the time occupied in making tests of certain materials. Wrought iron and soft steel can be made to show a higher than their actual apparent strength by keeping them under strain for a great length of time.

In testing soft alloys, copper, tin, zinc, and the like, which flow under constant strain, their highest apparent strength is obtained by testing them rapidly. In recording tests of such materials the length of time occupied in the test should be stated.

For very accurate measurements of elongation, corresponding to increments of load during the tests, the electric contact micrometer, described in *Trans. A. S. M. E.*, vol. vi. p. 479, will be found convenient. When readings of elongation are then taken during the test, a strain diagram may be plotted from the reading, which is useful in comparing the qualities of different specimens. Such strain diagrams are made automatically by the new Olsen testing-machine, described in *Jour. Frank. Inst.* 1891.

The coefficient of elasticity should be deduced from measurement

observed between fixed increments of load per unit section, say between 2000 and 12,000 pounds per square inch or between 1000 and 11,000 pounds instead of between 0 and 10,000 pounds.

COMPRESSIVE STRENGTH.

What is meant by the term "compressive strength" has not yet been settled by the authorities, and there exists more confusion in regard to this term than in regard to any other used by writers on strength of materials. The reason of this may be easily explained. The effect of a compressive stress upon a material varies with the nature of the material, and with the shape and size of the specimen tested. While the effect of a tensile stress is to produce rupture or separation of particles in the direction of the line of strain, the effect of a compressive stress on a piece of material may be either to cause it to fly into splinters, to separate into two or more wedge-shaped pieces and fly apart, to bulge, buckle, or bend, or to flatten out and utterly resist rupture or separation of particles. A piece of speculum metal (copper 2, tin 1) under compressive stress will exhibit no change of appearance until rupture takes place, and then it will fly to pieces as suddenly as if blown apart by gunpowder. A piece of cast iron or of stone will generally split into wedge-shaped fragments. A piece of wrought iron will buckle or bend. A piece of wood or zinc may bulge, but its action will depend upon its shape and size. A piece of lead will flatten out and resist compression till the last degree; that is, the more it is compressed the greater becomes its resistance.

Air and other gaseous bodies are compressible to any extent as long as they retain the gaseous condition. Water not confined in a vessel is compressed by its own weight to the thickness of a mere film, while when confined in a vessel it is almost incompressible.

It is probable, although it has not been determined experimentally, that solid bodies when confined are at least as incompressible as water. When they are not confined, the effect of a compressive stress is not only to shorten them, but also to increase their lateral dimensions or bulge them. Lateral stresses are therefore induced by compressive stresses.

The weight per square inch of original section required to produce any given amount or percentage of shortening of any material is not a constant quantity, but varies with both the length and the sectional area, with the shape of the sectional area, and with the relation of the area to the length. The "compressive strength" of a material, if this term be supposed to mean the weight in pounds per square inch necessary to cause rupture, may vary with every size and shape of specimen experimented upon. Still more difficult would it be to state what is the "compressive strength" of a material which does not rupture at all, but flattens out. Suppose we are testing a cylinder of a soft metal like lead, two inches in length and one inch in diameter, a certain weight will shorten it one per cent, another weight ten per cent, another fifty per cent, but no weight that we can place upon it will rupture it, for it will flatten out to a thin sheet. What, then, is its compressive strength? Again, a similar cylinder of soft wrought iron would probably compress a few per cent, bulging evenly all around; it would then commence to bend, but at first the bend would be imperceptible to the eye and too small to be measured. Soon this bend would be great enough to be noticed, and finally the piece might be bent nearly double, or otherwise distorted. What is the "compressive strength" of this piece of iron? Is it the weight per square inch which compresses the piece one per cent or five per cent, that which causes the first bending (impossible to be discovered), or that which causes a perceptible bend?

As showing the confusion concerning the definitions of compressive strength, the following statements from different authorities on the strength of wrought iron are of interest.

Wood's Resistance of Materials states, "Comparatively few experiments have been made to determine how much wrought iron will sustain at the point of crushing. Hodgkinson gives 65,000, Rondulet 70,800, Weisbach 72,000, Rankine 30,000 to 40,000. It is generally assumed that wrought

iron will resist about two thirds as much crushing as to tension, but the experiments fail to give a very definite ratio."

The following values, said to be deduced from the experiments of Major Wade, Hodgkinson, and Capt. Meigs, are given by Haswell:

| | |
|---|--------------|
| American wrought iron | 127,720 lbs. |
| " " (mean) | 85,500 " |
| English " " | 65,200 " |
| | 40,000 " |

Stoney states that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the piece is not less than one and does not exceed four or five diameters, and that the weight which will just crush a short prism whose base equals one square inch, and whose height is not less than 1 to $1\frac{1}{2}$ and does not exceed 4 or 5 diameters, is called the crushing strength of the material. It would be well if experimenters would all agree upon some such definition of the term "crushing strength," and insist that all experiments which are made for the purpose of testing the relative values of different materials in compression be made on specimens of exactly the same shape and size. An arbitrary size and shape should be assumed and agreed upon for this purpose. The size mentioned by Stoney is definite as regards area of section, viz., one square inch, but is indefinite as regards length, viz., from one to five diameters. In some metals a specimen five diameters long would bend, and give a much lower apparent strength than a specimen having a length of one diameter. The words "will just crush" are also indefinite for ductile materials, in which the resistance increases without limit if the piece tested does not bend. In such cases the weight which causes a certain percentage of compression, as five, ten, or fifty per cent, should be assumed as the crushing strength.

For future experiments on crushing strength three things are desirable: First, an arbitrary standard shape and size of test specimen for comparison of all materials. Secondly, a standard limit of compression for ductile materials, which shall be considered equivalent to fracture in brittle materials. Thirdly, an accurate knowledge of the relation of the crushing strength of a specimen of standard shape and size to the crushing strength of specimens of all other shapes and sizes. The latter can only be secured by a very extensive and accurate series of experiments upon all kinds of materials, and on specimens of a great number of different shapes and sizes.

The author proposes, as a standard shape and size, for a compressive test specimen for all metals, a cylinder one inch in length, and one half square inch in sectional area, or 0.798 inch diameter; and for the limit of compression equivalent to fracture, ten per cent of the original length. The term "compressive strength," or "compressive strength of standard specimen," would then mean the weight per square inch required to fracture by compressive stress a cylinder one inch long and 0.798 inch diameter, or to reduce its length to 0.9 inch if fracture does not take place before that reduction in length is reached. If such a standard, or any standard size whatever, had been used by the earlier authorities on the strength of materials, we never would have had such discrepancies in their statements in regard to the compressive strength of wrought iron as those given above.

The reasons why this particular size is recommended are: that the sectional area, one-half square inch, is as large as can be taken in the ordinary testing-machines of 100,000 pounds capacity, to include all the ordinary metals of construction, cast and wrought iron, and the softer steels; and that the length, one inch, is convenient for calculation of percentage of compression. If the length were made two inches, many materials would bend in testing, and give incorrect results. Even in cast iron Hodgkinson found as the mean of several experiments on various grades, tested in specimens $\frac{3}{4}$ inch in height, a compressive strength per square inch of 94,730 pounds, while the mean of the same number of specimens of the same irons tested in pieces $1\frac{1}{2}$ inches in height was

only 88,800 pounds. The best size and shape of standard specimen should, however, be settled upon only after consultation and agreement among several authorities.

The Committee on Standard Tests of the American Society of Mechanical Engineers say (vol. xi, p. 624):

"Although compression tests have heretofore been made on diminutive sample pieces, it is highly desirable that tests be also made on long pieces from 10 to 20 diameters in length, corresponding more nearly with actual practice, in order that elastic strain and change of shape may be determined by using proper measuring apparatus.

"The elastic limit, modulus or coefficient of elasticity, maximum and ultimate resistances, should be determined, as well as the increase of section at various points, viz., at bearing surfaces and at crippling point.

"The use of long compression-test pieces is recommended, because the investigation of short cubes or cylinders has led to no direct application of the constants obtained by their use in computation of actual structures, which have always been and are now designed according to empirical formulæ obtained from a few tests of long columns."

COLUMNS, PILLARS, OR STRUTS.

Notation. — P = crushing weight in pounds; d = exterior diameter in inches; a = area in square inches; L = length in feet; l = length in inches; S = compressive stress, lbs. per sq. in.; E = modulus of elasticity in tension or compression; r = least radius of gyration; ϕ , an experimental coefficient.

For a short column centrally loaded $S = P/a$, but for a long column which tends to bend under load, the stress on the concave side is greater, and on the convex side less than P/a .

Hodgkinson's Formula for Columns.

| Kind of Column. | Both ends rounded, the length of the column exceeding 15 times its diameter. | Both ends flat, the length of the column exceeding 30 times its diameter. |
|---|--|---|
| Solid cylindrical columns of cast iron... } | $P = 33,380 \frac{d^{3.76}}{L^{1.7}}$ | $P = 98,920 \frac{d^{3.55}}{L^{1.7}}$ |
| Solid cylindrical columns of wrought iron } | $P = 95,850 \frac{d^{3.76}}{L^2}$ | $P = 299,600 \frac{d^{3.55}}{L^2}$ |

These formulæ apply only in cases in which the length is so great that the column breaks by bending and not by simple crushing. Hodgkinson's tests were made on small columns, and his results are not now considered reliable.

Euler's Formula for Long Columns.

$P/a = \pi^2 E (r/l)^2$ for columns with round or hinged ends. For columns with fixed ends, multiply by 4; with one end round and the other fixed, multiply by $2\frac{1}{4}$; for one end fixed and the other free, as a post set in the ground, divide by 4. P is the load which causes a slight deflection; a load greater than P will cause an increase of deflection until the column fails by bending. The formula is now little used.

Christie's Tests (*Trans. A. S. C. E.* 1884; *Merriman's Mechanics of Materials*). — About 300 tests of wrought-iron struts were made, the quality of the iron being about as follows: tensile strength per sq. in., 49,600 lbs.; elastic limit 32,000 lbs., elongation 18% in 8 ins.

The following table gives the average results.

| Ratio l/r Length to Least Ra- dius of Gyration. | Ultimate Load, P/a , in Pounds per Square Inch. | | | |
|---|---|------------|--------------|-------------|
| | Fixed Ends. | Flat Ends. | Hinged Ends. | Round Ends. |
| 20 | 46,000 | 46,000 | 46,000 | 44,000 |
| 40 | 40,000 | 40,000 | 40,000 | 36,500 |
| 60 | 36,000 | 36,000 | 36,000 | 30,500 |
| 80 | 32,000 | 32,000 | 31,500 | 25,000 |
| 100 | 30,000 | 29,800 | 28,000 | 20,500 |
| 120 | 28,000 | 26,300 | 24,300 | 16,500 |
| 140 | 25,500 | 23,500 | 21,000 | 12,800 |
| 160 | 23,000 | 20,000 | 16,500 | 9,500 |
| 180 | 20,000 | 16,800 | 12,800 | 7,500 |
| 200 | 17,500 | 14,500 | 10,800 | 6,000 |
| 220 | 15,000 | 12,700 | 8,800 | 5,000 |
| 240 | 13,000 | 11,200 | 7,500 | 4,300 |
| 260 | 11,000 | 9,800 | 6,500 | 3,800 |
| 280 | 10,000 | 8,500 | 5,700 | 3,200 |
| 300 | 9,000 | 7,200 | 5,000 | 2,800 |
| 320 | 8,000 | 6,000 | 4,500 | 2,500 |
| 360 | 6,500 | 4,300 | 3,500 | 1,900 |
| 400 | 5,200 | 3,000 | 2,500 | 1,500 |

The results of Christie's tests agree with those computed by Euler's formula for round-end columns with l/r between 150 and 400, but differ widely from them in shorter columns, and still more widely in columns with fixed ends.

Rankine's Formula (sometimes called Gordon's), $S = \frac{P}{a} \left(1 + \phi \left(\frac{l}{r}\right)^2\right)$ or $\frac{P}{a} = \frac{S}{1 + \phi (l/r)^2}$. Applying Rankine's formula to the results of experiments, wide variations are found in the values of the empirical coefficient ϕ . Merriman gives the following values, which are extensively employed in practice.

VALUES OF ϕ FOR RANKINE'S FORMULA.

| Material. | Both Ends Fixed. | Fixed and Round. | Both Ends Round. |
|-------------------|---------------------|---------------------|---------------------|
| Timber..... | 1/3,000 | 1.78/3,000 | 4/3,000 |
| Cast Iron..... | 1/5,000 | 1.78/5,000 | 4/5,000 |
| Wrought Iron..... | 1/36,000 | 1.78/36,000 | 4/36,000 |
| Steel..... | 1/25,000 | 1.78/25,000 | 4/25,000 |

The value to be taken for S is the ultimate compressive strength of the material for cases of rupture, and the allowable compressive unit stress for cases of design.

Burr gives the following values as commonly taken for S and ϕ .

For solid wrought-iron columns, $S = 36,000$ to $40,000$, $\phi = 1/36,000$ to $1/40,000$.

For solid cast-iron columns, $S = 80,000$, $\phi = 1/6,400$.

For hollow cast-iron columns, $P/a = 80,000 \div 1 + \frac{1}{800} \frac{l^2}{d^2}$ (d = outside diam. in inches).

The coefficient of l^2/d^2 is given by different writers as $1/400$, $1/500$, $1/600$ and $1/800$. (See Strength of Cast-iron Columns, below.)

Sir Benjamin Baker gives for mild steel, $S = 67,000$ lbs., $\phi = 1/22,400$; for strong steel, $S = 114,000$ lbs., $\phi = 1/14,400$. Prof. Burr considers these only loose approximations. (See Straight-line Formula, below).

For dry timber, Rankine gives $S = 7200$ lbs., $\phi = 1/3000$.

The Straight-line Formula.—The results of computations by Euler's or Rankine's formulas give a curved line when plotted on a diagram with values of l/r as abscissas and value of P/a as ordinates. The average results of experiments on columns within the limits of l/r commonly used in practice, say from 50 to 200, can be represented by a straight line about as accurately as by a curve. Formulas derived from such plotted lines, of the general form $P/a = S - C l/r$, in which C is an experimental coefficient, are in common use, but Merriman says it is advisable that the use of this formula should be limited to cases in which the specifications require it to be employed, and for rough approximate computations. Values of S and C given by T. H. Johnson are as follows:

| | F | H | R | | F | H | R |
|-------------------|---|-----------------------|-------|--|---------------|---|---|
| Wrought Iron: | | | | | | | |
| | $S = 42,000$ lbs., $C = 128, 157, 203$; limit of $l/r = 218, 178, 138$ | | | | | | |
| Structural Steel: | | | | | | | |
| | $S = 52,500$ " | $C = 179, 220, 284$; | " " " | | 195, 159, 123 | | |
| Cast Iron: | $S = 80,000$ " | $C = 438, 537, 693$; | " " " | | 122, 99, 77 | | |
| Oak, flat ends: | | | | | | | |
| | $S = 5,400$ " | $C = 28$; | " " " | | 128 | | |

F , flat ends; H , hinged ends; R , round ends.

Merriman says: "The straight-line formula is not suitable for investigating a column, that is for determining values of S due to given loads, because S enters the formula in such a manner as to lead to a cubic equation when it is the only unknown quantity. It may be used to find the safe load for a given column to withstand a given unit stress, or to design a column for a given load and unit stress. When so used, it is customary to divide the values of S and C given in the table by an assumed factor of safety. For example, Cooper's specifications require that the sectional area a for a medium-steel post of a through railroad bridge shall be found from $P/a = 17,000 - 90 l/r$ lbs. per sq. in., in which P is the direct dead-load compression on the post plus twice the live-load compression; the values of S and C here used are a little less than one-third of those given in the table for round ends."

Working Formulæ for Wrought-iron and Steel Struts of Various Forms.—Burr gives the following practical formulæ:

| Kind of Strut. | p = Ultimate Strength, lbs. per sq. in. of Section. | p_1 = Working Strength = $\frac{1}{5}$ Ultimate, lbs. per sq. in. of Section. |
|---|---|---|
| Flat and fixed end iron angles and tees | $44000 - 140 \frac{l}{r}$ (1) | $8800 - 28 \frac{l}{r}$ (2) |
| Hinged-end iron angles and tees | $46000 - 175 \frac{l}{r}$ (3) | $9200 - 35 \frac{l}{r}$ (4) |
| Flat-end iron channels and I-beams | $40000 - 110 \frac{l}{r}$ (5) | $8000 - 22 \frac{l}{r}$ (6) |
| Flat-end mild-steel angles | $52000 - 180 \frac{l}{r}$ (7) | $10400 - 36 \frac{l}{r}$ (8) |
| Flat-end high-steel angles | $76000 - 290 \frac{l}{r}$ (9) | $15200 - 58 \frac{l}{r}$ (10) |
| Pin-end solid wrought-iron columns | $32000 - 80 \frac{l}{r}$ (11) | $6400 - 16 \frac{l}{r}$ (12) |
| | $32000 - 277 \frac{l}{d}$ | $6400 - 55 \frac{l}{d}$ |

Equations (1) to (4) are to be used only between $\frac{l}{r} = 40$ and $\frac{l}{r} = 200$
 " (5) and (6) " " " " " " " = 20 " " = 200
 " (7) to (10) " " " " " " " = 40 " " = 200
 " (11) and (12) " " " " " " " = 20 " " = 200
 or $\frac{l}{d} = 6$ and $\frac{l}{d} = 65$

Built Columns (Burr).—Steel columns, properly made, of steel ranging in specimens from 65,000 to 73,000 lbs. per square inch, should give a resistance 25 to 33 per cent in excess of that of wrought-iron columns with the same value of $l \div r$, provided that ratio does not exceed 140.

The unsupported width of a plate in a compression member should not exceed 30 times its thickness.

In built columns the transverse distance between centre lines of rivets securing plates to angles or channels, etc., should not exceed 35 times the plate thickness. If this width is exceeded, longitudinal buckling of the plate takes place, and the column ceases to fail as a whole, but yields in detail.

The thickness of the leg of an angle to which latticing is riveted should not be less than $\frac{1}{9}$ of the length of that leg or side if the column is purely a compression member. The above limit may be passed somewhat in stiff ties and compression members designed to carry transverse loads.

The panel points of latticing should not be separated by a greater distance than 60 times the thickness of the angle-leg to which the latticing is riveted, if the column is wholly a compression member.

The rivet pitch should never exceed 16 times the thickness of the thinnest metal pierced by the rivet, and if the plates are very thick it should never nearly equal that value.

Burr gives the following general principles which govern the resistance of built columns:

The material should be disposed as far as possible from the neutral axis of the cross-section, thereby increasing r ;

There should be no initial internal stress;

The individual portions of the column should be mutually supporting;

The individual portions of the column should be so firmly secured to each other that no relative motion can take place, in order that the column may fail as a whole, thus maintaining the original value of r .

Stoney says: "When the length of a rectangular wrought-iron tubular column does not exceed 30 times its least breadth, it fails by the bulging or buckling of a short portion of the plates, not by the flexure of the pillar as a whole."

WORKING STRAINS ALLOWED IN BRIDGE MEMBERS.

Theodore Cooper gives the following in his Bridge Specifications:

Compression members shall be so proportioned that the maximum load shall in no case cause a greater strain than that determined by the following formula:

$$P = \frac{8000}{1 + \frac{l^2}{40,000 r^2}} \text{ for square-end compression members;}$$

$$P = \frac{8000}{1 + \frac{l^2}{30,000 r^2}} \text{ for compression members with one pin and one square end;}$$

$$P = \frac{8000}{1 + \frac{l^2}{20,000 r^2}} \text{ for compression members with pin-bearings;}$$

(These values may be increased in bridges over 150 ft. span. See Cooper's Specifications.)

P = the allowed compression per square inch of cross-section;

l = the length of compression member, in inches;

r = the least radius of gyration of the section in inches.

No compression member, however, shall have a length exceeding 25 times its least width.

Tension Members. — All parts of the structure shall be so proportioned that the maximum loads shall in no case cause a greater tension than the following (except in spans exceeding 150 feet):

| | Pounds per sq. in. |
|--|-----------------------|
| On lateral bracing | 15,000 |
| On solid rolled beams, used as cross floor-beams and stringers | 9,000 |
| On bottom chords and main diagonals (forged eye-bars) | 10,000 |
| On bottom chords and main diagonals (plates or shapes), net section | 8,000 |
| On counter rods and long verticals (forged eye-bars) | 8,000 |
| On counter and long verticals (plates or shapes), net section | 6,500 |
| On bottom flange of riveted cross-girders, net section | 8,000 |
| On bottom flange of riveted longitudinal plate girders over 20 ft. long, net section | 8,000 |
| On bottom flange of riveted longitudinal plate girders under 20 ft. long, net section | 7,000 |
| On floor-beam hangers, and other similar members liable to sudden loading (bar iron with forged ends) | 6,000 |
| On floor-beam hangers, and other similar members liable to sudden loading (plates or shapes), net section | 5,000 |

Members subject to alternate strains of tension and compression shall be proportioned to resist each kind of strain. Both of the strains shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two strains, for determining the sectional area by the above allowed strains.

The Phoenix Bridge Co. (Standard Specifications, 1895) gives the following:

The greatest working stresses in pounds per square inch shall be as follows:

| Steel. | | <i>Tension.</i> | | Iron. | |
|---------------|--|---------------------------|---------------|--|--|
| $P=9,000$ | $\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$ | For bars, forged ends. | $P=7,500$ | $\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$ | |
| $P=8,500$ | $\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$ | Plates or shapes net. | $P=7,000$ | $\left[1 + \frac{\text{Min. stress}}{\text{Max. stress}}\right]$ | |
| 8,500 pounds. | Floor-beam hangers, forged ends | | 7,000 pounds. | | |
| 7,500 " | Floor-beam hangers, plates or shapes, net section | | 6,000 " | | |
| 10,000 " | Lower flanges of rolled beams | | 8,000 " | | |
| 20,000 " | Outside fibres of pins | | 15,000 " | | |
| 30,000 " | Pins for wind-bracing | | 22,500 " | | |
| 20,000 " | Lateral bracing | | 15,000 " | | |

Shearing.

| | | |
|---------------|---|---------------|
| 9,000 pounds. | Pins and rivets | 7,500 pounds. |
| | Hand-driven rivets 20% less unit stresses. For bracing increase unit stresses 50%. | |
| 6,000 pounds. | Webs of plate girders | 5,000 pounds. |

Bearing.

| | | |
|----------------|---|--|
| 16,000 pounds. | Projection semi-intrados pins and rivets, 12,000 pounds. Hand-driven rivets 20% less unit stresses. For bracing increase unit stresses 50%. | |
|----------------|---|--|

Compression.

Lengths less than forty times the least radius of gyration, P previously found. See Tension.

Lengths more than forty times the least radius of gyration, P reduced by following formulæ:

$$\text{For both ends fixed, } b = \frac{P}{1 + \frac{l^2}{36,000 r^2}}.$$

$$\text{For one end hinged, } b = \frac{P}{1 + \frac{l^2}{24,000 r^2}}.$$

$$\text{For both ends hinged, } b = \frac{P}{1 + \frac{l^2}{18,000 r^2}}.$$

P = permissible stress previously found (see Tension); b = allowable working stress per square inch; l = length of member in inches; r = least radius of gyration of section in inches. No compression member, however, shall have a length exceeding 45 times its least width.

| | Pounds per sq. in. |
|--|-----------------------|
| In counter web members..... | 10,500 |
| In long verticals..... | 10,000 |
| In all main-web and lower-chord eye-bars..... | 13,200 |
| In plate hangers (net section)..... | 9,000 |
| In tension members of lateral and transverse bracing..... | 19,000 |
| In steel-angle lateral ties (net section)..... | 15,000 |
| For spans over 200 feet in length the greatest allowed working stresses per square inch, in lower-chord and end main-web eye-bars, shall be taken at | |

$$10,000 \left(1 + \frac{\text{min. total stress}}{\text{max. total stress}} \right)$$

whenever this quantity exceeds 13,200.

The greatest allowable stress in the main-web eye-bars nearest the centre of such spans shall be taken at 13,200 pounds per square inch; and those for the intermediate eye-bars shall be found by direct interpolation between the preceding values.

The greatest allowable working stresses in steel plate and lattice girders and rolled beams shall be taken as follows:

| | Pounds per sq. in. |
|--|-----------------------|
| Upper flange of plate girders (gross section)..... | 10,000 |
| Lower flange of plate girders (net section)..... | 10,000 |
| In counters and long verticals of lattice girders (net section)..... | 9,000 |
| In lower chords and main diagonals of lattice girders (net section)..... | 10,000 |
| In bottom flanges of rolled beams..... | 10,000 |
| In top flanges of rolled beams..... | 10,000 |

THE STRENGTH OF CAST-IRON COLUMNS.

Hodgkinson's experiments (first published in Phil. Trans. Royal Socy., 1840, and condensed in Tredgold on Cast Iron, 4th ed., 1846), and Gordon's formula, based upon them, are still used (1898) in designing cast-iron columns. They are entirely inadequate as a basis of a practical formula suitable to the present methods of casting columns.

Hodgkinson's experiments were made on nine "long" pillars, about 7 1/2 ft. long, whose external diameters ranged from 1.74 to 2.23 in., and average thickness from 0.29 to 0.35 in., the thickness of each column also varying, and on 13 "short" pillars, 0.733 ft. to 2.251 ft. long, with exter-

nal diameters from 1.08 to 1.26 in., all of them less than $\frac{1}{4}$ in. thick. The iron used was Low Moor, Yorkshire, No. 3, said to be a good iron, not very hard, earlier experiments on which had given a tensile strength of 14,535 and a crushing strength of 109,801 lbs. per sq. in. Modern cast-iron columns, such as are used in the construction of buildings, are very different in size, proportions, and quality of iron from the slender "long" pillars used in Hodgkinson's experiments. There is usually no check, by actual tests or by disinterested inspection, upon the quality of the material. The tensile, compressive, and transverse strength of cast iron varies through a great range (the tensile strength ranging from less than 10,000 to over 40,000 lbs. per sq. in.), with variations in the chemical composition of the iron, according to laws which are as yet very imperfectly understood, and with variations in the method of melting and of casting. There is also a wide variation in the strength of iron of the same melt when cast into bars of different thicknesses.

Another difficulty in obtaining a practical formula for the strength of cast-iron columns is due to the uncertainty of the quality of the casting, and the danger of hidden defects, such as internal stresses due to unequal cooling, cinder or dirt, blow-holes, "cold-shuts," and cracks on the inner surface, which cannot be discovered by external inspection. Variation in thickness, due to rising of the core during casting, is also a common defect.

In addition to these objections to the use of Gordon's formula, for cast-iron columns, we have the data of experiments on full-sized columns, made by the Building Department of New York City (*Eng'g News*, Jan. 13 and 20, 1898). Ten columns in all were tested, six 15-inch, 190 $\frac{1}{4}$ inches long, two 8-inch, 160 inches long, and two 6-inch, 120 inches long. The tests were made on the large hydraulic machine of the Phoenix Bridge Co., of 2,000,000 pounds capacity, which was calibrated for frictional error by the repeated testing within the elastic limit of a large Phoenix column, and the comparison of these tests with others made on the government machine at the Watertown Arsenal. The average frictional error was calculated to be 15.4 per cent, but *Engineering News*, revising the data, makes it 17.1 per cent, with a variation of 3 per cent either way from the average with different loads. The results of the tests of the columns are given below.

TESTS OF CAST-IRON COLUMNS.

| Num- ber. | Diam. Inches. | Thickness. | | | Breaking Load. | |
|--------------|------------------|------------|--------|----------|----------------|-----------------------|
| | | Max. | Min. | Average. | Pounds. | Pounds per Sq. In. |
| 1 | 15 | 1 | 1 | 1 | 1,356,000 | 30,830 |
| 2 | 15 | 15/16 | 1 | 1 1/8 | 1,330,000 | 27,700 |
| 3 | 15 | 1 1/4 | 1 | 1 1/8 | 1,198,000 | 24,900 |
| 4 | 15 1/8 | 1 7/32 | 1 | 1 1/8 | 1,246,000 | 25,200 |
| 5 | 15 | 1 11/16 | 1 | 1 11/64 | 1,632,000 | 32,100 |
| 6 | 15 | 1 1/4 | 1 1/8 | 1 3/16 | 2,082,000 + | 40,400 + |
| 7 | 7 3/4 to 8 1/4 | 1 1/4 | 5/8 | 1 | 651,000 | 31,900 |
| 8 | 8 | 1 3/32 | 1 | 1 3/64 | 612,800 | 26,800 |
| 9 | 6 1/16 | 1 5/32 | 1 1/8 | 1 9/64 | 400,000 | 22,700 |
| 10 | 6 3/32 | 1 1/8 | 1 1/16 | 1 7/64 | 455,200 | 26,300 |

Column No. 6 was not broken at the highest load of the testing machine.

Columns Nos. 3 and 4 were taken from the Ireland Building, which collapsed on August 8, 1895; the other four 15-inch columns were made from drawings prepared by the Building Department, as nearly as possible duplicates of Nos. 3 and 4. Nos. 1 and 2 were made by a foundry in New York with no knowledge of their ultimate use. Nos. 5 and 6 were made

by a foundry in Brooklyn with the knowledge that they were to be tested. Nos. 7 to 10 were made from drawings furnished by the Department.

Applying Gordon's formula, as used by the Building Department, $S = \frac{80000 a}{1 + \frac{1}{400} \frac{l^2}{d^2}}$, to these columns gives for the breaking strength per square

inch of the 15-inch columns 57,143 pounds, for the 8-inch columns 40,000 pounds, and for the 6-inch columns 40,000. The strength of columns Nos. 3 and 4 as calculated is 128 per cent more than their actual strength; their actual strength is less than 44 per cent of their calculated strength; and the factor of safety, supposed to be 5 in the Building Law, is only 2.2 for central loading, no account being taken of the likelihood of eccentric loading.

Prof. Lanza, *Applied Mechanics*, p. 372, quotes the records of 14 tests of cast-iron mill columns, made on the Watertown testing-machine in 1887-88, the breaking strength per square inch ranging from 25,100 to 63,310 pounds, and showing no relation between the breaking strength per square inch and the dimensions of the columns. Only 3 of the 14 columns had a strength exceeding 33,500 pounds per square inch. The average strength of the other 11 was 29,600 pounds per square inch. Prof. Lanza says that it is evident that in the case of such columns we cannot rely upon a crushing strength of greater than 25,000 or 30,000 pounds per square inch of area of section.

He recommends a factor of safety of 5 or 6 with these figures for crushing strength, or 5000 pounds per square inch of area of section as the highest allowable safe load, and in addition makes the conditions that the length of the column shall not be greatly in excess of 20 times the diameter, that the thickness of the metal shall be such as to insure a good strong casting, and that the sectional area should be increased if necessary to insure that the extreme fibre stress due to probable eccentric loading shall not be greater than 5000 pounds per square inch.

Prof. W. H. Burr (*Eng'g News*, June 30, 1898) gives a formula derived from plotting the results of the Watertown and Phoenixville tests, above described, which represents the average strength of the columns in pounds per square inch. It is $p = 30,500 - 160 l/d$. It is to be noted that this is an average value, and that the actual strength of many of the columns was much lower. Prof. Burr says: "If cast-iron columns are designed with anything like a reasonable and real margin of safety, the amount of metal required dissipates any supposed economy over columns of mild steel."

Square Columns. — Square cast-iron columns should be abandoned. They are liable to have serious internal strains from difference in contraction on two adjacent sides. John F. Ward, *Eng. News*, Apr. 16, 1896.

Safe Load, in Tons of 2000 Lbs., for Round Cast-Iron Columns, with Turned Capitals and Bases.

Loads being not eccentric, and length of column not exceeding 20 times the diameter. Based on ultimate crushing strength of 25,000 lbs. per sq. in. and a factor of safety of 5.

| Thick- ness, Inches. | Diameter, Inches. | | | | | | | | | | | |
|----------------------------|-------------------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 18 |
| 5/8 | 26.4 | 31.3 | | | | | | | | | | |
| 3/4 | 30.9 | 36.8 | 42.7 | 48.6 | 54.5 | | | | | | | |
| 7/8 | 35.2 | 42.1 | 48.9 | 55.8 | 62.7 | 69.6 | 76.5 | | | | | |
| 1 | 39.2 | 47.1 | 55.0 | 62.8 | 70.7 | 78.5 | 86.4 | 94.2 | 102.1 | 110.0 | | |
| 1 1/8 | | | 60.8 | 69.6 | 78.4 | 87.2 | 96.1 | 104.9 | 113.8 | 122.6 | 131.4 | |
| 1 1/4 | | | | 76.1 | 85.9 | 95.7 | 105.5 | 115.3 | 125.2 | 135.0 | 144.8 | 164.4 |
| 1 3/8 | | | | | 93.1 | 103.9 | 114.7 | 125.5 | 136.3 | 147.1 | 157.9 | 179.5 |
| 1 1/2 | | | | | | | 123.7 | 135.5 | 147.3 | 159.0 | 170.8 | 194.4 |
| 1 3/4 | | | | | | | | | 168.4 | 182.1 | 195.8 | 223.3 |
| 2 | | | | | | | | | | 204.2 | 219.9 | 251.3 |

For lengths greater than 20 diameters the allowable loads should be decreased. How much they should be decreased is uncertain, since sufficient data of experiments on full-sized very long columns, from which a formula for the strength of such columns might be derived, are as yet lacking. There is, however, rarely, if ever, any need of proportioning cast-iron columns with a length exceeding 20 diameters.

Safe Loads in Tons of 2000 Pounds for Cast-iron Columns.

(By the Building Laws of New York City, Boston, and Chicago, 1897.)

| | New York. | Boston. | Chicago. |
|------------------------|--|---|--|
| Square columns | $\left\{ \begin{array}{l} \frac{8a}{1 + \frac{l^2}{500d^2}} \end{array} \right.$ | $\left\{ \begin{array}{l} \frac{5a}{1 + \frac{l^2}{1067d^2}} \end{array} \right.$ | $\left\{ \begin{array}{l} \frac{5a}{1 + \frac{l^2}{800d^2}} \end{array} \right.$ |
| Round columns | $\left\{ \begin{array}{l} \frac{8a}{1 + \frac{l^2}{400d^2}} \end{array} \right.$ | $\left\{ \begin{array}{l} \frac{5a}{1 + \frac{l^2}{800d^2}} \end{array} \right.$ | $\left\{ \begin{array}{l} \frac{5a}{1 + \frac{l^2}{600d^2}} \end{array} \right.$ |

a = sectional area in square inches; l = unsupported length of column in inches; d = side of square column or thickness of round column in inches.

The safe load of a 15-inch round column 11½ inches diameter, 16 feet long, according to the laws of these cities would be, in New York, 361 tons; in Boston, 264 tons; in Chicago, 250 tons.

The allowable stress per square inch of area of such a column would be, in New York, 11,350 pounds; in Boston, 8300 pounds; in Chicago, 7850 pounds. A safe stress of 5000 pounds per square inch would give for the safe load on the column 159 tons.

Strength of Brackets on Cast-iron Columns. — The columns tested by the New York Building Department referred to above had brackets cast upon them, each bracket consisting of a rectangular shelf supported by one or two triangular ribs. These were tested after the columns had been broken in the principal tests. In 17 out of 22 cases the brackets broke by tearing a hole in the body of the column, instead of by shearing or transverse breaking of the bracket itself. The results were surprisingly low and very irregular. Reducing them to strength per square inch of the total vertical section through the shelf and rib or ribs, they ranged from 2450 to 5600 lbs., averaging 4200 lbs., for a load concentrated at the end of the shelf, and 4100 to 10,900 lbs., averaging 8000 lbs., for a distributed load. (*Eng'g News*, Jan. 20, 1898.)

Maximum Permissible Stresses in columns used in buildings. (Building Ordinances of City of Chicago, 1893.)

For riveted or other forms of wrought-iron columns:

$$S = \frac{12000a}{1 + \frac{l^2}{36000r^2}} \quad \begin{array}{l} l = \text{length of column in inches;} \\ r = \text{least radius of gyration in inches;} \\ a = \text{area of column in square inches.} \end{array}$$

For riveted or other steel columns, if more than $60r$ in length:

$$S = 17,000 - \frac{60l}{r}$$

If less than $60r$ in length: $S = 13,500a$.

For wooden posts:

$$S = \frac{ac}{1 + \frac{l^2}{250d^2}} \quad \begin{array}{l} a = \text{area of post in square inches;} \\ d = \text{least side of rectangular post in inches;} \\ l = \text{length of post in inches;} \\ c = \begin{cases} 600 \text{ for white or Norway pine;} \\ 800 \text{ for oak;} \\ 900 \text{ for long-leaf yellow pine.} \end{cases} \end{array}$$

ECCENTRIC LOADING OF COLUMNS.

In a given rectangular cross-section, such as a masonry joint under pressure, the stress will be distributed uniformly over the section only when the resultant passes through the centre of the section; any deviation from such a central position will bring a maximum unit pressure to one edge and a minimum to the other; when the distance of the resultant from one edge is one third of the entire width of the joint, the pressure at the nearer edge is twice the mean pressure, while that at the farther edge is zero, and that when the resultant approaches still nearer to the edge the pressure at the farther edge becomes less than zero; in fact, becomes a tension, if the material (mortar, etc.) there is capable of resisting tension. Or, if, as usual in masonry joints, the material is practically incapable of resisting tension, the pressure at the nearer edge, when the resultant approaches it nearer than one third of the width, increases very rapidly and dangerously, becoming theoretically infinite when the resultant reaches the edge.

With a given position of the resultant relatively to one edge of the joint or section, a similar redistribution of the pressures throughout the section may be brought about by simply adding to or diminishing the width of the section.

Let P = the total pressure on any section of a bar of uniform thickness.

w = the width of that section = area of the section, when thickness = 1.

$p = P/w$ = the mean unit pressure on the section.

M = the maximum unit pressure on the section.

m = the minimum unit pressure on the section.

d = the eccentricity of the resultant = its distance from the centre of the section.

Then $M = p \left(1 + \frac{6d}{w} \right)$ and $m = p \left(1 - \frac{6d}{w} \right)$.

When $d = \frac{1}{6} w$ then $M = 2p$ and $m = 0$.

When d is greater than $1/6 w$, the resultant in that case being less than one third of the width from one edge, p becomes negative. (J. C. Trautwine, Jr., *Engineering News*, Nov. 23, 1893.)

Eccentric Loading of Cast-iron Columns. — Prof. Lanza writes the author as follows: The table on page 276 applies when the resultant of the loads upon the column acts along its central axis, *i.e.*, passes through the centre of gravity of every section. In buildings and other constructions, however, cases frequently occur when the resultant load does not pass through the centre of gravity of the section; and then the pressure is not evenly distributed over the section, but is greatest on the side where the resultant acts. (Examples occur when the loads on the floors are not uniformly distributed.) In these cases the outside fibre stresses of the column should be computed as follows, *viz.*:

Let P = total pressure on the section;

d = eccentricity of resultant = its distance from the centre of gravity of the section;

A = area of the section, and I its moment of inertia about an axis in its plane, passing through its centre of gravity, and perpendicular to d ;

c_1 = distance of most compressed and c_2 = that of least compressed fibre from above stated axis;

s_1 = maximum and s_2 = minimum pressure per unit of area. Then

$$s_1 = \frac{P}{A} + \frac{(Pd)c_1}{I} \quad \text{and} \quad s_2 = \frac{P}{A} - \frac{(Pd)c_2}{I}.$$

Having assumed a certain *trial* section for the column to be designed, s_1 should be computed, and, if it exceed the proper safe value, a different section should be used for which s_1 does not exceed this value.

The proper safe value, in the case of cast-iron columns whose ratio of length to diameter does not greatly exceed 20, is 5000 pounds per square inch when the eccentricity used in the computation of s_1 is liable to occur frequently in the ordinary uses of the structure; but when it is one which can only occur in rare cases the value 8000 lbs. per sq. in. may be used.

A long cap on a column is more conducive to the production of eccentricity of loading than a short one, hence a long cap is a source of weakness.

MOMENT OF INERTIA AND RADIUS OF GYRATION.

The moment of inertia of a section is the sum of the products of each elementary area of the section into the square of its distance from an assumed axis of rotation, as the neutral axis.

Assume the section to be divided into a great many equal small areas, a , and that each such area has its own radius, r , or distance from the assumed axis of rotation, then the sum of all the products derived by multiplying each a by the square of its r is the moment of inertia, I , or $I = \Sigma ar^2$, in which Σ is the sign of summation.

For moment of inertia of the weight or mass of a body see Mechanics.

The radius of gyration of the section equals the square root of the quotient of the moment of inertia divided by the area of the section. If R = radius of gyration, I = moment of inertia and A = area

$$R = \sqrt{I/A}. \quad I/A = R^2.$$

The center of gyration is the point where the entire area might be concentrated and have the same moment of inertia as the actual area. The distance of this center from the axis of rotation is the radius of gyration.

The moments of inertia of various sections are as follows.

d = diameter, or outside diameter; d_1 = inside diameter; b = breadth; h = depth; b_1 , h_1 , inside breadth and depth;

Solid rectangle $I = 1/12 bh^3$; Hollow rectangle $I = 1/12 (bh^3 - b_1 h_1^3)$;

Solid square $I = 1/12 b^4$; Hollow square $I = 1/12 (b^4 - b_1^4)$;

Solid cylinder $I = 1/64 \pi d^4$; Hollow cylinder $I = 1/64 \pi (d^4 - d_1^4)$.

Moment of Inertia about any Axis. — If b = breadth and h = depth of a rectangular section its moment of inertia about its central axis (parallel to the breadth) is $1/12 bh^3$; and about one side is $1/3 bh^3$. If a parallel axis exterior to the section is taken, and d = distance of this axis from the farthest side and d_1 = its distance from the nearest side, ($d - d_1 = h$), the moment of inertia about this axis is $1/3 b (d^3 - d_1^3)$.

The moment of inertia of a compound shape about any axis is equal to the sum of the moments of inertia, with reference to the same axis, of all the rectangular portions composing it.

Moment of Inertia of Compound Shapes. (Pencoyd Iron Works.) — The moment of inertia of any section about any axis is equal to the I about a parallel axis passing through its centre of gravity + (the area of the section \times the square of the distance between the axes).

By this rule, the moments of inertia or radii of gyration of any single sections being known, corresponding values may be obtained for any combination of these sections.

E. A. Dixon (*Am. Mach.*, Dec. 15, 1898) gives the following formula for the moment of inertia of any rectangular element of a built up beam: $I = 1/3 (h^3 - h_1^3)b$, I = moment of inertia about any axis parallel to the neutral axis, h = distance from the assumed axis to the farthest fiber, h_1 = distance to nearest fiber, b = breadth of element. The sum of the moments of inertia of all the elements, taken about the center of gravity or neutral axis of the section, is the moment of inertia of the section.

The polar moment of inertia of a surface is the sum of the products obtained by multiplying each elementary area by the square of its distance from the center of gravity of the surface; it is equal to the sum of the moments of inertia taken with respect to two axes at right angles to each other passing through the center of gravity. It is represented by J . For a solid shaft $J = 1/32 \pi d^4$; for a hollow shaft, $J = 1/32 \pi (d^4 - d_1^4)$, in which d is the outside and d_1 the inside diameter.

The polar radius of gyration, $R_p = \sqrt{J/A}$, is defined as the radius of a circumference along which the entire area might be concentrated and have the same polar moment of inertia as the actual area.

For a solid circular section $R_p^2 = 1/8 D^2$; for a hollow circular section $R_p^2 = 1/8 (d^2 + d_1^2)$.

Moments of Inertia and Radius of Gyration for Various Sections, and their Use in the Formulas for Strength of Girders and Columns. — The strength of sections to resist strains, either as girders or as columns, depends not only on the area but also on the form of the section, and the property of the section which forms the

basis of the constants used in the formulas for strength of girders and columns to express the effect of the form, is its moment of inertia about its neutral axis. The modulus of resistance of any section to transverse bending is its moment of inertia divided by the distance from the neutral axis to the fibres farthest removed from that axis; or

$$\text{Section modulus} = \frac{\text{Moment of inertia}}{\text{Distance of extreme fibre from axis}} \cdot \quad Z = \frac{I}{c}$$

Moment of resistance = section modulus \times unit stress on extreme fibre.

Radius of Gyration of Compound Shapes. — In the case of a pair of any shape without a web the value of R can always be found without considering the moment of inertia.

The radius of gyration for any section around an axis parallel to another axis passing through its centre of gravity is found as follows:

Let r = radius of gyration around axis through centre of gravity; R = radius of gyration around another axis parallel to above; d = distance between axes: $R = \sqrt{d^2 + r^2}$.

When r is small, R may be taken as equal to d without material error.

Graphical Method for Finding Radius of Gyration. — Benj. F. La Rue, *Eng. News*, Feb. 2, 1893, gives a short graphical method for finding the radius of gyration of hollow, cylindrical, and rectangular columns, as follows:

For cylindrical columns:

Lay off to a scale of 4 (or 40) a right-angled triangle, in which the base equals the outer diameter, and the altitude equals the inner diameter of the column, or *vice versa*. The hypotenuse, measured to a scale of unity (or 10), will be the radius of gyration sought.

This depends upon the formula

$$G = \sqrt{\text{Mom. of inertia} \div \text{Area}} = 1/4 \sqrt{D^2 + d^2}$$

in which A = area and D = diameter of outer circle, a = area and d = diameter of inner circle, and G = radius of gyration. $\sqrt{D^2 + d^2}$ is the expression for the hypotenuse of a right-angled triangle, in which D and d are the base and altitude.

The sectional area of a hollow round column is $0.7854(D^2 - d^2)$. By constructing a right-angled triangle in which D equals the hypotenuse and d equals the altitude, the base will equal $\sqrt{D^2 - d^2}$. Calling the value of this expression for the base B , the area will equal $0.7854B^2$.

Value of G for square columns:

Lay off as before, but using a scale of 10, a right-angled triangle of which the base equals D or the side of the outer square, and the altitude equals d , the side of the inner square. With a scale of 3 measure the hypotenuse, which will be, approximately, the radius of gyration.

This process for square columns gives an excess of slightly more than 4%. By deducting 4% from the result, a close approximation will be obtained.

A very close result is also obtained by measuring the hypotenuse with the same scale by which the base and altitude were laid off, and multiplying by the decimal 0.29; more exactly, the decimal is 0.28867.













The formula is

$$G = \sqrt{\frac{\text{Mom. of inertia}}{\text{Area}}} = \frac{1}{\sqrt{12}} \sqrt{D^2 + d^2} = 0.28867 \sqrt{D^2 + d^2}$$

This may also be applied to any rectangular column by using the lesser diameters of an unsupported column, and the greater diameters if the column is supported in the direction of its least dimensions.

ELEMENTS OF USUAL SECTIONS.

Moments refer to horizontal axis through centre of gravity. This table is intended for convenient application where extreme accuracy is not important. Some of the terms are only approximate; those marked * are correct. Values for radius of gyration in flanged beams apply to standard minimum sections only. A = area of section; b = breadth; h = depth; D = diameter.

| Shape of Section. | Moment of Inertia. | Section Modulus. | Square of Least Radius of Gyration. | Least Radius of Gyration. |
|--|--|--|---|--|
|  | $\frac{bh^3}{12}$ | $\frac{bh^2}{6}$ | (Least side) ² * 12 | Least side* 3.46 |
|  | $\frac{bh^3 - b_1h_1^3}{12}$ | $\frac{bh^3 - b_1h_1^3}{6h}$ | $\frac{h^2 + h_1^2}{12}$ | $\frac{h + h_1}{4.89}$ |
|  | $\frac{1}{64} \pi D^4$ $= 0.0491 D^4$ | $\frac{1}{32} \pi D^3$ $= 0.0982 D^3$ | $\frac{D^2}{16}$ | $\frac{D}{4}$ |
|  A, area of large section; a, area of small section. | $\frac{AD^2 - ad^2}{16}$ | $\frac{AD^2 - ad^2}{8D}$ | $\frac{D^2 + d^2}{16}$ | $\frac{D + d}{5.64}$ |
|  | $\frac{bh^3}{36}$ | $\frac{bh^2}{24}$ | The least of the two; $\frac{h^2}{18}$ or $\frac{b^2}{24}$ | The least of the two; $\frac{h}{4.24}$ or $\frac{b}{4.9}$ |
|  | $\frac{Ah^2}{10.2}$ | $\frac{Ah}{7.2}$ | $\frac{b^2}{25}$ | $\frac{b}{5}$ |
|  | $\frac{Ah^2}{9.5}$ | $\frac{Ah}{6.5}$ | $\frac{(hb)^2}{13(h^2 + b^2)}$ | $\frac{hb}{2.6(h + b)}$ |
|  | $\frac{Ah^2}{19}$ | $\frac{Ah}{9.5}$ | $\frac{h^2}{22.5}$ | $\frac{h}{4.74}$ |
|  | $\frac{Ah^2}{11.1}$ | $\frac{Ah}{8}$ | $\frac{b^2}{22.5}$ | $\frac{b}{4.74}$ |
|  | $\frac{Ah^2}{6.66}$ | $\frac{Ah}{3.2}$ | $\frac{b^2}{21}$ | $\frac{b}{4.58}$ |
|  | $\frac{Ah^2}{7.34}$ | $\frac{Ah}{3.67}$ | $\frac{b^2}{12.5}$ | $\frac{b}{3.54}$ |
|  | $\frac{Ah^2}{6.9}$ | $\frac{Ah}{4}$ | $\frac{b^2}{36.5}$ | $\frac{b}{6}$ |

Distance of base from centre of gravity, solid triangle, $\frac{h}{3}$; even angle, $\frac{h}{3.3}$; uneven angle, $\frac{h}{3.5}$; even tee, $\frac{h}{3.3}$; deck beam, $\frac{h}{2.3}$; all other shapes given in the table, $\frac{h}{2}$ or $\frac{D}{2}$.

TRANSVERSE STRENGTH.

In transverse tests the strength of bars of rectangular section is found to vary directly as the breadth of the specimen tested, as the square of its depth, and inversely as its length. The deflection under any load varies as the cube of the length, and inversely as the breadth and as the cube of the depth. Represented algebraically, if S = the strength and D the deflection, l the length, b the breadth, and d the depth,

$$S \text{ varies as } \frac{bd^2}{l} \text{ and } D \text{ varies as } \frac{l^3}{bd^3}.$$

For the purpose of reducing the strength of pieces of various sizes to a common standard, the term *modulus of rupture* (represented by R) is used. Its value is obtained by experiment on a bar of rectangular section supported at the ends and loaded in the middle and substituting numerical values in the following formula:

$$R = \frac{3Pl}{2bd^2}.$$

in which P = the breaking load in pounds, l = the length in inches, b the breadth, and d the depth.

The *modulus of rupture* is sometimes defined as the strain at the instant of rupture upon a unit of the section which is most remote from the neutral axis on the side which first ruptures. This definition, however, is based upon a theory which is yet in dispute among authorities, and it is better to define it as a numerical value, or experimental constant, found by the application of the formula above given.

From the above formula, making l 12 inches, and b and d each 1 inch, it follows that the modulus of rupture is 18 times the load required to break a bar one inch square, supported at two points one foot apart, the load being applied in the middle.

$$\begin{aligned} \text{Coefficient of transverse strength} &= \frac{\text{span in feet} \times \text{load at middle in lbs.}}{\text{breadth in inches} \times (\text{depth in inches})^2} \\ &= \frac{1}{18} \text{th of the modulus of rupture.} \end{aligned}$$

Fundamental Formulæ for Flexure of Beams (Merriman).

Resisting shear = vertical shear;

Resisting moment = bending moment;

Sum of tensile stresses = sum of compressive stresses;

Resisting shear = algebraic sum of all the vertical components of the internal stresses at any section of the beam.

If A be the area of the section and S_s the shearing unit stress, then resisting shear = AS_s ; and if the vertical shear = V , then $V = AS_s$.

The *vertical shear* is the algebraic sum of all the external vertical forces on one side of the section considered. It is equal to the reaction of one support, considered as a force acting upward, minus the sum of all the vertical downward forces acting between the support and the section.

The *resisting moment* = algebraic sum of all the moments of the internal horizontal stresses at any section with reference to a point in that section, = $\frac{SI}{c}$, in which S = the horizontal unit stress, tensile or compressive as the case may be, upon the fibre most remote from the neutral axis, c = the shortest distance from that fibre to said axis, and I = the moment of inertia of the cross-section with reference to that axis.

The *bending moment* M is the algebraic sum of the moment of the external forces on one side of the section with reference to a point in that section = moment of the reaction of one support minus sum of moments of loads between the support and the section considered.

$$M = \frac{SI}{c}.$$

The bending moment is a compound quantity = product of a force by the distance of its point of application from the section considered, the distance being measured on a line drawn from the section perpendicular to the direction of the action of the force.

GENERAL FORMULÆ FOR TRANSVERSE STRENGTH OF BEAMS OF UNIFORM CROSS-SECTION.

| Beam. (For notation see page 285.) | Rectangular Beam. | | Beam of any Section. | | |
|---|--|--|--|--------------------|--|
| | Breaking Load. | Deflection for Load P or W . | Maximum Moment of Stress. | Moment of Rupture. | Deflection. Δ |
| Fixed at one end, load at the other | $P = \frac{1}{6} \frac{Rbd^2}{l}$ | $\frac{4Pl^3}{Ebd^3}$ | Pl | $\frac{RI}{c}$ | $\frac{1}{3} \frac{Pl^3}{EI}$ |
| Same with load distributed uniformly | $W = \frac{1}{3} \frac{Rbd^2}{l}$ | $\frac{3}{2} \frac{Wl^3}{Ebd^3}$ | $\frac{1}{2} Wl$ | $\frac{RI}{c}$ | $\frac{1}{8} \frac{Wl^3}{EI}$ |
| Supported at ends, loaded in middle | $P = \frac{2}{3} \frac{Rbd^2}{l}$ | $\frac{Pl^3}{4Ebd^3}$ | $\frac{1}{4} Pl$ | $\frac{RI}{c}$ | $\frac{1}{48} \frac{Pl^3}{EI}$ |
| Same, loaded uniformly | $W = \frac{4}{3} \frac{Rbd^2}{l}$ | $\frac{5}{32} \frac{Wl^3}{Ebd^3}$ | $\frac{1}{8} Wl$ | $\frac{RI}{c}$ | $\frac{5}{384} \frac{Wl^3}{EI}$ |
| Same, loaded at middle, and also } with uniform load, } | $2P + W = \frac{4}{3} \frac{Rbd^2}{l}$ | $\frac{1}{4} \left(P + \frac{1}{8} W \right) \frac{l^3}{Ebd^3}$ | $\left(\frac{1}{4} P + \frac{1}{8} W \right) l$ | $\frac{RI}{c}$ | $\frac{1}{48} \left(P + \frac{5}{8} W \right) \frac{l^3}{EI}$ |
| Fixed at both ends, loaded in middle | $P = \frac{4}{3} \frac{Rbd^2}{l}$ | $\frac{1}{16} \frac{Pl^3}{Ebd^3}$ | $\frac{1}{8} Pl$ | $\frac{RI}{c}$ | $\frac{Pl^3}{192 EI}$ |
| Same, Barlow's Experiments | $P = \frac{Rbd^2}{l}$ | | $\frac{1}{6} Pl$ | $\frac{RI}{c}$ | |
| Same, uniformly loaded | $W = \frac{2Rbd^2}{l}$ | $\frac{1}{32} \frac{Wl^3}{Ebd^3}$ | $\frac{1}{12} Wl$ | $\frac{RI}{c}$ | $\frac{Wl^3}{384 EI}$ |
| Fixed at one end, supported at the other, } loaded at 0.634 <i>l</i> from fixed end, } | | $\frac{0.1148Pl^3}{Ebd^3}$ | $\frac{3}{8} (2\sqrt{3} - 3) Pl$ | $\frac{RI}{c}$ | $\frac{Pl^3}{105 EI}$ (nearly) |
| Same, uniformly loaded | $W = \frac{4}{3} \frac{Rbd^2}{l}$ | $\frac{0.0648Wl^3}{Ebd^3}$ | $\frac{1}{8} Wl$ | $\frac{RI}{c}$ | $\frac{Wl^3}{185 EI}$ (nearly) |

Concerning the formula, $M = SI/c$, p. 282, Prof. Merriman, *Eng. News*, July 21, 1894, says: The formula quoted is true when the unit-stress S on the part of the beam farthest from the neutral axis is within the elastic limit of the material. It is not true when this limit is exceeded, because then the neutral axis does not pass through the center of gravity of the cross-section, and because also the different longitudinal stresses are not proportional to their distances from that axis, these two requirements being involved in the deduction of the formula. But in all cases of design the permissible unit-stresses should not exceed the elastic limit, and hence the formula applies rationally, without regarding the ultimate strength of the material or any of the circumstances regarding rupture. Indeed, so great reliance is placed upon this formula that the practice of testing beams by rupture has been almost entirely abandoned, and the allowable unit-stresses are mainly derived from tensile and compressive tests.

APPROXIMATE GREATEST SAFE LOADS IN LBS. ON STEEL BEAMS. (Pencoyd Iron Works.)

Based on fiber strains of 16,000 lbs. for steel. (For iron the loads should be one-eighth less, corresponding to a fibre strain of 14,000 lbs. per square inch.) Beams supported at the ends and uniformly loaded.

L = length in feet between supports; a = interior area in square inches;
 A = sectional area of beam in square inches;
 d = interior depth in inches.
 D = depth of beam in inches. w = working load in net tons.

| Shape of Section. | Greatest Safe Load in Pounds. | | Deflection in Inches. | |
|---------------------------|-------------------------------|-------------------------|------------------------------|------------------------------|
| | Load in Middle. | Load Distributed. | Load in Middle. | Load Distributed. |
| Solid Rect-angle. | $\frac{890AD}{L}$ | $\frac{1780AD}{L}$ | $\frac{wL^3}{32AD^2}$ | $\frac{wL^3}{52AD^2}$ |
| Hollow Rectangle. | $\frac{890(AD-aa)}{L}$ | $\frac{1780(AD-ad)}{L}$ | $\frac{wL^3}{32(AD^2-aa^2)}$ | $\frac{wL^3}{52(AD^2-ad^2)}$ |
| Solid Cylinder. | $\frac{667AD}{L}$ | $\frac{1333AD}{L}$ | $\frac{wL^3}{24AD^2}$ | $\frac{wL^3}{38AD^2}$ |
| Hollow Cylinder. | $\frac{667(AD-aa)}{L}$ | $\frac{1333(AD-ad)}{L}$ | $\frac{wL^3}{24(AD^2-aa^2)}$ | $\frac{wL^3}{38(AD^2-ad^2)}$ |
| Even-legged Angle or Tee. | $\frac{885AD}{L}$ | $\frac{1770AD}{L}$ | $\frac{wL^3}{32AD^2}$ | $\frac{wL^3}{52AD^2}$ |
| Channel or Z bar | $\frac{1525AD}{L}$ | $\frac{3050AD}{L}$ | $\frac{wL^3}{53AD^2}$ | $\frac{wL^3}{85AD^2}$ |
| Deck Beam. | $\frac{1380AD}{L}$ | $\frac{2760AD}{L}$ | $\frac{wL^3}{50AD^2}$ | $\frac{wL^3}{80AD^2}$ |
| I Beam. | $\frac{1695AD}{L}$ | $\frac{3390AD}{L}$ | $\frac{wL^3}{58AD^2}$ | $\frac{wL^3}{95AD^2}$ |
| I | II | III | IV | V |

The above formulæ for the strength and stiffness of rolled beams of various sections are intended for convenient application in cases where strict accuracy is not required.

The rules for rectangular and circular sections are correct, while those for the flanged sections are approximate, and limited in their application to the standard shapes as given in the Pencoyd tables. When the section of any beam is increased above the standard minimum dimensions, the flanges remaining unaltered, and the web alone being thickened, the tendency will be for the load as found by the rules to be in excess of the actual; but within the limits that it is possible to vary any section in the rolling, the rules will apply without any serious inaccuracy.

The calculated safe loads will be approximately one half of loads that would injure the elasticity of the materials.

The rules for deflection apply to any load below the elastic limit, or less than double the greatest safe load by the rules.

If the beams are long without lateral support, reduce the loads for the ratios of width to span as follows:

| Length of Beam. | | | | Proportion of Calculated Load forming Greatest Safe Load. | |
|------------------------|---|---|---|---|-----|
| 20 times flange width. | | | | Whole calculated load. | |
| 30 | " | " | " | 9-10 | " " |
| 40 | " | " | " | 8-10 | " " |
| 50 | " | " | " | 7-10 | " " |
| 60 | " | " | " | 6-10 | " " |
| 70 | " | " | " | 5-10 | " " |

These rules apply to beams supported at each end. For beams supported otherwise, alter the coefficients of the table as described below, referring to the respective columns indicated by number.

Changes of Coefficients for Special Forms of Beams.

| Kind of Beam. | Coefficient for Safe Load. | Coefficient for Deflection. |
|--|---|--|
| Fixed at one end, loaded at the other. | One fourth of the coefficient, col. II. | One sixteenth of the coefficient of col. IV. |
| Fixed at one end, load evenly distributed. | One fourth of the coefficient of col. III. | Five forty-eighths of the coefficient of col. V. |
| Both ends rigidly fixed, or a continuous beam, with a load in middle. | Twice the coefficient of col. II. | Four times the coefficient of col. IV. |
| Both ends rigidly fixed, or a continuous beam, with load evenly distributed. | One and one-half times the coefficient of col. III. | Five times the coefficient of col. V. |

Formulae for Transverse Strength of Beams. — Referring to table on page 283,

P = load at middle;

W = total load, distributed uniformly;

l = length, b = breadth, d = depth, in inches;

E = modulus of elasticity;

R = modulus of rupture, or stress per square inch of extreme fiber;

I = moment of inertia;

c = distance between neutral axis and extreme fibre.

For breaking load of circular section, replace bd^2 by $0.59d^3$.

The value of R at rupture, or the modulus of rupture (see page 268), is about 60,000 for structural steel, and about 110,000 for strong steel. (Merriman.)

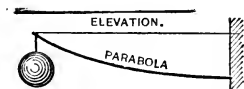
For cast iron the value of R varies greatly according to quality. Thurston found 45,740 and 67,980 in No. 2 and No. 4 cast iron, respectively.

For beams fixed at both ends and loaded in the middle, Barlow, by experiment, found the maximum moment of stress = $1/6 Pl$ instead of $1/8 Pl$, the result given by theory. Prof. Wood (Resist. Matls. p. 155) says of this case: The phenomena are of too complex a character to admit of a thorough and exact analysis, and it is probably safer to accept the results of Mr. Barlow in practice than to depend upon theoretical results.

BEAMS OF UNIFORM STRENGTH THROUGHOUT THEIR LENGTH.

The section is supposed in all cases to be rectangular throughout. The beams shown in plan are of uniform depth throughout. Those shown in elevation are of uniform breadth throughout.

B = breadth of beam. D = depth of beam.



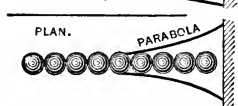
Fixed at one end, loaded at the other; curve parabola, vertex at loaded end; BD proportional to distance from loaded end. The beam may be reversed, so that the upper edge is parabolic, or both edges may be parabolic.



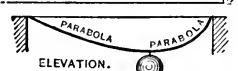
Fixed at one end, loaded at the other; triangle, apex at loaded end; BD^2 proportional to the distance from the loaded end.



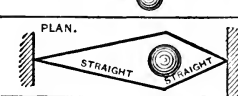
Fixed at one end; load distributed; triangle, apex at unsupported end; BD^2 proportional to square of distance from unsupported end.



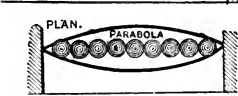
Fixed at one end; load distributed; curves two parabolas, vertices touching each other at unsupported end; BD^2 proportional to distance from unsupported end.



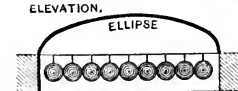
Supported at both ends; load at any one point; two parabolas, vertices at the points of support, bases at point loaded; BD^2 proportional to distance from nearest point of support. The upper edge or both edges may also be parabolic.



Supported at both ends; load at any one point; two triangles, apices at points of support, bases at point loaded; BD^2 proportional to distance from the nearest point of support.



Supported at both ends; load distributed; curves two parabolas, vertices at the middle of the beam; bases centre line of beam; BD^2 proportional to product of distances from points of support.



Supported at both ends; load distributed; curve semi-ellipse; BD^2 proportional to the product of the distances from the points of support.

PROPERTIES OF ROLLED STRUCTURAL STEEL.

Explanation of Tables of the Properties of I-Beams, Channels, Angles, Z-Bars, Tees, Trough and Corrugated Plates.

(The Carnegie Steel Co.)

The tables for I-beams and channels are calculated for all standard weights to which each pattern is rolled. The tables for angles are calculated for the minimum intermediate and maximum weights of the various shapes, while the properties of Z-bars are given for thicknesses differing by $\frac{1}{16}$ inch. For tees, each shape can be rolled to one weight only.

Columns headed *C* in the tables for I-beams and channels give coefficients by the help of which the safe uniformly distributed load may be readily determined. To do this, divide the coefficient given by the span or distance between supports in feet.

If a section is to be selected (as will usually be the case), intended to carry a certain load for a length of span already determined on, ascertain the coefficient which this load and span will require, and refer to the table for a section having a coefficient of this value. The coefficient is obtained by multiplying the load, in pounds uniformly distributed, by the span length in feet.

In case the load is not uniformly distributed, but is concentrated at the middle of the span, multiply the load by 2, and then consider it as uniformly distributed. The deflection will be $\frac{8}{10}$ of the deflection for the latter load.

For other cases of loading obtain the bending moment in foot-pounds; this multiplied by 8 will give the coefficient required.

If the loads are quiescent, the coefficients for a fiber stress of 16,000 pounds per square inch for steel may be used; but if moving loads are to be provided for, a coefficient of 12,500 pounds should be taken. Inasmuch as the effects of impact may be very considerable (the stresses produced in an unyielding inelastic material by a load suddenly applied being double those produced by the same load in a quiescent state), it will sometimes be advisable to use still smaller fiber stresses than those given in the tables. In such cases the coefficients may be determined by proportion. Thus, for a fiber stress of 8000 pounds per square inch the coefficient will equal the coefficient for 16,000 pounds fiber stress, from the table, divided by 2.

The section moduli are used to determine the fiber stress per square inch in a beam, or other shape, subjected to bending or transverse stresses, by simply dividing the bending moment expressed in inch-pounds by the section modulus.

In the case of T-shapes with the neutral axis parallel to the flange, there will be two section moduli, and the smaller is given. The fiber stress calculated from it will, therefore, give the larger of the two stresses in the extreme fibers, since these stresses are equal to the bending moment divided by the section modulus of the section.

For Z-bars the coefficient (*C*) may be applied for cases where the bars are subjected to transverse loading, as in the case of roof-purlins.

For angles, there will be two section moduli for each position of the neutral axis, since the distance between the neutral axis and the extreme fibers has a different value on one side of the axis from what it has on the other. The section modulus given in the table is the smaller of these two values.

Column headed *X*, in the table of the properties of standard channels, giving the distance of the center of gravity of channel from the outside of web, is used to obtain the radius of gyration for columns or struts consisting of two channels latticed, for the case of the neutral axis passing through the center of the cross-section parallel to the webs of the channels. This radius of gyration is equal to the distance between the center of gravity of the channel and the center of the section, i.e., neglecting the moments of inertia of the channels around their own axes, thereby introducing a slight error on the side of safety.

(For much other important information concerning rolled structural shapes, see the "Pocket Companion" of The Carnegie Steel Co., Pittsburg, Pa., price \$2.)

Properties of Carnegie Standard I-Beams—Steel.

| Section Index. | Depth of Beam. | Weight per Foot. | Area of Section. | Thickness of Web. | Width of Flange. | Moment of Inertia, Neutral Axis Perpendicular to Web at Center. | Moment of Inertia, Neutral Axis Coincident with Center Line of Web. | Radius of Gyration, Neutral Axis Perpendicular to Web at Center. | Radius of Gyration, Neutral Axis Coincident with Center Line of Web. | Section Modulus, Neutral Axis Perpendicular to Web at Center. | Coefficient of Strength for Fiber Stress of 16,000 lbs. per sq. in. |
|----------------|----------------|------------------|------------------|-------------------|------------------|---|---|--|--|---|---|
| | in. | lbs. | sq.in. | in. | in. | <i>I</i> | <i>I'</i> | <i>r</i> | <i>r'</i> | <i>S</i> | <i>C*</i> |
| B1 | 24 | 100 | 29.41 | 0.75 | 7.25 | 2380.3 | 48.56 | 9.00 | 1.28 | 198.4 | 2115800 |
| " | " | 95 | 27.94 | 0.69 | 7.19 | 2309.6 | 47.10 | 9.09 | 1.30 | 192.5 | 2052900 |
| " | " | 90 | 26.47 | 0.63 | 7.13 | 2239.1 | 45.70 | 9.20 | 1.31 | 186.6 | 1990300 |
| " | " | 85 | 25.00 | 0.57 | 7.07 | 2168.6 | 44.35 | 9.31 | 1.33 | 180.7 | 1927600 |
| " | " | 80 | 23.32 | 0.50 | 7.00 | 2087.9 | 42.86 | 9.46 | 1.36 | 174.0 | 1855900 |
| B2 | 20 | 100 | 29.41 | 0.88 | 7.28 | 1655.8 | 52.65 | 7.50 | 1.34 | 165.6 | 1766100 |
| " | " | 95 | 27.94 | 0.81 | 7.21 | 1606.8 | 50.78 | 7.58 | 1.35 | 160.7 | 1713900 |
| " | " | 90 | 26.47 | 0.74 | 7.14 | 1557.8 | 48.98 | 7.67 | 1.36 | 155.8 | 1661600 |
| " | " | 85 | 25.00 | 0.66 | 7.06 | 1508.7 | 47.25 | 7.77 | 1.37 | 150.9 | 1609300 |
| " | " | 80 | 23.73 | 0.60 | 7.00 | 1466.5 | 45.81 | 7.86 | 1.39 | 146.7 | 1564300 |
| B3 | 20 | 75 | 22.06 | 0.65 | 6.40 | 1268.9 | 30.25 | 7.58 | 1.17 | 126.9 | 1353500 |
| " | " | 70 | 20.59 | 0.58 | 6.33 | 1219.9 | 29.04 | 7.70 | 1.19 | 122.0 | 1301200 |
| " | " | 65 | 19.03 | 0.50 | 6.25 | 1169.6 | 27.86 | 7.83 | 1.21 | 117.0 | 1247600 |
| B80 | 18 | 70 | 20.59 | 0.72 | 6.26 | 921.3 | 24.62 | 6.69 | 1.09 | 102.4 | 1091900 |
| " | " | 65 | 19.12 | 0.64 | 6.18 | 881.5 | 23.47 | 6.79 | 1.11 | 97.9 | 1044800 |
| " | " | 60 | 17.65 | 0.56 | 6.10 | 841.8 | 22.38 | 6.91 | 1.13 | 93.5 | 997700 |
| " | " | 55 | 15.93 | 0.46 | 6.00 | 795.6 | 21.19 | 7.07 | 1.15 | 88.4 | 943000 |
| B4 | 15 | 100 | 29.41 | 1.18 | 6.77 | 900.5 | 50.98 | 5.53 | 1.31 | 120.1 | 1280700 |
| " | " | 95 | 27.94 | 1.09 | 6.68 | 872.9 | 48.37 | 5.59 | 1.32 | 116.4 | 1241500 |
| " | " | 90 | 26.47 | 0.99 | 6.58 | 845.4 | 45.91 | 5.65 | 1.32 | 112.7 | 1202300 |
| " | " | 85 | 25.00 | 0.89 | 6.48 | 817.8 | 43.57 | 5.72 | 1.32 | 109.0 | 1163000 |
| " | " | 80 | 23.81 | 0.81 | 6.40 | 795.5 | 41.76 | 5.78 | 1.32 | 106.1 | 1131300 |
| B5 | 15 | 75 | 22.06 | 0.88 | 6.29 | 691.2 | 30.68 | 5.60 | 1.18 | 92.2 | 983000 |
| " | " | 70 | 20.59 | 0.78 | 6.19 | 663.6 | 29.00 | 5.68 | 1.19 | 88.5 | 943800 |
| " | " | 65 | 19.12 | 0.69 | 6.10 | 636.0 | 27.42 | 5.77 | 1.20 | 84.8 | 904600 |
| " | " | 60 | 17.67 | 0.59 | 6.00 | 609.0 | 25.96 | 5.87 | 1.21 | 81.2 | 866100 |
| B7 | 15 | 55 | 16.18 | 0.66 | 5.75 | 511.0 | 17.06 | 5.62 | 1.02 | 68.1 | 726800 |
| " | " | 50 | 14.71 | 0.56 | 5.65 | 483.4 | 16.04 | 5.73 | 1.04 | 64.5 | 687500 |
| " | " | 45 | 13.24 | 0.46 | 5.55 | 455.8 | 15.00 | 5.87 | 1.07 | 60.8 | 648200 |
| " | " | 42 | 12.48 | 0.41 | 5.50 | 441.7 | 14.62 | 5.95 | 1.08 | 58.9 | 628300 |
| B8 | 12 | 55 | 16.18 | 0.82 | 5.61 | 321.0 | 17.46 | 4.45 | 1.04 | 53.5 | 570600 |
| " | " | 50 | 14.71 | 0.70 | 5.49 | 303.3 | 16.12 | 4.54 | 1.05 | 50.6 | 539200 |
| " | " | 45 | 13.24 | 0.58 | 5.37 | 285.7 | 14.89 | 4.65 | 1.06 | 47.6 | 507900 |
| " | " | 40 | 11.84 | 0.46 | 5.25 | 268.9 | 13.81 | 4.77 | 1.08 | 44.8 | 478100 |
| B9 | 12 | 35 | 10.29 | 0.44 | 5.09 | 228.3 | 10.07 | 4.71 | 0.99 | 38.0 | 405800 |
| " | " | 31.5 | 9.26 | 0.35 | 5.00 | 215.8 | 9.50 | 4.83 | 1.01 | 36.0 | 383700 |
| B11 | 10 | 40 | 11.76 | 0.75 | 5.10 | 158.7 | 9.50 | 3.67 | 0.90 | 31.7 | 338500 |
| " | " | 35 | 10.29 | 0.60 | 4.95 | 146.4 | 8.52 | 3.77 | 0.91 | 29.3 | 312400 |
| " | " | 30 | 8.82 | 0.46 | 4.81 | 134.2 | 7.65 | 3.90 | 0.93 | 26.8 | 286300 |
| " | " | 25 | 7.37 | 0.31 | 4.66 | 122.1 | 6.89 | 4.07 | 0.97 | 24.4 | 260500 |
| B13 | 9 | 35 | 10.29 | 0.73 | 4.77 | 111.8 | 7.31 | 3.29 | 0.84 | 24.8 | 265000 |
| " | " | 30 | 8.82 | 0.57 | 4.61 | 101.9 | 6.42 | 3.40 | 0.85 | 22.6 | 241500 |
| " | " | 25 | 7.35 | 0.41 | 4.45 | 91.9 | 5.65 | 3.54 | 0.88 | 20.4 | 217900 |
| " | " | 21 | 6.31 | 0.29 | 4.33 | 84.9 | 5.16 | 3.67 | 0.90 | 18.9 | 201300 |
| B15 | 8 | 25.5 | 7.50 | 0.54 | 4.27 | 68.4 | 4.75 | 3.02 | 0.80 | 17.1 | 182500 |
| " | " | 23 | 6.76 | 0.45 | 4.18 | 64.5 | 4.39 | 3.09 | 0.81 | 16.1 | 172000 |
| " | " | 20.5 | 6.03 | 0.36 | 4.09 | 60.6 | 4.07 | 3.17 | 0.82 | 15.1 | 161600 |
| " | " | 18 | 5.33 | 0.27 | 4.00 | 56.9 | 3.78 | 3.27 | 0.84 | 14.2 | 151700 |

* This coefficient used for buildings; for bridges use 12,500 pounds per square inch, or multiply value in this column by 0.78125.

Properties of Carnegie Standard I-Beams — Steel. *Continued.*

| Section Index. | Depth of Beam. | Weight per Foot. | Area of Section. | Thickness of Web. | Width of Flange. | Moment of Inertia, Neutral Axis Perpendicular to Web at Center. | Moment of Inertia, Neutral Axis Coincident with Center Line of Web. | Radius of Gyration, Neutral Axis, Perpendicular to Web at Center. | Radius of Gyration, Neutral Axis Coincident with Center Line of Web. | Section Modulus, Neutral Axis Perpendicular to Web at Center. | Coefficient of Strength for Fiber Stress of 16,000 lbs. per sq. in. |
|----------------|----------------|------------------|------------------|-------------------|------------------|---|---|---|--|---|---|
| | in. | lbs. | sq. in. | in. | in. | I | I' | r | r' | S | C^* |
| B17 | 7 | 20 | 5.88 | 0.46 | 3.87 | 42.2 | 3.24 | 2.68 | 0.74 | 12.1 | 128600 |
| " | " | 17.5 | 5.15 | 0.35 | 3.76 | 39.2 | 2.94 | 2.76 | 0.76 | 11.2 | 119400 |
| " | " | 15 | 4.42 | 0.25 | 3.66 | 36.2 | 2.67 | 2.86 | 0.78 | 10.4 | 110400 |
| B19 | 6 | 17 1/4 | 5.07 | 0.48 | 3.58 | 26.2 | 2.36 | 2.27 | 0.68 | 8.7 | 93100 |
| " | " | 14 3/4 | 4.34 | 0.35 | 3.45 | 24.0 | 2.09 | 2.35 | 0.69 | 8.0 | 85300 |
| " | " | 12 1/4 | 3.61 | 0.23 | 3.33 | 21.8 | 1.85 | 2.46 | 0.72 | 7.3 | 77500 |
| B21 | 5 | 14 3/4 | 4.34 | 0.50 | 3.29 | 15.2 | 1.70 | 1.87 | 0.63 | 6.1 | 64600 |
| " | " | 12 1/4 | 3.60 | 0.36 | 3.15 | 13.6 | 1.45 | 1.94 | 0.63 | 5.4 | 58100 |
| " | " | 9 3/4 | 2.87 | 0.21 | 3.00 | 12.1 | 1.23 | 2.05 | 0.65 | 4.8 | 51600 |
| B23 | 4 | 10.5 | 3.09 | 0.41 | 2.88 | 7.1 | 1.01 | 1.52 | 0.57 | 3.6 | 38100 |
| " | " | 9.5 | 2.79 | 0.34 | 2.81 | 6.7 | 0.93 | 1.55 | 0.58 | 3.4 | 36000 |
| " | " | 8.5 | 2.50 | 0.26 | 2.73 | 6.4 | 0.85 | 1.59 | 0.58 | 3.2 | 33900 |
| " | " | 7.5 | 2.21 | 0.19 | 2.66 | 6.0 | 0.77 | 1.64 | 0.59 | 3.0 | 31800 |
| B77 | 3 | 7.5 | 2.21 | 0.36 | 2.52 | 2.9 | 0.60 | 1.15 | 0.52 | 1.9 | 20700 |
| " | " | 6.5 | 1.91 | 0.26 | 2.42 | 2.7 | 0.53 | 1.19 | 0.52 | 1.8 | 19100 |
| " | " | 5.5 | 1.63 | 0.17 | 2.33 | 2.5 | 0.46 | 1.23 | 0.53 | 1.7 | 17600 |

Lightest weight in each section is standard; others are special.

 L = safe loads in pounds, uniformly distributed; l = span in feet.

 M = moments of forces in foot-pounds; C = coefficient given above.

$$L = \frac{C}{l}; M = \frac{C}{8}; C = Ll = 8M = \frac{8fS}{12}; f = \text{fiber stress.}$$

Properties of Carnegie Trough Plates — Steel.

| Section Index. | Size, in Inches. | Weight per Foot. | Area of Section. | Thickness in Inches. | Moment of Inertia, Neutral Axis Parallel to Length. | Section Modulus, Axis as before. | Radius of Gyration, Axis as before. |
|----------------|------------------|------------------|------------------|----------------------|---|----------------------------------|-------------------------------------|
| | | lb. | sq. in. | | I | S | r |
| M10 | 9 1/2 × 3 3/4 | 16.32 | 4.8 | 1/2 | 3.68 | 1.38 | 0.91 |
| M11 | 9 1/2 × 3 3/4 | 18.02 | 5.3 | 9/16 | 4.13 | 1.57 | 0.91 |
| M12 | 9 1/2 × 3 3/4 | 19.72 | 5.8 | 5/8 | 4.57 | 1.77 | 0.90 |
| M13 | 9 1/2 × 3 3/4 | 21.42 | 6.3 | 11/16 | 5.02 | 1.96 | 0.90 |
| M14 | 9 1/2 × 3 3/4 | 23.15 | 6.8 | 3/4 | 5.46 | 2.15 | 0.90 |

Properties of Carnegie Corrugated Plates — Steel.

| Section Index. | Size, in Inches. | Weight per Foot. | Area of Section. | Thickness in Inches. | Moment of Inertia, Neutral Axis Parallel to Length. | Section Modulus, Axis as before. | Radius of Gyration, Axis as before. |
|----------------|-------------------|------------------|------------------|----------------------|---|----------------------------------|-------------------------------------|
| | | lb. | sq. in. | | I | S | r |
| M30 | 8 3/4 × 1 1/2 | 8.01 | 2.4 | 1/4 | 0.64 | 0.80 | 0.52 |
| M31 | 8 3/4 × 1 9/16 | 10.10 | 3.0 | 5/16 | 0.95 | 1.13 | 0.57 |
| M32 | 8 3/4 × 1 5/8 | 12.00 | 3.5 | 3/8 | 1.25 | 1.42 | 0.62 |
| M33 | 12 3/16 × 2 3/4 | 17.75 | 5.2 | 3/8 | 4.79 | 3.33 | 0.96 |
| M34 | 12 3/16 × 2 13/16 | 20.71 | 6.1 | 7/16 | 5.81 | 3.90 | 0.98 |
| M35 | 12 3/16 × 2 7/8 | 23.67 | 7.0 | 1/2 | 6.82 | 4.46 | 0.99 |

Safe Loads, Uniformly Distributed, for Carnegie Standard Steel I-Beams.
In Tons of 2000 Lb.

| Distance between Supports in Feet. | 24" I. | | 20" I. | | 18" I. | | 15" I. | | | Distance between Supports in Feet. | 12" I. | | 10" I. | | 9" I. | | 8" I. | | 7" I. | | 6" I. | | 5" I. | | 4" I. | | 3" I. | | | | |
|--|--------|-------|--------|-------|--------|-------|--------|----|--------|---|--------|------|--------|-------|----------|------|--------|------|--------|--|--------|--|--------|--|-----------|--|----------|--|---------|--|---------|
| | 80 lb. | | 80 lb. | | 65 lb. | | 80 lb. | | 60 lb. | | 42 lb. | | 40 lb. | | 31.5 lb. | | 25 lb. | | 21 lb. | | 18 lb. | | 15 lb. | | 12.25 lb. | | 9.75 lb. | | 7.5 lb. | | 5.5 lb. |
| 12 | 77.33 | 65.18 | 51.98 | 39.29 | 47.14 | 36.09 | 26.18 | 12 | 19.92 | 15.99 | 10.85 | 8.39 | 5 | 15.17 | 11.04 | 7.75 | 5.16 | 3.18 | 1.76 | | | | | | | | | | | | |
| 13 | 71.38 | 60.16 | 47.98 | 36.27 | 43.51 | 33.31 | 24.17 | 13 | 18.39 | 14.76 | 10.02 | 7.74 | 6 | 12.64 | 9.20 | 6.46 | 4.30 | 2.65 | 1.47 | | | | | | | | | | | | |
| 14 | 66.28 | 55.87 | 44.56 | 33.68 | 40.40 | 30.93 | 22.44 | 14 | 17.08 | 13.70 | 9.30 | 7.19 | 7 | 10.84 | 7.89 | 5.54 | 3.69 | 2.27 | 1.26 | | | | | | | | | | | | |
| 15 | 61.86 | 52.14 | 41.59 | 31.43 | 37.71 | 28.87 | 20.94 | 15 | 15.94 | 12.79 | 8.68 | 6.71 | 8 | 9.48 | 6.90 | 4.84 | 3.23 | 1.99 | 1.10 | | | | | | | | | | | | |
| 16 | 58.00 | 48.88 | 38.99 | 29.47 | 35.35 | 27.07 | 19.63 | 16 | 14.94 | 11.99 | 8.14 | 6.29 | 9 | 8.43 | 6.13 | 4.31 | 2.87 | 1.77 | 0.98 | | | | | | | | | | | | |
| 17 | 54.58 | 46.01 | 36.69 | 27.74 | 33.27 | 25.47 | 18.48 | 17 | 14.06 | 11.29 | 7.66 | 5.92 | 10 | 7.59 | 5.52 | 3.88 | 2.58 | 1.59 | 0.88 | | | | | | | | | | | | |
| 18 | 51.56 | 43.45 | 34.66 | 26.19 | 31.42 | 24.06 | 17.45 | 18 | 13.28 | 10.66 | 7.24 | 5.59 | 11 | 6.90 | 5.02 | 3.52 | 2.35 | 1.45 | 0.80 | | | | | | | | | | | | |
| 19 | 48.84 | 41.17 | 32.83 | 24.82 | 29.77 | 22.79 | 16.53 | 19 | 12.58 | 10.10 | 6.86 | 5.30 | 12 | 6.32 | 4.60 | 3.23 | 2.15 | 1.33 | 0.73 | | | | | | | | | | | | |
| 20 | 46.40 | 39.11 | 31.19 | 23.58 | 28.28 | 21.65 | 15.71 | 20 | 11.95 | 9.59 | 6.51 | 5.03 | 13 | 5.83 | 4.25 | 2.98 | 1.98 | 1.22 | 0.68 | | | | | | | | | | | | |
| 21 | 44.19 | 37.24 | 29.70 | 22.45 | 26.94 | 20.62 | 14.96 | 21 | 11.38 | 9.14 | 6.20 | 4.79 | 14 | 5.42 | 3.94 | 2.77 | 1.84 | 1.14 | 0.63 | | | | | | | | | | | | |
| 22 | 42.18 | 35.55 | 28.35 | 21.43 | 25.71 | 19.68 | 14.28 | 22 | 10.87 | 8.72 | 5.92 | 4.58 | 15 | 5.06 | 3.68 | 2.58 | 1.72 | 1.06 | 0.59 | | | | | | | | | | | | |
| 23 | 40.35 | 34.01 | 27.12 | 20.50 | 24.59 | 18.83 | 13.66 | 23 | 10.39 | 8.34 | 5.66 | 4.38 | 16 | 4.74 | 3.45 | 2.42 | 1.61 | 0.99 | 0.55 | | | | | | | | | | | | |
| 24 | 38.67 | 32.59 | 25.99 | 19.65 | 23.57 | 18.04 | 13.09 | 24 | 9.96 | 7.99 | 5.43 | 4.19 | 17 | 4.46 | 3.25 | 2.28 | 1.52 | 0.94 | 0.52 | | | | | | | | | | | | |
| 25 | 37.12 | 31.29 | 24.95 | 18.86 | 22.63 | 17.32 | 12.57 | 25 | 9.56 | 7.67 | 5.21 | 4.03 | 18 | 4.21 | 3.07 | 2.15 | 1.43 | 0.88 | 0.49 | | | | | | | | | | | | |
| 26 | 35.69 | 30.08 | 23.99 | 18.14 | 21.76 | 16.66 | 12.08 | 26 | 9.19 | 7.38 | 5.01 | 3.87 | 19 | 3.99 | 2.91 | 2.04 | 1.36 | 0.84 | 0.46 | | | | | | | | | | | | |
| 27 | 34.37 | 28.97 | 23.10 | 17.46 | 20.95 | 16.04 | 11.64 | 27 | 8.85 | 7.11 | 4.82 | 3.73 | 20 | 3.79 | 2.76 | 1.94 | 1.29 | 0.80 | 0.44 | | | | | | | | | | | | |
| 28 | 33.14 | 27.93 | 22.28 | 16.84 | 20.20 | 15.47 | 11.22 | 28 | 8.54 | 6.85 | 4.65 | 3.59 | 21 | 3.61 | 2.63 | 1.85 | 1.23 | 0.76 | 0.42 | | | | | | | | | | | | |
| 29 | 32.00 | 26.97 | 21.51 | 16.26 | 19.51 | 14.93 | 10.83 | 29 | 8.24 | 6.62 | 4.49 | 3.47 | 22 | | | | | | | | | | | | | | | | | | |
| 30 | 30.93 | 26.07 | 20.79 | 15.72 | 18.86 | 14.43 | 10.47 | 30 | 7.97 | 6.40 | 4.34 | 3.36 | 23 | | | | | | | | | | | | | | | | | | |
| 31 | 29.94 | 25.23 | 20.12 | 15.21 | 18.25 | 13.97 | 10.13 | | | | | | | | | | | | | | | | | | | | | | | | |
| 32 | 29.00 | 24.44 | 19.49 | 14.73 | 17.68 | 13.53 | 9.82 | | | | | | | | | | | | | | | | | | | | | | | | |
| 33 | 28.12 | 23.70 | 18.90 | 14.29 | 17.14 | 13.12 | 9.52 | | | | | | | | | | | | | | | | | | | | | | | | |
| 34 | 27.29 | 23.00 | 18.35 | 13.87 | 16.64 | 12.74 | 9.24 | | | | | | | | | | | | | | | | | | | | | | | | |
| 35 | 26.51 | 22.35 | 17.82 | 13.47 | 16.16 | 12.37 | 8.98 | | | | | | | | | | | | | | | | | | | | | | | | |
| 36 | 25.78 | 21.73 | 17.33 | 13.10 | 15.71 | 12.03 | 8.73 | | | | | | | | | | | | | | | | | | | | | | | | |

Only figures above the cross-lines should be used with plastered ceilings, so that the deflection will not cause cracking of the plaster.

Only figures above the cross-lines should be used with plastered ceilings, so that the deflection will not cause cracking of the plaster.

Safe loads given include weight of beam. Maximum fiber stress, 16,000 pounds per square inch.

Spacing of Carnegie I-Beams for Uniform Load of 100 Lb. per Square Foot. STEEL.

(Proper distance in feet, center to center of beams.)

| Distance between Supports in Feet. | 24" I. | 20" I. | 18" I. | 15" I. | | | 12" I. | | 10" I. | Distance between Supports in Feet. | 9" I. | 8" I. | 7" I. | 6" I. | 5" I. | 4" I. | 3" I. |
|------------------------------------|--------|--------|--------|--------|--------|--------|--------|----------|--------|------------------------------------|--------|--------|--------|-----------|----------|---------|---------|
| | 80 lb. | 65 lb. | 55 lb. | 80 lb. | 60 lb. | 42 lb. | 40 lb. | 31.5 lb. | 25 lb. | | 21 lb. | 18 lb. | 15 lb. | 12.25 lb. | 9.75 lb. | 7.5 lb. | 5.5 lb. |
| 12 | 128.9 | 108.6 | 86.6 | 65.5 | 78.6 | 43.6 | 33.2 | 26.6 | 18.1 | 5 | 80.5 | 60.7 | 44.2 | 31.0 | 20.6 | 12.7 | 7.0 |
| 13 | 109.8 | 92.6 | 73.8 | 55.8 | 67.0 | 37.2 | 28.3 | 22.7 | 15.4 | 6 | 55.9 | 42.1 | 30.7 | 21.5 | 14.3 | 8.8 | 4.9 |
| 14 | 94.7 | 79.8 | 63.7 | 48.1 | 57.7 | 32.1 | 24.4 | 19.6 | 13.3 | 7 | 41.1 | 31.0 | 22.5 | 15.8 | 10.5 | 6.5 | 3.6 |
| 15 | 82.5 | 69.5 | 55.5 | 41.9 | 50.3 | 27.9 | 21.3 | 17.1 | 11.6 | 8 | 31.5 | 23.7 | 17.3 | 12.1 | 8.1 | 5.0 | 2.8 |
| 16 | 72.5 | 61.1 | 48.7 | 36.8 | 44.2 | 24.5 | 18.7 | 15.0 | 10.2 | 9 | 24.9 | 18.7 | 13.6 | 9.6 | 6.4 | 3.9 | 2.2 |
| 17 | 64.2 | 54.1 | 43.2 | 32.6 | 39.2 | 21.7 | 16.5 | 13.3 | 9.0 | 10 | 20.1 | 15.2 | 11.1 | 7.8 | 5.2 | 3.2 | 1.8 |
| 18 | 57.3 | 48.3 | 38.5 | 29.1 | 34.9 | 26.7 | 19.4 | 11.8 | 8.0 | 11 | 16.6 | 12.5 | 9.1 | 6.4 | 4.3 | 2.6 | 1.5 |
| 19 | 51.4 | 43.3 | 34.6 | 26.1 | 31.3 | 24.0 | 17.4 | 10.6 | 7.2 | 12 | 14.0 | 10.5 | 7.7 | 5.4 | 3.6 | 2.2 | 1.2 |
| 20 | 46.4 | 39.1 | 31.2 | 23.6 | 28.3 | 21.7 | 15.7 | 9.6 | 6.5 | 13 | 11.9 | 9.0 | 6.5 | 4.6 | 3.1 | 1.9 | 1.0 |
| 21 | 42.1 | 35.5 | 28.3 | 21.4 | 25.7 | 19.6 | 14.2 | 8.7 | 5.9 | 14 | 10.3 | 7.7 | 5.6 | 4.0 | 2.6 | 1.6 | 0.9 |
| 22 | 38.4 | 32.3 | 25.8 | 19.5 | 23.4 | 17.9 | 13.0 | 7.9 | 5.4 | 15 | 9.0 | 6.7 | 4.9 | 3.4 | 2.3 | 1.4 | ... |
| 23 | 35.1 | 29.6 | 23.6 | 17.8 | 21.4 | 16.4 | 11.9 | 7.3 | 4.9 | 16 | 7.9 | 5.9 | 4.3 | 3.0 | 2.0 | 1.2 | ... |
| 24 | 32.2 | 27.2 | 21.7 | 16.4 | 19.6 | 15.0 | 10.9 | 6.7 | 4.5 | 17 | 7.0 | 5.3 | 3.8 | 2.7 | 1.8 | 1.1 | ... |
| 25 | 29.7 | 25.0 | 20.0 | 15.1 | 18.1 | 13.9 | 10.1 | 7.7 | 4.2 | 18 | 6.2 | 4.7 | 3.4 | 2.4 | 1.6 | 0.98 | ... |
| 26 | 27.5 | 23.1 | 18.5 | 13.9 | 16.7 | 12.8 | 9.3 | 7.1 | 3.9 | | | | | | | | |
| 27 | 25.5 | 21.5 | 17.1 | 12.9 | 15.5 | 11.9 | 8.6 | 6.6 | 3.6 | 19 | 5.6 | 4.2 | 3.1 | 2.2 | 1.4 | ... | ... |
| 28 | 23.7 | 20.0 | 15.9 | 12.0 | 14.4 | 11.0 | 8.0 | 6.1 | 3.3 | 20 | 5.0 | 3.8 | 2.8 | 1.9 | 1.3 | ... | ... |
| 29 | 22.1 | 18.6 | 14.8 | 11.2 | 13.5 | 10.3 | 7.5 | 5.7 | 3.1 | 21 | 4.6 | 3.4 | 2.5 | 1.8 | 1.2 | ... | ... |
| 30 | 20.6 | 17.4 | 13.9 | 10.5 | 12.6 | 9.6 | 7.0 | 5.3 | 2.9 | 22 | 3.8 | 3.1 | 2.3 | 1.6 | 1.1 | ... | ... |

For any other load than 100 lb. per square foot, divide the spacing given by the ratio the given load per square foot bears to 100. Thus for a load of 150 lb. per square foot divide by 1.5. Maximum fiber stress, 16,000 lb. per square inch. Only figures above the cross-lines should be used for plastered ceilings, so that the deflection will not cause cracking of the plaster.

Properties of Carnegie Standard Channels—Steel.

| | Depth of Channel. | Weight per Foot. | Area of Section. | Thickness of Web. | Width of Flange. | Moment of Inertia, Neutral Axis Perpendicular to Web at Center. | Moment of Inertia, Neutral Axis Parallel with Center Line of Web. | Radius of Gyration, Neutral Axis Perpendicular to Web at Center. | Radius of Gyration, Neutral Axis Parallel with Center Line of Web. | Section Modulus, Neutral Axis Perpendicular to Web at Center. | Coefficient of Strength for Fiber Stress of 16,000 lbs. per sq. in. | Distance of Center of Gravity from Outside of Web. |
|--------|-------------------|------------------|------------------|-------------------|------------------|---|---|--|--|---|---|--|
| in. | lbs. | sq. in. | in. | in. | | I | I' | r | r' | S | C^* | x |
| 15 | 55. | 16.18 | 0.82 | 3.82 | | 430.2 | 12.19 | 5.16 | 0.868 | 57.4 | 611900 | 0.823 |
| 50. | 14.71 | 0.72 | 3.72 | | | 402.7 | 11.22 | 5.23 | 0.873 | 53.7 | 572700 | 0.803 |
| 45. | 13.24 | 0.62 | 3.62 | | | 375.1 | 10.29 | 5.32 | 0.882 | 50.0 | 533500 | 0.788 |
| 40. | 11.76 | 0.52 | 3.52 | | | 347.5 | 9.39 | 5.43 | 0.893 | 46.3 | 494200 | 0.783 |
| 35. | 10.29 | 0.43 | 3.43 | | | 320.0 | 8.48 | 5.58 | 0.908 | 42.7 | 455000 | 0.789 |
| 33. | 9.90 | 0.40 | 3.40 | | | 312.6 | 8.23 | 5.62 | 0.912 | 41.7 | 445500 | 0.794 |
| 12 | 40. | 11.76 | 0.76 | 3.42 | | 197.0 | 6.63 | 4.09 | 0.751 | 32.8 | 350200 | 0.722 |
| 35. | 10.29 | 0.64 | 3.30 | | | 179.3 | 5.90 | 4.17 | 0.757 | 29.9 | 318800 | 0.694 |
| 30. | 8.82 | 0.51 | 3.17 | | | 161.7 | 5.21 | 4.28 | 0.768 | 26.9 | 287400 | 0.677 |
| 25. | 7.35 | 0.39 | 3.05 | | | 144.0 | 4.53 | 4.43 | 0.785 | 24.0 | 256100 | 0.678 |
| 20.5 | 6.03 | 0.28 | 2.94 | | | 128.1 | 3.91 | 4.61 | 0.805 | 21.4 | 227800 | 0.704 |
| 10 | 35. | 10.29 | 0.82 | 3.18 | | 115.5 | 4.66 | 3.35 | 0.672 | 23.1 | 246400 | 0.695 |
| 30. | 8.82 | 0.68 | 3.04 | | | 103.2 | 3.90 | 3.42 | 0.672 | 20.6 | 220300 | 0.651 |
| 25. | 7.35 | 0.53 | 2.89 | | | 91.0 | 3.40 | 3.52 | 0.680 | 18.2 | 194100 | 0.620 |
| 20. | 5.88 | 0.38 | 2.74 | | | 78.7 | 2.85 | 3.66 | 0.696 | 15.7 | 168000 | 0.609 |
| 15. | 4.46 | 0.24 | 2.60 | | | 66.9 | 2.30 | 3.87 | 0.718 | 13.4 | 142700 | 0.639 |
| 9 | 25. | 7.35 | 0.62 | 2.82 | | 70.7 | 2.98 | 3.10 | 0.637 | 15.7 | 167600 | 0.615 |
| 20. | 5.88 | 0.45 | 2.65 | | | 60.8 | 2.45 | 3.21 | 0.646 | 13.5 | 144100 | 0.585 |
| 15. | 4.41 | 0.29 | 2.49 | | | 50.9 | 1.95 | 3.40 | 0.665 | 11.3 | 120500 | 0.590 |
| 13 1/4 | 3.89 | 0.23 | 2.43 | | | 47.3 | 1.77 | 3.49 | 0.674 | 10.5 | 112200 | 0.607 |
| 8 | 21 1/4 | 6.25 | 0.58 | 2.62 | | 47.8 | 2.25 | 2.77 | 0.600 | 11.9 | 127400 | 0.587 |
| 18 3/4 | 5.51 | 0.49 | 2.53 | | | 43.8 | 2.01 | 2.82 | 0.603 | 11.0 | 116900 | 0.567 |
| 16 1/4 | 4.78 | 0.40 | 2.44 | | | 39.9 | 1.78 | 2.89 | 0.610 | 10.0 | 106400 | 0.556 |
| 13 3/4 | 4.04 | 0.31 | 2.35 | | | 36.0 | 1.55 | 2.98 | 0.619 | 9.0 | 96000 | 0.557 |
| 11 1/4 | 3.35 | 0.22 | 2.26 | | | 32.3 | 1.33 | 3.11 | 0.630 | 8.1 | 86100 | 0.576 |
| 7 | 19 3/4 | 5.81 | 0.63 | 2.51 | | 33.2 | 1.85 | 2.39 | 0.565 | 9.5 | 101100 | 0.583 |
| 17 1/4 | 5.07 | 0.53 | 2.41 | | | 30.2 | 1.62 | 2.44 | 0.564 | 8.6 | 92000 | 0.555 |
| 14 3/4 | 4.34 | 0.42 | 2.30 | | | 27.2 | 1.40 | 2.50 | 0.568 | 7.8 | 82800 | 0.535 |
| 12 1/4 | 3.60 | 0.32 | 2.20 | | | 24.2 | 1.19 | 2.59 | 0.575 | 6.9 | 73700 | 0.528 |
| 9 3/4 | 2.85 | 0.21 | 2.09 | | | 21.1 | 0.98 | 2.72 | 0.586 | 6.0 | 66800 | 0.546 |
| 6 | 15.5 | 4.56 | 0.56 | 2.28 | | 19.5 | 1.28 | 2.07 | 0.529 | 6.5 | 69500 | 0.546 |
| 13. | 3.82 | 0.44 | 2.16 | | | 17.3 | 1.07 | 2.13 | 0.529 | 5.8 | 61600 | 0.517 |
| 10.5 | 3.09 | 0.32 | 2.04 | | | 15.1 | 0.88 | 2.21 | 0.534 | 5.0 | 53800 | 0.503 |
| 8. | 2.38 | 0.20 | 1.92 | | | 13.0 | 0.70 | 2.34 | 0.542 | 4.3 | 46200 | 0.517 |
| 5 | 11.5 | 3.38 | 0.48 | 2.04 | | 10.4 | 0.82 | 1.75 | 0.493 | 4.2 | 44400 | 0.508 |
| 9. | 2.65 | 0.33 | 1.89 | | | 8.9 | 0.64 | 1.83 | 0.493 | 3.5 | 37900 | 0.481 |
| 6.5 | 1.95 | 0.19 | 1.75 | | | 7.4 | 0.48 | 1.95 | 0.498 | 3.0 | 31600 | 0.489 |
| 4 | 7 1/4 | 2.13 | 0.33 | 1.73 | | 4.6 | 0.44 | 1.46 | 0.455 | 2.3 | 24400 | 0.463 |
| 6 1/4 | 1.84 | 0.25 | 1.65 | | | 4.2 | 0.38 | 1.51 | 0.454 | 2.1 | 22300 | 0.458 |
| 5 1/4 | 1.55 | 0.18 | 1.58 | | | 3.8 | 0.32 | 1.56 | 0.453 | 1.9 | 20200 | 0.464 |
| 3 | 6. | 1.76 | 0.36 | 1.60 | | 2.1 | 0.31 | 1.08 | 0.421 | 1.4 | 14700 | 0.459 |
| 5. | 1.47 | 0.26 | 1.50 | | | 1.8 | 0.25 | 1.12 | 0.415 | 1.2 | 13100 | 0.443 |
| 4. | 1.19 | 0.17 | 1.41 | | | 1.6 | 0.20 | 1.17 | 0.409 | 1.1 | 11600 | 0.443 |

* Used for buildings; for bridges use 12,500 pounds, or multiply coefficient in this column by 0.78125.

L = safe load in pounds, uniformly distributed; l = span in feet;
 M = moment of forces in foot-pounds; C = coefficient given above.

$$L = \frac{C}{l}; M = \frac{C}{8}; C = Ll = 8M = \frac{8fS}{12}; f = \text{fiber stress.}$$

Safe Loads Uniformly Distributed for Carnegie Standard and Special Channels, Tons of 2000 Lb. — Steel.

| Distance between Supports in Feet. | 15" C | | Add for every lb. increase in weight. | 12" C | | Add for every lb. increase in weight. | 10" C | | Add for every lb. increase in weight. | 9" C | | Add for every lb. increase in weight. | 8" C | | Add for every lb. increase in weight. | 7" C | | Add for every lb. increase in weight. | 6" C | | Add for every lb. increase in weight. | 5" C | | Add for every lb. increase in weight. | 4" C | | Add for every lb. increase in weight. | 3" C | | Add for every lb. increase in weight. |
|---------------------------------------|------------|------|--|--------------|------|--|------------|------|--|---------------|------|--|---------------|------|--|--------------|------|--|-----------|------|--|-------------|------|--|--------------|------|--|-----------|------|--|
| | 33 lbs. | ... | | 20.5 lbs. | ... | | 15 lbs. | ... | | 13.25 lbs. | ... | | 11.25 lbs. | ... | | 9.75 lbs. | ... | | 8 lbs. | ... | | 6.5 lbs. | ... | | 5.25 lbs. | ... | | 4 lbs. | ... | |
| 5 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 8.61 | 0.42 | 6.68 | 0.36 | 4.62 | 0.31 | 3.16 | 0.26 | 2.02 | 0.21 | 1.16 | 0.16 | 0.26 | 0.22 | 0.21 | 0.18 | 0.13 | 0.16 |
| 6 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 7.18 | 0.35 | 5.57 | 0.30 | 3.85 | 0.26 | 2.63 | 0.22 | 1.68 | 0.18 | 0.97 | 0.13 | 0.22 | 0.19 | 0.15 | 0.11 | 0.07 | 0.13 |
| 7 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 6.15 | 0.30 | 4.77 | 0.26 | 3.30 | 0.22 | 2.26 | 0.19 | 1.44 | 0.15 | 0.83 | 0.11 | 0.16 | 0.13 | 0.10 | 0.07 | 0.06 | 0.11 |
| 8 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 5.38 | 0.26 | 4.18 | 0.23 | 2.89 | 0.19 | 1.98 | 0.16 | 1.26 | 0.13 | 0.73 | 0.10 | 0.16 | 0.13 | 0.10 | 0.07 | 0.06 | 0.10 |
| 9 | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 4.78 | 0.23 | 3.71 | 0.20 | 2.57 | 0.17 | 1.76 | 0.14 | 1.12 | 0.12 | 0.64 | 0.09 | 0.14 | 0.12 | 0.11 | 0.08 | 0.05 | 0.09 |
| 10 | 22.23 | 0.39 | ... | 11.39 | 0.32 | 7.14 | 0.26 | 5.61 | 0.24 | 4.31 | 0.21 | 3.34 | 0.18 | 2.57 | 0.16 | 2.31 | 0.16 | 1.58 | 0.13 | 1.01 | 0.11 | 0.58 | 0.08 | 0.13 | 0.10 | 0.07 | 0.05 | 0.04 | 0.03 | 0.08 |
| 11 | 20.20 | 0.35 | ... | 10.35 | 0.29 | 6.49 | 0.24 | 5.10 | 0.21 | 3.91 | 0.19 | 3.04 | 0.16 | 2.10 | 0.14 | 1.44 | 0.10 | 1.44 | 0.10 | 0.92 | 0.10 | 0.53 | 0.07 | 0.12 | 0.10 | 0.07 | 0.05 | 0.04 | 0.03 | 0.07 |
| 12 | 18.52 | 0.33 | ... | 9.49 | 0.26 | 5.95 | 0.22 | 4.68 | 0.20 | 3.59 | 0.18 | 2.78 | 0.15 | 1.93 | 0.13 | 1.32 | 0.11 | 1.32 | 0.11 | 0.84 | 0.09 | 0.48 | 0.07 | 0.11 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.07 |
| 13 | 17.10 | 0.30 | ... | 8.76 | 0.24 | 5.49 | 0.20 | 4.32 | 0.18 | 3.31 | 0.16 | 2.57 | 0.14 | 1.78 | 0.12 | 1.22 | 0.10 | 1.22 | 0.10 | 0.78 | 0.08 | 0.45 | 0.07 | 0.10 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 14 | 15.87 | 0.28 | ... | 8.14 | 0.23 | 5.10 | 0.19 | 4.01 | 0.17 | 3.08 | 0.15 | 2.39 | 0.13 | 1.65 | 0.11 | 1.13 | 0.09 | 1.13 | 0.09 | 0.72 | 0.08 | 0.41 | 0.06 | 0.09 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 15 | 14.82 | 0.26 | ... | 7.59 | 0.21 | 4.76 | 0.17 | 3.74 | 0.16 | 2.87 | 0.14 | 2.23 | 0.12 | 1.54 | 0.10 | 1.05 | 0.09 | 1.05 | 0.09 | 0.67 | 0.07 | 0.39 | 0.05 | 0.09 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 16 | 13.89 | 0.24 | ... | 7.12 | 0.20 | 4.46 | 0.16 | 3.51 | 0.15 | 2.69 | 0.13 | 2.09 | 0.11 | 1.44 | 0.10 | 0.99 | 0.08 | 0.99 | 0.08 | 0.63 | 0.07 | 0.36 | 0.05 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 17 | 13.07 | 0.23 | ... | 6.70 | 0.18 | 4.20 | 0.15 | 3.30 | 0.14 | 2.53 | 0.12 | 1.96 | 0.11 | 1.36 | 0.09 | 0.93 | 0.08 | 0.93 | 0.08 | 0.59 | 0.06 | 0.34 | 0.05 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 18 | 12.35 | 0.22 | ... | 6.33 | 0.18 | 3.96 | 0.14 | 3.12 | 0.13 | 2.39 | 0.11 | 1.86 | 0.10 | 1.28 | 0.09 | 0.88 | 0.07 | 0.88 | 0.07 | 0.56 | 0.06 | 0.32 | 0.05 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 19 | 11.70 | 0.21 | ... | 5.99 | 0.17 | 3.76 | 0.14 | 2.95 | 0.12 | 2.27 | 0.11 | 1.76 | 0.09 | 1.22 | 0.08 | 0.83 | 0.07 | 0.83 | 0.07 | 0.53 | 0.06 | 0.31 | 0.04 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 20 | 11.11 | 0.20 | ... | 5.70 | 0.16 | 3.57 | 0.13 | 2.81 | 0.12 | 2.15 | 0.11 | 1.67 | 0.09 | 1.16 | 0.08 | 0.79 | 0.07 | 0.79 | 0.07 | 0.51 | 0.05 | 0.29 | 0.04 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 21 | 10.58 | 0.19 | ... | 5.42 | 0.15 | 3.40 | 0.12 | 2.67 | 0.11 | 2.05 | 0.10 | 1.59 | 0.09 | 1.10 | 0.07 | 0.75 | 0.06 | 0.75 | 0.06 | 0.48 | 0.05 | 0.28 | 0.04 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 22 | 10.10 | 0.18 | ... | 5.18 | 0.14 | 3.24 | 0.12 | 2.55 | 0.11 | 1.96 | 0.10 | 1.52 | 0.08 | 1.05 | 0.07 | 0.72 | 0.06 | 0.72 | 0.06 | 0.46 | 0.05 | 0.26 | 0.04 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 23 | 9.66 | 0.17 | ... | 4.95 | 0.14 | 3.10 | 0.11 | 2.44 | 0.10 | 1.87 | 0.09 | 1.45 | 0.08 | 1.00 | 0.07 | 0.69 | 0.06 | 0.69 | 0.06 | 0.44 | 0.05 | 0.25 | 0.04 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 24 | 9.26 | 0.16 | ... | 4.75 | 0.13 | 2.97 | 0.11 | 2.34 | 0.10 | 1.79 | 0.09 | 1.39 | 0.08 | 0.96 | 0.06 | 0.66 | 0.05 | 0.66 | 0.05 | 0.42 | 0.04 | 0.24 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 25 | 8.89 | 0.16 | ... | 4.56 | 0.13 | 2.85 | 0.10 | 2.24 | 0.09 | 1.72 | 0.08 | 1.34 | 0.07 | 0.92 | 0.06 | 0.63 | 0.05 | 0.63 | 0.05 | 0.40 | 0.04 | 0.23 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 26 | 8.55 | 0.15 | ... | 4.38 | 0.12 | 2.74 | 0.10 | 2.16 | 0.09 | 1.66 | 0.08 | 1.28 | 0.07 | 0.88 | 0.05 | 0.60 | 0.04 | 0.60 | 0.04 | 0.38 | 0.03 | 0.22 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 27 | 8.23 | 0.14 | ... | 4.22 | 0.12 | 2.64 | 0.10 | 2.08 | 0.09 | 1.60 | 0.08 | 1.22 | 0.07 | 0.84 | 0.05 | 0.58 | 0.04 | 0.58 | 0.04 | 0.36 | 0.03 | 0.21 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 28 | 7.94 | 0.14 | ... | 4.07 | 0.11 | 2.55 | 0.09 | 2.00 | 0.08 | 1.55 | 0.08 | 1.17 | 0.07 | 0.80 | 0.05 | 0.56 | 0.04 | 0.56 | 0.04 | 0.34 | 0.03 | 0.20 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 29 | 7.66 | 0.13 | ... | 3.93 | 0.11 | 2.46 | 0.09 | 1.93 | 0.08 | 1.50 | 0.08 | 1.13 | 0.07 | 0.78 | 0.05 | 0.54 | 0.04 | 0.54 | 0.04 | 0.32 | 0.03 | 0.19 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |
| 30 | 7.41 | 0.13 | ... | 3.80 | 0.11 | 2.38 | 0.09 | 1.87 | 0.08 | 1.47 | 0.08 | 1.10 | 0.07 | 0.76 | 0.05 | 0.52 | 0.04 | 0.52 | 0.04 | 0.30 | 0.03 | 0.18 | 0.03 | 0.08 | 0.07 | 0.05 | 0.04 | 0.03 | 0.06 | 0.07 |

Safe loads given include weight of channel. Maximum fiber stress, 16,000 lbs. per square inch.

Properties of Carnegie T-Shapes. — Steel.

| Size: Flange by Stem. | Weight per Foot. | Area of Section. | Distance of Center of Gravity from Outside of Flange. | Mom. of Inertia, Neutral Axis through Center of Gravity Parallel to Flange. | Least Section Modulus Neut. Axis through Center of Gravity Parallel to Flange. | Radius of Gyration, Neut. Axis through Center of Gravity Parallel to Flange. | Mom. of Inertia, Neut. Axis through C. of G. Coincident with Center Line of Stem. | Section Modulus, Neut. Axis through C. of G. Coincident with Center Line of Stem. | Radius of Gyration, Neut. Axis through C. of G. Coincident with Center Line of Stem. | Coeff. of Strength for Fiber Stress of 12,000 lb. per sq. in., Neutral Axis thro' C. of G. Parallel to Flange. |
|-----------------------|------------------|------------------|---|---|--|--|---|---|--|--|
| in. | lb. | sq. in. | in. | I | S | r | I' | S' | r' | C |
| 5 X 3 | 13.6 | 3.99 | 0.75 | 2.6 | 1.18 | 0.82 | 5.6 | 2.22 | 1.19 | 9410 |
| 5 X 2 1/2 | 11.0 | 3.24 | 0.65 | 1.6 | 0.86 | 0.71 | 4.3 | 1.70 | 1.16 | 6900 |
| 4 1/2 X 3 1/2 | 15.9 | 4.65 | 1.11 | 5.1 | 2.13 | 1.04 | 3.7 | 1.65 | 0.90 | 17020 |
| 4 1/2 X 3 | 8.6 | 2.55 | 0.73 | 1.8 | 0.81 | 0.87 | 2.6 | 1.16 | 1.03 | 6490 |
| 4 1/2 X 3 | 10.0 | 3.00 | 0.75 | 2.1 | 0.94 | 0.86 | 3.1 | 1.38 | 1.04 | 7540 |
| 4 1/2 X 2 1/2 | 8.0 | 2.40 | 0.58 | 1.1 | 0.56 | 0.69 | 2.6 | 1.16 | 1.07 | 4520 |
| 4 1/2 X 2 1/2 | 9.3 | 2.79 | 0.60 | 1.2 | 0.65 | 0.68 | 3.1 | 1.38 | 1.08 | 5220 |
| 4 X 5 | 15.7 | 4.56 | 1.56 | 10.7 | 3.10 | 1.54 | 2.8 | 1.41 | 0.79 | 24800 |
| 4 X 5 | 12.3 | 3.54 | 1.51 | 8.5 | 2.43 | 1.56 | 2.1 | 1.06 | 0.78 | 19410 |
| 4 X 4 1/2 | 14.8 | 4.29 | 1.37 | 8.0 | 2.55 | 1.37 | 2.8 | 1.41 | 0.81 | 20400 |
| 4 X 4 1/2 | 11.6 | 3.36 | 1.31 | 6.3 | 1.98 | 1.38 | 2.1 | 1.06 | 0.80 | 15840 |
| 4 X 4 | 13.9 | 4.02 | 1.18 | 5.7 | 2.02 | 1.20 | 2.8 | 1.40 | 0.84 | 16170 |
| 4 X 4 | 10.9 | 3.21 | 1.15 | 4.7 | 1.64 | 1.23 | 2.2 | 1.09 | 0.84 | 13100 |
| 4 X 3 | 9.3 | 2.73 | 0.78 | 2.0 | 0.88 | 0.86 | 2.1 | 1.05 | 0.88 | 7070 |
| 4 X 2 1/2 | 8.7 | 2.52 | 0.63 | 1.2 | 0.62 | 0.69 | 2.1 | 1.05 | 0.92 | 4980 |
| 4 X 2 1/2 | 7.4 | 2.16 | 0.60 | 1.0 | 0.55 | 0.70 | 1.8 | 0.88 | 0.91 | 4380 |
| 4 X 2 | 7.9 | 2.31 | 0.48 | 0.60 | 0.40 | 0.52 | 2.1 | 1.05 | 0.96 | 3180 |
| 4 X 2 | 6.7 | 1.95 | 0.51 | 0.54 | 0.34 | 0.51 | 1.8 | 0.88 | 0.95 | 2700 |
| 3 1/2 X 4 | 12.8 | 3.75 | 1.25 | 5.5 | 1.98 | 1.21 | 1.89 | 1.08 | 0.72 | 15870 |
| 3 1/2 X 4 | 10.0 | 2.91 | 1.19 | 4.3 | 1.55 | 1.22 | 1.42 | 0.81 | 0.70 | 12380 |
| 3 1/2 X 3 1/2 | 11.9 | 3.45 | 1.06 | 3.7 | 1.52 | 1.04 | 1.89 | 1.08 | 0.74 | 12160 |
| 3 1/2 X 3 1/2 | 9.3 | 2.70 | 1.01 | 3.0 | 1.19 | 1.05 | 1.42 | 0.81 | 0.73 | 9530 |
| 3 1/2 X 3 | 11.0 | 3.21 | 0.88 | 2.4 | 1.13 | 0.87 | 1.88 | 1.08 | 0.77 | 9050 |
| 3 1/2 X 3 | 8.7 | 2.49 | 0.83 | 1.9 | 0.88 | 0.88 | 1.41 | 0.81 | 0.75 | 7040 |
| 3 1/2 X 3 | 7.7 | 2.28 | 0.78 | 1.6 | 0.72 | 0.89 | 1.18 | 0.68 | 0.76 | 5790 |
| 3 X 4 | 11.9 | 3.48 | 1.32 | 5.2 | 1.94 | 1.23 | 1.21 | 0.81 | 0.59 | 15480 |
| 3 X 4 | 10.6 | 3.12 | 1.32 | 4.8 | 1.78 | 1.25 | 1.09 | 0.72 | 0.60 | 14270 |
| 3 X 4 | 9.3 | 2.73 | 1.29 | 4.3 | 1.57 | 1.26 | 0.93 | 0.62 | 0.59 | 12540 |
| 3 X 3 1/2 | 11.0 | 3.21 | 1.12 | 3.5 | 1.49 | 1.06 | 1.20 | 0.80 | 0.62 | 11910 |
| 3 X 3 1/2 | 9.8 | 2.88 | 1.11 | 3.3 | 1.37 | 1.08 | 1.31 | 0.88 | 0.68 | 10990 |
| 3 X 3 | 10.1 | 2.94 | 0.93 | 2.3 | 1.10 | 0.88 | 1.20 | 0.80 | 0.64 | 8780 |
| 3 X 3 | 9.0 | 2.67 | 0.92 | 2.1 | 1.01 | 0.90 | 1.08 | 0.72 | 0.64 | 8110 |
| 3 X 3 | 7.9 | 2.28 | 0.88 | 1.8 | 0.86 | 0.90 | 0.90 | 0.60 | 0.63 | 6900 |
| 3 X 2 1/2 | 7.2 | 2.10 | 0.71 | 1.1 | 0.60 | 0.72 | 0.89 | 0.60 | 0.66 | 4800 |
| 2 3/4 X 2 | 7.4 | 2.16 | 0.53 | 1.1 | 0.75 | 0.71 | 0.62 | 0.45 | 0.54 | 6000 |
| 2 1/2 X 3 | 7.2 | 2.10 | 0.97 | 1.8 | 0.87 | 0.92 | 0.54 | 0.43 | 0.51 | 6960 |
| 2 1/2 X 2 3/4 | 6.8 | 1.98 | 0.87 | 1.4 | 0.73 | 0.84 | 0.66 | 0.53 | 0.58 | 5860 |
| 2 1/2 X 2 1/2 | 6.5 | 1.89 | 0.76 | 1.0 | 0.59 | 0.74 | 0.52 | 0.42 | 0.53 | 4700 |
| 2 1/2 X 1 1/2 | 3.0 | 0.84 | 0.29 | 0.094 | 0.09 | 0.31 | 0.29 | 0.23 | 0.58 | 710 |
| 2 1/4 X 2 1/4 | 5.0 | 1.44 | 0.69 | 0.66 | 0.42 | 0.68 | 0.33 | 0.30 | 0.48 | 3360 |
| 2 X 2 | 4.4 | 1.26 | 0.63 | 0.45 | 0.33 | 0.60 | 0.23 | 0.23 | 0.43 | 2610 |
| 2 X 1 1/2 | 3.2 | 0.90 | 0.42 | 0.16 | 0.15 | 0.42 | 0.18 | 0.18 | 0.45 | 1200 |
| 1 3/4 X 1 3/4 | 3.2 | 0.90 | 0.54 | 0.23 | 0.19 | 0.51 | 0.12 | 0.14 | 0.37 | 1540 |
| 1 1/2 X 1 1/2 | 2.6 | 0.75 | 0.42 | 0.15 | 0.14 | 0.49 | 0.08 | 0.10 | 0.34 | 1150 |
| 1 1/4 X 1 1/4 | 2.1 | 0.60 | 0.40 | 0.08 | 0.10 | 0.36 | 0.05 | 0.07 | 0.27 | 760 |
| 1 X 1 | 1.3 | 0.36 | 0.32 | 0.03 | 0.05 | 0.29 | 0.02 | 0.04 | 0.21 | 370 |

Some light weight T's of the smaller sizes are omitted.

Properties of Carnegie Standard and Special Angles with Equal Legs. Minimum, Intermediate, and Maximum Thicknesses and Weights.

| Dimensions. — Inches. | | Thickness. — Inches. | Weight per Foot. — Lbs. | Area of Section. — Square Inches. | Distance of Center of Gravity from Back of Flange. — Inches. | Moment of Inertia, Neutral Axis through Center of Gravity Parallel to Flange. — I . | Section Modulus, Neutral Axis through Center of Gravity Parallel to Flange. — S . | Radius of Gyration, Neutral Axis through Center of Gravity Parallel to Flange. — r . | Least Radius of Gyration, Neutral Axis through Center of Gravity at Angle of 45° to Flanges. — r' . |
|-----------------------|---------|----------------------|-------------------------|-----------------------------------|--|---|---|--|---|
| 8 | × 8 | 1 1/8 | 56.9 | 16.73 | 2.41 | 97.97 | 17.53 | 2.42 | 1.55 |
| 8 | × 8 | 1 3/16 | 42.0 | 12.34 | 2.30 | 74.71 | 13.11 | 2.46 | 1.57 |
| 8 | × 8 | 1 1/2 | 26.4 | 7.75 | 2.19 | 48.63 | 8.37 | 2.50 | 1.58 |
| 6 | × 6 | 1 | 37.4 | 11.00 | 1.86 | 35.46 | 8.57 | 1.80 | 1.16 |
| 6 | × 6 | 1 1/16 | 26.5 | 7.78 | 1.75 | 26.19 | 6.17 | 1.83 | 1.17 |
| 6 | × 6 | 3/8 | 14.9 | 4.36 | 1.64 | 15.39 | 3.53 | 1.88 | 1.19 |
| * 5 | × 5 | 1 | 30.6 | 9.00 | 1.61 | 19.64 | 5.80 | 1.48 | 0.96 |
| * 5 | × 5 | 1 1/16 | 21.8 | 6.42 | 1.50 | 14.68 | 4.20 | 1.51 | 0.97 |
| * 5 | × 5 | 3/8 | 12.3 | 3.61 | 1.39 | 8.74 | 2.42 | 1.56 | 0.99 |
| 4 | × 4 | 13/16 | 19.9 | 5.84 | 1.29 | 8.14 | 3.01 | 1.18 | 0.77 |
| 4 | × 4 | 9/16 | 14.3 | 4.18 | 1.21 | 6.12 | 2.19 | 1.21 | 0.78 |
| 4 | × 4 | 5/16 | 8.2 | 2.40 | 1.12 | 3.71 | 1.29 | 1.24 | 0.79 |
| 3 1/2 | × 3 1/2 | 13/16 | 17.1 | 5.03 | 1.17 | 5.25 | 2.25 | 1.02 | 0.67 |
| 3 1/2 | × 3 1/2 | 9/16 | 12.4 | 3.62 | 1.08 | 3.99 | 1.65 | 1.05 | 0.68 |
| 3 1/2 | × 3 1/2 | 5/16 | 7.2 | 2.09 | 0.99 | 2.45 | 0.98 | 1.08 | 0.69 |
| 3 | × 3 | 5/8 | 11.5 | 3.36 | 0.98 | 2.62 | 1.30 | 0.88 | 0.57 |
| 3 | × 3 | 7/16 | 8.3 | 2.43 | 0.91 | 1.99 | 0.95 | 0.91 | 0.58 |
| 3 | × 3 | 1/4 | 4.9 | 1.44 | 0.84 | 1.24 | 0.58 | 0.93 | 0.59 |
| * 2 3/4 | × 2 3/4 | 1/2 | 8.5 | 2.50 | 0.87 | 1.67 | 0.89 | 0.82 | 0.52 |
| * 2 3/4 | × 2 3/4 | 3/8 | 6.6 | 1.92 | 0.82 | 1.33 | 0.69 | 0.83 | 0.53 |
| * 2 3/4 | × 2 3/4 | 1/4 | 4.5 | 1.31 | 0.78 | 0.93 | 0.48 | 0.85 | 0.55 |
| 2 1/2 | × 2 1/2 | 1/2 | 7.7 | 2.25 | 0.81 | 1.23 | 0.73 | 0.74 | 0.47 |
| 2 1/2 | × 2 1/2 | 3/8 | 5.9 | 1.73 | 0.76 | 0.98 | 0.57 | 0.75 | 0.48 |
| 2 1/2 | × 2 1/2 | 3/16 | 3.1 | 0.90 | 0.69 | 0.55 | 0.30 | 0.78 | 0.49 |
| * 2 1/4 | × 2 1/4 | 1/2 | 6.8 | 2.00 | 0.74 | 0.87 | 0.58 | 0.66 | 0.48 |
| * 2 1/4 | × 2 1/4 | 3/8 | 5.3 | 1.55 | 0.70 | 0.70 | 0.45 | 0.67 | 0.43 |
| * 2 1/4 | × 2 1/4 | 3/16 | 2.8 | 0.81 | 0.63 | 0.39 | 0.24 | 0.70 | 0.44 |
| 2 | × 2 | 7/16 | 5.3 | 1.56 | 0.66 | 0.54 | 0.40 | 0.59 | 0.39 |
| 2 | × 2 | 5/16 | 4.0 | 1.15 | 0.61 | 0.42 | 0.30 | 0.60 | 0.39 |
| 2 | × 2 | 3/16 | 2.5 | 0.72 | 0.57 | 0.28 | 0.19 | 0.62 | 0.40 |
| 1 3/4 | × 1 3/4 | 7/16 | 4.6 | 1.30 | 0.59 | 0.35 | 0.30 | 0.51 | 0.33 |
| 1 3/4 | × 1 3/4 | 5/16 | 3.4 | 1.00 | 0.55 | 0.27 | 0.23 | 0.52 | 0.34 |
| 1 3/4 | × 1 3/4 | 3/16 | 2.2 | 0.62 | 0.51 | 0.18 | 0.14 | 0.54 | 0.35 |
| 1 1/2 | × 1 1/2 | 3/8 | 3.4 | 0.99 | 0.51 | 0.19 | 0.19 | 0.44 | 0.29 |
| 1 1/2 | × 1 1/2 | 1/4 | 2.4 | 0.69 | 0.47 | 0.14 | 0.134 | 0.45 | 0.29 |
| 1 1/2 | × 1 1/2 | 1/8 | 1.3 | 0.36 | 0.42 | 0.08 | 0.070 | 0.46 | 0.30 |
| 1 1/4 | × 1 1/4 | 5/16 | 2.4 | 0.69 | 0.42 | 0.09 | 0.109 | 0.36 | 0.23 |
| 1 1/4 | × 1 1/4 | 1/4 | 2.0 | 0.56 | 0.40 | 0.077 | 0.091 | 0.37 | 0.24 |
| 1 1/4 | × 1 1/4 | 1/8 | 1.1 | 0.30 | 0.35 | 0.044 | 0.049 | 0.38 | 0.25 |
| 1 | × 1 | 1/4 | 1.5 | 0.44 | 0.34 | 0.037 | 0.056 | 0.29 | 0.19 |
| 1 | × 1 | 3/16 | 1.2 | 0.34 | 0.32 | 0.030 | 0.044 | 0.30 | 0.19 |
| 1 | × 1 | 1/8 | 0.8 | 0.24 | 0.30 | 0.022 | 0.031 | 0.31 | 0.20 |
| * 7/8 | × 7/8 | 3/16 | 1.0 | 0.29 | 0.29 | 0.019 | 0.033 | 0.26 | 0.18 |
| * 7/8 | × 7/8 | 1/8 | 0.7 | 0.21 | 0.26 | 0.014 | 0.023 | 0.26 | 0.19 |
| 3/4 | × 3/4 | 3/16 | 0.9 | 0.25 | 0.26 | 0.012 | 0.024 | 0.22 | 0.16 |
| 3/4 | × 3/4 | 1/8 | 0.6 | 0.17 | 0.23 | 0.009 | 0.017 | 0.23 | 0.17 |

Angles marked * are special.

**Properties of Carnegie Standard and Special Angles with
Unequal Legs; Minimum, Intermediate, and Maximum
Thicknesses, and Weights.**

| Dimensions. — Inches. | | Thickness. — Inches. | Weight per Foot. — Pounds. | Area of Section. — Square Inches. | Moment of Inertia. — <i>I</i> . | | Section Modulus. — <i>S</i> . | | Radius of Gyration. — <i>r</i> . | | |
|-----------------------|---------|----------------------|----------------------------|-----------------------------------|---|--|---|--|---|--|------------------------------|
| | | | | | Neutral Axis Parallel to Longer Flange. | Neutral Axis Parallel to Shorter Flange. | Neutral Axis Parallel to Longer Flange. | Neutral Axis Parallel to Shorter Flange. | Neutral Axis Parallel to Longer Flange. | Neutral Axis Parallel to Shorter Flange. | Least Radius. Axis Diagonal. |
| 8 | × 3 1/2 | 11/20 | 20.5 | 6.02 | 4.92 | 39.96 | 1.79 | 7.99 | 0.90 | 2.58 | 0.74 |
| *7 | × 3 1/2 | 1 | 32.3 | 9.50 | 7.53 | 45.37 | 2.96 | 10.58 | 0.89 | 2.19 | 0.88 |
| *7 | × 3 1/2 | 3/4 | 24.9 | 7.31 | 6.08 | 35.99 | 2.31 | 8.22 | 0.91 | 2.22 | 0.88 |
| *7 | × 3 1/2 | 7/16 | 15.0 | 4.40 | 3.95 | 22.56 | 1.47 | 5.01 | 0.95 | 2.26 | 0.89 |
| 6 | × 4 | 1 | 30.6 | 9.00 | 10.75 | 30.75 | 3.79 | 8.02 | 1.09 | 1.85 | 0.85 |
| 6 | × 4 | 11/16 | 21.8 | 6.41 | 8.11 | 22.82 | 2.76 | 5.78 | 1.13 | 1.89 | 0.86 |
| 6 | × 4 | 3/8 | 12.3 | 3.61 | 4.90 | 13.47 | 1.60 | 3.32 | 1.17 | 1.93 | 0.88 |
| 6 | × 3 1/2 | 1 | 28.9 | 8.50 | 7.21 | 29.24 | 2.90 | 7.83 | 0.92 | 1.85 | 0.74 |
| 6 | × 3 1/2 | 11/16 | 20.6 | 6.06 | 5.47 | 21.74 | 2.11 | 5.65 | 0.95 | 1.89 | 0.75 |
| 6 | × 3 1/2 | 3/8 | 11.7 | 3.42 | 3.34 | 12.86 | 1.23 | 3.25 | 0.99 | 1.94 | 0.77 |
| *5 | × 4 | 7/8 | 24.2 | 7.11 | 9.23 | 16.42 | 3.31 | 4.99 | 1.14 | 1.52 | 0.84 |
| *5 | × 4 | 5/8 | 17.8 | 5.23 | 7.14 | 12.61 | 2.48 | 3.73 | 1.17 | 1.55 | 0.84 |
| *5 | × 4 | 3/8 | 11.0 | 3.23 | 4.67 | 8.14 | 1.57 | 2.34 | 1.20 | 1.59 | 0.86 |
| 5 | × 3 1/2 | 7/8 | 22.7 | 6.67 | 6.21 | 15.67 | 2.52 | 4.88 | 0.96 | 1.53 | 0.75 |
| 5 | × 3 1/2 | 5/8 | 16.8 | 4.92 | 4.83 | 12.03 | 1.90 | 3.65 | 0.99 | 1.56 | 0.75 |
| 5 | × 3 1/2 | 5/16 | 8.7 | 2.56 | 2.72 | 6.60 | 1.02 | 1.94 | 1.03 | 1.61 | 0.76 |
| 5 | × 3 | 13/16 | 19.9 | 5.84 | 3.71 | 13.98 | 1.74 | 4.45 | 0.80 | 1.55 | 0.64 |
| 5 | × 3 | 9/16 | 14.3 | 4.18 | 2.83 | 10.43 | 1.27 | 3.23 | 0.82 | 1.58 | 0.65 |
| 5 | × 3 | 5/16 | 8.2 | 2.40 | 1.75 | 6.26 | 0.75 | 1.89 | 0.85 | 1.61 | 0.66 |
| *4 1/2 | × 3 | 13/16 | 18.5 | 5.43 | 3.60 | 10.33 | 1.71 | 3.62 | 0.81 | 1.38 | 0.64 |
| *4 1/2 | × 3 | 9/16 | 13.3 | 3.90 | 2.75 | 7.75 | 1.25 | 2.64 | 0.85 | 1.41 | 0.64 |
| *4 1/2 | × 3 | 5/16 | 7.7 | 2.25 | 1.73 | 4.69 | 0.76 | 1.54 | 0.88 | 1.44 | 0.66 |
| *4 | × 3 1/2 | 13/16 | 18.5 | 5.43 | 5.49 | 7.77 | 2.30 | 2.92 | 1.01 | 1.19 | 0.72 |
| *4 | × 3 1/2 | 9/16 | 13.3 | 3.90 | 4.17 | 5.86 | 1.68 | 2.15 | 1.03 | 1.23 | 0.72 |
| *4 | × 3 1/2 | 5/16 | 7.7 | 2.25 | 2.59 | 3.56 | 1.01 | 1.26 | 1.07 | 1.26 | 0.73 |
| 4 | × 3 | 13/16 | 17.1 | 5.03 | 3.47 | 7.34 | 1.68 | 2.87 | 0.83 | 1.21 | 0.64 |
| 4 | × 3 | 9/16 | 12.4 | 3.62 | 2.66 | 5.55 | 1.23 | 2.09 | 0.86 | 1.24 | 0.64 |
| 4 | × 3 | 5/16 | 7.2 | 2.09 | 1.65 | 3.38 | 0.74 | 1.23 | 0.89 | 1.27 | 0.65 |
| 3 1/2 | × 3 | 13/16 | 15.8 | 4.62 | 3.33 | 4.98 | 1.65 | 2.20 | 0.85 | 1.04 | 0.62 |
| 3 1/2 | × 3 | 5/16 | 6.6 | 1.93 | 1.58 | 2.33 | 0.72 | 0.96 | 0.90 | 1.10 | 0.63 |
| 3 1/2 | × 2 1/2 | 11/16 | 12.5 | 3.65 | 1.72 | 4.13 | 0.99 | 1.85 | 0.67 | 1.06 | 0.53 |
| 3 1/2 | × 2 1/2 | 1/4 | 4.9 | 1.44 | 0.78 | 1.80 | 0.41 | 0.75 | 0.74 | 1.12 | 0.54 |
| *3 1/4 | × 2 | 9/16 | 9.0 | 2.64 | 0.75 | 2.64 | 0.53 | 1.30 | 0.53 | 1.00 | 0.44 |
| *3 1/4 | × 2 | 1/4 | 4.3 | 1.25 | 0.40 | 1.36 | 0.26 | 0.63 | 0.57 | 1.04 | 0.45 |
| 3 | × 2 1/2 | 9/16 | 9.5 | 2.78 | 1.42 | 2.28 | 0.82 | 1.15 | 0.72 | 0.91 | 0.52 |
| 3 | × 2 1/2 | 1/4 | 4.5 | 1.31 | 0.74 | 1.17 | 0.40 | 0.56 | 0.75 | 0.95 | 0.53 |
| *3 | × 2 | 1/2 | 7.7 | 2.25 | 0.67 | 1.92 | 0.47 | 1.00 | 0.55 | 0.92 | 0.43 |
| *3 | × 2 | 1/4 | 4.1 | 1.19 | 0.39 | 1.09 | 0.25 | 0.54 | 0.57 | 0.95 | 0.43 |
| 2 1/2 | × 2 | 1/2 | 6.8 | 2.00 | 0.64 | 1.14 | 0.46 | 0.70 | 0.56 | 0.75 | 0.42 |
| 2 1/2 | × 2 | 3/16 | 2.8 | 0.81 | 0.29 | 0.51 | 0.20 | 0.29 | 0.60 | 0.79 | 0.43 |
| *2 1/4 | × 1 1/2 | 1/2 | 5.6 | 1.63 | 0.26 | 0.75 | 0.26 | 0.54 | 0.40 | 0.68 | 0.39 |
| *2 1/4 | × 1 1/2 | 3/8 | 4.4 | 1.27 | 0.21 | 0.61 | 0.20 | 0.42 | 0.41 | 0.69 | 0.39 |
| *2 1/4 | × 1 1/2 | 3/16 | 2.3 | 0.67 | 0.12 | 0.34 | 0.11 | 0.23 | 0.43 | 0.72 | 0.40 |
| *2 | × 1 3/8 | 1/4 | 2.7 | 0.78 | 0.12 | 0.37 | 0.12 | 0.23 | 0.39 | 0.63 | 0.30 |
| *2 | × 1 3/8 | 3/16 | 2.1 | 0.60 | 0.09 | 0.24 | 0.09 | 0.18 | 0.40 | 0.63 | 0.31 |
| *1 3/8 | × 1 | 1/4 | 1.9 | 0.53 | 0.04 | 0.09 | 0.05 | 0.09 | 0.27 | 0.41 | 0.22 |
| *1 3/8 | × 1 | 1/8 | 1.0 | 0.28 | 0.02 | 0.05 | 0.03 | 0.06 | 0.29 | 0.44 | 0.22 |

Angles marked * are special. A few of the smaller intermediate sizes are omitted.

Safe Loads (Tons, 2000 Lb.) Uniformly Distributed for
 Carnegie Standard and Special Angles With Equal Legs.

| Size of Angle. | Distance between Supports in Feet. | | | | | | | | | |
|-----------------------|------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8 × 8 × 1 1/8 | 93.49 | 46.74 | 31.16 | 23.37 | 18.70 | 15.58 | 13.36 | 11.69 | 10.39 | 9.35 |
| 8 × 8 × 1/2 | 44.64 | 22.32 | 14.88 | 11.16 | 8.93 | 7.44 | 6.38 | 5.58 | 4.96 | 4.46 |
| 6 × 6 × 1 | 45.72 | 22.86 | 15.24 | 11.43 | 9.14 | 7.62 | 6.53 | 5.72 | 5.08 | 4.57 |
| 6 × 6 × 3/8 | 18.82 | 9.41 | 6.27 | 4.70 | 3.76 | 3.14 | 2.69 | 2.35 | 2.09 | 1.88 |
| *5 × 5 × 1 | 30.91 | 15.45 | 10.30 | 7.73 | 6.18 | 5.15 | 4.42 | 3.86 | 3.43 | 3.09 |
| *5 × 5 × 3/8 | 12.91 | 6.45 | 4.30 | 3.23 | 2.58 | 2.15 | 1.84 | 1.61 | 1.43 | 1.29 |
| 4 × 4 × 13/16 | 16.05 | 8.03 | 5.35 | 4.01 | 3.21 | 2.68 | 2.29 | 2.01 | 1.78 | 1.61 |
| 4 × 4 × 5/16 | 6.88 | 3.44 | 2.29 | 1.72 | 1.38 | 1.15 | 0.98 | 0.86 | 0.76 | 0.69 |
| 3 1/2 × 3 1/2 × 13/16 | 12.00 | 6.00 | 4.00 | 3.00 | 2.40 | 2.00 | 1.71 | 1.50 | 1.33 | 1.20 |
| 3 1/2 × 3 1/2 × 5/16 | 5.20 | 2.60 | 1.73 | 1.30 | 1.04 | 0.87 | 0.74 | 0.65 | 0.58 | 0.52 |
| 3 × 3 × 5/8 | 6.93 | 3.47 | 2.31 | 1.73 | 1.39 | 1.16 | 0.99 | 0.87 | 0.77 | 0.69 |
| 3 × 3 × 1/4 | 3.09 | 1.55 | 1.03 | 0.77 | 0.62 | 0.52 | 0.44 | 0.39 | 0.34 | 0.31 |
| *23/4 × 23/4 × 1/2 | 4.75 | 2.37 | 1.58 | 1.19 | 0.95 | 0.79 | 0.68 | 0.59 | 0.53 | 0.47 |
| *23/4 × 23/4 × 1/4 | 2.56 | 1.28 | 0.85 | 0.64 | 0.51 | 0.43 | 0.37 | 0.32 | 0.28 | 0.26 |
| 2 1/2 × 2 1/2 × 1/2 | 3.89 | 1.95 | 1.29 | 0.97 | 0.78 | 0.65 | 0.56 | 0.49 | 0.43 | 0.39 |
| 2 1/2 × 2 1/2 × 3/16 | 1.61 | 0.81 | 0.54 | 0.40 | 0.32 | 0.27 | 0.23 | 0.20 | 0.18 | 0.16 |
| *2 1/4 × 2 1/4 × 1/2 | 3.09 | 1.55 | 1.03 | 0.77 | 0.62 | 0.52 | 0.44 | 0.39 | 0.34 | 0.31 |
| *2 1/4 × 2 1/4 × 3/16 | 1.30 | 0.65 | 0.43 | 0.32 | 0.26 | 0.22 | 0.19 | 0.16 | 0.14 | 0.13 |
| 2 × 2 × 7/16 | 2.13 | 1.07 | 0.71 | 0.53 | 0.43 | 0.36 | 0.30 | 0.27 | 0.24 | 0.21 |
| 2 × 2 × 3/16 | 1.01 | 0.51 | 0.34 | 0.25 | 0.20 | 0.17 | 0.14 | 0.13 | 0.11 | 0.10 |
| 1 3/4 × 1 3/4 × 7/16 | 1.60 | 0.80 | 0.53 | 0.40 | 0.32 | 0.27 | 0.23 | 0.20 | 0.18 | 0.16 |
| 1 3/4 × 1 3/4 × 3/16 | 0.75 | 0.37 | 0.25 | 0.19 | 0.15 | 0.12 | 0.11 | 0.093 | 0.083 | 0.075 |
| 1 1/2 × 1 1/2 × 3/8 | 1.01 | 0.51 | 0.34 | 0.25 | 0.20 | 0.17 | 0.14 | 0.130 | 0.110 | 0.100 |
| 1 1/2 × 1 1/2 × 1/8 | 0.38 | 0.19 | 0.13 | 0.096 | 0.077 | 0.064 | 0.055 | 0.048 | 0.043 | 0.038 |
| 1 1/4 × 1 1/4 × 5/16 | 0.58 | 0.29 | 0.19 | 0.150 | 0.120 | 0.097 | 0.083 | 0.073 | 0.065 | 0.058 |
| 1 1/4 × 1 1/4 × 1/8 | 0.26 | 0.13 | 0.087 | 0.065 | 0.052 | 0.044 | 0.037 | 0.033 | 0.029 | 0.026 |
| 1 × 1 × 1/4 | 0.30 | 0.15 | 0.100 | 0.075 | 0.060 | 0.050 | 0.043 | 0.037 | 0.033 | 0.030 |
| 1 × 1 × 1/8 | 0.17 | 0.083 | 0.055 | 0.041 | 0.033 | 0.028 | 0.024 | 0.021 | 0.018 | 0.017 |
| * 7/8 × 7/8 × 3/16 | 0.18 | 0.088 | 0.059 | 0.044 | 0.035 | 0.029 | 0.025 | 0.022 | 0.020 | 0.018 |
| * 7/8 × 7/8 × 1/8 | 0.12 | 0.061 | 0.041 | 0.031 | 0.025 | 0.020 | 0.018 | 0.015 | 0.014 | 0.012 |
| 3/4 × 3/4 × 3/16 | 0.13 | 0.064 | 0.043 | 0.032 | 0.026 | 0.021 | 0.018 | 0.016 | 0.014 | 0.013 |
| 3/4 × 3/4 × 1/8 | 0.091 | 0.045 | 0.030 | 0.023 | 0.018 | 0.015 | 0.013 | 0.011 | 0.010 | 0.009 |

Safe loads given include weight of angle. Maximum fiber stress, 16,000 pounds per square inch. Neutral axis through center of gravity parallel to one leg. Angles marked * are special.

Safe Loads in Tons (2000 Lb.) Uniformly Distributed for Standard Carnegie Angles with Unequal Legs.

(Short Leg Vertical.)

| Size of Angle. | Distance between Supports in Feet. | | | | | | | | | |
|-----------------------|------------------------------------|-------|------|------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6 × 4 × 1 | 20.21 | 10.11 | 6.74 | 5.05 | 4.04 | 3.37 | 2.89 | 2.53 | 2.25 | 2.02 |
| 6 × 4 × 3/8 | 8.53 | 4.27 | 2.84 | 2.13 | 1.71 | 1.42 | 1.22 | 1.07 | 0.95 | 0.85 |
| 6 × 3 1/2 × 1 | 15.47 | 7.74 | 5.16 | 3.87 | 3.09 | 2.58 | 2.21 | 1.93 | 1.72 | 1.55 |
| 6 × 3 1/2 × 3/8 | 6.56 | 3.28 | 2.19 | 1.64 | 1.31 | 1.09 | 0.94 | 0.82 | 0.73 | 0.66 |
| 5 × 3 1/2 × 7/8 | 13.44 | 6.72 | 4.48 | 3.36 | 2.69 | 2.24 | 1.92 | 1.68 | 1.49 | 1.34 |
| 5 × 3 1/2 × 5/16 | 5.44 | 2.72 | 1.81 | 1.36 | 1.09 | 0.91 | 0.78 | 0.68 | 0.60 | 0.54 |
| 5 × 3 × 13/16 | 9.28 | 4.64 | 3.09 | 2.32 | 1.86 | 1.55 | 1.33 | 1.16 | 1.03 | 0.93 |
| 5 × 3 × 7/16 | 4.00 | 2.00 | 1.33 | 1.00 | 0.80 | 0.67 | 0.57 | 0.50 | 0.44 | 0.40 |
| 4 × 3 × 13/16 | 8.96 | 4.48 | 2.99 | 2.24 | 1.79 | 1.49 | 1.28 | 1.12 | 1.00 | 0.90 |
| 4 × 3 × 7/16 | 3.95 | 1.97 | 1.32 | 0.99 | 0.79 | 0.65 | 0.56 | 0.49 | 0.44 | 0.39 |
| 3 1/2 × 3 × 13/16 | 8.80 | 4.40 | 2.93 | 2.20 | 1.76 | 1.47 | 1.26 | 1.10 | 0.98 | 0.88 |
| 3 1/2 × 3 × 5/16 | 3.84 | 1.92 | 1.28 | 0.96 | 0.77 | 0.64 | 0.55 | 0.48 | 0.43 | 0.38 |
| 3 1/2 × 2 1/2 × 11/16 | 5.28 | 2.64 | 1.76 | 1.32 | 1.06 | 0.88 | 0.75 | 0.66 | 0.59 | 0.53 |
| 3 1/2 × 2 1/2 × 1/4 | 2.19 | 1.09 | 0.73 | 0.55 | 0.44 | 0.36 | 0.31 | 0.27 | 0.24 | 0.22 |
| 3 × 2 1/2 × 9/16 | 4.37 | 2.19 | 1.46 | 1.09 | 0.87 | 0.73 | 0.62 | 0.55 | 0.49 | 0.44 |
| 3 × 2 1/2 × 1/4 | 2.13 | 1.07 | 0.71 | 0.53 | 0.43 | 0.36 | 0.30 | 0.27 | 0.24 | 0.21 |
| 2 1/2 × 2 × 1/2 | 2.45 | 1.23 | 0.82 | 0.61 | 0.49 | 0.41 | 0.35 | 0.31 | 0.27 | 0.25 |
| 2 1/2 × 2 × 3/16 | 1.07 | 0.53 | 0.36 | 0.27 | 0.21 | 0.18 | 0.15 | 0.13 | 0.12 | 0.11 |

Safe loads given include weight of angle. Maximum fiber stress, 16,000 lb. per sq. in. Neutral axis through center of gravity parallel to long leg.

Safe Loads in Tons (2000 Lb.) Uniformly Distributed for Standard Carnegie Angles with Unequal Legs.

(Long Leg Vertical.)

| Size of Angle. | Distance between Supports in Feet. | | | | | | | | | |
|-----------------------|------------------------------------|-------|-------|-------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 6 × 4 × 1 | 42.77 | 21.39 | 14.26 | 10.69 | 8.55 | 7.13 | 6.11 | 5.35 | 4.75 | 4.28 |
| 6 × 4 × 3/8 | 17.71 | 8.85 | 5.90 | 4.43 | 3.54 | 2.95 | 2.53 | 2.21 | 1.97 | 1.77 |
| 6 × 3 1/2 × 1 | 41.76 | 20.88 | 13.92 | 10.44 | 8.35 | 6.96 | 5.97 | 5.22 | 4.64 | 4.18 |
| 6 × 3 1/2 × 3/8 | 17.33 | 8.67 | 5.78 | 4.33 | 3.47 | 2.89 | 2.48 | 2.17 | 1.93 | 1.73 |
| 5 × 3 1/2 × 7/8 | 26.03 | 13.01 | 8.68 | 6.51 | 5.21 | 4.34 | 3.72 | 3.25 | 2.89 | 2.60 |
| 5 × 3 1/2 × 5/16 | 10.35 | 5.18 | 3.45 | 2.59 | 2.07 | 1.73 | 1.48 | 1.29 | 1.15 | 1.04 |
| 5 × 3 × 13/16 | 23.73 | 11.87 | 7.91 | 5.93 | 4.75 | 3.96 | 3.39 | 2.97 | 2.64 | 2.37 |
| 5 × 3 × 5/16 | 10.08 | 5.04 | 3.36 | 2.52 | 2.02 | 1.68 | 1.44 | 1.26 | 1.12 | 1.01 |
| 4 × 3 × 13/16 | 15.31 | 7.65 | 5.10 | 3.83 | 3.06 | 2.55 | 2.19 | 1.91 | 1.70 | 1.53 |
| 4 × 3 × 5/16 | 6.56 | 3.28 | 2.19 | 1.64 | 1.31 | 1.10 | 0.94 | 0.82 | 0.73 | 0.66 |
| 3 1/2 × 3 × 13/16 | 11.73 | 5.87 | 3.91 | 2.93 | 2.35 | 1.96 | 1.68 | 1.47 | 1.30 | 1.17 |
| 3 1/2 × 3 × 5/16 | 5.12 | 2.56 | 1.71 | 1.28 | 1.02 | 0.85 | 0.73 | 0.64 | 0.57 | 0.51 |
| 3 1/2 × 2 1/2 × 11/16 | 9.87 | 4.93 | 3.29 | 2.47 | 1.97 | 1.64 | 1.41 | 1.23 | 1.10 | 0.99 |
| 3 1/2 × 2 1/2 × 1/4 | 4.00 | 2.00 | 1.33 | 1.00 | 0.80 | 0.67 | 0.57 | 0.50 | 0.44 | 0.40 |
| 3 × 2 1/2 × 9/16 | 6.13 | 3.07 | 2.04 | 1.53 | 1.23 | 1.02 | 0.88 | 0.77 | 0.68 | 0.61 |
| 3 × 2 1/2 × 1/4 | 2.99 | 1.50 | 1.00 | 0.75 | 0.60 | 0.50 | 0.43 | 0.37 | 0.33 | 0.30 |
| 2 1/2 × 2 × 1/2 | 3.73 | 1.87 | 1.24 | 0.93 | 0.75 | 0.62 | 0.53 | 0.47 | 0.41 | 0.37 |
| 2 1/2 × 2 × 3/16 | 1.55 | 0.77 | 0.52 | 0.39 | 0.31 | 0.26 | 0.22 | 0.19 | 0.17 | 0.16 |

Safe loads given include weight of angle. Maximum fiber stress, 16,000 lb. per sq. in. Neutral axis through center of gravity parallel to short leg.

Properties of Carnegie Z-Bars.

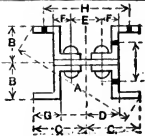
| Depth of Web. | Width of Flange. | Thickness of Metal. | Weight per Foot. | Area of Section. | Mom. of Inertia, Neut. Axis thro' C. of Gr. Perpendicular to Web. | Mom. of Inertia, Neut. Axis thro' C. of Gr. Coincident with Web. | Section Modulus, Neut. Axis thro' C. of Gr. Perpendicular to Web. | Section Modulus, Neut. Axis thro' C. of Gr. Coincident with Web. | Radius of Gyration, Neut. Axis thro' C. of Gr. Perpendicular to Web. | Radius of Gyration, Neut. Axis thro' C. of Gr. Coincident with Web. | Radius of Gyration, Least Radius, Neut. Axis Diag. | Coeff. of Strength for Fiber Stress of 12,000 lb. per sq. in., Axis Perpendicular to Web at Center. |
|---------------|------------------|---------------------|------------------|------------------|---|--|---|--|--|---|--|---|
| in. | in. | in. | lb. | sq. in. | I | I | S | S | r | r | r | C |
| 6 | 3 1/2 | 3/8 | 15.6 | 4.59 | 25.32 | 9.11 | 8.44 | 2.75 | 2.35 | 1.41 | 0.83 | 67,500 |
| 6 1/16 | 3 9/16 | 7/16 | 18.3 | 5.39 | 29.80 | 10.95 | 9.83 | 3.27 | 2.35 | 1.43 | 0.84 | 78,600 |
| 6 1/8 | 3 5/8 | 1/2 | 21.0 | 6.19 | 34.36 | 12.87 | 11.22 | 3.81 | 2.36 | 1.44 | 0.84 | 89,800 |
| 6 | 3 1/2 | 9/16 | 22.7 | 6.68 | 34.64 | 12.59 | 11.52 | 3.91 | 2.28 | 1.37 | 0.81 | 92,400 |
| 6 1/16 | 3 9/16 | 5/8 | 25.4 | 7.46 | 38.86 | 14.42 | 12.82 | 4.43 | 2.28 | 1.39 | 0.82 | 102,600 |
| 6 1/8 | 3 5/8 | 11/16 | 28.0 | 8.25 | 43.18 | 16.34 | 14.10 | 4.98 | 2.29 | 1.41 | 0.84 | 112,800 |
| 6 | 3 1/2 | 3/4 | 29.3 | 8.63 | 42.12 | 15.44 | 14.04 | 4.94 | 2.21 | 1.34 | 0.81 | 112,300 |
| 6 1/16 | 3 9/16 | 13/16 | 31.9 | 9.40 | 46.13 | 17.27 | 15.22 | 5.47 | 2.22 | 1.36 | 0.82 | 121,800 |
| 6 1/8 | 3 5/8 | 7/8 | 34.6 | 10.17 | 50.22 | 19.18 | 16.40 | 6.02 | 2.22 | 1.37 | 0.83 | 131,200 |
| 5 | 3 1/4 | 5/16 | 11.6 | 3.40 | 13.36 | 6.18 | 5.34 | 2.00 | 1.98 | 1.35 | 0.75 | 42,700 |
| 5 1/16 | 3 5/16 | 3/8 | 13.9 | 4.10 | 16.18 | 7.65 | 6.39 | 2.45 | 1.99 | 1.37 | 0.76 | 51,100 |
| 5 1/8 | 3 3/8 | 7/16 | 16.4 | 4.81 | 19.07 | 9.20 | 7.44 | 2.92 | 1.99 | 1.38 | 0.77 | 59,500 |
| 5 | 3 1/4 | 1/2 | 17.9 | 5.25 | 19.19 | 9.05 | 7.68 | 3.02 | 1.91 | 1.31 | 0.74 | 61,400 |
| 5 1/16 | 3 5/16 | 9/16 | 20.2 | 5.94 | 21.83 | 10.51 | 8.62 | 3.47 | 1.91 | 1.33 | 0.75 | 69,000 |
| 5 1/8 | 3 3/8 | 5/8 | 22.6 | 6.64 | 24.53 | 12.06 | 9.57 | 3.94 | 1.92 | 1.35 | 0.76 | 76,600 |
| 5 | 3 1/4 | 11/16 | 23.7 | 6.96 | 23.68 | 11.37 | 9.47 | 3.91 | 1.84 | 1.28 | 0.73 | 75,800 |
| 5 1/16 | 3 5/16 | 3/4 | 26.0 | 7.64 | 26.16 | 12.83 | 10.34 | 4.37 | 1.85 | 1.30 | 0.75 | 82,700 |
| 5 1/8 | 3 3/8 | 13/16 | 28.3 | 8.33 | 28.70 | 14.36 | 11.20 | 4.84 | 1.86 | 1.31 | 0.76 | 89,600 |
| 4 | 3 1/16 | 1/4 | 8.2 | 2.41 | 6.28 | 4.23 | 3.14 | 1.44 | 1.62 | 1.33 | 0.67 | 25,100 |
| 4 1/16 | 3 1/8 | 5/16 | 10.3 | 3.03 | 7.94 | 5.46 | 3.91 | 1.84 | 1.62 | 1.34 | 0.68 | 31,300 |
| 4 1/8 | 3 3/8 | 3/8 | 12.4 | 3.66 | 9.63 | 6.77 | 4.67 | 2.26 | 1.62 | 1.36 | 0.69 | 37,400 |
| 4 | 3 1/16 | 7/16 | 13.8 | 4.05 | 9.66 | 6.73 | 4.83 | 2.37 | 1.55 | 1.29 | 0.66 | 38,600 |
| 4 1/16 | 3 1/8 | 1/2 | 15.8 | 4.66 | 11.18 | 7.96 | 5.50 | 2.77 | 1.55 | 1.31 | 0.67 | 44,000 |
| 4 1/8 | 3 3/8 | 9/16 | 17.9 | 5.27 | 12.74 | 9.26 | 6.18 | 3.19 | 1.55 | 1.33 | 0.69 | 49,400 |
| 4 | 3 1/16 | 5/8 | 18.9 | 5.55 | 12.11 | 8.73 | 6.05 | 3.18 | 1.48 | 1.25 | 0.66 | 48,400 |
| 4 1/16 | 3 1/8 | 11/16 | 20.9 | 6.14 | 13.52 | 9.95 | 6.65 | 3.58 | 1.48 | 1.27 | 0.67 | 53,200 |
| 4 1/8 | 3 3/8 | 3/4 | 23.0 | 6.75 | 14.97 | 11.24 | 7.26 | 4.00 | 1.49 | 1.29 | 0.69 | 58,100 |
| 3 | 2 11/16 | 1/4 | 6.7 | 1.97 | 2.87 | 2.81 | 1.92 | 1.10 | 1.21 | 1.19 | 0.55 | 15,400 |
| 3 1/16 | 2 3/4 | 5/16 | 8.4 | 2.48 | 3.64 | 3.64 | 2.38 | 1.40 | 1.21 | 1.21 | 0.56 | 19,000 |
| 3 | 2 11/16 | 3/8 | 9.7 | 2.86 | 3.85 | 3.92 | 2.57 | 1.57 | 1.16 | 1.17 | 0.55 | 2,6000 |
| 3 1/16 | 2 3/4 | 7/16 | 11.4 | 3.36 | 4.57 | 4.75 | 2.98 | 1.88 | 1.17 | 1.19 | 0.56 | 23,800 |
| 3 | 2 11/16 | 1/2 | 12.5 | 3.69 | 4.59 | 4.85 | 3.06 | 1.99 | 1.12 | 1.15 | 0.55 | 24,500 |
| 3 1/16 | 2 3/4 | 9/16 | 14.2 | 4.18 | 5.26 | 5.70 | 3.43 | 2.31 | 1.12 | 1.17 | 0.56 | 27,400 |

Dimensions of 6, 8, and 10-Inch Carnegie Z-Bar Columns.

| Thickness of Metal. | A. | | | B. | | | C. | | | D. | | |
|------------------------|----------|----------|-----------|----------|----------|-----------|----------|----------|-----------|----------|----------|-----------|
| | 6 in. | 8 in. | 10 in. | 6 in. | 8 in. | 10 in. | 6 in. | 8 in. | 10 in. | 6 in. | 8 in. | 10 in. |
| in. | | | | | | | | | | | | |
| 1/4 | 12 3/4 | 15 5/16 | 16 11/16 | 3 1/8 | 4 1/8 | 5 5/32 | 5 9/16 | 6 7/16 | 6 9/16 | 3 1/8 | 3 5/8 | 3 5/8 |
| 5/16 | 12 7/8 | 15 3/8 | 16 11/16 | 3 7/32 | 4 7/32 | 5 5/32 | 5 9/16 | 6 7/16 | 6 9/16 | 3 1/8 | 3 5/8 | 3 5/8 |
| 3/8 | 12 5/8 | 15 1/2 | 16 13/16 | 3 3/16 | 4 5/16 | 5 1/4 | 5 7/16 | 6 7/16 | 6 9/16 | 3 1/8 | 3 5/8 | 3 5/8 |
| 7/16 | 12 11/16 | 15 1/16 | 16 15/16 | 3 9/32 | 4 7/32 | 5 11/32 | 5 7/16 | 6 1/4 | 6 9/16 | 3 1/8 | 3 5/8 | 3 5/8 |
| 1/2 | 12 7/16 | 15 3/16 | 16 1/2 | 3 1/4 | 4 5/16 | 5 1/4 | 5 5/16 | 6 1/4 | 6 3/8 | 3 1/8 | 3 5/8 | 3 5/8 |
| 9/16 | 12 9/16 | 15 5/16 | 16 5/8 | 3 11/32 | 4 13/32 | 5 11/32 | 5 5/16 | 6 1/4 | 6 3/8 | 3 1/8 | 3 5/8 | 3 5/8 |
| 5/8 | .. | 14 7/8 | 16 3/4 | .. | 4 5/16 | 5 7/16 | .. | 6 1/16 | 6 3/8 | .. | 3 5/8 | 3 5/8 |
| 11/16 | .. | 15 | 16 3/8 | .. | 4 13/32 | 5 11/32 | .. | 6 1/16 | 6 3/16 | .. | 3 5/8 | 3 5/8 |
| 3/4 | .. | 15 1/8 | 16 1/2 | .. | 4 1/2 | 5 7/16 | .. | 6 1/16 | 6 3/16 | .. | 3 5/8 | 3 5/8 |
| 13/16 | .. | .. | 16 5/8 | .. | .. | 5 17/32 | .. | .. | 6 3/16 | .. | .. | 3 5/8 |

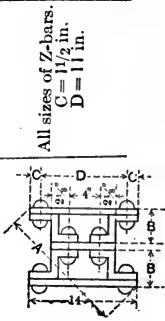
| Thickness of Metal. | E. | | | F. | | G. | | | H. | | I. | | |
|------------------------|----------|----------|-----------|----------|-----------------|----------|----------|-----------|----------|-----------------|----------|----------|-----------|
| | 6 in. | 8 in. | 10 in. | 6 in. | 8 and 10 in. | 6 in. | 8 in. | 10 in. | 6 in. | 8 and 10 in. | 6 in. | 8 in. | 10 in. |
| in. | | | | | | | | | | | | | |
| 1/4 | 3 | 3 1/2 | .. | 15/8 | 1 7/8 | 2 11/16 | 3 1/16 | .. | 8 1/2 | 10 | 3 1/4 | 4 1/4 | .. |
| 5/16 | 3 | 3 1/2 | 3 1/2 | 15/8 | 1 7/8 | 2 3/4 | 3 1/8 | 3 1/4 | 8 1/2 | 10 | 3 3/8 | 4 3/8 | 5 5/16 |
| 3/8 | 3 | 3 1/2 | 3 1/2 | 15/8 | 1 7/8 | 2 11/16 | 3 3/16 | 3 5/16 | 8 1/2 | 10 | 3 3/8 | 4 1/2 | 5 7/16 |
| 7/16 | 3 | 3 1/2 | 3 1/2 | 15/8 | 1 7/8 | 2 3/4 | 3 1/16 | 3 3/8 | 8 1/2 | 10 | 3 1/2 | 4 7/16 | 5 9/16 |
| 1/2 | 3 | 3 1/2 | 3 1/2 | 15/8 | 1 7/8 | 2 11/16 | 3 1/8 | 3 1/4 | 8 1/2 | 10 | 3 1/2 | 4 9/16 | 5 1/2 |
| 9/16 | 3 | 3 1/2 | 3 1/2 | 15/8 | 1 7/8 | 2 3/4 | 3 3/16 | 3 5/16 | 8 1/2 | 10 | 3 5/8 | 4 11/16 | 5 5/8 |
| 5/8 | .. | 3 1/2 | 3 1/2 | .. | 1 7/8 | .. | 3 1/16 | 3 3/8 | .. | 10 | .. | 4 5/8 | 5 3/4 |
| 11/16 | .. | 3 1/2 | 3 1/2 | .. | 1 7/8 | .. | 3 1/8 | 3 1/4 | .. | 10 | .. | 4 3/4 | 5 11/16 |
| 3/4 | .. | 3 1/2 | 3 1/2 | .. | 1 7/8 | .. | 3 3/16 | 3 5/16 | .. | 10 | .. | 4 7/8 | 5 13/16 |
| 13/16 | .. | .. | 3 1/2 | .. | 1 7/8 | .. | .. | 3 3/8 | .. | 10 | .. | .. | 5 15/16 |

- 6-in. col. { 4 Z-bars, 3-3 1/16 in. deep,
1 web plate 6 in. X thick. of Z-bars.
- 8-in. col. { 4 Z-bars, 4-4 1/8 in. deep,
1 web plate 7 in. X thick. of Z-bars.
- 10-in. col. { 4 Z-bars, 5-5 1/8 in. deep,
1 web plate 7 in. X thick. of Z-bars.
- All rivets or bolts 3/4 inch diameter.



Dimensions of 14-Inch Carnegie Z-Bar Columns.

| Thickness of Side Plates. | A. Inches. | | | | B. Inches. | | | |
|------------------------------|------------------------------|------------------------|-------------------------------|----------------------------|------------------------------|------------------------|-------------------------------|----------------------------|
| | 6 1/8 X 11/16 in. Z-bars. | 6 X 3/4 in. Z-bars. | 6 1/16 X 13/16 in. Z-bars. | 6 1/8 X 7/8 in. Z-bars. | 6 1/8 X 11/16 in. Z-bars. | 6 X 3/4 in. Z-bars. | 6 1/16 X 13/16 in. Z-bars. | 6 1/8 X 7/8 in. Z-bars. |
| 3/8 | 199/16 | 197/16 | 199/16 | 193/4 | 6 27/32 | 6 3/4 | 6 27/32 | 6 15/16 |
| 7/16 | 19 11/16 | 19 1/2 | 19 5/8 | 19 13/16 | 6 29/32 | 6 13/16 | 6 29/32 | 7 |
| 1/2 | 19 3/4 | 19 5/8 | 19 3/4 | 19 7/8 | 6 31/32 | 6 7/8 | 6 31/32 | 7 1/16 |
| 9/16 | 19 7/8 | 19 3/4 | 19 7/8 | 20 | 7 1/32 | 6 15/16 | 7 1/32 | 7 1/8 |
| 5/8 | 19 15/16 | 19 13/16 | 19 15/16 | 20 1/16 | 7 3/32 | 7 | 7 3/32 | 7 3/16 |
| 11/16 | 20 1/16 | 19 7/8 | 20 1/16 | 20 1/8 | 7 5/32 | 7 1/16 | 7 5/32 | 7 1/4 |
| 3/4 | 20 1/8 | 20 | 20 1/8 | 20 1/4 | 7 7/32 | 7 1/8 | 7 7/32 | 7 5/16 |
| 13/16 | 20 1/4 | 20 1/16 | 20 3/16 | 20 5/16 | 7 9/32 | 7 3/16 | 7 9/32 | 7 3/8 |
| 7/8 | 20 5/16 | 20 1/8 | 20 1/4 | 20 7/16 | 7 11/32 | 7 1/4 | 7 11/32 | 7 7/16 |



1 Web Plate, 8 in. X thick. of Z-bars.
2 Side Plates 14 in. wide 4 Z-bars.

Notes on Tables of Z-Bar and Channel Columns.

(Carnegie Steel Co., 1903.)

The tables of safe loads for steel Z-bar and channel columns are compiled on the basis of an allowable stress per square inch of 12,000 pounds, with a factor of safety of 4 for lengths of 90 radii and under and an allowable stress deduced from the formula $17,100 - 57 l \div r$ for lengths greater than 90 radii; l = length in feet; r = radius of gyration in inches. Calculations are made by means of Gordon's formula, modified for steel. The values used in these tables should be used only where the loads are mostly statical and equal or nearly so on opposite sides of the column. If the eccentricity is great or the load subject to sudden changes the values should be reduced according to circumstances. The safe loads given in the tables on channel columns range in value from $l \div r = 90$ to about $l \div r = 125$. The size and spacing of lattice bars of channel columns should be proportioned to the sections composing the column. They should not be less than $1\frac{1}{2}$ inch \times $\frac{5}{16}$ inch for 6-inch channels; $1\frac{3}{4}$ \times $\frac{5}{16}$ inch for 7- and 8-inch channels; $2 \times \frac{5}{16}$ inch for 9- and 10-inch channels; $2 \times \frac{3}{8}$ inch for 12-inch channels.

 Safe Loads in Tons (2000 Lb.) on Carnegie Z-Bar Columns
(Square Ends).

Dimensions and form of columns given in tables, p. 300.

6-INCH Z-BAR COLUMN.

| Length of Col. Feet. | Thickness of Metal, Inch. | | | | | | | | | |
|-------------------------------|---------------------------|------|------|------|-------|-------|-----|-------|-----|-------|
| | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 |
| r (min) = 1.86 | 1.90 | 1.88 | 1.93 | 1.90 | 1.95 | | | | | |
| 12 and under | 55.9 | 70.3 | 81.6 | 95.8 | 105.7 | 119.8 | ... | ... | ... | ... |
| 14 | 55.7 | 70.3 | 81.6 | 95.8 | 105.7 | 119.8 | ... | ... | ... | ... |
| 16 | 52.3 | 66.5 | 76.6 | 91.3 | 99.9 | 114.8 | ... | ... | ... | ... |
| 18 | 48.8 | 62.3 | 71.7 | 85.6 | 93.6 | 107.8 | ... | ... | ... | ... |
| 20 | 45.4 | 58.1 | 66.7 | 79.9 | 87.2 | 100.8 | ... | ... | ... | ... |
| 22 | 42.0 | 53.9 | 61.8 | 74.3 | 80.9 | 93.8 | ... | ... | ... | ... |
| 24 | 38.6 | 49.7 | 56.9 | 68.6 | 74.6 | 86.8 | ... | ... | ... | ... |
| 26 | 35.2 | 45.5 | 51.9 | 63.0 | 68.2 | 79.8 | ... | ... | ... | ... |
| 28 | 31.7 | 41.3 | 47.0 | 57.3 | 61.9 | 72.8 | ... | ... | ... | ... |
| 30 | 28.3 | 37.1 | 42.0 | 51.7 | 55.5 | 65.8 | ... | ... | ... | ... |

8-INCH Z-BAR COLUMN.

| | | | | | | | | | |
|------------------|------|------|-------|-------|-------|-------|-------|-------|-------|
| r (min) = 2.47 | 2.52 | 2.57 | 2.49 | 2.55 | 2.60 | 2.52 | 2.58 | 2.63 | |
| 18 and under | 67.5 | 84.8 | 102.4 | 114.2 | 131.2 | 148.5 | 157.5 | 174.3 | 191.2 |
| 20 | 65.0 | 82.5 | 100.5 | 110.5 | 128.2 | 146.4 | 153.3 | 171.3 | 189.6 |
| 22 | 61.9 | 78.7 | 95.9 | 105.3 | 122.4 | 139.9 | 146.2 | 163.5 | 181.3 |
| 24 | 58.8 | 74.8 | 91.3 | 100.1 | 116.5 | 133.4 | 139.1 | 155.8 | 173.0 |
| 26 | 55.7 | 71.0 | 86.8 | 94.8 | 110.6 | 126.9 | 132.0 | 148.1 | 164.7 |
| 28 | 52.6 | 67.1 | 82.3 | 89.6 | 104.7 | 120.3 | 124.8 | 140.4 | 156.4 |
| 30 | 49.4 | 63.3 | 77.7 | 84.4 | 98.8 | 113.8 | 117.7 | 132.7 | 148.2 |
| 32 | 46.3 | 59.5 | 73.2 | 79.2 | 93.0 | 107.3 | 110.6 | 125.0 | 139.9 |
| 34 | 43.2 | 55.6 | 68.7 | 74.0 | 87.1 | 100.8 | 103.5 | 117.3 | 131.6 |
| 36 | 40.1 | 51.8 | 64.1 | 68.7 | 81.2 | 94.3 | 96.4 | 109.6 | 123.3 |
| 38 | 37.0 | 48.0 | 59.6 | 63.5 | 75.3 | 87.8 | 89.4 | 101.9 | 115.0 |
| 40 | 33.9 | 44.1 | 55.0 | 58.3 | 69.5 | 81.3 | 82.2 | 94.2 | 106.7 |

**Safe Loads in Tons (2000 Lb.) on Carnegie Z-Bar Columns
(Square Ends). (Continued)**

10-INCH Z-BAR COLUMN.

| r (min.) = | | 3.08 | 3.13 | 3.18 | 3.10 | 3.15 | 3.21 | 3.13 | 3.18 | 3.25 |
|--------------|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 22 and under | ... | 94.7 | 114.2 | 133.9 | 147.0 | 166.2 | 185.6 | 196.0 | 214.9 | 234.0 |
| 24 | ... | 92.8 | 112.6 | 133.1 | 144.6 | 164.8 | 185.3 | 193.6 | 213.9 | 234.0 |
| 26 | ... | 89.3 | 108.6 | 128.3 | 139.2 | 158.7 | 178.7 | 186.5 | 206.2 | 226.6 |
| 28 | ... | 85.8 | 104.4 | 123.5 | 133.8 | 152.7 | 172.1 | 179.3 | 198.5 | 218.4 |
| 30 | ... | 82.3 | 100.2 | 118.7 | 128.4 | 146.7 | 165.5 | 172.2 | 190.8 | 210.2 |
| 32 | ... | 78.8 | 96.1 | 113.8 | 123.0 | 140.7 | 158.9 | 165.0 | 183.1 | 202.0 |
| 34 | ... | 75.3 | 91.9 | 109.1 | 117.6 | 134.7 | 152.3 | 157.9 | 175.4 | 193.8 |
| 36 | ... | 71.8 | 87.8 | 104.3 | 112.2 | 128.7 | 145.7 | 150.7 | 167.8 | 185.6 |
| 38 | ... | 68.3 | 83.6 | 99.5 | 106.8 | 122.7 | 139.1 | 143.6 | 160.0 | 177.4 |
| 40 | ... | 64.8 | 79.4 | 94.7 | 101.4 | 116.7 | 132.5 | 136.5 | 152.3 | 169.1 |
| 42 | ... | 61.3 | 75.3 | 89.9 | 96.0 | 110.6 | 125.9 | 129.4 | 144.6 | 160.9 |
| 44 | ... | 57.7 | 71.1 | 85.1 | 90.6 | 104.6 | 119.3 | 122.2 | 136.9 | 152.7 |
| 46 | ... | 54.2 | 67.0 | 80.3 | 85.2 | 98.6 | 112.7 | 115.1 | 129.2 | 144.5 |
| 48 | ... | 50.7 | 62.8 | 75.5 | 79.8 | 92.6 | 106.1 | 107.9 | 121.5 | 136.3 |
| 50 | ... | 47.2 | 58.6 | 70.7 | 74.4 | 86.6 | 99.5 | 100.8 | 113.8 | 128.1 |

**Safe Load in Tons (2000 Lb.) on 14-Inch Carnegie Z-Bar
Columns (Square Ends).**

Dimensions and form of column given in table, p. 300.

Section: 4 Z-bars $6\frac{1}{8} \times \frac{11}{16}$ in. 1 Web Plate $8 \times \frac{11}{16}$ in. 2 Side Plate
14 in. wide.

| Length of Column in Feet. | $14 \times \frac{3}{8}$ Plates = 166.6 lb. = 49.0 sq. in. | $14 \times \frac{7}{16}$ Plates = 172.6 lb. = 50.8 sq. in. | $14 \times \frac{1}{2}$ Plates = 178.5 lb. = 52.5 sq. in. | $14 \times \frac{9}{16}$ Plates = 184.5 lb. = 54.3 sq. in. | $14 \times \frac{5}{8}$ Plates = 190.4 lb. = 56.0 sq. in. | $14 \times \frac{11}{16}$ Plates = 196.4 lb. = 57.8 sq. in. | $14 \times \frac{3}{4}$ Plates = 202.3 lb. = 59.5 sq. in. | $14 \times \frac{13}{16}$ Plates = 208.4 lb. = 61.3 sq. in. | $14 \times \frac{7}{8}$ Plates = 214.2 lb. = 63.0 sq. in. |
|---------------------------------|--|---|--|---|--|--|--|--|--|
| r (min.) = | 3.80 | 3.81 | 3.82 | 3.82 | 3.83 | 3.84 | 3.85 | 3.85 | 3.85 |
| 28 and under | 294.0 | 304.5 | 315.0 | 325.5 | 336.0 | 346.5 | 357.0 | 367.5 | 378.0 |
| 30 | 286.6 | 297.2 | 307.7 | 318.3 | 328.9 | 339.5 | 350.0 | 360.4 | 370.9 |
| 32 | 277.8 | 288.1 | 298.3 | 308.6 | 318.9 | 329.2 | 339.4 | 349.5 | 359.7 |
| 34 | 269.0 | 278.9 | 288.9 | 298.9 | 308.9 | 318.9 | 328.8 | 338.6 | 348.6 |
| 36 | 260.1 | 269.8 | 279.5 | 289.2 | 298.9 | 308.6 | 318.2 | 327.7 | 337.4 |
| 38 | 251.3 | 260.7 | 270.1 | 279.5 | 289.0 | 298.3 | 307.6 | 316.8 | 326.2 |
| 40 | 242.5 | 251.6 | 260.7 | 269.7 | 278.9 | 288.0 | 297.0 | 306.0 | 315.0 |
| 42 | 233.7 | 242.5 | 251.3 | 260.1 | 269.0 | 277.8 | 286.4 | 295.1 | 303.8 |
| 44 | 224.9 | 233.3 | 241.9 | 250.4 | 258.9 | 267.4 | 275.8 | 284.2 | 292.3 |
| 46 | 216.0 | 224.3 | 232.4 | 240.7 | 249.0 | 257.2 | 265.2 | 273.3 | 281.6 |
| 48 | 207.2 | 215.1 | 223.0 | 230.9 | 238.9 | 246.9 | 254.6 | 262.4 | 270.5 |
| 50 | 198.4 | 206.0 | 213.6 | 221.3 | 229.0 | 236.5 | 244.0 | 251.5 | 259.1 |

Safe Load in Tons (2000 Lb.) on 14-Inch Carnegie Z-Bar Columns (Square Ends). (Continued)

 Section: 4 Z-bars $6 \times \frac{3}{4}$ in. 1 Web Plate $8 \times \frac{3}{4}$ in. 2 Side Plates 14 in. wide.

| Length of Column in Feet. | $14 \times \frac{3}{8}$ Plates = 173.4 lb. = 51.0 sq. in. | $14 \times \frac{7}{16}$ Plates = 179.4 lb. = 52.8 sq. in. | $14 \times \frac{1}{2}$ Plates = 185.3 lb. = 54.5 sq. in. | $14 \times \frac{9}{16}$ Plates = 191.4 lb. = 56.3 sq. in. | $14 \times \frac{5}{8}$ Plates = 197.2 lb. = 58.0 sq. in. | $14 \times \frac{11}{16}$ Plates = 203.2 lb. = 59.8 sq. in. | $14 \times \frac{3}{4}$ Plates = 209.1 lb. = 61.5 sq. in. | $14 \times \frac{13}{16}$ Plates = 215.1 lb. = 63.3 sq. in. | $14 \times \frac{7}{8}$ Plates = 221.0 lb. = 65.0 sq. in. |
|---------------------------|---|--|---|--|---|---|---|---|---|
| r (min.) = | 3.75 | 3.76 | 3.77 | 3.78 | 3.79 | 3.80 | 3.80 | 3.81 | 3.82 |
| 28 and under | 306.0 | 316.5 | 327.0 | 337.5 | 348.0 | 358.5 | 369.0 | 379.5 | 390.0 |
| 30 | 296.7 | 307.2 | 317.8 | 328.3 | 338.9 | 349.4 | 359.9 | 370.5 | 381.1 |
| 32 | 287.4 | 297.6 | 307.9 | 318.2 | 328.4 | 338.7 | 348.9 | 359.1 | 369.4 |
| 34 | 278.1 | 288.0 | 298.0 | 308.0 | 318.0 | 327.9 | 337.8 | 347.8 | 357.8 |
| 36 | 268.8 | 278.4 | 288.2 | 297.9 | 307.4 | 317.2 | 326.8 | 336.4 | 346.1 |
| 38 | 259.5 | 268.8 | 278.3 | 287.7 | 297.0 | 306.4 | 315.7 | 325.1 | 334.5 |
| 40 | 250.2 | 259.3 | 268.4 | 277.5 | 286.5 | 295.6 | 304.7 | 313.7 | 322.8 |
| 42 | 240.9 | 249.7 | 258.5 | 267.3 | 276.1 | 284.8 | 293.6 | 302.4 | 311.2 |
| 44 | 231.6 | 240.1 | 248.6 | 257.1 | 265.6 | 274.1 | 282.5 | 291.0 | 299.6 |
| 46 | 222.4 | 230.5 | 238.7 | 246.9 | 255.1 | 263.4 | 271.5 | 279.7 | 287.9 |
| 48 | 213.0 | 220.9 | 228.8 | 236.8 | 244.7 | 252.6 | 260.4 | 268.3 | 276.2 |
| 50 | 203.7 | 211.3 | 219.0 | 226.6 | 234.2 | 241.8 | 249.4 | 257.0 | 264.6 |

 Section: 4 Z-bars $6\frac{1}{16} \times \frac{13}{16}$ in. 1 Web Plate $8 \times \frac{13}{16}$ in. 2 Side Plates 14 in. wide.

| Length of Column in Feet. | $14 \times \frac{1}{8}$ Plates = 185.6 lb. = 54.6 sq. in. | $14 \times \frac{7}{16}$ Plates = 191.5 lb. = 56.3 sq. in. | $14 \times \frac{1}{2}$ Plates = 197.5 lb. = 58.1 sq. in. | $14 \times \frac{9}{16}$ Plates = 203.4 lb. = 59.8 sq. in. | $14 \times \frac{5}{8}$ Plates = 209.4 lb. = 61.6 sq. in. | $14 \times \frac{11}{16}$ Plates = 215.3 lb. = 63.3 sq. in. | $14 \times \frac{3}{4}$ Plates = 221.3 lb. = 65.1 sq. in. | $14 \times \frac{13}{16}$ Plates = 227.2 lb. = 66.8 sq. in. | $14 \times \frac{7}{8}$ Plate = 233.2 lb. = 68.6 sq. in. |
|---------------------------|---|--|---|--|---|---|---|---|--|
| (min.) = | 3.73 | 3.74 | 3.75 | 3.76 | 3.77 | 3.78 | 3.78 | 3.79 | 3.80 |
| 26 and under | 327.5 | 338.0 | 348.5 | 359.0 | 369.5 | 380.0 | 390.5 | 401.0 | 411.5 |
| 28 | 326.7 | 337.5 | 348.5 | 359.0 | 369.5 | 380.0 | 390.5 | 401.0 | 411.5 |
| 30 | 316.7 | 327.2 | 337.7 | 348.3 | 358.9 | 369.5 | 380.0 | 390.6 | 401.1 |
| 32 | 306.6 | 318.0 | 327.2 | 337.4 | 347.7 | 358.0 | 368.2 | 378.5 | 388.8 |
| 34 | 296.6 | 306.6 | 316.6 | 326.5 | 336.5 | 346.5 | 356.4 | 366.4 | 376.4 |
| 36 | 286.7 | 296.4 | 306.0 | 315.7 | 325.3 | 335.0 | 344.7 | 354.3 | 364.0 |
| 38 | 276.7 | 286.0 | 295.4 | 304.8 | 314.2 | 323.6 | 332.9 | 342.3 | 351.7 |
| 40 | 266.6 | 275.7 | 284.8 | 293.9 | 303.0 | 312.1 | 321.2 | 330.3 | 339.3 |
| 42 | 256.6 | 265.5 | 274.3 | 283.0 | 291.8 | 300.6 | 309.4 | 318.2 | 327.0 |
| 44 | 246.6 | 255.2 | 263.6 | 272.2 | 280.6 | 289.2 | 297.6 | 306.1 | 314.6 |
| 46 | 236.6 | 244.9 | 253.0 | 261.3 | 269.5 | 277.7 | 285.8 | 294.0 | 302.3 |
| 48 | 226.7 | 234.6 | 242.5 | 250.4 | 258.3 | 266.2 | 274.1 | 282.0 | 290.0 |
| 50 | 216.6 | 224.3 | 231.9 | 239.5 | 247.1 | 254.8 | 262.3 | 269.9 | 277.6 |

**Safe Load in Tons (2000 Lb.) on 14-Inch Carnegie Z-Bar
Columns (Square Ends). (Continued)**

Section: 4 Z-bars $6\frac{1}{8} \times 7\frac{7}{8}$ in. 1 Web Plate $8 \times 7\frac{7}{8}$ in. 2 Side Plates 14 in. wide.

| Length of Column in Feet. | $14 \times \frac{3}{8}$ Plates = 197.8 lb. = 56.2 sq. in. | $14 \times \frac{7}{16}$ Plates = 203.8 lb. = 59.9 sq. in. | $14 \times \frac{1}{2}$ Plates = 209.7 lb. = 61.7 sq. in. | $14 \times \frac{9}{16}$ Plates = 215.7 lb. = 63.4 sq. in. | $14 \times \frac{5}{8}$ Plates = 221.6 lb. = 65.2 sq. in. | $14 \times \frac{11}{16}$ Plates = 227.6 lb. = 66.9 sq. in. | $14 \times \frac{3}{4}$ Plates = 233.5 lb. = 68.7 sq. in. | $14 \times \frac{13}{16}$ Plates = 239.5 lb. = 70.4 sq. in. | $14 \times \frac{7}{8}$ Plates = 245.4 lb. = 72.2 sq. in. |
|---------------------------------|--|---|--|---|--|--|--|--|--|
| r (min.) = | 3.71 | 3.72 | 3.73 | 3.74 | 3.75 | 3.76 | 3.77 | 3.77 | 3.78 |
| 26 and under | 349.1 | 359.6 | 370.1 | 380.6 | 391.1 | 401.6 | 412.1 | 422.6 | 433.1 |
| 28 | 347.4 | 358.3 | 369.1 | 380.0 | 390.9 | 401.6 | 412.1 | 422.6 | 433.1 |
| 30 | 336.7 | 347.2 | 357.9 | 368.4 | 378.9 | 389.5 | 400.1 | 410.7 | 421.2 |
| 32 | 326.0 | 336.3 | 346.6 | 356.8 | 367.1 | 377.3 | 387.6 | 397.9 | 408.2 |
| 34 | 315.3 | 325.2 | 335.2 | 345.2 | 355.1 | 365.2 | 375.2 | 385.1 | 395.1 |
| 36 | 304.5 | 314.2 | 324.0 | 333.6 | 343.3 | 353.0 | 362.7 | 372.4 | 382.0 |
| 38 | 293.8 | 303.2 | 312.6 | 322.0 | 331.4 | 340.8 | 350.2 | 359.6 | 369.0 |
| 40 | 283.1 | 292.2 | 301.3 | 310.4 | 319.5 | 328.6 | 337.7 | 346.8 | 355.9 |
| 42 | 272.3 | 281.2 | 290.0 | 298.8 | 307.6 | 316.4 | 325.2 | 334.0 | 342.8 |
| 44 | 261.6 | 270.2 | 278.7 | 287.2 | 295.7 | 304.2 | 312.7 | 321.2 | 329.8 |
| 46 | 250.9 | 259.1 | 267.4 | 275.6 | 283.8 | 292.1 | 300.3 | 308.5 | 316.7 |
| 48 | 240.2 | 248.1 | 256.1 | 264.0 | 272.0 | 279.8 | 287.8 | 295.7 | 303.6 |
| 50 | 229.5 | 237.1 | 244.8 | 252.4 | 260.0 | 267.6 | 275.3 | 283.0 | 290.6 |



Dimensions of and Safe Loads on Carnegie Channel Columns, Tons (2000 Lb.).

Column comprises 2 Channels Latticed or with 2 Side Plates. (Square Ends.)

| Depth of Channel, Inches. | Wt. of Channel, lb. per ft. | Width of Side Pl., Inches. | B-inches. | C-inches. | Length of Col., Feet. | Latticed. | 1/4 Plates. | 5/16 Plates. | 3/8 Plates. | 7/16 Plates. | 1/2 Plates. | 9/16 Plates. | 5/8 Plates. | 11/16 Plates. | 3/4 Plates. |
|---------------------------|-----------------------------|----------------------------|-----------|-----------|-----------------------|-----------|-------------|--------------|-------------|--------------|-------------|--------------|-------------|---------------|-------------|
| 6 | 8 | 8 | 37/8 | 53/4 | r = | 2.33 | 2.32 | 2.32 | 2.32 | 2.32 | 2.32 | 2.32 | 2.32 | 2.32 | |
| | | | | | 16 | 28.6 | 52.6 | 58.6 | 64.6 | 70.6 | 76.6 | 82.6 | 88.6 | 94.6 | ... |
| | | | | | 18 | 28.1 | 51.7 | 57.5 | 63.4 | 69.3 | 75.2 | 81.1 | 87.0 | 92.9 | ... |
| | | | | | 20 | 26.7 | 49.1 | 54.7 | 60.3 | 65.8 | 71.4 | 77.0 | 82.6 | 88.2 | ... |
| | | | | | 22 | 25.3 | 46.5 | 51.8 | 57.1 | 62.4 | 67.7 | 73.0 | 78.2 | 83.5 | ... |
| | | | | | 24 | 23.9 | 43.9 | 48.9 | 53.9 | 58.9 | 63.9 | 68.9 | 73.9 | 78.9 | ... |
| | 15.5 | 8 | 37/8 | 53/4 | r = | 2.00 | 2.12 | 2.13 | 2.14 | 2.15 | 2.16 | 2.17 | 2.18 | 2.18 | |
| | | | | | 14 | 54.7 | 84.7 | 90.7 | 96.7 | 102.7 | 108.7 | 114.7 | 120.7 | 126.7 | ... |
| | | | | | 16 | 53.0 | 84.2 | 90.4 | 96.7 | 102.7 | 108.7 | 114.7 | 120.7 | 126.7 | ... |
| | | | | | 18 | 49.9 | 79.7 | 85.6 | 91.5 | 97.4 | 103.3 | 109.2 | 115.1 | 121.0 | ... |
| | | | | | 20 | 46.8 | 75.1 | 80.7 | 86.4 | 92.0 | 97.6 | 103.2 | 108.8 | 114.4 | ... |
| | | | | | 22 | ... | 70.5 | 75.9 | 81.2 | 86.5 | 91.8 | 97.1 | 102.5 | 107.8 | ... |
| 8 | 11 1/4 | 10 | 1 1/2 | 7 1/2 | r = | 3.11 | 3.03 | 3.02 | 3.01 | 3.00 | 2.99 | 2.98 | 2.98 | 2.97 | |
| | | | | | 22 | 40.2 | 70.2 | 77.7 | 85.2 | 92.7 | 100.2 | 107.7 | 115.2 | 122.7 | ... |
| | | | | | 24 | 39.6 | 68.4 | 75.5 | 82.7 | 89.8 | 97.0 | 104.1 | 111.2 | 118.4 | ... |
| | | | | | 26 | 38.1 | 65.7 | 72.6 | 79.4 | 86.3 | 93.1 | 100.1 | 106.8 | 113.7 | ... |
| | | | | | 28 | 36.6 | 63.1 | 69.7 | 76.2 | 82.8 | 89.3 | 95.9 | 102.4 | 109.0 | ... |
| | | | | | 30 | 35.2 | 60.5 | 66.7 | 73.0 | 79.2 | 85.5 | 91.8 | 98.0 | 104.2 | ... |
| | 21 1/4 | 10 | 5 1/2 | 7 1/2 | r = | 2.77 | 2.83 | 2.84 | 2.84 | 2.84 | 2.84 | 2.84 | 2.85 | 2.85 | |
| | | | | | 20 | 75.0 | 135.0 | 142.5 | 150.0 | 157.5 | 165.0 | 172.5 | 180.0 | 187.5 | ... |
| | | | | | 22 | 72.9 | 132.6 | 140.0 | 147.5 | 154.9 | 162.3 | 169.8 | 177.2 | 184.7 | ... |
| | | | | | 24 | 69.8 | 127.2 | 134.3 | 141.5 | 148.6 | 155.7 | 162.9 | 170.0 | 177.1 | ... |
| | | | | | 26 | 66.7 | 121.7 | 128.6 | 135.4 | 142.3 | 149.1 | 155.9 | 162.8 | 169.6 | ... |
| | | | | | 28 | 63.7 | 116.3 | 122.9 | 129.4 | 136.0 | 142.5 | 149.0 | 155.6 | 162.1 | ... |
| 10 | 15 | 12 | 7 | 9 1/2 | r = | 3.87 | | 3.74 | 3.72 | 3.70 | 3.68 | 3.67 | 3.65 | 3.64 | 3.63 |
| | | | | | 26 | ... | ... | ... | 107.5 | 116.5 | 125.5 | 134.5 | 143.5 | 152.5 | 161.5 |
| | | | | | 28 | 53.5 | ... | 98.5 | 107.0 | 115.7 | 124.4 | 133.1 | 141.8 | 150.5 | 159.2 |
| | | | | | 30 | 52.6 | ... | 95.3 | 103.7 | 112.2 | 120.5 | 128.9 | 137.3 | 145.7 | 154.2 |
| | | | | | 32 | 51.0 | ... | 92.3 | 100.4 | 108.6 | 116.7 | 124.8 | 132.9 | 141.0 | 149.1 |
| | | | | | 34 | 49.5 | ... | 89.3 | 97.1 | 105.0 | 112.8 | 120.6 | 128.4 | 136.2 | 144.0 |
| | 35 | 12 | 7 | 9 1/2 | 36 | 47.9 | ... | 86.3 | 93.8 | 101.4 | 108.9 | 116.4 | 123.9 | 131.4 | 139.0 |
| | | | | | 38 | 46.3 | ... | 83.3 | 90.5 | 97.8 | 105.0 | 112.2 | 119.4 | 126.7 | 133.9 |
| | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | |
| 10 | 35 | 12 | 7 | 9 1/2 | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | |
| | 35 | 12 | 7 | 9 1/2 | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | |

To above weights of column shaft add weights of rivets and lattice bars.

Dimensions of and Safe Loads on Carnegie Channel Columns, Tons (2000 Lb.).

Column comprises 2 Channels Latticed or with 2 Side Plates. (Square Ends.)

| Depth of Channel, Inches. | Wt. of Channel, lb. per ft. | Width of Side Pl., Inches. | B-inches. | C-inches. | Length of Col., Feet. | Latticed. | 1/4 Plates. | 5/16 Plates. | 3/8 Plates. | 7/16 Plates. | 1/2 Plates. | 9/16 Plates. | 5/8 Plates. | 11/16 Plates. | 3/4 Plates. |
|---------------------------|-----------------------------|----------------------------|-----------|-----------|-----------------------|-----------|-------------|--------------|-------------|--------------|-------------|--------------|-------------|---------------|-------------|
| | | | | | $r =$ | 4.61 | | 4.40 | 4.38 | 4.35 | 4.33 | 4.32 | 4.30 | 4.29 | 4.27 |
| 12 | 20 1/2 | 14 | 8 1/4 | 11 1/4 | 32 | ... | ... | 124.9 | 135.4 | 145.9 | 156.4 | 166.9 | 177.4 | 187.9 | 198.4 |
| | | | | | 34 | 72.4 | ... | 123.0 | 133.0 | 142.9 | 152.9 | 162.8 | 172.8 | 182.8 | 192.7 |
| | | | | | 36 | 70.9 | ... | 119.7 | 129.4 | 139.1 | 148.8 | 158.4 | 168.1 | 177.8 | 187.4 |
| | | | | | 38 | 69.1 | ... | 116.5 | 125.9 | 135.3 | 144.6 | 154.0 | 163.4 | 172.8 | 182.1 |
| | | | | | 40 | 67.3 | ... | 113.3 | 122.4 | 131.5 | 140.5 | 149.6 | 158.7 | 167.8 | 176.8 |
| | | | | | 42 | 65.5 | ... | 110.0 | 118.8 | 127.6 | 136.4 | 145.2 | 154.0 | 162.8 | 171.5 |
| | | | | | 44 | 63.7 | ... | 106.8 | 115.3 | 123.8 | 132.3 | 140.8 | 149.3 | 157.8 | 166.2 |
| | | | | | | | | | | | | | | | |
| 12 | 40 | 14 | 8 1/4 | 11 1/4 | $r =$ | 4.09 | Lat. | 13/16 Pl. | 7/8 Pl. | 15/16 Pl. | 1" Pl. | 1 1/8 Pl. | 1 1/4 Pl. | 1 3/8 Pl. | 1 1/2 Pl. |
| | | | | | 30 | 141.1 | 277.6 | 288.1 | 298.6 | 309.1 | 330.1 | 351.1 | 372.1 | 393.1 | ... |
| | | | | | 32 | 133.2 | 272.6 | 282.8 | 293.0 | 303.2 | 323.7 | 344.1 | 364.6 | 385.0 | ... |
| | | | | | 34 | 134.2 | 264.9 | 274.8 | 284.8 | 294.7 | 314.5 | 334.4 | 354.2 | 374.1 | ... |
| | | | | | 36 | 130.3 | 257.2 | 266.8 | 276.5 | 286.1 | 305.4 | 324.6 | 343.9 | 363.1 | ... |
| | | | | | 38 | 126.3 | 249.5 | 258.8 | 268.2 | 277.5 | 296.2 | 314.9 | 333.5 | 352.2 | ... |
| | | | | | 40 | 122.4 | 241.8 | 250.9 | 259.9 | 268.9 | 287.0 | 305.1 | 323.2 | 341.3 | ... |
| | | | | | 42 | 118.5 | 234.1 | 242.9 | 251.5 | 260.3 | 277.8 | 295.3 | 312.8 | 330.3 | ... |

To above weights of column shaft, add weights of rivets and lattice bars.

Bethlehem "Special," "Girder" and "H" Steel Beams. These beams are rolled on the Grey universal beam mill, and have wider flanges than the standard American forms of I-beams, which are rolled in grooved rolls. The special I-beams from 8 to 24 in. in depth have the same section modulus or coefficient of strength as the standard forms, but on account of putting a larger proportion of metal in the flanges they are 10% lighter. For equal weights of sections they have a coefficient of strength about 5% greater than the standard shapes. The 26, 28 and 30-in. beams are respectively equal in coefficient of strength to two 20-in. 65 lb., two 20-in. 80 lb., and two 24-in. 80 lb. standard beams.

The girder beams from 8 to 24 in. in depth have a coefficient of strength equal to that of two standard I-beams of minimum weight of the same depth, but weigh 12 1/2% less than the two combined.

The rolled H, or column sections are designed especially for columns of buildings. All shapes having the same section number are rolled from the same main rolls without change. Thus the 12-in. H column is rolled in 35 different weights, the sectional areas ranging from 11.76 to 79.06 sq. in.

The flanges of the special and girder beams have a uniform slope of 12 1/2%, and the flanges of the H sections a uniform slope of 2%.

The tables of special and girder beams give the sections and weights usually rolled. Intermediate and heavier weights may be obtained by special arrangement. The table of H columns gives only the minimum and maximum weights for each section number. Many intermediate weights are regularly made.

The coefficients of strength given in the tables are based on a maximum fiber stress of 16,000 lb. per sq. in., which is allowable for quiet loads, as in buildings. For moving loads the fiber stress of 12,500 lb. per sq. in. should be used, and the coefficients reduced proportionately. For suddenly applied loads, as in railroad bridges, they should be still further reduced. For a fiber stress of 8000 lb. per sq. in. the coefficients would be one half those given in the tables.

For further information see handbook of Structural Steel Shapes, Bethlehem Steel Co., South Bethlehem, Pa., 1907.

PROPERTIES OF BETHLEHEM GIRDER BEAMS.

| Depth of Beam, Inches. | Weight per Foot, Pounds. | Area of Section, Square Inches. | Thickness of Web, Inch. | Width of Flange, Inches. | Neutral Axis Perpendicular to Web at Center. | | | Coeff'ts of Strength for Fiber Stress of 16,000 Lbs. per Sq. In. for Buildings. | Maximum Safe Shear on Web, in Tons of 2000 Lbs. | Neutral Axis Coinci- dent with Center Line of Web. | |
|---------------------------|--------------------------------|------------------------------------|----------------------------|-----------------------------|--|----------------------------------|------------------------------|--|---|--|-----------------------------------|
| | | | | | Moment of Inertia. I | Radius of Gy- ration. r | Section Modu- lus S | | | Moment of Inertia. I' | Radius of Gy- ration. r' |
| 30 | 200.0 | 58.85 | 0.75 | 15.00 | 9154.7 | 12.47 | 610.3 | 6,510,000 | 95.2 | 599.7 | 3.19 |
| 30 | 175.0 | 51.35 | .68 | 12.00 | 7851.8 | 12.37 | 523.5 | 5,583,500 | 81.1 | 345.4 | 2.60 |
| 28 | 180.0 | 52.98 | .69 | 14.35 | 7269.0 | 11.72 | 519.2 | 5,538,200 | 81.3 | 507.6 | 3.09 |
| 28 | 162.5 | 47.81 | .65 | 12.00 | 6465.1 | 11.63 | 461.8 | 4,925,800 | 73.8 | 328.2 | 2.62 |
| 26 | 160.0 | 47.00 | .63 | 13.60 | 5618.7 | 10.93 | 432.2 | 4,610,200 | 68.3 | 414.5 | 2.97 |
| 26 | 150.0 | 44.13 | .62 | 12.00 | 5200.4 | 10.86 | 400.0 | 4,267,000 | 66.6 | 306.5 | 2.63 |
| 24 | 140.0 | 41.03 | .56 | 13.00 | 4241.9 | 10.17 | 353.5 | 3,770,700 | 54.9 | 338.3 | 2.87 |
| 24 | 120.0 | 35.31 | .51 | 12.00 | 3630.7 | 10.14 | 302.6 | 3,227,200 | 46.5 | 240.0 | 2.61 |
| 20 | 140.0 | 41.28 | .64 | 12.50 | 2938.3 | 8.44 | 293.8 | 3,134,200 | 62.4 | 334.3 | 2.85 |
| 20 | 112.0 | 32.88 | .52 | 12.00 | 2368.9 | 8.49 | 236.9 | 2,526,700 | 45.6 | 232.8 | 2.66 |
| 18 | 92.0 | 27.09 | .47 | 11.50 | 1595.3 | 7.67 | 177.3 | 1,890,800 | 37.1 | 172.4 | 2.52 |
| 15 | 140.0 | 41.28 | .80 | 11.75 | 1591.5 | 6.21 | 212.2 | 2,263,500 | 67.3 | 319.2 | 2.78 |
| 15 | 104.0 | 30.58 | .60 | 11.25 | 1219.7 | 6.32 | 162.6 | 1,734,700 | 47.4 | 203.3 | 2.58 |
| 15 | 73.0 | 21.52 | .42 | 10.50 | 886.5 | 6.42 | 118.2 | 1,260,900 | 28.8 | 116.6 | 2.33 |
| 12 | 70.0 | 20.60 | .445 | 10.00 | 540.9 | 5.12 | 90.2 | 961,600 | 28.0 | 109.5 | 2.31 |
| 12 | 55.0 | 16.12 | .35 | 9.75 | 432.0 | 5.18 | 72.0 | 768,000 | 19.7 | 76.1 | 2.17 |
| 10 | 44.0 | 12.95 | .30 | 9.00 | 244.3 | 4.34 | 48.9 | 521,200 | 14.3 | 53.6 | 2.03 |
| 9 | 38.0 | 11.18 | .29 | 8.50 | 169.8 | 3.90 | 37.7 | 402,500 | 12.8 | 40.7 | 1.91 |
| 8 | 32.5 | 9.52 | .28 | 8.00 | 113.9 | 3.46 | 28.5 | 303,800 | 11.4 | 30.3 | 1.78 |

W = Safe load in pounds uniformly distributed including weight of beam.

L = Span in feet. M = Moment of forces in foot-pounds. f = fiber stress.

$W = C/L$; $M = C/8$; $C = WL = 8M = 2/3 fS$.

Properties of Bethlehem Special I Beams.

| Depth of Beam, Inches. | Weight per Foot, Pounds. | Area of Section, Square Inches. | Thickness of Web, Inch. | Width of Flange, Inches. | Neutral Axis Perpendicular to Web at Center. | | | Coefficient of Strength for Fiber Stress of 16,000 Lbs. per Sq. In. for Buildings. | Maximum Safe Shear on Web, in Tons of 2000 Lbs. | Neutral Axis Coin- cident with Center Line of Web. | |
|---------------------------|-----------------------------|------------------------------------|----------------------------|-----------------------------|--|----------------------------------|-------------------------------|---|---|--|-----------------------------------|
| | | | | | Moment of Inertia. I | Radius of Gy- ration. r | Section Modu- lus. S | | | Moment of Inertia. I' | Radius of Gy- ration. r' |
| 30 | 120.0 | 35.25 | 0.52 | 10.00 | 5271 | 12.23 | 351.4 | 3,748,200 | 48.7 | 149.7 | 2.11 |
| 28 | 105.0 | 31.04 | .48 | 9.60 | 4089 | 11.43 | 292.1 | 3,115,700 | 41.5 | 122.6 | 1.98 |
| 26 | 90.0 | 26.63 | .44 | 9.15 | 3043 | 10.71 | 234.1 | 2,496,900 | 34.9 | 93.4 | 1.87 |
| 24 | 84.0 | 24.79 | .45 | 8.85 | 2392 | 9.82 | 199.3 | 2,125,900 | 36.3 | 82.0 | 1.82 |
| 24 | 82.0 | 24.33 | .50 | 8.83 | 2240 | 9.60 | 186.7 | 1,991,600 | 43.8 | 71.1 | 1.71 |
| 24 | 72.0 | 21.21 | .37 | 8.70 | 2091 | 9.93 | 174.2 | 1,858,100 | 24.4 | 67.7 | 1.79 |
| 20 | 82.0 | 24.23 | .57 | 8.51 | 1561 | 8.03 | 156.1 | 1,665,400 | 51.5 | 71.5 | 1.72 |
| 20 | 72.0 | 21.43 | .43 | 8.37 | 1468 | 8.28 | 146.8 | 1,565,800 | 32.7 | 67.6 | 1.78 |
| 20 | 68.0 | 19.95 | .49 | 7.69 | 1270 | 7.98 | 127.0 | 1,354,600 | 40.4 | 45.7 | 1.51 |
| 20 | 63.0 | 18.55 | .42 | 7.62 | 1223 | 8.12 | 122.3 | 1,304,500 | 31.1 | 44.3 | 1.54 |
| 20 | 60.0 | 17.65 | .375 | 7.58 | 1193 | 8.22 | 119.3 | 1,272,600 | 25.3 | 43.4 | 1.57 |
| 20 | 58.5 | 17.15 | .35 | 7.55 | 1176 | 8.28 | 117.6 | 1,254,800 | 22.2 | 43.0 | 1.58 |
| 18 | 58.5 | 17.29 | .48 | 7.47 | 883.6 | 7.15 | 98.2 | 1,047,500 | 37.4 | 35.9 | 1.44 |
| 18 | 52.5 | 15.40 | .375 | 7.37 | 832.9 | 7.35 | 92.5 | 987,200 | 24.8 | 34.4 | 1.49 |
| 18 | 48.5 | 14.23 | .31 | 7.30 | 801.3 | 7.50 | 89.0 | 949,800 | 17.4 | 33.4 | 1.53 |
| 15 | 72.0 | 21.27 | .54 | 7.15 | 797.9 | 6.13 | 106.4 | 1,134,800 | 41.2 | 55.1 | 1.61 |
| 15 | 64.0 | 18.85 | .60 | 7.20 | 666.8 | 5.95 | 88.9 | 948,100 | 46.6 | 40.8 | 1.47 |
| 15 | 54.0 | 15.85 | .40 | 7.00 | 610.5 | 6.21 | 81.4 | 868,100 | 26.5 | 37.2 | 1.53 |
| 15 | 46.0 | 13.46 | .43 | 6.81 | 484.6 | 5.99 | 64.6 | 689,200 | 29.1 | 24.2 | 1.34 |
| 15 | 42.0 | 12.41 | .36 | 6.74 | 464.9 | 6.12 | 62.0 | 661,200 | 22.1 | 23.4 | 1.37 |
| 15 | 38.0 | 11.21 | .28 | 6.66 | 442.4 | 6.28 | 59.0 | 629,200 | 14.2 | 22.5 | 1.42 |
| 12 | 36.0 | 10.63 | .31 | 6.30 | 270.2 | 5.04 | 45.0 | 480,300 | 16.2 | 20.4 | 1.38 |
| 12 | 31.0 | 9.13 | .31 | 6.16 | 225.2 | 4.97 | 37.5 | 400,300 | 16.0 | 14.7 | 1.27 |
| 12 | 28.5 | 8.41 | .25 | 6.10 | 216.6 | 5.07 | 36.1 | 385,000 | 11.2 | 14.2 | 1.30 |
| 10 | 27.5 | 8.05 | .34 | 5.94 | 134.6 | 4.09 | 26.9 | 287,300 | 16.7 | 11.7 | 1.20 |
| 10 | 24.5 | 7.15 | .25 | 5.85 | 127.1 | 4.22 | 25.4 | 271,300 | 10.6 | 11.1 | 1.24 |
| 10 | 22.5 | 6.65 | .20 | 5.80 | 122.8 | 4.27 | 24.6 | 262,000 | 7.3 | 10.8 | 1.27 |
| 9 | 23.0 | 6.76 | .31 | 5.50 | 92.4 | 3.70 | 20.5 | 219,100 | 13.8 | 8.5 | 1.12 |
| 9 | 21.0 | 6.22 | .25 | 5.44 | 88.8 | 3.78 | 19.7 | 210,300 | 10.0 | 8.2 | 1.15 |
| 9 | 19.0 | 5.68 | .19 | 5.38 | 85.1 | 3.87 | 18.9 | 201,800 | 6.5 | 7.9 | 1.18 |
| 8 | 21.25 | 6.25 | .36 | 5.37 | 64.7 | 3.22 | 16.2 | 172,500 | 15.3 | 6.8 | 1.05 |
| 8 | 18.00 | 5.37 | .25 | 5.26 | 60.0 | 3.34 | 15.0 | 160,000 | 9.5 | 6.4 | 1.09 |
| 8 | 16.25 | 4.81 | .18 | 5.19 | 57.0 | 3.44 | 14.3 | 152,000 | 5.7 | 6.1 | 1.12 |

W = Safe load in pounds uniformly distributed including weight of beam.

L = Span in feet. M = Moment of forces in foot-pounds. f = fiber stress.

C = Coefficients given in the table.

$W = C/L$; $M = C/S$; $C = WL = SM = 2/3 fS$.

**Dimensions and Properties of Bethlehem Rolled Steel.
14-Inch H Columns.**

Table greatly condensed from original.*

| Section Number. | Weight of Section, Lbs. per Foot. | Dimensions in Inches. | | | | | Axis Perpen. to Web. | | | Axis Center of Web. | | |
|------------------|--------------------------------------|--------------------------------|---------------------------------|-----------------------|----------------------|------------------------------------|-------------------------|---------------------|------------------------|------------------------|---------------------|------------------------|
| | | Depth. | Mean Thickness of Flange. | Breadth of Flange. | Thickness of Web. | Area of Section, Square Inches. | Moment of Inertia. | Section Modulus. | Radius of Gyration. | Moment of Inertia. | Section Modulus. | Radius of Gyration. |
| H14 _s | 42.6 | 13 ³ / ₈ | 1 ¹ / ₂ | 8.00 | 0.33 | 12.53 | 400.8 | 59.9 | 5.66 | 43.6 | 10.9 | 1.87 |
| | 93.7 | 14 | 1 ³ / ₁₆ | 13.00 | .51 | 27.56 | 1004.7 | 143.5 | 6.04 | 288.5 | 44.4 | 3.24 |
| H14 | 98.8 | 14 | 1 ³ / ₁₆ | 14.00 | .51 | 29.06 | 1070.6 | 153.0 | 6.07 | 355.9 | 50.8 | 3.50 |
| | 162.2 | 15 | 1 ⁵ / ₁₆ | 14.31 | .82 | 47.71 | 1894.0 | 252.5 | 6.31 | 625.1 | 87.4 | 3.62 |
| H14 _a | 164.4 | 15 | 1 ⁵ / ₁₆ | 14.57 | .82 | 48.36 | 1924.7 | 256.6 | 6.32 | 659.8 | 90.6 | 3.69 |
| | 222.3 | 15 ⁷ / ₈ | 1 ³ / ₄ | 14.84 | 1.09 | 65.39 | 2774.5 | 349.5 | 6.51 | 936.6 | 126.2 | 3.78 |
| H14 _b | 230.8 | 16 | 1 ¹³ / ₁₆ | 14.88 | 1.13 | 67.89 | 2905.9 | 363.2 | 6.55 | 978.7 | 131.5 | 3.80 |
| | 291.2 | 16 ⁷ / ₈ | 2 ¹ / ₄ | 15.16 | 1.41 | 85.63 | 3897.7 | 462.0 | 6.75 | 1290.7 | 170.3 | 3.88 |

13-Inch H Columns.

| | | | | | | | | | | | | |
|------------------|-------|--------------------------------|---------------------------------|-------|------|-------|--------|-------|------|--------|-------|------|
| H13 _s | 41.2 | 12 ³ / ₈ | 1 ¹ / ₂ | 8.00 | 0.33 | 12.12 | 334.5 | 54.1 | 5.25 | 43.2 | 10.8 | 1.89 |
| | 86.6 | 13 | 1 ³ / ₁₆ | 12.04 | .51 | 25.48 | 793.6 | 122.1 | 5.58 | 229.9 | 38.2 | 3.00 |
| H13 | 91.5 | 13 | 1 ³ / ₁₆ | 13.00 | .51 | 26.93 | 847.9 | 130.5 | 5.61 | 286.7 | 44.1 | 3.26 |
| | 150.5 | 14 | 1 ⁵ / ₁₆ | 13.31 | .82 | 44.27 | 1511.4 | 215.9 | 5.84 | 504.9 | 75.9 | 3.38 |
| H13 _a | 156.4 | 14 | 1 ⁵ / ₁₆ | 14.00 | .82 | 45.99 | 1581.6 | 225.9 | 5.86 | 585.1 | 83.6 | 3.57 |
| | 219.8 | 15 | 1 ¹³ / ₁₆ | 14.31 | 1.13 | 64.64 | 2404.9 | 320.7 | 6.10 | 870.2 | 121.6 | 3.67 |
| H13 _b | 226.5 | 15 | 1 ¹³ / ₁₆ | 14.88 | 1.13 | 66.62 | 2492.7 | 332.4 | 6.12 | 975.8 | 131.2 | 3.83 |
| | 285.9 | 15 ⁷ / ₈ | 2 ¹ / ₄ | 15.16 | 1.41 | 84.09 | 3361.9 | 423.6 | 6.32 | 1287.6 | 169.9 | 3.91 |

12-Inch H Columns.

| | | | | | | | | | | | | |
|------------------|-------|--------------------------------|-------------------------------|-------|------|-------|--------|-------|------|--------|-------|------|
| H12 _s | 40.0 | 11 ¹ / ₂ | 1 ¹ / ₂ | 8.00 | 0.33 | 11.76 | 282.1 | 41.9 | 4.90 | 42.8 | 10.7 | 1.91 |
| | 73.4 | 12 | 3 ⁴ / ₄ | 11.04 | .47 | 21.60 | 572.8 | 95.5 | 5.15 | 163.7 | 29.7 | 2.75 |
| H12 | 78.0 | 12 | 3 ⁴ / ₄ | 12.00 | .47 | 22.94 | 615.6 | 102.6 | 5.18 | 208.1 | 34.7 | 3.01 |
| | 132.5 | 13 | 1 ¹ / ₄ | 12.31 | .78 | 38.97 | 1141.3 | 175.6 | 5.41 | 380.7 | 61.9 | 3.13 |
| H12 _a | 138.1 | 13 | 1 ¹ / ₄ | 13.00 | .78 | 40.61 | 1198.8 | 184.4 | 5.43 | 446.4 | 68.7 | 3.32 |
| | 197.1 | 14 | 1 ³ / ₄ | 13.31 | 1.09 | 57.96 | 1862.2 | 266.0 | 5.67 | 676.6 | 101.7 | 3.42 |
| H12 _b | 204.9 | 14 | 1 ³ / ₄ | 14.00 | 1.09 | 60.27 | 1950.8 | 278.7 | 5.69 | 784.8 | 112.1 | 3.61 |
| | 268.8 | 15 | 2 ¹ / ₄ | 14.32 | 1.41 | 79.06 | 2777.0 | 370.3 | 5.93 | 1086.2 | 151.7 | 3.71 |

* Only the minimum and maximum weights of each section number are given here. The original table gives many intermediate weights.

Dimensions and Properties of Bethlehem Rolled Steel.

11-Inch H Columns.

| Section Number. | Weight of Section, Lbs. per Foot. | Dimensions in Inches. | | | | Area of Section, Square Inches. | Axis Perpen. to Web. | | | Axis Center of Web. | | |
|-----------------|--------------------------------------|--------------------------|---------------------------------|-----------------------|----------------------|------------------------------------|-------------------------|---------------------|------------------------|------------------------|---------------------|------------------------|
| | | Depth. | Mean Thickness of Flange. | Breadth of Flange. | Thickness of Web. | | Moment of Inertia. | Section Modulus. | Radius of Gyration. | Moment of Inertia. | Section Modulus. | Radius of Gyration. |
| H11 s | 38.4 | 10 ^{5/8} | 1/2 | 8.00 | 0.32 | 11.30 | 234.1 | 44.1 | 4.55 | 42.4 | 10.6 | 1.94 |
| | 61.3 | 11 | 11/16 | 10.03 | .43 | 18.02 | 401.2 | 73.0 | 4.72 | 112.6 | 22.4 | 2.50 |
| H11 | 65.5 | 11 | 11/16 | 11.00 | .43 | 19.26 | 434.6 | 79.0 | 4.75 | 147.0 | 26.7 | 2.76 |
| | 115.5 | 12 | 1 3/16 | 11.31 | .74 | 33.98 | 843.1 | 140.5 | 4.98 | 280.7 | 49.6 | 2.87 |
| H11 a | 120.9 | 12 | 1 3/16 | 12.00 | .74 | 35.54 | 889.4 | 148.2 | 5.00 | 333.5 | 55.6 | 3.06 |
| | 175.8 | 13 | 1 11/16 | 12.32 | 1.06 | 51.70 | 1417.0 | 218.0 | 5.24 | 517.9 | 84.1 | 3.17 |

10-Inch H Columns.

| | | | | | | | | | | | | |
|-------|-------|------------------|-------|-------|------|-------|--------|-------|------|-------|------|------|
| H10 s | 37.2 | 9 ^{3/4} | 1/2 | 8.00 | 0.32 | 10.95 | 192.0 | 39.4 | 4.19 | 41.9 | 10.5 | 1.96 |
| | 50.6 | 10 | 5/8 | 9.04 | .40 | 14.88 | 272.5 | 54.5 | 4.28 | 75.1 | 16.6 | 2.25 |
| H10 | 54.1 | 10 | 5/8 | 10.00 | .39 | 15.91 | 296.8 | 59.4 | 4.32 | 100.4 | 20.1 | 2.51 |
| | 99.7 | 11 | 1 1/8 | 10.31 | .70 | 29.32 | 607.0 | 110.4 | 4.55 | 201.7 | 39.1 | 2.62 |
| H10 a | 104.7 | 11 | 1 1/8 | 11.00 | .70 | 30.80 | 643.6 | 117.0 | 4.57 | 243.7 | 44.3 | 2.81 |
| | 155.2 | 12 | 1 5/8 | 11.32 | 1.02 | 45.64 | 1053.6 | 175.6 | 4.80 | 387.2 | 68.4 | 2.91 |

9-Inch H Columns.

| | | | | | | | | | | | | |
|------|-------|------------------|--------|-------|------|-------|-------|-------|------|-------|------|------|
| H9 s | 28.8 | 8 ^{3/4} | 7/16 | 7.00 | 0.28 | 8.46 | 119.3 | 27.3 | 3.76 | 24.7 | 7.0 | 1.71 |
| | 40.6 | 9 | 9/16 | 8.04 | .36 | 11.95 | 177.0 | 39.3 | 3.85 | 47.6 | 11.8 | 2.00 |
| H9 | 43.8 | 9 | 9/16 | 9.00 | .35 | 12.88 | 194.7 | 43.3 | 3.89 | 65.9 | 14.6 | 2.26 |
| | 85.3 | 10 | 1 1/16 | 9.32 | .67 | 25.08 | 424.6 | 84.9 | 4.11 | 140.9 | 30.2 | 2.37 |
| H9 a | 90.0 | 10 | 1 1/16 | 10.00 | .67 | 26.46 | 452.6 | 90.5 | 4.14 | 173.1 | 34.6 | 2.56 |
| | 135.6 | 11 | 1 9/16 | 10.31 | .98 | 39.87 | 762.8 | 138.7 | 4.38 | 281.6 | 54.6 | 2.66 |

8-Inch H Columns.

| | | | | | | | | | | | | |
|------|-------|------------------|-------|------|------|-------|-------|-------|------|-------|------|------|
| H8 s | 27.7 | 7 ^{7/8} | 7/16 | 7.00 | 0.28 | 8.15 | 93.6 | 23.8 | 3.39 | 24.4 | 7.0 | 1.73 |
| | 31.8 | 8 | 1/2 | 7.04 | .32 | 9.35 | 109.1 | 27.3 | 3.42 | 28.5 | 8.1 | 1.74 |
| H8 | 34.6 | 8 | 1/2 | 8.00 | .31 | 10.17 | 121.5 | 30.4 | 3.46 | 41.1 | 10.3 | 2.01 |
| | 71.6 | 9 | 1 | 8.32 | .63 | 21.05 | 285.6 | 63.5 | 3.68 | 94.4 | 22.7 | 2.12 |
| H8 a | 76.0 | 9 | 1 | 9.00 | .63 | 22.35 | 306.8 | 68.2 | 3.70 | 118.9 | 26.4 | 2.31 |
| | 117.1 | 10 | 1 1/2 | 9.31 | .94 | 34.45 | 535.9 | 107.2 | 3.94 | 199.3 | 42.8 | 2.41 |

TORSIONAL STRENGTH.

Let a horizontal shaft of diameter = d be fixed at one end, and at the other or free end, at a distance = l from the fixed end, let there be fixed a horizontal lever arm with a weight = P acting at a distance = a from the axis of the shaft so as to twist it; then Pa = moment of the applied force.

Resisting moment = twisting moment = SJ/c , in which S = unit shearing resistance, J = polar moment of inertia of the section with respect to the axis, and c = distance of the most remote fiber from the axis, in a cross-section. For a circle with diameter d

$$J = 1/32 \pi d^4; \quad c = 1/2 d;$$

$$Pa = \frac{SJ}{c} = \frac{\pi d^3 S}{16} = \frac{d^3 S}{5.1} = 0.1963 d^3 S; \quad d = \sqrt[3]{\frac{5.1 Pa}{S}}.$$

For hollow shafts of external diameter d and internal diameter d_1 ,

$$Pa = 0.1963 \frac{d^4 - d_1^4}{d} S; \quad d = \sqrt[3]{\frac{5.1 Pa}{\left(1 - \frac{d_1^4}{d^4}\right) S}}.$$

For a rectangular bar in which b and d are the long and short sides of the rectangle, $Pa = 0.2222 bd^2 S$; and for a square bar with side d , $Pa = 0.2222 d^3 S$. (Merriman, "Mechanics of Materials," 10th ed.)

The above formulæ are based on the supposition that the shearing resistance at any point of the cross-section is proportional to its distance from the axis; but this is true only within the elastic limit. In materials capable of flow, while the particles near the axis are strained within the elastic limit those at some distance within the circumference may be strained nearly to the ultimate resistance, so that the total resistance is something greater than that calculated by the formulæ. For working strength, however, the formulæ may be used, with S taken at the safe working unit resistance.

The ultimate torsional shearing resistance S is about the same as the direct shearing resistance, and may be taken at 20,000 to 25,000 lbs. per square inch for cast iron, 45,000 lbs. for wrought iron, and 50,000 to 150,000 lbs. for steel, according to its carbon and temper. Large factors of safety should be taken, especially when the direction of stress is reversed, as in reversing engines, and when the torsional stress is combined with other stresses, as is usual in shafting. (See "Shafting.")

Elastic Resistance to Torsion. — Let l = length of bar being twisted, d = diameter, P = force applied at the extremity of a lever arm of length = a , Pa = twisting moment, G = torsional modulus of elasticity, θ = angle through which the free end of the shaft is twisted, measured in arc of radius = 1.

For a cylindrical shaft,

$$Pa = \frac{\pi \theta G d^4}{32 l}; \quad \theta = \frac{32 Pa l}{\pi d^4 G}; \quad G = \frac{32 Pa l}{\theta \pi d^4}; \quad \frac{32}{\pi} = 10.186.$$

If α = angle of torsion in degrees,

$$\theta = \frac{\alpha \pi}{180}; \quad \alpha = \frac{180 \theta}{\pi} = \frac{180 \times 32 Pa l}{\pi^2 d^4 G} = \frac{583.6 Pa l}{d^4 G}.$$

The value of G is given by different authorities as from $1/3$ to $2/5$ of E , the modulus of elasticity for tension. For steel it is generally taken as 12,000,000 lbs. per sq. in.

COMBINED STRESSES.

Combined Tension and Flexure. — Let A = the area of a bar subjected to both tension and flexure, P = tensile stress applied at the ends, $P \div A$ = unit tensile stress, S = unit stress at the fiber on the tensile side most remote from the neutral axis, due to flexure alone, then maximum tensile unit stress = $(P \div A) + S$. A beam to resist combined tension and flexure should be designed so that $(P \div A) + S$ shall not exceed the proper allowable working unit stress.

Combined Compression and Flexure. — If $P \div A$ = unit stress due to compression alone, and S = unit compressive stress at fiber most remote from neutral axis, due to flexure alone, then maximum compressive unit stress = $(P \div A) + S$.

Combined Tension (or Compression) and Shear. — If applied tension (or compression) unit stress = p , applied shearing unit stress = v , then from the combined action of the two forces

Max. $S = \pm \sqrt{v^2 + 1/4p^2}$, Maximum shearing unit stress;

Max. $t = 1/2 p + \sqrt{v^2 + 1/4p^2}$, Maximum tensile (or compressive) unit stress.

Combined Flexure and Torsion. — If S = greatest unit stress due to flexure alone, and S_s = greatest torsional shearing unit stress due to torsion alone, then for the combined stresses

Max. tension or compression unit stress $t = 1/2 S + \sqrt{S_s^2 + 1/4 S^2}$;

Max. shear $s = \pm \sqrt{S_s^2 + 1/4 S^2}$.

Equivalent bending moment = $1/2 M + 1/2 \sqrt{M^2 + T^2}$, where M = bending moment and T = torsional moment.

Formula for diameter of a round shaft subjected to transverse load while transmitting a given horse-power (see also Shafts of Engines):

$$d^3 = \frac{16 M}{\pi t} + \frac{16}{t} \sqrt{\frac{M^2}{\pi^2} + \frac{402,500,000 H^2}{n^2}},$$

where M = maximum bending moment of the transverse forces in pound-inches, H = horse-power transmitted, n = No. of revs. per minute, and t = the safe allowable tensile or compressive working strength of the material.

Gust's Formula for maximum tension or compression unit stress is $t = \sqrt{S_s^2 + S^2}$ (*Phil. Mag.*, July, 1900). It is claimed by many writers to be more accurate than Rankine's formula, given above. Equivalent bending moment = $\sqrt{M^2 + T^2}$. (*Eng'g.*, Sept. 13 and 27, 1907; July 10, 1908; April 23, 1909.)

Combined Compression and Torsion. — For a vertical round shaft carrying a load and also transmitting a given horse-power, the resultant maximum compressive unit stress

$$t = \frac{4 P}{\pi d^2} + \sqrt{321,000^2 \frac{H^2}{n^2 d^2} + \frac{16 P^2}{\pi^2 d^4}},$$

In which P is the load. From this the diameter d may be found when t and the other data are given.

Stress due to Temperature. — Let l = length of a bar, A = its sectional area, c = coefficient of linear expansion for one degree, t = rise or fall in temperature in degrees, E = modulus of elasticity, λ the change of length due to the rise or fall t ; if the bar is free to expand or contract, $\lambda = c t l$.

If the bar is held so as to prevent its expansion or contraction the stress produced by the change of temperature = $S = A c t E$. The following are average values of the coefficients of linear expansion for a change in temperature of one degree Fahrenheit:

For brick and stone..... $\alpha = 0.0000050$,
For cast iron..... $\alpha = 0.0000056$,
For wrought iron and steel.... $\alpha = 0.0000065$.

The stress due to temperature should be added to or subtracted from the stress caused by other external forces according as it acts to increase or to relieve the existing stress.

What stress will be caused in a steel bar 1 inch square in area by a change of temperature of 100° F.? $S = ActE = 1 \times 0.0000065 \times 100 \times 30,000,000 = 19,500$ lbs. Suppose the bar is under tension of 19,500 lbs. between rigid abutments before the change in temperature takes place, a cooling of 100° F. will double the tension, and a heating of 100° will reduce the tension to zero.

STRENGTH OF FLAT PLATES.

For a circular plate supported at the edge, uniformly loaded, according to Grashof,

$$f = \frac{5}{6} \frac{r^2}{t^2} p; \quad t = \sqrt{\frac{5r^2 p}{6f}}; \quad p = \frac{6ft^2}{5r^2}.$$

For a circular plate fixed at the edge, uniformly loaded,

$$f = \frac{2}{3} \frac{r^2}{t^2} p; \quad t = \sqrt{\frac{2}{3} \frac{r^2 p}{f}}; \quad p = \frac{3ft^2}{2r^2};$$

in which f denotes the working stress; r , the radius in inches; t , the thickness in inches; and p , the pressure in pounds per square inch.

For mathematical discussion, see Lanza, "Applied Mechanics."

Lanza gives the following table, using a factor of safety of 8, with tensile strength of cast iron 20,000, of wrought iron 40,000, and of steel 80,000:

| | Supported. | Fixed. |
|-------------------|--------------------------------------|--------------------------------------|
| Cast iron..... | $t = 0.0182570 r \sqrt{\frac{p}{r}}$ | $t = 0.0163300 r \sqrt{\frac{p}{r}}$ |
| Wrought iron..... | $t = 0.0117850 r \sqrt{\frac{p}{r}}$ | $t = 0.0105410 r \sqrt{\frac{p}{r}}$ |
| Steel..... | $t = 0.0091287 r \sqrt{\frac{p}{r}}$ | $t = 0.0081649 r \sqrt{\frac{p}{r}}$ |

For a circular plate supported at the edge, and loaded with a concentrated load P applied at a circumference the radius of which is r_0 :

$$f = \left(\frac{4}{3} \log \frac{r}{r_0} + 1 \right) \frac{P}{\pi t^2} = c \frac{P}{\pi t^2};$$

$$\begin{aligned} \text{for } \frac{r}{r_0} &= 10 \quad 20 \quad 30 \quad 40 \quad 50; \\ c &= 4.07 \quad 5.00 \quad 5.53 \quad 5.92 \quad 6.22; \\ t &= \sqrt{\frac{cP}{\pi f}}; \quad P = \frac{\pi t^2 f}{c}. \end{aligned}$$

The above formulæ are deduced from theoretical considerations, and give thicknesses much greater than are generally used in steam-engine cylinder-heads. (See empirical formulæ under Dimensions of Parts of Engines.) The theoretical formulæ seem to be based on incorrect or incomplete hypotheses, but they err in the direction of safety.

Thickness of Flat Cast-iron Plates to resist Bursting Pressures.
—Capt. John Ericsson (Church's Life of Ericsson) gave the following rules: The proper thickness of a square cast-iron plate will be obtained by the following: Multiply the side in feet (or decimals of a foot) by $\frac{1}{4}$ of the pressure in pounds and divide by 850 times the side in inches; the quotient is the square of the thickness in inches.

For a circular plate, multiply 11-14 of the diameter in feet by $\frac{1}{4}$ of the pressure on the plate in pounds. Divide by 850 times 11-14 of the diameter in inches. [Extract the square root.]

Prof. Wm. Harkness, *Eng'g News*, Sept. 5, 1895, shows that these rules can be put in a more convenient form, thus: For square plates $T = 0.00495 S \sqrt{p}$, and for circular plates $T = 0.00439 D \sqrt{p}$, where T = thickness of plate, S = side of the square, D = diameter of the circle, and p = pressure in lbs. per sq. in. Professor Harkness, however, doubts the value of the rules, and says that no satisfactory theoretical solution has yet been obtained.

The Strength of Unstayed Flat Surfaces. — Robert Wilson (*Eng'g*, Sept. 24, 1877) draws attention to the apparent discrepancy between the results of theoretical investigations and of actual experiments on the strength of unstayed flat surfaces of boiler-plate, such as the unstayed flat crowns of domes and of vertical boilers.

On trying to make the rules given by the authorities agree with the results of his experience of the strength of unstayed flat ends of cylindrical boilers and domes that had given way after long use, Mr. Wilson was led to believe that the rules give the breaking strength much lower than it actually is. He describes a number of experiments made by Mr. Nichols of Kirkstall, which gave results varying widely from each other, as the method of supporting the edges of the plate was varied, and also varying widely from the calculated bursting pressures, the actual results being in all cases very much the higher. Some conclusions drawn from these results are:

1. Although the bursting pressure has been found to be so high, boiler-makers must be warned against attaching any importance to this, since the plates deflected almost as soon as any pressure was put upon them and sprang back again on the pressure being taken off. This springing of the plate in the course of time inevitably results in grooving or channeling, which, especially when aided by the action of the corrosive acids in the water or steam, will in time reduce the thickness of the plate, and bring about the destruction of an unstayed surface at a very low pressure.

2. Since flat plates commence to deflect at very low pressures, they should never be used without stays; but it is better to dish the plates when they are not stayed by flues, tubes, etc.

3. Against the commonly accepted opinion that the limit of elasticity should never be reached in testing a boiler or other structure, these experiments show that an exception should be made in the case of an unstayed flat end-plate of a boiler, which will be safer when it has assumed a permanent set that will prevent its becoming grooved by the continual variation of pressure in working. The hydraulic pressure in this case simply does what should have been done before the plate was fixed, that is, dishes it.

4. These experiments appear to show that the mode of attaching by flange or by an inside or outside angle-iron exerts an important influence on the manner in which the plate is strained by the pressure.

When the plate is secured to an angle-iron, the stretching under pressure is, to a certain extent, concentrated at the line of rivet-holes, and the plate partakes rather of a beam supported than fixed round the edge. Instead of the strength increasing as the square of the thickness, when the plate is attached by an angle-iron, it is probable that the strength does not increase even directly as the thickness, since the plate gives way simply by stretching at the rivet-holes, and the thicker the plate, the less uniformly is the strain borne by the different layers of which the plate may be considered to be made up. When the plate is flanged, the flange becomes compressed by the pressure against the body of the plate, and near the rim, as shown by the contrary flexure, the inside of the plate is stretched more than the outside, and it may be by a kind of shearing action that the plate gives way along the line where the crushing and stretching meet.

5. These tests appear to show that the rules deduced from the theoretical investigations of Lamé, Rankine, and Grashof are not confirmed by experiment, and are therefore not trustworthy.

The rules of Lamé, etc., apply only within the elastic limit. (*Eng'g*, Dec. 13, 1895.)

Unbraced Wrought-iron Heads of Boilers, etc. (*The Locomotive*, Feb., 1890). — Few experiments have been made on the strength of flat heads, and our knowledge of them comes largely from theory. Experiments have been made on small plates $\frac{1}{16}$ of an inch thick,

yet the data so obtained cannot be considered satisfactory when we consider the far thicker heads that are used in practice, although the results agreed well with Rankine's formula. Mr. Nichols has made experiments on larger heads, and from them he has deduced the following rule: "To find the proper thickness for a flat unstayed head, multiply the area of the head by the pressure per square inch that it is to bear safely, and multiply this by the desired factor of safety (say 8); then divide the product by ten times the tensile strength of the material used for the head." His rule for finding the bursting pressure when the dimensions of the head are given is: "Multiply the thickness of the end-plate in inches by ten times the tensile strength of the material used, and divide the product by the area of the head in inches."

In Mr. Nichols's experiments the average tensile strength of the iron used for the heads was 44,800 pounds. The results he obtained are given below, with the calculated pressure, by his rule, for comparison.

1. An unstayed flat boiler-head is $34\frac{1}{2}$ inches in diameter and $\frac{9}{16}$ inch thick. What is its bursting pressure? The area of a circle $34\frac{1}{2}$ inches in diameter is 935 square inches; then $\frac{9}{16} \times 44,800 \times 10 = 252,000$, and $252,000 \div 935 = 270$ pounds, the calculated bursting pressure. The head actually burst at 280 pounds.

2. Head $34\frac{1}{2}$ inches in diameter and $\frac{3}{8}$ inch thick. The area = 935 square inches; then, $\frac{3}{8} \times 44,800 \times 10 = 168,000$, and $168,000 \div 935 = 180$ pounds, calculated bursting pressure. This head actually burst at 200 pounds.

3. Head $26\frac{1}{4}$ inches in diameter, and $\frac{3}{8}$ inch thick. The area 541 square inches; then, $\frac{3}{8} \times 44,800 \times 10 = 168,000$, and $168,000 \div 541 = 311$ pounds. This head burst at 370 pounds.

4. Head $28\frac{1}{2}$ inches in diameter and $\frac{3}{8}$ inch thick. The area = 638 square inches; then, $\frac{3}{8} \times 44,800 \times 10 = 168,000$, and $168,000 \div 638 = 263$ pounds. The actual bursting pressure was 300 pounds.

In the third experiment, the amount the plate bulged under different pressures was as follows:

| | | | | | | | | |
|----------------------------|----------------|----------------|---------------|---------------|---------------|---------------|---------------|---------------|
| At pounds per sq. in. | 10 | 20 | 40 | 80 | 120 | 140 | 170 | 200 |
| Plate bulged | $\frac{1}{32}$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ |

The pressure was now reduced to zero, and the end sprang back $\frac{3}{16}$ inch, leaving it with a permanent set of $\frac{9}{16}$ inch. The pressure of 200 lbs. was again applied on 36 separate occasions during an interval of five days, the bulging and permanent set being noted on each occasion, but without any appreciable difference from that noted above.

The experiments described were confined to plates not widely different in their dimensions, so that Mr. Nichols's rule cannot be relied upon for heads that depart much from the proportions given in the examples.

Strength of Stayed Surfaces. — A flat plate of thickness t is supported uniformly by stays whose distance from center to center is a , uniform load p lbs. per square inch. Each stay supports pa^2 lbs. The greatest stress on the plate is

$$f = \frac{2}{9} \frac{a^2}{t^2} p. \quad (\text{Unwin.})$$

For additional matter on this subject see strength of Steam Boilers.

Stresses in Steel Plating due to Water-pressure, as in plating of vessels and bulkheads (*Engineering*, May 22, 1891, page 629).

Mr. J. A. Yates has made calculations of the stresses to which steel plates are subjected by external water-pressure, and arrives at the following conclusions:

Assume $2a$ inches to be the distance between the frames or other rigid supports, and let d represent the depth in feet, below the surface of the water, of the plate under consideration, t = thickness of plate in inches, D the deflection from a straight line under pressure in inches, and P = stress per square inch of section.

For outer bottom and ballast-tank plating, $a = 420 t/d$, D should not be greater than $0.05 \times 2a/12$, and $P/2$ not greater than 2 to 3 tons; while for bulkheads, etc., $a = 2352 t/d$, D should not be greater than

$0.1 \times 2a/12$, and $P/2$ not greater than 7 tons. To illustrate the application of these formulæ the following cases have been taken:

| For Outer Bottom, etc. | | | For Bulkheads, etc. | | |
|-------------------------------|--------------------------|---|-------------------------------|--------------------|--|
| Thick- ness of Plating. | Depth below Water. | Spacing of Frames should not exceed | Thick- ness of Plating. | Depth of Water. | Maximum Spac- ing of Rigid Stiffeners. |
| in. | ft. | in. | in. | ft. | ft. in. |
| $1/2$ | 20 | About 21 | $1/2$ | 20 | 9 10 |
| $1/2$ | 10 | " 42 | $3/8$ | 20 | 7 4 |
| $3/8$ | 18 | " 18 | $3/8$ | 10 | 14 8 |
| $3/8$ | 9 | " 36 | $1/4$ | 20 | 4 10 |
| $1/4$ | 10 | " 20 | $1/4$ | 10 | 9 8 |
| $1/4$ | 5 | " 40 | $1/8$ | 10 | 4 10 |

It would appear that the course which should be followed in stiffening bulkheads is to fit substantially rigid stiffening frames at comparatively wide intervals, and only work such light angles between as are necessary for making a fair job of the bulkhead.

SPHERICAL SHELLS AND DOMED BOILER-HEADS.

To find the Thickness of a Spherical Shell to resist a given Pressure. — Let d = diameter in inches, and p the internal pressure per square inch. The total pressure which tends to produce rupture around the great circle will be $\frac{1}{4}\pi d^2 p$. Let S = safe tensile stress per square inch, and t the thickness of metal in inches; then the resistance to the pressure will be $\pi d t S$. Since the resistance must be equal to the pressure,

$$\frac{1}{4}\pi d^2 p = \pi d t S. \quad \text{Whence } t = \frac{pd}{4S}.$$

The same rule is used for finding the thickness of a hemispherical head to a cylinder, as of a cylindrical boiler.

Thickness of a Domed Head of a Boiler. — If S = safe tensile stress per square inch, d = diameter of the shell in inches, and t = thickness of the shell, $t = pd \div 2S$; but the thickness of a hemispherical head of the same diameter is $t = pd \div 4S$. Hence if we make the *radius* of curvature of a domed head equal to the diameter of the boiler, we shall have $t = \frac{2pd}{4S} = \frac{pd}{2S}$, or the thickness of such a domed head will be equal to the thickness of the shell.

THICK HOLLOW CYLINDERS UNDER TENSION.

Lamé's formula, which is generally used, gives

$$t = r_1 \left\{ \left(\frac{h+p}{h-p} \right)^{\frac{1}{2}} - 1 \right\} \quad \begin{array}{l} t = \text{thickness; } r_1 = \text{inside and } r_2 = \text{outside radius;} \\ h = \text{maximum allowable hoop tension at the} \\ \text{interior of the cylinder;} \\ p = \text{intensity of interior pressure;} \\ s = \text{tension at the exterior of the cylinder.} \end{array}$$

$$h = p \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}; \quad s = p \frac{2r_1^2}{r_2^2 - r_1^2}; \quad r_2 = r_1 \left(\frac{h+p}{h-p} \right)^{\frac{1}{2}}.$$

EXAMPLE: Let maximum unit stress at the inner edge of the annulus = 8000 lbs. per square inch, radius of cylinder = 4 inches, interior pressure = 4000 lbs. per square inch. Required the thickness and the tension at the exterior surface.

$$t = 4 \left\{ \left(\frac{8000 + 4000}{8000 - 4000} \right)^{\frac{1}{2}} - 1 \right\} = 4 (\sqrt{3} - 1) = 2.928 \text{ inches.}$$

$$s = p \frac{2r_1^2}{r_2^2 - r_1^2} = 4000 \times \frac{2 \times 16}{48 - 16} = 4000 \text{ lbs. per sq. in.}$$

For short cast-iron cylinders, such as are used in hydraulic presses, it is doubtful if the above formulæ hold true, since the strength of the cylindrical portion is reinforced by the end. In that case the strength would be higher than that calculated by the formula. A rule used in practice for such presses is to make the thickness = $1/10$ of the inner circumference, for pressures of 3000 to 4000 lbs. per square inch.

Hooped Cylinders. — For very high pressures, as in large guns, hoops or outer tubes of forged steel are shrunk on inner tubes, thus bringing a compressive stress on the latter which assists in resisting the tension due to the internal pressure. For discussion of Lamé's, and other formulæ for built-up guns, see Merriman's "Mechanics of Materials."

THIN CYLINDERS UNDER TENSION.

Let p = safe working pressure in lbs. per sq. in.;

d = diameter in inches;

T = tensile strength of the material, lbs. per sq. in.;

t = thickness in inches;

f = factor of safety;

c = ratio of strength of riveted joint to strength of solid plate.

$$fpd = 2Ttc; \quad p = \frac{2Ttc}{df}; \quad t = \frac{fpd}{2Tc}.$$

If $T = 50,000$, $f = 5$, and $c = 0.7$; then

$$p = \frac{14,000t}{d}; \quad t = \frac{dp}{14,000}.$$

The above represents the strength resisting rupture along a longitudinal seam. For resistance to rupture in a circumferential seam, due to pressure on the ends of the cylinder, we have $\frac{p\pi d^2}{4} = \frac{Tt\pi dc}{f}$;

$$\text{whence } p = \frac{4Ttc}{df}.$$

Or the strength to resist rupture around a circumference is twice as great as that to resist rupture longitudinally; hence boilers are commonly single-riveted in the circumferential seams and double-riveted in the longitudinal seams.

CARRYING CAPACITY OF STEEL ROLLERS AND BALLS.

Carrying Capacity of a Steel Roller between Flat Plates. — (Merriman, *Mech. of Malls*.) Let S = maximum safe unit stress of the material, l = length of the roller in inches, d = diameter, E = modulus of elasticity, W = load, then $W = \frac{2}{3}ldS$ ($2S/E$) $^{\frac{1}{2}}$. Taking $w = W/l$, and $S = 15,000$ and $E = 30,000,000$ lbs. per sq. in. for steel the formula reduces to $w = 316d$. Cooper's specifications for bridges, 1901, gives $w = 300d$. (The rule given in some earlier specifications, $w = 1200\sqrt{d}$, is erroneous.) The formula assumes that only the roller is deformed by the load, but experiments show that the plates also are deformed, and that the formula errs on the side of safety. Experiments by Crandall

and Marston on steel rollers of diameters from 1 to 16 in. show that their crushing loads are closely given by the formula $W = 880 ld$. (See Roller Bearings.)

Spherical Rollers. — With the same notation as above, d being the diameter of the sphere, $S = \sqrt{WE/1/4 \pi d^3}$; $W = 1/4 \pi d^3 S^2/E$. The diameter of a sphere to carry a given load with an allowable unit-stress S is $d = 2 \sqrt{WE/\pi S^2}$. This rule assumes that there is no deformation of the plates between which the sphere acts, hence it errs on the side of safety. (See Ball Bearings.)

RESISTANCE OF HOLLOW CYLINDERS TO COLLAPSE.

Fairbairn's empirical formula (*Phil. Trans.*, 1858) is

$$p = 9,675,600 \frac{t^{2.19}}{ld}, \quad \dots \dots \dots (1)$$

where p = pressure in lbs. per square inch, t = thickness of cylinder, d = diameter, and l = length, all in inches; or,

$$p = 806,300 \frac{t^{2.19}}{Ld}, \text{ if } L \text{ is in feet } \dots \dots \dots (2)$$

He recommends the simpler formula

$$p = 9,675,600 \frac{t^2}{ld} \quad \dots \dots \dots (3)$$

as sufficiently accurate for practical purposes, for tubes of considerable diameter and length.

The diameters of Fairbairn's experimental tubes were 4, 6, 8, 10, and 12 inches, and their lengths ranged between 19 and 60 inches.

His formula (3) was until about '908 generally accepted as the basis of rules for strength of boiler-flues. In some cases, however, limits were fixed to its application by a supplementary formula.

Lloyd's Register contains the following formula for the strength of circular boiler-flues, viz.,

$$P = \frac{89,600 t^2}{Ld} \quad \dots \dots \dots (4)$$

The English Board of Trade prescribes the following formula for circular flues, when the longitudinal joints are welded, or made with riveted butt-straps, viz.,

$$P = \frac{90,000 t^2}{(L+1)d} \quad \dots \dots \dots (5)$$

For lap-joints and for inferior workmanship the numerical factor may be reduced as low as 60,000.

The rules of Lloyd's Register, and those of the Board of Trade, prescribe further, that in no case the value of P must exceed $800 t/d$. (6)

In formulæ (4), (5), (6) P is the highest working pressure in pounds per square inch, t and d are the thickness and diameter in inches, L is the length of the flue in feet measured between the strengthening rings, in case it is fitted with such. Formula (4) is the same as formula (3), with a factor of safety of 9. In formula (5) the length L is increased by 1; the influence which this addition has on the value of P is, of course, greater for short tubes than for long ones.

Nystrom has deduced from Fairbairn's experiments the following formula for the collapsing strength of flues:

$$p = \frac{4 T t^2}{d \sqrt{L}}, \quad \dots \dots \dots (7)$$

where p , t , and d have the same meaning as in formula (1), L is the length in feet, and T is the tensile strength of the metal in pounds per square inch.

If we assign to T the value 50,000, and express the length of the flue in inches, equation (7) assumes the following form, viz.,

$$p = 692,800 \frac{t^2}{d \sqrt{l}} \quad \dots \dots \dots (8)$$

Nystrom considers a factor of safety of 4 sufficient in applying his formula. (See "A New Treatise on Steam Engineering," by J. W. Nystrom, p. 106.)

Formulae (1), (4), and (8) have the common defect that they make the collapsing pressure decrease indefinitely with increase of length, and *vice versa*.

D. K. Clark, in his "Manual of Rules," etc., p. 696, gives the dimensions of six flues, selected from the reports of the Manchester Steam-Users Association, 1862-69, which collapsed while in actual use in boilers. These flues varied from 24 to 60 inches in diameter, and from $\frac{3}{16}$ to $\frac{3}{8}$ inch in thickness. They consisted of rings of plates riveted together, with one or two longitudinal seams, but all of them unfortified by intermediate flanges or strengthening rings. At the collapsing pressures the flues experienced compressions ranging from 1.53 to 2.17 tons, or a mean compression of 1.82 tons per square inch of section. From these data Clark deduced the following formula "for the average resisting force of common boiler-flues," viz.,

$$p = t^2 \left(\frac{50,000}{d} - 500 \right), \quad (9)$$

where p is the collapsing pressure in pounds per square inch, and d and t are the diameter and thickness expressed in inches.

Clark (S. E., vol. i, p. 643) says: The resistance to collapse of plain-riveted flues is directly as the square of the thickness of the plate, and inversely as the square of the diameter. The support of the two ends of the flue does not practically extend over a length of tube greater than twice or three times the diameter. The collapsing pressure of long tubes is therefore practically independent of the length. Instances of collapsed flues of Cornish and Lancashire boilers collated by Clark, showed that the resistance to collapse of flues of $\frac{3}{8}$ -inch plates, 18 to 43 feet long, and 30 to 50 inches diameter, varied as the 1.75 power of the diameter. Thus,

| | | | | | | |
|---|----|----|----|----|----|------------------|
| for diameters of | 30 | 35 | 40 | 45 | 50 | inches, |
| the collapsing pressures were | 76 | 58 | 45 | 37 | 30 | lbs. per sq. in. |
| for $\frac{7}{16}$ -inch plates the collapsing pressures were | | | 60 | 49 | 42 | lbs. per sq. in. |

C. R. Roelker, in *Van Nostrand's Magazine*, March, 1881, says that Nystrom's formula, (8), gives a closer agreement of the calculated with the actual collapsing pressures in experiments on flues of every description than any of the other formulæ.

For collapsing pressures of plain iron flue-tubes of Cornish and Lancashire steam-boilers, Clark gives:

$$p = \frac{200,000 t^2}{d^{1.75}}.$$

For short lengths the longitudinal tensile resistance may be effective in augmenting the resistance to collapse. Flues efficiently fortified by flange-joints or hoops at intervals of 3 feet may be enabled to resist from 50 lbs. to 60 lbs. or 70 lbs. pressure per square inch more than plain tubes, according to the thickness of the plates.

(For strength of Segmental Crowns of Furnaces and Cylinders see Clark, S. E., vol. i. pp. 649-651 and pp. 627, 628.)

Formula for Corrugated Furnaces (*Eng'g*, July 24, 1891, p. 102). — As the result of a series of experiments on the resistance to collapse of Fox's corrugated furnaces, the Board of Trade and Lloyd's Register altered their formulæ for these furnaces in 1891 as follows:

Board of Trade formula is altered from

$$\frac{12,500 \times T}{D} = WP \text{ to } \frac{14,000 \times T}{D} = WP.$$

T = thickness in inches;

D = mean diameter of furnace;

WP = working pressure in pounds per square inch.

Lloyd's formula is altered from

$$\frac{1000 \times (T - 2)}{D} = WP \text{ to } \frac{1234 \times (T - 2)}{D} = WP.$$

T = thickness in sixteenths of an inch;

D = greatest diameter of furnace;

WP = working pressure in pounds per square inch.

Stewart's Experiments. — Prof. Reid T. Stewart (*Trans. A.S.M.E.*, xxvii, 730) made two series of tests on Bessemer steel lap-welded tubes 3 to 10 ins. diam. One series was made on tubes $8\frac{5}{8}$ in. outside diam. with the different commercial thicknesses of wall, and in lengths of $2\frac{1}{2}$, 5, 10, 15 and 20 ft. between transverse joints tending to hold the tube in a circular form. A second series was made on single lengths of 20 ft. Seven sizes, from 3 to 10 in. outside diam., in all the commercial thicknesses obtainable, were tested. The tests showed that all the old formulæ were inapplicable to the wide range of conditions found in modern practice. The principal conclusions drawn from the research are as follows:

1. The length of tube, between transverse joints tending to hold it in circular form, has no practical influence upon the collapsing pressure of a commercial lap-welded tube so long as this length is not less than about six diameters of tube.

2. The formulæ, based upon this research, for the collapsing pressures of modern lap-welded Bessemer steel tubes, for all lengths greater than six diameters, are as follows:

$$P = 1,000 \left(1 - \sqrt{1 - 1600 \frac{t^2}{d^2}} \right) \dots \dots \dots (A)$$

$$P = 86,670 \frac{t}{d} - 1386 \dots \dots \dots (B)$$

Where P = collapsing pressure, pounds per sq. inch, d = outside diameter of tube in inches, t = thickness of wall in inches.

Formula A is for values of P less than 581 pounds, or for values of $\frac{t}{d}$ less than 0.023, while formula B is for values greater than these. When applying these formulæ, to practice, a suitable factor of safety must be applied.

3. The apparent fibre stress under which the different tubes failed varied from about 7000 lbs. for the relatively thinnest to 35,000 lbs. per sq. in. for the relatively thickest walls. Since the average yield point of the material was 37,000 and the tensile strength 58,000 lbs. per sq. in., it would appear that the strength of a tube subjected to a fluid collapsing pressure is not dependent alone upon either the elastic limit or ultimate strength of the material constituting it. The element of greatest weakness in a tube is its departure from roundness, even when this departure is relatively small.

The table on the following page is a condensed statement of the principal results of the tests.

Rational Formulæ for Collapse of Tubes. (S. E. Slocum, *Eng'g.*, Jan. 8, 1909.)

Heretofore designers have been forced to rely either upon the antiquated experiments of Fairbairn, which were known to be in error by as much as 100% in many cases, or else to apply the theoretical formulæ of Love and others, without knowing how far the assumptions on which these formulæ are based are actually realized.

A rational formula for thin tubes under external pressure, due to A. E. H. Love, is

$$P = [2E/(1 - m^2)] (t/D)^3, \dots \dots \dots (1)$$

in which P = collapsing pressure in lbs. per sq. in.

E = modulus of elasticity in lbs. per sq. in.

m = Poisson's ratio of lateral to transverse deformation.

t = thickness of tube wall in ins.

D = external tube diameter in ins.

COLLAPSING PRESSURE OF LAP-WELDED STEEL TUBES.

Outside Diameter, 85/8 In.; Length of Pipe, 20 Ft.

| Thick- ness, In. | Length, Ft. | Bursting Pressure, Lbs. per Sq. In. | Aver- age. | Outside Diam. In. | Thick- ness. | Bursting Pressure. | Aver- age. |
|------------------------|----------------|--|---------------|-------------------------|-----------------|-----------------------|---------------|
| 0.176 | 2.21 | 815-1085 | 977 | 3 | 0.112 | 1550-2175 | 1860 |
| 0.180 | 4.70 | 525-705 | 792 | 3 | 0.143 | 2575-3350 | 2962 |
| 0.181 | 10.08 | 455-650 | 565 | 3 | 0.188 | 3700-4200 | 4095 |
| 0.184 | 14.71 | 425-610 | 548 | 4 | 0.119 | 860-1030 | 964 |
| 0.185 | 19.72 | 450-625 | 536 | 4 | 0.175 | 2050-2540 | 2280 |
| 0.212 | 2.21 | 1240-1353 | 1314 | 4 | 0.212 | 3075-3375 | 3170 |
| 0.212 | 4.70 | 805-975 | 907 | 4 | 0.327 | 5425-5625 | 5560 |
| 0.217 | 10.50 | 700-960 | 841 | 6 | 0.130 | 450-640 | 524 |
| 0.219 | 12.79 | 750-1115 | 905 | 6 | 0.167 | 715-1110 | 928 |
| 0.268 | 2.14 | 1475-2200 | 1872 | 6 | 0.222 | 1200-2075 | 1797 |
| 0.274 | 4.64 | 1345-2030 | 1684 | 6 | 0.266 | 1750-2890 | 2441 |
| 0.272 | 9.64 | 1150-1908 | 1583 | 7 | 0.160 | 515-675 | 592 |
| 0.273 | 14.64 | 1250-1725 | 1485 | 7 | 0.242 | 1525-1850 | 1680 |
| 0.268 | 19.64 | 1250-1520 | 1419 | 7 | 0.279 | 1835-2445 | 2147 |
| 0.311 | 2.16 | 2290-2490 | 2397 | 8.64 | 0.185 | 450-625 | 536 |
| 0.306 | 4.64 | 1795-2325 | 2073 | 8.66 | 0.268 | 1250-1520 | 1419 |
| 0.306 | 9.64 | 1585-2055 | 1807 | 8.67 | 0.354 | 1830-2180 | 2028 |
| 0.309 | 14.64 | 1520-2025 | 1781 | 10 | 0.165 | 210-240 | 225 |
| 0.302 | 19.75 | 1575-1960 | 1762 | 10 | 0.194 | 305-425 | 383 |
| | | | | 10 | 0.316 | 1275-1385 | 1319 |

COLLAPSING PRESSURE OF LAP-WELDED STEEL TUBES (LBS. PER SQ. IN.)
Calculated by Stewart's Formulæ.

| Thickness. | Outside Diameters, Inches. | | | | | | | | | | |
|------------|----------------------------|-----------|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| | 2 In. | 2 1/2 In. | 3 In. | 4 In. | 5 In. | 6 In. | 7 In. | 8 In. | 9 In. | 10 In. | 11 In. |
| 0.10 | 2947 | 2081 | 1503 | 781 | | | | | | | |
| 0.12 | 3814 | 2774 | 2081 | 1214 | 694 | 400 | | | | | |
| 0.14 | 4671 | 3468 | 2659 | 1647 | 1041 | 636 | 400 | 286 | 217 | | |
| 0.16 | 5548 | 4161 | 3236 | 2081 | 1387 | 925 | 595 | 400 | 297 | 232 | 187 |
| 0.18 | 6414 | 4854 | 3814 | 3514 | 1734 | 1214 | 843 | 564 | 400 | 306 | 244 |
| 0.20 | 7281 | 5548 | 4392 | 2947 | 2081 | 1503 | 1090 | 781 | 542 | 400 | 314 |
| 0.22 | 8148 | 6241 | 4970 | 3381 | 2427 | 1792 | 1338 | 997 | 733 | 525 | 400 |
| 0.24 | 9014 | 6934 | 5548 | 3814 | 2774 | 2081 | 1586 | 1214 | 935 | 694 | 512 |
| 0.26 | 9881 | 7628 | 6125 | 4248 | 3121 | 2370 | 1833 | 1431 | 1118 | 867 | 633 |
| 0.28 | | 8321 | 6703 | 4681 | 3468 | 2669 | 2081 | 1647 | 1310 | 1041 | 820 |
| 0.30 | | 9014 | 7281 | 5114 | 3814 | 2947 | 2328 | 1864 | 1503 | 1214 | 978 |
| 0.32 | | 9708 | 7859 | 5548 | 4161 | 3236 | 2576 | 2081 | 1696 | 1387 | 1135 |
| 0.34 | | | 8437 | 5981 | 4508 | 3525 | 2824 | 2297 | 1888 | 1561 | 1293 |
| 0.36 | | | 9014 | 6414 | 4854 | 3814 | 3071 | 2514 | 2081 | 1734 | 1450 |
| 0.38 | | | 9592 | 6848 | 5201 | 4103 | 3319 | 2731 | 2273 | 1907 | 1608 |
| 0.40 | | | | 7281 | 5548 | 4392 | 3567 | 2947 | 2466 | 2081 | 1766 |
| 0.42 | | | | 7714 | 5894 | 4681 | 3814 | 3164 | 2659 | 2254 | 1923 |
| 0.44 | | | | 8148 | 6241 | 4970 | 4062 | 3381 | 2851 | 2427 | 2081 |
| 0.46 | | | | 8581 | 6588 | 5259 | 4309 | 3598 | 3044 | 2601 | 2238 |
| 0.48 | | | | 9014 | 6934 | 5548 | 4557 | 3814 | 3236 | 2774 | 2396 |
| 0.50 | | | | 9448 | 7281 | 5887 | 4805 | 4031 | 3429 | 2947 | 2554 |

For thick tubes a special case of Lamé's general formula is

$$P = 2u[(t/D) - (t/D)^2], \quad \dots \quad (2)$$

in which u = ultimate compressive strength in lbs. per sq. in.

The average values of the elastic constants are for steel, $E = 30,000,000$, $m = 0.295$, $u = 40,000$; and for brass, $E = 14,000,000$, $m = 0.357$, $u = 11,000$.

Hence, for thin steel tubes, $P = 65,720,000 (t/D)^3 \dots \dots \dots (3)$

For thick steel tubes, $P = 80,000 [(t/D) - (t/D)^2] \dots \dots \dots (4)$

For thin brass tubes, $P = 32,090,000 (t/D)^3 \dots \dots \dots (5)$

For thick brass tubes, $P = 22,000 [(t/D) - (t/D)^2] \dots \dots \dots (6)$

It is desirable to introduce a correction factor C in (1) which shall allow for the average ellipticity and variation in thickness. The correction for ellipticity = $C_1 = (D_{\min}/D_{\max})^3$, and that for variation in thickness = $C_2 = (t_{\min}/t_{\text{aver}})^3$. From Stewart's twenty-five experiments $C_1 = 0.967$ and $C_2 = 0.712$. The correction factor $C = C_1 C_2 = 0.69$; and (1) becomes

$$P = C[2E/(1 - m^2)](t/D)^3 \quad \dots \quad (7)$$

in which $C = 0.69$ for Stewart's lap-welded steel flues, t = average thickness in ins., and D = maximum diameter in ins.

The empirical formulas obtained by Carman (Univ. of Illinois, Bull. No. 17, 1906), are for thin cold-drawn seamless steel tubes,

$$P = 50,200,000 (t/D)^3,$$

and for thin seamless brass tubes,

$$P = 25,150,000 (t/D)^3.$$

Carman assigns 0.025 as the upper limit of t/D for thin tubes and 0.03 as the lower limit of t/D for thick tubes. Stewart assigns 0.023 as the limit of t/D between thin and thick tubes.

Comparing these with (3) and (5), it is evident that they correspond to a correction factor of 0.76 for the steel tubes and 0.78 for the brass tubes. Since Carman's experiments were performed on seamless drawn tubes, while Stewart used lap-welded tubes, it might have been anticipated that the latter would develop a smaller percentage of the theoretical strength for perfect tubes than the former.

Formula (2) for thick tubes when corrected for ellipticity and variation in thickness reads

$$P = 2u_c C (t/D) [1 - C (t/D)] \quad \dots \quad (8)$$

in which t = average thickness, and $C = C_1, C_2, C_1$ being equal to D_{\min}/D_{\max} ; $C_2 = t_{\text{average}}/t_{\min}$.

From Stewart's experiments, average ellipticity $C_1 = 0.9874$, and average variation in thickness $C_2 = 0.9022$; $\therefore C = 0.9874 \times 0.5022 = 0.89$.

We have then, for thick lap-welded steel flues,

$$P = 2u_c 0.89 (t/D) [1 - 0.89 (t/D)]$$

and for thin lap-welded steel flues,

$$P = 0.69 [2E/(1 - m^2)] (t/D)^3$$

in which $E = 30,000,000$, $m = 0.295$, and $u_c = 38,500$ lbs. per sq. in.

The experimental data of Stewart and Carman have made it possible to correct the rational formulas of Love and Lamé to conform to actual conditions; and the result is a pair of supplementary formulas (7) and (8), which cover the entire range of materials, diameters, and thicknesses for long tubes of circular section. All that now remains to be done is the experimental determination of the correction constants for other types of commercial tubes than those already tested.

HOLLOW COPPER BALLS.

Hollow copper balls are used as floats in boilers or tanks, to control feed and discharge valves, and regulate the water-level.

They are spun up in halves from sheet copper, and a rib is formed on one half. Into this rib the other half fits, and the two are then soldered or brazed together. In order to facilitate the brazing, a hole is left on one side of the ball, to allow air to pass freely in or out; and this hole is

made use of afterwards to secure the float to its stem. The original thickness of the metal may be anything up to about 1/16 of an inch, if the spinning is done on a hand lathe, though thicker metal may be used when special machinery is provided for forming it. In the process of spinning, the metal is thinned down in places by stretching; but the thinnest place is neither at the equator of the ball (i.e., along the rib) nor at the poles. The thinnest points lie along two circles, passing around the ball parallel to the rib, one on each side of it, from a third to a half of the way to the poles. Along these lines the thickness may be 10, 15, or 20 per cent less than elsewhere, the reduction depending somewhat on the skill of the workman.

The *Locomotive* for October, 1891, gives two empirical rules for determining the thickness of a copper ball which is to work under an external pressure, as follows:

- 1. Thickness = $\frac{\text{diameter in inches} \times \text{pressure in pounds per sq. in.}}{16,000}$.
- 2. Thickness = $\frac{\text{diameter} \times \sqrt{\text{pressure}}}{1240}$.

These rules give the same result for a pressure of 166 lbs. only. **EXAMPLE:** Required the thickness of a 5-inch copper ball to sustain

| | | | | | | |
|--------------------------|-------|-------|-------|-------|-------|---------------------|
| Pressures of..... | 50 | 100 | 150 | 166 | 200 | 250 lbs.per sq. in. |
| Answer by first rule.... | .0156 | .0312 | .0469 | .0519 | .0625 | .0781 inch. |
| Answer by second rule . | .0285 | .0403 | .0494 | .0518 | .0570 | .0637 " |

HOLDING-POWER OF NAILS, SPIKES, AND SCREWS.

(A. W. Wright, Western Society of Engineers, 1881.)

Spikes. — Spikes driven into dry cedar (cut 18 months):

| | | | | |
|---|-----------------|---------|---------|-----------|
| Size of spikes..... | 5 × 1/4 in. sq. | 6 × 1/4 | 6 × 1/2 | 5 × 3/8 |
| Length driven in..... | 4 1/4 in. | 5 in. | 5 in. | 4 1/4 in. |
| Pounds resistance to drawing. Av'ge. lbs. | 857 | 821 | 1691 | 1202 |
| From 6 to 9 tests each..... | { Max. " 1159 | 923 | 2129 | 1556 |
| | { Min. " 766 | 766 | 1120 | 687 |

A. M. Wellington found the force required to draw spikes 9/16 × 9/16 in., driven 4 1/4 inches into seasoned oak, to be 4281 lbs.; same spikes, etc., in unseasoned oak, 6523 lbs.

"Professor W. R. Johnson found that a plain spike 3/8 inch square driven 33/8 inches into seasoned Jersey yellow pine or unseasoned chestnut required about 2000 lbs. force to extract it; from seasoned white oak about 4000 and from well-seasoned locust 6000 lbs."

Experiments in Germany, by Funk, give from 2465 to 3940 lbs. (mean of many experiments about 3000 lbs.) as the force necessary to extract a plain 1/2-inch square iron spike 6 inches long, wedge-pointed for one inch and driven 4 1/2 inches into white or yellow pine. When driven 5 inches the force required was about 1/10 part greater. Similar spikes 9/16 inches square, 7 inches long, driven 6 inches deep, required from 3700 to 6745 lbs. to extract them from pine; the mean of the results being 4873 lbs. In all cases about twice as much force was required to extract them from oak. The spikes were all driven across the grain of the wood. When driven with the grain, spikes or nails do not hold with more than half as much force.

Boards of oak or pine nailed together by from 4 to 16 tenpenny common cut nails and then pulled apart in a direction lengthwise of the boards, and across the nails, tending to break the latter in two by a shearing action, averaged about 300 to 400 lbs. per nail to separate them, as the result of many trials.

Resistance of Drift-bolts in Timber. — Tests made by Rust and Coolidge, in 1878.

| | | | | | White Pine. | Norway Pine. |
|-------------------------|--------|-------------------------|-------|---|-------------|--------------|
| 1 in. square iron drove | 30 in. | in 15/16-in. hole, lbs. | | | 26,400 | 19,200 |
| 1 in. round " | " | 34 " " 13/16-in. | " | " | 16,800 | 18,720 |
| 1 in. square " | " | 18 " " 15/16-in. | " | " | 14,600 | 15,600 |
| 1 in. round " | " | 22 " " 13/16-in. | " | " | 13,200 | 14,400 |

Holding-power of Bolts in White Pine. (Eng'g News, Sept. 26, 1891.)

| | Round. | Square. |
|---|--------|---------|
| | Lbs. | Lbs. |
| Average of all plain 1-in. bolts..... | 8224 | 8200 |
| Average of all plain bolts, $\frac{5}{8}$ to $1\frac{1}{8}$ in..... | 7805 | 8110 |
| Average of all bolts | 8383 | 8598 |

Round drift-bolts should be driven in holes $\frac{13}{16}$ of their diameter, and square drift-bolts in holes whose diameter is $\frac{14}{16}$ of the side of the square.

Force required to draw Screws out of Norway Pine.

| | | |
|--|-------------------------|-----------|
| $\frac{1}{2}$ " diam. drive screw 4 in. in wood. | Power required, average | 2424 lbs. |
| " " 4 threads per in. 5 in. in wood. | " " " | 2743 " |
| " " D'ble thr'd, 3 per in., 4 in. in " | " " " | 2730 " |
| " " Lag-screw, 7 per in., $1\frac{1}{2}$ " | " " " | 1465 " |
| " " " 6 " " $\frac{21}{2}$ " " " | " " " | 2026 " |
| $\frac{1}{2}$ inch R.R. spike.....5 | " " " | 2191 " |

Force required to draw Wood Screws out of Dry Wood. — Tests made by Mr. Bevan. The screws were about two inches in length, 0.22 diameter at the exterior of the threads, 0.15 diameter at the bottom, the depth of the worm or thread being 0.035 and the number of threads in one inch equal 12. They were passed through pieces of wood half an inch in thickness and drawn out by the weights stated: Beech, 460 lbs.; ash, 790 lbs.; oak, 760 lbs.; mahogany, 770 lbs.; elm, 665 lbs.; sycamore, 830 lbs.

Tests of Lag-screws in Various Woods were made by A. J. Cox, University of Iowa, 1891:

| Kind of Wood. | Size Screw. | Size Hole bored. | Length in Tie. | Max. Resist. lbs. | No. Tests. |
|-----------------------------|-------------------|-------------------|--------------------|-------------------|------------|
| Seasoned white oak | $\frac{5}{8}$ in. | $\frac{1}{2}$ in. | $4\frac{1}{2}$ in. | 8037 | 3 |
| " " " | $\frac{9}{16}$ " | $\frac{7}{16}$ " | 3 | 6480 | 1 |
| " " " | $\frac{1}{2}$ " | $\frac{3}{8}$ " | $4\frac{1}{2}$ " | 8780 | 2 |
| Yellow-pine stick | $\frac{5}{8}$ " | $\frac{1}{2}$ " | 4 | 3800 | 2 |
| White cedar, unseasoned.... | $\frac{5}{8}$ " | $\frac{1}{2}$ " | 4 | 3405 | 2 |

Cut versus Wire Nails. — Experiments were made at the Watertown Arsenal in 1893 on the comparative direct tensile adhesion, in pine and spruce, of cut and wire nails. The results are stated by Prof. W. H. Burr as follows:

There were 58 series of tests, ten pairs of nails (a cut and a wire nail in each) being used. The tests were made in spruce wood in most instances. The nails were of all sizes, from $1\frac{1}{8}$ to 6 in. in length. In every case the cut nails showed the superior holding strength by a large percentage. In spruce, in nine different sizes of nails, both standard and light weight, the ratio of tenacity of cut to wire nail was about 3 to 2. With the "finishing" nails the ratio was roughly 3.5 to 2. With box nails ($1\frac{1}{2}$ to 4 inches long) the ratio was roughly 3 to 2. The mean superiority in spruce wood was 61%. In white pine, cut nails, driven with taper along the grain, showed a superiority of 100%, and with taper across the grain of 135%. Also when the nails were driven in the end of the stick, i.e., along the grain, the superiority of cut nails was 100%, or the ratio of cut to wire was 2 to 1. The total of the results showed the ratio of tenacity to be about 3.2 to 2 for the harder wood, and about 2 to 1 for the softer, and for the whole taken together the ratio was 3.5 to 2.

Nail-holding Power of Various Woods. — Tests at the Watertown Arsenal on different sizes of nails from 8d. to 60d., reduced to holding power per sq. in. of surface in wood, gave average results, in pounds, as follows: white pine, wire, 167; cut, 405. Yellow pine, wire, 318; cut, 662. White oak, wire, 940; cut, 1216. Chestnut, cut, 683. Laurel, wire, 651; cut, 1200.

Experiments by F. W. Clay. (*Eng'g News*, Jan. 11, 1894.)

| Wood. | Tenacity of 6d nails— | | | |
|------------------|-----------------------|---------|--------|-------|
| | Plain. | Barbed. | Blued. | Mean. |
| White pine..... | 106 | 94 | 135 | 111 |
| Yellow pine..... | 190 | 130 | 270 | 196 |
| Basswood..... | 78 | 132 | 219 | 143 |
| White oak..... | 226 | 300 | 555 | 360 |
| Hemlock..... | 141 | 201 | 319 | 220 |

STRENGTH OF WROUGHT IRON BOLTS.

(Computed by A. F. Nagle.)

| Dia. | No. Thr. | Dia. of Root. | Area at Root. | Stress upon Bolt upon Basis of | | | | | Probable Breaking Load. |
|-------|----------|---------------|---------------|--------------------------------|-----------|-----------|-----------|------------|-------------------------|
| | | | | 3,000 lbs. per Sq. In. | 4,000 lb. | 5,000 lb. | 7,500 lb. | 10,000 lb. | |
| 1/2 | 13 | 0.400 | 0.126 | 378 | 504 | 630 | 945 | 1,260 | 6,400 |
| 9/16 | 12 | 0.454 | 0.162 | 486 | 648 | 810 | 1,215 | 1,620 | 8,200 |
| 5/8 | 11 | 0.507 | 0.202 | 606 | 808 | 1,010 | 1,515 | 2,020 | 10,200 |
| 3/4 | 10 | 0.620 | 0.302 | 906 | 1,208 | 1,510 | 2,265 | 3,020 | 15,200 |
| 7/8 | 9 | 0.731 | 0.420 | 1,260 | 1,680 | 2,100 | 3,150 | 4,200 | 21,100 |
| 1 | 8 | 0.837 | 0.550 | 1,650 | 2,200 | 2,750 | 4,125 | 5,500 | 27,500 |
| 1 1/8 | 7 | 0.940 | 0.694 | 2,082 | 2,776 | 3,470 | 5,205 | 6,940 | 34,500 |
| 1 1/4 | 7 | 1.065 | 0.893 | 2,679 | 3,572 | 4,465 | 6,698 | 8,930 | 44,000 |
| 1 3/8 | 6 | 1.160 | 1.057 | 3,171 | 4,228 | 5,285 | 7,927 | 10,570 | 52,000 |
| 1 1/2 | 6 | 1.284 | 1.295 | 3,885 | 5,180 | 6,475 | 9,712 | 12,950 | 63,000 |
| 1 5/8 | 5 1/2 | 1.389 | 1.515 | 4,545 | 6,060 | 7,575 | 11,362 | 15,150 | 74,000 |
| 1 3/4 | 5 | 1.491 | 1.746 | 5,238 | 6,984 | 8,730 | 13,095 | 17,460 | 84,000 |
| 1 7/8 | 5 | 1.616 | 2.051 | 6,153 | 8,204 | 10,255 | 15,382 | 20,510 | 99,000 |
| 2 | 4 1/2 | 1.712 | 2.302 | 6,906 | 9,208 | 11,510 | 17,265 | 23,020 | 110,000 |
| 2 1/4 | 4 1/2 | 1.962 | 3.023 | 9,069 | 12,092 | 15,165 | 22,672 | 30,230 | 143,000 |
| 2 1/2 | 4 | 2.176 | 3.719 | 11,157 | 14,876 | 18,595 | 27,892 | 37,190 | 174,000 |
| 2 3/4 | 4 | 2.426 | 4.620 | 13,860 | 18,480 | 23,100 | 34,650 | 46,200 | 214,000 |
| 3 | 3 1/2 | 2.629 | 5.428 | 16,284 | 21,712 | 27,140 | 40,710 | 54,280 | 248,000 |
| 3 1/2 | 3 1/4 | 3.100 | 7.548 | 22,644 | 30,192 | 37,740 | 56,610 | 75,480 | 337,000 |
| 4 | 3 | 3.567 | 9.963 | 29,889 | 39,852 | 49,815 | 74,722 | 99,630 | 433,000 |

The U. S. or Sellers System of Screw Threads is used in the above table.

The "Probable Breaking Load" is based upon wrought iron running from 51,000 lbs. per sq. in. for 1/2 inch diam. down to 43,500 lbs. for 4 in. diam.

For soft steel bolts add 20% to this column.

When it is known what load is to be put upon a bolt, and the judgment of the engineer has determined what stress is safe to put upon the iron, look down in the proper column of said stress until the required load is found. The area at the bottom of the thread will give the equivalent area of a flat bar to that of the bolt.

Effect of Initial Strain in Bolts. — Suppose that bolts are used to connect two parts of a machine and that they are screwed up tightly before the effective load comes on the connected parts. Let P_1 = the initial tension on a bolt due to screwing up, and P_2 = the load afterwards added. The greatest load may vary but little from P_1 or P_2 , according as the former or the latter is greater, or it may approach the value $P_1 + P_2$, depending upon the relative rigidity of the bolts and of the parts connected. Where rigid flanges are bolted together, metal to metal, it is probable that the extension of the bolts with any additional tension relieves the initial tension, and that the total tension is P_1 or P_2 , but in cases where elastic packing, as india rubber, is interposed, the extension of the bolts may very little affect the initial tension, and the total strain may be nearly $P_1 + P_2$. Since the latter assumption is more unfavorable to the resistance of the bolt, this contingency should usually be provided for. (See Unwin, "Elements of Machine Design," for demonstration.)

Forrest E. Cardullo (*Machinery's Reference Series No. 22, 1908*) states the effect of initial stress in bolts due to screwing them tight as follows:

1. When the bolt is more elastic than the material it compresses, the stress in the bolt is either the initial stress or the force applied, whichever is greater.

2. When the material compressed is more elastic than the bolt, the stress in the bolt is the sum of the initial stress and the force applied.

Experiments on screwing up $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ in. bolts showed that the stress produced is often sufficient to break a $\frac{1}{2}$ -in. bolt, and that the stress varies about as the square of the diameter. From these experiments Prof. Cardullo calculates what he calls the "working section" of a bolt as equal to its area at the root of the thread, less the area of a $\frac{1}{2}$ -in. bolt at the root of the thread times twice the diameter of the bolt, and gives the following table based on this rule.

Working Strength of Bolts. U. S. Standard Threads.

| Diameter of Bolt, inches. | Area at Root of Thread, square inches. | Working Section, square inches. | Strength of Bolt, 5000 pounds Stress. | Strength of Bolt, 6000 pounds Stress. | Strength of Bolt, 7000 pounds Stress. | Strength of Bolt, 8000 pounds Stress. | Strength of Bolt, 10,000 pounds Stress. | Strength of Bolt, 12,000 pounds Stress. |
|---------------------------|--|---------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---|---|
| $\frac{1}{2}$ | 0.126 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{5}{8}$ | 0.202 | 0.044 | 220 | 264 | 308 | 352 | 440 | 528 |
| $\frac{3}{4}$ | 0.302 | 0.113 | 565 | 678 | 791 | 904 | 1,130 | 1,356 |
| $\frac{7}{8}$ | 0.420 | 0.200 | 1,000 | 1,200 | 1,400 | 1,600 | 2,000 | 2,400 |
| 1 | 0.550 | 0.298 | 1,490 | 1,788 | 2,086 | 2,384 | 2,980 | 3,476 |
| $1\frac{1}{8}$ | 0.694 | 0.411 | 2,055 | 2,466 | 2,877 | 3,288 | 4,110 | 4,932 |
| $1\frac{1}{4}$ | 0.893 | 0.578 | 2,890 | 3,468 | 4,046 | 4,624 | 5,780 | 6,936 |
| $1\frac{3}{8}$ | 1.057 | 0.710 | 3,550 | 4,260 | 4,970 | 5,680 | 7,100 | 8,520 |
| $1\frac{1}{2}$ | 1.295 | 0.917 | 4,585 | 5,502 | 6,419 | 7,336 | 9,170 | 10,504 |
| $1\frac{5}{8}$ | 1.515 | 1.105 | 5,525 | 6,630 | 7,735 | 8,840 | 11,050 | 13,260 |
| $1\frac{3}{4}$ | 1.746 | 1.305 | 6,525 | 7,830 | 9,135 | 10,440 | 13,050 | 15,660 |
| $1\frac{7}{8}$ | 2.051 | 1.578 | 7,890 | 9,468 | 11,046 | 12,624 | 15,780 | 18,936 |
| 2 | 2.302 | 1.798 | 8,990 | 10,788 | 12,586 | 14,384 | 17,980 | 21,576 |
| $2\frac{1}{4}$ | 3.023 | 2.456 | 12,280 | 14,736 | 17,192 | 19,648 | 24,560 | 29,472 |
| $2\frac{1}{2}$ | 3.719 | 3.089 | 15,445 | 18,534 | 21,623 | 24,712 | 30,890 | 37,068 |
| $2\frac{3}{4}$ | 4.620 | 3.927 | 19,635 | 23,562 | 27,489 | 31,416 | 39,270 | 47,124 |
| 3 | 5.428 | 4.672 | 23,360 | 28,032 | 32,704 | 37,376 | 46,720 | 56,064 |
| $3\frac{1}{4}$ | 6.510 | 5.690 | 28,450 | 34,140 | 39,830 | 45,520 | 56,900 | 68,280 |
| $3\frac{1}{2}$ | 7.548 | 6.666 | 33,330 | 39,996 | 46,664 | 53,328 | 66,660 | 79,992 |

The stresses on bolts caused by tightening the nuts by a wrench may be calculated as follows: Let L = the effective length of the wrench in inches, P = the force in pounds applied at the distance L , n = no. of threads per inch of the bolt, T = total tension on the bolt if there were no friction, then $T = 2\pi nLP$. Wilfred Lewis, *Trans. A. S. M. E.*, gives for the efficiency of a bolt $E = 1 \div (1 + nd)$, where d = external diameter of the screw. $T \times E = 2\pi nLP \div (1 + nd)$ is the tension corrected for friction. It also expresses the load that can be lifted by screwing a nut on a bolt or a bolt into a nut.

STRENGTH OF CHAINS.

Formulas for Safe Load on Chains.—Writing the formula for the safe load on chains $P = Kd^2$, P in pounds, d in inches, the following figures for K are given by the authorities named.

| | Open link | Stud link |
|----------|-----------------|-----------------|
| Unwin | 13,440; 11,200* | 20,160 |
| Weisbach | 13,350 | 17,800 |
| Bach | 13,750; 11,000* | 16,500; 13,200* |

* The lower figures are for much used chain, subject frequently to the maximum load. G. A. Goodenough and L. E. Moore, *Univ. of Illinois*

STAND-PIPES AND THEIR DESIGN.

(Freeman C. Coffin, New England Water Works Assoc., *Eng. News*, March 16, 1893.) See also papers by A. H. Howland, *Eng. Club of Phil.*, 1887; B. F. Stephens, *Amer. Water Works Assoc.*, *Eng. News*, Oct. 6 and 13, 1888; W. Kiersted, *Rensselaer Soc. of Civil Eng.*, *Eng'g Record*, April 25 and May 2, 1891, and W. D. Pence, *Eng. News*, April and May, 1894; also, J. N. Hazlehurst's "Towers and Tanks for Water Works."

The question of diameter is almost entirely independent of that of height. The efficient capacity must be measured by the length from the high-water line to a point below which it is undesirable to draw the water on account of loss of pressure for fire-supply, whether that point is the actual bottom of the stand-pipe or above it. This allowable fluctuation ought not to exceed 50 ft., in most cases. This makes the diameter dependent upon two conditions, the first of which is the amount of the consumption during the ordinary interval between the stopping and starting of the pumps. This should never draw the water below a point that will give a good fire stream and leave a margin for still further draught for fires. The second condition is the maximum number of fire streams and their size which it is considered necessary to provide for, and the maximum length of time which they are liable to have to run before the pumps can be relied upon to reinforce them.

Another reason for making the diameter large is to provide for stability against wind-pressure when empty.

The following table gives the height of stand-pipes beyond which they are not safe against wind-pressures of 40 and 50 lbs. per square foot. The area of surface taken is the height multiplied by one half the diameter.

| | | | | |
|-------------------------------|----|----|-----|-----|
| Diameter, feet..... | 20 | 25 | 30 | 35 |
| Max. height, wind 40 lbs..... | 45 | 70 | 150 | ... |
| 50 "..... | 35 | 55 | 80 | 160 |

Any form of anchorage that depends upon connections with the side plates near the bottom is unsafe. By suitable guys the wind-pressure is resisted by tension in the guys, and the stand-pipe is relieved from wind strains that tend to overthrow it. The guys should be attached to a band of angle or other shaped iron that completely encircles the tank, and rests upon some sort of bracket or projection, and not be riveted to the tank. They should be anchored at a distance from the base equal to the height of the point at which they are attached, if possible.

The best plan is to build the stand-pipe of such diameter that it will resist the wind by its own stability.

Thickness of the Side Plates.

The pressure on the sides tending to rupture the plates by tension, due to the weight of the water, increases in direct ratio to the height, and also to the diameter. The strain upon a section 1 inch in height at any point is the total strain at that point divided by two — for each side is supposed to bear the strain equally. The total pressure at any point is equal to the diameter in inches, multiplied by the pressure per square inch, due to the height at that point. It may be expressed as follows:

$$\begin{aligned}
 H &= \text{height in feet, and } f = \text{factor of safety;} \\
 d &= \text{diameter in inches;} \\
 p &= \text{pressure in lbs. per square inch;} \\
 0.434 &= p \text{ for 1 ft. in height;} \\
 s &= \text{tensile strength of material per square inch;} \\
 T &= \text{thickness of plate.}
 \end{aligned}$$

Bulletin, No. 18, 1907, after an extensive theoretical and experimental investigation, find that these values give maximum stresses in the external fibers of from 26,400 to 40,320 lbs. per sq. in., which they consider much too high for safety. Taking 20,000 as a permissible maximum stress, they give the formulæ for safe load $P = 8000 d^2$ for open links and $P = 10,000 d^2$ for stud links. They say that the stud link will within the elastic limit bear from 20 to 25% more load than the open link, but that the ultimate strength of the stud link is probably less than that of the open link. See also tables of Size and Strength of Chains, page 251.

Then the total strain on each side per vertical inch

$$= \frac{0.434 Hd}{2} = \frac{pd}{2}; \quad T = \frac{0.434 Hdf}{2s} = \frac{pdf}{2s}.$$

Mr. Coffin takes $f = 5$, not counting reduction of strength of joint, equivalent to an actual factor of safety of 3 if the strength of the riveted joint is taken as 60 per cent of that of the plate.

The amount of the wind strain per square inch of metal at any joint can be found by the following formula, in which

H = height of stand-pipe in feet above joint;
 T = thickness of plate in inches;
 p = wind-pressure per square foot;
 W = wind-pressure per foot in height above joint;
 $W = Dp$ where D is the diameter in feet;
 m = average leverage or movement about neutral axis
 or central points in the circumference; or,
 m = sine of 45° , or 0.707 times the radius in feet.

Then the strain per square inch of plate

$$(Hw) \frac{H}{2} \\ = \frac{\text{circ. in ft.} \times mT}{2}.$$

Mr. Coffin gives a number of diagrams useful in the design of stand-pipes, together with a number of instances of failures, with discussion of their probable causes.

Mr. Kiersted's paper contains the following: Among the most prominent strains a stand-pipe has to bear are: that due to the static pressure of the water, that due to the overturning effect of the wind on an empty stand-pipe, and that due to the collapsing effect, on the upper rings, of violent wind storms.

For the thickness of metal to withstand safely the static pressure of water, let t = thickness of the plate iron in inches; H = height of stand-pipe in feet; D = diameter of stand-pipe in feet.

Then, assuming a tensile strength of 48,000 lbs. per square inch, a factor of safety of 4, and efficiency of double-riveted lap-joint equaling 0.6 of the strength of the solid plate, $t = 0.00036 H \times D$; $H = 10,000 t \div 3.6 D$; which will give safe heights for thicknesses up to $5/8$ to $3/4$ of an inch. The same formula may also apply for greater heights and thicknesses within practical limits, if the joint efficiency be increased by triple riveting.

The conditions for the severest overturning wind strains exist when the stand-pipe is empty.

Formula for wind-pressure of 50 pounds per square foot, when d = diameter of stand-pipe in inches; x = any unknown height of stand-pipe; $x = \sqrt{80\pi dt} = 15.85 \sqrt{dt}$.

Failures of Stand-pipes. — A list showing 23 important failures inside of nine years is given in a paper by Prof. W. D. Pence, *Eng'g News*, April 5, 12, 19 and 26, May 3, 10 and 24, and June 7, 1894. His discussion of the probable causes of the failures is most valuable.

Water Tower at Yonkers, N.Y. — This tower, with a pipe 122 feet high and 20 feet diameter, is described in *Engineering News*, May 18, 1892.

The thickness of the lower rings is $11/16$ of an inch, based on a tensile strength of 60,000 lbs. per square inch of metal, allowing 65% for the strength of riveted joints, using a factor of safety of $3 1/2$ and adding a constant of $1/8$ inch. The plates diminish in thickness by $1/16$ inch to the last four plates at the top, which are $1/4$ inch thick.

The contract for steel requires an elastic limit of at least 33,000 lbs. per square inch; an ultimate tensile strength of from 56,000 to 66,000 lbs. per square inch; an elongation in 8 inches of at least 20%, and a reduction of area of at least 45%. The inspection of the work was made by the Pittsburgh Testing Laboratory. According to their report the actual conditions developed were as follows: Elastic limit from 34,020 to 39,420;

the tensile strength from 58,330 to 65,390; the elongation in 8 inches from 22½ to 32%; reduction in area from 52.72 to 71.32%; 17 plates out of 141 were rejected in the inspection.

The following table is calculated by Mr. Kiersted's formulæ. The stand-pipe is intended to be self-sustaining; that is, without guys or stiffeners.

Heights of Stand-pipes for Various Diameters and Thicknesses of Plates.

| Thickness of Plate in Fractions of an Inch. | Diameters in Feet. | | | | | | | | | | | | |
|---|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 16 | 18 | 20 | 25 |
| 3/16..... | 50 | 55 | 60 | 65 | 55 | 50 | 35 | | | | | | |
| 7/32..... | 55 | | | | 65 | 60 | 50 | 40 | 40 | | | | |
| 1/4..... | 60 | 65 | 70 | 75 | 75 | 70 | 55 | 50 | 45 | 40 | 35 | 35 | 25 |
| 5/16..... | 70 | 75 | 80 | 85 | 90 | 85 | 70 | 60 | 55 | 50 | 45 | 40 | 35 |
| 3/8..... | 75 | 80 | 90 | 95 | 100 | 100 | 85 | 75 | 70 | 65 | 55 | 50 | 40 |
| 7/16..... | 80 | 90 | 95 | 100 | 110 | 115 | 100 | 85 | 80 | 75 | 65 | 60 | 45 |
| 1/2..... | 85 | 95 | 100 | 110 | 115 | 120 | 115 | 100 | 90 | 85 | 75 | 70 | 55 |
| 9/16..... | | | | 115 | 125 | 130 | 130 | 110 | 100 | 95 | 85 | 80 | 60 |
| 5/8..... | | | | | 130 | 135 | 145 | 120 | 115 | 105 | 95 | 85 | 65 |
| 11/16..... | | | | | | 145 | 155 | 135 | 125 | 120 | 105 | 95 | 75 |
| 3/4..... | | | | | | 150 | 165 | 145 | 135 | 130 | 115 | 105 | 80 |
| 13/16..... | | | | | | | | 160 | 150 | 140 | 125 | 110 | 90 |
| 17/8..... | | | | | | | | | 160 | 150 | 135 | 120 | 95 |
| 5/1..... | | | | | | | | | | 160 | 145 | 130 | 105 |
| 1..... | | | | | | | | | | | 155 | 140 | 110 |

Heights to nearest 5 feet. Rings are to build 5 feet vertically.

WROUGHT-IRON AND STEEL WATER-PIPES.

Riveted Steel Water-pipes (*Engineering News*, Oct. 11, 1890, and Aug. 1, 1891). — The use of riveted wrought-iron pipe has been common in the Pacific States for many years, the largest being a 44-inch conduit in connection with the works of the Spring Valley Water Co., which supplies San Francisco. The use of wrought iron and steel pipe has been necessary in the West, owing to the extremely high pressures to be withstood and the difficulties of transportation. As an example: In connection with the water supply of Virginia City and Gold Hill, Nev., there was laid in 1872 an 11½-inch riveted wrought-iron pipe, a part of which is under a head of 1720 feet.

In the East, an important example of the use of riveted steel water pipe is that of the East Jersey Water Co., which supplies the city of Newark. The contract provided for a maximum high service supply of 25,000,000 gallons daily. In this case 21 miles of 48-inch pipe was laid, some of it under 340 feet head. The plates from which the pipe is made are about 13 feet long by 7 feet wide, open-hearth steel. Four plates are used to make one section of pipe about 27 feet long. The pipe is riveted longitudinally with a double row, and at the end joints with a single row of rivets. Before being rolled into the trench, two of the 27-foot lengths are riveted together, thus diminishing the number of joints to be made in the trench and the extra excavation to give room for joining.

The thickness of the plates varies with the pressure, but only three thicknesses are used, ¼, 5/16, and 3/8 inches, the pipe made of these thicknesses having a weight of 160, 185, and 225 lbs. per foot, respectively. At the works all the pipe was tested to pressure 1½ times that to which it is to be subjected when in place.

An important discussion of the design of large riveted steel pipes to

resist not only the internal pressure but also the external pressure from moist earth in which they are laid, together with notes on the design of a pipe 18 ft. diam. 6000 ft. long for the Ontario Water Power Co., Niagara Falls, by Joseph Mayer, will be found in *Eng. News*, April 26, 1906.

STRENGTH OF VARIOUS MATERIALS. EXTRACTS FROM KIRKALDY'S TESTS.

The publication, in a book by W. G. Kirkaldy, of the results of many thousand tests made during a quarter of a century by his father, David Kirkaldy, has made an important contribution to our knowledge concerning the range of variation in strength of numerous materials. A condensed abstract of these results was published in the *American Machinist*, May 11 and 18, 1893, from which the following still further condensed extracts are taken:

The figures for tensile and compressive strength, or, as Kirkaldy calls them, pulling and thrusting stress, are given in pounds per square inch of original section, and for bending strength in pounds of actual stress or pounds per BD^2 (breadth \times square of depth) for length of 36 inches between supports. The contraction of area is given as a percentage of the original area, and the extension as a percentage in a length of 10 inches, except when otherwise stated. The abbreviations T. S., E. L., Contr., and Ext. are used for the sake of brevity, to represent tensile strength, elastic limit, and percentages of contraction of area, and elongation, respectively.

Cast Iron. — 44 tests: T. S. 15,468 to 28,740 pounds; 17 of these were unsound, the strength ranging from 15,468 to 24,357 pounds. Average of all, 23,805 pounds.

Thrusting stress, specimens 2 inches long, 1.34 to 1.5 in. diameter; 43 tests, all sound, 94,352 to 131,912; one, unsound, 93,759; average of all, 113,825.

Bending stress, bars about 1 in. wide by 2 in. deep, cast on edge. Ultimate stress 2876 to 3854; stress per $BD^2 = 725$ to 892; average, 820. Average modulus of rupture, $R = \frac{3}{2}$ stress per $BD^2 \times$ length, = 44,280. Ultimate deflection, 0.29 to 0.40 in.; average, 0.34 inch.

Other tests of cast iron, 460 tests, 16 lots from various sources, gave results with total range as follows: Pulling stress, 12,688 to 33,616 pounds; thrusting stress, 66,363 to 175,950 pounds; bending stress, per BD^2 , 505 to 1128 pounds; modulus of rupture, R , 27,270 to 61,912. Ultimate deflection, 0.21 to 0.45 inch.

The specimen which was the highest in thrusting stress was also the highest in bending, and showed the greatest deflection, but its tensile strength was only 26,502.

The specimen with the highest tensile strength had a thrusting stress of 143,939 and a bending strength, per BD^2 , of 979 pounds with 0.41 deflection. The specimen lowest in T. S. was also lowest in thrusting and bending, but gave 0.38 deflection. The specimen which gave 0.21 deflection had T. S., 19,188; thrusting, 104,281; and bending, 561.

Iron Castings. — 69 tests; tensile strength, 10,416 to 31,652; thrusting stress, ultimate per square inch, 53,502 to 132,031.

Channel Irons. — Tests of 18 pieces cut from channel irons. T. S. 40,693 to 53,141 pounds per square inch; contr. of area from 3.9 to 32.5%. Ext. in 10 in. from 2.1 to 22.5%. The fractures ranged all the way from 100% fibrous to 100% crystalline. The highest T. S., 53,141, with 8.1% contr. and 5.3% ext., was 100% crystalline; the lowest T. S., 40,693, with 3.9 contr. and 2.1% ext., was 75% crystalline. All the fibrous irons showed from 12.2 to 22.5% ext., 17.3 to 32.5 contr., and T. S. from 43,426 to 49,615. The fibrous irons are therefore of medium tensile strength and high ductility. The crystalline irons are of variable T. S., highest to lowest, and low ductility.

Lowmoor Iron Bars. — Three rolled bars $2\frac{1}{2}$ inches diameter; tensile tests: elastic, 23,200 to 24,200; ultimate, 50,875 to 51,905; contraction, 44.4 to 42.5; extension, 29.2 to 24.3. Three hammered bars, $4\frac{1}{2}$ inches diameter, elastic 25,100 to 24,200; ultimate, 46,810 to 49,223; contraction, 20.7 to 46.5; extension, 10.8 to 31.6. Fractures of all, 100 per cent fibrous. In the hammered bars the lowest T. S. was accompanied by lowest ductility.

Iron Bars, Various. — Of a lot of 80 bars of various sizes, some rolled and some hammered (the above Lowmoor bars included), the lowest T. S. (except one) 40,808 pounds per square inch, was shown by the Swedish "hoop L" bar $3\frac{1}{4}$ inches diameter, rolled. Its elastic limit was 19,150 pounds; contraction 68.7% and extension 37.7% in 10 inches. It was also the most ductile of all the bars tested, and was 100% fibrous. The highest T. S., 60,780 pounds, with elastic limit, 29,400; contr., 36.6; and ext., 24.3%, was shown by a "Farnley" 2-inch bar, rolled. It was also 100% fibrous. The lowest ductility 2.6% contr., and 4.1% ext., was shown by a $3\frac{3}{4}$ -inch hammered bar, without brand. It also had the lowest T. S., 40,278 pounds, but rather high elastic limit, 25,700 pounds. Its fracture was 95% crystalline. Thus of the two bars showing the lowest T. S., one was the most ductile and the other the least ductile in the whole series of 80 bars.

Generally, high ductility is accompanied by low tensile strength, as in the Swedish bars, but the Farnley bars showed a combination of high ductility and high tensile strength.

Locomotive Forgings, Iron. — 17 tests average, E. L., 30,420; T. S., 50,521; contr., 36.5; ext. in 10 inches, 23.8.

Broken Anchor Forgings, Iron. — 4 tests: average, E. L., 23,825; T. S., 40,083; contr., 3.0; ext. in 10 inches, 3.8.

Kirkaldy places these two irons in contrast to show the difference between good and bad work. The broken anchor material, he says, is of a most treacherous character, and a disgrace to any manufacturer.

Iron Plate Girder. — Tensile tests of pieces cut from a riveted iron girder after twenty years' service in a railway bridge. Top plate, average of 3 tests, E. L., 26,600; T. S., 40,806; contr., 16.1; ext. in 10 inches, 7.8. Bottom plate, average of 3 tests, E. L., 31,200; T. S., 44,288; contr., 13.3; ext. in 10 inches, 6.3. Web-plate, average of 3 tests, E. L., 28,000; T. S., 45,902; contr., 15.9; ext. in 10 inches, 8.9. Fractures all fibrous. The results of 30 tests from different parts of the girder prove that the iron has undergone no change during twenty years of use.

Steel Plates. — Six plates 100 inches long, 2 inches wide, thickness various, 0.36 to 0.97 inch. T. S., 55,485 to 60,805; E. L., 29,600 to 33,200; contr., 52.9 to 59.5; ext., 17.05 to 18.57.

Steel Bridge Links. — 40 links from Hammersmith Bridge, 1886.

| | T. S. | E. L. | Contr. | Ext. in 100 in. | Fracture. | |
|----------------------------|--------|--------|--------|--------------------|-----------|----------------|
| | | | | | Silky. | Gran- ular. |
| Average of all..... | 67,294 | 38,294 | 34.5% | 14.11% | | |
| Lowest T. S..... | 60,753 | 36,030 | 30.1 | 15.51 | 30% | 70% |
| Highest T. S. and E. L.... | 75,936 | 44,166 | 31.2 | 12.42 | 15 | 85 |
| Lowest E. L..... | 64,044 | 32,441 | 34.7 | 13.43 | 30 | 70 |
| Greatest Contraction..... | 63,745 | 38,118 | 52.8 | 15.46 | 100 | 0 |
| Greatest Extension..... | 65,930 | 36,792 | 40.8 | 17.78 | 35 | 65 |
| Least Contr. and Ext..... | 63,980 | 39,017 | 6.0 | 6.62 | 0 | 100 |

The ratio of elastic to ultimate strength ranged from 50.6 to 65.2 per cent; average, 56.9 per cent.

Extension in lengths of 100 inches. At 10,000 lbs. per sq. in., 0.018 to 0.024; mean, 0.020 inch; at 20,000 lbs. per sq. in., 0.049 to 0.063; mean, 0.055 inch; at 30,000 lbs. per sq. in., 0.083 to 0.100; mean, 0.090; set at 30,000 pounds per sq. in., 0 to 0.062; mean, 0.

The mean extension between 10,000 to 30,000 lbs. per sq. in. increased regularly at the rate of 0.007 inch for each 2000 lbs. per sq. in. increment of strain. This corresponds to a modulus of elasticity of 28,571,429. The least increase of extension for an increase of load of 20,000 lbs. per sq. in., 0.065 inch, corresponds to a modulus of elasticity of 30,769,231, and the greatest, 0.076 inch, to a modulus of 26,315,789.

Steel Rails. — Bending tests, 5 feet between supports, 11 tests of flange rails 72 pounds per yard, 4.63 inches high.

| | Elastic stress. Pounds. | Ultimate stress. Pounds. | Deflection at 50,000 Pounds. | Ultimate Deflection. |
|-------------|----------------------------|-----------------------------|---------------------------------|-------------------------|
| Hardest... | 34,200 | 60,960 | 3.24 ins. | 8 ins. |
| Softest.... | 32,000 | 56,740 | 3.76 " | 8 " |
| Mean..... | 32,763 | 59,209 | 3.53 " | 8 " |

All uncracked at 8 inches deflection.

Pulling tests of pieces cut from same rails. Mean results.

| | Elastic Stress. per sq. in. | Ultimate Pounds. per sq. in. | Contraction of area of frac- ture. | Extension in 10 ins. |
|----------------------|-----------------------------------|------------------------------------|--|-------------------------|
| Top of rails..... | 44,200 | 83,110 | 19.9% | 13.5% |
| Bottom of rails..... | 40,900 | 77,820 | 30.9% | 22.8% |

Steel Tires. — Tensile tests of specimens cut from steel tires.

KRUPP STEEL. — 262 Tests.

| | E. L. | T. S. | Contr. | Ext. in 5 inches. |
|--------------|--------|---------|--------|----------------------|
| Highest..... | 69,250 | 119,079 | 31.9 | 18.1 |
| Mean..... | 52,869 | 104,112 | 29.5 | 19.7 |
| Lowest..... | 41,700 | 90,523 | 45.5 | 23.7 |

VICKERS, SONS & Co. — 70 Tests.

| | E. L. | T. S. | Contr. | Ext. in 5 inches. |
|--------------|--------|---------|--------|----------------------|
| Highest..... | 58,600 | 120,789 | 11.8 | 8.4 |
| Mean..... | 51,066 | 101,264 | 17.6 | 12.4 |
| Lowest..... | 43,700 | 87,697 | 24.7 | 16.0 |

Note the correspondence between Krupp's and Vickers' steels as to tensile strength and elastic limit, and their great difference in contraction and elongation. The fractures of the Krupp steel averaged 22 per cent silky, 78 per cent granular; of the Vicker steel, 7 per cent silky, 93 per cent granular.

Steel Axles. — Tensile tests of specimens cut from steel axles.

PATENT SHAFT AND AXLE TREE CO. — 157 Tests.

| | E. L. | T. S. | Contr. | Ext. in 5 inches. |
|--------------|--------|--------|--------|----------------------|
| Highest..... | 49,800 | 99,009 | 21.1 | 16.0 |
| Mean..... | 36,267 | 72,099 | 33.0 | 23.6 |
| Lowest..... | 31,800 | 61,382 | 34.8 | 25.3 |

VICKERS, SONS & Co. — 125 Tests.

| | E. L. | T. S. | Contr. | Ext. in 5 inches. |
|--------------|--------|--------|--------|----------------------|
| Highest..... | 42,600 | 83,701 | 18.9 | 13.2 |
| Mean..... | 37,618 | 70,572 | 41.6 | 27.5 |
| Lowest..... | 30,250 | 56,388 | 49.0 | 37.2 |

The average fracture of Patent Shaft and Axle Tree Co. steel was 33 per cent silky, 67 per cent granular.

The average fracture of Vickers' steel was 88 per cent silky, 12 per cent granular.

Steel Propeller Shafts. — Tensile tests of pieces cut from two shafts, mean of four tests each. Hollow shaft, Whitworth, T. S., 61,290; E. L., 30,575; contr., 52.8; ext. in 10 inches, 28.6. Solid shaft, Vickers', T. S., 46,870; E. L., 20,425; contr., 44.4; ext. in 10 inches, 30.7.

Thrusting tests, Whitworth, ultimate, 56,201; elastic, 29,300; set at 30,000 lbs., 0.18 per cent; set at 40,000 lbs., 2.04 per cent; set at 50,000 lbs., 3.82 per cent.

Thrusting tests, Vickers', ultimate, 44,602; elastic, 22,250; set at 30,000 lbs., 2.29 per cent; set at 40,000 lbs., 4.69 per cent.

Shearing strength of the Whitworth shaft, mean of four tests, 40,654 lbs. per square inch, or 66.3 per cent of the pulling stress. Specific gravity of the Whitworth steel, 7.867; of the Vickers', 7.856.

Spring Steel. — Untempered, 6 tests, average, E. L., 67,916; T. S., 115,668; contr., 37.8; ext. in 10 inches, 16.6. Spring steel untempered, 15 tests, average, E. L., 38,785; T. S., 69,496; contr., 19.1; ext. in 10 inches, 29.8. These two lots were shipped for the same purpose, viz., railway carriage leaf springs.

Steel Castings. — 44 tests, E. L., 31,816 to 35,567; T. S., 54,928 to 63,840; contr., 1.67 to 15.8; ext., 1.45 to 15.1. Note the great variation in ductility. The steel of the highest strength was also the most ductile.

Riveted Joints, Pulling Tests of Riveted Steel Plates, Triple Riveted Lap Joints, Machine Riveted, Holes Drilled.

Plates, width and thickness, inches:

13.50 × 0.25 13.00 × 0.51 11.75 × 0.78 12.25 × 1.01 14.00 × 0.77

Plates, gross sectional area square inches:

3.375 6.63 9.165 12.372 10.780

Stress, total, pounds:

199,320 332,640 423,180 528,000 455,210

Stress per square inch of gross area, joint:

59,058 50,172 46,173 42,696 42,227

Stress per square inch of plates, solid:

70,765 65,300 64,050 62,280 68,045

Ratio of strength of joint to solid plate:

83.46 76.83 72.09 68.55 62.06

Ratio net area of plate to gross:

73.4 65.5 62.7 64.7 72.9

Where fractured:

plate at plate at plate at plate at rivets
holes. holes. holes. holes. sheared

Rivets, diameter, area and number:

0.45, 0.159, 24 0.64, 0.321, 21 0.95, 0.708, 12 1.08, 0.916, 12 0.95, 0.708, 12

Rivets, total area:

3.816 6.741 8.496 10.992 8.496

Strength of Welds. — Tensile tests to determine ratio of strength of weld to solid bar.

IRON TIE BARS. — 28 Tests.

Strength of solid bars varied from 43,201 to 57,065 lbs.

Strength of welded bars varied from 17,816 to 44,586 lbs.

Ratio of weld to solid varied from 37.0 to 79.1%

IRON PLATES. — 7 Tests.

Strength of solid plate from 44,851 to 47,481 lbs.

Strength of welded plate from 26,442 to 38,931 lbs.

Ratio of weld to solid 57.7 to 83.9%

CHAIN LINKS. — 216 Tests.

Strength of solid bar from 49,122 to 57,875 lbs.

Strength of welded bar from 39,575 to 48,824 lbs.

Ratio of weld to solid 72.1 to 95.4%

IRON BARS. — Hand and Electric Machine Welded.

32 tests, solid iron, average 52,444

17 " electric welded, average 46,836 ratio 89.1%

19 " hand " " 46,899 " 89.3%

STEEL BARS AND PLATES. — 14 Tests.

Strength of solid 54,226 to 64,580

Strength of weld 28,553 to 46,019

Ratio weld to solid 52.6 to 82.1%

The ratio of weld to solid in all the tests ranging from 37.0 to 95.4 is proof of the great variation of workmanship in welding.

Cast Copper. — 4 tests, average, E. L. 5900; T. S., 24,781; contr., 24.5; ext., 21.8.

Copper Plates. — As rolled, 22 tests, 0.26 to 0.75 in. thick; E. L., 9766 to 18,650; T. S., 30,993 to 34,281; contr., 31.1 to 57.6; ext., 39.9 to 52.2. The variation in elastic limit is due to difference in the heat at which the plates were finished. Annealing reduces the T. S. only about 1000 pounds, but the E. L. from 3000 to 7000 pounds.

Another series, 0.38 to 0.52 in. thick; 148 tests, T. S., 29,099 to 31,924; contr., 28.7 to 56.7; ext. in 10 inches, 28.1 to 41.8. Note the uniformity in tensile strength.

Drawn Copper. — 74 tests (0.88 to 1.08 inch diameter); T. S., 31,634 to 40,557; contr., 37.5 to 64.1; ext. in 10 inches, 5.8 to 48.2.

Bronze from a Propeller Blade. — Means of two tests each from center and edge. Central portion (sp. gr. 8.320), E. L., 7550; T. S., 26,312; contr., 25.4; ext. in 10 inches, 32.8. Edge portion (sp. gr. 8.550), E. L., 8950; T. S., 35,960; contr., 37.8; ext. in 10 inches, 47.9.

Cast German Silver. — 10 tests: E. L., 13,400 to 29,100; T. S., 23,714 to 46,540; contr., 3.2 to 21.5; ext. in 10 inches, 0.6 to 10.2.

Thin Sheet Metal. — Tensile Strength.

| | |
|-------------------------------|------------------|
| German silver, 2 lots..... | 75,816 to 87,129 |
| Bronze, 4 lots..... | 73,380 to 92,086 |
| Brass, 2 lots..... | 44,398 to 58,188 |
| Copper, 9 lots..... | 30,470 to 48,450 |
| Iron, 13 lots, lengthway..... | 44,331 to 59,484 |
| Iron, 13 lots, crossway..... | 39,838 to 57,350 |
| Steel, 6 lots..... | 49,253 to 78,251 |
| Steel, 6 lots, crossway..... | 55,948 to 80,799 |

Wire Ropes.

Selected Tests Showing Range of Variation.

| Description. | Circumference, inches. | Weight per Fathom. | Strands. | | Diameter of Wires, inches. | Hemp Core. | Ultimate Strength, lbs. |
|-------------------|---------------------------|-----------------------|--------------------|------------------|-------------------------------|------------------|-------------------------------|
| | | | No. of Strands. | No. of Wires. | | | |
| Galvanized..... | 7.70 | 53.00 | 6 | 19 | 0.1563 | Main | 339,780 |
| Ungalvanized..... | 7.00 | 53.10 | 7 | 19 | 0.1495 | Main and Strands | 314,860 |
| Ungalvanized..... | 6.38 | 42.50 | 7 | 19 | 0.1347 | Wire Core | 295,920 |
| Galvanized..... | 7.10 | 37.57 | 6 | 30 | 0.1004 | Main and Strands | 272,750 |
| Ungalvanized..... | 6.18 | 40.46 | 7 | 19 | 0.1302 | Wire Core | 268,470 |
| Ungalvanized..... | 6.19 | 40.33 | 7 | 19 | 0.1316 | Wire Core | 221,820 |
| Galvanized..... | 4.92 | 20.86 | 6 | 30 | 0.0728 | Main and Strands | 190,890 |
| Galvanized..... | 5.36 | 18.94 | 6 | 12 | 0.1104 | Main and Strands | 136,550 |
| Galvanized..... | 4.82 | 21.50 | 6 | 7 | 0.1693 | Main | 129,710 |
| Ungalvanized..... | 3.65 | 12.21 | 6 | 19 | 0.0755 | Main | 110,180 |
| Ungalvanized..... | 3.50 | 12.65 | 7 | 7 | 0.122 | Wire Core | 101,440 |
| Ungalvanized..... | 3.82 | 14.12 | 6 | 7 | 0.135 | Main | 98,670 |
| Galvanized..... | 4.11 | 11.35 | 6 | 12 | 0.080 | Main and Strands | 75,110 |
| Galvanized..... | 3.31 | 7.27 | 6 | 12 | 0.068 | Main and Strands | 55,095 |
| Ungalvanized..... | 3.02 | 8.62 | 6 | 7 | 0.105 | Main | 49,555 |
| Ungalvanized..... | 2.68 | 6.26 | 6 | 6 | 0.0963 | Main and Strands | 41,205 |
| Galvanized..... | 2.87 | 5.43 | 6 | 12 | 0.0560 | Main and Strands | 38,555 |
| Galvanized..... | 2.46 | 3.85 | 6 | 12 | 0.0472 | Main and Strands | 28,075 |
| Ungalvanized..... | 1.75 | 2.80 | 6 | 7 | 0.0619 | Main | 24,552 |
| Galvanized..... | 2.04 | 2.72 | 6 | 12 | 0.0378 | Main and Strands | 20,415 |
| Galvanized..... | 1.76 | 1.85 | 6 | 12 | 0.0305 | Main | 14,634 |

Wire. — Tensile Strength.

| | |
|-----------------------------------|--------------------|
| German silver, 5 lots..... | 81,735 to 92,224 |
| Bronze, 1 lot..... | 78,049 |
| Brass, as drawn, 4 lots..... | 81,114 to 98,578 |
| Copper, as drawn, 3 lots..... | 37,607 to 46,494 |
| Copper annealed, 3 lots..... | 34,936 to 45,210 |
| Copper (another lot), 4 lots..... | 35,052 to 62,190 |
| Copper (extension 36.4 to 0.6%). | |
| Iron, 8 lots..... | 59,246 to 97,908 |
| Iron (extension 15.1 to 0.7%). | |
| Steel, 8 lots..... | 103,272 to 318,823 |

The steel of 318,823 T. S. was 0.047 inch diam., and had an extension of only 0.3 per cent; that of 103,272 T. S. was 0.107 inch diam., and had an extension of 2.2 per cent. One lot of 0.044 inch diam. had 267,114 T. S., and 5.2 per cent extension.

Hemp Ropes, Untarred. — 15 tests of ropes from 1.53 to 6.90 inches circumference, weighing 0.42 to 7.77 pounds per fathom, showed an ultimate strength of from 1670 to 33,808 pounds, the strength per fathom weight varying from 2872 to 5534 pounds.

Hemp Ropes, Tarred. — 15 tests of ropes from 1.44 to 7.12 inches circumference, weighing from 0.38 to 10.39 pounds per fathom, showed an ultimate strength of from 1046 to 31,549 pounds, the strength per fathom weight varying from 1767 to 5149 pounds.

Cotton Ropes. — 5 ropes, 2.48 to 6.51 inches circumference, 1.08 to 8.17 pounds per fathom. Strength 3089 to 23,258 pounds, or 2474 to 3346 pounds per fathom weight.

Manila Ropes. — 35 tests: 1.19 to 8.90 inches circumference, 0.20 to 11.40 pounds per fathom. Strength 1280 to 65,550 pounds, or 3003 to 7394 pounds per fathom weight.

Belting.

| No. of lots. | Tensile strength per square inch. |
|--|--------------------------------------|
| 11 Leather, single, ordinary tanned..... | 3248 to 4824 |
| 4 Leather, single, Helvetia..... | 5631 to 5944 |
| 7 Leather, double, ordinary tanned..... | 2160 to 3572 |
| 8 Leather, double Helvetia..... | 4078 to 5412 |
| 6 Cotton, solid woven..... | 5648 to 8869 |
| 14 Cotton, folded, stitched..... | 4570 to 7750 |
| 1 Flax, solid, woven..... | 9946 |
| 1 Flax, folded, stitched..... | 6389 |
| 6 Hair, solid, woven..... | 3852 to 5159 |
| 2 Rubber, solid, woven..... | 4271 to 4343 |

Canvas. — 35 lots: Strength, lengthwise, 113 to 408 pounds per inch; crossways, 191 to 468 pounds per inch.

The grades are numbered 1 to 6, but the weights are not given. The strengths vary considerably, even in the same number.

Marbles. — Crushing strength of various marbles. 38 tests, 8 kinds. Specimens were 6-inch cubes, or columns 4 to 6 inches diameter, and 6 and 12 inches high. Range 7542 to 13,720 pounds per square inch.

Granite. — Crushing strength, 17 tests: square columns 4 × 4 and 6 × 4, 4 to 24 inches high, 3 kinds. Crushing strength ranges 10,026 to 13,271 pounds per square inch. (Very uniform.)

Stones. — (Probably sandstone, local names only given.) 11 kinds, 42 tests, 6 × 6, columns 12, 18 and 24 inches high. Crushing strength ranges from 2105 to 12,122. The strength of the column 24 inches long is generally from 10 to 20 per cent less than that of the 6-inch cube.

Stones. — (Probably sandstone) tested for London & Northwestern Railway. 16 lots, 3 to 6 tests in a lot. Mean results of each lot ranged from 3785 to 11,956 pounds. The variation is chiefly due to the stones being from different lots. The different specimens in each lot gave results which generally agreed within 30 per cent.

Bricks. — Crushing strength, 8 lots; 6 tests in each lot; mean results ranged from 1835 to 9209 pounds per square inch. The maximum variation in the specimens of one lot was over 100 per cent of the lowest. In the most uniform lot the variation was less than 20 per cent.

Wood. — Transverse and Thrusting Tests.

| | Tests. | Sizes abt. in square. | Span, inches. | Ultimate Stress. | $S = \frac{LW}{4BD^2}$. | Thrusting Stress per sq. in. |
|-----------------------|--------|-----------------------|---------------|----------------------------------|---------------------------|------------------------------|
| Pitch pine..... | 10 | 11½ to 12½ | 144 | 45,856 to 80,520 37,948 | 1096 to 1403 657 | 3586 to 5438 2478 |
| Dantzic fir..... | 12 | 12 to 13 | 144 | to 54,152 32,856 | to 790 1505 | to 3423 2473 |
| English oak..... | 3 | 4½ × 12 | 120 | to 39,084 23,624 | to 1779 1190 | to 4437 2656 |
| American white oak... | 5 | 4½ × 12 | 120 | to 26,952 | 1372 | to 3899 |

| | |
|---|----------------|
| Demerara greenheart, 9 tests (thrusting)..... | 8169 to 10,785 |
| Oregon pine, 2 tests..... | 5888 and 7284 |
| Honduras mahogany, 1 test..... | 6769 |
| Tobasco mahogany, 1 test..... | 5978 |
| Norway spruce, 2 tests..... | 5259 and 5494 |
| American yellow pine, 2 tests..... | 3875 and 3993 |
| English ash, 1 test..... | 3025 |

Portland Cement. — (Austrian.) Cross-sections of specimens 2 × 2½ inches for pulling tests only; cubes, 3 × 3 inches for thrusting tests; weight, 98.8 pounds per imperial bushel; residue, 0.7 per cent with sieve 2500 meshes per square inch; 38.8 per cent by volume of water required for mixing; time of setting, 7 days; 10 tests to each lot. The mean results in lbs. per sq. in. were as follows:

| Age. | Cement alone, Pulling. | Cement alone, Thrusting. | 1 Cement, 2 Sand, Thrusting. | 1 Cement, 3 Sand, Thrusting. | 1 Cement, 4 Sand, Thrusting. |
|---------|------------------------|--------------------------|------------------------------|------------------------------|------------------------------|
| 10 days | 376 | 2910 | 893 | 407 | 228 |
| 20 days | 420 | 3342 | 1023 | 494 | 275 |
| 30 days | 451 | 3724 | 1172 | 594 | 338 |

Portland Cement. — Various samples pulling tests, 2 × 2½ inches cross-section, all aged 10 days, 180 tests; ranges 87 to 643 pounds per square inch.

TENSILE STRENGTH OF WIRE.

(From J. Bucknall Smith's Treatise on Wire.)

| | Tons per sq. in. sectional area. | Pounds per sq. in. sectional area. |
|--|----------------------------------|------------------------------------|
| Black or annealed iron wire..... | 25 | 56,000 |
| Bright hard drawn | 35 | 78,400 |
| Bessemer, steel wire..... | 40 | 89,600 |
| Mild Siemens-Martin steel wire | 60 | 134,000 |
| High carbon ditto (or "improved")..... | 80 | 179,200 |
| Crucible cast-steel "improved" wire..... | 100 | 224,000 |
| "Improved" cast-steel "plough" | 120 | 268,800 |
| Special qualities of tempered and improved cast steel wire may attain..... | 150 to 170 | 336,000 to 380,800 |

MISCELLANEOUS TESTS OF MATERIALS.

Reports of Work of the Watertown Testing-machine in 1883.

TESTS OF RIVETED JOINTS, IRON AND STEEL PLATES.

| | Thickness Plate. | Diameter, Rivets, inches. | Diameter, Punched Holes, inches. | Width Plate Tested, inches. | No. Rivets. | Pitch Rivets, inches. | Tensile Strength Joint in Net Sec- tion of Plate per square inch, pounds. | Tensile Strength Plate per square inch, pounds. | Efficiency of Joint, Per Cent. |
|---|------------------|------------------------------|--|--------------------------------|-------------|--------------------------|---|---|-----------------------------------|
| * | 3/8 | 11/16 | 3/4 | 10 1/2 | 6 | 13/4 | 39,300 | 47,180 | 47.0 |
| * | 3/8 | 11/16 | 3/4 | 10 1/2 | 6 | 13/4 | 41,000 | 47,180 | 49.0 |
| * | 1/2 | 3/4 | 13/16 | 10 | 5 | 2 | 35,650 | 44,615 | 45.6 |
| * | 1/2 | 3/4 | 13/16 | 10 | 5 | 2 | 35,150 | 44,615 | 44.9 |
| * | 3/8 | 11/16 | 3/4 | 10 | 5 | 2 | 46,360 | 47,180 | 59.9 |
| * | 3/8 | 11/16 | 3/4 | 10 | 5 | 2 | 46,875 | 47,180 | 60.5 |
| * | 1/2 | 3/4 | 13/16 | 10 | 5 | 2 | 46,400 | 44,615 | 59.4 |
| * | 1/2 | 3/4 | 13/16 | 10 | 5 | 2 | 46,140 | 44,615 | 59.2 |
| * | 5/8 | 1 | 1 1/16 | 10 1/2 | 4 | 2 5/8 | 44,260 | 44,635 | 57.2 |
| * | 5/8 | 1 | 1 1/16 | 10 1/2 | 4 | 2 5/8 | 42,350 | 44,635 | 54.9 |
| * | 3/4 | 1 1/8 | 1 3/16 | 11.9 | 4 | 2.9 | 42,310 | 46,590 | 52.1 |
| * | 3/4 | 1 1/8 | 1 3/16 | 11.9 | 4 | 2.9 | 41,920 | 46,590 | 51.7 |
| * | 3/8 | 3/4 | 13/16 | 10 1/2 | 6 | 13/4 | 61,270 | 53,330 | 59.5 |
| † | 3/8 | 3/4 | 13/16 | 10 1/2 | 6 | 13/4 | 60,830 | 53,330 | 59.1 |
| † | 1/2 | 15/16 | 1 | 10 | 5 | 2 | 47,530 | 57,215 | 40.2 |
| † | 1/2 | 15/16 | 1 | 10 | 5 | 2 | 49,840 | 57,215 | 42.3 |
| † | 3/8 | 11/16 | 3/4 | 10 | 5 | 2 | 62,770 | 53,330 | 71.7 |
| † | 3/8 | 11/16 | 3/4 | 10 | 5 | 2 | 61,210 | 53,330 | 69.8 |
| † | 1/2 | 15/16 | 1 | 10 | 5 | 2 | 68,920 | 57,215 | 57.1 |
| † | 1/2 | 15/16 | 1 | 10 | 5 | 2 | 66,710 | 57,215 | 55.0 |
| † | 5/8 | 1 | 1 1/16 | 9 1/2 | 4 | 2 3/8 | 62,180 | 52,445 | 63.4 |
| † | 5/8 | 1 | 1 1/16 | 9 1/2 | 4 | 2 3/8 | 62,590 | 52,445 | 63.8 |
| † | 3/4 | 1 1/8 | 1 3/16 | 10 | 4 | 2 1/2 | 54,650 | 51,545 | 54.0 |
| † | 3/4 | 1 1/8 | 1 3/16 | 10 | 4 | 2 1/2 | 54,200 | 51,545 | 53.4 |

* Iron.

† Steel.

‡ Lap-joint.

§ Butt-joint.

The efficiency of the joints is found by dividing the maximum tensile stress on the gross sectional area of plate by the tensile strength of the material.

COMPRESSION TESTS OF 3 × 3 INCH WROUGHT-IRON BARS.

| Length, inches. | Tested with Two Pin Ends, Pins 1 1/2 in. Diam. Com- pressive Strength, lbs. per sq. in. | Tested with Two Flat Ends. Com- pressive Strength, lbs. per sq. in. | Tested with One Flat and One Pin End. Compressive Strength, lbs. per sq. in. |
|-----------------|---|--|--|
| 30..... | { 28,260 31,990 | | |
| 60..... | { 26,310 26,640 | | |
| 90..... | { 24,030 25,380 | { 26,780 25,580 | { 25,120 25,190 |
| 120..... | { 20,660 20,200 | { 23,010 22,450 | { 22,450 21,870 |
| 150..... | { 16,520 17,840 | | |
| 180..... | { 13,010 15,700 | | |

| Tested with Two Pin Ends. Length of Bars 120 inches. | Diameter of Pins. | | Comp. Str., per sq. in., lbs. |
|--|----------------------|-------|----------------------------------|
| | | | |
| | | | |
| | | | |
| | 7/8 inch. | | 16,250 |
| | 1 1/8 inches. | | 17,740 |
| | 1 7/8 " | | 21,400 |
| | 2 1/4 " | | 22,210 |

COMPRESSION OF WROUGHT-IRON COLUMNS, LATTICED BOX AND SOLID WEB.

ALL TESTED WITH PIN ENDS.

| Columns made of | Length, feet. | Sectional Area, square inch. | Total Weight of Column, pounds. | Ultimate Strength, per square inch, pounds |
|---|---------------|---------------------------------|---------------------------------------|---|
| 6-inch channel, solid web..... | 10.0 | 9.831 | 432 | 30,220 |
| 6 " " " "..... | 15.0 | 9.977 | 592 | 21,050 |
| 6 " " " "..... | 20.0 | 9.762 | 755 | 16,220 |
| 8 " " " "..... | 20.0 | 16.281 | 1,290 | 22,540 |
| 8 " " " "..... | 26.8 | 16.141 | 1,645 | 17,570 |
| 8-inch channels, with 5/16-in. continuous plates..... | 26.8 | 19.417 | 1,940 | 25,290 |
| 5/16-inch continuous plates and angles.. Width of plates, 12 in., 1 in. and 7.35 in. | 26.8 | 16.168 | 1,765 | 28,020 |
| 7/16-inch continuous plates and angles.. Plates 12 in. wide..... | 26.8 | 20.954 | 2,242 | 25,770 |
| 8-inch channels, latticed..... | 13.3 | 7.628 | 679 | 33,910 |
| 8 " " " "..... | 20.0 | 7.621 | 924 | 34,120 |
| 8 " " " "..... | 26.8 | 7.673 | 1,255 | 29,870 |
| 8-inch channels, latticed, swelled sides.. | 13.4 | 7.624 | 684 | 33,530 |
| 8 " " " "..... | 20.0 | 7.517 | 921 | 33,390 |
| 8 " " " "..... | 26.8 | 7.702 | 1,280 | 30,770 |
| 10-inch channels, latticed, swelled sides. | 16.8 | 11.944 | 1,470 | 33,740 |
| 10 " " " "..... | 25.0 | 12.175 | 1,926 | 32,440 |
| 10 " " " "..... | 16.7 | 12.366 | 1,549 | 31,130 |
| 10 " " " "..... | 25.0 | 11.932 | 1,962 | 32,740 |
| * 10-inch channels, latticed one side; con- tinuous plate one side..... | 25.0 | 17.622 | 1,848 | 26,190 |
| † 10-inch channels, latticed one side; con- tinuous plate one side..... | 25.0 | 17.721 | 1,827 | 17,270 |

* Pins in center of gravity of channel bars and continuous plate, 1.63 inches from center line of channel bars.

† Pins placed in center of gravity of channel bars.

TENSILE TEST OF SIX STEEL EYE-BARS.

COMPARED WITH SMALL TEST INGOTS.

The steel was made by the Cambria Iron Company, and the eye-bar heads made by Keystone Bridge Company by upsetting and hammering. All the bars were made from one ingot. Two test pieces, 3/4-inch round, rolled from a test-ingot, gave elastic limit 48,040 and 42,210 pounds; tensile strength, 73,150 and 69,470 pounds, and elongation in 8 inches, 22.4 and 25.6 per cent respectively. The ingot from which the eye-bars were made was 14 inches square, rolled to billet, 7 × 6 inches. The eye-bars were rolled to 6 1/2 × 1 inch. Chemical tests gave carbon 0.27 to 0.30; manganese, 0.64 to 0.73; phosphorus, 0.074 to 0.098.

| Gauged Length, inches. | Elastic limit, lbs. per sq. in. | Tensile strength per sq. in., lbs. | Elongation per cent, in Gauged Length. |
|------------------------|---------------------------------|------------------------------------|--|
| 160 | 37,480 | 67,800 | 15.8 |
| 160 | 36,650 | 64,000 | 6.96 |
| 160 | | 71,560 | 8.6 |
| 200 | 37,600 | 68,720 | 12.3 |
| 200 | 35,810 | 65,850 | 12.0 |
| 200 | 33,230 | 64,410 | 16.4 |
| 200 | 37,640 | 68,290 | 13.9 |

The average tensile strength of the $\frac{3}{4}$ -inch test pieces was 71,310 lbs., that of the eye-bars 67,230 lbs., a decrease of 5.7%. The average elastic limit of the test pieces was 45,150 lbs., that of the eye-bars 36,402 lbs., a decrease of 19.4%. The elastic limit of the test pieces was 63.3% of the ultimate strength, that of the eye-bars 54.2% of the ultimate strength.

Tests of 11 full-sized eye bars, $15 \times 1\frac{1}{4}$ to $21\frac{1}{16}$ in., 20.5 to 21.4 ft. long between centers of pins, made by the Phoenix Iron Co., are reported in *Eng. News*, Feb. 2, 1905. The average T.S. of the bars was 58,300 lbs. per sq. in., E.L., 32,800. The average T.S. of small specimens was 63,900, E.L., 37,000. The T.S. of the full-sized bars averaged 8.8% and the E.L. 12.1% lower than the small specimens.

EFFECT OF COLD-DRAWING ON STEEL.

Three pieces cut from the same bar of hot-rolled steel:

1. Original bar, 2.03 in. diam., gauged length 30 in., tensile strength 55,400 lbs. per square in.; elongation 23.9%.
2. Diameter reduced in compression dies (one pass) .094 in.; T. S. 70,420; el. 2.7% in 20 in.
3. " " " " " " " 0.222 in.; T. S. 81,890; el. 0.075% in 20 in.

Compression test of cold-drawn bar (same as No. 3), length 4 in., diam. 1.808 in.: Compressive strength per sq. in., 75,000 lbs.; amount of compression 0.057 in.; set 0.04 in. Diameter increased by compression to 1.821 in. in the middle; to 1.813 in. at the ends.

MISCELLANEOUS TESTS OF IRON AND STEEL.

Tests of Cold-rolled and Cold-drawn Steel, made by the Cambria Iron Co. in 1897, gave the following results (averages of 12 tests of each):

| | E. L. | T. S. | El. in 8 in. | Red. |
|--------------------------|--------|--------|--------------|-------|
| Before cold-rolling..... | 35,390 | 59,980 | 28.3% | 58.5% |
| After cold-rolling..... | 72,530 | 79,830 | 9.6% | 34.9% |
| After cold-drawing..... | 76,350 | 83,860 | 8.9% | 34.2% |

The original bars were 2 in. and $\frac{7}{8}$ in. diameter. The test pieces cut from the bars were $\frac{3}{4}$ in. diam., 18 in. long. The reduction in diameter from the hot-rolled to the cold-rolled or cold-drawn bar was $\frac{1}{16}$ in. in each case.

Cold Rolled Steel Shafting (Jones & Laughlins) $1\frac{11}{16}$ in. diam. — Torsion tests of 12 samples gave apparent outside fiber stress, calculated from maximum twisting moment, 70,700 to 82,900 lbs. per sq. in.; fiber stress at elastic limit, 32,500 to 38,800 lbs. per sq. in.; shearing modulus of elasticity, 11,800,000 to 12,100,000; number of turns per foot before fracture, 1.60 to 2.06. — *Tech. Quar.*, vol. xii, Sept., 1899.

Torsion Tests on Cold Rolled Shafting. — (*Tech. Quar.* XIII, No. 3, 1900, p. 229.) 14 tests. Diameter about 1.69 in. Gauged length, 40 to 50 in. Outside fiber stress at elastic limit, 28,610 to 33,590 lbs. per sq. in.; apparent outside fiber stress at maximum load, 67,980 to 77,290. Shearing modulus of elasticity, 11,400,000 to 12,030,000 lbs. per sq. in. Turns per foot between jaws at fracture, 0.413 to 2.49.

Torsion Tests on Refined Iron. — $1\frac{3}{4}$ in. diam. 14 tests. Gauged length, 40 ins. Outside fiber stress at elastic limit, 12,790 to 19,140 lbs. per sq. in.; apparent outside fiber stress at maximum load, 45,350 to 58,340. Shearing modulus of elasticity, 10,220,000 to 11,700,000. Turns per foot between jaws at fracture, 1.08 to 1.42.

Tests of Steel Angles with Riveted End Connections. (F. P. McKibbin, *Proc. A.S.T.M.*, 1907.) — The angles broke through the rivet holes in all cases. The strength developed ranged from 62.5 to 79.1% of the ultimate strength of the gross area, or from 73.9 to 92% of the calculated strength of the net section at the rivet holes.

SHEARING STRENGTH.

H. V. Loss in *American Engineer and Railroad Journal*, March and April, 1893, describes an extensive series of experiments on the shearing of iron and steel bars in shearing machines. Some of his results are:

Depth of penetration at point of maximum resistance for soft steel bars is independent of the width, but varies with the thickness. If d = depth of penetration and t = thickness, $d = 0.3t$ for a flat knife, $d = 0.25t$ for a 4° bevel knife, and $d = 0.16\sqrt{t^3}$ for an 8° bevel knife. The ultimate pressure per inch of width in flat steel bars is approximately "0,000 lbs. $\times t$ ". The energy consumed in foot-pounds per inch width of steel bars is, approximately: 1" thick, 1300 ft.-lbs.; $1\frac{1}{2}$ ", 2500; $1\frac{3}{4}$ ", 3700; $1\frac{7}{8}$ ", 4500; the energy increasing at a slower rate than the square of the thickness. Iron angles require more energy than steel angles of the same size; steel breaks while iron has to be cut off. For hot-rolled steel the resistance per square inch for rectangular sections varies from 4400 lbs. to 20,500 lbs., depending partly upon its hardness and partly upon the size of its cross-area, which latter element indirectly but greatly indicates the temperature, as the smaller dimensions require a considerably longer time to reduce them down to size, which time again means loss of heat.

It is not probable that the resistance in practice can be brought very much below the lowest figures here given — viz., 4400 lbs. per square inch — as a decrease of 1000 lbs. will henceforth mean a considerable increase in cross-section and temperature.

Relation of Shearing to Tensile Strength of Different Metals. E. G. Izod, in a paper presented to the Institution of Mech. Engrs. (*Am. Mach.*, Jan. 18, 1906), describes a series of tests on bars and plates of different metals. The specimens were firmly clamped on two steel plates with opposed shearing edges 4 ins. apart, and a shearing block, which was a sliding fit between these edges, was brought down upon the specimen, so as to cut it in double shear, by a testing machine.

| | <i>a</i> | <i>b</i> | <i>c</i> | | <i>a</i> | <i>b</i> | <i>c</i> |
|-------------------|----------|----------|----------|------------------------|----------|----------|----------|
| Cast iron. A..... | 9.7 | | 152 | Rolled phosphor- | | | |
| Cast iron. B..... | 13.4 | | 111 | bronze..... | 39.5 | 11.7 | 61 |
| Cast iron. C..... | 11.3 | | 122 | Aluminum..... | 6.4 | 25.5 | 70 |
| Cast aluminum- | | | | Aluminum alloy..... | 12.7 | 9.6 | 59 |
| bronze..... | 33.1 | 12.5 | 60 | Wrought-iron bar.. | 26.0 | 22.5 | 75 |
| Cast phosphor- | | | | Mild-steel, 0.14 car- | | | |
| bronze..... | 13.4 | 2.2 | 128 | bon..... | 26.9 | 34.7 | 78 |
| Cast phosphor- | | | | Crucible steel, 0.12 C | 24.9 | 43.0 | 74 |
| bronze..... | 19.7 | 8.0 | 93 | 0.48 C..... | 42.1 | 26.0 | 68 |
| Gun metal..... | 12.1 | 7.8 | 103 | 0.71 C..... | 56.3 | 15.0 | 65 |
| Yellow brass..... | 7.5 | 6.5 | 126 | 0.77 C..... | 61.3 | 11.0 | 62 |
| Yellow brass..... | 16.0 | 35.0 | 74 | | | | |

a. Tensile strength of the metal, gross tons per sq. in.; *b.* elongation in 2 in.%; *c.* ratio shearing \div tensile strength. The results seem to point to the fact that there is no common law connecting the ultimate shearing stress with the ultimate tensile stress, the ratio varying greatly with different materials. The test figures from crystalline materials, such as cast iron or those with very little or no elongation, seem to indicate that the ultimate shear stress exceeds the ultimate tensile stress by as much as 20 or 25%, while from those with a fairly high measure of ductility, the ultimate shear stress may be anything from 0 to 50% less than the ultimate tensile stress.

For shearing strength of rivets, see pages 407 and 412.

STRENGTH OF IRON AND STEEL PIPE.

Tests of Strength and Threading of Wrought-Iron and Steel Pipe. T. N. Thomson, in Proc. Am. Soc. Heat and Vent. Engineers, vol. xii., p. 80, describes some experiments on welded wrought iron and steel pipes. Short rings of 6-in. pipe were pulled in the direction of a diameter so as to elongate the ring. Four wrought iron rings broke at 2400, 3000, 3100 and 4100 lbs. and four steel rings at 5300 (defective weld) 18,000, 29,000 and 35,000 lbs. Another series of 9 tests each were tested so as to show the tensile strength of the metal and of the weld. The average strength of the metal was, iron, 34,520, steel, 61,850 lbs. The strength of the weld in iron ranged from 49 to 84, averaging 71 per cent of the strength of the metal, and in steel from 50 to 93, averaging 72%.

A large number of iron and steel pipes of different sizes were tested by twisting, the force being applied at the end of a three-foot lever. The average pull on the steel pipes was: $\frac{1}{2}$ in. pipe, 109 lbs.; 1 in., 172 lbs.; $1\frac{1}{2}$ in., 300 lbs.; number of turns in 6 ft. length, respectively, 15, 8 and $5\frac{1}{2}$. Per cent failed in weld, 0, 13 and 13 respectively. For different lots of iron pipe the average pull was: $\frac{1}{2}$ in., 68, 81 and 65 lbs.; 1 in., 154, 136, 107 lbs.; $1\frac{1}{2}$ in. 256, 250, 258 lbs. The number of turns in 6 feet for the nine lots were respectively, $4\frac{1}{2}$, $5\frac{3}{4}$, $2\frac{1}{2}$; $6\frac{1}{4}$, $3\frac{1}{2}$, $2\frac{1}{2}$; $4\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{4}$. The failures in the weld ranged from 33 to 100% in the different lots.

The force required to thread $1\frac{1}{4}$ -in. pipe with two forms of die was tested by pulling on a lever 21 ins. long. The results were as follows:

Old form of die, iron pipe. . . 83 to 87 lbs. pull, steel pipe 100 to 111 lbs.
Improved die, iron pipe. . . . 58 to 62 lbs. pull, steel pipe, 60 to 65 lbs.

Mr. Thomson gives the following table showing approximately the steady pull in pounds required at the end of a 16-in. lever to thread twist and split iron and steel pipe of small sizes:

| | To Thread with Oiled Dies. | | | To Twist Lbs. | To Split Lbs. | Safety Margin Lbs. - |
|------------------------------|----------------------------|------------------|------------------|---------------|---------------|----------------------|
| | New Rake Dies. | New Common Dies. | Old Common Dies. | | | |
| $\frac{1}{2}$ in. steel..... | 34 | 56 | 60 | 122 | 152 | 74 |
| $\frac{1}{2}$ in. iron..... | 27 | 33 | 49 | 102 | 110 | 46 |
| $\frac{3}{4}$ in. steel..... | 44 | 60 | 91 | 150 | 240 | 112 |
| $\frac{3}{4}$ in. iron..... | 44 | 51 | 73 | 140 | 176 | 81 |
| 1 in. steel..... | 69 | 111 | 124 | 286 | 420 | 259 |
| 1 in. iron..... | 62 | 106 | 116 | 273 | 327 | 173 |

The margin of safety is computed by adding 30% to the pull required to thread with the old dies and subtracting the sum from the pull required to split the pipe. If the mechanic pulls on the dies beyond the limit, due to imperfect dies, or to a hard spot in the pipe, he will split the pipe.

Old Boiler Tubes used as Columns. (*Tech. Quar.* XIII, No. 3, 1900, p. 225.) Thirteen tests were made of old 4-in. tubes taken from worn-out boilers. The lengths were from 6 to 8 ft., ratio l/r 53 to 71, and thickness of metal 0.13 to 0.18 in. It is not stated whether the tubes were iron or steel. The maximum load ranged from 34,600 to 50,000 lbs., and the maximum load per sq. in. from 17,100 to 27,500 lbs. Six new tubes also were tested, with maximum loads 55,600 to 64,800 lbs., and maximum loads per sq. in. 31,600 to 38,100 lbs. The relation of the strength per sq. in. of the old tubes to the ratio l/r was very variable, being expressed approximately by the formula $S = 41,000 - 300 l/r \pm 5000$. That of the new tubes is approximately $S = 52,000 - 300 l/r \pm 2000$.

HOLDING-POWER OF BOILER-TUBES EXPANDED INTO TUBE-SHEETS.

Experiments by Chief Engineer W. H. Shock, U. S. N., on brass tubes, 2½ inches diameter, expanded into plates ¾ inch thick, gave results ranging from 5850 to 46,000 lbs. Out of 48 tests 5 gave figures under 10,000 lbs., 12 between 10,000 and 20,000 lbs., 18 between 20,000 and 30,000 lbs., 10 between 30,000 and 40,000 lbs., and 3 over 40,000 lbs.

Experiments by Yarrow & Co., on steel tubes, 2 to 2¼ inches diameter, gave results similarly varying, ranging from 7900 to 41,715 lbs., the majority ranging from 20,000 to 30,000 lbs. In 15 experiments on 4 and 5 inch tubes the strain ranged from 20,720 to 68,040 lbs. Beading the tube does not necessarily give increased resistance, as some of the lower figures were obtained with beaded tubes. (See paper on Rules Governing the Construction of Steam Boilers, Trans. Engineering Congress, Section G, Chicago, 1893.)

The Slipping Point of Rolled Boiler-Tube Joints.

(O. P. Hood and G. L. Christensen, *Trans. A. S. M. E.*, 1908).

When a tube has started from its original seat, the fit may be no longer continuous at all points and a leak may result, although the ultimate holding power of the tube may not be impaired. A small movement of the tube under stress is then the preliminary to a possible leak, and it is of interest to know at what stress this slipping begins.

As results of a series of experiments with tube sheets of from ½ in. to 1 in. in thickness and with straight and tapered tube seats, the authors found that the slipping point of a 3-in. 12-gage Shelby cold-drawn tube rolled into a straight, smooth machined hole in a 1-in. sheet occurs with a pull of about 7,000 lbs. The frictional resistance of such tubes is about 750 lbs. per sq. in. of tube-bearing area in sheets ⅝ in. and 1 in. thick.

Various degrees of rolling do not greatly affect the point of initial slip, and for higher resistances to initial slip other resistance than friction must be depended upon. Cutting a 10-pitch square thread in the seat, about 0.01 in. deep will raise the slipping point to three or four times that in a smooth hole. In one test this thread was made 0.015 in. deep in a sheet 1 in. thick, giving an abutting area of about 1.4 sq. in., and a resistance to initial slip of 45,000 lbs. The elastic limit of the tube was reached at about 34,000 lbs.

Where tubes give trouble from slipping and are required to carry an unusual load, the slipping point can be easily raised by serrating the tube seat by rolling with an ordinary flue expander, the rolls of which are grooved about 0.007 in. deep and 10 grooves to the inch. One tube thus serrated had its slipping point raised between three and four times its usual value.

METHODS OF TESTING THE HARDNESS OF METALS.

Brinell's Method. J. A. Brinell, a Swedish engineer, in 1900 published a method for determining the relative hardness of steel which has come into somewhat extensive use. A hardened steel ball, 10 mm. (0.3937 in.), is forced with a pressure of 3000 kg. (6614 lbs.) into a flat surface on the sample to be tested, so as to make a slight spherical indentation, the diameter of which may be measured by a microscope or the depth by a micrometer. The hardness is defined as the quotient of the pressure by the area of the indentation. From the measurement the "hardness number" is calculated by one of the following formulæ:

$$H = K (r + \sqrt{r^2 - R^2}) \div 2 \pi r R^2, \text{ or } H = K \div 2 \pi r d.$$

K = load, = 3000 kg., r = radius of ball, = 5 mm., R = radius and d = depth of indentation.

The following table gives the hardness number corresponding to different values of R and d .

| R | H | R | H | R | H | d | H | d | H | d | H |
|------|-----|------|-----|------|-----|------|-----|------|-----|------|-----|
| 1.00 | 955 | 2.40 | 398 | 3.80 | 251 | 2.00 | 946 | 3.20 | 364 | 4.60 | 170 |
| 1.20 | 796 | 2.60 | 367 | 4.00 | 239 | 2.10 | 857 | 3.40 | 321 | 4.80 | 156 |
| 1.40 | 682 | 2.80 | 341 | 4.20 | 227 | 2.20 | 782 | 3.60 | 286 | 5.00 | 143 |
| 1.60 | 597 | 3.00 | 318 | 4.40 | 217 | 2.40 | 652 | 3.80 | 255 | 5.50 | 116 |
| 1.80 | 531 | 3.20 | 298 | 4.60 | 208 | 2.60 | 555 | 4.00 | 228 | 6.00 | 95 |
| 2.00 | 477 | 3.40 | 281 | 4.80 | 199 | 2.80 | 477 | 4.20 | 207 | 6.50 | 80 |
| 2.20 | 434 | 3.60 | 265 | 4.95 | 193 | 3.00 | 418 | 4.40 | 187 | 6.95 | 68 |

The hardness of steel, as determined by the Brinell method, has a direct relation to the tensile strength, and is equal to the product of a coefficient, C , into the hardness number. Experiments made in Sweden with annealed steel showed that when the impression was made transversely to the rolling direction, with H below 175, $C = 0.362$; with H above 175, $C = 0.344$. When the impression was made in the rolling direction, with H below 175, $C = 0.354$; with H above 175, $C = 0.324$. The product, $C \times H$, or the tensile strength, is expressed in kilograms per square millimeter.

Electro-magnetic Method.—Several instruments have been devised for testing the hardness of steel by electrical methods. According to Prof. D. E. Hughes (*Cass. Mag.*, Sept., 1908), the magnetic capacity of iron and steel is directly proportional to the softness, and the resistance to a feeble external magnetic force is directly as the hardness. The electric conductivity of steel decreases with the increase of hardness. (See Electric Conductivity of Steel, p. .)

The Scleroscope.—This is the name of an instrument invented by A. F. Shore for determining the hardness of metals. It consists chiefly of a vertical glass tube in which slides freely a small cylinder of very hard steel, pointed on the lower end, called the hammer. This hammer is allowed to fall about 10 inches on to the sample to be tested, and the distance it rebounds is taken as a measure of the hardness of the sample. A scale on the tube is divided into 140 equal parts, and the hardness is expressed as the number on the scale to which the hammer rebounds. Measured in this way the hardness of different substances is as follows: Glass, 130; porcelain, 120; hardest steel, 110; tool steel, 1% C., may be as low as 31; mild steel, 0.5 C, 26 to 30; gray castings, 39; wrought iron, 18; babbitt metal, 4 to 10; soft brass, 12; zinc, 8; copper, 6; lead, 2. (*Cass. Mag.*, Sept., 1908.)

STRENGTH OF GLASS.

(Fairbairn's "Useful Information for Engineers," Second Series.)

| | Best Flint Glass. | Common Green Glass. | Extra White Crown Glass. |
|---|-------------------------|---------------------------|--------------------------------|
| Mean specific gravity | 3.078 | 2.528 | 2.450 |
| Mean tensile strength, lbs. per sq. in., bars | 2,413 | 2,896 | 2,546 |
| do. thin plates | 4,200 | 4,800 | 6,000 |
| Mean crush'g strength, lbs. p. sq. in., cyl'drs | 27,582 | 39,876 | 31,063 |
| do. cubes | 13,130 | 20,206 | 21,867 |

The bars in tensile tests were about $\frac{1}{2}$ inch diameter. The crushing tests were made on cylinders about $\frac{3}{4}$ inch diameter and from 1 to 2 inches high, and on cubes approximately 1 inch on a side. The mean transverse strength of glass, as calculated by Fairbairn from a mean tensile strength of 2560 lbs. and a mean compressive strength of 30,150 lbs. per sq. in., is, for a bar supported at the ends and loaded in the middle, $w = 3140 bd^2/l$, in which w = breaking weight in lbs., b = breadth, d = depth, and l = length, in inches. Actual tests will probably show wide variations in both directions from the mean calculated strength.

STRENGTH OF ICE.

Experiments at the University of Illinois in 1895 (*The Technograph*, vol. ix) gave 620 lbs. per sq. in. as the average crushing strength of cubes of manufactured ice tested at 23° F., and 906 lbs. for cubes tested at 14° F. Natural ice, at 12° F., tested with the direction of pressure parallel to the original water surface, gave a mean of 1070 lbs., and tested with the pressure perpendicular to this surface 1845 lbs. The range of variation in strength of individual pieces is about 50% above and below the mean figures, the lowest and highest figures being respectively 318 and 2818 lbs. per sq. in. The tensile strength of 34 samples tested at 19 to 23° F. was from 102 to 256 lbs. per sq. in.

STRENGTH OF COPPER AT HIGH TEMPERATURES.

The British Admiralty conducted some experiments at Portsmouth Dockyard in 1877, on the effect of increase of temperature on the tensile strength of copper and various bronzes. The copper experimented upon was in rods 0.72 in. diameter.

The following table shows some of the results:

| Temperature, Fahr. | Tensile Strength in lbs. per sq. in. | Temperature, Fahr. | Tensile Strength in lbs. per sq. in. |
|-----------------------|---|-----------------------|---|
| Atmospheric | 23,115 | 300° | 21,607 |
| 100° | 23,366 | 400° | 21,105 |
| 200° | 22,110 | 500° | 19,597 |

Up to a temperature of 400° F. the loss of strength was only about 10 per cent, and at 500° F. the loss was 16 per cent. The temperature of steam at 200 lbs. pressure is 382° F., so that according to these experiments the loss of strength at this point would not be a serious matter. Above a temperature of 500° the strength is seriously affected.

STRENGTH OF TIMBER.

Strength of Long-leaf Pine (Yellow Pine, *Pinus Palustris*) from Alabama (Bulletin No. 8, Forestry Div., Dept. of Agriculture, 1893. Tests by Prof. J. B. Johnson).

The following is a condensed table of the range of results of mechanical tests of over 2000 specimens, from 26 trees from four different sites in Alabama; reduced to 15 per cent moisture:

| | Butt Logs. | Middle Logs. | Top Logs. | Av'g of all Butt Logs. |
|---|-----------------|-----------------|-----------------|---------------------------------|
| Specific gravity | 0.449 to 1.039 | 0.575 to 0.859 | 0.484 to 0.907 | 0.767 |
| Transverse strength, $\frac{3WL}{2bh^2}$ | 4,762 to 16,200 | 7,640 to 17,128 | 4,268 to 15,554 | 12,614 |
| do. do. at elast. limit | 4,930 to 13,110 | 5,540 to 11,790 | 2,553 to 11,950 | 9,460 |
| Mod. of elast., thous. lbs. | 1,119 to 3,117 | 1,136 to 2,982 | 842 to 2,697 | 1,926 |
| Relative elast. resilience, inch-pounds per cub. in. | 0.23 to 4.69 | 1.34 to 4.21 | 0.09 to 4.65 | 2.98 |
| Crushing endwise, str. per sq. in.-lbs. | 4,781 to 9,850 | 5,030 to 9,300 | 4,587 to 9,100 | 7,452 |
| Crushing across grain, strength per sq. in., lbs. | 675 to 2,094 | 656 to 1,445 | 584 to 1,766 | 1,598 |
| Tensile strength per sq. in. | 8,600 to 31,890 | 6,330 to 29,500 | 4,170 to 23,280 | 17,359 |
| Shearing strength (with grain), mean per sq. in. | 464 to 1,299 | 539 to 1,230 | 484 to 1,156 | 866 |

Some of the deductions from the tests were as follows:

1. With the exception of tensile strength a reduction of moisture is accompanied by an increase in strength, stiffness, and toughness.
2. Variation in strength goes generally hand-in-hand with specific gravity.

3. In the first 20 or 30 feet in height the values remain constant; then occurs a decrease of strength which amounts at 70 feet to 20 to 40 per cent of that of the butt-log.

4. In shearing parallel with the grain and crushing across and parallel with the grain, practically no difference was found.

5. Large beams appear 10 to 20 per cent weaker than small pieces.

6. Compression tests endwise seem to furnish the best average statement of the value of wood, and if one test only can be made, this is the safest, as was also recognized by Bauschinger.

7. Bled timber is in no respect inferior to unbled timber.

The figures for crushing across the grain represent the load required to cause a compression of 15 per cent. The relative elastic resilience, in inch-pounds per cubic inch of the material, is obtained by measuring the area of the plotted strain-diagram of the transverse test from the origin to the point in the curve at which the rate of deflection is 50 per cent greater than the rate in the earlier part of the test where the diagram is a straight line. This point is arbitrarily chosen since there is no definite "elastic limit" in timber as there is in iron. The "strength at the elastic limit" is the strength taken at this same point. Timber is not perfectly elastic for any load if left on any great length of time.

The long-leaf pine is found in all the Southern coast states from North Carolina to Texas. Prof. Johnson says it is probably the strongest timber in large sizes to be had in the United States. In small selected specimens, other species, as oak and hickory, may exceed it in strength and toughness. The other Southern yellow pines, viz., the Cuban, short-leaf and the loblolly pines are inferior to the long-leaf about in the ratios of their specific gravities; the long-leaf being the heaviest of all the pines. It averages (kiln-dried) 48 pounds per cubic foot, the Cuban 47, the short-leaf 40, and the loblolly 34 pounds.

Strength of Spruce Timber. — The modulus of rupture of spruce is given as follows by different authors: Hatfield, 9900 lbs. per square inch; Rankine, 11,100; Laslett, 9045; Trautwine, 8100; Rodman, 6168. Trautwine advises for use to deduct one-third in the case of knotty and poor timber.

Prof. Lanza, in 25 tests of large spruce beams, found a modulus of rupture from 2995 to 5666 lbs.; the average being 4613 lbs. These were average beams, ordered from dealers of good repute. Two beams of selected stock, seasoned four years, gave 7562 and 8748 lbs. The modulus of elasticity ranged from 897,000 to 1,588,000, averaging 1,294,000.

Time tests show much smaller values for both modulus of rupture and modulus of elasticity. A beam tested to 5800 lbs. in a screw machine was left over night, and the resistance was found next morning to have dropped to about 3000, and it broke at 3500.

Prof. Lanza remarks that while it was necessary to use larger factors of safety, when the moduli of rupture were determined from tests with smaller pieces, it will be sufficient for most timber constructions, except in factories, to use a factor of four. For breaking strains of beams, he states that it is better engineering to determine as the safe load of a timber beam the load that will not deflect it more than a certain fraction of its span, say about $1/300$ to $1/400$ of its length.

Expansion of Timber Due to the Absorption of Water.

(De Volson Wood, A. S. M. E., vol. x.)

Pieces 36×5 in., of pine, oak, and chestnut, were dried thoroughly, and then immersed in water for 37 days.

The mean per cent of elongation and lateral expansion were:

| | Pine. | Oak. | Chestnut. |
|-----------------------------------|-------|-------|-----------|
| Elongation, per cent | 0.065 | 0.085 | 0.165 |
| Lateral expansion, per cent . . . | 2.6 | 3.5 | 3.65 |

Expansion of Wood by Heat. — Trautwine gives for the expansion, of white pine for 1 degree Fahr. 1 part in 440,530, or for 180 degrees 1 part in 2447, or about one-third of the expansion of iron.

TESTS OF AMERICAN WOODS. (Watertown Arsenal Tests, 1883.)

In all cases a large number of tests were made of each wood. Minimum and maximum results only are given. All of the test specimens had a sectional area of 1.575×1.575 inches. The transverse test specimens were 39.37 inches between supports, and the compressive test specimens were 12.60 inches long. Modulus of rupture calculated from formula $R = \frac{3 Pl}{2 bd^2}$; P = load in pounds at the middle, l = length, in inches, b = breadth, d = depth:

| Name of Wood. | Transverse Tests. Modulus of Rupture. | | Compression Parallel to Grain, pounds per square inch. | |
|---|--|--------|---|--------|
| | Min. | Max. | Min. | Max. |
| Cucumber tree (<i>Magnolia acuminata</i>) | 7,440 | 12,050 | 4,560 | 7,410 |
| Yellow poplar white wood (<i>Liriodendron tulipifera</i>) | 6,560 | 11,756 | 4,150 | 5,790 |
| White wood, Basswood (<i>Tilia Americana</i>) | 6,720 | 11,530 | 3,810 | 6,480 |
| Sugar-maple, Rock-maple (<i>Acer saccharinum</i>) | 9,680 | 20,130 | 7,460 | 9,940 |
| Red maple (<i>Acer rubrum</i>) | 8,610 | 13,450 | 6,010 | 7,500 |
| Locust (<i>Robinia pseudacacia</i>) | 12,200 | 21,730 | 8,330 | 11,940 |
| Wild cherry (<i>Prunus serotina</i>) | 8,310 | 16,800 | 5,830 | 9,120 |
| Sweet gum (<i>Liquidambar styraciflua</i>) | 7,470 | 11,130 | 5,630 | 7,620 |
| Dogwood (<i>Cornus florida</i>) | 10,190 | 14,560 | 6,250 | 9,400 |
| Sour gum, Pepperidge (<i>Nyssa sylvatica</i>) | 9,830 | 14,300 | 6,240 | 7,480 |
| Persimmon (<i>Diospyros Virginiana</i>) | 10,290 | 18,500 | 6,650 | 8,080 |
| White ash (<i>Fraxinus Americana</i>) | 5,950 | 15,800 | 4,520 | 8,830 |
| Sassafras (<i>Sassafras officinale</i>) | 5,180 | 10,150 | 4,050 | 5,970 |
| Slippery elm (<i>Ulmus fulva</i>) | 10,220 | 13,952 | 6,980 | 8,790 |
| White elm (<i>Ulmus Americana</i>) | 8,250 | 15,070 | 4,960 | 8,040 |
| Sycamore; Buttonwood (<i>Platanus occidentalis</i>) | 6,720 | 11,360 | 4,960 | 7,340 |
| Butternut; white walnut (<i>Juglans cinerea</i>) | 4,700 | 11,740 | 5,480 | 6,810 |
| Black walnut (<i>Juglans nigra</i>) | 8,400 | 16,320 | 6,940 | 8,850 |
| Shellbark hickory (<i>Carya alba</i>) | 14,870 | 20,710 | 7,650 | 10,280 |
| Pignut (<i>Carya porcina</i>) | 11,560 | 19,430 | 7,460 | 8,470 |
| White oak (<i>Quercus alba</i>) | 7,010 | 18,360 | 5,810 | 9,070 |
| Red oak (<i>Quercus rubra</i>) | 9,760 | 18,370 | 4,960 | 8,970 |
| Black oak (<i>Quercus tinctoria</i>) | 7,900 | 18,420 | 4,540 | 8,550 |
| Chestnut (<i>Castanea vulgaris</i>) | 5,950 | 12,870 | 3,680 | 6,650 |
| Beech (<i>Fagus ferruginea</i>) | 13,850 | 18,840 | 5,770 | 7,840 |
| Canoe-birch, paper-birch (<i>Betula papyracea</i>) | 11,710 | 17,610 | 5,770 | 8,590 |
| Cottonwood (<i>Populus monilifera</i>) | 8,390 | 13,430 | 3,790 | 6,510 |
| White cedar (<i>Thuja occidentalis</i>) | 6,310 | 9,530 | 2,660 | 5,810 |
| Red cedar (<i>Juniperus Virginiana</i>) | 5,640 | 15,100 | 4,400 | 7,040 |
| Cypress (<i>Saxodium Distichum</i>) | 9,530 | 10,030 | 5,060 | 7,140 |
| White pine (<i>Pinus strobus</i>) | 5,610 | 11,530 | 3,750 | 5,600 |
| Spruce pine (<i>Pinus glabra</i>) | 3,780 | 10,980 | 2,580 | 4,680 |
| Long-leaved pine, Southern pine (<i>Pinus palustris</i>) | 9,220 | 21,060 | 4,010 | 10,600 |
| White spruce (<i>Picea alba</i>) | 9,900 | 11,650 | 4,150 | 5,300 |
| Hemlock (<i>Tsuga Canadensis</i>) | 7,590 | 14,680 | 4,500 | 7,420 |
| Red fir, yellow fir (<i>Pseudotsuga Douglasii</i>) | 8,220 | 17,920 | 4,880 | 9,800 |
| Tamarack (<i>Larix Americana</i>) | 10,080 | 16,770 | 6,810 | 10,700 |

Shearing Strength of American Woods, adapted for Pins or Tree-nails.J. C. Trautwine (*Jour. Franklin Inst.*). (Shearing across the grain.)

| | per sq. in. | | per sq. in. |
|------------------------------|-------------|--------------------------------|-------------|
| Ash..... | 6280 | Hickory..... | 6045 |
| Beech..... | 5223 | Hickory..... | 7285 |
| Birch..... | 5595 | Maple..... | 6355 |
| Cedar (white)..... | 1372 | Oak..... | 4425 |
| Cedar (white)..... | 1519 | Oak (live)..... | 8480 |
| Cedar (Central American).... | 3410 | Pine (white)..... | 2480 |
| Cherry..... | 2945 | Pine (Northern yellow)..... | 4340 |
| Chestnut..... | 1536 | Pine (Southern yellow)..... | 5735 |
| Dogwood..... | 6510 | Pine (very resinous yellow)... | 5053 |
| Ebony..... | 7750 | Poplar..... | 4418 |
| Gum..... | 5890 | Spruce..... | 3255 |
| Hemlock..... | 2750 | Walnut (black)..... | 4728 |
| Locust..... | 7176 | Walnut (common)..... | 2830 |

Transverse Tests of Pine and Spruce Beams. (*Tech. Quar.* XIII, No. 3, 1900, p. 226.)—Tests of 37 hard pine beams, 4 to 10 ins. wide, 6 to 12 ins. deep, and 8 to 16 ft. length between supports, showed great variations in strength. The modulus of rupture of different beams was as follows: 1, 2970; 4, 4000 to 5000; 1, 5510; 1, 6220; 9, 7000 to 8000; 8, 8000 to 9000; 4, 9000 to 10,000; 5, 10,000 to 11,000; 3, 11,000 to 12,000; 1, 13,600.

Six tests of white pine beams gave moduli of rupture ranging from 1840 to 7810; and eighteen tests of spruce beams from 2750 to 7970 lbs. per sq. in.

Drying of Wood.—Circular 111, U. S. Forest Service, 1907. Sticks of Southern loblolly pine 11 to 13 inches diameter, 9 to 10 ft. long, were weighed every two weeks until seasoned, to find the weight of water evaporated. The loss, per cent of weight, was as follows:

| | | | | | | | | |
|----------------------------------|----|----|----|----|----|----|----|----|
| Weeks..... | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| Loss per cent of green wood..... | 16 | 21 | 26 | 31 | 32 | 34 | 35 | 35 |

Preservation of Timber.—U. S. Forest Service, Circular 111, 1907, discusses preservative treatment of timber by different methods, namely, brush treatment with creosote and with carbolinium; open tank treatment with salt solution, zinc chloride solution; and cylinder treatment with zinc chloride solution and creosote.

The increased life necessary to pay the cost of these several preservative treatments is respectively: 6, 16, 7, 13, 41, 27, and 55%. The results of the experiments prove that it will pay mining companies to peel their timber, to season it for several months and to treat it with a good preservative. Loblolly and pitch pine have been most successfully preserved by treatment with creosote in an open tank.

Circular No. 151 of the Forest Service describes experiments on the best method of treating loblolly pine cross-arms of telegraph poles. The arms after being seasoned in air are placed in a closed air-tight cylinder, a vacuum is applied sufficient to draw the oil (creosote, dead oil of coal tar) from the storage tank into the treating cylinder. Sufficient pressure is then applied to force the oil into the heartwood portion of the timber, and continued until the desired amount of oil is absorbed, then a vacuum is maintained until the surplus oil is drawn from the sapwood. It is recommended that heartwood should finally contain about 6 lbs. of oil per cubic foot, and sapwood about 10 lbs. The preliminary bath of live steam, formerly used, has been found unnecessary. Much valuable information concerning timber treatment and its benefits is contained in the several circulars on the subject issued by the Forest Service.

THE STRENGTH OF BRICK, STONE, ETC.

A great advance has recently (1895) been made in the manufacture of brick, in the direction of increasing their strength. Chas. P. Chase, in *Engineering News*, says: "Taking the tests as given in standard engineering books eight or ten years ago, we find in Trautwine the strength of brick given as 500 to 4200 lbs. per sq. in. Now, taking recent tests in

experiments made at Watertown Arsenal, the strength ran from 5000 to 22,000 lbs. per sq. in. In the tests on Illinois paving-brick, by Prof. I. O. Baker, we find an average strength in hard paving brick of over 5000 lbs. per square inch. The average crushing strength of ten varieties of paving-brick much used in the West, I find to be 7150 lbs. to the square inch."

A test of brick made by the dry-clay process at Watertown Arsenal, according to *Paving*, showed an average compressive strength of 3972 lbs. per sq. in. In one instance it reached 4973 lbs. per sq. in. A test was made at the same place on a "fancy pressed brick." The first crack developed at a pressure of 305,000 lbs., and the brick crushed at 364,300 lbs., or 11,130 lbs. per sq. in. This indicates almost as great compressive strength as granite paving-blocks, which is from 12,000 to 20,000 lbs. per sq. in.

The three following notes on bricks are from Trautwine's *Engineer's Pocket-book*:

Strength of Brick. — 40 to 300 tons per sq. ft., 622 to 4668 lbs. per sq. in. A soft brick will crush under 450 to 600 lbs. per sq. in., or 30 to 40 tons per square foot, but a first-rate machine-pressed brick will stand 200 to 400 tons per sq. ft. (3112 to 6224 lbs. per sq. in.).

Weight of Bricks. — Per cubic foot, best pressed brick, 150 lbs.; good pressed brick, 131 lbs.; common hard brick, 125 lbs.; good common brick, 118 lbs.; soft inferior brick, 100 lbs.

Absorption of Water. — A brick will in a few minutes absorb $\frac{1}{2}$ to $\frac{3}{4}$ lb. of water, the last being $\frac{1}{7}$ of the weight of a hand-molded one, or $\frac{1}{3}$ of its bulk.

Tests of Bricks, full size, on flat side. (Tests made at Watertown Arsenal in 1883.) — The bricks were tested between flat steel buttresses. Compressed surfaces (the largest surface) ground approximately flat. The bricks were all about 2 to 2.1 inches thick, 7.5 to 8.1 inches long, and 3.5 to 3.76 inches wide. Crushing strength per square inch: One lot ranged from 11,056 to 16,734 lbs.; a second, 12,995 to 22,351; a third, 10,390 to 12,709. Other tests gave results from 5960 to 10,250 lbs. per sq. in.

Tests of Brick. (*Tech. Quar.*, 1900.) — Different brands of brick tested on the broad surfaces, and on edge, gave results as follows, lbs. per sq. in.

(*Tech. Quar.* XII, No. 3, 1899.) 38 tests.

| | No. Test. | Aver- age. | Maxi- mum. | Mini- mum. | Per cent Water Absorbed. |
|--|--------------|---------------|---------------|---------------|-----------------------------|
| On broad surface | | | | | |
| Bay State, light hard | 71 | 7039 | 11,240 | 3587 | 15.15 to 19.3 av. 7.5 |
| Same, tested on edge .. | 67 | 6241 | 10,840 | 3325 | 13.67 to 18.2 " 7.4 |
| On broad surface | | | | | |
| Dover River, soft burned..... | 38 | 5350 | 8630 | 3930 | 14.0 to 18.6 " 11.6 |
| Dover River, hard burned..... | 36 | 8070 | 10,940 | 5850 | 4.7 to 10.1 " 7.0 |
| Central N. Y., soft burned..... | 36 | 2190 | 3060 | 1370 | 17.8 to 22.0 " 19.9 |
| Central N. Y., med- ium burned..... | 36 | 3600 | 4950 | 2080 | 16.6 to 23.4 " 13.6 |
| Central N. Y., hard burned..... | 36 | 5360 | 8810 | 3310 | 8.3 to 16.7 " 12.5 |
| Another lot,* hard burned..... | 16 | 7940 | 9770 | 6570 | 7.6 to 12.9 " 10.6 |
| Same,* tested on edge | 16 | 6430 | 10,230 | 3830 | 6.2 to 18.7 " 11.4 |

* Brand not named.

The per cent water absorbed in general seemed to have a relation to the strength, the greatest absorption corresponding to the lowest strength, and *vice versa*, but there were many exceptions to the rule.

Strength of Common Red Brick. — Tests of 67 samples of Hudson River machine-molded brick were made by I. H. Woolson, *Eng. News*, April 13, 1905. The crushing strength, in lbs. per sq. in., of 15 pale brick ranged from 1607 to 4546, average 3010; 44 medium, 2080 to 8944, av. 4080; 8 hard brick, 2396 to 6420, av. 4960. Five Philadelphia pressed brick gave from 3524 to 9425, av. 6361. The absorption ranged from 8.7 to 21.4% by weight. The relation of absorption to strength varied greatly, but on the average there was an increase of absorption up to 3000 lbs. per sq. in. crushing strength, and beyond that a decrease.

The Strongest Brick ever tested at the Watertown Arsenal was a paving brick from St. Louis, Mo., which showed a compressive strength of 38,446 lbs. per sq. in. The absorption was 0.21% by weight and 0.5% by volume. The sample was set on end, and measured 2.45×3.06 ins. in cross section. — *Eng. News*, Mar. 14, 1907.

Crushing Strength of Masonry Materials. (From Howe's "Retaining-Walls.") —

| | tons per sq. ft. | | tons per sq. ft. |
|-----------------------|------------------|------------------------|------------------|
| Brick, best pressed . | 40 to 300 | Limestones and marbles | 250 to 1000 |
| Chalk..... | 20 to 30 | Sandstone..... | 150 to 550 |
| Granite..... | 300 to 1200 | Soapstone..... | 400 to 800 |

Strength of Granite. — The crushing strength of granite is commonly rated at 12,000 to 15,000 lbs. per sq. in. when tested in two-inch cubes, and only the hardest and toughest of the commonly used varieties reach a strength above 20,000 lbs. Samples of granite from a quarry on the Connecticut River, tested at the Watertown Arsenal, have shown a strength of 35,965 lbs. per sq. in. (*Engineering News*, Jan. 12, 1893).

Ordinary granite ranges from 20,000 to 30,000 lbs. compressive strength per sq. in. A granite from Asheville, N.C., tested at the Watertown Arsenal, gave 51,900 lbs. — *Eng. News*, Mar. 14, 1907.

Strength of Avondale, Pa., Limestone. (*Engineering News*, Feb. 9, 1893.) — Crushing strength of 2-in. cubes: light stone 12,112, gray stone 18,040, lbs. per sq. in.

Transverse test of lintels, tool-dressed, 42 in. between knife-edge bearings, load with knife-edge brought upon the middle between bearings:

| | |
|--|-------------|
| Gray stone, section 6 in. wide \times 10 in. high, broke under a load of 20,950 lbs. | |
| Modulus of rupture | 2,200 " |
| Light stone, section $8\frac{1}{4}$ in. wide \times 10 in. high, broke under... | 14,720 " |
| Modulus of rupture | 1,170 " |
| Absorption. — Gray stone | 0.051 of 1% |
| Light stone..... | 0.052 of 1% |

Tests of Sand-lime Brick. (I. H. Woolson, *Eng. News*, June 14, 1906). — Eight varieties of brick in lots of 300 to 800 were received from different manufacturers. They were tested for transverse strength, on supports 7 in. apart, loaded in the middle; and half bricks were tested by compression, sheets of heavy fibrous paper being inserted between the specimen and the plates of the testing machine to insure an even bearing. Tests were made on the brick as received, and on other samples after drying at about 150° F. to constant weight, requiring from four to six days. The moisture in two bricks of each series was determined, and found to range from 1 to 10%, average 5.9%. The figures of results given below are the averages of 10 tests in each case. Other bricks of each lot were tested for absorption by being immersed $\frac{1}{2}$ in. in water for 48 hours, for resistance to 20 repeated freezings and thawings, and for resistance to fire by heating them in a fire testing room, the bricks being built in as 8-in. walls, to 1700° F. and maintaining that temperature three hours, then cooling them with a $1\frac{1}{8}$ -in. stream of cold water from a hydrant. Transverse and compressive tests were made after these treatments. The results given below are averages of five tests, except in the case of the bricks tested after firing, in which two samples are averaged.

EFFECT OF THE FIRE TEST. — Several large cracks developed in both the sand-lime and the clay brick walls during the test. These were no worse in one wall than in the other. With the exception of surface deterioration the walls were solid and in good condition. After they

were cooled the inside course of each wall was cut through and specimens of each series secured for examination and test. It was difficult to secure whole bricks, owing to the extreme brittleness.

In general the bricks were affected by fire about half way through. They were all brittle and many of them tender when removed from the wall. With the sand-lime brick, if a brick broke the remainder had to be chiseled out like concrete, whereas a clay brick under like conditions would chip out easily. The clay brick were so brittle and full of cracks that the wall could be broken down without trouble. The sand-lime bricks adhered to the mortar better, were cracked less, and were not so brittle.

| Designation of Brick. | | A | B | C | D | E | F | G |
|--|----------------|------|------|------|------|------|------|------|
| Modulus of Rupture | As received | 272 | 424 | 377 | 262 | 190 | 301 | 365 |
| | Dried | 320 | 505 | 406 | 334 | 197 | 570 | 494 |
| | Increase, % | 15.0 | 16.0 | 7.1 | 21.5 | 3.5 | 47.2 | 26.2 |
| | Wet | 248 | 349 | 345 | 241 | 243 | 250 | 485 |
| | After fire | 17 | 57 | 20 | 32 | 24 | 27 | 37 |
| Compressive Strength, lbs. per sq. in. | As received | 1875 | 2300 | 2871 | 1923 | 1610 | 2460 | 2669 |
| | Dried | 2604 | 2772 | 3240 | 2476 | 1870 | 3273 | 3190 |
| | Increase, % | 30.2 | 17.1 | 20.7 | 22.3 | 13.5 | 24.8 | 16.3 |
| | Wet | 1611 | 2174 | 2097 | 1923 | 1108 | 2063 | 2183 |
| | After freezing | 1596 | 1619 | 2265 | 1174 | 1167 | 1851 | 1739 |
| | After fire | 1807 | 2814 | 2573 | 2069 | 1089 | 2051 | 4885 |
| % of lime in brick | | 6 | 10 | 5 | 41/2 | 41/2 | 5 | 8 |
| Pressure for hardening, lbs. | | 120 | 135 | 150 | 125 | 120 | 150 | 125 |
| Hours in hardening, lbs. | | 10 | 8 | 7 | 10 | 10 | 7 | 10 |

Transverse Strength of Flagging.

(N. J. Steel & Iron Co.'s Book.)

EXPERIMENTS MADE BY R. G. HATFIELD AND OTHERS.

b = width of the stone in inches; d = its thickness in inches; l = distance between bearings in inches.

The *breaking loads* in tons of 2000 lbs., for a weight placed at the *center* of the space, will be as follows:

| $\frac{bd^2}{l} \times$ | $\frac{bd^2}{l} \times$ |
|--------------------------------------|---------------------------------|
| Bluestone flagging..... 0.744 | Dorchester freestone..... 0.264 |
| Quincy granite..... 0.624 | Aubigny freestone..... 0.216 |
| Little Falls freestone..... 0.576 | Caen freestone..... 0.144 |
| Belleville, N. J., freestone.. 0.480 | Glass..... 1.000 |
| Granite (another quarry)... 0.432 | Slate..... 1.2 to 2.7 |
| Connecticut freestone..... 0.312 | |

Thus a block of Quincy granite 80 inches wide and 6 inches thick, resting on beams 36 inches in the clear, would be broken by a load resting midway between the beams = $\frac{80 \times 36}{36} \times 0.624 = 49.92$ tons.

STRENGTH OF LIME AND CEMENT MORTAR.

(Engineering, October 2, 1891.)

Tests made at the University of Illinois on the effects of adding cement to lime mortar. In all the tests a good quality of ordinary fat lime was used, slaked for two days in an earthenware jar, adding two parts by weight of water to one of lime, the loss by evaporation being made up

by fresh additions of water. The cements used were a German Portland, Black Diamond (Louisville), and Rosendale. As regards fineness of grinding, 85 per cent of the Portland passed through a No. 100 sieve, as did 72 per cent of the Rosendale. A fairly sharp sand, thoroughly washed and dried, passing through a No. 18 sieve and caught on a No. 30, was used. The mortar in all cases consisted of two volumes of sand to one of lime paste. The following results were obtained on adding various percentages of cement to the mortar:

Tensile Strength, pounds per square inch.

| Age..... { | 4 Days. | 7 Days. | 14 Days. | 21 Days. | 28 Days. | 50 Days. | 84 Days. |
|-----------------------|------------|------------|-------------|-------------|-------------|-------------|-------------|
| Lime mortar..... | 4 | 8 | 10 | 13 | 18 | 21 | 26 |
| 20 per cent Rosendale | 5 | 8 1/2 | 9 1/2 | 12 | 17 | 17 | 18 |
| 20 " " Portland. | 5 | 8 1/2 | 14 | 20 | 25 | 24 | 26 |
| 30 " " Rosendale | 7 | 11 | 13 | 18 1/2 | 21 | 22 1/2 | 23 |
| 30 " " Portland. | 8 | 16 | 18 | 22 | 25 | 28 | 27 |
| 40 " " Rosendale | 10 | 12 | 16 1/2 | 21 1/2 | 22 1/2 | 24 | 36 |
| 40 " " Portland. | 27 | 39 | 38 | 43 | 47 | 59 | 57 |
| 60 " " Rosendale | 9 | 13 | 20 | 16 | 22 | 22 1/2 | 23 |
| 60 " " Portland. | 45 | 58 | 55 | 68 | 67 | 102 | 78 |
| 80 " " Rosendale | 12 | 18 1/2 | 22 1/2 | 27 | 29 | 31 1/2 | 33 |
| 80 " " Portland. | 87 | 91 | 103 | 124 | 94 | 210 | 145 |
| 100 " " Rosendale | 18 | 23 | 26 | 31 | 34 | 46 | 48 |
| 100 " " Portland. | 90 | 120 | 146 | 152 | 181 | 205 | 202 |

Tests of Portland Cement.

(Tech. Quar. XIII. No. 3, 1900, p. 236.)

| | 1 Day. | 2 Days. | 14 Days | 1 Mo. | 2 Mos. | 6 Mos. | 1 Year. |
|-----------------------------------|---------|---------|---------|---------|----------|-----------|----------|
| Neat cement: | | | | | | | |
| Tension, lbs. per sq. in... | 268-312 | 454-532 | 780-820 | 915-920 | 950-1100 | 1036-1190 | 996-1248 |
| Compression, lbs. per sq. in { | 8650 | 13,080 | 23,640 | | 34,000 | | 36,150 |
| | to | to | to | | to | | to |
| | 10,250 | 14,860 | 34,820 | | 38,500 | | 50,000 |
| 3 sand, 1 cem. Tens..... | 56-75 | 79-92 | 185-211 | 211-230 | 217-240 | 300-382 | 280-383 |
| 3 sand, 1 cem. { | 1200 | 1750 | 3790 | | 7850 | | 8000 |
| Comp. | to | to | to | | to | | to |
| | 1585 | 1885 | 4420 | | 8250 | | 10,000 |

MODULI OF ELASTICITY OF VARIOUS MATERIALS.

The modulus of elasticity determined from a tensile test of a bar of any material is the quotient obtained by dividing the tensile stress in pounds per square inch at any point of the test by the elongation per inch of length produced by that stress; or if P = pounds of stress applied, K = the sectional area, l = length of the portion of the bar in which the measurement is made, and λ = the elongation in that length, the modulus of elasticity $E = \frac{P}{K} \div \frac{\lambda}{l} = \frac{Pl}{K\lambda}$. The modulus is generally measured within the elastic limit only, in materials that have a well-defined elastic limit, such as iron and steel, and when not otherwise stated the modulus is understood to be the modulus within the elastic limit. Within this limit, for such materials the modulus is practically constant for any given bar, the elongation being directly proportional to the stress. In

other materials, such as cast iron, which have no well-defined elastic limit, the elongations from the beginning of a test increase in a greater ratio than the stresses, and the modulus is therefore at its maximum near the beginning of the test, and continually decreases. The moduli of elasticity of various materials have already been given above in treating of these materials, but the following table gives some additional values selected from different sources:

| | | | |
|------------------------------|------------|---------------------------|----------------|
| Brass, cast..... | 9,170,000 | | |
| Brass wire..... | 14,230,000 | | |
| Copper..... | 15,000,000 | to 18,000,000 | |
| Lead..... | 1,000,000 | | |
| Tin, cast..... | 4,600,000 | | |
| Iron, cast..... | 12,000,000 | to 27,000,000 (?) | |
| Iron, wrought..... | 22,000,000 | to 29,000,000 (?) | |
| Steel..... | 28,000,000 | to 32,000,000 (see below) | |
| Marble..... | 25,000,000 | | |
| Slate..... | 14,500,000 | | |
| Glass..... | 8,000,000 | | |
| Ash..... | 1,600,000 | | |
| Beech..... | 1,300,000 | | |
| Birch..... | 1,250,000 | to 1,500,000 | |
| Fir..... | 869,000 | to 2,191,000 | |
| Oak..... | 974,000 | to 2,283,000 | |
| Teak..... | 2,414,000 | | |
| Walnut..... | 306,000 | | |
| Pine, long-leaf (butt-logs). | 1,119,000 | to 3,117,000 | Avg. 1,926,000 |

The maximum figures given by some early writers for iron and steel, viz., 40,000,000 and 42,000,000, are undoubtedly erroneous. The modulus of elasticity of steel (within the elastic limit) is remarkably constant, notwithstanding great variations in chemical analysis, temper, etc. It rarely is found below 29,000,000 or above 31,000,000. It is generally taken at 30,000,000 in engineering calculations. Prof. J. B. Johnson, in his report on Long-leaf Pine, 1893, says: "The modulus of elasticity is the most constant and reliable property of all engineering materials. The wide range of value of the modulus of elasticity of the various metals found in public records must be explained by erroneous methods of testing."

In a tensile test of cast iron by the author (Van Nostrand's Science Series, No. 41, page 45), in which the ultimate strength was 23,285 lbs. per sq. in., the measurements of elongation were made to 0.0001 inch, and the modulus of elasticity was found to decrease from the beginning of the test, as follows: At 1000 lbs. per sq. in., 25,000,000; at 2000 lbs., 16,666,000; at 4000 lbs., 15,384,000; at 6000 lbs., 13,636,000; at 8000 lbs., 12,500,000; at 12,000 lbs., 11,250,000; at 15,000 lbs., 10,000,000; at 20,000 lbs., 8,000,000; at 23,000 lbs., 6,140,000.

FACTORS OF SAFETY.

A factor of safety is the ratio in which the load that is just sufficient to overcome instantly the strength of a piece of material is greater than the greatest safe ordinary working load. (Rankine.)

Rankine gives the following "examples of the values of those factors which occur in machines":

| | Dead Load. | Live Load, Greatest. | Live Load, Mean. |
|---------------------|------------|-------------------------|---------------------|
| Iron and steel..... | 3 | 6 | from 6 to 40 |
| Timber..... | 4 to 5 | 8 to 10 | |
| Masonry..... | 4 | 8 | |

The great factor of safety, 40, is for shafts in millwork which transmit very variable efforts,

Unwin gives the following "factors of safety which have been adopted in certain cases for different materials." They "include an allowance for ordinary contingencies."

| | Dead Load. | In Temporary Structures. | In Permanent Structures. | In Structures subj. to Shocks. |
|------------------------|------------|--------------------------|--------------------------|--------------------------------|
| Wrought iron and steel | 3 | 4 | 4 to 5 | 10 |
| Cast iron | 3 | 4 | 5 | 10 |
| Timber | ... | 4 | 10 | |
| Brickwork | ... | | 6 | |
| Masonry | 20 | | 20 to 30 | |

Unwin says that "these numbers fairly represent practice based on experience in many actual cases, but they are not very trustworthy."

Prof. Wood in his "Resistance of Materials" says: "In regard to the margin that should be left for safety, much depends upon the character of the loading. If the load is simply a dead weight, the margin may be comparatively small; but if the structure is to be subjected to percussive forces or shocks, the margin should be comparatively large on account of the indeterminate effect produced by the force. In machines which are subjected to a constant jar while in use, it is very difficult to determine the proper margin which is consistent with economy and safety. Indeed, in such cases, economy as well as safety generally consists in making them *excessively* strong, as a single breakage may cost much more than the extra material necessary to fully insure safety."

For discussion of the resistance of materials to repeated stresses and shocks, see pages 261 to 264.

Instead of using factors of safety, it is becoming customary in designing to fix a certain number of pounds per square inch as the maximum stress which will be allowed on a piece. Thus, in designing a boiler, instead of naming a factor of safety of 6 for the plates and 10 for the stay-bolts, the ultimate tensile strength of the steel being from 50,000 to 60,000 lbs. per sq. in., an allowable working stress of 10,000 lbs. per sq. in. on the plates and 6000 lbs. per sq. in. on the stay-bolts may be specified instead. So also in the use of formulæ for columns (see page 271) the dimensions of a column are calculated after assuming a maximum allowable compressive stress per square inch on the concave side of the column.

The factors for masonry under dead load as given by Rankine and by Unwin, viz., 4 and 20, show a remarkable difference, which may possibly be explained as follows: If the actual crushing strength of a pier of masonry is known from direct experiment, then a factor of safety of 4 is sufficient for a pier of the same size and quality under a steady load; but if the crushing strength is merely assumed from figures given by the authorities (such as the crushing strength of pressed brick, quoted above from Howe's Retaining Walls, 40 to 300 tons per square foot, average 170 tons), then a factor of safety of 20 may be none too great. In this case the factor of safety is really a "factor of ignorance."

The selection of the proper factor of safety or the proper maximum unit stress for any given case is a matter to be largely determined by the judgment of the engineer and by experience. No definite rules can be given. The customary or advisable factors in many particular cases will be found where these cases are considered throughout this book. In general the following circumstances are to be taken into account in the selection of a factor:

1. When the ultimate strength of the material is known within narrow limits, as in the case of structural steel when tests of samples have been made, when the load is entirely a steady one of a known amount, and there is no reason to fear the deterioration of the metal by corrosion, the lowest factor that should be adopted is 3.

2. When the circumstances of 1 are modified by a portion of the load being variable, as in floors of warehouses, the factor should be not less than 4.

3. When the whole load, or nearly the whole, is apt to be alternately put on and taken off, as in suspension rods of floors of bridges, the factor should be 5 or 6.

4. When the stresses are reversed in direction from tension to compression, as in some bridge diagonals and parts of machines, the factor should be not less than 6.

5. When the piece is subjected to repeated shocks, the factor should be not less than 10.

6. When the piece is subject to deterioration from corrosion the section should be sufficiently increased to allow for a definite amount of corrosion before the piece be so far weakened by it as to require removal.

7. When the strength of the material, or the amount of the load, or both are uncertain, the factor should be increased by an allowance sufficient to cover the amount of the uncertainty.

8. When the strains are of a complex character and of uncertain amount, such as those in the crank-shaft of a reversing engine, a very high factor is necessary, possibly even as high as 40, the figure given by Rankine for shafts in millwork.

Formulas for Factor of Safety. — (F. E. Cardullo, *Mach'y*, Jan., 1906.) The apparent factor of safety is the product of four factors, or,

$$F = a \times b \times c \times d.$$

a is the ratio of the ultimate strength of the material to its elastic limit, not the yield point, but the true elastic limit within which the material is, in so far as we can discover, perfectly elastic, and takes no permanent set. Two reasons for keeping the working stress within this limit are: (1) that the material will rupture if strained repeatedly beyond this limit; and (2) that the form and dimensions of the piece would be destroyed under the same circumstances.

The second factor, b , is one depending upon the character of the stress produced within the material. The experiments of Wohler proved that the repeated application of a stress less than the ultimate strength of a material would rupture it. Prof. J. B. Johnson's formula for the relation between the ultimate strength and the "carrying strength" under conditions of variable loads is as follows:

$$f = U \div (2 - p_1/p),$$

where f is the "carrying strength" when the load varies repeatedly between a maximum value, p , and a minimum value, p_1 , and U is the ultimate strength of the material. The quantities p and p_1 have plus signs when they represent loads producing tension, and minus signs when they represent loads producing compression.

If the load is variable the factor b must then have a value,

$$b = U/f = 2 - p_1/p.$$

Taking a load varying between zero and a maximum,

$$p_1/p = 0, \text{ and } b = 2 - p_1/p = 2.$$

Taking a load that produces alternately a tension and a compression equal in amount,

$$p' = -p \text{ and } p_1/p = -1, \text{ and } b = 2 - p_1/p = 2 - (-1) = 3.$$

The third factor, c , depends upon the manner in which the load is applied to the piece. When the load is suddenly applied $c = 2$. When not all of the load is applied suddenly, the factor 2 is reduced accordingly. If a certain fraction of the load, n/m , is suddenly applied, the factor is $1 + n/m$.

The last factor, d , we may call the "factor of ignorance." All the other factors have provided against known contingencies; this provides against the unknown. It commonly varies in value between $1\frac{1}{2}$ and 3, although occasionally it becomes as great as 10. It provides against excessive or accidental overload, unexpectedly severe service, unreliable or imperfect materials, and all unforeseen contingencies of manufacture or operation. When we know that the load will not be likely to be increased, that the material is reliable, that failure will not result disastrously, or even that the piece for some reason must be small or light, this factor will be reduced to its lowest limit, $1\frac{1}{2}$. When life or property would be endangered by the failure of the piece, this factor must be made larger. Thus, while it is $1\frac{1}{2}$ to 2 in most ordinary steel constructions, it is rarely less than $2\frac{1}{2}$ for steel in a boiler.

The reliability of the material in a great measure determines the value of this factor. For instance, in all cases where it would be $1\frac{1}{2}$ for mild steel, it is made 2 for cast iron. It will be larger for those materials subject to internal strains, for instance for complicated castings, heavy

forgings, hardened steel, and the like, also for materials subject to hidden defects, such as internal flaws in forgings, spongy places in castings, etc. It will be smaller for ductile and larger for brittle materials. It will be smaller as we are sure that the piece has received uniform treatment and as the tests we have give more uniform results and more accurate indications of the real strength and quality of the piece itself. In fixing the factor d , the designer must depend on his judgment, guided by the general rules laid down.

Table of Factors of Safety.

The following table may assist in a proper choice of the factor of safety. It shows the value of the four factors for various materials and conditions of service.

| CLASS OF SERVICE OR MATERIALS. | Factor | | | | F |
|---|---------|---|---|---------|-----------------|
| | a | b | c | d | |
| Boilers.. | 2 | 1 | 1 | 2 1/4-3 | 4 1/2- 6 |
| Piston and connecting rods for double-acting engines..... | 1 1/2-2 | 3 | 2 | 1 1/2 | 13 1/2-18 |
| Piston and connecting rod for single-acting engines..... | 1 1/2-2 | 2 | 2 | 1 1/2 | 9 -12 |
| Shaft carrying bandwheel, fly-wheel, or armature..... | 1 1/2-2 | 3 | 1 | 1 1/2 | 6 3/4- 9 |
| Lathe spindles..... | 2 | 2 | 2 | 1 1/2 | 12 |
| Mill shafting..... | 2 | 3 | 2 | 2 | 24 |
| Steel work in buildings..... | 2 | 1 | 1 | 2 | 4 |
| Steel work in bridges..... | 2 | 1 | 1 | 2 1/2 | 5 |
| Steel work for small work..... | 2 | 1 | 2 | 1 1/2 | 6 |
| Cast iron wheel rims..... | 2 | 1 | 1 | 10 | 20 |
| Steel wheel rims..... | 2 | 1 | 1 | 4 | 8 |
| | | | | | Minimum Values. |
| Cast iron and other castings..... | 2 | 1 | 1 | 2 | 4 |
| Wrought iron or mild steel..... | 2 | 1 | 1 | 1 1/2 | 3 |
| Oil tempered or nickel steel..... | 1 1/2 | 1 | 1 | 1 1/2 | 2 1/4 |
| Hardened steel..... | 1 1/2 | 1 | 1 | 2 | 3 |
| Bronze and brass, rolled or forged..... | 2 | 1 | 1 | 1 1/2 | 3 |

THE MECHANICAL PROPERTIES OF CORK.

Cork possesses qualities which distinguish it from all other solid or liquid bodies, namely, its power of altering its volume in a very marked degree in consequence of change of pressure. It consists, practically, of an aggregation of minute air-vessels, having thin, water-tight, and very strong walls, and hence, if compressed, the resistance to compression rises in a manner more like the resistance of gases than the resistance of an elastic solid such as a spring. In a spring the pressure increases in proportion to the distance to which the spring is compressed, but with gases the pressure increases in a much more rapid manner; that is, inversely as the volume which the gas is made to occupy. But from the permeability of cork to air, it is evident that, if subjected to pressure in one direction only, it will gradually part with its occluded air by effusion, that is, by its passage through the porous walls of the cells in which it is contained. The gaseous part of cork constitutes 53% of its bulk. Its elasticity has not only a very considerable range, but it is very persistent. Thus in the better kind of corks used in bottling the corks expand the instant they escape from the bottles. This expansion may amount to an increase of volume of 75%, even after the corks have been kept in a state of compression in the bottles for ten years. If the cork be steeped in hot water, the volume continues to increase till it attains nearly three times that which it occupied in the neck of the bottle.

When cork is subjected to pressure a certain amount of permanent deformation or "permanent set" takes place very quickly. This property is common to all solid elastic substances when strained beyond their elastic limits, but with cork the limits are comparatively low. Besides the permanent set, there is a certain amount of sluggish elasticity — that is, cork on being released from pressure springs back a certain amount at once, but the complete recovery takes an appreciable time.

Cork which had been compressed and released in water many thousand times had not changed its molecular structure in the least, and had continued perfectly serviceable. Cork which has been kept under a pressure of three atmospheres for many weeks appears to have shrunk to from 80% to 85% of its original volume. — *Van Nostrand's Eng'g Mag.*, 1886, xxxv. 307.

VULCANIZED INDIA-RUBBER.

The specific gravity of a rubber compound, or the number of cubic inches to the pound, is generally taken by buyers as a correct index of the value, though in reality such is often very far from being the case. In the rubber works the qualities of the rubber made vary from floating, the best quality, to densities corresponding to 11 or 12 cu. in. to the pound, the latter densities being in demand by consumers with whom price appears to be the main consideration. Such densities as these can only be obtained by utilizing to the utmost the quality that rubber exhibits of taking up a large bulk of added matters. — *Eng'g*, 1897.

Lieutenant L. Vladimiroff, a Russian naval officer, has recently carried out a series of tests at the St. Petersburg Technical Institute with view to establishing rules for estimating the quality of vulcanized india-rubber. The following, in brief, are the conclusions arrived at, recourse being had to physical properties, since chemical analysis did not give any reliable result: 1. India-rubber should not give the least sign of superficial cracking when bent to an angle of 180 degrees after five hours of exposure in a closed air-bath to a temperature of 125° C. The test-pieces should be 2.4 inches thick. 2. Rubber that does not contain more than half its weight of metallic oxides should stretch to five times its length without breaking. 3. Rubber free from all foreign matter, except the sulphur used in vulcanizing it, should stretch to at least seven times its length without rupture. 4. The extension measured immediately after rupture should not exceed 12% of the original length, with given dimensions. 5. Suppleness may be determined by measuring the percentage of ash formed in incineration. This may form the basis for deciding between different grades of rubber for certain purposes. 6. Vulcanized rubber should not harden under cold. These rules have been adopted for the Russian navy. — *Iron Age*, June 15, 1893.

Singular Action of India Rubber under Tension. — R. H. Thurston, *Am. Mach.*, Mar. 17, 1898, gives a diagram showing the stretch at different loads of a piece of partially vulcanized rubber. The results translated into figures are:

| | | | | | | | | |
|--|------|------|-----|------|------|------|------|------|
| Load, lbs..... | 30 | 50 | 80 | 120 | 150 | 200 | 320 | 430 |
| Stretch per in. of length, in..... | 0.5 | 1. | 2.2 | 4 | 5 | 6 | 7 | 7.5 |
| Stretch per 10 lbs. increase of load.... | 0.17 | 0.25 | 0.4 | 0.45 | 0.33 | 0.20 | 0.08 | 0.04 |

Up to about 30% of the breaking load the rubber behaves like a soft metal in showing an increasing rate of stretch with increase of load, then the rate of stretch becomes constant for a while and later decreases steadily until before rupture it is less than one-tenth of the maximum. Even when stretched almost to rupture it restores itself very nearly to its original dimensions on removing the load, and gradually recovers a part of the loss of form at that instant observable. So far as known, no other substance shows this curious relation of stretch to load.

Rubber Goods Analysis. Randolph Bolling. (*Iron Age*, Jan. 28, 1909.)

The loading of rubber goods used in manufacturing establishments with zinc oxide, lead sulphate, calcium sulphate, etc., and the employment of the so-called "rubber substitutes" mixed with good rubber call for close inspection of the works chemist in order to determine the value of the samples and materials received. The following method of analysis is recommended:

Thin strips of the rubber must be cut into small bits about the size of No. 7 shot. A half gram is heated in a 200 c.c. flask with red fuming nitric acid on the hot plate until all organic matter has been decomposed, and the total sulphur is determined by precipitation as barium sulphate. The difference between the total and combined sulphur gives the per cent that has been used for vulcanization. Free sulphur indicates either that improper methods were used in vulcanizing or that an excessive

per cent of substitutes was employed. Following is a scheme for the analysis of india-rubber articles:

1. Extraction with acetone: A. Solution: Resinous constituents of india-rubber, fatty oils, mineral oils, resin oils, solid hydrocarbons, resins free sulphur. B. Residue.
2. Extraction with pyridine: C. Extract: Tar, pitch, bituminous bodies, sulphur in above. D. Residue.
3. Extraction with alcoholic potash: E. Extract: Chlorosulphide substitutes, sulphide substitutes, oxidized (blown) oils, sulphur in substitutes, chlorine in substitutes. F. Residue.
4. Extraction with nitro-naphthalene: G. Extract: India-rubber, sulphur in india-rubber, chlorine in india-rubber, the total of the above three estimated by loss. H. Residue.
5. Extraction with boiling water: I. Extract: Starch (farina), dextrine. K. Residue: Mineral matter, free carbon, fibrous materials, sulphur in inorganic compounds.
6. Separate estimations: Total sulphur, chlorine in rubber.

NICKEL.

Properties of Nickel.—(F. L. Sperry, *Trans. A. I. M. E.*, 1895.) Nickel has similar physical properties to those of iron and copper. It is less malleable and ductile than iron, and less malleable and more ductile than copper. It alloys with these metals in all proportions. It has nearly the same specific gravity as copper, and is slightly heavier than iron. It melts at a temperature of about 2900° to 3200° F. A small percentage of carbon in metallic nickel lowers its melting-point perceptibly. Nickel is harder than either iron or copper; is magnetic, but will not take a temper. It has a grayish-white color, takes a fine polish, and may be rolled easily into thin plates or drawn into wire. It is unappreciably affected by atmospheric action, or by salt water. Commercial nickel is from 98 to 99 per cent pure. The impurities are iron, copper, silicon, sulphur, arsenic, carbon, and (in some nickel) a kernel of unreduced oxide. It is not difficult to cast, and acts like some iron in being cold-short. Cast bars are likely to be porous or spongy, but, after hammering or rolling, are compact and tough.

The average results of several tests are as follows: Castings, tensile strength, 85,000 lbs. per sq. in., elongation, 12%; wrought nickel, T. S., 96,000, El., 14%; wrought nickel, annealed, T. S., 95,000, El., 23%; hard rolled, T. S., 78,000, El., 10%. (See also page 473.)

Nickel readily takes up carbon, and the porous nature of the metal is undoubtedly due to occluded gases. According to Dr. Wedding, nickel may take up as much as 9% of carbon, which may exist either as amorphous or as graphitic carbon.

Dr. Fleitmann, of Germany, discovered that a small quantity of pure magnesium would free nickel from occluded gases and give a metal capable of being drawn or rolled perfectly free from blow-holes, to such an extent that the metal may be rolled into thin sheets 3 feet in width. Aluminum or manganese may be used equally as well as a purifying agent; but either, if used in excess, serves to make the nickel very much harder. Nickel will alloy with most of the useful metals, and generally adds the qualities of hardness, toughness, and ductility.

ALUMINUM — ITS PROPERTIES AND USES.

(By Alfred E. Hunt, Pres't of the Pittsburgh Reduction Co.)

The specific gravity of pure aluminum in a cast state is 2.58; in rolled bars of large section it is 2.6; in very thin sheets subjected to high compression under chilled rolls, it is as much as 2.7. Taking the weight of a given bulk of cast aluminum as 1, wrought iron is 2.90 times heavier; structural steel, 2.95 times; copper, 3.60; ordinary high brass, 3.45. Most wood suitable for use in structures has about one third the weight of aluminum, which weighs 0.092 lb. to the cubic inch.

Pure aluminum is practically not acted upon by boiling water or steam. Carbonic oxide or hydrogen sulphide does not act upon it at any temperature under 600° F. It is not acted upon by most organic secretions.

Hydrochloric acid is the best solvent for aluminum, and strong solutions of caustic alkalies readily dissolve it. Ammonia has a slight solvent action, and concentrated sulphuric acid dissolves aluminum upon heating, with evolution of sulphurous acid gas. Dilute sulphuric acid acts but slowly on

the metal, though the presence of any chlorides in the solution allows rapid decomposition. Nitric acid, either concentrated or dilute, has very little action upon the metal, and sulphur has no action unless the metal is at a red heat. Sea-water has very little effect on aluminum. Strips of the metal placed on the sides of a wooden ship corroded less than 1/1000 inch after six months' exposure to sea-water, corroding less than copper sheets similarly placed.

In malleability pure aluminum is only exceeded by gold and silver. In ductility it stands seventh in the series, being exceeded by gold, silver, platinum, iron, very soft steel, and copper. Sheets of aluminum have been rolled down to a thickness of 0.0005 inch, and beaten into leaf nearly as thin as gold leaf. The metal is most malleable at a temperature of between 400° and 600° F., and at this temperature it can be drawn down between rolls with nearly as much draught upon it as with heated steel. It has also been drawn down into the very finest wire. By the Mannesmann process aluminum tubes have been made in Germany.

Aluminum stands very high in the series as an electro-positive metal, and contact with other metals should be avoided, as it would establish a galvanic couple.

The electrical conductivity of aluminum is only surpassed by pure copper, silver, and gold. With silver taken at 100 the electrical conductivity of aluminum is 54.20; that of gold on the same scale is 78; zinc is 29.90; iron is only 16, and platinum 10.60. Pure aluminum has no polarity, and the metal in the market is absolutely non-magnetic.

Sound castings can be made of aluminum in either dry or "green" sand moulds, or in metal "chills." It must not be heated much beyond its melting-point, and must be poured with care, owing to the ready absorption of occluded gases and air. The shrinkage in cooling is 17/64 inch per foot, or a little more than ordinary brass. It should be melted in plumbago crucibles, and the metal becomes molten at a temperature of 1215° F.

The coefficient of linear expansion, as tested on 3/8-inch round aluminum rods, is 0.00002295 per degree centigrade between the freezing and boiling point of water. The mean specific heat of aluminum is higher than that of any other metal, excepting only magnesium and the alkali metals. From zero to the melting-point it is 0.2185; water being taken as 1, and the latent heat of fusion at 28.5 heat units. The coefficient of thermal conductivity of unannealed aluminum is 37.96; of annealed aluminum, 38.37. As a conductor of heat aluminum ranks fourth, being exceeded only by silver, copper, and gold.

Aluminum, under tension, and section for section, is about as strong as cast iron. The tensile strength of aluminum is increased by cold rolling or cold forging, and there are alloys which add considerably to the tensile strength without increasing the specific gravity to over 3 or 3.25.

The strength of commercial aluminum is given in the following table as the result of many tests:

| Form. | Elastic Limit per sq. in. in Tension, lbs. | Ultimate Strength per sq. in. in Tension, lbs. | Percentage of Reduct'n of Area in Tension. |
|---------------|---|---|---|
| Castings..... | 6,500 | 15,000 | 15 |
| Sheet..... | 12,000 | 24,000 | 35 |
| Wire..... | 16,000-30,000 | 30,000-65,000 | 60 |
| Bars..... | 14,000 | 28,000 | 40 |

The elastic limit per square inch under compression in cylinders, with length twice the diameter, is 3500. The ultimate strength per square inch under compression in cylinders of same form is 12,000. The modulus of elasticity of cast aluminum is about 11,000,000. It is rather an open metal in its texture, and for cylinders to stand pressure an increase in thickness must be given to allow for this porosity. Its maximum shearing stress in castings is about 12,000, and in forgings about 16,000, or about that of pure copper.

Pure aluminum is too soft and lacking in tensile strength and rigidity for many purposes. Valuable alloys are now being made which seem to give great promise for the future. They are alloys containing from 2% to 7% or 8% of copper, manganese, iron, and nickel.

Plates and bars of these alloys have a tensile strength of from 40,000 to

50,000 pounds per square inch, an elastic limit of 55% to 60% of the ultimate tensile strength, an elongation of 20% in 2 inches, and a reduction of area of 25%.

This metal is especially capable of withstanding the punishment and distortion to which structural material is ordinarily subjected. Some aluminum alloys have as much resilience and spring as the hardest of hard-drawn brass.

Their specific gravity is about 2.80 to 2.85, where pure aluminum has a specific gravity of 2.72.

In castings, more of the hardening elements are necessary in order to give the maximum stiffness and rigidity, together with the strength and ductility of the metal; the favorite alloy material being zinc, iron, manganese, and copper. Tin added to the alloy reduces the shrinkage, and alloys of aluminum and tin can be made which have less shrinkage than cast iron.

The tensile strength of hardened aluminum-alloy castings is from 20,000 to 25,000 pounds per square inch.

Alloys of aluminum and copper form two series, both valuable. The first is aluminum bronze, containing from 5% to 11½% of aluminum; and the second is copper-hardened aluminum, containing from 2% to 15% of copper. Aluminum-bronze is a very dense, fine-grained, and strong alloy, having good ductility as compared with tensile strength. The 10% bronze in forged bars will give 100,000 lbs. tensile strength per square inch, with 60,000 lbs. elastic limit per square inch, and 10% elongation in 8 inches. The 5% to 7½% bronze has a specific gravity of 8 to 8.30, as compared with 7.50 for the 10% to 11½% bronze, a tensile strength of 70,000 to 80,000 lbs., an elastic limit of 40,000 lbs. per square inch, and an elongation of 30% in 8 inches.

Aluminum is used by steel manufacturers to prevent the retention of the occluded gases in the steel, and thereby produce a solid ingot. The proportions of the dose range from ½ lb. to several pounds of aluminum per ton of steel. Aluminum is also used in giving extra fluidity to steel used in castings, making them sharper and sounder. Added to cast iron, aluminum causes the iron to be softer, free from shrinkage, and lessens the tendency to "chill."

With the exception of lead and mercury, aluminum unites with all metals, though it unites with antimony with great difficulty. A small percentage of silver whitens and hardens the metal, and gives it added strength; and this alloy is especially applicable to the manufacture of fine instruments and apparatus. The following alloys have been found recently to be useful in the arts: Nickel-aluminum, composed of 20 parts nickel to 80 of aluminum; rosine, made of 40 parts nickel, 10 parts silver, 30 parts aluminum, and 20 parts tin, for jewellers' work; mettaline, made of 35 parts cobalt, 25 parts aluminum, 10 parts iron, and 30 parts copper. The aluminum-bourbouze metal, shown at the Paris Exposition of 1889, has a specific gravity of 2.9 to 2.96, and can be cast in very solid shapes, as it has very little shrinkage. From analysis the following composition is deduced: Aluminum, 85.74%; tin, 12.94%; silicon, 1.32%; iron, none.

The metal can be readily electrically welded, but soldering is still not satisfactory. The high heat conductivity of the aluminum withdraws the heat of the molten solder so rapidly that it "freezes" before it can flow sufficiently. A German solder said to give good results is made of 80% tin to 20% zinc, using a flux composed of 80 parts stearic acid, 10 parts chloride of zinc, and 10 parts of chloride of tin. Pure tin, fusing at 250° C., has also been used as a solder. The use of chloride of silver as a flux has been patented, and used with ordinary soft solder has given some success. A pure nickel soldering-bit should be used, as it does not discolor aluminum as copper bits do.

Aluminum Wire. — Tension tests. Diam. 0.128 in. 14 tests. F.L. 12,500 to 19,100; T. S. 25,800 to 26,900 lbs. per sq. in.; el. 0.30 to 1.02% in 43 ins.; Red. of area, 75.0 to 83.4%. Mod. of el. 8,800,000 to 10,700,000. — *Tech. Quar.*, xii, 1899.

Aluminum Rod. — Torsion tests. 10 samples, 0.257 in. diam. Apparent outside fiber stress, lbs. per sq. in. 15,900 to 18,300 lbs. per sq. in. 11 samples, 0.367 in. diam. Apparent outside fiber stress, 18,400 to 19,200. 10 samples, 0.459 in. diam. Apparent outside fiber stress, 20,700 to 21,500 lbs. per sq. in. The average number of turns per inch for the three series were respectively, 1.58 to 3.65; 1.20 to 2.64; 0.87 to 1.06. *Ibid.*

ALLOYS.

ALLOYS OF COPPER AND TIN.

(Extract from Report of U. S. Test Board.*)

| Number. | Mean Com- position by Analysis. | | Tensile Strength, lbs. per sq. in. | Elastic Limit, lbs. per sq. in. | Elongation, per cent in 5 inches. | Transverse Test, Modulus of Rupture. | Deflection, 1" sq Bar 22 in. long, inches. | Crushing Strength, lbs. per sq. in. | Torsion Tests. | |
|---------|---------------------------------------|-------|---------------------------------------|------------------------------------|---|--|--|---|---------------------------------------|----------------------------------|
| | Cop- per. | Tin. | | | | | | | Maximum Tor. Mo- ment, ft.-lbs. | Angle of Torsion, degrees. |
| 1 | 100. | ... | 27,800 | 14,000 | 6.47 | 29,848 | bent. | 42,000 | 143 | 153 |
| 1a | 100. | ... | 12,760 | 11,000 | 0.47 | 21,251 | 2.31 | 39,000 | 65 | 40 |
| 2 | 97.89 | 1.90 | 24,580 | 10,000 | 13.33 | ... | ... | 34,000 | 150 | 317 |
| 3 | 96.06 | 3.76 | 32,000 | 16,000 | 14.29 | 33,232 | bent. | 42,048 | 157 | 247 |
| 4 | 94.11 | 5.43 | ... | ... | ... | 38,659 | ... | ... | ... | ... |
| 5 | 92.11 | 7.80 | 28,540 | 19,000 | 5.53 | 43,731 | " | 42,000 | 160 | 126° |
| 6 | 90.27 | 9.58 | 26,860 | 15,750 | 3.66 | 49,400 | " | 38,000 | 175 | 114 |
| 7 | 88.41 | 11.59 | ... | ... | ... | 60,403 | " | ... | ... | ... |
| 8 | 87.15 | 12.73 | 29,430 | 20,000 | 3.33 | 34,531 | 4.00 | 53,000 | 182 | 100° |
| 9 | 82.70 | 17.34 | ... | ... | ... | 67,930 | 0.63 | ... | ... | ... |
| 10 | 80.95 | 18.84 | 32,980 | ... | 0.04 | 56,715 | 0.49 | 78,000 | 190 | 16° |
| 11 | 77.56 | 22.25 | ... | ... | 0. | 29,926 | 0.16 | ... | ... | ... |
| 12 | 76.63 | 23.24 | 22,010 | 22,010 | 0. | 32,210 | 0.19 | 114,000 | 122 | 3.4 |
| 13 | 72.89 | 26.85 | ... | ... | 0. | 9,512 | 0.05 | ... | ... | ... |
| 14 | 69.84 | 29.88 | 5,585 | 5,585 | 0. | 12,076 | 0.06 | 147,000 | 18 | 1.5 |
| 15 | 68.58 | 31.26 | ... | ... | 0. | 9,152 | 0.04 | ... | ... | ... |
| 16 | 67.87 | 32.10 | ... | ... | 0. | 9,477 | 0.05 | ... | ... | ... |
| 17 | 65.34 | 34.47 | 2,201 | 2,201 | 0. | 4,776 | 0.02 | 84,700 | 16 | 1° |
| 18 | 56.70 | 43.17 | 1,455 | 1,455 | 0. | 2,126 | 0.02 | ... | ... | ... |
| 19 | 44.52 | 55.28 | 3,010 | 3,010 | 0. | 4,776 | 0.03 | 35,800 | 23 | 1° |
| 20 | 34.22 | 65.80 | 3,371 | 3,371 | 0. | 5,384 | 0.04 | 19,600 | 17 | 2 |
| 21 | 23.35 | 76.29 | 6,775 | 6,775 | 0. | 12,408 | 0.27 | ... | ... | ... |
| 22 | 15.08 | 84.62 | ... | ... | ... | 9,063 | 0.86 | 6,500 | 23 | 25° |
| 23 | 11.49 | 88.47 | 6,380 | 3,500 | 4.10 | 10,706 | 5.85 | 10,100 | 23 | 62 |
| 24 | 8.57 | 91.39 | 6,450 | 3,500 | 6.87 | 5,305 | bent. | 9,800 | 23 | 132 |
| 25 | 3.72 | 96.31 | 4,780 | 2,750 | 12.32 | 6,925 | " | 9,800 | 23 | 220 |
| 26 | 0. | 100. | 3,505 | ... | 35.51 | 3,740 | " | 6,400 | 12 | 557 |

* The tests of the alloys of copper and tin and of copper and zinc, the results of which are published in the Report of the U. S. Board appointed to test Iron, Steel, and other Metals, Vols. I and II, 1879 and 1881, were made by the author under direction of Prof. R. H. Thurston, chairman of the Committee on Alloys. See preface to the report of the Committee, in Vol. I.

Nos. 1a and 2 were full of blow-holes.

Tests Nos. 1 and 1a show the variation in cast copper due to varying conditions of casting. In the crushing tests Nos. 12 to 20, inclusive, crushed and broke under the strain, but all the others bulged and flattened out. In these cases the crushing strength is taken to be that which caused a decrease of 10% in the length. The test-pieces were 2 in. long and $\frac{5}{8}$ in. diameter. The torsional tests were made in Thurston's torsion-machine, on pieces $\frac{5}{8}$ in. diameter and 1 in. long between heads.

Specific Gravity of the Copper-tin Alloys. — The specific gravity of copper, as found in these tests, is 8.874 (tested in turnings from the ingot, and reduced to 39.1° F.). The alloy of maximum sp. gr. 8.956 contained 62.42 copper, 37.48 tin, and all the alloys containing less than

37% tin varied irregularly in sp. gr. between 8.65 and 8.93, the density depending not on the composition, but on the porosity of the casting. It is probable that the actual sp. gr. of all these alloys containing less than 37% tin is about 8.95, and any smaller figure indicates porosity in the specimen.

From 37% to 100% tin, the sp. gr. decreases regularly from the maximum of 8.956 to that of pure tin, 7.293.

Note on the Strength of the Copper-tin Alloys.

The bars containing from 2% to 24% tin, inclusive, have considerable strength, and all the rest are practically worthless for purposes in which strength is required. The dividing line between the strong and brittle alloys is precisely that at which the color changes from golden yellow to silver-white, viz., at a composition containing between 24% and 30% of tin.

It appears that the tensile and compressive strengths of these alloys are in no way related to each other, that the torsional strength is closely proportional to the tensile strength, and that the transverse strength may depend in some degree upon the compressive strength, but it is much more nearly related to the tensile strength. The modulus of rupture, as obtained by the transverse tests, is, in general, a figure between those of tensile and compressive strengths per square inch, but there are a few exceptions in which it is larger than either.

The strengths of the alloys at the copper end of the series increase rapidly with the addition of tin till about 4% of tin is reached. The transverse strength continues regularly to increase to the maximum, till the alloy containing about 17½% of tin is reached, while the tensile and torsional strengths also increase, but irregularly, to the same point. This irregularity is probably due to porosity of the metal, and might possibly be removed by any means which would make the castings more compact. The maximum is reached at the alloy containing 82.70 copper, 17.34 tin, the transverse strength, however, being very much greater at this point than the tensile or torsional strength. From the point of maximum strength the figures drop rapidly to the alloys containing about 27.5% of tin, and then more slowly to 37.5%, at which point the minimum (or nearly the minimum) strength, by all three methods of test, is reached. The alloys of minimum strength are found from 37.5% tin to 52.5% tin. The absolute minimum is probably about 45% of tin.

From 52.5% of tin to about 77.5% tin there is a rather slow and irregular increase in strength. From 77.5% tin to the end of the series, or all tin, the strengths slowly and somewhat irregularly decrease.

The results of these tests do not seem to corroborate the theory given by some writers, that peculiar properties are possessed by the alloys which are compounded of simple multiples of their atomic weights or chemical equivalents, and that these properties are lost as the compositions vary more or less from this definite constitution. It does appear that a certain percentage composition gives a maximum strength and another certain percentage a minimum, but neither of these compositions is represented by simple multiples of the atomic weights.

There appears to be a regular law of decrease from the maximum to the minimum strength which does not seem to have any relation to the atomic proportions, but only to the percentage compositions.

Hardness.—The pieces containing less than 24 % of tin were turned in the lathe without difficulty, a gradually increasing hardness being noticed, the last named giving a very short chip, and requiring frequent sharpening of the tool.

With the most brittle alloys it was found impossible to turn the test-pieces in the lathe to a smooth surface. No. 13 to No. 17 (26.85 to 34.47 tin) could not be cut with a tool at all. Chips would fly off in advance of the tool and beneath it, leaving a rough surface; or the tool would sometimes, apparently, crush off portions of the metal, grinding it to powder. Beyond 40% tin the hardness decreased so that the bars could be easily turned.

ALLOYS OF COPPER AND ZINC. (U. S. Test Board.)

| No. | Mean Com- position by Analysis. | | Tensile Str'gth, lbs. per sq. in. | Elastic Limit % of Break- ing Load, lbs. per sq. in. | Elongation % in 5 inches. | Trans- verse Test Modu- lus of Rup- ture. | Deflection 1" sq. bar 22" long, in. | Crush- ing Str'gth per sq. in., lbs. | Torsional Tests. | |
|-----|---------------------------------------|-------|--|---|------------------------------|---|---|--|----------------------------------|------------------------|
| | Cop- per. | Zinc. | | | | | | | Max. Tors. Moment ft.-lbs. | Angle of Tors. deg. |
| 1 | 97.83 | 1.88 | 27,240 | | | | | | 130 | 357 |
| 2 | 82.93 | 16.98 | 32,600 | 26.1 | 26.7 | 23,197 | Bent | ... | 155 | 329 |
| 3 | 81.91 | 17.99 | 32,670 | 30.6 | 31.4 | 21,193 | " | ... | 166 | 345 |
| 4 | 77.39 | 22.45 | 35,630 | 20.0 | 35.5 | 25,374 | " | ... | 169 | 311 |
| 5 | 76.65 | 23.08 | 30,520 | 24.6 | 35.8 | 22,325 | " | 42,000 | 165 | 267 |
| 6 | 73.20 | 26.47 | 31,580 | 23.7 | 38.5 | 25,694 | " | ... | 168 | 293 |
| 7 | 71.20 | 28.54 | 30,510 | 29.5 | 29.2 | 24,468 | " | ... | 164 | 269 |
| 8 | 69.74 | 30.06 | 28,120 | 28.7 | 20.7 | 26,930 | " | ... | 143 | 202 |
| 9 | 66.27 | 33.50 | 37,800 | 25.1 | 37.7 | 28,459 | " | ... | 176 | 257 |
| 10 | 63.44 | 36.36 | 48,300 | 32.8 | 31.7 | 43,216 | " | ... | 202 | 230 |
| 11 | 60.94 | 38.65 | 41,065 | 40.1 | 20.7 | 38,968 | " | 75,000 | 194 | 202 |
| 12 | 58.49 | 41.10 | 50,450 | 54.4 | 10.1 | 63,304 | " | ... | 227 | 93 |
| 13 | 55.15 | 44.44 | 44,280 | 44.0 | 15.3 | 42,463 | " | 78,000 | 209 | 109 |
| 14 | 54.86 | 44.78 | 46,400 | 53.9 | 8.0 | 47,955 | " | ... | 223 | 72 |
| 15 | 49.66 | 50.14 | 30,990 | 54.5 | 5.0 | 33,467 | 1.26 | 117,400 | 172 | 38 |
| 16 | 48.99 | 50.82 | 26,050 | 100 | 0.8 | 40,189 | 0.61 | ... | 176 | 16 |
| 17 | 47.56 | 52.28 | 24,150 | 100 | 0.8 | 48,471 | 1.17 | 121,000 | 155 | 13 |
| 18 | 43.36 | 56.22 | 9,170 | 100 | ... | 17,691 | 0.10 | ... | 88 | 2 |
| 19 | 41.30 | 58.12 | 3,727 | 100 | ... | 7,761 | 0.04 | ... | 18 | 2 |
| 20 | 32.94 | 66.23 | 1,774 | 100 | ... | 8,296 | 0.04 | ... | 29 | 1 |
| 21 | 29.20 | 70.17 | 6,414 | 100 | ... | 16,579 | 0.04 | ... | 40 | 2 |
| 22 | 20.81 | 77.63 | 9,000 | 100 | 0.2 | 22,972 | 0.13 | 52,152 | 65 | 1 |
| 23 | 12.12 | 86.67 | 12,413 | 100 | 0.4 | 35,026 | 0.31 | ... | 82 | 3 |
| 24 | 4.35 | 94.59 | 18,065 | 100 | 0.5 | 26,162 | 0.46 | ... | 81 | 22 |
| 25 | Cast. | Zinc. | 5,400 | 75 | 0.7 | 7,539 | 0.12 | 22,000 | 37 | 142 |

Variation in Strength of Gun-bronze, and Means of Improving the Strength. — The figures obtained for alloys of from 7.8% to 12.7% tin, viz., from 26,860 to 29,430 pounds, are much less than are usually given as the strength of gun-metal. Bronze guns are usually cast under the pressure of a head of metal, which tends to increase the strength and density. The strength of the upper part of a gun casting, or sinking head, is not greater than that of the small bars which have been tested in these experiments. The following is an extract from the report of Major Wade concerning the strength and density of gun-bronze (1850): — Extreme variation of six samples from different parts of the same gun (a 32-pounder howitzer): Specific gravity, 8.487 to 8.835; tenacity, 26,428 to 52,192. Extreme variation of all the samples tested: Specific gravity, 8.308 to 8.850; tenacity, 23,108 to 54,531. Extreme variation of all the samples from the gun heads: Specific gravity, 8.308 to 8.756; tenacity, 23,529 to 35,484.

Major Wade says: The general results on the quality of bronze as it is found in guns are mostly of a negative character. They expose defects in density and strength, develop the heterogeneous texture of the metal in different parts of the same gun, and show the irregularity and uncertainty of quality which attend the casting of all guns, although made from similar materials, treated in like manner.

Navy ordnance bronze containing 9 parts copper and 1 part tin, tested at Washington, D.C., in 1875-6, showed a variation in tensile strength from 29,800 to 51,400 lbs. per square inch, in elongation from 3% to 58%, and in specific gravity from 8.39 to 8.88.

That a great improvement may be made in the density and tenacity of gun-bronze by compression has been shown by the experiments of Mr. S. B. Dean in Boston, Mass., in 1869, and by those of General Uchatius in Austria in 1873. The former increased the density of the

metal next the bore of the gun from 8.321 to 8.875, and the tenacity from 27,238 to 41,471 pounds per square inch. The latter, by a similar process, obtained the following figures for tenacity:

| | Pounds per sq. in. |
|--------------------------|--------------------|
| Bronze with 10% tin..... | 72,053 |
| Bronze with 8% tin..... | 73,958 |
| Bronze with 6% tin..... | 77,656 |

ALLOYS OF COPPER, TIN, AND ZINC.

(Report of U. S. Test Board, Vol. II, 1881.)

| No. in Report. | Analysis, Original Mixture. | | | Transverse Strength. | | Tensile Strength per square inch. | | Elongation per cent in 5 inches. | |
|----------------|-----------------------------|-------|-------|----------------------|------------------|-----------------------------------|--------------|----------------------------------|---------|
| | Cu. | Sn. | Zn. | Modulus of Rupture. | Deflection, ins. | A. | B. | A. | B. |
| 72 | 90 | 5 | 5 | 41,334 | 2.63 | 23,660 | 30,740 | 2.34 | 9.68 |
| 5 | 88.14 | 1.86 | 10 | 31,986 | 3.67 | 32,000 | 33,000 | 17.6 | 19.5 |
| 70 | 85 | 5 | 10 | 44,457 | 2.85 | 28,840 | 28,560 | 6.80 | 5.28 |
| 71 | 85 | 10 | 5 | 62,470 | 2.56 | 35,680 | 36,000 | 2.51 | 2.25 |
| 89 | 85 | 12.5 | 2.5 | 62,405 | 2.83 | 34,500 | 32,800 | 1.29 | 2.79 |
| 88 | 82.5 | 12.5 | 5 | 69,960 | 1.61 | 36,000 | 34,000 | 0.86 | 0.92 |
| 77 | 82.5 | 15 | 2.5 | 69,045 | 1.09 | 33,600 | 33,800 | ... | 0.68 |
| 67 | 80 | 5 | 15 | 42,618 | 3.88 | 37,560 | 32,300 | 11.6 | 3.59 |
| 68 | 80 | 10 | 10 | 67,117 | 2.45 | 32,830 | 31,950 | 1.57 | 1.67 |
| 69 | 80 | 15 | 5 | 54,476 | 0.44 | 32,350 | 30,760 | 0.55 | 0.44 |
| 86 | 77.5 | 10 | 12.5 | 63,849 | 1.19 | 35,500 | 36,000 | 1.00 | 1.00 |
| 87 | 77.5 | 12.5 | 10 | 61,705 | 0.71 | 36,000 | 32,500 | 0.72 | 0.59 |
| 63 | 75 | 5 | 20 | 55,355 | 2.91 | 33,140 | 34,960 | 2.50 | 3.19 |
| 85 | 75 | 7.5 | 17.5 | 62,607 | 1.39 | 33,700 | 39,300 | 1.56 | 1.33 |
| 64 | 75 | 10 | 15 | 58,345 | 0.73 | 35,320 | 34,000 | 1.13 | 1.25 |
| 65 | 75 | 15 | 10 | 51,109 | 0.31 | 35,440 | 28,000 | 0.59 | 0.54 |
| 66 | 75 | 20 | 5 | 40,235 | 0.21 | 23,140 | 27,660 | 0.43 | ... |
| 83 | 72.5 | 7.5 | 20 | 51,839 | 2.86 | 32,700 | 34,800 | 3.73 | 3.78 |
| 84 | 72.5 | 10 | 17.5 | 53,230 | 0.74 | 30,000 | 30,000 | 0.48 | 0.49 |
| 59 | 70 | 5 | 25 | 57,349 | 1.37 | 38,000 | 32,940 | 2.06 | 0.99 |
| 82 | 70 | 7.5 | 22.5 | 48,836 | 0.36 | 38,000 | 32,400 | 0.84 | 0.40 |
| 60 | 70 | 10 | 20 | 36,520 | 0.18 | 33,140 | 26,300 | 0.31 | ... |
| 61 | 70 | 15 | 15 | 37,924 | 0.20 | 33,440 | 27,800 | 0.25 | ... |
| 62 | 70 | 20 | 10 | 15,126 | 0.08 | 17,000 | 12,900 | 0.03 | ... |
| 81 | 67.5 | 2.5 | 30 | 58,343 | 2.91 | 34,720 | 45,850 | 7.27 | 3.09 |
| 74 | 67.5 | 5 | 27.5 | 55,976 | 0.49 | 34,000 | 34,460 | 1.06 | 0.43 |
| 75 | 67.5 | 7.5 | 25 | 46,875 | 0.32 | 29,500 | 30,000 | 0.36 | 0.26 |
| 80 | 65 | 2.5 | 32.5 | 56,949 | 2.36 | 41,350 | 38,300 | 3.26 | 3.02 |
| 55 | 65 | 5 | 30 | 51,369 | 0.56 | 37,140 | 36,000 | 1.21 | 0.61 |
| 56 | 65 | 10 | 25 | 27,075 | 0.14 | 25,720 | 22,500 | 0.15 | 0.19 |
| 57 | 65 | 15 | 20 | 13,591 | 0.07 | 6,820 | 7,231 | ... | ... |
| 58 | 65 | 20 | 15 | 11,932 | 0.05 | 3,765 | 2,665 | ... | ... |
| 79 | 62.5 | 2.5 | 35 | 69,255 | 2.34 | 44,400 | 45,000 | 2.15 | 2.19 |
| 78 | 60 | 2.5 | 37.5 | 69,508 | 1.46 | 57,400 | 52,900 | 4.87 | 3.02 |
| 52 | 60 | 5 | 35 | 46,076 | 0.28 | 41,160 | 38,330 | 0.39 | 0.40 |
| 53 | 60 | 10 | 30 | 24,699 | 0.13 | 21,780 | 21,240 | 0.15 | ... |
| 54 | 60 | 15 | 25 | 18,248 | 0.09 | 18,020 | 12,400 | ... | ... |
| 12 | 58.22 | 2.30 | 39.48 | 95,623 | 1.99 | 66,500 | 67,600 | 3.13 | 3.15 |
| 3 | 58.75 | 8.75 | 32.5 | 35,752 | 0.18 | Broke | before test; | very | brittle |
| 4 | 57.5 | 21.25 | 21.25 | 2,752 | 0.02 | 725 | 1,300 | ... | ... |
| 73 | 55 | 0.5 | 44.5 | 72,308 | 3.05 | 68,900 | 68,900 | 9.43 | 2.88 |
| 50 | 55 | 5 | 40 | 38,174 | 0.22 | 27,400 | 30,500 | 0.46 | 0.43 |
| 51 | 55 | 10 | 35 | 28,258 | 0.14 | 25,460 | 18,500 | 0.29 | 0.10 |
| 49 | 50 | 5 | 45 | 20,814 | 0.11 | 23,000 | 31,300 | 0.66 | 0.45 |

The transverse tests were made in bars 1 in. square, 22 in. between supports. The tensile tests were made on bars 0.798 in. diam. turned from the two halves of the transverse-test bar, one half being marked A and the other B.

Ancient Bronzes. — The usual composition of ancient bronze was the same as that of modern gun-metal — 90 copper, 10 tin; but the proportion of tin varies from 5% to 15%, and in some cases lead has been found. Some ancient Egyptian tools contained 88 copper, 12 tin.

Strength of the Copper-zinc Alloys. — The alloys containing less than 15% of zinc by original mixture were generally defective. The bars were full of blow-holes, and the metal showed signs of oxidation. To insure good castings it appears that copper-zinc alloys should contain more than 15% of zinc.

From No. 2 to No. 8 inclusive, 16.98 to 30.06% zinc the bars show a remarkable similarity in all their properties. They have all nearly the same strength and ductility, the latter decreasing slightly as zinc increases, and are nearly alike in color and appearance. Between Nos. 8 and 10, 30.06 and 36.36% zinc, the strength by all methods of test rapidly increases. Between No. 10 and No. 15, 36.36 and 50.14% zinc, there is another group, distinguished by high strength and diminished ductility. The alloy of maximum tensile, transverse and torsional strength contains about 41% of zinc.

The alloys containing less than 55% of zinc are all yellow metals. Beyond 55% the color changes to white, and the alloy becomes weak and brittle. Between 70% and pure zinc the color is bluish gray, the brittleness decreases and the strength increases, but not to such a degree as to make them useful for constructive purposes.

Difference between Composition by Mixture and by Analysis. — There is in every case a smaller percentage of zinc in the average analysis than in the original mixture, and a larger percentage of copper. The loss of zinc is variable, but in general averages from 1 to 2%.

Liquation or Separation of the Metals. — In several of the bars a considerable amount of liquation took place, analysis showing a difference in composition of the two ends of the bar. In such cases the change in composition was gradual from one end of the bar to the other, the upper end in general containing the higher percentage of copper. A notable instance was bar No. 13, in the above table, turnings from the upper end containing 40.36% of zinc, and from the lower end 48.52%.

Specific Gravity. — The specific gravity follows a definite law, varying with the composition, and decreasing with the addition of zinc. From the plotted curve of specific gravities the following mean values are taken:

| | | | | | | | | | | | |
|-------------------------|------|------|------|------|------|------|------|------|------|------|------|
| Per cent zinc | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Specific gravity . . . | 8.80 | 8.72 | 8.60 | 8.40 | 8.36 | 8.20 | 8.00 | 7.72 | 7.40 | 7.20 | 7.14 |

Graphic Representation of the Law of Variation of Strength of Copper-Tin-Zinc Alloys. — In an equilateral triangle the sum of the perpendicular distances from any point within it to the three sides is equal to the altitude. Such a triangle can therefore be used to show graphically the percentage composition of any compound of three parts, such as a triple alloy. Let one side represent 0 copper, a second 0 tin, and the third 0 zinc, the vertex opposite each of these sides representing 100 of each element respectively. On points in a triangle of wood representing different alloys tested, wires were erected of lengths proportional to the tensile strengths, and the triangle then built up with plaster to the height of the wires. The surface thus formed has a characteristic topography representing the variations of strength with variations of composition. The cut shows the surface thus made. The vertical section to the left represents the law of tensile strength of the copper-tin alloys, the one to the right that of tin-zinc alloys, and the one at the rear that of the copper-zinc alloys. The high point represents the strongest possible alloys of the three metals. Its composition is copper 55, zinc 43, tin 2, and its strength about 70,000 lbs. The high ridge from this point to the point of maximum height of the section on the left is the line of the strongest alloys, represented by the formula $\text{zinc} + (3 \times \text{tin}) = 55$.

All alloys lying to the rear of the ridge, containing more copper and less tin or zinc are alloys of greater ductility than those on the line of

maximum strength, and are the valuable commercial alloys; those in front on the declivity toward the central valley are brittle, and those in the valley are both brittle and weak. Passing from the valley toward the section at the right the alloys lose their brittleness and become soft, the maximum softness being at tin=100, but they remain weak, as is shown by the low elevation of the surface. This model was planned and constructed by Prof. Thurston in 1877. (See *Trans. A. S. C. E.*, 1881. Report of the U. S. Board appointed to test Iron, Steel, etc., vol. ii, Washington, 1881, and Thurston's *Materials of Engineering*, vol. iii.)

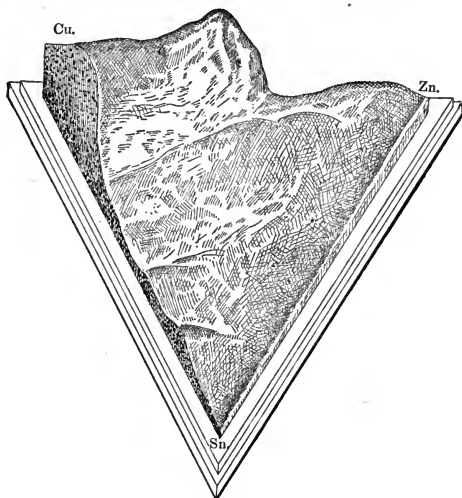


FIG. 79.

The best alloy obtained in Thurston's research for the U. S. Testing Board has the composition, copper 55, tin 0.5, zinc 44.5. The tensile strength in a cast bar was 68,900 lbs. per sq. in., two specimens giving the same result; the elongation was 47 to 51 per cent in 5 inches. Thurston's formula for copper-tin-zinc alloys of maximum strength (*Trans. A. S. C. E.*, 1881) is $z + 3t = 55$, in which z is the percentage of zinc and t that of tin. Alloys proportioned according to this formula should have a strength of about 40,000 lbs. per sq. in. + 500 z . The formula fails with alloys containing less than 1 per cent of tin.

The following would be the percentage composition of a number of alloys made according to this formula, and their corresponding tensile strength in castings:

| Tin. | Zinc. | Copper. | Tensile Strength, lbs. per sq. in. | Tin. | Zinc. | Copper. | Tensile Strength lbs. per sq. in. |
|------|-------|---------|------------------------------------|------|-------|---------|-----------------------------------|
| 1 | 52 | 47 | 66,000 | 8 | 31 | 61 | 55,500 |
| 2 | 49 | 49 | 64,500 | 9 | 28 | 63 | 54,000 |
| 3 | 46 | 51 | 63,000 | 10 | 25 | 65 | 52,500 |
| 4 | 43 | 53 | 61,500 | 12 | 19 | 69 | 49,500 |
| 5 | 40 | 55 | 60,000 | 14 | 13 | 73 | 46,500 |
| 6 | 37 | 57 | 58,500 | 16 | 7 | 77 | 43,500 |
| 7 | 34 | 59 | 57,000 | 18 | 1 | 81 | 40,500 |

These alloys, while possessing maximum tensile strength, would in general be too hard for easy working by machine tools. Another series made on the formula $z + 4t = 50$ would have greater ductility, together with considerable strength, as follows, the strength being calculated as before, tensile strength in lbs. per sq. in. = $40,000 + 500z$.

| Tin. | Zinc. | Copper. | Tensile Strength, lbs. per sq. in. | Tin. | Zinc. | Copper. | Tensile Strength, lbs. per sq. in. |
|------|-------|---------|------------------------------------|------|-------|---------|------------------------------------|
| 1 | 46 | 53 | 63,000 | 7 | 22 | 71 | 51,000 |
| 2 | 42 | 56 | 61,000 | 8 | 18 | 74 | 49,000 |
| 3 | 38 | 59 | 59,000 | 9 | 14 | 77 | 47,000 |
| 4 | 34 | 62 | 57,000 | 10 | 10 | 80 | 45,000 |
| 5 | 30 | 65 | 55,000 | 11 | 6 | 83 | 43,000 |
| 6 | 26 | 68 | 53,000 | 12 | 2 | 86 | 41,000 |

Composition of Alloys in Every-day Use in Brass Foundries.

(*American Machinist.*)

| | Cop- per. | Zinc. | Tin. | Lead. | |
|--------------------|--------------|-------|-------------|-------|--|
| | lbs. | lbs. | lbs. | lbs. | |
| Admiralty metal... | 87 | 5 | 8 | | For parts of engines on board naval vessels. |
| Bell metal..... | 16 | | 4 | | Bells for ships and factories. |
| Brass (yellow).... | 16 | 8 | | 1/2 | For plumbers, ship and house brass work. |
| Bush metal..... | 64 | 8 | 4 | 4 | For bearing bushes for shafting. |
| Gun metal..... | 32 | 1 | 3 | | For pumps and other hydraulic purposes. |
| Steam metal..... | 20 | 1 | 11/2 | 1 | Castings subjected to steam pressure. |
| Hard gun metal... | 16 | | 21/2 | | For heavy bearings. |
| Muntz metal..... | 60 | 40 | | | Metal from which bolts and nuts are forged, valve spindles, etc. |
| Phosphor bronze.. | 92 | | 8 phos. tin | | For valves, pumps and general work. |
| " " .. | 90 | | 10 | " " | For cog and worm wheels, bushes, axle bearings, slide valves, etc. |
| Brazing metal..... | 16 | 3 | | | Flanges for copper pipes. |
| " solder..... | 50 | 50 | | | Solder for the above flanges. |

Admiralty Metal, for surface condenser tubes where sea water is used for cooling, Cu, 70: Zn, 29: Sn, 1. *Power*, June 1, 1909.

Gurley's Bronze. — 16 parts copper, 1 tin, 1 zinc, 1/2 lead, used by W & L. E. Gurley of Troy for the framework of their engineer's transits. Tensile strength 41,114 lbs. per sq. in., elongation 27% in 1 inch, sp. gr. 8.696. (W. J. Keep, *Trans. A. I. M. E.*, 1890.)

Composition of Various Grades of Rolled Brass, Etc.

| Trade Name. | Copper. | Zinc. | Tin. | Lead. | Nickel. |
|--------------------------------|---------|--------|-------|------------|---------|
| Common high brass..... | 61.5 | 38.5 | ... | ... | ... |
| Yellow metal..... | 60 | 40 | ... | ... | ... |
| Cartridge brass..... | 66 2/3 | 33 1/3 | ... | ... | ... |
| Low brass..... | 80 | 20 | ... | ... | ... |
| Clock brass..... | 60 | 40 | ... | 1 1/2 | ... |
| Drill rod..... | 60 | 40 | ... | 1 1/2 to 2 | ... |
| Spring brass..... | 66 2/3 | 33 1/3 | 1 1/2 | ... | ... |
| 18 per cent German silver..... | 61 1/2 | 20 1/2 | ... | ... | 18 |

The above table was furnished by the superintendent of a mill in Connecticut in 1894. He says: While each mill has its own proportions for various mixtures, depending upon the purposes for which the product is intended, the figures given are about the average standard. Thus, between cartridge brass with 33 1/3 per cent zinc and common high brass with 38 1/2 per cent zinc, there are any number of different mixtures known generally as "high brass," or specifically as "spinning brass," "drawing brass," etc., wherein the amount of zinc is dependent upon the amount of scrap used in the mixture, the degree of working to which the metal is to be subjected, etc.

Useful Alloys of Copper, Tin, and Zinc.

(Selected from numerous sources.)

| | Copper. | Tin. | Zinc. |
|---|--------------|-----------|-----------------------------|
| U. S. Navy Dept. journal boxes } and guide-gibs..... } | { 6 82.8 | 1 13.8 | 1/4 parts. 3.4 per cent. |
| Tobin bronze..... | 58.22 | 2.30 | 39.48 " " |
| Naval brass..... | 62 | 1 | 37 " " |
| Composition, U. S. Navy..... | 88 | 10 | 2 " " |
| Brass bearings (J. Rose)..... | { 64 87.7 | 8 11.0 | 1 parts. 1.3 per cent |
| Gun metal..... | 92.5 | 5 | 2.5 " " |
| " "..... | 91 | 7 | 2 " " |
| " "..... | 87.75 | 9.75 | 2.5 " " |
| " "..... | 85 | 5 | 10 " " |
| " "..... | 83 | 2 | 15 " " |
| Tough brass for engines..... | { 13 76.5 | 2 11.8 | 2 parts. 11.7 per cent. |
| Bronze for rod-boxes (Lafond)..... | 82 | 16 | 2 slightly malleable. |
| " " pieces subject to shock.. | 83 | 15 | 1.50 0.50 lead. |
| Red brass..... parts | 20 | 1 | 1 " " |
| " " per cent | 87 | 4.4 | 4.3 4.3 " |
| Bronze for pump casings (Lafond).. | 88 | 10 | 2 |
| " " eccentric straps..... | 84 | 14 | 2 |
| " " shrill whistles..... | 80 | 18 | 2.0 antimony. |
| " " low-toned whistles..... | 81 | 17 | 2.0 " |
| Art bronze, dull red fracture..... | 97 | 2 | 1 |
| Gold bronze..... | 89.5 | 2.1 | 5.6 2.8 lead. |
| Bearing metal..... | 89 | 8 | 3 |
| " "..... | 89 | 21 1/2 | 81/2 |
| " "..... | 86 | 14 | |
| " "..... | 85 1/4 | 12 3/4 | 2 |
| " "..... | 80 | 18 | 2 |
| " "..... | 79 | 18 | 21 1/2 1/2 lead. |
| " "..... | 74 | 91/2 | 91/2 7 lead. |
| English brass of A.D. 1504..... | 64 | 3 | 29 1/2 31 1/2 lead. |

"Steam-metal." Alloys of copper and zinc are unsuitable for steam valves and other like purposes, since their strength is greatly reduced at high temperatures, and they appear to undergo a deterioration by continued heating. Alloys of copper with from 10 to 12% of tin, when cast without oxidation are good steam metals, and a favorite alloy is what is known as "government mixture," 88 Cu, 10 Sn, 2 Zn. It has a tensile strength of about 33,000 lbs. per sq. in., when cold, and about 30,600 lbs. when heated to 407° F., corresponding to steam of 250 lbs. pressure.

Tobin Bronze. — This alloy is practically a sterro or delta metal with the addition of a small amount of lead, which tends to render copper softer and more ductile. (F. L. Garrison, *J. F. I.*, 1891.)

The following analyses of Tobin bronze were made by Dr. Chas. B. Dudley:

| | Pig Metal, per cent. | Test Bar (Rolled), per cent. |
|-------------|----------------------|------------------------------|
| Copper..... | 59.00 | 61.20 |
| Zinc..... | 38.40 | 37.14 |
| Tin..... | 2.16 | 0.90 |
| Iron..... | 0.11 | 0.18 |
| Lead..... | 0.31 | 0.35 |

Dr. Dudley writes, "We tested the test bars and found 78,500 tensile strength with 40½% elongation in two inches, and 15% in eight inches. This high tensile strength can only be obtained when the metal is manipulated. Such high results could hardly be expected with cast metal."

The original Tobin bronze in 1875, as described by Thurston, *Trans. A. S. C. E.*, 1881, had copper 58.22, tin 2.30, zinc 39.48. As cast it had a tenacity of 66,000 lbs. per sq. in., and as rolled 79,000 lbs.; cold rolled it gave 104,000 lbs.

A circular of Ansonia Brass & Copper Co. gives the following: — The tensile strength of six Tobin bronze one-inch round rolled rods, turned down to a diameter of 5/8 of an inch, tested by Fairbanks, averaged 79,600 lbs. per sq. in., and the elastic limit obtained on three specimens averaged 54,257 lbs. per sq. in.

At a cherry-red heat Tobin bronze can be forged and stamped as readily as steel. Bolts and nuts can be forged from it, either by hand or by machinery. Its great tensile strength, and resistance to the corrosive action of sea-water, render it a most suitable metal for condenser plates, steam-launch shafting, ship sheathing and fastenings, nails, hull plates for steam yachts, torpedo and life boats, and ship deck fittings.

The Navy Department has specified its use for certain purposes in the machinery of the new cruisers. Its specific gravity is 8.071. The weight of a cubic inch is 0.291 lb.

Special Alloys. (*Engineering*, March 24, 1893.)

JAPANESE ALLOYS for art work:

| | Copper. | Silver. | Gold. | Lead. | Zinc. | Iron. |
|-----------------|---------|---------|---------|-------|--------|--------|
| Shaku-do..... | 94.50 | 1.55 | 3.73 | 0.11 | trace. | trace. |
| Shibu-ichi..... | 67.31 | 32.07 | traces. | 0.52 | | |

GILBERT'S ALLOY for *cera-perduta* process, for casting in plaster-of-paris.

Copper 91.4 Tin 5.7 Lead 2.9 Very fusible.

COPPER-ZINC-IRON ALLOYS.

(F. L. Garrison, *Jour. Frank. Inst.*, June and July, 1891.)

Delta Metal. — This alloy, which was formerly known as *sterro-metal*, is composed of about 60 copper, from 34 to 44 zinc, 2 to 4 iron, and 1 to 2 tin.

The peculiarity of all these alloys is the content of iron, which appears to have the property of increasing their strength to an unusual degree. In making delta metal the iron is previously alloyed with zinc in known and definite proportions. When ordinary wrought-iron is introduced into molten zinc, the latter readily dissolves or absorbs the former, and will take it up to the extent of about 5% or more. By adding the zinc-iron alloy thus obtained to the requisite amount of copper, it is possible to introduce any definite quantity of iron up to 5% into the copper alloy. Garrison gives the following as the range of composition of copper-zinc-iron, and copper-zinc-tin-iron alloys:

| I. | | II. | |
|-------------|------------|-------------|-----------|
| | Per cent. | | Per cent. |
| Iron..... | 0.1 to 5 | Iron..... | 0.1 to 5 |
| Copper..... | 50 to 65 | Tin..... | 0.1 to 10 |
| Zinc..... | 49.9 to 30 | Zinc..... | 1.8 to 45 |
| | | Copper..... | 98 to 40 |

The advantages claimed for delta metal are great strength and toughness. It produces sound castings of close grain. It can be rolled and forged hot, and can stand a certain amount of drawing and hammering when cold. It takes a high polish, and when exposed to the atmosphere tarnishes less than brass.

When cast in sand delta metal has a tensile strength of about 45,000 pounds per square inch, and about 10% elongation; when rolled, tensile strength of 60,000 to 75,000 pounds per square inch, elongation from 9% to 17% on bars 1.128 inch in diameter and 1 inch area.

Wallace gives the ultimate tensile strength 33,600 to 51,520 pounds per square inch, with from 10% to 20% elongation.

Delta metal can be forged, stamped and rolled hot. It must be forged at a dark cherry-red heat, and care taken to avoid striking when at a black heat.

According to Lloyd's Proving House tests, made at Cardiff, December 20, 1887, a half-inch delta metal-rolled bar gave a tensile strength of 88,400 pounds per square inch, with an elongation of 30% in three inches.

ALLOYS OF COPPER, TIN, AND LEAD.

G. H. Clamer, in *Castings*, July, 1908, describes some experiments on the use of lead in copper alloys. A copper and lead alloy does not make what would be called good castings; by the introduction of tin a more homogeneous product is secured. By the addition of nickel it was found that more than 15% of lead could be used, while maintaining tin at 8 to 10%, and also that the tin could be dispensed with. A good alloy for bearings was then made without nickel, containing Cu 65, Sn 5, Pb 30. This alloy is largely sold under the name of "plastic bronze." If the matrix of tin and copper were so proportioned that the tin remained below 9% then more than 20% of lead could be added with satisfactory results. As the tin is decreased more lead may be added. (See Bearing Metal Alloys, below.)

The Influence of Lead on Brass. — E. S. Sperry, *Trans. A.I.M.E.*, 1897. As a rule, the lower the brass (that is, the lower in zinc) the more difficult it is to cut. If the alloy is made from pure copper and zinc, the chips are long and tenacious, and a slow speed must be employed in cutting. For some classes of work, such as spinning or cartridge brass, these qualities are essential, but for others, such as clock brass or screw rod, they are almost prohibitory. To make an alloy which will cut easily, giving short chips, the best method is the addition of a small percentage of lead. Experiments were made on alloys con-

taining different percentages of lead. The following is a condensed statement of the chief results:

Cu, 60; Zn, 30; Pb, 10. Difficult to obtain a homogeneous alloy. Cracked badly on rolling.

Cu, 60; Zn, 35; Pb, 5. Good cutting qualities but cracked on rolling.

Cu, 60; Zn, 37.5; Pb, 2.5. Cutting qualities excellent, but could only be hot-rolled or forged with difficulty.

Cu, 60; Zn, 38.75; Pb, 1.25. Cutting qualities inferior to those of the alloy containing 2.5% of lead, but superior to those of pure brass.

Cu, 60; Zn, 40. Perfectly homogeneous. Rolls easily at a cherry red heat, and cracks but slightly in cold rolling. Chips long and tenacious, necessitating a slow speed in cutting.

Tensile tests of these alloys gave the following results:

| Copper, %..... | 60.0 | | | 60.0 | | | 60.0 | | | 60.0 | | |
|------------------------|-------|----|-----|------|----|----|------|----|----|------|----|----|
| Zinc, %..... | 40.0 | | | 37.5 | | | 35.0 | | | 30.0 | | |
| Lead, %..... | None. | | | 2.5 | | | 5.0 | | | 10.0 | | |
| | C | A | H | C | A | H | C | A | H | C | A | H |
| T. S. per sq. in.* .. | 16 | 60 | 107 | 39 | 51 | 88 | 33 | 42 | 61 | 36 | 35 | 63 |
| Elonga. in 1 in., % .. | 48 | 51 | 1 | 28 | 27 | 0 | 28 | 26 | 1 | 36 | 20 | 2 |
| Elonga. in 8 in., % .. | 27 | 33 | 0 | 27 | 23 | 0 | 27 | 22 | 0 | 30 | 16 | 3 |
| Red. of area, %..... | 61 | 44 | 13 | 30 | 33 | 0 | 26 | 33 | 0 | 29 | 25 | 4 |
| P. R..... | 92% | | | 65% | | | 61% | | | 38% | | |

* Thousands of pounds. C, casting; A, annealed sheet; H, hard rolled sheet; P. R., possible reduction in rolling.

The use of tin, even in small amounts, hardens and increases the tensile strength of brass, which is detrimental to free turning. Mr. Sperry gives analyses of several brasses which have given excellent results in turning, all included within the following range: Cu, 60 to 66%, Zn, 38 to 32%, Pb, 1.5 to 2.5%. For cartridge-brass sheet, anything over 0.10% of lead increases the liability of cracking in drawing.

PHOSPHOR-BRONZE AND OTHER SPECIAL BRONZES.

Phosphor-bronze. — In the year 1868, Montefiore & Kunzel of Liege, Belgium, found by adding small proportions of phosphorus or "phosphoret of tin or copper" to copper that the oxides of that metal, nearly always present as an impurity, more or less, were deoxidized and the copper much improved in strength and ductility, the grain of the fracture became finer, the color brighter, and a greater fluidity was attained.

Three samples of phosphor-bronze tested by Kirkaldy gave:

| | | | |
|--------------------------------------|--------|--------|--------|
| Elastic limit, lbs. per sq. in. | 23,800 | 24,700 | 16,100 |
| Tensile strength, lbs. per sq. in.. | 52,625 | 46,100 | 44,448 |
| Elongation, per cent. | 8.40 | 1.50 | 33.40 |

The strength of phosphor-bronze varies like that of ordinary bronze according to the percentages of copper, tin, zinc, lead, etc., in the alloy.

Phosphor-bronze Rod. — Torsion tests of 20 samples, 1/4 in. diam. Apparent outside fiber stress, 77,500 to 86,700 lbs. per sq. in.; average number of turns per inch of length, 0.76 to 1.50. — *Tech. Quar.*, vol. xii, Sept., 1899.

Penn. R. R. Co.'s Specifications for Phosphor-bronze (1902). — The metal desired is a homogeneous alloy of copper, 79.70; tin, 10.00; lead, 9.50; phosphorus, 0.80. Lots will not be accepted if samples do not show tin, between 9 and 11%; lead, between 8 and 11%; phosphorus, between 0.7 and 1%; nor if the metal contains a sum total of other substances than copper, tin, lead, and phosphorus in greater quantity than 0.50 per cent. (See also p. 381.)

Deoxidized Bronze. — This alloy resembles phosphor bronze somewhat in composition and also delta metal, in containing zinc and iron. The following analysis gives its average composition: Cu, 82.67; Sn, 12.40; Zn, 3.23; Pb, 2.14; Fe, 0.10; Ag, 0.07; P, 0.005.

Comparison of Copper, Silicon-bronze, and Phosphor-bronze Wires. (*Engineering*, Nov. 23, 1883.)

| Description of Wire. | Tensile Strength. | Relative Conductivity. |
|----------------------------------|-------------------------|------------------------|
| Pure copper | 39,827 lbs. per sq. in. | 100 per cent. |
| Silicon bronze (telegraph)..... | 41,696 " " " " | 96 " " |
| " " (telephone)..... | 108,080 " " " " | 34 " " |
| Phosphor bronze (telephone)..... | 102,390 " " " " | 26 " " |

Silicon Bronze. (*Aluminum World*, May, 1897.)

The most useful of the silicon bronzes are the 3% (97% copper, 3% silicon) and the 5% (95% copper, 5% silicon), although the hardness and strength of the alloy can be increased or decreased at will by increasing or decreasing silicon. A 3% silicon bronze has a tensile strength, in a casting, of about 55,000 lbs. per sq. in., and from 50% to 60% elongation. The 5% bronze has a tensile strength of about 75,000 lbs. and about 8% elongation. More than 5% or 5 1/2% of silicon in copper makes a brittle alloy. In using silicon, either as a flux or for making silicon bronze, the rich alloy of silicon and copper which is now on the market should be used. It should be free from iron and other metals if the best results are to be obtained. Ferro-silicon is not suitable for use in copper or bronze mixtures.

Copper and Vanadium Alloys. The Vanadium Sales Co. of America reports (1908) that the addition of vanadium to copper has given a tensile strength of 83,000 lbs. per sq. in.; with an elongation of over 60%.

ALLOYS FOR CASTING UNDER PRESSURE IN METAL MOLDS. E. L. Lake, *Am. Mach.*, Feb. 13, 1908.

| No. | Tin. | Copper. | Aluminum. | Zinc. | Lead. | Antimony. | Iron |
|-----|-------|---------|-----------|-------|-------|-----------|-------|
| 1 | 14.75 | 5.25 | 6.25 | 73.75 | | | |
| 2 | 19 | 5 | 1. | 72.7 | 2 | 0.3 | |
| 3 | 12 | 10.6 | 3.4 | 73.8 | | | 0.2 |
| 4 | 30.8 | 20.4 | 2.6 | 46.2 | | | |

Nos. 1 and 2 suitable for ordinary work, such as could be performed by average brass castings. No. 3 and 4 are harder.

ALUMINUM ALLOYS.

The useful alloys of aluminum so far found have been chiefly in two groups, the one of aluminum with not more than 35% of other metals, and the other of metals containing not over 15% of aluminum; in the one case the metals impart hardness and other useful qualities to the aluminum, and in the other the aluminum gives useful qualities to the metal with which it is alloyed.

Aluminum-Copper Alloys. — The useful aluminum-copper alloys can be divided into two classes, — the one containing less than 11% of aluminum, and the other containing less than 15% of copper. The first class is best known as Aluminum Bronze.

Aluminum Bronze. (Cowles Electric Smelting and Al. Co.'s circular.) The standard A No. 2 grade of aluminum bronze, containing 10% of aluminum and 90% of copper, has many remarkable characteristics which distinguish it from all other metals.

The tenacity of castings of A No. 2 grade metal varies between 75,000 and 90,000 lbs. to the square inch, with from 4% to 14% elongation.

Increasing the proportion of aluminum in bronze beyond 11% produces a brittle alloy; therefore nothing higher than the A No. 1, which contains 11%, is made.

The B, C, D, and E grades, containing 7 1/2%, 5%, 2 1/2%, and 1 1/4% of aluminum, respectively, decrease in tenacity in the order named, that of the former being about 65,000 pounds, while the latter is 25,000 pounds. While there is also a proportionate decrease in transverse and torsional strengths, elastic limit, and resistance to compression as the percentage of aluminum is lowered and that of copper raised, the ductility on the other hand increases in the same proportion. The specific gravity of the A No. 1 grade is 7.56.

Bell Bros., Newcastle, gave the specific gravity of the aluminum bronzes as follows:

3%, 8.691; 4%, 8.621; 5%, 8.369; 10%, 7.689.

The Thermit Process. — When finely divided aluminum is mixed with a metallic oxide and ignited the aluminum burns with great rapidity and intense heat, the chemical reaction being $\text{Al} + \text{Fe}_2\text{O}_3 = \text{Al}_2\text{O}_3 + \text{Fe}$. The heat thus generated may be used to fuse or weld iron and other metals. See the Thermit Process, under Welding of Steel, page 463.

Tests of Aluminum Bronzes.

(John H. J. Dagger, British Association, 1889.)

| Per cent of Aluminum. | Tensile Strength. | | Elonga- tion, per cent. | Specific Gravity. |
|-----------------------------|--------------------------|----------------------------|-------------------------------|----------------------|
| | Tons per square inch. | Pounds per square inch. | | |
| 11..... | 40 to 45 | 89,600 to 100,800 | 8 | 7.23 |
| 10..... | 33 " 40 | 73,920 " 89,600 | 14 | 7.69 |
| 7 1/2..... | 25 " 30 | 56,000 " 67,200 | 40 | 8.00 |
| 5-5 1/2..... | 15 " 18 | 33,600 " 40,320 | 40 | 8.37 |
| 2 1/2..... | 13 " 15 | 29,120 " 35,600 | 50 | 8.69 |
| 1 1/4..... | 11 " 13 | 24,640 " 29,120 | 55 | |

Both physical and chemical tests made of samples cut from various sections of 2 1/2%, 5%, 7 1/2%, or 10% aluminized copper castings tend to prove that the aluminum unites itself with each particle of copper with uniform proportion in each case, so that we have a product that is free from liquation and highly homogeneous. (R. C. Cole, *Iron Age*, Jan. 16, 1890.)

Casting. — The melting point of aluminum bronze varies slightly with the amount of aluminum contained, the higher grades melting at a somewhat lower temperature than the lower grades. The A No. 1 grades melt at about 1700° F., a little higher than ordinary bronze or brass.

Aluminum bronze shrinks more than ordinary brass. As the metal solidifies rapidly it is necessary to pour it quickly and to make the feeders amply large, so that there will be no "freezing" in them before the casting is properly fed. Baked-sand molds are preferable to green sand, except for small castings, and when fine skin colors are desired in the castings. (Thos. D. West, *Trans. A. S. M. E.*, 1886, vol. viii.)

All grades of aluminum bronze can be rolled, swedged, spun, or drawn cold except A 1 and A 2. They can all be worked at a bright red heat.

In rolling, swedging, or spinning cold, it should be annealed very often, and at a brighter red heat than is used for annealing brass.

Seamless Tubes. — Leonard Waldo, *Trans. A. S. M. E.*, vol. xviii, describes the manufacture of aluminum bronze seamless tubing. Many difficulties were met in all stages of the process. A cold drawn bar, 1.49 ins. outside diameter, 0.05 in. thick, showed a yield point of 68,700, and a tensile strength of 96,000 lbs. per sq. in. with an elongation of 4.9 in 10 in.; heated to bright red and plunged in water, the Y. P. reduced to 24,200 and the T. S. to 47,600 lbs. per sq. in., and the elongation in 10 ins. increased to 64.9 %.

Brazing. — Aluminum bronze will braze as well as any other metal, using one-quarter brass solder (zinc 500, copper 500) and three-quarters borax, or, better, three-quarters cryolite.

Soldering. — To solder aluminum bronze with ordinary soft (pewter) solder: Cleanse well the parts to be joined free from grease and dirt. Then place the parts to be soldered in a strong solution of sulphate of copper and place in the bath a rod of soft iron touching the parts to be joined. After a while a coppery-like surface will be seen on the metal. Remove from bath, rinse quite clean, and brighten the surfaces. These surfaces can then be tinned by using a fluid consisting of zinc dissolved in hydrochloric acid, in the ordinary way, with common soft solder.

Mierzinski recommends ordinary hard solder, and says that Hulot uses an alloy of the usual half-and-half lead-tin solder, with 12.5%, 25% or 50% of zinc amalgam.

Aluminum Brass. (E. H. Cowles, *Trans. A. I. M. E.*, vol. xviii.) — Cowles aluminum brass is made by fusing together equal weights of A 1 aluminum bronze, copper, and zinc. The copper and bronze are first thoroughly melted and mixed, and the zinc is finally added. The material is left in the furnace until small test-bars are taken from it and broken. When these bars show a tensile strength of 80,000 pounds or over, with 2 or 3 per cent ductility, the metal is ready to be poured. Tests of this brass, on small bars, have at times shown as high as 100,000 pounds tensile strength.

The screw of the United States gunboat *Petrel* is cast from this brass mixed with a trifle less zinc in order to increase its ductility.

Tests of Aluminum Brass.

(Cowles E. S. & Al. Co.)

| Specimen (Castings) | Diameter of Piece, Inch. | Area, sq. in. | Tensile Strength, lbs. per sq. in. | Elastic Limit, lbs. per sq. in. | Elongation, per ct. | Remarks. |
|---|--------------------------|---------------|------------------------------------|---------------------------------|---------------------|---|
| 15% A grade Bronze } 17% Zinc..... } 68% Copper..... } | 0.465 | 0.1698 | 41,225 | 17,668 | 41 1/2 | These test pieces were all 6" long between the shoulders. |
| 1 part A Bronze ... } 1 part Zinc } 1 part Copper } | 0.465 | 0.1698 | 78,327 | | 21 1/2 | |
| 1 part A Bronze ... } 1 part Zinc } 1 part Copper } | 0.460 | 0.1661 | 72,246 | | 21 1/2 | |

The first brass on the above list is an extremely tough metal with low elastic limit, made purposely so as to "upset" easily. The other, which is called Aluminum brass No. 2, is very hard.

We have not in this country or in England any official standard by which to judge of the physical characteristics of cast metals. There are two conditions that are absolutely necessary to be known before we can make a fair comparison of different materials; namely, whether the casting was made in dry or green sand or in a chill, and whether it was attached to a larger casting or cast by itself. It has also been found that chill-castings give higher results than sand-castings, and that bars cast by themselves purposely for testing almost invariably run higher than test-bars attached to castings. It is also a fact that bars cut out from castings are generally weaker than bars cast alone. (E. H. Cowles.)

Caution as to Reported Strength of Alloys. — The same variation in strength which has been found in tests of gun-metal (copper and tin) noted above, must be expected in tests of aluminum bronze and in fact of all alloys. They are exceedingly subject to variation in density and in grain, caused by differences in method of moulding and casting, temperature of pouring, size and shape of casting, depth of "sinking head," etc.

Aluminum Hardened by Addition of Copper.Tests of rolled sheets 0.04 inch thick. (*The Engineer*, Jan. 2, 1891.)

| Al. Per cent. | Cu. Per cent. | Sp. Gr. Calculated. | Sp. Gr. Determined. | Tensile Strength lbs. per sq. in. |
|------------------|------------------|------------------------|------------------------|---|
| 100 | | | 2.67 | 26,535 |
| 98 | 2 | 2.78 | 2.71 | 43,563 |
| 96 | 4 | 2.90 | 2.77 | 44,130 |
| 94 | 6 | 3.02 | 2.82 | 54,773 |
| 92 | 8 | 3.14 | 2.85 | 50,374 |

Tests of Aluminum Alloys.(Engineer Harris, U. S. N., *Trans. A. I. M. E.*, vol. xviii.)

| Composition. | | | | | Tensile Strength per sq. in., lbs. | Elastic Limit, lbs. per sq. in. | Elonga- tion, per ct. | Reduction of Area, per ct. |
|--------------|----------------|----------|--------|-------|---|--|-----------------------------|-------------------------------------|
| Copper. | Alumi- num. | Silicon. | Zinc. | Iron. | | | | |
| 91.50% | 6.56% | 1.7% | | 0.25% | 60,700 | 18,000 | 23.2 | 30.7 |
| 88.50 | 9.33 | 1.66 | | 0.50 | 66,000 | 27,000 | 3.8 | 7.8 |
| 91.50 | 6.50 | 1.75 | | 0.25 | 67,600 | 24,000 | 13 | 21.62 |
| 90.00 | 9.00 | 1.00 | | | 72,830 | 33,000 | 2.40 | 5.78 |
| 63.00 | 3.33 | 0.33 | 33.33% | | 82,200 | 60,000 | 2.33 | 9.88 |
| 63.00 | 3.33 | 0.33 | 33.33 | | 70,400 | 55,000 | 0.4 | 4.33 |
| 91.50 | 6.50 | 1.75 | | 0.25 | 59,100 | 19,000 | 15.1 | 23.59 |
| 93.00 | 6.50 | 0.50 | | | 53,000 | 19,000 | 6.2 | 15.5 |
| 88.50 | 9.33 | 1.66 | | 0.50 | 69,930 | 33,000 | 1.33 | 3.30 |
| 92.00 | 6.50 | 0.50 | | | 46,530 | 17,000 | 7.8 | 19.19 |

For comparison with the above 6 tests of "Navy Yard Bronze," Cu 88, Sn 10, Zn 2, are given in which the T. S. ranges from 18,000 to 24,590, E. L. from 10,000 to 13,000, El. 2.5 to 5.8%, Red. 4.7 to 10.89.

Alloys of Aluminum, Silicon and Iron.

M. and E. Bernard have succeeded in obtaining through electrolysis, by treating directly and without previous purification, the aluminum earths (red and white bauxites), the following:

Alloys such as ferro-aluminum, ferro-silicon-aluminum and silicon-aluminum, where the proportion of silicon may exceed 10%, which are employed in the metallurgy of iron for refining steel and cast-iron.

Also silicon-aluminum, where the proportion of silicon does not exceed 10%, which may be employed in mechanical constructions in a rolled or hammered condition, in place of steel, on account of their great resistance, especially where the lightness of the piece in construction constitutes one of the main conditions of success.

The following analyses are given:

1. Alloys applied to the metallurgy of iron, the refining of steel and cast iron: No. 1. Al, 70%; Fe, 25%; Si, 5%. No. 2. Al, 70; Fe, 20; Si, 10. No. 3. Al, 70; Fe, 15; Si, 15. No. 4. Al, 70; Fe, 10; Si, 20. No. 5. Al, 70; Fe, 10; Si, 10; Mn, 10. No. 6. Al, 70; Fe, trace; Si, 20; Mn, 10.

2. Mechanical alloys: No. 1. Al, 92; Si, 6.75; Fe, 1.25. No. 2. Al, 90; Si, 9.25; Fe, 0.75. No. 3. Al, 90; Si, 10; Fe, trace. The best results were with alloys where the proportion of iron was very low, and the proportion of silicon in the neighborhood of 10%. Above that proportion the alloy becomes crystalline and can no longer be employed. The density of the alloys of silicon is approximately the same as that of aluminum. — *La Metallurgie*, 1892.

Tungsten and Aluminum. — Mr. Leinhardt Mannesmann says that the addition of a little tungsten to pure aluminum or its alloys communicates a remarkable resistance to the action of cold and hot water, salt water and other reagents. When the proportion of tungsten is sufficient the alloys offer great resistance to tensile strains. An alloy of aluminum and tungsten called partinium, from the name of its inventor, M. Partin, has been used in France since 1898 for motor-car bodies. Its properties are stated as follows: Cast, sp. gr., 2.86; T. S., 17,000 to 24,000; elong., 12 to 6%. Rolled, sp. gr., 3.09; T. S., 45,500 to 53,600; elong., 8 to 6%.

Aluminum, Copper, and Tin. — Prof. R. C. Carpenter, *Trans. A. S. M. E.*, vol. xix., finds the following alloys of maximum strength in a series in which two of the three metals are in equal proportions:

Al, 85; Cu, 7.5; Sn, 7.5; tensile strength, 30,000 lbs. per sq. in.; elongation in 6 in., 4%; sp. gr., 3.02. Al, 6.25; Cu, 87.5; Sn, 6.25; T. S., 63,000; EL., 3.8; sp. gr., 7.35. Al, 5; Cu, 5; Sn, 90; T. S., 11,000; EL., 10.1; sp. gr., 6.82.

From 85 to 95% Cu the bars have considerable strength, are close grained and of a golden color. Between 78 and 80% the color changes to silver white and the bars become brittle. From 78 to 20% Cu the alloys are very hard and brittle, and worthless for practical purposes. Aluminum is strengthened by the addition of equal parts of copper and tin up to 7.5% of each, beyond which the strength decreases. All the alloys that contain between 20 and 60% of either one of the three metals are very weak.

Aluminum and Zinc. — Like the copper alloys, the zinc alloys can be divided into two classes, (1) those containing a relatively small amount of aluminum, and (2) those containing less than 55% of zinc. The first class is used largely in galvanizing baths to produce greater fluidity, while the second class embraces the zinc casting alloys. Prof. Carpenter finds that the strongest alloy of these metals consists of two parts of aluminum and one part of zinc. Its tensile strength is 24,000 to 26,000 lbs. per sq. in.; has but little ductility, is readily cut with machine-tools, and is a good substitute for hard cast brass.

Aluminum and Tin. — M. Bourbouze has compounded an alloy of aluminum and tin, by fusing together 100 parts of the former with 10 parts of the latter. This alloy is paler than aluminum, and has a specific gravity of 2.85. The alloy is not as easily attacked by several reagents as aluminum is, and it can also be worked more readily. Another advantage is that it can be soldered as easily as bronze, without further preliminary preparations. Prof. Carpenter found that aluminum-tin alloys with from 2 to 10% Al are as a rule weaker than pure aluminum and of little practical value.

Aluminum with Nickel, German Silver or Titanium. — J. W. Richards, *Jour. Frank. Inst.*, 1895, says that an addition of 5% of nickel or German silver, or 2% of titanium to aluminum increases the tensile strength to 20,000–30,000 lbs. per sq. in. in castings and to 40,000–50,000 lbs. in sheet. For purposes where the requirements are fine color, strength, hardness and springiness the German-silver alloy is recommended.

Aluminum-Antimony Alloys. — Dr. C. R. Alder Wright describes some aluminum-antimony alloys in a communication read before the Society of Chemical Industry. The results of his researches do not disclose the existence of a commercially useful alloy of these two metals, and have greater scientific than practical interest. A remarkable point is that the alloy with the chemical composition Al Sb has a higher melting-point than either aluminum or antimony alone, and that when aluminum is added to pure antimony the melting-point goes up from that of antimony (450° C.) to a certain temperature rather above that of silver (1000° C.).

Aluminum and Cast Iron. — Aluminum alloys readily with cast iron, up to 14 to 15% Al, but the metal decreases in strength as the Al is increased. Mixtures with greater percentages of Al are granular, and have practically no coherence. — *Trans. A. I. M. E.*, vol. xviii., *A. S. M. E.*, vol. xix.

Other Aluminum Alloys. — Al 75.7, Cu 3, Zn 20, Mn 1.3 is an excellent casting metal, having a tensile strength of over 35,000 lbs. per sq. in., and a sp. gr. slightly above 3. It has very little ductility.

Al 96.5, Cu 2, and chromium 1.5 is a little heavier than pure aluminum and has a tensile strength of 26,300 lbs. per sq. in. — *A. S. M. E.*, vol. xix.

Aluminum and Magnesium. — Magnalium. — An alloy containing 90 to 98% of aluminum, the balance being mainly magnesium, has been patented under the trade name of "magnalium." Its specific gravity is only 2.5; it is whiter, harder and stronger than aluminum, and can be forged, rolled, drawn, machined and filed. It takes a high polish and resists oxidation better than any other light metals or alloys. The tensile strength of cast magnalium, class X, is reported at 18,400 to 21,300 lbs. per sq. in., with a reduction of area of 3.75%; hard rolled plates, class Z, 52,200 lbs. per sq. in., with 3.7% reduction; annealed plates, 42,200 lbs. per sq. in., 17.8% reduction. Made by the Magnalium Syndicate of Berlin. The price is said to be about twice that of aluminum. — (*Mach'y*, July, 1908.)

Prof. Carpenter (*A. S. M. E.*, vol. xix) found that additions of Mn increased the strength of Al up to 10% Mn. Larger additions made brittle alloys.

Resistance of Aluminum Alloys to Corrosion. — J. W. Richards, *Jour. Frank. Inst.*, 1895, gives the following table showing the relative resistance to corrosion of aluminum (99% pure) and alloys of aluminum with different metals, when immersed in the liquids named. The figures are losses per day in milligrams per square centimeter of surface:

| | 3% Caustic potash. Cold. | 3% Hydro- chloric Acid. Cold. | Strong Nitric Acid. Cold. | Strong Salt Solu- tion. 150° F. | Strong Acetic Acid. 140° F. | Car- bonic Acid. Water. 77° F. |
|--------------------------------|-----------------------------------|---|------------------------------------|---|--------------------------------------|--|
| 3 per cent copper | 265.0 | 53.3 | 36.1 | 0.1 | 0.4 | 0.0 |
| 3 per cent German silver | 1534.4 | 130.6 | 97.7 | 0.05 | 0.6 | 0.01 |
| 3 per cent nickel | 580.3 | 180.0 | 83.0 | 0.13 | 0.75 | 0.04 |
| 2 per cent titanium | 73.4 | 4.3 | 18.6 | 0.06 | 0.20 | 0.0 |
| 99 per cent aluminum | 34.6 | 5.8 | 9.6 | 0.04 | 0.15 | 0.01 |

Aluminum Alloys used in Automobile Construction (*Am. Mach.*, Aug. 22, 1907.)

- (1) Al 2, Zn, 1, T.S. 35,000; Sp. gr. 3.1
 (2) Al 92, Cu, 8, T.S. 18,000; Sp. gr. 2.84 Ni, trace
 (3) Al 83, Zn, 15, Cu, 2, T.S. 23,000; Sp. gr. 3.1

(1) Unsatisfactory on account of failures under repeated vibration.
 (2) Generally used. Resists vibrations well. (3) Used to some extent. Many motor-car makers decline to use it because of uncertainty of its behavior under vibration.

ALLOYS OF MANGANESE AND COPPER.

Various Manganese Alloys. — E. H. Cowles, in *Trans. A. I. M. E.* vol. xviii, p. 495, states that as the result of numerous experiments on mixtures of the several metals, copper, zinc, tin, lead, aluminum, iron, and manganese, and the metalloid silicon, and experiments upon the same in ascertaining tensile strength, ductility, color, etc., the most important determinations appear to be about as follows:

1. That pure metallic manganese exerts a bleaching effect upon copper more radical in its action even than nickel. In other words, it was found that 18½% of manganese present in copper produces as white a color in the resulting alloy as 25% of nickel would do, this being the amount of each required to remove the last trace of red.

2. That upwards of 20% or 25% of manganese may be added to copper without reducing its ductility, although doubling its tensile strength and changing its color.

3. That manganese, copper, and zinc when melted together and poured into molds behave very much like the most "yeasty" German

silver, producing an ingot which is a mass of blow-holes, and which swells up above the mold before cooling.

4. That the alloy of manganese and copper by itself is very easily oxidized.

5. That the addition of 1.25% of aluminum to a manganese-copper alloy converts it from one of the most refractory of metals in the casting process into a metal of superior casting qualities, and the non-corrodibility of which is in many instances greater than that of either German or nickel silver.

A "silver-bronze" alloy especially designed for rods, sheets, and wire has the following composition: Mn, 18; Al, 1.20; Si, 0.5; Zn, 13; and Cu, 67.5%. It has a tensile strength of about 57,000 lbs. on small bars, and 20% elongation. It has been rolled into thin plate and drawn into wire 0.008 inch in diameter. A test of the electrical conductivity of this wire (of size No. 32) shows its resistance to be 41.44 times that of pure copper. This is far lower conductivity than that of German silver.

Manganese Bronze. (F. L. Garrison, *Jour. F. I.*, 1891.)—This alloy has been used extensively for casting propeller-blades. Tests of some made by B. H. Cramp & Co., of Philadelphia, gave an average elastic limit of 30,000 lbs. per sq. in., tensile strength of about 60,000 lbs. per sq. in. with an elongation of 8% to 10% in sand castings. When rolled, the E. L. is about 80,000 lbs. per sq. in., tensile strength 95,000 to 106,000 lbs. per sq. in., with an elongation of 12% to 15%.

Compression tests made at United States Navy Department from the metal in the pouring-gate of propeller-hub of U. S. S. Maine gave in two tests a crushing stress of 126,450 and 135,750 lb. per sq. in. The specimens were 1 inch high by 0.7 × 0.7 inch in cross-section = 0.49 square inch. Both specimens gave way by shearing, on a plane making an angle of nearly 45° with the direction of stress.

A test on a specimen 1 × 1 × 1 inch was made from a piece of the same pouring-gate. Under stress of 150,000 pounds it was flattened to 0.72 inch high by about 1 1/4 × 1 1/4 inches, but without rupture or any sign of distress.

One of the great objections to the use of manganese bronze, or in fact any alloy except iron or steel, for the propellers of iron ships is on account of the galvanic action set up between the propeller and the stern-posts. This difficulty has in great measure been overcome by putting strips of rolled zinc around the propeller apertures in the stern-frames.

The following analysis of Parsons' manganese bronze No. 2 was made from a chip from the propeller of Mr. W. K. Vanderbilt's yacht *Alva*.

Cu, 88.64; Zn, 1.57; Sn, 8.70; Fe, 0.72; Pb, 0.30; P, trace.

It will be observed there is no manganese present and the amount of zinc is very small.

E. H. Cowles, *Trans. A. I. M. E.*, vol. xviii, says: Manganese bronze, so called, is in reality a manganese brass, for zinc instead of tin is the chief element added to the copper. Mr. P. M. Parsons, the proprietor of this brand of metal, has claimed for it a tensile strength of from 24 to 28 tons per sq. in. in small bars when cast in sand.

E. S. Sperry, *Am. Mach.*, Feb. 1, 1906, gives the following analyses of manganese bronze:

| | Cu. | Zn. | Fe. | Sn. | Al. | Mn. | Pb. |
|------------|-------|-------|------|------|-------|------|-------|
| No. 1..... | 60.27 | 37.52 | 1.41 | 0.75 | | 0.01 | 0.0. |
| " 2..... | 56.11 | 41.34 | 1.30 | 0.75 | 0.47 | 0.01 | 0.02 |
| " 3..... | 60.00 | 38.00 | 1.25 | 0.65 | | 0.10 | |
| " 4..... | 56.00 | 42.38 | 1.25 | 0.75 | 0.50 | 0.12 | |

No. 1 is Parsons' alloy for sheet, No. 2 for sand casting. No. 3 is Mr. Sperry's formula for sheet, and No. 4 his formula for sand castings. The mixture for No. 3, allowing for volatilization of some zinc is: copper, 60 lbs.; zinc, 39 lbs.; "steel alloy," 2 lbs. That for No. 4 is: copper, 56 lbs.; zinc, 43 lbs.; "steel alloy," 2 lbs.; aluminum, 0.5 lb. The steel alloy is made by melting wrought iron, 18 lbs.; ferro-manganese (80 Fe, 20 Mn), 4 lbs.; tin, 10 lbs. The iron and ferro-manganese are first melted and then the tin is added. In making the bronzes about 15 lbs. of the copper is first melted under charcoal, the steel alloy is

added, melted and stirred, then the aluminum is added, melted and stirred, then the rest of the copper is added, and finally the zinc. The only function of the manganese is to act as a carrier to the iron, which is difficult to alloy with copper without such carrier. The iron is needed to give a high elastic limit. Green sand castings of No. 4 frequently give results as high as the following: T. S., 70,000; E. L., 30,000 lbs. per sq. in.; elongation in 6 ins., 18%; reduction of area, 26%.

Magnetic Alloys of Non-Magnetic Metals. (*El. World*, April 15, 1905; *Electrot.-Zeit.* Mar. 2, 1905.)—Dr. Heusler has discovered that alloys of manganese, aluminum, and copper are strongly magnetic. The best results have been obtained when the Mn and Al are in the proportions of their respective atomic weights, 55 and 27.1. Two such alloys are described (1) Mn, 26.8; Al, 13.2; Cu, 60. (2) Mn, 20.1; Al, 9.9; Cu, 70, with 1% Pb added. The first was too brittle to be workable. The second was machined without difficulty. These alloys have as yet no commercial importance, as they are far inferior magnetically (at most 1 to 4) to iron.

GERMAN-SILVER AND OTHER NICKEL ALLOYS.

German Silver.—The composition of German silver is a very uncertain thing and depends largely on the honesty of the manufacturer and the price the purchaser is willing to pay. It is composed of copper, zinc, and nickel in varying proportions. The best varieties contain from 18% to 25% of nickel and from 20% to 30% of zinc, the remainder being copper. The more expensive nickel silver contains from 25% to 33% of nickel and from 75% to 66% of copper. The nickel is used as a whitening element; it also strengthens the alloy and renders it harder and more non-corrodible than the brass made without it, of copper and zinc. Of all troublesome alloys to handle in the foundry or rolling-mill, German silver is the worst. It is unmanageable and refractory at every step in its transition from the crude elements into rods, sheets, or wire. (E. H. Cowles, *Trans. A. I. M. E.*, xviii, p. 494.)

The following list of copper-nickel alloys is from various sources:

| | Copper. | Nickel. | Tin. | Zinc. |
|---------------------------|----------|----------|-------|-----------|
| German silver..... | 51.6 | 25.8 | 22.6 | |
| “ “..... | 50.2 | 14.8 | 3.1 | 31.9 |
| “ “..... | 51.1 | 13.8 | 3.2 | 31.9 |
| “ “..... | 52 to 55 | 18 to 25 | | 20 to 30 |
| Nickel “..... | 75 to 66 | 25 to 33 | | |
| Chinese packfong..... | 40.4 | 31.6 | | 6.5 parts |
| “ tutenag..... | 8 | 3 | | 6.5 “ |
| German silver..... | 2 | 1 | | 1 “ |
| “ “ (cheaper)..... | 8 | 2 | | 3.3 “ |
| “ “ (resembles silver)... | 8 | 3 | | 3.5 “ |

Nickel-copper Alloys. — (F. L. Sperry, *A. I. M. E.*, 1895.)

| | Copper. | Nickel. | Zinc. | Iron. | Cobalt. |
|--------------------------|------------|------------|------------|----------|----------|
| Berlin..... | 52 to 63 | 22 to 6 | 26 to 31 | | |
| French, tableware..... | 50 | 18.7 to 20 | 31.3 to 30 | | |
| Maillechort..... | 65.4 | 16.8 | 13.4 | 3.4 | |
| Christofle..... | 50 | 50 | | | |
| Austrian, tableware..... | 50 to 60 | 25 to 20 | 25 to 20 | | |
| English, Sheffield..... | 45.7 to 60 | 31.6 to 15 | 25.4 to 17 | 0 to 2.6 | 0 to 3.4 |
| American, castings..... | 52.5 | 17.7 | 28.8 | | |
| “ bearings..... | 50 | 25 | 25 | | |
| “ one-cent coin..... | 88 | 12 | | | |
| Nickel coins..... | 75 | 25 | | | |

A refined copper-nickel alloy containing 50% copper and 49% nickel, with very small amounts of iron, silicon and carbon, is produced direct from Bessemer matte in the Sudbury (Canada) Nickel Works. German-silver manufacturers purchase a ready-made alloy, which melts at a low heat and requires only the addition of zinc, instead of buying the nickel and copper separately. This alloy, "50-50" as it is called, is almost indistinguishable from pure nickel. Its cost is less than nickel, its melting-point much lower, it can be cast solid in any form desired, and furnishes a casting which works easily in the lathe or planer, yielding a silvery-white surface unchanged by air or moisture. For bullet casings now used in various British and Continental rifles, a special alloy of 80% copper and 20% nickel is made.

Monel Metal. — An alloy of about 72% Ni, 1.5 Fe, 26.5 Cu, made from the Canadian copper-nickel ores, is described in the *Metal Worker*, Oct. 10, 1908. It has many valuable properties when rolled into sheets, making it especially suitable for roofing. It is ductile and flexible, is easily soldered, has a high resistance to corrosion, and a relatively small expansion and contraction under temperature changes. The tensile strength in castings is from 70,000 to 80,000 lbs. per sq. in., and in rolled sheets as high as 108,000 lbs.

Constantan is an alloy containing about 60% copper and 40% nickel, which is much used for resistance wire in electrical instruments. Its electrical resistance is about twenty-eight to thirty times that of copper, and it possesses a very low temperature coefficient, — approximately .00003. This same material is also much used to form one element of base-metal thermo-couples.

ALLOYS OF BISMUTH.

By adding a small amount of bismuth to lead the latter may be hardened and toughened. An alloy consisting of three parts of lead and two of bismuth has ten times the hardness and twenty times the tenacity of lead. The alloys of bismuth with both tin and lead are extremely fusible, and take fine impressions of casts and molds. An alloy of one part Bi, two parts Sn, and one part Pb is used by pewter-workers as a soft solder, and by soap-makers for molds. An alloy of five parts Bi, two parts Sn, and three parts Pb melts at 199° F., and is somewhat used for stereotyping, and for metallic writing-pencils. Thorpe gives the following proportions for the better-known fusible metals:

| Name of Alloy. | Bis- muth. | Lead. | Tin. | Cad- mium. | Mer- cury. | Melting- point. |
|------------------------|---------------|-------|-------|---------------|---------------|--------------------|
| Newton's..... | 50 | 31.25 | 18.75 | | | 202° F. |
| Rose's..... | 50 | 28.10 | 24.10 | | | 203° " |
| D'Arcet's..... | 50 | 25.00 | 25.00 | | | 201° " |
| D'Arcet's with mercury | 50 | 25.00 | 25.00 | | 250.0 | 113° " |
| Wood's..... | 50 | 25.00 | 12.50 | 12.50 | | 149° " |
| Lipowitz's..... | 50 | 26.90 | 12.78 | 10.40 | | 149° " |
| Guthrie's "Eutectic". | 50 | 20.55 | 21.10 | 14.03 | | "Very low." |

The action of heat upon some of these alloys is remarkable. Thus, Lipowitz's alloy, which solidifies at 149° F., contracts very rapidly at first, as it cools from this point. As the cooling goes on the contraction becomes slower and slower, until the temperature falls to 101.3° F. From this point the alloy *expands* as it cools, until the temperature falls to about 77° F., after which it again contracts, so that at 32° F. a bar of the alloy has the same length as at 115° F.

Alloys of bismuth have been used for making fusible plugs for boilers, but it is found that they are altered by the continued action of heat, so that one cannot rely upon them to melt at the proper temperature. Pure Banca tin is used by the U. S. Government for fusible plugs.

FUSIBLE ALLOYS.

(From various sources. Many of the figures are probably very inaccurate.)

| | |
|--|--------------|
| Sir Isaac Newton's, bismuth 5, lead 3, tin 2, melts at..... | 212° F. |
| Rose's, bismuth 2, lead 1, tin 1, melts at | 200 " |
| Wood's, cadmium 1, bismuth 4, lead 2, tin 1, melts at | 165 " |
| Guthrie's, cadmium 13.29, bismuth 47.38, lead 19.36, tin 19.97, melts at | 160 " |
| Lead 1, tin 1, bismuth 1, cadmium 1, melts at | 155 " |
| Lead 3, tin 5, bismuth 8, melts at | 208 " |
| Lead 1, tin 3, bismuth 5, melts at | 212 " |
| Lead 1, tin 4, bismuth 5, melts at | 240 " |
| Tin 1, bismuth 1, melts at | 286 " |
| Lead 2, tin 3, melts at | 334 to 367 " |
| Tin 2, bismuth 1, melts at | 336 " |
| Lead 1, tin 2, melts at | 340 to 360 " |
| Tin 8, bismuth 1, melts at | 392 " |
| Lead 2, tin 1, melts at | 440 to 475 " |
| Lead 1, tin 1, melts at | 370 to 400 " |
| Lead 1, tin 3, melts at | 356 to 383 " |
| Tin 3, bismuth 1, melts at | 392 " |
| Lead 1, bismuth 1, melts at | 257 " |
| Lead 1, tin 1, bismuth 4, melts at | 201 " |
| Lead 5, tin 3, bismuth 8, melts at | 202 " |
| Tin 3, bismuth 5, melts at | 202 " |

BEARING-METAL ALLOYS.

(C. B. Dudley, *Jour. F. I.*, Feb. and March, 1892.)

Alloys are used as bearings in place of wrought iron, cast iron, or steel, partly because wear and friction are believed to be more rapid when two metals of the same kind work together, partly because the soft metals are more easily worked and got into proper shape, and partly because it is desirable to use a soft metal which will take the wear rather than a hard metal, which will wear the journal more rapidly.

A good bearing-metal must have five characteristics: (1) It must be strong enough to carry the load without distortion. Pressures on car-journals are frequently as high as 350 to 400 lb. per square inch.

(2) A good bearing-metal should not heat readily. The old copper-tin bearing, made of seven parts copper to one part tin, is more apt to heat than some other alloys. In general, research seems to show that the harder the bearing-metal, the more likely it is to heat.

(3) Good bearing-metal should work well in the foundry. Oxidation while melting causes spongy castings. It can be prevented by a liberal use of powdered charcoal while melting. The addition of 1% to 2% of zinc or a small amount of phosphorus greatly aids in the production of sound castings. This is a principal element of value in phosphor-bronze.

(4) Good bearing-metals should show small friction. It is true that friction is almost wholly a question of the lubricant used; but the metal of the bearing has certainly some influence.

(5) Other things being equal, the best bearing-metal is that which wears slowest.

The principal constituents of bearing-metal alloys are copper, tin, lead, zinc, antimony, iron, and aluminum. The following table gives the constituents of most of the prominent bearing-metals as analyzed at the Pennsylvania Railroad laboratory at Altoona.

Analyses of Bearing-metal Alloys.

| Metal. | Copper. | Tin. | Lead. | Zinc. | Anti-mony. | Iron. |
|-----------------------------------|---------|-------|-------|-------|------------|-----------|
| Camelia metal..... | 70.20 | 4.25 | 14.75 | 10.20 | | 0.55 |
| Anti-friction metal..... | 1.60 | 98.13 | | | | trace |
| White metal..... | | | 87.92 | | 12.08 | |
| Car-brass lining..... | | trace | 84.87 | | 15.10 | |
| Salgee anti-friction..... | 4.01 | 9.91 | 1.15 | 85.57 | | |
| Graphite bearing-metal..... | | 14.38 | 67.73 | | 16.73 | ? (1) |
| Antimonial lead..... | | | 80.69 | | 18.83 | |
| Carbon bronze..... | 75.47 | 9.72 | 14.57 | | | (2) |
| Cornish bronze..... | 77.83 | 9.60 | 12.40 | trace | | trace (3) |
| Delta metal..... | 92.39 | 2.37 | 5.10 | | | 0.07 |
| * Magnolia metal..... | trace | | 83.55 | trace | 16.45 | trace (4) |
| American anti-friction metal..... | | | 78.44 | 0.98 | 19.60 | 0.65 |
| Tobin bronze..... | 59.00 | 2.16 | 0.31 | 38.40 | | 0.11 |
| Graney bronze..... | 75.80 | 9.20 | 15.06 | | | |
| Damascus bronze..... | 76.41 | 10.60 | 12.52 | | | |
| Manganese bronze..... | 90.52 | 9.58 | | | | (5) |
| Ajax metal..... | 81.24 | 10.98 | 7.27 | | | (6) |
| Anti-friction metal..... | | | 88.32 | | 11.93 | |
| Harrington bronze..... | 55.73 | 0.97 | | 42.67 | | 0.68 |
| Car-box metal..... | | | 84.33 | trace | 14.38 | 0.61 |
| Hard lead..... | | | 94.40 | | 6.03 | |
| Phosphor-bronze..... | 79.17 | 10.22 | 9.61 | | | (7) |
| Ex. B. metal..... | 76.80 | 8.00 | 15.00 | | | (8) |

Other constituents:

- | | |
|--------------------------------|----------------------------------|
| (1) No graphite. | (5) No manganese. |
| (2) Possible trace of carbon. | (6) Phosphorus or arsenic, 0.37. |
| (3) Trace of phosphorus. | (7) Phosphorus, 0.94. |
| (4) Possible trace of bismuth. | (8) Phosphorus, 0.20. |

* Dr. H. C. Torrey says this analysis is erroneous and that Magnolia metal always contains tin.

As an example of the influence of minute changes in an alloy, the Harrington bronze, which consists of a minute proportion of iron in a copper-zinc alloy, showed after rolling a tensile strength of 75,000 lb. and 20% elongation in 2 inches.

In experimenting on this subject on the Pennsylvania Railroad, a certain number of the bearings were made of a standard bearing-metal, and the same number were made of the metal to be tested. These bearings were placed on opposite ends of the same axle, one side of the car having the standard bearings, the other the experimental. Before going into service the bearings were carefully weighed, and after a sufficient time they were again weighed. The standard bearing-metal used is the "S bearing-metal" of the Phosphor-Bronze Smelting Co. It contains about 79.70% copper, 9.50% lead, 10% tin, and 0.80% phosphorus. A large number of experiments have shown that the loss of weight of a bearing of this metal is 1 lb. to each 18,000 to 25,000 miles traveled. Besides the measurement of wear, observations were made on the frequency of "hot boxes" with the different metals.

The results of the tests for wear, so far as given, are condensed into the following table:

| Metal. | Composition. | | | | | Rate of Wear. |
|------------------------------|--------------|-------|-------|-------|-----------|---------------|
| | Copper. | Tin. | Lead. | Phos. | Arseenic. | |
| Standard..... | 79.70 | 10.00 | 9.50 | 0.80 | | 100 |
| Copper-tin..... | 87.50 | 12.50 | | | | 148 |
| Same, second experiment..... | | | | | | 153 |
| Same, third experiment..... | | | | | | 147 |
| Arsenic-bronze..... | 89.20 | 10.00 | | | 0.80 | 142 |
| Arsenic-bronze..... | 79.20 | 10.00 | 7.00 | | 0.80 | 115 |
| Arsenic-bronze..... | 79.70 | 10.00 | 9.50 | | 0.80 | 101 |
| "K" bronze..... | 77.00 | 10.50 | 12.50 | | | 92 |
| Same, second experiment..... | | | | | | 92.7 |
| Alloy "B"..... | 77.00 | 8.00 | 15.00 | | | 86.5 |

The old copper-tin alloy of 7 to 1 has repeatedly proved its inferiority to the phosphor-bronze metal. Many more of the copper-tin bearings heated than of the phosphor-bronze. The showing of these tests was so satisfactory that phosphor-bronze was adopted as the standard bearing-metal of the Pennsylvania R.R., and was used for a long time.

The experiments, however, were continued. It was found that arsenic practically takes the place of phosphorus in a copper-tin alloy, and three tests were made with arsenic-bronzes as noted above. As the proportion to lead is increased to correspond with the standard, the durability increases as well. In view of these results the "K" bronze was tried, in which neither phosphorus nor arsenic were used, and in which the lead was increased above the proportion in the standard phosphor-bronze. The result was that the metal wore 7.30% slower than the phosphor-bronze. No trouble from heating was experienced with the "K" bronze more than with the standard. Dr. Dudley continues:

At about this time we began to find evidences that wear of bearing-metal alloys varied in accordance with the following law: "That alloy which has the greatest power of distortion without rupture (resilience), will best resist wear." It was now attempted to design an alloy in accordance with this law, taking first the proportions of copper and tin. 9 1/2 parts copper to 1 of tin was settled on by experiment as the standard, although some evidence since that time tends to show that 12 or possibly 15 parts copper to 1 of tin might have been better. The influence of lead on this copper-tin alloy seems to be much the same as a still further diminution of tin. However, the tendency of the metal to yield under pressure increases as the amount of tin is diminished, and the amount of the lead increased, so a limit is set to the use of lead. A certain amount of tin is also necessary to keep the lead alloyed with the copper.

Bearings were cast of the metal noted in the table as alloy "B," and it wore 13.5% slower than the standard phosphor-bronze. This metal is now the standard bearing-metal of the Pennsylvania Railroad, being slightly changed in composition to allow the use of phosphor-bronze scrap. The formula adopted is: Copper, 105 lbs.; phosphor-bronze, 60 lbs.; tin, 9 3/4 lbs.; lead, 25 1/4 lbs. By using ordinary care in the foundry, keeping the metal well covered with charcoal during the melting, no trouble is found in casting good bearings with this metal. The copper and the phosphor-bronze can be put in the pot before putting it in the melting-hole. The tin and lead should be added after the pot is taken from the fire.

It is not known whether the use of a little zinc, or possibly some other combination, might not give still better results. For the present, however, this alloy is considered to fulfill the various conditions required for good bearing-metal better than any other alloy. The phosphor-bronze had an ultimate tensile strength of 30,000 lb., with 6% elongation, whereas the alloy "B" had 24,000 lb. T. S. and 11% elongation.

Bearing Metal Practice, 1907. (G. H. Clamer, *Proc. A. S. T. M.*, vii, 302, discusses the history of bearing metal practice since the date of Dr. Dudley's paper quoted above. It was found that tin could be diminished and lead increased far beyond the figures formerly used, and a satisfactory bearing metal was made with 65% copper, 5% tin and 30% lead. This alloy is largely sold under the name of "plastic bronze." It has a compressive strength of about 15,000 lbs. per sq. in., and is found to operate without distortion in the bearings of the heaviest locomotives, not only for driving brasses, but also for rod brasses and bushings, and for bearings of cars of 100,000 lbs. capacity, the heaviest cars now in service. Specifications of different railroads cover bearing alloys with tin from 8 to 10% and lead from 10 to 15%. There is also used a vast quantity of bearings made from scrap. These contain copper, 65 to 75%, tin, 2 to 8%, lead, 10 to 18%, zinc, 5 to 20%, and they constitute from 50 to 75 per cent of the car bearings now in use.

White Metal for Engine Bearings. (Report of a British Naval Committee, *Eng'g*, July 18, 1902.) — For lining bearings, crankpin bushes, and other parts exclusive of cross-head bushes: Tin 12, copper 1, antimony 1. Melt 6 tin 1 copper, and 6 tin 1 antimony separately and mix the two together. For cross-head bushes a harder alloy, viz., 85% tin, 5% copper, 10% antimony, has given good results.

(For other bearing-metals, see "Alloys containing Antimony," below.)

ALLOYS CONTAINING ANTIMONY.

VARIOUS ANALYSES OF BABBITT METAL AND OTHER ALLOYS CONTAINING ANTIMONY.

| Tin. | Copper. | Antimony. | Zinc. | Lead. | Bismuth. |
|------------------------|---------|-------------|-------|-------|----------|
| Babbitt metal } 50 | 1 | 5 parts | | | |
| for light duty } =89.3 | 1.8 | 8.9 per ct. | | | |
| Harder Babbitt } 96 | 4 | 8 parts | | | |
| for bearings* } =88.9 | 3.7 | 7.4 per ct. | | | |
| Britannia85.7 | 1.0 | 10.1 | 2.9 | | |
| "81.9 | | 16.2 | 1.9 | | |
| "81.0 | 2 | 16 | 1 | | |
| "70.5 | 4 | 25.5 | | | |
| "22 | 10 | 62 | 6 | | |
| " Babbitt "45.5 | 1.5 | 13 | | 40.0 | |
| Plate pewter.....89.3 | 1.8 | 7.1 | | | 1.8 |
| White metal.....85 | 5 | 10 | | | |

Bearings on Ger. locomotives.

* It is mixed as follows: Twelve parts of copper are first melted and then 36 parts of tin are added; 24 parts of antimony are put in, and then 36 parts of tin, the temperature being lowered as soon as the copper is melted in order not to oxidize the tin and antimony, the surface of the bath being protected from contact with the air. The alloy thus made is subsequently remelted in the proportion of 50 parts of alloy to 100 tin. (Joshua Rose.)

White-metal Alloys. — The following alloys are used as lining metals by the Eastern Railroad of France (1890):

| Number. | Lead. | Antimony. | Tin. | Copper. |
|---------|-------|-----------|-------|---------|
| 1..... | 65 | 25 | 0 | 10 |
| 2..... | 0 | 11.12 | 83.33 | 5.55 |
| 3..... | 70 | 20 | 10 | 0 |
| 4..... | 80 | 8 | 12 | 0 |

No 1 is used for lining cross-head slides, rod-brasses and axle-bearings; No. 2 for lining axle-bearings and connecting-rod brasses of heavy engines; No. 3 for lining eccentric straps and for bronze slide-valves; and No. 4 for metallic rod-packing.

Some of the best-known white-metal alloys are the following (Circular of Hoveler & Dieckhaus, London, 1893):

| | Tin. | Antimony. | Lead. | Copper. | Zinc. |
|---------------------|------|-----------|-------|---------|-------|
| 1. Parsons'..... | 86 | 1 | 2 | 2 | 27 |
| 2. Richards'..... | 70 | 15 | 101½ | 41½ | 0 |
| 3. Babbitt's..... | 55 | 18 | 231½ | 31½ | 0 |
| 4. Fenton's..... | 16 | 0 | 0 | 5 | 79 |
| 5. French Navy..... | 71½ | 0 | 7 | 7 | 871½ |
| 6. German Navy..... | 85 | 7½ | 0 | 7½ | 0 |

"There are engineers who object to white metal containing lead or zinc. This is, however, a prejudice quite unfounded, inasmuch as lead and zinc often have properties of great use in white alloys.

It is a further fact that an "easy liquid" alloy must not contain more than 18% of antimony, which is an invaluable ingredient of white metal

for improving its hardness; but in no case must it exceed that margin, as this would reduce the plasticity of the compound and make it brittle.

Hardest tin-lead alloy: 6 tin, 4 lead. Hardest of all tin alloys (?) : 74 tin, 18 antimony, 8 copper.

Alloy for thin open-work, ornamental castings: Lead 2, antimony 1. White metal for patterns: Lead 10, bismuth 6, antimony 2, common brass 8, tin 10.

Type-metal is made of various proportions of lead and antimony, from 17% to 20% antimony according to the hardness desired.

Babbitt Metals. (C. R. Tompkins, *Mechanical News*, Jan., 1891.)

The practice of lining journal-boxes with a metal that is sufficiently fusible to be melted in a common ladle is not always so much for the purpose of securing anti-friction properties as for the convenience and cheapness of forming a perfect bearing in line with the shaft without the necessity of boring them. Boxes that are bored, no matter how accurate, require great care in fitting and attaching them to the frame or other parts of a machine.

It is not good practice, however, to use the shaft for the purpose of casting the bearings, especially if the shaft be steel, for the reason that the hot metal is apt to spring it; the better plan is to use a mandrel of the same size or a trifle larger for this purpose. For slow-running journals, where the load is moderate, almost any metal that may be conveniently melted and will run free will answer the purpose. For wearing properties, with a moderate speed, there is probably nothing superior to pure zinc, but when not combined with some other metal it shrinks so much in cooling that it cannot be held firmly in the recess, and soon works loose; and it lacks those anti-friction properties which are necessary in order to stand high speed.

For line-shafting, and all work where the speed is not over 300 or 400 r. p. m., an alloy of 8 parts zinc and 2 parts block-tin will not only wear longer than any composition of this class, but will successfully resist a heavy load. The tin counteracts the shrinkage, so that the metal, if not overheated, will firmly adhere to the box until it is worn out. But this mixture does not possess sufficient anti-friction properties to warrant its use in fast-running journals.

Among all the soft metals in use there are none that possess greater anti-friction properties than pure lead; but lead alone is impracticable, for it is so soft that it cannot be retained in the recess. But when by any process lead can be sufficiently hardened to be retained in the boxes without materially injuring its anti-friction properties, there is no metal that will wear longer in light fast-running journals. With most of the best and most popular anti-friction metals in use and sold under the name of the Babbitt metal, the basis is lead.

Lead and antimony have the property of combining with each other in all proportions without impairing the anti-friction properties of either. The antimony hardens the lead, and when mixed in the proportion of 80 parts lead by weight with 20 parts antimony, no other known composition of metals possesses greater anti-friction or wearing properties, or will stand a higher speed without heat or abrasion. It runs free in its melted state, has no shrinkage, and is better adapted to light high-speed machinery than any other known metal. Care, however, should be manifested in using it, and it should never be heated beyond a temperature that will scorch a dry pine stick.

Many different compositions are sold under the name of Babbitt metal. Some are good, but more are worthless; while but very little genuine Babbitt metal is sold that is made strictly according to the original formula. Most of the metals sold under that name are the refuse of type-foundries and other smelting-works, melted and cast into fancy ingots with special brands, and sold under the name of Babbitt metal.

It is difficult at the present time to determine the exact formulas used by the original Babbitt, the inventor of the recessed box, as a number of different formulas are given for that composition. Tin, copper,

and antimony were the ingredients, and from the best sources of information the original proportions were as follows:

| | | | |
|-----------------------|---------|-----------------------|-------|
| 50 parts tin..... | = 89.3% | Another writer gives: | 83.3% |
| 2 parts copper..... | = 3.6% | | 8.3% |
| 4 parts antimony..... | = 7.1% | | 8.3% |

The copper was first melted, and the antimony added first and then about ten or fifteen pounds of tin, the whole kept at a dull-red heat and constantly stirred until the metals were thoroughly incorporated, after which the balance of the tin was added, and after being thoroughly stirred again it was then cast into ingots. When the copper is thoroughly melted, and before the antimony is added, a handful of powdered charcoal should be thrown into the crucible to form a flux, in order to exclude the air and prevent the antimony from vaporizing; otherwise much of it will escape in the form of a vapor and consequently be wasted. This metal, when carefully prepared, is probably one of the best metals in use for lining boxes that are subjected to a heavy weight and wear; but for light fast-running journals the copper renders it more susceptible to friction, and it is more liable to heat than the metal composed of lead and antimony in the proportions just given.

SOLDERS.

Common solders, equal parts tin and lead; fine solder, 2 tin to 1 lead; cheap solder, 2 lead, 1 tin.

Fusing-point of tin-lead alloys (many figures probably inaccurate).

| Tin | 1 to lead | 25..... | 558° F. | Tin | 1 1/2 to lead | 1..... | 334° F. |
|-----|-----------|---------|---------|-----|---------------|--------|---------|
| " | 1 " " | 10..... | 541 | " | 2 " " | 1..... | 340 |
| " | 1 " " | 5..... | 511 | " | 3 " " | 1..... | 356 |
| " | 1 " " | 3..... | 482 | " | 4 " " | 1..... | 365 |
| " | 1 " " | 2..... | 441 | " | 5 " " | 1..... | 378 |
| " | 1 " " | 1..... | 370 | " | 6 " " | 1..... | 381 |

The melting point of the tin-lead alloys decreases almost proportionately to the increase of tin, from 619°F, the melting point of pure lead, to 356°F when the alloy contains 68% of tin, and then increases to 448°F, the melting point of pure tin. Alloys on either side of the 68% mixture begin to soften materially at 356°F, because at that temperature the eutectic alloy melts and permits the whole alloy to soften. (Dr. J. A. Mathews.)

Common pewter contains 4 lead to 1 tin.

The relative hardness of the various tin and lead solders has been determined by Brinell's method. The results are as follows:

| % Tin | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|----------|-------|-------|-------|-------|-------|-------|-------|
| Hardness | 3.90 | 10.10 | 12.16 | 14.46 | 15.76 | 14.90 | 14.58 |
| % Tin | 66 | 67 | 68 | 70 | 80 | 90 | 100 |
| Hardness | 16.66 | 15.40 | 14.58 | 15.84 | 15.20 | 13.25 | 4.14 |

The hardest solder is the one composed of 2 parts of tin and 1 part of lead. It is the eutectic alloy, or the one with the lowest melting point of all the mixtures. — *Mechanical World*.

Gold solder: 14 parts gold, 6 silver, 4 copper. Gold solder for 14-carat gold: 25 parts gold, 25 silver, 12 1/2 brass, 1 zinc.

Silver solder: Yellow brass 70 parts, zinc 7, tin 1 1/2. Another: Silver 145 parts, brass (3 copper, 1 zinc) 73, zinc 4.

German-silver solder: Copper 38, zinc 54, nickel 8.

Novel's solders for aluminum:

| | | | | |
|-----|------------|--------------------|----------|-----------------|
| Tin | 100 parts, | lead 5; | melts at | 536° to 572° F. |
| " | 100 | " zinc 5; | " | 536 to 612 |
| " | 1000 | " copper 10 to 15; | " | 662 to 842 |
| " | 1000 | " nickel 10 to 15; | " | 662 to 842 |

Novel's solder for aluminum bronze: Tin, 900 parts, copper 100, bismuth 2 to 3. It is claimed that this solder is also suitable for joining aluminum to copper, brass, zinc, iron, or nickel.

ROPES AND CABLES.

STRENGTH OF ROPES.

(A. S. Newell & Co., Birkenhead. Klein's Translation of Weisbach, vol. iii, part 1, sec. 2.)

| Hemp. | | Iron. | | Steel. | | Tensile Strength, Gross tons. |
|----------------|----------------------------|----------------|----------------------------|----------------|----------------------------|-------------------------------|
| Girth. Inches. | Weight per Fathom. Pounds. | Girth. Inches. | Weight per Fathom. Pounds. | Girth. Inches. | Weight per Fathom. Pounds. | |
| 23/4 | 2 | 1 | 1 | | | 2 |
| | | 1 1/2 | 1 1/2 | | | 3 |
| 33/4 | 4 | 1 5/8 | 2 | 1 | 1 | 4 |
| | | 1 3/4 | 2 1/2 | 1 1/2 | 1 1/2 | 5 |
| 4 1/2 | 5 | 1 7/8 | 3 | | | 6 |
| | | 2 | 3 1/2 | 1 5/8 | 2 | 7 |
| 5 1/2 | 7 | 2 1/8 | 4 | 1 3/4 | 2 1/2 | 8 |
| | | 2 1/4 | 4 1/2 | | | 9 |
| 6 | 9 | 2 3/8 | 5 | 1 7/8 | 3 | 10 |
| | | 2 1/2 | 5 1/2 | | | 11 |
| 6 1/2 | 10 | 2 5/8 | 6 | 2 | 3 1/2 | 12 |
| | | 2 3/4 | 6 1/2 | 2 1/8 | 4 | 13 |
| 7 | 12 | 2 7/8 | 7 | 2 1/4 | 4 1/2 | 14 |
| | | 3 | 7 1/2 | | | 15 |
| 7 1/2 | 14 | 3 1/8 | 8 | 2 3/8 | 5 | 16 |
| | | 3 1/4 | 8 1/2 | | | 17 |
| 8 | 16 | 3 3/8 | 9 | 2 1/2 | 5 1/2 | 18 |
| | | 3 1/2 | 10 | 2 5/8 | 6 | 20 |
| 8 1/2 | 18 | 3 5/8 | 11 | 2 3/4 | 6 1/2 | 22 |
| | | 3 3/4 | 12 | | | 24 |
| 9 1/2 | 22 | 3 7/8 | 13 | 3 1/4 | 8 | 26 |
| 10 | 26 | 4 | 14 | | | 28 |
| 11 | 30 | 4 1/4 | 15 | 3 3/8 | 9 | 30 |
| | | 4 3/8 | 16 | | | 32 |
| | | 4 1/2 | 18 | 3 1/2 | 10 | 36 |
| 12 | 34 | 4 5/8 | 20 | 3 3/4 | 12 | 40 |

Length Sufficient to Cause the Maximum Working Stress.
(Weisbach.)

| | |
|----------------------------------|------------|
| Hemp rope, dry and untarred..... | 2855 feet. |
| Hemp rope, wet or tarred..... | 1975 " |
| Wire rope..... | 4590 " |
| Open-link chain..... | 1360 " |
| Stud chain..... | 1660 " |

Sometimes, when the depths are very great, ropes are given approximately the form of a body of uniform strength, by making them of separate pieces, whose diameters diminish towards the lower end. It is evident that by this means the tensions in the fibres caused by the rope's own weight can be considerably diminished.

Rope for Hoisting or Transmission. Manila Rope. (C. W. Hunt Company, New York.) — Rope used for hoisting or for transmission of power is subjected to a very severe test. Ordinary rope chafes and grinds to powder in the center, while the exterior may look as though it was little worn.

In bending a rope over a sheave, the strands and the yarns of these strands slide a small distance upon each other, causing friction, and wear the rope internally.

The "Stevedore" rope used by the C. W. Hunt Company is made by lubricating the fibres with plumbago, mixed with sufficient tallow to hold it in position. This lubricates the yarns of the rope, and prevents internal chafing and wear. After running a short time the exterior of the rope gets compressed and coated with the lubricant.

In manufacturing rope, the fibres are first spun into a yarn, this yarn being twisted in a direction called "right hand." From 20 to 80 of these yarns, depending on the size of the rope, are then put together and twisted in the opposite direction, or "left hand," into a strand. Three of these strands, for a 3-strand, or four for a 4-strand rope, are then twisted together, the twist being again in the "right hand" direction. When the strand is twisted, it untwists each of the threads, and when the three strands are twisted together into rope, it untwists the strands, but again twists up the threads. It is this opposite twist that keeps the rope in its proper form. When a weight is hung on the end of a rope, the tendency is for the rope to untwist, and become longer. In untwisting the rope, it would twist the threads up, and the weight will revolve until the strain of the untwisting strands just equals the strain of the threads being twisted tighter. In making a rope it is impossible to make these strains exactly balance each other. It is this fact that makes it necessary to take out the "turns" in a new rope, that is, untwist it when it is put at work. The proper twist that should be put in the threads has been ascertained approximately by experience.

The amount of work that the rope will do varies greatly. It depends not only on the quality of the fibre and the method of laying up the rope, but also on the kind of weather when the rope is used, the blocks or sheaves over which it is run, and the strain in proportion to the strain put upon the rope. The principal wear comes in practice from defective or badly set sheaves, from excess of load and exposure to storms.

The loads put upon the rope should not exceed those given in the tables, for the most economical wear. The indications of excessive load will be the twist coming out of the rope, or one of the strands slipping out of its proper position. A certain amount of twist comes out in using it the first day or two, but after that the rope should remain substantially the same. If it does not, the load is too great for the durability of the rope. If the rope wears on the outside, and is good on the inside, it shows that it has been chafed in running over the pulleys or sheaves. If the blocks are very small, it will increase the sliding of the strands and threads, and result in a more rapid internal wear. Rope made for hoisting and for rope transmission is usually made with four strands, as experience has shown this to be the most serviceable.

The strength and weight of "Stevedore" rope is estimated as follows:

Breaking strength in pounds = 720 (circumference in inches)²;
Weight in pounds per foot = 0.032 (circumference in inches)².

Flat Ropes. (Weisbach.)

| Iron. | | Steel. | | Tensile Strength, Gross tons. | Iron. | | Steel. | | Tensile Strength, Gross tons. |
|---------------|-----------------------|---------------|-----------------------|-------------------------------------|---------------|-----------------------|---------------|-----------------------|-------------------------------------|
| Girth. In. | Weight per Fathom. | Girth. In. | Weight per Fathom. | | Girth. In. | Weight per Fathom. | Girth. In. | Weight per Fathom. | |
| | Lbs. | | Lbs. | | | Lbs. | | Lbs. | |
| 21/4 × 1/2 | 11 | | | 20 | 3 3/4 × 11/16 | 22 | 2 1/2 × 1/2 | 13 | 40 |
| 21/2 × 1/2 | 13 | | | 23 | 4 × 11/16 | 25 | 2 3/4 × 3/8 | 15 | 45 |
| 2 3/4 × 5/8 | 15 | | | 27 | 4 1/4 × 3/4 | 28 | 3 × 3/4 | 16 | 50 |
| 3 × 5/8 | 16 | 2 × 1/2 | 10 | 28 | 4 1/2 × 3/4 | 32 | 3 1/4 × 3/8 | 18 | 56 |
| 3 1/4 × 5/8 | 18 | 2 1/4 × 1/2 | 11 | 32 | 4 5/8 × 3/4 | 34 | 3 1/2 × 3/8 | 20 | 60 |
| 3 1/2 × 5/8 | 20 | 2 1/4 × 1/2 | 12 | 36 | | | | | |

The Technical Words relating to Cordage most frequently heard are:

YARN. — Fibres twisted together.

THREAD. — Two or more *small yarns* twisted together.

STRING. — The same as a thread but a little larger *yarns*.

STRAND. — Two or more *large yarns* twisted together.

CORD. — Several threads twisted together.

ROPE. — Several *strands* twisted together.

HAWSER. — A rope of three *strands*.

SHROUD-LAID. — A rope of four *strands*.

CABLE. — Three hawsers twisted together.

YARNS are laid up left-handed into *strands*.

STRANDS are laid up right-handed into rope.

HAWSERS are laid up left-handed into a cable.

A rope is:

LAID by twisting strands together in making the rope.

SPLICED by joining to another rope by interweaving the strands.

WHIPPED. — By winding a string around the end to prevent untwisting.

SERVED. — When covered by winding a yarn continuously and tightly around it.

PARCELED. — By wrapping with canvas.

SEIZED. — When two parts are bound together by a yarn, thread or string.

PAYED. — When painted, tarred or greased to resist wet.

HAUL. — To pull on a rope.

TAUT. — Drawn tight or strained.

Splicing of Ropes. — The splice in a transmission rope is not only the weakest part of the rope but is the first part to fail when the rope is worn out. If the rope is larger at the splice, the projecting part will wear on the pulleys and the rope fail from the cutting off of the strands. The following directions are given for splicing a 4-strand rope.

The engravings show each successive operation in splicing a 1¾-inch manila rope. Each engraving was made from a full-size specimen.

Tie a piece of twine, 9 and 10, around the rope to be spliced, about 6 feet from each end. Then unlay the strands of each end back to the twine.

Butt the ropes together and twist each corresponding pair of strands loosely, to keep them from being tangled, as shown in Fig. 80.

The twine 10 is now cut, and the strand 8 unlay and strand 7 carefully laid in its place for a distance of four and a half feet from the junction.

The strand 6 is next unlay about one and a half feet and strand 5 laid in its place.

The ends of the cores are now cut off so they just meet.

Unlay strand 1 four and a half feet, laying strand 2 in its place.

Unlay strand 3 one and a half feet, laying in strand 4.

Cut all the strands off to a length of about twenty inches for convenience in manipulation.

The rope now assumes the form shown in Fig. 81 with the meeting points of the strands three feet apart.

Each pair of strands is successively subjected to the following operation:

From the point of meeting of the strands 8 and 7, unlay each one three turns; split both the strand 8 and the strand 7 in halves as far back as they are now unlay and "whip" the end of each half strand with a small piece of twine.

The half of the strand 7 is now laid in three turns and the half of 8 also laid in three turns. The half strands now meet and are tied in a simple knot, 11, Fig. 82, making the rope at this point its original size.

The rope is now opened with a marlin spike and the half strand of 7 worked around the half strand of 8 by passing the end of the half strand 7 through the rope, as shown in the engraving, drawn taut, and again worked around this half strand until it reaches the half strand 13 that was not laid in. This half strand 13 is now split, and the half strand 7 drawn through the opening thus made, and then tucked under the two adjacent strands, as shown in Fig. 83. The other half of the strand 8 is now wound around the other half strand 7 in the same manner. After each pair of strands has been treated in this manner, the ends are cut off at 12, leaving them about four inches long. After a few days' wear they will

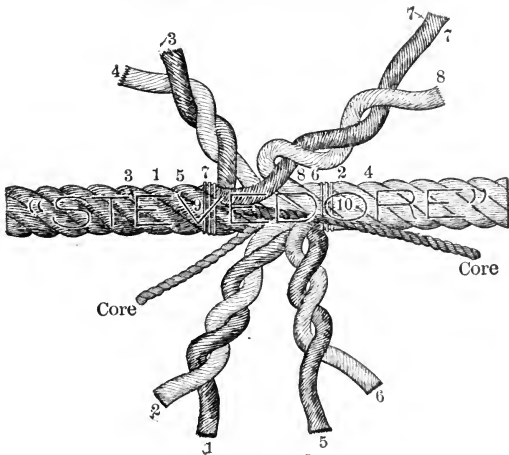


FIG. 80.

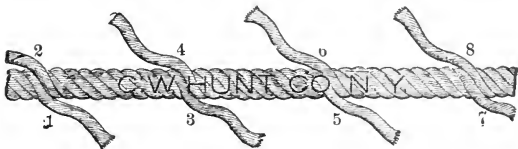


FIG. 81.

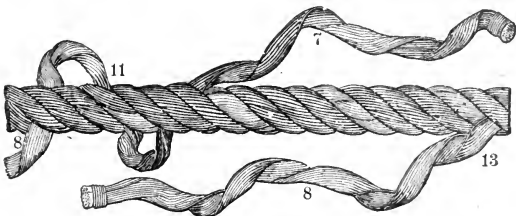


FIG. 82.

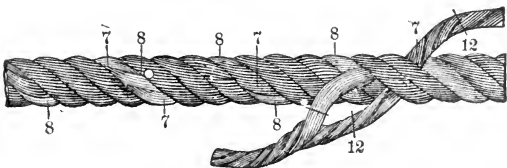


FIG. 83.

SPlicing OF ROPES.

draw into the body of the rope or wear off, so that the locality of the splice can scarcely be detected.

Cargo Hoisting. (C. W. Hunt Company.) — The amount of coal that can be hoisted with a rope varies greatly. Under the ordinary conditions of use a rope hoists from 5000 to 8000 tons. Where the circumstances are more favorable, the amounts run up frequently to 12,000 or 15,000 tons, occasionally to 20,000 and in one case 32,400 tons to a single fall.

When a hoisting rope is first put in use, it is likely from the strain put upon it to twist up when the block is loosened from the load. This occurs in the first day or two only. The rope should then be taken down and the "turns" taken out of the rope. When put up again the rope should give no further trouble until worn out.

It is necessary that the rope should be much larger than is needed to bear the strain from the load.

Practical experience for many years has substantially settled the most economical size of rope to be used which is given in the table below.

Hoisting ropes are not spliced, as it is difficult to make a splice that will not pull out while running over the sheaves, and the increased wear to be obtained in this way is very small.

Coal is usually hoisted with what is commonly called a "double whip;" that is, with a running block that is attached to the tub which reduces the strain on the rope to approximately one-half the weight of the load hoisted.

Hoisting rope is ordered by circumference, transmission rope by diameter.

Working Loads for Manila Rope (C. W. Hunt, *Trans. A. S. M. E.*, xlii, 125.)

| Diameter of Rope, Inches. | Ultimate Strength, Pounds. | Working Load in Pounds. | | | Minimum Diameter of Sheaves in Inches. | | |
|---------------------------|----------------------------|-------------------------|---------|-------|--|---------|-------|
| | | Rapid. | Medium. | Slow. | Rapid. | Medium. | Slow. |
| 1 | 7,100 | 200 | 400 | 1000 | 40 | 12 | 8 |
| 1 1/8 | 9,000 | 250 | 500 | 1250 | 45 | 13 | 9 |
| 1 1/4 | 11,000 | 300 | 600 | 1500 | 50 | 14 | 10 |
| 1 3/8 | 13,400 | 380 | 750 | 1900 | 55 | 15 | 11 |
| 1 1/2 | 15,800 | 450 | 900 | 2200 | 60 | 16 | 12 |
| 1 5/8 | 18,800 | 530 | 1100 | 2600 | 65 | 17 | 13 |
| 1 3/4 | 21,800 | 620 | 1250 | 3000 | 70 | 18 | 14 |

In this table the work required of the rope is, for convenience, divided into three classes — "rapid," "medium," and "slow," these terms being used in the following sense: "Slow" — Derrick, crane and quarry work; speed from 50 to 100 feet per minute. "Medium" — Wharf and cargo, hoisting 150 to 300 feet per minute. "Rapid" — 400 to 800 feet per minute.

The ultimate strength given in the table is materially affected by the age and condition of a rope in active service, and also it is said to be weaker when it is wet. Trautwine states that a few months of exposed work weakens rope 20 to 50 per cent. The ultimate strength of a new rope given in the table is the result of tests of full sized specimens of manila rope, purchased in the open market, and made by three independent rope walks.

The proper diameter of pulley-block sheaves for different classes of work given in the table is a compromise of the various factors affecting the case. An increase in the diameter of sheave will materially increase the life of a rope. The advantage, however, is gained by increased difficulty of installation, a clumsiness in handling, and an increase in first cost. The best size is one that considers the advantages and the drawbacks as they are found in practical use, and makes a fair balance between the conflicting elements of the problem.

Records covering many years have been kept by various coal dealers, of the diameter and cost of their rope per ton of coal hoisted from vessels, using sheaves of from 12 to 16 inches in diameter. These records show conclusively that, in hoisting a bucket that produces 900 pounds stress upon the rope, a 1 1/4-inch diameter rope is too small and a 1 3/4-inch rope is too large for economy. The Pennsylvania Railroad Company

uses 1 1/2-inch rope, running over 14-inch diameter sheaves for hoisting freight on lighters in New York harbor, and handle on a single part of the rope loads up to 3,000 pounds as a maximum. Greater weights are handled on a 6-part tackle.

Life of Hoisting and Transmission Rope. A rope 1 1/2-in. diam. usually hoists from a vessel from 7000 to 10,000 tons of coal, running with a working stress of 850 to 950 lbs. over three sheaves, one 12 in., and two 16-in. diam. In hoisting 10,000 tons it makes 20,000 trips, bending in that time from a straight line to the curve of the sheave 120,000 times, when it is worn out. A 1000 ft. transmission in a tin-plate mill, with 1 1/2 in. rope, sheaves 5 ft., 17 ft., and 36 ft. apart, center to center, runs 5000 ft. per minute making 13,900 bends per hour, or more bends in 9 hours than the hoisting rope made in its entire life, yet the life of a transmission rope is measured in years, not hours. This enormous difference in the life of ropes of the same size and quality is wholly gained by reducing the stresses on the rope and increasing the diameter of the sheaves.

Efficiency of Knots as a percentage of the full strength of the rope, and the factor of safety when used with the stresses given in the 5th column of the table of working loads.

| Kind of Knot. | Effy. | Fact. S |
|---|-------|---------|
| Eye splice over an iron thimble..... | 90 | 6.3 |
| Short splice in the rope..... | 80 | 5.6 |
| Timber hitch, round turn, half-hitch..... | 65 | 4.5 |
| Bowline slip knot, clove hitch..... | 60 | 4.2 |
| Square knot, weaver's knot sheet bend..... | 50 | 3.5 |
| Flemish loop, overhand knot..... | 45 | 3.1 |
| Full strength of dry rope, average of four tests..... | 100 | 7.0 |

Efficiency of Rope Tackles. Robert Grimshaw in 1893 tested a 3 3/4-in., 3-strand ordinary dry manila rope on a "cat and fish" tackle with a 6-fold purchase. The sheaves were 8-in. diam., the three upper ones having roller bearings and the three lower ones solid bushings. The results were as below:

| | | | | |
|--|-----|--------|-------|------|
| Net load on tackle, weight raised, lbs..... | 600 | 800 | 1000 | 1200 |
| Theoretical force required to raise the weight | 100 | 1333.3 | 166.7 | 200 |
| Actual force required..... | 158 | 198 | 243 | 288 |
| Percentage above the theoretical..... | 58 | 48 | 45.8 | 44 |

Weight and Strength of Manila Rope. Spencer Miller (*Eng'g News*, Dec. 6, 1890) gives a table of breaking strength of manila rope, which he considers more reliable than the strength computed by Mr. Hunt's formula: Breaking strength = $720 \times (\text{circumference in inches})^2$. Mr. Miller's formula is: Breaking weight lbs. = circumference² \times a coefficient which varies from 900 for 1/2" to 700 for 2" diameter rope, as below:

| | | | | | | | | | | | | |
|------------------|-------|-----|-------|-------|-----|-------|-------|-------|-------|-----|-------|-----|
| Circumference .. | 1 1/2 | 2 | 2 1/2 | 2 3/4 | 3 | 3 1/2 | 3 3/4 | 4 1/4 | 4 1/2 | 5 | 5 1/2 | 6 |
| Coefficient..... | 900 | 845 | 820 | 790 | 780 | 765 | 760 | 745 | 735 | 725 | 712 | 700 |

Knots. The principle of a knot is that no two parts, which would move in the same direction if the rope were to slip, should lay along side of and touching each other. (See illustrations on the next page.)

The bowline is one of the most useful knots, it will not slip, and after being strained is easily untied. Commence by making a bight in the rope, then put the end through the bight and under the standing part as shown in *G*, then pass the end again through the bight, and haul tight.

The square or reef knot must not be mistaken for the "granny" knot that slips under a strain. Knots *H*, *K* and *M* are easily untied after being under strain. The knot *M* is useful when the rope passes through an eye and is held by the knot, as it will not slip and is easily untied after being strained.

The timber hitch *S* looks as though it would give way, but it will not; the greater the strain the tighter it will hold. The wall knot looks complicated, but is easily made by proceeding as follows: Form a bight with strand 1 and pass the strand 2 around the end of it, and the strand 3 round the end of 2 and then through the bight of 1 as shown in the cut *Z*. Haul the ends taut when the appearance is as shown in *AA*. The end of the strand 1 is now laid over the center of the knot; strand 2 laid over 1 and 3 over 2, when the end of 3 is passed through the bight of 1 as shown in *BB*. Haul all the strands taut as shown in *CC*.

Varieties of Knots. — A great number of knots have been devised of which a few only are illustrated, but those selected are the most frequently used. In the cut, Fig. 84, they are shown open, or before being drawn taut, in order to show the position of the parts. The names usually given to them are:

- A. Bight of a rope.
- B. Simple or Overhand knot.
- C. Figure 8 knot.
- D. Double knot.
- E. Boat knot.
- F. Bowline, first step.
- G. Bowline, second step.
- H. Bowline completed.
- I. Square or reef knot.
- J. Sheet bend or weaver's knot.
- K. Sheet bend with a toggle.
- L. Carrick bend.
- M. Stevedore knot completed.
- N. Stevedore knot commenced.
- O. Slip knot.

- P. Flemish loop.
- Q. Chain knot with toggle.
- R. Half-hitch.
- S. Timber-hitch.
- T. Clove-hitch.
- U. Rolling-hitch.
- V. Timber-hitch and half-hitch.
- W. Blackwall-hitch.
- X. Fisherman's bend.
- Y. Round turn and half-hitch.
- Z. Wall knot commenced.
- AA. Wall knot completed.
- BB. Wall knot crown commenced.
- CC. Wall knot crown completed.

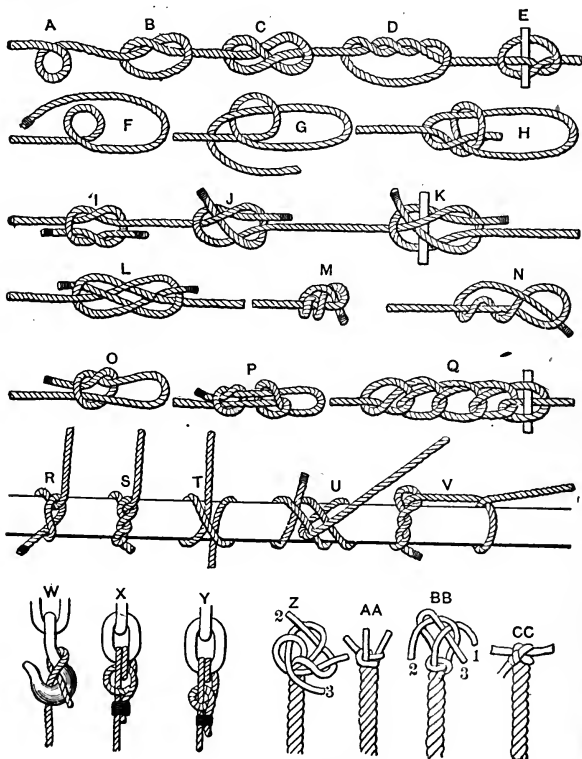


FIG. 84. — KNOTS.

To Splice a Wire Rope. — The tools required will be a small marline spike, nipping cutters, and either clamps or a small hemp-rope sling with which to wrap around and untwist the rope. If a bench-vise is accessible it will be found convenient.

In splicing rope, a certain length is used up in making the splice. An allowance of not less than 16 feet for $\frac{1}{2}$ -inch rope, and proportionately longer for larger sizes, must be added to the length of an endless rope in ordering.

Having measured, carefully, the length the rope should be after splicing, and marked the points M and M' , Fig. 85, unlay the strands from each end E and E' to M and M' and cut off the center at M and M' , and then:

(1). Interlock the six unlayed strands of each end alternately and draw them together so that the points M and M' meet, as in Fig. 86.

(2). Unlay a strand from one end, and following the unlay closely, lay into the seam or groove it opens, the strand opposite it belonging to the other end of the rope, until within a length equal to three or four times the length of one lay of the rope, and cut the other strand to about the same length from the point of meeting as at A , Fig. 87.

(3). Unlay the adjacent strand in the opposite direction, and following the unlay closely, lay in its place the corresponding opposite strand, cutting the ends as described before at B , Fig. 87.

There are now four strands laid in place terminating at A and B , with the eight remaining at MM' , as in Fig. 87.

It will be well after laying each pair of strands to tie them temporarily at the points A and B .

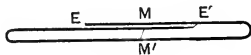


FIG. 85.

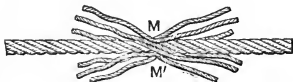


FIG. 86.

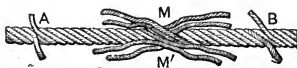


FIG. 87.

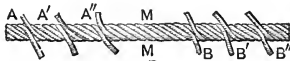
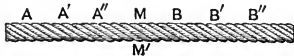


FIG. 88. SPLICING WIRE ROPE. FIG. 89.



Pursue the same course with the remaining four pairs of opposite strands, stopping each pair about eight or ten turns of the rope short of the preceding pair, and cutting the ends as before.

We now have all the strands laid in their proper places with their respective ends passing each other, as in Fig. 88.

All methods of rope-splicing are identical to this point: their variety consists in the method of tucking the ends. The one given below is the one most generally practiced.

Clamp the rope either in a vise at a point to the left of A , Fig. 88, and by a hand-clamp applied near A , open up the rope by untwisting sufficiently to cut the core at A , and seizing it with the nippers, let an assistant draw it out slowly, you following it closely, crowding the strand in its place until it is all laid in. Cut the core where the strand ends, and push the end back into its place. Remove the clamps and let the rope close together around it. Draw out the core in the opposite direction and lay the other strand in the center of the rope, in the same manner. Repeat the operation at the five remaining points, and hammer the rope lightly at the points where the ends pass each other at A , B , B , etc., with small wooden mallets, and the splice is complete, as shown in Fig. 89.

If a clamp and vise are not obtainable, two rope slings and short wooden levers may be used to untwist and open up the rope.

A rope spliced as above will be nearly as strong as the original rope and smooth everywhere. After running a few days, the splice, if well made, cannot be found except by close examination.

The above instructions have been adopted by the leading rope manufacturers of America.

SPRINGS.

Definitions.—A spiral spring is one which is wound around a fixed point or center, and continually receding from it, like a watch spring. A helical spring is one which is wound around an arbor, and at the same time advancing like the thread of a screw. An elliptical or laminated spring is made of flat bars, plates, or "leaves," of regularly varying lengths, superposed one upon the other.

Laminated Steel Springs.—Clark (Rules, Tables and Data) gives the following from his work on *Railway Machinery*, 1855:

$$\Delta = \frac{1.66 L^3}{bt^3n}; \quad s = \frac{bt^2n}{11.3 L}; \quad n = \frac{1.66 L^3}{\Delta bt^3};$$

Δ = elasticity, or deflection, in sixteenths of an inch per ton of load;

s = working strength, or load, in tons (2240 lbs.);

L = span, when loaded in inches;

b = breadth of plates, in inches, taken as uniform;

t = thickness of plates, in sixteenths of an inch;

n = number of plates.

NOTE.—1. The span and the elasticity are those due to the spring when weighted.

2. When extra thick back and short plates are used, they must be replaced by an equivalent number of plates of the ruling thickness, prior to the employment of the first two formulæ. This is found by multiplying the number of extra thick plates by the cube of their thickness, and dividing by the cube of the ruling thickness. Conversely, the number of plates of the ruling thickness given by the third formula, required to be deducted and replaced by a given number of extra thick plates, are found by the same calculation.

3. It is assumed that the plates are similarly and regularly formed, and that they are of uniform breadth, and but slightly taper at the ends.

Reuleaux's Constructor gives for semi-elliptic springs:

$$P = \frac{Snbh^2}{6l} \quad \text{and} \quad f = \frac{6Pl^3}{Enbh^3};$$

S = max. direct fiber-strain in plate;

n = number of plates in spring;

l = one-half length of spring;

P = load on one end of spring;

b = width of plates;

h = thickness of plates;

f = deflection of end of spring;

E = modulus of direct elasticity

The above formula for deflection can be relied upon where all the plates of the spring are regularly shortened; but in semi-elliptic springs, as used, there are generally several plates extending the full length of the spring, and the proportion of these long plates to the whole number is usually about one-fourth. In such cases $f = \frac{5.5 Pl^3}{Enbh^3}$. (G. R. Henderson,

Trans. A. S. M. E., vol. xvi.)

In order to compare the formulæ of Reuleaux and Clark we may make the following substitutions in the latter: s in tons = P in lbs. \div 1120; $\Delta s = 16 f$; $L = 2 l$; $t = 16 h$; then

$$\Delta s = 16 f = \frac{1.66 \times 8 l^3 \times P}{4096 \times 1120 \times nbh^3}, \quad \text{whence} \quad f = \frac{Pl^3}{5,527,133},$$

which corresponds with Reuleaux's formula for deflection if in the latter we take $E = 33,162,800$.

$$\text{Also} \quad s = \frac{P}{1120} = \frac{256 nbh^2}{11.3 \times 2l}, \quad \text{whence} \quad P = \frac{12,687 nbh^2}{l},$$

which corresponds with Reuleaux's formula for working load when S in the latter is taken at 76,120.

The value of E is usually taken at 30,000,000 and S at 80,000, in which case Reuleaux's formulæ become

$$[P = \frac{13,333 \, nbh^2}{l} \quad \text{and} \quad f = \frac{Pl^3}{5,000,000nbh^3}.$$

G. R. Henderson, in *Trans. A. S. M. E.*, vol. xvii, gives a series of tables for use in designing both elliptical and helical springs.

Helical Steel Springs.

NOTATION. Let d = diam. of wire or rod of which the spring is made.

D = outside diameter of coil, inches.

R = mean radius of coil, = $\frac{1}{2}(D - d)$.

n = number of coils.

P = load applied to the spring, lbs.

G = modulus of torsional elasticity.

S = stress on extreme fiber caused by load P .

F = extension or compression of one coil, in., for load P .

Fn = total extension or compression, for load P .

W = safe carrying capacity of spring, lbs.

$$F = \frac{64 PR^3}{Gd^4}; \quad Fn = \frac{64 PR^3n}{Gd^4}; \quad W = \frac{0.1963 Sd^3}{R} = \frac{\pi Sd^3}{16 R}.$$

Values of G according to different authorities range from 10,000,000 to 14,000,000.

The safe working value commonly taken for S = 60,000 lbs. per sq. in.

Taking G at 12,000,000 and S at 60,000 the above formulæ become

$$F = \frac{PR^3}{187,500 d^4}, \quad W = 11,781 \frac{d^3}{R}. \quad \text{If } P = W, \text{ then } F = 0.06285 \frac{R^2}{d}.$$

For square steel the values found for F and W are to be multiplied by 0.59 and 1.2 respectively, d being the side of the square.

The stress in a helical spring is almost wholly one of torsion. For method of deriving the formulæ for springs from torsional formulæ see paper by J. W. Cloud, *Trans. A. S. M. E.*, vol. 173. Mr. Cloud takes S = 80,000 and G = 12,600,000.

Taking from the Pennsylvania Railroad Specifications (1891) the capacity when closed, W_1 , of the following springs, and the total compression when closed $H - h$, in which H = height when free and h when closed, and assuming $n = h \div d$, we have the following comparison of the specified values of capacity and compression with those obtained from the formulæ.

| No. | d , in. | D | $D - d$ | W_1 | W | H | h | $H - h$ | Fn | n |
|-----|----------------|----------------|----------------|-------|------|-----------------|----------------|----------------|------|----------------|
| T. | $\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{1}{4}$ | 400 | 295 | 9 | 6 | 3 | 3.20 | 24 |
| S. | $\frac{1}{2}$ | 3 | $2\frac{1}{2}$ | 1900 | 1178 | 8 | 5 | 3 | 3.16 | 10 |
| K. | $\frac{3}{4}$ | $5\frac{3}{4}$ | 5 | 2100 | 1988 | 7 | $4\frac{1}{4}$ | $2\frac{3}{4}$ | 3.15 | $5\frac{2}{3}$ |
| D. | 1 | 5 | 4 | 8100 | 5890 | $10\frac{1}{2}$ | 8 | $2\frac{1}{2}$ | 2.76 | 8 |
| I. | $1\frac{1}{4}$ | 8 | $6\frac{3}{4}$ | 10000 | 6788 | 9 | $5\frac{3}{4}$ | $3\frac{1}{4}$ | 3.86 | $4\frac{3}{5}$ |
| C. | $1\frac{1}{8}$ | $4\frac{7}{8}$ | $3\frac{3}{4}$ | 16000 | 8946 | $4\frac{3}{8}$ | $3\frac{3}{8}$ | 1 | 1.05 | 3 |

The value of Fn in the table is calculated from the formula with $P = W_1$. Wilson Hartnell (*Proc. Inst. M. E.*, 1882, p. 426), says: "The size of a spiral spring may be calculated from the formula on page 304 of 'Rankine's Useful Rules and Tables;' but the experience with Salter's springs has shown that the safe limit of stress is more than twice as great as there given, namely 60,000 to 70,000 lbs. per square inch of section with $\frac{3}{8}$ -inch wire, and about 50,000 with $\frac{1}{2}$ -inch wire. Hence the work that can be done by springs of wire is four or five times as great as Rankine allows.

For $\frac{3}{8}$ -inch wire and under,

$$\text{Maximum load in lbs.} = \frac{12,000 \times (\text{diam. of wire})^3}{\text{Mean radius of springs}};$$

$$\text{Weight in lbs. to deflect spring 1 in.} = \frac{180,000 \times (\text{diam.})^4}{\text{Number of coils} \times (\text{rad.})^3}.$$

The work in foot-pounds that can be stored up in a spiral spring would lift it above 50 ft.

In a few rough experiments made with Salter's springs the coefficient of rigidity was noticed to be 12,600,000 to 13,700,000 with $\frac{1}{4}$ -inch wire; 11,000,000 for $\frac{11}{32}$ inch; and 10,600,000 to 10,900,000 for $\frac{3}{8}$ -inch wire.

Helical Springs. — J. Begtrup, in the *American Machinist* of Aug. 18, 1892, gives formulas for the deflection and carrying capacity of helical springs of round and square steel, as follow:

$$W = 0.3927 \frac{Sd^3}{D - d}, \quad F = 8 \frac{P(D - d)^3}{Ed^4}, \quad \text{for round steel.}$$

$$W = 0.471 \frac{Sd^3}{D - d}, \quad F = 4.712 \frac{P(D - d)^3}{Ed^4}, \quad \text{for square steel.}$$

W = carrying capacity in pounds,
 S = greatest shearing stress per square inch of material,
 d = diameter of steel,
 D = outside diameter of coil,
 F = deflection of one coil,
 E = torsional modulus of elasticity,
 P = load in pounds.

From these formulas the following table has been calculated by Mr. Begtrup. A spring being made of an elastic material, and of such shape as to allow a great amount of deflection, will not be affected by sudden shocks or blows to the same extent as a rigid body, and a factor of safety very much less than for rigid constructions may be used.

HOW TO USE THE TABLE.

When designing a spring for continuous work, as a car spring, use a greater factor of safety than in the table; for intermittent working, as in a steam-engine governor or safety valve, use figures given in table; for square steel multiply line W by 1.2 and line F by 0.59.

Example 1. — How much will a spring of $\frac{3}{8}$ " round steel and 3" outside diameter carry with safety? In the line headed D we find 3, and right underneath 473, which is the weight it will carry with safety. How many coils must this spring have so as to deflect 3" with a load of 400 pounds? Assuming a modulus of elasticity of 12 millions we find in the line headed F the figure 0.0610; this is deflection of one coil for a load of 100 pounds; therefore $0.061 \times 4 = 0.244$ " is deflection of one coil for 400 pounds load, and $3 \div 0.244 = 12\frac{1}{2}$ is the number of coils wanted. This spring will therefore be $4\frac{3}{4}$ " long when closed, counting working coils only, and stretch to $7\frac{3}{4}$ ".

Example 2. — A spring $3\frac{1}{4}$ " outside diameter of $\frac{7}{16}$ " steel is wound close; how much can it be extended without exceeding the limit of safety? We find maximum safe load for this spring to be 702 pounds, and deflection of one coil for 100 pounds load 0.0405 inches; therefore $7.02 \times 0.0405 = 0.284$ " is the greatest admissible opening between coils. We may thus, without knowing the load, ascertain whether a spring is overloaded or not.

Carrying Capacity and Deflection of Helical Springs of Round Steel.

d = diameter of steel. D = outside diameter of coil. W = safe working load in pounds — tensile stress not exceeding 60,000 pounds per square inch. F = deflection by a load of 100 pounds of one coil, with a modulus of elasticity of 12 millions. The ultimate carrying capacity will be about twice the safe load. (The original table gives three values

of F , corresponding respectively to a modulus of elasticity of 10, 12 and 14 millions. To find values of F for 10 million modulus increase the figures here given by one-sixth; for 14 million subtract one-sixth.)

| | | | | | | | | | | | | |
|------------|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| d in. | D | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | | | |
| .065 | W | 35 | 15 | 9 | 7 | 5 | 4.5 | 3.8 | 3.3 | | | |
| | F | 0.0236 | 0.3075 | 1.228 | 3.053 | 6.214 | 11.04 | 17.87 | 27.06 | | | |
| .120 | D | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | | |
| | W | 107 | 65 | 46 | 36 | 29 | 25 | 22 | 19 | 17 | | |
| | F | 0.0176 | 0.0804 | 0.2191 | 0.4639 | 0.8448 | 1.392 | 2.136 | 3.107 | 4.334 | | |
| .180 | D | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | |
| | W | 241 | 167 | 128 | 104 | 88 | 75 | 66 | 59 | 53 | 49 | |
| | F | 0.0118 | 0.0350 | 0.0778 | 0.1460 | 0.2457 | 0.3828 | 0.5632 | 0.7928 | 1.077 | 1.423 | |
| 1/4 | D | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | |
| | W | 368 | 294 | 245 | 210 | 184 | 164 | 147 | 134 | 123 | 113 | |
| | F | 0.0171 | 0.0333 | 0.0576 | 0.0914 | 0.1365 | 0.1944 | 0.2665 | 0.3548 | 0.4607 | 0.5859 | |
| 5/16 | D | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 |
| | W | 605 | 500 | 426 | 371 | 329 | 295 | 267 | 245 | 226 | 209 | 195 |
| | F | 0.0117 | 0.0207 | 0.0336 | 0.0508 | 0.0732 | 0.1012 | 0.1357 | 0.1771 | 0.2263 | 0.2839 | 0.3503 |
| 3/8 | D | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 |
| | W | 765 | 663 | 589 | 523 | 473 | 433 | 398 | 368 | 343 | 321 | 301 |
| | F | 0.0145 | 0.0222 | 0.0323 | 0.0452 | 0.0610 | 0.0801 | 0.1029 | 0.1297 | 0.1606 | 0.1963 | 0.2367 |
| 7/16 | D | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.50 | 5.00 |
| | W | 1263 | 1089 | 957 | 853 | 770 | 702 | 644 | 596 | 544 | 486 | 432 |
| | F | 0.0069 | 0.0108 | 0.0160 | 0.0225 | 0.0306 | 0.0405 | 0.0529 | 0.0661 | 0.0823 | 0.1220 | 0.1728 |
| 1/2 | D | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.50 | 5.00 |
| | W | 1963 | 1683 | 1472 | 1309 | 1178 | 1071 | 982 | 906 | 841 | 736 | 654 |
| | F | 0.0036 | 0.0057 | 0.0085 | 0.0121 | 0.0167 | 0.0222 | 0.0288 | 0.0366 | 0.0457 | 0.0683 | 0.0972 |
| 9/16 | D | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 5.00 | 5.50 |
| | W | 2163 | 1916 | 1720 | 1560 | 1427 | 1315 | 1220 | 1137 | 1065 | 945 | 849 |
| | F | 0.0048 | 0.0070 | 0.0096 | 0.0129 | 0.0169 | 0.0216 | 0.0271 | 0.0334 | 0.0406 | 0.0582 | 0.0801 |
| 5/8 | D | 2.50 | 2.75 | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 5.00 | 5.50 |
| | W | 3068 | 2707 | 2422 | 2191 | 2001 | 1841 | 1704 | 1587 | 1484 | 1315 | 1180 |
| | F | 0.0029 | 0.0042 | 0.0058 | 0.0079 | 0.0104 | 0.0133 | 0.0168 | 0.0208 | 0.0254 | 0.0366 | 0.0506 |
| 11/16 | D | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 5.50 | 6.00 |
| | W | 3311 | 2988 | 2723 | 2500 | 2311 | 2151 | 2009 | 1885 | 1776 | 1591 | 1441 |
| | F | 0.0037 | 0.0050 | 0.0066 | 0.0086 | 0.0108 | 0.0135 | 0.0165 | 0.0200 | 0.0239 | 0.0333 | 0.0447 |
| 3/4 | D | 3.00 | 3.25 | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 5.50 | 6.003 |
| | W | 4418 | 3976 | 3615 | 3313 | 3058 | 2840 | 2651 | 2485 | 2339 | 2093 | 1893 |
| | F | 0.0024 | 0.0033 | 0.0044 | 0.0057 | 0.0072 | 0.0090 | 0.0111 | 0.0135 | 0.0162 | 0.0226 | 0.005 |
| 7/8 | D | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 5.25 | 5.50 | 6.00 | 6.50 |
| | W | 6013 | 5490 | 5051 | 4676 | 4354 | 4073 | 3826 | 3607 | 3413 | 3080 | 2806 |
| | F | 0.0018 | 0.0024 | 0.0030 | 0.0038 | 0.0047 | 0.0058 | 0.0070 | 0.0083 | 0.0098 | 0.0134 | 0.0177 |
| 1 | D | 3.50 | 3.75 | 4.00 | 4.25 | 4.50 | 4.75 | 5.00 | 5.25 | 5.50 | 6.00 | 6.50 |
| | W | 9425 | 8568 | 7854 | 7250 | 6732 | 6283 | 5890 | 5544 | 5236 | 4712 | 4284 |
| | F | 0.0010 | 0.0014 | 0.0018 | 0.0023 | 0.0028 | 0.0035 | 0.0043 | 0.0051 | 0.0061 | 0.0083 | 0.0111 |

F. D. Howe, *Am. Mach.*, Dec. 20, 1906, using Begtrup's formulæ, computes a table for springs made from wire of Roebling's or Washburn and Moen gauges, Nos. 28 to 000. It is here given somewhat abridged, values of F corresponding to a torsional modulus of elasticity of 12,000,000 only being used.

| | | | | | | | | | | | |
|-------------------|-------------|-------------------------|-------------------------|------------------------|-------------------------|------------------------|------------------------|------------------------|-----------------------|-----------------------|-----------------------|
| No. 28 0.016" | D W F | 0.20 0.524 6.32 | 0.25 0.41 13.02 | 0.3125 0.31 30.2 | 0.375 0.27 47.0 | 0.4375 0.23 76.0 | 0.500 0.20 115 | 0.5625 0.175 166 | 0.625 0.16 230 | 0.75 0.13 402 | 0.875 0.11 695 |
| No. 24 0.0225" | D W F | 0.25 1.18 2.78 | 0.3125 0.92 6.31 | 0.375 0.76 11.35 | 0.4375 0.45 18.57 | 0.500 0.56 28.2 | 0.5625 0.50 40.8 | 0.625 0.45 56.9 | 0.75 0.37 97.5 | 0.875 0.31 166 | 1.00 0.28 242 |
| No. 22 0.028" | D W F | 0.25 2.35 1.19 | 0.3125 1.84 2.50 | 0.375 1.49 4.53 | 0.4375 1.26 7.42 | 0.50 1.095 11.40 | 0.5625 0.96 16.5 | 0.625 0.865 23.1 | 0.75 0.715 40.8 | 0.875 0.61 66.0 | 1.00 0.53 99.5 |
| No. 20 0.035" | D W F | 0.25 4.7 0.451 | 0.3125 3.64 0.952 | 0.375 2.97 1.75 | 0.4375 2.5 2.90 | 0.50 2.18 4.47 | 0.5625 1.92 6.51 | 0.625 1.72 9.14 | 0.75 1.42 16.3 | 0.875 1.20 26.4 | 1.00 1.05 40.0 |
| No. 18 0.047" | D W F | 0.25 12.05 0.1158 | 0.3125 9.2 0.294 | 0.375 74.5 0.488 | 0.4375 6.57 0.824 | 0.50 5.40 1.320 | 0.625 4.23 1.870 | 0.75 3.48 3.96 | 0.875 2.95 7.85 | 1.00 2.85 12.60 | 1.125 2.27 17.5 |
| No. 14 0.08" | D W F | 0.375 41 0.0418 | 0.5 28.8 0.128 | 0.625 22.2 0.342 | 0.75 18.1 0.572 | 0.875 15.2 0.82 | 1.00 13.15 1.27 | 1.125 11.6 1.86 | 1.25 10.35 2.60 | 1.50 8.52 5.48 | 1.75 7.25 7.57 |
| No. 12 0.105" | D W F | 0.625 52.5 0.069 | 0.75 42.25 0.1480 | 0.875 35.4 0.262 | 1.00 30.4 0.395 | 1.25 2.38 0.830 | 1.50 19.5 1.49 | 1.75 16.6 2.45 | 2.00 14.4 3.74 | 2.25 12.7 5.45 | 2.50 11.4 7.34 |
| No. 10 0.135" | D W F | 0.875 77 0.081 | 1.00 67 0.135 | 1.25 52 0.276 | 1.50 42.5 0.512 | 1.75 36 0.846 | 2.00 31 1.295 | 2.25 27 1.910 | 2.50 24 2.660 | 2.75 22 3.58 | 3.00 20 4.75 |
| No. 8 0.162" | D W F | 1.00 120 0.0570 | 1.25 98.5 0.124 | 1.50 76 0.199 | 1.75 64 0.554 | 2.00 55.5 0.597 | 2.25 48.8 0.880 | 2.50 43.5 1.26 | 2.75 39 1.68 | 3.00 36 2.20 | 3.25 33 2.85 |
| No. 7 0.177" | D W F | 1.00 159 0.0382 | 1.25 122 0.0828 | 1.50 99 0.156 | 1.75 83.5 0.265 | 2.00 72 0.416 | 2.25 63 0.603 | 2.50 56.4 0.830 | 2.75 51 1.15 | 3.00 46.5 1.54 | 3.25 42.5 1.96 |
| No. 6 0.192" | D W F | 1.25 158 0.0572 | 1.50 128 0.108 | 1.75 107 0.185 | 2.00 92.5 0.284 | 2.25 81 0.420 | 2.50 72 0.590 | 2.75 65 0.802 | 3.00 59.5 1.07 | 3.25 55.5 1.38 | 3.50 50 1.74 |
| No. 5 0.205" | D W F | 1.50 155 0.0820 | 1.75 131 0.139 | 2.00 113 0.218 | 2.25 99 0.321 | 2.50 88.5 0.412 | 2.75 80 0.6175 | 3.00 70 0.82 | 3.25 67 1.60 | 3.50 61.5 1.34 | 4.00 53.5 2.22 |
| No. 4 0.225" | D W F | 1.50 210 0.0536 | 1.75 175 0.093 | 2.00 150 0.147 | 2.25 132 0.220 | 2.50 118 0.303 | 2.75 106 0.412 | 3.00 97 0.652 | 3.25 89 0.715 | 3.50 82 0.91 | 4.00 71 1.30 |
| No. 2 0.263" | D W F | 1.50 345 0.0264 | 1.75 290 0.0458 | 2.00 250 0.0730 | 2.25 215 0.109 | 2.50 192 0.154 | 2.75 175 0.214 | 3.00 156 0.274 | 3.25 146 0.371 | 3.50 134 0.469 | 4.00 115 0.720 |
| No. 1 0.283" | D W F | 1.75 360 0.0328 | 2.00 310 0.0550 | 2.25 270 0.0778 | 2.50 240 0.112 | 2.75 215 0.155 | 3.00 195 0.208 | 3.25 180 0.270 | 3.50 165 0.344 | 4.00 145 0.530 | 4.50 127 0.775 |
| No. 0 0.307" | D W F | 1.75 470 0.0308 | 2.00 400 0.0380 | 2.25 350 0.0548 | 2.50 310 0.0788 | 2.75 280 0.109 | 3.00 250 0.149 | 3.25 230 0.199 | 3.50 212 0.244 | 4.00 185 0.327 | 4.50 162 0.550 |
| No. 00 0.331" | D W F | 2.00 510 0.0289 | 2.25 445 0.0388 | 2.50 390 0.0564 | 2.75 350 0.0780 | 3.00 320 0.105 | 3.25 290 0.137 | 3.50 270 0.176 | 4.00 230 0.273 | 4.50 205 0.414 | 5.00 183 0.562 |

To find deflection of one coil by one pound, divide the values of F by 100.

ELLIPTICAL SPRINGS, SIZES, AND PROOF TESTS.

Pennsylvania Railroad Specifications, 1896.

| Class. | Length between Centers, Ins. | Width over all, Inches. | Plates, No. Size, In | Tests. | | | | | |
|---------------------|------------------------------|-------------------------|----------------------|------------|--------|--------|-----------|--------|-------------------|
| | | | | Ins. high. | | lbs. | Ins. lbs. | | a. p. t. (a) ins. |
| | | | | (a) | (b) | | (a) | | |
| E 1, Triple..... | 40 | 11 3/4 | 5 3 × 11/32 | 3 3/4 | 9 3/8 | 4,800 | 3 | 5,500 | 2 |
| E 2, Quadruple.. | 40 | 15 1/2 | 5 3 × 3/8 | 3 3/4 | 9 3/4 | 6,650 | 3 | 8,000 | 2 |
| E 3, Triple..... | 36 | 11 3/4 | 6 3 × 11/32 | 4 | 9 5/8 | 6,000 | 3 | 8,000 | |
| E 4, Single †..... | 40 | | 8 3 × 11/32 | 5* | | free | 3 | 2,350 | |
| E 5, " †..... | 40 | | 7 3 × 3/8 | 15/16* | | 3,000 | 0 | 4,970 | |
| E 6, " †..... | 42 | | 8 3 1/2 × 3/8 | 1 1/8* | | 4,375 | 0 | 6,350 | |
| E 7, Triple..... | 36 | 11 3/4 | 8 3 × 11/32 | 2 1/2 | 9 1/2 | 11,800 | | | |
| E 8, Double..... | 32 | 7 1/2 | 6 3 × 3/8 | 3 | 9 | 8,000 | | | |
| E 9, "..... | 36 | 9 1/2 | 5 4 × 11/32 | 3 1/2 | 8 7/16 | 5,400 | 3 | 6,000 | |
| E 10, Quadruple.. | 40 | 15 1/2 | 5 3 × 3/8 | 4 | 10 | 8,000 | 3 | 10,000 | 2 |
| E 11, "..... | 40 | 15 1/2 | 5 3 × 3/8 | 3 3/4 | 9 3/4 | 10,600 | 3 | 12,200 | 2 |
| E 12, "..... | 34 | 15 1/2 | 5 3 × 3/8 | 3 3/4 | 9 3/4 | 13,100 | 3 | 15,780 | 2 |
| E 13, Double..... | 30 | 9 1/2 | 5 4 × 3/8 | 3 3/4 | 9 | 5,600 | 2 | 10,600 | |
| E 14, "..... | 40 | 9 1/2 | 6 4 × 11/32 | 3 3/8 | 9 | 6,840 | 2 | 8,600 | |
| E 15, Quadruple.. | 36 | 15 1/2 | 6 3 × 11/32 | 3 7/16 | 9 3/4 | 11,820 | 2 1/2 | 14,370 | 2 |
| E 16, "..... | 30 | 15 1/2 | 6 3 × 11/32 | 4 1/2 | 10 1/8 | 8,000 | 2 3/4 | 15,500 | |
| E 17, Double..... | 36 | 9 1/2 | 5 4 × 3/8 | 2 3/4 | 8 | 8,070 | 2 | 9,540 | |
| E 18, Single †..... | 42 | | 9 3 1/2 × 3/8 | 1* | | 5,250 | 0 | 7,300 | |
| E 19, Double..... | 22 | 10 1/2 | 6 4 1/2 × 11/32 | 13/16 | 6 7/16 | 13,800 | | | |
| E 20, "..... | 22 | 10 1/2 | 7 4 1/2 × 11/32 | 13/16 | 7 1/8 | 15,600 | | | |
| E 21, "..... | 24 | 10 1/2 | 7 4 1/2 × 3/8 | 1 | 7 1/4 | 15,750 | 0 | 28,800 | |
| E 22, "..... | 24 | 10 1/2 | 8 4 1/2 × 3/8 | 1 | 8 1/2 | 18,000 | 0 | 32,930 | |
| E 23, "..... | 36 | 10 | 5 4 × 3/8 | 2 1/4 | 8 | 8,750 | 1 1/4 | 10,750 | |
| E 24, "..... | 36 | 10 | 5 4 × 3/8 | 2 1/4 | 8 | 7,500 | 1 1/4 | 9,500 | |

(a) Between bands; (b) over all; a.p.t., auxiliary plates touching.

* Between bottom of eye and top of leaf. † Semi-elliptical.

Tracings are furnished for each class of spring.

SPRINGS TO RESIST TORSIONAL FORCE.

(Reuleaux's Constructor.)

$$\begin{aligned}
 \text{Flat spiral or helical spring } P &= \frac{S b h^2}{6 R}; & f = R\vartheta &= 12 \frac{P l R^2}{E b h^3}. \\
 \text{Round helical spring.....} P &= \frac{S \pi d^3}{32 R}; & f = R\vartheta &= \frac{64 P l R^2}{\pi E d^4}. \\
 \text{Round bar, in torsion.....} P &= \frac{S \pi d^3}{16 R}; & f = R\vartheta &= \frac{32 P R^2 l}{\pi G d^4}. \\
 \text{Flat bar, in torsion.....} P &= \frac{S}{3 R} \frac{b^2 h^2}{\sqrt{b^2 + h^2}}; & f = R\vartheta &= \frac{3 P R^2 l}{G} \frac{b^2 + h^2}{b^3 h^3}.
 \end{aligned}$$

P = force applied at end of radius or lever-arm R ; ϑ = angular motion at end of radius R ; S = permissible maximum stress, = $4/5$ of permissible stress in flexure; E = modulus of elasticity in tension; G = torsional modulus, = $2/5 E$; l = developed length of spiral, or length of bar; d = diameter of wire; b = breadth of flat bar; h = thickness.

Compare Elastic Resistance to Torsion, p. 311.

HELICAL SPRINGS—SIZES AND CAPACITIES.

(Selected from Specifications of Penna. R. R. Co., 1899.)

| P. R. R. Co.'s Class. | Diam. of Bar, ins. | Length of Bar, ins. | Tapered to ins. | Normal Weight. | Outside Diam. of Coil, ins. | Test. Height and Loads. | | | | Capacity of Single Coil, lbs. |
|--------------------------|--------------------|------------------------|-----------------|----------------|--------------------------------|----------------------------|-------------|-----------|--------------|----------------------------------|
| | | | | | | Free, ins. | Solid, ins. | Ins. with | Load of lbs. | |
| H 26 | 9/64 | 571/2 | 59 | lbs. oz. | 1 | 53/4 | 3 | 31/4 | 110 | 130 |
| H 18 | 11/64 | 75 | 761/4 | 0 4 | 1 | 8 | 5 | 6 | 170 | 270 |
| H 55 | 3/16 | 451/8 | 465/16 | 0 55/8 | 1 | 41/2 | 35/16 | 4 | 103 | 245 |
| H 73 | 3/16 | 426 | 4273/4 | 3 51/2 | 15/16 | 39 | 221/2 | 35 | 45 | 185 |
| H 29 | 7/32 | 201/2 | 227/16 | 0 31/2 | 115/32 | 111/16 | 19/64 | 13/8 | 110 | 200 |
| H 1 | 1/4 | 451/2 | 47 | 0 10 | 11/4 | 51/8 | 35/8 | 43/8 | 250 | 500 |
| H 5 | 1/4 | 251/4 | 281/4 | 0 6 | 21/4 | 21/4 | 11/8 | 11/2 | 164 | 240 |
| H 58 | 5/16 | 2531/2 | 2561/2 | 5 7 | 21/4 | 23 | 13 | 18 | 248 | 495 |
| H 74 | 5/16 | 180 | 1821/8 | 3 141/2 | 111/16 | 191/8 | 13 | 141/8 | 587 | 700 |
| H 68,* | 3/8 | 991/2 | 1031/4 | 3 11/2 | 23/4 | 9 | 5 | 7 | 350 | 700 |
| H 79 | 3/8 | 88 | 903/4 | 2 12 | 21/8 | 85/8 | 6 | 63/4 | 676 | 946 |
| H 80 ₂ | 13/32 | 1923/8 | 1953/4 | 7 11/2 | 29/16 | 18 | 119/16 | 151/2 | 380 | 975 |
| H 43 | 7/16 | 96 | 1025/16 | 4 1 | 47/16 | 815/16 | 33/8 | 51/8 | 450 | 660 |
| H 64 | 7/16 | 755/8 | 781/2 | 3 3 | 29/32 | 75/8 | 55/8 | 53/4 | 1350 | 1440 |
| H 53 ₂ | 15/32 | 1695/16 | 1729/16 | 8 4 | 217/32 | 161/2 | 121/4 | 151/2 | 330 | 1410 |
| H 27 ₂ | 1/2 | 903/4 | 951/8 | 5 0 | 31/4 | 81/2 | 51/4 | 63/4 | 810 | 1500 |
| H 61 | 1/2 | 151/2 | 213/8 | 0 133/4 | 41/4 | 13/8 | 05/8 | 1 | 532 | 1050 |
| H 19 | 17/32 | 811/2 | 851/2 | 5 2 | 31/32 | 8 | 59/16 | 67/16 | 1200 | 1900 |
| H 86 ₃ | 17/32 | 1535/8 | 159 | 9 10 | 4 | 133/4 | 71/2 | 87/16 | 1156 | 1360 |
| H 6 ₃ | 9/16 | 98 | 103 | 6 15 | 33/4 | 91/8 | 51/2 | 7 | 1050 | 1800 |
| H 33 ₃ | 9/16 | 801/4 | 847/8 | 5 101/2 | 31/4 | 8 | 53/8 | 613/16 | 1000 | 2200 |
| H 59 ₂ | 5/8 | 741/4 | 773/4 | 6 7 | 27/8 | 81/4 | 69/16 | 71/4 | 2100 | 3500 |
| H 80 ₁ | 5/8 | 1921/2 | 1973/4 | 16 11 | 315/16 | 18 | 119/16 | 151/2 | 900 | 2315 |
| H 72 ₂ | 21/32 | 601/8 | 631/2 | 5 117/8 | 23/4 | 75/16 | 6 | 63/8 | 3260 | 4240 |
| H 15 ₂ | 11/16 | 557/8 | 593/4 | 5 14 | 31/2 | 53/4 | 45/16 | 53/16 | 1400 | 3500 |
| H 41 | 11/16 | 1171/2 | 1231/2 | 12 10 | 41/2 | 107/8 | 63/4 | 85/8 | 1500 | 2720 |
| H 40 | 3/4 | 1771/2 | 1865/8 | 22 21/2 | 61/2 | 16 | 73/8 | 87/8 | 1900 | 2300 |
| H 70 | 3/4 | 62 | 66 | 7 12 | 33/8 | 7 | 55/8 | 61/4 | 2750 | 5050 |
| H 17 ₂ | 13/16 | 100 | 1063/4 | 14 12 | 51/8 | 91/8 | 6 | 75/8 | 1700 | 3700 |
| H 66 ₂ | 13/16 | 1051/4 | 1103/8 | 15 7 | 45/32 | 107/8 | 81/8 | 87/8 | 3670 | 5040 |
| H 37 | 27/32 | 77 | 817/8 | 12 21/2 | 315/16 | 81/2 | 611/16 | 71/2 | 3300 | 6250 |
| H 87 ₂ | 27/32 | 13013/16 | 13715/16 | 20 9 | 53/8 | 121/4 | 73/4 | 87/16 | 3540 | 4165 |
| H 12 ₂ | 7/8 | 85 | 911/2 | 14 7 | 5 | 81/2 | 53/4 | 73/8 | 2000 | 5200 |
| H 33 ₂ | 7/8 | 82 | 8811/16 | 13 15 | 51/8 | 8 | 53/8 | 613/16 | 2250 | 5000 |
| H 2 | 15/16 | 46 | 523/8 | 8 151/4 | 5 | 45/8 | 33/8 | 4 | 3250 | 7000 |
| H 16 | 15/16 | 85 | 927/8 | 16 10 | 6 | 8 | 5 | 6 | 3600 | 5100 |
| H 10 | 1 | 85 | 92 | 18 14 | 51/2 | 81/2 | 6 | 7 | 4500 | 7000 |
| H 42 ₁ | 1 | 36 | 427/8 | 8 0 | 53/8 | 35/8 | 25/8 | 33/8 | 1795 | 7180 |
| H 4 | 11/16 | 987/8 | 105 | 24 12 | 5 | 107/8 | 81/2 | 93/8 | 6000 | 9570 |
| H 86 ₁ | 11/16 | 1535/8 | 1641/2 | 38 9 | 8 | 133/4 | 71/2 | 87/16 | 4624 | 5440 |
| H 3 | 11/8 | 353/8 | 411/4 | 9 15 | 47/8 | 41/8 | 33/8 | 33/4 | 6000 | 12000 |
| H 14 ₁ | 11/8 | 51 | 587/8 | 14 4 | 61/8 | 51/8 | 311/16 | 43/16 | 5000 | 8950 |
| H 6 ₁ | 13/16 | 991/8 | 1093/4 | 31 1 | 8 | 91/8 | 51/2 | 7 | 4550 | 7750 |
| H 47 | 13/16 | 731/2 | 791/2 | 23 0 | 57/16 | 81/4 | 69/16 | 71/4 | 7400 | 12500 |
| H 9 | 11/4 | 971/2 | 108 | 33 12 | 8 | 9 | 53/4 | 71/2 | 4000 | 9100 |
| H 72 ₁ | 11/4 | 621/8 | 683/4 | 21 81/2 | 53/8 | 75/16 | 6 | 63/8 | 10700 | 14875 |
| H 8 | 15/16 | 96 | 1061/2 | 36 12 | 8 | 91/8 | 6 | 71/4 | 6350 | 10600 |
| H 62 | 15/16 | 70 | 771/16 | 26 12 | 513/16 | 8 | 61/2 | 71/4 | 7900 | 15800 |
| H 12 ₁ | 13/8 | 87 | 973/8 | 36 7 | 8 | 81/2 | 53/4 | 73/8 | 5000 | 12200 |
| H 39 ₁ | 13/8 | 755/8 | 831/2 | 31 11 | 63/8 | 83/8 | 65/8 | 71/2 | 8150 | 16300 |
| H 28 ₁ | 113/32 | 8411/16 | 95 | 37 3 | 8 | 81/4 | 53/4 | 67/8 | 7325 | 13250 |

* The subscript 1 means the outside coil of a concentric group or cluster; 2 and 3 are inner coils.

Phosphor-Bronze Springs. Wilfred Lewis (Engs' Club, Phila., 1887) made some tests of a helical spring of phosphor-bronze wire, 0.12 in. diameter, $1\frac{1}{4}$ in. diameter from center to center, making 52 coils.

Such a spring of steel, according to the practice of the P. R. R., might be used for 40 lbs. A load of 30 lbs. gradually applied gave a permanent set. With a load of 21 lbs. in 30 hours the spring lengthened from $20\frac{3}{8}$ inches to $21\frac{1}{8}$ inches, and in 200 hours to $21\frac{1}{4}$ inches. It was concluded that 21 lbs. was too great for durability. For a given load the extension of the bronze spring was just double the extension of a similar steel spring, that is, for the same extension the steel spring is twice as strong.

Chromium-Vanadium Spring Steel. (*Proc. Inst. M. E.*, 1904, pp 1263, 1305.) — A spring steel containing C, 0.44; Si, 0.173; Mn, 0.837; Cr, 1.044; Va, 0.188 was made into a spring with dimensions as follows: length unstretched 9.6 in., mean diam. of coils (D) 5.22; No. of coils (n) 4; diam. of wire, (d) 0.561. It was tempered in the usual way. When stretched it showed signs of permanent set at about 1900 lbs. Compared with two springs of ordinary steels the following formulæ are obtained:

Load at which Permanent Set begins. Extension for a load W .

Chrome-Vanadium Spring... $56,300 d^3/D$ lbs. $WnD^3 \div 1,468,000 d^4$

West Bromwich Spring.... $28,400 d^3/D$ " $WnD^3 \div 1,575,000 d^4$

Turton & Platt Spring..... $44,200 d^3/D$ " $WnD^3 \div 1,331,600 d^4$

Test of a Vanadium-steel Spring. (Circular of the American Vanadium Co., 1908). — Comparative tests of an ordinary carbon-steel locomotive flat spring and of a vanadium-steel spring, made by the American Locomotive Co., showed the following: The vanadium spring, on 36-in. centers tested to 94,000 lbs., reached its elastic limit at 85,000 lbs., or 234,000 lbs. per sq. in. fiber stress, and a permanent set of 0.48 in. The test was repeated three times without change in the deflection. The carbon spring was tested to 89,280 lbs. and reached an elastic limit at 65,000 lbs., or 180,000 lbs. fiber stress, with a permanent set of 1.12 in. On repeating the test it took an additional set of 0.25 in., and on the next test several of the plates failed.

RIVETED JOINTS.

Fairbairn's Experiments. — The earliest published experiments on riveted joints are contained in the memoir by Sir W. Fairbairn in the Transactions of the Royal Society. Making certain empirical allowances, he adopted the following ratios as expressing the relative strength of riveted joints:

| | |
|---------------------------|-----|
| Solid plate..... | 100 |
| Double-riveted joint..... | 70 |
| Single-riveted joint..... | 56 |

These celebrated ratios appear to rest on a very unsatisfactory analysis of the experiments on which they were based.

Loss of Strength in Punched Plates. (*Proc. Inst. M. E.*, 1881.) — A report by Mr. W. Parker and Mr. John, made in 1878 to Lloyd's Committee, on the effect of punching and drilling, showed that thin steel plates lost comparatively little from punching, but that in thick plates the loss was very considerable. The following table gives the results for plates punched and not annealed or reamed:

| | | | | |
|---------------------------------|----------------|----------------|----------------|----------------|
| Thickness of plates..... | $1\frac{1}{4}$ | $3\frac{3}{8}$ | $1\frac{1}{2}$ | $3\frac{1}{4}$ |
| Loss of tenacity, per cent..... | 8 | 18 | 26 | 33 |

When 7/8-in. punched holes were reamed out to $1\frac{1}{8}$ in. diameter, the loss of tenacity disappeared, and the plates carried as high a stress as drilled plates. Annealing also restores to punched plates their original tenacity.

The Report of the Research Committee of the Institution of Mechanical Engineers, on Riveted Joints (1881), and records of investigations by Prof. A. B. W. Kennedy (1881, 1882, and 1885), summarize the existing information regarding the comparative effects of punching and drilling upon iron and steel plates. An examination of the voluminous tables given in Professor Unwin's Report, of the experiments made on iron and steel plates, leads to the general conclusion that, while thin plates, even of steel, do not suffer very much from punching, yet in those of $\frac{1}{2}$ inch thickness and upwards the loss of tenacity due to punching ranges from 10% to 23% in iron plates, and from 11% to 33% in the case of mild steel. In drilled plates there is no appreciable loss of strength. It is

possible to remove the bad effects of punching by subsequent reaming or annealing. The introduction of a practicable method of drilling the plating of ships and other structures, after it has been bent and shaped, is a matter of great importance. In the modern English practice (1887) of the construction of steam-boilers with steel plates punching is almost entirely abolished, and all rivet-holes are drilled after the plates have been bent to the desired form.

Strength of Perforated Plates. (P. D. Bennett, *Eng'g*, Feb. 12, 1886, p. 155.) — Tests were made to determine the relative effect produced upon tensile strength of a flat bar of iron or steel: 1. By a $\frac{3}{4}$ -inch hole drilled to the required size; 2. By a hole punched $\frac{1}{8}$ inch smaller and then drilled to the size of the first hole; and, 3. By a hole punched in the bar to the size of the drilled hole. The relative results in strength per square inch of original area were as follows:

| | 1. | 2. | 3. | 4. |
|-------------------------------------|-------|-------|--------|--------|
| | Iron. | Iron. | Steel. | Steel. |
| Unperforated bar..... | 1.000 | 1.000 | 1.000 | 1.000 |
| Perforated by drilling..... | 1.029 | 1.012 | 1.068 | 1.103 |
| Perforated by punching and drilling | 1.030 | 1.008 | 1.059 | 1.110 |
| Perforated by punching only..... | 0.795 | 0.894 | 0.935 | 0.927 |

In tests 2 and 4 the holes were filled with rivets driven by hydraulic pressure. The increase of strength per square inch caused by drilling is a phenomenon of similar nature to that of the increased strength of a grooved bar over that of a straight bar of sectional area equal to the smallest section of the grooved bar. Mr. Bennett's tests on an iron bar 0.84 in. diameter, 10 in. long, and a similar bar turned to 0.84 in. diameter at one point only, showed that the relative strength of the latter to the former was 1.323 to 1.000.

Comparative Efficiency of Riveting done by Different Methods.

The Reports of Professors Unwin and Kennedy to the Institution of Mechanical Engineers (*Proc.* 1881, 1882, and 1885) tend to establish the four following points:

1. That the shearing resistance of rivets is not highest in joints riveted by means of the greatest pressure;
2. That the ultimate strength of joints is not affected to an appreciable extent by the mode of riveting; and, therefore,
3. That very great pressure upon the rivets in riveting is not the indispensable requirement that it has been sometimes supposed to be;
4. That the most serious defect of hand-riveted as compared with machine-riveted work consists in the fact that in hand-riveted joints visible slip commences at a comparatively small load, thus giving such joints a low value as regards tightness, and possibly also rendering them liable to failure under sudden strains after slip has once commenced.

The following figures of mean results give a comparative view of hand and hydraulic riveting, as regards their ultimate strengths in joints, and the periods at which in both cases visible slip commenced.

| | | | | | |
|----------------------------------|----------------|-------|-------|-------|-------|
| Total breaking load. Tons.... | Hand..... | 86.01 | 82.16 | 149.2 | 193.6 |
| | Hydraulic..... | 85.75 | 82.70 | 145.5 | 183.1 |
| Load at which visible slip began | Hand..... | 21.7 | 25.0 | 31.7 | 25.0 |
| | Hydraulic..... | 47.5 | 53.7 | 49.7 | 56.0 |

Some of the Conclusions of the Committee of Research on Riveted Joints.

(*Proc. Inst. M. E.*, April, 1885.)

The conclusions refer to joints made in soft steel plate with steel rivets, the holes drilled, and the plates in their natural state (unannealed). The rivet or shearing area has been assumed to be that of the holes, not the area of the rivets themselves. The strength of the metal in the joint has been compared with that of strips cut from the same plates.

The metal between the rivet-holes has a considerably greater tensile resistance per square inch than the unperforated metal. This excess tenacity amounted to more than 20% both in $\frac{3}{8}$ -inch and $\frac{3}{4}$ -inch plates, when the pitch of the rivet was about 1.9 diameters. In other cases $\frac{3}{8}$ -inch plate gave an excess of 15% at fracture with a pitch of 2 diameters, of 10% with a pitch of 3.6 diameters, and of 6.6%, with a pitch of 3.9 diameters; and $\frac{3}{4}$ -inch plate gave 7.8% excess with a pitch of 2.8 diameters.

In single-riveted joints it may be taken that about 22 tons per square inch is the shearing resistance of rivet steel, when the pressure on the rivets does not exceed about 40 tons per square inch. In double-riveted joints, with rivets of about $\frac{3}{4}$ -inch diameter, most of the experiments gave about 24 tons per square inch as the shearing resistance, but the joints in one series went at 22 tons. [Tons of 2240 lbs.]

The ratio of shearing resistance to tenacity is not constant, but diminishes very markedly and not very irregularly as the tenacity increases.

The size of the rivet heads and ends plays a most important part in the strength of the joints — at any rate in the case of single-riveted joints. An increase of about one-third in the weight of the rivets (all this increase, of course, going to the heads and ends) was found to add about $8\frac{1}{2}\%$ to the resistance of the joint, the plates remaining unbroken at the full shearing resistance of 22 tons per square inch, instead of tearing at a shearing stress of only a little over 20 tons. The additional strength is probably due to the prevention of the distortion of the plates by the great tensile stress in the rivets.

The intensity of bearing pressure on the rivet exercises, with joints proportioned in the ordinary way, a very important influence on their strength. So long as it does not exceed 40 tons per square inch (measured on the projected area of the rivets), it does not seem to affect their strength; but pressures of 50 to 55 tons per square inch seem to cause the rivets to shear in most cases at stresses varying from 16 to 18 tons per square inch. For ordinary joints, which are to be made equally strong in plate and in rivets, the bearing pressure should therefore probably not exceed 42 or 43 tons per square inch. For double-riveted butt-joints perhaps, as will be noted later, a higher pressure may be allowed, as the shearing stress may probably not be more than 16 or 18 tons per square inch when the plate tears.

A margin (or net distance from outside of holes to edge of plate) equal to the diameter of the drilled hole has been found sufficient in all cases hitherto tried.

To attain the maximum strength of a joint, the breadth of lap must be such as to prevent it from breaking zigzag. It has been found that the net metal measured zigzag should be from 30% to 35% in excess of that measured straight across, in order to insure a straight fracture. This corresponds to a diagonal pitch of $\frac{2}{3}p + \frac{d}{3}$, if p be the straight pitch and d the diameter of the rivet-hole.

Visible slip or "give" occurs always in a riveted joint at a point very much below its breaking load, and by no means proportional to that load. A collation of the results obtained in measuring the slip indicates that it depends upon the number and size of the rivets in the joint, rather than upon anything else; and that it is tolerably constant for a given size of rivet in a given type of joint. The loads per rivet at which a joint will commence to slip visibly are approximately as follows:

| Diameter of Rivet. | Type of Joint. | Riveting. | Slipping Load per Rivet. |
|--------------------|----------------|-----------|--------------------------|
| $\frac{3}{4}$ inch | Single-riveted | Hand | 2.5 tons |
| $\frac{3}{4}$ " | Double-riveted | Hand | 3.0 to 3.5 tons |
| $\frac{3}{4}$ " | Double-riveted | Machine | 7 tons |
| 1 inch | Single-riveted | Hand | 3.2 tons |
| 1 " | Double-riveted | Hand | 4.3 tons |
| 1 " | Double-riveted | Machine | 8 to 10 tons |

To find the probable load at which a joint of any breadth will commence to slip, multiply the number of rivets in the given breadth by the proper figure taken from the last column of the table above. The above figures are not given as exact; but they represent the results of the experiments.

The experiments point to simple rules for the proportioning of joints of maximum strength. Assuming that a bearing pressure of 43 tons per square inch may be allowed on the rivet, and that the excess tenacity of the plate is 10% of its original strength, the following table gives the values of the ratios of diameter d of hole to thickness t of plate ($d \div t$), and of pitch p to diameter of hole ($p \div d$) in joints of maximum strength in $\frac{3}{8}$ -inch plate.

For Single-riveted Plates.

| Original Tenacity of Plate. | | Shearing Resistance of Rivets. | | Ratio. $d \div t$ | Ratio. $p \div d$ | Ratio. $\frac{\text{Plate Area}}{\text{Rivet Area}}$ |
|-----------------------------|------------------|--------------------------------|------------------|----------------------|----------------------|---|
| Tons per Sq. In. | Lbs. per Sq. In. | Tons per Sq. In. | Lbs. per Sq. In. | | | |
| 30 | 67,200 | 22 | 49,200 | 2.48 | 2.30 | 0.667 |
| 28 | 62,720 | 22 | 49,200 | 2.48 | 2.40 | 0.785 |
| 30 | 67,200 | 24 | 53,760 | 2.28 | 2.27 | 0.713 |
| 28 | 62,720 | 24 | 53,760 | 2.28 | 2.36 | 0.690 |

This table shows that the diameter of the hole should be $2\frac{1}{3}$ times the thickness of the plate, and the pitch of the rivets $2\frac{3}{8}$ times the diameter of the hole. Also, it makes the mean plate area 71% of the rivet area. If a smaller rivet be used than that here specified, the joint will not be of uniform, and therefore not of maximum, strength; but with any other size of rivet the best result will be got by use of the pitch obtained from the simple formula $p = ad^2/t + d$, where, as before, d is the diameter of the hole.

The value of the constant a in this equation is as follows:

| | |
|---|-------|
| For 30-ton plate and 22-ton rivets, $a =$ | 0.524 |
| “ 28 “ “ “ 22 “ “ “ | 0.558 |
| “ 30 “ “ “ 24 “ “ “ | 0.570 |
| “ 28 “ “ “ 24 “ “ “ | 0.606 |

Or, in the mean, the pitch $p = 0.56 \frac{d^2}{t} + d$. With too small rivets this gives pitches often considerably smaller in proportion than $2\frac{3}{8}$ times the diameter.

For double-riveted lap-joints a similar calculation to that given above, but with a somewhat smaller allowance for excess tenacity, on account of the large distance between the rivet-holes, shows that for joints of maximum strength the ratio of diameter to thickness should remain precisely as in single-riveted joints; while the ratio of pitch to diameter of hole should be 3.64 for 30-ton plates and 22 or 24 ton rivets, and 3.82 for 28-ton plates with the same rivets.

Here, still more than in the former case, it is likely that the prescribed size of rivet may often be inconveniently large. In this case the diameter of rivet should be taken as large as possible; and the strongest joint for a given thickness of plate and diameter of hole can then be obtained by using the pitch given by the equation $p = ad^2/t + d$, where the values of the constant a for different strengths of plates and rivets may be taken as follows, for any thickness of plate from $\frac{3}{8}$ to $\frac{3}{4}$ -inch:

| | | |
|------------------------------------|---|-------------------------------|
| For 30-ton plate and 24-ton rivets | } | $p = 1.16 \frac{d^2}{t} + d;$ |
| “ 28 “ “ “ 22 “ “ “ | | |
| “ 30 “ “ “ 22 “ “ “ | | $p = 1.06 \frac{d^2}{t} + d;$ |
| “ 28 “ “ “ 24 “ “ “ | | $p = 1.24 \frac{d^2}{t} + d.$ |

In double-riveted butt-joints it is impossible to develop the full shearing resistance of the joint without getting excessive bearing pressure, because the shearing area is doubled without increasing the area on which the pressure acts. Considering only the plate resistance and the bearing pressure, and taking this latter as 45 tons per square inch, the best pitch would be about 4 times the diameter of the hole. We may probably say with some certainty that a pressure of from 45 to 50 tons per square inch on the rivets will cause shearing to take place at from 16 to 18 tons per square inch. Working out the equations as before, but allowing excess strength of only 5% on account of the large pitch, we find that the proportions of double-riveted butt-joints of maximum strength, under given conditions, are those of the following table:

Double-riveted Butt-joints.

| Original Tenacity of Plate, Tons per Sq. In. | Shearing Resistance of Rivets, Tons per Sq. In. | Bearing Pressure, Tons per Sq. In. | Ratio $\frac{d}{t}$ | Ratio $\frac{p}{d}$ |
|--|---|------------------------------------|---------------------|---------------------|
| 30 | 16 | 45 | 1.80 | 3.85 |
| 28 | 16 | 45 | 1.80 | 4.06 |
| 30 | 18 | 48 | 1.70 | 4.03 |
| 28 | 18 | 48 | 1.70 | 4.27 |
| 30 | 16 | 50 | 2.00 | 4.20 |
| 28 | 16 | 50 | 2.00 | 4.42 |

Practically, therefore, it may be said that we get a double-riveted butt-joint of maximum strength by making the diameter of hole about 1.8 times the thickness of the plate, and making the pitch 4.1 times the diameter of the hole.

The proportions just given belong to joints of maximum strength. But in a boiler the one part of the joint, the plate, is much more affected by time than the other part, the rivets. It is therefore not unreasonable to estimate the percentage by which the plates might be weakened by corrosion, etc., before the boiler would be unfit for use at its proper steam-pressure, and to add correspondingly to the plate area. Probably the best thing to do in this case is to proportion the joint, not for the actual thickness of plate, but for a nominal thickness less than the actual by the assumed percentage. In this case the joint will be approximately one of uniform strength by the time it has reached its final workable condition; up to which time the joint as a whole will not really have been weakened, the corrosion only gradually bringing the strength of the plates down to that of rivets.

Efficiencies of Joints.

The average results of experiments by the committee gave: For double-riveted lap-joints in $\frac{3}{8}$ -inch plates, efficiencies ranging from 67.1% to 81.2%. For double-riveted butt-joints (in double shear) 61.4% to 71.3%. These low results were probably due to the use of very soft steel in the rivets. For single-riveted lap-joints of various dimensions the efficiencies varied from 54.8% to 60.8%. The shearing resistance of steel did not increase nearly so fast as its tensile resistance. With very soft steel, for instance, of only 26 tons tenacity, the shearing resistance was about 80% of the tensile resistance, whereas with very hard steel of 52 tons tenacity the shearing resistance was only somewhere about 65% of the tensile resistance.

Proportions of Pitch and Overlap of Plates to Diameter of Rivet-Hole and Thickness of Plate.

(Prof. A. B. W. Kennedy, *Proc. Inst. M. E.*, April, 1885.)

t = thickness of plate;

d = diameter of rivet (actual) in parallel hole;

p = pitch of rivets, center to center;

s = space between lines of rivets;

l = overlap of plate.

The pitch is as wide as is allowable without impairing the tightness of the joint under steam.

For single-riveted lap-joints in the circular seams of boilers which have double-riveted longitudinal lap-joints,

$$d = t \times 2.25; p = d \times 2.25 = t \times 5 \text{ (nearly)}; l = t \times 6.$$

For double-riveted lap-joints:

$$d = 2.25t; p = 8t; s = 4.5t; l = 10.5t.$$

| Single-riveted Joints. | | | | Double-riveted Joints. | | | | |
|------------------------|-------|---------|-------|------------------------|-------|-------|--------|-------|
| t | d | p | l | t | d | p | s | l |
| 3/16 | 7/16 | 15/16 | 11/8 | 3/16 | 7/16 | 11/2 | 7/8 | 2 |
| 1/4 | 9/16 | 1 1/4 | 11/2 | 1/4 | 9/16 | 2 | 1 3/16 | 23/4 |
| 5/16 | 11/16 | 1 9/16 | 17/8 | 5/16 | 11/16 | 2 1/2 | 1 1/2 | 3 3/8 |
| 3/8 | 13/16 | 1 7/8 | 2 1/4 | 3/8 | 13/16 | 3 | 1 3/4 | 4 |
| 7/16 | 1 | 2 3/16 | 2 5/8 | 7/16 | 1 | 3 1/2 | 2 | 4 5/8 |
| 1/2 | 1 1/8 | 2 1/2 | 3 | 1/2 | 1 1/8 | 4 | 2 1/4 | 5 1/4 |
| 9/16 | 1 1/4 | 2 13/16 | 3 3/8 | 9/16 | 1 1/4 | 4 1/2 | 2 1/2 | 5 7/8 |

With these proportions and good workmanship there need be no fear of leakage of steam through the riveted joint.

The net diagonal area, or area of plate, along a zigzag line of fracture should not be less than 30% in excess of the net area straight across the joint, and 35% is better.

Mr. Theodore Cooper (*R. R. Gazette*, Aug. 22, 1890), referring to Prof. Kennedy's statement quoted above, gives as a sufficiently approximate rule for the proper pitch between the rows in staggered riveting, one-half of the pitch of the rivets in a row plus one-quarter the diameter of a rivet-hole.

Test of Double-riveted Lap and Butt Joints.

(*Proc. Inst. M. E.*, October, 1888.)

Steel plates of 25 to 26 tons per square inch T. S., steel rivets of 24.6 tons shearing strength per square inch.

| Kind of Joint. | Thickness of Plate. | Diameter of Rivet-holes. | Ratio of Pitch to Diameter. | Comparative Efficiency of Joint. |
|----------------|---------------------|--------------------------|-----------------------------|----------------------------------|
| Lap..... | 3/8" | 0.8" | 3.62 | 75.2 |
| Butt..... | 3/8 | 0.7 | 3.93 | 76.5 |
| Lap..... | 3/4 | 1.1 | 2.82 | 68.0 |
| Lap..... | 3/4 | 1.6 | 3.41 | 73.6 |
| Butt..... | 3/4 | 1.1 | 4.00 | 72.4 |
| Butt..... | 3/4 | 1.6 | 3.94 | 76.1 |
| Lap..... | 1 | 1.3 | 2.42 | 63.0 |
| Lap..... | 1 | 1.75 | 3.00 | 70.2 |
| Butt..... | 1 | 1.3 | 3.92 | 76.1 |

Diameter of Rivets for Different Thicknesses of Plates.

| Thickness of Plate. | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 | 15/16 | 1 |
|---------------------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| Diam. (1) .. | 5/8 | 5/8 | 5/8 | 3/4 | 3/4 | 3/4 | 3/8 | 7/8 | 7/8 | 1 | 1 | 1 |
| Diam. (2) .. | 5/8 | 5/8 | 3/4 | 13/16 | 13/16 | 7/8 | 7/8 | 15/16 | 1 | 1 1/8 | 13/16 | 1 1/4 |
| Diam. (3) .. | 1/2 | 5/8 | 3/4 | 3/4 | 7/8 | 7/8 | 7/8 | 1 | 1 | 1 1/8 | 1 1/8 | 1 1/8 |
| Diam. (4) .. | 5/8 | 5/8 | 5/8 | | 3/4 | | 13/16 | 7/8 | | 1 | | 1 1/16 |
| Diam. (5) .. | 3/4 | 7/8 | 15/16 | 1 | | | | | | | | |
| Diam. (6) .. | 11/16 | 3/4 | 7/8 | 15/16 | 1 | 1 | | | | | | |
| Diam. (7) .. | 3/8 | 1/2 | 9/16 | 11/16 | 3/4 | 13/16 | | | | | | |

(1) Lloyd's Rules. (2) Liverpool Rules. (3) English Dock-yards. (4) French Veritas. (5) Hartford Steam Boiler Inspection and Insurance Co., double-riveted lap-joints. (6) Ditto, triple-riveted butt-joints. (7) F. E. Cardullo. ($1/16$ less than diam. of hole.)

Calculated Efficiencies — Steel Plates and Steel Rivets.—The following table has been calculated by the author on the assumptions that the excess strength of the perforated plate is 10%, and that the shearing strength of the rivets per square inch is four-fifths of the tensile strength of the plate (or, if no allowance is made for excess strength of the perforated plate that the shearing strength is 72.7% of the tensile strength). If t = thickness of plate, d = diameter of rivet-hole, p = pitch, and T = tensile strength per square inch, then for single-riveted plates

$$(p - d)t \times 1.10T = \frac{\pi}{4} d^2 \times \frac{4}{5} T, \text{ whence } p = 0.571 \frac{d^2}{t} + d.$$

$$\text{For double-riveted lap-joints, } p = 1.142 \frac{d^2}{t} + d.$$

The coefficients 0.571 and 1.142 agree closely with the averages of those given in the report of the committee of the Institution of Mechanical Engineers, quoted on page 404, *ante*.

| Thickness. | Diam. of Rivet-hole. | Pitch. | | Efficiency. | | Thickness. | Diam. of Rivet-hole. | Pitch. | | Efficiency. | |
|------------|----------------------|------------------|------------------|------------------|------------------|------------|----------------------|------------------|------------------|------------------|------------------|
| | | Single Riveting. | Double Riveting. | Single Riveting. | Double Riveting. | | | Single Riveting. | Double Riveting. | Single Riveting. | Double Riveting. |
| in. | in. | in. | in. | % | % | in. | in. | in. | in. | % | % |
| 3/16 | 7/16 | 1.020 | 1.603 | 57.1 | 72.7 | 1/2 | 3/4 | 1.392 | 2.035 | 46.1 | 63.1 |
| 3/16 | 1/2 | 1.261 | 2.023 | 60.5 | 75.3 | 1/2 | 7/8 | 1.749 | 2.624 | 50.0 | 66.6 |
| 1/4 | 1/2 | 1.071 | 1.642 | 53.3 | 69.6 | 1/2 | 1 | 2.142 | 3.284 | 53.3 | 70.0 |
| 1/4 | 9/16 | 1.285 | 2.008 | 56.2 | 72.0 | 1/2 | 1 1/8 | 2.570 | 4.016 | 56.2 | 72.0 |
| 5/16 | 9/16 | 1.137 | 1.712 | 50.5 | 67.1 | 9/16 | 3/4 | 1.321 | 1.892 | 43.2 | 60.3 |
| 5/16 | 5/8 | 1.339 | 2.053 | 53.3 | 69.5 | 9/16 | 7/8 | 1.652 | 2.429 | 47.0 | 64.0 |
| 5/16 | 11/16 | 1.551 | 2.415 | 55.7 | 71.5 | 9/16 | 1 | 2.015 | 3.030 | 50.4 | 67.0 |
| 3/8 | 5/8 | 1.218 | 1.810 | 48.7 | 65.5 | 9/16 | 1 1/8 | 2.410 | 3.694 | 53.3 | 69.5 |
| 3/8 | 3/4 | 1.607 | 2.463 | 53.3 | 69.5 | 9/16 | 1 1/4 | 2.836 | 4.422 | 55.9 | 71.5 |
| 3/8 | 7/8 | 2.041 | 3.206 | 57.1 | 72.7 | 5/8 | 3/4 | 1.264 | 1.778 | 40.7 | 57.8 |
| 7/16 | 5/8 | 1.136 | 1.647 | 45.9 | 62.0 | 5/8 | 7/8 | 1.575 | 2.274 | 44.4 | 61.5 |
| 7/16 | 3/4 | 1.484 | 2.218 | 49.5 | 66.2 | 5/8 | 1 | 1.914 | 2.827 | 47.7 | 64.6 |
| 7/16 | 7/8 | 1.869 | 2.864 | 53.2 | 69.4 | 5/8 | 1 1/8 | 2.281 | 3.438 | 50.7 | 67.3 |
| 7/16 | 1 | 2.305 | 3.610 | 56.6 | 72.3 | 5/8 | 1 1/4 | 2.678 | 4.105 | 53.3 | 69.5 |

Apparent Shearing Resistance of Rivet Iron and Steel.

(*Proc. Inst. M. E.*, 1879, *Engineering*, Feb. 20, 1880.)

The true shearing resistance of the rivets cannot be ascertained from experiments on riveted joints (1) because the uniform distribution of the load to all the rivets cannot be insured; (2) because of the friction of the plates, which has the effect of increasing the apparent resistance to shearing in an element uncertain in amount. Probably in the case of single-riveted joints the shearing resistance is not much affected by the friction.

Fairbairn's experiments show that a rivet is $6\frac{1}{2}\%$ weaker in a drilled than in a punched hole. By rounding the edge of the rivet-hole, the apparent shearing resistance is increased 12%. Messrs. Greig and Eyth's experiments indicate a greater resistance of the rivets in punched holes than in drilled holes.

If the apparent shearing resistance is less for double than for single shear, it is probably due to unequal distribution of the stress on the two-rivet sections.

The shearing resistance of a bar, when sheared in circumstances which prevent friction, is usually less than the tenacity of the bar. The following results show the decrease:

| | | | |
|---------------------------|----------------|----------------|-------------|
| Harkort, iron | Tenacity, 26.4 | Shearing, 16.5 | Ratio, 0.62 |
| Lavalley, iron | " 25.4 | " 20.2 | " 0.79 |
| Greig and Eyth, iron. . . | " 22.2 | " 19.0 | " 0.85 |
| Greig and Eyth, steel . . | " 28.8 | " 22.1 | " 0.77 |

In Wöhler's researches (in 1870) the shearing strength of iron was found to be four-fifths of the tenacity. Later researches of Bauschinger confirm this result generally, but they show that for iron the ratio of the shearing resistance and tenacity depends on the direction of the stress relatively to the direction of rolling. The above ratio is valid only if the shear is in a plane perpendicular to the direction of rolling, and if the tension is applied parallel to the direction of rolling. If the plane of shear is parallel to the breadth of the bar, the resistance is only half as great as in a plane perpendicular to the fibers.

THE STRENGTH OF RIVETED JOINTS.

Joint of Maximum Efficiency. — (F. E. Cardullo.) If a riveted joint is made with sufficient lap, and a proper distance between the rows of rivets, it will break in one of the three following ways:

1. By tearing the plate along a line, through the outer row of rivets.
2. By shearing the rivets.
3. By crushing the plate or the rivets.

Let t = the thickness of the main plates.

d = the diameter of the rivet-holes.

f = the tensile strength of the plate in pounds per sq. in.

s = the shearing strength of the rivets in pounds per sq. in. when in single shear.

p = the distance between the centers of rivets of the outer row (see Figs. 90 and 91) = the pitch in single and double lap riveting = twice

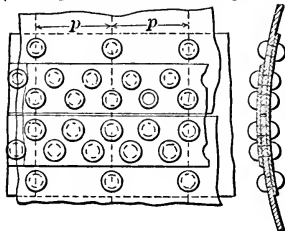


FIG. 90.

TRIPLE RIVETING.

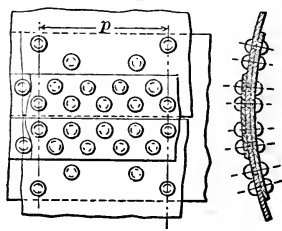


FIG. 91.

QUADRUPLE RIVETING.

the pitch of the inner rows in triple butt strap riveting, in which alternate rivets in the outer row are omitted, = four times the pitch in quadruple butt strap riveting, in which the outer row has one-fourth of the number of rivets of the two inner rows.

c = the crushing strength of the rivets or plates in pounds per sq. in.

n = the number of rivets in each group in single shear. (A group is the number of rivets on one side of a joint corresponding to the distance p ; = 1 rivet in single riveting, 2 in double riveting, 5 in triple butt strap riveting, and 11 in quadruple butt strap riveting.)

m = the number of rivets in each group in double shear.

s' = the shearing strength of rivets in double shear, in pounds per sq. in., the rivet section being counted once.

T = the strength of the plate at the weakest section. = $ft(p - d)$.

S = the strength of the rivets against shearing, = $0.7854 d^2 (ns + ms')$.

C = the strength of the rivets or the plates against crushing, = $dtc(n + m)$.

In order that the joint shall have the greatest strength possible, the tearing, shearing, and crushing strength must all be equal. In order to make it so,

1. Substitute the known numerical values, equate the expressions for shearing and crushing strength, and find the value of d , taking it to the nearest $1/16$ in.

2. Next find the value of S in the second equation, and substitute it for T in the first equation. Substitute numerical values for the other factors in the first equation, and solve for p .

The efficiency of a riveted joint in tearing, shearing and crushing, is equal to the tearing, shearing or crushing strength, divided by the quantity ftp , or the strength of the solid plate.

The efficiency in tearing is also equal to $(p - d) \div p$.

The maximum possible efficiency for a well-designed joint is

$$E = \frac{m + n}{m + n + (f \div c)}.$$

Empirical formula for the diameter of the rivet-hole when the crushing strength is unknown. Assuming that $c = 1.4f$, and $s'' = 1.75s$, we have by equating C and S , and substituting,

$$d = 1.782 t \frac{f(n + m)}{s(n + 1.75 m)}.$$

Margin. The distance from the center of any rivet-hole to the edge of the plate should be not less than $1\frac{1}{2}d$. The distance between two adjacent rivet centers should be not less than $2d$. It is better to increase each of these dimensions by $1/8$ in.

The distance between the rows of rivets should be such that the net section of plate material along any broken diagonal through the rivet-holes should be not less than 30 per cent greater than the plate section along the outer line of rivets.

The thickness of the inner cover strap of a butt joint should be $3/4$ of the thickness of the main plate or more. The thickness of the outer strap should be $5/8$ of the thickness of the main plate or more.

Steam Tightness. It is of great importance in boiler riveting that the joint be steam tight. It is therefore necessary that the pitch of the rivets nearest to the calked edge be limited to a certain function of the thickness of the plate. The Board of Trade rule for steam tightness is

$$p = Ct + 15/8 \text{ in.}$$

where p = the maximum allowable pitch in inches.

t = the thickness of main plate in inches.

C = a constant from the following table.

| No. of Rivets per Group... | 1 | 2 | 3 | 4 | 5 |
|----------------------------|------------|------|------|------|------|
| Lap Joints..... | $C = 1.31$ | 2.62 | 3.47 | 4.14 | ... |
| Double-strapped Joints.... | $C = 1.75$ | 3.50 | 4.63 | 5.52 | 6.00 |

The pitch should not exceed ten inches under any circumstances.

When the joint has been designed for strength, it should be checked by the above formula. Should the pitch for strength exceed the pitch for steam tightness, take the latter, substitute it in the formula

$$ft(p - d) = 0.7854 d^2 (ns + ms''),$$

and solve for d . If the value of d so obtained is not the diameter of some standard size rivet, take the next larger $1/16$ in.

Calculation of Triple-riveted Butt and Strap Joints. — Formulæ: $T = ft(p - d)$, $S = 0.7854 d^2 (ns + ms'')$, $C = dlc(m + n)$ (notation on preceding page), $n = 1$, $m = 4$.

Take $f = 55,000$; $s = 0.8f$, $= 44,000$; $s'' = 1.75s = 77,000$, $c = 1.4f = 77,000$.

Then $T = 55,000t(p - d)$, $S = 276,460d^2$, $C = 385,000dt$.

For maximum strength, $T = S = C$; dividing by $55,000t$, $(p - d) = 5.027d^2 = 7dt$; whence $d = 1.3925t$; $p = 8d$.

| Thickness of plate, $t = 5/16$ | $3/8$ | $7/16$ | $1/2$ | $9/16$ | $5/8$ | |
|--|--------|--------|--------|--------|--------|--------|
| Diam. rivet hole, $d = 1.3925t$ | 0.4353 | 0.5222 | 0.6092 | 0.6962 | 0.7833 | 0.8703 |

| | | | | | | |
|---|--------|--------|---------|---------|---------|---------|
| Pitch of outer row, $p = 8d$ | 3.4816 | 4.1776 | 4.8736 | 5.5696 | 6.2664 | 6.9624 |
| $T = 55,000t(p - d)$ | 52,360 | 75,390 | 102,610 | 134,020 | 169,630 | 209,420 |
| $S = 276,460d^2$ | 52,330 | 75,360 | 102,570 | 133,970 | 169,560 | 209,330 |
| $C = 385,000dt$ | 52,350 | 75,390 | 102,620 | 134,030 | 169,630 | 209,420 |

Calculations by logarithms, to nearest 10 pounds.

Efficiency of all joints $(p - d) \div p = 87.5$ per cent.

Maximum efficiency by Cardullo's formula, $\frac{n + m}{n + m + f/c} = \frac{5}{5 + 1/1.4} = 87.5$ per cent.

Diameter of rivet-hole, next largest 16th, $7/16$ $9/16$ $5/8$ $3/4$ $13/16$ $7/8$
 For the same thickness of plates the Hartford Steam-Boiler Inspection and Insurance Co. gives the following proportions:

| | | | | | |
|-----------------------------------|---------|---------|---------|----------|----------|
| Thickness, t , $5/16$ | $3/8$ | $7/16$ | $1/2$ | $9/16$ | $5/8$ |
| Diam. rivet-hole, d , $3/4$ | $13/16$ | $15/16$ | 1 | $1 1/16$ | $1 1/16$ |
| Pitch of outer row, p , $6 1/4$ | $6 1/2$ | $6 3/4$ | $7 1/2$ | $7 3/4$ | $7 3/4$ |

Using the same values for f , s , s'' and c , we obtain:

| | | | | | | |
|-----------------|---------|---------|---------|---------|---------|---------|
| $T =$ | 94,530 | 117,300 | 139,860 | 178,750 | 207,850 | 229,880 |
| $S =$ | 155,400 | 168,400 | 194,300 | 207,300 | 220,200 | 220,200 |
| $C =$ | 90,030 | 117,300 | 157,900 | 192,500 | 230,000 | 255,500 |

Strength of solid plate, $fpt =$ 107,360 134,060 162,420 206,250 239,770 266,400

Efficiency T, S or C , lowest $\div fpt$,
 per cent 83.9 87.5 86.1 86.7 86.7 82.6

The $5/16$ in. plate fails by crushing, the $5/8$ by shearing, the others by tearing.

Calculation of Quadruple Riveting. — In this case there are 11 rivets in the group. If the upper strap plate contains all the rivets except the outer row, then $n = 1$, $m = 10$. Using the same values for f , s , s'' and c as above, we have $ns + ms'' = 814,000$; $T = 55,000t(p - d)$; $S = 639,315d^2$; $C = 847,000dt$.

For maximum strength, $t(p - d) = 11.624d^2 = 15.4dt$; whence $d = 1.32485t$, $p = 16.4d$. Efficiency $(p - d) \div p = 93.9$ per cent. Check by

Cardullo's formula $\frac{n + m}{n + m + f/c} = \frac{11}{11 + 10/14} = 93.9$ per cent.

British Board of Trade and Lloyd's Rules for Riveted Joints. — *Board of Trade.* — Tensile strength of rivet bars between 26 and 30 tons, el. in 10" not less than 25%, and contr. of area not less than 50%.

The shearing resistance of the rivet steel to be taken at 23 tons per square inch, 5 to be used for the factor of safety independently of any addition to this factor for the plating. Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The diameter must not be less than the thickness of the plate, and the pitch never greater than $8 1/2$ ". The thickness of double butt-straps (each) not to be less than $5/8$ the thickness of the plate; single butt-straps not less than $9/8$.

Distance from center of rivet to edge of hole = diameter of rivet $\times 1 1/2$.

Distance between rows of rivets

= $2 \times$ diam. of rivet or = $[(\text{diam.} \times 4) + 1] \div 2$, if chain, and

= $\frac{\sqrt{[(\text{pitch} \times 11) + (\text{diam.} \times 4)] \times (\text{pitch} + \text{diam.} \times 4)}}{10}$ if zigzag.

Diagonal pitch = $(\text{pitch} \times 6 + \text{diam.} \times 4) \div 10$.

Lloyd's. — T. S. of rivet bars, 26 to 30 tons; el. not less than 20% in 8". The material must stand bending to a curve, the inner radius of which is

not greater than $1\frac{1}{2}$ times the thickness of the plate, after having been uniformly heated to a low cherry-red, and quenched in water at 82°F .

Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The shearing strength of rivet steel to be taken at 85% of the T. S. of the material of shell plates. In any case where the strength of the longitudinal joint is satisfactorily shown by experiment to be greater than given by the formula, the actual strength may be taken in the calculation.

Proportions of Riveted Joints. (Hartford S. B. Insp. and Ins. Co.)

Single-riveted Girth Seams of Boilers.

| Thickness. | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ |
|-------------------|---------------------------------|--------------------------------|---------------------------------|--------------------------------|--------------------------------|
| Diam. rivet-hole. | $\frac{3}{4}$ $\frac{11}{16}$ | $\frac{13}{16}$ $\frac{3}{4}$ | $\frac{15}{16}$ $\frac{13}{16}$ | 1 $\frac{15}{16}$ | $\frac{11}{16}$ 1 |
| Pitch..... | $\frac{21}{16}$ $\frac{21}{16}$ | $\frac{21}{8}$ $\frac{21}{8}$ | $\frac{23}{8}$ $\frac{21}{8}$ | $\frac{27}{16}$ $\frac{23}{8}$ | $\frac{21}{2}$ $\frac{21}{2}$ |
| Center to edge.. | $\frac{11}{8}$ $\frac{11}{32}$ | $\frac{17}{32}$ $\frac{11}{8}$ | $\frac{13}{32}$ $\frac{17}{32}$ | $\frac{11}{2}$ $\frac{13}{32}$ | $\frac{19}{32}$ $\frac{11}{2}$ |

Double-riveted Lap Joints.

| Thickness of plate..... | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ |
|------------------------------|------------------|------------------|-----------------|-----------------|------------------|
| Diam. rivet-hole..... | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{15}{16}$ | 1 | $\frac{11}{16}$ |
| Pitch..... | $\frac{27}{8}$ | $\frac{27}{8}$ | $\frac{31}{4}$ | $\frac{31}{4}$ | $\frac{3}{32}$ |
| Dist. bet. rows..... | $\frac{115}{16}$ | $\frac{115}{16}$ | $\frac{23}{16}$ | $\frac{23}{16}$ | $\frac{2}{2}$ |
| Dist. inner row to edge..... | $\frac{11}{8}$ | $\frac{17}{32}$ | $\frac{13}{32}$ | $\frac{11}{2}$ | $\frac{119}{32}$ |
| Efficiency..... | 0.74 | 0.72 | 0.70 | 0.70 | 0.68 |

Triple-riveted Lap Joints.

| Thickness..... | $\frac{1}{4}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ |
|------------------------|-----------------|-----------------|-----------------|------------------|------------------|
| Diam. rivet-hole..... | $\frac{11}{16}$ | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{15}{16}$ | 1 |
| Pitch..... | 3 | $\frac{31}{8}$ | $\frac{31}{4}$ | $\frac{33}{4}$ | $\frac{315}{16}$ |
| Dist. bet. rows..... | 2 | $\frac{21}{16}$ | $\frac{23}{16}$ | $\frac{21}{2}$ | $\frac{25}{8}$ |
| Inner row to edge..... | $\frac{11}{32}$ | $\frac{11}{8}$ | $\frac{17}{32}$ | $\frac{113}{32}$ | $\frac{11}{2}$ |
| Efficiency..... | 0.77 | 0.76 | 0.75 | 0.75 | 0.75 |

Triple-riveted Bull-strap Joints.

| Thickness..... | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{5}{8}$ |
|----------------------------|----------------|-----------------|-----------------|----------------|-----------------|-----------------|
| Diam. rivet-hole..... | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{15}{16}$ | 1 | $\frac{11}{16}$ | $\frac{11}{16}$ |
| Pitch, inner rows..... | $\frac{31}{8}$ | $\frac{31}{4}$ | $\frac{33}{8}$ | $\frac{33}{4}$ | $\frac{37}{8}$ | $\frac{37}{8}$ |
| Dist. bet. inner rows..... | $\frac{21}{8}$ | $\frac{23}{16}$ | $\frac{21}{4}$ | $\frac{23}{8}$ | $\frac{25}{8}$ | $\frac{25}{8}$ |
| Dist. outer to 2d row..... | $\frac{23}{8}$ | $\frac{21}{2}$ | $\frac{23}{4}$ | 3 | $\frac{33}{16}$ | $\frac{33}{16}$ |
| Edge to nearest row..... | $\frac{11}{4}$ | $\frac{17}{32}$ | $\frac{13}{32}$ | $\frac{11}{2}$ | $\frac{19}{32}$ | $\frac{19}{32}$ |
| Efficiency %..... | 88 (?) | 87.5 | 86 | 86.6 | 85.4 | 84 (?) |

The distance to the edge of the plate is from the center of rivet-holes.

Pressure Required to Drive Hot Rivets. — R. D. Wood & Co. Philadelphia, give the following table (1897):

POWER TO DRIVE RIVETS HOT.

| Size. | Girder-work. | Tank-work. | Boiler-work. | Size. | Girder-work. | Tank-work. | Boiler-work. |
|-------|--------------|------------|--------------|-------|--------------|------------|--------------|
| in. | tons. | tons. | tons. | in. | tons. | tons. | tons. |
| 1/2 | 9 | 15 | 20 | 1 1/8 | 38 | 60 | 75 |
| 5/8 | 12 | 18 | 25 | 1 1/4 | 45 | 70 | 100 |
| 3/4 | 15 | 22 | 33 | 1 1/2 | 60 | 85 | 125 |
| 7/8 | 22 | 30 | 45 | 1 3/4 | 75 | 100 | 150 |
| 1 | 30 | 45 | 60 | | | | |

The above is based on the rivet passing through only two thicknesses of plate which together exceed the diameter of the rivet but little, if any.

As the plate thickness increases the power required increases approximately in proportion to the square root of the increase of thickness. Thus, if the total thickness of plate is four times the diameter of the rivet, we should require twice the power given above in order to thoroughly fill the rivet-holes and do good work. Double the thickness of plate would increase the necessary power about 40%.

It takes about four or five times as much power to drive rivets cold as to drive them hot. Thus, a machine that will drive 3/4-in. rivets hot will usually drive 3/8-in. rivets cold (steel). Baldwin Locomotive Works drive 1/2-in. soft-iron rivets cold with 15 tons.

Riveting Pressure Required for Bridge and Boiler Work.

(Wilfred Lewis, Engineers' Club of Philadelphia, Nov., 1893.)

A number of 3/8-inch rivets were subjected to pressures between 10,000 and 60,000 lbs. At 10,000 lbs. the rivet swelled and filled the hole without forming a head. At 20,000 lbs. the head was formed and the plates were slightly pinched. At 30,000 lbs. the rivet was well set. At 40,000 lbs. the metal in the plate surrounding the rivet began to stretch, and the stretching became more and more apparent as the pressure was increased to 50,000 and 60,000 lbs. From these experiments the conclusion might be drawn that the pressure required for cold riveting was about 300,000 lbs. per square inch of rivet section. In hot riveting, until recently there was never any call for a pressure exceeding 60,000 lbs., but now pressures as high as 150,000 lbs. are not uncommon, and even 300,000 lbs. have been contemplated as desirable.

Pressure Required for Heading Cold Rivets. — Experiments made by the author in 1906 on 1/2 and 5/8 in. soft steel rivets showed that the pressure required to head a rivet cold, with a hemispherical heading die, was a function of the final or maximum diameter of the head. The metal began to flow and fill the hole at about 50,000 lbs. per sq. in. pressure, but it hardened and increased its resistance as it flowed until it reached a maximum of about 100,000 lbs. per sq. in. of the maximum area of the head.

Chemical and Physical Tests of Soft Steel Rivets. — Ten rivet bars and ten rivets selected from stock of the Champion Rivet Co., Cleveland, O., were analyzed by Oscar Textor, with results as follows:

P. 0.008 to 0.027, av. 0.015; Mn. 0.31 to 0.69, av. 0.46; S. 0.023 to 0.044, av. 0.033; Si. 0.001 to 0.008, av. 0.005; C. 0.06 to 0.19, av. 0.11. Only four of the 20 samples were over 0.14 C. and these were made for high strength. Ten bars and two rivets gave tensile strength, 46,735 to 55,380, av. 52,195 lbs. per sq. in.; elastic limit, 31,350 to 43,150, av. 35,954; elongation, bars only, 28 to 35, av. 31.9% in 8 ins.; reduction of area, 65.6%. Eight bars in single shear gave shearing strength 35,660 to 50,190, av. 44,478 lbs. per sq. in.; seven bars in double shear gave 39,170 to 53,900, av. 45,720 lbs. The shearing strength averaged 86.3% of the tensile strength.

IRON AND STEEL.

CLASSIFICATION OF IRON AND STEEL.

(W. Kent, *Railroad and Engineering Journal*, April, 1887.)

| Generic Term. | IRON. | | | |
|-------------------------|---|---|--|--|
| How Obtained. | <i>Cast</i> , Or obtained from a fluid mass. | | <i>Wrought</i> , Or welded from a pasty mass. | |
| Distinguishing Quality. | <i>Non-malleable</i> . | <i>Malleable</i> . | <i>Will Not Harden</i> . | <i>Will Harden</i> . |
| Species. | CAST IRON. | | (7) WROUGHT IRON. | (8†) WROUGHT STEEL. |
| Varieties. | (1) Ordinary cast-irons. | (2) Malleable cast iron obtained from No. 1 by annealing in oxides. | (3) Crucible, (4) Bessemer, and (5) Open-hearth steels. (6) Mitis.* | <i>a.</i> Obtained by direct process from ores, as Catalan, Chenot, and other process irons. <i>b.</i> Obtained by indirect process from cast iron, as finery-hearth and puddled irons. |
| | | | | Obtained by direct or indirect process, as German, shear, blister, and puddled steels. |

* No. 6. Mitis is the name given to a new product (having the same general properties and produced by the same processes as soft cast steels) made by adding an alloy of aluminum to melted wrought iron or soft steel before pouring.

† No. 8. Wrought steel is almost an obsolete product, having been replaced in commerce by cast steel. Blister steel, however, is still made as an intermediate product for retempering in the crucible. Sub-varieties of Nos. 3, 4, and 5, soft, mild, medium, and hard steels, according to percentage of carbon, the divisions between them not being well defined.

Cast iron usually contains over 3% of carbon; cast steel anywhere from 0.06% to 1.50%, according to the purpose for which it is used; wrought iron from 0.02% to 0.10%. The quality of hardening and tempering which formerly distinguished steel from wrought iron is now no longer the dividing line between them, since soft steels are now produced which, by the ordinary blacksmith's tests, will not harden. All products of the crucible, Bessemer, and open-hearth processes are now commercially known as steel.

CAST IRON.

The Manufacture of Cast Iron. — Pig iron is the name given to the crude form of iron as it is produced in the blast furnace. This furnace is a tall shaft, lined with fire brick, often as large as 100 ft. high and 20 ft. in diameter at its widest part, called the "bosh." The furnace is kept filled with alternate layers of fuel (coke, anthracite or charcoal), while a melting temperature is maintained at the bottom by a strong blast. The iron ore as it travels down the furnace is decarbonized by the carbon monoxide gas produced by the incomplete combustion of the fuel, and as it travels farther, into a zone of higher temperature, it absorbs carbon and silicon. The phosphorus originally in the ore remains in the iron. The sulphur present in the ore and in the fuel may go into combination with the lime in the slag, or into the iron, depending on the constitution of the slag and on the temperature. The silica and alumina in the ore unite with the lime to form a fusible slag, which rests on the melted iron in the hearth. The iron is tapped from the furnace several times a day, while in large furnaces the slag is usually run off continuously.

Grading of Pig Iron. — Pig iron is approximately graded according to its fracture, the number of grades varying in different districts. In Eastern Pennsylvania the principal grades recognized are known as No. 1 and 2 foundry, gray forge or No. 3, mottled or No. 4, and white or No. 5. Intermediate grades are sometimes made, as No. 2 X, between No. 1 and No. 2, and special names are given to irons more highly silicized than No. 1, as No. 1 X, silver-gray, and soft. Charcoal foundry pig iron is graded by numbers 1 to 5, but the quality is very different from the corresponding numbers in anthracite and coke pig. Southern coke pig iron is graded into ten or more grades. Grading by fracture is a fairly satisfactory method of grading irons made from uniform ore mixtures and fuel, but is unreliable as a means of determining quality of irons produced in different sections or from different ores. Grading by chemical analysis, in the latter case, is the only satisfactory method. The following analyses of the five standard grades of northern foundry and mill pig irons are given by J. M. Hartman (*Bull. I. & S. A.*, Feb., 1892):

| | No. 1. | No. 2. | No. 3. | No. 4. | No. 4 B. | No. 5. |
|-----------------------|--------|--------|--------|--------|----------|--------|
| Iron..... | 92.37 | 92.31 | 94.66 | 94.48 | 94.08 | 94.68 |
| Graphitic carbon..... | 3.52 | 2.99 | 2.50 | 2.02 | 2.02 | |
| Combined carbon..... | 0.13 | 0.37 | 1.52 | 1.98 | 1.43 | 3.83 |
| Silicon..... | 2.44 | 2.52 | 0.72 | 0.56 | 0.92 | 0.41 |
| Phosphorus..... | 1.25 | 1.08 | 0.26 | 0.19 | 0.04 | 0.04 |
| Sulphur..... | 0.02 | 0.02 | trace | 0.08 | 0.04 | 0.02 |
| Manganese..... | 0.28 | 0.72 | 0.34 | 0.67 | 2.02 | 0.98 |

CHARACTERISTICS OF THESE IRONS.

No. 1. Gray. — A large, dark, open-grain iron, softest of all the numbers and used exclusively in the foundry. Tensile strength low. Elastic limit low. Fracture rough. Turns soft and tough.

No. 2. Gray. — A mixed large and small dark grain, harder than No. 1 iron, and used exclusively in the foundry. Tensile strength and elastic limit higher than No. 1. Fracture less rough than No. 1. Turns harder, less tough, and more brittle than No. 1.

No. 3. Gray. — Small, gray, close grain, harder than No. 2 iron, used either in the rolling-mill or foundry. Tensile strength and elastic limit higher than No. 2. Turns hard, less tough, and more brittle than No. 2.

No. 4. Mottled. — White background, dotted closely with small black spots of graphitic carbon; little or no grain. Used exclusively in the rolling-mill. Tensile strength and elastic limit lower than No. 3. Turns with difficulty; less tough and more brittle than No. 3. The manganese in the B pig iron replaces part of the combined carbon, making the iron harder and closing the grain, notwithstanding the lower combined carbon.

No. 5. White. — Smooth, white fracture, no grain, used exclusively in the rolling mill. Tensile strength and elastic limit much lower than No. 4. Too hard to turn and more brittle than No. 4.

Southern pig irons are graded as follows, beginning with the highest in silicon: Nos. 1 and 2 silvery, Nos. 1 and 2 soft, all containing over 3% of silicon; Nos. 1, 2, and 3 foundry, respectively about 2.75%, 2.5% and 2% silicon; No. 1 mill, or "foundry forge;" No. 2 mill, or gray forge; mottled; white.

Chemistry of Cast Iron. — Abbreviations, TC, total carbon; GC, graphitic carbon; CC, combined carbon. Numerous researches have been made and many papers written, especially between the years 1895 and 1908, on the relation of the physical properties to the chemical constitution of cast iron. Much remains to be learned on the subject, but the following is a brief summary of prevailing opinions.

CARBON. — Carbon exists in three states in cast iron: 1, Combined carbon, which has the property of making iron white and hard; 2, Graphitic carbon or graphite, which is not alloyed with the iron, but exists in it as a separate body, since it may be removed from the fractured surface of pig iron by a brush; 3, a third form, called by Ledebur "tempering graphite carbon," into which combined carbon may be changed by prolonged heating. The relative percentages in which GC and CC may be found in cast iron differ with the rate of cooling from the liquid state, so that in a large casting, cooled slowly, nearly all the C may be GC, while in a small casting from the same ladle cooled quickly, it may be nearly all CC. The total C in cast iron usually is between 3 and 4%.

COMBINED CARBON. — CC increases hardness, brittleness and shrinkage. Up to about 1% it increases strength, then decreases it. The presence of S tends to increase the CC in a casting, while Si tends to change CC to GC.

GRAPHITE. — GC in a casting causes softness and weakness when above 3%; softness and strength when added to irons low in GC and over 1% in CC. It increases with the size of the casting, with slow cooling, or rather with holding a long time in the mold at a high temperature.

SILICON. — Si acts as a softener by counteracting the hardening effect of S, and by changing CC into GC, changes white iron to gray, increases fluidity and lessens shrinkage. When added to hard brittle iron, high in CC, it may increase strength by removing hard brittleness, but when it reduces the CC to 1% and less it weakens the iron. Above 3.5 or 4% it changes the fracture to silvery gray, and the iron becomes brittle and weak. The softening effect of Si is modified by S and Mn.

SULPHUR. — S causes the C to take the form of CC, increases hardness, brittleness, and shrinkage, and also has a weakening effect of its own. Above about 0.1% it makes iron very weak and brittle. When Si is below 1%, even 0.06 S makes the iron dangerously brittle.

MANGANESE. — Mn in small amount, less than 0.5%, counteracts the hardening influence of S; in larger amounts it changes GC into CC, and acts as a hardener. Above 2% it makes the iron very hard. Mn combines with iron in almost all proportions. When it is from 10 to 30% the alloy is called spiegeleisen, from the German word for mirror, and has large, bright crystalline faces. Above 50% it is known as ferro-manganese. Mn has the property of increasing the solubility of iron for carbon; ordinary pig iron containing rarely over 4.2% C, while spiegeleisen may have 5%, and ferro-manganese as high as 6%. Cast iron with 1% Mn is used in making chilled rolls, in which a hard chill is desired. When softness is required in castings, Mn over 0.4% has to be avoided. Mn increases shrinkage. It also decreases the magnetism of iron. Iron with 25% Mn loses all its magnetism. It therefore has to be avoided in castings for dynamo fields and other pieces of electrical machinery.

PHOSPHORUS. — P increases fluidity, and is therefore valuable for thin and ornamental castings in which strength is not needed. It increases softness and decreases shrinkage. Below 0.7% it does not appear to decrease strength, but above 1% it is a weakener.

COPPER. — Cu is found in pig irons made from ores containing Cu. From 0.1 to 1% it closes the grain of cast iron, but does not appreciably cause brittleness.

ALUMINUM. — Al from 0.2 to 1.0% (added to the ladle in the form of a FeAl alloy) increases the softness and strength of white iron; added to gray iron it softens and weakens it.

TITANIUM. — An addition of 2 to 3% of a TiFe alloy containing 10% Ti caused an increase of 20 to 30% in strength of cast iron. A. J. Rossi, *A.I.M.E.*, xxxiii, 194. Ti reacts with any O or N present in the metal and thus purifies it, and does not remain in the metal. After enough Ti for deoxidation has been added, further additions have no effect. R. Moldenke, *A.I.M.E.*, xxxv, 153.

VANADIUM. — Va to the extent of 0.15% added to the ladle in the form of a ground FeVa alloy greatly increases the strength of cast iron. It acts as a deoxidizer and also by alloying.

OXIDE OF IRON. — The cause of the difference in strength of charcoal and coke irons of identical composition is believed by Dr. Moldenke (*A.I.M.E.*, xxxi, 988) to be the degree of oxidation to which they have been subjected in making or remelting. Since Mn, Ti, and Va all act as deoxidizers, it should be possible by additions to the ladle of alloys of FeMn, FeVa, or FeTi, to make the two irons of equal strength.

Temper Carbon. The main part of the C in white cast iron is the carbide Fe_3C . This breaks down under annealing to what Ledebur calls "temper carbon," and in annealing in oxides, as in making malleable iron, it is oxidized to CO. The C remaining in the casting at the end of the process is nearly all GC, since the latter is very slowly oxidized.

Influence of Various Elements on Cast Iron. — W. S. Anderson, *Castings*, Sept., 1908, gives the following:

| | | | |
|-------------------------|-------------|------------|----------------|
| Fluidity, increased by | Si, P, G.C. | Reduced by | S, C.C. |
| Shrinkage, increased by | S, Mn, C.C. | Reduced by | Si, P, G.C. |
| Strength, increased by | Mn, C.C. | Reduced by | Si, S, P, G.C. |
| Hardness, increased by | S, Mn, C.C. | Reduced by | Si, G.C. |
| Chill, increased by | S, Mn, C.C. | Reduced by | Si, P, G.C. |

Microscopic Constituents. (See also Metallography, under Steel.)

Ferrite, iron free from carbon. It is found in mild steel in small amounts in gray cast iron, and in malleable cast iron.

Cementite, Fe_3C . Fe with 6.67% C. Harder than hardened steel. Hardness U on the mineralogical scale. Found in high C steel, and in white and mottled pig.

Pearlite, a compound made up of alternate laminae of ferrite and cementite, in the ratio of 7 ferrite to 1 cementite, and containing therefore 0.83% C. Found in iron and steel cooled very slowly from a high temperature. In steel of 0.83 C it composes the entire mass. Steels lower or higher than 0.83 C contain pearlite mixed with ferrite or with cementite respectively.

Martensite, the hardening component of steel. Found in iron and steel quenched above the recalescence point, and in tempered steel. It forms the entire structure of 0.83 C steel quenched.

Analyses of Cast Iron. (Notes of the table on page 417.)

1 to 7. R. Moldenke, Pittsburg Foundrymen's Assn., 1898; 1 to 5, pig irons; 6, white iron cast in chills; 7, gray iron cast in sand from the same ladle. The temperatures were taken with a Le Chatelier pyrometer. For comparison, steel, 1.18 C, melted at 2450° F.; silico-spiegel, 12.30 Si, 16.98 Mn, at 2190°; ferro-silicon, 12.01 Si, 2.17 C, at 2040°; ferro-tungsten, 39.02 W, at 2280°; ferro-manganese, 81.4 Mn, at 2255°; ferro-chrome, 62.7 Cr, at 2400°; ditto, 5.4 Cr., at 2180°.

8. Gray foundry Swedish pig, very strong. 9. Pig to be used in mixtures of gray pig and scrap, for castings requiring a hard close grain, machining to a fine surface, and resisting wear. 8 to 15, from paper by F. M. Thomas, *Castings*, July, 1908.

16. Specification by J. E. Johnston, Jr., *Am. Mach.*, Oct. 15, 1903. The results were excellent. Si might have been 0.75 to 1.25 if S had been kept below 0.035.

17 to 22. G. R. Henderson, *Trans. A.S.M.E.*, vol. xx. The chill is to be measured in a test bar 2 × 2 × 24 in., the chill piece being so placed as to form part of one side of the mold. The actual depth of white iron will be measured.

Analyses of Cast Iron.

(Abbreviations, TC, total carbon; GC, graphitic carbon; CC, combined carbon.)

| No. | TC | GC | CC | Silicon. | Man- ganese. | Phos- phorus. | Sul- phur. | |
|-----|------|----------|----------|----------|-----------------|------------------|---------------|----------------------------------|
| 1 | 3.98 | 0.39 | 3.59 | 0.38 | 0.13 | 0.20 | 0.038 | Melts at 2048° F. |
| 2 | 3.78 | 1.76 | 2.01 | 0.69 | 0.44 | 0.53 | 0.031 | Melts at 2156° F. |
| 3 | 3.88 | 2.60 | 1.28 | 1.52 | 0.49 | 0.45 | 0.035 | Melts at 2211° F. |
| 4 | 4.03 | 3.47 | 0.56 | 2.01 | 0.49 | 0.39 | 0.034 | Melts at 2248° F. |
| 5 | 3.56 | 3.43 | 0.13 | 2.40 | 0.90 | 0.08 | 0.032 | Melts at 2280° F. |
| 6 | 4.39 | 0.13 | 4.26 | 0.65 | 0.40 | 0.25 | 0.038 | Melts at 2000° F. |
| 7 | 4.45 | 2.99 | 1.46 | 0.67 | 0.41 | 0.26 | 0.039 | Melts at 2237° F. |
| 8 | 3.30 | 2.80 | 0.50 | 2.00 | 0.60 | 0.08 | 0.03 | Swedish char- coal pig. |
| 9 | | 2.25-2.5 | 0.6-0.8 | 0.8-1.2 | 0.4-0.8 | 0.15-0.4 | | For engine cylin- ders. |
| 10 | 3.40 | 3.40 | trace | 2.90 | 0.50 | 1.65 | 0.04 | English, high P. No. 1. |
| 11 | 3.40 | 3.20 | 0.20 | 2.60 | 0.50 | 1.58 | 0.04 | English, high P. No. 3. |
| 12 | | 3.2-3.6 | 0.1-0.15 | 2.5-2.8 | up to 1.0 | 1.3-1.5 | .03-.04 | For thin orna- mental work. |
| 13 | | 3.0-3.2 | 0.4-0.5 | 2-2.3 | up to 1.0 | 1-1.3 | .06-.08 | For medium size castings. |
| 14 | | 2.8-3.0 | 0.4-0.6 | 1.2-1.5 | 0.6-0.9 | 0.4-0.6 | .06-.08 | Heavy machin- ery castings. |
| 15 | | 2.5-2.8 | 0.6-0.8 | 1.0-1.3 | 0.5-0.7 | 0.4-0.7 | .08-.12 | Cylinders and hydraulic work. |
| 16 | | | | 1.2-1.8 | 0.4-1.0 | 0.4-0.7 | to .06 | For hydraulic cylinders. |
| 17 | | 2.7-3.0 | 0.5-0.8 | 0.5-0.7 | 0.3-0.5 | 0.3-0.5 | .05-.07 | For car wheels. |
| 18 | | 2.6-3.1 | 0.6-1.0 | 0.6-0.7 | 0.1-0.3 | 0.3-0.5 | .05-.08 | For car wheels. |
| 19 | | 2.5-3.0 | 0.4-0.9 | 1.3-1.7 | 0.5-1.0 | 0.3-0.4 | .03 max | Charcoal pig. 1/4 in. chill. |
| 20 | | 2.3-2.7 | 0.5-1.0 | 1.0-1.5 | 0.5-1.0 | 0.3-0.4 | .03 | " Ditto 1/2 in. chill. |
| 21 | | 2.0-2.5 | 0.8-1.2 | 0.8-1.2 | 0.5-1.0 | 0.3-0.4 | .035 | " Ditto 3/4 in. chill. |
| 22 | | 1.8-2.2 | 0.9-1.4 | 0.5-1.0 | 0.3-0.7 | 0.3-0.4 | .035 | " Ditto 1 in. chill. |
| 23 | 3.87 | 3.44 | 0.43 | 1.67 | 0.29 | 0.095 | 0.032 | Series A. Am. F'dmen's Assn. |
| 24 | 3.82 | 3.23 | 0.59 | 1.95 | 0.39 | 0.405 | 0.042 | Series B. ditto. |
| 25 | 3.84 | 3.52 | 0.32 | 2.04 | 0.39 | 0.578 | 0.044 | Series C. ditto. |
| 26 | | 2.8-3.2 | 0.5-0.7 | 1.3-1.5 | 0.3-0.6 | 0.5-0.8 | .06-.10 | For locomotive cylinders. |
| 27 | | 2.3-2.4 | 0.8-1.0 | 1.8-2.0 | 0.8-1.0 | 0.6-0.8 | .06-.10 | "Semi-steel." |
| 28 | | 2.4-2.6 | 0.8-1.0 | 0.9-1.0 | 0.6-0.7 | 0.1-0.3 | .04-.06 | "Semi-steel." |
| 29 | 4.33 | 3.08 | 1.25 | 0.73 | 0.44 | 0.43 | 0.08 | A strong car wheel, Cu, 0.03. |
| 30 | 3.17 | 2.72 | 0.45 | 1.99 | 0.39 | 0.65 | 0.13 | Automobile cyl- inders. |
| 31 | 3.34 | 2.57 | 0.77 | 1.89 | 0.39 | 0.70 | 0.09 | Ditto. |
| 32 | 3.5 | 2.9 | 0.6 | 0.7 | 0.4 | 0.5 | 0.08 | Good car wheel. |
| 33 | 3.55 | 3.0 | 0.55 | 2.75 | 2.39 | 0.86 | 0.014 | Scotch irons. |
| 34 | | | | 3.10 | 1.80 | 0.90 | | "Am. Scotch" Ohio irons. |
| 35 | | | | 0.75-1.5 | to 0.6 | to 0.22 | to 0.04 | Pig for malle- able castings. |
| 36 | | | | 2-25 | to 0.7 | to 0.7 | to 0.15 | Brake-shoes. |
| 37 | | | | 1.2-1.5 | 0.5-0.8 | 0.35-0.6 | to 0.09 | Hard iron for heavy work. |
| 38 | | | | 1.5-2 | 0.5-0.8 | 0.35-0.6 | to 0.08 | Medium iron for general work. |
| 39 | | | | 2.2-2.8 | to 0.7 | to 0.7 | to 0.085 | Soft iron cast'gs |

23 to 25. Series of bars tested by a committee of the association. See results of tests on page 419. Series A, soft Bessemer mixture; B, dynamo-frame iron; C, light machinery iron. Samples for analysis were taken from the 1-in. square dry sand bars.

26. Specifications by a committee of the Am. Ry. Mast. Mechs. Assn., 1906. T.S., 25,000; transverse test, 3000 lb. on $1\frac{1}{4}$ -in. round bar, 12 in. between supports; deflection, 0.1 in. minimum; shrinkage, $\frac{1}{8}$ in. max. 27, soft "semi-steel;" 28, harder do. They approach air-furnace iron in most respects, and excel it in strength; test bars $2 \times 1 \times 24$ in. of the low Si semi-steel showing 2800 to 3000 lb. transverse strength, with $\frac{7}{16}$ in. deflection. M. B. Smith, *Eng. Digest*, Aug., 1908. 29. J. M. Hartman, *Bull. I. & S. Assn.*, Feb., 1892. The chill was very hard, $\frac{1}{4}$ in. deep at root of flange, $\frac{1}{2}$ in. deep on tread. 30, 31. Strong and shock-resisting. T.S., 38,000. *Castings*, June, 1908. 32. Com. of A.S.T.M., 1905, *Proc.*, v. 65. Successful wheels varying quite considerably from these figures may be made. 33, 34. C. A. Meissner, *Iron Age*, 1890. Average of several. 35. R. Moldenke, *A.S.M.E.*, 1908. 36-39. J. W. Keep, *A.S.M.E.*, 1907.

A *Chilling Iron* is one which when cooled slowly has a gray fracture, but when cast in a mold one side of which is a thick mass of cast-iron, called a chill, the fractured surface shows white iron for some depth on the side that was rapidly cooled by the chill. See Table Nos. 19-22.

Specifications for Castings, recommended by a committee of the A.S.T.M., 1908. S in gray iron castings, light, not over 0.08; medium, not over 0.10; heavy, not over 0.12. A light casting is one having no section over $\frac{1}{2}$ in. thick, a heavy casting one having no section less than 2 in. thick, and a medium casting one not included in the classification of light or heavy. The transverse strength of the arbitration bar shall not be under 2500 lb. for light, 2900 lb. for medium, and 3300 lb. for heavy castings; in no case shall the deflection be under 0.10 in. When a tensile test is specified this shall run not less than 18,000 lb. per sq. in. for light, 21,000 lb. for medium, and 24,000 lb. for heavy castings.

The "arbitration bar" is $1\frac{1}{4}$ in. diam., 15 in. long, cast in a thoroughly dried and cold sand mold. The transverse test is made with supports 12 in. apart. The moduli of rupture corresponding to the figures for transverse strength are respectively 39115, 45373, and 51632, being the product of the figures given and the constant 15.646, the factor for R/P for a $1\frac{1}{4}$ -in. round bar 12 in. between supports.* The standard form of tensile test piece is 0.8 in. diam., 1 in. long between shoulders, with a fillet $\frac{7}{32}$ in. radius, and ends 1 in. long, $1\frac{1}{4}$ in. diam., cut with standard thread, to fit the holders of the testing machine.

Specifications by J. W. Keep, *A.S.M.E.*, 1907. See Table of Analyses, Nos. 37-39, page 417. Transverse test, $1 \times 1 \times 12$ -in. bar, hard iron castings. No. 37, 2400 to 2600 lb.; tensile test of same bar, 22,000 to 25,000 lb. No. 38, medium, transverse, 2200 to 2400; tensile, 20,000 to 23,000. No. 39, soft, transverse, 2000 to 2200; tensile, 18,000 to 20,000.

Standard Specifications for Foundry Pig Iron.

(American Foundrymen's Association, May, 1909.)

ANALYSIS.—It is recommended that foundry pig be bought by analysis.

SAMPLING.—Each carload or its equivalent shall be considered as a unit. One pig of machine-cast, or one-half pig of sand-cast iron shall be taken to every four tons in the car, and shall be so chosen from different parts of the car as to represent as nearly as possible the average quality of the iron. Drillings shall be taken so as to fairly represent the composition of the pig as cast. An equal quantity of the drillings from each pig shall be thoroughly mixed to make up the sample for analysis.

PERCENTAGE OF ELEMENTS.—When the elements are specified the following percentages and variations shall be used. Opposite each percentage of the different elements a syllable has been affixed so that buyers, by combining these syllables, can form a code word to be used in telegraphing.

* Formula, $\frac{1}{4}Pl = RI/c$; see page 283. $I = \frac{1}{64}\pi d^4$; $c = \frac{1}{2}d$; $d = 1\frac{1}{4}$ in.; $l = 12$ in.

| SILICON | | SULPHUR | | TOTAL CARBON | | MANGANESE | | PHOSPHORUS | |
|---------|------|---------|------|--------------|------|-----------|------|------------|------|
| % | Code | (max.) | Code | (min.) | Code | % | Code | % | Code |
| 1.00 | La | 0.04 | Sa | 3.00 | Ca | 0.20 | Ma | 0.20 | Pa |
| 1.50 | Le | 0.05 | Se | 3.20 | Ce | 0.40 | Me | 0.40 | Pe |
| 2.00 | Li | 0.06 | Si | 3.40 | Ci | 0.60 | Mi | 0.60 | Pi |
| 2.50 | Lo | 0.07 | So | 3.60 | Co | 0.80 | Mo | 0.80 | Po |
| 3.00 | Lu | 0.08 | Su | 3.80 | Cu | 1.00 | Mu | 1.00 | Pu |
| | | 0.09 | Sy | | | 1.25 | My | 1.25 | Py |
| | | 0.10 | Sh | | | 1.50 | Mh | 1.50 | Ph |

Percentages of any element specified one-half way between the above shall be designated by the addition of the letter *x* to the next lower symbol, thus Lex means 1.75 Si.

Allowed variation: Si, 0.25; P, 0.20; Mn, 0.20. The percentages of P and Mn may be used as maximum or minimum figures when so specified.

Example: — Le-sa-pi-me represents 1.50 Si, 0.04 S, 0.60 P, 0.40 Mn.

BASE OR QUOTING PRICE.—For market quotations an iron of 2.00 Si (with variation 0.25 either way) and S 0.05 (max.) shall be taken as the base. The following table may be filled out, and become a part of a contract; "B," or Base, represents the price agreed upon for a pig of 2.00 Si and under 0.05 S. "C" is a constant differential to be determined at the time the contract is made.

| Sul- | Silicon | | | | | | | | | |
|------|---------|------|------|------|------|------|------|------|------|------|
| phur | 3.25 | 3.00 | 2.75 | 2.50 | 2.25 | 2.00 | 1.75 | 1.50 | 1.25 | 1.00 |
| 0.04 | B+6C | B+5C | B+4C | B+3C | B+2C | B+C | B | B-1C | B-2C | B-3C |
| 0.05 | B+5C | B+4C | B+3C | B+2C | B+1C | B | B-1C | B-2C | B-3C | B-4C |
| 0.03 | B+4C | B+3C | B+2C | B+1C | B | B-1C | B-2C | B-3C | B-4C | B-5C |
| 0.07 | B+3C | B+2C | B+1C | B | B-1C | B-2C | B-3C | B-4C | B-5C | B-6C |
| 0.08 | B+2C | B+1C | B | B-1C | B-2C | B-3C | B-4C | B-5C | B-6C | B-7C |
| 0.09 | B+1C | B | B-1C | B-2C | B-3C | B-4C | B-5C | B-6C | B-7C | B-8C |
| 0.10 | B | B-1C | B-2C | B-3C | B-4C | B-5C | B-6C | B-7C | B-8C | B-9C |

Specifications for Metal for Cast-iron Pipe.—*Proc. A.S.T.M.*, 1905, *A.I.M.E.*, xxxv, 166. Specimen bars 2 in. wide × 1 in. thick × 24 in. between supports, loaded in the center, for pipes 12 in. or less in diam. shall support 1900 lb. and show a deflection of not less than 0.30 in. before breaking. For pipes larger than 12 in., 2000 lb. and 0.32 in. The corresponding moduli of rupture are respectively 34,200 and 36,000 lb. Four grades of pig are specified: No. 1, Si, 2.75; S, 0.035. No. 2, Si, 2.25; S, 0.045. No. 3, Si, 1.75; S, 0.055. No. 4, Si, 1.25; S, 0.065. A variation of 10% of the Si either way, and of 0.01 in the S above the standard, is allowed.

Tensile Tests of Cast-iron Bars.

(American Foundrymen's Association, 1899.)

| | Square Bars. | | | | Round Bars. | | | |
|-------------|--------------|--------|---------|--------|-------------|--------|--------|--------|
| Size, in... | 0.5×0.5 | 1×1 | 1.5×1.5 | 2×2 | 0.56 | 1.13 | 1.69 | 2.15 |
| (A) g. c. | 15,900 | 13,900 | 12,100 | 10,600 | 16,000 | 13,800 | 12,000 | 11,000 |
| " g. m. | 15,400 | 13,400 | 11,600 | 10,100 | 15,500 | 13,300 | 11,500 | 10,500 |
| " d. s. | 14,600 | 12,900 | 11,100 | 9,800 | 14,300 | 13,700 | 11,700 | 10,600 |
| " d. m. | 13,800 | 12,100 | 10,300 | 9,000 | 13,600 | 13,200 | 11,400 | 10,400 |
| (B) g. c. | 17,100 | 15,200 | 12,900 | 11,500 | 16,500 | 15,900 | 13,100 | 11,400 |
| " g. m. | 16,600 | 14,700 | 12,400 | 11,000 | 16,000 | 15,400 | 12,600 | 10,900 |
| " d. s. | 16,300 | 14,400 | 12,100 | 10,700 | 16,700 | 16,200 | 13,200 | 11,000 |
| " d. m. | 15,500 | 13,600 | 11,800 | 10,400 | 15,900 | 15,500 | 12,500 | 10,300 |
| (C) g. c. | 17,700 | 15,800 | 13,500 | 11,900 | 17,800 | 17,400 | 14,200 | 12,000 |
| " g. m. | 17,200 | 15,300 | 13,000 | 11,400 | 17,300 | 16,900 | 13,700 | 11,500 |
| " d. s. | 16,400 | 14,500 | 12,200 | 10,800 | 16,400 | 16,000 | 12,800 | 11,600 |
| " d. m. | 15,600 | 13,700 | 11,900 | 10,500 | 15,600 | 15,200 | 12,000 | 10,400 |
| av. g... | 13,600 | 16,100 | 13,400 | 11,300 | 13,400 | 16,000 | 13,900 | 11,600 |
| av. d... | 15,800 | 15,500 | 13,400 | 11,000 | 15,800 | 15,700 | 13,800 | 11,200 |
| av. c... | 14,700 | 14,800 | 12,500 | 10,900 | 16,300 | 15,200 | 13,000 | 11,200 |
| av. m... | 16,800 | 16,800 | 14,200 | 11,400 | 16,400 | 16,400 | 14,600 | 11,700 |

Transverse Tests of Cast-Iron Bars. Modulus of Rupture.

| Size *..... | 0.5×0.5 | 1×1 | 1.5×1.5 | 2×2 | 2.5×2.5 | 3×3 | 3.5×3.5 | 4×4 |
|---------------------|---------|--------|---------|--------|---------|--------|---------|---------|
| Diam. †..... | 0.56 | 1.13 | 1.69 | 2.15 | 2.82 | 3.38 | 3.95 | 4.51 |
| (A) r. d. c. | 31,100 | 33,400 | 33,900 | 31,700 | 27,000 | 26,600 | 23,400 | 22,600 |
| " r. d. m. | | 27,800 | 38,000 | 32,300 | 28,000 | 28,600 | 22,400 | 22,900 |
| (B) s. g. c. | 44,400 | 39,100 | 39,500 | 33,900 | 31,900 | 29,700 | 27,200 | 27,600 |
| " s. g. m. | | 37,400 | 40,300 | 34,700 | 35,800 | 33,500 | 30,100 | 27,100 |
| " s. d. c. | 35,500 | 38,300 | 34,000 | 32,900 | 31,900 | 30,200 | 29,300 | 25,900 |
| " s. d. m. | | 30,200 | 36,200 | 33,300 | 35,200 | 30,900 | 28,100 | 25,800 |
| " r. g. c. | 36,400 | 46,200 | 41,200 | 41,400 | 41,300 | 36,300 | 34,800 | 31,000 |
| " r. g. m. | | 40,000 | 44,800 | 38,800 | 37,100 | 32,900 | 32,700 | 32,300 |
| " r. d. c. | 37,800 | 49,000 | 44,300 | 39,200 | 40,700 | 31,800 | 35,300 | 31,100 |
| " r. d. m. | | 39,100 | 37,800 | 37,700 | 33,900 | 32,800 | 32,000 | 31,200 |
| (C) s. g. c. | 51,800 | 39,200 | 33,600 | 37,900 | 32,200 | 31,100 | 31,300 | 29,200 |
| " s. g. m. | | | 40,200 | 37,000 | 33,700 | 33,300 | 32,300 | 27,900 |
| " s. d. c. | 48,000 | 39,100 | 38,800 | 35,100 | 31,200 | 29,300 | 29,300 | 27,800 |
| " s. d. m. | | | 38,900 | 35,400 | 33,500 | 32,700 | 29,100 | 25,500 |
| " r. g. c. | 62,800 | 48,500 | 39,000 | 44,500 | 41,400 | 41,200 | 35,000 | 32,300 |
| " r. g. m. | | 55,700 | 49,200 | 42,900 | 41,500 | 36,500 | 34,100 | 36,000 |
| " r. d. c. | 53,000 | 50,400 | 44,000 | 40,200 | 39,500 | 37,800 | 35,200 | 32,100 |
| " r. d. m. | | 47,900 | 51,300 | 38,000 | 38,900 | 36,300 | 32,200 | 33,500 |
| Av. (B) s. | 39,900 | 36,200 | 37,500 | 33,700 | 33,700 | 31,100 | 28,700 | 26,600 |
| " r. | 37,100 | 43,600 | 42,000 | 39,300 | 38,200 | 33,400 | 33,700 | 31,400 |
| " (C) s. | 49,900 | 39,100 | 37,900 | 36,300 | 32,600 | 31,600 | 30,500 | 27,600 |
| " r. | 57,900 | 50,600 | 45,900 | 41,400 | 40,400 | 37,900 | 34,100 | 33,200 |
| " (B) & (C) g. | 48,800 | 43,100 | 41,000 | 38,800 | 36,800 | 33,900 | 32,200 | 30,400 |
| " " & " d. | 43,300 | 41,600 | 40,700 | 36,500 | 35,600 | 32,700 | 31,300 | 30,400 |
| Gen'l av. | 46,100 | 42,400 | 40,800 | 37,700 | 36,200 | 33,400 | 31,700 | 29,900 |
| Equiv. load. | 320 | 2356 | 7650 | 16,756 | 31,424 | 50,100 | 75,516 | 106,311 |

* Size of square bars as cast, in.

† Diam. of round bars as cast, in.

Compression Tests of Cast-Iron Bars.

| Size, in. | 0.5×0.5 | 1×1 | 1.5×1.5 | 2×2 | 2.5×2.5 | 3×3 | 3.5×3.5 | 4×4 |
|----------------|---------|--------|---------|--------|---------|--------|---------|--------|
| (A) (1).... | 29,570 | 20,010 | 17,180 | 13,810 | 10,950 | 9,830 | 9,350 | 9,100 |
| " (2).... | | 21,990 | 17,920 | 13,750 | 12,040 | 11,200 | 10,770 | 10,340 |
| " (3).... | | | 17,180 | 13,880 | 11,430 | 10,270 | 9,830 | 9,950 |
| " (4).... | | | | | 10,950 | 10,430 | 9,540 | 9,570 |
| (B) (1).... | 38,360 | 23,000 | 20,980 | 18,130 | 15,060 | 13,790 | 13,160 | 12,430 |
| " (2).... | | 12,440 | 24,820 | 21,640 | 18,270 | 17,000 | 15,970 | 16,140 |
| " (3).... | | | 20,980 | 18,740 | 15,940 | 14,410 | 15,200 | 13,950 |
| " (4).... | | | | 15,060 | | 13,900 | 13,560 | 13,760 |
| (C) (1).... | 38,360 | 24,890 | 20,750 | 18,010 | 17,840 | 15,950 | 15,880 | 14,220 |
| " (2).... | | 27,900 | 22,060 | 21,750 | 19,800 | 18,170 | 17,100 | 16,410 |
| " (3).... | | | 20,750 | 19,340 | 18,050 | 16,850 | 16,510 | 15,250 |
| " (4).... | | | | 17,840 | | 16,040 | 16,080 | 14,880 |

NOTES ON THE TABLES OF TESTS. — The machined bars were cut to the next size smaller than the size they were cast. The transverse bars were 12 in. long between supports. (A), (B), (C), three qualities of iron; for analyses see page 417; r, round bars; s, square bars; d, cast in dry sand; g, cast in green sand; c, bar tested as cast; m, bar machined to size. The general average (next to last line of the first table) is the average of the six lines preceding. The equivalent load (last line) is the calculated total load that would break a square bar whose modulus of rupture is that of the general average.

COMPRESSION TESTS. — The figures given are the crushing strengths, in pounds, of $\frac{1}{2}$ in. cubes cut from the bars. Multiply by 4 to obtain lbs. per sq. in. (1) Cube cut from the middle of the bar; (2) first $\frac{1}{2}$ in. from edge; (3) second $\frac{1}{2}$ in. from edge; (4) third $\frac{1}{2}$ in. from edge.

Some Tests of Cast Iron. (G. Lanza, *Trans. A.S.M.E.*, x, 187.) — The chemical analyses were as follows: Gun iron: TC, 3.51; GC, 2.80; S, 0.133; P, 0.155; Si, 1.140. Common iron: S, 0.173; P, 0.413; Si, 1.89.

The test specimens were 26 in. long; those tested with the skin on being very nearly 1 in. square, and those tested with the skin removed being cast nearly $1\frac{1}{4}$ in. square, and afterwards planed down to 1 in. square.

| | Tensile Strength. | Elastic Limit. | Modulus of Elasticity. |
|--|----------------------|-------------------|------------------------------|
|--|----------------------|-------------------|------------------------------|

| | | | | |
|-------------------|-----------------------|--------------|--------|------------|
| Unplaned common.. | 20,200 to 23,000 T.S. | Av. = 22,066 | 6,500 | 13,194,233 |
| Planed common.... | 20,300 to 20,800 " | " = 20,520 | 5,833 | 11,943,953 |
| Unplaned gun..... | 27,000 to 28,775 " | " = 28,175 | 11,000 | 16,130,300 |
| Planed gun..... | 29,500 to 31,000 " | " = 30,500 | 8,500 | 15,932,880 |

The elastic limit is not clearly defined in cast iron, the elongations increasing faster than the increase of the loads from the beginning of the test. The modulus of elasticity is therefore variable, decreasing as the loads increase.

The Strength of Cast Iron depends on many other things besides its chemical composition. Among them are the size and shape of the casting, the temperature at which the metal is poured, and the rapidity of cooling. Internal stresses are apt to be induced by rapid cooling, and slow cooling tends to cause segregation of the chemical constituents and opening of the grain of the metal, making it weak. The author recommends that in making experiments on the strength of cast iron, bars of several different sizes, such as $1\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2 in. square (or round), should be taken, and the results compared. Tests of bars of one size only do not furnish a satisfactory criterion of the quality of the iron of which they are made. *Trans. A.I.M.E.*, xxvi, 1017.

Theory of the Relation of Strength to Chemical Constitution. — J. E. Johnston, Jr. (*Am. Mach.*, April 5 and 12, 1900), and H. M. Howe (*Trans. A.I.M.E.*, 1901) have presented a theory to explain the variation in strength of cast iron with the variation in combined carbon. It is that cast iron is steel of CC ranging from 0 to 4%, with particles of graphite, which have no strength, enmeshed with it. The strength of the cast iron therefore is that of the steel or graphiteless iron containing the same percentage of CC, weakened in some proportion to the percentage of GC. The tensile strength of steel ranges approximately from 40,000 lb. per sq. in. with 0 C to 125,000 lb. with 1.20 C. With higher C it rapidly becomes weak and brittle. White cast iron with 3% CC is about 30,000 T.S., and with 4% about 18,000. The amount of weakening due to GC is not known, but by making a few assumptions we may construct a table of hypothetical strengths of different compositions, with which results of actual tests may be compared. Suppose the strength of the steel-white cast-iron series is as given below for different percentages of CC, that 6.25% GC entirely destroys the strength, and that the weakening effect of other percentages is proportional to the ratio of the square root of that percentage to the square root of 6.25, that the TC. in two irons is respectively 3% and 4%, then we have the following:

| | | | | | | | | | | | | | |
|-----------------|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|-----|----|
| Per cent CC.. | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.5 | 2.0 | 2.5 | 3 | 3.5 | 4 |
| Steel, T.S..... | 40 | 60 | 80 | 100 | 110 | 120 | 125 | 110 | 60 | 40 | 30 | 22 | 18 |
| Cast iron, 4% | | | | | | | | | | | | | |

| | | | | | | | | | | | | | |
|---------------|---|------|------|----|------|----|------|------|----|------|----|------|----|
| TC | 8 | 13.2 | 19.2 | 26 | 31.2 | 37 | 41.5 | 40.5 | 26 | 20.7 | 18 | 15.8 | 18 |
| Cast iron, 3% | | | | | | | | | | | | | |

| | | | | | | | | | | | | | |
|---------|------|------|------|----|------|------|----|------|----|------|----|------|----|
| TC..... | 15.4 | 19.9 | 28.5 | 38 | 42.9 | 52.1 | 58 | 56.1 | 36 | 28.7 | 30 | | .. |
|---------|------|------|------|----|------|------|----|------|----|------|----|------|----|

The figures for strength are in thousands of pounds per sq. in. The table is calculated as follows: Take 0.6 CC; with 4% TC., this leaves 3.4 GC, and with 3% TC, 2.4 GC. The sq. root of 3.4 is 1.9, and of 2.4 is 1.55. The ratio of these to $\sqrt{6.25}$ is respectively 74 and 62%, which subtracted from 100 leave 26 and 38% as the percentage of strength of the 0.6 C steel remaining after the effect of the GC is deducted. The table indicates that strength is increased as total C is diminished, and this agrees with general experience.

Relation of Strength to Size of Bar as Cast. — If it is desired that a test bar shall fairly represent a casting made from the same iron, then the dimensions of the bar as cast should correspond to the dimensions of the casting, so as to have about the same ratio of cooling surface to volume that the casting has. If the test bar is to represent the strength of a plate, it should be cut from the plate itself if possible or else cut from a cylindrical shell made of considerable diameter and of a thickness equal to that of the casting. If the test is for distinguishing the quality of the iron, then at least two test bars should be cast, one say $1\frac{1}{2}$ or $5\frac{1}{8}$ in. and one say 2 or $2\frac{1}{2}$ in. diameter, in order to show the effect of rapid and slow cooling.

In 1904 the author made some tests of four bars of "semi-steel" advertised to have a strength of over 30,000 lb. per sq. in. The bars were cast $\frac{1}{2}$, 1, 2, and 3 in. diam., and turned to 0.46, 0.69, 1.6, and 1.85 in. respectively. The results of transverse and tensile tests were:

Mod. of rupture. $\frac{1}{2}$ in., 100,000; 1 in., 61,613; 2 in., 67,619; 3 in., 58,543
T.S. per sq. in. 38,510; " 37,005; " 25,685; " 20,375

The $\frac{1}{2}$ -in. piece was so hard that it could not be turned in a lathe and had to be ground.

Influence of Length of Bar upon the Modulus of Rupture.—(R. Moldenke, *Jour. Am. Foundrymen's Assn.*, Sept., 1899.) Seven sets, each of five 2-in. square bars, made of a heavy machinery mixture, and cast on end, were broken transversely, the distance between supports ranging from 6 to 16 ins. The average results were:

Dist. bet. supports, ins. 6 8 10 12 14 16
Modulus of rupture 40,000 39,000 35,600 37,000 36,000 34,400

The 10-in. bar in six out of seven cases gave a lower result than the 12-in. It appears that the ordinary formulas used in calculating the cross breaking strength of beams are not only incorrect for cast iron, on account of the chemical differences in the iron itself when in different cross sections, but that with the cross sections identical the distance between the supports must be specially provided for by suitable constants in whatever formulæ may be developed. As seen from the above results, the doubling of the distance between supports means a drop in the modulus of rupture in the same sized bar of nearly 10 per cent.

Strength in Relation to Silicon and Cross-section.—In castings one half-inch square in section the strength increases as silicon increases from 1.00 to 3.50; in castings 1 in. square in section the strength is practically independent of silicon, while in larger castings the strength decreases as silicon increases.

The following table shows values taken from Mr. Keep's curves of the approximate transverse strength of cast bars of different sizes reduced to the equivalent strength of a $\frac{1}{2}$ -in. \times 12-in. bar.

| Silicon, Per cent. | Size of Square Cast Bars. | | | | | Silicon, Per cent. | Size of Square Cast Bars. | | | | |
|-----------------------|--|-------|-------|-------|-------|-----------------------|--|-------|-------|-------|-------|
| | 1/2 in. | 1 in. | 2 in. | 3 in. | 4 in. | | 1/2 in. | 1 in. | 2 in. | 3 in. | 4 in. |
| | Strength of a 1/2-in. × 12-in. Section, lb. | | | | | | Strength of a 1/2-in. × 12-in. Section, lb. | | | | |
| 1.00 | 290 | 260 | 232 | 222 | 220 | 2.50 | 392 | 278 | 212 | 190 | 184 |
| 1.50 | 324 | 272 | 228 | 212 | 208 | 3.00 | 426 | 276 | 202 | 180 | 172 |
| 2.00 | 358 | 278 | 220 | 202 | 196 | 3.50 | 446 | 264 | 192 | 168 | 160 |

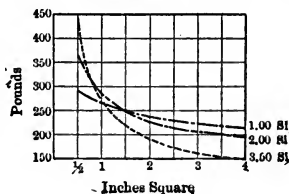


FIG. 92.

Fig. 92 shows the relation of the strength to the size of the cast-iron bar and to Si, according to the figures in the above table. Comparing the 2-in. bars with the $\frac{1}{2}$ -in. bars, we find

Si, per cent. 1 1.5 2 2.5 3 3.5
2-in. weaker than $\frac{1}{2}$ -in., per cent. 20 30 35 46 53 57

The fact that with the 1-in. bar the strength is nearly independent of Si, shows that it is the worst size of bar to use to distinguish the quality of the metal. If two bars were used, say 1/2-in. and 2-in., the drop in strength would be a better index to the quality than the test of any single bar could be.

Shrinkage of Cast Iron. — W. J. Keep (*A. S. M. E.* xvi., 1082) gives a series of curves showing that shrinkage depends on silicon and on the cross-section of the casting, decreasing as the silicon and the section increase. The following figures are obtained by inspection of the curves:

| Silicon, Per cent. | Size of Square Bars. | | | | | Silicon, Per cent. | Size of Square Bars. | | | | |
|-----------------------|--------------------------|-------|-------|-------|-------|-----------------------|--------------------------|-------|-------|-------|-------|
| | 1/2 in. | 1 in. | 2 in. | 3 in. | 4 in. | | 1/2 in. | 1 in. | 2 in. | 3 in. | 4 in. |
| | Shrinkage, In. per Foot. | | | | | | Shrinkage, In. per Foot. | | | | |
| 1.00 | 0.178 | 0.158 | 0.129 | 0.112 | 0.102 | 2.50 | 0.142 | 0.121 | 0.091 | 0.072 | 0.060 |
| 1.50 | .166 | .145 | .116 | .099 | .088 | 3.00 | .130 | .109 | .078 | .058 | .046 |
| 2.00 | .154 | .133 | .104 | .086 | .074 | 3.50 | .118 | .097 | .065 | .045 | .032 |

Mr. Keep says: "The measure of shrinkage is practically equivalent to a chemical analysis of silicon. It tells whether more or less silicon is needed to bring the quality of the casting to an accepted standard of excellence."

A shrinkage of 1/8 in. per ft. is commonly allowed by pattern makers. According to the table, this shrinkage will be obtained by varying the Si in relation to the size of the bar as follows: 1/2 in., 3.25 Si; 1 in., 2.4 Si; 2 in., 1.1 Si; 3 and 4, less than 1.0 Si.

Shrinkage and Expansion of Cast Iron in Cooling. (T. Turner, *Proc. I. & S. I.*, 1906.) — Some irons show the phenomenon of expanding immediately after pouring, and then contracting. Four irons were tested, analyzing as follows: (1) "Washed" white iron, CC 2.73; Si, 0.01; P, 0.01; Mn and S, traces. (2) Gray hematite, GC, 2.53; CC, 0.86; Si, 3.47; Mn, 0.55; P, 0.04; S, 0.03. (3) Northampton, GC, 2.60; CC, 0.15; Si, 3.98; Mn, 0.50; P, 1.25; S, 0.03. (4) Cast iron, GC, 2.73; CC, 0.79; Si, 1.41; Mn, 0.43; P, 0.96; S, 0.07. No. 1 was stationary for 5 seconds after pouring, shrunk 125 sec., stationary 10 sec., then shrunk till cold. No. 2 expanded 15 sec., shrunk 20 sec. to original size, continued shrinking 90 sec. longer, stationary 10 sec., expanded 30 sec., then shrunk till cold. No. 3 expanded irregularly with three expansions and two shrinkages, until 125 sec. after pouring the total expansion was 0.019 in. in 12 in., then shrunk till cold. No. 4 expanded 0.08 in. in 50 sec., then shrunk till cold.

Shrinkage Strains Relieved by Uniform Cooling. (F. Schumann, *A. S. M. E.*, xvii, 433.) — Mr. Jackson in 1873 cast a flywheel with a very large rim and extremely small straight arms. Cast in the ordinary way, the arms broke either at the rim or at the hub. Then the same pattern was molded so that large chunks of iron were cast between the arms, a thickness of sand separating them. Cast in this way, all the arms remained unbroken.

Deformation of Castings from Unequal Shrinkage. — (F. Schumann, *A. S. M. E.*, vol. xvii.) A prism cast in a sand mold will maintain its alignment, after cooling in the mold, provided all parts around its center of gravity of cross section cool at the same rate as to time and temperature. Deformation is due to unequal contraction, and this is due chiefly to unequal cooling.

Modifying causes that effect contraction are: Imperfect alloying of two or more different irons having different rates of contraction; variations in the thickness of sand forming the mold; unequal dissipation of heat, the upper surface dissipating the greater amount of heat; position and form of cores, which tend to resist the action of contraction, also the difference in conducting power between moist sand and dry-baked cores; differences in the degree of moisture of the sand; unequal expos-

ure by the removal of the sand while yet in the act of contracting; flanges, ribs, or gussets that project from the side of the prism, of sufficient area to cause the sand to act as a buttress, and thus prevent the natural longitudinal adjustment due to contraction; in light castings of sufficient length the unyielding sand between the flanges, etc., may cause rupture.

Irregular Distribution of Silicon in Pig Iron.—J. W. Thomas (*Iron Age*, Nov. 12, 1891) finds in analyzing samples taken from every other bed of a cast of pig iron that the silicon varies considerably, the iron coming first from the furnace having generally the highest percentage. In one series of tests the silicon decreased from 2.040 to 1.713 from the first bed to the eleventh. In another case the third bed had 1.260 Si, the seventh 1.718, and the eleventh 1.101. He also finds that the silicon varies in each pig, being higher at the point than at the butt. Some of his figures are: Point of pig, 2.328 Si; butt of same, 2.157; point of pig, 1.834; butt of same, 1.787.

White Iron Converted into Gray by Heating. (A. E. Outerbridge, Jr., *Proc. Am. Socy. for Testing Mat'ls*, 1902, p. 229.)—When white chilled iron containing a considerable amount of Si and low in GC is heated to about 1850° F. from 3½ to 10 hours the CC is changed into C, which differs materially from graphite, and a metal is formed which has properties midway between those of steel and cast iron. The specific gravity is raised from 7.2 to about 7.8; the fracture is of finer grain than normal gray iron; and the metal is capable of being forged, hardened, and taking a sharp cutting edge, so that it may be used for axes, hatchets, etc. It differs from malleable cast iron, since the latter has its carbon removed by oxidation, while the converted cast iron retains its original total carbon, although in a changed form. The tensile strength of the new metal is high, 40,000 to 50,000 lb. per sq. in., with very small elongation. The peculiar change from white to gray iron does not take place if Si is low. The analysis of the original castings should be about TC, 3.4 to 3.8; Si, 0.9 to 1.2; Mn, 0.35 to 0.20; S, 0.05 to 0.04; P, 0.04 to 0.03. The following shows the change effected by the heat treatment:

Before annealing, GC, 0.72; CC, 2.60; Si, 0.71; Mn, 0.11; S, 0.045; P, 0.04
After annealing, GC, 2.75; CC, 0.82; Si, 0.73; Mn, 0.11; S, 0.040; P, 0.04

The GC after annealing is, however, not ordinary graphite, but an allotropic form, evidently identical with what Ledebur calls "tempering graphite carbon."

Change of Combined to Graphitic Carbon by Heating.—(H. M. Howe, *Trans. A. I. M. E.*, 1908, p. 483.) On heating white cast iron to different temperatures for some hours, the carbon changes from the combined to the graphitic state to a degree which increases in general with the temperature and with the silicon-content. With 0.05 Si, a little graphite formed at 1832° F.; with 0.13 Si, at 1652° F.; with 2.12 Si, graphite formed at a moderate rate at 1112°, and with 3.15 Si, it formed rapidly at 1112° F. In iron free from Si, with 4.271 comb. C. and 0.255 graphitic, none of the C. was changed to graphite on long heating to from 1680° to 2040° F., but in iron with 0.75 Si the graphite, originally 0.938%, rose to 1.69% on heating to 1787°, and to 2.795% on heating to 2057° F. On the other hand, when carbon enters iron, as in the cementation process in making blister-steel, it appears chiefly as cementite (combined carbon). Also on heating iron containing graphite to high temperatures and cooling quickly, some of the graphite is changed to cementite.

Mobility of Molecules of Cast Iron. (A. E. Outerbridge, Jr., *A. I. M. E.*, xxvi, 176; xxxv, 223.)—Within limits, cast iron is materially strengthened by being subjected to repeated shocks or blows. Six bars 1 in. sq., 15 in. long, subjected for about 4 hours to incessant blows in a tumbling barrel, were 10 to 15% stronger than companion bars not thus treated. Six bars were struck 1000 blows on one end only with a hand hammer, and they showed a like gain in strength. The increase is greater in hard mixtures, or strong iron, than in soft mixtures, or weak iron; greater in 1-in. bars than in ½-in., and somewhat greater in 2-in. than in 1-in. bars. Bars were treated in a machine by dropping a 14-lb. weight on the middle of a 1-in. bar, supports 12 in. apart. Six bars

were first broken by having the weight fall a sufficient distance to break them at the first blow, then six companion bars were subjected to from 10 to 50 blows of the same weight falling one-half the former distance, and then the weight was allowed to fall from the height at which the first bars broke. Not one of the bars broke at the first blow; and from 2 to 10, and in one case 15 blows from the extreme height were required to break them. Mr. Outerbridge believes that every casting when first made is under a condition of strain, due to the difference in the rate of cooling at the surface and near the center, and that it is practicable to relieve these strains by repeatedly tapping the casting, allowing the particles to rearrange themselves and assume a new condition of molecular equilibrium. The results, first reported in 1896, were corroborated by other experimenters. A report in *Jour. Frank. Inst.*, 1898, gave tests of 82 bars, in which the maximum gain in strength compared with untreated bars was 40%, and the maximum increase in deflection was 41%.

In his second paper, 1904, Mr. Outerbridge describes another series of tests which showed that 1-in. sq. bars 15 in. long subjected to repeated heating and cooling grew longer and thicker with each successive operation. One bar heated about an hour each day to about 1450° F. in a gas furnace for 27 times increased its length $1\frac{11}{16}$ in. and its cross-section $\frac{1}{8}$ in. Soft iron expands more rapidly than hard iron. White iron does not expand sufficiently to cover the original shrinkage. Wrought iron and steel bars similarly treated in a closed tube all contracted slightly, the average contraction after 60 heatings being $\frac{1}{8}$ in. per foot. The strength and deflection of the cast-iron bars was greatly decreased by the treatment, 1250 as compared with 2150 lb., and 0.1 in. deflection as compared with 0.15 in. The specific gravity of the expanded bars was 5.49 to 6.01, as compared with 7.13 for the untreated bars.

Grate-bars of boiler furnaces grow longer in use, as do also cast-iron pipes in ovens for heating air.

Castings from Blast Furnace Metal. Castings are frequently made from iron run directly from the blast furnace, or from a ladle filled with furnace metal. Such metal, if high in Si, is more apt to throw out "kish" or loose particles of graphite than cupola metal. With the same percentage of Si, it is softer than cupola metal, which is due to two causes: 1, lower S; 2, higher temperature. T. D. West, *A.I.M.E.*, xxxv, 211, reports an example of furnace metal containing Si, 0.51; S, 0.045; Mn, 0.75; P, 0.094; which was easily planed, whereas if it had been cupola metal it would have been quite hard. J. E. Johnson, Jr., *ibid.*, p. 213, says that furnace metal with S, 0.03, and Si, 0.7, makes good castings, not too hard to be machined. Should the metal contain over 0.9 Si, difficulty is experienced in preventing holes and soft places in the castings, caused by the deposition of kish or graphite during or after pouring. The best way to prevent this is to pour the iron very hot when making castings of small or moderate size.

Effect of Cupola Melting. (G. R. Henderson, *A.S.M.E.*, xx, 621.) — 27 car-wheels were analyzed in the pig and also after remelting. The P remains constant, as does Si when under 1%. Some of the Mn always disappears. The total C remains the same, but the GC and CC vary in an erratic manner. The metal charged into the cupola should contain more GC, Si and Mn than are desired in the castings. Fairbairn (*Manufacture of Iron*, 1865) found that remelting up to 12 times increased the strength and the deflection, but after 18 remeltings the strength was only $\frac{5}{8}$ and the deflection $\frac{1}{3}$ of the original. The increase of strength in the first remeltings was probably due to the change of GC into CC, and the subsequent weakening to the increase of S absorbed from the fuel.

Hard Castings from Soft Pig. (B. F. Fackenthal, Jr., *A.I.M.E.*, xxxv, 993.) — Samples from a car load of pig gave Si, 2.61; S, 0.023. Castings from the same iron gave 2.33 and 2.26 Si, and 0.26 and 0.25 S, or 12 times the S in the original pig; probably due to fuel too high in S, but more probably to the use of too little fuel in remelting.

The loss of Si in remelting, and the consequent hardening, is affected by the amount of Mn, as shown below:

| | | | | |
|--------------------------------------|------|------|------|------|
| Mn, per cent. | 0.04 | 0.20 | 0.43 | 0.53 |
| Si lost in remelting, per cent. | 34 | 23 | 12 | 4 |

Difficult Drilling due to Low Mn.—H. Souther, *A.S.T.M.*, v. 219, reports a case where thin castings drilled easily while thick parts on the same castings rapidly dulled $\frac{1}{2}$ and $\frac{3}{4}$ -in. drills. The chemical constitution was normal except Mn; Si, 2.5; P, 0.7; S about 0.08; C, 3.5; Mn, 0.16. When the Mn was raised to 0.5 the trouble disappeared.

Addition of Ferro-silicon in the Ladle. (A. E. Outerbridge, *Proc. A.S.T.M.*, vi, 263.)—Half a pound of FeSi, containing 50% Si, added to a 200-lb. ladle of soft cast iron used for making pulleys with rims $\frac{1}{4}$ in. thick, prevented the chilling of the surface of the casting, and enabled the pulleys to be turned more rapidly. Analysis showed that the actual increase of the Si in the casting was less than the calculated increase. Tests of the metal treated with FeSi as compared with untreated metal showed a gain in strength of from 2 to 26%, and a gain in deflection of 2 to 3%. The reason assigned for the increase of strength with increase of softness is that cupola iron contains a small amount of iron oxide, which reacts with the Si added in the ladle, forming SiO_2 , which goes into the slag.

Experiments with Titanium added to cast iron in the ladle are reported by R. Moldenke, *Proc. Am. Fdrymen's Assn.*, 1908. Two irons were used: gray, with 2.58 Si, 0.042 S, 0.54 P, 0.74 Mn; and white, with 0.85 Si, 0.07 S, 0.42 P, 0.6 Mn. Two Fe Ti alloys with 10% Ti were used, one containing no C, and the other 5% C. The latter has the lower melting point. The results were as below:

| | | Gray Iron. | | | White Iron. Lbs. | |
|--|---------|------------|----------|----------|------------------|----------|
| Original iron..... | 9 tests | 1720-2260 | av. 2020 | 8 tests | 1920-2110 | av. 2050 |
| Plus 0.05 Ti..... | 4 tests | 2750-3140 | " 3100 | 11 tests | 2210-2660 | " 2400 |
| Plus 0.10 Ti..... | 3 tests | 2880-3150 | " 3030 | | | |
| Plus 0.05 Ti and C | 6 tests | 2850-3230 | " 3070 | 9 tests | 2230-2720 | " 2420 |
| Plus 0.10 Ti and C | 6 tests | 2850-3150 | " 2990 | 10 tests | 2320-2460 | " 2400 |
| Plus 0.15 Ti and C | 4 tests | 3030-3270 | " 3190 | 10 tests | 2280-2620 | " 2520 |
| Average of treated iron..... | | 3070 | | | 2430 | |
| Increase over original..... | | 52% | | | 18% | |
| Modulus of rupture, treated iron, | | | | 48,030 | 38,020 | |

The test bars were $1\frac{1}{4}$ in. diam. 12 in. between supports. The improvement is as marked whether 0.05, 0.10, or 0.15% Ti is used, which indicates that if sufficient Ti is used for deoxidation of the iron, any additional Ti is practically wasted.

Ti lessens the chilling action, yet whatever chill remains shows much harder iron. Test pieces made with iron which chilled $1\frac{1}{2}$ in. deep gave but 1 in. chill when the iron was treated in the ladle. The original iron crushed at 173,000 lbs. per sq. in. and stood 445 in Brinel's test for hardness, soft steel running about 105. The treated piece ran 298,000 lbs. per sq. in. and showed a hardness of 557. Testing the soft metal below the chilled portion for hardness gave 332 for the original and 322 for the treated piece.

Additions of Vanadium and Manganese.—R. Moldenke, *Am. Fdrymen's Assn.*, 1908, *Am. Mach.*, Feb. 20, '08. Experiments were made by adding to melted cast iron in the ladle a ground alloy of ferro-vanadium, containing 14.67 Va, 6.36 C, and 0.18 Si. In other experiments ferro-manganese (80% Mn) was added, together with the vanadium. Four kinds of iron were used: burnt gray iron (gratebars, stove iron, etc.), burnt white iron, gray machinery iron (Si, 2.72, S, 0.065, P, 0.068, Mn, 0.54) and remelted car wheels (white, two samples analyzed: Si, 0.60 and 0.53, S, 0.122, 0.138; P, 0.399, 0.374; Mn, 0.38, 0.44). The following are average results:

| Gray Machinery Iron. | | | | Remelted Car Wheels. | | | |
|-----------------------------|-------|-------------------------|-----------------|----------------------|-------|-------------------------|-----------------|
| Added Per cent. | | Breaking Strength, lbs. | Deflection, In. | Added Per cent. | | Breaking Strength, lbs. | Deflection, In. |
| Va. | Mn. | | | Va. | Mn. | | |
| 0.0 | 0.0 | 1980 | 0.105 | 0.0 | 0.0 | 1470 | 0.050 |
| 0.0 | 0.50 | 1970 | 0.100 | | 0.50 | 2790 | 0.070 |
| 0.05 | | 1980 | 0.100 | 0.05 | | 3020 | 0.060 |
| 0.05 | 0.50 | 2130 | 0.100 | 0.05 | 0.50 | 2970 | 0.090 |
| 0.10 | | 2372 | 0.090 | 0.10 | | 2800 | 0.055 |
| 0.10 | 0.50 | 2530 | 0.120 | 0.10 | 0.50 | 3030 | 0.090 |
| 0.15 | | 2360 | 0.100 | 0.15 | | 2950 | 0.070 |
| | bars | | | 0.15 | 0.50 | 3920 | 0.095 |
| Average treated | | 2224 | | | | 3069 | |
| Mod. of rupture..... 35,800 | | | | 48,020 | | | |

The bars were 1 1/4 in. diam. 12 in. between supports.

The burnt gray iron was increased in breaking strength from 1310 to 2220 lbs. by the addition of 0.05% Va, and the burnt white iron from 1440 to 1910 lbs. by the addition of 0.05 Va and 0.50 Mn.

Strength of Cast-Iron Beams. — C. H. Benjamin, *Mach'y*, May, 1906. Numerous tests were made of beams of different sections, including hollow rectangles and cylinders, I and T-shapes, etc. All the sections were made approximately the same area, about 4.4 sq. in., and all were tested by transverse loading, with supports 18 in. apart. The results, when reduced by the ordinary formula for stress on the extreme fiber, $S = My/I$, showed an extraordinary variation, some of the values being as follows: Square bar, 23,300; Round bar, 25,000. Hollow round, 3.4 in. outside and 2.5 in. inside diam., 26,450, and 35,800. Hollow ellipse, 3 in. wide, 3.9 in. high, 0.9 in. thick, 36,000. I-beam, 4 in. high, web 0.44 in. thick, 17,700. The hollow cylindrical and elliptical sections are much stronger than the solid sections. This is due to the thinner metal, the greater surface of hard skin, and freedom from shrinkage strains. Professor Benjamin's conclusions from these tests are:

(1) The commonly accepted formulas for the strength and stiffness of beams do not apply well to cored and ribbed sections of cast iron.

(2) Neither the strength nor the stiffness of a section increases in proportion to the increase in the section modulus or the moment of inertia.

(3) The best way to determine these qualities for a cast-iron beam is by experiment with the particular section desired and not by reasoning from any other section.

Bursting Strength of Cast-Iron Cylinders. — C. H. Benjamin, *A. S. M. E.*, XIX, 597; *Mach'y*, Nov., 1905. Four cylinders, 20 in. long, 10 1/8 in. int. diam., 3/4 in. thick, with flanged ends and bolted covers, burst at 1350, 1400, 1350, and 1200 lbs. per sq. in. hydraulic pressure, the corresponding fiber stress, from the formula $S = pd/2t$, being 9040, 10,200, 9735 and 9080. Pieces cut from the shell had an average tensile strength of 14,000 lbs. per sq. in., and a modulus of rupture in transverse tests of 30,000.

Transverse Strength of Cast-iron Water-pipe. (*Technology Quarterly*, Sept., 1897.) — Tests of 31 cast-iron pipes by transverse stress gave a maximum outside fibre stress, calculated from maximum load, assuming each half of pipe as a beam fixed at the ends, ranging from 12,800 lbs. to 23,300 lbs. per sq. in.

Bars 2 in. wide cut from the pipes gave moduli of rupture ranging from 28,400 to 51,400 lbs. per sq. in. Four of the tests, bars and pipes:

| | | | | |
|--------------------------------|--------|--------|--------|--------|
| Moduli of rupture of bar | 28,400 | 34,400 | 40,000 | 51,400 |
| Fiber stress of pipe..... | 18,300 | 12,800 | 14,500 | 26,300 |

These figures show a great variation in the strength of both bars and pipes, and also that the strength of the bar does not bear any definite relation to the strength of the pipe.

Bursting Strength of Flanged Fittings. — *Power*, Feb. 4, 1908. The Crane Company, Chicago, published in the *Valve World* records of tests of tees and ells, standard and extra heavy, which show that the bursting strength of such fittings is far less than is given by the standard formulæ for thick cylinders. As a result of the tests they give the following empirical formula: $B = TS/D$, in which B = bursting pressures, lbs. per sq. in., T = thickness of metal, D = inside diam., and S = 65% of the tensile strength of the metal for pipes up to 12 in. diam., for larger sizes use 60%. The pipes were made of "ferro-steel" of 33,000 lbs. T. S., and of cast iron of 22,000 lbs. as tested in bars. The following are the principal results of tests of extra heavy tees and ells compared with results of calculation by the Crane Company's formula:

BURSTING STRENGTH OF PIPE-FITTINGS. POUNDS PER SQUARE INCH.

| Inside Diam. Thickness. | 6 3/4 | 8 13/16 | 10 15/16 | 12 1 | 14 1 1/8 | 16 1 3/16 | 18 1 1/4 | 20 1 5/16 | 24 1 1/2 |
|----------------------------|----------|------------|-------------|---------|-------------|--------------|-------------|--------------|-------------|
| B, Ferro-steel..... | 2733 | 2250 | 2160 | 2033 | 1825 | 1700 | 1450 | 1275 | 1300 |
| calculated..... | 2680 | 2180 | 2010 | 1870 | 1570 | 1450 | 1350 | 1280 | 1220 |
| B, Cast iron..... | 1687 | 1350 | 1306 | 1380 | 1100 | 1025 | 600 | 750 | 700 |
| calculated..... | 1790 | 1450 | 1340 | 1190 | 1060 | 980 | 920 | 870 | 820 |
| Ells, ferro steel..... | 3266 | 2725 | 2350 | 2133 | | | | | |
| " cast-iron..... | 2275 | 1625 | 1541 | 1275 | 1075 | 1250 | | | |

Specific Gravity and Strength. (Major Wade, 1856.)

Third-class guns: Sp. Gr. 7.087, T. S. 20,148. Another lot: least Sp. Gr. 7.163, T. S. 22,402.

Second-class guns: Sp. Gr. 7.154, T. S. 24,767. Another lot: mean Sp. Gr. 7.302, T. S. 27,232.

First-class guns: Sp. Gr. 7.204, T. S. 28,805. Another lot: greatest Sp. Gr. 7.402, T. S. 31,027.

Strength of Charcoal Pig Iron. — Pig iron made from Salisbury ores, in furnaces at Wassaic and Millerton, N. Y., has shown over 40,000 lbs. T. S. per square inch, one sample giving 42,281 lbs. Muirkirk, Md., iron tested at the Washington Navy Yard showed: average for No. 2 iron, 21,601 lbs.; No. 3, 23,959 lbs.; No. 4, 41,329 lbs.; average density of No. 4, 7.336 (J. C. I. W., v. p. 44).

Nos. 3 and 4 charcoal pig iron from Chapinville, Conn., showed a tensile strength per square inch of from 34,761 lbs. to 41,882 lbs. Charcoal pig iron from Shelby, Ala. (tests made in August, 1891), showed a strength of 34,800 lbs. for No. 3; No. 4, 39,675 lbs.; No. 5, 46,450 lbs.; and a mixture of equal parts of Nos. 2, 3, 4, and 5, 41,470 lbs. (*Bull. I. & S. A.*)

Variation of Density and Tenacity of Gun-Irons. — An increase of density invariably follows the rapid cooling of cast iron, and as a general rule the tenacity is increased by the same means. The tenacity generally increases quite uniformly with the density, until the latter ascends to some given point; after which an increased density is accompanied by a diminished tenacity.

The turning-point of density at which the best qualities of gun-iron attain their maximum tenacity appears to be about 7.30. At this point of density, or near it, whether in proof-bars or gun-heads, the tenacity is greatest.

As the density of iron is increased its liquidity when melted is diminished. This causes it to congeal quickly, and to form cavities in the interior of the casting. (Pamphlet of Builders' Iron Foundry, 1893.)

"Semi-steel" is a trade name given by some founders to castings made from pig iron melted in the cupola with additions of from 20 to 30 per cent of steel scrap. Ferro-manganese is also added either in the cupola or in the ladle. The addition of the steel dilutes the Si of the pig iron, and changes some of the C from GC to CC, but the TC is unchanged, for any reduction made by the steel is balanced by absorption of C from the fuel.

Semi-steel therefore is nothing more than a strong cast iron, low in Si and containing some Mn, and the name given it is a misnomer.

Mixture of Cast Iron with Steel. — Car wheels are sometimes made from a mixture of charcoal iron, anthracite iron, and Bessemer steel. The following shows the tensile strength of a number of tests of wheel mixtures, the average tensile strength of the charcoal iron used being 22,000 lbs. (*Jour. C. I. W.*, iii, p. 184):

| | | lbs. per sq. in. |
|--------------------|--|------------------|
| Charcoal iron with | 21 1/2% steel | 22,467 |
| " | " | " |
| " | 33 3/4% steel | 26,733 |
| " | 61 1/4% steel and 61 1/4% anthracite | 24,400 |
| " | 71 3/8% steel and 71 1/2% anthracite | 28,150 |
| " | 21 1/2% steel, 21 1/2% wro't iron, and 61 1/4% anth. | 25,550 |
| " | 5 % steel, 5% wro't iron, and 10% anth. | 26,500 |

Cast Iron Partially Bessemerized. — Car wheels made of partially Bessemerized iron (blown in a Bessemer converter for 3 1/2 minutes), chilled in a chill test mold over an inch deep, just as a test of cold blast charcoal iron for car wheels would chill. Car wheels made of this blown iron have run 250,000 miles. (*Jour. C. I. W.*, vi, p. 77.)

Bad Cast Iron. — On October 15, 1891, the cast-iron fly-wheel of a large pair of Corliss engines belonging to the Amoskeag Mfg. Co., of Manchester, N.H., exploded from centrifugal force. The fly-wheel was 30 feet diameter and 110 inches face, with one set of 12 arms, and weighed 116,000 lbs. After the accident, the rim castings, as well as the ends of the arms, were found to be full of flaws, caused chiefly by the drawing and shrinking of the metal. Specimens of the metal were tested for tensile strength, and varied from 15,000 lbs. per square inch in sound pieces to 1000 lbs. in spongy ones. None of these flaws showed on the surface, and a rigid examination of the parts before they were erected failed to give any cause to suspect their true nature. Experiments were carried on for some time after the accident in the Amoskeag Company's foundry in attempting to duplicate the flaws, but with no success in approaching the badness of these castings.

Permanent Expansion of Cast Iron by Heating. (*Valve World*, Sept., 1908.) — Cast iron subjected to continued temperatures of approximately 500° to 600° took a permanent expansion and did not return to its original volume when cooled.

As steam is being superheated quite commonly to temperatures above 575°, this fact is of great interest inasmuch as it modifies our ideas about the proper material to be used in the construction of valves and fittings for service under high temperatures. A permanent volumetric expansion is followed by a loss of strength, the loss in cast iron being fully 40 per cent in four years.

Crane Co. made an attempt to determine whether cast steel was affected in the same manner as cast iron. Three flanges were taken, one of cast iron, one of ferrosteel, and the third of cast steel. These flanges were exposed for a total period of 130 hours to temperatures ranging as follows:

Less than 500°, 18 hours; 500° to 700°, 97 hours; 710° to 800°, 12 hours; over 800°, 3 hours. Average temp., 583°.

The outside diameter in each case was 12 1/2 in. and the bore 6 29/64 in.

The results were: Cast-steel flange, no change. Cast-iron flange, outside diam. increased 0.019 in., inside diam. increased 0.007 in. Ferro-steel flange, outside diam. increased 0.033 in., inside diam. increased 0.017 in.

If the permanent expansion of cast iron stopped at the figures given above, it would not be a serious matter; but all evidence points toward a steady increase as time goes on, as was shown by one of Crane Co.'s 14-in. valves, which originally was 22 1/2 in. face to face, and increased 5/16 in. in length in four years under an average temperature of about 590°.

MALLEABLE CAST IRON.*

There are four great classes of work for whose requirements malleable cast iron (commonly called "malleable iron" in America) is especially

* References. — R. Moldenke, *Cass. Mag.*, 1907, and *Iron Trade Review*, 1908; E. C. Wheeler, *Iron Age*, Nov. 9, 1899; C. H. Gale, *Indust. World*, April 13, 1908; W. H. Hatfield, *ibid.* G. A. Akerlund, *Iron Tr. Rev.*, Aug. 23, 1906; C. H. Day, *Am. Mach.*, April 5, 1906.

adapted. These are agricultural implements, railway supplies, carriage and harness castings and pipe fittings. Besides these main classes there are innumerable other unclassified uses. The malleable casting is seldom over 175 lbs. in weight, or 3 ft. in length, or $\frac{3}{4}$ in. in thickness. The great majority of even the heavier castings do not exceed 10 lbs.

When properly made, malleable cast iron should have a tensile strength of 42,000 to 48,000 lbs. per sq. in., with an elongation of 5% in 2 in. Bars 1 in. square and on supports 12 in. apart should show a transverse strength of 2500 to 3500 lbs., with a deflection of at least $\frac{1}{2}$ in.

While the strength of malleable iron should be as stated, much of it will fall as low as 35,000 lbs. per sq. in., and this will still be good for such work as pipe fittings, hardware castings and the like. On the other hand, even 63,000 lbs. per sq. in. has been reached, with a load of 5000 lbs. and a deflection of $2\frac{1}{2}$ in. in the transverse test. This high strength is not desirable, as the softness of the casting is sacrificed, and its resistance to continued shock is lessened. For the repeated stresses of severe service the malleable casting ranks ahead of steel, and only where a high tensile strength is essential must it be replaced by that material.

The process of making malleable iron may be summarized as follows: The proper cast irons are melted in either the crucible, the air furnace, the open-hearth furnace or the cupola. The metal when cast into the sand molds must chill white or not more than just a little mottled. After removing the sand from the hard castings they are packed in iron scale, or other materials containing iron oxide, and subjected to a red heat (1250 to 1350° F.) for over 60 hours. They are then cooled slowly, cleaned from scale, chipped or ground, and straightened.

When hard, or just from the sand, the composition of the iron should be about as follows: Si, from 0.35 up to 1.00, depending upon the thickness and the purpose the casting is to be used for; P not over 0.225, Mn not over 0.20, S not over 0.05. The total carbon can be from 2.75 upward, 4.15 being about the highest that can be carried. The lower the carbon the stronger the casting subsequently. Below 2.75 there is apt to be trouble in the anneal, the black-heart structure may not appear, and the castings remain weak. A casting 1 in. thick would necessitate silicon at 0.35, and the use of chills in the mold in addition, to get the iron white. For a casting $\frac{1}{2}$ in. thick, Si about 0.60 is the proper limit, except where great strength is desired, when it can be dropped to 0.45. Above 0.60 there is danger of getting heavily-mottled if not gray iron from the sand molds, and this material, when annealed the long time required for the white castings, would be ruined. For very thin castings, Si can run up to 1.00 and still leave the metal white in fracture.

Pig Iron for Malleable Castings. — The specifications run as follows: Si, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00%, as required; Mn, not over 0.60; P, not over 0.225; S, not over 0.05.

Works making heavy castings almost exclusively, specify Si to include 0.75 up to 1.50%. Makers of very light work take 1.25 to 2.00%.

The Melting Furnace. — Malleable iron is melted in the reverberatory furnace, the open-hearth furnace and the cupola; the reverberatory being the most extensively used, about 85 per cent of the entire output of the United States being melted by this process. Prior to about 1885, the standard furnace was one of 5 tons capacity. At present (1908) we have furnaces of 25 and 30 tons capacity, though furnaces of from 10 to 15 tons are the most popular and give more uniform results than those of larger capacity.

The adoption of the open-hearth furnace for malleable iron dates back to about 1893. It is used largely in the Pittsburgh district.

Cupola melted iron does not possess the tensile strength nor ductility of iron melted in the reverberatory or open-hearth furnace, due partly to the higher carbon and sulphur caused by the metal being in contact with the fuel. This feature is rather an advantage than otherwise, as most of the product of cupola melted iron consists of pipe fittings; castings that are not subjected to any great stress or shock. The castings are threaded, and a strong, tough malleable iron does not cut a clean, smooth thread, but rather will rough up under the cutting tool.

In the reverberatory and open-hearth furnaces the metal may be partly desiliconized at will, by an oxidizing flame or by additions of scrap or other low-silicon material. Manganese is also oxidized in the furnace,

The composition of good castings in American practice is: Si, from 0.45 to 1.00%; Mn, up to 0.30%; P, up to 0.225%; S, up to 0.07%; total carbon in the hard casting, above 2.75%.

In special cases, especially for very small castings, the silicon may go up as high as 1.25%, while for very heavy work it may drop down to 0.35% with very good results. In the case of charcoal iron this figure gives the strongest castings. With coke irons, however, especially when steel scrap additions are the rule, 0.45 should be the lower limit, and 0.65 is the best silicon for all-around medium and heavy work, such as railroad castings.

In American practice phosphorus is required not to exceed 0.225%, and is preferred lower. In European practice it is required as low as 0.10%, but castings have been made successfully with P as high as 0.40%.

The heat treatment of metal during melting has an important bearing upon its tensile strength, elongation, etc. Excessive temperatures promote the chances of burning. Iron is burnt mainly through the generation in melting furnaces of higher temperatures than those prevailing during the initial casting at blast furnaces and an excess of air in the flame. The choicest irons may thus turn out poor material.

Shrinkage of the Casting. — The shrinkage of the hard casting is about $\frac{1}{4}$ in. to the foot, or double that of gray iron. In annealing about half of this is recovered, and hence the net result is the same as in ordinary foundry pattern practice. The effect of this great shrinkage is to cause shrinkage cracks or sponginess in the interior of the casting. As soon as the liquid metal sets against the surface of the mold and the source of supply is cut off, the contraction of the metal in the interior as it cools causes the particles to be torn apart and to form minute cracks or cavities. "Every test bar, and for that matter every casting may be regarded as a shell of fairly continuous metal with an interior of slight planes of separation at right angles to the surface. This characteristic of malleable iron forms the basis of many a mysterious failure." (*Moldenke*.)

Packing for Annealing. — After the castings have been chipped and sorted they are packed in iron annealing pots, holding about 800 pounds of iron, together with a packing composed of iron ore, hammer and rolling mill scale, turnings, borings, etc. The turnings, etc., were formerly treated with a solution of salammoniac or muriatic acid to form a heavy coating of oxide, but such treatment is now considered unnecessary. Blast furnace slag, coke, sand, and fire clay have also been used for packing. The changes in chemical composition of the castings when annealed in slag and in coke are given as follows by C. H. Gale:

| | Si. | S. | P. | Mn. | C. C. | G. C. |
|-----------------------|------|-------|-------|------|-------|-------|
| Hard iron..... | 0.63 | 0.043 | 0.147 | 0.21 | 2.54 | Trace |
| Annealed in slag..... | 0.61 | 0.049 | 0.145 | 0.21 | 0.24 | 1.65 |
| Annealed in coke..... | 0.61 | 0.065 | 0.150 | 0.21 | 0.25 | 2.00 |

The Annealing Process. — The effect of the annealing is to oxidize and remove the carbon from the surface of the casting, to remove it to a greater or less degree below the surface, and to convert the remaining carbon from the combined form into the amorphous form called a "temper carbon" by Professor Ledebur, the German metallurgist. It differs from the graphite found in pig iron, but is usually reported as graphitic carbon by the chemists. In the original malleable process, invented by Reaumur, in 1722, the castings were packed in iron ore and annealed thoroughly, so that most of the carbon was probably oxidized, but in American practice the annealing process is rather a heat treatment than an oxidizing process, and its effect is to precipitate the carbon rather than to eliminate it. According to the analysis quoted above, the metal annealed in slag lost 0.65% of its total C, while that annealed in coke lost only 0.29%. In the former, S increased 0.006% and in the latter 0.022%. The Si decreased 0.02% in both cases, while the P and Mn remained constant.

As to the distribution of carbon in an annealed casting, Dr. Moldenke says: "Take a flat piece of malleable and plane off the skin, say $\frac{1}{16}$ in. deep and gather the chips for analysis. The carbon will be found, say, 0.15% perhaps even less. Cut in another $\frac{1}{16}$ in. and the total C will be nearer 0.60%. Now go down successively by sixteenths and the total C will range from, say, 1.70 to 3.65% and will then remain constant until the center is reached." "The malleable casting is for practical purposes a poor steel casting with a lot of graphite, not crystallized, between the crystals or groups of crystals of the steel."

The heat in the annealing process must be maintained for from two to four days, depending upon the thickness of sections of the castings and the compactness with which the castings or annealing boxes are placed in the furnace. An annealing temperature 1550° to 1600° Fahr. is often used, but it is not essential, as the annealing can be accomplished at 1300°, but the time required will be longer than that at the higher temperature. Burnt iron in the anneal is no uncommon feature, and, generally speaking, it is the result of carelessness. The most carefully prepared metal from melting furnaces can here be turned into worthless castings by some slight inattention of detail. The highest temperature for annealing should be registered in each foundry, and kept there by the daily and frequent use of a thermometer constructed for that sole purpose. Steady, continued heat insures soft castings, while unequal temperatures destroy all chances for successful work, although the initial metal was of the most excellent quality.

After annealing, the castings are cleaned by tumblers or the sand blast; they are carefully examined for cracks or other defects, and if sprung out of shape are hammered or forced by hydraulic power to the correct shape. Such parts as are produced in great quantities are placed in a drop hammer and one or two blows will insure a correct form. They may be drop-forged or even welded when the iron has been made for that purpose. Castings are sometimes dipped into asphaltum diluted with benzine to give them a better finish.

Malleable castings must never be straightened hot, especially when thick. In the case of very thin castings there is some latitude, as the material is so decarbonized that it is nearer a steel than genuine malleable cast iron. In heating portions of castings that were badly warped, it seems that the amorphous carbon in them was combined again, and while the balance of the casting remained black and sound, the heated parts became white and brittle, as in the original hard casting. Hence the advice to straighten the castings cold, preferably with a drop hammer and suitable dies, or still better in the hydraulic press. (R. Moldenke. *Proc. A. S. T. M.*, vi, 244.)

Physical Characteristics. — The characteristic that gives malleable iron its greatest value as compared with gray iron is its ability to resist shocks. Malleability in a light casting $\frac{1}{4}$ in. thick and less means a soft, pliable condition and the ability to withstand considerable distortion without fracture, while in the heavy sections, $\frac{1}{2}$ in. and over, it means the ability to resist shocks without bending or breaking.

For general purposes it is not altogether desirable to have a metal very high in tensile strength, but rather one which has a high transverse strength, and especially a good deflection. It is not always that a strong and at the same time soft material can be produced in a foundry operating on the lighter grades of castings. The purchaser, therefore, unless he requires very stiff material, should rather look upon the deflection of the metal coupled with the weight it took to do this bending before failure, than for a high tensile strength.

The ductility of the malleable casting permits the driving of rivets, which cannot so readily be done with gray cast iron; and for certain parts of cars, like the journal boxes, malleable cast iron may be considered supreme, leaving cast iron and "semi-steel" far behind.

It was formerly the general belief that the strength of malleable iron was largely in the white skin always found on this material, but it has been demonstrated that the removal of the skin does not proportionately lessen the strength of the casting.

Test Bars. — The rectangular shape is used for test bars in preference to the round section, because the latter is more apt to have serious cracks in the center, due to shrinkage, especially if the diameter is large. A round section, unless in very light hardware, is to be avoided, as the

shrinkage crack in the center may have an outlet to the skin, and cause failure in service.

It is customary to provide for two sizes of test bars, the heavy and the light. Thus the 1-in. square bar represents work $\frac{1}{2}$ an inch thick and over, and a $1 \times \frac{1}{2}$ -in. section bar cares for the lighter castings. Both are 14 inches long. They should be cast at the beginning and at the end of each heat.

Design of Malleable Castings.—As white cast iron shrinks a great deal more than gray iron, and as the sections of malleable castings are lighter than those of similar castings of gray iron, fractures are very common. It is therefore the designer's aim to distribute the metal so as to meet these conditions. In long pieces the stiffening ribs should extend lengthways so as to produce as little resistance as possible to the contraction of the metal at the time of solidification. If this be not possible, the mold provides a "crush core" whose interior is filled with crushed coke. When the metal solidifies in the flask the core is crushed by the casting and thus prevents shrinkage cracks. At other times a certain corner or juncture of ribs in the casting will be found cracked. In order to prevent this a small piece of cast iron (chill) is embedded in the sand at this critical point, and the metal will cool here more quickly than elsewhere, and thus fortify this point, although it may happen that some other part of the casting will be found fractured instead, and in many cases the locations and the shape of strengthening ribs in the casting must be altered until a casting is procured free from shrinkage cracks. In designing of malleable cast-iron details the following rules should be observed:

(1) Endeavor to keep the metal in different parts of the casting at a uniform thickness. In a small casting, of, say, 10 lbs. weight, $\frac{1}{4}$ -in. metal is about the practical thickness, $\frac{5}{16}$ in. for a casting of 15 to 20 lbs., and $\frac{3}{8}$ to $\frac{1}{2}$ in. for castings of 40 lbs. and over. (2) Endeavor to avoid sharp junctions of ribs or parts, and if the casting is long, say 24 inches or more, the ends should be made of such shape as to offer as little resistance as possible to the contraction of metal when cooling in the mold.

Specifications for Malleable Iron.—The tensile strength of malleable iron varies with the thickness of the metal, the lighter sections having a greater strength per square inch than the heavier sections. An Eastern railroad designates the tensile strength desired as follows: Sections $\frac{3}{8}$ in. thick or less should have a tensile strength of not less than 40,000 lbs. per sq. in.; $\frac{3}{8}$ to $\frac{3}{4}$ in. thick, not less than 38,000; and over $\frac{3}{4}$ in., not less than 36,000 lbs. per sq. in. Test bars $\frac{5}{8}$ and $\frac{7}{8}$ in. diam. were made in the same mold and poured from the same ladle, and annealed together. The average tensile strength of five pairs of bars so treated, representing five heats, was, $\frac{5}{8}$ -in. bars, 45,095; $\frac{7}{8}$ -in. bars, 41,316 lbs. per sq. in. Average elongation in 6 in.: $\frac{5}{8}$ -in. bars 5.3%; $\frac{7}{8}$ -in. bars 4.2%.

A very high tensile strength can be obtained approaching that of cast steel but at the expense of the malleability of the product. Malleable test bars have been made with a tensile strength of between 60,000 and 70,000 lbs. per sq. in., but the ductility and ability to resist shocks of these bars was not equal to that of bars breaking at 40,000 to 45,000 pounds per sq. in.

The British Admiralty specification is 18 tons (40,320 lbs.) per square inch, a minimum elongation of $4\frac{1}{2}\%$ in three inches and a bending angle of at least 90° over a 1-in. radius, the bar being $1 \times \frac{3}{8}$ in. in section.

The specifications of the American Society for Testing Materials include the following:

Cupola iron is not recommended for heavy or important castings.

Castings for which physical requirements are specified shall not contain over 0.06 sulphur or over 0.225 phosphorus.

The Standard Test Bar is 1 in. square and 14 in. long, cast without chills and left perfectly free in the mold. Three bars shall be cast in one mold, heavy risers insuring sound bars. Where the full heat goes into castings which are subject to specification, one mold shall be poured two minutes after tapping into the first ladle, and another mold from the last iron of the heat.

The tensile strength of a standard test bar shall not be less than 40,000 lbs. per sq. in. The elongation in 2 in. shall not be less than $2\frac{1}{2}\%$.

The transverse strength of a standard test bar on supports 12 inches apart shall not be less than 3000 lbs., deflection being at least $\frac{1}{2}$ in.

Improvement in Quality of Castings. (Moldenke.)—The history of improvement in the malleable casting is admirably reflected in the test records of any works that has them. Going back to the early 90's, the average tensile strength of malleable cast iron was about 35,000 lbs. per sq. in., with an elongation of about 2% in 2 in. The transverse strength was perhaps 2800 lbs., with a deflection of $\frac{1}{2}$ in. Toward the close of the 90's a fair average of the castings then made would run about 44,000 lbs. per sq. in., with an elongation of 5% in 2 in., and the transverse strength, about 3500 lbs., with a deflection of $\frac{1}{2}$ inch. These average figures were greatly exceeded in establishments where special attention was given to the niceties of the process. The tensile strength here would run 52,000 lbs. per sq. in. regularly, with 7% elongation in 2 in., and the transverse strength, 5000 and over, with $1\frac{1}{2}$ in. deflection.

Further Progress Desirable. (Moldenke.)—We do not know at the present time why cupola malleables require an annealing heat several hundred degrees higher than air or open-hearth furnace iron. The underlying principles of the oxidation of the bath, which is a frequent cause of defective iron, is practically unknown to the majority of those engaged in this industry. Heats are frequently made that will not pour nor anneal properly, but the causes are still being sought. To produce castings from successive heats, so that with the same composition they will have the same physical strength regardless of how they are tested, is a problem partially solved for steel, but not yet approached for malleable cast iron.

Sufficient progress in the study of iron with the microscope has been made to warrant the belief that in the not distant future we may be able to distinguish the constituents of the material by means of etching with various chemicals. When the sulphides and phosphides of iron, or the manganese-sulphur compounds, can be seen directly under the microscope, it is probable that a method may be found by which the dangerous ingredients may be so scattered or arranged that they will do the least harm.

The high sulphur in European malleable accounts to some extent for the comparatively low strength when contrasted with our product. Their castings being all very light, so long as they bend and twist properly, the purpose is served, and hence until heavier castings become the rule instead of the exception, "white heart" and steely-looking fractures will remain the characteristic feature of European work.

Strength of Malleable Cast Iron.

Bars cast by Buhl Malleable Co., Detroit, Mich. Reported by Chas. H. Day, *Am. Mach.*, April 5, 1906. The castings were all made at the same time. The figures here given are the maximum and minimum results from three bars of each section.

TENSILE TESTS.

COMPRESSION TESTS.

| Section. | Area, sq. in. | Tensile St'gth, lbs. per sq. in. | Elong. in 8 in., %. | Red. of Area, %. | Area, sq. in. | L'gth, in. | Comp. Str., lbs. per sq. in. | Final Area, sq. in. |
|----------|---------------|----------------------------------|---------------------|------------------|---------------|------------|------------------------------|---------------------|
| Round | 0.817 | 43,000 | 5.87 | 4.76 | 0.847 | 15 | 31,700 | 0.901 |
| " | 0.801 | 43,400 | 6.21 | 3.98 | 0.801 | 15 | 33,240 | 0.886 |
| " | 0.219 | 41,130 | 7.70 | 3.40 | 0.209 | 7.5 | 32,600 | 0.221 |
| " | 0.202 | 44,700 | 13.00 | 3.63 | 0.204 | 7.5 | 34,600 | 0.215 |
| Square | 0.277 | 36,700 | 4.70 | 2.20 | 0.263 | 7.5 | 33,200 | 0.272 |
| " | 0.277 | 38,100 | 3.72 | 3.00 | 0.254 | 7.5 | 31,870 | 0.278 |
| " | 1.040 | 38,460 | 4.10 | 3.30 | 1.051 | 15 | 29,650 | 1.070 |
| " | 1.050 | 37,860 | 2.38 | 2.94 | 1.040 | 15 | 30,450 | 1.066 |
| Rect. | 0.239 | 31,200* | 5.19 | 1.50 | 0.436 | 15 | 32,200 | 0.448 |
| " | 0.244 | 37,600 | 3.87 | 3.80 | 0.457 | 15 | 30,400 | 0.467 |
| Star | 0.584 | 34,600 | 4.20 | 3.10 | | | | |
| " | 0.575 | 37,200 | 4.80 | 3.50 | | | | |

* Broke in flaw.

The rectangular sections were approximately $\frac{1}{4} \times \frac{3}{4}$ in. The star sections were square crosses, 1 inch wide, with arms about $\frac{1}{4}$ in. thick.

Tests of Rectangular Cast Bars, made by a committee of the Master Car-builders' Assn. in 1891 and 1892, gave the following results (selected to show range of variation):

| Size of Section, in. | Tensile St'gth, lbs. per sq. in. | Elastic Limit, lbs. per sq. in. | Elongation, % in 4 in. | Size of Section, in. | Tensile St'gth, lbs. per sq. in. | Elastic Limit, lbs. per sq. in. | Elong. in 8 in., %. |
|----------------------|----------------------------------|---------------------------------|------------------------|----------------------|----------------------------------|---------------------------------|---------------------|
| 0.25 × 1.52 | 34,700 | 21,100 | 2 | 0.29 × 2.78 | 28,160 | 22,650 | 0.6 |
| 0.5 × 1.53 | 32,800 | 17,000 | 2 | 0.39 × 2.82 | 32,060 | 20,595 | 1.5 |
| 0.78 × 2 | 25,100 | 15,400 | 1.5 | 0.53 × 2.76 | 27,875 | 19,520 | 1.1 |
| 0.88 × 1.54 | 33,600 | 19,300 | 1.5 | 0.8 × 2.76 | 25,120 | 18,390 | 1.1 |
| 1.52 × 1.54 | 28,200 | | 1.5 | 1.03 × 2.82 | 28,720 | 18,220 | 1.5 |

Tests of Square Bars, $\frac{1}{2}$ in. and 1 in., by tension, compression and transverse stress, by M. H. Miner and F. E. Blake (*Railway Age*, Jan. 25, 1901).

TENSION. Six $\frac{1}{2}$ -in. and six 1-in. round bars, also two 1-in. bars turned to remove the skin, from each of four makers. Average results:

T. S., $\frac{1}{2}$ -in. bars, 37,470–42,950, av. 40,960; E. L., 16,500–21,100, av. 19,176.

T. S., 1-in. bars, 35,750–40,530, av. 38,300; E. L., 14,860–19,900, av. 17,181.

Tensile strength, turned bars, av. 35,090; Elastic limit, av. 15,660.

Elong. in 8 in., $\frac{1}{2}$ -in. bars, 4.75%; 1-in. bars, 4.32%; turned bars, 3.73%.

Modulus of elasticity, $\frac{1}{2}$ -in. bars, 22,289,000; 1-in. bars, 21,677,000.

COMPRESSION. 16 short blocks, 2 in. long, 1 in. and $\frac{1}{2}$ in. square respectively.

8 long columns, 15 in. long, 1 in. sq., and 7.5 in. long, $\frac{1}{2}$ in. sq. respectively.

Averages of blocks from each of four makers:

Short blocks, $\frac{1}{2}$ -in. sq., 93,000 to 114,500 lbs. per sq. in. Mean, 101,900 lbs. per sq. in.

Short blocks, 1 in. sq., 137,600 to 165,300 lbs. per sq. in. Mean, 152,800 lbs. per sq. in.

Ratio of final to original length, $\frac{1}{2}$ in., 61.7%; 1 in., 52.6%. A small part of the shortening was due to sliding on the 45° plane of fracture.

Long columns: $\frac{1}{2}$ in. × 7.5 in. Mean, 29,400 lbs. per sq. in.; 1 in. × 15 in., 27,500 lbs. per sq. in. Ratio of final to original length, $\frac{1}{2}$ in., 98.5%; 1 in., 98.8%. The long columns did not rupture, but reached the maximum stress after bending into a permanent curve.

TRANSVERSE TESTS. Maximum fiber stress, mean of 8 tests, $\frac{1}{2}$ -in. bars, 34,163 lbs. per sq. in. 1-in. bars, 36,125 lbs. per sq. in. Length between supports, 20 in. The bars did not break, but failed by bending. The $\frac{1}{2}$ -in. bars could be bent nearly double.

WROUGHT IRON.

The Manufacture of Wrought Iron. — When iron ore, which is an oxide of iron, Fe_2O_3 or Fe_3O_4 , containing silica, phosphorus, sulphur, etc., as impurities, is heated to a yellow heat in contact with charcoal or other fuel, the oxygen of the ore combines with the carbon of the fuel, part of the iron combines with silica to form a fusible cinder or slag, and the remainder of the iron agglutinates into a pasty mass which is intermingled with the cinder. Depending upon the time and the temperature of the operation, and on the kind and quality of the impurities present in the ore and the fuel, more or less of the sulphur and phosphorus may remain in the iron or may pass into the slag; a small amount of carbon may also be absorbed by the iron. By squeezing, hammering, or rolling the lump of iron while it is highly heated, the cinder may be

nearly all expelled from it, but generally enough remains to give a bar after being rolled, cooled and broken across, the appearance of a fibrous structure. The quality of the finished bar depends upon the extent to which the chemical impurities and the intermingled slag have been removed from the iron.

The process above described is known as the direct process. It is now but little used, having been replaced by the indirect process known as puddling or boiling. In this process pig iron which has been melted in a reverberatory furnace is desiliconized and decarbonized by the oxygen derived from iron ore or iron scale in the bottom of the furnace, and from the oxidizing flame of the furnace. The temperature being too low to maintain the iron, when low in carbon, in a melted condition, it gradually "comes to nature" by the formation of pasty particles in the bath, which adhere to each other, until at length all the iron is decarbonized and becomes of a pasty condition, and the lumps so formed when gathered together make the "puddle-ball" which is consolidated into a bloom by the squeezer and then rolled into "muck-bar." By cutting the muck-bar into short lengths and making a "pile" of them, heating the pile to a welding heat and rerolling, a bar is made which is freer from cinder and more homogeneous than the original bar, and it may be further "refined" by another piling and rerolling. The quality of the iron depends on the quality of the pig-iron, on the extent of the decarbonization, on the extent of dephosphorization which has been effected in the furnace, on the greater or less contamination of the iron by sulphur derived from the fuel, and on the amount of work done on the piles to free the iron from slag. Iron insufficiently decarbonized is irregular, and hard or "steely." Iron thoroughly freed from impurities is soft and of low tensile strength. Iron high in sulphur is "hot-short," liable to break when being forged. Iron high in phosphorus is "cold-short," of low ductility when cold, and breaking with an apparently crystalline fracture.

See papers on Manufacture and Characteristics of Wrought Iron, by J. P. Roe, *Trans. A. I. M. E.*, xxxiii, p. 551; xxxvi, pp. 203, 807.

Influence of Chemical Composition on the Properties of Wrought Iron. (Beardslee on Wrought Iron and Chain Cables. Abridgment by W. Kent. Wiley & Sons, 1879.) — A series of 2000 tests of specimens from 14 brands of wrought iron, most of them of high repute, was made in 1877 by Capt. L. A. Beardslee, U.S.N., of the United States Testing Board. Forty-two chemical analyses were made of these irons, with a view to determine what influence the chemical composition had upon the strength, ductility, and welding power. From the report of these tests by A. L. Holley the following figures are taken:

| Brand. | Average Tensile Strength. | Chemical Composition. | | | | | |
|--------|---------------------------------|-----------------------|------------------|----------------|----------------|----------------|----------------|
| | | S. | P. | Si. | C. | Mn. | Slag. |
| L | 66,598 | trace | { 0.065 0.084 | 0.080 0.105 | 0.212 0.512 | 0.005 0.029 | 0.192 0.452 |
| P | 54,363 | { 0.009 0.001 | 0.250 0.095 | 0.182 0.028 | 0.033 0.066 | 0.033 0.009 | 0.848 1.214 |
| B | 52,764 | 0.008 | 0.231 | 0.156 | 0.015 | 0.017 | |
| J | 51,754 | { 0.003 0.005 | 0.140 0.291 | 0.182 0.321 | 0.027 0.051 | trace 0.053 | 0.678 1.724 |
| O | 51,134 | { 0.004 0.005 | 0.067 0.078 | 0.065 0.073 | 0.045 0.042 | 0.007 0.005 | 1.168 0.974 |
| C | 50,765 | 0.007 | 0.169 | 0.154 | 0.042 | 0.021 | |

Where two analyses are given, they are the extremes of two or more analyses of the brand. Where one is given, it is the only analysis. Brand L should be classed as a puddled steel.

ORDER OF QUALITIES GRADED FROM NO. 1 TO NO. 19.

| Brand. | Tensile Strength. | Reduction of Area. | Elongation. | Welding Power. |
|--------|-------------------|--------------------|-------------|-----------------|
| L | 1 | 18 | 19 | most imperfect. |
| P | 6 | 6 | 3 | badly. |
| B | 12 | 16 | 15 | best. |
| J | 16 | 19 | 18 | rather badly. |
| O | 18 | 1 | 4 | very good. |
| C | 19 | 12 | 16 | |

The reduction of area varied from 54.2 to 25.9 per cent, and the elongation from 29.9 to 8.3 per cent.

Brand O, the purest iron of the series, ranked No. 18 in tensile strength, but was one of the most ductile; brand B, quite impure, was below the average both in strength and ductility, but was the best in welding power; P, also quite impure, was one of the best in every respect except welding, while L, the highest in strength, was not the most pure, it had the least ductility, and its welding power was most imperfect. The evidence of the influence of chemical composition upon quality, therefore, is quite contradictory and confusing. The irons differing remarkably in their mechanical properties, it was found that a much more marked influence upon their qualities was caused by different treatment in rolling than by differences in composition.

In regard to slag Mr. Holley says: "It appears that the smallest and most worked iron often has the most slag. It is hence reasonable to conclude that an iron may be dirty and yet thoroughly condensed."

In his summary of "What is learned from chemical analysis," he says: "So far, it may appear that little of use to the makers or users of wrought iron has been learned. . . . The character of steel can be surely predicated on the analyses of the materials; that of wrought iron is altered by subtle and unobserved causes."

Influence of Reduction in Rolling from Pile to Bar on the Strength of Wrought Iron.—The tensile strength of the irons used in Beardslee's tests ranged from 46,000 to 62,700 lbs. per sq. in., brand L, which was really a steel, not being considered. Some specimens of L gave figures as high as 70,000 lbs. The amount of reduction of sectional area in rolling the bars has a notable influence on the strength and elastic limit; the greater the reduction from pile to bar, the higher the strength.

The following are a few figures from tests of one of the brands:

| | | | | | | |
|-------------------------|--------|--------|--------|--------|--------|--------|
| Size of bar, in. diam.: | 4 | 3 | 2 | 1 | 1/2 | 1/4 |
| Area of pile, sq. in.: | 80 | 80 | 72 | 25 | 9 | 3 |
| Bar per cent of pile: | 15.7 | 8.83 | 4.36 | 3.14 | 2.17 | 1.6 |
| Tensile strength, lb.: | 46,322 | 47,761 | 48,280 | 51,128 | 52,275 | 59,585 |
| Elastic limit, lb.: | 23,430 | 26,400 | 31,892 | 36,467 | 39,126 | — |

Specifications for Wrought Iron. (F. H. Lewis, Engineers' Club of Philadelphia, 1891.)—1. All wrought iron must be tough, ductile, fibrous, and of uniform quality for each class, straight, smooth, free from cinder-pockets, flaws, buckles, blisters, and injurious cracks along the edges, and must have a workmanlike finish. No specific process or provision of manufacture will be demanded, provided the material fulfills the requirements of these specifications.

2. The tensile strength, limit of elasticity, and ductility shall be determined from a standard test-piece not less than 1/4 inch thick, cut from the full-sized bar, and planed or turned parallel. The area of cross-section shall not be less than 1/2 square inch. The elongation shall be measured after breaking on an original length of 8 inches.

3. The tests shall show not less than the following results:

| | |
|--------------------------------|--|
| For bar iron in tension. . . . | T. S. = 50,000; E. L. = 26,000; E. L., 18% |
| For shape iron in tension. . . | " = 48,000; " = 26,000; " 15% |
| For plates under 36 in. wide | " = 48,000; " = 26,000; " 12% |
| For plates over 36 in. wide. | " = 46,000; " = 25,000; " 10% |

4. When full-sized tension members are tested to prove the strength of their connections, a reduction in their ultimate strength of (500 × width of bar) pounds per square inch will be allowed.

5. All iron shall bend, cold, 180 degrees around a curve whose diameter is twice the thickness of piece for bar iron, and three times the thickness for plates and shapes.

6. Iron which is to be worked hot in the manufacture must be capable of bending sharply to a right angle at a working heat without sign of fracture.

7. Specimens of tensile iron upon being nicked on one side and bent shall show a fracture nearly all fibrous.

8. All rivet iron must be tough and soft, and be capable of bending cold until the sides are in close contact without sign of fracture on the convex side of the curve.

Penna. R. R. Co.'s Specifications for Merchant-bar Iron (1904).—One bar will be selected for test from each 100 bars in a pile.

All the iron of one size in the shipment will be rejected if the average tensile strength of the specimens tested full size as rolled falls below 47,000 lbs. or exceeds 53,000 lbs. per sq. in., or if a single specimen falls below 45,000 lbs. per sq. in.; or when the test specimen has been reduced by machining if the average tensile strength exceeds 53,000 or falls below 46,000, or if a single specimen falls below 44,000 lbs. per sq. in.

All the iron of one size in the shipment will be rejected if the average elongation in 8 in. falls below the following limits: Flats and rounds, tested as rolled, $\frac{1}{2}$ in. and over, 20%; less than $\frac{1}{2}$ in., 16%. Flats and rounds reduced by machining 16%.

Nicking and Bending Tests.—When necessary to make nicking and bending tests, the iron will be nicked lightly on one side and then broken by holding one end in a vise, or steam hammer, and breaking the iron by successive blows. It must when thus broken show a generally fibrous structure, not more than 25% crystalline, and must be free from admixture of steel.

Stay-bolt Iron. (Penna. R. R. Co.'s specifications, 1902).—Sample bars must show a tensile strength of not less than 48,000 lbs. per sq. in. and an elongation of not less than 25% in 8 in. One piece from each lot will be threaded in dies with a sharp V thread, 12 to 1 in. and firmly screwed through two holders having a clear space between them of 5 in. One holder will be rigidly secured to the bed of a suitable machine, and the other vibrated at right angles to the axis over a space of $\frac{1}{4}$ in. or $\frac{1}{8}$ in. each side of the center line. Acceptable iron should stand 2800 double vibrations before breakage.

Mr. Vauclain, of the Baldwin Locomotive Works, at a meeting of the American Railway Master Mechanics' Association, in 1892, says: Many advocate the softest iron in the market as the best for stay-bolts. He believed in an iron as hard as was consistent with heading the bolt nicely. The higher the tensile strength of the iron, the more vibrations it will stand, for it is not so easily strained beyond the yield-point. The Baldwin specifications for stay-bolt iron call for a tensile strength of 50,000 to 52,000 lbs. per square inch, the upper figure being preferred, and the lower being insisted upon as the minimum.

Specifications for Wrought Iron for the World's Fair Buildings. (*Eng'g News*, March 26, 1892.)—All iron to be used in the tensile members of open trusses, laterals, pins and bolts, except plate iron over 8 inches wide, and shaped iron, must show by the standard test-pieces a tensile strength in lbs. per square inch of:

$$52,000 - \frac{7000 \times \text{area of original bar in sq. in.}}{\text{circumference of original bar in inches}}$$

with an elastic limit not less than half the strength given by this formula, and an elongation of 20% in 8 in.

Plate iron 8 to 24 inches wide, T. S. 48,000, E. L. 26,000 lbs. per sq. in., elong. 12%. Plates over 24 inches wide, T. S. 46,000, E. L. 26,000 lbs. per sq. in. Plates 24 to 36 in. wide, elong. 10%; 36 to 48 in., 8%; over 48 in., 5%.

All shaped iron, flanges of beams and channels, and other iron not hereinbefore specified, must show a T. S. in lbs. per sq. in. of:

$$50,000 - \frac{7000 \times \text{area of original bar}}{\text{circumference of original bar}}$$

with an elastic limit of not less than half the strength given by this formula, and an elongation of 15% for bars $\frac{5}{8}$ inch and less in thickness, and of 12% for bars of greater thickness. For webs of beams and channels, specifications for plates will apply.

All rivet iron must be tough and soft, and pieces of the full diameter of the rivet must be capable of bending cold, until the sides are in close contact, without sign of fracture on the convex side of the curve.

TENACITY OF METALS AT VARIOUS TEMPERATURES.

The British Admiralty made a series of experiments to ascertain what loss of strength and ductility takes place in gun-metal compositions when raised to high temperatures. It was found that all the varieties of gun metal suffer a gradual but not serious loss of strength and ductility up to a certain temperature, at which, within a few degrees, a great change takes place, the strength falls to about one-half the original, and the ductility is wholly gone. At temperatures above this point, up to 500° F., there is little, if any, further loss of strength; the temperature at which this great change and loss of strength takes place, although uniform in the specimens cast from the same pot, varies about 100° in the same composition cast at different temperatures, or with some varying conditions in the foundry process. The temperature at which the change took place in No. 1 series was ascertained to be about 370°, and in that of No. 2, at a little over 250°. Rolled Muntz metal and copper are satisfactory up to 500°, and may be used as securing-bolts with safety. Wrought iron increases in strength up to 500°, but loses slightly in ductility up to 300°, where an increase begins and continues up to 500°, where it is still less than at the ordinary temperature of the atmosphere. The strength of Landore steel is not affected by temperature up to 500°, but its ductility is reduced more than one-half. (*Iron*, Oct. 6, 1877.)

Tensile Strength of Iron and Steel at High Temperatures. — James E. Howard's tests (*Iron Age*, April 10, 1890) show that the tensile strength of steel diminishes as the temperature increases from 0° until a minimum is reached between 200° and 300° F., the total decrease being about 4000 lbs. per square inch in the softer steels, and from 6000 to 8000 lbs. in steels of over 80,000 lbs. tensile strength. From this minimum point the strength increases up to a temperature of 400° to 650° F., the maximum being reached earlier in the harder steels, the increase amounting to from 10,000 to 20,000 lbs. per square inch above the minimum strength at from 200° to 300°. From this maximum, the strength of all the steel decreases steadily at a rate approximating 10,000 lbs. decrease per 100° increase of temperature. A strength of 20,000 lbs. per square inch is still shown by 0.10 C. steel at about 1000° F., and by 0.60 to 1.00 C. steel at about 1600° F.

The strength of wrought iron increases with temperature from 0° up to a maximum at from 400 to 600° F., the increase being from 8000 to 10,000 lbs. per square inch, and then decreases steadily till a strength of only 6000 lbs. per square inch is shown at 1500° F.

Cast iron appears to maintain its strength, with a tendency to increase, until 900° is reached, beyond which temperature the strength gradually diminishes. Under the highest temperatures, 1500° to 1600° F., numerous cracks on the cylindrical surface of the specimen were developed prior to rupture. It is remarkable that cast iron, so much inferior in strength to the steels at atmospheric temperature, under the highest temperatures has nearly the same strength the high-temper steels then have.

Strength of Iron and Steel Boiler-plate at High Temperatures. (Chas. Huston, *Jour. F. I.*, 1877.)

AVERAGE OF THREE TESTS OF EACH.

| Temperature F. | 68° | 575° | 925° |
|---|--------|--------|--------|
| Charcoal iron plate, tensile strength, lbs. | 55,366 | 63,080 | 65,343 |
| " " " contr. of area %..... | 26 | 23 | 21 |
| Soft open-hearth steel, tensile strength, lbs. | 54,600 | 66,083 | 64,350 |
| " " " contr. %..... | 47 | 38 | 33 |
| " Crucible steel, tensile strength, lbs. | 64,000 | 69,266 | 68,600 |
| " " " contr. %..... | 36 | 30 | 21 |

Strength of Wrought Iron and Steel at High Temperatures. (*Jour. F. I.*, cxii, 1881, p. 241.) — Kollmann's experiments at Oberhausen

included tests of the tensile strength of iron and steel at temperatures ranging between 70° and 2000° F. Three kinds of metal were tested, viz., fibrous iron of 52,464 lbs. T. S., 38,280 lbs. E. L., and 17.5% elong.; fine-grained iron of 56,892 lbs. T. S., 39,113 lbs. E. L., and 20% elong.; and Bessemer steel of 84,826 lbs. T. S., 55,029 lbs. E. L., and 14.5% elong. The mean ultimate tensile strength of each material expressed in per cent of that at ordinary atmospheric temperature is given in the following table, the fifth column of which exhibits, for purposes of comparison, the results of experiments by a committee of the Franklin Institute in the years 1832-36.

| Temperature Degrees F. | Fibrous Iron, %. | Fine-grained Iron, %. | Bessemer Steel, %. | Franklin In- stitute, %. |
|---------------------------|---------------------|--------------------------|-----------------------|-----------------------------|
| 0 | 100.0 | 100.0 | 100.0 | 96.0 |
| 100 | 100.0 | 100.0 | 100.0 | 102.0 |
| 200 | 100.0 | 100.0 | 100.0 | 105.0 |
| 300 | 97.0 | 100.0 | 100.0 | 106.0 |
| 400 | 95.5 | 100.0 | 100.0 | 106.0 |
| 500 | 92.5 | 98.5 | 98.5 | 104.0 |
| 600 | 88.5 | 95.5 | 92.0 | 99.5 |
| 700 | 81.5 | 90.0 | 68.0 | 92.5 |
| 800 | 67.5 | 77.5 | 44.0 | 75.5 |
| 900 | 44.5 | 51.5 | 36.5 | 53.5 |
| 1000 | 26.0 | 36.0 | 31.0 | 36.0 |
| 1100 | 20.0 | 30.5 | 26.5 | |
| 1200 | 18.0 | 28.0 | 22.0 | |
| 1400 | 13.5 | 19.0 | 15.0 | |
| 1600 | 7.0 | 12.5 | 10.0 | |
| 1800 | 4.5 | 8.5 | 7.5 | |
| 2099 | 3.5 | 5.0 | 5.0 | |

Effect of Cold on the Strength of Iron and Steel. — The following conclusions were arrived at by Mr. Styffe in 1865:

(1) The absolute strength of iron and steel is not diminished by cold, even at the lowest temperature which ever occurs in Sweden.

(2) Neither in steel nor in iron is the extensibility less in severe cold than at the ordinary temperature.

(3) The limit of elasticity in both steel and iron lies higher in severe cold.

(4) The modulus of elasticity in both steel and iron is increased on reduction of temperature, and diminished on elevation of temperature; but that these variations never exceed 0.05% for a change of 1.8° F.

W. H. Barlow (*Proc. Inst. C. E.*) made experiments on bars of wrought iron, cast iron, malleable cast iron, Bessemer steel, and tool steel. The bars were tested with tensile and transverse strains, and also by impact; one-half of them at a temperature of 50° F., and the other half at 5° F.

The results of the experiments were summarized as follows:

1. When bars of wrought iron or steel were submitted to a tensile strain and broken, their strength was not affected by severe cold (5° F.), but their ductility was increased about 1% in iron and 3% in steel.

2. When bars of cast iron were submitted to a transverse strain at a low temperature, their strength was diminished about 3% and their flexibility about 16%.

3. When bars of wrought iron, malleable cast iron, steel, and ordinary cast iron were subjected to impact at 5° F., the force required to break them, and their flexibility, were reduced as follows:

| | Reduction of Force of Im- pact, %. | Reduction of Flexibility, %. |
|------------------------------------|--|------------------------------------|
| Wrought iron, about..... | 3 | 18 |
| Steel (best cast tool), about..... | 3 1/2 | 17 |
| Malleable cast iron, about..... | 4 1/2 | 15 |
| Cast iron, about..... | 21 | not taken |

The experience of railways in Russia, Canada, and other countries where the winter is severe, is that the breakages of rails and tires are far more numerous in the cold weather than in the summer. On this account a softer class of steel is employed in Russia for rails than is usual in more temperate climates.

The evidence extant in relation to this matter leaves no doubt that the capability of wrought iron or steel to resist impact is reduced by cold. On the other hand, its static strength is not impaired by low temperatures.

Increased Strength of Steel at very Low Temperature.—Steel of 72,300 lb. T. S. and 52,800 lb. elastic limit when tested at 76° F. gave 97,600 T. S. and 80,000 E. L. when tested at the temperature of liquid air. — Watertown Arsenal Tests, *Eng. Rec.*, July 21, 1906.

Prof. R. C. Carpenter (*Proc. A. A. S.* 1897) found that the strength of wrought iron at - 70° F. was 20% greater than at 70° F.

Effect of Low Temperatures on Strength of Railroad Axles. (Thos. Andrews, *Proc. Inst. C. E.*, 1891.) — Axles 6 ft. 6 in. long between centers of journals, total length 7 ft. 3 1/2 in., diameter at middle 4 1/2 in., at wheel-sets 5 1/8 in., journals 3 3/4 × 7 in., were tested by impact at temperatures of 0° and 100° F. Between the blows each axle was half turned over, and was also replaced for 15 minutes in the water-bath.

The mean force of concussion resulting from each impact was ascertained as follows:

Let h = height of free fall in feet, w = weight of test ball, $hw = W$ = "energy," or work in foot-tons, x = extent of deflections between bearings

then F (mean force) = $W/x = hw/x$.

The results of these experiments show that whereas at 0° F. a total average mean force of 179 tons was sufficient to cause the breaking of the axles, at 100° F. a total average mean force of 428 tons was required. In other words, the resistance to concussion of the axles at 0° F. was only about 42% of what it was at 100° F.

The average total deflection at 0° F. was 6.48 in., as against 15.06 in. with the axles at 100° F. under the conditions stated; this represents an ultimate reduction of flexibility, under the test of impact, of about 57% for the cold axles at 0° F., compared with the warm axles at 100° F.

EXPANSION OF IRON AND STEEL BY HEAT.

James E. Howard, engineer in charge of the U. S. testing-machine at Watertown, Mass., gives the following results of tests made on bars 35 inches long (*Iron Age*, April 10, 1890):

| | C. | Mn. | Si. | Coeffi. of Expansion per degree F. | | C. | Mn. | Si. | Coeffi. of Expansion per degree F. |
|--------------|------|------|-----|------------------------------------|-----------------|------|------|-----|------------------------------------|
| Wrought iron | ... | ... | ... | 0.0000067302 | Steel | 0.57 | 0.93 | .07 | 0.0000063691 |
| Steel | 0.09 | 0.11 | ... | .0000067561 | " | .71 | .58 | .08 | .0000064716 |
| " | .20 | .45 | ... | .0000066259 | " | .81 | .56 | .17 | .0000062167 |
| " | .31 | .57 | ... | .0000065149 | " | .89 | .57 | .19 | .0000062335 |
| " | .37 | .70 | ... | .0000066597 | " | .97 | .80 | .28 | .0000061700 |
| " | .51 | .58 | .02 | .0000066202 | Cast (gun) iron | ... | ... | ... | .0000059261 |

DURABILITY OF IRON, CORROSION, ETC.

Crystallization of Iron by Fatigue. — Wrought iron of the best quality is very tough, and breaks, on being pulled in a testing machine or bent after nicking, with a fibrous fracture. Cold-short iron, however, is more brittle, and breaks square across the fibers with a fracture which is commonly called crystalline although no real crystals are present. Iron which has been repeatedly overstrained, and especially iron subjected to repeated vibrations and shocks, also becomes brittle, and breaks with an apparently crystalline fracture. See "Resistance of Metals to Repeated Shocks," p. 262.

Walter H. Finley (*Am. Mach.*, April 27, 1905) relates a case of failures of 1 1/8-in. wrought-iron coupling pins on a train of 1-ton mine cars, apparently due to crystallization. After two pins were broken after a year's hard service, "several hitchings were laid on an anvil and the pin broken by a single blow from a sledge. Pieces of the broken pins were then heated to a bright red, and, after cooling slowly, were again put under the hammer, which failed entirely to break them. After cutting with a cleaver, the pins were broken, and the fracture showed a complete restoration of the fibrous structure. This annealing process was then applied to the whole supply of hitchings. Piles of twenty-five or thirty were covered by a hot wood fire, which was allowed to die down and go out, leaving the hitchings in a bed of ashes to cool off slowly. By repeating this every six months the danger from brittle pins was entirely avoided."

Durability of Cast Iron. — Frederick Graff, in an article on the Philadelphia water-supply, says that the first cast-iron pipe used there was laid in 1820. These pipes were made of charcoal iron, and were in constant use for 53 years. They were uncoated, and the inside was well filled with tubercles. In salt water good cast iron, even uncoated, will last for a century at least; but it often becomes soft enough to be cut by a knife, as is shown in iron cannon taken up from the bottom of harbors after long submersion. Close-grained, hard white metal lasts the longest in sea water. (*Eng'g News*, April 23, 1887, and March 26, 1892.)

Tests of Iron after Forty Years' Service. — A square link 12 inches broad, 1 inch thick and about 12 feet long was taken from the Kieff bridge, then 40 years old, and tested in comparison with a similar link which had been preserved in the stock-house since the bridge was built. The following is the record of a mean of four longitudinal test-pieces, 1 × 1 1/8 × 8 inches, taken from each link (*Stahl und Eisen*, 1890):

| | | | |
|----------------|-------------------|-------------------|----------------|
| Old Link..... | T. S., 21.8 tons; | E. L., 11.1 tons; | Elong., 14.05% |
| New Link | 22.2 " | 11.9 " | 13.42% |

Durability of Iron in Bridges. (G. Lindenthal, *Eng'g*, May 2, 1884, p. 139.) — The Old Monongahela suspension bridge in Pittsburg, built in 1845, was taken down in 1882. The wires of the cables were frequently strained to half of their ultimate strength, yet on testing them after 37 years' use they showed a tensile strength of from 72,700 to 100,000 lbs. per sq. in. The elastic limit was from 67,100 to 78,600 lbs. per sq. in. Reduction at point of fracture, 35% to 75%. Their diameter was 0.13 in.

A new ordinary telegraph wire of same gauge tested for comparison showed: T. S., of 100,000 lbs.; E. L., 81,550 lbs.; reduction, 57%. Iron rods used as stays or suspenders showed: T. S., 43,770 to 49,720 lbs. E. L., 26,380 to 29,200. Mr. Lindenthal draws these conclusions:

"The above tests indicate that iron highly strained for a long number of years, but still within the elastic limit, and exposed to slight vibration, will not deteriorate in quality.

"That if subjected to only one kind of strain it will not change its texture, even if strained beyond its elastic limit, for many years. It will stretch and behave much as in a testing-machine during a long test.

"That iron will change its texture only when exposed to alternate severe straining, as in bending in different directions. If the bending is slight but very rapid, as in violent vibrations, the effect is the same."

Durability of Iron in Concrete. — In Paris a sewer of reinforced concrete 40 years old was removed and the metal was found in a perfect state of preservation. In excavating for the foundations of the new General Post Office in London some old Roman brickwork had to be removed, and the hoop-iron bonds were still perfectly bright and good. (*Eng'g*, Aug. 16, 1907, p. 227.)

Corrosion of Iron Bolts. — On bridges over the Thames in London, bolts exposed to the action of the atmosphere and rain-water were eaten away in 25 years from a diameter of 7/8 in. to 1/2 in., and from 5/8 in. diameter to 5/16 in.

Wire ropes exposed to drip in colliery shafts are very liable to corrosion.

Corrosive Agents in the Atmosphere. — The experiments of F. Crace Calvert (*Chemical News*, March 3, 1871) show that carbonic acid, in the presence of moisture, is the agent which determines the oxidation of iron in the atmosphere. He subjected perfectly cleaned blades of iron

and steel to the action of different gases for a period of four months, with results as follows:

Dry oxygen, dry carbonic acid, a mixture of both gases, dry and damp oxygen and ammonia: no oxidation. Damp oxygen: in three experiments one blade only was slightly oxidized.

Damp carbonic acid: slight appearance of a white precipitate upon the iron, found to be carbonate of iron. Damp carbonic acid and oxygen: oxidation very rapid. Iron immersed in water containing carbonic acid oxidized rapidly.

Iron immersed in distilled water deprived of its gases by boiling rusted the iron in spots that were found to contain impurities.

Sulphurous acid (the product of the combustion of the sulphur in coal) is an exceedingly active corrosive agent, especially when the exposed iron is coated with soot. This accounts for the rapid corrosion of iron in railway bridges exposed to the smoke from locomotives. (See account of experiments by the author on action of sulphurous acid in *Jour. Frank. Inst.*, June, 1875, p. 437.) An analysis of sooty iron rust from a railway bridge showed the presence of sulphurous, sulphuric, and carbonic acids, chlorine, and ammonia. Bloxam states that ammonia is formed from the nitrogen of the air during the process of rusting.

Galvanic Action is a most active agent of corrosion. It takes place when two metals, one electro-negative to the other, are placed in contact and exposed to dampness.

Corrosion in Steam-boilers. — Internal corrosion may be due either to the use of water containing free acid, or water containing sulphate or chloride of magnesium, which decompose when heated, liberating the acid, or to water containing air or carbonic acid in solution. External corrosion rarely takes place when a boiler is kept hot, but when cold it is apt to corrode rapidly in those portions where it adjoins the brick-work or where it may be covered by dust or ashes, or wherever dampness may lodge. (See Impurities of Water, p. 691, and Incrustation and Corrosion, p. 897.)

Corrosion of Iron and Steel. — Experiments made at the Riverside Iron Works, Wheeling, W. Va., on the comparative liability to rust of iron and soft Bessemer steel: A piece of iron plate and a similar piece of steel, both clean and bright, were placed in a mixture of yellow loam and sand, with which had been thoroughly incorporated some carbonate of soda, nitrate of soda, ammonium chloride, and chloride of magnesium. The earth as prepared was kept moist. At the end of 33 days the pieces of metal were taken out, cleaned, and weighed, when the iron was found to have lost 0.84% of its weight and the steel 0.72%. The pieces were replaced and after 28 days weighed again, when the iron was found to have lost 2.06% of its original weight and the steel 1.79%. (*Eng'g*, June 26, 1891.)

Internal Corrosion of Iron and Steel Pipes by Warm Water. (T. N. Thomson, *Proc. A. S. H. V. E.*, 1908.) — Three short pieces of iron and three of steel pipes, 2 in. diam., were connected together by nipples and made part of a pipe line conveying water at a temperature varying from 160° to 212° F. In one year 9¹³/₃₂ lbs. of wrought iron lost 20³/₄ oz., and 9¹³/₃₂ lbs. of steel 24⁷/₈ oz. The pipes were sawed in two lengthwise, and the deepest pittings were measured by a micrometer. Assuming that the pitting would have continued at a uniform rate the wrought-iron pipes would have been corroded through in from 686 to 780 days, and the steel pipes from 760 to 850 days, the average being 742 days for iron and 797 days for steel. Two samples each of galvanized iron and steel pipe were also included in the pipe line, and their calculated life was: iron 770 and 1163 days; steel 619 and 1163 days. Of numerous samples of corroded pipe received from heating engineers ten had given out within four years of service, and of these six were steel and four were iron.

To ascertain whether Pipe is made of Wrought Iron or Steel, cut off a short piece of the pipe and suspend it in a solution of 9 parts of water, 3 of sulphuric acid, and 1 of hydrochloric acid in a porcelain or glass dish in such a way that the end will not touch the bottom of the dish. After 2 to 3 hours' immersion remove the pipe and wash off the acid. If the pipe is steel the end will present a bright, solid, unbroken surface, while if made of iron it will show faint ridges or rings, like the year rings in a tree, showing the different layers of iron and streaks of cinder. In order that the scratches made by the cutting-off tool may not be mistaken for

the cinder marks, file the end of the pipe straight across or grind on an emery wheel until the marks of the cutting-off tool have disappeared before putting it in the acid.

Relative Corrosion of Wrought Iron and Steel. (H. M. Howe, *Proc. A. S. T. M.*, 1906.) — On one hand we have the very general opinion that steel corrodes very much faster than wrought iron, an opinion held so widely and so strongly that it cannot be ignored. On the other hand we have the results of direct experiments by a great many observers, in different countries and under widely differing conditions; and these results tend to show that there is no very great difference between the corrosion of steel and wrought iron. Under certain conditions steel seems to rust a little faster than wrought iron, and under others wrought iron seems to rust a little faster than steel. Taking the tests in unconfined sea water as a whole wrought iron does constantly a little better than steel, and its advantage seems to be still greater in the case of boiling sea water. In the few tests in alkaline water wrought iron seems to have the advantage over steel, whereas in acidulated water steel seems to rust more slowly than wrought iron.

Steel which in the first few months may rust faster than wrought iron may, on greatly prolonging the experiments, or pushing them to destruction, actually rust more slowly, and *vice versa*.

Carelessly made steel, containing blowholes, may rust faster than wrought iron, yet carefully made steel, free from blowholes, may rust more slowly. Any difference between the two may be due not to the inherent and intrinsic nature of the material, but to defects to which it is subject if carelessly made. Care in manufacture, and special steps to lessen the tendency to rust, might well make steel less corrodible than wrought iron, even if steel carelessly made should really prove more corrodible than wrought iron.

For extensive discussions on this subject see *Trans. A. I. M. E.*, 1905, and *Proc. A. S. T. M.*, 1906.

Corrosion of Fence Wire. (A. S. Cushman, *Farmers' Bulletin*, No. 239, U. S. Dept. of Agriculture, 1905.) — "A large number of letters were received from all over the country in response to official inquiry, and all pointed in the same direction. As far as human testimony is capable of establishing a fact, there need be not the slightest question that modern steel does not serve the purpose as well as the older metal manufactured twenty or more years ago."

Electrolytic Theory, and Prevention of Corrosion. (A. S. Cushman, *Bulletin* No. 30, U. S. Dept. of Agriculture, Office of Public Roads, 1907. *The Corrosion of Iron*.) — The various kinds of merchantable iron and steel differ, within wide limits, in their resistance, not only to the ordinary processes of oxidation known as rusting, but also in other corrosive influences. Different specimens of one and the same kind of iron or steel will show great variability in resistance to corrosion under the conditions of use and service. The causes of this variability are numerous and complex, and the subject is not nearly so well understood at the present time as it should be. All investigators are agreed that iron cannot rust in air or oxygen unless water is present, and on the other hand it cannot rust in water unless oxygen is present.

From the standpoint of the modern theory of solutions, all reactions which take place in the wet way are attended with certain readjustments of the electrical states of the reacting ions. The electrolytic theory of rusting assumes that before iron can oxidize in the wet way it must first pass into solution as a ferrous ion.

Dr. Cushman then gives an account of his experiments which he considers demonstrate that iron goes into solution up to a certain maximum concentration in pure water, without the aid of oxygen, carbonic acid or other reacting substances. It is apparent that the rusting of iron is primarily due, not to attack by oxygen, but by hydrogen ions.

Solutions of chromic acid and potassium bichromate inhibit the rusting of iron. If a rod or strip of bright iron or steel is immersed for a few hours in a 5 to 10 per cent solution of potassium bichromate, and is then removed and thoroughly washed, a certain change has been produced on the surface of the metal. The surface may be thoroughly washed and wiped with a clean cloth without disturbing this new surface condition. No visible change has been effected, for the polished surfaces

examined under the microscope appear to be untouched. If, however, the polished strips are immersed in water it will be found that rusting is inhibited. An ordinary untreated polished specimen of steel will show rusting in a few minutes when immersed in the ordinary distilled water of the laboratory. Chromated specimens will stand immersion for varying lengths of time before rust appears. In some cases it is a matter of hours, in others of days or even weeks before the inhibiting effect is overcome.

It would follow from the electrolytic theory that in order to have the highest resistance to corrosion a metal should either be as free as possible from certain impurities, such as manganese, or should be so homogeneous as not to retain localized positive and negative nodes for a long time without change. Under the first condition iron would seem to have the advantage over steel, but under the second much would depend upon care exercised in manufacture, whatever process was used.

There are two lines of advance by which we may hope to meet the difficulties attendant upon rapid corrosion. One is by the manufacture of better metal, and the other is by the use of inhibitors and protective coverings. Although it is true that laboratory tests are frequently unsuccessful in imitating the conditions in service, it nevertheless appears that chromic acid and its salts should under certain circumstances come into use to inhibit extremely rapid corrosion by electrolysis.

Chrome Paints.—G. B. Heckel (*Jour. F. I., Eng. Dig.*, Sept., 1908) quotes a letter from Mr. Cushman as follows: "My observation that chromic acid and certain of its compounds act as inhibitors has led to many experiments by other workers along the same line. I have found that the chrome compounds on the market vary very much in their action. Some of them show up as strong inhibitors, while others go to the opposite extreme and stimulate corrosion. Referring only to the labeled names of the pigments, I find among the good ones, in the order cited: Zinc chromate, American vermilion, chrome yellow orange, chrome yellow dd. Among the bad ones, also in the order given, I find: Chrome yellow medium, chrome green, chrome red. Much the worst of all is chrome yellow lemon. I presume that the difference is due to impurities that are present in the bad pigments."

Mr. Heckel suggests the following formula for a protective paint: 40 lbs. American vermilion, 10 lbs. red lead, 5 lbs. Venetian red. Zinc oxide and lamp-black to produce the required tint or shade. Grind in $1\frac{1}{3}$ gal. of raw linseed oil — increasing the quantity as required for added zinc oxide or lamp-black — and $\frac{1}{8}$ gal. crusher's drier. For use, thin with raw oil and very little turpentine or benzene.

He states that the substitution of zinc chrome for the American vermilion; of any high-grade finely ground iron oxide for the Venetian red; and of American vermilion for the red lead, would probably improve the protective value of the formula; that the addition of a very little kauri gum varnish, if zinc oxide is used, might be found advantageous; and that the substitution of a certain proportion of China wood oil for some of the linseed oil might improve the wearing qualities of the paint.

Dr. Cushman points out two dangers confronting us when we attempt to base an inhibitive formula on commercial products. The first is that all carbon pigments, excepting pure graphite, may contain sulphur compounds easily oxidizable to sulphuric acid when spread out as in a paint film. The second is the probability of variation in the composition of basic lead chromate or American vermilion. Because of these facts, it is necessary, before selecting any particular pigment for its inhibitive quality, to ascertain that it is free from acids or acid-forming impurities. As a result of his experiments he recommends the substitution of Prussian blue for the lamp-black in Mr. Heckel's formula, and lays down as a safe rule in the formulation of inhibitive paints, a careful avoidance of all potential stimulators of the hydrogen ions and consequently of any substance which might develop acid; preference being given to chromate pigments which are to some extent soluble in water, and to other pigments which in undergoing change tend to develop an alkaline rather than an acid reaction. Calcium sulphate, for example, in any form (as a constituent of Venetian red, for example), he deems dangerous to use because of the possibility of its developing acid. Barium sulphate, on the other hand, he regards as practically safe, because of its well-known chemical stability.

Corrosion caused by Stray Electric Currents. (W. W. Churchill, *Science*, Sept. 28, 1906). — Surface condensers in electric lighting and other plants were abandoned on account of electrolytic corrosion. The voltage of the rails in the freight yard of the Long Island railroad at the peak of the load was 9 volts above the potential of the river, decreasing to 2 volts or less at light loads. This caused a destruction of water pipes and other things in the railroad yards. Experiments with various metal plates immersed in samples of East River water showed that it gave a more violent action than ordinary sea water. It was further observed that there was a local galvanic action going on, and that the amount of stray currents had something to do with the polarization of the surfaces, making the galvanic action exceedingly violent and destroying thin copper tubes at a very rapid rate. There was a violent local action between the zinc and the copper of the brass tubes which were in contact with the electrolyte, and this increased in the reaction as it progressed in stagnant conditions. By interposing a counter electromotive force against the galvanic couple which should exceed in pressure the voltage of the couple, the actions of the electrolytic corrosion ceased. When unconnected, or electrically separated, plates were placed in the electrolyte, if they were of composite construction and had sharp projections into the fluid, raised by cutting and prying up with a knife, they would have these projections promptly destroyed, and if an electric battery having a pressure exceeding that of the couple in the East River water was caused to act to produce a counter current, and having a pressure exceeding that of the galvanic couple (0.42 volt), the capacity of this electrolyte to drive off atoms of the mechanically combined metals in the alloys used was overcome and corrosion was arrested.

It, therefore, became desirable not only to carefully provide the balancing quantity of current to equal the stray traction currents arising from the ground returns of railway and other service, but to add to this the necessary voltage through a cathode placed in the circulating water in such a way as to bring to bear electrolytic action which would prevent the galvanic action due to this current coming into contact with alloys of mechanically combined metals such as the brass tubes (60% copper, 40% zinc).

In order to accomplish these two things, it was first necessary to so install the condensers as to prevent undue amounts of stray currents flowing through them, thus tending to reduce the amount of power required to prevent injurious action of these currents and otherwise to neutralize them. This was done by insulating the joints in the piping and from ground connections, and even lining the large water connections with glass melted on to the surface.

To furnish electromotive force, a 3-K.W. motor generator was provided. By means of a system of wiring, with ammeters and voltmeters, and a connection to an outlying anode in the condensing supply intake at its harbor end, this generator was planned to provide current to neutralize the stray currents in the condenser structure to any extent that they had passed the insulated joints in the supports and connections, as well as through the columns of water in the pipe connections, and then to adjust the additional voltage needed to counteract and prevent the galvanic action. All connections were made in a manner to insure a uniform voltage of the various parts of the condenser to prevent local action, each connection being so made and provided with such measuring instruments as to insure ready adjustment to effect this. The apparatus was designed in accordance with the above statements. Its operation has extended over fourteen months (to date, 1906), and with the exception of about ten tubes which have become pitted, the results have been satisfactory. The efficiency of the apparatus amply justifies the expense of its installation, while its operation is not expensive, and the plant described will be followed by other protecting plants of the same character.

Electrolytic Corrosion due to Overstrain. (C. F. Burgess, *El. Rev.*, Sept. 19, 1908.) — Mild steel bars overstrained in their middle portion were subjected to corrosion by suspension in dilute hydrochloric acid solutions, and others by making them the anode in neutral solutions of ammonium chloride and causing current to flow under low current density. In all cases a marked difference was noted in the rate at which the strained portions corroded as compared with the unstrained.

Differences of potential of from five to nine millivolts were noted between two electrodes, one of which constituted the strained portion and one the unstrained.

The more rapid electrolytic corrosion of the strained portion appears to be due to the fact that the strained metal is electropositive to the unstrained, the current finding the easier path through the surface of the electropositive metal. That the strained metal is the more electropositive is also shown by a liberation of hydrogen bubbles on the unstrained portion.

PRESERVATIVE COATINGS.

The following notes have been furnished to the author by Prof. A. H. Sabin. (Revised, 1908.)

Cement. — Iron-work is often bedded in concrete; if free from cracks and voids it is an efficient protection. The metal should be cleaned and then washed with neat cement before embedding.

Asphaltum. — This is applied either by dipping (as water-pipe) or by pouring it on (as bridge floors). The asphalt should be slightly elastic when cold, with a high melting-point, not softening much at 100° F., applied at 300° to 400°; the surface must be dry and should be hot; the coating should be of considerable thickness.

Paint. — Composed of a vehicle or binder, usually linseed oil or some inferior substitute, or varnish (enamel paints); and a pigment, which is a more or less inert solid in the form of a powder, either mixed or ground together. Nearly all paint contains paint drier or japan, which is a lead or (and) manganese compound soluble in oil, and acts as a carrier of oxygen; as little should be used as possible. Boiled oil contains drier; no additional drier is needed. None should be used with varnish paints, nor with "ready-mixed paints" in general.

The principal pigments are white lead (carbonate or oxy-sulphate) and white zinc (oxide), red lead (peroxide), oxides of iron, hydrated and anhydrous, graphite, lampblack, bone black, chrome yellow, chrome green, ultramarine and Prussian blue, and various tinting colors. White lead has the greatest body or opacity of white pigments; three coats of it equal five of white zinc; zinc is more brilliant and permanent, but it is liable to peel, and it is customary to mix the two. These are the standard white paints for all uses, and the basis of all light-colored paints. Anhydrous iron oxides are brown and purplish brown, hydrated oxides are yellowish red to reddish yellow, with more or less brown; most iron oxides are mixtures of both sorts, and often contain a little manganese and much clay. They are cheap, and are serviceable paints on wood and are often used on iron, but for the latter use are falling into disrepute. Graphite used for painting iron contains from 10 to 90% foreign matter, usually silicates. It is very opaque, hence has great covering power and may be applied in a very thin coat, which is to be avoided. The best graphite paints give very good results. There are many grades of lampblack; the cheaper sorts contain oily matter and are especially hard to dry; all lampblack is slow to dry in oil. In a less degree this is true of all paints containing carbon, including graphite. Lampblack is used with advantage with red lead; it is also an ingredient of many "carbon" paints, the base of which is either bone black or artificial graphite. Red lead dries by uniting chemically with the oil to form a cement; it is heavy, and makes an expensive paint, and is often highly adulterated. Pure red lead has long had a high reputation as a paint for iron and steel, and is still used extensively, especially as a first coat; but of late years some of the new paints and varnish-like preparations have displaced it to a considerable extent even, on the most important work.

Varnishes. — These are made by melting fossil resin, to which is then added from half its weight to three times its weight of refined linseed oil, and the compound is thinned with turpentine; they usually contain a little drier. They are chiefly used on wood, being more durable and more brilliant than oil, and are often used over paint to preserve it. Asphaltum is sometimes substituted in part or in whole for the fossil resin, and in this way are made black varnishes which have been used on iron and steel with good results. Asphaltum and substances like it have

also been simply dissolved in solvents, as benzine or carbon disulphide, and used for the same purpose.

All these preservative coatings are supposed to form impervious films, keeping out air and moisture; but in fact all are somewhat porous. On this account it is necessary to have a film of appreciable thickness, best formed by successive coats, so that the pores of one will be closed by the next. The pigment is used to give an agreeable color, to help fill the pores of the oil film, to make the paint harder, so that it will resist abrasion, and to make a thicker film. In varnishes these results are sought to be attained by the resin which is dissolved in the oil. There is no sort of agreement among practical men as to which coating is best for any particular case; this is probably because so much depends on the preparation of the surface and the care with which the coating is applied, and also because the conditions of exposure vary so greatly.

Methods of Application. — From the surface of the metal mud and dirt must be first removed, then any rusty spots must be cleaned thoroughly; loose scale may be removed with wire brushes, but thick and closely adherent rust must be removed with steel scrapers, or with hammer and chisel if necessary. The sand-blast is used largely and increasingly to clean before painting, and is the best method known. Pickling is usually done with 10% sulphuric acid; the solution is made more active by heating. All traces of acid must be removed by washing, and the metal must be immediately dried and painted. Less than two coats of paint should never be used, and three or four are better. The first painting of metal is the most important. Paint is always thin on angles and edges, also on bolt and rivet heads; after the first full coat apply a partial or stripping coat, covering the angles and edges for at least an inch back from the edge, also all bolt and rivet heads. After this is dry apply the second full coat. At least a week should elapse between coats.

Cast-iron water pipes are usually coated by dipping in a hot mixture of coal-tar and coal-tar pitch; riveted steel pipes by dipping in hot asphalt or by a japan enamel which is baked on at about 400° F. Ships' bottoms are coated with a varnish paint to prevent rusting, over which is a similar paint containing a poison, as mercury chloride, or a copper compound, or else for this second coat a greasy copper soap is applied hot; this prevents the accumulation of marine growths. Galvanized iron and tin surfaces should be thoroughly cleaned with benzine and scrubbed before painting. When new they are partly covered with grease and chemicals used in coating the plates, and these must be removed or the paint will not adhere.

Quantity of Paint for a Given Surface. — One gallon of paint will cover 250 to 400 sq. ft. as a first coat, depending on the character of the surface, and from 350 to 500 sq. ft. as a second coat.

Qualities of Paints. — *The Railroad and Engineering Journal*, vols. liv. and lv., 1890 and 1891, has a series of articles on paint as applied to wooden structures, its chemical nature, application, adulteration, etc., by Dr. C. B. Dudley, chemist, and F. N. Pease, assistant chemist, of the Penna. R. R. They give the results of a long series of experiments on paints as applied to railway purposes.

Inoxydation Processes. (Contributed by Alfred Sang, Pittsburg, Pa., 1908.) — The black oxide of iron (Fe_3O_4) as a continuous coating affords excellent protection against corrosion. Lavoisier (1781) noted its artificial production and its stable qualities. Faraday (1858) observed the protective properties of the coating formed by the action of steam in superheating tubes. Berthier discovered its formation by the action of highly heated air.

Bower-Barff Process. — Dr. Barff's method was to heat articles to be coated to about 1800° F. and inject steam heated to 1000° F. into the muffle. George and A. S. Bower used air instead of steam, then carbon monoxide (producer gas) to reduce the red oxide. In the combined process, the articles are heated to 1600° F. in a closed retort; superheated steam is injected for 20 min., then producer gas for 15 to 25 min.; the treatment can be repeated to increase the depth of oxidation. Less heat is required for wrought than for cast iron or steel. By a later improvement, steam heated above the temperature of the articles was injected during the last 1 to 2 hours. By a further improvement known as the "Wells Process," the work is finished in one operation, the steam

and producer-gas being injected together. Articles are slightly increased in size by the treatment. The surface is gray, changing to black when oiled; it will chip off if too thin; it will take paint or enamel and may be polished, but cannot be either bent or machined; the coating itself is incorrodible and resists sea-water, mine-water and acid fumes; the strength of the metal is slightly reduced. The process is extensively used for small hardware. (See F. S. Barff, *Jour. I. & S. Inst.*, 1877, p. 356; A. S. Bower, *Trans. A. I. M. E.* 1882, p. 329; B. H. Thwaite, *Proc. Inst. C. E.* 1883, p. 255; George W. Maynard, *Trans. A. S. M. E.* iv, 351.)

Gesner Process. — Dr. George W. Gesner's process is in commercial operation since 1890. The coating retort is kept at 1200° F. for 20 minutes after charging, then steam, partially decomposed by passing through a red-hot pipe, is allowed to act at intervals during 35 min.; finally, a small quantity of naphtha, or other hydrocarbon, is introduced and allowed to act for 15 min. The work is withdrawn when the heat has fallen to 800° F. The articles are neither increased in size nor distorted; the loss of strength and reduction of elongation are only slight. Large pieces can be treated. (See *Jour. I. & S. Inst.*, 1890 (ii), p. 850; *Iron Age*, 1890, p. 544.)

Hydraesfer Process. — An improvement of the Gesner process patented by J. J. Bradley and in commercial operation. As its name implies, the coating is thought to be an alloy of hydrogen, copper and iron. The sulphides and phosphides are claimed to be burned out of the surface of the metal by the action of hydrogen at a high temperature, giving additional rust-proof qualities. The appearance of the finished work is that of genuine Bower Barffing.

Russia and Planished Iron. — Russia iron is made by cementation and slight oxidation. W. Dewees Wood (U. S. Pat. No. 252,166 of 1882) treated planished sheets with hydrocarbon vapors or gas and superheated steam within an air-tight and heated chamber.

Niter Process. — An old process improved by Col. A. R. Buffington in 1884. The articles are stirred about in a mixture of fused potassium nitrate (saltpeter) and manganese dioxide, then suspended in the vapors and finally dipped and washed in boiling water. Pure chemicals are essential. Used for small arms and pieces which cannot stand the high heat of other processes. (*Trans. A. S. M. E.*, vol. vi, p. 628.)

Electric Process. — A. de Meritens connected polished articles as anodes in a bath of warm distilled water and used a current as weak as would be conducted. A black film of oxide was formed; too strong a current produced rust. It being essential that hydrogen be occluded in the surface of the metal, it was found necessary, as a rule, to connect the articles as cathodes for a short time previous to inoxidation. (*Bull. Soc. Intle. des Electr.*, 1886, p. 239.)

Aluminum Coatings. — Aluminum can be deposited electrically, the main difficulties being the high voltage required and the readiness of the coating to redissolve. The metal-work of the tower of City Hall, Philadelphia, was coated by the Tacony Iron & Metal Co., Tacony, Pa., with 14 oz. per sq. ft. of copper on which was deposited 2½ oz. of an alloy of tin and aluminum. The Reeves Mfg. Co., Canal Dover, Ohio, makes aluminum-coated conductor pipes, etc., said to be as durable as copper and as rust-proof as aluminum. The Aluminum Co. of America makes "bi-metallic" tubing composed of aluminum and other metal tubes placed one inside the other and drawn down together to the required size.

Galvanizing is a method of coating articles, usually of iron or steel, with zinc. Galvanized iron resists ordinary corroding agencies, the zinc becoming covered with a film of zinc carbonate, which protects the metal from further chemical action. The coating is, however, quickly destroyed by mine-water, tunnel gases, sea water and conditions that commonly exist in tropical countries. If the work is badly done and the coating does not adhere properly, and if any acid from the pickle or any chloride from the flux remains on the iron, corrosion takes place under the zinc coating. (See M. P. Wood: *Trans. A. S. M. E.* xvi. 350. Alfred Sang: *Trans. Am. Foundrymen's Assoc.*, 1907, *Iron Age*, May 23d and 30th, 1907, and *Proc. Eng. Soc. of W. Penna.*, Nov., 1907.)

The Penna. R. R. Specifications for galvanized sheets for car roofs

(1907) prescribe that the black sheets before galvanizing should weigh 16 oz. per sq. ft., the galvanized sheet 18 oz. Sheets will not be accepted if a chemical determination shows less than 1.5 oz. of zinc per sq. ft.

Hot Galvanizing. — The articles to be galvanized are first cleaned by pickling and then dipped in a solution of hydrochloric acid and immersed in a bath of molten zinc at a temperature of from 800 to 900° F.; when they have reached the temperature of the bath, they are withdrawn and the coating is set in water; sal-ammoniac is used on the pot as a flux, either alone or as an emulsion with glycerine or some other fatty medium. Wire, bands and similar articles are drawn continuously through the bath, and may be passed through asbestos wipers to remove the surplus metal; in this case it is advisable to use a very soft spelter free from iron. If wire is treated slowly and passed through charcoal dust instead of wipers the product is known as "double-galvanized." Tin can be added to the bath to help bring out the spangles, but it gives a less durable coating. Aluminum is added as a Zn-Al alloy, with about 20% Al, to give fluidity. Sheets are galvanized continuously, and except in the case of so-called "flux sheets," are put through rolls as they emerge from the bath, to squeeze off the excess of zinc and improve the adherence.

Test for Galvanized Wire. — Sir W. Preece devised the following standard test for the British Post Office: dip for one minute in a saturated neutral solution of sulphate of copper, wash and wipe; to pass, the material must stand 3 dips.

The American standard test is as follows: prepare a neutral solution of sulphate of copper of sp. gr. 1.185, dip for one minute, wash and wipe dry; the wire must stand 4 dips without a permanent coating of copper showing on any part of the wire.

Galvanizing by Cementation; Sherardizing. — The alloying of metals at temperatures below their melting points has been known since 1820 or earlier. Berry (1838) invented a process of depositing zinc, in which the objects to be coated were placed in a closed retort and covered with a mixture of charcoal and powder of zinc; the retort was heated to cherry-red for a longer or shorter period, according to the bulk of the article and to the desired thickness of the coating. Dumas gave iron articles a slight coating of copper by dipping them in a solution of sulphate of copper and then heated them in a closed retort with oxide of zinc and charcoal dust. Sheet steel cowbells are coated with brass by placing them in a mixture of finely divided brass and charcoal dust and heating them to redness in an air-tight crucible.

S. Cowper-Coles's process, known as Sherardizing, patented in 1902, consists in packing the objects which are to be coated in zinc dust or pulverized zinc to which zinc oxide with a small percentage of charcoal dust is added, and heating in a closed retort to a temperature below the melting point of zinc. A large proportion of sand can be used to reduce the amount of zinc dust carried in the retort, to prevent caking and give a brighter finish; motion of the retort is in most cases necessary to obtain an even coating. The operation lasts from 30 minutes to several hours, depending on the size of the drum. Tempered steel is not affected by the process, but surfaces are hardened, there being a zinc-iron alloy formed to a depth varying with the time of treatment. This process is suitable for small work, giving a superior quality of zinc coating. (See Cowper-Coles, "Preservation and Ornamentation of Iron and Steel Surfaces," *Trans. Soc. Engrs.* 1905, p. 183; "Sherardizing," *Iron Age*, 1904, p. 12. Alfred Sang, "Theory and Practice of Sherardizing," *El. Chem. and Metall. Ind.*, May, 1907.)

Lead Coatings. — Lead is a good protection for iron and steel provided it is perfectly gas-tight. Electrically deposited lead does not bond well and the coating is porous. Sheets having a light coating of lead, produced by dipping in the molten metal, are known as *terne* plates; they have no lasting qualities. Lead-lined wrought pipe, fittings and valves are made for conveying acids and other corroding liquids.

STEEL.

The Manufacture of Steel. (See Classification of Iron and Steel, p. 413.) Cast steel is a malleable alloy of iron, cast from a fluid mass. It is distinguished from cast iron, which is not malleable, by being much lower in carbon, and from wrought iron, which is welded from a pasty mass, by being free from intermingled slag. Blister steel is a highly carbonized wrought iron, made by the "cementation" process, which consists in keeping wrought-iron bars at a red heat for some days in contact with charcoal. Not over 2% of C is usually absorbed. The surface of the iron is covered with small blisters, supposedly due to the action of carbon on slag. Other wrought steels were formerly made by direct processes from iron ore, and by the puddling process from wrought iron, but these steels are now replaced by cast steels. Blister steel is, however, still used as a raw material in the manufacture of crucible steel. Case-hardening is a process of surface cementation.

Crucible Steel is commonly made in pots or crucibles holding about 80 pounds of metal. The raw material may be steel scrap; blister steel bars; wrought iron with charcoal; cast iron with wrought iron or with iron ore; or any mixture that will produce a metal having the desired chemical constitution. Manganese in some form is usually added to prevent oxidation of the iron. Some silicon is usually absorbed from the crucible, and carbon also if the crucible is made of graphite and clay. The crucible being covered, the steel is not affected by the oxygen or sulphur in the flame. The quality of crucible steel depends on the freedom from objectionable elements, such as phosphorus, in the mixture, on the complete removal of oxide, slag and blowholes by "dead-melting" or "killing" before pouring, and on the kind and quantity of different elements which are added in the mixture, or after melting, to give particular qualities to the steel, such as carbon, manganese, chromium, tungsten and vanadium.

Bessemer Steel is made by blowing air through a bath of melted pig iron. The oxygen of the air first burns away the silicon, then the carbon, and before the carbon is entirely burned away, begins to burn the iron. Spiegeleisen or ferro-manganese is then added to deoxidize the metal and to give it the amount of carbon desired in the finished steel. In the ordinary or "acid" Bessemer process the lining of the converter is a silicious material, which has no effect on phosphorus, and all the phosphorus in the pig iron remains in the steel. In the "basic" or Thomas and Gilchrist process the lining is of magnesian limestone, and limestone additions are made to the bath, so as to keep the slag basic, and the phosphorus enters the slag. By this process ores that were formerly unsuited to the manufacture of steel have been made available.

Open-hearth Steel.—Any mixture that may be used for making steel in a crucible may also be melted on the open hearth of a Siemens regenerative furnace, and may be desiliconized and decarbonized by the action of the flame and by additions of iron ore, deoxidized by the addition of spiegeleisen or ferro-manganese, and recarbonized by the same additions or by pig iron. In the most common form of the process pig iron and scrap steel are melted together on the hearth, and after the manganese has been added to the bath it is tapped into the ladle. In the Talbot process a large bath of melted material is kept in the furnace, melted pig iron, taken from a blast furnace, is added to it, and iron ore is added which contributes its iron to the melted metal while its oxygen decarbonizes the pig iron. When the decarbonization has proceeded far enough, ferro-manganese is added to destroy iron oxide, and a portion of the metal is tapped out, leaving the remainder to receive another charge of pig iron, and thus the process is continued indefinitely. In the Duplex Process melted cast iron is desiliconized in a Bessemer converter, and then run into an open hearth, where the steel-making operation is finished.

The open-hearth process, like the Bessemer, may be either acid or basic, according to the character of the lining. The basic process is a dephosphorizing one, and is the one most generally available, as it can use pig irons that are either low or high in phosphorus.

Relation between the Chemical Composition and Physical Character of Steel.

W. R. Webster (*Trans. A. I. M. E.*, vols. xxi and xxii, 1893-4) gives results of several hundred analyses and tensile tests of basic Bessemer steel plates, and from a study of them draws conclusions as to the relation of chemical composition to strength, the chief of which are condensed as follows:

The indications are that a pure iron, without carbon, phosphorus, manganese, silicon, or sulphur, if it could be obtained, would have a tensile strength of 34,750 lbs. per sq. in., if tested in a $\frac{3}{8}$ -in. plate. With this as a base, a table is constructed by adding the following hardening effects, as shown by increase of tensile strength, for the several elements named.

Carbon, a constant effect of 800 lbs. for each 0.01%.

Sulphur, " " 500 " " 0.01%.

Phosphorus, the effect is higher in high-carbon than in low-carbon steels.

With carbon hundredths %..... 9 10 11 12 13 14 15 16 17

Each 0.01% P has an effect of lbs.. 900 1000 1100 1200 1300 1400 1500 1500 1500

Manganese, the effect decreases as the per cent of manganese increases.

Mn being per cent. { .00 .15 .20 .25 .30 .35 .40 .45 .50 .55
to to to to to to to to to to
.15 .20 .25 .30 .35 .40 .45 .50 .55 .65

Strength incr. for 0.01%... 240 240 220 200 180 160 140 120 100 100 lbs.

Total increase from 0 Mn.. 3600 4800 5900 6900 7800 8600 9300 9900 10,400 11,400

Silicon is so low in this steel that its hardening effect has not been considered.

With the above additions for carbon and phosphorus the following table has been constructed (abridged from the original by Mr. Webster). To the figures given the additions for sulphur and manganese should be made as above.

Estimated Ultimate Strengths of Basic Bessemer-steel Plates.

For Carbon, 0.06 to 0.24; Phosphorus, .00 to .10; Manganese and Sulphur, .00 in all cases.

| Carbon. | 0.06 | .08 | .10 | .12 | .14 | .16 | .18 | .20 | .22 | .24 |
|------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Phos. .005 | 39,950 | 41,550 | 43,250 | 44,950 | 46,650 | 48,300 | 49,900 | 51,500 | 53,100 | 54,700 |
| " .01 | 40,350 | 41,950 | 43,750 | 45,550 | 47,350 | 49,050 | 50,650 | 52,250 | 53,850 | 55,450 |
| " .02 | 41,150 | 42,750 | 44,750 | 46,750 | 48,750 | 50,550 | 52,150 | 53,750 | 55,350 | 56,950 |
| " .03 | 41,950 | 43,550 | 45,750 | 47,950 | 50,150 | 52,050 | 53,650 | 55,250 | 56,850 | 58,450 |
| " .04 | 42,750 | 44,350 | 46,750 | 49,150 | 51,550 | 53,550 | 55,150 | 56,750 | 58,350 | 59,950 |
| " .05 | 43,550 | 45,150 | 47,750 | 50,350 | 52,950 | 55,050 | 56,650 | 58,250 | 59,850 | 61,450 |
| " .06 | 44,350 | 45,950 | 48,750 | 51,550 | 54,350 | 56,550 | 58,150 | 59,750 | 61,350 | 62,950 |
| " .07 | 45,150 | 46,750 | 49,750 | 52,750 | 55,750 | 58,050 | 59,650 | 61,250 | 62,850 | 64,450 |
| " .08 | 45,950 | 47,550 | 50,750 | 53,950 | 57,150 | 59,550 | 61,150 | 62,750 | 64,350 | 65,950 |
| " .09 | 46,750 | 48,350 | 51,750 | 55,150 | 58,550 | 61,050 | 62,650 | 64,250 | 65,850 | 67,450 |
| " .10 | 47,550 | 49,150 | 52,750 | 56,350 | 59,950 | 62,550 | 64,150 | 65,750 | 67,350 | 68,950 |
| 0.001 P. = | 80 lbs. | 80 lbs. | 100 lb. | 120 lb. | 140 lb. | 150 lb. | 150 lb. | 150 lb. | 150 lb. | 150 lb. |

In all rolled steel the quality depends on the size of the bloom or ingot from which it is rolled, the work put on it, and the temperature at which it is finished, as well as the chemical composition.

The above table is based on tests of plates $\frac{3}{8}$ inch thick and under 70 inches wide; for other plates Mr. Webster gives the following corrections for thickness and width. They are made necessary only by the effect of thickness and width on the finishing temperature in ordinary practice. Steel is frequently spoiled by being finished at too high a temperature.

| Thickness, in..... | 3/4* | 11/16 | 5/8 | 9/16 | 1/2 | 7/16 | 3/8 | 5/16 |
|---------------------|-------|-------|-------|-------|-------|------|-------|-------|
| Correction (1)..... | -2000 | -1750 | -1500 | -1250 | -1000 | -500 | 0 | +3000 |
| Correction (2)..... | -1000 | -750 | -500 | -250 | 0 | ±500 | +1000 | +5000 |

* And over. (1) Plates up to 70 in. wide. (2) Over 70 in. wide.

Comparing the actual result of tests of 408 plates with the calculated results, Mr. Webster found the variation to range as below.

| Within lbs. | 1000 | 2000 | 3000 | 4000 | 5000 |
|-------------------|------|------|------|------|------|
| Per cent. | 28.4 | 55.1 | 74.7 | 89.9 | 94.9 |

The last figure would indicate that if specifications were drawn calling for steel plates not to vary more than 5000 lbs. T. S. from a specified figure (equal to a total range of 10,000 lbs.), there would be a probability of the rejection of 5% of the blooms rolled, even if the whole lot was made from steel of identical chemical analysis.

Campbell's Formulæ. (H. H. Campbell, *The Manufacture and Properties of Iron and Steel*, p. 387.) —

Acid steel, $40,000 + 1000 C + 1000 P + xMn = \text{Ultimate strength.}$

Basic steel, $41,500 + 770 C + 1000 P + yMn = \text{Ultimate strength.}$

The values of xMn and yMn are given by Mr. Campbell in a table, but they may be found from the formulæ $xMn = 8 CMn - 320 C$ and $yMn = 90 Mn + 4 CMn - 2700 - 120 C$, or, combining the formulæ we have:

Ult. strength, acid steel, $40,000 + 680 C + 1000 P + 8 CMn.$

basic " $38,800 + 650 C + 1000 P + 90 Mn + 4 CMn.$

In these formulæ the unit of each chemical element is 0.01%.

Examples. Required the tensile strength of two steels containing respectively C, 0.10, P, 0.10, Mn, 0.30, and C, 0.20, P, 0.10, Mn, 0.65.

Answers, by Webster, 59,650 and 77,150; by Campbell, 57,700 and 72,850.

Low Tensile Strength of Very Pure Steel. — Swedish nail-rod open-hearth steel, tested by the author in 1881, showed a tensile strength of only 42,591 lbs. per sq. in. A piece of American nail-rod steel showed 45,021 lbs. per sq. in. Both steels contained about 0.10 C and 0.015 P, and were very low in S, Mn, and Si. The pieces tested were bars about $2 \times 3/8$ in. section.

R. A. Hadfield (*Jour. Iron and Steel Inst.*, 1894) gives the strength of very pure Swedish iron, remelted and tested as cast, 45,024 lbs. per sq. in.; remelted and forged, 47,040 lbs. The analysis of the cast bar was: C, 0.08; Si, 0.04; S, 0.02; P, 0.02; Mn, 0.01; Fe, 99.82.

Effect of Oxygen upon Strength of Steel. — A. Lantz, of the Peine works, Germany, in a letter to Mr. Webster, says that oxygen plays an important rôle — such that, given a like content of C, P, and Mn, a blow with greater oxygen content gives a greater hardness and less ductility than a blow with less oxygen content. The method used for determining oxygen is that of Prof. Ledebur, given in *Stahl und Eisen*, May, 1892, p. 193. The variation in O may make a difference in strength of nearly $1/2$ ton per sq. in. (*Jour. I. and S. I.*, 1894.)

Electric Conductivity of Steel. — Louis Campredon reports in *Le Genie Civil* [prior to 1895] the results of experiments on the electric resistance of steel wires of different composition, ranging from 0.09 to 0.14 C; 0.21 to 0.54 Mn; Si, S, and P low. The figures show that the purer and softer the steel the better is its electric conductivity, and, furthermore, that manganese is the element which most influences the conductivity. The results may be expressed by the formula $R = 5.2 + 6.2S \pm 0.3$; in which R = relative resistance, copper being taken as 1, and S = the sum of the percentages of C, P, S, Si, and Mn. The conclusions are confirmed by J. A. Capp, in 1903, *Trans. A. I. M. E.*, vol. xxxiv, who made forty-five experiments on steel of a wide range of composition. His results may be expressed by the formula $R = 5.5 + 4S \pm 1$. High manganese increases the resistance at an increasing rate. Mr. Capp proposes the following specification for steel to make a satisfactory third rail, having a resistance eight times that of copper: C, 0.15; Mn, 0.30; P, 0.06; S, 0.06; Si, 0.05; none of these figures to be exceeded,

Range of Variation in Strength of Bessemer and Open-Hearth Steels.

The Carnegie Steel Co. in 1888 published a list of 1057 tests of Bessemer and open-hearth steel, from which the following figures are selected:

| Kind of Steel. | No. of Tests. | Elastic Limit. | | Ultimate Strength. | | Elongation per cent in 8 Inches. | |
|------------------------|---------------|----------------|---------|--------------------|---------|----------------------------------|---------|
| | | High't. | Lowest. | High't. | Lowest. | High't. | Lowest. |
| (a) Bess. structural. | 100 | 46,570 | 39,230 | 71,300 | 61,450 | 33.00 | 23.75 |
| (b) " " | 170 | 47,690 | 39,970 | 73,540 | 65,200 | 30.25 | 23.15 |
| (c) Bess. angles..... | 72 | 41,890 | 32,630 | 63,450 | 56,130 | 34.30 | 26.25 |
| (d) O. H. fire-box.... | 25 | | | 62,790 | 50,350 | 36.00 | 25.62 |
| (e) O. H. bridge..... | 20 | | | 69,940 | 63,970 | 30.00 | 22.75 |

REQUIREMENTS OF SPECIFICATIONS.

- (a) E. L., 35,000; T. S., 62,000 to 70,000; elong., 22% in 8 in.
- (b) E. L., 40,000; T. S., 67,000 to 75,000.
- (c) E. L., 30,000; T. S., 56,000 to 64,000; elong., 20% in 8 in.
- (d) T. S., 50,000 to 62,000; elong., 26% in 4 in.
- (e) T. S., 64,000 to 70,000; elong., 20% in 8 in.

Bending Tests of Steel. (Pencoyd Iron Works.) — Steel below 0.10 C should be capable of doubling flat without fracture, after being chilled from a red heat in cold water. Steel of 0.15 C will occasionally submit to the same treatment, but will usually bend around a curve whose radius is equal to the thickness of the specimen; about 90% of specimens stand the latter bending test without fracture. As the steel becomes harder its ability to endure this bending test becomes more exceptional, and when the carbon becomes 0.20 little over 25% of specimens will stand the last-described bending test. Steel having about 0.40% C will usually harden sufficiently to cut soft iron and maintain an edge.

EFFECT OF HEAT TREATMENT AND OF WORK ON STEEL.

Low Strength Due to Insufficient Work. (A. E. Hunt, *Trans. A. I. M. E.*, 1886.) — Soft steel ingots, made in the ordinary way for boiler plates, have only from 10,000 to 20,000 lbs. tensile strength per sq. in., an elongation of only about 10% in 8 in., and a reduction of area of less than 20%. Such ingots, properly heated and rolled down from 10 in. to 1/2 in. thickness, will give from 55,000 to 65,000 lbs. tensile strength, an elongation in 8 in. of from 23% to 33%, and a reduction of area of from 55% to 70%. Any work stopping short of the above reduction in thickness ordinarily yields intermediate results in tensile tests.

Effect of Finishing Temperature in Rolling. — The strength and ductility of steel depend to a high degree upon fineness of grain, and this may be obtained by having the temperature of the steel rather low, say at a dull red heat, 1300° to 1400° F., during the finishing stage of rolling. In the manufacture of steel rails a great improvement in quality has been obtained by finishing at a low temperature. An indication of the finishing temperature is the amount of shrinkage by cooling after leaving the rolls. The Phila. & Reading Railway Co.'s specification for rails (1902) says, "The temperature of the ingot or bloom shall be such that with rapid rolling and without holding before or in the finishing passes or subsequently, and without artificial cooling after leaving the last pass, the distance between the hot saws shall not exceed 30 ft. 6 in. for a 30-ft. rail."

Fining the Grain by Annealing. — Steel which is coarse-grained on account of leaving the rolls at too high a temperature may be made fine-grained and have its ductility greatly increased without lowering its tensile strength by reheating to a cherry-red and cooling at once in air. (See paper on "Steel Rails," by Robert Job, *Trans. A. I. M. E.*, 1902.)

Effect of Cold Rolling. — Cold rolling of iron and steel increases the elastic limit and the ultimate strength, and decreases the ductility. Major Wade's experiments on bars rolled and polished cold by Lauth's process showed an average increase of load required to give a slight permanent set as follows: Transverse, 162%; torsion, 130%; compression, 161% on short columns 1½ in. long, and 64% on columns 8 in. long; tension, 95%. The hardness, as measured by the weight required to produce equal indentations, was increased 50%; and it was found that the hardness was as great in the center of the bars as elsewhere. Sir W. Fairbairn's experiments showed an increase in ultimate tensile strength of 50%, and a reduct on in the elongation in 10 in. from 2 in. or 20% to 0.79 in. or 7.9%.

Hardening of Soft Steel. — A. E. Hunt (*Trans. A. I. M. E.*, 1883, vol. xii) says that soft steel, no matter how low in carbon, will harden to a certain extent upon being heated red-hot and plunged into water, and that it hardens more when plunged into brine and less when quenched in oil.

A heat of open-hearth steel of 0.15% C and 0.29% Mn gave the following results upon test-pieces from the same ¼ in. thick plate.

| | | | | | |
|------------------------|--------------|--------------|-----|--------------|-----|
| Unhardened..... | T. S. 55,000 | El. in 8 in. | 27% | Red. of Area | 62% |
| Hardened in water..... | " 74,000 | " | 25% | " | 50% |
| Hardened in brine..... | " 84,000 | " | 22% | " | 43% |
| Hardened in oil..... | " 67,000 | " | 26% | " | 49% |

The greatly increased tenacity after hardening indicates that there must be a considerable molecular change in the steel thus hardened, and that if such a hardening should be created locally in a steel plate, there must be very dangerous internal strains caused thereby.

Comparative Tests of Full-sized Eye-bars and Small Samples. (G. G. S. Morison, *A. S. C. E.*, 1893.) — 17 full-sized eye-bars, of the steel used in the Memphis bridge, sections 10 in. wide × 1 to 23/16 in. thick, and sample bars from the same melts. Average results:

Eye-bars: E. L., 32,350; T. S., 63,330; El. in full length, 13.7%; Red. of area, 36.3%.

Small bars: E. L., 40,650; T. S., 71,640; El. in 8 ins., 26.2%; Red. of area, 46.7%.

Effect of Annealing on Rolled Bars. (Campbell, *Mfr. of Iron and Steel*, p. 275.) —

| Ultimate Strength. | | Elastic Limit. | | Elong. in 8 in., %. | | Red. Area, %. | | Elas. Ratio. | |
|----------------------------|-----------|----------------|-----------|---------------------|-----------|---------------|-----------|--------------|-----------|
| Natural. | Annealed. | Natural. | Annealed. | Natural. | Annealed. | Natural. | Annealed. | Natural. | Annealed. |
| 2 × 3/8 in. flats, rounds. | | | | | | | | | |
| 58,568 | 54,098 | 40,300 | 31,823 | 29.7 | 28.8 | 60.8 | 62.7 | 68.8 | 58.8 |
| 62,187 | 58,364 | 42,606 | 35,120 | 28.0 | 28.6 | 62.2 | 63.5 | 68.5 | 60.2 |
| 70,530 | 65,500 | 49,000 | 37,685 | 26.9 | 23.4 | 61.1 | 55.3 | 69.5 | 57.5 |
| 76,616 | 69,402 | 51,108 | 40,505 | 24.5 | 23.0 | 53.7 | 56.5 | 66.7 | 58.4 |
| 58,130 | 51,418 | 40,400 | 30,393 | 30.1 | 31.1 | 61.8 | 60.5 | 69.5 | 59.1 |
| 62,089 | 55,021 | 42,441 | 31,576 | 30.1 | 30.4 | 60.9 | 60.0 | 68.4 | 57.4 |
| 69,420 | 60,850 | 45,090 | 34,000 | 25.6 | 26.5 | 59.3 | 52.1 | 65.0 | 55.9 |
| 75,865 | 67,618 | 49,691 | 39,403 | 24.7 | 26.3 | 54.4 | 51.4 | 65.5 | 58.3 |

The bars were rolled from 4 × 4-in. billets of open-hearth steel. The figures are averages of from 2 to 12 tests of each heat. In annealing the bars were heated in a muffle and withdrawn when they had reached a dull yellow heat.

"Recalcence" of Steel. — If we heat a bar of copper by a flame of constant strength, and note carefully the interval of time occupied in passing from each degree to the next higher degree, we find that these intervals increase regularly, i.e., that the bar heats more and more slowly, as its temperature approaches that of the flame. If we substitute a bar of steel for one of copper, we find that these intervals increase regularly up to a certain point, when the rise of temperature is suddenly and in most

cases greatly retarded or even completely arrested. After this the regular rise of temperature is resumed, though other like retardations may recur as the temperature rises farther. So if we cool a bar of steel slowly the fall of temperature is greatly retarded when it reaches a certain point in dull redness. If the steel contains much carbon, and if certain favoring conditions be maintained, the temperature, after descending regularly, suddenly rises spontaneously very abruptly, remains stationary a while and then redescends. This spontaneous reheating is known as "recalescence."

These retardations indicate that some change which absorbs or evolves heat occurs within the metal. A retardation while the temperature is rising points to a change which absorbs heat; a retardation during cooling points to some change which evolves heat. (Henry M. Howe, on "Heat Treatment of Steel," *Trans. A. I. M. E.*, vol. xxii.)

Critical Point. (Campbell, p. 287.)—If a piece of steel containing over 0.50 C be allowed to cool slowly from a high temperature the cooling at first proceeds at a uniformly retarded rate, but when about 700° C. is reached there is an interruption of this regularity. In some cases the rate of cooling may be very slow, in other cases the bar may not decrease in temperature at all, while in still other cases the bar may actually grow hotter for a moment. When this "critical point" is passed, the bar cools as before until it reaches the temperature of the atmosphere.

In metallography such a critical point is denoted by the letter A, and the particular one just described is known as Ar. In heating a piece of steel an opposite phenomenon is observed, there being an absorption of heat by internal molecular action, with a consequent retardation in the rise of temperature, and this point, which is some 30° C. higher than Ar, is called Ac.

In soft steels, below 0.30 C, three critical points are found in cooling a bar from a high temperature, called Ar₃, Ar₂, Ar₁, Ar₁ being the lowest, and in heating the bar there are also three points, Ac₁, Ac₂, Ac₃, the first named being the lowest. At each of the points there is a change in the micro-structure of the steel.

Metallography.—This is a name given to a study of the micro-structure of metals. The steel metallographist designates the different structures that are found in a polished and etched section by the names austenite, martensite, pearlite, cementite, ferrite, troostite, and sorbite. Austenite is produced by quenching steel of over 1.40 C in ice water from above 1050° C. Martensite is produced by quenching this steel from temperatures between 1050° C. and Ar₁. It is also found together with cementite or ferrite in carbon steels below 1.30 C quenched at any point above Ar₁. It is the constituent which confers hardness on steel. In steels cooled slowly to below Ar₁ the structure is composed entirely of ferrite, or entirely of pearlite, or of pearlite mixed with ferrite or cementite. Ferrite is iron free from carbon and forms almost the whole of a low-carbon steel, while cementite is considered to be a compound of iron and carbon, Fe₃C, the C of this form being known as cement carbon. Pearlite is an intimate mixture of definite proportions of ferrite and cementite, corresponding to a pure steel of about 0.80 C, which, unhardened, consists of pearlite alone. Steels lower in C contain pearlite with ferrite, and steels higher in C contain pearlite and cementite. Troostite is a structure found when steel is quenched while cooling through the critical range, and sorbite when it is quenched at the end of the critical range. Quenching in lead or reheating quenched steel to a purple tint may also produce sorbite. (Campbell, p. 296.)

Effect of Work on the Structure of Soft and Medium Steel.—Steel as usually cast, cooling slowly, forms in crystals or grains. Rolling tends to break up this grain, but immediately after the cessation of work the formation of grains begins and continues until the metal has cooled to the lower critical point. Hence the lower the temperature to which the steel is worked, the more broken up the structure will be, but on the other hand if the rolling be continued below the critical point, the effect of cold work will be shown and strains will be set up which will make the piece unfit for use without annealing.

Effect of Heat Treatment.—In heating steel through the lowest critical point the crystalline structure is obliterated, the metal assuming the finest condition of which it is capable. Above this point the size of grain increases with the temperature.

Effect of Heating on Crucible Steel. (W. Campbell, *Proc. A. S. T. M.*, vi, 213.)—Six steels, containing carbon as follows: (1) 2.04, (2) 1.94, (3) 1.72, (4) 1.61, (5) 1.04, and (6) 0.70, were heated in a small gas furnace to the temperatures given in the table and allowed to cool slowly in the furnace, and were then tested, with results as below.

| | As Rolled. | 650° C | 715° C | 760° C | 800° C | 855° C | 905° C | 950° C | 1070° C | 1200° C |
|--------------|---------------|--------|--------|--------|--------|--------|--------|--------|------------|------------|
| (1) T. S... | 144000 | 115400 | 114500 | 98800 | 95650 | 93800 | 95250 | 95200 | 99000 | 57400 |
| E. L. | 104200 | 84600 | 83900 | 57700 | 57800 | 55500 | 55350 | 49350 | 49600 | 56000 |
| El. in 2 in. | 4.0 | 6.0 | 7.0 | 11.5 | 12.5 | 12.0 | 11.5 | 6.0 | 4.5 | 1.0 |
| (2) T. S... | 146400 | 115200 | 104100 | 95000 | 92000 | 89000 | 95350 | 91800 | 97000 | 61350 |
| E. L. | 91000 | 91500 | 72600 | 68650 | 50500 | 51000 | 49450 | 49800 | 41750 | 47000 |
| El. in 2 in. | 6.3 | 8.0 | 9.5 | 15.0 | 17.0 | 12.5 | 7.0 | 9.5 | 8.5 | 2.0 |
| (3) T. S... | 153100 | 126000 | 114100 | 100300 | 98000 | 94000 | 94350 | 95000 | 92350 | 65300 |
| E. L. | 98100 | 78300 | 75700 | 50500 | 48750 | 47900 | 48600 | 45200 | 43100 | 50600 |
| El. in 2 in. | 7.2 | 8.0 | 11.5 | 16.5 | 10.0 | 13.5 | 11.0 | 7.5 | 6.0 | 2.0 |
| (4) T. S... | 157700 | 128100 | 117000 | 98650 | 97700 | 95000 | 97350 | 96350 | 94400 | 69800 |
| E. L. | 105200 | 85300 | 81300 | 52300 | 53350 | 51350 | 51350 | 48500 | 51400 | |
| El. in 2 in. | 6.5 | | 14.5 | | 18.5 | 15.0 | 11.5 | 7.5 | 3.5 | 3.0 |
| (5) T. S... | 141100 | 105400 | 97800 | 86800 | 96600 | 111800 | 115900 | 111500 | 106100 | 112600 |
| E. L. | 75800 | 57700 | 55200 | 44850 | 46600 | 47200 | 50600 | 46800 | 56500 | 89600 |
| El. in 2 in. | 12.8 | 18.0 | 22.0 | 26.5 | 19.0 | 13.0 | 13.0 | 10.5 | 11.0 | 11.5 |
| (6) T. S... | 117000 | 95200 | 88700 | 85600 | 94300 | 91350 | 90300 | 90500 | 89500 | 90000 |
| E. L. | 64700 | 53250 | 49700 | 40200 | 42150 | 42100 | 41400 | 39700 | 57350 | 58500 |
| El. in 2 in. | 17.0 | 23.0 | 27.5 | 27.0 | 19.0 | 18.5 | 18.0 | 16.5 | 18.0 | 16.0 |

The critical points A_{r1} and A_{c1} were determined, and the six steels gave practically identical results; thus A_{r1} ranged from 696 to 708, averaging 704° C., and A_{c1} ranged from 730 to 737, averaging 733° C.

The temperatures at which the finest-grained and a very coarse-grained fracture were found are as follows:

| Steel No..... | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------------|------|------|---------|------|-----|--------|
| Finest fracture..... | 800 | 760 | 715-760 | 760 | 715 | 715° C |
| Very coarse fracture.... | 1070 | 1070 | 1070 | 1070 | 855 | 800° C |

Mr. Campbell's paper gives a list of fourteen papers by different authorities on the micro-structure and the heat treatment of steel.

Burning, Overheating, and Restoring Steel. (G. B. Waterhouse, *A. S. T. M.*, vi, 247.)—Burnt metal is defined as coarsely crystalline and exceedingly brittle iron or steel, in consequence of excessive heating, often with some layers of oxide of iron. It cannot be effectively restored by heat treatment or mechanical work. Overheated metal is coarsely crystalline from excessive heating, but with no inter-crystalline spaces. It can be restored by heat treatment or mechanical work. Seven lots of nickel steel bars, containing 3.8% Ni, and C as in the table, were heated to various temperatures in a muffle furnace, with results as below.

| % C. | Heated to..... | 1000a | 1000b | 1100b | 1200b | 1300b | 1200c | 1200d |
|------|--------------------|--------|--------|--------|--------|-------|--------|--------|
| 0.41 | T. S..... | 90245 | 71800 | 71700 | 74000 | 71320 | 71487 | 74989 |
| | El. % in 2 in..... | 26.0 | 26.0 | 25.5 | 11.0 | 7.0 | 10.5 | 25.0 |
| 0.51 | T. S..... | 99109 | 78600 | 78800 | 84900 | 79600 | 81487 | 80795 |
| | El. % in 2 in..... | 21.0 | 25.0 | 24.0 | 11.5 | 5.0 | 15.5 | 22.5 |
| 0.63 | T. S..... | 115421 | 89000 | 89400 | 99600 | 85200 | 96040 | 89842 |
| | El. % in 2 in..... | 16.5 | 20.5 | 19.0 | 7.0 | 2.0 | 10.0 | 21.0 |
| 0.79 | T. S..... | 135194 | 108950 | 111840 | 109600 | 66800 | 102705 | 90214 |
| | El. % in 2 in..... | 14.0 | 15.0 | 14.0 | 3.0 | 0.5 | 6.0 | 21.0 |
| 0.97 | T. S..... | 156827 | 130336 | 138112 | 83117 | 46648 | 114107 | 103476 |
| | El. % in 2 in..... | 7.5 | | 3.5 | 0.5 | 0.0 | 5.5 | 18.0 |
| 1.24 | T. S..... | 168697 | 97510 | 98183 | 90729 | 60600 | 95103 | 106304 |
| | El. in 2 in..... | 3.5 | 15.0 | 1.0 | 0.5 | 0.0 | 1.5 | 3.5 |
| 1.48 | T. S..... | 145642 | 63950 | 66640 | 97894 | 35480 | 89045 | 74592 |
| | El. in 2 in..... | 10.5 | 23.6 | 25.0 | 8.0 | 1.0 | 17.5 | 24.0 |

a. Heated to 1000 C., which took 1 hr. 25 min., held there 25 min. and cooled in air. b. The time required to heat to the temperatures named was respectively 1 h. 10 m., 1 h. 45 m., 2 h. 35 m., and 2 h. 35 m. The bars were kept at the desired temperature for an hour and then cooled slowly in place. c. Reheated to 700 C. d. Reheated to 775 C.

In the steels below 1% C heating to 1200° is accompanied by an increase in ultimate strength and a drop in ductility. Heating above 1200° produces a very coarse crystallization and a great loss in strength and ductility. Reheating the overheated bars to 700° does not materially affect their structure, but reheating to 775° restores the structure nearly to that found before overheating, and completely restores the ductility. Similar results are found with carbon steel.

Working Steel at a Blue Heat. — Not only are wrought iron and steel much more brittle at a blue heat (i.e., the heat that would produce an oxide coating ranging from light straw to blue on bright steel, 430° to 600° F.), but while they are probably not seriously affected by simple exposure to blueness, even if prolonged, yet if they be worked in this range of temperature they remain extremely brittle after cooling, and may indeed be more brittle than when at blueness; this last point, however, is not certain. (Howe, *Metallurgy of Steel*, p. 534.)

Tests by Prof. Krohn, for the German State Railways, show that working at blue heat has a decided influence on all materials tested, the injury done being greater on wrought iron and harder steel than on the softer steel. The fact that wrought iron is injured by working at a blue heat was reported by Stromeyer. (*Engineering News*, Jan. 9, 1892.)

A practice among boiler-makers for guarding against failures due to working at a blue heat consists in the cessation of work as soon as a plate which had been red-hot becomes so cool that the mark produced by rubbing a hammer-handle or other piece of wood will not glow. A plate which is not hot enough to produce this effect, yet too hot to be touched by the hand, is most probably blue hot, and should under no circumstances be hammered or bent. (C. E. Stromeyer, *Proc. Inst. C. E.*, 1886.)

Oil-tempering and Annealing of Steel Forgings. — H. F. J. Porter says (1897) that all steel forgings above 0.1% carbon should be annealed, to relieve them of forging and annealing strains, and that the process of annealing reduces the elastic limit to 47% of the ultimate strength. Oil-tempering should only be practiced on thin sections, and large forgings should be hollow for the purpose. This process raises the elastic limit above 50% of the ultimate tensile strength, and in some alloys of steel, notably nickel steel, will bring it up to 60% of the ultimate.

Heat Treatment of Armor Plates. (Hadfield Process, *Iron Tr. Rev.*, Dec. 7, 1905.) — A cast armor plate of nickel-chromium steel is heated to from 950° C. to 1100° C., then cooled, preferably in air, then reheated to about 700° and cooled slowly, preferably in the furnace in which the heating was previously effected, again heated to about 700° and allowed to cool slowly to 640° C., whereupon it is suddenly cooled by spraying with water or by an air blast, but preferably in water. It is then reheated to about 600° and again suddenly cooled, preferably by quenching in water. Steel treated as described is suitable for armor plates and other articles, including parts of safes. Satisfactory results have been obtained by thus treating cast 6-in. armor plates containing about 0.3 to 0.4 C, 0.25 Mn, 1.8 Cr, and 3.3 Ni cast in a sand mold. Such a 6-in. plate attacked by armor-piercing projectiles of 4.7-in. and 6-in. calibers, stood over 15,000 foot-tons of energy without showing a crack. Also a 4-in. plate treated as described and having a carbonized or cemented face has withstood the attack of a 5.7-in. armor-piercing shell.

Brittleness Due to Long-continued Heating. If low-carbon steel, (say under 0.15%) is held for a very long time at temperatures between 500 and 750° C. (930 and 1380° F.), the crystals become enormous and the steel loses a large part of its strength and ductility. It takes a long time, in fact days, to produce this effect to any alarming degree, so that it is not liable to occur during manufacture or mechanical treatment, but steel is sometimes placed in positions where it may suffer this injury, for example, in the case of the tie-rods of furnaces, supports of boilers, etc., so that the danger should be borne in mind by all engineers and users of steel. A wrought-iron chain that supported one side of a 50-ton open-hearth ladle, which was heated many times to a temperature above 500° C., finally reached a condition of coarse crystallization, so that it was unable

to bear the strain upon it. This phenomenon of coarse crystallization in low-carbon steel is known as "Stead's Brittleness," after J. E. Stead, who has explained its cause. The effect seems to begin at a temperature of about 500° C. and proceeds more rapidly with an increase in temperature until we reach 750° C. The damage may be repaired completely by heating the steel to a temperature between 800 and 900° C. The remedy is the same as that for coarse crystallization, due to overheating, and all steel which is placed in positions where it is liable to reach these temperatures frequently should be restored at intervals of a week or a month, or as often as may be necessary. (Stoughton.)

Influence of Annealing upon Magnetic Capacity.

Prof. D. E. Hughes (*Eng'g*, Feb. 8, 1884, p. 130) has invented a "Magnetic Balance," for testing the condition of iron and steel, which consists chiefly of a delicate magnetic needle suspended over a graduated circular index, and a magnet coil for magnetizing the bar to be tested. He finds that the following laws hold with every variety of iron and steel:

1. The magnetic capacity is directly proportional to the softness, or molecular freedom.

2. The resistance to a feeble external magnetizing force is directly as the hardness, or molecular rigidity.

The magnetic balance shows that annealing not only produces softness in iron, and consequent molecular freedom, but it entirely frees it from all strains previously introduced by drawing or hammering. Thus a bar of iron drawn or hammered has a peculiar structure, say a fibrous one, which gives a greater mechanical strength in one direction than another. This bar, if thoroughly annealed at high temperatures, becomes homogeneous in all directions, and has no longer even traces of its previous strains, provided that there has been no actual separation into a distinct series of fibers.

TREATMENT OF STRUCTURAL STEEL.

(James Christie, *Trans. A. S. C. E.*, 1893.)

Effect of Punching and Shearing. — The physical effects of punching and shearing as denoted by tensile test are for iron or steel:

Reduction of ductility; elevation of tensile strength at elastic limit; reduction of ultimate tensile strength.

In very thin material the disturbance described is less than in thick; in fact, a degree of thinness is reached where this disturbance practically ceases. On the contrary, as thickness is increased the injury becomes more evident.

The effects described do not invariably ensue; for unknown reasons there are sometimes marked deviations from what seems to be a general result.

By thoroughly annealing sheared or punched steels the ductility is to a large extent restored and the exaggerated elastic limit reduced, the change being modified by the temperature of reheating and the method of cooling.

It is probable that the best results combined with least expenditure can be obtained by punching all holes where vital strains are not transferred by the rivets, and by reaming for important joints where strains on riveted joints are vital, or wherever perforation may reduce sections to a minimum. The reaming should be sufficient to thoroughly remove the material disturbed by punching; to accomplish this it is best to enlarge punched holes at least $\frac{1}{8}$ in. diameter with the reamer.

Riveting. — It is the current practice to perforate holes $\frac{1}{16}$ in. larger than the rivet diameter. For work to be reamed it is also a usual requirement to punch the holes from $\frac{1}{8}$ to $\frac{3}{16}$ in. less than the finished diameter, the holes being reamed to the proper size after the various parts are assembled.

It is also excellent practice to remove the sharp corner at both ends of the reamed holes, so that a fillet will be formed at the junction of the body and head of the finished rivets.

The rivets of either iron or mild steel should be heated to a bright red or yellow heat and subjected to a pressure of not less than 50 tons per square inch of sectional area.

For rivets of ordinary length this pressure has been found sufficient to completely fill the hole. If, however, the holes and the rivets are excep-

tionally long, a greater pressure and a slower movement of the closing tool than is used for shorter rivets has been found advantageous.

Welding. — No welding should be allowed on any steel that enters into structures. [See page 463.]

Upsetting. — Enlarged ends on tension bars for screw-threads, eye-bars, etc., are formed by upsetting the material. With proper treatment and a sufficient increment of enlarged sectional area over the body of the bar the result is entirely satisfactory. The upsetting process should be performed so that the properly heated metal is compelled to flow without folding or lapping.

Annealing. — The object of annealing structural steel is for the purpose of securing homogeneity of structure that is supposed to be impaired by unequal heating, or by the manipulation necessarily attendant on certain processes. The objects to be annealed should be heated throughout to a uniform temperature and uniformly cooled.

The physical effects of annealing, as indicated by tensile tests, depend on the grade of steel, or the amount of hardening elements associated with it; also on the temperature to which the steel is raised, and the method or rate of cooling the heated material.

The physical effects of annealing medium-grade steel, as indicated by tensile test, are reported very differently by different observers, some claiming directly opposite results from others. It is evident, when all the attendant conditions are considered, that the obtained results must vary both in kind and degree.

The temperatures employed will vary from 1000° to 1500° F. In some cases the heated steel is withdrawn at full temperature from the furnace and allowed to cool in the atmosphere; in others the mass is removed from the furnace, but covered under a muffle, to lessen the free radiation; or, again, the charge is retained in the furnace, and the whole mass cooled with the furnace, and more slowly than by either of the other methods.

The best general results from annealing will probably be obtained by introducing the material into a uniformly heated oven in which the temperature is not so high as to cause a possibility of cracking by sudden and unequal changing of temperature, then gradually raising the temperature of the material until it is uniformly about 1200° F., then withdrawing the material after the temperature is somewhat reduced and cooling under shelter of a muffle sufficiently to prevent too free and unequal cooling on the one hand or excessively slow cooling on the other.

G. G. Mehlertens, *Trans. A. S. C. E.*, 1893, says: "Annealing is of advantage to all steel above 64,000 lbs. strength per square inch, but it is questionable whether it is necessary in softer steels. The distortions due to heating cause trouble in subsequent straightening, especially of thin plates.

"In a general way all unannealed mild steel for a strength of 56,000 to 64,000 lbs. may be worked in the same way as wrought iron. Rough treatment or working at a blue heat must, however, be prohibited. Shearing is to be avoided, except to prepare rough plates, which should afterwards be smoothed by machine tools or files before using. Drifting is also to be avoided, because the edges of the holes are thereby strained beyond the yield-point. Reaming drilled holes is not necessary, particularly when sharp drills are used and neat work is done. A slight counter-sinking of the edges of drilled holes is all that is necessary. Working the material while heated should be avoided as far as possible, and the engineer should bear this in mind when designing structures. Upsetting, cranking, and bending ought to be avoided, but when necessary the material should be annealed after completion.

"The riveting of a mild-steel rivet should be finished as quickly as possible, before it cools to the dangerous heat. For this reason machine work is the best. There is a special advantage in machine work from the fact that the pressure can be retained upon the rivet until it has cooled sufficiently to prevent elongation and the consequent loosening of the rivet."

Punching and Drilling of Steel Plates. (*Proc. Inst. M. E.*, Aug., 1887, p. 326.) — In Prof. Unwin's report the results of the greater number of the experiments made on iron and steel plates lead to the general conclusion that while thin plates, even of steel, do not suffer very much from punching, yet in those of $\frac{1}{2}$ in. thickness and upwards the loss of tenacity due to punching ranges from 10% to 23% in iron plates and from 11% to 33% in the case of mild steel.

MISCELLANEOUS NOTES ON STEEL.

May Carbon be Burned Out of Steel? — Experiments made at the Laboratory of the Penna. Railroad Co. (Specifications for Springs, 1888) with the steel of spiral springs, show that the place from which the borings are taken for analysis has a very important influence on the amount of carbon found. If the sample is a piece of the round bar, and the borings are taken from the end of this piece, the carbon is always higher than if the borings are taken from the side of the piece. It is common to find a difference of 0.10% between the center and side of the bar, and in some cases the difference is as high as 0.23%. Apparently during the process of reducing the metal from the ingots to the round bar, with successive heatings, the carbon in the outside of the bar is burned out.

Effect of Nicking a Steel Bar. — The statement is sometimes made that, owing to the homogeneity of steel, a bar with a surface crack or nick in one of its edges is liable to fail by the gradual spreading of the nick, and thus break under a very much smaller load than a sound bar. With iron it is contended this does not occur, as this metal has a fibrous structure. Sir Benjamin Baker has, however, shown that this theory, at least so far as statical stress is concerned, is opposed to the facts, as he purposely made nicks in specimens of the mild steel used at the Forth Bridge, but found that the tensile strength of the whole was thus reduced by only about one ton per square inch of section. In an experiment by the Union Bridge Company a full-sized steel counter-bar, with a screw-turned buckle connection, was tested under a heavy statical stress, and at the same time a weight weighing 1040 lbs. was allowed to drop on it from various heights. The bar was first broken by ordinary statical strain, and showed a breaking stress of 66,800 lbs. per square inch. The longer of the broken parts was then placed in the machine and put under the following loads, whilst a weight, as already mentioned, was dropped on it from various heights at a distance of five feet from the sleeve-nut of the turn-buckle, as shown below:

| | | | | | |
|-----------------------------------|---------|---------|---------|---------|---------|
| Stress in pounds per sq. in. | 50,000 | 55,000 | 60,000 | 63,000 | 65,000 |
| | ft. in. | ft. in. | ft. in. | ft. in. | ft. in. |
| Height of fall. | 2 1 | 2 6 | 3 0 | 4 0 | 5 0 |

The weight was then shifted so as to fall directly on the sleeve-nut, and the test proceeded as follows:

| | | | |
|---|--------|--------|--------|
| Stress on specimen in lbs. per square inch | 65,350 | 65,350 | 68,800 |
| Height of fall, feet. | 3 | 6 | 6 |

It will be seen that under this trial the bar carried more than when originally tested statically, showing that the nicking of the bar by screwing had not appreciably weakened its power of resisting shocks. — *Eng'g News*.

Specific Gravity of Soft Steel. (W. Kent, *Trans. A. I. M. E.*, xiv, 585.) — Five specimens of boiler-plate of C 0.14, P 0.03 gave an average sp. gr. of 7.932, maximum variation 0.008. The pieces were first planed to remove all possible scale indentations, then filed smooth, then cleaned in dilute sulphuric acid, and then boiled in distilled water, to remove all traces of air from the surface.

The figures of specific gravity thus obtained by careful experiment on bright, smooth pieces of steel are, however, too high for use in determining the weights of rolled plates for commercial purposes. The actual average thickness of these plates is always a little less than is shown by the calipers, on account of the oxide of iron on the surface, and because the surface is not perfectly smooth and regular. A number of experiments on commercial plates, and comparison of other authorities, led to the figure 7.854 as the average specific gravity of open-hearth boiler-plate steel. This figure is easily remembered as being the same figure with change of position of the decimal point (.7854) which expresses the relation of the area of a circle to that of its circumscribed square. Taking the weight of a cubic foot of water at 62° F. as 62.36 lbs. (average of several authorities), this figure gives 489.775 lbs. as the weight of a cubic foot of steel, or the even figure, 490 lbs., may be taken as a convenient figure, and accurate within the limits of the error of observation.

A common method of approximating the weight of iron plates is to consider them to weigh 40 lbs. per square foot one inch thick. Taking this

weight and adding 2% gives almost exactly the weight of steel boiler-plate given above ($40 \times 12 \times 1.02 = 489.6$ lbs. per cubic foot).

Occasional Failures of Bessemer Steel.—G. H. Clapp and A. E. Hunt, in their paper on "The Inspection of Materials of Construction in the United States" (*Trans. A. I. M. E.*, vol. xix), say: Numerous instances could be cited to show the unreliability of Bessemer steel for structural purposes. One of the most marked, however, was the following: A 12-in. I-beam weighing 30 lbs. to the foot, 20 feet long, on being unloaded from a car broke in two about 6 feet from one end.

The analyses and tensile tests made do not show any cause for the failure.

The cold and quench bending tests of both the original $\frac{3}{4}$ -in. round test-pieces, and of pieces cut from the finished material, gave satisfactory results; the cold-bending tests closing down on themselves without sign of fracture.

Numerous other cases of angles and plates that were so hard in places as to break off short in punching, or, what was worse, to break the punches, have come under our observation, and although makers of Bessemer steel claim that this is just as likely to occur in open-hearth as in Bessemer steel, we have as yet never seen an instance of failure of this kind in open-hearth steel having a composition such as C 0.25%, Mn 0.70%, P 0.08%.

J. W. Wailes, in a paper read before the Chemical Section of the British Association for the Advancement of Science, in speaking of mysterious failures of steel, states that investigation shows that "these failures occur in steel of one class, viz., soft steel made by the Bessemer process."

Segregation in Steel Ingots. (A. Pourcel, *Trans. A. I. M. E.*, 1893.)—H. M. Howe, in his "Metallurgy of Steel," gives a *résumé* of observations, with the results of numerous analyses, bearing upon the phenomena of segregation.

A test-piece taken 24 inches from the head of an ingot 7.5 feet in length gave by analysis very different results from those of a test-piece taken 30 inches from the bottom.

| | C. | Mn. | Si. | S. | P. |
|-------------|------|-------|-------|-------|-------|
| Top..... | 0.92 | 0.535 | 0.043 | 0.161 | 0.261 |
| Bottom..... | 0.37 | 0.498 | 0.006 | 0.025 | 0.096 |

Segregation is less marked in ingots of extra-soft metal cast in cast-iron molds of considerable thickness. It is, however, still important, and explains the difference often shown by the results of tests on pieces taken from different portions of a plate. Two samples, taken from the sound part of a flat ingot, one on the outside and the other in the center, 7.9 inches from the upper edge, gave:

| | C. | S. | P. | Mn. |
|---------------|------|-------|-------|-------|
| Center..... | 0.14 | 0.053 | 0.072 | 0.576 |
| Exterior..... | 0.11 | 0.036 | 0.027 | 0.610 |

Manganese is the element most uniformly disseminated in hard or soft steel.

For cannon of large caliber, if we reject, in addition to the part cast in sand and called the *masselotte* (sinking-head), one-third of the upper part of the ingot, we can obtain a tube practically homogeneous in composition, because the central part is naturally removed by the boring of the tube. With extra-soft steels, destined for ship- or boiler-plates, the solution for practically perfect homogeneity lies in the obtaining of a metal more closely deserving its name of extra-soft metal.

The injurious consequences of segregation must be suppressed by reducing, as far as possible, the elements subject to liquation.

Segregation in Steel Plates. (C. L. Huston, *Proc. A. S. T. M.*, vi, 182.)

A plate $370 \times 76 \times \frac{5}{16}$ in. was rolled from a 16×18 -in. ingot, weighing

2800 lbs., the ladle test of which showed 0.18 C. Test pieces from the plate gave the following:

Top of Ingot:

| | | | | | |
|-----------------------|--------|--------|--------|--------|--------|
| Tensile Strength..... | 56,730 | 67,420 | 67,050 | 66,980 | 56,440 |
| Carbon..... | 0.13 | 0.25 | 0.27 | 0.25 | 0.13 |

Bottom of Ingot:

| | | | | | |
|-----------------------|--------|--------|--------|--------|--------|
| Tensile Strength..... | 56,120 | 57,720 | 58,400 | 58,140 | 56,900 |
| Carbon..... | 0.13 | 0.13 | 0.16 | 0.16 | 0.14 |
| | 1 | 2 | 3 | 4 | 5 |

Columns 1 and 5, edge of plate; 3, middle; 2 and 4, half way between middle and edge.

Other tests of low-carbon steel showed a lower degree of segregation. A plate from an ingot of 0.23 C gave minimum 0.18 C T. S., 64,580; maximum 0.38 C, T. S., 70,340. One from an ingot of 0.26 C gave maximum 0.20 C, T. S., 59,600; maximum 0.50 C, T. S., 78,600. (See also paper on this subject by H. M. Howe in vol. vii, p. 75.)

Endurance of Steel under Repeated Alternate Stresses. (J. E. Howard, A. S. T. M., 1907, p. 252.)—Small bars were rapidly rotated in a machine while being subjected to a transverse strain. Two steels gave results as follows: (1) 0.55 C, T. S., 111,200; E. L., 59,000; Elong., 12%; Red. of area, 33.5%. (2) 0.82 C, T. S., 142,000; E. L., 64,000; Elong., 7%; Red. of area, 11.8%.

| | | | | | | |
|----------------------------------|---------------------------|-------------------|--------------------|------------------------|---------|---------------------------|
| Fiber stress..... | 60,000 | 50,000 | 45,000 | 40,000 | 35,000 | 30,000 |
| No. of rotations before rupture. | {(1) 12,490 (2) 37,250 | 33,160 213,150 | 166,240 605,640 | 455,000 202,000,000 | 900,720 | 76,326,240 Not broken. |

Welding of Steel.—H. H. Campbell (*Manuf. of Iron and Steel*, p. 402) had numerous bars of steel welded by different skilled blacksmiths. The record of results, he says, "is extremely unsatisfactory." The worst weld by each of four workmen showed respectively 70, 54, 58, and 44% of the strength of the original bar. Forging steel showed one weld with only 48%, common soft steel 44%, and pure basic steel 59%. In a series of tests by the Royal Prussian Testing Institute, the average strength of welded bars of medium steel was 58% of the natural, the poorest bar showing only 23%. In softer steel the average was 71%, and the poorest 33%, while in puddled iron the average was 81% and the poorest 62%. Mr. Campbell concludes: "A weld as performed by ordinary blacksmiths, whether on iron or steel, is not nearly as good as the rest of the bar; and it is still more certain that welds of large rods of common forging steel are unreliable and should not be employed in structural work. Electric methods do not offer a solution of the problem, for the metal is heated beyond the critical temperature of crystallization, and only by heavy reductions under the hammer or press can much be done towards restoring the ductility of the piece."

Welding of Steel.—A. E. Hunt (A. I. M. E., 1892) says: "I have never seen so-called 'welded' pieces of steel pulled apart in a testing-machine or otherwise broken at the joint which have not shown a smooth cleavage plane, as it were, such as in iron would be condemned as an imperfect weld. My experience in this matter leads me to agree with the position taken by Mr. William Metcalf in his paper upon Steel in the *Trans. A. S. C. E.*, vol. xvi, p. 301. Mr. Metcalf says, 'I do not believe steel can be welded.'"

The Thermit Welding Process. (Goldschmidt Thermit Co., New York.)—When powdered or finely divided aluminum is mixed with a metallic oxide and ignited, the aluminum burns with great rapidity and intense heat, reducing the oxide to a metal and fusing it. It is said that iron oxide and aluminum will make a temperature of 5400° F., producing fused iron which will melt any iron or steel with which it comes in contact. The process is largely used for repairing breaks of large castings or forgings, such as the stern post of a steamship, a locomotive frame, etc. In the operation of welding a large fractured piece, the fracture is drilled out with a series of 3/4-in. holes close together, making a clear opening. A mold of fire-clay and sand is then made to fit all around the fracture, leaving a collar or ring surrounding it, baked in a furnace

and then placed in position. The fractured section is then heated by a blow-torch inserted in the riser of the mold. A conical sheet iron crucible, lined with magnesia tar, is then inserted in the riser, and thermit (the mixture of aluminum and oxide of iron) poured into it. An ignition powder is placed on top of the thermit, and lighted with a storm match. The mixture begins to burn with great agitation; when this ceases the crucible is tapped, and white-hot fused iron or steel runs into the mold and thoroughly fuses with the pieces to be joined.

Oxy-acetylene Welding and Cutting of Metals. — Autogenous Welding. — By means of acetylene gas and oxygen, stored in tanks under pressure, and a properly constructed nozzle or torch in which the two gases are united and fired, an intense temperature said to be 6000° F., is generated, and it may be used to weld or fuse together iron, steel, aluminum, brass, copper, or other metals. The process of uniting metals by heat without using either flux or compression is called autogenous welding. The oxy-acetylene torch may also be used for cutting metals, such as steel plates, beams and large forgings, and for repairing flaws or defects, or filling cavities by melting a strip of metal and flowing it into place. The apparatus, with instruction in its use, is furnished by the Davis-Bournonville Co., Jersey City, N. J.

Electric Welding. — For description see Electrical Engineering.

Hydraulic Forging. — In the production of heavy forgings from cast ingots of mild steel it is essential that the mass of metal should be operated on as equally as possible throughout its entire thickness. When employing a steam-hammer for this purpose it has been found that the external surface of the ingot absorbs a large proportion of the sudden impact of the blow, and that a comparatively small effect only is produced on the central portions of the ingot, owing to the resistance offered by the inertia of the mass to the rapid motion of the falling hammer — a disadvantage that is entirely overcome by the slow, though powerful, compression of the hydraulic forging-press, which appears destined to supersede the steam-hammer for the production of massive steel forgings.

Fluid-compressed Steel by the "Whitworth Process." (*Proc. Inst. M. E.*, May, 1887, p. 167.) — In this system a gradually increasing pressure up to 6 or 8 tons per square inch is applied to the fluid ingot, and within half an hour or less after the application of the pressure the column of fluid steel is shortened 1 1/2 inches per foot or one-eighth of its length; the pressure is then kept on for several hours, the result being that the metal is compressed into a perfectly solid and homogeneous material, free from blow-holes.

In large gun-ring ingots during cooling the carbon is driven to the center, the center containing 0.8 carbon and the outer ring 0.3. The center is bored out until a test shows that the inside of the ring contains the same percentage of carbon as the outside.

Fluid-compressed steel is made by the Bethlehem Steel Co. for gun and other heavy forgings.

Putting sufficient pressure upon the outside of the ingot when the walls are solid but the interior is still liquid will prevent the formation of a pipe. In Whitworth's system the ingot is raised and compressed lengthwise against a solid ram situated above it, during and shortly after solidification. In Harmet's method the ingot is forced upward during solidification into its tapered mold. This causes a large radial pressure on its sides. In Lilienberg's method the ingots are stripped and then run on their ears between a solid and movable wall. The movable wall is then pressed against one side of the ingots. (*Stoughton's Metallurgy of Iron and Steel*.)

For other methods of compressing ingots see paper by A. J. Capron in *Jour. I. & S. I.*, 1906, *Iron Tr. Rev.*, May 24, 1906.

STEEL CASTINGS.

(E. S. Cramp, *Proc. Eng'g Congress*, Dept. of Marine Eng'g, Chicago, 1893.)

In 1891 American steel-founders had successfully produced a considerable variety of heavy and difficult castings, of which the following are the most noteworthy specimens:

Bed-plates up to 24,000 lbs.; stern-posts up to 54,000 lbs.; stems up to 21,000 lbs.; hydraulic cylinders up to 11,000 lbs.; shaft-struts up to 32,000 lbs.; hawse-pipes up to 7500 lbs.; stern-pipes up to 8000 lbs.

The percentage of success in these classes of castings since 1890 has ranged from 65% in the more difficult forms to 90% in the simpler ones; the tensile strength has been from 62,000 to 78,000 lbs., elongation from 15% to 25%.

The first steel castings of which anything is generally known were crossing-frogs made for the Philadelphia & Reading R. R. in July, 1867, by the William Butcher Steel Works, now the Midvale Steel Co. The molds were made of a mixture of ground fire-brick, black-lead crucible-pots ground fine, and fire-clay, and washed with a black-lead wash. The steel was melted in crucibles, and was about as hard as tool steel. The surface of these castings was very smooth, but the interior was very much honey-combed. This was before the days when the use of silicon was known for solidifying steel. The sponginess, which was almost universal, was a great obstacle to their general adoption.

The next step was to leave the ground pots out of the molding mixture and to wash the mold with finely ground fire-brick. This was a great improvement, especially in very heavy castings; but this mixture still clung so strongly to the casting that only comparatively simple shapes could be made with certainty. A mold made of such a mixture became almost as hard as fire-brick, and was such an obstacle to the proper shrinkage of castings that, when at all complicated in shape, they had so great a tendency to crack as to make their successful manufacture almost impossible. By this time the use of silicon had been discovered, and the only obstacle in the way of making good castings was a suitable molding mixture. This was ultimately found in mixtures having the various kinds of silica sand as the principal constituent.

One of the most fertile sources of defects in castings is a bad design. Very intricate shapes can be cast successfully if they are so designed as to cool uniformly. Mr. Cramp says while he is not yet prepared to state that anything that can be cast successfully in iron can be cast in steel, indications seem to point that way in all cases where it is possible to put on suitable sinking-heads for feeding the casting.

H. L. Gantt (*Trans. A. S. M. E.*, xii, 710) says: Steel castings not only shrink much more than iron ones, but with less regularity. The amount of shrinkage varies with the composition and the heat of the metal; the hotter the metal the greater the shrinkage; and, as we get smoother castings from hot metal, it is better to make allowance for large shrinkage and pour the metal as hot as possible. Allow $\frac{3}{16}$ or $\frac{1}{4}$ in. per ft. in length for shrinkage, and $\frac{1}{4}$ in. for finish on machined surfaces, except such as are cast "up." Cope surfaces which are to be machined should, in large or hard castings, have an allowance of from $\frac{3}{8}$ to $\frac{1}{2}$ in. for finish, as a large mass of metal slowly rising in a mold is apt to become crusty on the surface, and such a crust is sure to be full of imperfections. On small, soft castings $\frac{1}{8}$ in. on drag side and $\frac{1}{4}$ in. on cope side will be sufficient. No core should have less than $\frac{1}{4}$ in. finish on a side and very large ones should have as much as $\frac{1}{2}$ in. on a side. Blow-holes can be entirely prevented in castings by the addition of manganese and silicon in sufficient quantities; but both of these cause brittleness, and it is the object of the conscientious steel-maker to put no more manganese and silicon in his steel than is just sufficient to make it solid. The best results are arrived at when all portions of the castings are of a uniform thickness, or very nearly so.

The following table will illustrate the effect of annealing on tensile strength and elongation of steel castings:

| Carbon. | Tensile Strength. | | Elongation. | |
|---------|-------------------|-----------|-------------|-----------|
| | Unannealed. | Annealed. | Unannealed. | Annealed. |
| 0.23% | 68,738 | 67,210 | 22.40% | 31.40% |
| 0.37 | 85,540 | 82,228 | 8.20 | 21.80 |
| 0.53 | 90,121 | 106,415 | 2.35 | 9.80 |

The proper annealing of large castings takes nearly a week.

The proper steel for roll pinions, hammer dies, etc., seems to be that containing about 0.60% of carbon. Such castings, properly annealed, have worn well and seldom broken. Miscellaneous gearing should contain

carbon 0.40% to 0.60%, gears larger in diameter being softest. General machinery castings should, as a rule, contain less than 0.40% of carbon, those exposed to great shocks containing as low as 0.20% of carbon. Such castings will give a tensile strength of from 60,000 to 80,000 lbs. per sq. in. and at least 15% extension in 2 in. Machinery and hull castings for war-vessels for the United States Navy, as well as carriages for naval guns, contain from 0.20% to 0.30% of carbon.

For description of methods of manufacture of steel castings by the Bessemer, open-hearth, and crucible processes, see paper by P. G. Salom, *Trans. A. I. M. E.*, xiv. 118.

CRUCIBLE STEEL.

Selection of Grades by the Eye, and Effect of Heat Treatment. (J. W. Langley, *Amer. Chemist*, Nov., 1876.) — In the early days of steel making the grades were determined by inspection of the fractured surfaces of the cast ingots. The method of selection is described as follows:

The steel when thoroughly fluid is poured into cast-iron molds, and when cold the top of the ingot is broken off, exposing a freshly fractured surface. The appearance presented is that of confused groups of crystals, all appearing to have started from the outside and to have met in the center; this general form is common to all ingots of whatever composition, but to the trained eye, and only to one long and critically exercised, a minute but indescribable difference is perceived between varying samples of steel, and this difference is now known to be owing almost wholly to variations in the amount of combined carbon, as the following table will show. Twelve samples selected by the eye alone, and analyses of drillings taken direct from the ingot before it had been heated or hammered, gave results as below:

| Ingot Nos. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|------------|-------|-------|------|------|------|------|------|------|------|-------|-------|-------|
| C | 0.302 | .490 | .529 | .649 | .801 | .841 | .867 | .871 | .955 | 1.005 | 1.058 | 1.079 |
| Diff. of C | | 0.188 | .039 | .120 | .152 | .040 | .026 | .004 | .084 | .050 | .053 | .021 |

The C is seen to increase in quantity in the order of the numbers. The other elements, with the exception of total iron, bear no relation to the number on the samples. The mean difference of C is 0.071.

In mild steels the discrimination is less perfect.

The appearance of the fracture by which the above twelve selections were made can only be seen in the cold ingot before any operation, except the original one of casting, has been performed upon it. As soon as it is hammered, the structure changes, so that all trace of the primitive condition appears to be lost.

The specific gravity of steel is influenced not only by its chemical analysis but by the heat to which it is subjected.

The sp. gr. of the ingots in the above list ranged from 7.855 for No. 1 down to 7.803 for No. 12. Rolling into bars produced a very slight difference, — 0.005 in Nos. 5 and 6 and +0.020 in No. 12, but overheating reduced the sp. gr. of the bar 0.023 in No. 3 to 0.135 in No. 12, the sp. gr. of the burnt sample of No. 12 being only 7.690.

Effect of Heat on the Grain of Steel. (W. Metcalf, — *Jeans on Steel*, p. 642.) — A simple experiment will show the alteration produced in a high-carbon steel by different methods of hardening. If a bar of such steel be nicked at about 9 or 10 places, and about half an inch apart, a suitable specimen is obtained for the experiment. Place one end of the bar in a good fire, so that the first nicked piece is heated to whiteness, while the rest of the bar, being out of the fire, is heated up less and less as we approach the other end. As soon as the first piece is at a good white heat, which of course burns a high-carbon steel, and the temperature of the rest of the bar gradually passes down to a very dull red, the metal should be taken out of the fire and suddenly plunged in cold water, in which it should be left till quite cold. It should then be taken out and carefully dried. An examination with a file will show that the first piece has the greatest hardness, while the last piece is the softest, the intermediate pieces gradually passing from one condition to the other. On now breaking off the pieces at each nick it will be seen that very considerable and characteristic changes have been produced in the appearance of the metal. The first burnt piece is very open or crystalline in fracture; the succeeding pieces become closer and closer in the grain until one piece is found to possess that perfectly even grain and velvet-like appearance

which is so much prized by experienced steel users. The first pieces also, which have been too much hardened, will probably be cracked; those at the other end will not be hardened through. Hence if it be desired to make the steel hard and strong, the temperature used must be high enough to harden the metal through, but not sufficient to open the grain.

Heating Tool Steel. (Crescent Steel Co., Pittsburg, Pa.) — There are three distinct stages or times of heating: First, for forging; second, for hardening; third, for tempering.

The first requisite for a good heat for forging is a clean fire and plenty of fuel, so that jets of hot air will not strike the corners of the piece; next, the fire should be regular, and give a good uniform heat to the whole part to be forged. It should be keen enough to heat the piece as rapidly as may be, and allow it to be thoroughly heated through, without being so fierce as to overheat the corners.

Steel should not be left in the fire any longer than is necessary to heat it clear through, as "soaking" in fire is very injurious; and, on the other hand, it is necessary that it should be hot through, to prevent surface cracks.

By observing these precautions a piece of steel may always be heated safely, up to even a bright yellow heat, when there is much forging to be done on it.

The best and most economical of welding fluxes is clean, crude borax, which should be first thoroughly melted and then ground to fine powder.

After the steel is properly heated, it should be forged to shape as quickly as possible; and just as the red heat is leaving the parts intended for cutting edges, these parts should be refined by rapid, light blows, continued until the red disappears.

For the second stage of heating, for hardening, great care should be used: first, to protect the cutting edges and working parts from heating more rapidly than the body of the piece; next, that the whole part to be hardened be heated uniformly through, without any part becoming visibly hotter than the other. A uniform heat, as low as will give the required hardness, is the best for hardening.

For every variation of heat which is great enough to be seen there will result a variation in grain, which may be seen by breaking the piece; and for every such variation in temperature there is a very good chance for a crack to be seen. Many a costly tool is ruined by inattention to this point.

The effect of too high heat is to open the grain; to make the steel coarse. The effect of an irregular heat is to cause irregular grain, irregular strains, and cracks.

As soon as the piece is properly heated for hardening, it should be promptly and thoroughly quenched in plenty of the cooling medium, water, brine, or oil, as the case may be.

An abundance of the cooling bath, to do the work quickly and uniformly all over, is very necessary to good and safe work.

To harden a large piece safely a running stream should be used.

Much uneven hardening is caused by the use of too small baths.

For the third stage of heating, to temper, the first important requisite is again uniformity. The next is time; the more slowly a piece is brought down to its temper, the better and safer is the operation.

When expensive tools are to be made it is a wise precaution to try small pieces of the steel at different temperatures, so as to find out how low a heat will give the necessary hardness. The lowest heat is the best for any steel. [This is true of carbon steel but not of "high speed" alloy steels.]

Heating in a Lead Bath. — A good method of heating steel to a uniform temperature is by means of a bath of lead kept at a red heat by a gas furnace. See Heat Treatment by the Taylor-White Process, under Machine Shop.

Heating Steel in Melted Salts by Electric Current. — L. M. Cohn (*Electrol. Z.*, Aug., 1906, *Mach'g*, Dec., 1906) describes a furnace patented by Gebr. Körting, Berlin, in which steel may be heated uniformly to any desired temperature up to 1300° C. (2372° F.) without danger of oxidizing.

The furnace consists mainly of a cast-iron box, lined inside with fire-clay, a second lining of fire-bricks, lined again with asbestos, and inclosing the crucible made of one piece of fireproof material. Two electrodes lead into the crucible, through which alternating current is sent. The crucible is filled with metal salts. For temperatures above

1000° C. pure chloride of barium is used, the melting-point of which is at about 950° C. (1742 F.); for lower temperatures a mixture of chloride of barium and chloride of potassium, 2 to 1, is used, melting at about 670° C. (1238 F.). Any other suitable salts may be used. A special regulating transformer serves to regulate the current, and thus also the temperature.

A test was made with a furnace, the bath of which was $6\frac{1}{2} \times 6\frac{1}{2} \times 7$ in. A 50-period alternating current of 190-volt primary tension was used. This tension had to be reduced to from 50 to 55 volts by the regulating transformer for starting the furnace, and lowered later on. The heating lasted about half an hour. For temperatures from 750 to 1300° C., the secondary tension amounted to from 13 to 18 volts. The consumption of energy was as follows: 880° C., 5.4 Kw.; 1140° C., 8.5 Kw.; 1300° C., 12.25 Kw.

A milling cutter 5 in. diameter, $1\frac{1}{4}$ in. bore, 1 in. thick, was heated in 62 seconds to 1300° C. A bushing of tool steel $2\frac{3}{4}$ in. diam., $2\frac{3}{4}$ in. long, $\frac{5}{8}$ in. bore, was heated in 243 seconds to 850° C.

Heating to Forge. (Crescent Steel Co.) — The trouble in the forge fire is usually uneven heat, and not too high heat. Suppose the piece to be forged has been put into a very hot fire, and forced as quickly as possible to a high yellow heat, so that it is almost up to the scintillating point. If this be done, in a few minutes the outside will be quite soft and in a nice condition for forging, while the middle parts will not be more than red-hot. Now let the piece be placed under the hammer and forged, and the soft outside will yield so much more readily than the hard inside, that the outer particles will be torn asunder, while the inside will remain sound.

Suppose the case to be reversed and the inside to be much hotter than the outside; that is, that the inside shall be in a state of semi-fusion, while the outside is hard and firm. Now let the piece be forged, and the outside will be all sound and the whole piece will appear perfectly good until it is cropped, and then it is found to be hollow inside.

In either case, if the piece had been heated soft all through, or if it had been only red-hot all through, it would have forged perfectly sound.

In some cases a high heat is more desirable to save heavy labor, but in every case where a fine steel is to be used for cutting purposes it must be borne in mind that very heavy forging refines the bars as they slowly cool, and if the smith heats such refined bars until they are soft, he raises the grain, makes them coarse, and he cannot get them fine again unless he has a very heavy steam-hammer at command and knows how to use it well.

Annealing. (Crescent Steel Co.) — Annealing or softening is accomplished by heating steel to a red heat and then cooling it very slowly, to prevent it from getting hard again.

The higher the degree of heat, the more will steel be softened, until the limit of softness is reached, when the steel is melted.

It does not follow that the higher a piece of steel is heated the softer it will be when cooled, no matter how slowly it may be cooled; this is proved by the fact that an ingot is always harder than a rolled or hammered bar made from it.

Therefore there is nothing gained by heating a piece of steel hotter than a good, bright, cherry-red; on the contrary, a higher heat has several disadvantages: First. If carried too far, it may leave the steel actually harder than a good red heat would leave it. Second. If a scale is raised on the steel, this scale will be harsh, granular oxide of iron, and will spoil the tools used to cut it. Third. A high scaling heat continued for a little time changes the structure of the steel, makes it brittle, liable to crack in hardening, and impossible to refine.

To anneal any piece of steel, heat it red-hot; heat it uniformly and heat it through, taking care not to let the ends and corners get too hot.

As soon as it is hot, take it out of the fire, the sooner the better, and cool it as slowly as possible. A good rule for heating is to heat it at so low a red that when the piece is cold it will still show the blue gloss of the oxide that was put there by the hammer or the rolls.

Steel annealed in this way will cut very soft: it will harden very hard, without cracking; and when tempered it will be very strong, nicely refined, and will hold a keen, strong edge.

Tempering. — Tempering steel is the act of giving it, after it has been shaped, the hardness necessary for the work it has to do. This is done by

first hardening the piece, generally a good deal harder than is necessary, and then toughening it by slow heating and gradual softening until it is just right for work.

A piece of steel properly tempered should always be finer in grain than the bar from which it is made. If it is necessary, in order to make the piece as hard as is required, to heat it so hot that after being hardened the grain will be as coarse as or coarser than the grain in the original bar, then the steel itself is of too low carbon for the desired work.

If a great degree of hardness is not desired, as in the case of taps and most tools of complicated form, and it is found that at a moderate heat the tools are too hard and are liable to crack, the smith should first use a lower heat in order to save the tools already made, and then notify the steel-maker that his steel is too high, so as to prevent a recurrence of the trouble.

For descriptions of various methods of tempering steel, see "Tempering of Metals," by Joshua Rose, in App. Cyc. Mech., vol. ii, p. 863; also, "Wrinkles and Recipes," from the *Scientific American*. In both of these works Mr. Rose gives a "color scale," lithographed in colors, by which the following is a list of the tools in their order on the color scale, together with the approximate color and the temperature at which the color appears on brightened steel when heated in the air:

| | |
|--|---|
| Scrapers for brass; <i>very pale yellow</i> , 430° F. | Hand-plane irons. |
| Steel-engraving tools. | Twist-drills. |
| Slight turning tools. | Flat drills for brass. |
| Hammer faces. | Wood-boring cutters. |
| Planer tools for steel. | Drifts. |
| Ivory-cutting tools. | Coopers' tools. |
| Planer tools for iron. | Edging cutters; <i>light purple</i> , 530° F. |
| Paper-cutters. | Augers. |
| Wood-engraving tools. | Dental and surgical instruments. |
| Bone-cutting tools. | Cold chisels for steel. |
| Milling-cutters; <i>straw yellow</i> , 460° F. | Axes; <i>dark purple</i> , 550° F. |
| Wire-drawing dies. | Gimlets. |
| Boring-cutters. | Cold chisels for cast iron. |
| Leather-cutting dies. | Saws for bone and ivory. |
| Screw-cutting dies. | Needles. |
| Inserted saw-teeth. | Firmer-chisels. |
| Taps. | Hack-saws. |
| Rock-drills. | Framing-chisels. |
| Chasers. | Cold chisels for wrought iron. |
| Punches and dies. | Molding and planing cutters to be filed. |
| Penknives. | Circular saws for metal. |
| Reamers. | Screw-drivers. |
| Half-round bits. | Springs. |
| Planing and molding cutters. | Saws for wood. |
| Stone-cutting tools; <i>brown yellow</i> , 500° F. | <i>Dark blue</i> , 570° F. |
| Gauges. | <i>Pale blue</i> , 610°. |
| | <i>Blue tinged with green</i> , 630°. |

Uses of Crucible Steel of Different Carbons. (Metcalf on Steel.) —

- 0.50 to 0.60 C, for hot work and for battering tools.
- 0.60 to 0.70 C, ditto, and for tools of dull edge.
- 0.70 to 0.80 C, battering tools, cold-sets, and some forms of reamers and taps.
- 0.80 to 0.90 C, cold-sets, hand-chisels, drills, taps, reamers and dies.
- 0.90 to 1.00 C, chisels, drills, dies, axes, knives, etc.
- 1.00 to 1.10 C, axes, hatchets, knives, large lathe-tools, and many kinds of dies and drills if care be used in tempering them.
- 1.10 to 1.50 C, lathe-tools, graving tools, scribes, scrapers, little drills, and many similar purposes.

The best all-around tool steel is found between 0.90 and 1.10 C; steel that can be adapted safely and successfully to more uses than any other.

High-speed Tool Steel. (A. L. Valentine, *Am. Mach.*, July 2, 1908.) — Eight brands of high-speed steel were analyzed with the following results:

| Steel. | C. | W. | Cr. | Mn. | Si. | Mo. | P. | S. |
|--------|------|-------|-------|-------|-------|-------|-------|-------|
| a | 0.70 | 14.91 | 2.95 | 0.01 | | | 0.013 | 0.008 |
| b | 0.25 | 17.27 | 2.69 | Trace | 0.179 | | 0.035 | Trace |
| c | 0.75 | 14.83 | 2.90 | 0.08 | | 5.19 | 0.02 | 0.01 |
| d | 0.49 | 17.60 | 5.11 | | | | 0.01 | 0.007 |
| e | 0.65 | | | 0.19 | 0.039 | 9.60 | 0.016 | 0.005 |
| f | 0.60 | 13.00 | 2.88 | | | | 0.019 | 0.01 |
| g | 0.55 | 17.81 | 2.48 | 0.11 | 0.090 | | | |
| h | 0.66 | 19.03 | | | 0.036 | | 0.015 | |

W, Wolfram, symbol for tungsten.

Where blanks appear in the table, the steel was not analyzed for these ingredients.

Many different brands of high-speed steel are being made. Some that have been marketed are almost worthless. From some of these steels a tool can be made from one end of a bar that is easily forged, machined and hardened, while the other end of the bar would resist almost any cutting tool and would invariably crack in hardening. Different bars of the same make also give very different results. These faults are sometimes caused by non-uniform annealing in the steels which are sent out as thoroughly annealed, and in many cases they are caused by the use of impure ingredients. A good high-speed steel will stand a temperature as high as 1200° F., or over double that of carbon steel, without losing its hardness, and experience has proven that the higher the temperature is raised over the white-heat point, the higher a temperature caused by friction the tool will withstand, before losing its intense hardness. The higher the percentage of carbon is, the more brittle and hard to work the steel will be, especially to forge. The steel which has given the best all-around results has contained about 0.40 C. The analysis of this same steel showed nearly 3% of chromium. The higher the percentage of tungsten in the steel, the better has been its cutting qualities. (See Best High-Speed Tool Steel, and description of the Taylor-White process of heat treatment, under "The Machine-Shop.")

MANGANESE, NICKEL, AND OTHER "ALLOY" STEELS.

Manganese Steel. (H. M. Howe, *Trans. A. S. M. E.*, vol. xii.) — Manganese steel is an alloy of iron and manganese, incidentally, and probably unavoidably, containing a considerable proportion of carbon.

The effect of small proportions of manganese on the hardness, strength, and ductility of iron is probably slight. The point at which manganese begins to have a predominant effect is not known; it may be somewhere about 2.5%.

Manganese steel is very free from blow-holes; it welds with great difficulty; its toughness is increased by quenching from a yellow heat; its electric resistance is enormous, and very constant with changing temperature; it is low in thermal conductivity. Its remarkable combination of great hardness, which cannot be materially lessened by annealing, and great tensile strength, with astonishing toughness and ductility, at once creates and limits its usefulness.

The hardness of manganese steel seems to be of an anomalous kind. The alloy is hard, but under some conditions not rigid. It is very hard in its resistance to abrasion; it is not always hard in its resistance to impact.

Manganese steel forges readily at a yellow heat, though at a bright white heat it crumbles under the hammer. But it offers greater resistance to deformation, i.e., it is harder when hot, than carbon steel.

The most important single use for manganese steel is for the pins which hold the buckets of elevator dredges. Here abrasion chiefly is to be resisted. Another important use is for the links of common chain-elevators. As a material for stamp-shoes, for horse-shoes, for the knuckles of an automatic car-coupler, it has not met expectations.

Manganese steel has been regularly adopted for the blades of the Cyclone pulverizer. Some manganese-steel wheels are reported to have run over 300,000 miles each without turning, on a New England railroad.

Manganese Steel and its Uses. (E. F. Lake, *Am. Mach.*, May 16, 1907.) — When more than 2% and less than 6% of Mn is added, with C less than 0.5%, it makes steel very brittle, so that it can be powdered under a hand hammer. From 6% Mn up, this brittleness gradually disappears until 12% is reached, when the former strength returns and reaches its maximum at 15%. After this, a decrease in toughness, but not in transverse strength, takes place until 20% is reached, after which a rapid decrease in strength again takes place.

Steel with from 12 to 15% Mn and less than 0.5% of C is very hard and cannot be machined or drilled in the ordinary way; yet it is so tough that it can be twisted and bent into peculiar shapes without breaking. It is malleable enough to be used for rivets that are to be headed cold.

This hardness, toughness and malleability make manganese steel the most durable metal known, in its ability to resist wear, for such parts as the teeth on steam-shovel dippers, where they will outwear about three teeth made of the best tool steel; for plow points on road-building work; for frogs, switches and crossings in railroad construction; for fluted or toothed crushing rolls used on ore, coal and stone crushers; for screen shells to screen these crushings; gears, sprockets, link belts, etc., when used in the vicinity of ore, stone and coal crushers or other places where they are subjected to the hard, grinding wear of the gritty particles of dust with which they are usually covered.

The higher the percentage of C in the steel, the less percentage of Mn will be required to produce brittleness. Si, however, neutralizes the injurious tendencies of Mn, and in Europe the Si-Mn alloy is used for automobile springs and gears. This steel is not high in Mn and can be rolled, while the peculiar properties given to steel by the addition of from 12 to 15% of manganese make such steel impossible to roll; therefore all parts made of this steel have to be cast, after which it can be forged and rendered tougher by quenching from a white heat.

One of its peculiarities is that it is softened by rapid cooling and can be restored to its former hardness by heating to a bright red.

It is more difficult to mold in the foundry than the ordinary cast steel, as it must be poured at a very high temperature, and in cooling it shrinks nearly twice as much. The shrinkage allowed for patterns to be cast of the ordinary cast steel is $\frac{3}{16}$ in. per foot, and for manganese-steel castings $\frac{5}{16}$ in. per foot.

This enormous shrinkage makes it impossible to cast in any intricate or delicate shapes, and as it is too hard to machine or drill successfully, all holes must be cored in the casting. If a close fit is desired in these they must be ground out with an emery wheel. These properties limit its use to a large extent.

The composition that seems to give the best results is:

Mn, from 12 to 15%; C, not over 0.5%; P, not over 0.04%; S, not over 0.04%.

Manganese-steel castings should be annealed in order to remove any internal strains which may be caused by its high shrinkage and the fact that the outer surface cools so much quicker than the core, which leaves the center of the casting strained. This can be done by heating to 1500° F. and quenching in water, after which it can be hardened by heating to 900° and allowed to cool slowly.

Manganese-steel castings, when tested in a $\frac{7}{8}$ -inch round bar, should show:

T. S. per sq. in., not less than 140,000 lbs.; E. L., not less than 90,000 lbs.; Red. of area, not less than 50%; Elong. in 2 in., not less than 20%.

Chrome Steel. (F. L. Garrison, *Jour. F. I.*, Sept., 1891.) — Chromium increases the hardness of iron, perhaps also the tensile strength and elastic limit, but it lessens its weldability.

Chromium does not appear to give steel the power of becoming harder when quenched or chilled. Howe states that chrome steels forge more readily than tungsten steels, and when not containing over 0.5 of chromium nearly as well as ordinary carbon steels of like percentage of carbon. On the whole the status of chrome steel is not satisfactory. There are other steel alloys coming into use, which are so much better, that it would seem to be only a question of time when it will drop entirely out of the race. Howe states that many experienced chemists have found no chromium, or but the merest traces, in chrome steel sold in the markets.

J. W. Langley) *Trans. A. S. C. E.*, 1892) says: Chromium, like manganese,

is a true hardener of iron even in the absence of carbon. The addition of 1% or 2% of chromium to a carbon steel will make a metal which gets excessively hard. Hitherto its principal employment has been in the production of chilled shot and shell. Powerful molecular stresses result during cooling, and the shells frequently break spontaneously months after they are made.

Tungsten Steel — Mushet Steel. (J. B. Nau, *Iron Age*, Feb. 11, 1892.) — By incorporating simultaneously carbon and tungsten in iron, it is possible to obtain a much harder steel than with carbon alone, without danger of an extraordinary brittleness in the cold metal or an increased difficulty in the working of the heated metal.

When a special grade of hardness is required, it is frequently the custom to use a high tungsten steel, known in England as special steel. A specimen from Sheffield, used for chisels, contained 9.3% of tungsten, 0.7% of silver, and 0.6% of carbon. This steel, though used with advantage in its untempered state to turn chilled rolls, was not brittle; nevertheless it was hard enough to scratch glass.

A sample of Mushet's special steel contained 8.3% of tungsten and 1.73% of manganese.

According to analyses made by the Duc de Luynes of ten specimens of the celebrated Oriental damasked steel, eight contained tungsten, two of them in notable quantities (0.518% to 1%), while in all of the samples analyzed nickel was discovered ranging from traces to nearly 4%.

Stein & Schwartz, of Philadelphia, in a circular say: It is stated that tungsten steel is suitable for the manufacture of steel magnets, since it retains its magnetism longer than ordinary steel. Cast steel to which tungsten has been added needs a higher temperature for tempering than ordinary steel, and should be hardened only between yellow, red, and white. Chisels made of tungsten steel should be drawn between cherry-red and blue, and stand well on iron and steel. Tempering is best done in a mixture of 5 parts of yellow rosin, 3 parts of tar, and 2 parts of tallow, and then the article is once more heated and then tempered as usual in water of about 15° C.

Aluminum Steel. — R. A. Hadfield (*Trans. A. I. M. E.*, 1890) says: Aluminum appears to be of service as an addition to baths of molten iron or steel unduly saturated with oxides, and these in properly regulated steel manufacture should not often occur. Speaking generally, its rôle appears to be similar to that of silicon. The statement that aluminum lowers the melting-point of iron seems to have no foundation in fact. If any increase of heat or fluidity takes place by the addition of small amounts of aluminum, it may be due to evolution of heat from oxidation of the aluminum, as the calorific value of this metal is very high — in fact, higher than silicon. According to Berthollet, the conversion of aluminum to Al_2O_3 equals 7900 cal.; silicon to SiO_2 is stated as 7800.

The action of aluminum may be classed along with that of silicon, sulphur, phosphorus, arsenic, and copper, as giving no increase of hardness to iron, in contradistinction to carbon, manganese, chromium, tungsten, and nickel. Its special advantage seems to be that it combines in itself the advantages of both silicon and manganese; but so long as alloys containing these metals are so cheap and aluminum dear, its extensive use seems hardly probable.

J. E. Stead, in discussion of Mr. Hadfield's paper, said: Every one of our trials has indicated that aluminum can kill the most fiery steel, providing, of course, that it is added in sufficient quantity to combine with all the oxygen which the steel contains. The metal will then be absolutely dead, and will pour like dead-melted silicon steel. If the aluminum is added as metallic aluminum, and not as a compound, and if the addition is made just before the steel is cast, 0.1% is ample to obtain perfect solidity in the steel.

Nickel Steel. — The remarkable tensile strength and ductility of nickel steel, as shown by the test-bars and the behavior of nickel-steel armor-plate under shot tests, are witness of the valuable qualities conferred upon steel by the addition of a few per cent of nickel.

Nickel steel has shown itself to be possessed of some exceedingly valuable properties; these are, resistance to cracking, high elastic limit, and homogeneity. Resistance to cracking, a property to which the name of non-fissibility has been given, is shown more remarkably as the percentage of nickel increases. Bars of 27% nickel illustrate this property. A 1 1/4-in. square

bar was nicked $\frac{1}{4}$ in. deep and bent double on itself without further fracture than the splintering off, as it were, of the nicked portion. Sudden failure or rupture of this steel would be impossible; it seems to possess the toughness of rawhide with the strength of steel. With this percentage of nickel the steel is practically non-corrodible and non-magnetic. The resistance to cracking shown by the lower percentages of nickel is best illustrated in the many trials of nickel-steel armor.

In such places (shafts, axles, etc.) where failure is the result of the fatigue of the metal this higher elastic limit of nickel steel will tend to prolong indefinitely the life of the piece, and at the same time, through its superior toughness, offer greater resistance to the sudden strains of shock.

Howe states that the hardness of nickel steel depends on the proportion of nickel and carbon jointly, nickel up to a certain percentage increasing the hardness, beyond this lessening it. Thus while steel with 2% of nickel and 0.90% of carbon cannot be machined, with less than 5% nickel it can be worked cold readily, provided the proportion of carbon be low. As the proportion of nickel rises higher, cold-working becomes less easy. It forges easily whether it contain much or little nickel.

The presence of manganese in nickel steel is most important, as it appears that without the aid of manganese in proper proportions the conditions of treatment would not be successful.

Properties of Nickel Steel. — D. H. Browne, in *Proc. A. I. M. E.*, 1899, gives a paper of 79 pages, entitled "Nickel Steel: a synopsis of experiment and opinion," including a bibliography containing 50 titles. Some extracts from this paper are here given.

Commercially pure nickel, containing 98.13 Ni, 1.15 Co, 0.43 Fe, 0.08 Si, 0.11 Mn, showed the following physical properties:

| | L. P.* | E. L. | T. S. | M. E.† | El., % in 2 in. |
|-------------------|--------|--------|--------|------------|--------------------|
| Cast bars..... | 5,119 | 12,557 | 40,669 | 23,989,140 | 18.2 |
| Rolled { Raw..... | 9,243 | 21,045 | 72,522 | 29,506,500 | 43.9 |
| { Annealed..... | 17,064 | 18,059 | 72,806 | 26,870,800 | 48.6 |
| { Quenched..... | | 16,921 | 71,860 | | 45.0 |

* Limit of Proportionality.

† Modulus of Elasticity.

ANNEALED CAST BARS OF NICKEL STEEL WITH C 0.15 to 0.20. (Hadfield.) — The proportion of Ni used in soft steels for armor and for engine-forgings is from 3 to 3.5%. With 0.25 C this produces an E. L. and T. S. equal to open-hearth steel of 0.45 C without Ni, with a ductility equal to that of the lower-carbon steel.

NICKEL STEEL, 3.25 Ni, AND SIMPLE STEEL FORGINGS COMPARED. (Bethlehem Steel Co.)

| C. | Ni. | T. S. | E. L. | El., % | Red. Area, % | C. | Ni. | T. S. | E. L. | El., % | Red. Area, % |
|------|-----|-------|-------|--------|--------------|------|-----|--------|-------|--------|--------------|
| 0.20 | 0 | 55000 | 28000 | 34 | 60 | 0.20 | 3.5 | 85000 | 48000 | 26 | 55 |
| 0.30 | 0 | 75000 | 37000 | 30 | 50 | 0.30 | 3.5 | 95000 | 60000 | 22 | 48 |
| 0.40 | 0 | 85000 | 43000 | 25 | 45 | 0.40 | 3.5 | 110000 | 72000 | 18 | 40 |
| 0.50 | 0 | 95000 | 48000 | 21 | 40 | 0.50 | 3.5 | 125000 | 85000 | 13 | 32 |

As compared with simple steels of the same tensile strength, a 3% nickel steel will have from 10 to 20% higher E. L. and from 20 to 30% greater elongation, while as compared with simple steels of the same carbon, the nickel steel, up to 5% Ni, will have about 40% greater tensile strength, with practically the same elongation and reduction of area.

Cholat and Harmet found with 0.30 C and 15% Ni a T. S. of 213,400 lbs. per sq. in.; when oil-tempered a T. S. of 277,290 and an E. L. of 166,300.

Riley states that steel of 25% Ni and 0.27 C gave a T. S. of 102,600 and elong. 29%, while steel of 25% Ni gave 94,300 T. S. and 40% elong. Steels high in Ni are entirely different in physical properties from low-nickel steels.

EFFECT OF NI ON HARDNESS. — Gun barrels with 4.5% Ni and 0.30 C are soft and very ductile; T. S. 80,000, elong. 25%, red. of area 45%. Rolls with 5% Ni and 1% C turned easier than simple steel of 1% C. If a steel contains less than 6% Ni the influence of the C present on the hardness produced by water quenching is strongly marked. Above 8% Ni the effect of the C seems to be masked by the Ni; steel with 18% Ni is as hard and elastic with 0.30 as with 0.75 C. If steel with 18% Ni and 0.60 C be heated and plunged in water it will be perceptibly softened, and if the Ni is raised to 25% this softening is very noticeable.

COMPRESSION TESTS OF LOW-CARBON NICKEL STEELS. (Hadfield.)

| | | | | | | | | | | | |
|-------------------|------|------|------|------|------|------|-------|-------|-------|-------|-------|
| Carbon..... | 0.13 | 0.14 | 0.19 | 0.18 | 0.17 | 0.16 | 0.18 | 0.23 | 0.19 | 0.16 | 0.14 |
| Nickel..... | 0.95 | 1.92 | 3.82 | 5.81 | 7.65 | 9.51 | 11.39 | 13.48 | 19.64 | 24.51 | 29.07 |
| E. L., tons | 20 | 27 | 28 | 40 | 40 | 70 | 100 | 80 | 80 | 50 | 24 |
| Shortening* ... | 49 | 47 | 41 | 37 | 33 | 3 | 1 | 1 | 3 | 16 | 41 |

* Shortening by 100-ton load, %.

SPECIFIC GRAVITY. — The sp. gr. of low-carbon nickel steels containing up to 15% Ni is about the same as that of carbon steel, from 7.86 to 7.90; from 19 to 39% Ni it is from 7.91 to 8.08; one sample of wire of 29% Ni, however, being reported at 8.4. A 44% Ni steel, according to Guillaume, has a sp. gr. of 8.12.

THE RESISTANCE OF CORROSION of nickel steel increases with the percentage of Ni up to 18. "This alloy is practically non-corrodible." "Tico" resistance wire, 27.5% Ni, was very slightly rusted after a year's exposure in a wet cellar; iron wire under the same conditions was entirely changed to oxide. With the ordinary nickel steels, 3 to 3.5% Ni, corrosion is slightly less than in simple steels.

ELECTRICAL RESISTANCE. — All nickel steels have a high electrical resistance which does not seem to vary much with the percentage of Ni. The resistance wires, "Tico," "Superior," and "Climax," containing from 25 to 30% Ni, have about 48 times, while German silver has about 18 times the resistance of copper.

MAGNETIC PROPERTIES. — According to Guillaume all nickel steels below 25.7% Ni can be, at the same temperature, either magnetic or non-magnetic, according to their previous heat-treatment, and they show different properties at ascending and at descending temperatures. The low-nickel steels, 3 to 5% Ni, possess a magnetic permeability greater than that of wrought iron.

Nickel Steel for Bridges. — J. A. L. Waddell, *Trans. A. S. C. E.*, 1908, presents at length an argument in favor of the use of nickel steel in long-span bridges.

Some Uses of Nickel Steel. (F. L. Sperry, *A. I. M. E.*, xxv, 51.) — The propeller shaft of the U. S. cruiser Brooklyn was made of hollow-forged, oil-tempered nickel steel, 17 in. outside, 11 in. inside diam., length 38 ft. 11 in., weight per foot, 449 lbs. Test bars cut from the tube gave T. S., 90,350 to 94,245; E. L., 56,470 to 60,770; El. in 2 in., 25.5 to 28.0%; Red. of area, 59.8 to 61.3%. A solid shaft of the same elastic strength of simple steel, having an E. L. of 3/5 of that of the nickel steel, would be 18.9 in. diam., and would have weighed 920 lbs. per foot.

The rotating field of the 5000 H.P. electric generators of the Niagara Falls Power Co. is inclosed in a ring of forged nickel steel, outside diam. 139 3/8 in.; inside, 130 in.; width, 50 3/4 in.; weight, 28,840 lbs. It travels at the rate of nearly two miles per minute.

Nickel steel wire with 27.7% Ni and 0.40 C used for torpedo defense netting, 0.116 in. diam., gave a T. S. of 198,700; El. in 2 in., 6.25%; Red. of area, 16.5%.

Flange plate of soft nickel steel, Ni, 2.69; C, 0.08; Mn, 0.36; P, 0.045; S, 0.038, gave, average of 6 tests, T. S., 65,760; E. L., 47,080; El. in 8 in., 24.8%; Red. of area, 52.0%. For comparison: Soft carbon steel, C, 0.10; Mn, 0.27; P, 0.048; S, 0.039; T. S., 54,450; E. L., 35,240; El., 27.4%; Red. of area, 55.3%.

Coefficients of Expansion of Nickel Steel. (D. H. Browne, *A. I. M. E.*, 1899.) — Per degree C. (Prefix 0.0000 to the figures here given.)

| | | | | | | | | | | |
|--------|------|------|------|------|------|------|------|------|------|------|
| % Ni. | 26. | 28. | 28.7 | 30.4 | 31.4 | 34.6 | 35.6 | 37.3 | 39.4 | 44.4 |
| Coeff. | 1312 | 1131 | 1041 | 0458 | 0340 | 0137 | 0087 | 0356 | 0537 | 0856 |

For comparison: Brass, 1878; Hard steel, 1239; Soft steel, 1078; Platinum, 0884; Glass, 0861; Nickel, 1252. Ordinary commercial nickel steels, containing 3 to 4% Ni, have coefficients about the same as carbon steel. See also page 540.

Invar is a nickel-iron alloy, which is characterized by an extraordinarily low coefficient of expansion at ordinary temperatures. The analysis is about as follows:—carbon, 0.18; nickel, 35.5%; manganese, 0.42, — the other elements being low. Guillaume gives the mean coefficient of expansion for an alloy containing 35.6% nickel as $(0.877 + 0.00117 t)^{10-6}$ between temperatures 0°C. and $t^{\circ}\text{C.}$ where t does not exceed 200°C. This material is used in measuring instruments and for standards of length, chronometers, etc. Its expansion as compared with ordinary steel is about as 1:11.5; with brass, as 1:17.2; with glass, as 1:8.5. Alloys either richer or poorer in nickel show much greater expansion, and the alloy containing 47.5% nickel, known as “Platinite,” has the same coefficient of expansion as platinum and glass. See also page 540.

Copper Steels.—Pierre Breuil (*Jour. I. and S. I.*, 1907) gives an account of experiments on four series of copper steels containing respectively 0.15, 0.40, 0.65, and 1% of C with Cu in each ranging from 0 to 34%. An abstract of his principal conclusions is as follows:

Copper steel does not yield a metal capable of being rolled in practice, if Cu exceeds 4%.

When in the ingot state copper hardens steel in proportion as there is less C present.

Copper steels as rolled appear to be stronger in proportion as they contain more Cu. This difference is the more manifest in proportion as the C is lower.

Annealing leaves the steels with the same characteristics, but greatly reduces the differences observed in the case of the untreated steels. Quenching restores the differences encountered in the case of the steels as cast.

Copper steels equal nickel steels in tensile strength and would be less costly than the latter. They are no more brittle than nickel steels containing equivalent percentages of Ni. The steel containing 0.16% C and 4% Cu is remarkable in this respect.

The presence of copper makes the constituents of the steel finer, approximating them to classes containing higher percentages of C. While hardening the steel the presence of Cu does not render it brittle. It confers upon it a very fair degree of elasticity, while leaving the elongation good, thus conducing to the production of a most valuable metal.

Cutting tests were carried on with steels containing C about 1% and Cu 0%, 1%, and 3% respectively. The presence of Cu in no wise altered the cutting properties.

The presence of Cu was found to increase the electrical resistance, and a well-defined maximum was shown, coinciding with 2% Cu in 0.15 C, with 1.7% in 0.35% C, and with 0.5% Cu in 0.7 to 1% carbon steels.

Nickel-Vanadium Steels. (*Eng. Mag.*, April, 1906.)—M. Leon Guillet has investigated the influence of Ni and Va when used jointly.

In steels containing 0.20 C and from 2 to 12% of Ni, the tensile strength and the elastic limit are both materially increased by the addition of small percentages of Va. In no case should the Va exceed 1%, the best results being secured by the use of 0.7 to 1%. A steel containing 0.20 C, 2% of Ni, and 0.7% Va showed a tensile strength of 91,000 lbs., an elastic limit of 70,000 lbs., and an elongation of 23.5%. With 1% Va, the T. S. increased to 119,500 lbs., and the E. L. to 91,000 lbs., the elong. falling to 22%. A nickel steel of 0.20% C and 12% Ni gave, with 0.7 Va, a T. S. of over 200,000 lbs. and an E. L. of 172,000 lbs. per sq. in., the elong. being 6%, while with 1% Va the T. S. rose to 220,000 lbs. and the E. L. to 176,000 lbs., the elongation remaining unchanged. When the Va is increased above 1% the tensile strength falls off, and the material begins to show evidence of brittleness.

Similar effects are produced for steels of the higher carbon, but in a lesser degree.

When the nickel-vanadium steels are subjected to a tempering process the beneficial effects of the Va are still further emphasized. The tempering experiments of M. Guillet were conducted by heating the steel to a temperature of 850°C. , and cooling in water at 20°C. The T. S. and

the E. L. were increased, being nearly doubled for the low nickel content. Thus while the 0.20 C steel with 2% of Ni, untempered, and containing 0.7% of Va, gave a T. S. of 91,000 lbs., with an E. L. of 70,000 lbs., the same steel, tempered from 850° C., showed a T. S. of 168,000 lbs. and an E. L. of 150,000 lbs., the resistance to shock and the hardness being also increased.

Static and Dynamic Properties of Steels. (W. L. Turner, *Iron Age*, July 2, 1908.)—The term "crystallization" is a name given to designate phenomena due to the influences of shock and alternating stresses, whether pure or combined. The name has been advantageously altered to "intermolecular disintegration," but, whatever we choose to call it, there remains the evidence that some modification takes place in the structure of steel when the above-named forces are to be dealt with.

Resistance to fatigue is not a function of static strength.

An example of our knowledge of the "life" properties of ordinary steel is the case of the staying of a locomotive fire-box. Something is required which will possess considerable strength combined with the power to withstand a moderate degree of flexure in all directions. Experience has shown that the use of anything but the mildest steel for this work is prohibitive, and that wrought iron, or even copper, is still more satisfactory.

The writer has completed a preliminary investigation into the relative dynamic properties of iron and the various ordinary and alloy steels, the results being given in the accompanying table. The conditions of the "dynamic" tests were as follows:

A cylindrical test-piece, 6 in. long, $\frac{3}{8}$ in. diam., finished with emery to remove all tool marks, is clamped at one end in a vise. A tool-steel head, in which there is cut a slot, is placed over the other end, the distance from the striking center of this head to the vise line being 4 in. A crank and connecting rod furnished the reciprocating motion for this head, thereby causing the test-piece to be deflected $\frac{3}{8}$ in. each side of the neutral position. In addition to this alternating flexure, the test-piece is also subjected, at each reversal, to an impact, due to the slot on the reciprocating head. The sample undergoes 650 alternations per minute. A deflection of $\frac{3}{8}$ in. on each side has the effect of imparting a permanent set to the test-piece.

On each class of steel a large number of dynamic tests were made, an average being taken of the results after elimination of those figures which were apparently abnormal.

It is apparent that the action of nickel is twofold: 1. It statically intensifies. 2. It dynamically "poisons." As an instance of this, take tests Nos. 13 and 15, the former being a 3.7% nickel steel and the latter a chrome-vanadium steel. In the annealed condition, the elastic limits of the two are almost identical, but at the same time the alternations of stress endured by the latter are $2\frac{1}{4}$ times the number sustained by the nickel steel. Take again Nos. 17 and 18. The dynamic figures are more than three to one in favor of the chrome-vanadium product, whereas the difference in elastic limit is only about 3%.

It is manifest that the static action of vanadium is similar to that of nickel, but that its dynamic effects are the exact converse. The differences are markedly brought out in the quality figures, which invite attention as to comparison with those of ordinary carbon steel. Taking the latter as standard, the chrome-vanadium steels are as much above it as the nickel steels are below it.

Chromium, *per se*, does not appear to exert appreciable influence other than statically, but it is possible that the effect of this metal in a ternary steel might be very marked.

The dynamic attributes of plain carbon steel reach a maximum with about 0.25% C, falling away on both sides of this amount.

The quality figure in the case of the chrome-vanadium steel does not appear to undergo much alteration in the process of oil tempering, but there are considerable variations in other cases. The dynamic test may eventually act as a reliable guide to the correct methods for the heat treatment of individual steels.

Strength for strength, the chrome-vanadium steels also have the advantage over all others as regards machining properties. Chrome-vanadium steel may be forged with the same ease as ordinary steel of similar contents, no special precaution being necessary as to temperatures.

| No. | Material. | Heat Treatment. | C. | Approximate Analysis. | | | | Elastic Limit. (E.) | Tensile Str'ngth | Elong. in 2 in. %. | Red. of Area, % (R.) | Alter-nations. (A.) | Quality. $E \times R \times A \div 10^6$ |
|-----|------------------------|-----------------|------|------------------------------------|-------|-------|-------|---------------------|------------------|--------------------|----------------------|---------------------|--|
| | | | | Mn. | Cr. | Ni | V. | | | | | | |
| | | | | Elastic Limits, 30,000 to 60,000. | | | | | | | | | |
| 1 | Wrought iron..... | R. | 0.05 | 0.05 | | | | 32,020 | 49,450 | 42.5 | 53.8 | 583 | 1004 |
| 2 | Mild Va steel..... | A. | 0.11 | 0.31 | | | | 32,120 | 54,400 | 44.0 | 60.1 | 1111 | 2143 |
| 3 | Old plate..... | R. | 0.24 | 0.42 | | | 0.19 | 58,160 | 70,840 | 25.5 | 53.4 | 612 | 1901 |
| 4 | Mild steel..... | A. | 0.18 | 0.40 | | | | 39,460 | 60,650 | 35.0 | 62.6 | 871 | 2152 |
| 5 | Mild steel..... | O. T. | 0.18 | 0.40 | | | | 45,390 | 70,030 | 33.0 | 64.0 | 777 | 2258 |
| 6 | Forging steel..... | A. | 0.26 | 0.28 | | | | 43,010 | 61,850 | 34.0 | 63.4 | 1415 | 3858 |
| 7 | Forging steel..... | O. T. | 0.26 | 0.28 | | | | 52,230 | 77,310 | 28.0 | 65.3 | 1175 | 4008 |
| 8 | Va case hardening..... | A. | 0.15 | 0.25 | 0.30 | | 0.12 | 44,790 | 55,990 | 45.0 | 69.0 | 1958 | 6051 |
| 9 | Cr-Ni steel..... | A. | 0.36 | 0.34 | 0.95 | 1.70 | | 56,520 | 81,370 | 32.0 | 68.5 | 978 | 3787 |
| 10 | Steel casting..... | A. | 0.18 | 0.65 | | | | 34,690 | 58,800 | 28.0 | 44.9 | 269 | 419 |
| 11 | Va steel casting..... | A. | 0.19 | 0.60 | | | 0.076 | 44,340 | 70,250 | 25.5 | 44.9 | 850 | 1671 |
| 12 | Cr-Va casting..... | A. | 0.57 | 0.68 | 0.75 | | 0.16 | 52,330 | 92,900 | 16.0 | 20.5 | 656 | 704 |
| | | | | Elastic Limits, 60,000 to 100,000. | | | | | | | | | |
| 13 | Ni forging steel..... | A. | 0.21 | 0.45 | | 3.70 | | 61,140 | 79,700 | 30.0 | 62.5 | 746 | 2851 |
| 14 | Ni forging steel..... | O. T. | 0.21 | 0.45 | | 3.70 | | 77,140 | 98,470 | 25.0 | 65.3 | 664 | 3345 |
| 15 | Cr-Va forging..... | A. | 0.26 | 0.50 | 1.00 | | 0.16 | 61,920 | 92,900 | 25.0 | 57.3 | 1608 | 5706 |
| 16 | Spring steel..... | A. | 1.00 | 0.30 | | | | 63,800 | 125,000 | 8.5 | 15.2 | 1260 | 1222 |
| 17 | Cr-Va spring..... | A. | 0.40 | 0.77 | 1.22 | | 0.19 | 67,520 | 100,600 | 26.0 | 61.7 | 1406 | 5858 |
| 18 | Cr-Ni-Va steel..... | A. | 0.30 | 0.27 | 1.51 | 3.45 | 0.085 | 69,140 | 96,880 | 28.5 | 68.5 | 507 | 2402 |
| 19 | Ni-Va steel..... | A. | 0.24 | 0.72 | | 3.40 | 0.15 | 79,260 | 99,700 | 25.0 | 64.0 | 798 | 4048 |
| 20 | Cr-Ni-Va steel..... | A. | 0.57 | 0.27 | 0.93 | 2.04 | 0.07 | 95,150 | 129,100 | 21.0 | 49.8 | 983 | 4659 |
| 21 | Cr-Ni steel..... | T. | 0.37 | 0.34 | 0.89 | 1.70 | | 86,080 | 102,700 | 22.0 | 63.8 | 702 | 3855 |
| | | | | Elastic Limits, 100,000 and Over. | | | | | | | | | |
| 22 | Spring steel..... | O. T. | 1.00 | 0.30 | | | | 101,000 | 186,300 | 9.5 | 16.1 | 561 | 912 |
| 23 | Cr-Va spring..... | O. T. | 0.40 | 0.77 | 1.22 | | 0.19 | 195,300 | 208,500 | 10.0 | 36.3 | 480 | 3403 |
| 24 | Cr-Va forging..... | O. T. | 0.30 | 0.50 | 1.00 | | 0.16 | 141,600 | 151,750 | 16.0 | 56.2 | 717 | 5705 |
| 25 | Ni-Va steel..... | O. T. | 0.24 | 0.72 | | 3.40 | 0.15 | 129,900 | 134,600 | 18.0 | 64.8 | 626 | 5270 |
| 26 | Cr-Ni-Va steel..... | O. T. | 0.30 | 0.27 | 1.51 | 3.45 | 0.085 | 152,300 | 159,900 | 17.0 | 58.9 | 487 | 4369 |
| 27 | Cr-Va spring..... | O. T. | 0.40 | 0.77 | 1.22 | | 0.19 | 183,400 | 187,600 | 14.0 | 50.6 | 634 | 5883 |
| 28 | Cr-Ni steel..... | T. | 0.36 | 0.34 | 0.95 | 1.70 | | 134,500 | 150,300 | 15.5 | 53.5 | 579 | 4166 |

NOTES. — Static tests made on samples $1\frac{1}{2}$ in. diam. \times 2 in. long. Dynamic tests made on the Landgraf-Turner alternating impact machine (old form). The quality figure is the product of: Elastic limit, representing useful strength; reduction of area, representing static ductility; dynamic figure, representing fatigue-resisting property — divided by $1,000,000 = E \times R \times A \div 10^6$. Heat treatment, R., raw; A., annealed; O. T., oil tempered; T., tempered. Nos. 9, 17, 18, 20, 21, 26, 27, 28 were crucible steels, the others open-hearth.

Comparative Effects of Cr and Va. Sankey and J. Kent Smith, *Proc. Inst. M. E.*, 1904.

| Cr. | Va. | T.S.* | E.L.* | El. in 2 in. | Red. A. | Cr. | Va. | T.S.* | E.L.* | El. in 2 in. | Red. A. |
|-----|-------|-------|-------|-----------------|---------|------|------|-------|-------|-----------------|---------|
| 0.5 | | 34.0 | 22.9 | 33% | 60.6% | 1.0 | 0.15 | 48.6 | 36.2 | 24. | 56.6 |
| 1.0 | | 38.2 | 25.0 | 30 | 57.3 | 1.0 | 0.15 | †52.6 | 34.4 | 25.0 | 55.5 |
| ... | 0.1 | 34.8 | 28.5 | 31 | 60.0 | 1.0 | 0.25 | 60.4 | 49.4 | 18.5 | 46.3 |
| ... | 0.15 | 36.5 | 30.4 | 26 | 59.0 | C-Mn | | 27.0 | 16.0 | 35. | 60.0 |
| ... | 0.25 | 39.3 | 34.1 | 24 | 59.0 | C-Mn | | †32.2 | 17.7 | 34. | 52.6 |

* Tons, of 2240 lbs., per sq. in. † Open-hearth steels; all the others are crucible. The last two steels in the table are ordinary carbon steels.

Effect of Heat Treatment on Cr-Va Steel. (H. R. Sankey and J. Kent Smith, *Proc. Inst. M. E.*, 1904, p. 1235.) — Various kinds of heat treatment were given to several Cr-Va steels, the results of which are recorded at length. The following is selected as a sample of the results obtained. Steel with C, 0.297; Si, 0.086; Mn, 0.29; Cr, 1.02; Va, 0.17, gave:

| | Tens. Str. | Yield Point. | El. in 2 in. | Red. Area. | Im- pact. | Alter- na- tions. |
|--|---------------|-----------------|-----------------|---------------|--------------|-------------------------|
| As rolled..... | 121,200 | 82,650 | 24.0% | 44.9% | 3.1 | 1906 |
| Annealed 1/2 hr. at 800° C..... | 87,360 | 47,260 | 34.5 | 53.1 | 15.6 | 2237 |
| Soaked 12 hours at 800° C..... | 86,020 | 68,100 | 33.7 | 51.5 | 11.2 | |
| Water quenched at 800° C..... | 167,100 | 135,070 | 7.5 | 16.6 | 1.2 | 174 |
| Oil quenched at 800° C..... | 122,080 | 82,880 | 22.0 | 35.2 | 2.4 | 296 |
| Oil quenched at 800°, reheated to 350°..... | 132,830 | 111,550 | 23.0 | 50.8 | 9.0 | 1314 |
| Water quenched at 1200° C..... | 209,440 | 191,520 | 1.2 | 1.5 | * | * |
| Oil quenched at 1200° C..... | 140,220 | 118,500 | 8.5 | 21.5 | 3.0 | |

* Too hard to machine.

The impact tests were made on a machine described in *Eng'g*, Sept. 25, 1903, p. 431. The test-piece was 3/4 in. broad, notched so that 0.137 in. in depth remained to be broken through. The figures represent ft.-lbs. of energy absorbed. The piece was broken in one blow. The alternations-of-stress tests were made on Prof. Arnold's machine, described in *The Engineer*, Sept. 2, 1904, p. 227. The pieces were 3/8 in. square, one end was gripped in the machine and the free end, 4 in. long, was bent forwards and backwards about 710 times a minute, the motion of the free end being 3/4 in. on each side of the center line.

Tests by torsion of the same steel were made. The test-piece was 6 in. long, 3/4 in. diam. The results were:

| | Shearing Stress. | | Twist Angle. | No. of Twists. |
|------------------------------------|------------------|----------------|-----------------|-------------------|
| | Elastic. | Ulti- mate. | | |
| As rolled..... | 45,700 | 99,900 | 1410° | 3.92 |
| Annealed 1/2 hr. at 800° C., | 38,528 | 90,272 | 1628° | 4.52 |

Heat-treatment of Alloy Steels. (E. F. Lake, *Am. Mach.*, Aug. 1, 1907.) — In working the high-grade alloy steels it is very important that they be properly heat treated, as poor workmanship in this regard will produce working parts that are no better than ordinary steel, although the stock used be the highest grade procurable. By improperly heat-treating them it is possible to make these high-grade steels more brittle than ordinary carbon steels.

The theory of heat treatment rests upon the influence of the rate of cooling on certain molecular changes in structure occurring at different temperatures. These changes are of two classes, critical and progressive; the former occur periodically between certain narrow temperature limits, while the latter proceed gradually with the rise in temperature, each change producing alterations in the physical characteristics. By controlling the rate of cooling, these changes can be given a permanent set, and the characteristics can thus be made different from those in the metal in its normal state.

The results obtained are influenced by certain factors: 1. The original chemical and physical properties of the metal; 2. The composition of the gases and other substances which come in contact with the metal in heating and cooling. 3. The time in which the temperature is raised between certain degrees. 4. The highest temperature attained. 5. The length of time the metal is maintained at the highest temperature. 6. The time consumed in allowing the temperature to fall to atmospheric.

The highest temperature that it is safe to submit a steel to for heat-treating is governed by the chemical composition of the steel. Thus pure carbon steel should be raised to about 1300° F., while some of the high-grade alloy steels may safely be raised to 1750°. The alloy steels must be handled very carefully in the processes of annealing, hardening, and tempering; for this reason special apparatus has been installed to aid in performing these operations with definite results.

The baths for quenching are composed of a large variety of materials. Some of the more commonly used are as follows, being arranged according to their intensity on 0.85% carbon steel: Mercury; water with sulphuric acid added; nitrate of potassium; sal ammoniac; common salt; carbonate of lime; carbonate of magnesia; pure water; water containing soap, sugar, dextrine or alcohol; sweet milk; various oils; beef suet; tallow; wax.

With many of these alloy steels a dual quenching gives the best results, that is, the metal is quenched to a certain temperature in one bath and then immersed in the second one until completely cooled, or it may be cooled in the air after being quenched in the first bath. For this a lead bath, heated to the proper temperature, is sometimes used for the first quenching.

With the exception of the oils and some of the greases, the quenching effect increases as the temperature of the bath lowers. Sperm and linseed oils, however, at all temperatures between 32° and 250°, act about the same as distilled water at 160°.

The more common materials used for annealing are powdered charcoal, charred bone, charred leather, fire clay, magnesia or refractory earth. The piece to be annealed is usually packed in a cast-iron box in some of these materials or combinations of them, the whole heated to the proper temperature and then set aside, with the cover left on, to cool gradually to the atmospheric temperature. For certain grades of steel these materials give good results; but for all kinds of steels and for all grades of annealing, the slow-cooling furnace no doubt gives the best satisfaction, as the temperature can be easily raised to the right point, kept there as long as necessary, and then regulated to cool down as slowly as is desired. The gas furnace is the easiest to handle and regulate.

A high-grade alloy steel should be annealed after every process in manufacturing which tends to throw it out of its equilibrium, such as forging, rolling and rough machining, so as to return it to its natural state of repose. It should also be annealed before quenching, case-hardening or carbonizing.

The wide range of strength given to some of the alloy steels by heat

treatment is shown by the table below. The composition of the alloy was: Ni, 2.43; Cr, 0.42; Si, 0.26; C, 0.23; Mn, 0.43; P, 0.025; S, 0.022.

| | Quenched at 1550° F. | Tempered at 575° F. | Tempered at 800° F. | Tempered at 925° F. | Tempered at 1025° F. | Tempered at 1125° F. | Tempered at 1550° F. |
|--------------------|-------------------------|------------------------|------------------------|------------------------|-------------------------|-------------------------|-------------------------|
| Tensile Strength . | 227,000 | 219,000 | 195,500 | 172,000 | 156,500 | 141,000 | 109,500 |
| E. L..... | 208,000 | 203,500 | 150,000 | 148,500 | 125,000 | 102,000 | 70,500 |
| Elong., % in 2 in. | 4 | 6 | 8 | 11 | 13 | 15 | 22 |

VARIOUS SPECIFICATIONS FOR STEEL.

Structural Steel. — There has been a change during the ten years from 1880 to 1890, in the opinions of engineers, as to the requirements in specifications for structural steel, in the direction of a preference for metal of low tensile strength and great ductility. The following specifications for tension members at different dates are given by A. E. Hunt and G. H. Clapp, *Trans. A. I. M. E.*, xix, 926:

| | 1879. | 1881. | 1882. | 1885. | 1887. | 1888. |
|---------------------|--------|-------------|--------|--------|-------------|-------------|
| Elastic limit.... | 50,000 | 40 @ 45,000 | 40,000 | 40,000 | 40,000 | 38,000 |
| Tensile strength | 80,000 | 70 @ 80,000 | 70,000 | 70,000 | 67 @ 75,000 | 63 @ 70,000 |
| Elongation in 8 in. | 12% | 18% | 18% | 18% | 20% | 22% |
| Reduction of area | 20% | 30% | 45% | 42% | 42% | 45% |

F. H. Lewis (*Iron Age*, Nov. 3, 1892) says: Regarding steel to be used under the same conditions as wrought iron, that is, to be punched without reaming, there seems to be a decided opinion (and a growing one) among engineers, that it is not safe to use steel in this way, when the ultimate tensile strength is above 65,000 lbs. The reason for this is not so much because there is any marked change in the material of this grade, but because all steel, especially Bessemer steel, has a tendency to segregations of carbon and phosphorus, producing places in the metal which are harder than they normally should be. As long as the percentages of carbon and phosphorus are kept low, the effect of these segregations is inconsiderable; but when these percentages are increased, the existence of these hard spots in the metal becomes more marked, and it is therefore less adapted to the treatment to which wrought iron is subjected.

There is a wide consensus of opinion that at an ultimate of 64,000 to 65,000 lbs. the percentages of carbon and phosphorus reach a point where the steel has a tendency to crack when subjected to rough treatment.

A grade of steel, therefore, running in ultimate strength from 54,000 to 62,000 lbs., or in some cases to 64,000 lbs., is now generally considered a proper material for this class of work.

A. E. Hunt, *Trans. A. I. M. E.*, 1892, says: Why should the tests for steel be so much more rigid than for iron destined for the same purpose? Some of the reasons are as follows: Experience shows that the acceptable qualities of one melt of steel offer no absolute guarantee that the next melt to it, even though made of the same stock, will be equally satisfactory.

It is now almost universally recognized that soft steel, if properly made and of good quality, is for many purposes a safe and satisfactory substitute for wrought iron, being capable of standing the same shop-treatment as wrought iron. But the conviction is equally general, that poor steel, or an unsuitable grade of steel, is a very dangerous substitute for wrought iron even under the same unit strains.

For this reason it is advisable to make more rigid requirements in selecting material which may range between the brittleness of glass and a ductility greater than that of wrought iron.

Specifications for Structural Steel for Bridges. (*Proc. A. S. T. M.*, 1905.)—Steel shall be made by the open-hearth process. The chemical and physical properties shall conform to the following limits:

| Elements Considered. | Structural Steel. | Rivet Steel. | Steel Castings. |
|---|---------------------|-------------------|---------------------------|
| Phosphorus. { Basic... | 0.04% | 0.04% | 0.05% |
| Max. { Acid.... | 0.08% | 0.04% | 0.08% |
| Sulphur, Max..... | 0.05% | 0.04% | 0.05% |
| Tensile strength, lbs. per sq. in..... | { Desired 60,000 | Desired 50,000 | Not less than 65,000 |
| Elong.: Min. % in 8 in. | { 1,500,000* | 1,500,000 | |
| Elong.: Min. % in 2 in. | { tens. str. 22 | tens. str. | 18 |
| Fracture..... | Silky | Silky | Silky or fine granular |
| Cold bend without fracture..... | 180° flat† | 180° flat‡ | 90°. $d=3t$ |

* The following modifications will be allowed in the requirements for elongation for structural steel: For each $\frac{1}{16}$ inch in thickness below $\frac{5}{16}$ inch, a deduction of $2\frac{1}{2}$ will be allowed from the specified percentage. For each $\frac{1}{8}$ inch in thickness above $\frac{3}{4}$ inch, a deduction of 1 will be allowed from the specified percentage.

† Plates, shapes and bars less than 1 in. thick shall bend as called for. Full-sized material for eye-bars and other steel 1 in. thick and over, tested as rolled, shall bend cold 180° around a pin of a diameter twice the thickness of the bar, without fracture on the outside of bend. When required by the inspector, angles $\frac{3}{4}$ in. and less in thickness shall open flat, and angles $\frac{1}{2}$ in. and less in thickness shall bend shut, cold, under blows of a hammer, without sign of fracture.

‡ Rivet steel, when nicked and bent around a bar of the same diameter as the rivet rod, shall give a gradual break and a fine, silky, uniform fracture.

If the ultimate strength varies more than 4000 lbs. from that desired, a retest may be made, at the discretion of the inspector, on the same gauge, which, to be acceptable, shall be within 5000 lbs. of the desired strength.

Chemical determinations of C, P, S, and Mn shall be made from a test ingot taken at the time of the pouring of each melt of steel. Check analyses shall be made from finished material, if called for by the purchaser, in which case an excess of 25% above the required limits will be allowed.

Specimens for tensile and bending tests for plates, shapes and bars shall be made by cutting coupons from the finished product, which shall have both faces rolled and both edges milled with edges parallel for at least 9 in.; or they may be turned $\frac{3}{4}$ in. diam. for a length of at least 9 in., with enlarged ends. Rivet rods shall be tested as rolled. Specimens shall be cut from the finished rolled or forged bar in such manner that the center of the specimen shall be 1 in. from the surface of the bar. The specimen for tensile test shall be turned with a uniform section 2 in. long, with enlarged ends. The specimen for bending test shall be $1 \times \frac{1}{2}$ in. in section.

Specifications for Steel for the Manhattan Bridge. (*Eng. News*, Aug. 3, 1905.)—

MATERIAL FOR CABLES, SUSPENDERS AND HAND ROPES. Open-heart steel. (The wire for serving the cables shall be made of Norway iron of approved quality.) The ladle tests of the steel shall contain not more than: C, 0.85; Mn, 0.55; Si, 0.20; P, 0.04; S, 0.04; Cu, 0.02%. The wire shall have an ultimate strength of not less than 215,000 lbs. per sq. in. before galvanizing, and an elongation of not less than 2% in 12 in. The bright wire shall be capable of bending cold around a rod $1\frac{1}{2}$ times its own diam. without sign of fracture. The cable wire before galvanizing shall be 0.192 in. \pm 0.003 in. in diam.; after galvanizing, the wire shall have an ultimate strength of not less than 200,000 lbs. per sq. in. of gross section.

CARBON STEEL. The ladle tests as usually taken shall contain not more than: P, 0.04; S, 0.04; Mn, 0.60; Si, 0.10%. The ladle tests of the carbon rivet steel shall contain not more than: P, 0.035; S, 0.03. Rivet steel shall be used for all bolts and threaded rods.

NICKEL STEEL. The ladle test shall contain not less than 3.25 Ni, and not more than: P, 0.04; S, 0.04; Mn, 0.60; Si, 0.10; nickel rivet steel not more than: P, 0.035; S, 0.03%.

Nickel steel for plates and shapes in the finished material must show: T. S., 85,000 to 95,000 lbs. per sq. in.; E. L., 55,000 lbs. min.; elong. in 8 ins., min., = $1,600,000 \div T. S.$; min. red. of area, 40%.

Specimens cut from the finished material shall show the following physical properties:

| Material. | T. S., lbs. per sq. in. | Min. E. L., lbs. per sq. in. | Min. Elong., % in 8 in. | Min. Red. of Area, %. |
|---------------------------------------|-------------------------|------------------------------|-------------------------------------|-----------------------|
| Shapes and universal mill plates..... | 60,000 to 68,000 | 33,000 | $\frac{1,500,000}{\text{ultimate}}$ | 44 |
| Eye-bars, pins and rollers..... | 64,000 to 72,000 | 35,000 | | 40 |
| Sheared plates..... | 60,000 to 68,000 | 33,000 | | 44 |
| Rivet rods..... | 50,000 to 58,000 | 30,000 | | 50 |
| High-carbon steel for trusses..... | 85,000 to 95,000 | 45,000 | | 35 |

Nickel rivet steel: T. S., 70,000 to 80,000; E. L., min., 45,000; elong., min., $1,600,000 \div T. S.$, % in 8 ins.

STEEL CASTINGS. The ladle test of steel for castings shall contain not more than: P, 0.05; S, 0.05; Mn, 0.80; Si, 0.35%. Test-pieces taken from coupons on the annealed castings shall show T. S., 65,000; E. L., 35,000; elong. 20% in 8 ins. They shall bend without cracking around a rod three times the thickness of the test-piece.

Specifications for Steel. (*Proc. A. S. T. M.*, 1905.)

| Steel Forgings. | Kind of Steel. | Tensile Strength. | Elast. Limit. | El. in 2 in., %. | Red Area, %. |
|--|----------------|-------------------|---------------|------------------|--------------|
| Solid or hollow forgings, no diam. or thickness of section to exceed 10 in. | S. | 58,000 | 29,000* | 28 | 35 (a) |
| | C. | 75,000 | 37,500* | 18 | 30 (c) |
| | C. A. | 80,000 | 40,000 | 22 | 35 (b) |
| | N. A. | 80,000 | 50,000 | 25 | 45 (a) |
| Solid or hollow forgings, diam. not to exceed 20 in. or thickness of section 15 in. | C. A. | 75,000 | 37,500 | 23 | 35 (b) |
| | N. A. | 80,000 | 45,000 | 25 | 45 (a) |
| Solid forgings, over 20 in..... | C. A. | 70,000 | 35,000 | 24 | 30 (c) |
| Solid forgings..... | N. A. | 80,000 | 45,000 | 24 | 40 (a) |
| Solid or hollow forgings, diam. or thickness not over 3 in. | C. O. | 90,000 | 55,000 | 20 | 45 (b) |
| | N. O. | 95,000 | 65,000 | 21 | 50 (b) |
| Solid rectangular sections, thickness not over 6 in., or hollow with walls not over 6 in. thick. | C. O. | 85,000 | 50,000 | 22 | 45 (b) |
| | N. O. | 90,000 | 60,000 | 22 | 50 (b) |
| Solid rect. sections, thickness not over 10 in., or hollow with walls not over 10 in. thick. | C. O. | 80,000 | 45,000 | 23 | 40 (b) |
| | N. O. | 85,000 | 55,000 | 24 | 45 (b) |
| Locomotive forgings | | 80,000 | 40,000 | 20 | 25 (d) |

* The yield point, instead of the elastic limit, is specified for soft steel and carbon steel not annealed. It is determined by the drop of the beam or halt in the gauge of the testing machine. The elastic limit, specified for all other steels, is determined by an extensometer, and is defined as that point where the proportionality changes. The standard test specimen is $\frac{1}{2}$ in. turned diam. with a gauge length of 2 inches.

Kind of steel: S., soft or low carbon. C., carbon steel, not annealed. C. A., carbon steel, annealed. C. O., carbon steel, oil tempered. N. A., nickel steel, annealed. N. O., nickel steel, oil tempered. Bending tests: A specimen $1 \times \frac{1}{2}$ in. shall bend cold 180° without fracture on outside of bent portion, as follows: (a) around a diam. of $\frac{1}{2}$ in.; (b) around a diam. of 1 in.; (c) around a diam. of $\frac{1}{2}$ in.; (d) no bending test required.

Chemical composition: P and S not to exceed 0.10 in low-carbon steel, 0.06 in carbon steel not annealed, 0.04 in carbon or nickel steel oil tempered or annealed, 0.05 in locomotive forgings. Mn not to exceed 0.60 in locomotive forgings. Ni 3 to 4% in nickel steel.

Specifications for Steel Ship Material. (Amer. Bureau of Shipping, 1900. *Proc. A. S. T. M.*, 1906, p. 175.) —

| For Hull Construction. | Tens. Strength. | E. L. | El. in 8 in., %. |
|--------------------------------|------------------|---------------------|------------------|
| Plates, angles and shapes..... | 58,000 to 60,000 | $\frac{1}{2}$ T. S. | 22*, 18† |
| Castings..... | 60,000 to 75,000 | | 15 |
| Forgings..... | 55,000 to 65,000 | | 20 |

* In plates 18 lbs. per sq. ft. and over.

† In plates under 18 lbs.

FOR MARINE BOILERS: Open-hearth steel; Shell: P and S, each not over 0.04%. Fire-box, not over 0.035%. Tensile Strength: Rivet steel, 45,000 to 55,000; Fire-box, 52,000 to 62,000; Shell, 55,000 to 73,000; Braces and stays, 55,000 to 65,000; Tubes and all other steel, 52,000 to 62,000 lbs. per sq. in.

Elongation in 8 in.: Rivet steel, 28%; Plates $\frac{3}{8}$ in. and under, 20%; $\frac{3}{8}$ to $\frac{3}{4}$ in., 22%; $\frac{3}{4}$ in. and over, 25%.

COLD BENDING AND QUENCHING TESTS. Rivet steel and all steel of 52,000 to 62,000 lbs. T. S., $\frac{1}{2}$ in. thick and under, must bend 180° flat on itself without fracture on outside of bent portion; over $\frac{1}{2}$ in. thick, 180° around a mandrel $1\frac{1}{2}$ times the thickness of the test-piece. For hull construction a specimen must stand bending on a radius of half its thickness, without fracture on the convex side, either cold or after being heated to cherry-red and quenched in water at 80° F.

High-strength Steel for Shipbuilding. (*Eng'g*, Aug. 2, 1907, p. 137.)—The average tensile strength of the material selected for the *Lusitania* was 82,432 lbs. per sq. in. for normal high-tensile steel, and 81,984 lbs. for the same annealed, as compared with 66,304 lbs. for ordinary mild steel. The metal was subjected to tup tests as well as to other severe punishments, including the explosion of heavy charges of dynamite against the plates, and in every instance the results were satisfactory. It was not deemed prudent to adopt the high-tensile steel for the rivets, a point upon which there seems some difference of opinion.

Penna. R. R. Specifications for Steel.

| | Note. | Date. | C. | Mn. | Si. | P. | S. | Cu. |
|-------------------------------|-------|-------|------|-------|-------|-------|-------|-------|
| Plates for steel cars..... | (1) | 1899 | 0.12 | 0.35 | 0.05 | 0.04— | 0.03— | |
| Bar spring steel..... | | 1901 | 1.00 | 0.25 | 0.15— | 0.03— | 0.03— | 0.03— |
| Steel for axles..... | (2) | 1899 | 0.40 | 0.50 | 0.05 | 0.05— | 0.04— | |
| Steel for crank pins..... | (3) | 1904 | 0.45 | 0.60— | 0.05— | 0.03— | 0.04— | |
| Billets or blooms for forging | (4) | 1902 | 0.45 | 0.50 | 0.05 | 0.03— | 0.02— | 0.03— |
| Boiler-shell sheets..... | (5) | 1906 | 0.18 | 0.40— | 0.05— | 0.04— | 0.03— | 0.03— |
| Fire-box sheets..... | (6) | 1906 | 0.18 | 0.40— | 0.02— | 0.03— | 0.02— | 0.03— |

The minus sign after a figure means "or less." The figures without the minus sign represent the composition desired.

Steel castings. Desired T. S., 70,000 lbs. per sq. in.; elong. in 2 in., 15%. Will be rejected if T. S. is below 60,000, or elong. below 12%, or if the castings show blow-holes or shrinkage cracks on machining.

NOTES. (1) Tensile strength, 52,000 lbs. per sq. in.; elong. in 8 ins. = $1,500,000 \div \text{T. S.}$ (2) Axles are also subjected to a drop test, similar to that of the A. S. T. M. specifications. Axles will be rejected if they contain C below 0.35 or above 0.50, Mn above 0.60, P above 0.07%. (3) T. S. desired, 85,000 lbs. per sq. in.; elong. in 8 ins., 18%. Pins will be rejected if the T. S. is below 80,000 or above 95,000, if the elongation is less than 12%, or if the P is above 0.05%. (4) The steel will be rejected if the C is below 0.35 or above 0.50, Si above 0.25, S above 0.05, P above 0.05, or Mn above 0.60%. (5) T. S. desired, 60,000; elong. in 8 ins., 26%. Sheets will be rejected if the T. S. is less than 55,000 or over 65,000, or if the elongation is less than the quotient of 1,400,000 divided by the T. S., or if P is over 0.05%. (6) T. S. desired, 60,000, with elong. of 28% in 8 in. Sheets will be rejected if the T. S. is less than 55,000 or above 65,000 (but if the elong. is 30% or over plates will not be rejected for high T. S.), if the elongation is less than $1,450,000 \div \text{T. S.}$, if a single seam or cavity more than $\frac{1}{4}$ in. long is shown in either one of the three fractures obtained in the test for homogeneity, described below, or if on analysis C is found below 0.15 or over 0.25, P over 0.035, Mn over 0.45, Si over 0.03, S over 0.045, or Cu over 0.05%.

Homogeneity Test for Fire-box Steel.—This test is made on one of the broken tensile-test specimens, as follows:

A portion of the test-piece is nicked with a chisel, or grooved on a machine, transversely about a sixteenth of an inch deep, in three places about 2 in. apart. The first groove should be made on one side, 2 in. from the square end of the piece; the second, 2 in. from it on the opposite side; and the third, 2 in. from the last, and on the opposite side from it. The test-piece is then put in a vise, with the first groove about $\frac{1}{4}$ in. above the jaws, care being taken to hold it firmly. The projecting end of the test-piece is then broken off by means of a hammer, a number of light blows being used, and the bending being away from the groove. The piece is broken at the other two grooves in the same way. The object of this treatment is to open and render visible to the eye any seams due to failure to weld up, or to foreign interposed matter, or cavities due to gas bubbles in the ingot. After rupture, one side of each fracture is examined, a pocket lens being used if necessary, and the length of the seams and cavities is determined. The sample shall not show any single seam or cavity more than $\frac{1}{4}$ in. long in either of the three fractures.

Dr. Chas. B. Dudley, chemist of the P. R. R. (*Trans. A. I. M. E.*, 1892), referring to tests of crank-pins, says: In testing a recent shipment, the piece from one side of the pin showed 88,000 lbs. strength and 22% elongation, and the piece from the opposite side showed 106,000 lbs. strength and 14% elongation. Each piece was above the specified strength and ductility, but the lack of uniformity between the two sides of the pin was so marked that it was finally determined not to put the lot of 50 pins in use. To guard against trouble of this sort in future, the specifications are to be amended to require that the difference in ultimate strength of the two specimens shall not be more than 3000 lbs.

Specifications for Steel Rails. (Adopted by the manufacturers of the U. S. and Canada. In effect Jan. 1, 1909).— Bessemer rails:

| | | | | | |
|--------------------|-----------|-----------|-----------|-----------|-----------|
| Wt. per yard, lbs. | 50 to 60 | 61 to 70 | 71 to 80 | 81 to 90 | 91 to 100 |
| Carbon, % | 0.35-0.45 | 0.35-0.45 | 0.40-0.50 | 0.43-0.53 | 0.45-0.55 |
| Manganese, % | 0.70-1.00 | 0.70-1.00 | 0.75-1.05 | 0.80-1.10 | 0.84-1.14 |

Phosphorus not over 0.10%; silicon not over 0.20%. Drop Test: A piece of rail 4 to 6 ft. long, selected from each blow, is placed head upwards on supports 3 ft. apart. The anvil weighs at least 20,000 lbs., and the tup, or falling weight, 2000 lbs. The rail should not break when the drop is as follows:

| | | | |
|-----------------------|----------|----------|-----------|
| Weight per yard, lbs. | 71 to 80 | 81 to 90 | 91 to 100 |
| Height of drop, feet | 16 | 17 | 18 |

If any rail breaks when subjected to the drop test, two additional tests will be made of other rails from the same blow of steel, and if either of

these latter tests fail, all the rails of the blow which they represent will be rejected; but if both of these additional test-pieces meet the requirements, all the rails of the blow which they represent will be accepted.

Shrinkage: The number of passes and the speed of the roll train shall be so regulated that for sections 75 lbs. per yard and heavier the temperature on leaving the rolls will not exceed that which requires a shrinkage allowance at the hot saws of $6\frac{11}{16}$ inches for a 33-ft. 75-lb. rail, with an increase of $\frac{1}{16}$ in. for each increase of 5 lbs. in the weight of the section.

Open-hearth rails; chemical specifications:

Weight per yard, lbs. 50 to 60 61 to 70 71 to 80 81 to 90 90 to 100
Carbon, % 0.46-0.59 0.46-0.59 0.52-0.65 0.59-0.72 0.62-0.75

Manganese, 0.60 to 0.90; Phosphorus, not over 0.04; Silicon, not over 0.20. Drop Tests: 50 to 60-lb., 15 ft.; 61 to 70-lb., 16 ft.; heavier sections same as Bessemer.

Specifications for Steel Axles. (*Proc. A. S. T. M.*, 1905 p. 56.) —

| | P. & S. ‡ | Tens. Str. | Yield Pt. | El. in 2 in. | Red. Area. |
|--|--------------|---------------|--------------|-----------------|---------------|
| Car and tender truck | 0.06 | | | | |
| Driving and engine truck, C. S.* | 0.06 | 80,000 | 40,000 | 20% | 25% |
| Driving and engine truck, N. S.† | 0.04 | 80,000 | 50,000 | 25% | 45% |

* Carbon steel.

† Nickel steel, 3 to 4% Ni.

‡ Each not to exceed. Mn in carbon steel not over 0.60%.

Drop Tests. — One drop test to be made from each melt. The axle rests on supports 3 ft. apart, the tup weighs 1640 lbs., the anvil supported on springs, 17,500 lbs.; the radius of the striking face of the tup is 5 in. The axle is turned over after the first, third and fifth blows. It must stand the number of blows named below without rupture and without exceeding, as the result of the first blow, the deflection given.

| | | | | | | | |
|-----------------------------------|-------|-------|--------|------|------|------|-------|
| Diam. axle at center, in. | 41/4 | 43/8 | 47/16 | 45/8 | 43/4 | 53/8 | 57/8 |
| Number of blows | 5 | 5 | 5 | 5 | 5 | 5 | 7 |
| Height of drop, ft. | 24 | 26 | 28 1/2 | 31 | 34 | 43 | 43 |
| Deflection, in. | 8 1/4 | 8 1/4 | 8 1/4 | 8 | 8 | 7 | 5 1/2 |

Specifications for Tires. (*A. S. T. M.*, 1901.) — Physical requirements of test-piece $\frac{1}{2}$ in. diam. Tires for passenger engines: T. S., 100,000; El. in 2 in., 12%. Tires for freight engines and car wheels: T. S., 110,000; El., 10%. Tires for switching engines: T. S., 120,000; El., 8%.

Drop Test. — If a drop test is called for, a selected tire shall be placed vertically under the drop on a foundation at least 10 tons in weight and subjected to successive blows from a tup weighing 2240 lbs. falling from increasing heights until the required deflection is obtained, without breaking or cracking. The minimum deflection must equal $D^2 \div (40T^2 + 2D)$, D being internal diameter and T thickness of tire at center of tread.

Splice-bars. (*A. S. T. M.*, 1901.) — Tensile strength of a specimen cut from the head of the bar, 54,000 to 64,000 lbs.; yield point, 32,000 lbs. Elongation in 8 in., not less than 25 per cent. A test specimen cut from the head of the bar shall bend 180° flat on itself without fracture on the outside of the bent portion. If preferred, the bending test may be made on an unpunched splice-bar, which shall be first flattened and then bent. One tensile test and one bending test to be made from each blow or melt of steel.

Specifications for Steel Used in Automobile Construction.
(E. F. Lake, *Am. Mach.*, March 14, 1907.) —

| | C. | Mn. | Cr. | Ni. | P. | S. | T. S. | E. L. | El. in 2 in. | R. of A. |
|-----|-----------|----------|-------|-------|--------|-------|---------------------|---------------------|-----------------|--------------|
| (1) | 0.40-0.55 | 0.40- | 0.80+ | 1.50+ | 0.04- | 0.04- | { 90000+ 180000+ | { 65000+ 140000+ | 18+ 8+ | 35+a 20+b |
| (2) | 0.20-0.35 | 0.40- | 0.80+ | 1.50+ | 0.04- | 0.04- | { 85000+ 130000+ | { 65000+ 100000+ | 20+ 12+ | 50+a 30+b |
| (3) | 0.25 | 0.40 | 1.50 | 3.50 | 0.015 | 0.025 | { 120000 105000 | { 20 58c | | |
| (4) | 0.25-0.35 | 0.60 | | 1.50+ | 0.03 | 0.04 | { 85000+ 100000+ | { 60000+ 70000+ | 25+ 20+ | 50+a 50+b |
| (5) | 0.45-0.55 | 1.1-1.3 | | | 0.065- | 0.06- | { 85000+ 75000+ | { 55000+ 40000+ | 15+ 25+ | 45+c 40+c |
| (6) | 0.28-0.36 | 0.3-0.6 | | | 0.05- | 0.06- | | | | |
| (7) | 0.85-1.00 | 0.25-0.5 | | | 0.03- | 0.03- | | | | |
| (8) | 0.50 | 1.50- | | 30.0 | 0.04- | 0.06- | | | | |

The plus sign means "or over"; the minus sign "or less."

a, fully annealed; b, heat-treated, that is oil-quenched and partly annealed; c, as rolled.

(1) 45% carbon chrome-nickel steel, for gears of high-grade cars. When annealed this steel can be machined with a high-speed tool at the rate of 35 ft. per min., with a $\frac{1}{16}$ -in. feed and a $\frac{3}{16}$ -in. cut. It is annealed at 1400° F. 4 or 5 hours, and cooled slowly. In heat-treating it is heated to 1500°, quenched in oil or water and drawn at 500° F.

(2) 25% carbon chrome-nickel steel, for shafts, axles, pivots, etc. This steel may be machined at the same rate as (1), and it forges more easily.

(3) A foreign steel used for forgings that have to withstand severe alternating shocks, such as differential shafts, transmission parts, universal joints, axles, etc.

(4) Nickel steel, used instead of (1) in medium and low-priced cars.

(5) "Gun-barrel" steel, used extensively for rifle barrels, also in low-priced automobiles, for shafts, axles, etc. It is used as it comes from the maker, without heat-treating.

(6) Machine steel. Used for parts that do not require any special strength.

(7) Spring steel used in automobiles.

(8) Nickel steel for valves. Used for its heat-resisting qualities in valves of internal-combustion engines.

Carbonizing or Case-hardening.—Some makers carbonize the surface of gears made from steel (1) above. They are packed in cast-iron boxes with a mixture of bone and powdered charcoal and heated four hours at nearly the melting-point of the boxes, then cooled slowly in the boxes. They are then taken out, heated to 1400° F. for four hours to break up the coarse grain produced by the carbonizing temperature. After this the work is heat-treated as above described.

The machine steel (6) case-hardens well by the use of this process.

Specifications for Steel Castings. (*Proc. A. S. T. M.*, 1905, p. 53.)—Open-hearth, Bessemer, or crucible. Castings to be annealed unless otherwise specified. Ordinary castings, in which no physical requirements are specified, shall contain not over 0.04 C and not over 0.08 P. Castings subject to physical test shall contain not over 0.05 P and not over 0.05 S. The minimum requirements are:

| | T. S. | Y. P. | El. in 2 in. | Red. Area. |
|----------------------|--------|--------|-----------------|---------------|
| Hard castings..... | 85,000 | 38,250 | 15 % | 20 % |
| Medium castings..... | 70,000 | 31,500 | 18 % | 25 % |
| Soft castings..... | 60,000 | 27,000 | 22 % | 30 % |

For small or unimportant castings a test to destruction may be substituted. Three samples are selected from each melt or blow, annealed in the same furnace charge, and shall show the material to be ductile and free from injurious defects, and suitable for the purpose intended. Large castings are to be suspended and hammered all over. No cracks, flaws, defects nor weakness shall appear after such treatment. A specimen $1 \times \frac{1}{2}$ in. shall bend cold around a diam. of 1 in. without fracture on outside of bent portion, through an angle of 120° for soft and 90° for medium castings.

Specifications for steel castings issued by the U. S. Navy Department, 1889 (abridged): Steel for castings must be made by either the open-hearth or the crucible process, and must not show more than 0.06% of phosphorus. All castings must be annealed, unless otherwise directed. The tensile strength of steel castings shall be at least 60,000 lbs., with an elongation of at least 15% in 8 in. for all castings for moving parts of machinery, and at least 10% in 8 in. for other castings. Bars 1 in. sq. shall be capable of bending cold, without fracture, through an angle of 90° , over a radius not greater than $1\frac{1}{2}$ in. All castings must be sound, free from injurious roughness, sponginess, pitting, shrinkage, or other cracks, cavities, etc.

Pennsylvania Railroad specifications, 1888: Steel castings should have a tensile strength of 70,000 lbs. per sq. in. and an elongation of 15% in section originally 2 in. long. Steel castings will not be accepted if tensile strength falls below 60,000 lbs., nor if the elongation is less than 12%, nor if castings have blow-holes and shrinkage cracks. Castings weighing 80 lbs. or more must have cast with them a strip to be used as a test-piece. The dimensions of this strip must be $\frac{3}{4}$ in. sq. by 12 in. long.

MECHANICS.

FORCE, STATICAL MOMENT, EQUILIBRIUM, ETC.

MECHANICS is the science that treats of the action of force upon bodies. Statics is the mechanics of bodies at rest relatively to the earth's surface. Dynamics is the mechanics of bodies in motion. Hydrostatics and hydrodynamics are the mechanics of liquids, and Pneumatics the mechanics of air and other gases. These are treated in other chapters.

There are four elementary quantities considered in Mechanics: *Matter, Force, Space, Time.*

Matter. — Any substance or material that can be weighed or measured. It exists in three forms: solid, liquid, and gaseous. A definite portion of matter is called a body.

The Quantity of Matter in a body may be determined either by measuring its bulk or by weighing it, but as the bulk varies with temperature, with porosity, with size, shape and method of piling its particles, etc., weighing is generally the more accurate method of determining its quantity.

Weight. Mass. — The word "weight" is commonly used in two senses: 1. As the measure of quantity of matter in a body, as determined by weighing it in an even balance scale or on a lever or platform scale, and thus comparing its quantity with that of certain pieces of metal called standard weights, such as the pound avoirdupois. 2. As the measure of the force which the attraction of gravitation of the earth exerts on the body, as determined by measuring that force with a spring balance. As the force of gravity varies with the latitude and elevation above sea level of different parts of the earth's surface, the weight determined in this second method is a variable, while that determined by the first method is a constant. For this reason, and also because spring balances are generally not as accurate instruments as even balances, or lever or platform scales, the word "weight," in engineering, unless otherwise specified, means the quantity of matter as determined by weighing it by the first method. The standard unit of weight is the pound.

The word "mass" is used in three senses by writers on physics and engineering: 1. As a general expression of an indefinite quantity, synonymous with lump, piece, portion, etc., as in the expression "a mass whose weight is one pound." 2. As the quotient of the weight, as

determined by the first method of weighing given above, by 32.2, the value of g , the acceleration due to gravity, at London, expressed by the formula $M = W/g$. This value is merely the arithmetical ratio of the weight in pounds to the acceleration in feet per second per second, and it has no unit. 3. As a measure of the quantity of matter, exactly synonymous with the first meaning of the word "weight," given above. In this sense the word is used in many books on physics and theoretical mechanics, but it is not so used by engineers. The statement in such books that the engineers' unit of mass is 32.2 lbs. is an error. There is no such unit. Whenever the term "mass" is represented by M in engineering calculations it is equivalent to W/g , in which W is the quantity of matter in pounds, and $g = 32.2$.

A Force is anything that tends to change the state of a body with respect to rest or motion. If a body is at rest, anything that tends to put it in motion is a force; if a body is in motion, anything that tends to change either its direction or its rate of motion is a force.

A force should always mean the pull, pressure, rub, attraction (or repulsion) of one body upon another, and always implies the existence of a simultaneous equal and opposite force exerted by that other body on the first body, i.e., the *reaction*. In no case should we call anything a force unless we can conceive of it as capable of measurement by a spring balance, and are able to say from what other body it comes. (I. P. Church.)

Forces may be divided into two classes, extraneous and molecular: extraneous forces act on bodies from without; molecular forces are exerted between the neighboring particles of bodies.

Extraneous forces are of two kinds, pressures and moving forces: pressures simply tend to produce motion; moving forces actually produce motion. Thus, if gravity act on a fixed body, it creates pressure; if on a free body, it produces motion.

Molecular forces are of two kinds, attractive and repellent: attractive forces tend to bind the particles of a body together; repellent forces tend to thrust them asunder. Both kinds of molecular forces are continually exerted between the molecules of bodies, and on the predominance of one or the other depends the physical state of a body, as solid, liquid, or gaseous.

The Unit of Force used in engineering, by English writers, is the pound avoirdupois. For some scientific purposes, as in electro-dynamics, forces are sometimes expressed in "absolute units." The absolute unit of force is that force which acting on a unit of mass during a unit of time produces a unit of velocity. In the French C. G. S., or centimeter-gram-second system, it is the force which acting on the mass whose weight is one gram at Paris will produce in one second a velocity of one centimeter per second. This unit is called a "dyne" = $1/981$ gram at Paris.

An attempt has been made by some writers on physics to introduce the so-called "absolute system" into English weights and measures, and to define the "absolute unit" of force as that force which acting on the mass whose weight is one pound at London will in one second produce a velocity of one foot per second, and they have given this unit the name "poundal." The use of this unit only makes confusion for students, and it is to be hoped that it will soon be abandoned in high-school textbooks. Professor Perry in his "Calculus for Engineers," p. 26, says, "One might as well talk Choctaw in the shops as to speak about . . . so many poundals of force and so many foot-poundals of work." *

Inertia is that property of a body by virtue of which it tends to continue in the state of rest or motion in which it may be placed, until acted on by some force.

Newton's Laws of Motion. — 1st Law. If a body be at rest, it will remain at rest; or if in motion, it will move uniformly in a straight line till acted on by some force.

* Professor Perry himself, however, makes a slip on the same page in saying: "Force in pounds is the space-rate at which work in foot-pounds is done; it is also the time-rate at which momentum is produced or destroyed." He gets this idea, no doubt, from the equations $FT = MV$, $F = MV/T$, $F = 1/2 MV^2 \div S$. Force is not these things: it is merely numerically equivalent, when certain units are chosen, to these last two quotients. We might as well say, since $T = MV/F$, that time is the force-rate of momentum.

2d Law. If a body be acted on by several forces, it will obey each as though the others did not exist, and this whether the body be at rest or in motion.

3d Law. If a force act to change the state of a body with respect to rest or motion, the body will offer a resistance equal and directly opposed to the force. Or, to every action there is opposed an equal and opposite reaction.

Graphic Representation of a Force. — Forces may be represented geometrically by straight lines, proportional to the forces. A force is given when we know its intensity, its point of application, and the direction in which it acts. When a force is represented by a line, the length of the line represents its intensity; one extremity represents the point of application; and an arrow-head at the other extremity shows the direction of the force.

Composition of Forces is the operation of finding a single force whose effect is the same as that of two or more given forces. The required force is called the resultant of the given forces.

Resolution of Forces is the operation of finding two or more forces whose combined effect is equivalent to that of a given force. The required forces are called components of the given force.

The resultant of two forces applied at a point, and acting in the same direction, is equal to the sum of the forces. If two forces act in opposite directions, their resultant is equal to their difference, and it acts in the direction of the greater.

If any number of forces be applied at a point, some in one direction and others in a contrary direction, their resultant is equal to the sum of those that act in one direction, diminished by the sum of those that act in the opposite direction; or, the resultant is equal to the algebraic sum of the components.

Parallelogram of Forces. — If two forces acting on a point be represented in direction and intensity by adjacent sides of a parallelogram, their resultant will be represented by that diagonal of the parallelogram which passes through the point. Thus *OR*, Fig. 93, is the resultant of *OQ* and *OP*.

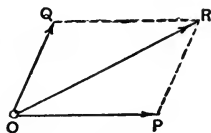


FIG. 93.

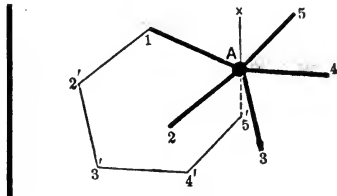


FIG. 94.

Polygon of Forces. — If several forces are applied at a point and act in a single plane, their resultant is found as follows:

Through the point draw a line representing the first force; through the extremity of this draw a line representing the second force; and so on, throughout the system; finally, draw a line from the starting-point to the extremity of the last line drawn, and this will be the resultant required.

Suppose the body *A*, Fig. 94, to be urged in the directions *A1*, *A2*, *A3*, *A4*, and *A5* by forces which are to each other as the lengths of those lines. Suppose these forces to act successively and the body to first move from *A* to 1; the second force *A2* then acts and finding the body at 1 would take it to 2'; the third force would then carry it to 3', the fourth to 4', and the fifth to 5'. The line *A5'* represents in magnitude and direction the resultant of all the forces considered. If there had been an additional force, *A_x*, in the group, the body would be returned by that force to its original position, supposing the forces to act successively, but if they had acted simultaneously the body would never have moved at all; the tendencies to motion balancing each other.

It follows, therefore, that if the several forces which tend to move a body can be represented in magnitude and direction by the sides of a closed polygon taken in order, the body will remain at rest; but if the forces are represented by the sides of an open polygon, the body will move

and the direction will be represented by the straight line which closes the polygon.

Twisted Polygon.—The rule of the polygon of forces holds true even when the forces are not in one plane. In this case the lines $A1, 1-2', 2'-3',$ etc., form a twisted polygon, that is, one whose sides are not in one plane.

Parallelepipedon of Forces.—If three forces acting on a point be represented by three edges of a parallelepipedon which meet in a common point, their resultant will be represented by the diagonal of the parallelepipedon that passes through their common point.

Thus OR , Fig. 95, is the resultant of OQ, OS and OP . OM is the resultant of OP and OQ , and OR is the resultant of OM and OS .

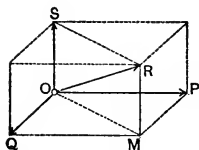


FIG. 95.

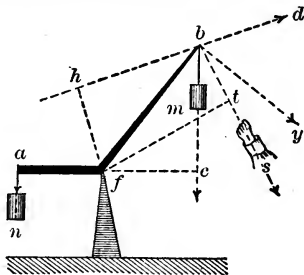


FIG. 96.

Moment of a Force.—The moment of a force (sometimes called statical moment), with respect to a point, is the product of the force by the perpendicular distance from the point to the direction of the force. The fixed point is called the center of moments; the perpendicular distance is the lever-arm of the force; and the moment itself measures the tendency of the force to produce rotation about the center of moments.

If the force is expressed in pounds and the distance in feet, the moment is expressed in foot-pounds. It is necessary to observe the distinction between foot-pounds of statical moment and foot-pounds of work or energy. (See Work.)

In the bent lever, Fig. 96 (from Trautwine), if the weights n and m represent forces, their moments about the point f are respectively $n \times af$ and $m \times fc$. If instead of the weight m a pulling force to balance the weight n is applied in the direction bs , or by or bd , s , y , and d being the amounts of these forces, their respective moments are $s \times ft$, $y \times fb$, $d \times fh$.

If the forces acting on the lever are in equilibrium it remains at rest, and the moments on each side of f are equal, that is, $n \times af = m \times fc$, or $s \times ft$, or $y \times fb$, or $d \times hf$.

The moment of the resultant of any number of forces acting together in the same plane is equal to the algebraic sum of the moments of the forces taken separately.

Statical Moment. Stability.—The statical moment of a body is the product of its weight by the distance of its line of gravity from some assumed line of rotation. The line of gravity is a vertical line drawn from its center of gravity through the body. The stability of a body is that resistance which its weight alone enables it to oppose against forces tending to overturn it or to slide it along its foundation.

To be safe against turning on an edge the moment of the forces tending to overturn it, taken with reference to that edge, must be less than the statical moment. When a body rests on an inclined plane, the line of gravity, being vertical, falls toward the lower edge of the body, and the condition of its not being overturned by its own weight is that the line of gravity must fall within this edge. In the case of an inclined tower resting on a plane the same condition holds—the line of gravity must fall within the base. The condition of stability against sliding along a horizontal plane is that the horizontal component of the force exerted tending to cause it to slide shall be less than the product of the weight of

the body into the coefficient of friction between the base of the body and its supporting plane. This coefficient of friction is the tangent of the angle of repose, or the maximum angle at which the supporting plane might be raised from the horizontal before the body would begin to slide. (See Friction.)

The Stability of a Dam against overturning about its lower edge is calculated by comparing its statical moment referred to that edge with the resultant pressure of the water against its upper side. The horizontal pressure on a square foot at the bottom of the dam is equal to the weight of a column of water of one square foot in section, and of a height equal to the distance of the bottom below water-level; or, if H is the height, the pressure at the bottom per square foot = $62.4 \times H$ lbs. At the water-level the pressure is zero, and it increases uniformly to the bottom, so that the sum of the pressures on a vertical strip one foot in breadth may be represented by the area of a triangle whose base is $62.4 \times H$ and whose altitude is H , or $62.4 H^2 \div 2$. The center of gravity of a triangle being $\frac{1}{3}$ of its altitude, the resultant of all the horizontal pressures may be taken as equivalent to the sum of the pressures acting at $\frac{1}{3} H$, and the moment of the sum of the pressures is therefore $62.4 \times H^3 \div 6$.

Parallel Forces. — If two forces are parallel and act in the same direction, their resultant is parallel to both, and lies between them, and the intensity of the resultant is equal to the sum of the intensities of the two forces. Thus in Fig. 96 the resultant of the forces n and m acts vertically downward at f , and is equal to $n + m$.

If two parallel forces act at the extremities of a straight line and in the same direction, the resultant divides the line joining the points of appli-

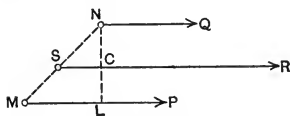


FIG. 97.

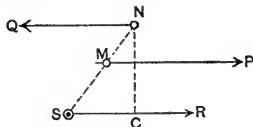


FIG. 98.

cation of the components, inversely as the components. Thus in Fig. 96, $m:n::af:fc$; and in Fig. 97, $P:Q::SN:SM$.

The resultant of two parallel forces acting in opposite directions is parallel to both, lies without both, on the side and in the direction of the greater, and its intensity is equal to the difference of the intensities of the two forces.

Thus the resultant of the two forces Q and P , Fig. 98, is equal to $Q - P = R$. Of any two parallel forces and their resultant each is proportional to the distance between the other two; thus in both Figs. 97 and 98, $P:Q:R::SN:SM:MN$.

Couples. — If P and Q be equal and act in opposite directions, $R = 0$; that is, they have no resultant. Two such forces constitute what is called a couple.

The tendency of a couple is to produce rotation; the measure of this tendency, called the *moment of the couple*, is the product of one of the forces by the distance between the two.

Since a couple has no single resultant, no single force can balance a couple. To prevent the rotation of a body acted on by a couple the application of two other forces is required, forming a second couple. Thus in Fig. 99, P and Q , forming a couple, may be balanced by a second couple formed by R and S . The point of application of either R or S may be a fixed pivot or axis.

Moment of the couple $PQ = P(c + b + a) = \text{moment of } RS = Rb$. Also, $P + R = Q + S$.

The forces R and S need not be parallel to P and Q , but if not, then their components parallel to PQ are to be taken instead of the forces themselves.

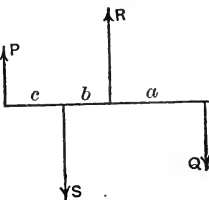


FIG. 99.

Equilibrium of Forces. — A system of forces applied at points of a solid body will be in equilibrium when they have no tendency to produce motion, either of translation or of rotation.

The conditions of equilibrium are: 1. The algebraic sum of the components of the forces in the direction of any three rectangular axes must be separately equal to 0.

2. The algebraic sum of the moments of the forces, with respect to any three rectangular axes, must be separately equal to 0.

If the forces lie in a plane: 1. The algebraic sum of the components of the forces, in the direction of any two rectangular axes, must be separately equal to 0.

2. The algebraic sum of the moments of the forces, with respect to any point in the plane, must be equal to 0.

If a body is restrained by a fixed axis, as in case of a pulley, or wheel and axle, the forces will be in equilibrium when the algebraic sum of the moments of the forces with respect to the axis is equal to 0.

CENTER OF GRAVITY.

The center of gravity of a body, or of a system of bodies rigidly connected together, is that point about which, if suspended, all the parts will be in equilibrium, that is, there will be no tendency to rotation. It is the point through which passes the resultant of the efforts of gravitation on each of the elementary particles of a body. In bodies of equal heaviness throughout, the center of gravity is the center of magnitude.

(The center of magnitude of a figure is a point such that if the figure be divided into equal parts the distance of the center of magnitude of the whole figure from any given plane is the mean of the distances of the centers of magnitude of the several equal parts from that plane.)

If a body be suspended at its center of gravity, it will be in equilibrium in all positions. If it be suspended at a point out of its center of gravity, it will swing into a position such that its center of gravity is vertically beneath its point of suspension.

To find the center of gravity of any plane figure mechanically, suspend the figure by any point near its edge, and mark on it the direction of a plumb-line hung from that point; then suspend it from some other point, and again mark the direction of the plumb-line in like manner. Then the center of gravity of the surface will be at the point of intersection of the two marks of the plumb-line.

The Center of Gravity of Regular Figures, whether plane or solid, is the same as their geometrical center; for instance, a straight line, parallelogram, regular polygon, circle, circular ring, prism, cylinder, sphere, spheroid, middle frustums of spheroid, etc.

Of a triangle: On a line drawn from any angle to the middle of the opposite side, at a distance of one-third of the line from the side; or at the intersection of such lines drawn from any two angles.

Of a trapezium or trapezoid: Draw a diagonal, dividing it into two triangles. Draw a line joining their centers of gravity. Draw the other diagonal, making two other triangles, and a line joining their centers of gravity. The intersection of the two lines is the center of gravity required.

Of a sector of a circle: On the radius which bisects the arc, $2cr + 3l$ from the center, c being the chord, r the radius, and l the arc.

Of a semicircle: On the middle radius, $0.4244 r$ from the center.

Of a quadrant: On the middle radius, $0.6002 r$ from the center.

Of a segment of a circle: $c^3 + 12a$ from the center. c = chord, a = area.

Of a parabolic surface: In the axis, $\frac{3}{5}$ of its length from the vertex.

Of a semi-parabola (surface): $\frac{3}{5}$ length of the axis from the vertex, and $\frac{3}{8}$ of the semi-base from the axis.

Of a cone or pyramid: In the axis, $\frac{1}{4}$ of its length from the base.

Of a paraboloid: In the axis, $\frac{2}{3}$ of its length from the vertex.

Of a cylinder, or regular prism: In the middle point of the axis.

Of a frustum of a cone or pyramid: Let a = length of a line drawn from the vertex of the cone when complete to the center of gravity of the base, and a' that portion of it between the vertex and the top of the frustum; then distance of center of gravity of the frustum from center of gravity of

its base = $\frac{a}{4} - \frac{3a'^3}{4(a^2 + aa' + a'^2)}$

For two bodies, fixed one at each end of a straight bar, the common center of gravity is in the bar, at that point which divides the distance between their respective centers of gravity in the inverse ratio of the weights. In this solution the weight of the bar is neglected. But it may be taken as a third body, and allowed for as in the following directions:

For more than two bodies connected in one system: Find the common center of gravity of two of them; and find the common center of these two jointly with a third body, and so on to the last body of the group.

Another method, by the principle of moments: To find the center of gravity of a system of bodies, or a body consisting of several parts, whose several centers are known. If the bodies are in a plane, refer their several centers to two rectangular coördinate axes. Multiply each weight by its distance from one of the axes, add the products, and divide the sum by the sum of the weights; the result is the distance of the center of gravity from that axis. Do the same with regard to the other axis. If the bodies are not in a plane, refer them to three planes at right angles to each other, and determine the mean distance of the sum of the weights from each of the three planes.

MOMENT OF INERTIA.

The moment of inertia of the weight of a body with respect to an axis is the algebraic sum of the products obtained by multiplying the weight of each elementary particle by the square of its distance from the axis. If the moment of inertia with respect to any axis = I , the weight of any element of the body = w , and its distance from the axis = r , we have $I = \sum (wr^2)$.

The moment of inertia varies, in the same body, according to the position of the axis. It is the least possible when the axis passes through the center of gravity. To find the moment of inertia of a body, referred to a given axis, divide the body into small parts of regular figure. Multiply the weight of each part by the square of the distance of its center of gravity from the axis. The sum of the products is the moment of inertia. The value of the moment of inertia thus obtained will be more nearly exact, the smaller and more numerous the parts into which the body is divided.

MOMENTS OF INERTIA OF REGULAR SOLIDS. — Rod, or bar, of uniform thickness, with respect to an axis perpendicular to the length of the rod,

$$I = W \left(\frac{l^2}{3} + d^2 \right), \quad (1)$$

W = weight of rod, $2l$ = length, d = distance of center of gravity from axis.

Thin circular plate, axis in its own plane, $\left. \vphantom{\begin{array}{l} \text{Thin circular plate, axis in its} \\ \text{own plane,} \end{array}} \right\} I = W \left(\frac{r^2}{4} + d^2 \right), \quad (2)$

r = radius of plate.

Circular plate, axis perpendicular to the plate, $\left. \vphantom{\begin{array}{l} \text{Circular plate, axis perpendicular to} \\ \text{the plate,} \end{array}} \right\} I = W \left(\frac{r^2}{2} + d^2 \right) \quad (3)$

Circular ring, axis perpendicular to its own plane, $\left. \vphantom{\begin{array}{l} \text{Circular ring, axis perpendicular to} \\ \text{its own plane,} \end{array}} \right\} I = W \left(\frac{r^2 + r'^2}{2} + d^2 \right), \quad (4)$

r and r' are the exterior and interior radii of the ring.

Cylinder, axis perpendicular to the axis of the cylinder, $\left. \vphantom{\begin{array}{l} \text{Cylinder, axis perpendicular to the} \\ \text{axis of the cylinder,} \end{array}} \right\} I = W \left(\frac{r^2}{4} + \frac{l^2}{3} + d^2 \right). \quad (5)$

r = radius of base, $2l$ = length of the cylinder.

By making $d = 0$ in any of the above formulæ, we find the moment of inertia for a parallel axis through the center of gravity.

The moment of inertia, $\sum wr^2$, numerically equals the weight of a body which, if concentrated at the distance unity from the axis of rotation, would require the same work to produce a given increase of angular velocity that the actual body requires. It bears the same relation to angular acceleration which weight does to linear acceleration (Rankine). The term moment of inertia is also used in regard to areas, as the cross-sections of beams under strain. In this case $I = \sum ar^2$, in which a is any elementary area, and r its distance from the center. (See under Strength of Materials, p. 279.) Some writers call $\sum mr^2 = \sum wr^2 \div g$ the moment of inertia.

CENTERS OF OSCILLATION AND OF PERCUSSION.

Center of Oscillation. — If a body oscillate about a fixed horizontal axis, not passing through its center of gravity, there is a point in the line drawn from the center of gravity perpendicular to the axis whose motion is the same as it would be if the whole mass were collected at that point and allowed to vibrate as a pendulum about the fixed axis. This point is called the center of oscillation.

The Radius of Oscillation, or distance of the center of oscillation from the point of suspension = the square of the radius of gyration \div distance of the center of gravity from the point of suspension or axis. The centers of oscillation and suspension are convertible.

If a straight line, or uniform thin bar or cylinder, be suspended at one end, oscillating about it as an axis, the center of oscillation is at $\frac{2}{3}$ the length of the rod from the axis. If the point of suspension is at $\frac{1}{3}$ the length from the end, the center of oscillation is also at $\frac{2}{3}$ the length from the axis, that is, it is at the other end. In both cases the oscillation will be performed in the same time. If the point of suspension is at the center of gravity, the length of the equivalent simple pendulum is infinite, and therefore the time of vibration is infinite.

For a sphere suspended by a cord, r = radius, h = distance of axis of motion from the center of the sphere, h' = distance of center of oscillation from center of the sphere, l = radius of oscillation = $h + h' = h + \frac{2}{5} \frac{r^2}{h}$.

If the sphere vibrate about an axis tangent to its surface, $h = r$, and $l = r + \frac{2}{5}r$. If $h = 10r$, $l = 10r + \frac{r}{25}$.

Lengths of the radius of oscillation of a few regular plane figures or thin plates, suspended by the vertex or uppermost point.

1st. When the vibrations are flatwise, or perpendicular to the plane of the figure:

In an isosceles triangle the radius of oscillation is equal to $\frac{3}{4}$ of the height of the triangle.

In a circle, $\frac{5}{8}$ of the diameter.

In a parabola, $\frac{5}{7}$ of the height.

2d. When the vibrations are edgewise, or in the plane of the figure:

In a circle the radius of oscillation is $\frac{3}{4}$ of the diameter.

In a rectangle suspended by one angle, $\frac{2}{3}$ of the diagonal.

In a parabola, suspended by the vertex, $\frac{5}{7}$ of the height plus $\frac{1}{3}$ of the parameter.

In a parabola, suspended by the middle of the base, $\frac{4}{7}$ of the height plus $\frac{1}{2}$ the parameter.

Center of Percussion. — The center of percussion of a body oscillating about a fixed axis is the point at which, if a blow is struck by the body, the percussive action is the same as if the whole mass of the body were concentrated at the point. This point is identical with the center of oscillation.

CENTER AND RADIUS OF GYRATION.

The center of gyration, with reference to an axis, is a point at which, if the entire weight of a body be concentrated, its moment of inertia will remain unchanged; or, in a revolving body, the point in which the whole weight of the body may be conceived to be concentrated, as if a pound of platinum were substituted for a pound of revolving feathers, the angular velocity and the accumulated work remaining the same. The distance of this point from the axis is the *radius of gyration*. If W = the weight of a body, $I = \sum wr^2$ = its moment of inertia, and k = its radius of gyration,

$$I = Wk^2 = \sum wr^2; \quad k = \sqrt{\frac{\sum wr^2}{W}}.$$

The moment of inertia = the weight \times the square of the radius of gyration.

To find the radius of gyration divide the body into a considerable number of equal small parts, — the more numerous the more nearly exact is the result, — then take the mean of all the squares of the distances of the parts from the axis of revolution, and find the square root of the mean square. Or, if the moment of inertia is known, divide it by the weight and extract the square root. For radius of gyration of an area, as a cross-section of a beam, divide the moment of inertia of the area by the area and extract the square root.

The radius of gyration is the least possible when the axis passes through the center of gravity. This minimum radius is called the principal radius of gyration. If we denote it by k and any other radius of gyration by k' , we have for the five cases given under the head of moment of inertia above the following values:

- (1) Rod, axis perpen. to length, $\left\{ \begin{array}{l} k = l \sqrt{\frac{1}{3}}; k' = \sqrt{\frac{l^2}{3} + d^2}. \end{array} \right.$
- (2) Circular plate, axis in its plane, $\left\{ \begin{array}{l} k = \frac{r}{2}; k' = \sqrt{\frac{r^2}{4} + d^2}. \end{array} \right.$
- (3) Circular plate, axis perpen. to plane, $\left\{ \begin{array}{l} k = r \sqrt{\frac{1}{2}}; k' = \sqrt{\frac{r^2}{2} + d^2}. \end{array} \right.$
- (4) Circular ring, axis perpen. to plane, $\left\{ \begin{array}{l} k = \sqrt{\frac{r^2 + r'^2}{2}}; k' = \sqrt{\frac{r^2 + r'^2}{2} + d^2}. \end{array} \right.$
- (5) Cylinder, axis perpen. to length, $\left\{ \begin{array}{l} k = \sqrt{\frac{r^2}{4} + \frac{l^2}{3}}; k' = \sqrt{\frac{r^2}{4} + \frac{l^2}{3} + d^2}. \end{array} \right.$

Principal Radii of Gyration and Squares of Radii of Gyration.

(For radii of gyration of sections of columns, see page 281.)

| Surface or Solid. | Rad. of Gyration. | Square of R. of Gyration. |
|--|---|--|
| Parallelogram: $\left\{ \begin{array}{l} \text{axis at its base} \dots\dots\dots \\ \text{height } h \text{ " mid-height} \dots\dots\dots \end{array} \right.$ | $0.5773 h$ $0.2886 h$ | $\frac{1}{3} h^2$ $\frac{1}{12} h^2$ |
| Straight rod: $\left\{ \begin{array}{l} \text{length } l, \text{ or thin " axis at end} \dots\dots\dots \\ \text{rectang. plate " mid-length} \dots\dots\dots \end{array} \right.$ | $0.5773 l$ $0.2886 l$ | $\frac{1}{3} l^2$ $\frac{1}{12} l^2$ |
| Rectangular prism: axes $2a, 2b, 2c$, referred to axis $2a \dots\dots$ | $0.577 \sqrt{b^2 + c^2}$ | $(b^2 + c^2) \div 3$ |
| Parallelopiped: length l , base b , axis at one end, at mid-breadth. $\dots\dots\dots$ | $0.289 \sqrt{4l^2 + b^2}$ | $\frac{4l^2 + b^2}{12}$ |
| Hollow square tube: out. side h , inner h' , axis mid-length. $\dots\dots$ very thin, side $= h$, axis mid-length. $\dots\dots$ | $0.289 \sqrt{h^2 + h'^2}$ $.403 h$ | $(h^2 + h'^2) \div 12$ $\frac{h^2}{6}$ |
| Thin rectangular tube: sides b, h , axis mid-length. $\dots\dots\dots$ | $0.289 h \sqrt{\frac{h + 3b}{h + b}}$ | $\frac{h^2}{12} \cdot \frac{h + 3b}{h + b}$ |
| Thin circ. plate: rad. r , diam. h , ax. diam. $\dots\dots$ | $\frac{1}{2} r$ | $\frac{1}{4} r^2 = h^2 \div 16$ |
| Flat circ. ring: diams. h, h' , axis diam. $\dots\dots$ | $\frac{1}{4} \sqrt{h^2 + h'^2}$ | $(h^2 + h'^2) \div 16$ |
| Solid circular cylinder: length l , axis diameter at mid-length. $\dots\dots\dots$ | $0.289 \sqrt{l^2 + 3r^2}$ | $\frac{l^2}{12} + \frac{r^2}{4}$ |
| Circular plate: solid wheel of uniform thickness, or cylinder of any length, referred to axis of cyl. $\dots\dots\dots$ | $0.7071 r$ | $\frac{1}{2} r^2$ |
| Hollow circ. cylinder, or flat ring: l , length; R, r , outer and inner radii. Axis, 1, longitudinal axis; 2, diam. at mid-length. $\dots\dots\dots$ | $0.7071 \sqrt{R^2 + r^2}$ $.289 \sqrt{l^2 + 3(R^2 + r^2)}$ | $(R^2 + r^2) \div 2$ $\frac{l^2}{12} + \frac{R^2 + r^2}{2}$ |
| Same: very thin, axis its diameter. $\dots\dots\dots$ | $0.289 \sqrt{l^2 + 6R^2}$ | $\frac{l^2}{12} + \frac{R^2}{2}$ |
| " radius r ; axis, longitudinal axis $\dots\dots$ | r | r^2 |
| Circumf. of circle, axis its center. $\dots\dots\dots$ | $\frac{r}{2}$ $0.7071 r$ | $\frac{r^2}{2}$ $\frac{1}{2} r^2$ |
| Sphere: radius r , axis its diam. $\dots\dots\dots$ | $0.6325 r$ | $\frac{2}{5} r^2$ |
| Spheroid: equatorial radius r , revolving polar axis $a \dots\dots\dots$ | $0.6325 r$ | $\frac{2}{5} r^2$ |
| Paraboloid: r =rad. of base, rev. on axis. $\dots\dots\dots$ | $0.5773 r$ | $\frac{1}{3} r^2$ |
| Ellipsoid: semi-axes a, b, c ; revolving on axis $2a \dots\dots\dots$ | $0.4472 \sqrt{b^2 + c^2}$ | $\frac{b^2 + c^2}{5}$ |
| Spherical shell: radii R, r , revolving on its diam. $\dots\dots\dots$ | $0.6325 \sqrt{R^2 - r^2}$ $\sqrt{\frac{R^3 - r^3}{3}}$ | $\frac{2}{5} R^2 - r^2$ $\frac{2}{5} \frac{R^3 - r^3}{3}$ |
| Same: very thin, radius $r \dots\dots\dots$ | $0.8165 r$ | $\frac{2}{3} r^2$ |
| Solid cone: r =rad. of base, rev. on axis. $\dots\dots$ | $0.5477 r$ | $\frac{0.3}{5} r^2$ |

THE PENDULUM.

A body of any form suspended from a fixed axis about which it oscillates by the force of gravity is called a *compound pendulum*. The ideal body concentrated at the center of oscillation, suspended from the center of suspension by a string without weight, is called a *simple pendulum*. This equivalent simple pendulum has the same weight as the given body, and also the same moment of inertia, referred to an axis passing through the point of suspension, and it oscillates in the same time.

The ordinary pendulum of a given length vibrates in equal times when the angle of the vibrations does not exceed 4 or 5 degrees, that is, 2° or $2\frac{1}{2}^\circ$ each side of the vertical. This property of a pendulum is called its *isochronism*.

The time of vibration of a pendulum varies directly as the square root of the length, and inversely as the square root of the acceleration due to gravity at the given latitude and elevation above the earth's surface.

If T = the time of vibration, l = length of the simple pendulum, g = acceleration = 32.16, $T = \pi \sqrt{\frac{l}{g}}$; since π is constant, $T \propto \frac{\sqrt{l}}{\sqrt{g}}$. At a given location g is constant and $T \propto \sqrt{l}$. If l be constant, then for any location $T \propto \frac{1}{\sqrt{g}}$. If T be constant, $gT^2 = \pi^2 l$; $l \propto g$; $g = \frac{\pi^2 l}{T^2}$. From this equation

the force of gravity at any place may be determined if the length of the simple pendulum, vibrating seconds, at that place is known. At New York this length is 39.1017 inches = 3.2585 ft., whence $g = 32.16$ ft. At London the length is 39.1393 inches. At the equator 39.0152 or 39.0168 inches, according to different authorities.

Time of vibration of a pendulum of a given length at New York

$$= t = \sqrt{\frac{l}{39.1017}} = \frac{\sqrt{l}}{6.253},$$

t being in seconds and l in inches. Length of a pendulum having a given time of vibration, $l = t^2 \times 39.1017$ inches.

The time of vibration of a pendulum may be varied by the addition of a weight at a point above the center of suspension, which counteracts the lower weight, and lengthens the period of vibration. By varying the height of the upper weight the time is varied.

To find the weight of the upper bob of a compound pendulum, vibrating seconds, when the weight of the lower bob and the distances of the weights from the point of suspension are given:

$$w = W \frac{(39.1 \times D) - D^2}{(39.1 \times d) + d^2}.$$

W = the weight of the lower bob, w = the weight of the upper bob; D = the distance of the lower bob and d = the distance of the upper bob from the point of suspension, in inches.

Thus, by means of a second bob, short pendulums may be constructed to vibrate as slowly as longer pendulums.

By increasing w or d until the lower weight is entirely counterbalanced, the time of vibration may be made infinite.

Conical Pendulum. — A weight suspended by a cord and revolving at a uniform speed in the circumference of a circular horizontal plane whose radius is r , the distance of the plane below the point of suspension being h , is held in equilibrium by three forces — the tension in the cord, the centrifugal force, which tends to increase the radius r , and the force of gravity acting downward. If v = the velocity in feet per second of the center of gravity of the weight, as it describes the circumference, $g = 32.16$, and r and h are taken in feet, the time in seconds of performing one revolution is

$$t = \frac{2\pi r}{v} = 2\pi \sqrt{\frac{h}{g}}; \quad h = \frac{gt^2}{4\pi^2} = 0.8146 t^2.$$

If $t = 1$ second, $h = 0.8146$ foot = 9.775 inches.

The principle of the conical pendulum is used in the ordinary fly-ball governor for steam-engines. (See Governors.)

CENTRIFUGAL FORCE.

A body revolving in a curved path of radius = R in feet exerts a force, called centrifugal force, F , upon the arm or cord which restrains it from moving in a straight line, or "flying off at a tangent." If W = weight of the body in pounds, N = number of revolutions per minute, v = linear velocity of the center of gravity of the body, in feet per second, $g = 32.16$, then

$$v = \frac{2\pi RN}{60}; F = \frac{Wv^2}{gR} = \frac{Wv^2}{32.16R} = \frac{W4\pi^2 RN^2}{3600g} = \frac{WRN^2}{2933} = .0003410 WRN^2 \text{ lbs.}$$

If n = number of revolutions per second, $F = 1.2276 WRn^2$.
(For centrifugal force in fly-wheels, see Fly-wheels.)

VELOCITY, ACCELERATION, FALLING BODIES.

Velocity is the rate of motion, or the speed of a body at any instant.

If s = space in feet passed over in t seconds, and v = velocity in feet per second, if the velocity is uniform,

$$v = \frac{s}{t}; s = vt; t = \frac{s}{v}.$$

If the velocity varies uniformly, the mean velocity $v_m = \frac{1}{2}(v_1 + v_2)$, in which v_1 is the velocity at the beginning and v_2 the velocity at the end of the time t .

$$s = \frac{1}{2}(v_1 + v_2)t. \quad \dots \quad (1)$$

If $v_1 = 0$, then $s = \frac{1}{2}v_2t$. $v_2 = 2s/t$.

If the velocity varies, but not uniformly, v for an exceedingly short interval of time = s/t , or in calculus $v = ds/dt$.

Acceleration is the change in velocity which takes place in a unit of time. Unit of acceleration = $a = 1$ foot per second in one second. For uniformly varying velocity, the acceleration is a constant quantity, and

$$a = \frac{v_2 - v_1}{t}; v_2 = v_1 + at; v_1 = v_2 - at; t = \frac{v_2 - v_1}{a}. \quad \dots \quad (2)$$

If the body start from rest, $v_1 = 0$; then if v_m = mean velocity

$$v_m = \frac{v_2}{2}; v_2 = 2v_m; a = \frac{v_2}{t}; v_2 = at; v_2 - at = 0; t = \frac{v_2}{a}.$$

Combining (1) and (2), we have

$$s = \frac{v_2^2 - v_1^2}{2a}; s = v_1t + \frac{at^2}{2}; s = v_2t - \frac{at^2}{2}.$$

If $v_1 = 0$, $s = \frac{1}{2}v_2t$.

Retarded Motion. — If the body start with a velocity v_1 and come to rest, $v_2 = 0$; then $s = \frac{1}{2}v_1t$.

In any case, if the change in velocity is v ,

$$s = \frac{v}{2}t; s = \frac{v^2}{2a}; s = \frac{a}{2}t^2.$$

For a body starting from or ending at rest, we have the equations

$$v = at; s = \frac{v}{2}t; s = \frac{at^2}{2}; v^2 = 2as.$$

Falling Bodies. — In the case of falling bodies the acceleration due to gravity, at 40° latitude, is 32.16 feet per second in one second, = g . Then if v = velocity acquired at the end of t seconds, or final velocity, and h = height or space in feet passed over in the same time,

$$v = gt = 32.16t = \sqrt{2gh} = 8.02\sqrt{h} = \frac{2h}{t};$$

$$h = \frac{gt^2}{2} = 16.08t^2 = \frac{v^2}{2g} = \frac{v^2}{64.32} = \frac{vt}{2}.$$

$$t = \frac{v}{g} = \frac{v}{32.16} = \sqrt{\frac{2h}{g}} = \frac{\sqrt{h}}{4.01} = \frac{2h}{v};$$

u = space fallen through in the T th second = $g (T - 1/2)$.

From the above formulæ for falling bodies we obtain the following:

During the first second the body starting from a state of rest (resistance of the air neglected) falls $g \div 2 = 16.08$ feet; the acquired velocity is $g = 32.16$ ft. per sec.; the distance fallen in two seconds is $h = \frac{gt^2}{2} = 16.08 \times 4 = 64.32$ ft.; and the acquired velocity is $v = gt = 64.32$ ft. The acceleration, or increase of velocity in each second, is constant, and is 32.16 ft. per second. Solving the equations for different times, we find for

| | | | | | | | |
|--|--------------------------|---|---|---|----|----|----|
| Seconds, t | | 1 | 2 | 3 | 4 | 5 | 6 |
| Acceleration, g | $32.16 \times$ | 1 | 1 | 1 | 1 | 1 | 1 |
| Velocity acquired at end of time, v | $32.16 \times$ | 1 | 2 | 3 | 4 | 5 | 6 |
| Height of fall in each second, u | $\frac{32.16}{2} \times$ | 1 | 3 | 5 | 7 | 9 | 11 |
| Total height of fall, h | $\frac{32.16}{2} \times$ | 1 | 4 | 9 | 16 | 25 | 36 |

Value of g . — The value of g increases with the latitude, and decreases with the elevation. At the latitude of Philadelphia, 40° , its value is 32.16. At the sea-level, Everett gives $g = 32.173 - .082 \cos 2 \text{ lat.} - .000003$ height in feet. At Paris, lat. $48^\circ 50' \text{ N.}$, $g = 980.87 \text{ cm.} = 32.181 \text{ ft.}$

Values of $\sqrt{2g}$, calculated by an equation given by C. S. Pierce, are given in a table in Smith's Hydraulics, from which we take the following:

| | | | | | | | |
|-------------------------|-----------|------------|------------|------------|------------|------------|------------|
| Latitude..... | 0° | 10° | 20° | 30° | 40° | 50° | 60° |
| Value of $\sqrt{2g}$.. | 8.0112 | 8.0118 | 8.0137 | 8.0165 | 8.0199 | 8.0235 | 8.0269 |
| Value of g | 32.090 | 32.094 | 32.105 | 32.132 | 32.160 | 32.189 | 32.216 |

The value of $\sqrt{2g}$ decreases about .0004 for every 1000 feet increase in elevation above the sea-level.

For all ordinary calculations for the United States, g is generally taken at 32.16, and $\sqrt{2g}$ at 8.02. In England $g = 32.2$, $\sqrt{2g} = 8.025$. Practical limiting values of g for the United States, according to Pierce, are:

| | |
|---------------------------------------|--------------|
| Latitude 49° at sea-level..... | $g = 32.186$ |
| " 25° 10,000 feet above the sea | $g = 32.089$ |

Fig. 100 represents graphically the velocity, space, etc., of a body falling for six seconds. The vertical line at the left is the time in seconds, the horizontal lines represent the acquired velocities at the end of each second = 32.16 t . The area of the small triangle at the top represents the height fallen through in the first second = $1/2 g = 16.08$ feet, and each of the other triangles is an equal space. The number of triangles between each pair of horizontal lines represents the height of fall in each second, and the number of triangles between any horizontal line and the top is the total height fallen during the time. The figures under h , u and v adjoining the cut are to be multiplied by 16.08 to obtain the actual velocities and heights for the given times.

Angular and Linear Velocity of a Turning Body. — Let r = radius of a turning body in feet, n = number of revolutions per minute, v = linear velocity of a point on the circumference in feet per second, and $60 v$ = velocity in feet per minute.

$$v = \frac{2\pi rn}{60}; \quad 60 v = 2\pi rn$$

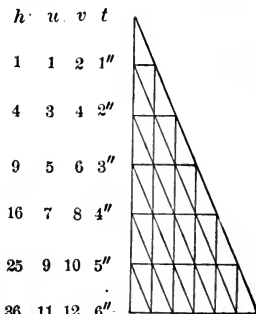


FIG. 100.

Angular velocity is a term used to denote the angle through which any radius of a body turns in a second, or the rate at which any point in it having a radius equal to unity is moving, expressed in feet per second. The unit of angular velocity is the angle which at a distance = radius from the center is subtended by an arc equal to the radius. This unit angle = $\frac{180}{\pi}$ degrees = 57.3° . $2\pi \times 57.3^\circ = 360^\circ$, or the circumference.

If A = angular velocity, $v = Ar$, $A = \frac{v}{r} = \frac{2\pi n}{60}$. The unit angle $\frac{180}{\pi}$ is called a *radian*.

Height Corresponding to a Given Acquired Velocity.

| Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. |
|---------------------|---------|---------------------|---------|---------------------|---------|---------------------|---------|---------------------|---------|---------------------|---------|
| feet per sec. | feet. | feet per sec. | feet. | feet per sec. | feet. | feet per sec. | feet. | feet per sec. | feet. | feet per sec. | feet. |
| .25 | 0.0010 | 13 | 2.62 | 34 | 17.9 | 55 | 47.0 | 76 | 89.8 | 97 | 146 |
| .50 | 0.0039 | 14 | 3.04 | 35 | 19.0 | 56 | 48.8 | 77 | 92.2 | 98 | 149 |
| .75 | 0.0087 | 15 | 3.49 | 36 | 20.1 | 57 | 50.5 | 78 | 94.6 | 99 | 152 |
| 1.00 | 0.016 | 16 | 3.98 | 37 | 21.3 | 58 | 52.3 | 79 | 97.0 | 100 | 155 |
| 1.25 | 0.024 | 17 | 4.49 | 38 | 22.4 | 59 | 54.1 | 80 | 99.5 | 105 | 171 |
| 1.50 | 0.035 | 18 | 5.03 | 39 | 23.6 | 60 | 56.0 | 81 | 102.0 | 110 | 188 |
| 1.75 | 0.048 | 19 | 5.61 | 40 | 24.9 | 61 | 57.9 | 82 | 104.5 | 115 | 205 |
| 2 | 0.062 | 20 | 6.22 | 41 | 26.1 | 62 | 59.8 | 83 | 107.1 | 120 | 224 |
| 2.5 | 0.097 | 21 | 6.85 | 42 | 27.4 | 63 | 61.7 | 84 | 109.7 | 130 | 263 |
| 3 | 0.140 | 22 | 7.52 | 43 | 28.7 | 64 | 63.7 | 85 | 112.3 | 140 | 304 |
| 3.5 | 0.190 | 23 | 8.21 | 44 | 30.1 | 65 | 65.7 | 86 | 115.0 | 150 | 350 |
| 4 | 0.248 | 24 | 8.94 | 45 | 31.4 | 66 | 67.7 | 87 | 117.7 | 175 | 476 |
| 4.5 | 0.314 | 25 | 9.71 | 46 | 32.9 | 67 | 69.8 | 88 | 120.4 | 200 | 622 |
| 5 | 0.388 | 26 | 10.5 | 47 | 34.3 | 68 | 71.9 | 89 | 123.2 | 300 | 1399 |
| 6 | 0.559 | 27 | 11.3 | 48 | 35.8 | 69 | 74.0 | 90 | 125.9 | 400 | 2488 |
| 7 | 0.761 | 28 | 12.2 | 49 | 37.3 | 70 | 76.2 | 91 | 128.7 | 500 | 3887 |
| 8 | 0.994 | 29 | 13.1 | 50 | 38.9 | 71 | 78.4 | 92 | 131.6 | 600 | 5597 |
| 9 | 1.26 | 30 | 14.0 | 51 | 40.4 | 72 | 80.6 | 93 | 134.5 | 700 | 7618 |
| 10 | 1.55 | 31 | 14.9 | 52 | 42.0 | 73 | 82.9 | 94 | 137.4 | 800 | 9952 |
| 11 | 1.83 | 32 | 15.9 | 53 | 43.7 | 74 | 85.1 | 95 | 140.3 | 900 | 12,593 |
| 12 | 2.24 | 33 | 16.9 | 54 | 45.3 | 75 | 87.5 | 96 | 143.3 | 1000 | 15,547 |

Parallelogram of Velocities. — The principle of the composition and resolution of forces may also be applied to velocities or to distances moved in given intervals of time. Referring to Fig. 93, page 489, if a body at O has a force applied to it which acting alone would give it a velocity represented by OQ per second, and at the same time it is acted on by another force which acting alone would give it a velocity OP per second, the result of the two forces acting together for one second will carry it to R , OR being the diagonal of the parallelogram of OQ and OP , and the resultant velocity. If the two component velocities are uniform, the resultant will be uniform and the line OR will be a straight line; but if either velocity is a varying one, the line will be a curve. Fig. 101 shows the resultant velocities, also the path traversed by a body acted on by two forces, one of which would carry it at a uniform velocity over the intervals 1, 2, 3, B and the other of which would carry it by an accelerated motion over the intervals a , b , c , D in the same times. At

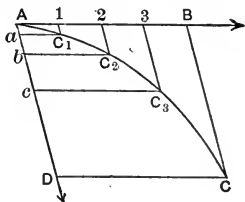


FIG. 101.

Falling Bodies: Velocity Acquired by a Body Falling a Given Height.

| Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. | Height. | Velocity. |
|---------|----------------|---------|----------------|---------|----------------|---------|----------------|---------|----------------|---------|----------------|
| feet. | feet p.sec. | feet. | feet p.sec. | feet. | feet p.sec. | feet. | feet p.sec. | feet. | feet p.sec. | feet. | feet p.sec. |
| 0.005 | .57 | 0.39 | 5.01 | 1.20 | 8.79 | 5. | 17.9 | 23. | 38.5 | 72 | 68.1 |
| 0.010 | .80 | 0.40 | 5.07 | 1.22 | 8.87 | .2 | 18.3 | .5 | 38.9 | 73 | 68.5 |
| 0.015 | .98 | 0.41 | 5.14 | 1.24 | 8.94 | .4 | 18.7 | 24. | 39.3 | 74 | 69.0 |
| 0.020 | 1.13 | 0.42 | 5.20 | 1.26 | 9.01 | .6 | 19.0 | .5 | 39.7 | 75 | 69.5 |
| 0.025 | 1.27 | 0.43 | 5.26 | 1.28 | 9.08 | .8 | 19.3 | 25. | 40.1 | 76 | 69.9 |
| 0.030 | 1.39 | 0.44 | 5.32 | 1.30 | 9.15 | 6. | 19.7 | 26 | 40.9 | 77 | 70.4 |
| 0.035 | 1.50 | 0.45 | 5.38 | 1.32 | 9.21 | .2 | 20.0 | 27 | 41.7 | 78 | 70.9 |
| 0.040 | 1.60 | 0.46 | 5.44 | 1.34 | 9.29 | .4 | 20.3 | 28 | 42.5 | 79 | 71.3 |
| 0.045 | 1.70 | 0.47 | 5.50 | 1.36 | 9.36 | .6 | 20.6 | 29 | 43.2 | 80 | 71.8 |
| 0.050 | 1.79 | 0.48 | 5.56 | 1.38 | 9.43 | .8 | 20.9 | 30 | 43.9 | 81 | 72.2 |
| 0.055 | 1.88 | 0.49 | 5.61 | 1.40 | 9.49 | 7. | 21.2 | 31 | 44.7 | 82 | 72.6 |
| 0.060 | 1.97 | 0.50 | 5.67 | 1.42 | 9.57 | .2 | 21.5 | 32 | 45.4 | 83 | 73.1 |
| 0.065 | 2.04 | 0.51 | 5.73 | 1.44 | 9.62 | .4 | 21.8 | 33 | 46.1 | 84 | 73.5 |
| 0.070 | 2.12 | 0.52 | 5.78 | 1.46 | 9.70 | .6 | 22.1 | 34 | 46.8 | 85 | 74.0 |
| 0.075 | 2.20 | 0.53 | 5.84 | 1.48 | 9.77 | .8 | 22.4 | 35 | 47.4 | 86 | 74.4 |
| 0.080 | 2.27 | 0.54 | 5.90 | 1.50 | 9.82 | 8. | 22.7 | 36 | 48.1 | 87 | 74.8 |
| 0.085 | 2.34 | 0.55 | 5.95 | 1.52 | 9.90 | .2 | 23.0 | 37 | 48.8 | 88 | 75.3 |
| 0.090 | 2.41 | 0.56 | 6.00 | 1.54 | 9.96 | .4 | 23.3 | 38 | 49.4 | 89 | 75.7 |
| 0.095 | 2.47 | 0.57 | 6.06 | 1.56 | 10.0 | .6 | 23.5 | 39 | 50.1 | 90 | 76.1 |
| 0.100 | 2.54 | 0.58 | 6.11 | 1.58 | 10.1 | .8 | 23.8 | 40 | 50.7 | 91 | 76.5 |
| 0.105 | 2.60 | 0.59 | 6.16 | 1.60 | 10.2 | 9. | 24.1 | 41 | 51.4 | 92 | 76.9 |
| 0.110 | 2.66 | 0.60 | 6.21 | 1.65 | 10.3 | .2 | 24.3 | 42 | 52.0 | 93 | 77.4 |
| 0.115 | 2.72 | 0.62 | 6.32 | 1.70 | 10.5 | .4 | 24.6 | 43 | 52.6 | 94 | 77.8 |
| 0.120 | 2.78 | 0.64 | 6.42 | 1.75 | 10.6 | .6 | 24.8 | 44 | 53.2 | 95 | 78.2 |
| 0.125 | 2.84 | 0.66 | 6.52 | 1.80 | 10.8 | .8 | 25.1 | 45 | 53.8 | 96 | 78.6 |
| 0.130 | 2.89 | 0.68 | 6.61 | 1.90 | 11.1 | 10. | 25.4 | 46 | 54.4 | 97 | 79.0 |
| 0.14 | 3.00 | 0.70 | 6.71 | 2. | 11.4 | .5 | 26.0 | 47 | 55.0 | 98 | 79.4 |
| 0.15 | 3.11 | 0.72 | 6.81 | 2.1 | 11.7 | 11. | 26.6 | 48 | 55.6 | 99 | 79.8 |
| 0.16 | 3.21 | 0.74 | 6.90 | 2.2 | 11.9 | .5 | 27.2 | 49 | 56.1 | 100 | 80.2 |
| 0.17 | 3.31 | 0.76 | 6.99 | 2.3 | 12.2 | 12. | 27.8 | 50 | 56.7 | 125 | 89.7 |
| 0.18 | 3.40 | 0.78 | 7.09 | 2.4 | 12.4 | .5 | 28.4 | 51 | 57.3 | 150 | 98.3 |
| 0.19 | 3.50 | 0.80 | 7.18 | 2.5 | 12.6 | 13. | 28.9 | 52 | 57.8 | 175 | 106 |
| 0.20 | 3.59 | 0.82 | 7.26 | 2.6 | 12.0 | .5 | 29.5 | 53 | 58.4 | 200 | 114 |
| 0.21 | 3.68 | 0.84 | 7.35 | 2.7 | 13.2 | 14. | 30.0 | 54 | 59.0 | 225 | 120 |
| 0.22 | 3.76 | 0.86 | 7.44 | 2.8 | 13.4 | .5 | 30.5 | 55 | 59.5 | 250 | 126 |
| 0.23 | 3.85 | 0.88 | 7.53 | 2.9 | 13.7 | 15. | 31.1 | 56 | 60.0 | 275 | 133 |
| 0.24 | 3.93 | 0.90 | 7.61 | 3. | 13.9 | .5 | 31.6 | 57 | 60.6 | 300 | 139 |
| 0.25 | 4.01 | 0.92 | 7.69 | 3.1 | 14.1 | 16. | 32.1 | 58 | 61.1 | 350 | 150 |
| 0.26 | 4.09 | 0.94 | 7.78 | 3.2 | 14.3 | .5 | 32.6 | 59 | 61.6 | 400 | 160 |
| 0.27 | 4.17 | 0.96 | 7.86 | 3.3 | 14.5 | 17. | 33.1 | 60 | 62.1 | 450 | 170 |
| 0.28 | 4.25 | 0.98 | 7.94 | 3.4 | 14.8 | .5 | 33.6 | 61 | 62.7 | 500 | 179 |
| 0.29 | 4.32 | 1.00 | 8.02 | 3.5 | 15.0 | 18. | 34.0 | 62 | 63.2 | 550 | 188 |
| 0.30 | 4.39 | 1.02 | 8.10 | 3.6 | 15.2 | .5 | 34.5 | 63 | 63.7 | 600 | 197 |
| 0.31 | 4.47 | 1.04 | 8.18 | 3.7 | 15.4 | 19. | 35.0 | 64 | 64.2 | 700 | 212 |
| 0.32 | 4.54 | 1.06 | 8.26 | 3.8 | 15.6 | .5 | 35.4 | 65 | 64.7 | 800 | 227 |
| 0.33 | 4.61 | 1.08 | 8.34 | 3.9 | 15.8 | 20. | 35.9 | 66 | 65.2 | 900 | 241 |
| 0.34 | 4.68 | 1.10 | 8.41 | 4. | 16.0 | .5 | 36.3 | 67 | 65.7 | 1000 | 254 |
| 0.35 | 4.74 | 1.12 | 8.49 | .2 | 16.4 | 21. | 36.8 | 68 | 66.1 | 2000 | 359 |
| 0.36 | 4.81 | 1.14 | 8.57 | .4 | 16.8 | .5 | 37.2 | 69 | 66.6 | 3000 | 439 |
| 0.37 | 4.88 | 1.16 | 8.64 | .6 | 17.2 | 22. | 37.6 | 70 | 67.1 | 4000 | 507 |
| 0.38 | 4.94 | 1.18 | 8.72 | .8 | 17.6 | .5 | 38.1 | 71 | 67.6 | 5000 | 567 |

the end of the respective intervals the body will be found at C_1, C_2, C_3, C , and the mean velocity during each interval is represented by the distances between these points. Such a curved path is traversed by a shot, the impelling force from the gun giving it a uniform velocity in the direction the gun is aimed, and gravity giving it an accelerated velocity downward. *The path of a projectile is a parabola.* The distance it will travel is greatest when its initial direction is at an angle 45° above the horizontal.

Mass — Force of Acceleration. — The mass of a body, $m = w/g$, is a constant quantity. If g = the acceleration due to gravity, and w = weight, then the mass $m = \frac{w}{g}$; $w = mg$. If the weight w is taken to be the

resultant of the force of gravity on the particles of a body, such as may be measured by a spring balance, or by the extension or deflection of a rod of metal loaded with the given weight, then the weight varies according to the variation in the force of gravity at different places, and the value of g is that at the place where the body is weighed; but if w is the weight as weighed on a platform scale, then $g = 32.2$, the English value. In either case $m = w/g$ is a constant.

Force has been defined as that which causes, or tends to cause, or to destroy, motion. It may also be defined as the cause of acceleration; and the unit of force, the pound, as the force required to produce an acceleration of 32.2 ft. per second per second in a pound of free mass.

Force equals the product of the mass by the acceleration, or $f = ma$.

Also, if v = the velocity acquired in the time t , $ft = mv$; $f = mv \div t$; the acceleration being uniform.

The force required to produce an acceleration of g (that is, 32.16 ft. per sec. in one second) is $f = mg = \frac{w}{g} g = w$, or the weight of the body. Also,

$f = ma = m \frac{v_2 - v_1}{t}$, in which v_2 is the velocity at the end, and v_1 the

velocity at the beginning of the time t , and $f = mg = \frac{w}{g} \frac{(v_2 - v_1)}{t} = \frac{w}{g} a$;

$\frac{f}{w} = \frac{a}{g}$; or, the force required to give any acceleration to a body is to the weight of the body as that acceleration is to the acceleration produced by gravity. (The weight w is the weight where g is measured.)

EXAMPLE. — Tension in a cord lifting a weight. A weight of 100 lbs. is lifted vertically by a cord a distance of 80 feet in 4 seconds, the velocity uniformly increasing from 0 to the end of the time. What tension must be maintained in the cord? Mean velocity = $v_m = 20$ ft. per sec.; final

velocity = $v_2 = 2 v_m = 40$; acceleration $a = \frac{v_2}{t} = \frac{40}{4} = 10$. Force $f =$

$ma = \frac{wa}{g} = \frac{100}{32.16} \times 10 = 31.1$ lbs. This is the force required to produce the acceleration only; to it must be added the force required to lift the weight without acceleration, or 100 lbs., making a total of 131.1 lbs.

The Resistance to Acceleration is the same as the force required to produce the acceleration = $\frac{w}{g} \frac{(v_2 - v_1)}{t}$.

Formulæ for Accelerated Motion. — For cases of uniformly accelerated motion other than those of falling bodies, we have the formulæ already given, $f = \frac{w}{g} a$, = $\frac{w}{g} \frac{v_2 - v_1}{t}$. If the body starts from rest, $v_1 = 0$.

$v_1 = v$, and $f = \frac{w}{g} \frac{v}{t}$; $fgt = wv$. We also have $s = \frac{vt}{2}$. Transforming and substituting for g its value 32.16, we obtain

$$f = \frac{wv^2}{64.32 s} = \frac{wv}{32.16 t} = \frac{ws}{16.08 t^2}; \quad w = \frac{32.16 ft}{v} = \frac{64.32 fs}{v^2};$$

$$s = \frac{wv^2}{64.32 f} = \frac{16.08 ft^2}{w} = \frac{vt}{2}; \quad v = 8.02 \sqrt{\frac{fs}{w}} = \frac{32.16 ft}{w};$$

$$t = \frac{wv}{32.16 f} = \frac{1}{4.01} \sqrt{\frac{ws}{f}}.$$

For any change in velocity, $f = w \left(\frac{v_2^2 - v_1^2}{64.32 s} \right)$.

(See also Work of Acceleration, under Work.)

Motion on Inclined Planes. — The velocity acquired by a body descending an inclined plane by the force of gravity (friction neglected) is equal to that acquired by a body falling freely from the height of the plane.

The times of descent down different inclined planes of the same height vary as the length of the planes.

The rules for uniformly accelerated motion apply to inclined planes. If a is the angle of the plane with the horizontal, $\sin a$ = the ratio of the height to the length = $\frac{h}{l}$, and the constant accelerating force is $g \sin a$.

The final velocity at the end of t seconds is $v = gt \sin a$. The distance passed over in t seconds is $l = \frac{1}{2} gt^2 \sin a$. The time of descent is

$$t = \sqrt{\frac{2l}{g \sin a}} = \frac{l}{4.01 \sqrt{h}}.$$

FUNDAMENTAL EQUATIONS IN DYNAMICS.

(1) $FS = \frac{1}{2} MV^2 = WH$. Force into space equals energy, or work.

(2) $FT = MV$. Force into time equals momentum.

(3) $F = MA = MV/T$. Force equals mass into acceleration.

(4) $V = \sqrt{2gH}$. Falling bodies.

The sign = here means "numerically equivalent to," the proper units of each elementary quantity being chosen.

M = mass = W/g ; W = weight in pounds, $g = 32.2$; F = force in pounds, exerted on a mass free to move; S = space, or distance in feet through which F is exerted; T = time in seconds; H = height in feet through which a body falls, or in eq. (1) is lifted; A = acceleration in feet per second per second, = V/T ; V = velocity in feet per second acquired at the end of the time T , the space S , or the height of fall H .

By these four equations and their algebraic transformations practically all problems in dynamics (except those relating to impact) may be solved.

MOMENTUM, VIS-VIVA.

Momentum, in many books erroneously defined as the quantity of motion in a body, is the product of the mass by the velocity at any instant,

$$= mv = \frac{w}{g} v.$$

Since the moving force = product of mass by acceleration, $f = ma$;

and if the velocity acquired in t seconds = v , or $a = \frac{v}{t}$, $f = \frac{mv}{t}$; $ft = mv$; that is, the product of a constant force into the time in which it acts equals numerically the momentum.

Since $ft = mv$, if $t = 1$ second $mv = f$, whence momentum might be defined as numerically equivalent to the number of pounds of force that will stop a moving body in 1 second, or the number of pounds of force which acting during 1 second will give it the given velocity.

Vis-viva, or living force, is a term used by early writers on Mechanics to denote the energy stored in a moving body. Some defined it as the product of the mass into the square of the velocity, mv^2 , = $\frac{w}{g} v^2$; others as one-half of this quantity, or $\frac{1}{2} mv^2$, or the same as what is now known as energy. The term is now obsolete, its place being taken by the word energy.

WORK, ENERGY, POWER.

Work is the overcoming of resistance through a certain distance. It is measured by the product of the resistance into the space through which it is overcome. It is also measured by the product of the moving force into the distance through which the force acts in overcoming the resistance. Thus in lifting a body from the earth against the attraction of gravity,

the resistance is the weight of the body, and the product of this weight into the height the body is lifted is the work done.

The Unit of Work, in British measures, is the *foot-pound*, or the amount of work done in overcoming a pressure or weight equal to one pound through one foot of space.

The work performed by a piston in driving a fluid before it, or by a fluid in driving a piston before it, may be expressed in either of the following ways:

$$\begin{aligned} & \text{Resistance} \times \text{distance traversed} \\ &= \text{intensity of pressure} \times \text{area} \times \text{distance traversed}; \\ &= \text{intensity of pressure} \times \text{volume traversed.} \end{aligned}$$

By intensity of pressure is meant pressure per unit of area, as lbs. per sq. in.

The work performed in lifting a body is the product of the weight of the body into the height through which its center of gravity is lifted.

If a machine lifts the centers of gravity of several bodies at once to heights either the same or different, the whole quantity of work performed in so doing is the sum of the several products of the weights and heights; but that quantity can also be computed by multiplying the sum of all the weights into the height through which their common center of gravity is lifted. (Rankine.)

Power is the rate at which work is done, and is expressed by the quotient of the work divided by the time in which it is done, or by units of work per second, per minute, etc., as foot-pounds per second. The most common unit of power is the *horse-power*, established by James Watt as the power of a strong London draught-horse to do work during a short interval, and used by him to measure the power of his steam-engines. This unit is 33,000 foot-pounds per minute = 550 foot-pounds per second = 1,980,000 foot-pounds per hour.

Expressions for Force, Work, Power, etc.

The fundamental conceptions in Dynamics are:

Mass, Force, Time, Space, represented by the letters M, F, T, S .

Mass = weight $\div g$. If the weight of a body is determined by a spring balance standardized at London it will vary with the latitude, and the value of g to be taken in order to find the mass is that of the latitude where the weighing is done. If the weight is determined by a balance or by a platform scale, as is customary in engineering and in commerce, the London value of g , = 32.2, is to be taken.

Velocity = space divided by time, $V = S \div T$, if V be uniform. $V = 2S \div T$ if V be uniformly accelerated.

Work = force multiplied by space = $FS = \frac{1}{2} MV^2 = FVT$ (V uniform).

Power = rate of work = work divided by time = $FS \div T = P =$ product of force into uniform velocity = FV .

Power exerted for a certain time produces work; $PT = FS = FVT$.

Effort is a force which acts on a body in the direction of its motion.

Resistance is that which is opposed to an acting force. It is equal and opposite to the force.

Horse-power Hours, an expression for work measured as the product of a power into the time during which it acts, = PT . Sometimes it is the summation of a variable power for a given time, or the average power multiplied by the time.

Energy, or stored work, is the capacity for performing work. It is measured by the same unit as work, that is, in foot-pounds. It may be either *potential*, as in the case of a body of water stored in a reservoir, capable of doing work by means of a water-wheel, or *actual*, sometimes called *kinetic*, which is the energy of a moving body. Potential energy is measured by the product of the weight of the stored body into the distance through which it is capable of acting, or by the product of the pressure it exerts into the distance through which that pressure is capable of acting. Potential energy may also exist as stored heat, or as stored chemical energy, as in fuel, gunpowder, etc., or as electrical energy, the measure of these energies being the amount of work that they are capable of performing. Actual energy of a moving body is the work which it is capable of performing against a retarding resistance before being brought to rest, and is equal to the work which must be done upon it to bring it from a state of rest to its actual velocity.

The measure of actual energy is the product of the weight of the body into the height from which it must fall to acquire its actual velocity. If v = the velocity in feet per second, according to the principle of falling bodies, h , the height due to the velocity, $= \frac{v^2}{2g}$; and if w = the weight, the energy $= \frac{1}{2} mv^2 = wv^2 \div 2g = wh$. Since energy is the capacity for performing work, the units of work and energy are equivalent, or $FS = \frac{1}{2} mv^2 = wh$. Energy exerted = work done.

The actual energy of a rotating body whose angular velocity is A and moment of inertia $\Sigma wr^2 = I$ is $\frac{A^2 I}{2g}$, that is, the product of the moment of inertia into the height due to the velocity, A , of a point whose distance from the axis of rotation is unity; or it is equal to $\frac{wv^2}{2g}$, in which w is the weight of the body and v is the velocity of the center of gyration.

Work of Acceleration. — The work done in giving acceleration to a body is equal to the product of the force producing the acceleration, or of the resistance to acceleration, into the distance moved in a given time. This force, as already stated, equals product of the mass into the acceleration, or $f = ma = \frac{w}{g} \frac{v_2 - v_1}{t}$. If the distance traversed in the time $t = s$,

$$\text{then work} = fs = \frac{w}{g} \frac{v_2 - v_1}{t} s.$$

EXAMPLE. — What work is required to move a body weighing 100 lbs. horizontally a distance of 80 ft. in 4 seconds, the velocity uniformly increasing, friction neglected?

Mean velocity $v_m = 20$ ft. per second; final velocity $= v_2 = 2 v_m = 40$; initial velocity $v_1 = 0$; acceleration, $a = \frac{v_2 - v_1}{t} = \frac{40}{4} = 10$; force $= \frac{w}{g} a = \frac{100}{32.16} \times 10 = 31.1$ lbs.; distance 80 ft.; work $= fs = 31.1 \times 80 = 2488$ foot-pounds.

The energy stored in the body moving at the final velocity of 40 ft. per second is

$$\frac{1}{2} mv^2 = \frac{1}{2} \frac{w}{g} v^2 = \frac{100 \times 40^2}{2 \times 32.16} = 2488 \text{ foot-pounds,}$$

which equals the work of acceleration,

$$fs = \frac{w}{g} \frac{v_2}{t} s = \frac{w}{g} \frac{v_2}{t} \frac{v_2}{2} t = \frac{1}{2} \frac{w}{g} v_2^2.$$

If a body of the weight W falls from a height H , the work of acceleration is simply WH , or the same as the work required to raise the body to the same height.

Work of Accelerated Rotation. — Let A = angular velocity of a solid body rotating about an axis, that is, the velocity of a particle whose radius is unity. Then the velocity of a particle whose radius is r is $v = Ar$. If the angular velocity is accelerated from A_1 to A_2 , the increase of the velocity of the particle is $v_2 - v_1 = r(A_2 - A_1)$, and the work of accelerating it is

$$\frac{w}{g} \times \frac{v_2^2 - v_1^2}{2} = \frac{wr^2}{g} \frac{A_2^2 - A_1^2}{2},$$

in which w is the weight of the particle. A is measured in radians.

The work of acceleration of the whole body is

$$\Sigma \left\{ \frac{w}{g} \times \frac{v_2^2 - v_1^2}{2} \right\} = \frac{A_2^2 - A_1^2}{2g} \times \Sigma wr^2.$$

The term Σwr^2 is the moment of inertia of the body.

"Force of the Blow" of a Steam Hammer or Other Falling Weight. — The question is often asked: "With what force does a falling hammer strike?" The question cannot be answered directly, and it is based upon a misconception or ignorance of fundamental mechanical

laws. The energy, or capacity of doing work, of a body raised to a given height and let fall cannot be expressed in pounds, simply, but only in foot-pounds, which is the product of the weight into the height through which it falls, or the product of its weight $\div 64.32$ into the square of the velocity, in feet per second, which it acquires after falling through the given height. If F = weight of the body, M its mass, g the acceleration due to gravity, S the height of fall, and v the velocity at the end of the fall, the energy in the body just before striking is $FS = \frac{1}{2} Mv^2 = Wv^2 \div 2g = Wv^2 \div 64.32$, which is the general equation of energy of a moving body. Just as the energy of the body is a product of a force into a distance, so the work it does when it strikes is not the manifestation of a force, which can be expressed simply in pounds, but it is the overcoming of a resistance through a certain distance, which is expressed as the product of the average resistance into the distance through which it is exerted. If a hammer weighing 100 lbs. falls 10 ft., its energy is 1000 foot-pounds. Before being brought to rest it must do 1000 foot-pounds of work against one or more resistances. These are of various kinds, such as that due to motion imparted to the body struck, penetration against friction, or against resistance to shearing or other deformation, and crushing and heating of both the falling body and the body struck. The distance through which these resisting forces act is generally indeterminate, and therefore the average of the resisting forces, which themselves generally vary with the distance, is also indeterminate.

Impact of Bodies. — If two inelastic bodies collide, they will move on together as one mass, with a common velocity. The momentum of the combined mass is equal to the sum of the momenta of the two bodies before impact. If m_1 and m_2 are the masses of the two bodies and v_1 and v_2 their respective velocities before impact, and v their common velocity after impact, $(m_1 + m_2)v = m_1v_1 + m_2v_2$,

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}.$$

If the bodies move in opposite directions, $v = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}$, or the velocity of two inelastic bodies after impact is equal to the algebraic sum of their momenta before impact, divided by the sum of their masses.

If two inelastic bodies of equal momenta impinge directly upon one another from opposite directions they will be brought to rest.

Impact of Inelastic Bodies Causes a Loss of Energy, and this loss is equal to the sum of the energies due to the velocities lost and gained by the bodies, respectively.

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} (m_1 + m_2) v^2 = \frac{1}{2} m_1 (v_1 - v)^2 + \frac{1}{2} m_2 (v_2 - v)^2$; in which $v_1 - v$ is the velocity lost by m_1 and $v - v_2$ the velocity gained by m_2 .

EXAMPLE. — Let $m_1 = 10$, $m_2 = 8$, $v_1 = 12$, $v_2 = 15$.

If the bodies collide they will come to rest, for $v = \frac{10 \times 12 - 8 \times 15}{10 + 8} = 0$.

The energy loss is

$\frac{1}{2} 10 \times 144 + \frac{1}{2} 8 \times 225 - \frac{1}{2} 18 \times 0 = \frac{1}{2} 10 (12 - 0)^2 + \frac{1}{2} 8 (15 - 0)^2 = 1620$ ft.-lbs.

What becomes of the energy lost? Ans. It is used doing internal work on the bodies themselves, changing their shape and heating them.

For *imperfectly elastic bodies*, let e = the elasticity, that is, the ratio which the force of restitution, or the internal force tending to restore the shape of a body after it has been compressed, bears to the force of compression; and let m_1 and m_2 be the masses, v_1 and v_2 their velocities before impact, and v_1' , v_2' their velocities after impact; then

$$v_1' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} - \frac{m_2e(v_1 - v_2)}{m_1 + m_2};$$

$$v_2' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} + \frac{m_1e(v_1 - v_2)}{m_1 + m_2}.$$

If the bodies are perfectly elastic, their relative velocities before and after impact are the same. That is, $v_1' - v_2' = v_2 - v_1$.

In the impact of bodies, the sum of their momenta after impact is the same as the sum of their momenta before impact.

$$m_1 v_1' + m_2 v_2' = m_1 v_1 + m_2 v_2.$$

For demonstration of these and other laws of impact, see Smith's Mechanics; also, Weisbach's Mechanics.

Energy of Recoil of Guns. (*Eng'g*, Jan. 25, 1884, p. 72.) —

Let W = the weight of the gun and carriage;

V = the maximum velocity of recoil;

w = the weight of the projectile;

v = the muzzle velocity of the projectile.

Then, since the momentum of the gun and carriage is equal to the momentum of the projectile (because both are acted on by equal force, the pressure of the gases in the gun, for equal time), we have $WV = wv$, or $V = wv \div W$.

Taking the case of a 10-inch gun firing a 400-lb. projectile with a muzzle velocity of 2000 feet per second, the weight of the gun and carriage being 22 tons = 50,000 lbs., we find the velocity of recoil =

$$V = \frac{2000 \times 400}{50,000} = 16 \text{ feet per second.}$$

Now the energy of a body in motion is $WV^2 \div 2g$.

Therefore the energy of recoil = $\frac{50,000 \times 16^2}{2 \times 32.2} = 198,800$ foot-pounds.

The energy of the projectile is $\frac{400 \times 2000^2}{2 \times 32.2} = 24,844,000$ foot-pounds.

Conservation of Energy. — No form of energy can ever be produced except by the expenditure of some other form, nor annihilated except by being reproduced in another form. Consequently the sum total of energy in the universe, like the sum total of matter, must always remain the same. (S. Newcomb.) Energy can never be destroyed or lost; it can be transformed, can be transferred from one body to another, but no matter what transformations are undergone, when the total effects of the exertion of a given amount of energy are summed up the result will be exactly equal to the amount originally expended from the source. This law is called the Conservation of Energy. (Cotterill and Slade.)

A heavy body sustained at an elevated position has potential energy. When it falls, just before it reaches the earth's surface it has actual or kinetic energy, due to its velocity. When it strikes, it may penetrate the earth a certain distance or may be crushed. In either case friction results by which the energy is converted into heat, which is gradually radiated into the earth or into the atmosphere, or both. Mechanical energy and heat are mutually convertible. Electric energy is also convertible into heat or mechanical energy, and either kind of energy may be converted into the other.

Sources of Energy. — The principal sources of energy on the earth's surface are the muscular energy of men and animals, the energy of the wind, of flowing water, and of fuel. These sources derive their energy from the rays of the sun. Under the influence of the sun's rays vegetation grows and wood is formed. The wood may be used as fuel under a steam-boiler, its carbon being burned to carbon dioxide. Three-tenths of its heat energy escapes in the chimney and by radiation, and seven-tenths appears as potential energy in the steam. In the steam-engine, of this seven-tenths six parts are dissipated in heating the condensing water and are wasted; the remaining one-tenth of the original heat energy of the wood is converted into mechanical work in the steam-engine, which may be used to drive machinery. This work is finally, by friction of various kinds, or possibly after transformation into electric currents, transformed into heat which is radiated into the atmosphere, increasing its temperature. Thus

all the potential heat energy of the wood is, after various transformations, converted into heat, which, mingling with the store of heat in the atmosphere, apparently is lost. But the carbon dioxide generated by the combustion of the wood is, again, under the influence of the sun's rays, absorbed by vegetation, and more wood may thus be formed having potential energy equal to the original.

Perpetual Motion. — The law of the conservation of energy, than which no law of mechanics is more firmly established, is an absolute barrier to all schemes for obtaining by mechanical means what is called "perpetual motion," or a machine which will do an amount of work greater than the equivalent of the energy, whether of heat, of chemical combination, of electricity, or mechanical energy, that is put into it. Such a result would be the creation of an additional store of energy in the universe, which is not possible by any human agency.

The Efficiency of a Machine is a fraction expressing the ratio of the useful work to the whole work performed, which is equal to the energy expended. The limit to the efficiency of a machine is unity, denoting the efficiency of a perfect machine in which no work is lost. The difference between the energy expended and the useful work done, or the loss, is usually expended either in overcoming friction or in doing work on bodies surrounding the machine from which no useful work is received. Thus in an engine propelling a vessel part of the energy exerted in the cylinder does the useful work of giving motion to the vessel, and the remainder is spent in overcoming the friction of the machinery and in making currents and eddies in the surrounding water.

A common and useful definition of efficiency is "output divided by input."

ANIMAL POWER.

Work of a Man against Known Resistances. (Rankine.)

| Kind of Exertion. | R , lbs. | V , ft. per sec. | $\frac{T''}{3600}$ (hours per day). | RV , ft.-lbs. per sec. | RVT , ft.-lbs. per day. |
|---|---------------|--------------------------|--|--------------------------------|---------------------------------|
| 1. Raising his own weight up stair or ladder..... | 143 | 0.5 | 8 | 71.5 | 2,059,200 |
| 2. Hauling up weights with rope, and lowering the rope un- loaded..... | 40 | 0.75 | 6 | 30 | 648,000 |
| 3. Lifting weights by hand..... | 44 | 0.55 | 6 | 24.2 | 522,720 |
| 4. Carrying weights up-stairs and returning unloaded.... | 143 | 0.13 | 6 | 18.5 | 399,600 |
| 5. Shoveling up earth to a height of 5 ft. 3 in..... | 6 | 1.3 | 10 | 7.8 | 280,800 |
| 6. Wheeling earth in barrow up slope of 1 in 12, $\frac{1}{2}$ horiz. veloc. 0.9 ft. per sec., and re- turning unloaded..... | 132 | 0.075 | 10 | 9.9 | 356,400 |
| 7. Pushing or pulling horizon- tally (capstan or oar)..... | 26.5 | 2.0 | 8 | 53 | 1,526,400 |
| 8. Turning a crank or winch.... | 12.5 | 5.0 | ? | 62.5 | |
| | 18.0 | 2.5 | 8 | 45 | 1,296,000 |
| | 20.0 | 14.4 | 2 min. | 288 | |
| 9. Working pump..... | 13.2 | 2.5 | 10 | 33 | 1,188,000 |
| 10. Hammering..... | 15 | ? | 8? | ? | 480,000 |

EXPLANATION. — R , resistance; V , effective velocity = distance through which R is overcome \div total time occupied, including the time of moving unloaded, if any; T'' , time of working, in seconds per day; $\frac{T''}{3600}$, same time, in hours per day; RV , effective power, in foot-pounds per second; RVT , daily work.

Performance of a Man in Transporting Loads Horizontally.
(Rankine.)

| Kind of Exertion. | L , lbs. | V , ft.-sec. | $\frac{T''}{3600}$ (hours per day). | LV , lbs. con- veyed 1 foot. | LVT , lbs. con- veyed 1 foot. |
|--|---------------|-------------------|--|--|--|
| 11. Walking unloaded, trans- porting his own weight.... | 140 | 5 | 10 | 700 | 25,200,000 |
| 12. Wheeling load L in 2-whld. barrow, return unloaded... | 224 | $12\frac{2}{3}$ | 10 | 373 | 13,428,000 |
| 13. Ditto in 1-wh. barrow, ditto.. | 132 | $12\frac{2}{3}$ | 10 | 220 | 7,920,000 |
| 14. Traveling with burden..... | 90 | $21\frac{1}{2}$ | 7 | 225 | 5,670,000 |
| 15. Carrying burden, returning unloaded..... | 140 | $12\frac{2}{3}$ | 6 | 233 | 5,032,800 |
| 16. Carrying burden, for 30 sec- onds only..... | 252 | 0 | | 0 | |
| | 126 | 11.7 | | 1474.2 | |
| | 0 | 23.1 | | 0 | |

EXPLANATION. — L , load; V , effective velocity, computed as before; T'' , time of working, in seconds per day; $T'' \div 3600$, same time in hours per day; LV , transport per second, in lbs. conveyed one foot; LVT , daily transport.

In the first line only of each of the two tables above is the weight of the man taken into account in computing the work done.

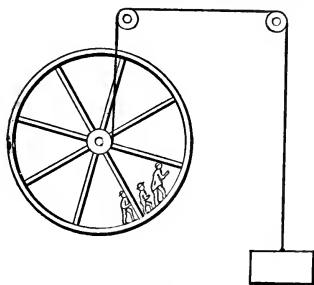


FIG. 102.

Clark says that the average net daily work of an ordinary laborer at a pump, a winch, or a crane may be taken at 3300 foot-pounds per minute, or one-tenth of a horse-power, for 8 hours a day; but for shorter periods from four to five times this rate may be exerted.

Mr. Glynn says that a man may exert a force of 25 lbs. at the handle of a crane for short periods; but that for continuous work a force of 15 lbs. is all that should be assumed, moving through 220 feet per minute.

Man-wheel.—Fig. 102 is a sketch of a very efficient man-power hoisting-machine which the author saw in Berne, Switzerland, in 1889.

The face of the wheel was wide enough for three men to walk abreast, so that nine men could work in it at one time.

Work of a Horse against a Known Resistance. (Rankine.)

| Kind of Exertion. | R . | V . | $\frac{T''}{3600}$ | RV . | RVT . |
|---|---|------------------|--------------------|-------------------|-----------|
| 1. Cantering and trotting, draw- ing a light railway carriage (thoroughbred)..... | { min. $221\frac{1}{2}$ mean $301\frac{1}{2}$ max. 50 } | $142\frac{2}{3}$ | 4 | $4471\frac{1}{2}$ | 6,444,000 |
| 2. Horse drawing cart or boat, walking (draught-horse)... | | | | | |
| 3. Horse drawing a gin or mill, walking..... | | | | | |
| 4. Ditto, trotting..... | 66 | 6.5 | $41\frac{1}{2}$ | 429 | 6,950,000 |

EXPLANATION. — *R*, resistance, in lbs.; *V*, velocity, in feet per second; *T*" ÷ 3600, hours work per day; *RV*, work per second; *RV**T*, work per day.

The average power of a draught-horse, as given in line 2 of the above table, being 432 foot-pounds per second, is $\frac{432}{550} = 0.785$ of the conventional value assigned by Watt to the ordinary unit of the rate of work of prime movers. It is the mean of several results of experiments, and may be considered the average of ordinary performance under favorable circumstances.

Performance of a Horse in Transporting Loads Horizontally.
(Rankine.)

| Kind of Exertion. | <i>L.</i> | <i>V.</i> | <i>T.</i> | <i>LV.</i> | <i>LVT.</i> |
|--|-----------|-----------|-----------|------------|-------------|
| 5. Walking with cart, always loaded..... | 1500 | 3.6 | 10 | 5400 | 194,400,000 |
| 6. Trotting, ditto..... | 750 | 7.2 | 4½ | 5400 | 87,480,000 |
| 7. Walking with cart, going loaded, returning empty; <i>V</i> , mean velocity..... | 1500 | 2.0 | 10 | 3000 | 108,000,000 |
| 8. Carrying burden, walking.. | 270 | 3.6 | 10 | 972 | 34,992,000 |
| 9. Ditto, trotting..... | 180 | 7.2 | 7 | 1296 | 32,659,200 |

EXPLANATION. — *L*, load in lbs.; *V*, velocity in feet per second; *T*, working hours per day; *LV*, transport per second; *LVT*, transport per day.

This table has reference to conveyance on common roads only, and those evidently in bad order as respects the resistance to traction upon them.

Horse-Gin. — In this machine a horse works less advantageously than in drawing a carriage along a straight track. In order that the best possible results may be realized with a horse-gin, the diameter of the circular track in which the horse walks should not be less than about forty feet.

Oxen, Mules, Asses. — Authorities differ considerably as to the power of these animals. The following may be taken as an approximative comparison between them and draught-horses (Rankine):

Ox. — Load, the same as that of average draught-horse; best velocity and work, two-thirds of horse.

Mule. — Load, one-half of that of average draught-horse; best velocity, the same as horse; work, one-half.

Ass. — Load, one-quarter that of average draught-horse; best velocity, the same; work, one-quarter.

Reduction of Draught of Horses by Increase of Grade of Roads. (*Engineering Record*, Prize Essays on Roads, 1892.) — Experiments on English roads by Gayffier & Parnell:

Calling load that can be drawn on a level 100:

On a rise of..... 1 in 100. 1 in 50. 1 in 40. 1 in 30. 1 in 26. 1 in 20. 1 in 10.
A horse can draw only 90 81 72 64 54 40 25

The Resistance of Carriages on Roads is (according to Gen. Morin) given approximately by the following empirical formula:

$$R = \frac{W}{r} [a + b (u - 3.28)].$$

In this formula *R* = total resistance; *r* = radius of wheel in inches; *W* = gross load; *u* = velocity in feet per second; while *a* and *b* are constants, whose values are: For good broken-stone road, *a* = 0.4 to 0.55, *b* = 0.024 to 0.026; for paved roads, *a* = 0.27, *b* = 0.0684.

Rankine states that on gravel the resistance is about double, and on sand five times, the resistance on good broken-stone roads.

ELEMENTS OF MACHINES.

The object of a machine is usually to transform the work or mechanical energy exerted at the point where the machine receives its motion into work at the point where the final resistance is overcome. The specific result may be to change the character or direction of motion, as from circular to rectilinear, or *vice versa*, to change the velocity, or to overcome a great resistance by the application of a moderate force. In all cases the total energy exerted equals the total work done; the latter including the overcoming of all the frictional resistances of the machine as well as the useful work performed. No increase of power can be obtained from any machine, since this is impossible according to the law of conservation of energy. In a frictionless machine the product of the force exerted at the driving-point into the velocity of the driving-point, or the distance it moves in a given interval of time, equals the product of the resistance into the distance through which the resistance is overcome in the same time.

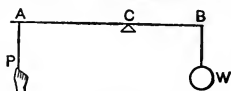


FIG. 103.

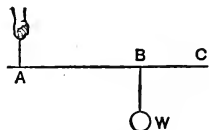


FIG. 104.

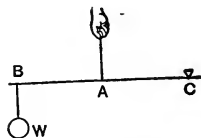


FIG. 105.

The most simple machines, or elementary machines, are reducible to three classes, viz., the Lever, the Cord, and the Inclined Plane.

The first class includes every machine consisting of a solid body capable of revolving on an axis, as the Wheel and Axle.

The second class includes every machine in which force is transmitted by means of flexible threads, ropes, etc., as the Pulley.

The third class includes every machine in which a hard surface inclined to the direction of motion is introduced, as the Wedge and the Screw.

A **Lever** is an inflexible rod capable of motion about a fixed point, called a fulcrum. The rod may be straight or bent at any angle, or curved.

It is generally regarded, at first, as without weight, but its weight may be considered as another force applied in a vertical direction at its center of gravity.

The arms of a lever are the portions of it intercepted between the force, P , and fulcrum, C , and between the weight or load, W , and fulcrum.

Levers are divided into three kinds or orders, according to the relative positions of the applied force, load, and fulcrum.

In a lever of the first order, the fulcrum lies between the points at which the force and load act. (Fig. 103.)

In a lever of the second order, the load acts at a point between the fulcrum and the point of action of the force. (Fig. 104.)

In a lever of the third order, the point of action of the force is between that of the load and the fulcrum. (Fig. 105.)

In all cases of levers the relation between the force exerted or the pull, P , and the load lifted, or resistance overcome, W , is expressed by the equation $P \times AC = W \times BC$, in which AC is the lever-arm of P , and BC is the lever-arm of W , or moment of the force = the moment of the resistance. (See Moment.)

In cases in which the direction of the force (or of the resistance) is not at right angles to the arm of the lever on which it acts, the "lever-arm" is the length of a perpendicular from the fulcrum to the line of direction of the force (or of the resistance). $W : P :: AC : BC$, or, the ratio of the resistance to the applied force is the inverse ratio of their lever-arms. Also, if V_w is the velocity of W , and V_p is the velocity of P , $W : P :: V_p : V_w$, and $P \times V_p = W \times V_w$.

If Sp is the distance through which the applied force acts, and Sw is the distance the load is lifted or through which the resistance is overcome, $W : P :: Sp : Sw$, or $W \times Sw = P \times Sp$, or the load into the dis-

tance it is lifted equals the force into the distance through which it is exerted.

These equations are general for all classes of machines as well as for levers, it being understood that friction, which in actual machines increases the resistance, is not at present considered.

The Bent Lever. — In the bent lever (see Fig. 96, p. 490), the lever-arm of the weight m is cf instead of bf . The lever is in equilibrium when $n \times af = m \times cf$, but it is to be observed that the action of a bent lever may be very different from that of a straight lever. In the latter, so long as the force and the resistance act in lines parallel to each other, the ratio of the lever-arms remains constant, although the lever itself changes its inclination with the horizontal. In the bent lever, however, this ratio changes: thus, in the cut, if the arm bf is depressed to a horizontal direction, the distance cf lengthens while the horizontal projection of af shortens, the latter becoming zero when the direction of af becomes vertical. As the arm af approaches the vertical, the weight m which may be lifted with a given force s is very great, but the distance through which it may be lifted is very small: In all cases the ratio of the weight m to the weight n is the inverse ratio of the horizontal projection of their respective lever-arms.

The Moving Strut (Fig. 106) is similar to the bent lever, except that one of the arms is missing, and that the force and the resistance to be overcome act at the same end of the single arm. The resistance in the case shown in the cut is not the load W , but its resistance to being moved, R , which may be simply that due to its friction on the horizontal plane, or some other opposing force. When the angle between the strut and the horizontal plane changes, the ratio of the resistance to the applied force changes. When the angle becomes very small, a moderate force will overcome a very great resistance, which tends to become infinite as the angle approaches zero. If a = the angle, $P \times \cos a = R \times \sin a$. If $a = 5$ degrees, $\cos a = 0.99619$, $\sin a = 0.08716$, $R = 11.44 P$.

The stone-crusher (Fig. 107) shows a practical example of the use of two moving struts.

The Toggle-joint is an elbow or knee-joint consisting of two bars so connected that they may be brought into a straight line and made to produce great endwise pressure when a force is applied to bring them into this position. It is a case of two moving struts placed end to end,

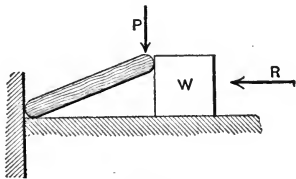


FIG. 106.

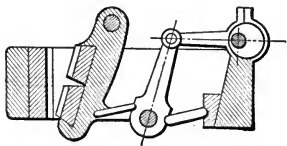


FIG. 107.

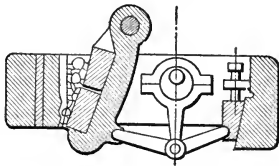


FIG. 108.

the moving force being applied at their point of junction, in a direction at right angles to the direction of the resistance, the other end of one of the struts resting against a fixed abutment, and that of the other against the body to be moved. If a = the angle each strut makes with the straight line joining the points about which their outer ends rotate, the ratio of the resistance to the applied force is $R : P :: \cos a : 2 \sin a$; $2 R \sin a = P \cos a$. The ratio varies when the angle varies, becoming infinite when the angle becomes zero.

The toggle-joint is used where great resistances are to be overcome through very small distances, as in stone-crushers (Fig. 108).

The Inclined Plane, as a mechanical element, is supposed perfectly hard and smooth, unless friction be considered. It assists in sustaining a heavy body by its reaction. This reaction, however, being normal to the plane, cannot entirely counteract the weight of the body, which acts vertically downward. Some other force must therefore be made to act upon the body, in order that it may be sustained.

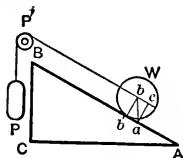


FIG. 109.

If the sustaining force act parallel to the plane (Fig. 109), the force is to the weight as the height of the plane is to its length, measured on the incline.

If the force act parallel to the base of the plane, the force is to the weight as the height is to the base.

If the force act at any other angle, let i = the angle of the plane with the horizon, and e = the angle of the direction of the applied force with the angle of the plane.
 $P : W :: \sin i : \cos e$; $P \times \cos e = W \sin i$.

Problems of the inclined plane may be solved by the parallelogram of forces thus:

Let the weight W be kept at rest on the incline by the force P , acting in the line bP' , parallel to the plane. Draw the vertical line ba to represent the weight; also bb' perpendicular to the plane, and complete the parallelogram $b'c$. Then the vertical weight ba is the resultant of bb' , the measure of support given by the plane to the weight, and bc , the force of gravity tending to draw the weight down the plane. The force required to maintain the weight in equilibrium is represented by this force bc . Thus the force and the weight are in the ratio of bc to ba . Since the triangle of forces abc is similar to the triangle of the incline ABC , the latter may be substituted for the former in determining the relative magnitude of the forces, and

$$P : W :: bc : ab :: BC : AB.$$

The Wedge is a pair of inclined planes united by their bases. In the application of pressure to the head or butt end of the wedge, to cause it to penetrate a resisting body, the applied force is to the resistance as the thickness of the wedge is to its length. Let t be the thickness, l the length, W the resistance, and P the applied force or pressure on the head of the wedge. Then, friction neglected, $P : W :: t : l$; $P = \frac{Wt}{l}$; $W = \frac{Pl}{t}$.

The Screw is an inclined plane wrapped around a cylinder in such a way that the height of the plane is parallel to the axis of the cylinder. If the screw is formed upon the internal surface of a hollow cylinder, it is usually called a nut. When force is applied to raise a weight or overcome a resistance by means of a screw and nut, either the screw or the nut may be fixed, the other being movable. The force is generally applied at the end of a wrench or lever-arm, or at the circumference of a wheel. If r = radius of the wheel or lever-arm, and p = pitch of the screw, or distance between threads, that is, the height of the inclined plane for one revolution of the screw, P = the applied force, and W = the resistance overcome, then, neglecting resistance due to friction, $2\pi r \times P = Wp$; $W = 6.283 Pr \div p$. The ratio of P to W is thus independent of the diameter of the screw.

In actual screws, much of the power transmitted is lost through friction.

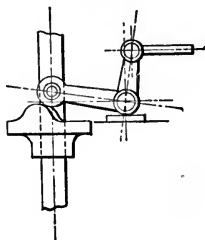


FIG. 111.

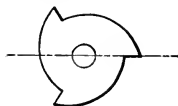


FIG. 110.

The Cam is a revolving inclined plane. It may be either an inclined plane wrapped around a cylinder in such a way that the height of the plane is radial to the cylinder, such as the ordinary lifting-cam, used in stamp-mills (Fig. 110),

or it may be an inclined plane curved edgewise, and rotating in a plane parallel to its base (Fig. 111). The relation of the weight to the applied force is calculated in the same manner as in the case of the screw.

Pulleys or Blocks. — P = force applied, or pull; W = load lifted, or resistance. In the simple pulley A (Fig. 112) the point P on the pulling rope descends the same amount that the load is lifted, therefore $P = W$. In B and C the point P moves twice as far as the load is lifted, therefore $W = 2P$. In B and C there is one movable block, and two plies of the rope engage with it. In D there are three sheaves in the movable block, each with two plies engaged, or six in all. Six plies of the rope are therefore shortened by the same amount that the load is lifted, and the point P moves six times as far as the load, consequently

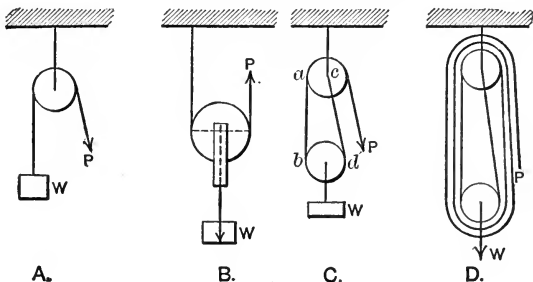


FIG. 112.

$W = 6P$. In general, the ratio of W to P is equal to the number of plies of the rope that are shortened, and also is equal to the number of plies that engage the lower block. If the lower block has 2 sheaves and the upper 3, the end of the rope is fastened to a hook in the top of the lower block, and then there are 5 plies shortened instead of 6, and $W = 5P$. If V = velocity of W , and v = velocity of P , then in all cases $VW = vP$, whatever the number of sheaves or their arrangement. If the hauling rope, at the pulling end, passes first around a sheave in the upper or stationary block, it makes no difference in what direction the rope is led from this block to the point at which the pull on the rope is applied; but if it first passes around the movable block, it is necessary that the pull be exerted in a direction parallel to the line of action of the resistance, or a line joining the centers of the two blocks, in order to obtain the maximum effect. If the rope pulls on the lower block at an angle, the block will be pulled out of the line drawn between the load and the upper block, and the effective pull will be less than the actual pull on the rope in the ratio of the cosine of the angle the pulling rope makes with the vertical, or line of action of the resistance, to unity.

Differential Pulley. (Fig. 113.) — Two pulleys, B and C , of different radii, rotate as one piece about a fixed axis, A . An endless chain, $BDECLKH$, passes over both pulleys. The rims of the pulleys are shaped so as to hold the chain and prevent it from slipping. One of the bights or loops in which the chain hangs, DE , passes under and supports the running block F . The other loop or bight, HKL , hangs freely, and is called the hauling part. It is evident that the velocity of the hauling part is equal to that of the pitch-circle of the pulley B .

In order that the velocity-ratio may be exactly uniform, the radius of the sheave F should be an exact mean between the radii of B and C .

Consider that the point B of the cord BD moves through an arc whose length = AB , during the same time the point C or the cord CE will

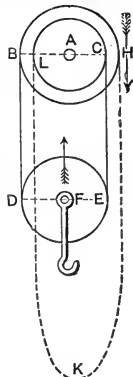


FIG. 113.

move downward a distance $= AC$. The length of the bight or loop $BDEC$ will be shortened by $AB - AC$, which will cause the pulley F to be raised half of this amount. If P = the pulling force on the cord HK , and W the weight lifted at F , then $P \times AB = W \times \frac{1}{2}(AB - AC)$.

To calculate the length of chain required for a differential pulley, take the following sum: Half the circumference of A + half the circumference of B + half the circumference of F + twice the greatest distance of F from A + the least length of loop HKL . The last quantity is fixed according to convenience.

The Differential Windlass (Fig. 114) is identical in principle with the differential pulley, the difference in construction being that in the differential windlass the running block hangs in the bight of a rope whose two parts are wound round, and have their ends respectively made fast to two barrels of different radii, which rotate as one piece about the axis A . The differential windlass is little used in practice, because of the great length of rope which it requires.

The Differential Screw (Fig. 115) is a compound screw of different pitches, in which the threads wind the same way. N_1 and N_2 are the two nuts; S_1S_1 , the longer-pitched thread; S_2S_2 , the shorter-pitched thread; in the figure both these threads are left-handed. At



FIG. 115.

each turn of the screw the nut N_2 advances relatively to N_1 through a distance equal to the difference of the pitches. The use of the differential screw is to combine the slowness of advance due to a fine pitch with the strength of thread which can be obtained by means of a coarse pitch only.

A Wheel and Axle, or Windlass, resembles two pulleys on one axis, having different diameters. If a weight be lifted by means of a rope wound over the axle, the force being applied at the rim of the wheel, the action is like that of a lever of which the shorter arm is equal to the radius of the axle plus half the thickness of the rope, and the longer arm is equal to the radius of the wheel. A wheel and axle is therefore sometimes classed as a perpetual lever. If P = the applied force, D = diameter of the wheel, W = the weight lifted, and d the diameter of the axle + the diameter of the rope, $PD = Wd$.

Toothed-wheel Gearing is a combination of two or more wheels and axles (Fig. 116). If a series of wheels and pinions gear into each other, as in the cut, friction neglected, the weight lifted, or resistance overcome, is to the force applied inversely as the distances through which they act in a given time. If R, R_1, R_2 be the radii of the successive wheels, measured to the pitch-line of the teeth, and r, r_1, r_2 the radii of the corresponding pinions, P the applied force, and W the weight lifted, $P \times R \times R_1 \times R_2 = W \times r \times r_1 \times r_2$, or the applied force is to the weight as the product of the radii of the pinions is to the product of the radii of the wheels; or, as the product of the numbers expressing the teeth in each pinion is to the product of the numbers expressing the teeth in each wheel.

Endless Screw, or Worm-gear. (Fig. 117.) — This gear is commonly used to convert motion at high speed into motion at very slow speed. When the handle P describes a complete circumference, the pitch-line of the cog-wheel moves through a distance equal to the pitch of the screw, and the weight W is lifted a distance equal to the pitch of the screw multiplied by the ratio of the diameter of the axle to the diameter of the pitch-circle of the wheel. The ratio of the applied force to the weight lifted is inversely as their velocities, friction not being considered; but the friction in the worm-gear is usually very great, amounting sometimes to three or four times the useful work done.

If v = the distance through which the force P acts in a given time, say 1 second, and V = distance the weight W is lifted in the same time, r = radius of the crank or wheel through which P acts, t = pitch of the screw,

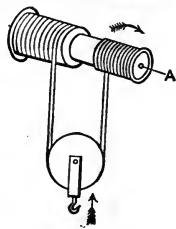


FIG. 114.

and also of the teeth on the cog-wheel, d = diameter of the axle, and D = diameter of the pitch-line of the cog-wheel, $v = \frac{6.283 r D}{t d} \times V$; $V = v \times td \div 6.283 r D$. $Pv = WV + \text{friction}$.

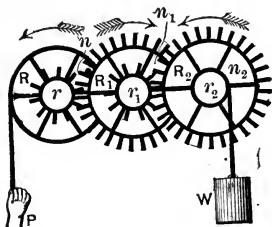


FIG. 116.

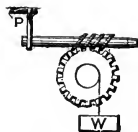


FIG. 117.

STRESSES IN FRAMED STRUCTURES.

Framed structures in general consist of one or more triangles, for the reason that the triangle is the one polygonal form whose shape cannot be changed without distorting one of its sides. Problems in stresses of simple framed structures may generally be solved either by the application of the triangle, parallelogram, or polygon of forces, by the principle of the lever, or by the method of moments. We shall give a few examples, referring the student to the works of Burr, Dubois, Johnson, and others for more elaborate treatment of the subject.

1. **A Simple Crane.** (Figs. 118 and 119.) — A is a fixed mast, B a brace or boom, T a tie, and P the load. Required the strains in B and T . The weight P , considered as acting at the end of the boom, is held in equilibrium by three forces: first, gravity acting downwards; second, the tension in T ; and third, the thrust of B . Let the length of the line p represent the magnitude of the downward force exerted by the load, and draw a parallelogram with sides bt parallel, respectively, to B and T , such that p is the diagonal of the parallelogram. Then b and t are the components drawn to the same scale as p , p being the resultant. Then if the length p represents the load, t is the tension in the tie, and b is the compression in the brace.

Or, more simply, T , B , and that portion of the mast included between them or A' may represent a triangle of forces, and the forces are proportional to the length of the sides of the triangle; that is, if the height of the

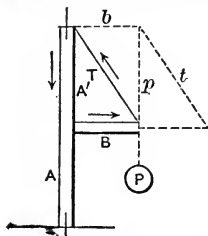


FIG. 118.

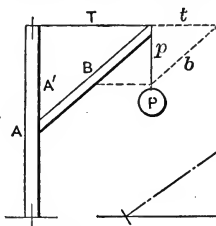


FIG. 119.

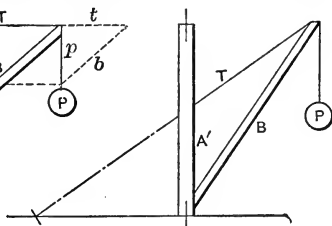


FIG. 120.

triangle $A' =$ the load, then $B =$ the compression in the brace, and $T =$ the tension in the tie; or if $P =$ the load in pounds, the tension in $T = P \times \frac{T}{A'}$, and the compression in $B = P \times \frac{B}{A'}$. Also, if $a =$ the angle the inclined member makes with the mast, the other member being

are of equal length, in which case $\frac{1}{2}$ of P is supported by each abutment C and D . If they are unequal in length (Fig. 124), then, by the principle of the lever, find the reactions of the abutments R_1 and R_2 . If P is the load applied at the point B on the lever CD , the fulcrum being D , then $R_1 \times CD = P \times BD$ and $R_2 \times CD = P \times BC$; $R_1 = P \times BD \div CD$; $R_2 = P \times BC \div CD$.

The strain on $AC = R_1 \times AC \div AB$, and on $AD = R_2 \times AD \div AB$.

The strain on the tie = $R_1 \times CB \div AB = R_2 \times BD \div AB$.

When $CB = BD$, $R_1 = R_2$. The strain on CB and BD is the same, whether the braces are of equal length or not, and is equal to $\frac{1}{2} P \times \frac{1}{2} CD \div AB$.

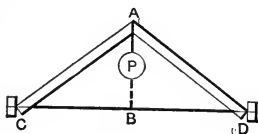


FIG. 123.

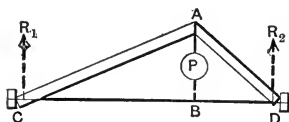


FIG. 124.

If the braces support a uniform load, as a pair of rafters, the strains caused by such a load are equivalent to that caused by one-half of the load applied at the center. The horizontal thrust of the braces against each other at the apex equals the tensile strain in the tie.

King-post Truss or Bridge. (Fig. 125.) — If the load is distributed over the whole length of the truss, the effect is the same as if half the load were placed at the center, the other half being carried by the abutments. Let P = one-half the load on the truss, then tension in the vertical tie $AB = P$. Compression in each of the inclined braces = $\frac{1}{2} P \times AD \div AB$. Tension in the tie $CD = \frac{1}{2} P \times BD \div AB$. Horizontal thrust of inclined brace AD at D = the tension in the tie. If W = the total load on one truss uniformly distributed, l = its length and d = its depth, then

the tension on the horizontal tie = $Wl \div 8d$.

Inverted King-post Truss. (Fig. 126.) — If P = a load applied at B , or one-half of a uniformly distributed load, then compression on $AB = P$ (the floor-beam CD not being considered to have any resistance to a slight bending). Tension on AC or $AD = \frac{1}{2} P \times AD \div AB$. Compression on $CD = \frac{1}{2} P \times BD \div AB$.

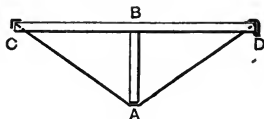


FIG. 126.

Queen-post Truss. (Fig. 127.) — If uniformly loaded, and the queen-posts divide the length into three equal bays, the load may be considered to be divided into three equal parts, two parts of which, P_1 and P_2 , are concentrated at the panel joints and the remainder

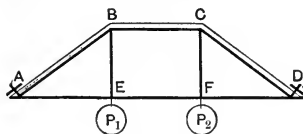


FIG. 127.

is equally divided between the abutments and supported by them directly. The two parts P_1 and P_2 only are considered to affect the members of the truss. Strain in the vertical ties BE and CF each equals P_1 or P_2 . Strain on AB and CD each = $P_1 \times CD \div CF$. Strain on the tie AE or EF or $ED = P_1 \times FD \div CF$. Thrust on BC = tension on EF .

For stability to resist heavy unequal loads the queen-post truss should have diagonal braces from B to F and from C to E .

Inverted Queen-post Truss. (Fig. 128.) — Compression on EB and FC each = P_1 or P_2 . Compression on AB or BC or $CD = P_1 \times AB \div EB$.

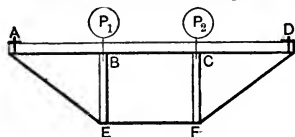


FIG. 128.

Tension on AE or $FD = P_1 \times AE \div EB$. Tension on EF = compression on BC . For stability to resist unequal loads, ties should be run from C to E and from B to F .

Burr Truss of Five Panels. (Fig. 129.) — Four-fifths of the load may be taken as concentrated at the points E, K, L and F , the other fifth being supported directly by the two abutments. For the strains in BA

and CD the truss may be considered as a queen-post truss, with the loads P_1, P_2 concentrated at E , and the loads P_3, P_4 concentrated at F . Then compressive strain on $AB = (P_1 + P_2) \times AB \div BE$. The strain on CD is the same if the loads and panel lengths are equal. The tensile

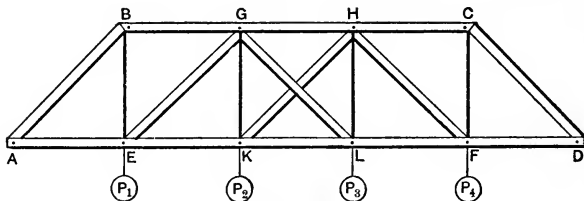


FIG. 129.

strain on BE or $CF = P_1 + P_2$. That portion of the truss between E and F may be considered as a smaller queen-post truss, supporting the loads P_2, P_3 at K and L . The strain on EG or $HF = P_2 \times EG \div GK$. The diagonals GL and KH receive no strain unless the truss is unequally loaded. The verticals GK and HL each receive a tensile strain equal to P_2 or P_3 .

For the strain in the horizontal members: BG and CH receive a thrust equal to the horizontal component of the thrust in AB or CD , $= (P_1 + P_2) \times \tan \text{angle } ABE$, or $(P_1 + P_2) \times AE \div BE$. GH receives this thrust, and also, in addition, a thrust equal to the horizontal component of the thrust in EG or HF , or, in all, $(P_1 + P_2 + P_3) \times AE \div BE$.

The tension in AE or FD equals the thrust in BG or HC , and the tension in EK, KL , and LF equals the thrust in GH .

Pratt or Whipple Truss. (Fig. 130.) — In this truss the diagonals are ties, and the verticals are struts or columns.

Calculation by the method of distribution of strains: Consider first the load P_1 . The truss having six bays or panels, $5/6$ of the load is transmitted to the abutment H , and $1/6$ to the abutment O , on the principle of the lever. As the five-sixths must be transmitted through JA and AH , write on these members the figure 5. The one-sixth is transmitted successively through JC, CK, KD, DL , etc., passing alternately through a tie and a strut. Write on these members, up to the strut GO inclusive, the figure 1. Then consider the load P_2 , of which $4/6$ goes to AH and $2/6$ to GO . Write on KB, BJ, JA , and AH the figure 4, and on KD, DL, LE , etc., the figure 2. The load P_2 transmits $3/6$ in each direction; write 3 on each of the members through which this stress passes, and so on for all the loads, when the figures on the several members will appear as on the cut. Adding them up, we have the following totals:

Tension on diagonals $\begin{Bmatrix} AJ & BH & BK & CJ & CL & DK & DM & EL & EN & FM & FO & GN \\ 15 & 0 & 10 & 1 & 6 & 3 & 3 & 6 & 1 & 10 & 0 & 15 \end{Bmatrix}$

Compression on verticals $\begin{Bmatrix} AH & BJ & CK & DL & EM & FN & GO \\ 15 & 10 & 7 & 6 & 7 & 10 & 15 \end{Bmatrix}$

Each of the figures in the first line is to be multiplied by $1/6 P \times \secant$ of angle HAF , or $1/6 P \times AJ \div AH$, to obtain the tension, and each

figure in the lower line is to be multiplied by $\frac{1}{6}P$ to obtain the compression. The diagonals HB and FO receive no strain.

It is common to build this truss with a diagonal strut at HB instead of the post HA and the diagonal AJ ; in which case $\frac{5}{6}$ of the load P is carried through JB and the strut BH , which latter then receives a strain $= \frac{15}{6}P \times \secant \text{ of } HBJ$.

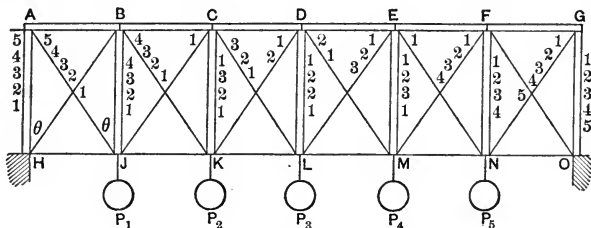


FIG. 130.

The strains in the upper and lower horizontal members or chords increase from the ends to the center, as shown in the case of the Burr truss. AB receives a thrust equal to the horizontal component of the tension in AJ , or $\frac{15}{6}P \times \tan \angle AJB$. BC receives the same thrust + the horizontal component of the tension in BK , and so on. The tension in the lower chord of each panel is the same as the thrust in the upper chord of the same panel. (For calculation of the chord strains by the method of moments, see below.)

The maximum thrust or tension is at the center of the chords and is equal to $\frac{WL}{8D}$, in which W is the total load supported by the truss, L is the length, and D the depth. This is the formula for maximum stress in the chords of a truss of any form whatever.

The above calculation is based on the assumption that all the loads P_1, P_2 , etc., are equal. If they are unequal, the value of each has to be taken into account in distributing the strains. Thus the tension in AJ , with unequal loads, instead of being $15 \times \frac{1}{6}P \secant \theta$ would be $\sec \theta \times (\frac{5}{6}P_1 + \frac{4}{6}P_2 + \frac{3}{6}P_3 + \frac{2}{6}P_4 + \frac{1}{6}P_5)$. Each panel load, P_1 , etc., includes its fraction of the weight of the truss.

General Formula for Strains in Diagonals and Verticals. — Let n = total number of panels, x = number of any vertical considered from the nearest end, counting the end as 1, r = rolling load for each panel, P = total load for each panel,

$$\text{Strain on verticals} = \frac{[(n-x) + (n-x)^2 - (x-1) + (x-1)^2] P}{2n} + \frac{r(x-1) + (x-1)^2}{2n}.$$

For a uniformly distributed load, leave out the last term,

$$[r(x-1) + (x-1)^2] \div 2n.$$

Strain on principal diagonals (AJ, GN , etc.) = strain on verticals $\times \secant \theta$, that is secant of the angle the diagonal makes with the vertical.

Strain on the counterbraces (BH, CJ, FO , etc.): The strain on the counterbrace in the first panel is 0, if the load is uniform. On the 2d, 3d, 4th, etc., it is $P \secant \theta \times \frac{1}{n}, \frac{1+2}{n}, \frac{1+2+3}{n}$, etc., P being the total load in one panel.

Strain in the Chords — Method of Moments. — Let the truss be uniformly loaded, the total load acting on it = W . Weight supported at each end, or reaction of the abutment = $W/2$. Length of the truss = L . Weight on a unit of length = W/L . Horizontal distance from the nearest abutment to the point (say M in Fig. 130) in the chord where the strain is to be determined = x . Horizontal strain at that point (tension on the lower chord, compression in the upper) = H . Depth of the truss = D .

By the method of moments we take the difference of the moments, about the point M , of the reaction of the abutment and of the load between M and the abutments, and equate that difference with the moment of the resistance, or of the strain in the horizontal chord, considered with reference to a point in the opposite chord, about which the truss would turn if the first chord were severed at M .

The moment of the reaction of the abutment is $Wx/2$. The moment of the load from the abutment to M is $(W/Lx) \times$ the distance of its center of gravity from M , which is $x/2$, or moment = $Wx^2 \div 2L$. Moment of the stress in the chord = $HD = \frac{Wx}{2} - \frac{Wx^2}{2L}$, whence $H = \frac{W}{2D} \left(x - \frac{x^2}{L} \right)$.

If $x = 0$ or L , $H = 0$. If $x = L/2$, $H = \frac{WL}{8D}$, which is the horizontal strain at the middle of the chords, as before given.

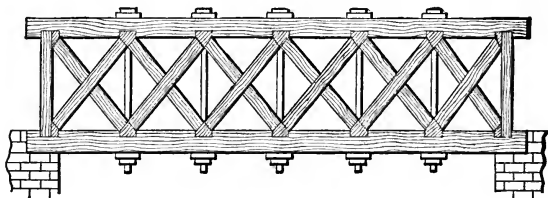


FIG. 131.

The Howe Truss. (Fig. 131.) — In the Howe truss the diagonals are struts, and the verticals are ties. The calculation of strains may be made in the same method as described above for the Pratt truss.

The Warren Girder. (Fig. 132.) — In the Warren girder, or triangular truss, there are no vertical struts, and the diagonals may transmit either tension or compression. The strains in the diagonals may be calculated by the method of distribution of strains as in the case of the rectangular truss.

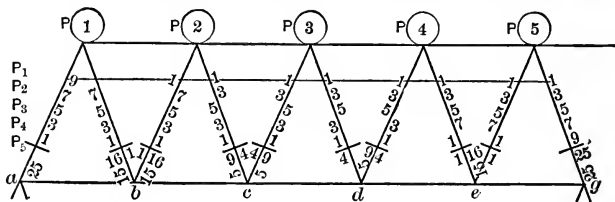


FIG. 132.

On the principle of the lever, the load P_1 being $1/10$ of the length of the span from the line of the nearest support a , transmits $9/10$ of its weight to a and $1/10$ to g . Write 9 on the right hand of the strut $1a$, to represent the compression, and 1 on the right hand of $1b$, $2c$, $3d$, etc., to represent tension, and on the left hand of $b2$, $c3$, etc., to represent tension. The load P_2 transmits $7/10$ of its weight to a and $3/10$ to g . Write 7 on each member from 2 to a , and 3 on each member from 2 to g , placing the figures representing compression on the right hand of the member, and those representing tension on the left. Proceed in the same manner with all the loads, then sum up the figures on each side of each diagonal, and write the difference of each sum beneath, and on the side of the greater sum, to show whether the difference represents tension or compression. The results are as follows: Compression, $1a$, 25; $2b$, 15; $3c$, 5; $3d$, 5; $4e$, 15; $5g$, 25. Tension, $1b$, 15; $2c$, 5; $4d$, 5; $5e$, 15. Each of these figures is to

be multiplied by $1/10$ of one of the loads as P_1 , and by the secant of the angle the diagonals make with a vertical line.

The strains in the horizontal chords may be determined by the method of moments as in the case of rectangular trusses.

Roof-truss. — *Solution by Method of Moments.* — The calculation of strains in structures by the method of statical moments consists in taking a cross-section of the structure at a point where there are not more than three members (struts, braces, or chords).

To find the strain in either one of these members take the moment about the intersection of the other two as an axis of rotation. The sum of the moments of these members must be 0 if the structure is in equilibrium. But the moments of the two members that pass through the point of reference or axis are both 0, hence one equation containing one unknown quantity can be found for each cross-section.

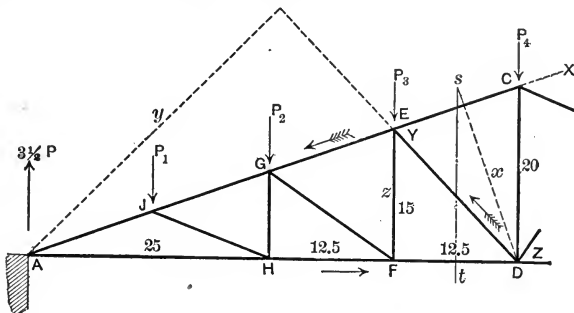


FIG. 133.

In the truss shown in Fig. 133 take a cross-section at ts , and determine the strain in the three members cut by it, viz., CE , ED , and DF . Let X = force exerted in direction CE , Y = force exerted in direction DE , Z = force exerted in direction FD .

For X take its moment about the intersection of Y and Z at $D = Xx$.
 For Y take its moment about the intersection of X and Z at $A = Yy$.
 For Z take its moment about the intersection of X and Y at $E = Zz$.
 Let $z = 15$, $x = 18.6$, $y = 38.4$, $AD = 50$, $CD = 20$ ft. Let P_1 , P_2 , P_3 , P_4 be equal loads, as shown, and $3\frac{1}{2}P$ the reaction of the abutment A .

The sum of all the moments taken about D or A or E will be 0 when the structure is at rest. Then $-Xx + 3.5P \times 50 - P_3 \times 12.5 - P_2 \times 25 - P_1 \times 37.5 = 0$.

The + signs are for moments in the direction of the hands of a watch or "clockwise" and - signs for the reverse direction or anti-clockwise. Since $P = P_1 = P_2 = P_3$, $-18.6 X + 175 P - 75 P = 0$; $-18.6 X = -100 P$;

$$\begin{aligned} -Yy + P_3 \times 37.5 + P_2 \times 25 \div P_1 \times 12.5 &= 0; \quad 38.4 Y = 75; \quad Y = \\ &= 75 P \div 38.4 = 1.953 P. \\ -Zz + 3.5 P \times 37.5 - P_1 \times 25 - P_2 \times 12.5 - P_3 \times 0 &= 0; \quad 15 Z = \\ &= 93.75 P; \quad Z = 6.25 P. \end{aligned}$$

In the same manner the forces exerted in the other members have been found as follows: $EG = 6.73 P$; $GJ = 8.07 P$; $JA = 9.42 P$; $JH = 1.35 P$; $GF = 1.59 P$; $AH = 8.75 P$; $HF = 7.50 P$.

The Fink Roof-truss. (Fig. 134.)—An analysis by Prof. P. H. Hilbrich (*Van N., Mag.*, Aug., 1880) gives the following results:

W = total load on roof:

N = No. of panels on both rafters:

$$W/N = P = \text{load at each joint } b, d, f, \text{ etc.:}$$
$$V = \text{reaction at } A = \frac{1}{2} W = \frac{1}{2} NP = 4P;$$
 $AD = S; AC = L; CD = D;$ $t_1, t_2, t_3 =$ tension on De, eg, gA , respectively; $c_1, c_2, c_3, c_4 =$ compression on Cb, bd, df , and fA .

Strains in

- | | |
|---------------------------------------|--|
| 1, or $De = t_1 = 2 PS \div D$; | 7, or $bC = c_1 = 7/2 PL/D - 3 PD/L$; |
| 2, " $eg = t_2 = 3 PS \div D$; | 8, " bc or $fg = PS \div L$; |
| 3, " $gA = t_3 = 7/2 PS \div D$; | 9, " $de = 2 PS \div L$; |
| 4, " $Af = c_4 = 7/2 PL \div D$; | 10, " cd or $dg = 1/2 PS \div D$; |
| 5, " $fd = c_3 = 7/2 PL/D - PD/L$; | 11, " $ec = PS \div D$; |
| 6, " $db = c_2 = 7/2 PL/D - 2 PD/L$; | 12, " $cC = 3/2 PS \div D$. |

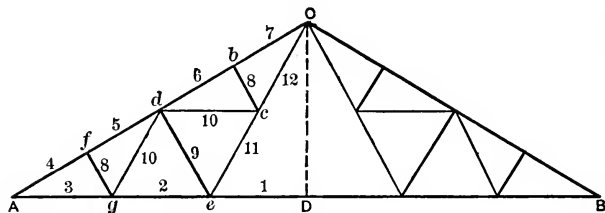


FIG. 134.

EXAMPLE. — Given a Fink roof-truss of span 64 ft., depth 16 ft., with four panels on each side, as in hecut; total load 32 tons, or 4 tons each at the points f, d, b, C , etc. (and 2 tons each at A and B , which transmit no strain to the truss members). Here $W = 32$ tons, $P = 4$ tons, $S = 32$ ft., $D = 16$ ft., $L = \sqrt{S^2 + D^2} = 2.236 \times D$. $L \div D = 2.236$, $D \div L = 0.4472$, $S \div D = 2$, $S \div L = 0.8944$. The strains on the numbered members then are as follows:

| | |
|---|---|
| 1, $2 \times 4 \times 2 = 16$ tons; | 7, $31.3 - 12 \times 0.447 = 25.94$ tons. |
| 2, $3 \times 4 \times 2 = 24$ " | 8, $4 \times 0.8944 = 3.58$ " |
| 3, $7/2 \times 4 \times 2 = 28$ " | 9, $8 \times 0.8944 = 7.16$ " |
| 4, $7/2 \times 4 \times 2.236 = 31.3$ " | 10, $2 \times 2 = 4$ " |
| 5, $31.3 - 4 \times 0.447 = 29.52$ " | 11, $4 \times 2 = 8$ " |
| 6, $31.3 - 8 \times 0.447 = 27.72$ " | 12, $6 \times 2 = 12$ " |

The Economical Angle. — A structure of triangular form, Fig. 135, is supported at a and b . It sustains any load L , the elements cc being in compression and t in tension. Required the angle θ so that the total weight of the structure shall be a minimum. F. R. Honey (*Sci. Am. Supp.*, Jan. 17, 1891) gives a solution of this problem, with the

result $\tan \theta = \sqrt{\frac{C+T}{T}}$, in which C and T represent

the crushing and the tensile strength respectively of the material employed. It is applicable to any material. For $C = T$, $\theta = 54\frac{3}{4}^\circ$. For $C = 0.4 T$ (yellow pine), $\theta = 49\frac{3}{4}^\circ$. For $C = 0.8 T$ (soft steel), $\theta = 53\frac{1}{4}^\circ$. For $C = 6 T$ (cast iron), $\theta = 69\frac{1}{4}^\circ$.

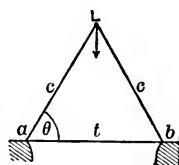


FIG. 135.

HEAT.

THERMOMETERS.

The Fahrenheit thermometer is generally used in English-speaking countries, and the Centigrade, or Celsius, thermometer in countries that use the metric system. In many scientific treatises in English, however, the Centigrade temperatures are also used, either with or without their Fahrenheit equivalents. The Réaumur thermometer is used to some extent on the Continent of Europe.

In the Fahrenheit thermometer the freezing-point of water is taken at 32°, and the boiling-point of water at mean atmospheric pressure at the sea-level, 14.7 lbs. per sq. in., is taken at 212°, the distance between these two points being divided into 180°. In the Centigrade and Réaumur thermometers the freezing-point is taken at 0°. The boiling-point is 100° in the Centigrade scale, and 80° in the Réaumur.

| | | |
|------------------------|------------------------|------------------------|
| 1 Fahrenheit degree | = 5/9 deg. Centigrade | = 4/9 deg. Réaumur. |
| 1 Centigrade degree | = 9/5 deg. Fahrenheit | = 4/5 deg. Réaumur. |
| 1 Réaumur degree | = 9/4 deg. Fahrenheit | = 5/4 deg. Centigrade. |
| Temperature Fahrenheit | = 9/5 × temp. C. + 32° | = 9/4 R. + 32°. |
| Temperature Centigrade | = 5/9 (temp. F. - 32°) | = 5/4 R. |
| Temperature Réaumur | = 4/5 temp. C. | = 4/9 (F. - 32°). |

HANDY RULE FOR CONVERTING CENTIGRADE TEMPERATURE TO FAHRENHEIT. — Multiply by 2, subtract a tenth, add 32.

EXAMPLE. — 100° C. × 2 = 200, - 20 = 180, + 32 = 212° F.

Mercurial Thermometer. (Rankine, *S. E.*, p. 234.) — The rate of expansion of mercury with rise of temperature increases as the temperature becomes higher; from which it follows, that if a thermometer showing the dilatation of mercury simply were made to agree with an air thermometer at 32° and 212°, the mercurial thermometer would show lower temperatures than the air thermometer between those standard points, and higher temperatures beyond them.

For example, according to Regnault, when the air thermometer marked 350° C. (= 662° F.), the mercurial thermometer would mark 362.16° C. (= 683.89° F.), the error of the latter being in excess 12.16° C. (= 21.89° F.).

Actual mercurial thermometers indicate intervals of temperature proportional to the difference between the expansion of mercury and that of glass.

The inequalities in the rate of expansion of the glass (which are very different for different kinds of glass) correct, to a greater or less extent, the errors arising from the inequalities in the rate of expansion of the mercury.

For practical purposes connected with heat engines, the mercurial thermometer made of common glass may be considered as sensibly coinciding with the air-thermometer at all temperatures not exceeding 500° F.

If the mercury is not throughout its whole length at the same temperature as that being measured, a correction, k , must be added to the temperature t in Fahrenheit degrees; $k = 95 D (t - t') \div 1,000,000$, where D is the length of the mercury column exposed, measured in Fahrenheit degrees, and t' is the temperature of the exposed part of the thermometer. When long thermometers are used in shallow wells in high-pressure steam pipes this correction is often 5° to 10° F. (Moyer on Steam Turbines.)

PYROMETRY.

Principles Used in Various Pyrometers.

Pyrometers may be classified according to the principles upon which they operate, as follows:

1. Expansion of mercury in a glass tube. When the space above the mercury is filled with compressed nitrogen, and a specially hard glass is used for the tube, mercury thermometers may be made to indicate temperatures as high as 1000° F.

TEMPERATURES, CENTIGRADE AND FAHRENHEIT.

| C. | F. | C. | F. | C. | F. | C. | F. | C. | F. | C. | F. | C. | F. |
|-----|-------|----|-------|-----|-------|-----|-------|-----|-------|-----|------|------|------|
| -40 | -40. | 26 | 78.8 | 92 | 197.6 | 158 | 316.4 | 224 | 435.2 | 290 | 554 | 950 | 1742 |
| -39 | -38.2 | 27 | 80.6 | 93 | 199.4 | 159 | 318.2 | 225 | 437. | 300 | 572 | 960 | 1760 |
| -38 | -36.4 | 28 | 82.4 | 94 | 201.2 | 160 | 320. | 226 | 438.8 | 310 | 590 | 970 | 1778 |
| -37 | -34.6 | 29 | 84.2 | 95 | 203. | 161 | 321.8 | 227 | 440.6 | 320 | 608 | 980 | 1796 |
| -36 | -32.8 | 30 | 86. | 96 | 204.8 | 162 | 323.6 | 228 | 442.4 | 330 | 626 | 990 | 1814 |
| -35 | -31. | 31 | 87.8 | 97 | 206.6 | 163 | 325.4 | 229 | 444.2 | 340 | 644 | 1000 | 1832 |
| -34 | -29.2 | 32 | 89.6 | 98 | 208.4 | 164 | 327.2 | 230 | 446. | 350 | 662 | 1010 | 1850 |
| -33 | -27.4 | 33 | 91.4 | 99 | 210.2 | 165 | 329. | 231 | 447.8 | 360 | 680 | 1020 | 1868 |
| -32 | -25.6 | 34 | 93.2 | 100 | 212. | 166 | 330.8 | 232 | 449.6 | 370 | 698 | 1030 | 1886 |
| -31 | -23.8 | 35 | 95. | 101 | 213.8 | 167 | 332.6 | 233 | 451.4 | 380 | 716 | 1040 | 1904 |
| -30 | -22. | 36 | 96.8 | 102 | 215.6 | 168 | 334.4 | 234 | 453.2 | 390 | 734 | 1050 | 1922 |
| -29 | -20.2 | 37 | 98.6 | 103 | 217.4 | 169 | 336.2 | 235 | 455. | 400 | 752 | 1060 | 1940 |
| -28 | -18.4 | 38 | 100.4 | 104 | 219.2 | 170 | 338. | 236 | 456.8 | 410 | 770 | 1070 | 1958 |
| -27 | -16.6 | 39 | 102.2 | 105 | 221. | 171 | 339.8 | 237 | 458.6 | 420 | 788 | 1080 | 1976 |
| -26 | -14.8 | 40 | 104. | 106 | 222.8 | 172 | 341.6 | 238 | 460.4 | 430 | 806 | 1090 | 1994 |
| -25 | -13. | 41 | 105.8 | 107 | 224.6 | 173 | 343.4 | 239 | 462.2 | 440 | 824 | 1100 | 2012 |
| -24 | -11.2 | 42 | 107.6 | 108 | 226.4 | 174 | 345.2 | 240 | 464. | 450 | 842 | 1110 | 2030 |
| -23 | -9.4 | 43 | 109.4 | 109 | 228.2 | 175 | 347. | 241 | 465.8 | 460 | 860 | 1120 | 2048 |
| -22 | -7.6 | 44 | 111.2 | 110 | 230. | 176 | 348.8 | 242 | 467.6 | 470 | 878 | 1130 | 2066 |
| -21 | -5.8 | 45 | 113. | 111 | 231.8 | 177 | 350.6 | 243 | 469.4 | 480 | 896 | 1140 | 2084 |
| -20 | -4. | 46 | 114.8 | 112 | 233.6 | 178 | 352.4 | 244 | 471.2 | 490 | 914 | 1150 | 2102 |
| -19 | -2.2 | 47 | 116.6 | 113 | 235.4 | 179 | 354.2 | 245 | 473. | 500 | 932 | 1160 | 2120 |
| -18 | -0.4 | 48 | 118.4 | 114 | 237.2 | 180 | 356. | 246 | 474.8 | 510 | 950 | 1170 | 2138 |
| -17 | + | 49 | 120.2 | 115 | 239. | 181 | 357.8 | 247 | 476.6 | 520 | 968 | 1180 | 2156 |
| -16 | 3.2 | 50 | 122. | 116 | 240.8 | 182 | 359.6 | 248 | 478.4 | 530 | 986 | 1190 | 2174 |
| -15 | 5. | 51 | 123.8 | 117 | 242.6 | 183 | 361.4 | 249 | 480.2 | 540 | 1004 | 1200 | 2192 |
| -14 | 6.8 | 52 | 125.6 | 118 | 244.4 | 184 | 363.2 | 250 | 482. | 550 | 1022 | 1210 | 2210 |
| -13 | 8.6 | 53 | 127.4 | 119 | 246.2 | 185 | 365. | 251 | 483.8 | 560 | 1040 | 1220 | 2228 |
| -12 | 10.4 | 54 | 129.2 | 120 | 248. | 186 | 366.8 | 252 | 485.6 | 570 | 1058 | 1230 | 2246 |
| -11 | 12.2 | 55 | 131. | 121 | 249.8 | 187 | 368.6 | 253 | 487.4 | 580 | 1076 | 1240 | 2264 |
| -10 | 14. | 56 | 132.8 | 122 | 251.6 | 188 | 370.4 | 254 | 489.2 | 590 | 1094 | 1250 | 2282 |
| -9 | 15.8 | 57 | 134.6 | 123 | 253.4 | 189 | 372.2 | 255 | 491. | 600 | 1112 | 1260 | 2300 |
| -8 | 17.6 | 58 | 136.4 | 124 | 255.2 | 190 | 374. | 256 | 492.8 | 610 | 1130 | 1270 | 2318 |
| -7 | 19.4 | 59 | 138.2 | 125 | 257. | 191 | 375.8 | 257 | 494.6 | 620 | 1148 | 1280 | 2336 |
| -6 | 21.2 | 60 | 140. | 126 | 258.8 | 192 | 377.6 | 258 | 496.4 | 630 | 1166 | 1290 | 2354 |
| -5 | 23. | 61 | 141.8 | 127 | 260.6 | 193 | 379.4 | 259 | 498.2 | 640 | 1184 | 1300 | 2372 |
| -4 | 24.8 | 62 | 143.6 | 128 | 262.4 | 194 | 381.2 | 260 | 500. | 650 | 1202 | 1310 | 2390 |
| -3 | 26.6 | 63 | 145.4 | 129 | 264.2 | 195 | 383. | 261 | 501.8 | 660 | 1220 | 1320 | 2408 |
| -2 | 28.4 | 64 | 147.2 | 130 | 266. | 196 | 384.8 | 262 | 503.6 | 670 | 1238 | 1330 | 2426 |
| -1 | 30.2 | 65 | 149. | 131 | 267.8 | 197 | 386.6 | 263 | 505.4 | 680 | 1256 | 1340 | 2444 |
| 0 | 32. | 66 | 150.8 | 132 | 269.6 | 198 | 388.4 | 264 | 507.2 | 690 | 1274 | 1350 | 2462 |
| + | 33.8 | 67 | 152.6 | 133 | 271.4 | 199 | 390.2 | 265 | 509. | 700 | 1292 | 1360 | 2480 |
| 1 | 35.6 | 68 | 154.4 | 134 | 273.2 | 200 | 392. | 266 | 510.8 | 710 | 1310 | 1370 | 2498 |
| 2 | 37.4 | 69 | 156.2 | 135 | 275. | 201 | 393.8 | 267 | 512.6 | 720 | 1328 | 1380 | 2516 |
| 3 | 39.2 | 70 | 158. | 136 | 276.8 | 202 | 395.6 | 268 | 514.4 | 730 | 1346 | 1390 | 2534 |
| 4 | 41. | 71 | 159.8 | 137 | 278.6 | 203 | 397.4 | 269 | 516.2 | 740 | 1364 | 1400 | 2552 |
| 5 | 42.8 | 72 | 161.6 | 138 | 280.4 | 204 | 399.2 | 270 | 518. | 750 | 1382 | 1410 | 2570 |
| 6 | 44.6 | 73 | 163.4 | 139 | 282.2 | 205 | 401. | 271 | 519.8 | 760 | 1400 | 1420 | 2588 |
| 7 | 46.4 | 74 | 165.2 | 140 | 284. | 206 | 402.8 | 272 | 521.6 | 770 | 1418 | 1430 | 2606 |
| 8 | 48.2 | 75 | 167. | 141 | 285.8 | 207 | 404.6 | 273 | 523.4 | 780 | 1436 | 1440 | 2624 |
| 9 | 50. | 76 | 168.8 | 142 | 287.6 | 208 | 406.4 | 274 | 525.2 | 790 | 1454 | 1450 | 2642 |
| 10 | 51.8 | 77 | 170.6 | 143 | 289.4 | 209 | 408.2 | 275 | 527. | 800 | 1472 | 1460 | 2660 |
| 11 | 53.6 | 78 | 172.4 | 144 | 291.2 | 210 | 410. | 276 | 528.8 | 810 | 1490 | 1470 | 2678 |
| 12 | 55.4 | 79 | 174.2 | 145 | 293. | 211 | 411.8 | 277 | 530.6 | 820 | 1508 | 1480 | 2696 |
| 13 | 57.2 | 80 | 176. | 146 | 294.8 | 212 | 413.6 | 278 | 532.4 | 830 | 1526 | 1490 | 2714 |
| 14 | 59. | 81 | 177.8 | 147 | 296.6 | 213 | 415.4 | 279 | 534.2 | 840 | 1544 | 1500 | 2732 |
| 15 | 60.8 | 82 | 179.6 | 148 | 298.4 | 214 | 417.2 | 280 | 536. | 850 | 1562 | 1510 | 2750 |
| 16 | 62.6 | 83 | 181.4 | 149 | 300.2 | 215 | 419. | 281 | 537.8 | 860 | 1580 | 1520 | 2768 |
| 17 | 64.4 | 84 | 183.2 | 150 | 302. | 216 | 420.8 | 282 | 539.6 | 870 | 1598 | 1530 | 2786 |
| 18 | 66.2 | 85 | 185. | 151 | 303.8 | 217 | 422.6 | 283 | 541.4 | 880 | 1616 | 1540 | 2804 |
| 19 | 68. | 86 | 186.8 | 152 | 305.6 | 218 | 424.4 | 284 | 543.2 | 890 | 1634 | 1550 | 2822 |
| 20 | 69.8 | 87 | 188.6 | 153 | 307.4 | 219 | 426.2 | 285 | 545. | 900 | 1652 | 1600 | 2912 |
| 21 | 71.6 | 88 | 190.4 | 154 | 309.2 | 220 | 428. | 286 | 546.8 | 910 | 1670 | 1650 | 3002 |
| 22 | 73.4 | 89 | 192.2 | 155 | 311. | 221 | 429.8 | 287 | 548.6 | 920 | 1688 | 1700 | 3092 |
| 23 | 75.2 | 90 | 194. | 156 | 312.8 | 222 | 431.6 | 288 | 550.4 | 930 | 1706 | 1750 | 3182 |
| 24 | 77. | 91 | 195.8 | 157 | 314.6 | 223 | 433.4 | 289 | 552.2 | 940 | 1724 | 1800 | 3272 |

TEMPERATURES, FAHRENHEIT AND CENTIGRADE.

| F. | C. | F. | C. | F. | C. | F. | C. | F. | C. | F. | C. | F. | C. |
|-----|-------|----|------|-----|------|-----|-------|-----|-------|-----|-------|------|-------|
| -40 | -40. | 26 | -3.3 | 92 | 33.3 | 158 | 70. | 224 | 106.7 | 290 | 143.3 | 360 | 182.2 |
| -39 | -39.4 | 27 | -2.8 | 93 | 33.9 | 159 | 70.6 | 225 | 107.2 | 291 | 143.9 | 370 | 187.8 |
| -38 | -38.9 | 28 | -2.2 | 94 | 34.4 | 160 | 71.1 | 226 | 107.8 | 292 | 144.4 | 380 | 193.3 |
| -37 | -38.3 | 29 | -1.7 | 95 | 35. | 161 | 71.7 | 227 | 108.3 | 293 | 145. | 390 | 198.9 |
| -36 | -37.8 | 30 | -1.1 | 96 | 35.6 | 162 | 72.2 | 228 | 108.9 | 294 | 145.6 | 400 | 204.4 |
| -35 | -37.2 | 31 | -0.6 | 97 | 36.1 | 163 | 72.8 | 229 | 109.4 | 295 | 146.1 | 410 | 210. |
| -34 | -36.7 | 32 | 0. | 98 | 36.7 | 164 | 73.3 | 230 | 110. | 296 | 146.7 | 420 | 215.6 |
| -33 | -36.1 | 33 | +0.6 | 99 | 37.2 | 165 | 73.9 | 231 | 110.6 | 297 | 147.2 | 430 | 221.1 |
| -32 | -35.6 | 34 | 1.1 | 100 | 37.8 | 166 | 74.4 | 232 | 111.1 | 298 | 147.8 | 440 | 226.7 |
| -31 | -35. | 35 | 1.7 | 101 | 38.3 | 167 | 75. | 233 | 111.7 | 299 | 148.3 | 450 | 232.2 |
| -30 | -34.4 | 36 | 2.2 | 102 | 38.9 | 168 | 75.6 | 234 | 112.2 | 300 | 148.9 | 460 | 237.8 |
| -29 | -33.9 | 37 | 2.8 | 103 | 39.4 | 169 | 76.1 | 235 | 112.8 | 301 | 149.4 | 470 | 243.3 |
| -28 | -33.3 | 38 | 3.3 | 104 | 40. | 170 | 76.7 | 236 | 113.3 | 302 | 150. | 480 | 248.9 |
| -27 | -32.8 | 39 | 3.9 | 105 | 40.6 | 171 | 77.2 | 237 | 113.9 | 303 | 150.6 | 490 | 254.4 |
| -26 | -32.2 | 40 | 4.4 | 106 | 41.1 | 172 | 77.8 | 238 | 114.4 | 304 | 151.1 | 500 | 260. |
| -25 | -31.7 | 41 | 5. | 107 | 41.7 | 173 | 78.3 | 239 | 115. | 305 | 151.7 | 510 | 265.6 |
| -24 | -31.1 | 42 | 5.6 | 108 | 42.2 | 174 | 78.9 | 240 | 115.6 | 306 | 152.2 | 520 | 271.1 |
| -23 | -30.6 | 43 | 6.1 | 109 | 42.8 | 175 | 79.4 | 241 | 116.1 | 307 | 152.8 | 530 | 276.7 |
| -22 | -30. | 44 | 6.7 | 110 | 43.3 | 176 | 80. | 242 | 116.7 | 308 | 153.3 | 540 | 282.2 |
| -21 | -29.4 | 45 | 7.2 | 111 | 43.9 | 177 | 80.6 | 243 | 117.2 | 309 | 153.9 | 550 | 287.8 |
| -20 | -28.9 | 46 | 7.8 | 112 | 44.4 | 178 | 81.1 | 244 | 117.8 | 310 | 154.4 | 560 | 293.3 |
| -19 | -28.3 | 47 | 8.3 | 113 | 45. | 179 | 81.7 | 245 | 118.3 | 311 | 155. | 570 | 298.9 |
| -18 | -27.8 | 48 | 8.9 | 114 | 45.6 | 180 | 82.2 | 246 | 118.9 | 312 | 155.6 | 580 | 304.4 |
| -17 | -27.2 | 49 | 9.4 | 115 | 46.1 | 181 | 82.8 | 247 | 119.4 | 313 | 156.1 | 590 | 310. |
| -16 | -26.7 | 50 | 10. | 116 | 46.7 | 182 | 83.3 | 248 | 120. | 314 | 156.7 | 600 | 315.6 |
| -15 | -26.1 | 51 | 10.6 | 117 | 47.2 | 183 | 83.9 | 249 | 120.6 | 315 | 157.2 | 610 | 321.1 |
| -14 | -25.6 | 52 | 11.1 | 118 | 47.8 | 184 | 84.4 | 250 | 121.1 | 316 | 157.8 | 620 | 326.7 |
| -13 | -25. | 53 | 11.7 | 119 | 48.3 | 185 | 85. | 251 | 121.7 | 317 | 158.3 | 630 | 332.2 |
| -12 | -24.4 | 54 | 12.2 | 120 | 48.9 | 186 | 85.6 | 252 | 122.2 | 318 | 158.9 | 640 | 337.8 |
| -11 | -23.9 | 55 | 12.8 | 121 | 49.4 | 187 | 86.1 | 253 | 122.8 | 319 | 159.4 | 650 | 343.3 |
| -10 | -23.3 | 56 | 13.3 | 122 | 50. | 188 | 86.7 | 254 | 123.3 | 320 | 160. | 660 | 348.9 |
| -9 | -22.8 | 57 | 13.9 | 123 | 50.6 | 189 | 87.2 | 255 | 123.9 | 321 | 160.6 | 670 | 354.4 |
| -8 | -22.2 | 58 | 14.4 | 124 | 51.1 | 190 | 87.8 | 256 | 124.4 | 322 | 161.1 | 680 | 360. |
| -7 | -21.7 | 59 | 15. | 125 | 51.7 | 191 | 88.3 | 257 | 125. | 323 | 161.7 | 690 | 365.6 |
| -6 | -21.1 | 60 | 15.6 | 126 | 52.2 | 192 | 88.9 | 258 | 125.6 | 324 | 162.2 | 700 | 371.1 |
| -5 | -20.6 | 61 | 16.1 | 127 | 52.8 | 193 | 89.4 | 259 | 126.1 | 325 | 162.8 | 710 | 376.7 |
| -4 | -20. | 62 | 16.7 | 128 | 53.3 | 194 | 90. | 260 | 126.7 | 326 | 163.3 | 720 | 382.2 |
| -3 | -19.4 | 63 | 17.2 | 129 | 53.9 | 195 | 90.6 | 261 | 127.2 | 327 | 163.9 | 730 | 387.8 |
| -2 | -18.9 | 64 | 17.8 | 130 | 54.4 | 196 | 91.1 | 262 | 127.8 | 328 | 164.4 | 740 | 393.3 |
| -1 | -18.3 | 65 | 18.3 | 131 | 55. | 197 | 91.7 | 263 | 128.3 | 329 | 165. | 750 | 398.9 |
| 0 | -17.8 | 66 | 18.9 | 132 | 55.6 | 198 | 92.2 | 264 | 128.9 | 330 | 165.6 | 760 | 404.4 |
| + 1 | -17.2 | 67 | 19.4 | 133 | 56.1 | 199 | 92.8 | 265 | 129.4 | 331 | 166.1 | 770 | 410. |
| 2 | -16.7 | 68 | 20. | 134 | 56.7 | 200 | 93.3 | 266 | 130. | 332 | 166.7 | 780 | 415.6 |
| 3 | -16.1 | 69 | 20.6 | 135 | 57.2 | 201 | 93.9 | 267 | 130.6 | 333 | 167.2 | 790 | 421.1 |
| 4 | -15.6 | 70 | 21.1 | 136 | 57.8 | 202 | 94.4 | 268 | 131.1 | 334 | 167.8 | 800 | 426.7 |
| 5 | -15. | 71 | 21.7 | 137 | 58.3 | 203 | 95. | 269 | 131.7 | 335 | 168.3 | 810 | 432.2 |
| 6 | -14.4 | 72 | 22.2 | 138 | 58.9 | 204 | 95.6 | 270 | 132.2 | 336 | 168.9 | 820 | 437.8 |
| 7 | -13.9 | 73 | 22.8 | 139 | 59.4 | 205 | 96.1 | 271 | 132.8 | 337 | 169.4 | 830 | 443.3 |
| 8 | -13.3 | 74 | 23.3 | 140 | 60. | 206 | 96.7 | 272 | 133.3 | 338 | 170. | 840 | 448.9 |
| 9 | -12.8 | 75 | 23.9 | 141 | 60.6 | 207 | 97.2 | 273 | 133.9 | 339 | 170.6 | 850 | 454.4 |
| 10 | -12.2 | 76 | 24.4 | 142 | 61.1 | 208 | 97.8 | 274 | 134.4 | 340 | 171.1 | 860 | 460. |
| 11 | -11.7 | 77 | 25. | 143 | 61.7 | 209 | 98.3 | 275 | 135. | 341 | 171.7 | 870 | 465.6 |
| 12 | -11.1 | 78 | 25.6 | 144 | 62.2 | 210 | 98.9 | 276 | 135.6 | 342 | 172.2 | 880 | 471.1 |
| 13 | -10.6 | 79 | 26.1 | 145 | 62.8 | 211 | 99.4 | 277 | 136.1 | 343 | 172.8 | 890 | 476.7 |
| 14 | -10. | 80 | 26.7 | 146 | 63.3 | 212 | 100. | 278 | 136.7 | 344 | 173.3 | 900 | 482.2 |
| 15 | -9.4 | 81 | 27.2 | 147 | 63.9 | 213 | 100.6 | 279 | 137.2 | 345 | 173.9 | 910 | 487.8 |
| 16 | -8.9 | 82 | 27.8 | 148 | 64.4 | 214 | 101.1 | 280 | 137.8 | 346 | 174.4 | 920 | 493.3 |
| 17 | -8.3 | 83 | 28.3 | 149 | 65. | 215 | 101.7 | 281 | 138.3 | 347 | 175. | 930 | 498.9 |
| 18 | -7.8 | 84 | 28.9 | 150 | 65.6 | 216 | 102.2 | 282 | 138.9 | 348 | 175.6 | 940 | 504.4 |
| 19 | -7.2 | 85 | 29.4 | 151 | 66.1 | 217 | 102.8 | 283 | 139.4 | 349 | 176.1 | 950 | 510. |
| 20 | -6.7 | 86 | 30. | 152 | 66.7 | 218 | 103.3 | 284 | 140. | 350 | 176.7 | 960 | 515.6 |
| 21 | -6.1 | 87 | 30.6 | 153 | 67.2 | 219 | 103.9 | 285 | 140.6 | 351 | 177.2 | 970 | 521.1 |
| 22 | -5.6 | 88 | 31.1 | 154 | 67.8 | 220 | 104.4 | 286 | 141.1 | 352 | 177.8 | 980 | 526.7 |
| 23 | -5. | 89 | 31.7 | 155 | 68.3 | 221 | 105. | 287 | 141.7 | 353 | 178.3 | 990 | 532.2 |
| 24 | -4.4 | 90 | 32.2 | 156 | 68.9 | 222 | 105.6 | 288 | 142.2 | 354 | 178.9 | 1000 | 537.8 |
| 25 | -3.9 | 91 | 32.8 | 157 | 69.4 | 223 | 106.1 | 289 | 142.8 | 355 | 179.4 | 1010 | 543.3 |

2. Contraction of clay, as in the old Wedgwood pyrometer, at one time used by potters. This instrument was very inaccurate, as the contraction of clay varied with its nature.

3. Expansion of air, as in the air-thermometer, Wiborgh's pyrometer, Uehling and Steinbart's pyrometer, etc.

4. Pressure of vapors, as in some forms of Bristol's recording pyrometer.

5. Relative expansion of two metals or other substances, as in Brown's, Bulkley's and other metallic pyrometers, consisting of a copper rod or tube inside of an iron tube, or *vice versa*, with the difference of expansion multiplied by gearing and indicated on a dial.

6. Specific heat of solids, as in the copper-ball and platinum-ball pyrometers.

7. Melting-points of metals, alloys, or other substances, as in approximate determination of temperature by melting pieces of zinc, lead, etc., or as in Seger's fire-clay pyrometer.

8. Time required to heat a weighed quantity of water inclosed in a vessel, as in one form of water pyrometer.

9. Increase in temperature of a stream of water or other liquid flowing at a given rate through a tube inserted into the heated chamber.

10. Changes in the electric resistance of platinum or other metal, as in the Siemens pyrometer.

11. Measurement of an electric current produced by heating the junction of two metals, as in the Le Chatelier pyrometer.

12. Dilution by cold air of a stream of hot air or gas flowing from a heated chamber and determination of the temperature of the mixture by a mercury thermometer, as in Hobson's hot-blast pyrometer.

13. Polarization and refraction by prisms and plates of light radiated from heated surfaces, as in Mesuré and Nouel's pyrometric telescope or optical pyrometer, and Wanner's pyrometer.

14. Heating the filament of an electric lamp to the same color as that of an incandescent body, so that when the latter is observed through a telescope containing the lamp the filament becomes invisible, as in Holborn and Kurlbaum's and Morse's optical pyrometers. The current required to heat the filament is a measure of the temperature.

15. The radiation pyrometer. The radiation from an incandescent surface is received in a telescope containing a thermo-couple, and the electric current generated therein is measured, as in Féry's radiation pyrometer.

(See "Optical Pyrometry" by C. W. W. Waidner and G. K. Burgess, Bulletin No. 2, Bureau of Standards, Department of Commerce and Labor; also *Eng'g*, Mar. 1, 1907.)

Platinum or Copper Ball Pyrometer. — A weighed piece of platinum, copper, or iron is allowed to remain in the furnace or heated chamber till it has attained the temperature of its surroundings. It is then suddenly taken out and dropped into a vessel containing water of a known weight and temperature. The water is stirred rapidly and its maximum temperature taken. Let W = weight of the water, w the weight of the ball, t = the original and T the final heat of the water, and S the specific heat of the metal; then the temperature of fire may be found from the formula

$$x = \frac{W(T - t)}{wS} + T.$$

The mean specific heat of platinum between 32° and 446° F. is 0.03333 or $\frac{1}{30}$ that of water, and it increases with the temperature about 0.000305 for each 100° F. For a fuller description, by J. C. Hoadley, see *Trans. A. S. M. E.*, vi, 702. Compare also Henry M. Howe, *Trans. A. I. M. E.*, xviii, 728.

For accuracy corrections are required for variations in the specific heat of the water and of the metal at different temperatures, for loss of heat by radiation from the metal during the transfer from the furnace to the water, and from the apparatus during the heating of the water; also for the heat-absorbing capacity of the vessel containing the water.

Fire-clay or fire-brick may be used instead of the metal ball.

Le Chatelier's Thermo-electric Pyrometer. — For a very full description, see paper by Joseph Struthers, *School of Mines Quarterly*, vol. xii, 1891; also, paper read by Prof. Roberts-Austen before the Iron and Steel Institute, May 7, 1891,

The principle upon which this pyrometer is constructed is the measurement of a current of electricity produced by heating a couple composed of two wires, one platinum and the other platinum with 10% rhodium — the current produced being measured by a galvanometer.

The composition of the gas which surrounds the couple has no influence on the indications.

When temperatures above 2500° F. are to be studied, the wires must have an isolating support and must be of good length, so that all parts of a furnace can be reached. The wires are supported in an iron tube $\frac{1}{2}$ inch interior diameter and held in place by a cylinder of refractory clay having two holes bored through, in which the wires are placed. The shortness of time (five seconds) allows the temperature to be taken without deteriorating the tube.

Tests made by this pyrometer in measuring furnace temperatures under a great variety of conditions show that the readings of the scale uncorrected are always within 45° F. of the correct temperature, and in the majority of industrial measurements this is sufficiently accurate.

Graduation of Le Chatelier's Pyrometer. — W. C. Roberts-Austen in his *Researches on the Properties of Alloys*, *Proc. Inst. M. E.*, 1892, says: The electromotive force produced by heating the thermo-junction to any given temperature is measured by the movement of the spot of light on the scale graduated in millimeters. The scale is calibrated by heating the thermo-junction to temperatures which have been carefully determined by the aid of the air-thermometer, and plotting the curve from the data so obtained. Many fusion and boiling-points have been established by concurrent evidence of various kinds, and are now generally accepted. The following table contains certain of these:

| Deg. F. | Deg. C. | | Deg. F. | Deg. C. | |
|---------|---------|-----------------|---------|---------|---------------------------|
| 212 | 100 | Water boils. | 1733 | 945 | Silver melts. |
| 618 | 326 | Lead melts. | 1859 | 1015 | Potassium sulphate melts. |
| 676 | 358 | Mercury boils. | 1913 | 1045 | Gold melts. |
| 779 | 415 | Zinc melts. | 1929 | 1054 | Copper melts. |
| 838 | 448 | Sulphur boils. | 2732 | 1500 | Palladium melts. |
| 1157 | 625 | Aluminum melts. | 3227 | 1775 | Platinum melts. |
| 1229 | 665 | Selenium boils. | | | |

The Temperatures Developed in Industrial Furnaces. — M. Le Chatelier states that by means of his pyrometer he has discovered that the temperatures which occur in melting steel and in other industrial operations have been hitherto overestimated. He finds the melting heat of white cast iron 1135° (2075° F.), and that of gray cast iron 1220° (2228° F.). Mild steel melts at 1475° (2687° F.), and hard steel at 1410° (2570° F.). The furnace for hard porcelain at the end of the baking has a heat of 1370° (2498° F.). The heat of a normal incandescent lamp is 1800° (3272° F.), but it may be pushed to beyond 2100° (3812° F.).

Prof. Roberts-Austen (Recent Advances in Pyrometry, *Trans. A.I.M.E.*, Chicago Meeting, 1893) gives an excellent description of modern forms of pyrometers. The following are some of his temperature determinations.

TEN-TON OPEN-HEARTH FURNACE, WOOLWICH ARSENAL.

| | Degrees Centigrade. | Degrees Fahr. |
|--|---------------------|---------------|
| Temperature of steel, 0.3% carbon, pouring into ladle... | 1645 | 2993 |
| Steel, 0.3% carbon, pouring into large mold..... | 1580 | 2876 |
| Reheating furnace, interior..... | 930 | 1706 |
| Cupola furnace, No. 2 cast iron, pouring into ladle..... | 1600 | 2912 |

The following determinations have been effected by M. Le Chatelier:

BESSEMER PROCESS. SIX-TON CONVERTER.

| | | |
|------------------------------------|------|------|
| A. Bath of slag..... | 1580 | 2876 |
| B. Metal in ladle..... | 1640 | 2984 |
| C. Metal in ingot mold..... | 1580 | 2876 |
| D. Ingot in reheating furnace..... | 1200 | 2192 |
| E. Ingot under the hammer..... | 1080 | 1976 |

OPEN-HEARTH FURNACE (Semi-mild Steel). Deg. C. Deg. F.

| | | |
|---|------|------|
| A. Fuel gas near gas generator..... | 720 | 1328 |
| B. Fuel gas entering into bottom of regenerator chamber.. | 400 | 752 |
| C. Fuel gas issuing from regenerator chamber..... | 1200 | 2192 |
| Air issuing from regenerator chamber..... | 1000 | 1832 |
| Chimney gases. Furnace in perfect condition..... | 300 | 590 |
| End of the melting of pig charge..... | 1420 | 2588 |
| Completion of conversion..... | 1500 | 2732 |
| Molten steel. In the ladle — Commencement of casting... | 1580 | 2876 |
| End of casting..... | 1490 | 2714 |
| In the molds..... | 1520 | 2768 |

For very mild (soft) steel the temperatures are higher by 50° C.

BLAST-FURNACE (Gray-Bessemer Pig).

| | | |
|--|------|------|
| Opening in face of tuyere..... | 1930 | 3506 |
| Molten metal — Commencement of fusion..... | 1400 | 2552 |
| End, or prior to tapping..... | 1570 | 2858 |

HOFFMAN RED-BRICK KILN.

| | | |
|---------------------------|------|------|
| Burning temperatures..... | 1100 | 2012 |
|---------------------------|------|------|

R. Moldenke (*The Foundry*, Nov., 1898) determined with a Le Chatelier pyrometer the melting-point of 42 samples of pig iron of different grades. The range was from 2030° F. for pig containing 3.98% combined carbon to 2280 for pig containing 0.13 combined carbon and 3.43% graphite. The results of the whole series may be expressed within 30° F. by the formula $\text{Temp.} = 2300^\circ - 70 \times \% \text{ of combined carbon}$.

Hobson's Hot-blast Pyrometer consists of a brass chamber having three hollow arms and a handle. The hot blast enters one of the arms and induces a current of atmospheric air to flow into the second arm. The two currents mix in the chamber and flow out through the third arm, in which the temperature of the mixture is taken by a mercury thermometer. The openings in the arms are adjusted so that the proportion of hot blast to the atmospheric air remains the same.

The Wiborgh Air-pyrometer. (E. Trotz, *Trans. A.I.M.E.*, 1892.) — The inventor using the expansion-coefficient of air, as determined by Gay-Lussac, Dulon, Rudberg, and Regnault, bases his construction on the following theory: If an air-volume, V , inclosed in a porcelain globe and connected through a capillary pipe with the outside air, be heated to the temperature T (which is to be determined) and thereupon the connection be discontinued, and there be then forced into the globe containing V another volume of air V' of known temperature t , which was previously under atmospheric pressure H , the additional pressure h , due to the addition of the air-volume V' to the air-volume V , can be measured by a manometer. But this pressure is of course a function of the temperature T . Before the introduction of V' , we have the two separate air-volumes, V at the temperature T , and V' at the temperature t , both under the atmospheric pressure H . After the forcing in of V' into the globe, we have, on the contrary, only the volume V of the temperature T , but under the pressure $H + h$.

Seger Cones. (Catalog, Stowe-Fuller Co., 1907.) — Seger cones were developed in Germany by Dr. Herman A. Seger. They comprise a series of triangular pyramids about 3 in. high and $\frac{5}{8}$ in. wide at the base, each a trifle less fusible than the next. When the series is placed in a furnace whose temperature is gradually raised, one cone after another will bend as its temperature of plasticity is reached. The temperature at which it bends so far that its apex touches the surface supporting it, determines a point on Seger's scale. Seger used as his standard, Zettlitz kaolin and Rackonitz shale clay of the following analyses:

| | Silica. | Alu- mina. | Lime. | Iron Oxide. | Mag- nesia. | { Potash } { Soda. } | Loss on Ig- nition. |
|--------------------|---------|---------------|-------|----------------|----------------|-------------------------|---------------------------|
| Zettlitz kaolin... | 46.87 | 38.56 | trace | 0.83 | trace | 1.06 | 12.73 |
| Rackonitz clay.. | 52.50 | 45.22 | 0.50 | 0.81 | 0.54 | trace | 0.78 |

Rackonitz shale clay consists of 99.27% clay substance and 0.73% sand. The melting-point of a cone depends on the ratio of alumina to silica and the amount of fluxes contained. The following table shows the chemical formulæ, mixtures and melting-points of Seger cones from 1 to 36. The temperatures corresponding to the melting-points of cones 21 to 26 are attained in the iron and steel industries. Cones 26 to 36 serve to determine the refractoriness of clays.

| Cone. | Chemical Composition. | | | | | Mixture. | | | | | Melting-Point. | |
|-------|-----------------------|-------|----------------------------------|----------------------------------|--------------------|----------------------|---------|---------|-------------|------------------|----------------|-------|
| | K ₂ O. | CaO. | Fe ₂ O ₃ . | Al ₂ O ₃ . | SiO ₂ . | Feldspar. | Marble. | Quartz. | Iron Oxide. | Zettlitz Kaolin. | Fahr. | Cent. |
| 1 | 0.3 | 0.7 | 0.2 | 0.3 | 4 | 83.55 | 35 | 66 | 16.0 | | 2102 | 1150 |
| 2 | 0.3 | 0.7 | 0.1 | 0.4 | 4 | 83.55 | 35 | 60 | 8.0 | 12.95 | 2138 | 1170 |
| 3 | 0.3 | 0.7 | 0.05 | 0.45 | 4 | 83.55 | 35 | 57 | 4.0 | 19.43 | 2174 | 1190 |
| 4 | 0.3 | 0.7 | | 0.5 | 4 | 83.55 | 35 | 54 | | 25.90 | 2210 | 1210 |
| 5 | 0.3 | 0.7 | | 0.5 | 5 | 83.55 | 35 | 84 | | 25.90 | 2246 | 1230 |
| 6 | 0.3 | 0.7 | | 0.6 | 6 | 83.55 | 35 | 108 | | 38.85 | 2282 | 1250 |
| 7 | 0.3 | 0.7 | | 0.7 | 7 | 83.55 | 35 | 132 | | 51.80 | 2318 | 1270 |
| 8 | 0.3 | 0.7 | | 0.8 | 8 | 83.55 | 35 | 156 | | 64.75 | 2354 | 1290 |
| 9 | 0.3 | 0.7 | | 0.9 | 9 | 83.55 | 35 | 180 | | 77.70 | 2390 | 1310 |
| 10 | 0.3 | 0.7 | | 1.0 | 10 | 83.55 | 35 | 204 | | 90.65 | 2426 | 1330 |
| 11 | 0.3 | 0.7 | | 1.2 | 12 | 83.55 | 35 | 252 | | 116.55 | 2462 | 1350 |
| 12 | 0.3 | 0.7 | | 1.4 | 14 | 83.55 | 35 | 300 | | 142.45 | 2498 | 1370 |
| 13 | 0.3 | 0.7 | | 1.6 | 16 | 83.55 | 35 | 348 | | 168.35 | 2534 | 1390 |
| 14 | 0.3 | 0.7 | | 1.8 | 18 | 83.55 | 35 | 396 | | 194.25 | 2570 | 1410 |
| 15 | 0.3 | 0.7 | | 2.1 | 21 | 83.55 | 35 | 468 | | 233.10 | 2606 | 1430 |
| 16 | 0.3 | 0.7 | | 2.4 | 24 | 83.55 | 35 | 540 | | 271.95 | 2642 | 1450 |
| 17 | 0.3 | 0.7 | | 2.7 | 27 | 83.55 | 35 | 612 | | 310.80 | 2678 | 1470 |
| 18 | 0.3 | 0.7 | | 3.1 | 31 | 83.55 | 35 | 708 | | 362.60 | 2714 | 1490 |
| 19 | 0.3 | 0.7 | | 3.5 | 35 | 83.55 | 35 | 804 | | 414.40 | 2750 | 1510 |
| 20 | 0.3 | 0.7 | | 3.9 | 39 | 83.55 | 35 | 900 | | 466.20 | 2786 | 1530 |
| 21 | 0.3 | 0.7 | | 4.4 | 44 | 83.55 | 35 | 1020 | | 530.95 | 2822 | 1550 |
| 22 | 0.3 | 0.7 | | 4.9 | 49 | 83.55 | 35 | 1140 | | 595.70 | 2858 | 1570 |
| 23 | 0.3 | 0.7 | | 5.4 | 54 | 83.55 | 35 | 1260 | | 660.45 | 2894 | 1590 |
| 24 | 0.3 | 0.7 | | 6.0 | 60 | 83.55 | 35 | 1404 | | 738.15 | 2930 | 1610 |
| 25 | 0.3 | 0.7 | | 6.6 | 66 | 83.55 | 35 | 1548 | | 815.85 | 2966 | 1630 |
| 26 | 0.3 | 0.7 | | 7.2 | 72 | 83.55 | 35 | 1692 | | 893.55 | 3002 | 1650 |
| 27 | 0.3 | 0.7 | | 20.0 | 200 | 83.55 | 35 | 4764 | | 2551.13 | 3038 | 1670 |
| 28 | | | | 1.0 | 10 | | | 240 | | 129.50 | 3074 | 1690 |
| 29 | | | | 1.0 | 8 | | | 180 | | 129.5 | 3110 | 1710 |
| 30 | | | | 1.0 | 6 | | | 120 | | 129.5 | 3146 | 1730 |
| 31 | | | | 1.0 | 5 | | | 90 | | 129.5 | 3182 | 1750 |
| 32 | | | | 1.0 | 4 | | | 60 | | 129.5 | 3218 | 1770 |
| 33 | | | | 1.0 | 3 | | | 30 | | 129.5 | 3254 | 1790 |
| 34 | | | | 1.0 | 2.5 | | | 15 | | 129.5 | 3290 | 1810 |
| 35 | | | | 1.0 | 2 | Zettlitz kaolin | | | | | 3326 | 1830 |
| 36 | | | | | | Rackonitz shale clay | | | | | 3362 | 1850 |

Mesuré and Nouel's Pyrometric Telescope. (H. M. Howe, *E. and M. J.*, June 7, 1890.)—Mesuré and Nouel's telescope gives an immediate determination of the temperature of incandescent bodies, and is therefore better adapted to cases where a great number of observations are to be made, and at short intervals, than Seger's. The little telescope, carried in the pocket or hung from the neck, can be used by foreman or heater at any moment.

It is based on the fact that a plate of quartz, cut at right angles to the axis, rotates the plane of polarization of polarized light to a degree nearly inversely proportional to the square of the length of the waves; and, further, on the fact that while a body at dull redness merely emits red

light, as the temperature rises, the orange, yellow, green, and blue waves successively appear.

If, now, such a plate of quartz is placed between two Nicol prisms at right angles, "a ray of monochromatic light which passes the first, or polarizer, and is watched through the second, or analyzer, is not extinguished as it was before interposing the quartz. Part of the light passes the analyzer, and, to again extinguish it, we must turn one of the Nicols a certain angle," depending on the length of the waves of light, and hence on the temperature of the incandescent object which emits this light. Hence the angle through which we must turn the analyzer to extinguish the light is a measure of the temperature of the object observed.

The Uehling and Steinhart Pyrometer. (For illustrated description see *Engineering*, Aug. 24, 1894.) — The action of the pyrometer is based on a principle which involves the law of the flow of gas through minute apertures in the following manner: If a closed tube or chamber be supplied with a minute inlet and a minute outlet aperture, and air be caused by a constant suction to flow in through one and out through the other of these apertures, the tension in the chamber between the apertures will vary with the difference of temperature between the inflowing and outflowing air. If the inflowing air be made to vary with the temperature to be measured, and the outflowing air be kept at a certain constant temperature, then the tension in the space or chamber between the two apertures will be an exact measure of the temperature of the inflowing air, and hence of the temperature to be measured.

In operation it is necessary that the air be sucked into it through the first minute aperture at the temperature to be measured, through the second aperture at a lower but constant temperature, and that the suction be of a constant tension. The first aperture is therefore located in the end of a platinum tube in the bulb of a porcelain tube over which the hot blast sweeps, or inserted into the pipe or chamber containing the gas whose temperature is to be ascertained.

The second aperture is located in a coupling, surrounded by boiling water, and the suction is obtained by an aspirator and regulated by a column of water of constant height.

The tension in the chamber between the apertures is indicated by a manometer.

The Air-thermometer. (Prof. R. C. Carpenter, *Eng'g News*, Jan. 5, 1893.) — Air is a perfect thermometric substance, and if a given mass of air be considered, the product of its pressure and volume divided by its absolute temperature is in every case constant. If the volume of air remain constant, the temperature will vary with the pressure; if the pressure remain constant, the temperature will vary with the volume. As the former condition is more easily attained, air-thermometers are usually constructed of constant volume, in which case the absolute temperature will vary with the pressure.

If we denote pressures by p and p' , and the corresponding absolute temperatures by T and T' , we should have

$$p : p' :: T : T' \text{ and } T' = p' \frac{T}{p}.$$

The absolute temperature T is to be considered in every case 460 higher than the thermometer-reading expressed in Fahrenheit degrees. From the form of the above equation, if the pressure p corresponding to a known absolute temperature T be known, T' can be found. The quotient T/p is a constant which may be used in all determinations with the instrument. The pressure on the instrument can be expressed in inches of mercury, and is evidently the atmospheric pressure b as shown by a barometer, plus or minus an additional amount h shown by a manometer attached to the air-thermometer. That is, in general, $p = b \pm h$.

The temperature of 32° F. is fixed as the point of melting ice, in which case $T = 460 + 32 = 492^\circ \text{ F}$. This temperature can be produced by surrounding the bulb in melting ice and leaving it several minutes, so that the temperature of the confined air shall acquire that of the surrounding ice. When the air is at that temperature, note the reading of the attached manometer h , and that of a barometer; the sum will be the value of p corresponding to the absolute temperature of 492° F. The constant of the instrument, $K = 492 \div p$, once obtained, can be used in all future determinations.

High Temperatures judged by Color. — The temperature of a body can be approximately judged by the experienced eye unaided. M. Pouillet in 1836 constructed a table, which has been generally quoted in the text-books, giving the colors and their corresponding temperature, but which is now replaced by the tables of H. M. Howe and of Maunsel White and F. W. Taylor (*Trans. A. S. M. E.*, 1899), which are given below.

| Howe. | ° C. | ° F. | White and Taylor. | ° C. | ° F. |
|--------------------------------------|-------------|--------------|-----------------------------------|-------|------|
| Lowest red visible in dark.. | 470 | 878 | Dark blood-red, black-red | | 990 |
| Lowest red visible in day-light..... | 475 | 887 | Dark red, blood-red, low red..... | 556 | 1050 |
| Dull red..... | 550 to 625 | 1022 to 1157 | Dark cherry-red..... | 635 | 1175 |
| Full cherry..... | 700 | 1292 | Medium cherry-red..... | | 1250 |
| Light red..... | 850 | 1562 | Cherry, full red..... | 746 | 1375 |
| Full yellow..... | 950 to 1000 | 1742 to 1832 | Light cherry, light red*. | 843 | 1550 |
| Light yellow... | 1050 | 1922 | Orange, free scaling heat | 899 | 1650 |
| White..... | 1150 | 2102 | Light orange..... | 941 | 1725 |
| | | | Yellow..... | 996 | 1825 |
| | | | Light yellow..... | 1079 | 1975 |
| | | | White..... | 1205 | 2200 |

* Heat at which scale forms and adheres on iron and steel, i.e., does not fall away from the piece when allowed to cool in air.

Skilled observers may vary 100° F. or more in their estimation of relatively low temperatures by color, and beyond 2200° F. it is practically impossible to make estimations with any certainty whatever. (Bulletin No. 2, Bureau of Standards, 1905.)

In confirmation of the above paragraph we have the following, in a booklet published by the Halcomb Steel Co., 1908.

| ° C. | ° F. | Colors. | ° C. | ° F. | Colors. |
|------|------|--------------------------------|------|------|--------------------------------|
| 400 | 752 | Red, visible in the dark. | 1000 | 1832 | Bright cherry-red. |
| 474 | 885 | Red, visible in the twilight. | 1100 | 2012 | Orange-red. |
| 525 | 975 | Red, visible in the day-light. | 1200 | 2192 | Orange-yellow. |
| 581 | 1077 | Red, visible in the sun-light. | 1300 | 2372 | Yellow-white. |
| 700 | 1292 | Dark red. | 1400 | 2552 | White welding heat. |
| 800 | 1472 | Dull cherry-red. | 1500 | 2732 | Brilliant white. |
| 900 | 1652 | Cherry-red. | 1600 | 2912 | Dazzling white (bluish white). |

Different substances heated to the same temperature give out the same color tints. Objects which emit the same tint and intensity of light cannot be distinguished from each other, no matter how different their texture, surface, or shape may be. When the temperature at all parts of a furnace at a low yellow heat is the same, different objects inside the furnace (firebrick, sand, platinum, iron) become absolutely invisible. (H. M. Howe.)

A bright bar of iron, slowly heated in contact with air, assumes the following tints at annexed temperatures (Claudel):

| | Cent. | Fahr. | | Cent. | Fahr. |
|----------------|-------|-------|-----------------|-------|-------|
| Yellow at..... | 225 | 437 | Indigo at..... | 288 | 550 |
| Orange at..... | 243 | 473 | Blue at..... | 293 | 559 |
| Red at..... | 265 | 509 | Green at..... | 332 | 630 |
| Violet at..... | 277 | 531 | "Oxide-gray"... | 400 | 752 |

The Halcomb Steel Co. (1908) gives the following heats and temper colors of steel:

| Cent. Fahr. | Colors. | Cent. Fahr. | Colors. |
|-------------|------------------------|-------------|------------------------|
| 221.1 | 430 Very pale yellow. | 265.6 | 510 Spotted red-brown. |
| 226.7 | 440 Light yellow. | 271.1 | 520 Brown-purple. |
| 232.2 | 450 Pale straw-yellow. | 276.7 | 530 Light purple. |
| 237.8 | 460 Straw-yellow. | 282.2 | 540 Full purple. |
| 243.3 | 470 Deep straw-yellow. | 287.8 | 550 Dark purple. |
| 248.9 | 480 Dark yellow. | 293.3 | 560 Full blue. |
| 254.4 | 490 Yellow-brown. | 298.9 | 570 Dark blue. |
| 260.0 | 500 Brown-yellow. | 315.6 | 600 Very dark blue. |

BOILING-POINTS AT ATMOSPHERIC PRESSURE.

14.7 lbs. per square inch.

| | | | |
|------------------------|---------|------------------------|---------|
| Ether, sulphuric..... | 100° F. | Saturated brine..... | 226° F. |
| Carbon bisulphide..... | 118 | Nitric acid..... | 248 |
| Ammonia..... | 140 | Oil of turpentine..... | 315 |
| Chloroform..... | 140 | Aniline..... | 363 |
| Bromine..... | 145 | Naphthaline..... | 428 |
| Wood spirit..... | 150 | Phosphorus..... | 554 |
| Alcohol..... | 173 | Sulphur..... | 833 |
| Benzine..... | 176 | Sulphuric acid..... | 590 |
| Water..... | 212 | Linseed oil..... | 597 |
| Average sea-water..... | 213.2 | Mercury..... | 676 |

The boiling-points of liquids increase as the pressure increases.

MELTING-POINTS OF VARIOUS SUBSTANCES.

The following figures are given by Clark (on the authority of Pouillet, Claudel, and Wilson), except those marked *, which are given by Prof. Roberts-Austen, and those marked †, which are given by Dr. J. A. Harker. These latter are probably the most reliable figures.

| | | | |
|--------------------------|-------------|------------------------|---------------------|
| Sulphurous acid..... | - 148° F. | Cadmium..... | 442° F. |
| Carbonic acid..... | - 108 | Bismuth..... | 504 to 507 |
| Mercury..... | - 39, - 38† | Lead..... | 618*, 620† |
| Bromine..... | + 9.5 | Zinc..... | 779*, 786† |
| Turpentine..... | 14 | Antimony..... | 1150, 1169† |
| Hyponitric acid..... | 16 | Aluminum..... | 1157*, 1214† |
| Ice..... | 32 | Magnesium..... | 1200 |
| Nitro-glycerine..... | 45 | NaCl, common salt..... | 1472† |
| Tallow..... | 92 | Calcium..... | Full red heat. |
| Phosphorus..... | 112 | Bronze..... | 1692 |
| Acetic acid..... | 113 | Silver..... | 1733*, 1751† |
| Stearine..... | 109 to 120 | Potassium sulphate.. | 1859*, 1958† |
| Spermaceti..... | 120 | Gold..... | 1913*, 1947† |
| Margaric acid.... | 131 to 140 | Copper..... | 1929*, 1943† |
| Potassium..... | 136 to 144 | Nickel..... | 2600† |
| Wax..... | 142 to 154 | Cast iron, white..... | 1922, 2075† |
| Stearic acid..... | 158 | " gray..... | 2012 to 2786, 2228* |
| Sodium..... | 194 to 208 | Steel..... | 2372 to 2532* |
| Iodine..... | 225 | " hard... .. | 2570*; mild, 2687 |
| Sulphur..... | 239 | Wrought iron..... | 2732 to 2912, 2737* |
| Alloy, 1 1/2 tin, 1 lead | 334, 367† | Palladium..... | 2732* |
| Tin..... | 446, 449† | Platinum..... | 3227*, 3110† |

Cobalt and manganese, fusible in highest heat of a forge. Tungsten and chromium, not fusible in forge, but soften and agglomerate. Platinum and iridium, fusible only before the oxyhydrogen blowpipe, or in an electrical furnace. For melting-point of fusible alloys see Alloys. For boiling and freezing points of air and other gases see p. 580.

QUANTITATIVE MEASUREMENT OF HEAT.

Unit of Heat. — The British thermal unit, or heat unit (B.T.U.), is the quantity of heat required to raise the temperature of 1 lb. of pure water from 62° to 63° F. (Peabody), or $\frac{1}{180}$ of the heat required to raise the temperature of 1 lb. of water from 32° to 212° F. (Marks and Davis, see Steam, p. 840).

The French thermal unit, or *calorie*, is the quantity of heat required to raise the temperature of 1 kilogram of pure water from 15° to 16° C.

1 French calorie = 3.968 British thermal units; 1 B.T.U. = 0.252 calorie. The "pound calorie" is sometimes used by English writers; it is the quantity of heat required to raise the temperature of 1 lb. of water 1° C. 1 lb. calorie = $\frac{9}{5}$ B.T.U. = 0.4536 calorie. The heat of combustion of carbon, to CO₂, is said to be 8080 calories. This figure is used either for French calories or for pound calories, as it is the number of pounds of water that can be raised 1° C. by the complete combustion of 1 lb. of carbon, or the number of kilograms of water that can be raised 1° C. by the combustion of 1 kilo. of carbon; assuming in each case that all the heat generated is transferred to the water.

The Mechanical Equivalent of Heat is the number of foot-pounds of mechanical energy equivalent to one British thermal unit, heat and

mechanical energy being mutually convertible. Joule's experiments, 1843-50, gave the figure 772, which is known as Joule's equivalent. More recent experiments by Prof. Rowland (*Proc. Am. Acad. Arts and Sciences*, 1880; see also Wood's *Thermodynamics*) give higher figures, and the most probable average is now considered to be 778.

1 heat-unit is equivalent to 778 ft.-lbs. of energy. $1 \text{ ft.-lb.} = 1/778 = 0.0012852 \text{ heat-unit.}$ 1 horse-power = 33,000 ft.-lbs. per minute = 2545 heat-units per hour = $42.416 \div \text{per minute} = 0.70694 \text{ per second.}$ 1 lb. carbon burned to $\text{CO}_2 = 14,600 \text{ heat-units.}$ 1 lb. C per H.P. per hour = $2545 \div 14,600 = 17.43\% \text{ efficiency.}$

Heat of Combustion of Various Substances in Oxygen.

| | Heat-units. | | Authority. |
|--|------------------------------|------------------------------|---|
| | Cent. | Fahr. | |
| Hydrogen to liquid water at 0°C. | { 34,462 33,808 34,342 | { 62,032 60,854 61,816 | Favre and Silbermann. Andrews. Thomsen. |
| " to steam at 100°C. | 28,732 | 51,717 | Favre and Silbermann. |
| Carbon (wood charcoal) to carbonic acid, CO_2 ; ordinary temperatures | { 8,080 7,900 8,137 | { 14,544 14,220 14,647 | " Andrews. Berthelot. |
| Carbon, diamond to CO_2 | 7,859 | 14,146 | " |
| " black diamond to CO_2 | 7,861 | 14,150 | " |
| " graphite to CO_2 | 7,901 | 14,222 | " |
| Carbon to carbonic oxide, CO | 2,473 | 4,451 | Favre and Silbermann. |
| Carbonic oxide to CO_2 per unit of CO | { 2,403 2,431 2,385 | { 4,325 4,376 4,293 | " Andrews. Thomsen. |
| CO to CO_2 per unit of $\text{C} = 21/3 \times 2403$ | 5,607 | 10,093 | Favre and Silbermann. |
| Marsh-gas, Methane, CH_4 , to water and CO_2 | { 13,120 13,108 13,063 | { 23,616 23,594 23,513 | Thomsen. Andrews. Favre and Silbermann. |
| Olefiant gas, Ethylene, C_2H_4 , to water and CO_2 | { 11,858 11,942 11,957 | { 21,344 21,496 21,523 | " Andrews. Thomsen. |
| Benzole gas, C_6H_6 , to water and CO_2 | { 10,102 9,915 | { 18,184 17,847 | " Favre and Silbermann. |

In calculations of the heating value of mixed fuels the value for carbon is commonly taken at 14,600 B.T.U., and that of hydrogen at 62,000. Taking the heating value of C burned to CO_2 at 14,000, and that of C to CO at 4450, the difference, 10,150 B.T.U., is the heat lost by the imperfect combustion of each lb. of C burned to CO instead of to CO_2 . If the CO formed by this imperfect combustion is afterwards burned to CO_2 the lost heat is regained.

In burning 1 pound of hydrogen with 8 pounds of oxygen to form 9 pounds of water, the units of heat evolved are 62,000; but if the resulting product is not cooled to the initial temperature of the gases, part of the heat is rendered latent in the steam. The total heat of 1 lb. of steam at 212°F. is 1150.0 heat-units above that of water at 32° , and $9 \times 1150 = 10,350 \text{ heat-units,}$ which deducted from 62,000 gives 51,650 as the heat evolved by the combustion of 1 lb. of hydrogen and 8 lbs. of oxygen at 32°F. to form steam at 212°F.

Some writers subtract from the total heating value of hydrogen only the latent heat of the 9 lbs. of steam, or $9 \times 969.7 = 8727 \text{ B.T.U.,}$ leaving as the "low" heating value 53,273 B.T.U.

The use of heating values of hydrogen "burned to steam," in computations relating to combustion of fuel, is inconvenient, since it necessitates a statement of the conditions upon which the figures are based; and it is, moreover, misleading, if not inaccurate, since hydrogen in fuel is not often burned in pure oxygen, but in air; the temperature of the gases before burning is not often the assumed standard temperature, and the products of combustion are not often discharged at 212° . In steam-

boiler practice the chimney gases are usually discharged above 300°; but if economizers are used, and the water supplied to them is cold, the gases may be cooled to below 212°, in which case the steam in the gases is condensed and its latent heat of evaporation is utilized. If there is any need at all of using figures of the "available" heating value of hydrogen, or its heating value when "burned to steam," the fact that the gas is burned in air and not in pure oxygen should be taken into consideration. The resulting figures will then be much lower than those above given, and they will vary with different conditions. (Kent, "Steam Boiler Economy," p. 23.)

Suppose that 1 lb. of H is burned in twice the quantity of air required for complete combustion, or $2 \times (8 \text{ O} + 26.56 \text{ N}) = 69.12 \text{ lbs. air}$ supplied at 62° F., and that the products of combustion escape at 562° F. The heat lost in the products of combustion will be

| | | |
|--|--------|--------|
| 9 lbs. water heated from 62° to 212° | 1352 | B.T.U. |
| Latent heat of 9 lbs. H ₂ O at 212°, 9×969.7 | 8727 | " |
| Superheated steam, 9 lbs. $\times (562^\circ - 212^\circ) \times 0.48$ (sp. ht.) | 1512 | " |
| Nitrogen, $26.56 \times (562^\circ - 62^\circ) \times 0.2438$ | 3238 | " |
| Excess air, $34.56 \times (562^\circ - 62^\circ) \times 0.2375$ | 4104 | " |
| Total | 18,933 | " |

which subtracted from 62,000 gives 43,067 B.T.U. as the net available heating value under the conditions named.

Heating Value of Compound or Mixed Fuels. — The heating value of a solid compound or mixed fuel is the sum of its elementary constituents, and is calculated as follows by Dulong's formula:

$$\text{B.T.U.} = \frac{1}{100} \left[14,600 \text{ C} + 62,000 \left(\text{H} - \frac{\text{O}}{8} \right) + 4500 \text{ S} \right];$$

in which C, H, O, and S are respectively the percentages of the several elements. The term $\text{H} - \frac{1}{8} \text{O}$ is called the "available" or "disposable" hydrogen, or that which is not combined with oxygen in the fuel. For all the common varieties of coal, canal coal and some lignites excepted, the formula is accurate within the limits of error of chemical analyses and calorimetric determinations.

Heat Absorbed by Decomposition. — By the decomposition of a chemical compound as much heat is absorbed or rendered latent as was evolved when the compound was formed. If 1 lb. of carbon is burned to CO₂, generating 14,600 B.T.U., and the CO₂ thus formed is immediately reduced to CO in the presence of glowing carbon, by the reaction $\text{CO}_2 + \text{C} = 2 \text{ CO}$, the result is the same as if the 2 lbs. C had been burned directly to 2 CO, generating $2 \times 4450 = 8900 \text{ B.T.U.}$ The 2 lbs. C burned to CO₂ would generate $2 \times 14,600 = 29,200 \text{ B.T.U.}$, the difference, $29,200 - 8900 = 20,300 \text{ B.T.U.}$, being absorbed or rendered latent in the 2 CO, or 10,150 B.T.U. for each pound of carbon.

In like manner if 9 lbs. of water be injected into a large bed of glowing coal, it will be decomposed into 1 lb. H and 8 lbs. O. The decomposition will absorb 62,000 B.T.U., cooling the bed of coal this amount, and the same quantity of heat will again be evolved if the H is subsequently burned with a fresh supply of O. The 8 lbs. of O will combine with 6 lbs. C, forming 14 lbs. CO (since CO is composed of 12 parts C to 16 parts O), generating $6 \times 4450 = 26,700 \text{ B.T.U.}$, and $6 \times 10,150 = 60,900 \text{ B.T.U.}$ will be latent in this 14 lbs. CO, to be evolved later if it is burned to CO₂ with an additional supply of 8 lbs. O.

SPECIFIC HEAT.

Thermal Capacity. — The thermal capacity of a body between two temperatures T_0 and T_1 is the quantity of heat required to raise the temperature from T_0 to T_1 . The ratio of the heat required to raise the temperature of a certain weight of a given substance one degree to that required to raise the temperature of the same weight of water from 62° to 63° F. is commonly called the *specific heat* of the substance. Some writers object to the term as being an inaccurate use of the words "specific" and "heat." A more correct name would be "coefficient of thermal capacity."

Determination of Specific Heat.—Method by Mixture. — The body whose specific heat is to be determined is raised to a known temperature, and is then immersed in a mass of liquid of which the weight, specific

heat, and temperature are known. When both the body and the liquid have attained the same temperature, this is carefully ascertained.

Now the quantity of heat lost by the body is the same as the quantity of heat absorbed by the liquid.

Let c , w , and t be the specific heat, weight, and temperature of the hot body, and c' , w' , and t' of the liquid. Let T be the temperature the mixture assumes.

Then, by the definition of specific heat, $c \times w \times (t - T)$ = heat-units lost by the hot body, and $c' \times w' \times (T - t')$ = heat-units gained by the cold liquid. If there is no heat lost by radiation or conduction, these must be equal, and

$$cw(t - T) = c'w'(T - t') \text{ or } c = \frac{c'w'(T - t')}{w(t - T)}.$$

Electrical Method. This method is believed to be more accurate in many cases than the method by mixture. It consists in measuring the quantity of current in watts required to heat a unit weight of a substance one degree in one minute, and translating the result into heat-units. 1 Watt = 0.0569 B.T.U. per minute.

Specific Heats of Various Substances.

The specific heats of substances, as given by different authorities show considerable lack of agreement, especially in the case of gases.

The following tables give the mean specific heats of the substances named according to Regnault. (From Röntgen's Thermodynamics, p. 134.) These specific heats are average values, taken at temperatures which usually come under observation in technical application. The actual specific heats of all substances, in the solid or liquid state, increase slowly as the body expands or as the temperature rises. It is probable that the specific heat of a body when liquid is greater than when solid. For many bodies this has been verified by experiment.

SOLIDS.

| | | | |
|-------------------|--------|-------------------|--------|
| Antimony..... | 0.0508 | Steel (soft)..... | 0.1165 |
| Copper..... | 0.0951 | Steel (hard)..... | 0.1175 |
| Gold..... | 0.0324 | Zinc..... | 0.0956 |
| Wrought iron..... | 0.1138 | Brass..... | 0.0939 |
| Glass..... | 0.1937 | Ice..... | 0.5040 |
| Cast iron..... | 0.1298 | Sulphur..... | 0.2026 |
| Lead..... | 0.0314 | Charcoal..... | 0.2410 |
| Platinum..... | 0.0324 | Alumina..... | 0.1970 |
| Silver..... | 0.0570 | Phosphorus..... | 0.1887 |
| Tin..... | 0.0562 | | |

LIQUIDS.

| | | | |
|---------------------|--------|-------------------------|--------|
| Water..... | 1.0000 | Mercury..... | 0.0333 |
| Lead (melted)..... | 0.0402 | Alcohol (absolute)..... | 0.7000 |
| Sulphur "..... | 0.2340 | Fusel oil..... | 0.5640 |
| Bismuth "..... | 0.0308 | Benzine..... | 0.4500 |
| Tin "..... | 0.0637 | Ether..... | 0.5034 |
| Sulphuric acid..... | 0.3350 | | |

GASES.

Constant Pressure. Constant Volume.

| | | |
|--------------------------------------|---------|---------|
| Air..... | 0.23751 | 0.16847 |
| Oxygen..... | 0.21751 | 0.15507 |
| Hydrogen..... | 3.40900 | 2.41226 |
| Nitrogen..... | 0.24380 | 0.17273 |
| Superheated steam*..... | 0.4805 | 0.346 |
| Carbonic acid..... | 0.217 | 0.1535 |
| Olefiant gas (CH ₂)..... | 0.404 | 0.173 |
| Carbonic oxide..... | 0.2479 | 0.1758 |
| Ammonia..... | 0.508 | 0.299 |
| Ether..... | 0.4797 | 0.3411 |
| Alcohol..... | 0.4534 | 0.3200 |
| Acetic acid..... | 0.4125 | |
| Chloroform..... | 0.1567 | |

* See Superheated Steam, page 838.

In addition to the above, the following are given by other authorities. (Selected from various sources.)

METALS.

| | | | |
|--------------------------------------|--------|---|--------|
| Platinum, 32° to 446° F.... | 0.0333 | Wrought iron (Petit & Dulong). | |
| (increased .000305 for each 100° F.) | | " 32° to 212° .. | 0.1098 |
| Cadmium..... | 0.0567 | " 32° to 392° .. | 0.115 |
| Brass..... | 0.0939 | " 32° to 572° .. | 0.1218 |
| Copper, 32° to 212° F..... | 0.094 | " 32° to 662° .. | 0.1255 |
| " 32° to 572° F..... | 0.1013 | Iron at high temperatures. | |
| Zinc, 32° to 212° F..... | 0.0927 | (Pionchon, <i>Comptes Rendus</i> , 1887.) | |
| " 32° to 572° F..... | 0.1015 | 1382° to 1832° F..... | 0.213 |
| Nickel..... | 0.1086 | 1749° to 1843° F..... | 0.218 |
| Aluminum, 0° F. to melting- | | 1922° to 2192° F..... | 0.199 |
| point (A. E. Hunt)..... | 0.2185 | | |

Dr.-Ing. P. Oberhoffer, in *Zeit. des Vereines Deutscher Ingenieure* (*Eng. Digest*, Sept., 1908), describes some experiments on the specific heat of nearly pure iron. The following mean specific heats were obtained:

| | | | | | | |
|----------|--------|--------|--------|--------|--------|--------|
| Temp. F. | 500 | 600 | 800 | 1000 | 1200 | 1300 |
| Sp. Ht. | 0.1228 | 0.1266 | 0.1324 | 0.1388 | 0.1462 | 0.1601 |
| Temp. F. | 1500 | 1800 | 2100 | 2400 | 2700 | |
| Sp. Ht. | 0.1698 | 0.1682 | 0.1667 | 0.1662 | 0.1666 | |

The specific heat increases steadily between 500 and 1200 F. Then it increases rapidly to 1400, after which it remains nearly constant.

OTHER SOLIDS.

| | | | |
|------------------------------|-------------|-----------------------|---------------|
| Brickwork and masonry, about | 0.20 | Coal | 0.20 to 0.241 |
| Marble..... | 0.210 | Coke..... | 0.203 |
| Chalk..... | 0.215 | Graphite..... | 0.202 |
| Quicklime..... | 0.217 | Sulphate of lime..... | 0.197 |
| Magnesian limestone..... | 0.217 | Magnesia..... | 0.222 |
| Silica..... | 0.191 | Soda..... | 0.231 |
| Corundum..... | 0.198 | Quartz..... | 0.188 |
| Stones generally | 0.2 to 0.22 | River sand..... | 0.195 |

WOODS.

| | | | |
|------------------------|-------|-----------|-------|
| Pine (turpentine)..... | 0.467 | Oak..... | 0.570 |
| Fir..... | 0.650 | Pear..... | 0.500 |

LIQUIDS.

| | | | |
|--------------------------------|-------|-----------------------------|-------|
| Alcohol, density 0.793..... | 0.622 | Olive oil..... | 0.310 |
| Sulphuric acid, density 1.87.. | 0.335 | Benzine..... | 0.393 |
| " 1.30.. | 0.661 | Turpentine, density 0.872.. | 0.472 |
| Hydrochloric acid..... | 0.600 | Bromine..... | 1.111 |

GASES.

| | At Constant Pressure. | At Constant Volume. |
|--|-----------------------|---------------------|
| Sulphurous acid..... | 0.1553 | 0.1246 |
| Light carbureted hydrogen, marsh gas (CH ₄) .. | 0.5929 | 0.4683 |
| Blast-furnace gases..... | 0.2277 | |

Specific Heat of Water. (Peabody's Steam Tables, from Barnes and Regnault.)

| °C. | °F. | Sp. Ht. | °C. | °F. | Sp. Ht. | °C. | °F. | Sp. Ht. | °C. | °F. | Sp. Ht. |
|-----|-----|---------|-----|-----|---------|-----|-----|---------|-----|-----|---------|
| 0 | 32 | 1.0094 | 35 | 95 | 0.99735 | 70 | 158 | 1.00150 | 120 | 248 | 1.01620 |
| 5 | 41 | 1.00530 | 40 | 104 | 0.99735 | 75 | 167 | 1.00275 | 140 | 284 | 1.02230 |
| 10 | 50 | 1.00230 | 45 | 113 | 0.99760 | 80 | 176 | 1.00415 | 160 | 320 | 1.02850 |
| 15 | 59 | 1.00030 | 50 | 122 | 0.99800 | 85 | 188 | 1.00557 | 180 | 356 | 1.03475 |
| 20 | 68 | 0.99895 | 55 | 131 | 0.99850 | 90 | 194 | 1.00705 | 200 | 392 | 1.04100 |
| 25 | 77 | 0.99806 | 60 | 140 | 0.99940 | 95 | 203 | 1.00855 | 220 | 428 | 1.04760 |
| 30 | 86 | 0.99759 | 65 | 149 | 1.00040 | 100 | 212 | 1.01010 | | | |

Specific Heat of Salt Solution. (Schuller.)

| | | | | | |
|---------------------------------|--------|--------|--------|--------|--------|
| Per cent salt in solution . . . | 5 | 10 | 15 | 20 | 25 |
| Specific heat | 0.9306 | 0.8909 | 0.8606 | 0.8490 | 0.8073 |

Specific Heat of Air.—Regnault gives for the mean value at constant pressure

| | |
|---------------------------------------|---------|
| Between - 30° C. and + 10° C. | 0.23771 |
| “ 0° C. “ 100° C. | 0.23741 |
| “ 0° C. “ 200° C. | 0.23751 |

Hanssen uses 0.1686 for the specific heat of air at constant volume. The value of this constant has never been found to any degree of accuracy by direct experiment. Prof. Wood gives $0.2375 \div 1.406 = 0.1689$. The ratio of the specific heat of a fixed gas at constant pressure to the sp. ht. at constant volume is given as follows by different writers (*Eng'g*, July 12, 1889): Regnault, 1.3953; Moll and Beck, 1.4085; Szathmari, 1.4027; J. Macfarlane Gray, 1.4. The first three are obtained from the velocity of sound in air. The fourth is derived from theory. Prof. Wood says: The value of the ratio for air, as found in the days of La Place, was 1.41, and we have $0.2377 \div 1.41 = 0.1686$, the value used by Clausius, Hanssen, and many others. But this ratio is not definitely known. Rankine in his later writings used 1.408, and Tait in a recent work gives 1.404, while some experiments give less than 1.4, and others more than 1.41. Prof. Wood uses 1.406.

Specific Heat of Gases.—Experiments by Mallard and Le Chatelier indicate a continuous increase in the specific heat at constant volume of steam, CO₂, and even of the perfect gases, with rise of temperature. The variation is inappreciable at 100° C., but increases rapidly at the high temperatures of the gas-engine cylinder. (Robinson's Gas and Petroleum Engines.)

Thermal Capacity and Specific Heat of Gases. (From Damour's "Industrial Furnaces.")—The specific heat of a gas at any temperature is the first derivative of the function expressing the thermal capacity. It is not possible to derive from the specific heat of a gas at a given temperature, or even from the mean specific heat between 0° and 100° C., the thermal capacity at a temperature above 100° C. The specific heats of gases under constant pressure between 0° and 100° C., given by Regnault, are not sufficient to calculate the quantity of heat absorbed by a gas in heating or radiated in cooling, hence all calculations based on these figures are subject to a more or less grave error.

The thermal capacities of a molecular volume (22.32 liters) of gases from absolute 0° (- 273° C.) to a temperature T ($= 273^\circ + t$) may be expressed by the formula $Q = 0.001 aT + 0.000,001 bT^2$, in which a is a constant, 6.5, for all gases, and b has the following values for different gases: O₂, N₂, H₂, CO, 0.6; H₂O vapor, 2.9; CO₂, 3.7; CH₄, 6.0.

SPECIFIC HEATS OF GASES PER KILOGRAM.

| Gases. | Under Constant Pressure. | Under Constant Volume. |
|--|-------------------------------|-------------------------------|
| Oxygen | $0.213 + 38 \times 10^{-6}t$ | $0.150 + 38 \times 10^{-6}t$ |
| Nitrogen and Carbon Monoxide | $0.243 + 42 \times 10^{-6}t$ | $0.171 + 42 \times 10^{-6}t$ |
| Hydrogen | $3.400 + 600 \times 10^{-6}t$ | $2.400 + 600 \times 10^{-6}t$ |
| Water Vapor | $0.447 + 324 \times 10^{-6}t$ | $0.335 + 324 \times 10^{-6}t$ |
| Carbon Dioxide | $0.193 + 168 \times 10^{-6}t$ | $0.150 + 168 \times 10^{-6}t$ |
| Methane | $0.608 + 748 \times 10^{-6}t$ | $0.491 + 748 \times 10^{-6}t$ |

THERMAL CAPACITIES OF GASES PER KILOGRAM IN CENTIGRADE DEGREES.

| Gases. | Under Constant Pressure. | Under Constant Volume. |
|-----------------------------------|------------------------------------|------------------------------------|
| Oxygen..... | $0.213 t + 19 \times 10^{-6} t^2$ | $0.150 t + 19 \times 10^{-6} t^2$ |
| Nitrogen and Carbon Monoxide..... | $0.243 t + 21 \times 10^{-6} t^2$ | $0.243 t + 21 \times 10^{-6} t^2$ |
| Hydrogen..... | $3.400 t + 300 \times 10^{-6} t^2$ | $2.400 t + 300 \times 10^{-6} t^2$ |
| Water Vapor..... | $0.447 t + 162 \times 10^{-6} t^2$ | $0.335 t + 162 \times 10^{-6} t^2$ |
| Carbon Dioxide..... | $0.193 t + 84 \times 10^{-6} t^2$ | $0.150 t + 84 \times 10^{-6} t^2$ |
| Methane or Marsh Gas..... | $0.608 t + 374 \times 10^{-6} t^2$ | $0.491 t + 3.4 \times 10^{-6} t^2$ |

THERMAL CAPACITIES OF GASES PER KILOGRAM.

| Temperatures. | O ₂ | N ₂ , CO | H ₂ | H ₂ O | CO ₂ | CH ₄ |
|---------------------|----------------|---------------------|----------------|------------------|-----------------|-----------------|
| Degrees Centigrade. | 0 | 0 | 0 | 0 | 0 | 0 |
| 200..... | 47.3 | 50 | 700 | 100 | 43.1 | 136.6 |
| 400..... | 88.0 | 100 | 1400 | 203 | 91.0 | 303.0 |
| 600..... | 134.0 | 154 | 2150 | 326 | 145.0 | 499.0 |
| 800..... | 181.0 | 207 | 2900 | 461 | 208.0 | 726.0 |
| 1000..... | 232.0 | 264 | 3700 | 609 | 277.0 | 982.0 |
| 1200..... | 284.0 | 325 | 4550 | 770 | 354.0 | 1269.0 |
| 1400..... | 334.0 | 383 | 5350 | 943 | 435.0 | 1584.0 |
| 1600..... | 391.0 | 445 | 6250 | 1130 | 523.0 | 1931.0 |
| 1800..... | 444.0 | 508 | 7100 | 1330 | 618.0 | 2307.0 |
| 2000..... | 503.0 | 575 | 8050 | 1541 | 728.0 | 2712.0 |
| 2200..... | 558.0 | 637 | 8950 | 1751 | 840.0 | 3148.0 |
| 2400..... | 6 0.0 | 708 | 9900 | 1985 | 950.0 | 3614.0 |
| 2600..... | 681.0 | 777 | 10900 | 2241 | 1070.0 | 4109.0 |
| 2800..... | 735.0 | 850 | 11900 | 2520 | 1200.0 | 4635.0 |
| 3000..... | 810.0 | 921 | 12950 | 2799 | 1355.0 | 5190.0 |

EXPANSION BY HEAT.

In the centigrade scale the coefficient of expansion of air per degree is $0.003665 = 1/273$; that is, the pressure being constant, the volume of a perfect gas increases $1/273$ of its volume at 0°C . for every increase in temperature of 1°C . In Fahrenheit units it increases $1/491.2 = 0.003620$ of its volume at 32°F . for every increase of 1°F .

Expansion of Gases by Heat from 32° to 212°F . (Regnault.)

| | Increase in Volume, Pressure Constant. Volume at $32^\circ \text{Fahr.} = 1.0$, for | | Increase in Pressure, Volume Constant. Pressure at $32^\circ \text{Fahr.} = 1.0$, for | |
|----------------------|--|----------------------|--|----------------------|
| | 100°C . | 1°F . | 100°C . | 1°F . |
| Hydrogen..... | 0.3661 | 0.002034 | 0.3667 | 0.002037 |
| Atmospheric air..... | 0.3670 | 0.002039 | 0.3665 | 0.002036 |
| Nitrogen..... | 0.3670 | 0.002039 | 0.3668 | 0.002039 |
| Carbon monoxide..... | 0.3669 | 0.002038 | 0.3667 | 0.002037 |
| Carbon dioxide..... | 0.3710 | 0.002061 | 0.3688 | 0.002039 |
| Sulphur dioxide..... | 0.3903 | 0.002168 | 0.3845 | 0.002136 |

If the volume is kept constant, the pressure varies directly as the absolute temperature.

Lineal Expansion of Solids at Ordinary Temperatures.

(Mostly British Board of Trade; from Clark.)

| | For 1° Fahr. Length = 1. | For 1° Cent. Length = 1. | Expan- sion from 32° to 212° F. | Accord- ing to Other Author- ities. |
|---|-----------------------------------|-----------------------------------|---|---|
| Aluminum (cast)..... | 0.00001234 | 0.00002221 | 0.002221 | |
| Antimony (cryst.)..... | 0.00000627 | 0.00001129 | 0.001129 | 0.001083 |
| Brass, cast..... | 0.00000957 | 0.00001722 | 0.001722 | 0.001868 |
| Brass, plate..... | 0.00001052 | 0.00001894 | 0.001894 | |
| Brick..... | 0.00000306 | 0.00000550 | 0.000550 | |
| Brick (fire)..... | 0.00000300 | 0.00000540 | 0.000540 | |
| Bronze (Copper, 17; Tin, 2½; Zinc, 1)..... | 0.00000386 | 0.00001774 | 0.001774 | |
| Bismuth..... | 0.00000375 | 0.00001755 | 0.001755 | 0.001392 |
| Cement, Portland (mixed), pure..... | 0.00000594 | 0.00001070 | 0.001070 | |
| Concrete: cement-mortar and pebbles..... | 0.000003795 | 0.00001430 | 0.001430 | |
| Copper..... | 0.00000887 | 0.00001596 | 0.001596 | 0.001718 |
| Ebonite..... | 0.00004278 | 0.00007700 | 0.007700 | |
| Glass, English flint..... | 0.00000451 | 0.00000812 | 0.000812 | |
| Glass, thermometer..... | 0.00000499 | 0.00000897 | 0.000897 | |
| Glass, hard..... | 0.00000397 | 0.00000714 | 0.000714 | |
| Granite, gray, dry..... | 0.00000438 | 0.00000789 | 0.000789 | |
| Granite, red, dry..... | 0.00000498 | 0.00000897 | 0.000897 | |
| Gold, pure..... | 0.00000786 | 0.00001415 | 0.001415 | |
| Iridium, pure..... | 0.00000356 | 0.00000641 | 0.000641 | |
| Iron, wrought..... | 0.00000648 | 0.00001166 | 0.001166 | 0.001235 |
| Iron, cast..... | 0.00000556 | 0.00001001 | 0.001001 | 0.001110 |
| Lead..... | 0.00001571 | 0.00002828 | 0.002828 | |
| Magnesium..... | | | | 0.002694 |
| Marbles, various { from..... | 0.00000308 | 0.00000554 | 0.000554 | |
| { to..... | 0.00000786 | 0.00001415 | 0.001415 | |
| Masonry, brick { from..... | 0.00000256 | 0.00000460 | 0.000460 | |
| { to..... | 0.00000494 | 0.00000890 | 0.000890 | |
| Mercury (cubic expansion)..... | 0.00009984 | 0.00017971 | 0.017971 | 0.018018 |
| Nickel..... | 0.00000695 | 0.00001251 | 0.001251 | 0.001279 |
| Pewter..... | 0.00001129 | 0.00002033 | 0.002033 | |
| Plaster, white..... | 0.00000922 | 0.00001660 | 0.001660 | |
| Platinum..... | 0.00000479 | 0.00000863 | 0.000863 | |
| Platinum, 85 %, Iridium, 15 %..... | 0.00000453 | 0.00000815 | 0.000815 | 0.000884 |
| Porcelain..... | 0.00000200 | 0.00000360 | 0.000360 | |
| Quartz, parallel to maj. axis, 0° to 40° C..... | 0.00000434 | 0.00000781 | 0.000781 | |
| Quartz, perpend. to maj. axis, 0° to 40° C..... | 0.00000788 | 0.00001419 | 0.001419 | |
| Silver, pure..... | 0.00001079 | 0.00001943 | 0.001943 | 0.001908 |
| Slate..... | 0.00000577 | 0.00001038 | 0.001038 | |
| Steel, cast..... | 0.00000636 | 0.00001144 | 0.001144 | 0.001079 |
| Steel, tempered..... | 0.00000689 | 0.00001240 | 0.001240 | |
| Stone (sandstone), dry..... | 0.00000652 | 0.00001174 | 0.001174 | |
| Stone (sandstone), Rauville..... | 0.00000417 | 0.00000750 | 0.000750 | |
| Tin..... | 0.00001163 | 0.00002094 | 0.002094 | 0.001938 |
| Wedgwood ware..... | 0.00000489 | 0.00000881 | 0.000881 | |
| Wood, pine..... | 0.00000276 | 0.00000496 | 0.000496 | |
| Zinc..... | 0.00001407 | 0.00002532 | 0.002532 | 0.002942 |
| Zinc, 8, Tin, 1..... | 0.00001496 | 0.00002692 | 0.002692 | |

Invar (see next page), 0.000,000,374 to 0.000,000,44 for 1° C.

Cubical expansion, or expansion of volume = linear expansion \times 3.

Expansion of Steel at High Temperatures. (Charpy and Grenet, *Comptes Rendus*, 1902.) — Coefficients of expansion (for 1° C.) of annealed carbon and nickel steels at temperatures at which there is no transforma-

tion of the steel. The results seem to show that iron and carbide of iron have appreciably the same coefficient of expansion. [See also p. 474.]

| Composition of Steels. | | | | Mean Coefficients of Expansion from | | | Coeffs. between | |
|------------------------|------|------|-------|-------------------------------------|-----------------------|-----------------------|-----------------------|-------------|
| C | Mn | Si | P | 1.5° to 200° | 200° to 500° | 500° to 650° | | |
| 0.03 | 0.01 | 0.03 | 0.013 | 11.8×10^{-6} | 14.3×10^{-6} | 17.0×10^{-6} | 24.5×10^{-6} | 880° & 950° |
| 0.25 | 0.04 | 0.05 | 0.010 | 11.5 | 14.5 | 17.5 | 23.3 | 800° & 950° |
| 0.64 | 0.12 | 0.14 | 0.009 | 12.1 | 14.1 | 16.5 | 23.3 | 720° & 950° |
| 0.93 | 0.10 | 0.05 | 0.005 | 11.6 | 14.9 | 16.0 | 27.5 | " " |
| 1.23 | 0.10 | 0.08 | 0.005 | 11.9 | 14.3 | 16.5 | 33.8 | " " |
| 1.50 | 0.04 | 0.09 | 0.010 | 11.5 | 14.9 | 16.5 | 36.7 | " " |
| 3.50 | 0.03 | 0.07 | 0.005 | 11.2 | 14.2 | 18.0 | 33.3 | " " |

| Nickel Steels. | | | Mean Coefficients of Expansion from | | | | |
|----------------|------|------|-------------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Ni | C | Mn | 15° to 100° | 100° to 200° | 200° to 400° | 400° to 600° | 600° to 900° |
| 26.9 | 0.35 | 0.30 | 11.0×10^{-6} | 18.0×10^{-6} | 18.7×10^{-6} | 22.0×10^{-6} | 23.0×10^{-6} |
| 28.9 | 0.35 | 0.36 | 10.0 | 21.5 | 19.0 | 20.0 | 22.7 |
| 30.1 | 0.35 | 0.34 | 9.5 | 14.0 | 19.5 | 19.0 | 21.3 |
| 34.7 | 0.36 | 0.36 | 2.0 | 2.5 | 11.75 | 19.5 | 20.7 |
| 36.1 | 0.39 | 0.39 | 1.5 | 1.5 | 11.75 | 17.0 | 20.3 |
| 32.8 | 0.29 | 0.66 | 8.0 | 14.0 | 18.0 | 21.5 | 22.3 |
| 35.8 | 0.31 | 0.69 | 2.5 | 2.5 | 12.5 | 13.75 | 14.3 |
| 37.4 | 0.30 | 0.69 | 2.5 | 1.5 | 8.5 | 19.75 | 18.3 |
| 25.4 | 1.01 | 0.79 | 12.5 | 18.5 | 19.75 | 21.0 | 35.0 |
| 29.4 | 0.99 | 0.89 | 11.0 | 12.5 | 19.0 | 20.5 | 31.7 |
| 34.5 | 0.97 | 0.84 | 3.0 | 3.5 | 13.0 | 18.75 | 26.7 |

Invar, an alloy of iron with 36 per cent of nickel, has a smaller coefficient of expansion with the ordinary atmospheric changes of temperature than any other metal or alloy known. This alloy is sold under the name of "Invar," and is used for scientific instruments, pendulums of clocks, steel tape-measures for accurate survey work, etc. The Bureau of Standards found its coefficient of expansion to range from 0.000,000,374 to 0.000,000,44 for 1° C., or about $\frac{1}{23}$ of that of steel. For all surveys except in the most precise geodetic work a tape of invar may be used without correction for temperature. (*Eng. News*, Aug. 13, 1908.)

Platinite, an alloy of iron with 42 per cent of nickel, has the same coefficient of expansion and contraction at atmospheric temperatures as has glass. It can, therefore, be used for the manufacture of armored glass, that is, a plate of glass into which a network of steel wire has been rolled, and which is used for fire-proofing, etc. It can also be used instead of platinum for the electric connections passing through the glass plugs in the base of incandescent electric lights. (Stoughton's "Metallurgy of Steel.")

Expansion of Liquids from 32° to 212° F. — Apparent expansion in glass (Clark). Volume at 212°, volume at 32° being 1:

| | | | |
|----------------------------|--------|---------------------------------------|------|
| Water..... | 1.0466 | Nitric acid..... | 1.11 |
| Water saturated with salt. | 1.05 | Olive and linseed oils..... | 1.08 |
| Mercury..... | 1.0182 | Turpentine and ether..... | 1.07 |
| Alcohol..... | 1.11 | Hydrochloric and sulphuric acids..... | 1.06 |

For water at various temperatures, see Water.

For air at various temperatures, see Air.

ABSOLUTE TEMPERATURE — ABSOLUTE ZERO.

The absolute zero of a gas is a theoretical consequence of the law of expansion by heat, assuming that it is possible to continue the cooling of a perfect gas until its volume is diminished to nothing.

If the volume of a perfect gas increases $\frac{1}{273}$ of its volume at 0° C. for every increase of temperature of 1° C., and decreases $\frac{1}{273}$ of its volume for every decrease of temperature of 1° C., then at -273° C. the volume of the imaginary gas would be reduced to nothing. This point -273° C., or 491.2° F. below the melting-point of ice on the air-thermometer, or 492.66° F. below on a perfect gas-thermometer $= -459.2^{\circ}$ F. (or -460.66°), is called the absolute zero; and absolute temperatures are temperatures measured, on either the Fahrenheit or Centigrade scale, from this zero. The freezing-point, 32° F., corresponds to 491.2° F. absolute. If p_0 be the pressure and v_0 the volume of a gas at the temperature of 32° F. $= 491.2^{\circ}$ on the absolute scale $= T_0$, and p the pressure, and v the volume of the same quantity of gas at any other absolute temperature T , then

$$\frac{pv}{p_0v_0} = \frac{T}{T_0} = \frac{t + 459.2}{491.2}; \quad \frac{pv}{T} = \frac{p_0v_0}{T_0}.$$

The value of $p_0v_0 \div T_0$ for air is 53.37, and $pv = 53.37T$, calculated as follows by Prof. Wood:

A cubic foot of dry air at 32° F. at the sea-level weighs 0.080728 lb. The volume of one pound is $v_0 = \frac{1}{.080728} = 12.387$ cubic feet. The pressure per square foot is 2116.2 lbs.

$$\frac{p_0v_0}{T_0} = \frac{2116.2 \times 12.387}{491.13} = \frac{26214}{491.13} = 53.37.$$

The figure 491.13 is the number of degrees that the absolute zero is below the melting-point of ice, by the air-thermometer. On the absolute scale, whose divisions would be indicated by a perfect gas-thermometer, the calculated value approximately is 492.66, which would make $pv = 53.21T$. Prof. Thomson considers that -273.1° C., $= -459.4^{\circ}$ F., is the most probable value of the absolute zero. See *Heat* in *Ency. Brit.*

LATENT HEATS OF FUSION AND EVAPORATION.

Latent Heat means a quantity of heat which has disappeared, having been employed to produce some change other than elevation of temperature. By exactly reversing that change, the quantity of heat which has disappeared is reproduced. Maxwell defines it as the quantity of heat which must be communicated to a body in a given state in order to convert it into another state without changing its temperature.

Latent Heat of Fusion. — When a body passes from the solid to the liquid state, its temperature remains stationary, or nearly stationary, at a certain melting-point during the whole operation of melting; and in order to make that operation go on, a quantity of heat must be transferred to the substance melted, being a certain amount for each unit of weight of the substance. This quantity is called the latent heat of fusion.

When a body passes from the liquid to the solid state, its temperature remains stationary or nearly stationary during the whole operation of freezing; a quantity of heat equal to the latent heat of fusion is produced in the body and rejected into the atmosphere or other surrounding bodies.

The following are examples in British thermal units per pound, as given in Landolt and Bornstein's *Physikalische-Chemische Tabellen* (Berlin, 1894).

| Substances. | Latent Heat of Fusion. | Substances. | Latent Heat of Fusion. |
|--------------------|---------------------------|-----------------|---------------------------|
| Bismuth..... | 22.75 | Silver..... | 37.93 |
| Cast iron, gray... | 41.4 | Beeswax..... | 76.14 |
| Cast iron, white.. | 59.4 | Paraffine..... | 63.27 |
| Lead..... | 9.66 | Spermaceti..... | 66.56 |
| Tin..... | 25.65 | Phosphorus..... | 9.06 |
| Zinc..... | 50.63 | Sulphur..... | 16.86 |

Prof. Wood considers 144 heat-units as the most reliable value for the latent heat of fusion of ice. Person gives 142.65.

Latent Heat of Evaporation.—When a body passes from the solid or liquid to the gaseous state, its temperature during the operation remains stationary at a certain boiling-point, depending on the pressure of the vapor produced; and in order to make the evaporation go on, a quantity of heat must be transferred to the substance evaporated, whose amount for each unit of weight of the substance evaporated depends on the temperature. That heat does not raise the temperature of the substance, but disappears in causing it to assume the gaseous state, and it is called the latent heat of evaporation.

When a body passes from the gaseous state to the liquid or solid state, its temperature remains stationary, during that operation, at the boiling-point corresponding to the pressure of the vapor: a quantity of heat equal to the latent heat of evaporation at that temperature is produced in the body; and in order that the operation of condensation may go on, that heat must be transferred from the body condensed to some other body.

The following are examples of the latent heat of evaporation in British thermal units, of one pound of certain substances, when the pressure of the vapor is one atmosphere of 14.7 lbs. on the square inch:

| Substance. | Boiling-point under one atm. Fahr. | Latent Heat in British units. |
|---------------------------|---------------------------------------|----------------------------------|
| Water..... | 212.0 | 965.7 (Regnault). |
| Alcohol..... | 172.2 | 364.3 (Andrews). |
| Ether..... | 95.0 | 162.8 " |
| Bisulphide of carbon..... | 114.8 | 156.0 " |

The latent heat of evaporation of water at a series of boiling-points extending from a few degrees below its freezing-point up to about 375 degrees Fahrenheit has been determined experimentally by M. Regnault. The results of those experiments are represented approximately by the formula, in British thermal units per pound,

$$l \text{ nearly} = 1091.7 - 0.7 (t - 32^\circ) = 965.7 - 0.7 (t - 212^\circ).$$

Henning (*Ann. der Physik*, 1906) gives for t from 0° to 100° C.

For 1 kg., $l = 94.210 (365 - t^\circ \text{C.}) 0.31249$.

For 1 lb., $l = 141.124 (689 - t^\circ \text{F.}) 0.31249$.

The last formula gives for the latent heat at 212°F. , 969.7 B.T.U.

The Total Heat of Evaporation is the sum of the heat which disappears in evaporating one pound of a given substance at a given temperature (or latent heat of evaporation) and of the heat required to raise its temperature, before evaporation, from some fixed temperature up to the temperature of evaporation. The latter part of the total heat is called the sensible heat.

In the case of water, the experiments of M. Regnault show that the total heat of steam from the temperature of melting ice increases at a uniform rate as the temperature of evaporation rises. The following is the formula in British thermal units per pound:

$$h = 1091.7 + 0.305 (t - 32^\circ).$$

H. N. Davis (*Trans. A. S. M. E.*, 1908) gives, in British units,
 $h = 1150 + 0.3745 (t - 212) - 0.000550 (t - 212)^2$.

For the total heat, latent heat, etc., of steam at different pressures, see table of the Properties of Saturated Steam. For tables of total heat, latent heat, and other properties of steams of ether, alcohol, acetone, chloroform, chloride of carbon, and bisulphide of carbon, see Röntgen's Thermodynamics (Dubois's translation). For ammonia and sulphur dioxide, see Wood's Thermodynamics; also, tables under Refrigerating Machinery, in this book.

EVAPORATION AND DRYING.

In evaporation, the formation of vapor takes place on the surface; in boiling, within the liquid: the former is a slow, the latter a quick, method of evaporation.

If we bring an open vessel with water under the receiver of an air-pump and exhaust the air, the water in the vessel will commence to boil, and if we keep up the vacuum the water will actually boil near its freezing-point. The formation of steam in this case is due to the heat which the water takes out of the surroundings.

Steam formed under pressure has the same temperature as the liquid in which it was formed, provided the steam is kept under the same pressure.

By properly cooling the rising steam from boiling water, as in the multiple-effect evaporating systems, we can regulate the pressure so that the water boils at low temperatures.

Evaporation of Water in Reservoirs. — Experiments at the Mount Hope Reservoir, Rochester, N. Y., in 1891, gave the following results:

| | July. | Aug. | Sept. | Oct. |
|---|-------|------|-------|------|
| Mean temperature of air in shade..... | 70.5 | 70.3 | 68.7 | 53.3 |
| “ “ water in reservoir... | 68.2 | 70.2 | 66.1 | 54.4 |
| “ humidity of air, per cent..... | 67.0 | 74.6 | 75.2 | 74.7 |
| Evaporation in inches during month..... | 5.59 | 4.93 | 4.05 | 3.23 |
| Rainfall in inches during month..... | 3.44 | 2.95 | 1.44 | 2.16 |

Evaporation of Water from Open Channels. (Flynn's Irrigation Canals and Flow of Water.) — Experiments from 1881 to 1885 in Tulare County, California, showed an evaporation from a pan in the river equal to an average depth of $\frac{1}{8}$ in. per day throughout the year.

When the pan was in the air the average evaporation was less than $\frac{3}{16}$ in. per day. The average for the month of August was $\frac{1}{3}$ in. per day, and for March and April $\frac{1}{12}$ in. per day. Experiments in Colorado show that evaporation ranges from 0.088 to 0.16 in. per day during the irrigating season.

In Northern Italy the evaporation was from $\frac{1}{12}$ to $\frac{1}{9}$ inch per day, while in the south, under the influence of hot winds, it was from $\frac{1}{8}$ to $\frac{1}{5}$ inch per day.

In the hot season in Northern India, with a decidedly hot wind blowing, the average evaporation was $\frac{1}{2}$ inch per day. The evaporation increases with the temperature of the water.

Evaporation by the Multiple System. — A multiple effect is a series of evaporating vessels each having a steam chamber, so connected that the heat of the steam or vapor produced in the first vessel heats the second, the vapor or steam produced in the second heats the third, and so on. The vapor from the last vessel is condensed in a condenser. Three vessels are generally used, in which case the apparatus is called a *Triple Effect*. In evaporating in a triple effect the vacuum is graduated so that the liquid is boiled at a constant and low temperature.

A series distilling apparatus of high efficiency is described by W. F. M. Goss in *Trans. A. S. M. E.*, 1903. It has seven chambers in series, and is designed to distill 500 gallons of water per hour with an efficiency of approximately 60 lbs. of water per pound of coal.

Tests of Yaryan six-effect machines have shown as high as 44 lbs. of water evaporated per pound of fuel consumed. — *Mach'y*, April, 1905. A description of a large distilling apparatus, using three 125-H.P. boilers and a Lillie triple effect, with record of tests, is given in *Eng. News*, Mar. 29, 1900, and in *Jour. Am. Soc'y of Naval Engineers*, Feb., 1900.

Resistance to Boiling. — **Brine.** (Rankine.) — The presence in a liquid of a substance dissolved in it (as salt in water) resists ebullition, and raises the temperature at which the liquid boils, under a given pressure; but unless the dissolved substance enters into the composition of the vapor, the relation between the temperature and pressure of saturation of the vapor remains unchanged. A resistance to ebullition is also offered by a vessel of a material which attracts the liquid (as when water boils in a glass vessel), and the boiling takes place by starts. To avoid the errors which causes of this kind produce in the measurement of boiling-points, it is advisable to place the thermometer, not in the liquid, but in the vapor, which shows the true boiling-point, freed from the disturbing effect of the attractive nature of the vessel. The boiling-point of saturated brine under one atmosphere is 226° F., and that of weaker brine is higher than the boiling-point of pure water by 1.2° F., for each $\frac{1}{32}$ of salt that the water contains. Average sea-water contains $\frac{1}{32}$; and the brine in marine boilers is not suffered to contain more than from $\frac{2}{32}$ to $\frac{3}{32}$.

Methods of Evaporation Employed in the Manufacture of Salt. (F. E. Engelhardt, Chemist Onondaga Salt Springs; Report for 1889.) — 1. Solar heat — solar evaporation. 2. Direct fire, applied to the heating surface of the vessels containing brine — kettle and pan methods. 3. The steam-grainer system — steam-pans, steam-kettles, etc. 4. Use

of steam and a reduction of the atmospheric pressure over the boiling brine — vacuum system.

When a saturated salt solution boils, it is immaterial whether it is done under ordinary atmospheric pressure at 228° F., or under four atmospheres with a temperature of 320° F., or in a vacuum under $\frac{1}{10}$ atmosphere, the result will always be a fine-grained salt.

The fuel consumption is stated to be as follows: By the kettle method, 40 to 45 bu. of salt evaporated per ton of fuel, anthracite dust burned on perforated grates; evaporation, 5.53 lbs. of water per pound of coal. By the pan method, 70 to 75 bu. per ton of fuel. By vacuum pans, single effect, 86 bu. per ton of anthracite dust (2000 lbs.). With a double effect nearly double that amount can be produced.

Solubility of Common Salt in Pure Water. (Andree.)

| | | | | | | |
|-----------------------------------|-------|-------|-------|-------|-------|-------|
| Temp. of brine, F..... | 32 | 50 | 86 | 104 | 140 | 176 |
| 100 parts water dissolve parts.. | 35.63 | 35.69 | 36.03 | 36.32 | 37.06 | 38.00 |
| 100 parts brine contain salt | 26.27 | 26.30 | 26.49 | 26.64 | 27.04 | 27.54 |

According to Poggial, 100 parts of water dissolve at 229.66° F., 40.35 parts of salt, or in per cent of brine, 28.749. Gay-Lussac found that at 229.72° F., 100 parts of pure water would dissolve 40.38 parts of salt, in per cent of brine, 28.764 parts.

The solubility of salt at 229° F. is only 2.5% greater than at 32°. Hence we cannot, as in the case of alum, separate the salt from the water by allowing a saturated solution at the boiling-point to cool to a lower temperature.

Strength of Salt Brines. — The following table is condensed from one given in U. S. Mineral Resources for 1888, on the authority of Dr. Engelhardt.

Relations between Salinometer Strength, Specific Gravity, Solid Contents, etc., of Brines of Different Strengths.

| Salinometer, degrees. | Baumé, degrees. | Specific gravity. | Per cent of salt. | Weight of a gallon of this brine in pounds. | Pounds of salt in a gallon of brine of 231 cubic inches. | Gallons of brine required for a bushel of salt. | Pounds of water to be evaporated to produce a bushel of salt. | Lbs. of coal required to produce a bushel of salt, 1 lb. coal evaporating 6 lbs. of water. | Bushels of salt that can be made with a ton of coal of 2000 pounds. |
|-----------------------|-----------------|-------------------|-------------------|---|--|---|---|--|---|
| 1..... | 0.26 | 1.002 | 0.265 | 8.347 | 0.022 | 2,531 | 21,076 | 3,513 | 0.569 |
| 2..... | 0.52 | 1.003 | 0.530 | 8.356 | 0.044 | 1,264 | 10,510 | 1,752 | 1.141 |
| 4..... | 1.04 | 1.007 | 1.060 | 8.389 | 0.088 | 629.7 | 5,227 | 871.2 | 2.295 |
| 6..... | 1.56 | 1.010 | 1.590 | 8.414 | 0.133 | 418.6 | 3,466 | 577.7 | 3.462 |
| 8..... | 2.08 | 1.014 | 2.120 | 8.447 | 0.179 | 312.7 | 2,585 | 430.9 | 4.641 |
| 10..... | 2.60 | 1.017 | 2.650 | 8.472 | 0.224 | 249.4 | 2,057 | 342.9 | 5.833 |
| 12..... | 3.12 | 1.021 | 3.180 | 8.506 | 0.270 | 207.0 | 1,705 | 284.2 | 7.038 |
| 14..... | 3.64 | 1.025 | 3.710 | 8.539 | 0.316 | 176.8 | 1,453 | 242.2 | 8.256 |
| 16..... | 4.16 | 1.028 | 4.240 | 8.564 | 0.364 | 154.2 | 1,265 | 210.8 | 9.488 |
| 18..... | 4.68 | 1.032 | 4.770 | 8.597 | 0.410 | 136.5 | 1,118 | 186.3 | 10.73 |
| 20..... | 5.20 | 1.035 | 5.300 | 8.622 | 0.457 | 122.5 | 1,001 | 176.8 | 11.99 |
| 30..... | 7.80 | 1.054 | 7.950 | 8.781 | 0.698 | 80.21 | 648.4 | 108.1 | 18.51 |
| 40..... | 10.40 | 1.073 | 10.600 | 8.939 | 0.947 | 59.09 | 472.3 | 78.71 | 25.41 |
| 50..... | 13.00 | 1.093 | 13.250 | 9.105 | 1.206 | 46.41 | 366.6 | 61.10 | 32.73 |
| 60..... | 15.60 | 1.114 | 15.900 | 9.280 | 1.475 | 37.94 | 296.2 | 49.36 | 40.51 |
| 70..... | 18.20 | 1.136 | 18.550 | 9.464 | 1.755 | 31.89 | 245.9 | 40.98 | 48.80 |
| 80..... | 20.80 | 1.158 | 21.200 | 9.647 | 2.045 | 27.38 | 208.1 | 34.69 | 57.65 |
| 90..... | 23.40 | 1.182 | 23.850 | 9.847 | 2.348 | 23.84 | 178.8 | 29.80 | 67.11 |
| 100..... | 26.00 | 1.205 | 26.500 | 10.039 | 2.660 | 21.04 | 155.3 | 25.88 | 77.26 |

Solubility of Sulphate of Lime in Pure Water. (Marignac.)

| | | | | | | | | |
|---|-------|------|------|-------|-------|-------|-------|-----|
| Temperature F. degrees.. | 32 | 64.5 | 89.6 | 100.4 | 105.8 | 127.4 | 186.8 | 212 |
| Parts water to dissolve 1 part gypsum | } 415 | 386 | 371 | 368 | 370 | 375 | 417 | 452 |
| Parts water to dissolve 1 part anhydrous CaSO_4 | | | | | | | | |
| | 525 | 488 | 470 | 466 | 468 | 474 | 528 | 572 |

In salt brine sulphate of lime is much more soluble than in pure water. In the evaporation of salt brine the accumulation of sulphate of lime tends to stop the operation, and it must be removed from the pans to avoid waste of fuel.

The average strength of brine in the New York salt districts in 1889 was 69.38 degrees of the salinometer.

Concentration of Sugar Solutions.* (From "Heating and Concentrating Liquids by Steam," by John G. Hudson; *The Engineer*, June 13, 1890.) — In the early stages of the process, when the liquor is of low density, the evaporative duty will be high, say two to three (British) gallons per square foot of heating surface with 10 lbs. steam pressure, but will gradually fall to an almost nominal amount as the final stage is approached. As a generally safe basis for designing, Mr. Hudson takes an evaporation of one gallon per hour for each square foot of gross heating surface, with steam of the pressure of about 10 lbs.

As examples of the evaporative duty of a vacuum pan when performing the earlier stages of concentration, during which all the heating surface can be employed, he gives the following:

COIL VACUUM PAN. — $4\frac{3}{4}$ in. copper coils, 528 square feet of surface; steam in coils, 15 lbs.; temperature in pan, 141° to 148° ; density of feed, 25° Baumé, and concentrated to 31° Baumé.

First Trial. — Evaporation at the rate of 2000 gallons per hour = 3.8 gallons per square foot; transmission, 376 units per degree of difference of temperature.

Second Trial. — Evaporation at the rate of 1503 gallons per hour = 2.8 gallons per square foot; transmission, 265 units per degree.

As regards the total time needed to work up a charge of massecuite from liquor of a given density, the following figures, obtained by plotting the results from a large number of pans, form a guide to practical working. The pans were all of the coil type, some with and some without jackets, the gross heating surface probably averaging, and not greatly differing from, 0.25 square foot per gallon capacity, and the steam pressure 10 lbs. per square inch. Both plantation and refining pans are included, making various grades of sugar:

| | | | | | |
|--|------------|------------|------------|------------|------------|
| Density of feed (deg. Baumé)..... | 10° | 15° | 20° | 25° | 30° |
| Evaporation required per gallon massecuite discharged..... | 6.123 | 3.6 | 2.26 | 1.5 | .97 |
| Average working hours required per charge . | 12. | 9. | 6.5 | 5. | 4. |
| Equivalent average evaporation per hour per square foot of gross surface, assuming 0.25 sq. ft. per gallon capacity..... | 2.04 | 1.6 | 1.39 | 1.2 | 0.97 |
| Fastest working hours required per charge . | 8.5 | 5.5 | 3.8 | 2.75 | 2.0 |
| Equivalent average evaporation per hour per square foot..... | 2.88 | 2.6 | 2.38 | 2.18 | 1.9 |

The quantity of heating steam needed is practically the same in vacuum as in open pans. The advantages proper to the vacuum system are primarily the reduced temperature of boiling, and incidentally the possibility of using heating steam of low pressure.

In a solution of sugar in water, each pound of sugar adds to the volume of the water to the extent of 0.061 gallon at a low density to 0.0638 gallon at high densities.

A Method of Evaporating by Exhaust Steam is described by Albert Stearns in *Trans. A. S. M. E.*, vol. viii. A pan $17' 6'' \times 11' \times 1' 6''$,

* For other sugar data, see Bagasse as Fuel, under *Fuel*.

fitted with cast-iron condensing pipes of about 250 sq. ft. of surface, evaporated 120 gallons per hour from clear water, condensing only about one-half of the steam supplied by a plain slide-valve engine of 14" X 32" cylinder, making 65 revs. per min., cutting off about two-thirds stroke, with steam at 75 lbs. boiler pressure.

It was found that keeping the pan-room warm and letting only sufficient air in to carry the vapor up out of a ventilator adds to its efficiency, as the average temperature of the water in the pan was only about 165° F.

Experiments were made with coils of pipe in a small pan, first with no agitator, then with one having straight blades, and lastly with troughed blades; the evaporative results being about the proportions of one, two, and three respectively.

In evaporating liquors whose boiling-point is 220° F., or much above that of water, it is found that exhaust steam can do but little more than bring them up to saturation strength, but on weak liquors, sirups, glues, etc., it should be very useful.

Drying in Vacuum.—An apparatus for drying grain and other substances in vacuum is described by Mr. Emil Passburg in *Proc. Inst. Mech. Engrs.*, 1889. The three essential requirements for a successful and economical process of drying are: 1. Cheap evaporation of the moisture; 2. Quick drying at a low temperature; 3. Large capacity of the apparatus.

The removal of the moisture can be effected in either of two ways: either by slow evaporation, or by quick evaporation—that is, by boiling.

Slow Evaporation.—The principal idea carried into practice in machines acting by slow evaporation is to bring the wet substance repeatedly into contact with the inner surfaces of the apparatus, which are heated by steam, while at the same time a current of hot air is also passing through the substances for carrying off the moisture. This method requires much heat, because the hot-air current has to move at a considerable speed in order to shorten the drying process as much as possible; consequently a great quantity of heated air passes through and escapes unused. As a carrier of moisture hot air cannot in practice be charged beyond half its full saturation; and it is in fact considered a satisfactory result if even this proportion be attained. A great amount of heat is here produced which is not used; while, with scarcely half the cost for fuel, a much quicker removal of the water is obtained by heating it to the boiling-point.

Quick Evaporation by Boiling.—This does not take place until the water is brought up to the boiling-point and kept there, namely, 212° F., under atmospheric pressure. The vapor generated then escapes freely. Liquids are easily evaporated in this way, because by their motion consequent on boiling the heat is continuously conveyed from the heating surfaces through the liquid, but it is different with solid substances, and many more difficulties have to be overcome, because convection of the heat ceases entirely in solids. The substance remains motionless, and consequently a much greater quantity of heat is required than with liquids for obtaining the same results.

Evaporation in Vacuum.—All the foregoing disadvantages are avoided if the boiling-point of water is lowered, that is, if the evaporation is carried out under vacuum.

This plan has been successfully applied in Mr. Passburg's vacuum drying apparatus, which is designed to evaporate large quantities of water contained in solid substances.

The drying apparatus consists of a top horizontal cylinder, surmounted by a charging vessel at one end, and a bottom horizontal cylinder with a discharging vessel beneath it at the same end. Both cylinders are incased in steam-jackets heated by exhaust steam. In the top cylinder works a revolving cast-iron screw with hollow blades, which is also heated by exhaust steam. The bottom cylinder contains a revolving drum of tubes, consisting of one large central tube surrounded by 24 smaller ones, all fixed in tube-plates at both ends; this drum is heated by live steam direct from the boiler. The substance to be dried is fed into the charging vessel through two manholes, and is carried along the top cylinder by the screw creeper to the back end, where it drops through a valve into the bottom cylinder, in which it is lifted by blades attached to the drum and travels forward in the reverse direction; from the front end of the bottom cylinder it falls into a discharging vessel through another

valve, having by this time become dried. The vapor arising during the process is carried off by an air-pump, through a dome and air-valve on the top of the upper cylinder, and also through a throttle-valve on the top of the lower cylinder; both of these valves are supplied with strainers.

As soon as the discharging vessel is filled with dried material the valve connecting it with the bottom cylinder is shut, and the dried charge taken out without impairing the vacuum in the apparatus. When the charging vessel requires replenishing, the intermediate valve between the two cylinders is shut, and the charging vessel filled with a fresh supply of wet material; the vacuum still remains unimpaired in the bottom cylinder, and has to be restored only in the top cylinder after the charging vessel has been closed again.

In this vacuum the boiling-point of the water contained in the wet material is brought down as low as 110° F. The difference between this temperature and that of the heating surfaces is amply sufficient for obtaining good results from the employment of exhaust steam for heating all the surfaces except the revolving drum of tubes. The water contained in the solid substance to be dried evaporates as soon as the latter is heated to about 110° F., and as long as there is any moisture to be removed the solid substance is not heated above this temperature.

Wet grains from a brewery or distillery, containing from 75% to 78% of water, have by this drying process been converted from a worthless incumbrance into a valuable food-stuff. The water is removed by evaporation only, no previous mechanical pressing being resorted to.

At Guinness's brewery in Dublin two of these machines are employed. In each of these the top cylinder is 20 ft. 4 in. long and 2 ft. 8 in. diam., and the screw working inside it makes 7 revs. per min.; the bottom cylinder is 19 ft. 2 in. long and 5 ft. 4 in. diam., and the drum of the tubes inside it makes 5 revs. per min. The drying surfaces of the two cylinders amount together to a total area of about 1000 sq. ft., of which about 40% is heated by exhaust steam direct from the boiler. There is only one air-pump, which is made large enough for three machines; it is horizontal, and has only one air-cylinder, which is double-acting, $17\frac{3}{4}$ in. diam. and $17\frac{3}{4}$ in. stroke; and it is driven at about 45 revs. per min. As the result of about eight months' experience, the two machines have been drying the wet grains from about 500 cwt. of malt per day of 24 hours.

Roughly speaking, 3 cwt. of malt gave 4 cwt. of wet grains, and the latter yield 1 cwt. of dried grains; 500 cwt. of malt will therefore yield about 670 cwt. of wet grains, or 335 cwt. per machine. The quantity of water to be evaporated from the wet grains is from 75% to 78% of their total weight, or, say, about 512 cwt. altogether, being 256 cwt. per machine.

Driers and Drying.

(Contributed by W. B. Ruggles, 1909.)

Materials of different physical and chemical properties require different types of drying apparatus. It is therefore necessary to classify materials into groups, as below, and design different machines for each group.

Group A: Materials which may be heated to a high temperature and are not injured by being in contact with products of combustion. These include cement rock, sand, gravel, granulated slag, clay, marl, chalk, ore, graphite, asbestos, phosphate rock, slacked lime, etc.

The most simple machine for drying these materials is a single revolving shell with lifting flights on the inside, the shell resting on bearing wheels and having a furnace at one end and a stack or fan at the other. The advantage of this style of machine is its low cost of installation and the small number of parts. The disadvantages are great cost of repairs and excessive fuel consumption, due to radiation and high temperature of the stack gases. If the material is fed from the stack and towards the furnace end, the shell near the furnace gets red-hot, causing excessive radiation and frequent repairs. Should the feed be reversed the exhaust temperature

must be kept above 212° F., or recondensation will take place, wetting the material.

In order to economize fuel the shell is sometimes supported at the ends and brickwork is erected around the shell, the hot gases passing under the shell and back through it. Although this method is more economical in the use of fuel, the cost of installation and the cost of repairs are greater.

Group B: Materials such as will not be injured by the products of combustion but cannot be raised to a high temperature on account of driving off water of crystallization, breaking up chemical combinations, or on account of danger from ignition. Included in these are gypsum, fluor-spar, iron pyrites, coal, coke, lignite, sawdust, leather scraps, cork chips, tobacco stems, fish scraps, tankage, peat, etc. Some of these materials may be dried in a single-shell drier and some in a bricked-in machine, but none of them in a satisfactory way on account of the difficulty of regulating the temperature and, in some cases, the danger of explosion of dust.

Group C: Materials which are not injured by a high temperature but which cannot be allowed to come into contact with products of combustion. These are kaolin, ochre and other pigments, fuller's earth, which is to be used in filtering vegetable or animal oils, whitening and similar earthy materials, a large proportion of which would be lost as dust in direct-heat drying. These may be dried by passing through a single-shell drier incased in brickwork and allowing heat to come into contact with the shell only, but this is an uneconomical machine to operate, due to the high temperature of the escaping gases.

Group D: Organic materials which are used for food either by man or the lower animals, such as grain which has been wet, cotton-seed, starch feed, corn germs, brewers' grains, and breakfast foods, which must be dried after cooking. These, of course, cannot be brought into contact with furnace gases and must be kept at a low temperature. For these materials a drier using either exhaust or live steam is the only practical one. This is generally a revolving shell in which are arranged steam pipes. Care should be exercised in selecting a steam drier which has perfect and automatic drainage of the pipes. The condensed steam always amounts to more than the water evaporated from the material.

Group E: Materials which are composed wholly or contain a large proportion of soluble salts, such as nitrate of soda, nitrate of potash, carbonates of soda or potash, chlorates of soda or potash, etc. These in drying form a hard scale which adheres to the shell, and a rotary drier cannot be profitably used on account of frequent stops for cleaning. The only practical machine for such materials is a semicircular cast-iron trough having a shaft through the center carrying paddles that constantly stir up the material and feed it through the drier. This machine has brick side walls and an exterior furnace; the heat from the furnace passing under the shell and back through the drying material or out through a stack or fan without passing through the material, as may be desired. Should the material also require a low temperature, the same type of drier can be used by substituting steam-jacketed steel sections instead of cast iron.

The efficiency of a drier is the ratio of the theoretical heat required to do the drying to the total heat supplied. The greatest loss is the heat carried out by the exhaust or waste gases; this may be as great as 40% of the total heat from the fuel, or with a properly designed drier may be as small as 8%. The radiation from the shell or walls may be as high as 25% or as low as 4%. The heat carried away by the dried material may amount under conditions of careless operation to as much as 25% or may be as low as nothing.

A properly designed drier of the direct-heat type for either group "A" or "B" will give an efficiency of from 75% to 85%; a bricked-in return-draught single-shell drier, from 60% to 70%; and a single-shell straight-draught dryer, from 45% to 55%. A properly designed indirect-heat drier for group "C" will give an efficiency of 50% to 60%, and a poorly designed one may not give more than 30%. The best designed steam drier for group "D," in which the losses in the boiler producing the steam must be considered, will not often give an efficiency of more than

42%; and, while a poorly designed one may have an equal efficiency, its capacity may be not more than one-half of a good drier of equal size. The drier described for group "E" will not give an efficiency of more than 55%.

Performance of Different Types of Driers.

(W. B. Ruggles.)

| Type of drier | Double shell; direct heat. | Indirect heat; 705 sq. ft. | Single shell, bricked in; direct heat. | Single shell; direct heat. | Stationary, with paddles; direct heat. |
|--|-------------------------------|-------------------------------|--|-------------------------------|--|
| Material..... | Sand. | Coal. | Cement slurry. | Lime stone. | Nitrate of soda. |
| Moisture, initial, per cent..... | 4.58 | 10.2 | 61.2 | 3.6 | 7.2 |
| Moisture, final, per cent..... | 0 | 0 | 40.7 | 0.5 | 0.3 |
| Calorific value of fuel, B.T.U. | 12100 | 12290 | 13200 | 13180 | 13600 |
| Fuel consumed per hour, lbs. | 398 | 213.6 | 667 | 460 | 87 |
| Water evaporated per hour, lbs.. | 2196 | 924.2 | 4057 | 1325 | 349 |
| Water evap. per pound fuel, lbs.. | 5.3 | 4.3 | 6.1 | 2.3 | 4.0 |
| Material dried per hour, lbs. | 36460 | 8300 | 7680 | 41400 | 4581 |
| Fuel per ton dried material, lbs. ... | 21.8 | 51.3 | 17.3 | 22.2 | 38.0 |
| Heat lost in exhaust air, per cent | 11.3 | 42.8 | 38.4 | 38.2 | 40.7 |
| Heat lost by radiation, etc., per cent..... | 7.6 | 7.7 | 12.5 | 15.6 | 13.8 |
| Heat used to evaporate water, per cent..... | 52.5 | 39.4 | 52.0 | 24.4 | 33.1 |
| Heat used to raise temperature of material, per cent..... | 28.6 | 10.1 | 7.1 | 21.8 | 12.4 |
| Total efficiency, per cent..... | 81.1 | 49.5 | 59.1 | 46.2 | 45.5 |

PERFORMANCE OF A STEAM DRIER.

Material: Starch feed. Moisture, initial 39.8%, final 0.22%. Dried material per hour, 831 lbs. Water evaporated per hour, 548 lbs. Steam consumed per hour, 793 lbs. Water evaporated per pound steam, 0.691 lb. Temperature of material, moist, 58°, dry, 212°. Steam pressure, 98 lbs. gauge.

Total heat to evaporate 548 lbs. water at 58° into steam,

$$548 \times (154.2 + 969.7) = 615,897 \text{ B.T.U.}$$

Heat supplied by 793 lbs. steam condensed to water at 212°,

$$793 \times (1188.2 - 180.3) = 799,265 \text{ B.T.U.}$$

Heat used to evaporate water,

$$(615,897 \div 799,265) = 77.1\%.$$

Heat used to raise temp. of material,

$$(831 \times 154 \times 0.492) = 62,963 = 7.9\%.$$

Loss by radiation . . . 100 - (77.1 + 7.9) = 15%.

Total efficiency . . . , , , , , 85.0%.

WATER EVAPORATED AND HEAT REQUIRED FOR DRYING,

M = percentage of moisture in material to be dried.

Q = lbs. water evaporated per ton (2000 lbs.) of dry material.

H = British thermal units required for drying, per ton of dry material.

| M | Q | H | M | Q | H | M | Q | H |
|----|-------|---------|----|-------|-----------|----|--------|------------|
| 1 | 20.2 | 85,624 | 14 | 325.6 | 424,884 | 35 | 1,077 | 1,269,240 |
| 2 | 40.8 | 108,696 | 15 | 352.9 | 458,248 | 40 | 1,333 | 1,555,960 |
| 3 | 61.9 | 130,424 | 16 | 381.0 | 489,720 | 45 | 1,636 | 1,895,320 |
| 4 | 83.3 | 156,296 | 17 | 409.6 | 521,752 | 50 | 2,000 | 2,303,000 |
| 5 | 105.3 | 180,936 | 18 | 439.0 | 554,680 | 55 | 2,444 | 2,800,280 |
| 6 | 127.7 | 206,024 | 19 | 469.1 | 588,392 | 60 | 3,000 | 3,423,000 |
| 7 | 150.5 | 231,560 | 20 | 500.0 | 623,000 | 65 | 3,714 | 4,222,680 |
| 8 | 173.9 | 257,768 | 21 | 531.6 | 658,392 | 70 | 4,667 | 5,290,040 |
| 9 | 197.8 | 284,536 | 22 | 564.1 | 694,792 | 75 | 6,000 | 6,783,000 |
| 10 | 222.2 | 311,864 | 23 | 597.4 | 732,088 | 80 | 8,000 | 9,023,000 |
| 11 | 247.2 | 339,864 | 24 | 631.6 | 770,392 | 85 | 11,333 | 12,755,960 |
| 12 | 272.7 | 368,424 | 25 | 666.7 | 809,704 | 90 | 18,000 | 20,223,000 |
| 13 | 298.9 | 397,768 | 30 | 857.0 | 1,022,840 | 95 | 38,000 | 42,623,000 |

$$\text{Formula: } Q = \frac{2000 M}{100 - M}; \quad H = 1120 Q + 63,000.$$

The value of H is found on the assumption that the moisture is heated from 62° to 212° and evaporated at that temperature, and that the specific heat of the material is 0.21. $[2000 \times (212 - 62) \times 0.21] = 63,000$.

Calculations for Design of Drying Apparatus. — A most efficient system of drying of moist materials consists in a continuous circulation of a volume of warm dry air over or through the moist material, then passing the air charged with moisture over the cold surfaces of condenser coils to remove the moisture, then heating the same air by steam-heating coils or other means, and again passing it over the material. In the design of apparatus to work on this system it is necessary to know the amount of moisture to be removed in a given time, and to calculate the volume of air that will carry that moisture at the temperature at which it leaves the material, making allowance for the fact that the moist, warm air on leaving the material may not be fully saturated, and for the fact that the cooled air is nearly or fully saturated at the temperature at which it leaves the cooling coils. A paper by Wm. M. Grosvenor, read before the Am. Inst. of Chemical Engineers (*Heating and Ventilating Mag.*, May, 1909) contains a "humidity table" and a "humidity chart" which greatly facilitate the calculations required. The table is given in a condensed form below. It is based on the following data: Density of air + 0.04% CO₂ = 0.001293052

(in Kg. per cu. m.). Density of water vapor $1 + 0.00367 \times \text{Temp. C.} = 0.62186 \times \text{density of air}$. Density at partial pressure ÷ density at 760 m.m. = partial pressure ÷ 760 m.m. Specific heat of water vapor = 0.475; sp. ht. of air = 0.2373. Kg. per cu. meter $\times 0.062428 = \text{lbs. per cu. ft.}$ The results given in the table agree within 1/4% with the figures of the U. S. Weather Bureau. (Compare also the tables of H. M. Prevost Murphy, given under "Air," page 586.) The term "humid heat" in the heading of the table is defined as the B.T.U. required to raise 1° F. one pound of air plus the vapor it may carry when saturated at the given temperature and pressure; and "humid volume" is the volume of one pound of air when saturated at the given temperature and pressure.

Humidity Table.

| Temp. F. | Vapor Tension, Milli- meters of Mercury. | Lbs. Water Vapor per lb. Air. | Humid Heat, B.T.U. | Humid Volume cu. ft. | Density, lbs. per cu. ft. at 760 Millimeters. | | Volume in cu. ft. per lb. of | |
|-------------|--|---|--------------------------|----------------------------|---|---------------|---------------------------------|---------------|
| | | | | | Dry Air. | Sat'd Mix. | Dry Air. | Sat'd Mix. |
| 32 | 4.569 | .003761 | .2391 | 12.462 | .080726 | .080556 | 12.388 | 12.414 |
| 35 | 5.152 | .0042435 | .2393 | 12.549 | .080231 | .080085 | 12.464 | 12.496 |
| 40 | 6.264 | .0050463 | .2398 | 12.695 | .079420 | .079181 | 12.590 | 12.629 |
| 45 | 7.582 | .0062670 | .2403 | 12.843 | .078641 | .078348 | 12.718 | 12.763 |
| 50 | 9.140 | .0075697 | .2409 | 12.999 | .077867 | .077511 | 12.842 | 12.901 |
| 55 | 10.980 | .0091163 | .2416 | 13.159 | .077109 | .076685 | 12.968 | 13.041 |
| 60 | 13.138 | .010939 | .2425 | 13.326 | .076363 | .075865 | 13.095 | 13.180 |
| 65 | 15.660 | .013081 | .2435 | 13.501 | .075635 | .075039 | 13.222 | 13.325 |
| 70 | 18.595 | .015597 | .2447 | 13.683 | .074921 | .074219 | 13.348 | 13.471 |
| 75 | 22.008 | .018545 | .2461 | 13.876 | .074218 | .073471 | 13.474 | 13.624 |
| 80 | 25.965 | .021998 | .2478 | 14.081 | .073531 | .072644 | 13.600 | 13.777 |
| 85 | 30.573 | .026026 | .2497 | 14.301 | .072852 | .071744 | 13.726 | 13.938 |
| 90 | 35.774 | .030718 | .2519 | 14.539 | .072189 | .070894 | 13.852 | 14.106 |
| 95 | 41.784 | .036174 | .2545 | 14.793 | .071535 | .070051 | 13.979 | 14.275 |
| 100 | 48.679 | .042116 | .2575 | 15.071 | .070894 | .069179 | 14.106 | 14.455 |
| 105 | 56.534 | .049973 | .2610 | 15.376 | .070264 | .068288 | 14.232 | 14.643 |
| 110 | 65.459 | .058613 | .2651 | 15.711 | .069647 | .067383 | 14.358 | 14.840 |
| 115 | 75.591 | .068662 | .2699 | 16.084 | .069040 | .066447 | 14.484 | 15.050 |
| 120 | 87.010 | .080402 | .2755 | 16.499 | .068443 | .065477 | 14.611 | 15.272 |
| 125 | 99.024 | .094147 | .2820 | 16.968 | .067857 | .064480 | 14.736 | 15.509 |
| 130 | 114.437 | .11022 | .2896 | 17.499 | .067380 | .063449 | 14.863 | 15.761 |
| 135 | 130.702 | .12927 | .2987 | 18.103 | .066713 | .062374 | 14.989 | 16.032 |
| 140 | 148.885 | .15150 | .3093 | 18.800 | .066156 | .061255 | 15.116 | 16.325 |
| 145 | 169.227 | .17816 | .3219 | 19.609 | .065601 | .060104 | 15.242 | 16.643 |
| 150 | 191.860 | .21005 | .3371 | 20.559 | .065154 | .058865 | 15.368 | 16.993 |
| 155 | 216.983 | .24534 | .3553 | 21.687 | .064539 | .057570 | 15.494 | 17.370 |
| 160 | 244.803 | .29553 | .3776 | 23.045 | .064016 | .056218 | 15.621 | 17.788 |
| 165 | 275.592 | .35286 | .4054 | 24.708 | .063502 | .054795 | 15.748 | 18.250 |
| 170 | 309.593 | .42756 | .4405 | 26.790 | .062997 | .053305 | 15.874 | 18.761 |
| 175 | 347.015 | .52285 | .4856 | 29.454 | .062500 | .051708 | 16.000 | 19.339 |
| 180 | 388.121 | .64942 | .5458 | 32.967 | .062015 | .050035 | 16.126 | 19.987 |
| 185 | 433.194 | .82430 | .6288 | 37.796 | .061529 | .048265 | 16.253 | 20.719 |
| 190 | 482.668 | 1.00805 | .7519 | 44.918 | .061053 | .046391 | 16.379 | 21.557 |
| 195 | 536.744 | 1.4994 | .9494 | 56.302 | .060588 | .044405 | 16.505 | 22.521 |
| 200 | 595.771 | 2.2680 | 1.3147 | 77.304 | .060127 | .042308 | 16.631 | 23.638 |
| 205 | 660.116 | 4.2272 | 2.1562 | 131.028 | .059674 | .040075 | 16.758 | 24.954 |
| 210 | 730.267 | 15.8174 | 15.9148 | 562.054 | .059228 | .037323 | 16.884 | 26.796 |

RADIATION OF HEAT.

Radiation of heat takes place between bodies at all distances apart, and follows the laws for the radiation of light.

The heat rays proceed in straight lines, and the intensity of the rays radiated from any one source varies inversely as the square of their distance from the source.

This statement has been erroneously interpreted by some writers, who have assumed from it that a boiler placed two feet above a fire would receive by radiation only one-fourth as much heat as if it were only one foot above. In the case of boiler furnaces the side walls reflect those rays that are received at an angle,—following the law of optics, that the angle of incidence is equal to the angle of reflection,—with the result that the intensity of heat two feet above the fire is practically the same as at one foot above, instead of only one-fourth as much.

The rate at which a hotter body radiates heat, and a colder body absorbs heat, depends upon the state of the surfaces of the bodies as well as on their temperatures. The rate of radiation and of absorption are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish. For this reason the covering

of steam pipes and boilers should be smooth and of a light color: uncovered pipes and steam-cylinder covers should be polished.

The quantity of heat radiated by a body is also a measure of its heat-absorbing power under the same circumstances. When a polished body is struck by a ray of heat, it absorbs part of the heat and reflects the rest. The reflecting power of a body is therefore the complement of its absorbing power, which latter is the same as its radiating power.

The relative radiating and reflecting power of different bodies has been determined by experiment, as shown in the table below, but as far as quantities of heat are concerned, says Prof. Trowbridge (Johnson's Cyclopædia, art. Heat), it is doubtful whether anything further than the said relative determinations can, in the present state of our knowledge, be depended upon, the actual or absolute quantities for different temperatures being still uncertain. The authorities do not even agree on the relative radiating powers. Thus, Leslie gives for tin plate, gold, silver, and copper the figure 12, which differs considerably from the figures in the table below, given by Clark, stated to be on the authority of Leslie, De La Provostaye and Desains, and Melloni..

Relative Radiating and Reflecting Power of Different Substances.

| | Radiating or Absorbing Power. | Reflecting Power. | | Radiating or Absorbing Power. | Reflecting Power. |
|------------------------|-------------------------------------|----------------------|----------------------|-------------------------------------|----------------------|
| Lampblack..... | 100 | 0 | Zinc, polished..... | 19 | 81 |
| Water..... | 100 | 0 | Steel, polished..... | 17 | 83 |
| Carbonate of lead... | 100 | 0 | Platinum, polished. | 24 | 76 |
| Writing-paper..... | 98 | 2 | Platinum in sheet.. | 17 | 83 |
| Ivory, jet, marble... | 93 to 98 | 7 to 2 | Tin..... | 15 | 85 |
| Ordinary glass..... | 90 | 10 | Brass, cast, dead | | |
| Ice..... | 85 | 15 | polished..... | 11 | 89 |
| Gum lac..... | 72 | 28 | Brass, bright pol- | | |
| Silver-leaf on glass.. | 27 | 73 | ished..... | 7 | 93 |
| Cast iron, bright pol- | | | Copper, varnished.. | 14 | 86 |
| ished..... | 25 | 75 | Copper, hammered.. | 7 | 93 |
| Mercury, about..... | 23 | 77 | Gold, plated..... | 5 | 95 |
| Wrought iron, pol- | | | Gold on polished | | |
| ished..... | 23 | 77 | steel..... | 3 | 97 |
| | | | Silver, polished | | |
| | | | bright..... | 3 | 97 |

Experiments of Dr. A. M. Mayer give the following: The relative radiations from a cube of cast iron, having faces rough, as from the foundry, planed, "drawfiled," and polished, and from the same surfaces oiled, are as below (Prof. Thurston, in *Trans. A. S. M. E.*, vol. xvi):

| | Rough. | Planed. | Drawfiled. | Polished. |
|--------------------|--------|---------|------------|-----------|
| Surface oiled..... | 100 | 60 | 49 | 45 |
| Surface dry..... | 100 | 32 | 20 | 18 |

It here appears that the oiling of smoothly polished castings, as of cylinder-heads of steam-engines, more than doubles the loss of heat by radiation, while it does not seriously affect rough castings.

"Black Body" Radiation. Stefan and Boltzman's Law. (*Eng'g*, March 1, 1907.) — Kirchhoff defined a black body as one that would absorb all radiations falling on it, and would neither reflect nor transmit any. The radiation from such a body is a function of the temperature alone,

and is identical with the radiation inside an inclosure all parts of which have the same temperature. By heating the walls of an inclosure as uniformly as possible, and observing the radiation through a very small opening, a practical realization of a black body is obtained. Stefan and Boltzman's law is: The energy radiated by a black body is proportional to the fourth power of the absolute temperature, or $E = K(T^4 - T_0^4)$, where E = total energy radiated by the body at T to the body at T_0 , and K is a constant. The total radiation from other than black bodies increases more rapidly than the fourth power of the absolute temperature, so that as the temperature is raised the radiation of all bodies approaches that of the black body. A confirmation of the Stefan and Boltzman law is given in the results of experiments by Lummer and Kurlbaum, as below ($T_0 = 290$ degrees C., abs. in all cases).

| | $T = 492.$ | 654. | 795. | 1108. | 1481. | 1761. |
|--|------------|-------|-------|-------|-------|-------|
| $\frac{E}{T^4 - T_0^4}$ { Black body | 109.1 | 108.4 | 109.9 | 109.0 | 110.7 | |
| Polished platinum.. | 4.28 | 6.56 | 8.14 | 12.18 | 16.69 | 19.64 |
| Iron oxide..... | 33.1 | 33.1 | 36.6 | 46.9 | 65.3 | |

CONDUCTION AND CONVECTION OF HEAT.

Conduction is the transfer of heat between two bodies or parts of a body which touch each other. Internal conduction takes place between the parts of one continuous body, and external conduction through the surface of contact of a pair of distinct bodies.

The rate at which conduction, whether internal or external, goes on, being proportional to the area of the section or surface through which it takes place, may be expressed in thermal units per square foot of area per hour.

Internal Conduction varies with the *heat conductivity*, which depends upon the nature of the substance, and is directly proportional to the difference between the temperatures of the two faces of a layer, and inversely as its thickness. The reciprocal of the conductivity is called the *internal thermal resistance* of the substance. If r represents this resistance, x the thickness of the layer in inches, T' and T the temperatures on the two faces, and q the quantity in thermal units transmitted per hour per square foot of area, $q = \frac{T' - T}{rx}$. (Rankine.)

Péclet gives the following values of r :

| | | | |
|------------------------------|--------|-------------|--------|
| Gold, platinum, silver | 0.0016 | Lead..... | 0.0090 |
| Copper..... | 0.0018 | Marble..... | 0.0716 |
| Iron..... | 0.0043 | Brick..... | 0.1500 |
| Zinc..... | 0.0045 | | |

Relative Heat-conducting Power of Metals.

| Metals. | *C.&J. | †W.&F. | Metals. | *C.&J. | †W.&F. |
|---------------------------------|--------|--------|---------------------|--------|--------|
| Silver..... | 1000 | 1000 | Cadmium..... | 577 | ... |
| Gold..... | 981 | 532 | Wrought iron..... | 436 | 119 |
| Gold, with 1% of silver. | 840 | ... | Tin..... | 422 | 145 |
| Copper, rolled..... | 845 | 736 | Steel..... | 397 | 116 |
| Copper, cast..... | 811 | ... | Platinum..... | 380 | 84 |
| Mercury..... | 677 | ... | Sodium..... | 365 | ... |
| Mercury, with 1.25% of tin..... | 412 | ... | Cast iron..... | 359 | ... |
| Aluminum..... | 665 | ... | Lead..... | 287 | 85 |
| Zinc: | | | Antimony: | | |
| cast vertically..... | 628 | ... | cast horizontally.. | 215 | ... |
| cast horizontally.... | 608 | ... | cast vertically.... | 192 | ... |
| rolled..... | 641 | ... | Bismuth..... | 61 | 18 |

* Calvert & Johnson.

† Weidemann & Franz.

INFLUENCE OF A NON-METALLIC SUBSTANCE IN COMBINATION ON THE CONDUCTING POWER OF A METAL.

| | | | |
|------------------------------|-----|-------------------------------|-----|
| Influence of carbon on iron: | | Cast copper..... | 811 |
| Wrought iron..... | 436 | Copper with 1% of arsenic.... | 570 |
| Steel..... | 397 | " with 0.5% of arsenic.. | 669 |
| Cast iron..... | 359 | " with 0.25% of arsenic. | 771 |

The Rate of External Conduction through the bounding surface between a solid body and a fluid is approximately proportional to the difference of temperature, when that is small; but when that difference is considerable, the rate of conduction increases faster than the simple ratio of that difference. (Rankine.)

If r , as before, is the coefficient of internal thermal resistance, e and e' the coefficient of external resistance of the two surfaces, x the thickness of the plate, and T' and T the temperatures of the two fluids in contact with the two surfaces, the rate of conduction is $q = \frac{T' - T}{e + e' + rx}$. Accord-

ing to Péclet, $e + e' = \frac{1}{A [1 + B (T' - T)]}$, in which the constants A and B have the following values:

| | |
|--|--------|
| B for polished metallic surfaces..... | 0.0028 |
| B for rough metallic surfaces and for non-metallic surfaces .. | 0.0037 |
| A for polished metals, about..... | 0.90 |
| A for glassy and varnished surfaces..... | 1.34 |
| A for dull metallic surfaces..... | 1.58 |
| A for lampblack..... | 1.78 |

When a metal plate has a liquid at each side of it, it appears from experiments by Péclet that $B = 0.058$, $A = 8.8$.

The results of experiments on the evaporative power of boilers agree very well with the following approximate formula for the thermal resistance of boiler plates and tubes:

$$e + e' = \frac{a}{(T' - T)},$$

which gives for the rate of conduction, per square foot of surface per hour,

$$q = \frac{(T' - T)^2}{a}.$$

This formula is proposed by Rankine as a rough approximation, near enough to the truth for its purpose. The value of a lies between 160 and 200. Experiments on modern boilers usually give higher values.

Convection, or carrying of heat, means the transfer and diffusion of the heat in a fluid mass by means of the motion of the particles of that mass.

The conduction, properly so called, of heat through a stagnant mass of fluid is very slow in liquids, and almost, if not wholly, inappreciable in gases. It is only by the continual circulation and mixture of the particles of the fluid that uniformity of temperature can be maintained in the fluid mass, or heat transferred between the fluid mass and a solid body.

The free circulation of each of the fluids which touch the side of a solid plate is a necessary condition of the correctness of Rankine's formulæ for the conduction of heat through that plate; and in these formulæ it is implied that the circulation of each of the fluids by currents and eddies is such as to prevent any considerable difference of temperature between the fluid particles in contact with one side of the solid plate and those at considerable distances from it.

When heat is to be transferred by convection from one fluid to another, through an intervening layer of metal, the motions of the two fluid masses should, if possible, be in opposite directions, in order that the hottest particles of each fluid may be in communication with the hottest particles of the other, and that the minimum difference of temperature between the adjacent particles of the two fluids may be the greatest possible.

Thus, in the surface condensation of steam, by passing it through metal tubes immersed in a current of cold water or air, the cooling fluid should be made to move in the opposite direction to the condensing steam.

Coefficients of Heat Conduction of Different Materials. (W. Nusselt, *Zeit des Ver. Deut. Ing.*, June, 1908. *Eng. Digest*, Aug., 1908.)—The materials were inclosed between two concentric metal vessels, the inner of which contained an electric heating device.

It was found that the materials tested all followed Fourier's law, the quantity of heat transmitted being directly proportional to the extent of surface, the duration of flow and the temperature difference between the inner and outer surfaces; and inversely proportional to the thickness of the mass of material. It was also found that the coefficient of conduction increased as the temperature increased. The table gives the British equivalents of the average coefficients obtained.

COEFFICIENTS OF HEAT CONDUCTION AT DIFFERENT TEMPERATURES FOR VARIOUS INSULATING MATERIALS.

(B.T.U. per hour = Area of surface in square feet \times coefficient \div thickness in inch. s.)

| Lb. per cu. ft. | Materials. | 32° F. | 212° F. | 392° F. | 572° F. | 752° F. |
|-----------------|--|--------|---------|---------|---------|---------|
| 10. | Ground cork | 0.250 | 0.387 | 0.443 | | |
| 8.5 | Sheep's wool* | 0.266 | 0.403 | | | |
| 6.3 | Silk waste | 0.306 | 0.411 | | | |
| 9.18 | Silk, tufted | 0.314 | 0.419 | | | |
| 5.06 | Cotton wool | 0.379 | 0.476 | | | |
| 11.86 | Charcoal (carbonized cabbage leaves) | 0.403 | 0.508 | | | |
| 13.42 | Sawdust (0.443 at 112° F.) | | | | | |
| 10. | Peat refuse† (0.443 at 77° F.) | | | | | |
| 21.85 | Kieselguhr (infusorial earth), loose | 0.419 | 0.532 | 0.596 | 0.629 | |
| 12.49 | Asphalt-cork composition (0.492 at 65° F.) | | | | | |
| 25.28 | Composition,‡ loose | 0.484 | 0.613 | 0.653 | | |
| 12.49 | Kieselguhr stone§ | 0.516 | 0.629 | 0.742 | 0.854 | 0.961 |
| 12.17 | Peat refuse† (0.564 at 68° F.) | | | | | |
| 36.2 | Kieselguhr, dry and compacted (0.669 at 302° F.; 0.991 at 662° F.) | | | | | |
| 43.07 | Composition,§§ compacted (0.806 at 302° F.; 0.967 at 428° F.) | | | | | |
| 22.47 | Porous blast-furnace slag (0.766 at 112° F.) | | | | | |
| 35.96 | Asbestos (1.644 at 1112° F.) | 1.048 | 1.346 | 1.451 | 1.499 | 1.548 |
| 34.33 | Slag concrete (1.532 at 112° F.) | | | | | |
| 18.23 | Pumice stone gravel (1.612 at 112° F.) | | | | | |
| 128.5 | Portland cement, neat (6.287 at 95° F.) | | | | | |

* Tufted, oily, and containing foreign matter. Used in Linde's apparatus. † Hygroscopic; measurements made in moist zones. ‡ Cork, asbestos, kieselguhr and chopped straw, mixed with a binder and made in sheets for application to steam pipes in successive layers, the whole being wrapped in canvas and painted. § Kieselguhr, mixed with a binder and burned; very porous and hygroscopic. §§ Ingredients of (†) mixed with water and compacted. || 1 part cement, 9 parts porous blast-furnace slag, by volume.

Heat Resistance, the Reciprocal of Heat Conductivity. (W. Kent, *Trans. A. S. M. E.*, xxiv, 278.)—The resistance to the passage of heat through a plate consists of three separate resistances; viz., the resistances of the two surfaces and the resistance of the body of the plate, which latter is proportional to the thickness of the plate. It is probable also that the resistance of the surface differs with the nature of the body or medium with which it is in contact.

A complete set of experiments on the heat-resisting power of heat-insulating substances should include an investigation into the difference in surface resistance when a surface is in contact with air and when it is in contact with another solid body. Suppose we find that the total resistance of a certain non-conductor may be represented by the figure 10, and that similar pieces all give the same figure. Two pieces in contact give 16. One piece of half the thickness of the others gives 8. What is the resistance of the surface exposed to the air in either piece, of the surface in contact with another surface, and of the interior of the body itself? Let the resistance of the material itself, of the regular thickness, be represented by A , that of the surface exposed to the air by a , and that of the surface in contact with another surface by c .

We then have for the three cases,

| | |
|--------------------------------|-------------------------|
| Resistance of one piece..... | $A + 2a = 10$ |
| " of two pieces in contact.... | $2A + 2c + 2a = 16$ |
| " of the thin piece..... | $\frac{1}{2}A + 2a = 8$ |

These three equations contain three unknown quantities. Solving the equations we find $A = 4$, $a = 3$, and $c = 1$. Suppose that another experiment be made with the two pieces separated by an air space, and that the total resistance is then 22. If the resistance of the air space be represented by s we have the two equations: Resistance of one piece, $A + 2a = 10$; resistance of two pieces and air space, $2A + 4a + s = 22$, from which we find $s = 2$. Having these results we can easily estimate what will be the resistance to heat transfer of any number of layers of the material, whether in contact or separated by air spaces.

The writer has computed the figures for heat resistance of several insulating substances from the figures of conducting power given in a table published by John E. Starr, in *Ice and Refrigeration*, Nov., 1901. Mr. Starr's figures are given in terms of the B.T.U. transmitted per sq. ft. of surface per day per degree of difference of temperatures of the air adjacent to each surface. The writer's figures, those in the last column of the table given herewith, are calculated by dividing Mr. Starr's figures by 24, to obtain the hourly rate, and then taking their reciprocals. They may be called "coefficients of heat resistance" and defined as the reciprocals of the B.T.U. per sq. ft. per hour per degree of difference of temperature.

HEAT CONDUCTING AND RESISTING VALUES OF DIFFERENT INSULATING MATERIALS.

| Insulating Material. | Conductance, B.T.U. per sq. ft. per Day per De- gree of Differ- ence of Tem- perature. | Coefficient of Heat Resistance. C. |
|---|--|---|
| 1. 5/8-in. oak board, 1 in. lampblack, 7/8-in. pine board (ordinary family refrigerator)..... | 5.7 | 4.21 |
| 2. 7/8-in. board, 1 in. pitch, 7/8-in. board..... | 4.89 | 4.91 |
| 3. 7/8-in. board, 2 in. pitch, 7/8-in. board..... | 4.25 | 5.65 |
| 4. 7/8-in. board, paper, 1 in. mineral wool, paper, 7/8-in. board..... | 4.6 | 5.22 |
| 5. 7/8-in. board, paper, 2 1/2 in. mineral wool, paper, 7/8-in. board..... | 3.62 | 6.63 |
| 6. 7/8-in. board, paper, 2 1/2 in. calcined pumice, 7/8-in. board..... | 3.38 | 7.10 |
| 7. Same as above, when wet..... | 3.90 | 6.15 |
| 8. 7/8-in. board, paper, 3 in. sheet cork, 7/8-in. board..... | 2.10 | 11.43 |
| 9. Two 7/8-in. boards, paper, solid, no air space, paper, two 7/8-in. boards..... | 4.28 | 5.61 |
| 10. Two 7/8-in. boards, paper, 1 in. air space, paper, two 7/8-in. boards..... | 3.71 | 6.47 |
| 11. Two 7/8-in. boards, paper, 1 in. hair felt, paper, two 7/8-in. boards..... | 3.32 | 7.23 |
| 12. Two 7/8-in. boards, paper, 8 in. mill shavings, paper, two 7/8-in. boards..... | 1.35 | 17.78 |
| 13. The same, slightly moist..... | 1.80 | 13.33 |
| 14. The same, damp..... | 2.10 | 11.43 |
| 15. Two 7/8-in. boards, paper, 3 in. air, 4 in. sheet cork, paper, two 7/8-in. boards..... | 1.20 | 20.00 |
| 16. Same, with 5 in. sheet cork..... | 0.90 | 26.67 |
| 17. Same, with 4 in. granulated cork..... | 1.70 | 14.12 |
| 18. Same, with 1 in. sheet cork..... | 3.30 | 7.27 |
| 19. Four double 7/8-in. boards (8 boards), with paper between, three 8-in. air spaces..... | 2.70 | 8.89 |
| 20. Four 7/8-in. boards, with three quilts of 1/4-in. hair between, papers separating boards..... | 2.52 | 9.52 |
| 21. 7/8-in. board, 6 in. patented silicated straw-board, finished inside with thin cement..... | 2.48 | 9.68 |

Analyzing some of the results given in the last column of the table, we observe that, comparing Nos. 2 and 3, 1 in. added thickness of pitch increased the coefficient 0.74; comparing Nos. 4 and 5, 1½ in. of mineral wool increased the coefficient 1.11. If we assume that the 1 in. of mineral wool in No. 4 was equal in heat resistance to the additional 1½ in. added in No. 5, or 1.11 reciprocal units, and subtract this from 5.22, we get 4.11 as the resistance of two 7/8-in. boards and two sheets of paper. This would indicate that one 7/8-in. board and one sheet of paper give nearly twice as much resistance as 1 in. of mineral wool. In like manner any number of deductions may be drawn from the table, and some of them will be rather questionable, such as the comparison of No. 15 and No. 16, showing that 1 in. additional sheet cork increased the resistance given by four sheets 6.67 reciprocal units, or one-third the total resistance of No. 15. This result is extraordinary, and indicates that there must have been considerable differences of conditions during the two tests.

For comparison with the coefficients of heat resistance computed from Mr. Starr's results we may take the reciprocals of the figures given by Mr. Alfred R. Wolff as the result of German experiments on the heat transmitted through various building materials, as below:

K = B.T.U. transmitted per hour per sq. ft. of surface, per degree F. difference of temperature.

C = coefficient of heat resistance = reciprocal of K .

The irregularity of the differences of C computed from the original values of K for each increase of 4 inches in thickness of the brick walls indicates a difference in the conditions of the experiments. The average difference of C for each 4 inches of thickness is about 0.80. Using this average difference to even up the figures we find the value of C is expressed by the approximate formula $C = 0.70 + 0.20 t$, in which t is the thickness in inches. The revised values of C , computed by this formula, and the corresponding revised values of K , are as follows:

| Thickness, in. | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| C | 1.50 | 2.30 | 3.10 | 3.90 | 4.70 | 5.50 | 6.30 | 7.10 | 7.90 | 8.70 |
| K , revised. | 0.667 | 0.435 | 0.323 | 0.256 | 0.213 | 0.182 | 0.159 | 0.141 | 0.127 | 0.115 |
| K , original. | 0.68 | 0.46 | 0.32 | 0.26 | 0.23 | 0.20 | 0.174 | 0.15 | 0.129 | 0.115 |
| Difference.. | 0.013 | 0.025 | 0.003 | 0.004 | 0.017 | 0.018 | 0.015 | 0.009 | 0.002 | 0.0 |

The following additional values of C are computed from Mr. Wolff's figures for K :

| | K | C |
|--|-------|-------|
| Wooden beam construction, planked over or ceiled: | | |
| As flooring..... | 0.083 | 12.05 |
| As ceiling..... | 0.104 | 9.71 |
| Fireproof construction, floored over: | | |
| As flooring..... | 0.124 | 8.06 |
| As ceiling..... | 0.145 | 6.90 |
| Single window..... | 1.030 | 0.97 |
| Single skylight..... | 1.118 | 0.89 |
| Double window..... | 0.518 | 1.93 |
| Double skylight..... | 0.621 | 1.61 |
| Door..... | 0.414 | 2.42 |

It should be noted that the coefficient of resistance thus defined will be approximately a constant quantity for a given substance under certain fixed conditions, only when the difference of temperature of the air on its two sides is small — say less than 100° F. When the range of temperature is great, experiments on heat transmission indicate that the quantity of heat transmitted varies, not directly as the difference of temperature, but as the square of that difference. In this case a coefficient

of resistance with a different definition may be found — viz., that obtained from the formula $a = (T - t)^2 \div q$, in which a is the coefficient, $T - t$ the range of temperature, and q the quantity of heat transmitted, in British thermal units per square foot per hour.

Steam-pipe Coverings.

Experiments by Prof. Ordway, *Trans. A. S. M. E.*, vi, 168; also Circular No. 27 of Boston Mfrs. Mutual Fire Ins. Co., 1890.

| Substance 1 inch thick. Heat applied, 310° F. | Pounds of Water heated 10° F., per hour, through 1 sq. ft. | British Thermal Units per sq. ft. per minute. | Solid Mat- ter in 1 sq. ft., 1 inch thick, parts in 1000. | Air included, parts in 100. |
|---|---|---|--|--------------------------------|
| 1. <i>Loose wool</i> | 8.1 | 1.35 | 56 | 944 |
| 2. <i>Live-geese feathers</i> | 9.6 | 1.60 | 50 | 950 |
| 3. <i>Carded cotton wool</i> | 10.4 | 1.73 | 20 | 980 |
| 4. <i>Hair felt</i> | 10.3 | 1.72 | 185 | 815 |
| 5. <i>Loose lampblack</i> | 9.8 | 1.63 | 56 | 944 |
| 6. <i>Compressed lampblack</i> | 10.6 | 1.77 | 244 | 756 |
| 7. <i>Cork charcoal</i> | 11.9 | 1.98 | 53 | 947 |
| 8. <i>White-pine charcoal</i> | 13.9 | 2.32 | 119 | 881 |
| 9. <i>Anthracite-coal powder</i> | 35.7 | 5.95 | 506 | 494 |
| 10. <i>Loose calcined magnesia</i> | 12.4 | 2.07 | 23 | 977 |
| 11. <i>Compressed calcined magnesia</i> ... | 42.6 | 7.10 | 285 | 715 |
| 12. <i>Light carbonate of magnesia</i> ... | 13.7 | 2.28 | 60 | 940 |
| 13. <i>Compressed carb. of magnesia</i> ... | 15.4 | 2.57 | 150 | 850 |
| 14. <i>Loose fossil-meal</i> | 14.5 | 2.42 | 60 | 940 |
| 15. <i>Crowded fossil-meal</i> | 15.7 | 2.62 | 112 | 888 |
| 16. <i>Ground chalk (Paris white)</i> | 20.6 | 3.43 | 253 | 747 |
| 17. <i>Dry plaster of Paris</i> | 30.9 | 5.15 | 368 | 632 |
| 18. <i>Fine asbestos</i> | 49.0 | 8.17 | 81 | 919 |
| 19. <i>Air alone</i> | 48.0 | 8.00 | 0 | 1000 |
| 20. <i>Sand</i> | 62.1 | 10.35 | 529 | 471 |
| 21. <i>Best slag-wool</i> | 13. | 2.17 | | |
| 22. <i>Paper</i> | 14. | 2.33 | | |
| 23. <i>Blotting-paper wound tight</i> | 21. | 3.50 | | |
| 24. <i>Asbestos paper wound tight</i> | 21.7 | 3.62 | | |
| 25. <i>Cork strips bound on</i> | 14.6 | 2.43 | | |
| 26. <i>Straw rope wound spirally</i> | 18. | 3. | | |
| 27. <i>Loose rice chaff</i> | 18.7 | 3.12 | | |
| 28. <i>Paste of fossil-meal with hair</i> ... | 16.7 | 2.78 | | |
| 29. <i>Paste of fossil-meal with asbestos</i> ... | 22. | 3.67 | | |
| 30. <i>Loose bituminous-coal ashes</i> | 21. | 3.50 | | |
| 31. <i>Loose anthracite-coal ashes</i> | 27. | 4.50 | | |
| 32. <i>Paste of clay and vegetable fiber</i> | 30.9 | 5.15 | | |

It will be observed that several of the incombustible materials are nearly as efficient as wool, cotton, and feathers, with which they may be compared in the preceding table. The materials which may be considered wholly free from the danger of being carbonized or ignited by slow contact with pipes or boilers are printed in Roman type. Those which are more or less liable to be carbonized are printed in italics.

The results Nos. 1 to 20 inclusive were from experiments with the various non-conductors each used in a mass one inch thick, placed on a flat surface of iron kept heated by steam to 310° F. The substances Nos. 21 to 32 were tried as coverings for two-inch steam-pipe; the results being reduced to the same terms as the others for convenience of comparison.

Experiments on still air gave results which differ little from those of Nos. 3, 4, and 6. The bulk of matter in the best non-conductors is relatively too small to have any specific effect except to trap the air and keep it stagnant. These substances keep the air still by virtue of the roughness of their fibers or particles. The asbestos, No. 18, had smooth fibers. Asbestos with exceedingly fine fiber made a somewhat better showing, but asbestos is really one of the poorest non-conductors. It may be used advantageously to hold together other incombustible substances, but the less of it the better. A "magnesia" covering, made of carbonate of magnesia with a small percentage of good asbestos fiber and containing 0.25 of solid matter, transmitted 2.5 B.T.U. per square foot per minute, and one containing 0.396 of solid matter transmitted 3.33 B.T.U.

Any suitable substance which is used to prevent the escape of steam heat should not be less than one inch thick.

Any covering should be kept perfectly dry, for not only is water a good carrier of heat, but it has been found that still water conducts heat about eight times as rapidly as still air.

Tests of Commercial Coverings were made by Mr. Geo. M. Brill and reported in *Trans. A. S. M. E.*, xvi, 827. A length of 60 feet of 8-inch steam-pipe was used in the tests, and the heat loss was determined by the condensation. The steam pressure was from 109 to 117 lbs. gauge, and the temperature of the air from 58° to 81° F. The difference between the temperature of steam and air ranged from 263° to 286°, averaging 272°.

The following are the principal results:

| Kind of Covering. | Thickness of Covering, inches. | Lbs. Steam condensed per sq. ft. per hour. | B.T.U. per sq. ft. per minute. | B.T.U. per sq. ft. per hour per degree of average difference of temperature. | Saving due to covering, lbs. steam per hour per sq. ft. | Ratio of Heat lost, Bare to Covered Pipe, %. | H. P. lost per 100 sq. ft. of pipe (30 lbs. per hour = 1 H.P.). |
|--------------------------|--------------------------------|--|--------------------------------|--|---|--|---|
| Bare pipe..... | | 0.846 | 12.27 | 2.706 | | 100. | 2.819 |
| Magnesia..... | 1.25 | 0.120 | 1.74 | 0.384 | 0.725 | 14.2 | 0.400 |
| Rock wool..... | 1.60 | 0.080 | 1.16 | 0.256 | 0.766 | 9.5 | 0.267 |
| Mineral wool..... | 1.30 | 0.089 | 1.29 | 0.285 | 0.757 | 10.5 | 0.297 |
| Fire-felt..... | 1.30 | 0.157 | 2.28 | 0.502 | 0.689 | 18.6 | 0.523 |
| Manville sectional..... | 1.70 | 0.109 | 1.59 | 0.350 | 0.737 | 12.9 | 0.564 |
| Manv. sect and hair-felt | 2.40 | 0.056 | 0.96 | 0.212 | 0.780 | 7.8 | 0.221 |
| Manville wool-cement... | 2.20 | 0.108 | 1.56 | 0.345 | 0.738 | 12.7 | 0.359 |
| Champion mineral wool. | 1.44 | 0.099 | 1.44 | 0.317 | 0.747 | 11.7 | 0.330 |
| Hair-felt..... | 0.82 | 0.132 | 1.91 | 0.422 | 0.714 | 15.6 | 0.439 |
| Riley cement..... | 0.75 | 0.298 | 4.32 | 0.953 | 0.548 | 35.2 | 0.993 |
| Fossil-meal..... | 0.75 | 0.275 | 3.99 | 0.879 | 0.571 | 32.5 | 0.919 |

Tests of Pipe Coverings by an Electrical Method. (H. G. Stott, *Power*, 1902.) — A length of about 200 ft. of 2-in. pipe was heated to a known temperature by an electrical current. The pipe was covered with different materials, and the heat radiated by each covering was determined by measuring the current required to keep the pipe at a constant temperature. A brief description of the various coverings is given below.

No. 2. Solid sectional covering, 1½ in. thick, of granulated cork molded under pressure and then baked at a temperature of 500° F.; ⅛ in. asbestos paper next to pipe.

No. 3. Solid 1-in. molded sectional, 85% carbonate of magnesia.

No. 4. Solid 1-in. sectional, granulated cork molded under pressure and baked at 500° F.; $\frac{1}{8}$ in. asbestos next to pipe.

No. 5. Solid 1-in. molded sectional, 85% carbonate of magnesia; outside of sections covered with canvas pasted on.

No. 6. Laminated 1-in. sectional, nine layers of asbestos paper with granulated cork between; outside of sections covered with canvas, $\frac{1}{8}$ in. asbestos paper next to pipe.

No. 7. Solid 1-in. molded sectional, of 85% carbonate of magnesia; outside of sections covered with light canvas.

No. 8. Laminated 1-in. sectional, seven layers of asbestos paper indented with $\frac{1}{4}$ -in. square indentations, which serve to keep the asbestos layers from coming in close contact with one another; $\frac{1}{8}$ in. asbestos paper next to pipe.

No. 9. Laminated 1-in. sectional, 64 layers of asbestos paper, in which were embedded small pieces of sponge; outside covered with canvas.

No. 10. Laminated $1\frac{1}{2}$ -in. sectional, 12 plain layers of asbestos paper with corrugated layers between, forming longitudinal air cells; $\frac{1}{8}$ in. asbestos paper next to pipe; sections wired on.

No. 11. Laminated 1-in. sectional, 8 layers of asbestos paper with corrugated layers between, forming small air ducts radially around the covering.

No. 12. Laminated $1\frac{1}{4}$ -in. sectional, 6 layers of asbestos paper with corrugated layers; outside of sections covered with two layers of canvas.

No. 15. "Remanit," composed of 2 layers wound in reverse direction with ropes of carbonized silk. Inner layer $2\frac{1}{2}$ in. wide and $\frac{1}{2}$ in. thick; outer layer 2 in. wide and $\frac{3}{4}$ in. thick, over which was wound a network of fine wire; $\frac{1}{8}$ in. asbestos next to pipe. Made in Germany.

No. 16. $2\frac{1}{2}$ -in. covering, 85% carbonate of magnesia, $\frac{1}{2}$ -in. blocks about 3 in. wide and 18 in. long next to pipe and wired on; over these blocks were placed solid 2-in. molded sectional covering.

No. 17. $2\frac{1}{2}$ -in. covering, 85% magnesia. Put on in a 2-in. molded section wired on; next to the pipe and over this a $\frac{1}{2}$ -in. layer of magnesia plaster.

No. 18. $2\frac{1}{2}$ -in. covering, 85% carbonate of magnesia. Put on in two solid 1-in. molded sections with $\frac{1}{2}$ -in. layer of magnesia plaster between; two 1-in. coverings wired on and placed so as to break joints.

No. 19. 2-in. covering, of 85% carbonate of magnesia, put on in two 1-in. layers so as to break joints.

No. 20. Solid 2-in. molded sectional, 85% magnesia.

No. 21. Solid 2-in. molded sectional, 85% magnesia.

Two samples covered with the same thickness of similar material give different results; for example, Nos. 3 and 5, and also Nos. 20 and 21. The cause of this difference was found to be in the care with which the joints between sections were made. A comparison between Nos. 19 and 20, having the same total thickness, but one applied in a solid 2-in. section, and the other in two 1-in. sections, proved the desirability of breaking joints.

An attempt was made to determine the law governing the effect of increasing the thickness of the insulating material, and for all the 85% magnesia coverings the efficiency varied directly as the square root of the thickness, but the other materials tested did not follow this simple law closely, each one involving a different constant.

To determine which covering is the most economical the following quantities must be considered: (1) Investment in covering. (2) Cost of coal required to supply lost heat. (3) Five per cent interest on capital invested in boilers and stokers rendered idle through having to supply lost heat. (4) Guaranteed life of covering. (5) Thickness of covering.

The coverings Nos. 2 to 15 were finished on the outside with resin paper and 8-ounce canvas; the others had canvas pasted on outside of the sections, and an 8-oz. canvas finish. The following is a condensed statement of the results with the temperature of the pipe corresponding to 160 lb. steam pressure.

ELECTRICAL TEST OF STEAM-PIPE COVERINGS.

| No. | Covering. | Aver. Thickness. | B.T.U. Loss per sq. ft. at 160 lb. Pres. | B.T.U. per sq. ft. per Hr. per Deg. Diff. of Temp. | Per cent Heat Saved by Covering. |
|-----|--|------------------|--|--|----------------------------------|
| 2 | Solid cork..... | 1.68 | 1.672 | 0.348 | 87.1 |
| 3 | 85% magnesia..... | 1.18 | 2.008 | 0.418 | 84.5 |
| 4 | Solid cork..... | 1.20 | 2.048 | 0.427 | 84.2 |
| 5 | 85% magnesia..... | 1.19 | 2.130 | 0.444 | 83.6 |
| 6 | Laminated asbestos cork..... | 1.48 | 2.123 | 0.442 | 83.7 |
| 7 | 85% magnesia..... | 1.12 | 2.190 | 0.456 | 83.2 |
| 8 | Asbestos air cell [indent]..... | 1.26 | 2.333 | 0.486 | 83.1 |
| 9 | Asbestos sponge felted..... | 1.24 | 2.552 | 0.532 | 80.3 |
| 10 | Asbestos air cell [long]..... | 1.70 | 2.750 | 0.573 | 78.8 |
| 11 | "Asbestoscel" [radial]..... | 1.22 | 2.801 | 0.584 | 78.5 |
| 12 | Asbestos air cell [long]..... | 1.29 | 2.812 | 0.586 | 78.4 |
| 15 | "Remanit" [silk] wrapped..... | 1.51 | 1.452 | 0.302 | 88.8 |
| 16 | 85% magnesia, 2" sectional and 1/2" block..... | 2.71 | 1.381 | 0.288 | 89.4 |
| 17 | 85% magnesia, 2" sectional and 1/2" plaster..... | 2.45 | 1.387 | 0.289 | 88.7 |
| 18 | 85% magnesia, two 1" sectional..... | 2.50 | 1.412 | 0.294 | 89.0 |
| 19 | 85% magnesia, two 1" sectional..... | 2.24 | 1.465 | 0.305 | 88.7 |
| 20 | 85% magnesia, 2" sectional..... | 2.34 | 1.555 | 0.324 | 88.0 |
| 21 | 85% magnesia, 2" sectional..... | 2.20 | 1.568 | 0.314 | 87.9 |
| | Bare pipe [from outside tests]..... | | 13. | 2.708 | |

Transmission of Heat, through Solid Plates, from Water to Water. (Clark, S. E.) — M. Péclel found, from experiments made with plates of wrought iron, cast iron, copper, lead, zinc, and tin, that when the fluid in contact with the surface of the plate was not circulated by artificial means, the rate of conduction was the same for different metals and for plates of the same metal of different thicknesses. But when the water was thoroughly circulated over the surfaces, and when these were perfectly clean, the quantity of transmitted heat was inversely proportional to the thickness, and directly as the difference in temperature of the two faces of the plate. When the metal surface became dull, the rate of transmission of heat through all the metals was very nearly the same.

It follows, says Clark, that the absorption of heat through metal plates is more active whilst evaporation is in progress — when the circulation of the water is more active — than while the water is being heated up to the boiling-point.

Transmission from Steam to Water. — M. Péclel's principle is supported by the results of experiments made in 1867 by Mr. Isherwood on the conductivity of different metals. Cylindrical pots, 10 inches in diameter, 21 1/4 inches deep inside, and 1/8 inch, 1/4 inch, and 3/8 inch thick, turned and bored, were formed of pure copper, brass (60 copper and 40 zinc), rolled wrought iron, and remelted cast iron. They were immersed in a steam bath, which was varied from 220° to 320° F. Water at 212° was supplied to the pots, which were kept filled. It was ascertained that the rate of evaporation was in the direct ratio of the difference of the temperatures inside and outside of the pots; that is, that the rate of evaporation per degree of difference of temperatures was the same for all temperatures; and that the rate of evaporation was exactly the same for different thicknesses of the metal. The respective rates of conductivity of the several metals were as follows, expressed in weight of water evaporated from and at 212° F. per square foot of the interior surface of the pots per degree of difference of temperature per hour, together with the equivalent quantities of heat-units:

| | Water at 212°. | Heat-units. | Ratio. |
|-------------------|----------------|-------------|--------|
| Copper..... | 0.665 lb. | 642.5 | 1.00 |
| Brass..... | .577 " | 556.8 | 0.87 |
| Wrought iron..... | .387 " | 373.6 | .58 |
| Cast iron..... | .327 " | 315.7 | .49 |

Whitham, "Steam Engine Design," p. 283, also *Trans. A. S. M. E.*, ix, 425, in using these data in deriving a formula for surface condensers, calls these figures those of perfect conductivity, and multiplies them by a coefficient *C*, which he takes at 0.323, to obtain the efficiency of condenser surface in ordinary use, i.e., coated with saline and greasy deposits.

Transmission of Heat from Steam to Water through Coils of Iron Pipe. — H. G. C. Kopp and F. J. Meystre (*Stevens Indicator*, Jan., 1894) give an account of some experiments on transmission of heat through coils of pipe. They collate the results of earlier experiments as follows, for comparison:

| Experimenter. | Character of Surface. | Steam condensed per square foot per degree difference of temperature per hour. | | Heat transmitted per square foot per degree difference of temperature per hour. | | Remarks. |
|---------------|-----------------------|--|-----------------------|---|----------------------|---|
| | | Heating, pounds. | Evapo-rating, pounds. | Heating, B.T.U. | Evapo-rating, B.T.U. | |
| Laurens. | Copper coils .. | 0.292 | 0.981 | 315 | 974 | { Steam pressure = 100. Steam pressure = 10. |
| " | 2 Copper coils | | 1.20 | | 1120 | |
| Havrez .. | Copper coil ... | 0.268 | 1.26 | 280 | 1200 | |
| Perkins .. | Iron coil..... | | 0.24 | | 215 | |
| " | " " | | 0.22 | | 208.2 | |
| Box..... | Iron tube..... | 0.235 | | 230 | | |
| " | " " | 0.196 | | 207 | | |
| " | " " | 0.206 | | 210 | | |
| Havrez .. | Cast-iron boiler | 0.077 | 0.105 | 82 | 100 | |

From the above it would appear that the efficiency of iron surfaces is less than that of copper coils, plate surfaces being far inferior.

In all experiments made up to the present time, it appears that the temperature of the condensing water was allowed to rise, a mean between the initial and final temperatures being accepted as the effective temperature. But as water becomes warmer it circulates more rapidly, thereby causing the water surrounding the coil to become agitated and replaced by cooler water, which allows more heat to be transmitted.

Again, in accepting the mean temperature as that of the condensing medium, the assumption is made that the rate of condensation is in direct proportion to the temperature of the condensing water.

In order to correct and avoid any error arising from these assumptions and approximations, experiments were undertaken, in which all the conditions were constant during each test.

The pressure was maintained uniform throughout the coil, and provision was made for the free outflow of the condensed steam, in order to obtain at all times the full efficiency of the condensing surface. The condensing water was continually stirred to secure uniformity of temperature, which was regulated by means of a steam-pipe and a cold-water pipe entering the tank in which the coil was placed.

The following is a condensed statement of the results.

HEAT TRANSMITTED PER SQUARE FOOT OF COOLING SURFACE, PER HOUR, PER DEGREE OF DIFFERENCE OF TEMPERATURE. (British Thermal Units.)

| Temperature of Condensing Water. | 1-in. Iron Pipe; Steam inside, 60 lbs. Gauge Pressure. | 1½-in. Pipe; Steam inside, 10 lbs. Pressure. | 1½-in. Pipe; Steam outside, 10 lbs. Pressure. | 1½-in. Pipe; Steam inside, 60 lbs. Pressure. |
|----------------------------------|--|--|---|--|
| 80 | 265 | 128 | 200 | |
| 100 | 269 | 130 | 230 | 239 |
| 120 | 272 | 137 | 260 | 247 |
| 140 | 277 | 145 | 267 | 276 |
| 160 | 281 | 158 | 271 | 306 |
| 180 | 299 | 174 | 270 | 349 |
| 200 | 313 | | | 419 |

The results indicate that the heat transmitted per degree of difference of temperature in general increases as the temperature of the condensing water is increased.

The amount transmitted is much larger with the steam on the outside of the coil than with the steam inside the coil. This may be explained in part by the fact that the condensing water when inside the coil flows over the surface of conduction very rapidly, and is more efficient for cooling than when contained in a tank outside of the coil.

This result is in accordance with that found by Mr. Thomas Craddock, which indicated that the rate of cooling by transmission of heat through metallic surfaces was almost wholly dependent on the rate of circulation of the cooling medium over the surface to be cooled.

Transmission of Heat in Condenser Tubes. (*Eng'g*, Dec. 10, 1875, p. 449.) — In 1874 B. C. Nichol made experiments for determining the rate at which heat was transmitted through a condenser tube. The results went to show that the amount of heat transmitted through the walls of the tube per estimated degree of mean difference of temperature increased considerably with this difference. For example:

| Estimated mean difference of temperature between inside and outside of tube, degrees Fahr.... | Vertical Tube. | | | Horizontal Tube. | | |
|---|----------------|-------|-------|------------------|-------|-------|
| | 128 | 151.9 | 152.9 | 111.6 | 146.2 | 150.4 |
| Heat-units transmitted per hour per square foot of surface per degree of mean diff. of temp.... | 422 | 531 | 561 | 610 | 737 | 823 |

These results seem to throw doubt upon Mr. Isherwood's statement that the rate of evaporation per degree of difference of temperature is the same for all temperatures.

Mr. Thomas Craddock found that water was enormously more efficient than air for the abstraction of heat through metallic surfaces in the process of cooling. He proved that the rate of cooling by transmission of heat through metallic surfaces depends upon the rate of circulation of the cooling medium over the surface to be cooled. A tube filled with hot water, moved by rapid rotation at the rate of 59 ft. per second, through air, lost as much heat in one minute as it did in still air in 12 minutes. In water, at a velocity of 3 ft. per second, as much heat was abstracted in half a minute as was abstracted in one minute when it was at rest in the water. Mr. Craddock concluded, further, that the circulation of the cooling fluid became of greater importance as the difference of temperature on the two sides of the plate became less. (Clark, *R. T. D.*, p. 461.)

G. A. Orrok (*Power*, Aug. 11, 1908) gives a diagram showing the relation of the B.T.U. transmitted per hour per sq. ft. of surface per degree of difference of temperature to the velocity of the water in the condenser tubes, in feet per second, as obtained by different experimenters. Approximate figures taken from the several curves are given below.

| Authority. | Tubes. | Velocity of Water, Feet per Second. | | | | | | |
|------------------|------------------------------|---|-------|-------|-------|-------|-------|-------|
| | | 0.5 | 1 | 2 | 3 | 4 | 5 | 6 |
| | | B.T.U. per sq. ft. per hr. per deg. diff. | | | | | | |
| 1. Stanton..... | 1/2-in. vert. copper..... | | 325 | 400 | 465 | 520 | 550 | ... |
| 2. Stanton..... | 1/2-in. vert. copper..... | | 420 | 470 | 525 | 560 | 585 | ... |
| 3. Nichols..... | 3/4-in. vert. brass..... | | 340 | 370 | 405 | 435 | 460 | 470 |
| 4. Nichols..... | 3/4-in. horiz. brass..... | | 500 | 530 | 560 | 585 | 615 | 650 |
| 5. Hepburn..... | 1 1/4-in. horiz. copper..... | 250 | 365 | 590 | | | | |
| 6. Hepburn..... | 1 1/4-in. horiz. corrugated | 360 | 560 | | | | | |
| 7. Richter..... | 1 1/2-in. horiz. corrugated | 460 | | | | | | |
| 8. Weighton..... | 5/8-in. plain tubes..... | | 380 | 615 | 760 | 865 | 940 | |
| 9. Allen..... | 5/8-in. horizontal..... | | 225 | 290 | 365 | | | |

No. 1, water flowing up. Nos. 2 and 3, water flowing down.

Transmission of Heat in Feed-water Heaters. (W. R. Billings, *The National Engineer*, June, 1907.)—Experiments show that the rate of transmission of heat through metal surfaces from steam to water increases rapidly with the increased rate of flow of the water. Mr. Billings therefore recommends the use of small tubes in heaters in which the water is inside of the tubes. He says: A high velocity through the tubes causes friction between the water and the walls of the tubes; this friction is not the same as the friction between the particles of water themselves, and it tends to break up the column of water and bring fresh and cooler particles against the hot walls of the tubes.

The following results were obtained in tests:

| | | | | |
|----------------------------|------------|-----|-----|-----|
| 1 1/4-in. smooth tubes | $V = 22.5$ | 114 | 137 | ... |
| | $U = 185$ | 570 | 670 | ... |
| 1 1/2-in. corrugated tubes | $V = 24$ | 34 | 43 | 82 |
| | $U = 318$ | 444 | 465 | 735 |

V = velocity of the water, ft. per min. U = B.T.U. transmitted per sq. ft. per hour per degree difference of temperature. (See Condensers.)

In calculations of heat transmission in heaters it is customary to take as the mean difference of temperature the difference between the temperature of the steam and the arithmetical mean of the initial and final temperatures of the water; thus if S = steam temperature, I = initial and F = final temperature of the water, and D = mean difference, then $D = S - 1/2(I + F)$. Mr. Billings shows that this is incorrect, and on the assumption that the rate of transmission through any portion of the surface is directly proportional to the difference he finds the true mean to be $D = \frac{F - I}{\text{hyp. log} [(S - I) \div (S - F)]}$. (This formula was derived by

Cecil P. Poole in 1899, *Power*, Dec., 1906.)

The following table is calculated from the formula:

DEGREES OF DIFFERENCE BETWEEN STEAM TEMPERATURE AND ACTUAL AVERAGE TEMPERATURE OF WATER.

| Initial Temperature of Water. | Vacuum Heaters Between Engine and Condenser. | | | | | | | | | |
|-------------------------------|--|------|------|------|--|------------------------|------|------|------|-----------|
| | 26" Vac. Temp. 126° F. | | | | | 24" Vac. Temp. 141° F. | | | | |
| | Final Temp. of Water. | | | | | Final Temp. of Water. | | | | |
| | 105 | 110 | 115 | 120 | | 105 | 110 | 115 | 120 | 125 130 |
| 40..... | 46.1 | 41.6 | 36.9 | 30.1 | | 62.9 | 60.2 | 55.3 | 50.9 | 46.1 40.6 |
| 50..... | 42.8 | 38.4 | 33.6 | 27.6 | | 59.2 | 56.6 | 51.8 | 47.7 | 43.2 37.9 |
| 60..... | 39.3 | 35.3 | 30.7 | 25.0 | | 55.5 | 52.1 | 48.4 | 44.4 | 40.1 35.0 |
| 70..... | 35.6 | 31.9 | 27.6 | 22.4 | | 51.6 | 48.2 | 45.0 | 41.0 | 36.9 32.2 |
| 80..... | 31.8 | 28.3 | 24.5 | 19.6 | | 47.6 | 44.2 | 41.2 | 37.5 | 33.6 29.2 |

| Initial Temp. of Water. | Atmospheric Heaters. | | | | | | | | | | | | |
|----------------------------|-----------------------|------|------|------|------|------|----------------------------|-----------------------|------|------|------|------|------|
| | Atmos. Press. 212° F. | | | | | | Initial Temp. of Water. | Atmos. Press. 212° F. | | | | | |
| | Temp. | | | | | | | Temp. | | | | | |
| | Final Temp. of Water. | | | | | | | Final Temp. of Water. | | | | | |
| | 192 | 196 | 200 | 204 | 208 | 210 | | 192 | 196 | 200 | 204 | 208 | 210 |
| 40..... | 70.6 | 65.7 | 60.1 | 53.5 | 44.8 | 38.0 | 105 | 51.9 | 47.9 | 43.4 | 38.2 | 31.4 | 26.4 |
| 50..... | 67.9 | 63.1 | 57.6 | 51.2 | 42.8 | 36.4 | 110 | 50.3 | 46.4 | 42.1 | 36.9 | 30.2 | 25.5 |
| 60..... | 65.1 | 60.4 | 55.2 | 48.9 | 40.7 | 34.7 | 115 | 48.8 | 45.0 | 40.6 | 35.7 | 29.2 | 24.5 |
| 70..... | 62.2 | 57.7 | 52.6 | 46.6 | 38.7 | 32.9 | 120 | 47.2 | 43.5 | 39.2 | 34.4 | 28.0 | 23.5 |
| 80..... | 59.4 | 54.9 | 50.0 | 44.2 | 36.6 | 31.0 | 125 | 45.6 | 41.9 | 37.8 | 33.1 | 26.9 | 22.5 |

The error in using the arithmetic mean for the value of D is not important if F is very much lower than S , but if it is within 10° of S then the error may be a large one. With $S = 212$, $I = 40$, $F = 110$, the arithmetic mean difference is 137, and the value by the logarithmic formula 131, an error of less than 5%; but if F is 204, the arithmetic mean is 90, and the value by the formula 53.5.

It should be observed, however, that the formula is based on an assumption that is probably greatly in error for high temperature differences, i.e., that the transmission of heat is directly proportional to the temperature difference. It may be more nearly proportional to the square of the difference, as stated by Rankine. This seems to be indicated by the results of heating water by steam coils, given below.

Heating Water by Steam Coils.—A catalogue of the American Radiator Co. (1908) gives a chart showing the pounds of steam condensed per hour per sq. ft. of iron, brass and copper pipe surface, for different mean or average differences of temperature between the steam and the water. Taking the latent heat of the steam at 966 B.T.U. per lb., the following figures are derived from the table.

| Mean Temp. Diff. | Lb. Steam Condensed per Hour per Sq. Ft. of Pipe. | | | Lb. Steam Condensed per Hour per Sq. Ft. per Deg. Diff. | | | B.T.U. per Sq. Ft. per Hour per Deg. Diff. | | |
|------------------|---|--------|--------|---|--------|--------|--|--------|------|
| | Iron. | Brass. | Copper | Iron. | Brass. | Copper | Iron. | Brass. | Cop. |
| 50 | 7.5 | 12.5 | 14.5 | 0.150 | 0.250 | 0.290 | 101 | 198 | 283 |
| 100 | 18.5 | 38 | 43.5 | 0.185 | 0.380 | 0.435 | 179 | 367 | 415 |
| 150 | 32.2 | 76.5 | 87.8 | 0.215 | 0.510 | 0.585 | 208 | 493 | 565 |
| 200 | 48 | 128 | 144 | 0.240 | 0.640 | 0.720 | 232 | 618 | 695 |

The chart is said to be plotted from a large number of tests with pipes placed vertically in a tank of water, about 20 per cent being deducted from the actual results as a margin of safety.

W. R. Billings (*Eng. Rec.*, Feb., 1898) gives as the results of one set of experiments with a closed feed-water heater:

| | | | | | | |
|---|----|----|----|-----|-----|-----|
| Mean temp. diff., deg. F..... | 5 | 6 | 8 | 11 | 15 | 18 |
| B.T.U. per sq. ft. per hr. per deg. diff. | 67 | 79 | 89 | 114 | 129 | 139 |

Heat Transmission through Cast-iron Plates Pickled in Nitric Acid.—Experiments by R. C. Carpenter (*Trans. A. S. M. E.*, xii, 179) show a marked change in the conducting power of the plates (from steam to water), due to prolonged treatment with dilute nitric acid.

The action of the nitric acid, by dissolving the free iron and not attacking the carbon, forms a protecting surface to the iron, which is largely composed of carbon. The following is a summary of results:

| Character of Plates, each plate 8.4 in. by 5.4 in., exposed surface 27 sq. ft. | Increase in Temperature of 3.125 lbs. of Water each Minute. | Proportionate Thermal Units Transmitted for each Degree of Difference of Temperature per Square Foot per Hour. | Relative Transmission of Heat. |
|--|---|--|--------------------------------|
| Cast iron — untreated skin on, but clean, free from rust..... | 13.90 | 113.2 | 100.0 |
| Cast iron — nitric acid, 1% sol., 9 days.. | 11.5 | 97.7 | 86.3 |
| “ “ “ 1% sol., 18 days | 9.7 | 80.08 | 70.7 |
| “ “ “ 1% sol., 40 days | 9.6 | 77.8 | 68.7 |
| “ “ “ 5% sol., 9 days.. | 9.93 | 87.0 | 76.8 |
| “ “ “ 5% sol., 40 days | 10.6 | 77.4 | 68.5 |
| Plate of pine wood, same dimensions as the plate of cast iron..... | 0.33 | 1.9 | 1.6 |

The effect of covering cast-iron surfaces with varnish has been investigated by P. M. Chamberlain. He subjected the plate to the action of strong acid for a few hours, and then applied a non-conducting varnish. One surface only was treated. Some of his results are as follows:

| | | |
|--|------|--|
| Heat units per sq. ft. per hour, for each degree, $\tau - \tau'$. | 170. | As finished — greasy. |
| | 152. | “ “ washed with benzine and dried. |
| | 169. | Oiled with lubricating oil. |
| | 162. | After exposure to nitric acid sixteen hours, then oiled (linseed oil). |
| | 166. | After exposure to hydrochloric acid twelve hours, then oiled (linseed oil). |
| | 113. | { After exposure to sulphuric acid 1, water 2, for 48 hours, then oiled, varnished, and allowed to dry for 24 hours. |
| | 117. | { |

Transmission of Heat through Solid Plates from Air or other Dry Gases to Water. (From Clark on the Steam Engine.) — The law of the transmission of heat from hot air or other gases to water, through metallic plates, has not been exactly determined by experiment. The general results of experiments on the evaporative action of different portions of the heating surface of a steam-boiler point to the general law that the quantity of heat transmitted per degree difference of temperature is practically uniform for various differences of temperature.

The communication of heat from the gas to the plate surface is much accelerated by mechanical impingement of the gaseous products upon the surface.

Clark says that when the surfaces are perfectly clean, the rate of transmission of heat through plates of metal from air or gas to water is greater for copper, next for brass, and next for wrought iron. But when the surfaces are dimmed or coated, the rate is the same for the different metals.

With respect to the influence of the conductivity of metals and of the thickness of the plate on the transmission of heat from burnt gases to water, Mr. Napier made experiments with small boilers of iron and copper placed over a gas-flame. The vessels were 5 inches in diameter and 2 $\frac{1}{2}$ inches deep. From three vessels, one of iron, one of copper, and one of iron sides and copper bottom, each of them $\frac{1}{30}$ inch in thickness,

equal quantities of water were evaporated to dryness, in the times as follows:

| Water. | Iron Vessel. | Copper Vessel. | Iron and Copper Vessel. |
|----------|--------------|----------------|-------------------------|
| 4 ounces | 19 minutes | 18.5 minutes | |
| 11 " | 33 " | 30.75 " | |
| 5 1/2 " | 50 " | 44 " | |
| 4 " | 35.7 " | | 36.83 minutes |

Two other vessels of iron sides $\frac{1}{30}$ inch thick, one having a $\frac{1}{4}$ -inch copper bottom and the other a $\frac{1}{4}$ -inch lead bottom, were tested against the iron and copper vessel, $\frac{1}{3}$ inch thick. Equal quantities of water were evaporated in 54, 55, and 53 $\frac{1}{2}$ minutes respectively. Taken generally, the results of these experiments show that there are practically but slight differences between iron, copper, and lead in evaporative activity, and that the activity is not affected by the thickness of the bottom.

Mr. W. B. Johnson formed a like conclusion from the results of his observations of two boilers of 160 horse-power each, made exactly alike, except that one had iron flue-tubes and the other copper flue-tubes. No difference could be detected between the performances of these boilers.

Divergencies between the results of different experimenters are attributable probably to the difference of conditions under which the heat was transmitted, as between water or steam and water, and between gaseous matter and water. On one point the divergence is extreme: the rate of transmission of heat per degree of difference of temperature. Whilst from 400 to 600 units of heat are transmitted from water to water through iron plates, per degree of difference per square foot per hour, the quantity of heat transmitted between water and air, or other dry gas, is only about from 2 to 5 units, according as the surrounding air is at rest or in movement. In a locomotive boiler, where radiant heat was brought into play, 17 units of heat were transmitted through the plates of the fire-box per degree of difference of temperature per square foot per hour.

Transmission of Heat through Plates from Flame to Water.—Much controversy has arisen over the assertion by some makers of live-steam feed-water heaters that if the water fed to a boiler was first heated to the boiling point before being fed into the boiler, by means of steam taken from the boiler, an economy of fuel would result; the theory being that the rate of transmission through a plate to water was very much greater when the water was boiling than when it was being heated to the boiling point, on account of the greatly increased rapidity of circulation of the water when boiling. (See *Eng'g*, Nov. 16, 1906, and *Eng. Review* [London], Jan., 1908.) Two experiments by Sir Wm. Anderson (1872), with a steam-jacketed pan, are quoted, one of which showed an increased transmission when boiling of 133%, and the other of 80%; also an experiment by Sir F. Bramwell, with a steam-heated copper pan, which showed a gain of 164% with boiling water. On the other hand, experiments by S. B. Bilbrough (Transvaal Inst. Mining Engineers, Feb., 1908) showed in tests with a flame-heated pan that there was no difference in the rate of transmission whether the water was cold or boiling. W. M. Sawdon (*Pover*, Jan. 12, 1909) objects to Mr. Bilbrough's conclusions on the ground that no corrections for radiation were made, and finds by a similar experiment, with corrections, that the increased rate of transmission with boiling water is at least 38%. All of these experiments were on a small scale, and in view of their conflict no conclusions can be drawn from them as to the value of live-steam feed-water heating in improving the economy of a steam boiler.

A. Blechynden's Tests.—A series of steel plates from 0.125 in. to 1.187 in. thick were tested with hot gas on one side and water on the other with differences of temperature ranging from 373° to 1318° F. (*Trans. Inst. Naval Architects*, 1894.) Mr. Blechynden found that the heat transmitted is proportional to the square of the difference between the temperatures at the two sides of the plate, or: Heat transmitted per sq. ft. \div (diff. of temp.)² = a constant. A study of the results of these tests is made in Kent's "Steam Boiler Economy," p. 235, and it is shown that the value of a in Rankine's formula $q = (T_1 - T)^2 \div a$, which a is the reciprocal of Mr. Blechynden's constant and is a function of the thickness of the plate. One of the plates, A , originally 1.187 in. thick, was reduced

in four successive operations, by machining to 0.125 in. Another, *B*, was tested in four thicknesses. The other plates were tested in one or two thicknesses. Each plate was found to have a law of transmission of its own. For plate *A* the value of *a* is represented closely by the formula $a = 40 + 20t$, in which *t* is the thickness in inches. The formula $a = 40 + 20t \pm 10$ covers the whole range of the experiments. The whole range of values is 38.6 to 71.9, which are very low when compared with values of *a* computed from the results of boiler tests, which are usually from 200 to 400, the low values obtained by Blechynden no doubt being due to the exceptionally favorable conditions of his tests as compared with those of boiler tests. Rankine says the value of *a* lies between 160 and 200, but values below 200 are rarely found in tests of modern types of boilers. (See Steam-Boilers.)

Cooling of Air. — H. F. Benson (*Am. Mach.*, Aug. 31, 1905) derives the following formula for transmission of heat from air to water through copper tubes. It is assumed that the rate of transmission at any point of the surface is directly proportional to the difference of temperature between the air and water.

Let *A* = cooling surface, sq. ft.; *K* = lb. of air per hour; *S_a* = specific heat of air; *T_{a1}* = temp. of hot inlet air; *T_{a2}* = temp. of cooled outlet air; *d* = actual average diff. of temp. between the air and the water; *U* = B.T.U. absorbed by the water per degree of diff. of temp. per sq. ft. per hour. *W* = lb. of water per hour; *T_{w1}* = temp. of inlet water; *T_{w2}* = temp. of outlet water. Then

$$AdU = KS_a(T_{a1} - T_{a2}); \quad A = KS_a(T_{a1} - T_{a2}) \div dU.$$

$$d = [(T_{a1} - T_{a2}) - (T_{w2} - T_{w1})] \div \log [(T_{a1} - T_{w2}) \div (T_{a2} - T_{w1})].$$

$$AU = \frac{KS_a W}{W - KS_a} \log_e \frac{T_{a1} - T_{w2}}{T_{a2} - T_{w1}}.$$

$$T_{w2} = (S_a K \div W)(T_{a1} - T_{a2}) + T_{w1}.$$

The more cooling water used, the lower is the temperature *T_{w2}*. Also the less *T_{w2}* is, the larger *d* becomes and the less surface is needed. About 10 is the largest value of *W/K* that it is economical to use, as there is a saving of less than 0.5% in increasing it from 10 to 15. When desirable to save water it will be advisable to make *W/K* = 5. Values of *U* obtained by experiment with a Wainwright cooler made with corrugated copper tubes are given in the following table. *K* and *W* are in lb. per minute, *B_a* = B.T.U. from air per min., *B_w* = B.T.U. from water per min., *V_w* = velocity of water, ft. per min.

| <i>T_{a1}</i> | <i>T_{a2}</i> | <i>T_{w1}</i> | <i>T_{w2}</i> | <i>K</i> | <i>W</i> | <i>B_a</i> | <i>B_w</i> | <i>V_w</i> | <i>U</i> |
|-----------------------|-----------------------|-----------------------|-----------------------|----------|----------|----------------------|----------------------|----------------------|----------|
| 221.0 | 76.3 | 50.0 | 169.0 | 125.2 | 28.50 | 4303 | 3392 | 2.20 | 6.75 |
| 217.0 | 64.3 | 45.8 | 146.4 | 122.8 | 36.73 | 4452 | 3695 | 2.84 | 7.12 |
| 224.0 | 63.3 | 45.7 | 149.2 | 126.3 | 40.30 | 4819 | 4171 | 3.11 | 7.91 |
| 209.6 | 54.0 | 43.8 | 125.9 | 122.1 | 50.00 | 4511 | 4105 | 3.86 | 8.81 |
| 214.5 | 46.3 | 43.0 | 106.2 | 124.6 | 68.95 | 4976 | 4357 | 5.32 | 10.55 |
| 234.6 | 63.6 | 52.6 | 120.2 | 124.4 | 73.25 | 5051 | 4.52 | 5.65 | 8.41 |
| 214.2 | 43.5 | 43.0 | 94.7 | 117.3 | 79.84 | 4753 | 4128 | 6.16 | 14.32 |
| 242.9 | 61.7 | 55.3 | 114.0 | 133.6 | 92.72 | 5649 | 5443 | 7.15 | 10.01 |
| 223.0 | 46.0 | 40.1 | 79.1 | 130.5 | 114.80 | 5484 | 4477 | 8.86 | 7.85 |
| 239.3 | 57.5 | 51.0 | 95.2 | 130.0 | 125.70 | 612 | 5556 | 9.70 | 9.33 |
| 246.0 | 58.0 | 52.3 | 95.1 | 133.8 | 145.90 | 5977 | 6244 | 11.26 | 10.57 |

Sixteen other tests were made besides those given above, and their plotted results all come within the field covered by those in the table.

There is apparently an error in the last line of the table, for the heat gained by the water could not be greater than that lost by the air. The excess lost by the air may be due to radiation, but it shows a great irregularity. It appears that for velocities of water between 2.2 and 5.3 ft. per min. the value of U increases with the velocity, but for higher velocities the value of U is very irregular, and the cause of the irregularity is not explained.

Chas. L. Hubbard (*The Engineer*, Chicago, May 18, 1902) made some tests by blowing air through a tight wooden box which contained a nest of 30 $1\frac{1}{2}$ -in. tin tubes, of a total surface of about 20 sq. ft., through which cold water flowed. The results were as follows:

| | | | | | | |
|--|-----|-----|------|------|------|------|
| Cu. ft. of air per minute..... | 268 | 268 | 469 | 469 | 636 | 636 |
| Velocity over cooling surface..... | 638 | 638 | 1116 | 1116 | 1514 | 1514 |
| Initial temperature of air..... | 72° | 72° | 72° | 74° | 74° | 74° |
| Drop in temperature..... | 8° | 12° | 8° | 10° | 8° | 10° |
| Average temp. of water..... | 50° | 43° | 48° | 48° | 50° | 44° |
| Average temp. of air..... | 68° | 66° | 68° | 69° | 70° | 68° |
| Difference..... | 18° | 23° | 20° | 21° | 20° | 24° |
| B.T.U. per hour per sq. ft. per degree difference..... | 6.5 | 7.6 | 10.2 | 12.1 | 13.8 | 14.4 |

Transmission of Heat through Plates and Tubes from Steam or Hot Water to Air. — The transfer of heat from steam or water through a plate or tube into the surrounding air is a complex operation, in which the internal and external conductivity of the metal, the radiating power of the surface, and the convection of heat in the surrounding air, are all concerned. Since the quantity of heat radiated from a surface varies with the condition of the surface and with the surroundings, according to laws not yet determined, and since the heat carried away by convection varies with the rate of the flow of the air over the surface, it is evident that no general law can be laid down for the total quantity of heat emitted.

The following is condensed from an article on "Loss of Heat from Steam-pipes," in *The Locomotive*, Sept. and Oct., 1892.

A hot steam-pipe is radiating heat constantly off into space, but at the same time it is cooling also by convection. Experimental data on which to base calculations of the heat radiated and otherwise lost by steam-pipes are neither numerous nor satisfactory.

In Box's "Practical Treatise on Heat" a number of results are given for the amount of heat radiated by different substances when the temperature of the air is 1° Fahr. lower than the temperature of the radiating body. A portion of this table is given below. It is said to be based on Petit's experiments.

HEAT UNITS RADIATED PER HOUR, PER SQUARE FOOT OF SURFACE,
FOR 1° FAHRENHEIT EXCESS IN TEMPERATURE.

| | | | |
|--------------------------------|--------|-------------------------------|--------|
| Copper, polished..... | 0.0327 | Glass..... | 0.5948 |
| Tin, polished..... | 0.0440 | Cast iron, new..... | 0.6480 |
| Zinc and brass, polished... .. | 0.0491 | Common steam-pipe, in- | |
| Tinned iron, polished..... | 0.0858 | ferred..... | 0.6400 |
| Sheet iron, polished..... | 0.0920 | Cast and sheet iron, rusted.. | 0.6868 |
| Sheet lead..... | 0.1329 | Wood, building stone, and | |
| Sheet iron, ordinary..... | 0.5662 | brick..... | 0.7358 |

When the temperature of the air is about 50° or 60° Fahr., and the radiating body is not more than about 30° hotter than the air, we may calculate the radiation of a given surface by assuming the amount of heat given off by it in a given time to be proportional to the difference in temperature between the radiating body and the air. This is "Newton's law of cooling." But when the difference in temperature is great, Newton's law does not hold good; the radiation is no longer proportional to the difference in temperature, but must be calculated by a complex formula established experimentally by Dulong and Petit. Box has computed a table from this

formula, which greatly facilitates its application, and which is given below:

FACTORS FOR REDUCTION TO DULONG'S LAW OF RADIATION.

| Differences in Temperature between Radiating Body and the Air. | Temperature of the Air on the Fahrenheit Scale. | | | | | | | | | | | | |
|--|---|------|------|------|------|------|------|------|------|------|------|------|------|
| | 32° | 50° | 59° | 68° | 86° | 104° | 122° | 140° | 158° | 176° | 194° | 212° | 212° |
| Deg. Fahr. | | | | | | | | | | | | | |
| 18 | 1.00 | 1.07 | 1.12 | 1.16 | 1.25 | 1.36 | 1.47 | 1.58 | 1.70 | 1.85 | 1.99 | 2.15 | |
| 36 | 1.03 | 1.11 | 1.16 | 1.21 | 1.30 | 1.40 | 1.52 | 1.68 | 1.76 | 1.91 | 2.06 | 2.23 | |
| 54 | 1.07 | 1.16 | 1.20 | 1.25 | 1.35 | 1.45 | 1.58 | 1.70 | 1.83 | 1.97 | 2.14 | 2.31 | |
| 72 | 1.12 | 1.20 | 1.25 | 1.30 | 1.40 | 1.52 | 1.64 | 1.76 | 1.90 | 2.07 | 2.23 | 2.40 | |
| 90 | 1.16 | 1.25 | 1.31 | 1.36 | 1.46 | 1.58 | 1.71 | 1.84 | 1.98 | 2.15 | 2.33 | 2.51 | |
| 108 | 1.21 | 1.31 | 1.36 | 1.42 | 1.52 | 1.65 | 1.78 | 1.92 | 2.07 | 2.28 | 2.42 | 2.62 | |
| 126 | 1.26 | 1.36 | 1.42 | 1.48 | 1.60 | 1.72 | 1.86 | 2.00 | 2.16 | 2.34 | 2.52 | 2.72 | |
| 144 | 1.32 | 1.42 | 1.48 | 1.54 | 1.65 | 1.79 | 1.94 | 2.08 | 2.24 | 2.44 | 2.64 | 2.83 | |
| 162 | 1.37 | 1.48 | 1.54 | 1.60 | 1.73 | 1.86 | 2.02 | 2.17 | 2.34 | 2.54 | 2.74 | 2.96 | |
| 180 | 1.44 | 1.55 | 1.61 | 1.68 | 1.81 | 1.95 | 2.11 | 2.27 | 2.46 | 2.66 | 2.87 | 3.10 | |
| 198 | 1.50 | 1.62 | 1.69 | 1.75 | 1.89 | 2.04 | 2.21 | 2.38 | 2.56 | 2.78 | 3.00 | 3.24 | |
| 216 | 1.58 | 1.69 | 1.76 | 1.83 | 1.97 | 2.13 | 2.32 | 2.48 | 2.68 | 2.91 | 3.13 | 3.38 | |
| 234 | 1.64 | 1.77 | 1.84 | 1.90 | 2.06 | 2.23 | 2.43 | 2.52 | 2.80 | 3.03 | 3.28 | 3.46 | |
| 252 | 1.71 | 1.85 | 1.92 | 2.00 | 2.15 | 2.33 | 2.52 | 2.71 | 2.92 | 3.18 | 3.43 | 3.70 | |
| 270 | 1.79 | 1.93 | 2.01 | 2.09 | 2.26 | 2.44 | 2.64 | 2.84 | 3.06 | 3.32 | 3.58 | 3.87 | |
| 288 | 1.89 | 2.03 | 2.12 | 2.20 | 2.37 | 2.56 | 2.78 | 2.99 | 3.22 | 3.50 | 3.77 | 4.07 | |
| 306 | 1.98 | 2.13 | 2.22 | 2.31 | 2.49 | 2.69 | 2.90 | 3.12 | 3.37 | 3.66 | 3.95 | 4.26 | |
| 324 | 2.07 | 2.23 | 2.33 | 2.42 | 2.62 | 2.81 | 3.04 | 3.28 | 3.53 | 3.84 | 4.14 | 4.46 | |
| 342 | 2.17 | 2.34 | 2.44 | 2.54 | 2.73 | 2.95 | 3.19 | 3.44 | 3.70 | 4.02 | 4.34 | 4.68 | |
| 360 | 2.27 | 2.45 | 2.56 | 2.66 | 2.86 | 3.09 | 3.35 | 3.60 | 3.88 | 4.22 | 4.55 | 4.91 | |
| 378 | 2.39 | 2.57 | 2.68 | 2.79 | 3.00 | 3.24 | 3.51 | 3.78 | 4.08 | 4.42 | 4.77 | 5.15 | |
| 396 | 2.50 | 2.70 | 2.81 | 2.93 | 3.15 | 3.40 | 3.68 | 3.97 | 4.28 | 4.64 | 5.01 | 5.40 | |
| 414 | 2.63 | 2.84 | 2.95 | 3.07 | 3.31 | 3.56 | 3.87 | 4.12 | 4.48 | 4.87 | 5.26 | 5.67 | |
| 432 | 2.76 | 2.98 | 3.10 | 3.23 | 3.47 | 3.76 | 4.10 | 4.32 | 4.61 | 5.12 | 5.53 | 6.04 | |

The loss of heat by *convection* appears to be independent of the nature of the surface, that is, it is the same for iron, stone, wood, and other materials. It is different for bodies of different shape, however, and it varies with the position of the body. Thus a vertical steam-pipe will not lose so much heat by convection as a horizontal one will: for the air heated at the lower part of the vertical pipe will rise along the surface of the pipe, protecting it to some extent from the chilling action of the surrounding cooler air. For a similar reason the shape of a body has an important influence on the result, those bodies losing most heat whose forms are such as to allow the cool air free access to every part of their surface. The following table from Box gives the number of heat units that horizontal cylinders or pipes lose by convection per square foot of surface per hour, for one degree difference in temperature between the pipe and the air.

HEAT UNITS LOST BY CONVECTION FROM HORIZONTAL PIPES, PER SQUARE FOOT OF SURFACE PER HOUR, FOR A TEMPERATURE DIFFERENCE OF 1° FAHR.

| External Diameter of Pipe in Inches. | Heat Units Lost. | External Diameter of Pipe in Inches. | Heat Units Lost. | External Diameter of Pipe in Inches. | Heat Units Lost. |
|--------------------------------------|------------------|--------------------------------------|------------------|--------------------------------------|------------------|
| 2 | 0.728 | 7 | 0.509 | 18 | 0.455 |
| 3 | 0.626 | 8 | 0.498 | 24 | 0.447 |
| 4 | 0.574 | 9 | 0.489 | 36 | 0.438 |
| 5 | 0.544 | 10 | 0.482 | 48 | 0.434 |
| 6 | 0.523 | 12 | 0.472 | | |

The loss of heat by convection is nearly proportional to the difference in temperature between the hot body and the air, but the experiments of Dulong and Péclet show that this is not exactly true, and we may here also resort to a table of factors for correcting the results obtained by sample proportion.

FACTORS FOR REDUCTION TO DULONG'S LAW OF CONVECTION.

| Difference in Temp. between Hot Body and Air. | Factor. | Difference in Temp. between Hot Body and Air. | Factor. | Difference in Temp. between Hot Body and Air. | Factor. |
|---|---------|---|---------|---|---------|
| 18° F. | 0.94 | 180° F. | 1.62 | 342° F. | 1.87 |
| 36° | 1.11 | 198° | 1.65 | 360° | 1.90 |
| 54° | 1.22 | 216° | 1.68 | 378° | 1.92 |
| 72° | 1.30 | 234° | 1.72 | 396° | 1.94 |
| 90° | 1.37 | 252° | 1.74 | 414° | 1.96 |
| 108° | 1.43 | 270° | 1.77 | 432° | 1.98 |
| 126° | 1.49 | 288° | 1.80 | 450° | 2.00 |
| 144° | 1.53 | 306° | 1.83 | 468° | 2.02 |
| 162° | 1.58 | 324° | 1.85 | | |

EXAMPLE IN THE USE OF THE TABLES. — Required the total loss of heat by both radiation and convection, per foot of length of a steam-pipe $2\frac{11}{32}$ in. external diameter, steam pressure 60 lbs., temperature of the air in the room 68° Fahr.

Temperature corresponding to 60 lbs. equals 307°; temperature difference = $307^\circ - 68 = 239^\circ$.

Area of one foot length of steam-pipe = $2\frac{11}{32} \times 3.1416 \div 12 = 0.614$ sq. ft.

Heat radiated per hour per square foot per degree of difference, from table, 0.64.

Radiation loss per hour by Newton's law = $239^\circ \times 0.614 \text{ ft.} \times 0.64 = 93.9$ heat units. Same reduced to conform with Dulong's law of radiation: factor from table for temperature difference of 239° and temperature of air $68^\circ = 1.93$. $93.9 \times 1.93 = 181.2$ heat units, total loss by radiation.

Convection loss per square foot per hour from a $2\frac{11}{32}$ -inch pipe: by interpolation from table, $2'' = 0.728$, $3'' = 0.626$, $2\frac{11}{32}'' = 0.693$.

Area, $0.614 \times 0.693 \times 239^\circ = 101.7$ heat units. Same reduced to conform with Dulong's law of convection: 101.7×1.73 (from table) = 175.9 heat units per hour. Total loss by radiation and convection = $181.2 + 175.9 = 357.1$ heat units per hour. Loss per degree of difference of temperature per linear foot of pipe per hour = $357.1 \div 239 = 1.494$ heat units = 2.433 per sq. ft.

It is not claimed, says *The Locomotive*, that the results obtained by this method of calculation are strictly accurate. The experimental data are not sufficient to allow us to compute the heat-loss from steam-pipes with any great degree of refinement; yet it is believed that the results obtained as indicated above will be sufficiently near the truth for most purposes. An experiment by Prof. Orday, in a pipe $2\frac{11}{32}$ in. diam. under the above conditions (*Trans. A. S. M. E.*, v. 73), showed a condensation of steam of 181 grams per hour, which is equivalent to a loss of heat of 358.7 heat units per hour, or within half of one per cent of that given by the above calculation.

The quantity of heat given off by steam and hot-water radiators in ordinary practice of heating buildings by direct radiation varies from 1.25 to about 3.25 heat units per hour per square foot per degree of difference of temperature. (See Heating and Ventilation.)

THERMODYNAMICS.

Thermodynamics, the science of heat considered as a form of energy, is useful in advanced studies of the theory of steam, gas, and air engines, refrigerating machines, compressed air, etc. The method of treatment adopted by the standard writers is severely mathematical, involving constant application of the calculus. The student will find the subject

thoroughly treated in the works by Rontgen (Dubois's translation), Wood, Peabody, and Zeuner.

First Law of Thermodynamics. — Heat and mechanical energy are mutually convertible in the ratio of about 778 foot-pounds for the British thermal unit. (Wood.)

Second Law of Thermodynamics. — The second law has by different writers been stated in a variety of ways, and apparently with ideas so diverse as not to cover a common principle. (Wood, Therm., p. 389.)

It is impossible for a self-acting machine, unaided by any external agency, to convert heat from one body to another at a higher temperature. (Clausius.)

If all the heat absorbed be at one temperature, and that rejected be at one lower temperature, then will the heat which is transmuted into work be to the entire heat absorbed in the same ratio as the difference between the absolute temperature of the source and refrigerator is to the absolute temperature of the source. In other words, the second law is an expression for the efficiency of the perfect elementary engine. (Wood.)

The expression $\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$ may be called the symbolical or algebraic enunciation of the second law, — the law which limits the efficiency of heat engines, and which does not depend on the nature of the working medium employed. (Trowbridge.) Q_1 and T_1 = quantity and absolute temperature of the heat received; Q_2 and T_2 = quantity and absolute temperature of the heat rejected.

The expression $\frac{T_1 - T_2}{T_1}$ represents the efficiency of a perfect heat engine which receives all its heat at the absolute temperature T_1 , and rejects heat at the temperature T_2 , converting into work the difference between the quantity received and rejected.

EXAMPLE. — What is the efficiency of a perfect heat engine which receives heat at 388°F . (the temperature of steam of 200 lbs. gauge pressure) and rejects heat at 100°F . (temperature of a condenser, pressure 1 lb. above vacuum)?

$$\frac{388 + 459.2 - (100 + 459.2)}{388 + 459.2} = 34\%, \text{ nearly.}$$

In the actual engine this efficiency can never be attained, for the difference between the quantity of heat received into the cylinder and that rejected into the condenser is not all converted into work, much of it being lost by radiation, leakage, etc. In the steam engine the phenomenon of cylinder condensation also tends to reduce the efficiency.

The Carnot Cycle. — Let one pound of gas of a pressure p_1 , volume v_1 and absolute temperature T_1 be enclosed in an ideal cylinder, having non-conducting walls but the bottom a perfect conductor, and having a moving non-conducting frictionless piston. Let the pressure and volume of the gas be represented by the point A on the pv or pressure-volume diagram, Fig. 136, and let it pass through four operations, as follows:

1. Apply heat at a temperature of T_1 to the bottom of the cylinder and let the gas expand, doing work against the piston, at the constant temperature T_1 , or isothermally, to p_2v_2 , or B .
2. Remove the source of heat and put a non-conducting cover on the bottom, and let the gas expand adiabatically, or without transmission of heat, to p_3v_3 , or C , while its temperature is being reduced to T_2 .

3. Apply to the bottom of the cylinder a cold body, or refrigerator, of the temperature T_2 , and let the gas be compressed by the piston isothermally to the point D , or p_4v_4 , rejecting heat into the cold body.

4. Remove the cold body, restore the non-conducting bottom, and compress the gas adiabatically to A , or the original p_1v_1 , while its temperature is being raised to the original T_1 . The point D on the isothermal line CD is chosen so that an adiabatic line passing through it will also pass through A , and so that $v_4/v_1 = v_3/v_2$.

The area $aABCc$ represents the work done by the gas on the piston;

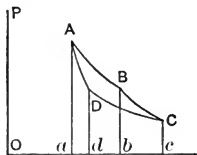


FIG. 136.

the area $CDAac$ the negative work, or the work done by the piston on the gas; the difference, $ABCD$, is the net work.

1a. The area $aABB$ represents the work done during isothermal expansion. It is equal in foot-pounds to $W_1 = p_1 v_1 \log_e (v_2/v_1)$, where p_1 = the initial absolute pressure in lbs. per sq. ft. and v_1 = the initial volume in cubic feet. It is also equal to the quantity of heat supplied to the gas, $= U_1 = RT_1 \log_e (v_2/v_1)$. R is a constant for a given gas, $= 53.35$ for air.

2a. The area $bBCc$ is the work done during adiabatic expansion, $= W_2 = \frac{p_2 v_2}{\gamma - 1} \left\{ 1 - \left(\frac{v_2}{v_3} \right)^{\gamma-1} \right\}$, γ being the ratio of the specific heat at constant pressure to the specific heat at constant volume. For air $\gamma = 1.406$. The loss of intrinsic energy $= K_v(T_1 - T_2)$ ft.-lbs. K_v = specific heat at constant volume $\times 778$.

3a. $CDdc$ is the work of isothermal compression, $= W_3 = p_4 v_4 \log_e (v_3/v_4)$ = heat rejected $= U_2 = RT_2 \log_e (v_3/v_4)$.

4a. $DAad$ is the work of adiabatic compression

$$= W_4 = \frac{p_1 v_1}{\gamma - 1} \left\{ 1 - \left(\frac{v_1}{v_4} \right)^{\gamma-1} \right\},$$

which is the same as W_2 and therefore, being negative, cancels it, and the net work $ABCD = W_1 - W_3$. The gain of intrinsic energy is $K_v(T_1 - T_2)$.

Comparing 1a and 3a, we have $p_1 v_1 = p_2 v_2$; $p_3 v_3 = p_4 v_4$; $v_2/v_3 = v_1/v_4 = r$.

$$W_1 = p_1 v_1 \log_e r = RT_1 \log_e r; \quad W_3 = p_4 v_4 \log_e r = RT_2 \log_e r.$$

$$\begin{aligned} \text{Efficiency } \frac{W_1 - W_3}{W_1} &= \frac{R(T_1 - T_2) \log_e r}{RT_1 \log_e r} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \\ &= 1 - \left(\frac{v_2}{v_3} \right)^{\gamma-1} = \frac{U_1 - U_2}{U_1}. \end{aligned}$$

Entropy.—In the pv or pressure-volume diagram, energy exerted or expended is represented by an area the lines of which show the changes of the values of p and v . In the Carnot cycle these changes are shown by curved lines. If a given quantity of heat Q is added to a substance at a constant temperature, we may represent it by a rectangular area in which the temperature is represented by a vertical line, and the base is the quotient of the area divided by the length of the vertical line. To this quotient is given the name entropy. When the temperature at which the heat is added is not constant a more general definition is needed, viz.: *Entropy is length on a diagram the area of which represents a quantity of heat, and the height at any point represents absolute temperature.* The value of the increase of entropy is given in the language of

calculus, $E = \int_{T_2}^{T_1} \frac{dQ}{T}$, which may be interpreted thus: increase of entropy

between the temperatures T_2 and T_1 equals the summation of all the quotients arising by dividing each small quantity of heat added by the absolute temperature at which it is added. It is evident that if the several small quantities of heat added are equal, while the values of T constantly increase, the quotients are not equal, but are constantly decreasing. The diagram, called the temperature-entropy diagram, or the $\theta\phi$, theta-phi, diagram, is one in which the abscissas, or horizontal distances, represent entropy, and vertical distances absolute temperature. The horizontal distances are measured from an arbitrary vertical line representing entropy at 32° F. , and values of entropy are given as values beyond that point, while the temperatures are measured above absolute zero. Horizontal lines are isothermals, vertical lines adiabatics. The usefulness of entropy in thermodynamic studies is due to the fact that in many cases it simplifies calculations and makes it possible to use algebraic or graphical methods instead of the more difficult methods of the calculus.

The Carnot Cycle in the Temperature-Entropy Diagram.—Let a pound of gas having a temperature T_1 and entropy E be subjected to the

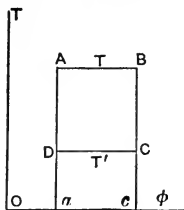


FIG. 137.

four operations described above. (1) T_1 being constant, heat (area $aABc$, Fig. 137) is added and the entropy increases from A to B ; isothermal expansion. (2) No heat is transferred, as heat, but the temperature is reduced from T_1 to T_2 ; entropy constant; adiabatic expansion. (3) Heat is rejected at the constant temperature T_2 , the area $CcAD$ being subtracted; entropy decreases from C to D ; isothermal compression. (4) Entropy constant, temperature increases from D to A , or from T_2 to T_1 ; no heat transferred as heat; adiabatic compression. The area $aABc$ represents the total heat added during the cycle, the area $cCDa$ the heat rejected; the difference, or the area $ABCD$, is the heat utilized or converted into work. The ratio of this area to the whole area

$aABc$ is the efficiency; it is the same as the ratio $(T_1 - T_2) \div T_1$. It appears from this diagram that the efficiency may be increased by increasing T_1 or by decreasing T_2 ; also that since T_2 cannot be lowered by any self-acting engine below the temperature of the surrounding atmosphere, say $460^\circ + 62^\circ \text{ F.} = 522^\circ \text{ F.}$, it is not possible even in a perfect engine to obtain an efficiency of 50 per cent unless the temperature of the source of heat is above 1000° F. It is shown also by this diagram that the Carnot cycle gives the highest possible efficiency of a heat engine working between any given temperatures T_1 and T_2 , and that the admission and rejection of heat each at a constant temperature gives a higher efficiency than the admission or rejection at any variable temperatures within the range $T_1 - T_2$.

The Reversed Carnot Cycle—Refrigeration.—Let a pound of cool gas whose temperature and entropy are represented by the "state-point" D on the diagram (1) receive heat at a constant temperature T_2 (the temperature of a refrigerating room) until its entropy is C ; (2) then let it be compressed adiabatically (no heat transmission, CB) to a high temperature T_1 ; (3) then let it reject heat into the atmosphere at this temperature T_1 (isothermal compression); (4) then let it expand adiabatically, doing work, as through a throttled expansion cock, or by pushing a piston, it will then cool to a temperature which may be far below that of the atmosphere and be used to absorb heat from the atmosphere. (See Refrigeration.)

Principal Equations of a Perfect Gas.—Notation: P = pressure in lbs. per sq. ft. V = volume in cu. ft. P_0V_0 , pressure and volume at 32° F. T , absolute temperature $= t^\circ \text{ F.} + 459.4$. C_p , specific heat at constant pressure. C_v , specific heat at constant volume. $K_p = C_p \times 778$; $K_v = C_v \times 778$; specific heats taken in foot-pounds of energy. R , a constant, $= K_p - K_v$. $\gamma = C_p/C_v$. r = ratio of isothermal expansion or compression $= P_2/P_1$ or V_1/V_2 .

For air: $C_p = 0.2375$; $C_v = 0.1689$; $K_p = 184.8$; $K_v = 131.4$; $R = 53.35$; $\gamma = 1.406$.

Boyle's Law, PV = constant when T is constant. $P_1V_1 = P_2V_2$. For 1 lb. air $P_0V_0 = 2116.2 \times 12.387 = 26,224 \text{ ft.-lbs.}$

Charles's Law, $P_1V_1/T_1 = P_2V_2/T_2$; $P_1V_1 = P_0V_0 \times T_1/T_0$; $T_0 = 32 + 459.4 = 491.4$; P_1V_1 for air $= 26,224 \div 491.4 = 53.35$.

General Equation, $PV = RT$. R is a constant which is different for different gases.

Internal or Intrinsic Energy $K_v (T_1 - T_0) = R (T_1 - T_0) \div (\gamma - 1) = P_1V_1 \div (\gamma - 1)$ = amount of heat in a body, measured above absolute zero. For air at 32° F. , $K_v (T_1 - T_0) = 131.4 \times 491.4 = 64,570 \text{ ft.-lbs.}$ When air is expanded or compressed isothermally, PV = constant, and the internal energy remains constant, the work done in expansion = the heat added, and the work done in compression = the heat rejected.

Work done by Adiabatic Expansion, no transmission of heat, from $P_1 V_1$ to $P_2 V_2 = P_1 V_1 \left\{ 1 - (V_1/V_2)^{\gamma-1} \right\} \div (\gamma - 1)$, $= (P_1 V_1 - P_2 V_2) \div (\gamma - 1)$

$$= P_1 V_1 \left\{ 1 - (P_2/P_1)^{\frac{\gamma-1}{\gamma}} \right\} \div (\gamma - 1).$$

Work of Adiabatic Compression from $P_1 V_1$ to $P_2 V_2$ (P_2 here being the higher pressure) $= P_1 V_1 \left\{ (V_1/V_2)^{\gamma-1} - 1 \right\} \div (\gamma - 1) = (P_2 V_2 - P_1 V_1) \div \gamma - 1$

$$= P_1 V_1 \left\{ (P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \div (\gamma - 1).$$

Loss of Intrinsic Energy in adiabatic expansion, or gain in compression $= K_v(T_1 - T_2)$, T_1 being the higher temperature.

Work of Isothermal Expansion, temperature constant, = heat expended $= P_1 V_1 \log_e V_2/V_1 = P_1 V_1 \log_e r = RT \log_e r$.

Work of Isothermal Compression from P_1 to $P_2 = P_1 V_1 \log_e P_1/P_2 = RT \log_e r$ = heat discharged.

Relation between Pressure, Volume and Temperature:

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}}, \quad V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma}} = V_1 \left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}}.$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}, \quad P_1 V_1^\gamma = P_2 V_2^\gamma.$$

For air, $\gamma = 1.406$; $\gamma - 1 = 0.406$; $1/\gamma = 0.711$; $1/(\gamma - 1) = 2.463$; $\gamma/(\gamma - 1) = 3.463$; $(\gamma - 1)/\gamma = 0.289$.

Differential Equations of a Perfect Gas. Q = quantity of heat. ϕ = entropy.

$$dQ = C_v dT + (C_p - C_v) \frac{T}{V} dV, \quad d\phi = C_v \frac{dT}{T} + (C_p - C_v) \frac{dV}{V}.$$

$$dQ = C_p dT + (C_v - C_p) \frac{T}{P} dP, \quad d\phi = C_p \frac{dT}{T} + (C_v - C_p) \frac{dP}{P}.$$

$$dQ = C_v \frac{T}{P} dP + C_p \frac{T}{P} dV, \quad d\phi = C_v \frac{dP}{P} + C_p \frac{dV}{V}.$$

$$\phi_2 - \phi_1 = C_v \log_e \frac{T_2}{T_1} + (C_p - C_v) \log_e \frac{V_2}{V_1}.$$

$$\phi_2 - \phi_1 = C_p \log_e \frac{T_2}{T_1} + (C_v - C_p) \log_e \frac{P_2}{P_1}$$

$$\phi_2 - \phi_1 = C_v \log_e \frac{P_2}{P_1} + C_p \log_e \frac{V_2}{V_1}.$$

Work of Isothermal Expansion, $W = P_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} = P_1 V_1 \log_e \frac{V_2}{V_1}$.

Heat supplied during isothermal expansion,

$$Q = (C_p - C_v) T_1 \int_{V_1}^{V_2} \frac{dV}{V} = (C_p - C_v) T_1 \log_e \frac{V_2}{V_1}.$$

Heat added = work done $= A R T_1 \log_e V_2/V_1 = A P_1 V_1 \log_e V_2/V_1$; ($A = 1/778$).

Work of adiabatic expansion,

$$W = \int_{V_1}^{V_2} P dV = V_1^\gamma P_1 \int_{V_1}^{V_2} \frac{dV}{V^\gamma} = \frac{P_1 V_1}{\gamma - 1} \left\{ 1 - \left(\frac{V_1}{V_2} \right)^{\gamma-1} \right\}.$$

stant temperature of 212° F. until it is evaporated. The quantity of heat added will be the latent heat of evaporation at 212° (see Steam Table) or $L = 969.7$ B.T.U., and it will be represented on the diagram by the rectangle $bBff$. Dividing by $T_2 = 672$, the absolute temperature, gives $\phi_2 - \phi_1 = 1.443 = BF$. Adding $\phi_1 = 0.312$ gives $\phi_2 = 1.755$, the entropy of 1 lb. steam at 212° F. measured from water at 32° F.

In like manner if we take $L = 835$ for steam at 852° abs., $\phi_2 - \phi_1 = 0.980 = CE$, and $\phi_1 =$ entropy of water at $852^{\circ} = 0.558$, the sum $\phi_2 = 1.538 = Oe$ on the diagram.

E is the state point of dry saturated steam at 852° abs. and F the state point at 672° . The line EF is the line of saturated steam and the line ABC the water line. The line CE represents the increase of entropy in the evaporation of water at 852° abs. If entropy CD only is added, or $cCdd$ of heat, then a part of the water will remain unevaporated, viz.: the fraction DE/CE of 1 lb. The state point D thus represents wet steam having a dryness fraction of CD/DE .

If steam having a state point E is expanded adiabatically to 672° abs. its state point is then e_1 , having the same entropy as at E , a total heat less by the amount represented by the area BCe_1e , and a dryness fraction Be/BF . If it is expanded while remaining saturated, heat must be added equal to $eEff$, and the entropy increases by ef .

If heat is added to the steam at E , the temperature and the entropy both increase, the line EH representing the superheating, and the area EH , down to the line Og , is the heat added. If from the state point H the steam is expanded adiabatically, the state point follows the line FJ until it cuts the line EFg_1 , when the steam is dry saturated, and if it crosses this line the steam becomes wet.

If the state point follows a horizontal line to the left, it represents condensation at a constant temperature, the amount of heat rejected being shown by the area under the horizontal line. If heat is rejected at a decreasing temperature, corresponding with the decreasing pressure at release in a steam engine, or condensation in a cylinder at a decreasing pressure, the state point follows a curved line to the left, as shown in the dotted curved line on the diagram.

In practical calculations with the entropy-temperature diagram it is necessary to have at hand tables or charts of entropy, total heat, etc., such as are given in Peabody's Steam Tables, Ripper's Steam Engine, and other works. The diagram is of especial service in the study of steam turbines, and an excellent chart for this purpose will be found in Moyer's Steam Turbine. It gives for all pressures of steam from 0.5 to 300 lbs. absolute, and for different degrees of dryness up to 300° of superheating, the total heat contents in B.T.U. per pound, the entropy, and the velocity of steam through nozzles.

PHYSICAL PROPERTIES OF GASES.

(Additional matter on this subject will be found under Heat, Air, Gas and Steam.)

When a mass of gas is inclosed in a vessel it exerts a pressure against the walls. This pressure is uniform on every square inch of the surface of the vessel; also, at any point in the fluid mass the pressure is the same in every direction.

In small vessels containing gases the increase of pressure due to weight may be neglected, since all gases are very light; but where liquids are concerned, the increase in pressure due to their weight must always be taken into account.

Expansion of Gases, Mariotte's Law. — The volume of a gas diminishes in the same ratio as the pressure upon it is increased, if the temperature is unchanged.

This law is by experiment found to be very nearly true for all gases, and is known as Boyle's or Mariotte's law.

If p = pressure at a volume v , and p_1 = pressure at a volume v_1 , $p_1 v_1 = pv$; $p_1 = \frac{v}{v_1} p$; $pv =$ a constant.

The constant, C , varies with the temperature, everything else remaining the same.

Air compressed by a pressure of seventy-five atmospheres has a volume about 2% less than that computed from Boyle's law, but this is the greatest divergence that is found below 160 atmospheres pressure.

Law of Charles. — The volume of a perfect gas at a constant pressure is proportional to its absolute temperature. If v_0 be the volume of a gas at 32°F. , and v_1 the volume at any other temperature, t_1 , then

$$v_1 = v_0 \left(\frac{t_1 + 459.2}{491.2} \right); \quad v_1 = \left(1 + \frac{t_1 - 32^\circ}{491.2} \right) v_0,$$

$$\text{or} \quad v_1 = [1 + 0.002036 (t_1 - 32^\circ)] v_0.$$

If the pressure also change from p_0 to p_1 ,

$$v_1 = v_0 \frac{p_0}{p_1} \left(\frac{t_1 + 459.2}{491.2} \right).$$

The Densities of the elementary gases are simply proportional to their atomic weights. The density of a compound gas, referred to hydrogen as 1, is one-half its molecular weight; thus the relative density of CO_2 is $\frac{1}{2} (12 + 32) = 22$.

Avogadro's Law. — Equal volumes of all gases, under the same conditions of temperature and pressure, contain the same number of molecules.

To find the weight of a gas in pounds per cubic foot at 32°F. , multiply half the molecular weight of the gas by 0.00559. Thus 1 cu. ft. marsh-gas, CH_4 ,

$$= \frac{1}{2} (12 + 4) \times 0.00559 = 0.0447 \text{ lb.}$$

When a certain volume of hydrogen combines with one-half its volume of oxygen, there is produced an amount of water vapor which will occupy the same volume as that which was occupied by the hydrogen gas when at the same temperature and pressure.

Saturation Point of Vapors. — A vapor that is not near the saturation point behaves like a gas under changes of temperature and pressure; but if it is sufficiently compressed or cooled, it reaches a point where it begins to condense: it then no longer obeys the same laws as a gas, but its pressure cannot be increased by diminishing the size of the vessel containing it, but remains constant, except when the temperature is changed. The only gas that can prevent a liquid evaporating seems to be its own vapor.

Dalton's Law of Gaseous Pressures. — Every portion of a mass of gas inclosed in a vessel contributes to the pressure against the sides of the vessel the same amount that it would have exerted by itself had no other gas been present.

Mixtures of Vapors and Gases. — The pressure exerted against the interior of a vessel by a given quantity of a perfect gas inclosed in it is the sum of the pressures which any number of parts into which such quantity might be divided would exert separately, if each were inclosed in a vessel of the same bulk alone, at the same temperature. Although this law is not exactly true for any actual gas, it is very nearly true for many. Thus if 0.080728 lb. of air at 32°F. , being inclosed in a vessel of one cubic foot capacity, exerts a pressure of one atmosphere, or 14.7 pounds, on each square inch of the interior of the vessel, then will each additional 0.080728 lb. of air which is inclosed, at 32° , in the same vessel, produce very nearly an additional atmosphere of pressure. The same law is applicable to mixtures of gases of different kinds. For example, 0.12344 lb. of carbonic-acid gas, at 32° , being inclosed in a vessel of one cubic foot in capacity, exerts a pressure of one atmosphere; consequently, if 0.080728 lb. of air and 0.12344 lb. of carbonic acid, mixed, be inclosed at the temperature of 32° , in a vessel of one cubic foot of capacity, the mixture will exert a pressure of two atmospheres. As a second example: Let 0.080728 lb.

of air, at 212° , be inclosed in a vessel of one cubic foot; it will exert a pressure of

$$\frac{212 + 459.2}{32 + 459.2} = 1.366 \text{ atmospheres.}$$

Let 0.03797 lb. of steam, at 212° , be inclosed in a vessel of one cubic foot; it will exert a pressure of one atmosphere. Consequently, if 0.080728 lb. of air and 0.03797 lb. of steam be mixed and inclosed together, at 212° , in a vessel of one cubic foot, the mixture will exert a pressure of 2.366 atmospheres. It is a common but erroneous practice, in elementary books on physics, to describe this law as constituting a difference between mixed and homogeneous gases; whereas it is obvious that for mixed and homogeneous gases the law of pressure is exactly the same, viz., that the pressure of the whole of a gaseous mass is the sum of the pressures of all its parts. This is one of the laws of mixture of gases and vapors.

A second law is that the presence of a foreign gaseous substance in contact with the surface of a solid or liquid does not affect the density of the vapor of that solid or liquid unless there is a tendency to chemical combination between the two substances, in which case the density of the vapor is slightly increased. (Rankine, S. E., p. 239.)

If 0.591 lb. of air, = 1 cu. ft. at 212° and atmospheric pressure, is contained in a vessel of 1 cu. ft. capacity, and water at 212° is introduced, heat at 212° being furnished by a steam jacket, the pressure will rise to two atmospheres.

If air is present in a condenser along with water vapor, the pressure is that due to the temperature of the vapor plus that due to the quantity of air present.

Flow of Gases. — By the principle of the conservation of energy, it may be shown that the velocity with which a gas under pressure will escape into a vacuum is inversely proportional to the square root of its density; that is, oxygen, which is sixteen times as heavy as hydrogen, would, under exactly the same circumstances, escape through an opening only one fourth as fast as the latter gas.

Absorption of Gases by Liquids. — Many gases are readily absorbed by water. Other liquids also possess this power in a greater or less degree. Water will, for example, absorb its own volume of carbonic-acid gas, 430 times its volume of ammonia, $2\frac{1}{3}$ times its volume of chlorine, and only about $\frac{1}{20}$ of its volume of oxygen.

The weight of gas that is absorbed by a given volume of liquid is proportional to the pressure. But as the volume of a mass of gas is less as the pressure is greater, the volume which a given amount of liquid can absorb at a certain temperature will be constant, whatever the pressure. Water, for example, can absorb its own volume of carbonic-acid gas at atmospheric pressure; it will also dissolve its own volume if the pressure is twice as great, but in that case the gas will be twice as dense, and consequently twice the weight of gas is dissolved.

Liquefaction of Gases.—**Liquid Air.** (A. L. Rice, *Trans. A. S. M. E.*, xxi, 156.)—Oxygen was first liquefied in 1877 by Cailletet and Pictet, working independently. In 1884 Dewar liquefied air, and in 1898 he liquefied hydrogen at a temperature of -396.4° F., or only 65° above the absolute zero. The method of obtaining the low temperatures required for liquefying gases was suggested by Sir W. Siemens, in 1857. It consists in expanding a compressed gas in a cylinder doing work, or through a small orifice, to a lower pressure, and using the cold gas thereby produced to cool, before expansion, the gas coming to the apparatus. Hampson claims to have condensed about 1.2 quarts of liquid air per hour at an expenditure of 3.5 H.P. for compression, using a pressure of 120 atmospheres expanded to 1, and getting 6.6 per cent of the air handled as liquid.

The following table gives some physical constants of the principal gases that have been liquefied. The critical temperature is that at which the properties of a liquid and its vapor are indistinguishable, and above which the vapor cannot be liquefied by compression. The critical pressure is the pressure of the vapor at the critical temperature.

| | | Criti- cal Temp. Deg. F. | Criti- cal Pres- sure in Atmos- pheres | Temp. of Satu- rated Vapor at Atmos. Pres- sure Deg. F. | Freez- ing Point. Deg. F. | Density of Liquid at Temperature Given. |
|--------------------|-------------------------------|-----------------------------------|---|--|------------------------------------|--|
| Water..... | H ₂ O | 689 | 200 | 212 | 32 | 1 at 39° F. |
| Ammonia..... | NH ₄ | 266 | 115 | - 27 | -107 | 0.6364 at 32° F. |
| Acetylene..... | C ₂ H ₂ | 98.6 | | -121 | -113.8 | |
| Carbon Dioxide.... | CO ₂ | 88 | 75 | -112 | - 69 | 0.83 at 32° F. |
| Ethylene..... | C ₂ H ₄ | 50 | 51.7 | -150 | -272 | |
| Methane..... | CH ₄ | -115.2 | 54.9 | -263.4 | -302.4 | { 0.415 at -263° F. } |
| Oxygen..... | O ₂ | -182- | 50.8 | -294.5 | | { 1.124 at -294° F. } |
| Argon..... | A | -185.8 | 50.6 | -304.6 | -309.3 | { about 1.5 at -305° F. } |
| Carbon Monoxide.. | CO | -219.1 | 35.5 | -310 | -340.6 | |
| Air..... | | -220 | 39 | -312.6 | | { 0.933 at -313° F. } |
| Nitrogen..... | N ₂ | -231 | 35 | -318 | -353.2 | { 0.885 at -318° F. } |
| Hydrogen..... | H ₂ | -389 | 20 | -403 | | |

AIR.

Properties of Air. — Air is a mechanical mixture of the gases oxygen and nitrogen, with about 1% by volume of argon. Atmospheric air of ordinary purity contains about 0.04% of carbon dioxide. The composition of air is variously given as follows:

| | By Volume. | | | By Weight. | | |
|--------|------------|--------|-------|------------|--------|-------|
| | N | O | Ar | N | O | Ar |
| 1..... | 79.3 | 23.7 | | 77 | 23 | |
| 2..... | 79.09 | 20.91 | | 76.85 | 23.15 | |
| 3..... | 78.122 | 20.941 | 0.937 | 75.539 | 23.024 | 1.437 |
| 4..... | 78.06 | 21. | 0.94 | 75.5 | 23.2 | 1.3 |

(1) Values formerly given in works on physics. (2) Average results of several determinations, Hempel's Gas Analysis. (3) Sir Win. Ramsay, *Bull. U. S. Geol. Survey*, No. 330. (4) A. Leduc, *Comptes Rendus*, 1896, *Jour. F. I.*, Jan., 1898. Leduc gives for the density of oxygen relatively to air 1.10523; for nitrogen 0.9671; for argon, 1.376.

The weight of pure air at 32° F. and a barometric pressure of 29.92 inches of mercury, or 14.6963 lbs. per sq. in., or 2116.3 lbs. per sq. ft., is 0.080728 lb. per cubic foot. Volume of 1 lb. = 12.387 cu. ft. At any other temperature and barometric pressure its weight in lbs. per cubic

foot is $W = \frac{1.3253 \times B}{459.2 + T}$, where B = height of the barometer, T = temperature Fahr., and 1.3253 = weight in lbs. of 459.2 cu. ft. of air at 0° F. and one inch barometric pressure. Air expands $1/491.2$ of its volume at 32° F. for every increase of 1° F., and its volume varies inversely as the pressure.

The **Air-manometer** consists of a long, vertical glass tube, closed at the upper end, open at the lower end, containing air, provided with a scale, and immersed, along with a thermometer, in a transparent liquid, such as water or oil, contained in a strong cylinder of glass, which communicates with the vessel in which the pressure is to be ascertained. The scale shows the volume occupied by the air in the tube.

Let v_0 be that volume, at the temperature of 32° Fahrenheit, and mean pressure of the atmosphere, p_0 ; let v_1 be the volume of the air at the temperature t , and under the absolute pressure to be measured p_1 ; then

$$p_1 = \frac{(t + 459.2^\circ) p_0 v_0}{491.2^\circ v_1}.$$

Pressure of the Atmosphere at Different Altitudes.

At the sea level the pressure of the air is 14.7 pounds per square inch; at $1/4$ of a mile above the sea level it is 14.02 pounds; at $1/2$ mile, 13.33 ; at $3/4$ mile, 12.66 ; at 1 mile, 12.02 ; at $1 1/4$ mile, 11.42 ; at $1 1/2$ mile, 10.88 ; and at 2 miles, 9.80 pounds per square inch. For a rough approximation we may assume that the pressure decreases $1/2$ pound per square inch for every 1000 feet of ascent.

It is calculated that at a height of about $3 1/2$ miles above the sea level the weight of a cubic foot of air is only one-half what it is at the surface of the earth, at seven miles only one-fourth, at fourteen miles only one-sixteenth, at twenty-one miles only one sixty-fourth, and at a height of over forty-five miles it becomes so attenuated as to have no appreciable weight.

The pressure of the atmosphere increases with the depth of shafts, equal to about one inch rise in the barometer for each 900 feet increase in depth: this may be taken as a rough-and-ready rule for ascertaining the depth of shafts.

Pressure of the Atmosphere per Square Inch and per Square Foot at Various Readings of the Barometer.

RULE. — Barometer in inches $\times 0.4908$ = pressure per square inch; pressure per square inch $\times 144$ = pressure per square foot.

| Barometer. | Pressure per Sq. In. | Pressure per Sq. Ft. | Barometer. | Pressure per Sq. In. | Pressure per Sq. Ft. |
|------------|-------------------------|-------------------------|------------|-------------------------|-------------------------|
| in. | lbs. | lbs.* | in. | lbs. | lbs.* |
| 28.00 | 13.74 | 1978 | 29.75 | 14.60 | 2102 |
| 28.25 | 13.86 | 1995 | 30.00 | 14.72 | 2119 |
| 28.50 | 13.98 | 2013 | 30.25 | 14.84 | 2136 |
| 28.75 | 14.11 | 2031 | 30.50 | 14.96 | 2154 |
| 29.00 | 14.23 | 2049 | 30.75 | 15.09 | 2172 |
| 29.25 | 14.35 | 2066 | 31.00 | 15.21 | 2190 |
| 29.50 | 14.47 | 2083 | | | |

* Decimals omitted.

For lower pressures see table of the Properties of Steam.

**Barometric Readings corresponding with Different
Altitudes, in French and English Measures.**

| Altitude. | Reading of Barometer. | Altitude. | Reading of Barometer. | Altitude. | Reading of Barometer. | Altitude. | Reading of Barometer. |
|-----------|-----------------------|-----------|-----------------------|-----------|-----------------------|-----------|-----------------------|
| meters | mm. | feet. | inches. | meters. | mm. | feet. | inches. |
| 0 | 762 | 0. | 30. | 1147 | 660 | 3763.2 | 25.98 |
| 21 | 760 | 68.9 | 29.92 | 1269 | 650 | 4163.3 | 25.59 |
| 127 | 750 | 416.7 | 29.52 | 1393 | 640 | 4568.3 | 25.19 |
| 234 | 740 | 767.7 | 29.13 | 1519 | 630 | 4983.1 | 24.80 |
| 342 | 730 | 1122.1 | 28.74 | 1647 | 620 | 5403.2 | 24.41 |
| 453 | 720 | 1486.2 | 28.35 | 1777 | 610 | 5830.2 | 24.01 |
| 564 | 710 | 1850.4 | 27.95 | 1909 | 600 | 6243. | 23.62 |
| 678 | 700 | 2224.5 | 27.55 | 2043 | 590 | 6702.9 | 23.22 |
| 793 | 690 | 2599.7 | 27.16 | 2180 | 580 | 7152.4 | 22.83 |
| 909 | 680 | 2962.1 | 26.77 | 2318 | 570 | 7605.1 | 22.44 |
| 1027 | 670 | 3369.5 | 26.38 | 2460 | 560 | 8071. | 22.04 |

Boiling Point of Water.—Temperature in degrees F., barometer in in. of mercury.

| In. | .0 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 28 | 208.7 | 208.9 | 209.1 | 209.2 | 209.4 | 209.5 | 209.7 | 209.9 | 210.1 | 210.3 |
| 29 | 210.5 | 210.6 | 210.8 | 210.9 | 211.1 | 211.3 | 211.4 | 211.6 | 211.8 | 212.0 |
| 30 | 212.1 | 212.3 | 212.4 | 212.6 | 212.8 | 212.9 | 213.1 | 213.3 | 213.5 | 213.6 |

Leveling by the Barometer and by Boiling Water. (Trautwine.) — Many circumstances combine to render the results of this kind of leveling unreliable where great accuracy is required. It is difficult to read off from an aneroid (the kind of barometer usually employed for engineering purposes) to within from two to five or six feet, depending on its size. The moisture or dryness of the air affects the results; also winds, the vicinity of mountains, and the daily atmospheric tides, which cause incessant and irregular fluctuations in the barometer. A barometer hanging quietly in a room will often vary $\frac{1}{4}$ of an inch within a few hours, corresponding to a difference of elevation of nearly 100 feet. No formula can possibly be devised that shall embrace these sources of error.

To Find the Difference in Altitude of Two Places. — Take from the table the altitudes opposite to the two boiling temperatures, or to the two barometer readings. Subtract the one opposite the lower reading from that opposite the upper reading. The remainder will be the required height, as a rough approximation. To correct this, add together the two thermometer readings, and divide the sum by 2, for their mean. From table of corrections for temperature, take out the number under this mean. Multiply the approximate height just found by this number.

At 70° F. pure water will boil at 1° less of temperature for an average of about 550 feet of elevation above sea level, up to a height of $\frac{1}{2}$ a mile. At the height of 1 mile, 1° of boiling temperature will correspond to about 560 feet of elevation. In the table the mean of the temperatures at the two stations is assumed to be 32° F., at which no correction for temperature is necessary in using the table.

| Boiling-point in Deg. Fahr. | Barom., In. | Altitude above Sea level, Feet. | Boiling-point in Deg. Fahr. | Barom., In. | Altitude above Sea level, Feet. | Boiling-point in Deg. Fahr. | Barom., In. | Altitude above Sea level, Feet. |
|-----------------------------------|----------------|--|-----------------------------------|----------------|--|-----------------------------------|----------------|--|
| 184° | 16.79 | 15,221 | 196 | 21.71 | 8,471 | 208 | 27.73 | 2,063 |
| 185 | 17.16 | 14,649 | 197 | 22.17 | 7,932 | 208.5 | 28.00 | 1,809 |
| 186 | 17.54 | 14,075 | 198 | 22.64 | 7,381 | 209 | 28.29 | 1,539 |
| 187 | 17.93 | 13,498 | 199 | 23.11 | 6,843 | 209.5 | 28.56 | 1,290 |
| 188 | 18.32 | 12,934 | 200 | 23.59 | 6,304 | 210 | 28.85 | 1,025 |
| 189 | 18.72 | 12,367 | 201 | 24.08 | 5,764 | 210.5 | 29.15 | 754 |
| 190 | 19.13 | 11,799 | 202 | 24.58 | 5,225 | 211 | 29.42 | 512 |
| 191 | 19.54 | 11,243 | 203 | 25.08 | 4,697 | 211.5 | 29.71 | 255 |
| 192 | 19.96 | 10,685 | 204 | 25.59 | 4,169 | 212 | 30.00 | S.L. = 0 |
| 193 | 20.39 | 10,127 | 205 | 26.11 | 3,642 | 212.5 | 30.30 | —261 |
| 194 | 20.82 | 9,579 | 206 | 26.64 | 3,115 | 213 | 30.59 | —511 |
| 195 | 21.26 | 9,031 | 207 | 27.18 | 2,589 | | | |

CORRECTIONS FOR TEMPERATURE.

| Mean temp. F. in shade. | 0 | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° | 100° |
|-------------------------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|
| Multiply by | .933 | .954 | .975 | .996 | 1.016 | 1.036 | 1.058 | 1.079 | 1.100 | 1.121 | 1.142 |

Moisture in the Atmosphere. — Atmospheric air always contains a small quantity of carbonic acid (see Ventilation), and a varying quantity of aqueous vapor or moisture. The relative humidity of the air at any time is the percentage of moisture contained in it as compared with the amount it is capable of holding at the same temperature.

The degree of saturation or relative humidity of the air is determined by the use of the dry and wet bulb thermometer. The degree of saturation for a number of different readings of the thermometer is given in the following table, condensed from the Hygrometric Tables of the U. S. Weather Bureau:

RELATIVE HUMIDITY, PER CENT.

| Dry Ther- mometer, Deg. F. | Difference between the Dry and Wet Thermometers, Deg. F. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------------------------------|--|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|--|--|--|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 26 | 23 | 30 | | | |
| | Relative Humidity, Saturation being 100. (Barometer = 30 ins.) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 32 | 89 | 79 | 69 | 59 | 49 | 39 | 30 | 20 | 11 | 2 | | | | | | | | | | | | | | | | | | | | |
| 40 | 92 | 83 | 75 | 68 | 60 | 52 | 45 | 37 | 29 | 23 | 15 | 7 | 0 | | | | | | | | | | | | | | | | | |
| 50 | 93 | 87 | 80 | 74 | 67 | 61 | 55 | 49 | 43 | 38 | 32 | 27 | 21 | 16 | 11 | 5 | 0 | | | | | | | | | | | | | |
| 60 | 94 | 89 | 83 | 78 | 73 | 68 | 63 | 58 | 53 | 48 | 43 | 39 | 34 | 30 | 26 | 21 | 17 | 13 | 9 | 5 | 1 | | | | | | | | | |
| 70 | 95 | 90 | 86 | 81 | 77 | 72 | 68 | 64 | 59 | 55 | 51 | 48 | 44 | 40 | 36 | 33 | 29 | 25 | 22 | 19 | 15 | 12 | 9 | 6 | | | | | | |
| 80 | 96 | 91 | 87 | 83 | 79 | 75 | 72 | 68 | 64 | 61 | 57 | 54 | 50 | 47 | 44 | 41 | 38 | 35 | 32 | 29 | 26 | 23 | 20 | 18 | 12 | 7 | | | | |
| 90 | 96 | 92 | 89 | 85 | 81 | 78 | 74 | 71 | 68 | 65 | 61 | 58 | 55 | 52 | 49 | 47 | 44 | 41 | 39 | 36 | 34 | 31 | 29 | 26 | 22 | 17 | 13 | | | |
| 100 | 96 | 93 | 89 | 86 | 83 | 80 | 77 | 73 | 70 | 68 | 65 | 62 | 59 | 56 | 54 | 51 | 49 | 46 | 44 | 41 | 39 | 37 | 35 | 33 | 28 | 24 | 21 | | | |
| 110 | 97 | 93 | 90 | 87 | 84 | 81 | 78 | 75 | 73 | 70 | 67 | 65 | 62 | 60 | 57 | 55 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 38 | 34 | 30 | 26 | | | |
| 120 | 97 | 94 | 91 | 88 | 85 | 82 | 80 | 77 | 74 | 72 | 69 | 67 | 65 | 62 | 60 | 58 | 55 | 53 | 51 | 49 | 47 | 45 | 43 | 41 | 38 | 34 | 31 | | | |
| 140 | 97 | 95 | 92 | 89 | 87 | 84 | 81 | 79 | 77 | 75 | 73 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 53 | 51 | 49 | 47 | 44 | 41 | 38 | | | |

Moisture in Air at Different Pressures and Temperatures. (H. M. Prevost Murphy, *Eng. News*, June 18, 1908.) — 1. The maximum amount of moisture that pure air can contain depends only on its temperature and pressure, and has an unvarying value for each condition.

2. The higher the temperature of the air, the greater is the amount of moisture that it can contain.

3. The higher the pressure of the air, the smaller is the amount of moisture that it can contain.

4. When air is compressed, the rise of temperature due to the compression, in all cases found in practice, far more than offsets the opposite effect of the rise of pressure on the moisture-carrying capacity of the air. Water is deposited, therefore, by compressed air as it passes from the compressor to the various portions of the system.

Suppose that a certain amount of atmospheric air enters a compressor and that it contains all the moisture possible at the existing outside temperature and pressure. As this air is compressed its moisture-carrying capacity rapidly increases, consequently all the moisture is retained by the air and passes with it into the main or storage reservoir. Now if this air is permitted to pass from the reservoir into the various parts of the system before being cooled to the outside temperature, it will carry more moisture than it is capable of holding when the temperature finally drops to the normal point, and this excess quantity will be deposited, because, the pressure being high, the air cannot hold as much moisture as it did at the same temperature and only atmospheric pressure.

In order to reduce the moisture to a minimum, it is desirable to cool the air to the outside temperature before it leaves the reservoir, thereby causing it to deposit all of its excess moisture, which may be easily removed by drain cocks.

Although compressed air may be properly dried before leaving the main reservoirs, some moisture may be temporarily deposited when the air is subsequently expanded to lower pressures, as its moisture-carrying capacity is usually affected more by the drop in temperature, resulting from the expansion, than by the drop in pressure, but when the air again attains the outside temperature, the moisture thus deposited will be re-absorbed if it is freely exposed to the compressed air.

In order to determine what percentage of moisture pure air can contain at various pressures and temperatures, to ascertain how low the "relative humidity" of the atmosphere must be in order that no water will be deposited in any part of a compressed-air system and also to find to what temperature air drawn from a saturated atmosphere must be cooled in order to cause the deposition of moisture to commence, the following formulæ and tables are used, based on Dalton's law of gaseous pressures, which may be stated as follows:

The total pressure exerted against the interior of a vessel by a given quantity of a mixed gas enclosed in it is the sum of the pressures which each of the component gases, or vapors, would exert separately if it were enclosed alone in a vessel of the same bulk, at the same temperature. [The derivation of the formulæ is given at length in the original paper.]

Formulæ for the Weight, in Lbs., of 1 Cu. Ft. of Dry Air, of 1 Cu. Ft. of Saturated Steam or Water Vapor and the Maximum Weight of Water Vapor that 1 Lb. of Pure Air Can Carry at Any Pressure and Temperature. (Copyright, 1908, by H. M. Prevost Murphy.)

The values K and H being given in the table for various temperatures, t , in Fahrenheit degrees, the formulæ are:

$$\text{Weight of 1 cu. ft. saturated steam} = \frac{1.325271 KH}{459.2 + t}.$$

H = elastic force or tension of water vapor or saturated steam, in in. of mercury corresponding to the temperature t (Fahr.) = $2.036 \times$ (gauge pressure + atmospheric pressure, in pounds per square inch).

K = the ratio of the weight of a volume of saturated steam to an equal volume of pure dry air at the same temperature and pressure,

$$= 0.6113 + \frac{0.092 t}{850 - t}.$$

Values of K and H corresponding to the various temperatures t are given in the table below.

$$\text{Weight of 1 cu. ft. pure dry air} = \frac{1.325271 M}{459.2 + t} = \frac{2.698192 P}{459.2 + t}.$$

M = absolute pressure in inch of mercury.

P = absolute pressure in pounds per square inch.

W = maximum weight, in lbs., of water vapor, that 1 lb. of pure air can contain, when the temperature of the mixture is t , and the total, or observed, absolute pressure in pounds per square inch is P ,

$$= \frac{KH}{2.036 P - H}$$

NOTE. — The results obtained by the use of any of the above formulæ agree exactly with the average data for air and steam weights as given by the most reliable authorities and careful experiments, for all pressures and temperatures; the value of K being correct for all temperatures up to the critical steam temperature of 689° F.

VALUES OF " K " AND " H " CORRESPONDING TO TEMPERATURES t FROM
— 30° TO 434° F.

| t | K | H | t | K | H | t | K | H | t | K | H | t | K | H |
|-----|-------|-------|-----|-------|-------|-----|-------|-------|-----|-------|-------|-----|-------|-------|
| -30 | .6082 | .0099 | 64 | .6188 | .5962 | 158 | .6323 | 9.177 | 252 | .6501 | 62.97 | 344 | .6739 | 254.2 |
| -28 | .6084 | .0111 | 66 | .6190 | .6393 | 160 | .6326 | 9.628 | 254 | .6505 | 65.21 | 346 | .6745 | 261.0 |
| -26 | .6086 | .0123 | 68 | .6193 | .6848 | 162 | .6330 | 10.10 | 256 | .6510 | 67.49 | 348 | .6751 | 268.0 |
| -24 | .6088 | .0137 | 70 | .6196 | .7332 | 164 | .6333 | 10.59 | 258 | .6514 | 69.85 | 350 | .6757 | 275.0 |
| -22 | .6090 | .0152 | 72 | .6198 | .7846 | 166 | .6336 | 11.10 | 260 | .6518 | 72.26 | 352 | .6763 | 282.2 |
| -20 | .6092 | .0168 | 74 | .6201 | .8391 | 168 | .6340 | 11.63 | 262 | .6523 | 74.75 | 354 | .6770 | 289.6 |
| -18 | .6094 | .0186 | 76 | .6203 | .8969 | 170 | .6343 | 12.18 | 264 | .6528 | 77.30 | 356 | .6776 | 297.1 |
| -16 | .6096 | .0206 | 78 | .6206 | .9585 | 172 | .6346 | 12.75 | 266 | .6532 | 79.93 | 358 | .6783 | 304.8 |
| -14 | .6098 | .0227 | 80 | .6209 | 1.024 | 174 | .6350 | 13.34 | 268 | .6537 | 82.62 | 360 | .6789 | 312.6 |
| -12 | .6100 | .0250 | 82 | .6211 | 1.092 | 176 | .6353 | 13.96 | 270 | .6541 | 85.39 | 362 | .6795 | 320.6 |
| -10 | .6102 | .0275 | 84 | .6214 | 1.165 | 178 | .6357 | 14.60 | 272 | .6546 | 88.26 | 364 | .6803 | 328.7 |
| -8 | .6104 | .0303 | 86 | .6217 | 1.242 | 180 | .6360 | 15.27 | 274 | .6551 | 91.18 | 366 | .6809 | 337.0 |
| -6 | .6107 | .0332 | 88 | .6219 | 1.324 | 182 | .6364 | 15.97 | 276 | .6555 | 94.18 | 368 | .6816 | 345.4 |
| -4 | .6109 | .0365 | 90 | .6222 | 1.410 | 184 | .6367 | 16.68 | 278 | .6560 | 97.26 | 370 | .6822 | 354.0 |
| -2 | .6111 | .0400 | 92 | .6225 | 1.501 | 186 | .6371 | 17.43 | 280 | .6565 | 100.4 | 372 | .6829 | 362.8 |
| 0 | .6113 | .0439 | 94 | .6227 | 1.597 | 188 | .6374 | 18.20 | 282 | .6570 | 103.7 | 374 | .6836 | 371.8 |
| 2 | .6115 | .0481 | 96 | .6230 | 1.698 | 190 | .6377 | 19.00 | 284 | .6575 | 107.0 | 376 | .6843 | 380.9 |
| 4 | .6117 | .0526 | 98 | .6233 | 1.805 | 192 | .6381 | 19.83 | 286 | .6580 | 110.4 | 378 | .6850 | 390.2 |
| 6 | .6120 | .0576 | 100 | .6236 | 1.918 | 194 | .6385 | 20.69 | 288 | .6584 | 113.9 | 380 | .6857 | 399.6 |
| 8 | .6122 | .0630 | 102 | .6238 | 2.036 | 196 | .6389 | 21.58 | 290 | .6590 | 117.5 | 382 | .6865 | 409.3 |
| 10 | .6124 | .0690 | 104 | .6241 | 2.161 | 198 | .6393 | 22.50 | 292 | .6594 | 121.2 | 384 | .6871 | 419.1 |
| 12 | .6125 | .0754 | 106 | .6244 | 2.294 | 200 | .6396 | 23.46 | 294 | .6600 | 125.0 | 386 | .6879 | 429.1 |
| 14 | .6128 | .0824 | 108 | .6247 | 2.432 | 202 | .6400 | 24.44 | 296 | .6604 | 128.8 | 388 | .6886 | 439.3 |
| 16 | .6131 | .0900 | 110 | .6250 | 2.578 | 204 | .6404 | 25.47 | 298 | .6610 | 132.8 | 390 | .6893 | 449.6 |
| 18 | .6133 | .0983 | 112 | .6253 | 2.731 | 206 | .6407 | 26.53 | 300 | .6615 | 136.8 | 392 | .6901 | 460.2 |
| 20 | .6135 | .1074 | 114 | .6256 | 2.892 | 208 | .6411 | 27.62 | 302 | .6620 | 141.0 | 394 | .6908 | 470.9 |
| 22 | .6137 | .1172 | 116 | .6258 | 3.061 | 210 | .6415 | 28.75 | 304 | .6625 | 145.3 | 396 | .6915 | 481.9 |
| 24 | .6140 | .1279 | 118 | .6261 | 3.239 | 212 | .6419 | 29.92 | 306 | .6631 | 149.6 | 398 | .6923 | 493.0 |
| 26 | .6142 | .1396 | 120 | .6264 | 3.425 | 214 | .6423 | 31.14 | 308 | .6636 | 154.1 | 400 | .6931 | 504.4 |
| 28 | .6144 | .1523 | 122 | .6267 | 3.621 | 216 | .6426 | 32.38 | 310 | .6641 | 158.7 | 402 | .6939 | 515.9 |
| 30 | .6147 | .1661 | 124 | .6270 | 3.826 | 218 | .6430 | 33.67 | 312 | .6647 | 163.3 | 404 | .6947 | 527.6 |
| 32 | .6149 | .1811 | 126 | .6273 | 4.042 | 220 | .6434 | 35.01 | 314 | .6652 | 168.1 | 406 | .6955 | 539.5 |
| 34 | .6151 | .1960 | 128 | .6276 | 4.267 | 222 | .6438 | 36.38 | 316 | .6658 | 173.0 | 408 | .6962 | 551.6 |
| 36 | .6154 | .2120 | 130 | .6279 | 4.503 | 224 | .6442 | 37.80 | 318 | .6663 | 178.0 | 410 | .6970 | 564.0 |
| 38 | .6156 | .2292 | 132 | .6282 | 4.750 | 226 | .6446 | 39.27 | 320 | .6669 | 183.1 | 412 | .6979 | 576.5 |
| 40 | .6158 | .2476 | 134 | .6285 | 5.008 | 228 | .6451 | 40.78 | 322 | .6674 | 188.3 | 414 | .6987 | 589.3 |
| 42 | .6161 | .2673 | 136 | .6288 | 5.280 | 230 | .6455 | 42.34 | 324 | .6680 | 193.7 | 416 | .6995 | 602.2 |
| 44 | .6163 | .2883 | 138 | .6291 | 5.563 | 232 | .6458 | 43.95 | 326 | .6686 | 199.2 | 418 | .7003 | 615.4 |
| 46 | .6166 | .3109 | 140 | .6294 | 5.859 | 234 | .6463 | 45.61 | 328 | .6691 | 204.8 | 420 | .7012 | 628.8 |
| 48 | .6168 | .3350 | 142 | .6298 | 6.167 | 236 | .6467 | 47.32 | 330 | .6697 | 210.5 | 422 | .7021 | 642.5 |
| 50 | .6170 | .3608 | 144 | .6301 | 6.490 | 238 | .6471 | 49.08 | 332 | .6703 | 216.4 | 424 | .7029 | 656.3 |
| 52 | .6173 | .3883 | 146 | .6304 | 6.827 | 240 | .6475 | 50.89 | 334 | .6709 | 222.4 | 426 | .7037 | 670.4 |
| 54 | .6175 | .4176 | 148 | .6307 | 7.178 | 242 | .6479 | 52.77 | 336 | .6715 | 228.5 | 428 | .7046 | 684.7 |
| 56 | .6178 | .4490 | 150 | .6310 | 7.545 | 244 | .6484 | 54.69 | 338 | .6721 | 234.7 | 430 | .7055 | 699.2 |
| 58 | .6180 | .4824 | 152 | .6313 | 7.929 | 246 | .6488 | 56.67 | 340 | .6727 | 241.1 | 432 | .7064 | 713.9 |
| 60 | .6183 | .5180 | 154 | .6317 | 8.328 | 248 | .6492 | 58.71 | 342 | .6733 | 247.6 | 434 | .7073 | 728.9 |
| 62 | .6185 | .5559 | 156 | .6320 | 8.744 | 250 | .6496 | 60.81 | | | | | | |

Applications of the Formulæ and Tables.

EXAMPLE 1. — How low must the relative humidity be, when the atmospheric pressure is 14.7 lb. per sq. in. and the outside temperature is 60°, in order that no moisture may be deposited in any part of a compressed air system carrying a constant gauge pressure of 90 lb. per sq. in.?

Ans. — The maximum amount of moisture that 1 lb. of pure air can contain at 90 lb. gauge, = 104.7 lb. (absolute pressure) and 60° F., is

$$W = \frac{KH}{2.036 P - H} = \frac{0.6183 \times 0.5180}{2.036 \times 104.7 - 0.5180} = 0.001506 \text{ lb.}$$

The maximum weight of moisture that 1 lb. of air can contain at 60° F. and 14.7 lb. (absolute pressure) is

$$W \text{ (at 14.7)} = \frac{0.6183 \times 0.5180}{2.036 \times 14.7 - 0.5180} = 0.01089 \text{ lb.}$$

In order that no moisture may be deposited, the relative humidity must not be above

$$(0.001506 \div 0.01089) \times 100 = 13.83\%.$$

Weights in Pounds, of Pure Dry Air, Water Vapor and Saturated Mixtures of Air and Water Vapor at Various Temperatures, at Atmospheric Pressure, 29.921 In. of Mercury or 14.6963 Lb. Per Sq. In. Also the Elastic Force or Pressure of the Air and Vapor Present in Saturated Mixtures.

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| Temperatures in Fahrenheit Degrees. | Weight of One Cubic Foot of Pure Dry Air, Lb. | Saturated Mixtures of Air and Water Vapor. | | | | | |
|---|---|---|--|---|--|---|--|
| | | Elastic Force of the Vapor, In. of Mercury. | Elastic Force of the Air alone, when Saturated, Ins. of Mercury. | Weight of the Vapor in 1 Cu. Ft. of the Mix- ture, or Wt. of 1 Cu. Ft. of Satu- rated Steam. | Weight of the Air in 1 Cu. Ft. of the Mixture. | Total Weight of 1 Cu. Ft. of the Mixture. | Weight of Water Vapor Mixed with 1 lb. of Air. |
| 0 | 0.086354 | 0.0439 | 29.877 | 0.000077 | 0.086226 | 0.086303 | 0.000898 |
| 12 | 0.084154 | 0.0754 | 29.846 | 0.000130 | 0.083943 | 0.084073 | 0.001548 |
| 22 | 0.082405 | 0.1172 | 29.804 | 0.000198 | 0.082083 | 0.082281 | 0.002413 |
| 32 | 0.080728 | 0.1811 | 29.740 | 0.000300 | 0.080239 | 0.080539 | 0.003744 |
| 42 | 0.079117 | 0.2673 | 29.654 | 0.000435 | 0.078411 | 0.078846 | 0.005554 |
| 52 | 0.077569 | 0.3883 | 29.533 | 0.000621 | 0.076563 | 0.077184 | 0.008116 |
| 62 | 0.076081 | 0.5559 | 29.365 | 0.000874 | 0.074667 | 0.075541 | 0.011709 |
| 72 | 0.074649 | 0.7846 | 29.136 | 0.001213 | 0.072690 | 0.073903 | 0.016691 |
| 82 | 0.073270 | 1.092 | 28.829 | 0.001661 | 0.070595 | 0.072256 | 0.023526 |
| 92 | 0.071940 | 1.501 | 28.420 | 0.002247 | 0.068331 | 0.070578 | 0.032877 |
| 102 | 0.070658 | 2.036 | 27.885 | 0.002999 | 0.065850 | 0.068849 | 0.045546 |
| 112 | 0.069421 | 2.731 | 27.190 | 0.003962 | 0.063085 | 0.067047 | 0.062806 |
| 122 | 0.068227 | 3.621 | 26.300 | 0.005175 | 0.059970 | 0.065145 | 0.086285 |
| 132 | 0.067073 | 4.750 | 25.171 | 0.006689 | 0.056425 | 0.063114 | 0.118548 |
| 142 | 0.065957 | 6.167 | 23.754 | 0.008562 | 0.052363 | 0.060925 | 0.163508 |
| 152 | 0.064878 | 7.929 | 21.992 | 0.010854 | 0.047686 | 0.058540 | 0.227609 |
| 162 | 0.063834 | 10.097 | 19.824 | 0.013636 | 0.042293 | 0.055929 | 0.322407 |
| 172 | 0.062822 | 12.749 | 17.172 | 0.016987 | 0.036055 | 0.053042 | 0.471146 |
| 182 | 0.061843 | 15.965 | 13.956 | 0.021000 | 0.028845 | 0.049845 | 0.728012 |
| 192 | 0.060893 | 19.826 | 10.095 | 0.025746 | 0.020545 | 0.046291 | 1.25319 |
| 202 | 0.059972 | 24.442 | 5.479 | 0.031354 | 0.010982 | 0.042336 | 2.85507 |
| 212 | 0.059079 | 29.921 | 0.000 | 0.037922 | 0.000000 | 0.037922 | Infinite. |

NOTE. — Air is said to be saturated with water vapor when it contains the maximum amount possible at the existing temperature and pressure.

EXAMPLE 2. — When compressing air into a reservoir carrying a constant gauge pressure of 75 lb., from a saturated atmosphere of 14.7 lb. abs. press. and 70° F., to what temperature must the air be cooled after compression in order to cause the deposition of moisture to commence?

Ans. — First find the maximum weight of moisture contained in 1 lb. of pure air at 14.7 lb. pressure and 70° F.

$$W = \frac{KH}{2.036 P - H} = \frac{0.6196 \times 0.7332}{2.036 \times 14.7 - 0.7332} = 0.01556 \text{ lb.}$$

The temperature to which the air must be cooled in order to cause the deposition of moisture may be found by placing this value of 0.01556 together with P equal to 75 + 14.7 in the equation thus:

$$0.01556 = \frac{KH}{2.036 \times 89.7 - H} = \frac{KH}{182.63 - H}$$

or $H = \frac{2.842}{0.01556 + K}$, and the temperature which satisfies this equation is found by aid of the table [by trial and error] to be approximately 129° F.

EXAMPLE 3. — When the outside temperature is 82° F., and the pressure of the atmosphere is 14.6963 lb. per sq. in., the relative humidity being 100%, how many cu. ft. of free air must be compressed and delivered into a reservoir at 100 lb. gauge in order to cause 1 lb. of water to be deposited when the air is cooled to 82° F.?

Ans. — Weight of moisture mixed with 1 lb. of air at 82° F., and atmospheric pressure = 0.023526 lb. For 100 lb. gauge pressure,

$$W = \frac{KH}{2.036 P - H} = \frac{0.6211 \times 1.092}{2.036 \times 114.6963 - 1.092} = 0.002918 \text{ lb.}$$

Weight of moisture deposited by each lb. of compressed air = 0.023526 - 0.002918 = 0.020608 lb. Each cu. ft. of the moist atmosphere contains 0.070595 lb. of pure air, therefore the number of cu. ft. that must be delivered to cause 1 lb. of water to be deposited is

$$\frac{1}{0.070595} \times \frac{1}{0.020608} = 687.37 \text{ cu. ft.}$$

EXAMPLE 4. — Under the same conditions as stated in Example 3, what is the loss in volumetric efficiency of the plant when the excess moisture is properly trapped in the main reservoirs?

Ans. — Before compression, each pound of air is mixed with 0.023526 lb. of water vapor and the weight of 1 cu. ft. of the mixture is 0.072256 lb., consequently the volume of the mixture is

$$1.023526 \div 0.072256 = 14.165 \text{ cu. ft.}$$

For 100 lb. gauge pressure and 82° F. as shown in Example 3, 1 lb. of air can hold 0.002918 lb. of water in suspension, having deposited 0.020608 lb. in the reservoir. The weight of 1 cu. ft. of water vapor at 82° is 0.001661 lb., consequently by Dalton's law the volume of the mixture of 1 lb. of air and 0.002918 lb. of water vapor at 100 lb. gauge pressure is the same as that of the vapor or saturated steam alone; that is,

$$0.002918 \div 0.001661 = 1.757 \text{ cu. ft.}$$

By Mariotte's law, the volume of the 1.757 cu. ft. of mixed gas at 114.6963 lb. absolute when expanded to atmospheric pressure will be

$$(114.6963 \div 14.6963) \times 1.757 = 13.712 \text{ cu. ft.};$$

consequently the decrease of volume, that is, the loss of volumetric efficiency, is

$$14.165 - 13.712 = 0.453 \text{ cu. ft., or } (0.453 \div 14.165) \times 100 = 3.2\%.$$

This example shows that, particularly in warm, moist climates, there is a very appreciable loss in the efficiency of compressors, due to the condensation of water vapor.

Specific Heat of Air at Constant Volume and at Constant Pressure. — Volume of 1 lb. of air at 32° F. and pressure of 14.7 lbs. per sq. in. = 12.387 cu. ft. = a column 1 sq. ft. area \times 12.387 ft. high. Raising tem-

perature 1° F. expands it 1/492, or to 12.4122 ft. high — a rise of 0.02522 foot.

Work done = 2116 lbs. per sq. ft. \times .02522 = 53.37 foot-pounds, or $53.37 \div 778 = 0.0686$ heat units.

The specific heat of air at constant pressure, according to Regnault, is 0.2375; but this includes the work of expansion, or 0.0686 heat units; hence the specific heat at constant volume = $0.2375 - 0.0686 = 0.1689$.

Ratio of specific heat at constant pressure to specific heat at constant volume = $0.2375 \div 0.1689 = 1.406$. (See Specific Heat, p. 534.)

Flow of Air through Orifices. — The theoretical velocity in feet per second of flow of any fluid, liquid, or gas through an orifice is $v = \sqrt{2gh} = 8.02 \sqrt{h}$, in which h is the "head" or height of the fluid in feet required to produce the pressure of the fluid at the level of the orifice. (For gases the formula holds good only for small differences of pressure on the two sides of the orifice.) The quantity of flow in cubic feet per second is equal to the product of this velocity by the area of the orifice, in square feet, multiplied by a "coefficient of flow," which takes into account the contraction of the vein or flowing stream, the friction of the orifice, etc.

For air flowing through an orifice or short tube, from a reservoir of the pressure p_1 into a reservoir of the pressure p_2 , Weisbach gives the following values for the coefficient of flow, obtained from his experiments.

FLOW OF AIR THROUGH AN ORIFICE.

Coefficient c in formula $v = c \sqrt{2gh}$.

| | | | | | | |
|-----------------------------|------|------|------|------|------|------|
| Diam. 1 cm. = 0.394 in.: | | | | | | |
| Ratio of pressures... | 1.05 | 1.09 | 1.43 | 1.65 | 1.89 | 2.15 |
| Coefficient..... | .555 | .589 | .692 | .724 | .754 | .788 |
| Diam. 2.14 cm. = 0.843 in.: | | | | | | |
| Ratio of pressures... | 1.05 | 1.09 | 1.36 | 1.67 | 2.01 | |
| Coefficient..... | .558 | .573 | .634 | .678 | .723 | |

FLOW OF AIR THROUGH A SHORT TUBE.

| | | | | | | |
|--|------|------|------|------|------|------|
| Diam. 1 cm., = 0.394 in., length 3 cm. = 1.181 in. | | | | | | |
| Ratio of pressures $p_1 \div p_2$... | 1.05 | 1.10 | 1.30 | | | |
| Coefficient..... | .730 | .771 | .830 | | | |
| Diam. 1.414 cm. = 0.557 in., length 4.242 cm. = 1.670 in.: | | | | | | |
| Ratio of pressures..... | 1.41 | 1.69 | | | | |
| Coefficient..... | .813 | .822 | | | | |
| Diam. 1 cm. = 0.394 in., length 1.6 cm. = 0.630 in. Orifice rounded: | | | | | | |
| Ratio of pressures..... | 1.24 | 1.38 | 1.59 | 1.85 | 2.14 | |
| Coefficient..... | .979 | .986 | .965 | .971 | .978 | |

Clark (Rules, Tables, and Data, p. 891) gives, for the velocity of flow of air through an orifice due to small differences of pressure,

$$V = C \sqrt{\frac{2gh}{12}} \times 773.2 \times \left(1 + \frac{t - 32}{493}\right) \times \frac{29.92}{p}$$

or, simplified,

$$V = 352 C \sqrt{(1 + .00203(t - 32)) \frac{h}{p}};$$

in which V = velocity in feet per second; $2g = 64.4$; h = height of the column of water in inches, measuring the difference of pressure; t = the temperature Fahr.; and p = barometric pressure in inches of mercury. 773.2 is the volume of air at 32° under a pressure of 29.92 inches of mercury when that of an equal weight of water is taken as 1.

For 62° F., the formula becomes $V = 363 C \sqrt{h/p}$, and if $p = 29.92$ inches, $V = 66.35 C \sqrt{h}$.

The coefficient of efflux C , according to Weisbach, is:

| | |
|--|----------------------|
| For conoidal mouthpiece, of form of the contracted vein, | |
| with pressures of from 0.23 to 1.1 atmospheres.... | $C = 0.97$ to 0.99 |
| Circular orifices in thin plates..... | $C = 0.56$ to 0.79 |
| Short cylindrical mouthpieces..... | $C = 0.81$ to 0.84 |
| The same rounded at the inner end..... | $C = 0.92$ to 0.93 |
| Conical converging mouthpieces..... | $C = 0.90$ to 0.99 |

R. J. Durley, *Trans. A. S. M. E.*, xxvii, 193, gives the following:

The consideration of the adiabatic flow of a perfect gas through a frictionless orifice leads to the equation

$$W = A \sqrt{2g \frac{\gamma}{\gamma-1} \cdot \frac{P_1}{V_1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad (1)$$

W = weight of gas discharged per second in pounds.

A = area of cross section of jet in square feet.

P_1 = pressure inside orifice in pounds per square foot.

P_2 = pressure outside orifice.

V_1 = specific volume of gas inside orifice in cu. ft. per lb.

γ = ratio of the specific heat at constant pressure to that at constant volume.

For air, where $\gamma = 1.404$, we have for a circular orifice of diameter d inches, the initial temperature of the air being 60° Fahr. (or 521° abs.),

$$W = 0.000491 d^2 P_1 \sqrt{\left(\frac{P_2}{P_1} \right)^{1.425} - \left(\frac{P_2}{P_1} \right)^{1.712}} \quad (2)$$

In practice the flow is not frictionless, nor is it perfectly adiabatic, and the amount of heat entering or leaving the gas is not known. Hence the weight actually discharged is to be found from the formulas by introducing a coefficient of discharge (generally less than unity) depending on the conditions of the experiment and on the construction of the particular form of orifice employed.

If we neglect the changes of density and temperature occurring as the air passes through the orifice, we may obtain a simpler though approximate formula for the ideal discharge:

$$W = 0.01369 d^2 \sqrt{\frac{iP}{T}} \quad (3)$$

in which d = diam. in inches, i = difference of pressures measured in inches of water, P = mean absolute pressure in lbs. per sq. ft., and T = absolute temperature on the Fahrenheit scale = degrees F. + 461. In the usual case, in which the discharge takes place into the atmosphere, P is approximately 2117 pounds per square foot and

$$W = 0.6299 d^2 \sqrt{\frac{i}{T}} \quad (4)$$

To obtain the actual discharge the values found by the formula are to be multiplied by an experimental coefficient C , values of which are given in the table below.

Up to a pressure of about 20 ins. of water (or 0.722 lbs. per sq. in.) above the atmospheric pressure, the results of formulæ (2) and (4) agree very closely. At higher differences of pressure divergence becomes noticeable.

They hold good only for orifices of the particular form experimented with, and bored in plates of the same thickness, viz.: iron plates 0.057 in. thick.

The experiments and curves plotted from them indicate that:—

(1) The coefficient for small orifices increases as the head increases, but at a lesser rate the larger the orifices, till for the 2-in. orifice it is almost constant. For orifices larger than 2 ins. it decreases as the head increases, and at a greater rate the larger the orifice.

(2) The coefficient decreases as the diameter of the orifice increases, and at a greater rate the higher the head.

(3) The coefficient does not change appreciably with temperature (between 40° and 100° F.).

(4) The coefficient (at heads under 6 ins.) is not appreciably affected by the size of the box in which the orifice is placed if the ratio of the areas of the box and orifice is at least 20 : 1.

**MEAN DISCHARGE IN POUNDS PER SQUARE FOOT OF ORIFICE PER SECOND
AS FOUND FROM EXPERIMENTS.**

| Diameter Orifice, Inches. | 1-inch Head Discharge per Sq. Ft. | 2-inch Head Discharge per Sq. Ft. | 3-inch Head Discharge per Sq. Ft. | 4-inch Head Discharge per Sq. Ft. | 5-inch Head Discharge per Sq. Ft. |
|---------------------------------|--|---|---|--|--|
| 0.3125 | 3.060 | 4.336 | 5.395 | 6.188 | 7.024 |
| 0.5005 | 3.012 | 4.297 | 5.242 | 6.129 | 6.821 |
| 1.002 | 3.058 | 4.341 | 5.348 | 6.214 | 6.838 |
| 1.505 | 3.050 | 4.257 | 5.222 | 6.071 | 6.775 |
| 2.002 | 2.983 | 4.286 | 5.284 | 6.107 | 6.788 |
| 2.502 | 3.041 | 4.303 | 5.224 | 5.991 | 6.762 |
| 3.001 | 3.078 | 4.297 | 5.219 | 6.033 | 6.802 |
| 3.497 | 3.051 | 4.258 | 5.202 | 5.966 | 6.814 |
| 4.002 | 3.046 | 4.325 | 5.264 | 5.951 | 6.774 |
| 4.506 | 3.075 | 4.383 | 5.508 | 6.260 | 7.028 |

**COEFFICIENTS OF DISCHARGE FOR VARIOUS HEADS AND DIAMETERS OF
ORIFICE.**

| Diameter of Orifice, Inches. | 1-inch Head. | 2-inch Head. | 3-inch Head. | 4-inch Head. | 5-inch Head. |
|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 5/16 | 0.603 | 0.606 | 0.610 | 0.613 | 0.616 |
| 1/2 | 0.602 | 0.605 | 0.608 | 0.610 | 0.613 |
| 1 | 0.601 | 0.603 | 0.605 | 0.606 | 0.607 |
| 1 1/2 | 0.601 | 0.601 | 0.602 | 0.603 | 0.603 |
| 2 | 0.600 | 0.600 | 0.600 | 0.600 | 0.600 |
| 2 1/2 | 0.599 | 0.599 | 0.599 | 0.598 | 0.598 |
| 3 | 0.599 | 0.598 | 0.597 | 0.596 | 0.596 |
| 3 1/2 | 0.599 | 0.597 | 0.596 | 0.595 | 0.594 |
| 4 | 0.598 | 0.597 | 0.595 | 0.594 | 0.593 |
| 4 1/2 | 0.598 | 0.596 | 0.594 | 0.593 | 0.592 |

**CORRECTED ACTUAL DISCHARGE IN POUNDS PER SECOND AT 60° F. AND
14.7 LBS. BAROMETRIC PRESSURE FOR CIRCULAR ORIFICES IN
PLATE 0.057 IN. THICK.**

| Head, In. of Water. | Diameter of Orifice in Inches. | | | | | | | | | | |
|------------------------|--------------------------------|---------|--------|--------|--------|--------|-------|-------|-------|-------|-------|
| | 0.3125 | 0.500 | 1.000 | 1.500 | 2.000 | 2.500 | 3.000 | 3.500 | 4.000 | 4.500 | 5.000 |
| 1/2 | 0.00114 | 0.00293 | 0.0117 | 0.0263 | 0.0468 | 0.0732 | 0.105 | 0.143 | 0.187 | 0.237 | 0.292 |
| 1 | 0.00162 | 0.00416 | 0.0166 | 0.0373 | 0.0663 | 0.103 | 0.149 | 0.202 | 0.264 | 0.334 | 0.413 |
| 1 1/2 | 0.00199 | 0.00510 | 0.0203 | 0.0457 | 0.0811 | 0.127 | 0.182 | 0.248 | 0.323 | 0.409 | 0.505 |
| 2 | 0.00231 | 0.00590 | 0.0235 | 0.0528 | 0.0937 | 0.146 | 0.210 | 0.285 | 0.373 | 0.471 | 0.582 |
| 2 1/2 | 0.00259 | 0.00662 | 0.0263 | 0.1591 | 0.105 | 0.163 | 0.235 | 0.319 | 0.416 | 0.526 | 0.649 |
| 3 | 0.00285 | 0.00726 | 0.0289 | 0.0648 | 0.115 | 0.179 | 0.257 | 0.349 | 0.455 | 0.575 | 0.710 |
| 3 1/2 | 0.00308 | 0.00786 | 0.0312 | 0.0700 | 0.124 | 0.193 | 0.277 | 0.377 | 0.491 | 0.621 | 0.766 |
| 4 | 0.00330 | 0.00842 | 0.0334 | 0.0749 | 0.133 | 0.206 | 0.296 | 0.402 | 0.525 | 0.663 | 0.817 |
| 4 1/2 | 0.00351 | 0.00895 | 0.0355 | 0.0794 | 0.141 | 0.219 | 0.314 | 0.426 | 0.556 | 0.702 | 0.865 |
| 5 | 0.00371 | 0.00945 | 0.0375 | 0.0838 | 0.148 | 0.231 | 0.331 | 0.449 | 0.586 | 0.739 | 0.912 |
| 5 1/2 | 0.00390 | 0.00993 | 0.0393 | 0.0879 | 0.155 | 0.242 | 0.347 | 0.471 | 0.613 | 0.774 | 0.953 |
| 6 | 0.00408 | 0.01049 | 0.0411 | 0.0918 | 0.162 | 0.252 | 0.362 | 0.492 | 0.640 | 0.808 | 0.995 |

Flow of Air in Pipes. — Hawksley (*Proc. Inst. C. E.*, xxxiii, 55) states that his formula for flow of water in pipes, $v = 48 \sqrt{\frac{HD}{L}}$, may also be employed for flow of air. In this case H = height in feet of a column of air required to produce the pressure causing the flow, or the loss of head for a given flow; v = velocity in feet per second, D = diameter in feet, L = length in feet.

If the head is expressed in inches of water, h , the air being taken at 62°F., its weight per cubic foot at atmospheric pressure = 0.0761 lb. Then $H = \frac{62.36}{0.0761 \times 12} = 68.3 h$. If d = diameter in inches, $D = \frac{d}{12}$, and the formula becomes $v = 114.5 \sqrt{\frac{hd}{L}}$, in which h = inches of water column, d = diameter in inches, and L = length in feet; $h = \frac{Lv^2}{13110 d}$; $d = \frac{Lv^2}{13110 h}$.

The quantity in cubic feet per second is

$$Q = 0.7854 \frac{d^2}{144}; v = 0.6245 \sqrt{\frac{hd^5}{L}}; d = \sqrt{\frac{Q^2 L}{0.39 h}}; h = \frac{Q^2 L}{.39 d^5}.$$

The horse-power required to drive air through a pipe is the volume Q in cubic feet per second multiplied by the pressure in pounds per square foot and divided by 550. Pressure in pounds per square foot = P = inches of water column $\times 5.196$, whence horse-power =

$$\text{H.P.} = \frac{QP}{550} = \frac{Qh}{105.9} = \frac{Q^3 L}{41.3 d^5}.$$

Volume of Air Transmitted in Cubic Feet per Minute in Pipes of Various Diameters.

$$\text{Formula } Q = \frac{0.7854}{144} d^2 v \times 60.$$

| Velo'y of Flow, Ft. per Sec. | Actual Diameter of Pipe in Inches. | | | | | | | | | | | |
|---------------------------------|------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
| 1 | 0.327 | 1.31 | 2.95 | 5.24 | 8.18 | 11.78 | 20.94 | 32.73 | 47.12 | 83.77 | 130.9 | 188.5 |
| 2 | 0.655 | 2.62 | 5.89 | 10.47 | 16.36 | 23.56 | 41.89 | 65.45 | 94.25 | 167.5 | 261.8 | 377.0 |
| 3 | 0.982 | 3.93 | 8.84 | 15.7 | 24.5 | 35.3 | 62.8 | 98.2 | 141.4 | 251.3 | 392.7 | 565.5 |
| 4 | 1.31 | 5.24 | 11.78 | 20.9 | 32.7 | 47.1 | 83.8 | 131 | 188 | 335 | 523 | 754 |
| 5 | 1.64 | 6.54 | 14.7 | 26.2 | 41.0 | 59.0 | 104 | 163 | 235 | 419 | 654 | 942 |
| 6 | 1.96 | 7.85 | 17.7 | 31.4 | 49.1 | 70.7 | 125 | 196 | 283 | 502 | 785 | 1131 |
| 7 | 2.29 | 9.16 | 20.6 | 36.6 | 57.2 | 82.4 | 146 | 229 | 330 | 586 | 916 | 1319 |
| 8 | 2.62 | 10.5 | 23.5 | 41.9 | 65.4 | 94 | 167 | 262 | 377 | 670 | 1047 | 1508 |
| 9 | 2.95 | 11.78 | 26.5 | 47 | 73 | 106 | 188 | 294 | 424 | 754 | 1178 | 1696 |
| 10 | 3.27 | 13.1 | 29.4 | 52 | 82 | 118 | 209 | 327 | 471 | 833 | 1307 | 1885 |
| 12 | 3.93 | 15.7 | 35.3 | 63 | 98 | 141 | 251 | 393 | 565 | 1005 | 1571 | 2262 |
| 15 | 4.91 | 19.6 | 44.2 | 78 | 122 | 177 | 314 | 491 | 707 | 1256 | 1963 | 2827 |
| 18 | 5.89 | 23.5 | 53 | 94 | 147 | 212 | 377 | 589 | 848 | 1508 | 2356 | 3393 |
| 20 | 6.54 | 26.2 | 59 | 105 | 164 | 235 | 419 | 654 | 942 | 1675 | 2618 | 3770 |
| 24 | 7.85 | 31.4 | 71 | 125 | 196 | 283 | 502 | 785 | 1131 | 2010 | 3141 | 4524 |
| 25 | 8.18 | 32.7 | 73 | 131 | 204 | 294 | 523 | 818 | 1178 | 2094 | 3272 | 4712 |
| 28 | 9.16 | 36.6 | 82 | 146 | 229 | 330 | 586 | 916 | 1319 | 2346 | 3665 | 5278 |
| 30 | 9.8 | 39.3 | 88 | 157 | 245 | 353 | 628 | 982 | 1414 | 2513 | 3927 | 5655 |

In Hawksley's formula and its derivatives the numerical coefficients are constant. It is scarcely possible, however, that they can be accurate except within a limited range of conditions. In the case of water it is found that the coefficient of friction, on which the loss of head depends, varies with the length and diameter of the pipe, and with the velocity, as well as with the condition of the interior surface. In the case of air and other gases we have, in addition, the decrease in density and consequent increase in volume and in velocity due to the progressive loss of head from one end of the pipe to the other.

Clark states that according to the experiments of D'Aubuisson and those of a Sardinian commission on the resistance of air through long conduits or pipes, the diminution of pressure is very nearly directly as the length, and as the square of the velocity and inversely as the diameter. The resistance is not varied by the density.

If these statements are correct, then the formulæ $h = \frac{Lv^2}{cd}$ and $h = \frac{Q^2L}{c'd^5}$ and their derivatives are correct in form, and they may be used when the numerical coefficients c and c' are obtained by experiment.

If we take the forms of the above formulæ as correct, and let C be a variable coefficient, depending upon the length, diameter, and condition of surface of the pipe, and possibly also upon the velocity, the temperature and the density, to be determined by future experiments, then for h = head in inches of water, d = diameter in inches, L = length in feet, v = velocity in feet per second, and Q = quantity in cubic feet per second:

$$\begin{aligned} v &= C \sqrt{\frac{hd}{L}}; & d &= \frac{Lv^2}{C^2h}; & h &= \frac{Lv^2}{C^2d}; \\ Q &= 0.005454 C \sqrt{\frac{hd^5}{L}}; & d &= \sqrt[5]{\frac{33683 Q^2L}{C^2h}}; & h &= \frac{33683 Q^2L}{C^2d^5}. \end{aligned}$$

For difference or loss of pressure p in pounds per square inch,

$$\begin{aligned} h &= 27.71 p; & \sqrt{h} &= 5.264 \sqrt{p}; \\ v &= 5.264 C \sqrt{\frac{pd}{L}}; & d &= \frac{Lv^2}{27.71 C^2p}; & p &= \frac{Lv^2}{27.71 C^2d}; \\ Q &= 0.02871 C \sqrt{\frac{pd^5}{L}}; & d &= \sqrt[5]{\frac{1213 Q^2L}{C^2p}}; & p &= \frac{1213 Q^2L}{C^2d^5}. \end{aligned}$$

(For other formulæ for flow of air, see Mine Ventilation.)

Loss of Pressure in Ounces per Square Inch. — B. F. Sturtevant Company uses the following formulæ:

$$p_1 = \frac{Lv^2}{25000 d}; \quad v = \sqrt{\frac{25000 dp_1}{L}}; \quad d = \frac{Lv^2}{25000 p_1};$$

in which p_1 = loss of pressure in ounces per square inch, v = velocity of air in feet per second, and L = length of pipe in feet. If p is taken in pounds per square inch, these formulæ reduce to

$$p = 0.0000025 \frac{Lv^2}{d}; \quad v = 632.5 \sqrt{\frac{dp_1}{L}}; \quad d = \frac{0.0000025 Lv^2}{p}.$$

These are deduced from the common formula (Weisbach's), $p = f \frac{l v^2}{d 2g}$, in which $f = 0.0001608$. They correspond to the formulæ given above when C is taken at 120.15, Hawksley's formula for the same notation giving 114.5. Using the notation given in the formulæ for compressed air, where Q is taken in cu. ft. per minute, Sturtevant's formula gives a value of $C = 57.1$, Hawksley's 54.4. The figure 60 is commonly used, assuming a density of air of 0.761 lb. per cu. ft.

The following table is condensed from one given in the catalogue of B. F. Sturtevant Company.

LOSS OF PRESSURE IN PIPES 100 FT. LONG,* IN OUNCES PER SQ. IN.

| Velocity ft. per min. | Diameter of Pipe in Inches. | | | | | | | | | | | |
|--------------------------|-----------------------------|-------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| | 14 | 16 | 18 | 20 | 22 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 600 | 0.400 | 0.200 | 0.133 | 0.100 | 0.080 | 0.067 | 0.057 | 0.050 | 0.044 | 0.040 | 0.036 | 0.033 |
| 1200 | 1.600 | 0.800 | 0.533 | 0.400 | 0.320 | 0.267 | 0.229 | 0.200 | 0.178 | 0.160 | 0.145 | 0.133 |
| 1800 | 3.600 | 1.800 | 1.200 | 0.900 | 0.720 | 0.600 | 0.514 | 0.450 | 0.400 | 0.360 | 0.327 | 0.300 |
| 2400 | 6.400 | 3.200 | 2.133 | 1.600 | 1.280 | 1.067 | 0.914 | 0.800 | 0.711 | 0.640 | 0.582 | 0.533 |
| 3000 | 10.0 | 5.0 | 3.333 | 2.5 | 2.0 | 1.667 | 1.429 | 1.250 | 1.111 | 1.000 | 0.909 | 0.833 |
| 3600 | 14.4 | 7.2 | 4.8 | 3.6 | 2.88 | 2.4 | 2.057 | 1.8 | 1.6 | 1.44 | 1.309 | 1.200 |
| 4200 | | 9.8 | 6.553 | 4.9 | 3.92 | 3.267 | 2.8 | 2.45 | 2.178 | 1.96 | 1.782 | 1.633 |
| 4800 | | 12.8 | 8.533 | 6.4 | 5.12 | 4.267 | 3.657 | 3.2 | 2.844 | 2.56 | 2.327 | 2.133 |
| 6000 | | 20. | 13.333 | 10.0 | 8.0 | 6.667 | 5.714 | 5.0 | 4.444 | 4.0 | 3.636 | 3.333 |
| 600 | .029 | .026 | .022 | .020 | .018 | .017 | .014 | .012 | .011 | .010 | .009 | .008 |
| 1200 | .114 | .100 | .089 | .080 | .073 | .067 | .057 | .050 | .044 | .040 | .036 | .033 |
| 1800 | .257 | .225 | .200 | .180 | .164 | .156 | .129 | .112 | .100 | .090 | .082 | .075 |
| 2400 | .457 | .400 | .356 | .320 | .291 | .267 | .239 | .200 | .178 | .160 | .145 | .133 |
| 3600 | 1.029 | .900 | .800 | .720 | .655 | .600 | .514 | .450 | .400 | .360 | .327 | .300 |
| 4200 | 1.400 | 1.225 | 1.089 | .980 | .891 | .817 | .700 | .612 | .544 | .490 | .445 | .408 |
| 4800 | 1.829 | 1.600 | 1.422 | 1.280 | 1.164 | 1.067 | .914 | .800 | .711 | .640 | .582 | .533 |
| 6000 | 2.857 | 2.500 | 2.222 | 2.000 | 1.818 | 1.667 | 1.429 | 1.250 | 1.111 | 1.000 | .909 | .833 |

* For any other length the loss is proportional to the length.

Effect of Bends in Pipes. (Norwalk Iron Works Co.)

Radius of elbow, in diameter
of pipe =

Equivalent lengths of straight
pipe, diams.

| | | | | | | | |
|------|------|------|-------|-------|-------|-------|-------|
| 5 | 3 | 2 | 1 1/2 | 1 1/4 | 1 | 3/4 | 1/2 |
| 7.85 | 8.24 | 9.03 | 10.36 | 12.72 | 17.51 | 35.09 | 121.2 |

Friction of Air in Passing through Valves and Elbows. W. L. Saunders, *Compressed Air*, Dec., 1902. — The following figures give the length in feet of straight pipe which will cause a reduction in pressure equal to that caused by globe valves, elbows, and tees in different diameters of pipe.

| | | | | | | | | | | | | |
|---------------------|---|-------|---|-------|----|-------|----|----|----|----|----|----|
| Diam. of pipe, in.. | 1 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 5 | 6 | 7 | 8 | 10 |
| Globe Valves..... | 2 | 4 | 7 | 10 | 13 | 16 | 20 | 28 | 36 | 44 | 53 | 70 |
| Elbows and Tees .. | 2 | 3 | 5 | 7 | 9 | 11 | 13 | 19 | 24 | 30 | 35 | 47 |

Compressed-air Transmission. (Frank Richards, *Am. Mach.*, March 8, 1894.) — The volume of free air transmitted may be assumed to be directly as the number of atmospheres to which the air is compressed. Thus, if the air transmitted be at 75 pounds gauge-pressure, or six atmospheres, the volume of free air will be six times the amount given in the table (page 591). It is generally considered that for economical transmission the velocity in main pipes should not exceed 20 feet per second. In the smaller distributing pipes the velocity should be decidedly less than this.

The loss of power in the transmission of compressed air in general is not a serious one, or at all to be compared with the losses of power in the operation of compression and in the re-expansion or final application of the air.

The formulas for loss by friction are all unsatisfactory. The statements of observed facts in this line are in a more or less chaotic state, and self-evidently unreliable.

A statement of the friction of air flowing through a pipe involves at least all the following factors: Unit of time, volume of air, pressure of air, diam-

To apply the formula given above to air of different pressures it may be given other forms, as follows:

Let Q = the volume in cubic feet per minute of the compressed air; Q_1 = the volume before compression, or "free air," both being taken at mean atmospheric temperature of 62° F.; w_1 = weight per cubic foot of Q_1 = 0.0761 lb.; r = atmospheres, or ratio of absolute pressures, = (gauge-pressure + 14.7) ÷ 14.7; w = weight per cu. ft. of Q ; p = difference of pressure, in lbs. per sq. in., causing the flow; d = diam. of pipe in in.; L = length of pipe in ft.; c = experimental constant. Then

$$Q = c \sqrt{\frac{pd^5}{wL}}; \quad Q_1 = rQ; \quad w = rw_1 = 0.0761 r;$$

$$Q = 3.625 c \sqrt{\frac{pd^5}{rL}}; \quad Q_1 = 3.625 c \sqrt{\frac{pd^5 r}{L}};$$

$$d = \sqrt[5]{0.0761 \frac{LQ^2 r}{c^2 p}} = 0.597 \sqrt[5]{\frac{LQ^2 r}{c^2 p}} = \sqrt[5]{0.0761 \frac{LQ_1^2}{c^2 p r}} = 0.597 \sqrt[5]{\frac{LQ_1^2}{c^2 p r}};$$

$$p = 0.0761 \frac{LQ^2 r}{c^2 d^5} = 0.0761 \frac{LQ_1^2}{c^2 d^5 r}.$$

The value of c according to the Mt. Ceniz experiments is about 58 for pipes 4, 8, and 14 in. diameter, 3281 ft. long. In the St. Gothard experiments it ranged from 62.8 to 73.2 (see table below) for pipes 5.91 and 7.87 in. diameter, 1713 and 15,092 ft. long. Values derived from Darcy's formula for flow of water in pipes, ranging from 45.3 for 1 in. diameter to 63.2 for 24 in., are given under "Flow of Steam," p. 845. For approximate calculations the value 60 may be used for all pipes of 4 in. diameter and upwards. Using $c = 60$, the above formulæ become

$$Q = 217.5 \sqrt{\frac{pd^5}{rL}}; \quad Q_1 = 217.5 \sqrt{\frac{pd^5 r}{L}};$$

$$d = 0.1161 \sqrt[5]{\frac{LQ^2 r}{p}} = 0.1161 \sqrt[5]{\frac{LQ_1^2}{p r}};$$

$$p = 0.00002114 \frac{LQ^2 r}{a^5} = 0.00002114 \frac{LQ_1^2}{d^5 r}.$$

Loss of Pressure in Compressed Air Pipe-main, at St. Gothard Tunnel. (E. Stockalper.)

| Experiment. | Air Main Diameter. | | Volume per second of free air, or equivalent volume at atmospheric pressure and 32° F. | Volume per second of compressed air at mean density. | Mean density of compressed air. (Water = 1.) | Weight of air flowing per second. | Mean velocity in feet per second. | Observed Pressures. | | | | Value of c in formula $Q = c \sqrt{\frac{pd^5}{wL}}$. |
|-------------|--------------------|--------|--|--|--|-----------------------------------|-----------------------------------|--------------------------------|--------------------------|-------------------|-------|--|
| | | | | | | | | Pressure at beginning of pipe. | Pressure at end of pipe. | Loss of Pressure. | | |
| | | | | | | | | | | lbs. per sq. in. | % | |
| No. 1 | 7.87 | 33.056 | 6.534 | .00650 | 2.669 | 19.32 | 5.60 | 5.24 | 5.292 | 6.4 | 73.2 | |
| | 5.91 | | 7.063 | .00603 | 2.669 | 37.14 | 5.24 | 5.00 | 3.528 | 4.6 | 63.9 | |
| 2 | 7.87 | 22.002 | 5.509 | .00514 | 1.776 | 16.30 | 4.35 | 4.13 | 3.234 | 5.1 | 70.7 | |
| | 5.91 | | 5.863 | .00482 | 1.776 | | 4.13 | | | | | |
| 3 | 7.87 | 18.364 | 5.252 | .00449 | 1.483 | 15.58 | 3.84 | 3.65 | 2.793 | 5.0 | 67.6 | |
| | 5.91 | | 5.580 | .00423 | 1.483 | 29.34 | 3.65 | 3.54 | 1.617 | 3.0 | 62.8 | |

The length of the pipe 7.87 in. in diameter was 15,092 ft., and of the smaller pipe 1712.6 ft. The mean temperature of the air in the large pipe was 70° F. and in the small pipe 80° F.

Flow of Air in Long Pipes with Large Differences of Pressure.—The formulæ given above are applicable strictly only to cases in which the

difference of pressure at the two ends of the pipe is small, and the density of the air, therefore, nearly constant. For long pipes with considerable difference of pressure the density decreases and the velocity increases during the flow from one end of the pipe to the other. Church (*Mechs. of Eng'g*, p. 790) develops a formula for flow in long pipes under the assumptions of uniform decrease of density and of constant temperature, the loss of heat by adiabatic expansion being in great part made up by the heat generated by friction. Using the same notation as above Church's

formula is $\frac{1}{2}[p_1^2 - p_2^2] = \frac{4}{2} \frac{fl}{2gd} \frac{W^2}{A^2} \frac{p_1}{w_1}$, f being the coefficient of friction,

A the area of the pipe in square inches, and w the density of air at the entrance. The value of f is given at 0.004 to 0.005.

J. E. Johnson, Jr. (*Am. Mach.*, July 27, 1899) gives Church's formula in a simpler form as follows: $p_1^2 - p_2^2 = KQ^2L \div d^5$, in which p_1 and p_2 are the initial and final pressures in lbs. per sq. in., Q the volume of free air (that is the volume reduced to atmospheric pressure) in cubic feet per minute, d the diameter of the pipe in inches, L the length in feet, and K a numerical coefficient, which from the Mt. Ceniz and St. Gothard experiments has a value of about 0.0006. E. A. Rix, in a paper on the Compression and Transmission of Illuminating Gas, read before the Pacific Coast Gas Ass'n, 1905, says he uses Johnson's formula, with a coefficient of 0.0005, which he considers more nearly correct than 0.0006. For gas the velocity varies inversely as the square root of the density, and for gas of a density G , relative to air as 1, Rix gives the formula $p_1^2 - p_2^2 = 0.0005 \sqrt{G} \times Q^2L/d^5$.

Measurement of the Velocity of Air in Pipes by an Anemometer.

— Tests were made by B. Donkin, Jr. (*Inst. Civil Engrs.*, 1892), to compare the velocity of air in pipes from 8 in. to 24 in. diam., as shown by an anemometer $2\frac{3}{4}$ in. diam. with the true velocity as measured by the time of descent of a gas-holder holding 1622 cubic feet. A table of the results with discussion is given in *Eng'g News*, Dec. 22, 1892. In pipes from 8 in. to 20 in. diam. with air velocities of from 140 to 690 feet per minute the anemometer showed errors varying from 14.5% fast to 10% slow. With a 24-inch pipe and a velocity of 73 ft. per minute, the anemometer gave from 44 to 63 feet, or from 13.6 to 39.6% slow. The practical conclusion drawn from these experiments is that anemometers for the measurement of velocities of air in pipes of these diameters should be used with great caution. The percentage of error is not constant, and varies considerably with the diameter of the pipes and the speeds of air. The use of a baffle consisting of a perforated plate, which tended to equalize the velocity in the center and at the sides in some cases diminished the error.

The impossibility of measuring the true quantity of air by an anemometer held stationary in one position is shown by the following figures, given by Wm. Daniel (*Proc. Inst. M. E.*, 1875), of the velocities of air found at different points in the cross-sections of two different airways in a mine.

DIFFERENCES OF ANEMOMETER READINGS IN AIRWAYS.
8 ft. square.

| | | | |
|------|------|------|------|
| 1712 | 1795 | 1859 | 1329 |
| 1622 | 1685 | 1782 | 1091 |
| 1477 | 1344 | 1524 | 1049 |
| 1262 | 1356 | 1293 | 1333 |

Average 1469.

5 × 8 ft.

| | | |
|------|------|------|
| 1170 | 1209 | 1288 |
| 948 | 1104 | 1177 |
| 1134 | 1049 | 1106 |

Average 1132.

Equalization of Pipes.— It is frequently desired to know what number of pipes of a given size are equal in carrying capacity to one pipe of a larger size. At the same velocity of flow the volume delivered by two pipes of different sizes is proportional to the squares of their diameters;

thus, one 4-inch pipe will deliver the same volume as four 2-inch pipes. With the same head, however, the velocity is less in the smaller pipe, and the volume delivered varies about as the square root of the fifth power (i.e., as the 2.5 power). The following table has been calculated on this basis. The figures opposite the intersection of any two sizes is the number of the smaller-sized pipes required to equal one of the larger. Thus one 4-inch pipe is equal to 5.7 two-inch pipes.

| Diam. In. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 | 24 |
|--------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 2 | 5.7 | 1 | | | | | | | | | | | | | | |
| 3 | 15.6 | 2.8 | 1 | | | | | | | | | | | | | |
| 4 | 32.0 | 5.7 | 2.1 | 1 | | | | | | | | | | | | |
| 5 | 55.9 | 9.9 | 3.6 | 1.7 | 1 | | | | | | | | | | | |
| 6 | 88.2 | 15.6 | 5.7 | 2.8 | 1.6 | 1 | | | | | | | | | | |
| 7 | 130 | 22.9 | 8.3 | 4.1 | 2.3 | 1.5 | 1 | | | | | | | | | |
| 8 | 181 | 32.0 | 11.7 | 5.7 | 3.2 | 2.1 | 1.4 | 1 | | | | | | | | |
| 9 | 243 | 43.0 | 15.6 | 7.6 | 4.3 | 2.8 | 1.9 | 1.3 | 1 | | | | | | | |
| 10 | 316 | 55.9 | 20.3 | 9.9 | 5.7 | 3.6 | 2.4 | 1.7 | 1.3 | 1 | | | | | | |
| 11 | 401 | 70.9 | 25.7 | 12.5 | 7.2 | 4.6 | 3.1 | 2.2 | 1.7 | 1.3 | | | | | | |
| 12 | 499 | 88.2 | 32.0 | 15.6 | 8.9 | 5.7 | 3.8 | 2.8 | 2.1 | 1.6 | 1 | | | | | |
| 13 | 609 | 108 | 39.1 | 19.0 | 10.9 | 7.1 | 4.7 | 3.4 | 2.5 | 1.9 | 1.2 | | | | | |
| 14 | 733 | 130 | 47.0 | 22.9 | 13.1 | 8.3 | 5.7 | 4.1 | 3.0 | 2.3 | 1.5 | 1 | | | | |
| 15 | 871 | 154 | 55.9 | 27.2 | 15.6 | 9.9 | 6.7 | 4.8 | 3.6 | 2.8 | 1.7 | 1.2 | | | | |
| 16 | ... | 181 | 65.7 | 32.0 | 18.3 | 11.7 | 7.9 | 5.7 | 4.2 | 3.2 | 2.1 | 1.4 | 1 | | | |
| 17 | ... | 211 | 76.4 | 37.2 | 21.3 | 13.5 | 9.2 | 6.6 | 4.9 | 3.8 | 2.4 | 1.6 | 1.2 | | | |
| 18 | ... | 243 | 88.2 | 43.0 | 24.6 | 15.6 | 10.6 | 7.6 | 5.7 | 4.3 | 2.8 | 1.9 | 1.3 | 1 | | |
| 19 | ... | 278 | 101 | 49.1 | 28.1 | 17.8 | 12.1 | 8.7 | 6.5 | 5.0 | 3.2 | 2.1 | 1.5 | 1.1 | | |
| 20 | ... | 316 | 115 | 55.9 | 32.0 | 20.3 | 13.8 | 9.9 | 7.4 | 5.7 | 3.6 | 2.4 | 1.7 | 1.3 | 1 | |
| 22 | ... | 401 | 146 | 70.9 | 40.6 | 25.7 | 17.5 | 12.5 | 9.3 | 7.2 | 4.6 | 3.1 | 2.2 | 1.7 | 1.3 | |
| 24 | ... | 499 | 181 | 88.2 | 50.5 | 32.0 | 21.8 | 15.6 | 11.6 | 8.9 | 5.7 | 3.8 | 2.8 | 2.1 | 1.6 | 1 |
| 26 | ... | 609 | 221 | 108 | 61.7 | 39.1 | 26.6 | 19.0 | 14.2 | 10.9 | 7.1 | 4.7 | 3.4 | 2.5 | 1.9 | 1.2 |
| 28 | ... | 733 | 266 | 130 | 74.2 | 47.0 | 32.0 | 22.9 | 17.1 | 13.1 | 8.3 | 5.7 | 4.1 | 3.0 | 2.3 | 1.5 |
| 30 | ... | 871 | 316 | 154 | 88.2 | 55.9 | 38.0 | 27.2 | 20.3 | 15.6 | 9.9 | 6.7 | 4.8 | 3.6 | 2.8 | 1.7 |
| 36 | ... | ... | 499 | 243 | 130 | 88.2 | 60.0 | 43.0 | 32.0 | 24.6 | 15.6 | 10.6 | 7.6 | 5.7 | 4.3 | 2.8 |
| 42 | ... | ... | 733 | 357 | 205 | 130 | 88.2 | 63.2 | 47.0 | 36.2 | 19.0 | 15.6 | 11.2 | 8.3 | 6.4 | 4.1 |
| 48 | ... | ... | ... | 499 | 286 | 181 | 123 | 88.2 | 62.7 | 50.5 | 32.0 | 21.8 | 15.6 | 11.6 | 8.9 | 5.7 |
| 54 | ... | ... | ... | ... | 670 | 383 | 243 | 165 | 118 | 88.2 | 67.8 | 43.0 | 29.2 | 20.9 | 15.6 | 12.0 |
| 60 | ... | ... | ... | ... | 871 | 499 | 316 | 215 | 154 | 115 | 88.2 | 55.9 | 38.0 | 27.2 | 20.3 | 15.6 |

WIND.

Force of the Wind. — Smeaton in 1759 published a table of the velocity and pressure of wind, as follows:

VELOCITY AND FORCE OF WIND, IN POUNDS PER SQUARE INCH.

| Miles per Hour. | Feet per Second. | Force per Sq. Ft., Pounds. | Common Appellation of the Force of Wind. | Miles per Hour. | Feet per Second. | Force per Sq. Ft., Pounds. | Common Appellation of the Force of Wind. |
|-----------------|------------------|----------------------------|--|-----------------|------------------|----------------------------|--|
| 1 | 1.47 | 0.005 | Hardly perceptible. | 18 | 26.4 | 1.55 | Very brisk. |
| 2 | 2.93 | 0.020 | Just perceptible. | 20 | 29.34 | 1.968 | |
| 3 | 4.4 | 0.044 | | 25 | 36.67 | 3.075 | |
| 4 | 5.87 | 0.079 | Gentle, pleasant wind. | 30 | 44.00 | 4.429 | High wind. |
| 5 | 7.33 | 0.123 | | 35 | 51.34 | 6.027 | |
| 6 | 8.8 | 0.177 | | 40 | 58.68 | 7.873 | Very high storm. |
| 7 | 10.25 | 0.241 | Pleasant, brisk gale | 45 | 66.01 | 9.963 | |
| 8 | 11.75 | 0.315 | | 50 | 73.35 | 12.30 | |
| 9 | 13.2 | 0.400 | | 55 | 80.7 | 14.9 | Great storm. |
| 10 | 14.67 | 0.492 | | 60 | 88.00 | 17.71 | |
| 12 | 17.6 | 0.708 | | 65 | 95.3 | 20.85 | |
| 14 | 20.5 | 0.964 | | 70 | 102.5 | 24.1 | Hurricane. |
| 15 | 22.00 | 1.107 | | 75 | 110.00 | 27.7 | |
| 16 | 23.45 | 1.25 | | 80 | 117.36 | 31.49 | |
| | | | | 100 | 146.67 | 49.2 | Immense hurricane. |

The pressures per square foot in the above table correspond to the formula $P = 0.005 V^2$, in which V is the velocity in miles per hour. *Eng'g News*, Feb. 9, 1893, says that the formula was never well established, and has floated chiefly on Smeaton's name and for lack of a better. It was put forward only for surfaces for use in windmill practice. The trend of modern evidence is that it is approximately correct only for such surfaces, and that for large, solid bodies it often gives greatly too large results. Observations by others are thus compared with Smeaton's formula:

| | |
|-------------------------------------|------------------|
| Old Smeaton formula | $P = 0.005 V^2$ |
| As determined by Prof. Martin | $P = 0.004 V^2$ |
| “ “ “ Whipple and Dines | $P = 0.0029 V^2$ |

At 60 miles per hour these formulas give for the pressure per square foot, 18, 14.4, and 10.44 lbs., respectively, the pressure varying by all of them as the square of the velocity. Lieut. Crosby's experiments (*Eng'g*, June 13, 1890), claiming to prove that $P = fV$ instead of $P = fV^2$, are discredited.

Experiments by M. Eiffel on plates let fall from the Eiffel tower in Paris gave coefficients of V^2 ranging from 0.0027 for small plates to 0.0032 for plates 10 sq. ft. area. For plates larger than 10 sq. ft. the coefficient remained constant at 0.0032. — *Eng'g*, May 8, 1908.

A. R. Wolff ("The Windmill as a Prime Mover," p. 9) gives as the theoretical pressure per sq. ft. of surface, $P = dQv/g$, in which d = density of air in pounds per cu. ft. = $\frac{0.018743 (p + P)}{t}$; p being the barometric pres-

sure per square foot at any level, and temperature of 32° F., t any absolute temperature, Q = volume of air carried along per square foot in one second, v = velocity of the wind in feet per second, $g = 32.16$. Since $Q = v$ cu. ft. per sec., $P = dv^2/g$. Multiplying this by a coefficient 0.93 found by experiment, and substituting the above value of d , he obtains

$$P = \frac{0.017431 \times p}{t \times 32.16 - 0.018743} \times v^2, \text{ and when } p = 2116.5 \text{ lb. per sq. ft., or average}$$

atmospheric pressure at the sea-level, $P = \frac{36.8929}{t \times 32.16 - 0.018743} \times v^2$, an ex-

pression in which the pressure is shown to vary with the temperature; and he gives a table showing the relation between velocity and pressure for temperatures from 0° to 100° F., and velocities from 1 to 80 miles per hour. For a temperature of 45° F. the pressures agree with those in Smeaton's table, for 0° F. they are about 10 per cent greater, and for 100°, 10 per cent less.

Prof. H. Allen Hazen, *Eng'g News*, July 5, 1890, says that experiments with whirling arms, by exposing plates to direct wind, and on locomotives with velocities running up to 40 miles per hour, have invariably shown the resistance to vary with V^2 . The coefficient of V^2 has been found in some experiments with very short whirling arms and low velocities to vary with the perimeter of the plate, but this entirely disappears with longer arms or straight line motion, and the only question now to be determined is the value of the coefficient. Perhaps some of the best experiments for determining this value were tried in France in 1886 by carrying flat boards on trains. The resulting formula in this case was, for 44.5 miles per hour, $p = 0.00535 SV^2$.

Prof. Kernot, of Melbourne (*Eng. Rec.*, Feb. 20, 1894), states that experiments at the Forth Bridge showed that the average pressure on surfaces as large as railway carriages, houses, or bridges never exceeded two-thirds of that upon small surfaces of one or two square feet, and also that an inertia effect, which is frequently overlooked, may cause some forms of anemometer to give false results enormously exceeding the correct indication. Experiments made by Prof. Kernot at speeds varying from 2 to 15 miles per hour agreed with the earlier authorities. The pressure upon one side of a cube, or of a block proportioned like an ordinary carriage, was found to be 0.9 of that upon a thin plate of the same area. The same result was obtained for a square tower. A square pyramid, whose height was three times its base, experienced 0.8 of the pressure upon a thin plate equal to one of its sides, but if an angle was turned to

the wind the pressure was increased by fully 20%. A bridge consisting of two plate-girders connected by a deck at the top was found to experience 0.9 of the pressure on a thin plate equal in size to one girder, when the distance between the girders was equal to their depth, and this was increased by one-fifth when the distance between the girders was double the depth. A lattice-work in which the area of the openings was 55% of the whole area experienced a pressure of 80% of that upon a plate of the same area. The pressure upon cylinders and cones was proved to be equal to half that upon the diametral planes, and that upon an octagonal prism to be 20% greater than upon the circumscribing cylinder. A sphere was subject to a pressure of 0.36 of that upon a thin circular plate of equal diameter. A hemispherical cup gave the same result as the sphere; when its concavity was turned to the wind the pressure was 1.15 of that on a flat plate of equal diameter. When a plane surface parallel to the direction of the wind was brought nearly into contact with a cylinder or sphere, the pressure on the latter bodies was augmented by about 20%, owing to the lateral escape of the air being checked. Thus it is possible for the security of a tower or chimney to be impaired by the erection of a building nearly touching it on one side.

Pressures of Wind Registered in Storms. — Mr. Frizell has examined the published records of Greenwich Observatory from 1849 to 1869; and reports that the highest pressure of wind he finds recorded is 41 lb. per sq. ft., and there are numerous instances in which it was between 30 and 40 lb. per sq. ft. Prof. Henry says that on Mount Washington, N. H., a velocity of 150 miles per hour has been observed, and at New York City 60 miles an hour, and that the highest winds observed in 1870 were of 72 and 63 miles per hour, respectively. Lieut. Dunwoody, U. S. A., says, in substance, that the New England coast is exposed to storms which produce a pressure of 50 lb. per sq. ft. — *Eng. News*, Aug. 20, 1880.

WINDMILLS.

Power and Efficiency of Windmills. — Rankine, S. E., p. 215, gives the following: Let Q = volume of air which acts on the sail, or part of a sail, in cubic feet per second, v = velocity of the wind in feet per second, s = sectional area of the cylinder, or annular cylinder of wind, through which the sail, or part of the sail, sweeps in one revolution, c = a coefficient to be found by experience; then $Q = cvs$. Rankine, from experimental data given by Smeaton, and taking c to include an allowance for friction, gives for a wheel with four sails, proportioned in the best manner, $c = 0.75$. Let A = weather angle of the sail at any distance from the axis, i.e., the angle the portion of the sail considered makes with its plane of revolution. This angle gradually diminishes from the inner end of the sail to the tip; u = the velocity of the same portion of the sail, and E = the efficiency. The efficiency is the ratio of the useful work performed to the whole energy of the stream of wind acting on the surface s of the wheel, which energy is $Dsv^3 \div 2g$, D being the weight of a cubic foot of air. Rankine's formula for efficiency is

$$E = \frac{Ru}{Dsv^3/2g} = c \left\{ \frac{u}{v} \sin 2A - \frac{u^2}{v^2} (1 - \cos 2A + f) - f \right\},$$

in which $c = 0.75$ and f is a coefficient of friction found from Smeaton's data = 0.016. Rankine gives the following from Smeaton's data:

| | | | |
|---|-------------|------------|------------|
| A = weather-angle..... | $= 7^\circ$ | 13° | 19° |
| $V \div v$ = ratio of speed of greatest efficiency, for a given weather-angle, to that of the wind..... | $= 2.63$ | 1.86 | 1.41 |
| E = efficiency..... | $= 0.24$ | 0.29 | 0.31 |

Rankine gives the following as the best values for the angle of weather at different distances from the axis:

| | | | | | | |
|------------------------------------|------------|------------|------------|------------|-----------------------|-----------|
| Distance in sixths of total radius | 1 | 2 | 3 | 4 | 5 | 6 |
| Weather angle..... | 18° | 19° | 18° | 16° | $12\frac{1}{2}^\circ$ | 7° |

But Wolff (p. 125) shows that Smeaton did not term these the best angles, but simply says they "answer as well as any," possibly any that

were in existence in his time. Wolff says that they "cannot in the nature of things be the most desirable angles." Mathematical considerations, he says, conclusively show that the angle of impulse depends on the relative velocity of each point of the sail and the wind, the angle growing larger as the ratio becomes greater. Smeaton's angles do not fulfil this condition. Wolff develops a theoretical formula for the best angle of weather, and from it calculates a table of the best angles for different relative velocities of the blades and the wind, which differ widely from those given by Rankine.

A. R. Wolff, in an article in the *American Engineer*, gives the following (see also his treatise on Windmills):

Let c = velocity of wind in feet per second;

n = number of revolutions of the windmill per minute;

b_0, b_1, b_2, b_x be the breadth of the sail or blade at distances l_0, l_1, l_2, l_3 , and l , respectively, from the axis of the shaft;

l_0 = distance from axis of shaft to beginning of sail or blade proper.

l = distance from axis of shaft to extremity of sail proper;

v_0, v_1, v_2, v_3, v_x = the velocity of the sail in feet per second at distances l_0, l_1, l_2, l_3, l , respectively, from the axis of the shaft;

a_0, a_1, a_2, a_3, a_x = the angles of impulse for maximum effect at distances l_0, l_1, l_2, l_3, l , respectively, from the axis of the shaft;

a = the angle of impulse when the sails or blocks are plane surfaces so that there is but one angle to be considered;

N = number of sails or blades of windmill;

$K = 0.93$;

d = density of wind (weight of a cubic foot of air at average temperature and barometric pressure where mill is erected);

W = weight of wind-wheel in pounds;

f = coefficient of friction of shaft and bearings;

D = diameter of bearing of windmill in feet.

The effective horse-power of a windmill with plane sails will equal

$$\frac{(l - l_0) K c^2 d N}{550 g} \times \text{mean of } \left\{ v_0 \left(\sin a - \frac{v_0}{c} \cos a \right) b_0 \cos a \right. \\ \left. v_x \left(\sin a - \frac{v_x}{c} \cos a \right) b_x \cos a \right\} - \frac{f W \times 0.05236 n D}{550}.$$

The effective horse-power of a windmill of shape of sail for maximum effect equals

$$\frac{N (l - l_0) K d c^3}{2200 g} \times \text{mean of } \left(\frac{2 \sin^2 a_0 - 1}{\sin^2 a_0} b_0, \frac{2 \sin^2 a_1 - 1}{\sin^2 a_1} b_1 \dots \right. \\ \left. \dots \frac{2 \sin^2 a_x - 1}{\sin^2 a_x} b_x \right) - \frac{f W \times 0.05236 n D}{550}.$$

The mean value of quantities in brackets is to be found according to Simpson's rule. Dividing l into 7 parts, finding the angles and breadths corresponding to these divisions by substituting them in quantities within brackets will be found satisfactory. Comparison of these formulæ with the only fairly reliable experiments in windmills (Coulomb's) showed a close agreement of results.

Approximate formulæ of simpler form for windmills of present construction can be based upon the above, substituting actual average values for a, c, d , and e , but since improvement in the present angles is possible, it is better to give the formulæ in their general and accurate form.

Wolff gives the following table, based on the practice of an American manufacturer. Since its preparation, he says, over 1500 windmills have been sold on its guaranty (1885), and in all cases the results obtained did not vary sufficiently from those presented to cause any complaint. The actual results obtained are in close agreement with those obtained by theoretical analysis of the impulse of wind upon windmill blades.

Capacity of the Windmill.

| Designation of Mill. | Velocity of Wind, in Miles per Hour. | Revolutions of Wheel per Minute. | Gallons of Water raised per Minute to an Elevation of | | | | | | Equivalent Actual Useful Horse-power developed. | Average No. of Hours per Day during which this Result will be obtained. |
|----------------------|--------------------------------------|----------------------------------|---|----------|----------|-----------|-----------|-----------|---|---|
| | | | 25 feet. | 50 feet. | 75 feet. | 100 feet. | 150 feet. | 200 feet. | | |
| wheel 8 1/2 ft. | 16 | 70 to 75 | 6.162 | 3.016 | | | | | 0.04 | 8 |
| 10 " | 16 | 60 to 65 | 19.179 | 9.563 | 6.638 | 4.750 | | | 0.12 | 8 |
| 12 " | 16 | 55 to 60 | 33.941 | 17.952 | 11.851 | 8.485 | 5.680 | | 0.21 | 8 |
| 14 " | 16 | 50 to 55 | 45.139 | 22.569 | 15.304 | 11.246 | 7.807 | 4.998 | 0.28 | 8 |
| 16 " | 16 | 45 to 50 | 64.600 | 31.654 | 19.542 | 16.150 | 9.771 | 8.075 | 0.41 | 8 |
| 18 " | 16 | 40 to 45 | 97.682 | 52.165 | 32.513 | 24.421 | 17.485 | 12.211 | 0.61 | 8 |
| 20 " | 16 | 35 to 40 | 124.950 | 63.750 | 40.800 | 31.248 | 19.284 | 15.938 | 0.78 | 8 |
| 25 " | 16 | 30 to 35 | 212.381 | 106.964 | 71.604 | 49.725 | 37.349 | 26.741 | 1.34 | 8 |

These windmills are made in regular sizes, as high as sixty feet diameter of wheel; but the experience with the larger class of mills is too limited to enable the presentation of precise data as to their performance.

If the wind can be relied upon in exceptional localities to average a higher velocity for eight hours a day than that stated in the above table, the performance or horse-power of the mill will be increased, and can be obtained by multiplying the figures in the table by the ratio of the cube of the higher average velocity of wind to the cube of the velocity above recorded.

He also gives the following table showing the economy of the windmill. All the items of expense, including both interest and repairs, are reduced to the hour by dividing the costs per annum by $365 \times 8 = 2920$; the interest, etc., for the twenty-four hours being charged to the eight hours of actual work. By multiplying the figures in the 5th column by 584, the first cost of the windmill, in dollars, is obtained.

Economy of the Windmill.

| Designation of Mill. | Gallons of Water raised 25 ft. per Hour. | Equivalent Actual Useful Horse-power developed. | Average Number of Hours per Day during which this Quantity will be raised. | Expense of Actual Useful Power Developed. in Cents, per Hour. | | | | | Expense per Horse-power, in Cents, per Hour. |
|----------------------|--|---|--|---|--|-----------------|----------|--------|--|
| | | | | For Interest on First Cost (First Cost, including Cost of Windmill, Pump, and Tower, 5% per Annum). | For Repairs and Depreciation (5% of First Cost per Annum). | For Attendance. | For Oil. | Total. | |
| wheel 8 1/2 ft. | 370 | 0.04 | 8 | 0.25 | 0.25 | 0.06 | 0.04 | 0.60 | 15.0 |
| 10 " | 1151 | 0.12 | 8 | 0.30 | 0.30 | 0.06 | 0.04 | 0.70 | 5.8 |
| 12 " | 2036 | 0.21 | 8 | 0.36 | 0.36 | 0.06 | 0.04 | 0.82 | 5.9 |
| 14 " | 2708 | 0.28 | 8 | 0.75 | 0.75 | 0.06 | 0.07 | 1.63 | 5.8 |
| 16 " | 3876 | 0.41 | 8 | 1.15 | 1.15 | 0.06 | 0.07 | 2.43 | 5.9 |
| 18 " | 5861 | 0.61 | 8 | 1.35 | 1.35 | 0.06 | 0.07 | 2.83 | 4.6 |
| 20 " | 7497 | 0.79 | 8 | 1.70 | 1.70 | 0.06 | 0.10 | 3.56 | 4.5 |
| 25 " | 12743 | 1.34 | 8 | 2.05 | 2.05 | 0.06 | 0.10 | 4.26 | 3.2 |

Prof. De Volson Wood (*Am. Mach.*, Oct. 29, 1896) quotes some results by Thos. O. Perry on three wheels, each 5 ft. diam.: *A*, a good "stock" wheel, *B* and *C*, improved wheels. Each wheel was tested with a dynamometer placed 1 ft. from the axis of the wheel, and it registered a constant load at that point of 1.9 lbs. The velocity of the wind in each test was 8.45 miles per hour = 12.4 ft. per second. The number of turns per minute was: *A*, 30.67; *B*, 38.13; *C*, 56.50. The efficiency was: *A*, 0.142; *B*, 0.176; *C*, 0.261. The work of wheel *C* was 674.5 ft. lb. per min. = 0.020 H.P. Assuming that the power increases as the square of the diameter and as the cube of the velocity, a wheel of the quality of *C*, 12½ ft. diam., with a wind velocity of 17 miles per hour, would be required for 1 H.P.; but wheel *C* had an exceptionally high efficiency, and such a high delivery would not likely be obtained in practice.

Prof. O. P. Hood (*Am. Mach.*, April 22, 1897) quotes the following results of experiments by E. C. Murphy; the mills were tested by pumping water:

| | | | | | | |
|---|------|------|------|------|------|----|
| Wind, miles per hour | 8 | 12. | 16. | 20. | 25. | 30 |
| Strokes per min., Mill No. 1, 8-ft. wheel .. | 10.2 | 19.3 | 25.3 | 28.1 | 25 | |
| Strokes per min., Mill No. 2, 8-ft. wheel .. | 8 | 20.2 | 26.1 | 28. | 27.5 | |
| Strokes per min., Mill No. 3, 12-ft. wheel .. | 4.8 | 12.7 | 18.8 | 23.3 | 25 | |
| Strokes per min., Mill No. 4, 12-ft. wheel .. | 6.2 | 11.9 | 14.7 | 16. | | |

Mill No. 3 was loaded nearly 90% heavier than mill No. 4.

In a 25-mile wind, seven 12-ft. mills developed, respectively, 0.379, 0.291, 0.309, 0.6, 0.247, 0.219, and 0.184 H.P.; and five 8-ft. mills, 0.043, 0.099, 0.059, 0.099, and 0.005 H.P. These effects include the effects of pumps of unknown and variable efficiency. The variations are largely due to the variable relation of the fixed load on the mill to the most favorable load which that mill might carry at each wind velocity. With each mill the efficiency is a maximum only for a certain load and a certain velocity, and for different loads and velocities the efficiency varies greatly. The useful work of mill No. 3 was equal to 0.6 H.P. in a 25-mile wind, and its efficiency was 5.8%. In a 16-mile wind the efficiency rose to 12.1%, and in a 12-mile wind it fell to 10.9%. The rule of the power developed, varying as the cube of the velocity, is far from true for a single wheel fitted with a single non-adjustable pump, and can only be true when the work of the pump per stroke is adjusted by varying the stroke of the pump, or by other means, for each change of velocity.

R. M. Dyer (*The Iowa Engineer*, July, 1906; also *Mach'y*, Aug., 1907) gives a brief review of the history of windmills, and quotes experiments by T. O. Perry, E. C. Murphy, Prof. F. H. King, and the Aermotor Co. Mr. Perry's experiments are reported in pamphlet No. 20 of the Water Supply and Irrigation Papers of the U. S. Geological Survey, Mr. Murphy's in pamphlets Nos. 41 and 42 of the same Papers, and Prof. King's, in Bulletin No. 82 of the Agricultural Experiment Station of the University of Wisconsin. The Aermotor Co.'s experiments are described in catalogues of that company. Some of Mr. Dyer's conclusions are as follows:

Experiments showed that 7/8 of the zone of interruption could be covered with sails; that the gain in power in from 3/4 to 7/8 of the surface was so small that the use of the additional material was not justifiable; that the sail surface should extend only two-thirds the distance from the outer diameter to the center; that a wheel running behind the carrying mast is not nearly as efficient as one running in front of the mast; that there should be the least possible obstruction behind the wheel; that to be efficient the velocity of the travel of the vertical circumference of the wheel should be from 1 to 1¼ times the velocity of the wind, hence the necessity of back gearing to reduce the pump speed to 40 strokes per minute as a maximum, which is the limit of safety at which ordinary pumps can be operated.

I hold that no manufacturer will be able to produce a marketable motor which will absorb and deliver, when acted upon by an elastic fluid, like air, in which it is entirely surrounded and submerged, more than 35% of the kinetic energy of the impinging current.

Theoretical demonstrations show that the kinetic energy of the air, impinging on the intercepted area of a wheel, varies as the cube of the wind velocity; consequently, the power of windmills of the same type

varies theoretically as the square of the diameter, and as the cube of the wind velocity; but as a wheel is designed to give its best efficiency in low winds, say 10 to 15 miles per hour, we cannot expect that the same angle of sail would obtain the same percentage of efficiency in winds of considerably higher velocity.

The ordinary wheel works most efficiently under wind velocities of from 10 to 12 miles per hour; such wheels will give reasonable efficiency in from 5- to 6-mile winds, while, if the wind blows more than 12 miles per hour, there will be power to spare. Our wheel must work in light winds, such being nearly always present, while the higher velocities only occur at intervals. Mills built for grinding purposes, or geared mills, will develop power almost approaching to the cube of the wind velocity, within reasonable limits, as their speed need not be kept down to a certain number of revolutions per minute, as in the case of the pumping mill.

Should this theoretic condition hold, the following table, showing the amount of power for different sizes of mills at different wind velocities, would apply: Figures show Horse Power.

| Size. | 5 mile. | 10 mile. | 15 mile. | 20 mile. | 25 mile. | 30 mile. | 35 mile. | 40 mile. |
|------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 8 ft..... | 0.011 | 0.088 | 0.297 | 0.704 | 1.375 | 2.176 | | |
| 12 ft..... | 0.025 | 0.20 | 0.675 | 1.6 | 3.125 | 5.4 | 8.57 | 12.8 |
| 16 ft..... | 0.045 | 0.36 | 1.215 | 2.88 | 5.52 | 9.75 | 15.3 | 21.04 |

These figures have been proven by laboratory tests at velocities ranging from 10 to 25 miles per hour and more practically by the Murphy tests on mills actually in use, which show very close relation at the wind velocities at which the mills are best adapted.

The Murphy figures are as follows:

| Size of mill. | 10 mile. | 15 mile. | 20 mlie. |
|---------------|-----------|-----------|-----------|
| 12 ft. | 0.21 H.P. | 0.58 H.P. | 1.05 H.P. |
| 16 ft. | 0.29 | 0.82 | 1.55 |

For higher wind velocities the Murphy values fall much under the theoretical values, but the range of velocities over which his experiments extend does not justify any change in the general law except inasmuch as common sense teaches us that theoretic conditions can rarely be attained in actual practice.

In view of the fact that a windmill does not work as efficiently in high winds as in winds under 20 miles per hour my experience would lead me to believe that the following figures (H.P.) would be the probable extension of the Murphy tests:

| Size of mill. | 25-mile wind. | 30-mile wind. | 35-mile wind. | 40-mile wind. |
|---------------|------------------|------------------|------------------|------------------|
| 12 ft. | 2.5 | 4 | 5 | 6 |
| 16 ft. | 4. | 6 | 8 | 10 |

A 20-ft. mill would deliver approximately 50% greater than a 16-ft.

The foregoing table must be translated with reasonable allowances for conditions under which wind wheels must work and which cannot well be avoided, *e.g.*: Pumping mills must be made to regulate off at a certain maximum speed to prevent damage to the attached pumping devices. The regulating point is usually between 20- and 25-mile wind velocities, so that no matter how much higher the wind velocity may be the power absorbed and delivered by the wheel will be no greater than that indicated at the regulating point.

Electric storage and lighting from the power of a windmill has been tested on a large scale for several years by Charles F. Brush, at Cleveland, Ohio. In 1887 he erected on the grounds of his dwelling a windmill 56 ft. in diameter, that operates with ordinary wind a dynamo at 500 revolutions per minute, with an output of 12,000 watts — 16 electric horse-power — charging a storage system that gives a constant lighting capacity of 100 16 to 20 candle-power lamps. The current from the dynamo is auto-

matically regulated to commence charging at 330 revolutions and 70 volts, and cutting the circuit at 75 volts. Thus, by its 24 hours' work, the storage system of 408 cells in 12 parallel series, each cell having a capacity of 100 ampere-hours, is kept in constant readiness for all the requirements of the establishment, it being fitted up with 350 incandescent lamps, about 100 being in use each evening. The plant runs at a mere nominal expense for oil, repairs, and attention. (For a fuller description of this plant, and of a more recent one at Marblehead Neck, Mass., see Lieut. Lewis's paper in *Engineering Magazine*, Dec., 1894, p. 475.)

COMPRESSED AIR.

Heating of Air by Compression. — Kimball, in his treatise on Physical Properties of Gases, says: When air is compressed, all the work which is done in the compression is converted into heat, and shows itself in the rise in temperature of the compressed gas. In practice many devices are employed to carry off the heat as fast as it is developed, and keep the temperature down. But it is not possible in any way to totally remove this difficulty. But, it may be objected, if all the work done in compression is converted into heat, and if this heat is got rid of as soon as possible, then the work may be virtually thrown away, and the compressed air can have no more energy than it had before compression. It is true that the compressed gas has no more energy than the gas had before compression, if its temperature is no higher, but the advantage of the compression lies in bringing its energy into more available form.

The total energy of the compressed and uncompressed gas is the same at the same temperature, but the available energy is much greater in the former.

When the compressed air is used in driving a rock-drill, or any other piece of machinery, it gives up energy equal in amount to the work it does, and its temperature is accordingly greatly reduced.

Causes of Loss of Energy in Use of Compressed Air. (Zahner, on Transmission of Power by Compressed Air.) — 1. The compression of air always develops heat, and as the compressed air always cools down to the temperature of the surrounding atmosphere before it is used, the mechanical equivalent of this dissipated heat is work lost.

2. The heat of compression increases the volume of the air, and hence it is necessary to carry the air to a higher pressure in the compressor in order that we may finally have a given volume of air at a given pressure, and at the temperature of the surrounding atmosphere. The work spent in effecting this excess of pressure is work lost.

3. Friction of the air in the pipes, leakage, dead spaces, the resistance offered by the valves, insufficiency of valve-area, inferior workmanship, and slovenly attendance, are all more or less serious causes of loss of power.

The first cause of loss of work, namely, the heat developed by compression, is entirely unavoidable. The whole of the mechanical energy which the compressor-piston spends upon the air is converted into heat. This heat is dissipated by conduction and radiation, and its mechanical equivalent is work lost. The compressed air, having again reached thermal equilibrium with the surrounding atmosphere, expands and does work in virtue of its intrinsic energy.

The intrinsic energy of a fluid is the energy which it is capable of exerting against a piston in changing from a given state as to temperature and volume to a total privation of heat and indefinite expansion.

Adiabatic and Isothermal Compression. — Air may be compressed either *adiabatically*, in which all the heat resulting from compression is retained in the air compressed, or *isothermally*, in which the heat is removed as rapidly as produced, by means of some form of refrigerator.

Volumes, Mean Pressures per Stroke, Temperatures, etc., in the Operation of Air-compression from 1 Atmosphere and 60° Fahr.
(F. Richards, *Am. Mach.*, March 30, 1893.)

| Gauge-pressure. | Atmospheres. | Volume with Air at Constant Temp. | Volume with Air not Cooled. | Mean Pressure per Stroke; Air Constant Temp. | Mean Pressure per Stroke; Air not Cooled. | Temp. of Air; not Cooled. | Gauge-pressure. | Atmospheres. | Volume with Air at Constant Temp. | Volume with Air not Cooled. | Mean Pressure per Stroke; Air Constant Temp. | Mean Pressure per Stroke; Air not Cooled. | Temp. of Air; not Cooled. |
|-----------------|--------------|-----------------------------------|-----------------------------|--|---|---------------------------|-----------------|--------------|-----------------------------------|-----------------------------|--|---|---------------------------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | 1 | 1 | 1 | 0 | 0 | 60° | 80 | 6.442 | .1552 | .266 | 27.38 | 36.64 | 432 |
| 1 | 1.068 | .9363 | .95 | .96 | .975 | 71 | 85 | 6.782 | .1474 | .2566 | 28.16 | 37.94 | 447 |
| 2 | 1.136 | .8803 | .91 | 1.87 | 1.91 | 80.4 | 90 | 7.122 | .1404 | .248 | 28.89 | 39.18 | 459 |
| 3 | 1.204 | .8305 | .876 | 2.72 | 2.8 | 88.9 | 95 | 7.462 | .134 | .24 | 29.57 | 40.4 | 472 |
| 4 | 1.272 | .7861 | .84 | 3.53 | 3.67 | 98 | 100 | 7.802 | .1281 | .2324 | 30.21 | 41.6 | 485 |
| 5 | 1.34 | .7462 | .81 | 4.3 | 4.5 | 106 | 105 | 8.142 | .1228 | .2254 | 30.81 | 42.78 | 496 |
| 10 | 1.68 | .5952 | .69 | 7.62 | 8.27 | 145 | 110 | 8.483 | .1178 | .2189 | 31.39 | 43.91 | 507 |
| 15 | 2.02 | .495 | .606 | 10.33 | 11.51 | 178 | 115 | 8.823 | .1133 | .2129 | 31.98 | 44.98 | 518 |
| 20 | 2.36 | .4237 | .543 | 12.62 | 14.4 | 207 | 120 | 9.163 | .1091 | .2073 | 32.54 | 46.04 | 529 |
| 25 | 2.7 | .3703 | .494 | 14.59 | 17.01 | 234 | 125 | 9.503 | .1052 | .2020 | 33.07 | 47.06 | 540 |
| 30 | 3.04 | .3289 | .4538 | 16.34 | 19.4 | 252 | 130 | 9.843 | .1015 | .1969 | 33.57 | 48.1 | 550 |
| 35 | 3.381 | .2957 | .42 | 17.92 | 21.6 | 281 | 135 | 10.183 | .0981 | .1922 | 34.05 | 49.1 | 560 |
| 40 | 3.721 | .2687 | .393 | 19.32 | 23.66 | 302 | 140 | 10.523 | .095 | .1878 | 34.57 | 50.02 | 570 |
| 45 | 4.061 | .2462 | .37 | 20.57 | 25.59 | 321 | 145 | 10.864 | .0921 | .1837 | 35.09 | 51. | 580 |
| 50 | 4.401 | .2272 | .35 | 21.69 | 27.39 | 339 | 150 | 11.204 | .0892 | .1796 | 35.48 | 51.89 | 589 |
| 55 | 4.741 | .2109 | .331 | 22.76 | 29.11 | 357 | 160 | 11.88 | .0841 | .1722 | 36.29 | 53.65 | 607 |
| 60 | 5.081 | .1968 | .3144 | 23.78 | 30.75 | 375 | 170 | 12.56 | .0796 | .1657 | 37.2 | 55.39 | 624 |
| 65 | 5.422 | .1844 | .301 | 24.75 | 32.32 | 389 | 180 | 13.24 | .0755 | .1595 | 37.96 | 57.01 | 640 |
| 70 | 5.762 | .1735 | .288 | 25.67 | 33.83 | 405 | 190 | 13.93 | .0718 | .154 | 38.68 | 58.57 | 657 |
| 75 | 6.102 | .1639 | .276 | 26.55 | 35.27 | 420 | 200 | 14.61 | .0685 | .149 | 39.42 | 60.14 | 672 |

Column 3 gives the volume of air after compression to the given pressure and after it is cooled to its initial temperature. After compression air loses its heat very rapidly, and this column may be taken to represent the volume of air after compression available for the purpose for which the air has been compressed.

Column 4 gives the volume of air more nearly as the compressor has to deal with it. In any compressor the air will lose some of its heat during compression. The slower the compressor runs the cooler the air and the smaller the volume.

Column 5 gives the mean effective resistance to be overcome by the air-cylinder piston in the stroke of compression, supposing the air to remain constantly at its initial temperature. Of course it will not so remain, but this column is the ideal to be kept in view in economical air-compression.

Column 6 gives the mean effective resistance to be overcome by the piston, supposing that there is no cooling of the air. The actual mean effective pressure will be somewhat less than as given in this column; but for computing the actual power required for operating air-compressor cylinders, the figures in this column may be taken and a certain percentage added — say 10 per cent — and the result will represent very closely the power required by the compressor.

The mean pressures given being for compression from one atmosphere upward, they will not be correct for computations in compound compression or for any other initial pressure.

Loss due to Excess of Pressure caused by Heating in the Compression-cylinder. — If the air during compression were kept at a constant temperature, the compression-curve of an indicator-diagram taken from the cylinder would be an isothermal curve, and would follow the law

of Boyle and Mariotte, $p v = \text{a constant}$, or $p_1 v_1 = p_0 v_0$, or $p_1 = p_0 \frac{v_0}{v_1}$, $p_0 v_0$

being the pressure and volume at the beginning of compression, and $p_1 v_1$ the pressure and volume at the end, or at any intermediate point. But as the air is heated during compression the pressure increases faster than the volume decreases, causing the work required for any given pressure to be increased. If none of the heat were abstracted by radiation or by injection of water, the curve of the diagram would be an adiabatic curve, with the equation $p_1 = p_0 \left(\frac{v_0}{v_1} \right)^{1.405}$. Cooling the air during compression, or compressing it in two cylinders, called compounding, and cooling the air as it passes from one cylinder to the other, reduces the exponent of this equation, and reduces the quantity of work necessary to effect a given compression. F. T. Gause (*Am. Mach.*, Oct. 20, 1892), describing the operations of the Popp air-compressors in Paris, says: The greatest saving realized in compressing in a single cylinder was 33 per cent of that theoretically possible. In cards taken from the 2000 H.P. compound compressor at Quai De La Gare, Paris, the saving realized is 85 per cent of the theoretical amount. Of this amount only 8 per cent is due to cooling during compression, so that the increase of economy in the compound compressor is mainly due to cooling the air between the two stages of compression. A compression-curve with exponent 1.25 is the best result that was obtained for compression in a single cylinder and cooling with a very fine spray. The curve with exponent 1.15 is that which must be realized in a single cylinder to equal the present economy of the compound compressor at Quai De La Gare.

Horse-power required to compress and deliver One Cubic Foot of Free Air per minute to a given pressure with no cooling of the air during the compression; also the horse power required, supposing the air to be maintained at constant temperature during the compression.

H.P. required to compress and deliver One Cubic Foot of Compressed Air per minute at a given pressure (the air being measured at the atmospheric temperature) with no cooling of the air during the compression; also supposing the air to be maintained at constant temperature during the compression.

| Gauge-pressure. | Air not cooled. | Air constant temperature. |
|-----------------|-----------------|---------------------------|
| 5 | 0.0196 | 0.0188 |
| 10 | 0.0361 | 0.0333 |
| 20 | 0.0628 | 0.0551 |
| 30 | 0.0846 | 0.0713 |
| 40 | 0.1032 | 0.0843 |
| 50 | 0.1195 | 0.0946 |
| 60 | 0.1342 | 0.1036 |
| 70 | 0.1476 | 0.1120 |
| 80 | 0.1599 | 0.1195 |
| 90 | 0.1710 | 0.1261 |
| 100 | 0.1815 | 0.1318 |

| Gauge-pressure. | Air not cooled. | Air constant temperature. |
|-----------------|-----------------|---------------------------|
| 5 | 0.0263 | 0.0251 |
| 10 | 0.0606 | 0.0559 |
| 20 | 0.1483 | 0.1300 |
| 30 | 0.2573 | 0.2168 |
| 40 | 0.3842 | 0.3138 |
| 50 | 0.5261 | 0.4166 |
| 60 | 0.6318 | 0.5266 |
| 70 | 0.8508 | 0.6456 |
| 80 | 1.0302 | 0.7700 |
| 90 | 1.2177 | 0.8979 |
| 100 | 1.4171 | 1.0291 |

The horse-power given above is the theoretical power, no allowance being made for friction of the compressor or other losses, which may amount to 10 per cent or more.

Formulae for Adiabatic Compression or Expansion of Air (or Other Sensibly Perfect Gas).

Let air at an absolute temperature T_1 , absolute pressure p_1 , and volume v_1 be compressed to an absolute pressure p_2 and corresponding volume v_2 and absolute temperature T_2 ; or let compressed air of an initial pressure, volume, and temperature p_1 , v_1 , and T_1 be expanded to p_2 , v_2 , and T_2 being no transmission of heat from or into the air during the operation.

Then the following equations express the relations between pressure, volume, and temperature (see works on Thermodynamics):

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{0.71}; \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{1.41}; \quad \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{2.46};$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{0.41}; \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.29}; \quad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{3.46}.$$

The exponents are derived from the ratio $c_p \div c_v = k$ of the specific heats of air at constant pressure and constant volume. Taking $k = 1.406$, $1 \div k = 0.711$; $k - 1 = 0.406$; $1 \div (k - 1) = 2.463$; $k + (k - 1) = 3.463$; $(k - 1) \div k = 0.289$.

Work of Adiabatic Compression of Air. — If air is compressed in a cylinder without clearance from a volume v_1 and pressure p_1 to a smaller volume v_2 and higher pressure p_2 , work equal to $p_1 v_1$ is done by the external air on the piston while the air is drawn into the cylinder. Work is then done by the piston on the air, first, in compressing it to the pressure p_2 and volume v_2 , and then in expelling the volume v_2 from the cylinder against the pressure p_2 . If the compression is adiabatic, $p_1 v_1^k = p_2 v_2^k = \text{constant}$. $k = 1.406$.

The work of compression of a given quantity of air is

$$\frac{p_1 v_1}{k - 1} \left\{ \left(\frac{v_1}{v_2}\right)^{k-1} - 1 \right\} = \frac{p_1 v_1}{k - 1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right\},$$

or
$$2.463 p_1 v_1 \left\{ \left(\frac{v_1}{v_2}\right)^{0.41} - 1 \right\} = 2.463 p_1 v_1 \left\{ \left(\frac{p_2}{p_1}\right)^{0.29} - 1 \right\}.$$

The work of expulsion is $p_2 v_2 = p_1 v_1 \left(\frac{p_2}{p_1}\right)^{0.29}$.

The total work is the sum of the work of compression and expulsion less the work done on the piston during admission, and it equals

$$p_1 v_1 \left\{ \frac{k}{k - 1} \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right\} = 3.463 p_1 v_1 \left\{ \left(\frac{p_2}{p_1}\right)^{0.29} - 1 \right\}.$$

The mean effective pressure during the stroke is

$$p_1 \frac{k}{k - 1} \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right\} = 3.463 p_1 \left\{ \left(\frac{p_2}{p_1}\right)^{0.29} - 1 \right\}.$$

p_1 and p_2 are absolute pressures above a vacuum in atmospheres or in pounds per square inch or per square foot.

EXAMPLE. — Required the work done in compressing 1 cubic foot of air per second from 1 to 6 atmospheres, including the work of expulsion from the cylinder.

$p_2 \div p_1 = 6$; $6^{0.29} - 1 = 0.681$; $3.463 \times 0.681 = 2.358$ atmospheres $\times 14.7 = 34.66$ lb. per sq. in. mean effective pressure, $\times 144 = 4991$ lb. per sq. ft.; $\times 1$ ft. stroke = 4991 ft.-lb., $\div 550$ ft.-lb. per second = 9.08 H.P.

If R = ratio of pressures = $p_2 \div p_1$, and if $v_1 = 1$ cubic foot, the work done in compressing 1 cubic foot from p_1 to p_2 is, in foot-pounds,

$$3.463 p_1 (R^{0.29} - 1),$$

p_1 being taken in lb. per sq. ft. For compression at the sea level p_1 may be taken at 14 lbs. per sq. in. = 2016 lb. per sq. ft., as there is some loss of pressure due to friction of valves and passages.

Horse-power required to compress and deliver 100 cubic feet of free air per minute = $1.511 P_1 (R^{0.29} - 1)$; P_1 being the pressure of the free air in pounds per sq. in., absolute.

EXAMPLE. To compress 100 cu. ft. from 1 to 6 atmospheres. $P_1 = 1.47$; $R = 6$; $1.511 \times 14.7 \times 0.681 = 15.13$ H.P.

Indicator-cards from compressors in good condition and under working-speeds usually follow the adiabatic line closely. A low curve indicates piston leakage. Such cooling as there may be from the cylinder-jacket and the re-expansion of the air in clearance-spaces tends to reduce the mean effective pressure, while the "camel-backs" in the expulsion-line, due to resistance to opening of the discharge-valve, tend to increase it.

Work of one stroke of a compressor, with adiabatic compression, in foot-pounds,

$$W = 3.463 P_1 V_1 (R^{0.29} - 1),$$

in which P_1 = initial absolute pressure in lb. per sq. ft. and V_1 = volume traversed by piston in cubic feet.

The work done during adiabatic compression (or expansion) of 1 pound of air from a volume v_1 and pressure p_1 to another volume v_2 and pressure p_2 is equal to the mechanical equivalent of the heating (or cooling). If t_1 is the higher and t_2 the lower temperature, Fahr., the work done is $c_v J (t_1 - t_2)$ foot-pounds, c_v being the specific heat of air at constant volume = 0.1689, and $J = 778$, $c_v J = 131.4$.

The work during compression also equals

$$\frac{c_v J}{R_a} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{0.29} - 1 \right] = 2.463 p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{0.29} - 1 \right],$$

R_a being the value of $p v \div$ absolute temperature for 1 pound of air = 53.37.

The work during expansion is

$$2.463 p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{0.29} \right] = 2.463 p_2 v_2 \left[\left(\frac{p_1}{p_2} \right)^{0.29} - 1 \right],$$

in which $p_1 v_1$ are the initial and $p_2 v_2$ the final pressures and volumes.

Compressed-air Engines, Adiabatic Expansion. — Let the initial pressure and volume taken into the cylinder be p_1 lb. per sq. ft. and v_1 cubic feet; let expansion take place to p_2 and v_2 according to the adiabatic law $p_1 v_1^{1.41} = p_2 v_2^{1.41}$; then at the end of the stroke let the pressure drop to the back-pressure p_3 , at which the air is exhausted. Assuming no clearance, the work done by one pound of air during admission, measured above vacuum, is $p_1 v_1$, the work during expansion is $2.463 p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{0.29} \right]$, and the negative or back pressure work is $-p_3 v_2$. The total work is $p_1 v_1 + 2.463 p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{0.29} \right] - p_3 v_2$, and the mean effective pressure is the total work divided by v_2 .

If the air is expanded down to the back-pressure p_3 the total work is

$$3.463 p_1 v_1 \left\{ 1 - \left(\frac{p_3}{p_1} \right)^{0.29} \right\},$$

or, in terms of the final pressure and volume,

$$3.463 p_3 v_2 \left\{ \left(\frac{p_1}{p_3} \right)^{0.29} - 1 \right\},$$

and the mean effective pressure is

$$3.463 p_3 \left\{ \left(\frac{p_1}{p_3} \right)^{0.29} - 1 \right\}.$$

The actual work is reduced by clearance. When this is considered, the product of the initial pressure p_1 by the clearance volume is to be subtracted from the total work calculated from the initial volume v_1 , including clearance. (See p. 931, under "Steam-engine.")

Mean Effective Pressures of Air Compressed Adiabatically.(F. A. Halsey, *Am. Mach.*, Mar. 10, 1898.)

| R . | $R^{0.29}$. | M.E.P. from 14 lbs. Initial. | R . | $R^{0.29}$. | M.E.P. from 14 lbs. Initial. |
|-------|--------------|------------------------------------|-------|--------------|------------------------------------|
| 1.25 | 1.067 | 3.24 | 4.75 | 1.570 | 27.5 |
| 1.50 | 1.125 | 6.04 | 5 | 1.594 | 28.7 |
| 1.75 | 1.176 | 8.51 | 5.25 | 1.617 | 29.8 |
| 2 | 1.223 | 10.8 | 5.5 | 1.639 | 30.8 |
| 2.25 | 1.265 | 12.8 | 5.75 | 1.660 | 31.8 |
| 2.5 | 1.304 | 14.7 | 6 | 1.681 | 32.8 |
| 2.75 | 1.341 | 16.4 | 6.25 | 1.701 | 33.8 |
| 3 | 1.375 | 18.1 | 6.5 | 1.720 | 34.7 |
| 3.25 | 1.407 | 19.6 | 6.75 | 1.739 | 35.6 |
| 3.5 | 1.438 | 21.1 | 7 | 1.757 | 36.5 |
| 3.75 | 1.467 | 22.5 | 7.25 | 1.775 | 37.4 |
| 4 | 1.495 | 23.9 | 7.5 | 1.793 | 38.3 |
| 4.25 | 1.521 | 25.2 | 8 | 1.827 | 39.9 |
| 4.5 | 1.546 | 26.4 | | | |

 R = final \div initial absolute pressure.

M.E.P. = mean effective pressure, lb. per sq. in., based on 14 lb. initial.

Compound Compression, with Air Cooled between the Two Cylinders. (*Am. Mach.*, March 10 and 31, 1898.) — Work in low-pressure cylinder = W_1 , in high-pressure cylinder W_2 . Total work

$$W_1 + W_2 = 3.46 P_1 V_1 [r_1^{0.29} + R^{0.29} \times r_1^{-0.29} - 2].$$

r_1 = ratio of pressures in l. p. cyl., r_2 = ratio in h.p. cyl., $R = r_1 r_2$. When $r_1 = r_2 = \sqrt{R}$, the sum $W_1 + W_2$ is a minimum. Hence for a given total ratio of pressures, R , the work of compression, will be least when the ratios of the pressures in each of the two cylinders are equal.

The equation may be simplified, when $r_1 = \sqrt{R}$, to the following:

$$W_1 + W_2 = 6.92 P_1 V_1 [R^{0.145} - 1].$$

Dividing by V_1 gives the mean effective pressure reduced to the low-pressure cylinder M.E.P. = $6.92 P_1 [R^{0.145} - 1]$.

In the above equation the compression in each cylinder is supposed to be adiabatic, but the intercooler is supposed to reduce the temperature of the air to that at which compression began.

Horse-power required to compress adiabatically 100 cu. ft. of free air per minute in two stages with intercooling, and with equal ratio of compression in each cylinder, = $3.022 P_1 (R^{0.145} - 1)$; P_1 being the pressure in lbs. per sq. in., absolute, of the free air, and R the total ratio of compression.**EXAMPLE.** To compress 100 cu. ft. per min. from 1 to 6 atmospheres, $P = 14.7$; $R = 6$; $3.022 \times 14.7 \times 0.2964 = 13.17$ H.P.**Mean Effective Pressures of Air Compressed in Two Stages, assuming the Intercooler to Reduce the Temperature to that at which Compression Began.** (F. A. Halsey, *Am. Mach.*, Mar. 31, 1898.)

| R . | $R^{0.145}$. | M.E.P. from 14 lbs. Initial. | Ultimate Saving by Com- pound- ing, %. | R . | $R^{0.145}$. | M.E.P. from 14 lbs. Initial. | Ultimate Saving by Com- pound- ing, %. |
|-------|---------------|------------------------------------|--|-------|---------------|------------------------------------|--|
| 5.0 | 1.263 | 25.4 | 11.5 | 9.0 | 1.375 | 36.3 | 15.8 |
| 5.5 | 1.280 | 27.0 | 12.3 | 9.5 | 1.386 | 37.3 | 16.2 |
| 6.0 | 1.296 | 28.6 | 12.8 | 10 | 1.396 | 38.3 | 16.6 |
| 6.5 | 1.312 | 30.1 | 13.2 | 11 | 1.416 | 40.2 | 17.2 |
| 7.0 | 1.326 | 31.5 | 13.7 | 12 | 1.434 | 41.9 | 17.8 |
| 7.5 | 1.336 | 32.8 | 14.3 | 13 | 1.451 | 43.5 | 18.4 |
| 8.0 | 1.352 | 34.0 | 14.8 | 14 | 1.466 | 45.0 | 19.0 |
| 8.5 | 1.364 | 35.2 | 15.3 | 15 | 1.481 | 46.4 | 19.4 |

R = final + initial absolute pressure.

M.E.P. = mean effective pressure, lb. per sq. in., based on 14 lb. absolute initial pressure reduced to the low-pressure cylinder.

Table for Adiabatic Compression or Expansion of Air.

(*Proc. Inst. M.E.*, Jan., 1881, p. 123.)

| Absolute Pressure. | | Absolute Temperature. | | Volume. | |
|--|--|--|--|--|--|
| Ratio of Greater to Less. (Expansion.) | Ratio of Less to Greater. (Compression.) | Ratio of Greater to Less. (Expansion.) | Ratio of Less to Greater. (Compression.) | Ratio of Greater to Less. (Compression.) | Ratio of Less to Greater. (Expansion.) |
| 1.2 | 0.833 | 1.054 | 0.948 | 1.138 | 0.879 |
| 1.4 | 0.714 | 1.102 | 0.907 | 1.270 | 0.788 |
| 1.6 | 0.625 | 1.146 | 0.873 | 1.396 | 0.716 |
| 1.8 | 0.556 | 1.186 | 0.843 | 1.518 | 0.659 |
| 2.0 | 0.500 | 1.222 | 0.818 | 1.636 | 0.611 |
| 2.2 | 0.454 | 1.257 | 0.796 | 1.750 | 0.571 |
| 2.4 | 0.417 | 1.289 | 0.776 | 1.862 | 0.537 |
| 2.6 | 0.385 | 1.319 | 0.758 | 1.971 | 0.507 |
| 2.8 | 0.357 | 1.348 | 0.742 | 2.077 | 0.481 |
| 3.0 | 0.333 | 1.375 | 0.727 | 2.182 | 0.458 |
| 3.2 | 0.312 | 1.401 | 0.714 | 2.284 | 0.438 |
| 3.4 | 0.294 | 1.426 | 0.701 | 2.384 | 0.419 |
| 3.6 | 0.278 | 1.450 | 0.690 | 2.483 | 0.403 |
| 3.8 | 0.263 | 1.473 | 0.679 | 2.580 | 0.388 |
| 4.0 | 0.250 | 1.495 | 0.669 | 2.676 | 0.374 |
| 4.2 | 0.238 | 1.516 | 0.660 | 2.770 | 0.361 |
| 4.4 | 0.227 | 1.537 | 0.651 | 2.863 | 0.349 |
| 4.6 | 0.217 | 1.557 | 0.642 | 2.955 | 0.338 |
| 4.8 | 0.208 | 1.576 | 0.635 | 3.046 | 0.328 |
| 5.0 | 0.200 | 1.595 | 0.627 | 3.135 | 0.319 |
| 6.0 | 0.167 | 1.681 | 0.595 | 3.569 | 0.280 |
| 7.0 | 0.143 | 1.758 | 0.569 | 3.981 | 0.251 |
| 8.0 | 0.125 | 1.828 | 0.547 | 4.377 | 0.228 |
| 9.0 | 0.111 | 1.891 | 0.529 | 4.759 | 0.210 |
| 10.0 | 0.100 | 1.950 | 0.513 | 5.129 | 0.195 |

Mean Effective Pressures for the Compression Part only of the Stroke when Compressing and Delivering Air from One Atmosphere to given Gauge-pressure in a Single Cylinder. (F. Richards, *Am. Mach.*, Dec. 14, 1893.)

| Gauge-Pressure. | Adiabatic Compression. | Isothermal Compression. | Gauge-Pressure. | Adiabatic Compression. | Isothermal Compression. |
|-----------------|------------------------|-------------------------|-----------------|------------------------|-------------------------|
| 1 | 0.44 | 0.43 | 45 | 13.95 | 12.62 |
| 2 | 0.96 | 0.95 | 50 | 15.05 | 13.48 |
| 3 | 1.41 | 1.4 | 55 | 15.98 | 14.3 |
| 4 | 1.86 | 1.84 | 60 | 16.89 | 15.05 |
| 5 | 2.26 | 2.22 | 65 | 17.88 | 15.76 |
| 10 | 4.26 | 4.14 | 70 | 18.74 | 16.43 |
| 15 | 5.99 | 5.77 | 75 | 19.54 | 17.09 |
| 20 | 7.58 | 7.2 | 80 | 20.5 | 17.7 |
| 25 | 9.05 | 8.49 | 85 | 21.22 | 18.3 |
| 30 | 10.39 | 9.66 | 90 | 22.0 | 18.87 |
| 35 | 11.59 | 10.72 | 95 | 22.77 | 19.4 |
| 40 | 12.8 | 11.7 | 100 | 23.43 | 19.92 |

The mean effective pressure for compression only is always lower than the mean effective pressure for the whole work.

To find the **Index of the Curve of an Air-diagram**. If P_1V_1 be pressure and volume at one point on the curve, and PV the pressure and volume at another point, then $\frac{P}{P_1} = \left(\frac{V_1}{V}\right)^x$, in which x is the index to be

found. Let $P \div P_1 = R$, and $V_1 \div V = r$; then $R = r^x$; $\log R = x \log r$, whence $x = \log R \div \log r$. (See also graphic method on page 576.)

Mean and Terminal Pressures of Compressed Air used Expansively for Gauge Pressures from 60 to 100 lb.

(Frank Richards, *Am. Mach.*, April 13, 1893.)

Initial Pressure.

| Point of Cut-off. | | | | | | | | | | |
|-------------------|--------------------|------------------------|--------------------|------------------------|--------------------|------------------------|--------------------|------------------------|--------------------|------------------------|
| | 60 | | 70 | | 80 | | 90 | | 100 | |
| | Mean Air-pressure. | Terminal Air-pressure. | Mean Air-pressure. | Terminal Air-pressure. | Mean Air-pressure. | Terminal Air-pressure. | Mean Air-pressure. | Terminal Air-pressure. | Mean Air-pressure. | Terminal Air-pressure. |
| .25 | 23.6 | <i>10.65</i> | 28.74 | <i>12.07</i> | 33.89 | <i>13.49</i> | 39.04 | <i>14.91</i> | 44.19 | 1.33 |
| .30 | 28.9 | <i>13.77</i> | 34.75 | 0.6 | 40.61 | 2.44 | 46.46 | 4.27 | 53.32 | 6.11 |
| | 32.13 | 0.96 | 38.41 | 3.09 | 44.69 | 5.22 | 50.98 | 7.35 | 57.26 | 9.48 |
| .35 | 33.66 | 2.33 | 40.15 | 4.38 | 46.64 | 6.66 | 53.13 | 8.95 | 59.62 | 11.23 |
| | 35.85 | 3.85 | 42.63 | 6.36 | 49.41 | 7.88 | 56.2 | 11.39 | 62.98 | 13.89 |
| .40 | 37.93 | 5.64 | 44.99 | 8.39 | 52.05 | 11.14 | 59.11 | 13.88 | 66.16 | 16.64 |
| .45 | 41.75 | 10.71 | 49.31 | 12.61 | 56.9 | 15.86 | 64.45 | 19.11 | 72.02 | 22.36 |
| .50 | 45.14 | 13.26 | 53.16 | 17. | 61.18 | 20.81 | 69.19 | 24.56 | 77.21 | 28.33 |
| .60 | 50.75 | 21.53 | 59.51 | 26.4 | 68.28 | 31.27 | 77.05 | 36.14 | 85.82 | 41.01 |
| | 51.92 | 23.69 | 60.84 | 28.85 | 69.76 | 34.01 | 78.69 | 39.16 | 87.61 | 44.32 |
| | 53.67 | 27.94 | 62.83 | 33.03 | 71.99 | 38.68 | 81.14 | 44.33 | 90.32 | 49.97 |
| .70 | 54.93 | 30.39 | 64.25 | 36.44 | 73.57 | 42.49 | 82.9 | 48.54 | 92.22 | 54.59 |
| .75 | 56.52 | 35.01 | 66.05 | 41.68 | 75.59 | 48.35 | 85.12 | 55.02 | 94.66 | 61.69 |
| .80 | 57.79 | 39.78 | 67.5 | 47.08 | 77.2 | 54.38 | 86.91 | 61.69 | 96.61 | 68.99 |
| | 59.15 | 47.14 | 69.03 | 55.43 | 78.92 | 63.81 | 88.81 | 72. | 98.7 | 80.28 |
| .90 | 59.46 | 49.65 | 69.38 | 58.27 | 79.31 | 66.89 | 89.24 | 75.52 | 99.17 | 87.82 |

Pressures in italics are absolute; all others are gauge pressures.

AIR COMPRESSION AT ALTITUDES.

(Ingersoll-Rand Co. Copyright, 1906, by F. M. Hitchcock.)

Multipliers to Determine the Volume of Free Air which, when Compressed, is Equivalent in Effect to a Given Volume of Free Air at Sea Level.

| Altitude, Feet. | Barometric Pressure. | | Gauge Pressure (Pounds). | | | | |
|-----------------|----------------------|-----------------|--------------------------|-------|-------|-------|-------|
| | In. of Mercury. | Lb. per Sq. In. | 60 | 80 | 100 | 125 | 150 |
| 1,000 | 28.88 | 14.20 | 1.032 | 1.033 | 1.034 | 1.035 | 1.036 |
| 2,000 | 27.80 | 13.67 | 1.064 | 1.066 | 1.068 | 1.071 | 1.072 |
| 3,000 | 26.76 | 13.16 | 1.097 | 1.102 | 1.105 | 1.107 | 1.109 |
| 4,000 | 25.76 | 12.67 | 1.132 | 1.139 | 1.142 | 1.147 | 1.149 |
| 5,000 | 24.79 | 12.20 | 1.168 | 1.178 | 1.182 | 1.187 | 1.190 |
| 6,000 | 23.86 | 11.73 | 1.206 | 1.218 | 1.224 | 1.231 | 1.234 |
| 7,000 | 22.97 | 11.30 | 1.245 | 1.258 | 1.267 | 1.274 | 1.278 |
| 8,000 | 22.11 | 10.87 | 1.287 | 1.300 | 1.310 | 1.319 | 1.326 |
| 9,000 | 21.29 | 10.46 | 1.329 | 1.346 | 1.356 | 1.366 | 1.374 |
| 10,000 | 20.49 | 10.07 | 1.373 | 1.394 | 1.404 | 1.416 | 1.424 |

Horse-power Developed in Compressing One Cubic Foot of Free Air at Various Altitudes from Atmospheric to Various Pressures.

Initial Temperature of the Air in Each Cylinder Taken as 60° F.; Jacket Cooling not Considered; Allowance made for usual losses.

| Altitude, Feet. | Simple Compression. | | | Two Stage Compression. | | | | |
|--------------------|-----------------------------|--------|--------|--------------------------|--------|--------|--------|--------|
| | Gauge Pressure (Pounds). | | | Gauge Pressure (Pounds). | | | | |
| | 60 | 80 | 100 | 60 | 80 | 100 | 125 | 150 |
| 0 | 0.1533 | 0.1824 | 0.2075 | 0.1354 | 0.1580 | 0.1765 | 0.1964 | 0.2138 |
| 1,000 | 0.1511 | 0.1795 | 0.2040 | 0.1332 | 0.1553 | 0.1734 | 0.1926 | 0.2093 |
| 2,000 | 0.1489 | 0.1766 | 0.2006 | 0.1310 | 0.1524 | 0.1700 | 0.1887 | 0.2048 |
| 3,000 | 0.1469 | 0.1739 | 0.1971 | 0.1286 | 0.1493 | 0.1666 | 0.1848 | 0.2003 |
| 4,000 | 0.1448 | 0.1712 | 0.1939 | 0.1263 | 0.1464 | 0.1635 | 0.1810 | 0.1963 |
| 5,000 | 0.1425 | 0.1685 | 0.1906 | 0.1241 | 0.1438 | 0.1600 | 0.1772 | 0.1921 |
| 6,000 | 0.1402 | 0.1656 | 0.1872 | 0.1218 | 0.1409 | 0.1566 | 0.1737 | 0.1879 |
| 7,000 | 0.1379 | 0.1628 | 0.1839 | 0.1197 | 0.1383 | 0.1536 | 0.1700 | 0.1838 |
| 8,000 | 0.1358 | 0.1600 | 0.1807 | 0.1173 | 0.1358 | 0.1504 | 0.1662 | 0.1797 |
| 9,000 | 0.1337 | 0.1572 | 0.1774 | 0.1151 | 0.1329 | 0.1473 | 0.1627 | 0.1758 |
| 10,000 | 0.1316 | 0.1547 | 0.1743 | 0.1132 | 0.1303 | 0.1442 | 0.1592 | 0.1717 |

EXAMPLE.— Required the volume of free air which when compressed to 100 lb. gauge at 9,000 ft. altitude will be equivalent to 1,000 cu. ft. of free air at sea level; also the power developed in compressing this volume to 100 lb. gauge in two stage compression at this altitude.

From first table the multiplier is 1.356. Equivalent free air = $1,000 \times 1.356 = 1,356$ cu. ft.

From second table, power developed in compressing 1 cu. ft. of free air is 0.1473 H.P.; $1,356 \times 0.1473 = 199.73$ H.P.

The Popp Compressed-air System in Paris.— A most extensive system of distribution of power by means of compressed air is that of M. Popp, in Paris. One of the central stations is laid out for 24,000 horse-power. For a very complete description of the system, see *Engineering*, Feb. 15, June 7, 21, and 28, 1889, and March 13 and 20, April 10, and May 1, 1891. Also *Proc. Inst. M. E.*, July, 1889. A condensed description will be found in *Modern Mechanism*, p. 12.

Utilization of Compressed Air in Small Motors.— In the earliest stages of the Popp system in Paris it was recognized that no good results could be obtained if the air were allowed to expand direct into the motor; not only did the formation of ice due to the expansion of the air rapidly accumulate and choke the exhaust, but the percentage of useful work obtained, compared with that put into the air at the central station, was so small as to render commercial results hopeless.

After a number of experiments M. Popp adopted a simple form of cast-iron stove lined with fire-clay, heated either by a gas jet or by a small coke fire. This apparatus answered the desired purpose until a better arrangement was perfected, and the type was accordingly adopted throughout the whole system. The economy resulting from the use of the improved form was very marked.

It was found that more than 70% of the total heating value of the fuel employed was absorbed by the air and transformed into useful work. The efficiency of fuel consumed in this way is at least six times greater than when utilized in a boiler and steam-engine. According to Prof. Riedler, from 15% to 20% above the power at the central station can be obtained by means at the disposal of the power users. By heating the air to 480° F. an increased efficiency of 30% can be obtained.

A large number of motors in use among the subscribers to the Compressed Air Company of Paris are rotary engines developing 1 H.P. and less, and these in the early times of the industry were very extravagant in their consumption. Small rotary engines, working cold air without expansion, used as high as 2330 cu. ft. of air per brake H.P. per hour, and with heated air 1624 cu. ft. Working expansively, a 1-H.P. rotary engine used 1469 cu. ft. of cold air, or 960 cu. ft. of heated air, and a

2-H.P. rotary engine 1059 cu. ft. of cold air, or 847 cu. ft. of air, heated to about 122° F.

The efficiency of this type of rotary motors, with air heated to 122° F., may now be assumed at 43%.

Tests of a small Riedinger rotary engine, used for driving sewing-machines and indicating about 0.1 H.P., showed an air-consumption of 1377 cu. ft. per H.P. per hour when the initial pressure of the air was 86 lb. per sq. in. and its temperature 54° F., and 988 cu. ft. when the air was heated to 338° F., its pressure being 72 lb. With a 1/2-H.P. variable-expansion rotary engine the air-consumption was from 800 to 900 cu. ft. per H.P. per hour for initial pressures of 54 to 85 lb. per sq. in. with the air heated from 336° to 388° F., and 1148 cu. ft. with cold air, 46° F., and an initial pressure of 72 lb. The volumes of air were all taken at atmospheric pressure.

Trials made with an old single-cylinder 80-horse-power Farcot steam-engine, indicating 72 H.P., gave a consumption of air per brake H.P. as low as 465 cu. ft. per hour. The temperature of admission was 320° F., and of exhaust 95° F.

Prof. Elliott gives the following as typical results of efficiency for various systems of compressors and air-motors:

| | |
|---|-------|
| Simple compressor and simple motor, efficiency..... | 39.1% |
| Compound compressor and simple motor, "..... | 44.9 |
| " " compound motor, efficiency. 50.7 | |
| Triples compressor and triples motor, "..... | 55.3 |

The efficiency is the ratio of the I.H.P. in the motor cylinders to the I.H.P. in the steam-cylinders of the compressor. The pressure assumed is 6 atmospheres absolute, and the losses are equal to those found in Paris over a distance of 4 miles.

Summary of Efficiencies of Compressed-air Transmission at Paris, between the Central Station at St. Fargeau and a 10-horse-power Motor Working with Pressure Reduced to 4 1/2 Atmospheres.

(The figures below correspond to mean results of two experiments cold and two heated.)

One indicated horse-power at central station gives 0.845 I.H.P. in compressors, and corresponds to the compression of 348 cu. ft. of air per hour from atmospheric pressure to 6 atmospheres absolute.

0.845 I.H.P. in compressors delivers as much air as will do 0.52 I.H.P. in adiabatic expansion after it has fallen to the normal temperature of the mains.

The fall of pressure in mains between central station and Paris (say 5 kilometres) reduces the possibility of work from 0.52 to 0.51 I.H.P.

The further fall of pressure through the reducing valve to 4 1/2 atmospheres (absolute) reduces the possibility of work from 0.51 to 0.50.

Incomplete expansion, wire-drawing, and other such causes reduce the actual I.H.P. of the motor from 0.50 to 0.39.

By heating the air before it enters the motor to about 320° F., the actual I.H.P. at the motor is, however, increased to 0.54. The ratio of gain by heating the air is, therefore, $0.54 \div 0.39 = 1.38$.

In this process additional heat is supplied by the combustion of about 0.39 lb. of coke per I.H.P. per hour, and if this be taken into account, the real indicated efficiency of the whole process becomes 0.47 instead of 0.54.

Working with cold air the work spent in driving the motor itself reduces the available horse-power from 0.39 to 0.26.

Working with heated air the work spent in driving the motor itself reduces the available horse-power from 0.54 to 0.44.

A summary of the efficiencies is as follows:

Efficiency of main engines 0.845.

Efficiency of compressors $0.52 \div 0.845 = 0.61$.

Efficiency of transmission through mains $0.51 \div 0.52 = 0.98$.

Efficiency of reducing valve $0.50 \div 0.51 = 0.98$.

The combined efficiency of the mains and reducing valve between 5 and 4 1/2 atmospheres is thus $0.98 \times 0.98 = 0.96$. If the reduction had been

to 4, 3½, or 3 atmospheres, the corresponding efficiencies would have been 0.93, 0.89, and 0.85 respectively.

Indicated efficiency of motor $0.39 \div 0.50 = 0.78$.

Indicated efficiency of whole process with cold air 0.39. Apparent indicated efficiency of whole process with heated air 0.54.

Real indicated efficiency of whole process with heated air 0.47.

Mechanical efficiency of motor, cold, 0.67.

Mechanical efficiency of motor, hot, 0.81.

Ingersoll-Sergeant Standard Air Compressors.

(Ingersoll-Rand Co., 1908.)

| Class and Type. | Diam. of Cyl., In. | | | | Stroke, In. | Rev. per Min. | Capacity, Cu. Ft. Free Air per Min. | Working Air Pressure. | Horse-power. | Cu. Ft. in Foundation. |
|--|--------------------|------|--------|--------|-------------|---------------|---|--------------------------|--------------|---------------------------|
| | Steam. | | Air. | | | | | | | |
| | High. | Low. | High. | Low. | | | | | | |
| A-1* Straight Line Steam Driven. | 10 | | 10 1/4 | | 12 | 160 | 177 | 50-100 | 23- 35 | 113 |
| | 12 | | 12 1/4 | | 14 | 155 | 285 | 50-100 | 37- 57 | 200 |
| | 14 | | 14 1/4 | | 18 | 120 | 381 | 50-100 | 50- 76 | 340 |
| | 16 | | 16 1/4 | | 18 | 120 | 498 | 50-100 | 65-100 | 340 |
| | 18 | | 18 1/4 | | 24 | 94 | 656 | 50-100 | 86-131 | 520 |
| | 20 | | 20 1/4 | | 24 | 94 | 807 | 50-100 | 106-161 | 520 |
| | 22 | | 22 1/4 | | 24 | 94 | 973 | 50-100 | 127-194 | 520 |
| | 24 | | 24 1/4 | | 30 | 80 | 1223 | 50-100 | 161-242 | 710 |
| A-2* Straight Line Steam Driven Compound Air. | 12 | | 7 1/2 | 12 1/4 | 12 | 160 | 252 | 90-110 | 40- 45 | 145 |
| | 14 | | 9 1/4 | 14 1/4 | 14 | 155 | 375 | 90-110 | 60- 66 | 230 |
| | 16 | | 10 1/4 | 16 1/4 | 18 | 135 | 550 | 90-110 | 89- 97 | 435 |
| | 18 | | 12 1/4 | 18 1/4 | 18 | 135 | 702 | 90-110 | 113-124 | 435 |
| | 20 | | 13 1/4 | 20 1/4 | 24 | 110 | 940 | 90-110 | 151-166 | 640 |
| | 22 | | 14 1/4 | 22 1/4 | 24 | 110 | 1131 | 90-110 | 182-193 | 640 |
| | 24 | | 15 1/4 | 24 1/4 | 24 | 110 | 1333 | 90-110 | 214-236 | 640 |
| | 26 | | 16 1/4 | 26 1/4 | 30 | 90 | 1606 | 90-110 | 258-284 | 950 |

B,* Straight line, belt driven. Same as A-1 in sizes up to 16 1/4 × 18 in.

C, Duplex Corliss Steam, Duplex air.

C-2, Compound Corliss Steam, Compound air.† } Designed to suit conditions, not made to standard sizes.

| | | | | | | | | | | |
|---|------|------|--------|--------|----|-----|------|--------|---------|------|
| D-1* Duplex and Half Duplex Belt Driven. | | | 10 1/4 | | 12 | 160 | 352 | 60-100 | 50-67 | 240 |
| | | | 12 1/4 | | 14 | 155 | 568 | 60-100 | 81-108 | 400 |
| | | | 14 1/4 | | 18 | 120 | 763 | 65-100 | 113-146 | 625 |
| | | | 16 1/4 | | 18 | 120 | 994 | 70-100 | 154-189 | 625 |
| | | | 18 1/4 | | 24 | 100 | 1338 | 70-100 | 207-256 | 1050 |
| | | | 20 1/4 | | 24 | 100 | 1674 | 70-100 | 259-320 | 1050 |
| D-2† Duplex Compound Belt Driven. | | | 10 1/4 | 16 1/4 | 12 | 160 | 444 | 80-100 | 65-72 | 240 |
| | | | 11 1/4 | 18 1/4 | 14 | 155 | 638 | 80-100 | 93-104 | 400 |
| | | | 14 1/4 | 22 1/4 | 18 | 120 | 925 | 80-100 | 134-150 | 625 |
| | | | 15 1/4 | 25 1/4 | 18 | 120 | 1205 | 80-100 | 174-194 | 625 |
| | | | 17 1/4 | 28 1/4 | 24 | 100 | 1622 | 80-100 | 235-263 | 1050 |
| | | | 18 1/4 | 30 1/4 | 24 | 100 | 1857 | 80-100 | 269-300 | 1050 |
| | | | 20 1/4 | 32 1/4 | 24 | 100 | 2130 | 80 | 309 | 1050 |

E,* Straight line, belt driven; same sizes as F-1.

| | | | | | | | | | | |
|--|----|------|--------|------|----|-----|-----|--------|----------|----|
| F-1* Straight Line Steam Driven. | 6 | | 6 | | 6 | 150 | 29 | 45-100 | 4-6 | 21 |
| | 8 | | 8 | | 8 | 150 | 69 | 50-100 | 9 1/2-14 | 32 |
| | 10 | | 10 | | 10 | 150 | 134 | 55-100 | 19-27 | 46 |
| | 12 | | 12 1/4 | | 12 | 150 | 233 | 60-100 | 35-47 | 63 |

* Built in intermediate sizes for lower pressures.

† Most economical form of compressor. ‡ For sea level; also built with larger low pressure cylinders for altitudes of 5,000 and 10,000 ft.

Ingersoll-Sergeant Standard Air Compressors.—Continued.

| Class and Type. | Diam. of Cyl., In. | | | | Stroke, In. | Rev. per Min. | Capacity, Cu. Ft. Free Air per Min. | Working Air Pressure. | Horse-power. | Cu. Ft. in Foundation. |
|--|--------------------|------|--------|--------|-------------|---------------|---|--------------------------|-----------------|---------------------------|
| | Steam. | | Air. | | | | | | | |
| | High. | Low. | High. | Low. | | | | | | |
| G-1* Duplex and Half Duplex Steam Driven. | 10 | | 10 1/4 | | 12 | 160 | 352 | 60-100 | 55- 70 | 330 |
| | 12 | | 12 1/4 | | 14 | 155 | 568 | 60-100 | 85-114 | 480 |
| | 14 | | 14 1/4 | | 18 | 120 | 763 | 65-100 | 119-152 | 800 |
| | 16 | | 16 1/4 | | 18 | 120 | 994 | 70-100 | 160-200 | 800 |
| | 18 | | 18 1/4 | | 24 | 100 | 1338 | 70-100 | 218-267 | 1450 |
| | 20 | | 20 1/4 | | 24 | 100 | 1674 | 70-100 | 273-335 | 1450 |
| | 22 | | 22 1/4 | | 24 | 100 | 2010 | 70-100 | 328-402 | 1450 |
| G-2† Duplex Steam, Compound Air. | 10 | | 10 1/4 | 16 1/4 | 12 | 160 | 444 | 80-100 | 67- 75 | 330 |
| | 12 | | 11 1/4 | 18 1/4 | 14 | 155 | 638 | 80-100 | 97-108 | 480 |
| | 14 | | 14 1/4 | 22 1/4 | 18 | 120 | 925 | 80-100 | 140-157 | 800 |
| | 16 | | 15 1/4 | 25 1/4 | 18 | 120 | 1205 | 80-100 | 182-204 | 800 |
| | 18 | | 17 1/4 | 28 1/4 | 24 | 100 | 1622 | 80-100 | 245-274 | 1475 |
| | 20 | | 18 1/4 | 30 1/4 | 24 | 100 | 1857 | 100 | 314 | 1475 |
| | 22 | | 20 1/4 | 32 1/4 | 24 | 100 | 2130 | 100 | 360 | 1475 |
| H-1* Duplex Steam, Duplex Air. | 6 | | 6 | | 6 | 150 | 58 | 50-100 | 7 1/2-11 1/2 | 115 |
| | 8 | | 8 1/4 | | 8 | 150 | 140 | 55-100 | 20- 28 | 150 |
| | 10 | | 10 1/4 | | 10 | 150 | 272 | 60-100 | 40- 54 | 180 |
| | 12 | | 12 1/4 | | 12 | 150 | 472 | 60-100 | 70- 94 | 220 |
| | 14 | | 14 1/4 | | 14 | 140 | 680 | 65-100 | 106-136 | 383 |
| | 16 | | 16 1/4 | | 16 | 135 | 986 | 70-100 | 160-197 | 585 |
| H-2† Duplex Steam, Compound Air. | 6 | | 7 | 10 | 6 | 150 | 81 | 80-100 | 121 1/2-141 1/2 | 115 |
| | 8 | | 9 1/4 | 14 1/4 | 8 | 150 | 215 | 80-100 | 33-37 | 150 |
| | 10 | | 10 1/4 | 16 1/4 | 10 | 150 | 348 | 80-100 | 53-59 | 180 |
| | 12 | | 12 1/4 | 18 1/4 | 12 | 150 | 526 | 80-100 | 80-90 | 220 |
| | 14 | | 14 1/4 | 22 1/4 | 14 | 140 | 841 | 80-100 | 129-144 | 383 |
| | 16 | | 16 1/4 | 25 1/4 | 16 | 135 | 1205 | 80-100 | 182-204 | 585 |
| J-1* Duplex Belt Driven. | | | 6 | | 6 | 150 | 58 | 50-100 | 7-11 | 83 |
| | | | 8 1/4 | | 8 | 150 | 140 | 55-100 | 19-27 | 125 |
| | | | 10 1/4 | | 10 | 150 | 272 | 60-100 | 39-53 | 135 |
| | | | 12 1/4 | | 12 | 150 | 472 | 60-100 | 67-90 | 172 |
| | | | 14 1/4 | | 14 | 140 | 680 | 65-100 | 101-130 | 315 |
| | | | 16 1/4 | | 16 | 135 | 986 | 70-100 | 153-190 | 429 |
| J-2† Duplex Compound Belt Driven. | | | 7 | 10 | 6 | 150 | 81 | 80-100 | 12-14 | 83 |
| | | | 9 1/4 | 14 1/4 | 8 | 150 | 215 | 80-100 | 31-35 | 125 |
| | | | 10 1/4 | 16 1/4 | 10 | 150 | 348 | 80-100 | 51-57 | 135 |
| | | | 12 1/4 | 18 1/4 | 12 | 150 | 526 | 80-100 | 77-86 | 117 |
| | | | 14 1/4 | 22 1/4 | 14 | 140 | 841 | 80-100 | 12-138 | 315 |
| | | | 16 1/4 | 25 1/4 | 16 | 135 | 1205 | 80-100 | 176-198 | 429 |
| | | | | | | | | | | |

* Built in intermediate sizes for lower pressures.

† For sea level; also built with larger low pressure cylinders for altitudes of 5,000 and 10,000 ft.

‡ For sea level; also built in the 4 largest sizes with larger low pressure cylinders for altitudes of 5,000 and 10,000 ft.

Many other styles of compressors are also built. Among them are the following:

Rand-Corliss, compound condensing steam, compound air; capacities, 750 to 7670 cu. ft. of free air per min.; steam cylinders, 10 and 18 to 28 and 52 in.; air cylinders, 11 1/2 and 18 to 33 and 52 in.; stroke 30 to 48 in.; I.H.P., from 114 to 1166.

Vertical duplex single acting, belt driven; capacities, 16.6 to 321 cu. ft. of free air per min.; air cylinders, 4½ to 12 in.; stroke 4½ to 14 in.; I.H.P., 2.5 to 66.

Duplex steam, non condensing, compound air; capacities, 343 to 2209 cu. ft. of free air per min.; steam cylinders, 10 to 20 in.; air cylinders, 9 and 14 to 19 and 30 in.; stroke, 16 to 30 in.; I.H.P., 53 to 380.

Compound steam, non condensing, duplex air; capacities, 349 to 1962 cu. ft. of free air per min.; steam cylinders, 10 and 16 to 20 and 32 in.; air cylinders, 10 to 20 in.; stroke, 16 to 30 in.; I.H.P., 62 to 392.

Straight line, steam driven; capacities, 42 to 630 cu. ft. of free air per min.; steam cylinders, 6 to 12 in.; air cylinders, 6 to 19 in.; stroke, 8 to 16 in.; I.H.P., 8.2 to 54.

Cubic Feet of Air Required to Run Rock Drills at Various Pressures and Altitudes.

(Ingersoll-Rand Co., 1908.)

TABLE I. — CUBIC FEET OF FREE AIR REQUIRED TO RUN ONE DRILL.

| Gauge Pressure, Lb. per Sq. In. | Size and Cylinder Diameter of Drill. | | | | | | | | | | | | |
|------------------------------------|--------------------------------------|--------------|--------|--------|-----|--------|---------|--------|--------|--------|--------|-----|--------|
| | A 35 | A 32 A 86 | B | C | D | D | D | E | F | F | G | H | H 9 |
| | 2" | 2 1/4" | 2 1/2" | 2 3/4" | 3" | 3 1/8" | 3 3/16" | 3 1/4" | 3 1/2" | 3 5/8" | 4 1/4" | 5" | 5 1/2" |
| 60 | 50 | 60 | 68 | 82 | 90 | 95 | 97 | 100 | 108 | 113 | 130 | 150 | 164 |
| 70 | 56 | 68 | 77 | 93 | 102 | 108 | 110 | 113 | 124 | 129 | 147 | 170 | 181 |
| 80 | 63 | 76 | 86 | 104 | 114 | 120 | 123 | 127 | 131 | 143 | 164 | 190 | 207 |
| 90 | 70 | 84 | 95 | 115 | 126 | 133 | 136 | 141 | 152 | 159 | 182 | 210 | 230 |
| 100 | 77 | 92 | 104 | 126 | 138 | 146 | 149 | 154 | 166 | 174 | 199 | 240 | 252 |

TABLE II. — MULTIPLIERS TO GIVE CAPACITY OF COMPRESSOR TO OPERATE FROM 1 TO 70 ROCK DRILLS AT VARIOUS ALTITUDES.

| Altitude Above Sea Level. | Number of Drills. | | | | | | | | | | | | | | | |
|------------------------------|-------------------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 25 | 30 | 40 | 50 |
| 0 | 1. | 1.8 | 2.7 | 3.4 | 4.1 | 4.8 | 5.4 | 6.0 | 6.5 | 7.1 | 9.5 | 11.7 | 13.7 | 15.8 | 21.4 | 25.5 |
| 1000 | 1.03 | 1.85 | 2.78 | 3.5 | 4.22 | 4.94 | 5.56 | 6.18 | 6.69 | 7.3 | 9.78 | 12.05 | 14.1 | 16.3 | 22.0 | 26.26 |
| 2000 | 1.07 | 1.92 | 2.89 | 3.64 | 4.39 | 5.14 | 5.78 | 6.42 | 6.95 | 7.60 | 10.17 | 12.52 | 14.66 | 16.9 | 22.9 | 27.28 |
| 3000 | 1.10 | 1.98 | 2.97 | 3.74 | 4.51 | 5.28 | 5.94 | 6.6 | 7.15 | 7.81 | 10.45 | 12.87 | 15.07 | 17.38 | 23.54 | 28.05 |
| 5000 | 1.17 | 2.10 | 3.16 | 3.98 | 4.8 | 5.62 | 6.32 | 7.02 | 7.61 | 8.31 | 11.12 | 13.69 | 16.03 | 18.49 | 25.04 | 29.84 |
| 8000 | 1.26 | 2.27 | 3.40 | 4.28 | 5.17 | 6.05 | 6.8 | 7.56 | 8.19 | 8.95 | 11.97 | 14.74 | 17.26 | 19.9 | 26.96 | 32.13 |
| 10000 | 1.32 | 2.38 | 3.56 | 4.49 | 5.41 | 6.34 | 7.13 | 7.92 | 8.58 | 9.37 | 12.54 | 15.44 | 18.08 | 20.86 | 28.25 | 33.66 |
| 15000 | 1.43 | 2.57 | 3.86 | 4.86 | 5.86 | 6.86 | 7.72 | 8.58 | 9.3 | 10.15 | 13.58 | 16.73 | 19.59 | 22.59 | 30.6 | 36.49 |

EXAMPLE. — Required the amount of free air to operate thirty 5-inch "H" drills at 8,000 ft. altitude, using air at a gauge pressure of 80 lb. per sq. in. From Table I, we find that one 5-inch "H" drill operating at 80 lb. gauge pressure requires 190 cu. ft. of free air per minute. From Table II, the factor for 30 drills at 8,000 feet altitude is 19.9; $190 \times 19.9 = 3781$ = the displacement of a compressor under average conditions, to which must be added pipe line losses.

The tables above are for fair conditions in ordinary hard rock. In soft material, where the drilling time is short more drills can be run with a given compressor than when working in hard material. In tunnel work, more rapid progress can be made if the drills are run at high air pressure, and it is advisable to have an excess of compressor capacity of about 25%. No allowance has been made in the tables for friction or pipe line losses.

Steam Required to Compress 100 Cu. Ft. of Free Air. (O. S. Shantz, *Power*, Feb. 4, 1908.)—The following tables show the number of pounds of steam required to compress 100 cu. ft. of free air to different gauge pressures, by means of steam engines using from 12 to 40 lbs. of steam per I.H.P. per hour. The figures assume adiabatic compression in the air cylinders, with intercooling to atmospheric temperature in the case of two-stage compression, and 90% mechanical efficiency of the compressor.

STEAM CONSUMPTION OF AIR COMPRESSORS—SINGLE-STAGE COMPRESSION.

| Air, Gauge Pres- sure. | Steam per I.H.P. Hour. Lbs. | | | | | | | | | | | | |
|---------------------------------|-----------------------------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|
| | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 36 | 40 |
| 20 | 1.36 | 1.58 | 1.82 | 2.04 | 2.26 | 2.49 | 2.72 | 2.94 | 3.17 | 3.40 | 3.61 | 4.08 | 4.54 |
| 30 | 1.84 | 2.14 | 2.45 | 2.76 | 3.06 | 3.37 | 3.68 | 3.98 | 4.29 | 4.60 | 4.90 | 5.51 | 6.12 |
| 40 | 2.26 | 2.64 | 3.02 | 3.39 | 3.77 | 4.15 | 4.52 | 4.90 | 5.26 | 5.65 | 6.03 | 6.78 | 7.50 |
| 50 | 2.62 | 3.06 | 3.50 | 3.93 | 4.36 | 4.80 | 5.25 | 5.68 | 6.10 | 6.55 | 7.00 | 8.86 | 8.71 |
| 60 | 2.92 | 3.41 | 3.90 | 4.38 | 4.80 | 5.36 | 5.85 | 6.32 | 6.80 | 7.30 | 7.80 | 8.76 | 9.71 |
| 70 | 3.22 | 3.76 | 4.30 | 4.83 | 5.36 | 5.90 | 6.45 | 6.97 | 7.50 | 8.05 | 8.60 | 9.66 | 10.70 |
| 80 | 3.50 | 4.08 | 4.67 | 5.25 | 5.84 | 6.42 | 7.00 | 7.59 | 8.15 | 8.75 | 9.34 | 10.50 | 11.61 |
| 90 | 3.72 | 4.34 | 4.96 | 5.58 | 6.20 | 6.82 | 7.45 | 8.05 | 8.66 | 9.30 | 9.94 | 11.15 | 12.35 |
| 100 | 3.96 | 4.61 | 5.29 | 5.95 | 6.60 | 7.25 | 7.92 | 8.58 | 9.22 | 9.90 | 10.56 | 11.88 | 13.15 |
| 110 | 4.18 | 4.87 | 5.58 | 6.26 | 6.96 | 7.66 | 8.36 | 9.05 | 9.75 | 10.45 | 11.15 | 12.52 | 13.90 |
| 120 | 4.38 | 5.11 | 5.85 | 6.57 | 7.30 | 8.04 | 8.76 | 9.50 | 10.20 | 10.95 | 11.66 | 13.13 | 14.55 |

TWO-STAGE COMPRESSION.

| | | | | | | | | | | | | | |
|-----|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| 70 | 2.82 | 3.25 | 3.76 | 4.23 | 4.69 | 5.16 | 5.63 | 6.10 | 6.56 | 7.04 | 7.50 | 8.45 | 9.35 |
| 80 | 3.01 | 3.51 | 4.03 | 4.52 | 5.02 | 5.53 | 6.03 | 6.53 | 7.03 | 7.53 | 8.03 | 9.05 | 10.01 |
| 90 | 3.19 | 3.72 | 4.26 | 4.79 | 5.32 | 5.85 | 6.38 | 6.91 | 7.44 | 7.98 | 8.50 | 9.57 | 10.60 |
| 100 | 3.37 | 3.93 | 4.50 | 5.05 | 5.61 | 6.19 | 6.74 | 7.30 | 7.85 | 8.42 | 8.99 | 10.10 | 11.20 |
| 110 | 3.54 | 4.14 | 4.74 | 5.32 | 5.91 | 6.51 | 7.10 | 7.70 | 8.27 | 8.86 | 9.46 | 10.64 | 11.80 |
| 120 | 3.69 | 4.30 | 4.93 | 5.54 | 6.15 | 6.78 | 7.38 | 8.00 | 8.61 | 9.24 | 9.85 | 11.05 | 12.27 |
| 130 | 3.83 | 4.46 | 5.11 | 5.75 | 6.38 | 7.03 | 7.66 | 8.30 | 8.92 | 9.57 | 10.20 | 11.48 | 12.72 |
| 140 | 3.96 | 4.62 | 5.29 | 5.94 | 6.60 | 7.26 | 7.92 | 8.60 | 9.23 | 9.90 | 10.56 | 11.88 | 13.15 |
| 150 | 4.10 | 4.76 | 5.46 | 6.14 | 6.81 | 7.50 | 8.18 | 8.86 | 9.55 | 10.20 | 10.90 | 12.26 | 13.60 |

Compressed-air Table for Pumping Plants.

(Ingersoll-Rand Co., 1908.)

The following table shows the pressure and volume of air required for any size pump for pumping by compressed air. Reasonable allowances have been made for loss due to clearances in pump and friction in pipe.

To find the amount of air and pressure required to pump a given quantity of water a given height, find the ratio of diameters between water and air cylinders, and multiply the number of gallons of water by the figure found in the column for the required lift. The result is the number of *cubic feet of free air*. The pressure required on the pump will be found directly above in the same column. For example: The ratio between cylinders being 2 to 1, required to pump 100 gallons, height of lift 250

feet. We find under 250 feet at ratio 2 to 1 the figures 2.11; $2.11 \times 100 = 211$ cubic feet of free air. The pressure required is 34.38 pounds delivered at the pump piston.

| Ratio of Diameters. | | Perpendicular Height, in Feet, to which the Water is to be Pumped. | | | | | | | | | | |
|------------------------|---|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 250 | 300 | 400 |
| 1 to 1 | A | 13.75 | 27.5 | 41.25 | 55.0 | 68.25 | 82.5 | 96.25 | 110.0 | | | |
| | B | 0.21 | 0.45 | 0.60 | 0.75 | 0.89 | 1.04 | 1.20 | 1.34 | | | |
| 1½ to 1 | A | | 12.22 | 18.33 | 24.44 | 30.33 | 36.66 | 42.76 | 48.88 | 61.11 | 73.32 | 97.66 |
| | B | | 0.65 | 0.80 | 0.95 | 1.09 | 1.24 | 1.39 | 1.53 | 1.83 | 2.12 | 2.70 |
| 1¾ to 1 | A | | | 13.75 | 19.8 | 22.8 | 27.5 | 32.1 | 36.66 | 45.83 | 55.0 | 73.33 |
| | B | | | 0.94 | 1.14 | 1.24 | 1.30 | 1.54 | 1.69 | 1.99 | 2.39 | 2.88 |
| 2 to 1 | A | | | | 13.75 | 17.19 | 20.63 | 24.06 | 27.5 | 34.38 | 41.25 | 55.0 |
| | B | | | | 1.23 | 1.37 | 1.52 | 1.66 | 1.81 | 2.11 | 2.40 | 2.98 |
| 2¼ to 1 | A | | | | | 13.75 | 16.5 | 19.25 | 22.0 | 27.5 | 33.0 | 44.0 |
| | B | | | | | 1.53 | 1.68 | 1.83 | 1.97 | 2.26 | 2.56 | 3.15 |
| 2½ to 1 | A | | | | | | 13.2 | 15.4 | 17.6 | 22.0 | 26.4 | 35.2 |
| | B | | | | | | 1.79 | 1.98 | 2.06 | 2.34 | 2.62 | 3.18 |

A = air-pressure at pump. B = cubic feet of free air per gallon of water.

Compressed-air Table for Hoisting-engines.

(Ingersoll-Rand Co., 1908.)

The following table gives an approximate idea of the volume of free air required for operating hoisting-engines, the air being delivered to the engine at 60 lbs. gauge. There are so many variable conditions to the operation of hoisting-engines in common use that accurate computations can only be offered when fixed data are given. In the table the engine is assumed to actually run but one-half of the time for hoisting, while the compressor runs continuously. If the engine runs less than one-half the time, the volume of air required will be proportionately less, and *vice versa*. The table is computed for maximum loads, which also in practice may vary widely. From the intermittent character of the work of a hoisting-engine the parts are able to resume their normal temperature between the hoists, and there is little probability of freezing up the exhaust-passages.

Volume of Free Air Required for Operating Hoisting-engines, the Air Compressed to 60 Pounds Gauge Pressure.

SINGLE-CYLINDER HOISTING-ENGINE.

| Diam. of Cylinder, Inches. | Stroke, Inches. | Revolutions per Minute. | Normal Horse- power. | Actual Horse- power. | Weight Lifted, Single Rope. | Cubic Ft. of Free Air Required. |
|----------------------------------|--------------------|-------------------------------|----------------------------|----------------------------|--------------------------------------|---------------------------------------|
| 5 | 6 | 200 | 3 | 5.9 | 600 | 75 |
| 5 | 8 | 160 | 4 | 6.3 | 1,000 | 80 |
| 6 1/4 | 8 | 160 | 6 | 9.9 | 1,500 | 125 |
| 7 | 10 | 125 | 10 | 12.1 | 2,000 | 151 |
| 8 1/4 | 10 | 125 | 15 | 16.8 | 3,000 | 170 |
| 8 1/2 | 12 | 110 | 20 | 18.9 | 5,000 | 238 |
| 10 | 12 | 110 | 25 | 26.2 | 6,000 | 330 |

DOUBLE-CYLINDER HOISTING-ENGINE.

| Diam. of Cylinder, Inches. | Stroke, Inches. | Revolutions per Minute. | Normal Horse-power. | Actual Horse-power. | Weight Lifted, Single Rope. | Cubic Ft. of Free Air Required. |
|----------------------------|-----------------|-------------------------|---------------------|---------------------|-----------------------------|---------------------------------|
| 5 | 6 | 200 | 6 | 11.8 | 1,000 | 150 |
| 5 | 8 | 160 | 8 | 12.6 | 1,650 | 160 |
| 6 1/4 | 8 | 160 | 12 | 19.8 | 2,500 | 250 |
| 7 | 10 | 125 | 20 | 24.2 | 3,500 | 302 |
| 8 1/4 | 10 | 125 | 30 | 33.6 | 6,000 | 340 |
| 8 1/2 | 12 | 110 | 40 | 37.8 | 8,000 | 476 |
| 10 | 12 | 110 | 50 | 52.4 | 10,000 | 660 |
| 12 1/4 | 15 | 100 | 75 | 89.2 | | 1,125 |
| 14 | 18 | 90 | 100 | 125. | | 1,587 |

Practical Results with Compressed Air. — *Compressed-air System at the Chapin Mines, Iron Mountain, Mich.* — These mines are three miles from the falls which supply the power. There are four turbines at the falls, one of 1000 horse-power and three of 900 horse-power each. The pressure is 60 pounds at 60° Fahr. Each turbine runs a pair of compressors. The pipe to the mines is 24 ins. diameter. The power is applied at the mines to Corliss engines, running pumps, hoists, etc., and direct to rock-drills.

A test made in 1888 gave 1430.27 H.P. at the compressors, and 390.17 H.P. as the sum of the horse-power of the engines at the mines. Therefore, only 27% of the power generated was recovered at the mines. This includes the loss due to leakage and the loss of energy in heat, but not the friction in the engines or compressors. (F. A. Pocock, *Trans. A. I. M. E.*, 1890.)

W. L. Saunders (*Jour. F. I.*, 1892) says: "There is not a properly designed compressed-air installation in operation to-day that loses over 5% by transmission alone. The question is altogether one of the size of pipe; and if the pipe is large enough, the friction loss is a small item.

"The loss of power in common practice, where compressed air is used to drive machinery in mines and tunnels, is about 70%. In the best practice, with the best air-compressors, and without reheating, the loss is about 60%. These losses may be reduced to a point as low as 20% by combining the best systems of reheating with the best air-compressors."

Gain due to Reheating. — Prof. Kennedy says compressed-air transmission system is now being carried on, on a large commercial scale, in such a fashion that a small motor four miles away from the central station can indicate in round numbers 10 horse-power, for 20 horse-power at the station itself, allowing for the value of the coke used in heating the air.

The limit to successful reheating lies in the fact that air-engines cannot work to advantage at temperatures over 350°.

The efficiency of the common system of reheating is shown by the results obtained with the Popp system in Paris. Air is admitted to the reheater at about 83°, and passes to the engine at about 315°, thus being increased in volume about 42%. The air used in Paris is about 11 cubic feet of free air per minute per horse-power. The ordinary practice in America with cold air is from 15 to 25 cubic feet per minute per horse-power. When the Paris engines were worked without reheating the air consumption was increased to about 15 cubic feet per horse-power per minute. The amount of fuel consumed during reheating is trifling.

Effect of Temperature of Intake upon the Discharge of a Compressor. — Air should be drawn from outside the engine-room, and from as cool a place as possible. The gain in efficiency amounts to one per cent for every five degrees that the air is taken in lower than the temperature of the engine-room. The inlet conduit should have an area at least 50% of the area of the air-piston, and should be made of wood, brick, or other non-conductor of heat.

Discharge of a compressor having an intake capacity of 1000 cubic feet per minute, and volumes of the discharge reduced to cubic feet at atmospheric pressure and at temperature of 62 degrees Fahrenheit:

| | | | | | | | | |
|---------------------------------|------|------|------|-----|-----|-----|------|------|
| Temperature of Intake, F. . . . | 0° | 32° | 62° | 75° | 80° | 90° | 100° | 110° |
| Volume discharged, cubic ft. | 1135 | 1060 | 1000 | 975 | 966 | 949 | 932 | 916 |

Compressed-Air Motors with a Return-Air Circuit. — In the ordinary use of motors, such as rock-drills, the air, after doing its work in the motor, is allowed to escape into the atmosphere. In some systems, however, notably in the electric air-drill, the air exhausted from the cylinder of the motor is returned to the air compressor. A marked increase in economy is claimed to have been effected in this way (*Cass. Mag.*, 1907).

Intercoolers for Air Compressors. — H. V. Haight (*Am. Mach.*, Aug. 30, 1906). In multi-stage air compressors, the efficiency is greater the more nearly the temperature of the air leaving the intercooler approaches that of the air entering it. The difference of these temperatures for given temperatures of the entering water and air is diminished by increasing the surface of the intercooler and thereby decreasing the ratio of the quantity of air cooled to the area of cooling surface. Numerous tests of intercoolers with different ratios of quantity of air to area of surface, on being plotted, approximate to a straight-line diagram, from which the following figures are taken:

Cu. ft. of free air per min. per sq. ft. of air cooling surface 5 10 15
Diff. of temp. F°. between water entering and air leaving 12.5° 25° 37.5°.

Centrifugal Air Compressors. — (*Eng. News*, Nov. 19, 1908.) The General Electric Co. has placed on the market a line of centrifugal air compressors with pressure ratings from 0.75 to 4.0 lbs. per sq. in. and capacities from 750 to 28,000 cu. ft. of free air per minute. The compressor consists essentially of a rotating impeller surrounded by a suitable casing with an intake opening at the center and a discharge opening at the circumference. It is similar to the centrifugal pump, the efficiency depending largely upon the design of the impeller and casing.

The compressors are driven by Curtis steam turbines or by electric motors especially designed for them. With "squirrel-cage" induction motors, since the speed cannot be varied, care must be taken to specify a pressure sufficiently high to cover the operating requirements, because at constant speed the pressure cannot be varied without altering the design of the impeller. For foundry cupola service direct-current motors can be compound wound so as to automatically increase the speed should the volume of air delivered decrease, thus increasing the pressure of the air and preventing undue reduction of flow of air through the cupola when it chokes up. Further adjustments of pressure can be made by changing the speed of the motor by means of the field rheostat.

Standard Single-Stage Centrifugal Air Compressors (1909).

| R.P.M. | Standard Conditions. | | Minimum Speed Conditions. | | Maximum Speed Conditions. | | Pipe Diameter Inches. |
|------------------|----------------------|------------------|---------------------------|------------------|---------------------------|------------------|-----------------------|
| | Lbs. per Sq. In. | Cu. Ft. per Min. | Lbs. per Sq. In. | Cu. Ft. per Min. | Lbs. per Sq. In. | Cu. Ft. per Min. | |
| 3450 | 1.0 | 800 | 0.75 | 1,100 | 1.25 | 600 | 10 |
| 3450 | 1.0 | 1,600 | 0.75 | 2,100 | 1.25 | 1,300 | 12 |
| 3450 | 1.0 | 3,200 | 0.75 | 4,100 | 1.25 | 2,600 | 12 |
| 3450 | 1.0 | 4,500 | 0.75 | 5,900 | 1.25 | 3,800 | 16 |
| 3450 | 1.0 | 7,200 | 0.75 | 8,800 | 1.25 | 6,000 | 20 |
| 3450 | 1.0 | 10,200 | 0.75 | 12,000 | 1.25 | 8,700 | 26 |
| 1725 | 1.0 | 25,000 | 0.75 | 31,000 | 1.25 | 21,000 | 36 |
| 3450 | 2.0 | 750 | 1.5 | 1,000 | 2.50 | 500 | 8 |
| 3450 | 2.0 | 1,600 | 1.5 | 2,100 | 2.50 | 1,200 | 10 |
| 3450 | 2.0 | 2,500 | 1.5 | 3,300 | 2.50 | 1,900 | 12 |
| 3450 | 2.0 | 4,200 | 1.5 | 5,400 | 2.50 | 3,300 | 16 |
| 3450 | 2.0 | 6,200 | 1.5 | 8,000 | 2.50 | 5,000 | 20 |
| 1725 | 2.0 | 15,000 | 1.5 | 19,000 | 2.50 | 11,000 | 26 |
| 1725 | 2.0 | 28,000 | 1.5 | 36,000 | 2.50 | 24,000 | 36 |
| 3450 | 3.25 | 1,250 | 2.5 | 1,800 | 4.00 | 900 | 8 |
| 3450 | 3.25 | 2,400 | 2.5 | 3,200 | 4.00 | 1,900 | 12 |
| 3450 | 3.25 | 3,800 | 2.5 | 5,000 | 4.00 | 3,000 | 14 |
| 3450 A.C. & tur. | 3.25 | 9,000 | 2.5 | 11,500 | 4.00 | 7,500 | 24 |
| 1725 D.C. | 3.25 | 9,000 | 2.5 | 11,000 | 4.00 | 6,400 | 24 |
| 3450 A.C. & tur. | 3.25 | 18,000 | 2.5 | 23,000 | 4.00 | 15,000 | 26 |
| 1725 D.C. | 3.25 | 18,000 | 2.5 | 23,500 | 4.00 | 14,000 | 26 |

Multi-stage compressors have been built of the following sizes.

| Cu. ft. free air per min. | Pressures. | Rated speed. |
|---------------------------|---------------|--------------|
| 22,500 | 10 to 25 lbs. | 1,800 r.p.m. |
| 8,000 | 8 to 15 lbs. | 3,750 r.p.m. |
| 3,450 | 25 to 35 lbs. | 3,450 r.p.m. |

From a curve of the load characteristics of a compressor rated at 1.7 lbs. pressure and 750 cu. ft. per min. the following figures are derived. The actual efficiency is not given:

| | | | | | | | | |
|---------------------------------------|------|------|------|------|------|------|------|------|
| Delivery, cu. ft. per min.* | 0 | 200 | 400 | 600 | 700 | 800 | 900 | 1000 |
| Discharge pressure, lbs. per sq. in. | 1.64 | 1.75 | 1.82 | 1.81 | 1.80 | 1.72 | 1.60 | 1.46 |
| Effy. per cent of maximum | 0 | 49 | 77 | 95 | 99 | 100 | 99 | 96 |

* Reduced to atmospheric pressure and 60° F.

As in the case of centrifugal pumps, the pressure depends on the peripheral velocity of the impeller. The volume of free air delivered is limited, however, by the capacity of the driver, and hence must be reduced proportionately to the increase in pressure, otherwise the driver might become overloaded.

The power required to drive centrifugal compressors varies approximately with the volume of air delivered when operating at a constant speed. This gives flexibility and economy to the centrifugal type where variable loads are required, satisfactory efficiency being obtained between the limits of 25% and 125% of the rated load.

When the compressor is operated as an exhaustor against atmospheric pressure, the rated pressure P in lbs. per square inch must be multiplied by 14.7 and then divided by $14.7 + P$. The result represents the vacuum obtained in lbs. per square inch below atmosphere.

High-Pressure Centrifugal Fans. — (A. Rateau, *Engg.*, Aug. 16, 1907.) In 1900, a single wheel fan driven by a steam turbine at 20,200 revs. per min. gave an air pressure of $8\frac{1}{4}$ lbs. per sq. in.; an output of 26.7 cu. ft. free air per second; useful work in H.P. adiabatic compression, 45.5; theoretical work in H.P. of steam-flow, 162; efficiency of the set, fan and turbine, 28%. An efficiency of 30.7% was obtained with an output of 23 cu. ft. per sec. and 132 theoretical H.P. of steam. The pressure obtained with a fan is — all things being equal — proportional to the specific weight of the gas which flows through it; therefore, if, instead of air at atmospheric pressure, air, the pressure of which has already been raised, or a gas of higher density, such as carbonic acid, be used, comparatively higher pressures still will be obtained, or the engine can run at lower speeds for the same increase of pressure.

Multiple Wheel Fans. — The apparatus having a single impeller gives satisfaction only when the duty and speed are sufficiently high. The speed is limited by the resistance of the metal of which the impeller is made, and also by the speed of the motor driving the fan. But by connecting several fans in series, as is done with high-lift centrifugal pumps, it is possible to obtain as high a pressure as may be desired.

Turbo-Compressor, Bethune Mines, 1906. — This machine compresses air to 6 and 7 atmospheres by utilizing the exhaust steam from the winding-engines. It consists of four sets of multi-cellular fans through which the air flows in succession. They are fitted on two parallel shafts, and each shaft is driven by a low-pressure turbine. A high-pressure turbine is also mounted on one of the shafts, but supplies no work in ordinary times. An automatic device divides the load equally between the two shafts. Between the two compressors are fitted refrigerators, in which cold water is made to circulate by the action of a small centrifugal pump keyed at the end of the shaft. In tests at a speed of 5000 r.p.m., the volume of air drawn per second was 31.7 cu. ft. and the discharge pressure 119.5 lb. per sq. in. absolute. These conditions of working correspond to an effective work in isothermal compression of 252 H.P. The efficiency of the compressor has been as high as 70%. The results of two tests of the compressor are given below. In the first test the air discharged, reduced to atmospheric pressure, was 26 cu. ft. per sec.; in the second test it was 46 cu. ft.

FIRST TEST.

| Stages. | 1st. | 2d. | 3d. | 4th. |
|--|-------|-------|-------|--------|
| Abs. pressure at inlet, lbs. per sq. in. ... | 15.18 | 23.37 | 38.69 | 66.44 |
| Abs. pressure at discharge | 24.10 | 39.98 | 66.44 | 102.60 |
| Speed, revs. per min. | 4660 | 4660 | 4660 | 4660 |
| Temperature of air at inlet, deg. F. ... | 57.2 | 67.8 | 63. | 66. |
| Temperature of air at discharge, deg. F. ... | 171. | 205. | 216. | 215.6 |
| Adiabatic rise in temp., deg. F. | 106. | 122. | 114.8 | 105.8 |
| Actual rise in temperature, deg. F. ... | 113.8 | 137.2 | 153. | 149.6 |
| Efficiency, per cent | 60.5 | 60.5 | 54. | 46.2 |

SECOND TEST.

| Stages. | 1st. | 2d. | 3d. | 4th. |
|--|-------|-------|-------|-------|
| Abs. pressure at inlet, lbs. per sq. in. ... | 15.18 | 21.31 | 37.33 | 65.12 |
| Abs. pressure at discharge | 23.52 | 38.22 | 65.12 | 99.66 |
| Speed, revs. per min. | 5000 | 5000 | 4840 | 4840 |
| Temp. of air at inlet, deg. F. | 55. | 69.8 | 64.4 | 68.5 |
| Temp. of air at discharge, deg. F. | 160.7 | 208.4 | 208.4 | 199.6 |
| Adiabatic rise in temp., deg. F. | 102.2 | 131. | 123.8 | 100.4 |
| Efficiency, per cent | 62.3 | 66.6 | 58.7 | 48.6 |

The Gutehoffnungshütte Co. in Germany have in course of construction several centrifugal blowing-machines to be driven by an electric motor, and up to 2000 H.P. Several machines are now being designed for Bessemer converters, some of which will develop up to 4000 H.P. The multicellular centrifugal compressors are identical in every point with centrifugal pumps. In the new machines cooling water is introduced inside the diaphragms, which are built hollow for this purpose, and also inside the diffuser vanes. By this means it is hoped to reduce proportionally the heating of the air: thus approaching isothermal compression much more nearly than is done in the case of reciprocating compressors.

Test of a Hydraulic Air Compressor. — (W. O. Webber, *Trans. A. S. M. E.*, xxii, 599.) The compressor embodies the principles of the old trompe used in connection with the Catalan forges some centuries ago, modified according to principles first described by J. P. Frizell, in *Jour. F. I.*, Sept., 1880, and improved by Charles H. Taylor, of Montreal. (Patent July 23, 1895.) It consists principally of a down-flow passage having an enlarged chamber at the bottom and an enlarged tank at the top. A series of small air pipes project into the mouth of the water inlet and the large chamber at the upper end of the vertically descending passage, so as to cause a number of small jets of air to be entrained by the water. At the lower end of the apparatus, deflector plates in connection with a gradually enlarging section of the lower end of the down-flow pipe are used to decrease the velocity of the air and water, and cause a partial separation to take place. The deflector plates change the direction of the flow of the water and are intended to facilitate the escape of the air, the water then passing out at the bottom of the enlarged chamber into an ascending shaft, maintaining upon the air a pressure due to the height of the water in the uptake, the compressed air being led on from the top of the enlarged chamber by means of a pipe. The general dimensions of the compressor plant are:

Supply penstock, 60 ins. diam.; supply tank at top, 8 ft. diam. \times 10 ft. high; air inlets (feeding numerous small tubes), 34 2-in. pipes; down tube, 44 ins. diam.; down tube, at lower end, 60 ins. diam.; length of taper in down tube, 20 ft.; air chamber in lower end of shaft, 16 ft. diam.; total depth of shaft below normal level of head water, about 150 ft.; normal head and fall, about 22 ft.; air discharge pipe, 7 ins. diam.

It is used to supply power to engines for operating the printing department of the Dominion Cotton Mills, Magog, P. Q., Canada.

There were three series of tests, viz.: (1) Three tests at different rates of flow of water, the compressor being as originally constructed. (2) Four tests at different rates of flow of water, the compressor inlet tubes for air being increased by 30 $\frac{3}{4}$ -in. pipes. (3) Four tests at different rates of flow of water, the compressor inlet tubes for air being increased by 15 $\frac{3}{4}$ -in. pipes.

The water used was measured by a weir, and the compressed air by air meters. The table on p. 623 shows the principal results:

Test 1, when the flow was about 3800 cu. ft. per min., showed a decided advantage by the use of 30 3/4-in. extra air inlet pipes. Test 5 shows, when the flow of water is about 4200 cu. ft. per min., that the economy is highest when only 15 extra air tubes are employed. Tests 8 and 9 show, when the flow is about 4600 cu. ft. per min., that there is no advantage in increasing the air-inlet area. Tests 10 and 11 show that a flow of 5000 or more cu. ft. of water is in excess of the capacity of the plant. These four tests may be summarized as follows:

The tests show: (1) That the most economic rate of flow of water with this particular installation is about 4300 cu. ft. per min. (2) That this plant has shown an efficiency of 70.7 % under such a flow, which is excellent for a first installation. (3) That the compressed air contains only from 30 to 20 % as much moisture as does the atmosphere. (4) That the air is compressed at the temperature of the water.

Using an old Corliss engine without any changes in the valve gear as a motor there was recovered 81 H.P. This would represent a total efficiency of work recovered from the falling water, of 51.2%. When the compressed air was preheated to 267° F. before being used in the engine, 111 H.P. was recovered, using 115 lbs. coke per hour, which would equal about 23 H.P. The efficiency of work recovered from the falling water and the fuel burned would be, therefore, about 61 1/2%. On the basis of Prof. Riedler's experiments, which require only about 425 cu. ft. of air per B.H.P. per hour, when preheated to 300° F. and used in a hot-air jacketed cylinder, the total efficiency secured would have been about 87 1/2%.

| Test No..... | 1 | 3 | 4 | 5 | 7 | 8 | 10 |
|--|-------|-------|-------|--------|-------|-------|-------|
| Flow of water, cu. ft. per min... | 3772 | 3623 | 4066 | 4,292 | 4408 | 4700 | 5058 |
| Available head in ft..... | 20.54 | 20.00 | 20.35 | 19.51 | 19.93 | 19.31 | 18.75 |
| Gross water, H.P..... | 146.3 | 136.5 | 156.2 | 158.1 | 165.8 | 171.4 | 179.1 |
| Cu. ft. air, at atmos. press., per minute..... | 864 | 901 | 967 | 1148 | 1091 | 1103 | 1165 |
| Pressure of air at comp., lbs..... | 51.5 | 53.7 | 53.2 | 53.3 | 53.7 | 52.9 | 53.3 |
| Effective work in compressing, H.P..... | 83.3 | 88.2 | 94.5 | 111.74 | 107 | 106.8 | 113.4 |
| Efficiency of compressor, %..... | 56.8 | 64.4 | 60.5 | 70.7 | 64.5 | 62.2 | 63.3 |
| Temp. of external air, deg. F.... | 68.3 | 57.7 | 60.4 | 55.2 | 59.7 | 65 | 64.2 |
| Temp. of water and comp. air, deg. F..... | 66 | 65.5 | 66.4 | 66.5 | 67 | 66.5 | 66 |
| Ratio of water to air, volumes... | 4.37 | 4.03 | 4.20 | 3.74 | 4.04 | 4.26 | 4.34 |
| Moisture in external air, p. c. of saturation..... | 61 | 77.5 | 71 | 68 | 90 | 60.5 | 63 |
| Moisture in comp. air, p. c. of saturation..... | 51.5 | 44 | 38.5 | 35 | 29 | 31.2 | 30 |

Tests 1, 4, and 7 were made with the original air inlets; 2, 5, 8 and 10 with the inlets increased by 15 3/4-in. pipes, and 3, 6, 9 and 11 with the inlets increased by 30 3/4-in. pipes. Tests 2, 6, 9 and 11 are omitted here. They gave, respectively, 55.5, 61.3, 62, and 55.4% efficiency.

Three other hydraulic air-compressor plants are mentioned in Mr. Webber's paper, some of the principal data of which are given below:

| | Peterboro, Ont. | Norwich, Conn. | Cascade Range, Wash. |
|--------------------------------|--------------------|-------------------|----------------------------|
| Head of water | 14 ft. | 18 1/2 ft. | 45 ft. |
| Gauge pressure | 25 lbs. | 85 lbs. | 85 lbs. |
| Diam. of shaft | 42 in. | 24 ft. | ... |
| Diam. of compressor pipe | 18 ft. | 13 ft. | 3 ft. |
| Depth below tailrace | 64 ft. | 215 ft. | ... |
| Horse-power | ... | 1365 | 200 |

In the Cascade Range plant there is no shaft, as the apparatus is constructed against the vertical walls of a canyon. The diameter of the up-flow pipe is 4 ft. 9 in.

A description of the Norwich plant is given by J. Herbert Shedd in a paper read before the New England Water Works Assn., 1905 (*Compressed Air*, April, 1906). The shaft, 24 ft. diam., is enlarged at the bottom into a chamber 52 ft. diam., from which leads an air reservoir 100 ft. long, 18 ft. wide and 15 to 20 ft. high. Suspended in the shaft is a downflow pipe 14 ft. diam. connected at the top with a head tank, and at the bottom with the air-chamber, from which a 16-in. main conveys the air four miles to Norwich, where it is used in engines in several establishments.

Pneumatic Postal Transmission.—A paper by A. Falkenau (Eng'rs Club of Philadelphia, April, 1894), entitled the "First United States Pneumatic Postal System," gives a description of the system used in London and Paris, and that recently introduced in Philadelphia between the main post-office and a substation. In London the tubes are 2 1/4 and 3-inch lead pipes laid in cast-iron pipes for protection. The carriers used in 2 1/4-inch tubes are but 1 1/4 inches diameter, the remaining space being taken up by packing. Carriers are despatched singly. First, vacuum alone was used; later, vacuum and compressed air. The tubes used in the Continental cities in Europe are wrought iron, the Paris tubes being 2 1/2 inches diameter. There the carriers are despatched in trains of six to ten, propelled by a piston. In Philadelphia the size of tube adopted is 6 1/8 inches, the tubes being of cast iron bored to size. The lengths of the outgoing and return tubes are 2928 feet each. The pressure at the main station is 7 lb., at the substation 4 lb., and at the end of the return pipe atmospheric pressure. The compressor has two air-cylinders 18 × 24 in. Each carrier holds about 200 letters, but 100 to 150 are taken as an average. Eight carriers may be despatched in a minute, giving a delivery of 48,000 to 72,000 letters per hour. The time required in transmission is about 57 seconds.

Pneumatic postal transmission tubes were laid in 1898 by the Batcheller Pneumatic Tube Co. between the general post-offices in New York and Brooklyn, crossing the East River on the Brooklyn bridge. The tubes are cast iron, 12-ft. lengths, bored to 8 1/8 in. diameter. The joints are bells, calked with lead and yarn. There are two tubes, one operating in each direction. Both lines are operated by air-pressure above the atmospheric pressure. One tube is operated by an air-compressor in the New York office and the other by one located in the Brooklyn office.

The carriers are 24 in. long, in the form of a cylinder 7 in. diameter, and are made of steel, with fibrous bearing-rings which fit the tube. Each carrier will contain about 600 ordinary letters, and they are despatched at intervals of 10 seconds in each direction, the time of transit between the two offices being 3 1/2 minutes, the carriers travelling at a speed of from 30 to 35 miles per hour.

One of the air-compressors is of the duplex type and has two steam-cylinders 10 × 20 in. and two air-cylinders 24 × 20 in., delivering 1570 cu. ft. of free air per minute, at 75 r.p.m. The power is about 50 H.P.

Two other duplex air-compressors have steam-cylinders 14 × 18 in. and air-cylinders 26 1/4 × 18 in. They are designed for 80 to 90 r.p.m. and to compress to 20 lb. per sq. in.

Another double line of pneumatic tubes has been laid between the main office and Postal Station H, Lexington Ave. and 44th St., in New York City. This line is about 3 1/2 miles in length. There are three intermediate stations. The carriers can be so adjusted when they are put into the tube that they will traverse the line and be discharged automatically from the tube at the station for which they are intended. The tubes are of the same size as those of the Brooklyn line and are operated in a similar manner. The initial air-pressure is about 12 to 15 lb. On the Brooklyn line it is about 7 lb.

There is also a tube system between the New York Post-office and the Produce Exchange. For a very complete description of the system and its machinery see "The Pneumatic Despatch Tube System," by B. C. Batcheller, J. B. Lippincott Co., Philadelphia, 1897.

The Mekarski Compressed-air Tramway at Berne, Switzerland. (*Eng'g News*, April 20, 1893.)—The Mekarski system has been introduced in Berne, Switzerland, on a line about two miles long, with grades of 0.25% to 3.7% and 5.2%. The air is heated by passing it through superheated water at 330° F. It thus becomes saturated with steam, which subsequently partly condenses, its latent heat being absorbed by the expanding air. The pressure in the car reservoirs is 440 lb. per sq. in.

The engine is constructed like an ordinary steam tramway locomotive, and drives two coupled axles, the wheel-base being 5.2 ft. It has a pair of outside horizontal cylinders, 5.1×8.6 in.; four coupled wheels, 27.5 in. diameter. The total weight of the car including compressed air is 7.25 tons, and with 30 passengers, including the driver and conductor, about 9.5 tons.

The authorized speed is about 7 miles per hour. Taking the resistance due to the grooved rails and to curves under unfavorable conditions at 30 lb. per ton of car weight, the engine has to overcome on the steepest grade, 5%, a total resistance of about 0.63 ton, and has to develop 25 H.P. At the maximum authorized working pressure in cylinders of 176 lb. per sq. in. the motors can develop a tractive force of 0.64 ton. This maximum is, therefore, just sufficient to take the car up the 5.2% grade, while on the flatter sections of the line the working pressure does not exceed 73 to 147 lb. per sq. in. Sand has to be frequently used to increase the adhesion on the 2% to 5% grades.

Between the two car frames are suspended ten horizontal compressed-air storage-cylinders, varying in length according to the available space, but of uniform inside diameter of 17.7 in., composed of riveted 0.27-in. sheet iron, and tested up to 588 lb. per sq. in., and having a collective capacity of 64.25 cu. ft., and two further small storage-cylinders of 5.3 cu. ft. capacity each, a total capacity for the 12 storage-cylinders per car of 75 cu. ft., divided into two groups, the working and the reserve battery, of 49 cu. ft. and 26 cu. ft. capacity respectively.

From the results of six official trips, the pressure and the mean consumption of air during a double trip per motor car are as follows:

Pressure of air in storage-cylinders at starting, 440 lb. per sq. in.; at end of up-trip, 176 lb., reserve, 260 lb.; at end of down-trip, 103 lb., reserve, 176 lb. Consumption of air during up-trip, 92 lb., during down-trip, 31 lb. The working experience of 1891 showed that the air consumption per motor car for a double trip was from 103 to 154 lb., mean 123 lb., and per car mile from 28 to 42 lb., mean 35 lb.

The disadvantages of this system consist in the extremely delicate adjustment of the different parts of the system, in the comparatively small supply of air carried by one motor car, which necessitates the car returning to the depot for refilling after a run of only four miles or 40 minutes, although on the Nogent and Paris lines the cars, which are, moreover, larger, and carry outside passengers on the top, run seven miles, and the loading pressure is 547 lb. per sq. in. as against only 440 lb. at Berne.

For description of the Mekarski system as used at Nantes, France, see paper by Prof. D. S. Jacobus, *Trans. A. S. M. E.*, xix, 553.

American Experiments on Compressed Air for Street Railways.

— Experiments have been made in Washington, D. C., and in New York City on the use of compressed air for street-railway traction. The air was compressed to 2000 lb. per sq. in. and passed through a reducing-valve and a heater before being admitted to the engine. The system has since been abandoned. For an extended discussion of the relative merits of compressed air and electric traction, with an account of a test of a four-stage compressor giving a pressure of 2500 lb. per sq. in., see *Eng'g News*, Oct. 7 and Nov. 4, 1897. A summarized statement of the probable efficiency of compressed-air traction is given as follows: Efficiency of compression to 2000 lb. per sq. in. 65%. By wire-drawing to 100 lbs. 57.5% of the available energy of the air will be lost, leaving $65 \times 0.425 = 27.625\%$ as the net efficiency of the air. This may be doubled by heating, making 55.25%, and if the motor has an efficiency of 80% the net efficiency of traction by compressed air will be $55.25 \times 0.80 = 44.2\%$. For a description of the Hardie compressed-air locomotive, designed for street-railway work, see *Eng'g News*, June 24, 1897. For use of compressed air in mine haulage, see *Eng'g News*, Feb. 10, 1898.

Operation of Mine Pumps by Compressed Air. — The advantages of compressed air over steam for the operation of mine pumps are: Absence of condensation and radiation losses in pipe lines; high efficiency of compressed-air transmission; ease of disposal of exhaust; absence of danger from broken pipes. The disadvantage is that, at a given initial pressure without reheating, a cylinder full of air develops less power than steam. The power end of the pump should be designed for the use of air, with low clearances and with proper proportions of air and water ends, with regard to the head under which the pump is to operate. Wm. Cox (*Comp.*

Air Mag., Feb., 1899) states the relations of simple or single-cylinder pumps to be $A/W = 1/2 h/p$, where A = area of air cylinder, sq. in., W = area of water cylinder, sq. in., h = head, ft., and p = air pressure, lb. per sq. in. Mr. Cox gives the volume V of free air in cu. ft. per minute to operate a direct-acting, single-cylinder pump, working without cut off, to be

$$V = 0.093 W_2 hG/P.$$

Where W_2 = volume of 1 cu. ft. of free air corresponding to 1 cu. ft. of free air at pressure P , G = gallons of water to be raised per minute, P = receiver-gauge pressure of air to be used, and h = head in feet under which pump works. This formula is based on a piston speed of 100 ft. per minute and 15% has been added to the volume of air to cover losses. The useful work done in a pump using air at full pressure is greater at low pressures than at high, and the efficiency is increased. High pressures are not so economical for simple pumps as low pressures. As high-pressure air is required for drills, etc., and as the air for pumps is drawn from the same main, the air must either be wire-drawn into the pumps, or a reducing valve be inserted between the pump and main. Wire-drawing causes a low efficiency in the pump. If a reducing valve is used, the increase of volume will be accompanied with a drop in temperature, so that the full value of the increase is not realized. Part of the lost heat may be regained by friction, and from external sources. The efficiency of the system may be increased by the use of underground receivers for the expanded air before it passes to the pump. If the receiver be of ample size, the air will regain nearly its normal temperature, the entrained moisture will be deposited and freezing troubles avoided. By compounding the pumps, the efficiency may be increased to about 25 per cent. In simple pumps it ranges from 7 to 16 per cent. For much further information on this subject, see Peele's "Compressed-Air Plant for Mines," 1908.

FANS AND BLOWERS.

Centrifugal Fans. — The ordinary centrifugal fan consists of a number of blades fixed to arms revolving at high speed. The width of the blade is parallel to the shaft. The experiments of W. Buckle (*Proc. Inst. M. E.*, 1847) are often quoted as still standard. Mr. Buckle's conclusions, however, do not agree with those of modern experimenters, nor do the proportions of fans as determined by him have any similarity to those of modern fans. His results are presented here merely for purposes of reference and comparison. The experiments were made on fans of the "paddle-wheel" type, and have no bearing on the more modern multivane fans of the "Sirocco" type.

From his experiments Mr. Buckle deduced the following proportions for a fan: 1. The width of the vanes should be one-fourth the diameter; 2. The diameter of the inlet opening in the sides of the fan chest should be one-half the diameter of the fan; 3. The length of the vanes should be one-fourth the diameter of the fan. These rules do not agree with those adopted by modern manufacturers, nor do the rules adopted by different manufacturers agree among themselves. An examination of 18 commercial sizes of fans, of the ordinary steel-plate type, built by two prominent manufacturers, A and B , shows the following proportions based on the diameter of the fan wheel, D , in inches:

PROPORTIONS OF FANS, RECTANGULAR BLADES.

| | A Max. | A Min. | A Av. | B Max. | B Min. | B Av. | Buckle. |
|------------------|-----------|-----------|----------|-----------|-----------|----------|---------|
| Diam. inlet ... | 0.666D | 0.618D | 0.636D | 0.495D | 0.430D | 0.476D | 0.5D |
| Width of blade . | 0.435D | 0.380D | 0.398D | 0.366D | 0.333D | 0.356D | 0.25D |

The rules laid down by Buckle do not give a fan the highest commercial efficiency without loss of mechanical efficiency. By commercial efficiency is meant the ratio of the volume of air delivered per revolution to the cubical contents of the wheel, if the wheel be considered a solid whose dimensions are those of the wheel. This ratio is also known as the *volumetric efficiency*. Inasmuch as the loss due to friction of the air entering the fan will be less with a large inlet than with a small one, in a wheel of

given diameter, more power will be consumed in delivering a given volume of air with a small inlet than with a larger one.

In the ordinary fan the number of vanes varies from 4 to 8, while with multivane fans it is 60 or more. The number of vanes has a direct relation to the size of the inlet. This is made as large as possible for the reason given above. Any increase in the diameter of the inlet necessarily decreases the depth of the blade, thus diminishing the capacity and pressure. To overcome this decrease, the number of blades is increased to the limit placed by constructional considerations. A properly proportioned fan is one in which a balance is obtained between these two features of maximum inlet and maximum number of blades. Generally speaking, in a purely centrifugal fan, increased pressure is obtained with the increase in depth of the blade. This appears to be due to the greater area of blade working on the air. A smaller wheel, with a greater number of blades, aggregating a larger blade area, gives a higher pressure than a larger wheel with less total blade area.

In some cases two fans mounted on one shaft may be more useful than a single wide one, as in such an arrangement twice the area of inlet opening is obtained, as compared with a single wide fan. Such an arrangement may be adopted where occasionally half the full quantity of air is required, as one of the fans may be put out of gear and thus save power.

Rules for Fan Design. — It is impossible to give any general rules or formulæ covering the proportions of parts of fans and blowers. There are no less than 14 variables involved in the construction and operation of fans, a slight change in any one producing wide variations in the performance. The design of a new fan by manufacturers is largely a matter of trial and error, based on experiments, until a compromise with all the variables is obtained which most nearly conforms to the given conditions.

Pressure Due to Velocity of the Fan Blades. — The pressure of the air due to the velocity of the fan blades may be determined by the formula

$H = \frac{v^2}{2g}$, deduced from the law of falling bodies, in which H is the "head"

or height of a homogeneous column of air one-inch square whose weight is equal to the pressure per square inch of the air leaving the fan, v is the velocity of the air leaving the fan in feet per second, and g the acceleration due to gravity. The pressure of the air is increased by increasing the number of revolutions per minute of the fan. Wolff, in his "The Windmill as a Prime Mover," p. 17, argues that it is an error to take $H = v^2 \div 2g$, the formula according to him being $H = v^2 \div g$. See also Trowbridge (*Trans. A. S. M. E.*, vii., 536). This law is analogous to that of the pressure of a fluid jet striking a plane surface perpendicularly and escaping at right angles to its original path, this pressure being twice that due the height calculated from the formula $h = v^2 \div 2g$. (See Hawksley, *Proc. Inst. M. E.*, 1882.) Later authorities and manufacturers, however, base all their calculations on the former formula.

Buckle says: "From the experiments it appears that the velocity of the tips of the fan is equal to nine-tenths of the velocity a body would acquire in falling the height of a homogeneous column of air equivalent to the density." D. K. Clark (*R. T. & D.*, p. 924), paraphrasing Buckle, apparently, says: "It further appears that the pressure generated at the circumference is one-ninth greater than that which is due to the actual circumferential velocity of the fan." The two statements, however, are not in

harmony, for if $v = 0.9\sqrt{2gH}$, $H = \frac{v^2}{0.81 \times 2g} = 1.234 \frac{v^2}{2g}$ and not $\frac{10}{9} \frac{v^2}{2g}$.

If we take the pressure as that equal to a head or column of air of twice the height due the velocity, as stated by Trowbridge, the paradoxical statements of Buckle and Clark — which would indicate that the actual pressure is greater than the theoretical — are explained, and the formula

becomes $H = 0.617 \frac{v^2}{g}$ and $v = 1.273 \sqrt{gH} = 0.9 \sqrt{2gH}$, in which H is

the head of a column producing the pressure, which is equal to twice the theoretical head due the velocity of a falling body ($h = v^2/2g$), multiplied by the coefficient 0.617. The difference between 1 and this coefficient expresses the loss of pressure due to friction, to the fact that the inner portions of the blade have a smaller velocity than the outer edge, and probably to other causes. The coefficient 1.273 means that the tip of the blade must be given a velocity 1.273 times that theoretically required to produce the head H .

Commenting on the above paragraphs and the formulæ below, the B. F. Sturtevant Co., in a letter to the author, says: "Let us assume that the fan considered is of the centrifugal type, which is a wheel in a spiral casing. In any case of centrifugal fan the pressure at the fan outlet is wholly dependent upon the load on the fan, and, therefore, the pressure cannot well be expressed by a formula, unless it includes some term which is an expression in some way of the load upon the fan. The actual pressure depends upon the design of both wheel and housing, upon the blade area and also upon the form of the blades. With a curved blade running with the concave side forward it is possible to obtain a much higher pressure than if the blade is running with the convex side forward. This can only be shown by tests, and can be figured out by blade-velocity diagrams."

It should be noted, however, that while the fan with a blade concaved in the direction of rotation has the highest efficiency, all other things being equal, the noise of operation is increased. A blade convex in the direction of rotation runs more quietly, and in most situations it is necessary to sacrifice efficiency in order to obtain quiet operation.

To convert the head H expressed in feet to pressure in lb. per sq. in. multiply it by the weight of a cubic foot of air at the pressure and temperature of the air expelled from the fan (about 0.08 lb. usually) and divide by 144. Multiply this by 16 to obtain pressure in ounces per sq. in. or by 2.035 to obtain inches of mercury, or by 27.71 to obtain pressure in inches of water column. Taking 0.08 as the weight of 1 cu. ft. of air, and $v = 0.9 \sqrt{2 gH}$,

$$p \text{ lb. per sq. in.} = 0.00001066 v^2; v = 310 \sqrt{p} \text{ nearly};$$

$$p_1 \text{ ounces per sq. in.} = 0.0001706 v^2; v = 80 \sqrt{p_1} \text{ "}$$

$$p_2 \text{ inches of mercury} = 0.00002169 v^2; v = 220 \sqrt{p_2} \text{ "}$$

$$p_3 \text{ inches of water} = 0.0002954 v^2; v = 60 \sqrt{p_3} \text{ "}$$

in which v = velocity of tips of blades in feet per second.

Testing the above formula by one of Buckle's experiments with a vane 14 inches long, we have $p = 0.00001066 v^2 = 9.56 \text{ oz.}$ The experiment gave 9.4 oz.

Testing it by the experiment of H. I. Snell, given below, in which the circumferential speed was about 150 ft. per second, we obtain 3.85 ounces, while the experiment gave from 2.38 to 3.50 ounces, according to the amount of opening for discharge.

Taking the formula $v = 80 \sqrt{p_1}$, we have for different pressures in ounces per square inch the following velocities of the tips of the blades in feet per second:

| | | | | | | | | | | |
|---------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| p_1 = ounces per square inch. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 | 14 |
| v = feet per second..... | 113 | 139 | 160 | 179 | 196 | 212 | 226 | 253 | 277 | 299 |

A rule in *App. Cyc. Mech.*, article "Blowers," gives the following velocities of circumference for different densities of blast in ounces: 3, 170; 4, 180; 5, 195; 6, 205; 7, 215.

The same article gives the following tables, the first of which shows that the density of blast is not constant for a given velocity, but depends on the ratio of area of nozzle to area of blades:

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| Velocity of circumference, feet per second... | 150 | 150 | 150 | 170 | 200 | 200 | 220 |
| Area of nozzle ÷ area of blades..... | 2 | 1 | 1/2 | 1/4 | 1/2 | 1/6 | 1/8 |
| Density of blast, oz. per square inch..... | 1 | 2 | 3 | 4 | 4 | 6 | 6 |

QUANTITY OF AIR OF A GIVEN DENSITY DELIVERED BY A FAN.

Total area of nozzles in square feet × velocity in feet per minute corresponding to density (see table) = air delivered in cubic feet per minute, discharging freely into the atmosphere (approximate). See p. 642.

| Density, ounces per sq. in. | Velocity, feet per minute. | Density, ounces per sq. in. | Velocity, feet per minute. | Density, ounces per sq. in. | Velocity, feet per minute. |
|-----------------------------------|----------------------------------|-----------------------------------|----------------------------------|-----------------------------------|----------------------------------|
| 1 | 5,000 | 5 | 11,000 | 9 | 15,000 |
| 2 | 7,000 | 6 | 12,250 | 10 | 15,800 |
| 3 | 8,600 | 7 | 13,200 | 11 | 16,500 |
| 4 | 10,000 | 8 | 14,150 | 12 | 17,300 |

"Blast Area," or "Capacity Area." When the fan outlet is small the velocity of the outflow is equal to the peripheral velocity of the fan.

Start with the outlet closed; then if the opening be slowly increased while the speed of the fan remains constant the air will continue to flow with the same velocity as the fan tips until a certain size of outlet is reached. If the outlet is still further increased the pressure within the casing will drop, and the velocity of outflow will become less than the tip velocity. The size of the outlet at which this change takes place is called the *blast area*, or *capacity area*, of the fan. This varies somewhat with different types and makes of fans, but for the common form of blower it is approximately, $DW \div 3$, in which D is the diameter of the fan wheel and W its width at the circumference. — (C. L. Hubbard.)

This established capacity area has no relation to the area of the outlet in the casing, which may be of any size, but is usually about twice the capacity area. The velocity of the air discharged through this latter area is practically that of the circumference of the wheel, and the pressure created is that corresponding thereto. — W. B. Snow.

Experiments with Blowers. (Henry I. Snell, *Trans. A. S. M. E.*, ix. 51.) — The following tables give velocities of air discharging through an aperture of any size under the given pressures into the atmosphere. The volume discharged can be obtained by multiplying the area of discharge opening by the velocity, and this product by the coefficient of contraction: 0.65 for a thin plate and 0.93 when the orifice is a conical tube with a convergence of about 3.5 degrees, as determined by the experiments of Weisbach.

The tables are calculated for a barometric pressure of 14.69 lb. (= 235 oz.), and for a temperature of 50° Fahr., from the formula $V = \sqrt{2gh}$.

Allowances have been made for the effect of the compression of the air, but none for the heating effect due to the compression.

At a temperature of 50 degrees, a cubic foot of air weighs 0.078 lb., and calling $g = 32.1602$, the above formula may be reduced to

$$V_1 = 60 \sqrt{31.5812 \times (235 + P) \times P},$$

where V_1 = velocity in feet per minute, P = pressure above atmosphere, or the pressure shown by gauge, in oz. per square inch.

| Pressure per sq. in., in. of water. | Corre- sponding Pressure, oz. per sq. in. | Velocity due to Pressure, ft. per min. | Pressure per sq. in., in. of water. | Corre- sponding Pressure, oz. per sq. in. | Velocity due to Pressure, ft. per min. |
|---|---|--|---|---|--|
| 1/32 | 0.01817 | 696.78 | 5/8 | 0.36340 | 3118.38 |
| 1/16 | 0.03634 | 987.66 | 3/4 | 0.43608 | 3416.64 |
| 1/8 | 0.07268 | 1393.75 | 7/8 | 0.50870 | 3690.62 |
| 3/16 | 0.10902 | 1707.00 | 1 | 0.58140 | 3946.17 |
| 1/4 | 0.14536 | 1971.30 | 1 1/4 | 0.7267 | 4362.62 |
| 5/16 | 0.18170 | 2204.16 | 1 1/2 | 0.8721 | 4836.06 |
| 3/8 | 0.21804 | 2414.70 | 1 3/4 | 1.0174 | 5224.98 |
| 1/2 | 0.29072 | 2788.74 | 2 | 1.1628 | 5587.58 |

| Pres- sure, oz. per sq. in. | Velocity due to Pressure, ft. per min. | Pres- sure, oz. per sq. in. | Velocity due to Pressure, ft. per min. | Pres- sure, oz. per sq. in. | Velocity due to Pressure, ft. per min. | Pres- sure, oz. per sq. in. | Velocity due to Pressure, ft. per min. |
|--------------------------------------|--|--------------------------------------|--|--------------------------------------|--|--------------------------------------|--|
| 0.25 | 2,582 | 2.25 | 7,787 | 5.50 | 12,259 | 11.00 | 17,534 |
| 0.50 | 3,658 | 2.50 | 8,213 | 6.00 | 12,817 | 12.00 | 18,350 |
| 0.75 | 4,482 | 2.75 | 8,618 | 6.50 | 13,354 | 13.00 | 19,138 |
| 1.00 | 5,178 | 3.00 | 9,006 | 7.00 | 13,873 | 14.00 | 19,901 |
| 1.25 | 5,792 | 3.50 | 9,739 | 7.50 | 14,374 | 15.00 | 20,641 |
| 1.50 | 6,349 | 4.00 | 10,421 | 8.00 | 14,861 | 16.00 | 21,360 |
| 1.75 | 6,861 | 4.50 | 11,065 | 9.00 | 15,795 | | |
| 2.00 | 7,338 | 5.00 | 11,676 | 10.00 | 16,684 | | |

| Pressure in ounces per square inch. | Velocity in feet per minute. | Pressure in ounces per square inch. | Velocity in feet per minute. |
|-------------------------------------|------------------------------|-------------------------------------|------------------------------|
| 0.01 | 516.90 | 0.06 | 1266.24 |
| 0.02 | 722.64 | 0.07 | 1367.76 |
| 0.03 | 895.26 | 0.08 | 1462.20 |
| 0.04 | 1033.86 | 0.09 | 1550.70 |
| 0.05 | 1155.90 | 0.10 | 1635.00 |

**Experiments on a Fan with Varying Discharge-opening.
Revolutions nearly constant.**

| Revolutions per minute. | Area of Discharge in square inches. | Observed Pressure in ounces. | Volume of Air discharged per min., cubic feet. | Horse-power. | Actual Number of cu.ft. of Air delivered per H.P. | Theoret. Vol. per min. that may be discharged with 1 H.P. at corresp. Pressure. | Efficiency of Blowers as per experiment. |
|-------------------------|-------------------------------------|------------------------------|--|--------------|---|---|--|
| 1519 | 0 | 3.50 | 0 | 0.80 | | 1048 | |
| 1479 | 6 | 3.50 | 406 | 1.15 | 353 | 1048 | 0.337 |
| 1480 | 10 | 3.50 | 676 | 1.30 | 520 | 1048 | 0.496 |
| 1471 | 20 | 3.50 | 1353 | 1.95 | 694 | 1048 | 0.66 |
| 1485 | 28 | 3.50 | 1894 | 2.55 | 742 | 1048 | 0.709 |
| 1485 | 36 | 3.40 | 2400 | 3.10 | 774 | 1078 | 0.718 |
| 1465 | 40 | 3.25 | 2605 | 3.30 | 790 | 1126 | 0.70 |
| 1468 | 44 | 3.00 | 2752 | 3.55 | 775 | 1222 | 0.635 |
| 1500 | 48 | 3.00 | 3002 | 3.80 | 790 | 1222 | 0.646 |
| 1426 | 89.5 | 2.38 | 3972 | 4.80 | 827 | 1544 | 0.536 |

The fan wheel was 23 in. diam., $6\frac{5}{8}$ in. wide at its periphery, and had an inlet $12\frac{1}{2}$ in. diam. on either side, which was partially obstructed by the pulleys, which were $5\frac{9}{16}$ in. diam. It had eight blades, each of an area of 45.49 sq. in. The discharge of air was through a conical tin tube with sides tapered at an angle of $31\frac{1}{2}$ degrees. The actual area of opening was 7% greater than given in the tables, to compensate for the *vena contracta*.

In the last experiment, 89.5 sq. in. represents the actual area of the mouth of the blower less a deduction for a narrow strip of wood placed across it for the purpose of holding the pressure-gauge. In calculating the volume of air discharged in the last experiment the value of *vena contracta* is taken at 0.80.

Experiments were undertaken for the purpose of showing the results obtained by running the same fan at different speeds with the discharge-opening the same throughout the series. The discharge-pipe was a conical tube $8\frac{1}{2}$ in. inside diam. at the end, having an area of 56.74 sq. in., which is 7% larger than 53 sq. in.; therefore 53 sq. in., equal to 0.368 square feet, is called the area of discharge, as that is the practical area by which the volume of air is computed.

Experiments on a Fan with Constant Discharge-opening and Varying Speed. — The first four columns are given by Mr. Snell, the others are calculated by the author.

| Revs. per min. | Pressure in ounces, p | Vol. of Air in cu. ft. per minute, V . | Horse-power. | Velocity of Tips of Blades, ft. per sec. | Velocity due Pres- sure From For- mula $v = 80 \sqrt{p}$. | Coefficient of for- mula $v = x \sqrt{p}$ from Experiment. | Velocity of Air per minute in Efflux Pipe, $V \div 0.368$. | Theoretical Horse- power. | Efficiency per cent. |
|----------------|----------------------------|---|--------------|---|--|--|---|------------------------------|----------------------|
| 600 | 0.50 | 1336 | 0.25 | 60.2 | 56.6 | 85.1 | 3,630 | 0.182 | 73 |
| 800 | 0.88 | 1787 | 0.70 | 80.3 | 75.0 | 85.6 | 4,856 | 0.429 | 61 |
| 1000 | 1.38 | 2245 | 1.35 | 100.4 | 94 | 85.4 | 6,100 | 0.845 | 63 |
| 1200 | 2.00 | 2712 | 2.20 | 120.4 | 113 | 85.1 | 7,370 | 1.479 | 67 |
| 1400 | 2.75 | 3177 | 3.45 | 140.5 | 133 | 84.8 | 8,633 | 2.283 | 66 |
| 1600 | 3.80 | 3670 | 5.10 | 160.6 | 156 | 82.4 | 9,973 | 3.803 | 74 |
| 1800 | 4.80 | 4172 | 8.00 | 180.6 | 175 | 82.4 | 11,337 | 5.462 | 68 |
| 2000 | 5.95 | 4674 | 11.40 | 200.7 | 195 | 85.6 | 12,701 | 7.586 | 67 |

Mr. Snell has not found any practical difference between the mechanical efficiencies of blowers with curved blades and those with straight radial ones. From these experiments, says Mr. Snell, it appears that we may expect to receive back 65% to 75% of the power expended, and no more. The great amount of power often used to run a fan is not due to the fan itself, but to the method of selecting, erecting, and piping it. (For opinions on the relative merits of fans and positive rotary blowers, see discussion of Mr. Snell's paper, *Trans. A. S. M. E.*, ix. 66, etc.)

Comparative Efficiency of Fans and Positive Blowers. (H. M. Howe, *Trans. A. I. M. E.*, x. 482.) — Experiments with fans and positive (Baker) blowers working at moderately low pressures, under 20 ounces, show that they work more efficiently at a given pressure when delivering large volumes (*i.e.*, when working nearly up to their maximum capacity) than when delivering comparatively small volumes. Therefore, when great variations in the quantity and pressure of blast required are liable to arise, the highest efficiency would be obtained by having a number of blowers, always driving them up to their full capacity, and regulating the amount of blast by altering the number of blowers at work, instead of having one or two very large blowers and regulating the amount of blast by the speed of the blowers.

There appears to be little difference between the efficiency of fans and of Baker blowers when each works under favorable conditions as regards quantity of work, and when each is in good order.

For a given speed of fan any diminution in the size of the blast-orifice decreases the consumption of power and at the same time raises the pressure of the blast; but it increases the consumption of power per unit of orifice for a given pressure of blast. When the orifice has been reduced to the normal size for any given fan, further diminishing it causes but slight elevation of the blast pressure; and, when the orifice becomes comparatively small, further diminishing it causes no sensible elevation of the blast pressure, which remains practically constant, even when the orifice is entirely closed.

Many of the failures of fans have been due to too low speed, to too small pulleys, to improper fastening of belts, or to the belts being too nearly vertical in brief, to bad mechanical arrangement, rather than to inherent defects in the principles of the machine.

If several fans are used, it is probably essential to high efficiency to provide a separate blast pipe for each (at least if the fans are of different size or speed), while any number of positive blowers may deliver into the same pipe without lowering their efficiency.

Capacity of Fans and Blowers.—The following tables supplied (1909) by the American Blower Co., Detroit, show the capacities of exhaust fans and volume and pressure blowers. The tables are all based on curves established by experiment. The pressures, volumes and horse-powers were all actually measured with the apparatus working against maintained resistances formed by restrictions equivalent to those found in actual practice, and which experience shows will produce the best results.

Speed, Capacity and Horse-power of Steel Plate Exhaust Fans.

(American Blower Co., Type E, 1908.)

| No. of fan. | Diameter of wheel, in. | Width periphery, in. | Diameter inlet (inside), in. | 1/2 oz. pressure. | | | 3/4 oz. pressure. | | | 1 oz. pressure. | | | 2 oz. pressure. | | |
|-------------|------------------------|----------------------|------------------------------|-------------------|-----------------------|--------------------|-------------------|-----------------------|--------------------|-----------------|-----------------------|--------------------|-----------------|-----------------------|--------------------|
| | | | | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. |
| 25 | 16 | 61/8 | 10 | 985 | 1.09 | 0.30 | 1200 | 1,345 | 0.56 | 1390 | 1,555 | 0.85 | 1966 | 2,200 | 2.40 |
| 30 | 19 | 71/8 | 12 | 830 | 1,580 | 0.43 | 1012 | 1,940 | 0.80 | 1170 | 2,240 | 1.22 | 1655 | 3,175 | 3.46 |
| 35 | 22 | 81/8 | 14 | 715 | 2,155 | 0.59 | 876 | 2,635 | 1.08 | 1010 | 3,040 | 1.66 | 1430 | 4,310 | 4.70 |
| 40 | 25 | 93/8 | 16 | 630 | 2,820 | 0.77 | 772 | 3,450 | 1.41 | 1890 | 3,980 | 2.17 | 1260 | 5,640 | 6.15 |
| 45 | 28 | 107/8 | 18 | 563 | 3,560 | 0.97 | 689 | 4,360 | 1.78 | 1795 | 5,030 | 2.74 | 1125 | 7,140 | 7.79 |
| 50 | 31 | 123/8 | 20 | 508 | 4,400 | 1.20 | 622 | 5,390 | 2.20 | 1719 | 6,220 | 3.39 | 1015 | 8,820 | 9.63 |
| 55 | 34 | 131/2 | 22 | 464 | 5,330 | 1.45 | 567 | 6,525 | 2.66 | 1655 | 7,530 | 4.10 | 927 | 10,650 | 11.60 |
| 60 | 38 | 141/2 | 24 | 415 | 6,350 | 1.73 | 509 | 7,775 | 3.18 | 1587 | 8,960 | 4.89 | 830 | 12,700 | 13.85 |
| 70 | 44 | 151/8 | 27 | 375 | 7,440 | 2.02 | 459 | 9,120 | 3.72 | 1530 | 10,500 | 5.72 | 750 | 14,875 | 16.20 |
| 80 | 50 | 161/2 | 29 | 328 | 10,050 | 2.75 | 402 | 12,100 | 4.94 | 1464 | 13,980 | 7.62 | 656 | 19,800 | 21.60 |

Speed, Capacity and Horse-power of Volume Blowers.

(American Blower Co., Type V, 1909.)

| No. of fan. | Diameter of wheel, in. | Width periphery, in. | Diameter inlet (inside), in. | 1/2 oz. pressure. | | | 3/4 oz. pressure. | | | 1 oz. pressure. | | | 1 1/2 oz. pressure. | | |
|-------------|------------------------|----------------------|------------------------------|-------------------|-----------------------|--------------------|-------------------|-----------------------|--------------------|-----------------|-----------------------|--------------------|---------------------|-----------------------|--------------------|
| | | | | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. | R.P.M. | Cubic ft. per minute. | Brake horse-power. |
| 1 | 81/2 | 2 | 41/2 | 1850 | 223 | 0.06 | 2270 | 273 | 0.11 | 2620 | 315 | 0.17 | 3210 | 386 | 0.32 |
| 2 | 101/4 | 23/8 | 51/2 | 1535 | 332 | 0.09 | 1880 | 407 | 0.17 | 2170 | 469 | 0.26 | 2660 | 576 | 0.48 |
| 3 | 12 | 31/4 | 61/2 | 1310 | 464 | 0.13 | 1600 | 569 | 0.23 | 1850 | 656 | 0.36 | 2275 | 805 | 0.66 |
| 4 | 151/2 | 43/2 | 81/2 | 1015 | 795 | 0.22 | 1240 | 975 | 0.40 | 1435 | 1122 | 0.61 | 1760 | 1377 | 1.13 |
| 5 | 19 | 51/8 | 103/8 | 830 | 1185 | 0.32 | 1013 | 1450 | 0.59 | 1170 | 1675 | 0.92 | 1435 | 2055 | 1.68 |
| 6 | 221/2 | 61/2 | 123/8 | 700 | 1686 | 0.46 | 858 | 2065 | 0.84 | 990 | 2385 | 1.30 | 1215 | 2930 | 2.40 |
| 7 | 26 | 71/2 | 141/4 | 606 | 2235 | 0.61 | 742 | 2740 | 1.12 | 858 | 3160 | 1.72 | 1050 | 3880 | 3.18 |
| 8 | 291/2 | 81/2 | 161/4 | 534 | 2910 | 0.79 | 654 | 3560 | 1.45 | 755 | 4110 | 2.24 | 928 | 5040 | 4.13 |
| 9 | 33 | 91/2 | 181/4 | 477 | 3660 | 1.00 | 585 | 4490 | 1.83 | 675 | 5175 | 2.82 | 825 | 6350 | 5.20 |

NOTE: This table also applies to Type V, cast-iron exhaust fans.

Steel Pressure Blowers for Cupolas (Average Application).

(American Blower Co., 1909.)

| No. of blower. | Dia. of wheel. in. | Width periph'y. in. | Circum. of wheel, ft. | Dia. outlet pipes, in. | Area of outlet, sq. ft. | Oz. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------------|-----------------------|------------------------|--------------------------|---------------------------|----------------------------|--------------------------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|
| | | | | | | In. | 3.46 | 5.19 | 6.92 | 8.65 | 10.38 | 12.12 | 13.83 | 15.56 |
| | | | | | | H.P. const. at 1000 cu. ft. | 1.242 | 1.86 | 2.48 | 3.10 | 3.73 | 4.35 | 4.95 | 5.58 |
| 1 | 14 1/2 | 1 3/8 | 3.80 | 5 3/4 | 0.18 | R.P.M. C.F. H.P. | 1960 361 0.45 | 2400 434 0.81 | 2770 500 1.24 | 3095 560 1.74 | 3390 610 2.28 | 3666 665 2.89 | 3915 708 3.51 | 4150 752 4.20 |
| 2 | 17 | 1 5/8 | 4.45 | 6 3/4 | 0.2485 | R.P.M. C.F. H.P. | 1675 498 0.62 | 2050 600 1.12 | 2362 691 1.72 | 2645 774 2.40 | 2895 843 3.15 | 3130 916 3.99 | 3340 978 4.84 | 3540 1038 5.79 |
| 3 | 19 1/2 | 1 7/8 | 5.11 | 7 3/4 | 0.327 | R.P.M. C.F. H.P. | 1460 655 0.82 | 1785 789 1.47 | 2060 910 2.26 | 2300 1018 3.16 | 2520 1110 4.15 | 2730 1207 5.25 | 2910 1286 6.36 | 3085 1365 7.62 |
| 4 | 22 | 2 1/8 | 5.76 | 8 3/4 | 0.4176 | R.P.M. C.F. H.P. | 1292 838 1.04 | 1582 1006 1.87 | 1825 1162 2.88 | 2040 1300 4.03 | 2235 1415 5.28 | 2420 1540 6.70 | 2585 1643 8.14 | 2740 1746 9.74 |
| 5 | 24 1/2 | 2 3/8 | 6.41 | 9 3/4 | 0.519 | R.P.M. C.F. H.P. | 1162 1040 1.30 | 1422 1250 2.33 | 1640 1442 3.58 | 1835 1612 5.00 | 2010 1760 6.57 | 2175 1915 8.34 | 2320 2040 10.10 | 2460 2166 12.10 |
| 6 | 27 | 2 7/8 | 7.06 | 10 3/4 | 0.63 | R.P.M. C.F. H.P. | 1055 1262 1.57 | 1290 1520 2.83 | 1490 1750 4.34 | 1665 1960 6.08 | 1825 2135 7.96 | 1975 2375 10.10 | 2105 2475 12.25 | 2233 2630 14.12 |
| 7 | 32 | 3 3/8 | 8.39 | 12 1/2 | 0.852 | R.P.M. C.F. H.P. | 889 1705 2.12 | 1087 2055 3.83 | 1255 2366 5.86 | 1405 2650 8.23 | 1535 2890 10.78 | 1660 3140 13.66 | 1775 3350 16.60 | 1880 3555 19.83 |
| 8 | 37 | 3 7/8 | 9.70 | 14 | 1.069 | R.P.M. C.F. H.P. | 769 2140 2.66 | 940 2575 4.79 | 1085 2970 7.36 | 1212 3325 10.3 | 1328 3620 13.5 | 1446 3940 17.15 | 1533 4200 20.00 | 1625 4460 24.90 |
| 9 | 42 | 4 3/8 | 10.98 | 16 | 1.396 | R.P.M. C.F. H.P. | 679 2800 3.48 | 830 3370 6.27 | 958 3880 9.63 | 1072 4340 13.46 | 1172 4730 17.65 | 1270 5150 22.40 | 1355 5500 27.25 | 1435 5825 32.50 |
| 10 | 47 | 4 7/8 | 12.30 | 17 1/2 | 1.67 | R.P.M. C.F. H.P. | 606 3350 4.17 | 742 4025 7.5 | 855 4640 11.5 | 956 5200 16.12 | 1048 5660 21.12 | 1133 6160 26.80 | 1210 6570 32.55 | 1280 6970 38.90 |
| 11 | 52 | 5 3/8 | 13.6 | 19 1/4 | 2.02 | R.P.M. C.F. H.P. | 548 4050 5.03 | 670 4870 9.06 | 774 5610 13.9 | 865 6290 19.5 | 947 6850 25.55 | 1025 7450 32.40 | 1093 7950 39.33 | 1160 8440 47.10 |
| 12 | 57 | 5 7/8 | 14.92 | 21 | 2.405 | R.P.M. C.F. H.P. | 500 4820 6.00 | 611 5800 10.78 | 705 6700 16.62 | 789 7490 23.25 | 863 8160 30.45 | 934 8870 38.60 | 996 9460 46.85 | 1056 10040 56.10 |

Steel Pressure Blowers for Cupolas (Average Application).—
Continued.

| No. of blower. | Dia. of wheel. in. | Width per y. in. | Circum. of wheel, ft. | Dia. outlet pipes, in. | Area of outlet, sq. ft. | Oz. | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----------------|-----------------------|---------------------|--------------------------|---------------------------|----------------------------|--------------------------------------|------------------------|------------------------|-----------------------|-----------------------|------------------------|-------------------------|-------|
| | | | | | | In. | 17.28 | 19.02 | 20.75 | 22.5 | 24.22 | 25.95 | 27.66 |
| | | | | | | H.P. const. at 1000 cu. ft. | 6.20 | 6.82 | 7.44 | 8.07 | 8.69 | 9.30 | 9.92 |
| 2 17 | 15/8 | 4.45 | 63/4 | 0.2485 | R.P.M. C.F. H.P. | 3740 1093 6.78 | 3920 1148 7.83 | 4090 1196 8.9 | | | | | |
| 3 19 1/2 | 17/8 | 5.11 | 73/4 | 0.327 | R.P.M. C.F. H.P. | 3255 1440 8.93 | 3415 1510 10.3 | 3570 1575 11.72 | 3710 1642 13.26 | 3955 1700 14.75 | 3985 1762 16.4 | 4120 1820 18.05 | |
| 4 22 | 21/8 | 5.76 | 83/4 | 0.4176 | R.P.M. C.F. H.P. | 2890 1840 11.40 | 3030 1930 13.16 | 3163 2012 14.96 | 3290 2095 16.9 | 3420 2175 18.9 | 3535 2250 20.9 | 3650 2325 23.1 | |
| 5 24 1/2 | 23/8 | 6.41 | 93/4 | 0.519 | R.P.M. C.F. H.P. | 2595 2280 14.13 | 2720 2395 16.33 | 2845 2500 18.6 | 2960 2605 21.05 | 3075 2700 23.45 | 3180 2800 26.05 | 3280 2885 23.66 | |
| 6 27 | 27/8 | 7.06 | 103/4 | 0.63 | R.P.M. C.F. H.P. | 2355 2770 17.18 | 2470 2910 19.85 | 2580 3033 22.6 | 2685 3165 25.55 | 2790 3280 28.50 | 2885 3395 31.55 | 2980 3500 34.7 | |
| 7 32 | 33/8 | 8.39 | 121/2 | 0.852 | R.P.M. C.F. H.P. | 1983 3750 23.25 | 2080 3930 26.80 | 2170 4110 30.6 | 2260 4276 34.5 | 2345 4430 38.5 | 2430 4590 42.7 | 2510 4730 47. | |
| 8 37 | 37/8 | 9.70 | 14 | 1.069 | R.P.M. C.F. H.P. | 1715 4700 29.15 | 1800 4930 33.66 | 1880 5150 38.33 | 1955 5360 43.25 | 2030 5560 48.30 | 2100 5760 53.55 | 2170 5940 59. | |
| 9 42 | 43/8 | 10.98 | 16 | 1.396 | R.P.M. C.F. H.P. | 1515 6150 38.15 | 1590 6450 44.00 | 1660 6730 50.15 | 1728 7010 56.60 | 1792 7270 63.2 | 1855 7525 70. | 1916 7760 77. | |
| 10 47 | 47/8 | 12.30 | 17 1/2 | 1.67 | R.P.M. C.F. H.P. | 1352 7350 45.60 | 1418 7715 52.66 | 1480 8055 60. | 1540 8390 67.66 | 1600 8700 75.6 | 1655 9010 83.9 | 1710 9300 92.25 | |
| 11 52 | 53/8 | 13.6 | 19 1/4 | 2.02 | R.P.M. C.F. H.P. | 1222 8900 55.20 | 1282 9330 63.6 | 1340 9750 72.5 | 1393 10140 82. | 1447 10520 91.5 | 1498 10890 101.2 | 1546 11220 111.33 | |
| 12 57 | 57/8 | 14.92 | 21 | 2.405 | R.P.M. C.F. H.P. | 1113 10580 65.5 | 1168 11100 75.70 | 1220 11600 86.33 | 1270 12080 97.5 | 1318 12520 109 | 1363 12960 120.5 | 1410 13380 132.75 | |

Caution in Regard to Use of Fan and Blower Tables.—Many engineers report that some manufacturers' tables overrate the capacity of their fans and underestimate the horse-power required to drive them. In some cases the complaints may be due to restricted air outlets, long and crooked pipes, slipping of belts, too small engines, etc. It may also be due to the fact that the volumes are stated without being accompanied by information as to the maintained resistance, and the volumes given

may be those delivered with an unrestricted inlet and outlet. As this condition is not a practical one, the volume delivered in an installation is much smaller than that given in the tables. The underestimating of horse-power required may be due to the fact that the volumes given in tables are for operation against a practical resistance, and in an installation it might be that the resistance was low, consequently the volume and also the horse-power required would be greater.

Capacity of Sturtevant High-Pressure Blowers (1908).

| Number of blower. | Capacity in cubic feet per minute, 1/2 lb. pressure. | Revolutions per minute. | Inside dia. of inlet and outlet, inches. | Approx. weight, pounds.* |
|-------------------|--|-------------------------|--|--------------------------|
| 000 | 1 to 5 | 200 to 1000 | 13/8 | 40 |
| 00 | 5 to 25 | 375 to 800 | 11/2 | 80 |
| 0 | 25 to 45 | 370 to 800 | 21/2 | 140 |
| 1 | 45 to 130 | 240 to 600 | 3 | 330 |
| 2 | 130 to 225 | 300 to 500 | 4 | 550 |
| 3 | 225 to 325 | 380 to 525 | 4 | 760 |
| 4 | 325 to 560 | 350 to 565 | 6 | 1,080 |
| 5 | 560 to 1,030 | 300 to 475 | 8 | 1,670 |
| 6 | 1,030 to 1,540 | 290 to 415 | 10 | 2,500 |
| 7 | 1,540 to 2,300 | 280 to 410 | 10 | 3,200 |
| 8 | 2,300 to 3,300 | 265 to 375 | 12 | 4,700 |
| 9 | 3,300 to 4,700 | 250 to 350 | 16 | 6,100 |
| 10 | 4,700 to 6,000 | 260 to 330 | 16 | 8,000 |
| 11 | 6,000 to 8,500 | 220 to 310 | 20 | 12,100 |
| 12 | 8,500 to 11,300 | 190 to 250 | 24 | 18,700 |
| 13 | 11,300 to 15,500 | 190 to 260 | 30 | 22,700 |

* Of blower for 1/2 lb. pressure.

Performance of a No. 7 Steel Pressure Blower under Varying Conditions of Outlet.

| | | | | | | | | | | | | | |
|---------------------------------|------|------|------|------|------|------|------|------|-----|-----|-----|-----|-----|
| Per cent of Rated Capacity..... | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| Per cent of Rated H.P. 28 | 42 | 57 | 72 | 86 | 100 | 116 | 130 | 144 | 159 | 173 | 187 | 202 | |
| Total pressure, oz. ... | 10.2 | 11.4 | 11.9 | 12.0 | 11.9 | 11.4 | 10.9 | 10.3 | 9.7 | 9.1 | 8.5 | 7.9 | 7.2 |
| Static pressure, oz. .. | 10.2 | 11.2 | 11.6 | 11.4 | 11.0 | 10.2 | 9.2 | 8.0 | 6.6 | 5.0 | 3.5 | 1.9 | 0.3 |
| Efficiency, per cent | 0 | 26 | 40 | 50 | 56 | 60 | 62 | 61 | 59 | 56 | 52 | 48 | 45 |

The above figures are taken from a plotted curve of the results of a test by the Buffalo Forge Co. in 1905. A letter describing the test says :

The object was to determine the variation of pressure, power and efficiency obtained at a constant speed with capacities varying from zero discharge to free delivery. A series of capacity conditions were secured by restricting the outlet of the blower by a series of converging cones, so arranged as to make the convergence in each case very slight, and of sufficient length to avoid any noticeable inequality in velocities at the discharge orifice. The fan was operated as nearly at constant speed as possible. The velocity of the air at the point of discharge was measured by a Pitot tube and draft gauge of usual construction. Readings were taken over several points of the outlet and the average taken, although

the variation under nearly all conditions was scarcely perceptible. A coefficient of 93% was assumed for the discharge orifice. The pressure was taken as the reading given by the Pitot tube and draft gauge at outlet. The agreement of this reading with the static pressure in a chamber from which a nozzle was conducted had been checked by a previous test in which the two readings, i.e., velocity and static pressure, were found to agree exactly within the limit of accuracy of the draft gauge, which was about 0.01 in., or, in this case, within 1%. The horsepower was determined by means of a motor which had been previously calibrated by a series of brake tests. Variations in speed were assumed to produce variation in capacity in proportion to the speed, variation in pressure to the square of the speed, and variation in H.P. in proportion to the cube of the speed. These relations had been previously shown to hold true for fans in other tests. They were also checked up by operating the fan at various speeds and plotting the capacities directly with the speed as abscissa, the pressure with the square of the speed as abscissa, and the horse power with the cube of the speed as abscissa. These were found, as in previous cases, to have a practically straight-line relation, in which the line passed through the origin.

Effect of Resistance upon the Capacity of a Fan. — A study of the figures in the above table shows the importance of having ample capacity in the air mains and delivery pipes, and of the absence of sharp bends or other obstructions to the flow which may increase the resistance or pressure against which the fan operates. The fan delivering its rated capacity against a static pressure of 10.2 ounces delivers only 40% of that capacity, with the same number of revolutions, if the pressure is increased to 11.6 ounces; the power is reduced only to 57%, instead of 40%, and the efficiency drops from 60% to 40%.

Dimensions of Sirocco Fans.

(American Blower Co., 1909.)

| Diameter of Wheel, in. | Width at Periphery, in. | No. of Blades. | Total Blade Area, sq. in. | Height of Housing, (Approx.) | Width bet. Sides, in. | Length of Housing. | Max. Area of Inlet, sq. ft. | Min. Area of Inlet, sq. ft. | Area of Square Outlet, sq. ft. | Area of Circular Evasé Outlet sq. ft. | Length of Evasé Cone. |
|------------------------|-------------------------|----------------|---------------------------|------------------------------|-----------------------|--------------------|-----------------------------|-----------------------------|--------------------------------|---------------------------------------|-----------------------|
| 6 | 3 | 48 | 56 | 11" | 4 | 10" | .23 | .123 | .11 | .12 | 3" |
| 9 | 4 1/2 | 48 | 127 | 1' 4" | 6 | 1' 3" | .49 | .349 | .25 | .35 | 4 1/4" |
| 12 | 6 | 64 | 226 | 1' 9" | 8 | 1' 7" | .85 | .616 | .44 | .60 | 5 3/4" |
| 15 | 7 1/2 | 64 | 353 | 2' 4" | 10 | 2' 0" | 1.46 | .957 | .69 | .92 | 7 1/4" |
| 18 | 9 | 64 | 509 | 2' 10" | 12 | 2' 5" | 1.87 | 1.37 | 1.00 | 1.40 | 8 1/2" |
| 21 | 10 1/2 | 64 | 693 | 3' 4" | 14 | 2' 10" | 2.40 | 1.87 | 1.34 | 1.87 | 10" |
| 24 | 12 | 64 | 904 | 3' 8" | 16 | 3' 3" | 3.14 | 2.46 | 1.78 | 2.40 | 11 1/2" |
| 27 | 13 1/2 | 64 | 1144 | 4' 3" | 18 | 3' 7" | 4.59 | 3.11 | 2.25 | 3.14 | 13" |
| 30 | 15 | 64 | 1413 | 4' 7" | 20 | 4' 0" | 5.58 | 3.83 | 2.78 | 3.83 | 14 1/2" |
| 36 | 18 | 64 | 2036 | 5' 6" | 24 | 4' 10" | 7.87 | 5.50 | 4.00 | 5.58 | 17" |
| 42 | 21 | 64 | 2770 | 6' 5" | 28 | 5' 7" | 10.56 | 7.47 | 5.44 | 7.47 | 20" |
| 48 | 24 | 64 | 3617 | 7' 3" | 32 | 6' 5" | 13.6 | 9.79 | 7.11 | 9.85 | 23" |
| 54 | 27 | 64 | 4578 | 8' 2" | 36 | 7' 3" | 17.0 | 12.3 | 9.00 | 12.3 | 26" |
| 60 | 30 | 64 | 5652 | 9' 1" | 40 | 8' 0" | 20.9 | 15.2 | 11.11 | 15.3 | 28 1/2" |
| 66 | 33 | 64 | 6839 | 9' 11" | 44 | 8' 10" | 25.2 | 18.4 | 13.41 | 18.3 | 31 1/2" |
| 72 | 36 | 64 | 8144 | 10' 10" | 48 | 9' 7" | 29.8 | 22.2 | 16.00 | 22.3 | 34 1/2" |

Sirocco or Multivane Fans. — There has recently (1909) come into use a fan of radically different proportions and characteristics from the ordinary centrifugal fan. This fan is composed of a great number of shallow vanes, ranging from 48 to 64, set close together around the periphery of the fan wheel. Over a large range of sizes, 64 vanes appear to give the

Speed, Capacities and Horse-power of Sirocco Fans. (American Blower Co., 1909.)

The figures given represent dynamic pressures in oz. per sq. in. For static pressure, deduct 28.8%; for velocity pressure, deduct 71.2%.

| Diam. Wheel. | | 1/4 oz. | 1/2 oz. | 3/4 oz. | 1 oz. | 1 1/4 oz. | 1 1/2 oz. | 1 3/4 oz. | 2 oz. | 2 1/2 oz. | 3 oz. |
|--------------|---------|---------|---------|---------|--------|-----------|-----------|-----------|--------|-----------|---------|
| 6 | Cu. ft. | 155 | 220 | 270 | 310 | 350 | 380 | 410 | 440 | 490 | 540 |
| | R.P.M. | 1,145 | 1,615 | 1,980 | 2,290 | 2,560 | 2,800 | 3,025 | 3,230 | 3,616 | 3,960 |
| | B.H.P. | .0185 | .052 | .095 | .147 | .205 | .270 | .34 | .42 | .58 | .76 |
| 9 | Cu. ft. | 350 | 500 | 610 | 700 | 790 | 860 | 930 | 1,000 | 1,110 | 1,220 |
| | R.P.M. | 762 | 1,076 | 1,320 | 1,524 | 1,700 | 1,866 | 2,020 | 2,152 | 2,408 | 2,640 |
| | B.H.P. | .042 | .118 | .216 | .333 | .463 | .610 | .77 | .95 | 1.32 | 1.73 |
| 12 | Cu. ft. | 625 | 880 | 1,080 | 1,250 | 1,400 | 1,530 | 1,650 | 1,770 | 1,970 | 2,170 |
| | R.P.M. | 572 | 808 | 990 | 1,145 | 1,280 | 1,400 | 1,512 | 1,615 | 1,808 | 1,980 |
| | B.H.P. | .074 | .208 | .381 | .588 | .82 | 1.08 | 1.36 | 1.66 | 2.32 | 3.05 |
| 15 | Cu. ft. | 975 | 1,380 | 1,690 | 1,950 | 2,180 | 2,400 | 2,590 | 2,760 | 3,090 | 3,390 |
| | R.P.M. | 456 | 645 | 790 | 912 | 1,020 | 1,120 | 1,210 | 1,290 | 1,444 | 1,580 |
| | B.H.P. | .115 | .326 | .600 | .923 | 1.29 | 1.69 | 2.14 | 2.61 | 3.65 | 4.8 |
| 18 | Cu. ft. | 1,410 | 1,990 | 2,440 | 2,820 | 3,160 | 3,450 | 3,720 | 3,980 | 4,450 | 4,880 |
| | R.P.M. | 381 | 538 | 660 | 762 | 850 | 933 | 1,010 | 1,076 | 1,204 | 1,320 |
| | B.H.P. | .167 | .470 | .862 | 1.33 | 1.85 | 2.43 | 3.07 | 3.75 | 5.25 | 6.9 |
| 21 | Cu. ft. | 1,925 | 2,710 | 3,310 | 3,850 | 4,290 | 4,700 | 5,070 | 5,420 | 6,060 | 6,620 |
| | R.P.M. | 326 | 462 | 565 | 652 | 730 | 800 | 864 | 924 | 1,032 | 1,130 |
| | B.H.P. | .227 | .640 | 1.17 | 1.81 | 2.53 | 3.33 | 4.18 | 5.11 | 7.15 | 9.4 |
| 24 | Cu. ft. | 2,500 | 3,540 | 4,340 | 5,000 | 5,600 | 6,120 | 6,620 | 7,080 | 7,900 | 8,680 |
| | R.P.M. | 286 | 404 | 495 | 572 | 640 | 700 | 756 | 807 | 904 | 990 |
| | B.H.P. | .296 | .832 | 1.53 | 2.35 | 3.28 | 4.32 | 5.44 | 6.64 | 9.3 | 12.2 |
| 27 | Cu. ft. | 3,175 | 4,490 | 5,500 | 6,350 | 7,100 | 7,780 | 8,400 | 8,980 | 10,050 | 11,000 |
| | R.P.M. | 254 | 359 | 440 | 508 | 568 | 622 | 672 | 718 | 804 | 880 |
| | B.H.P. | .373 | 1.05 | 1.94 | 2.98 | 4.16 | 5.48 | 6.90 | 8.44 | 11.8 | 15.5 |
| 30 | Cu. ft. | 3,910 | 5,520 | 6,770 | 7,820 | 8,750 | 9,600 | 10,350 | 11,050 | 12,350 | 13,550 |
| | R.P.M. | 228 | 322 | 395 | 456 | 510 | 560 | 604 | 645 | 722 | 790 |
| | B.H.P. | .460 | 1.30 | 2.40 | 3.68 | 5.15 | 6.75 | 8.53 | 10.4 | 14.5 | 19.1 |
| 36 | Cu. ft. | 5,650 | 7,950 | 9,750 | 11,300 | 12,640 | 13,800 | 14,900 | 15,900 | 17,800 | 19,500 |
| | R.P.M. | 190 | 269 | 330 | 381 | 425 | 466 | 504 | 538 | 602 | 660 |
| | B.H.P. | .665 | 1.87 | 3.44 | 5.30 | 7.40 | 9.72 | 12.25 | 15.0 | 20.9 | 27.5 |
| 42 | Cu. ft. | 7,700 | 10,850 | 13,300 | 15,400 | 17,170 | 18,800 | 20,300 | 21,700 | 24,250 | 26,600 |
| | R.P.M. | 163 | 231 | 283 | 326 | 365 | 400 | 432 | 462 | 516 | 566 |
| | B.H.P. | .903 | 2.55 | 4.69 | 7.24 | 10.1 | 13.3 | 16.7 | 20.4 | 28.5 | 37.5 |
| 48 | Cu. ft. | 10,000 | 14,150 | 17,350 | 20,000 | 22,400 | 24,500 | 26,500 | 28,300 | 31,600 | 34,700 |
| | R.P.M. | 143 | 202 | 248 | 286 | 320 | 350 | 378 | 403 | 452 | 495 |
| | B.H.P. | 1.18 | 3.32 | 6.10 | 9.40 | 13.1 | 17.2 | 21.75 | 26.6 | 37.1 | 48.8 |
| 54 | Cu. ft. | 12,700 | 17,950 | 22,000 | 25,400 | 28,400 | 31,100 | 33,600 | 35,900 | 40,200 | 44,000 |
| | R.P.M. | 127 | 179 | 220 | 254 | 284 | 311 | 336 | 359 | 402 | 440 |
| | B.H.P. | 1.49 | 4.20 | 7.75 | 11.9 | 16.6 | 21.9 | 27.6 | 33.7 | 47.1 | 62. |
| 60 | Cu. ft. | 15,650 | 22,100 | 27,100 | 31,300 | 35,000 | 38,400 | 41,400 | 44,200 | 49,400 | 54,200 |
| | R.P.M. | 114 | 161 | 198 | 228 | 255 | 280 | 302 | 322 | 361 | 396 |
| | B.H.P. | 1.84 | 5.20 | 9.58 | 14.7 | 20.6 | 27.0 | 34.1 | 41.6 | 58.2 | 76.5 |
| 66 | Cu. ft. | 18,950 | 26,800 | 32,850 | 37,900 | 42,300 | 46,400 | 50,100 | 53,600 | 60,000 | 65,700 |
| | R.P.M. | 104 | 147 | 180 | 208 | 232 | 254 | 275 | 294 | 328 | 360 |
| | B.H.P. | 2.23 | 6.30 | 11.6 | 17.8 | 24.9 | 32.7 | 41.2 | 50.4 | 70.4 | 92.6 |
| 72 | Cu. ft. | 22,600 | 31,800 | 39,000 | 45,200 | 50,600 | 55,200 | 59,600 | 63,600 | 71,200 | 78,000 |
| | R.P.M. | 95 | 134 | 165 | 190 | 212 | 233 | 252 | 269 | 301 | 330 |
| | B.H.P. | 2.66 | 7.48 | 13.7 | 21.2 | 29.6 | 38.9 | 49.0 | 59.8 | 83.6 | 110. |
| 78 | Cu. ft. | 26,400 | 37,350 | 45,800 | 52,800 | 59,100 | 64,700 | 70,000 | 74,700 | 83,500 | 91,600 |
| | R.P.M. | 88 | 124 | 153 | 176 | 197 | 215 | 233 | 248 | 278 | 305 |
| | B.H.P. | 3.10 | 8.77 | 16.1 | 24.8 | 34.7 | 45.6 | 57.5 | 70.2 | 98. | 129. |
| 84 | Cu. ft. | 30,800 | 43,400 | 53,200 | 61,600 | 68,700 | 75,200 | 81,200 | 86,800 | 97,100 | 106,400 |
| | R.P.M. | 81 | 115 | 142 | 163 | 182 | 200 | 216 | 231 | 258 | 283 |
| | B.H.P. | 3.61 | 10.2 | 18.7 | 28.9 | 40.4 | 53.0 | 66.8 | 81.7 | 114. | 150. |
| 90 | Cu. ft. | 35,250 | 49,800 | 61,000 | 70,500 | 78,800 | 86,400 | 93,300 | 99,600 | 111,200 | 122,000 |
| | R.P.M. | 76 | 107 | 132 | 152 | 170 | 186 | 201 | 214 | 241 | 264 |
| | B.H.P. | 4.14 | 11.7 | 21.5 | 33.1 | 46.2 | 60.7 | 76.7 | 93.6 | 131. | 172. |

best results. The vanes, measured radially, have a depth $\frac{1}{16}$ the fan diameter. Axially, they are much longer than those of the ordinary fan, being $\frac{3}{5}$ the fan diameter. The fan occupies about $\frac{1}{2}$ the space, and is about $\frac{2}{3}$ the weight of the ordinary fan. The vanes are concaved in the direction of rotation and the outer edge is set forward of the inner edge. The inlet area is of the same diameter as the inner edge of the blades. Usually the inlet is on one side of the fan only, and is unobstructed, the wheel being overhung from a bearing at the opposite end. A peculiarity of this type of fan is that the air leaves it at a velocity about 80 per cent in excess of the peripheral speed of the blades. The velocity of the air through the inlet is practically uniform over the entire inlet area. The power consumption is relatively low. This type of fan was invented by S. C. Davidson of Belfast, Ireland, and is known as the "Sirocco" fan. It is made under that name in this country by the American Blower Co., to which the author is indebted for the preceding tables.

A Test of a "Sirocco" Mine Fan at Llwynypia, Wales, is reported in *Eng'g.*, April 16, 1909. The fan is 11 ft. 8 in. diam., double inlet, direct-coupled to a 3-phase motor. Average of three tests: Revs. per min., 184; peripheral speed, 6,705 ft. per min.; water-gauge in fan drift and in main drift, each 6 in.; area of drift, 184.6 sq. ft.; av. velocity of air, 1842 ft. per min.; volume of air, 340,033 cu. ft. per min.; H.P. input at motor, 420; Brake H.P. on fan shaft, 390; Indicated H.P. in air, 321.5; efficiency of motor, 93%; mechanical efficiency of fan, 82.43%; combined mechanical efficiency of fan and motor, 76.6%.

The Sturtevant Multivane Fan. A modification of the Sirocco fan has been developed by the B. F. Sturtevant Co., in which the blades are made with spoon-shaped serrations along their length. The advantage claimed for this construction is that the air is discharged more evenly along the length of the blade. The following table shows the sizes; capacities and horse-power required by the fan.

Sizes, Capacities and Horse-power of Multivane Fans.

(B. F. Sturtevant Co., 1909.)

| Height of Fan Casing inches * | Resistance, $\frac{1}{2}$ In. | | | Resistance, 1 In. | | | Resistance, $1\frac{1}{2}$ In. | | |
|-------------------------------|-------------------------------|--------|------|-------------------|--------|------|--------------------------------|--------|------|
| | Vol. | R.P.M. | H.P. | Vol. | R.P.M. | H.P. | Vol. | R.P.M. | H.P. |
| 30 | 1,800 | 695 | 0.45 | 2,560 | 985 | 1.2 | 3,100 | 1,200 | 2.3 |
| 35 | 2,600 | 580 | 0.65 | 3,700 | 820 | 1.8 | 4,500 | 1,000 | 3.4 |
| 40 | 3,550 | 500 | 0.90 | 5,000 | 700 | 2.5 | 6,200 | 860 | 4.5 |
| 50 | 4,620 | 435 | 1.15 | 6,500 | 615 | 3.3 | 8,000 | 750 | 6.0 |
| 60 | 7,220 | 350 | 1.8 | 10,200 | 490 | 5.0 | 12,500 | 600 | 9.3 |
| 70 | 10,400 | 290 | 2.6 | 14,700 | 410 | 7.3 | 18,000 | 500 | 13.4 |
| 80 | 14,000 | 250 | 3.5 | 20,000 | 350 | 10.0 | 24,500 | 430 | 18.0 |
| 100 | 23,500 | 190 | 5.8 | 33,300 | 275 | 16.5 | 40,800 | 335 | 30.0 |
| 120 | 35,000 | 160 | 8.8 | 49,700 | 225 | 25 | 61,000 | 275 | 45.0 |
| 150 | 48,800 | 135 | 12.0 | 69,000 | 190 | 34 | 85,000 | 233 | 63.0 |
| 170 | 65,000 | 115 | 16.0 | 92,000 | 165 | 46 | 112,500 | 200 | 85.0 |

* Full housing. Bottom horizontal discharge.

The above table gives the volumes and horse-powers of Sturtevant multivane fans operating against a continuously maintained resistance, handling air at 65° F. The table is compiled for single-inlet fans, but when used with double inlet the volumes will be considerably increased (about 15-20%), and the power will also be greater (about 25-35%). It is possible to handle any of the volumes given against any stated pressure with quite an appreciable saving in power as compared with the table horse-power by using a larger fan, and by so doing obtaining lower veloci-

ties through the fan. It is also possible to handle any stated volume against any pressure given in the table with a considerably smaller fan, but when this is done it requires an increase in horse-power due to the greater velocity, which is increased in proportion to the decrease in size and to the lower mechanical efficiency of an overloaded fan. By maintained resistance is meant a static pressure existing in the air after it leaves the fan outlet, if the fan is applied to a blowing system. With the suction system, maintained resistance is the static suction existing in the duct just outside the fan inlet. If the fan is so placed in the system that there is resistance to the flow of air on both inlet and outlet, the maintained resistance against which the fan operates is the sum of the static suction existing in the air just before entering the inlet and the static pressure in the air just outside the fan outlet. In ordinary draw-through heating systems a maintained suction is encountered in the fan inlet due to the resistance of the heater, and the maintained pressure is created in the fan outlet due to the piping system. The volumes given are computed from tests in which the average velocity over rectangular or circular pipes is taken as 91% of that velocity (not velocity head) which is read at the center of the pipe by means of the Pitot tube. This method of computing velocity is conservative, especially for pipes having large sectional area.

High-Pressure Centrifugal Fans. (See page 620.)

Methods of Testing Fans.

(Compiled by B. F. Sturtevant Co., 1909.)

Various methods are used in testing centrifugal fans, some of which, being crude, credit fans with performances somewhat different from the true performance. Some of the formulæ used in determining the performances of a fan are given below:

h_v = Velocity head, in. of water; h_t = Total or Impact head, in. of water; h_s = Static head, in. of water; Q = Cu. ft. per min.; v = Velocity, ft. per min.; w = Density of air, lb. per cu. ft.; A = Area of outlet pipe, sq. ft.; A.H.P._s = Air horse-power crediting the fan with the energy due to static pressure only; A.H.P._t = Air horse-power, crediting the fan with both the energy due to static pressure and the kinetic energy in the discharge; B.H.P. = Brake horse-power.

$$V = 1097 \sqrt{\frac{h_v}{w}} \quad Q = 1097 \sqrt{\frac{h_v}{w}} \times A.$$

$$\text{A.H.P.}_s = Q \times h_s \times 0.0001575; \text{A.H.P.}_t = Q \times h_t \times 0.0001575.$$

$$\text{Mechanical Efficiency} = \text{A.H.P.} \div \text{B.H.P.}$$

$$\text{Volumetric Eff'y} = \text{Volume per Revolution} \div \text{Cubical Contents of wheel.}$$

Anemometer Method. Anemometers are subject to considerable error as they are very delicate and must be handled with care. Should they be placed in a draft where the velocity is much over 1000 ft. per min. they are apt to be damaged by bending the blades. The methods of calibrating these instruments are faulty, and give some chance of error, even though the instrument be in the same condition as when calibrated. Unless it is frequently calibrated, the instrument may not be true to its calibration curve, which is often a source of considerable error. An anemometer is seldom adapted to taking readings at the fan outlet, or within pipes, as the velocity in most cases exceeds the limitations of the instrument. Therefore, readings are usually taken at a point where the velocity is lower, and consequently over areas of various shapes with unknown coefficients, thus introducing another source of error. Unless the flow of air is constant, faulty readings are obtained, due to the inertia of the instrument, which results in the fan being credited with a volume greater than the true volume.

Water-Gauge Readings at End of Tapered Cone. In this method, cones are placed on the fan outlet, or on the end of a short outlet pipe. The readings at the end of the cone vary widely, due to the large number of variable eddies. The pressure reading at the end of the cone is a total of two components, static pressure and velocity pressure. Unless the static pressure is deducted from the total pressure the true velocity pressure is not obtained.

Air-tight Room with Sliding Door. This method consists of the fan discharging its air into a closed room whose outlet is a sliding door. In this method, the readings generally take into account not only the volumetric performance but also the static pressure in the room, against which the fan delivers air. All tests by this method must be corrected for leakage of air from the room, the leakage factor being much larger than would be supposed. A variable coefficient of orifice is encountered, since at no two positions of the sliding door is either the area or shape of orifice the same. Readings taken at the door, by anemometers, are subject to the errors of these instruments. If water-gauge readings are taken at the door, the results are in error if it is assumed that all pressure at the door is velocity pressure. Static readings should be made at each station and deducted from the total observed pressure in order to get the velocity head. Even then it is difficult to get a true static reading at the door, as the stream lines are not all perpendicular to the plane of the orifice.

Pitot Tube in Center of Discharge Pipe. This method requires a discharge pipe of the same size as the outlet of the fan. In the center of this pipe and at such a distance from the fan outlet that eddies are practically eliminated, is placed a Pitot tube. The discharge pipe is of such length beyond the tube that when restricted at its end, the stream lines in the vicinity of the tube are not materially affected. By this method the static and total pressures are observed with considerable accuracy. The velocity pressure is determined by subtracting the static pressure from the total pressure. By applying a proper coefficient to the readings at the center the average velocity over the full discharge area is obtained. It is possible to make a more complete test by placing several Pitot tubes in the discharge pipe at different points in a cross-section, thereby obtaining an average. But it is found that by taking readings at a distance of eight or ten diameters from the fan outlet very good results are obtained with one tube placed in the center of the section of the pipe, whose readings are corrected by a proper coefficient. For medium-size pipes it is found that a coefficient of 0.91 applied to the velocity read at the center of the discharge pipe gives good and conservative results. [Other authorities give 0.87 as the value of this coefficient. See Pitot Tube, under Illuminating Gas.]

Experiments with the tapered cone method and the Pitot tube in the center of pipe method show that the former credits a fan with greater volume than the latter, and also show that there is a variable relation between these two methods as regards the volume of air credited to the fan when it is handling a certain volume of air. The difference in volumes credited the fan becomes greater as the size of the discharge pipe increases. In tests on two fans of different sizes, but of symmetrical design, the Pitot tube in the center of the pipe will record symmetrical results under given conditions, while with the tapered cone the results obtained with the larger fan and larger discharge pipe are beyond those which would have been expected from the symmetry of the fan.

From the above formulæ the air horse-power is a function of two variables, volume and pressure. Opinions vary as to the pressure which should be credited to the fan. It is claimed that the fan should be credited with the difference between the static pressure in the medium from which the fan is drawing air and the static pressure in the discharge pipe. It is also claimed that the fan should also be credited with the kinetic energy in the air in the discharge pipe or with the difference between the static pressure in the medium from which the fan is drawing air and the total or impact pressure in the discharge pipe. Efficiencies determined by crediting the fan with the former pressure may be called static efficiencies, and those determined by crediting the fan with the latter pressure may be called impact efficiencies.

The work of compression is negligible, as these methods have to do with air under low pressure. When readings are taken on the suction side of

the fan, for the purpose of determining static efficiency, the fan is often erroneously credited with a pressure equal to the difference between the medium into which the fan is discharging and the negative static pressure in the pipe leading to the fan inlet, whereas it should be credited only with the difference between the static pressure in the discharging medium and the impact pressure in the inlet pipe. The static suction has a greater negative value than the impact pressure at the same point, which is the result of the reduction of pressure caused by the air entering the system changing from rest, or zero velocity, to a finite velocity which it has at the point of measurement. If the object is to determine the impact efficiency where readings are taken at the suction side of the fan, the pressure with which the fan should be credited is the difference between the impact reading at the fan discharge and the impact reading obtained in the inlet pipe. This total pressure with which the fan is credited may also be expressed as the difference between the static pressure in the discharge pipe and the static suction in the inlet pipe, plus the increase of the velocity pressure in the outlet pipe over the velocity pressure in the inlet pipe.

From the above methods it is seen that volumetric and mechanical efficiencies of wide variety are obtained, and that where a test is of any importance it is essential that it be made on the most correct lines. Using a Pitot tube in the center of the pipe through which air flows, affords the best means of getting the true pressures as a whole and their separate components, and, consequently, is most accurate in determining the

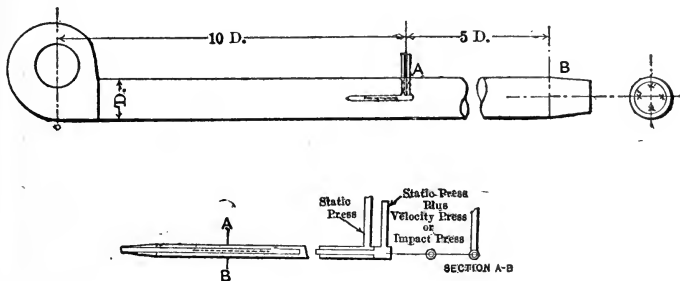


FIG. 140

volume flowing. Fig. 140 shows diagrammatically the method of test where the Pitot tube is used in the center of the discharge pipe. It also shows how readings could be taken by the cone method at the end of a discharge pipe. The details of the Pitot tube in what is considered its best form are also shown. The impact or total pressure is obtained at the end of the horizontal tube nearest the fan, and read by a water gauge connected to the vertical tube communicating with this point. The static pressure is obtained at the slots in the side of the outer horizontal tube which communicates with the second vertical tube, to which a water gauge may be connected.

Efficiency of Fans. — Much useful information on the theory and practice of fans and blowers, with results of tests of various forms, will be found in *Heating and Ventilation*, June to Dec. 1897, in papers by Prof. R. C. Carpenter and Mr. W. G. Walker. It is shown by theory that the volume of air delivered is directly proportional to the speed of rotation, that the pressure varies as the square of the speed, and that the horse-power varies as the cube of the speed. For a given volume of air moved the horse-power varies as the square of the speed, showing the great advantage of large fans at slow speeds over small fans at high speeds delivering the same volume. The theoretical values are greatly modified by variations in practical conditions. Professor Carpenter found that with three fans running at a speed of 6200 ft. per minute at the tips of the vanes, and

an air-pressure of $2\frac{1}{2}$ in. of water column, the mechanical efficiency, or the horse-power of the air delivered divided by the power required to drive the fan, ranged from 32% to 47%, under different conditions, but with slow speeds it was much less, in some cases being under 20%. Mr. Walker in experiments on disk fans found efficiencies ranging all the way from 7.4% to 43%, the size of the fans and the speed being constant, but the shape and angle of the blades varying. It is evident that there is a wide margin for improvements in the forms of fans and blowers, and a wide field for experiment to determine the conditions that will give maximum efficiency.

Flow of Air through an Orifice.

VELOCITY, VOLUME, AND H.P. REQUIRED WHEN AIR UNDER GIVEN PRESSURE IN OUNCES PER SQ. IN. IS ALLOWED TO ESCAPE INTO THE ATMOSPHERE.

(B. F. Sturtevant Co.)

| Pressure in ounces per sq. in. | Pressure, inches of water. | Velocity, ft. per min. | Volume through 1 sq. in. effec- tive area, cu. ft. per min. | Horse-power to move the given volume of air. | Horse-power per 1000 cu. ft. per min. | Pressure in ounces per sq. in. | Velocity, ft. per min. | Volume through 1 sq. in. effec- tive area, cu. ft. per min. | Horse-power to move the given volume of air. | Horse-power per 1000 cu. ft. per min. |
|--------------------------------------|-------------------------------|---------------------------|--|---|---|--------------------------------------|---------------------------|--|---|---|
| $\frac{1}{8}$ | 0.216 | 1,828 | 12.69 | 0.00043 | 0.0340 | 2 | 7,284 | 50.59 | 0.02759 | 0.5454 |
| $\frac{1}{4}$ | 0.432 | 2,585 | 17.95 | 0.00122 | 0.0680 | $2\frac{1}{8}$ | 7,507 | 52.13 | 0.03021 | 0.5795 |
| $\frac{3}{8}$ | 0.648 | 3,165 | 21.98 | 0.00225 | 0.1022 | $2\frac{1}{4}$ | 7,722 | 53.63 | 0.03291 | 0.6136 |
| $\frac{1}{2}$ | 0.864 | 3,654 | 25.37 | 0.00346 | 0.1363 | $2\frac{3}{8}$ | 7,932 | 55.08 | 0.03568 | 0.6476 |
| $\frac{5}{8}$ | 1.080 | 4,084 | 28.36 | 0.00483 | 0.1703 | $2\frac{1}{2}$ | 8,136 | 56.50 | 0.03852 | 0.6818 |
| $\frac{3}{4}$ | 1.295 | 4,473 | 31.06 | 0.00635 | 0.2044 | $2\frac{5}{8}$ | 8,334 | 57.88 | 0.04144 | 0.7160 |
| $\frac{7}{8}$ | 1.512 | 4,830 | 33.54 | 0.00800 | 0.2385 | $2\frac{3}{4}$ | 8,528 | 59.22 | 0.04442 | 0.7500 |
| 1 | 1.728 | 5,162 | 35.85 | 0.00978 | 0.2728 | $2\frac{7}{8}$ | 8,718 | 60.54 | 0.04747 | 0.7841 |
| $1\frac{1}{8}$ | 1.944 | 5,473 | 38.01 | 0.01166 | 0.3068 | 3 | 8,903 | 61.83 | 0.05058 | 0.8180 |
| $1\frac{1}{4}$ | 2.160 | 5,768 | 40.06 | 0.01366 | 0.3410 | $3\frac{1}{8}$ | 9,084 | 63.08 | 0.05376 | 0.8522 |
| $1\frac{3}{8}$ | 2.376 | 6,048 | 42.00 | 0.01575 | 0.3750 | $3\frac{1}{4}$ | 9,262 | 64.32 | 0.05701 | 0.8863 |
| $1\frac{1}{2}$ | 2.592 | 6,315 | 43.86 | 0.01794 | 0.4090 | $3\frac{3}{8}$ | 9,435 | 65.52 | 0.06031 | 0.9205 |
| $1\frac{5}{8}$ | 2.808 | 6,571 | 45.63 | 0.02022 | 0.4431 | $3\frac{1}{2}$ | 9,606 | 66.71 | 0.06368 | 0.9546 |
| $1\frac{3}{4}$ | 3.024 | 6,818 | 47.34 | 0.02260 | 0.4772 | $3\frac{5}{8}$ | 9,773 | 67.87 | 0.06710 | 0.9887 |
| $1\frac{7}{8}$ | 3.240 | 7,055 | 49.00 | 0.02505 | 0.5112 | $3\frac{3}{4}$ | 9,938 | 69.01 | 0.07058 | 1.0227 |
| (1) | (2) | (3) | (4) | (5) | (6) | $3\frac{7}{8}$ | 10,100 | 70.14 | 0.07412 | 1.0567 |
| | | | | | | (1) | (3) | (4) | (5) | (6) |

The headings of the 3d and 4th columns in the above table have been abridged from the original, which read as follows: Velocity of dry air, 50° F., escaping into the atmosphere through any shaped orifice in any pipe or reservoir in which the given pressure is maintained. Volume of air in cubic feet which may be discharged in one minute through an orifice having an effective area of discharge of one square inch. The 6th column, not in the original, has been calculated by the author. The figures represent the horse-power theoretically required to move 1000 cu. ft. of air of the given pressures through an orifice, without allowance for the work of compression or for friction or other losses of the fan. These losses may amount to 60% or more of the given horse-power.

The change in density which results from a change in pressure has been taken into account in the calculations of the table. The volume of air at a given velocity discharged through an orifice depends upon its shape, and is always less than that measured by its full area. For a given *effective* area the volume is proportional to the velocity. The power required to move air through an orifice is measured by the product of the velocity and the total resisting pressure. This power for a given orifice varies as the cube of the velocity. For a given volume it varies as the square of the velocity. In the movement of air by means of a fan there are unavoidable resistances which, in proportion to their amount, increase the actual power considerably above the amount here given.

Pipe Lines for Fans and Blowers. — In installing fans and blowers careful consideration should be given to the pipe line conducting the air from the fan or blower. Bends and turns in the pipe, even of long radii, will cause considerable drop in pressure, and in straight pipe the friction of the moving air is a source of considerable loss. The friction increases with the length of the pipe and is inversely as the diameter. It also varies as the square of the velocity. In long runs of pipe, the increased cost of a larger pipe can often be compensated by the decreased cost of the motor and power for operating the blower.

The advisability of using a large pipe for conveying the air is shown by the following table which gives the size of pipe which should be used for pressure losses not exceeding one-fourth and one-half ounce per square inch, for various lengths of pipe.

Diameters of Blast Pipes.

(B. F. Sturtevant Co., 1908.)

| Tons of iron per hour. | Inside dia. of cupola, inches. | Cubic ft. of air per minute. | Length of Pipe in Feet. | | | | | | | | | | | | | |
|------------------------|--------------------------------|------------------------------|-------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | | | Diameter of Pipe with Drop of | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| | | | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. | 1/4 Oz. | 1/2 Oz. |
| 1 | 23 | 500 | 6 | 5 | 7 | 6 | 7 | 6 | 8 | 7 | 9 | 8 | 9 | 8 | 9 | 8 |
| 2 | 27 | 1,000 | 8 | 7 | 9 | 8 | 10 | 9 | 11 | 9 | 11 | 10 | 12 | 11 | 12 | 11 |
| 3 | 30 | 1,500 | 10 | 8 | 11 | 10 | 11 | 10 | 12 | 11 | 13 | 11 | 13 | 12 | 14 | 12 |
| 4 | 32 | 2,000 | 11 | 9 | 12 | 11 | 13 | 12 | 14 | 12 | 15 | 13 | 15 | 14 | 16 | 14 |
| 5 | 36 | 2,500 | 12 | 10 | 14 | 12 | 15 | 13 | 15 | 14 | 16 | 14 | 17 | 15 | 17 | 15 |
| 6 | 39 | 3,000 | 13 | 11 | 15 | 13 | 16 | 14 | 17 | 15 | 18 | 15 | 18 | 16 | 18 | 16 |
| 7 | 42 | 3,500 | 13 | 12 | 15 | 13 | 17 | 15 | 17 | 15 | 18 | 16 | 19 | 17 | 20 | 18 |
| 8 | 45 | 4,000 | 15 | 12 | 16 | 15 | 18 | 15 | 18 | 16 | 19 | 17 | 20 | 18 | 21 | 18 |
| 9 | 48 | 4,500 | 15 | 13 | 17 | 15 | 18 | 16 | 19 | 17 | 20 | 18 | 21 | 19 | 22 | 19 |
| 10 | 54 | 5,000 | 15 | 13 | 18 | 15 | 19 | 17 | 20 | 18 | 21 | 18 | 22 | 19 | 23 | 20 |
| 11 | 54 | 5,500 | 16 | 14 | 18 | 16 | 20 | 17 | 21 | 18 | 22 | 19 | 23 | 20 | 23 | 20 |
| 12 | 60 | 6,000 | 17 | 14 | 19 | 17 | 20 | 17 | 21 | 19 | 22 | 20 | 23 | 21 | 24 | 21 |
| 13 | 60 | 6,500 | 17 | 14 | 19 | 17 | 21 | 18 | 23 | 19 | 23 | 20 | 24 | 21 | 25 | 22 |
| 14 | 60 | 7,000 | 18 | 15 | 20 | 18 | 22 | 19 | 23 | 20 | 24 | 21 | 25 | 22 | 26 | 23 |
| 15 | 66 | 7,500 | 18 | 16 | 21 | 18 | 22 | 19 | 24 | 21 | 25 | 22 | 26 | 22 | 27 | 23 |
| 16 | 66 | 8,000 | 18 | 16 | 22 | 18 | 23 | 20 | 24 | 22 | 26 | 22 | 26 | 23 | 27 | 24 |
| 17 | 66 | 8,500 | 18 | 16 | 22 | 18 | 23 | 20 | 24 | 22 | 26 | 22 | 27 | 24 | 28 | 24 |
| 18 | 72 | 9,000 | 18 | 17 | 22 | 18 | 24 | 21 | 25 | 22 | 27 | 23 | 27 | 24 | 28 | 25 |
| 19 | 72 | 9,500 | 20 | 17 | 23 | 20 | 24 | 22 | 26 | 23 | 28 | 23 | 28 | 25 | 29 | 26 |
| 20 | 72 | 10,000 | 20 | 18 | 23 | 20 | 25 | 22 | 27 | 23 | 28 | 24 | 29 | 25 | 30 | 26 |
| 21 | 78 | 10,500 | 21 | 18 | 24 | 21 | 26 | 23 | 27 | 23 | 29 | 25 | 30 | 26 | 30 | 26 |
| 22 | 78 | 11,000 | 21 | 18 | 24 | 21 | 27 | 23 | 28 | 24 | 29 | 26 | 30 | 27 | 31 | 27 |
| 23 | 78 | 11,500 | 21 | 19 | 25 | 21 | 27 | 24 | 28 | 25 | 30 | 26 | 30 | 27 | 31 | 27 |
| 24 | 84 | 12,000 | 22 | 19 | 25 | 22 | 28 | 24 | 28 | 25 | 31 | 26 | 31 | 27 | 32 | 28 |
| 25 | 84 | 12,500 | 22 | 19 | 26 | 22 | 28 | 24 | 29 | 26 | 31 | 27 | 32 | 28 | 33 | 28 |
| 26 | 84 | 13,000 | 22 | 19 | 26 | 22 | 28 | 24 | 29 | 26 | 31 | 27 | 32 | 28 | 33 | 28 |
| 27 | 90 | 13,500 | 23 | 20 | 26 | 23 | 28 | 24 | 30 | 26 | 31 | 27 | 32 | 28 | 34 | 28 |
| 28 | 90 | 14,000 | 23 | 20 | 27 | 23 | 29 | 25 | 30 | 27 | 32 | 28 | 33 | 29 | 34 | 29 |
| 29 | 90 | 14,500 | 23 | 20 | 27 | 23 | 29 | 26 | 31 | 27 | 32 | 28 | 33 | 29 | 34 | 30 |
| 30 | 90 | 15,000 | 24 | 21 | 27 | 24 | 29 | 26 | 31 | 27 | 32 | 28 | 34 | 30 | 35 | 30 |

The minimum radius of each turn should be equal to the diameter of the pipe. For each turn thus made add three feet in length, when using this table. If the turns are of less radius, the length added should be increased proportionately.

The above table has been constructed on the following basis: A loss of, say, $\frac{1}{2}$ oz. pressure was allowed as a standard for the transmission of a given quantity of air through a given length of pipe of any diameter. The increased loss due to increasing the length of pipe was compensated for by increasing the diameter sufficiently to keep the loss still at $\frac{1}{2}$ oz. Thus, if 2500 cu. ft. of air is to be delivered per minute through 100 ft. of pipe with a loss of not more than $\frac{1}{2}$ oz., a 14-in. pipe will be required. If it is necessary to increase the length of pipe to 140 ft., a pipe 15 in. diameter will be required if the loss in pressure is not to exceed $\frac{1}{2}$ oz. In deciding the size of pipe the loss in pressure in the pipe must be added to the pressure to be maintained at the fan or blower, if the tabulated efficiency of the latter is to be secured at the delivery end of the pipe.

Centrifugal Ventilators for Mines.—Of different appliances for ventilating mines various forms of centrifugal machines having proved their efficiency have now almost completely replaced all others. Most if not all of the machines in use in this country are of this class, being either open-periphery fans, or closed, with chimney and spiral casing, of a more or less modified Guibal type. The theory of such machines has been demonstrated by Mr. Daniel Murgue in "Theories and Practices of Centrifugal Ventilating Machines," translated by A. L. Stevenson, and is discussed in a paper by R. Van A. Norris, *Trans. A. I. M. E.*, xx. 637. From this paper the following formulæ are taken:

Let a = area in sq. ft. of an orifice in a thin plate, of such area that its resistance to the passage of a given quantity of air equals the resistance of the mine;

o = orifice in a thin plate of such area that its resistance to the passage of a given quantity of air equals that of the machine;

Q = quantity of air passing in cubic feet per minute;

V = velocity of air passing through a in feet per second;

V_0 = velocity of air passing through o in feet per second;

h = head in feet air-column to produce velocity V ;

h_0 = head in feet air-column to produce velocity V_0 .

$$Q = 0.65 a V; \quad V = \sqrt{2gh}; \quad Q = 0.65 a \sqrt{2gh};$$

$$a = \frac{Q}{0.65 \sqrt{2gh}} = \text{equivalent orifice of mine};$$

or, reducing to water-gauge in inches and quantity in thousands of cubic feet per minute,

$$a = \frac{0.403 Q}{\sqrt{W.G.}}; \quad Q = 0.65 o V_0; \quad V_0 = \sqrt{2gh_0}; \quad Q = 0.65 o \sqrt{2gh_0};$$

$$o = \sqrt{\frac{Q^2}{0.65^2 h_0 2g}} = \text{equivalent orifice of machine}.$$

The theoretical depression which can be produced by any centrifugal ventilator is double that due to its tangential speed. The formula

$$H = \frac{T^2}{2g} - \frac{V^2}{2g},$$

in which T is the tangential speed, V the velocity of exit of the air from the space between the blades, and H the depression measured in feet of air-column, is an expression for the theoretical depression which can be produced by an uncovered ventilator; this reaches a maximum when the air leaves the blades without speed, that is, $V = 0$, and $H = \frac{T^2}{2g}$.

Hence the theoretical depression which can be produced by any uncovered ventilator is equal to the height due to its tangential speed, and one-

half that which can be produced by a covered ventilator with expanding chimney. Practical considerations in the design of the fan wheel and casing will probably cause the actual results obtained with fans to vary considerably from these formulæ.

So long as the condition of the mine remains constant:

(1) The volume produced by any ventilator varies directly as the speed of rotation.

(2) The depression produced by any ventilator varies as the square of the speed of rotation.

(3) For the same tangential speed with decreased resistance the quantity of air increases and the depression diminishes.

The following table shows a few results, selected from Mr. Norris's paper, giving the range of efficiency which may be expected under different circumstances. Details of these and other fans, with diagrams of the results, are given in the paper.

Experiments on Mine-Ventilating Fans.

| Fan. | Revolutions per minute, fan. | Peripheral speed, feet per min. | Cubic feet of air per minute. | Cubic feet of air per revolution. | Cubical contents of fan-blades. | Cubic feet of air per 100 ft. periphery motion. | Water-gauge, inches. | Horse-power in air. | Indicated horse-power of engine. | Efficiency of engine and fan. | Equivalent orifice of mine, square feet. |
|------|------------------------------|---------------------------------|-------------------------------|-----------------------------------|---------------------------------|---|----------------------|---------------------|----------------------------------|-------------------------------|--|
| A | 84 | 5517 | 236,684 | 2818 | 3040 | 4290 | 1.80 | 67.13 | 88.40 | 75.9 | } Av'ge 80 |
| | 100 | 6282 | 336,862 | 3369 | 3040 | 5393 | 2.50 | 132.70 | 155.43 | 85.4 | |
| | 111 | 6973 | 347,396 | 3130 | 3040 | 5002 | 3.20 | 175.17 | 209.64 | 83.6 | |
| | 123 | 7727 | 394,100 | 3204 | 3040 | 5100 | 3.60 | 223.56 | 295.21 | 75.7 | |
| B | 100 | 6282 | 188,888 | 1889 | 1520 | 3007 | 1.40 | 41.67 | 97.99 | 42.5 | } 22 |
| | 130 | 8167 | 274,876 | 2114 | 1520 | 3366 | 2.00 | 86.63 | 194.95 | 44.6 | |
| C | 59 | 3702 | 59,587 | 1010 | 1520 | 1610 | 1.20 | 11.27 | 16.76 | 67.83 | } 32 |
| | 83 | 5208 | 82,969 | 1000 | 1520 | 1593 | 2.15 | 27.86 | 48.54 | 57.38 | |
| D | 40 | 3140 | 49,611 | 1240 | 3096 | 1580 | 0.87 | 6.80 | 13.82 | 49.2 | } 83 |
| | 70 | 5495 | 137,760 | 1825 | 3096 | 2507 | 2.55 | 55.35 | 67.44 | 82.07 | |
| E | 50 | 2749 | 147,232 | 2944 | 1522 | 5356 | 0.50 | 11.60 | 28.55 | 40.63 | } 26.9 |
| | 69 | 3793 | 205,761 | 2982 | 1522 | 5451 | 1.00 | 32.42 | 45.98 | 70.50 | |
| F | .96 | 5278 | 299,600 | 3121 | 1522 | 5676 | 2.15 | 101.50 | 120.64 | 84.10 | } 38.3 |
| | 200 | 7540 | 133,198 | 666 | 746 | 1767 | 3.35 | 70.30 | 102.79 | 68.40 | |
| G | 200 | 7540 | 180,809 | 904 | 746 | 2398 | 3.05 | 86.89 | 129.07 | 67.30 | } 46.3 |
| | 200 | 7540 | 209,150 | 1046 | 746 | 2774 | 2.80 | 92.50 | 150.08 | 61.70 | |
| G | 10 | 785 | 28,896 | 2890 | 3022 | 3680 | 0.10 | 0.45 | 1.30 | 35. | } 52 |
| | 20 | 1570 | 57,120 | 2856 | 3022 | 3637 | 0.20 | 1.80 | 3.70 | 49. | |
| | 25 | 1962 | 66,640 | 2665 | 3022 | 3399 | 0.29 | 2.90 | 6.10 | 48. | |
| | 30 | 2355 | 73,080 | 2436 | 3022 | 3103 | 0.40 | 4.60 | 9.70 | 47. | |
| | 35 | 2747 | 94,080 | 2688 | 3022 | 3425 | 0.50 | 7.40 | 15.00 | 48. | |
| | 40 | 3140 | 112,000 | 2800 | 3022 | 3567 | 0.70 | 12.30 | 24.90 | 49. | |
| | 50 | 3925 | 132,700 | 2654 | 3022 | 3381 | 0.90 | 18.80 | 38.80 | 48. | |
| | 60 | 4710 | 173,600 | 2893 | 3022 | 3686 | 1.35 | 36.90 | 66.40 | 55. | |
| | 70 | 5495 | 203,280 | 2904 | 3022 | 3718 | 1.80 | 57.70 | 107.10 | 54. | |
| | 80 | 6280 | 222,320 | 2779 | 3022 | 3540 | 2.25 | 78.80 | 152.60 | 52. | |

| Type of fan. | Diam. | Width. | No. inlets. | Diam. inlets. |
|---------------------------------|--------|--------|-------------|---------------|
| A. Guibal, double | 20 ft. | 6 ft. | 4 | 8 ft. 10 in. |
| B. Same, only left hand running | 20 | 6 | 4 | 8 10 |
| C. Guibal | 20 | 6 | 2 | 8 10 |
| D. Guibal | 25 | 8 | 1 | 11 6 |
| E. Guibal, double | 17 1/2 | 4 | 4 | 8 |
| F. Capell | 12 | 10 | 2 | 7 |
| G. Guibal | 25 | 8 | 1 | 12 |

An examination of the detailed results of each test in Mr. Norris's table shows a mass of contradictions from which it is exceedingly difficult to draw any satisfactory conclusions. The following, he states, appear to be more or less warranted by some of the figures:

1. *Influence of the Condition of the Airways on the Fan.* — Mines with varying equivalent orifices give air per 100 ft. speed of tip of fan, within limits as follows, the quantity depending on the resistance of the mine:

| Equivalent orifice. sq. ft. | Cu. ft. air per 100 ft. speed of fan. | Average. | Equivalent orifice. sq. ft. | Cu. ft. air per 100 ft. speed of fan. | Average. |
|--------------------------------|---|----------|--------------------------------|---|----------|
| Under 20 | 1100 to 1700 | 1300 | 60 to 70 | 3300 to 5100 | 4000 |
| 20 to 30 | 1300 to 1800 | 1600 | 70 to 80 | 4000 to 4700 | 4400 |
| 30 to 40 | 1500 to 2500 | 2100 | 80 to 90 | 3000 to 5600 | 4300 |
| 40 to 50 | 2300 to 3500 | 2700 | 90 to 100 | | |
| 50 to 60 | 2700 to 4800 | 3500 | 100 to 114 | 5200 to 6200 | 5700 |

The influence of the mine on the efficiency of the fan does not seem to be very clear. Eight fans, with equivalent orifices over 50 square feet, give efficiencies over 70%; four, with smaller equivalent mine-orifices, give about the same figures; while, on the contrary, six fans, with equivalent orifices of over 50 square feet, give lower efficiencies, as do ten fans, all drawing from mines with small equivalent orifices. It would seem that, on the whole, large airways tend to assist somewhat in attaining high efficiency.

2. *Influence of the Diameter of the Fan.* — This seems to be practically *nil*, the only advantage of large fans being in their greater width and the lower speed required of the engines.

3. *Influence of the Width of a Fan.* — This appears to be small as regards the efficiency of the machine; but the wider fans are, as a rule, exhausting more air. However, increasing the width of the fan of a given diameter causes an increase in the velocity of the air through the wheel inlet, and this increased velocity will become at a certain point a serious loss and will decrease the mechanical efficiency.

4. *Influence of Shape of Blades.* — This appears, within reasonable limits, to be practically *nil*. Thus, six fans with tips of blades curved forward, three fans with flat blades, and one with blades curved back to a tangent with the circumference, all give very high efficiencies — over 70 per cent. A prominent manufacturer claims, however, that his tests show a higher efficiency with vanes curved forward as compared with straight or backwardly curved vanes.

5. *Influence of the Shape of the Spiral Casing.* — This appears to be considerable. The shapes of spiral casing in use fall into two classes, the first presenting a large spiral, beginning at or near the point of cut-off, and the second a circular casing reaching around three-quarters of the circumference of the fan, with a short spiral reaching to the *evasée* chimney.

Fans having the first form of casing appear to give in almost every case high efficiencies.

Fans that have a spiral belonging to the first class, but very much contracted, give only medium efficiencies. It seems probable that the proper shape of spiral casing would be one of such form that the air between each pair of blades could constantly and freely discharge into the space between the fan and casing, the whole being swept along to the *evasée* chimney. This would require a spiral beginning near the point of cut-off, enlarging by gradually increasing increments, to allow for the slowing of the air caused by its friction against the casing, and reaching the chimney with an area such that the air could make its exit with its then existing speed — somewhat less than the periphery-speed of the fan.

6. *Influence of the Shutter.* — The shutter certainly appears to be an advantage, as by it the exit area can be regulated to suit the varying quantity of air given by the fan, and in this way re-entries can be prevented. It is not uncommon to find shutterless fans, into the chimneys of which bits of paper may be dropped, which are drawn *into* the fan, make the circuit, and are again thrown out. This peculiarity has not been noticed with fans provided with shutters.

7. *Influence of the Speed at which a Fan is Run.* — It is noticeable that most of the fans giving high efficiency were running at a rather high periphery velocity. The best speed seems to be between 5000 and 6000 feet per minute. The fans appear to reach a maximum efficiency at somewhere about the speed given, and to decrease rapidly in efficiency when this maximum point is passed. The same manufacturer mentioned in note 4 states that the efficiency is not affected by the tip speed, providing that the comparison is always made at the same point in the efficiency curve.

In discussion of Mr. Norris's paper, Mr. A. H. Storrs says: From the "cubic feet per revolution" and "cubical contents of fan-blades," as given in the table, we find that the enclosed fans empty themselves from one-half to twice per revolution, while the open fans are emptied from one and three-quarters to nearly three times; this for fans of both types, on mines covering the same range of equivalent orifices. One open fan, on a very large orifice, was emptied nearly four times, while a closed fan, on a still larger orifice, only shows one and one-half times. For the open fans the "cubic feet per 100 ft. motion" is greater, in proportion to the fan width and equivalent orifice, than for the enclosed type. Notwithstanding this apparently free discharge of the open fans, they show very low efficiencies.

As illustrating the very large capacity of centrifugal fans to pass air, if the conditions of the mine are made favorable, a 16-ft. diam. fan, 4 ft. 6 in. wide, at 130 revolutions, passed 360,000 cu. ft. per min., and another, of same diameter, but slightly wider and with larger intake circles, passed 500,000 cu. ft., the water-gauge in both instances being about $\frac{1}{2}$ in.

T. D. Jones says: The efficiency reported in some cases by Mr. Norris is larger than I have ever been able to determine by experiment. My own experiments, recorded in the Pennsylvania Mine Inspectors' Reports from 1875 to 1881, did not show more than 60% to 65%.

DISK FANS.

Efficiency of Disk Fans. — Prof. A. B. W. Kennedy (*Industries*, Jan. 17, 1890) made a series of tests on two disk fans, 2 and 3 ft. diameter, known as the Verity Silent Air-propeller. The principal results and conclusions are condensed below.

In each case the efficiency of the fan, that is, the quantity of air delivered per effective horse-power, increases very rapidly as the speed diminishes, so that lower speeds are much more economical than higher ones. On the other hand, as the quantity of air delivered per revolution is very nearly constant, the actual useful work done by the fan increases almost directly with its speed. Comparing the large and small fans with about the same air delivery, the former (running at a much lower speed, of course) is much the more economical. Comparing the two fans running at the same speed, however, the smaller fan is very much the more economical. The delivery of air per revolution of fan is very nearly directly proportional to the area of the fan's diameter.

The air delivered per minute by the 3-ft. fan is nearly $12.5R$ cubic feet (R being the number of revolutions made by the fan per minute). For the 2-ft. fan the quantity is $5.7R$ cubic feet. For either of these or any other similar fans of which the area is A square feet, the delivery will be about $1.8AR$ cubic feet. Of course any change in the pitch of the blades might entirely change these figures.

The net H.P. taken up is not far from proportional to the square of the number of revolutions above 100 per minute. Thus for the 3-ft. fan the net H.P. is $\frac{(R-100)^2}{200,000}$, while for the 2-ft. fan the net H. P is $\frac{(R-100)^2}{1,000,000}$.

The denominators of these two fractions are very nearly proportional inversely to the square of the fan areas or the fourth power of the fan diameters. The net H.P. required to drive a fan of diameter D feet or area A square feet, at a speed of R revolutions per minute, will therefore be approximately $\frac{D^4(R-100)^2}{17,000,000}$ or $\frac{A^2(R-100)^2}{10,400,000}$.

The 2-ft. fan was noiseless at all speeds. The 3-ft. fan was also noiseless up to over 450 revolutions per minute.

| | Propeller, 2 ft. diam. | | | Propeller, 3 ft. diam. | | |
|---|---------------------------|--------|--------|---------------------------|--------|--------|
| Speed of fan, revolutions per minute.. | 750 | 676 | 577 | 576 | 459 | 373 |
| Net H.P. to drive fan and belt..... | 0.42 | 0.32 | 0.227 | 1.02 | 0.575 | 0.324 |
| Cubic feet of air per minute..... | 4,183 | 3,830 | 3,410 | 7,400 | 5,800 | 4,470 |
| Mean velocity of air in 3-ft. flue, feet per minute..... | 593 | 543 | 482 | 1,046 | 820 | 632 |
| Mean velocity of air in flue, same diameter as fan..... | 1,330 | 1,220 | 1,085 | | | |
| Cu. ft. of air per min. per effective H.P | 9,980 | 11,970 | 15,000 | 7,250 | 10,070 | 13,800 |
| Motion given to air per rev. of fan, ft.. | 1.77 | 1.81 | 1.88 | 1.82 | 1.79 | 1.70 |
| Cubic feet of air per rev. of fan | 5.58 | 5.66 | 5.90 | 12.8 | 12.6 | 12.0 |

Experiments made with a Blackman Disk Fan, 4 ft. diam. by Geo. A. Suter, to determine the volumes of air delivered under various conditions, and the power required; with calculations of efficiency and ratio of increase of power to increase of velocity, by G. H. Babcock. (*Trans. A. S. M. E.*, vii. 547):

| Rev. per min. | Cu. ft. of Air delivered per min., V. | Horse-power, HP. | Water- gauge, in., h. | Ratio of In- crease of Speed. | Ratio of In- crease of Delivery | Ratio of In- crease of Power. | Exponent x , $HP \propto V^x$. | Exponent y , $h \propto V^y$. | Efficiency of Fan. |
|---------------|--|---------------------|-----------------------------|-------------------------------------|---------------------------------------|-------------------------------------|--------------------------------------|-------------------------------------|-----------------------|
| 350 | 25,797 | 0.65 | | | | | | | 1.682 |
| 440 | 32,575 | 2.29 | | 1.257 | 1.262 | 3.523 | 5.4 | | .9553 |
| 534 | 41,929 | 4.42 | | 1.186 | 1.287 | 1.843 | 2.4 | | 1.062 |
| 612 | 47,756 | 7.41 | | 1.146 | 1.139 | 1.677 | 3.97 | | .9358 |
| For series | | | | 1.749 | 1.851 | 11.140 | 4. | | |
| 340 | 20,372 | 0.76 | | | | | | | .7110 |
| 453 | 26,660 | 1.99 | | 1.332 | 1.308 | 2.618 | 3.55 | | .6063 |
| 536 | 31,649 | 3.86 | | 1.183 | 1.187 | 1.940 | 3.86 | | .5205 |
| 627 | 36,543 | 6.47 | | 1.167 | 1.155 | 1.676 | 3.59 | | .4802 |
| For series | | | | 1.761 | 1.794 | 8.513 | 3.63 | | |
| 340 | 9,983 | 1.12 | 0.28 | | | | | | .3939 |
| 430 | 13,017 | 3.17 | 0.47 | 1.265 | 1.304 | 2.837 | 3.93 | 1.95 | .3046 |
| 534 | 17,018 | 6.07 | 0.75 | 1.242 | 1.307 | 1.915 | 2.25 | 1.74 | .3319 |
| 570 | 18,649 | 8.46 | 0.87 | 1.068 | 1.096 | 1.394 | 3.63 | 1.60 | .3027 |
| For series | | | | 1.676 | 1.704 | 7.554 | 3.24 | 1.81 | |
| 330 | 8,399 | 1.31 | 0.26 | | | | | | .2631 |
| 437 | 10,071 | 3.27 | 0.45 | 1.324 | 1.199 | 3.142 | 6.31 | 3.06 | .2188 |
| 516 | 11,157 | 6.00 | 0.75 | 1.181 | 1.108 | 1.457 | 3.66 | 4.96 | .2202 |
| For series | | | | 1.563 | 1.329 | 4.580 | 5.35 | 3.72 | |

Nature of the Experiments. — First Series: Drawing air through 30 ft. of 48-in. diam. pipe on inlet side of the fan.

Second Series: Forcing air through 30 ft. of 48-in. diam. pipe on outlet side of the fan.

Third Series: Drawing air through 30 ft. of 48-in. pipe on inlet side of the fan — the pipe being obstructed by a diaphragm of cheese-cloth.

Fourth Series: Forcing air through 30 ft. of 48-in. pipe on outlet side of fan — the pipe being obstructed by a diaphragm of cheese-cloth.

Mr. Babcock says concerning these experiments: The first four experiments are evidently the subject of some error, because the efficiency is such as to prove on an average that the fan was a source of power sufficient to overcome all losses and help drive the engine besides. The second series is less questionable, but still the efficiency in the first two experiments is larger than might be expected. In the third and fourth series the resistance of the cheese-cloth in the pipe reduces the efficiency largely, as would be expected. In this case the value has been calculated from

the height equivalent to the water-pressure, rather than the actual velocity of the air.

This record of experiments made with the disk fan shows that this kind of fan is not adapted for use where there is any material resistance to the flow of the air. In the centrifugal fan the power used is nearly proportioned to the amount of air moved under a given head, while in this fan the power required for the same number of revolutions of the fan increases very materially with the resistance, notwithstanding the quantity of air moved is at the same time considerably reduced. In fact from the inspection of the third and fourth series of tests, it would appear that the power required is very nearly the same for a given pressure, whether more or less air be in motion. It would seem that the main advantage, if any, of the disk fan over the centrifugal fan for slight resistances consists in the fact that the delivery is the full area of the disk, while with centrifugal fans intended to move the same quantity of air the opening is much smaller.

It will be seen by columns 8 and 9 of the table that the power used increased much more rapidly than the cube of the velocity, as in centrifugal fans. The different experiments do not agree with each other, but a general average may be assumed as about the cube root of the eleventh power.

Capacity of Disk Fans. (C. L. Hubbard, *The Metal Worker*, Sept. 5, 1908.) — The rated capacities given in catalogues are for fans revolving in free air — that is, mounted in an opening without being connected with ducts or subject to other frictional resistance.

The following data, based upon tests, apply to fans working against a resistance equivalent to that of a shallow heater of open pattern, and connecting with ducts of medium length through which the air flows at a velocity not greater than 600 or 800 ft. per minute. Under these conditions a good type of fan will propel the air in a direction parallel to the shaft, a distance equal to about 0.7 of its diameter at each revolution. From this we have the equation $Q = 0.7 D \times R \times A$, in which Q = cu. ft. of air discharged per minute; D = diam. of fan, in ft.; R = revs. per min.; A = area of fan, in sq. ft. The following table is calculated on this basis.

| Diam. of fan, in. | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 72 | 84 | 96 |
|--|------|------|------|------|------|------|------|------|-------|-------|-------|
| Cu. ft per rev. | 1.85 | 4.40 | 8.59 | 14.8 | 23.6 | 35.2 | 50.1 | 68.7 | 118.7 | 188.6 | 281.5 |
| Revolutions per min. for velocity of air through fan = 1000 ft. per min. | 952 | 714 | 571 | 476 | 408 | 357 | 317 | 286 | 238 | 204 | 179 |

The velocity of the air through the fan is proportional to the number of revolutions. For the conditions stated the H.P. required per 1000 cu. ft. of air moved will be about 0.16 when the velocity through the fan is 1000 ft. per min., 0.14 for a velocity of 800 ft., and 0.18 for 1200 ft. For a fan moving in free air the required speed for moving a given volume of air will be about 0.6 of the number of revolutions given above and the H.P. about 0.3 of that required when moving against the resistance stated.

POSITIVE ROTARY BLOWERS.

Rotary Blowers, Centrifugal Fans, and Piston Blowers. (Catalogue of the Connersville Blower Co.) — In ordinary work the advantage of a positive blower over a fan begins at about 8 oz. pressure, and the efficiency of the positive blower increases from 8 oz. as the pressure goes up to a point where the ordinary centrifugal fan fails entirely. The highest efficiency of rotary blowers is when they are working against pressures ranging between 1 and 8 lbs.

Fans, when run at constant speed, cannot be made to handle a constant volume of fluid when the pressure is variable; and they cannot give a high efficiency except for low and uniform pressures.

When a fan blower is used to furnish blast for a cupola it is driven at a constant speed, and the amount of air discharged by it varies according to the resistance met with in the cupola. With a positive blower running at a constant speed, however, there is a constant volume of air forced into the cupola, regardless of changing resistance.

A rotary blower of the two-impeller type is not an economical compressor, because the impellers are working against the full pressure at all times, while in an ideal blowing engine the theoretical mean effective pressure on the piston, when discharging air at 15 lbs. pressure, is 11½ lbs. For high pressures, on account of the increase of leakage and the increase of power required because it does not compress gradually, the rotary blower must give way to the piston type of machine. Commercially, the line is crossed at about 8 lbs. pressure.

1. A fan is the cheapest in first cost, and if properly applied may be used economically for pressures up to 8 oz.

2. A rotary blower costs more than a fan, but much less than a blowing engine; is more economical than either between 8 oz. and 8 lbs. pressure, and can be arranged to give a constant pressure or a constant volume.

3. Piston machines cost much more than rotary blowers, but should be used for continuous duty for pressures above 8 lbs., and may be economical if they are properly constructed and not run at too high a piston speed.

The horse-power required to operate rotary blowers is proportional to the volume and pressure of air discharged. In making estimates for power it is safe to assume that for each 1000 cu. ft. of free air discharged, at one pound pressure, 5 H.P. should be provided.

Test of a Rotary Blower. (Connersville Blower Co.) — The test was made in 1904 on two 39 × 84 in. blowers coupled direct to two 12 and 24 × 36 in. compound Corliss engines. The results given below are for the combined units.

| | | | | | | | | | |
|---------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Air pressure, lbs.... | 0 | 0.05 | 0.5 | 1.0 | 1.5 | 2. | 2.5 | 3. | 3.5 |
| Engine, I.H.P..... | 19.30 | 23.76 | 52.83 | 100.91 | 132.67 | 176.11 | 223.20 | 256.87 | 287.56 |
| Displacement, cu.ft. | 19,212 | 18,727 | 18,508 | 18,344 | 18,200 | 18,028 | 17,966 | 17,863 | 17,863 |
| Efficiency..... | | | 68.5 | 79 | 84 | 85.6 | 86 | 86 | 85.9 |

In calculating the efficiency the theoretical horse-power was taken as the power required to compress adiabatically and to discharge the net amount of air at the different pressures and at the same altitude. The test was made up to 3.5 lbs. only. Estimated efficiencies for higher pressures from an extension of the plotted curve are: 6 lbs. 84%, 8 lbs. 82%, 10 lbs. 79.5%. The theoretical discharge of the blower was 19,250 cu. ft.

CAPACITY OF ROTARY BLOWERS FOR CUPOLAS.

| Cu. ft. per rev. | Revs. per min. | Tons per hour. | Suitable for cupola in. diam.* | Cu. ft. per rev. | Revs. per min. | Tons per hour. | Suitable for cupola in. diam. |
|------------------|---------------------|----------------|--------------------------------|------------------|---------------------|----------------|-------------------------------|
| 1.5 | { 200 400 | 1 2 | { 18 to 20 | 45 | { 135 165 200 | 12 15 18 | { 54 to 66 |
| 3.3 | { 175 335 | 1 2 | { 24 to 27 | | { 130 155 | 15 18 | { 60 to 72 |
| 6 | { 185 275 | 2 3 | { 28 to 32 | 57 | { 185 140 | 21 18 | { 66 to 84 |
| 10 | { 200 250 | 4 5 | { 32 to 38 | 65 | { 160 185 | 21 24 | { 72 to 90 |
| 13 | { 150 190 | 4 5 | { 32 to 40 | 84 | { 125 145 160 | 21 24 27 | { 84 to 96 |
| 17 | { 175 150 | 6½ 5 | { 36 to 45 | 100 | { 135 160 115 | 27 30 27 | { 96 to 108 |
| 24 | { 205 250 | 8½ 10 | { 42 to 54 | 118 | { 130 140 | 30 33 | { 108 to 120 |
| 33 | { 166 200 240 | 8 10 12 | { 48 to 60 | | | | { Two cupolas 60 to 66 |
| | { 150 180 210 | 10 12 14 | | | | | |

* Inside diam. The capacity in tons per hour is based on 30,000 cu. ft. of air per ton of iron melted.

For smith fires; an ordinary fire requires about 60 cu. ft. per min.

For oil furnaces; an ordinary furnace burns about 2 gallons of oil per hour and 1800 cu. ft. of air should be provided for each gallon of oil. For each 100 cu. ft. of air discharged per minute at 16 oz. pressure, $\frac{1}{2}$ H.P. should be provided.

| | | | |
|-------------------------|------------------|------------------|----------------------|
| Sizes of small blowers. | 173 | 288 | 576 cu. in. per rev. |
| Revs. per min. | 800 to 1500 | 500 to 900 | 300 to 600 |
| Diam. of outlet, in. | 21 $\frac{1}{2}$ | 21 $\frac{1}{2}$ | 3 |

ROTARY GAS EXHAUSTERS.

| | | | | | | | | | |
|------------------------|-----------------|------------------|-----|-----|-----|-----|-----|-----|-----|
| Cu. ft. per rev. | 2 $\frac{3}{4}$ | 11 $\frac{1}{2}$ | 3.3 | 6 | 10 | 13 | 17 | 24 | 33 |
| Rev. per min. | 200 | 180 | 170 | 160 | 150 | 150 | 140 | 130 | 120 |
| Diam. of pipe opening. | 4 | 6 | 8 | 10 | 12 | 12 | 16 | 16 | 20 |
| Cu. ft. per rev. | 45 | 57 | 65 | 84 | 100 | 118 | 155 | 200 | 300 |
| Rev. per min. | 110 | 100 | 95 | 90 | 85 | 82 | 80 | 80 | 75 |
| Diam. pipe opening | 20 | 24 | 24 | 30 | 30 | 30 | 36 | 36 | 42 |

There is no gradual compressing of air in a rotary machine, and the unbalanced areas of the impellers are working against the full difference of pressure at all times. The possible efficiency of such a machine under ordinary temperature and conditions of atmosphere, assuming no mechanical friction, leakage, nor radiation of heat of compression, would be as follows:

| | | | | | | | |
|-----------------|------|------|------|------|----|------|------|
| Gauge pres. lb. | 1 | 2 | 3 | 4 | 5 | 10 | 15 |
| Efficiency % | 97.5 | 95.5 | 93.3 | 91.7 | 90 | 82.7 | 76.7 |

The proper application of rotary positive machines when operating in air or gas under differences of pressures from 8 oz. to 5 lbs. is where constant quantities of fluid are required to be delivered against a variable resistance, or where a constant pressure is required and the volume is variable. These are the requirements of gas works, pneumatic-tube transmission (both the vacuum and pressure systems), foundry cupolas, smelting furnaces, knobbling fires, sand blast, burning of fuel oil, conveying granular substances, the operation of many kinds of metallurgical furnaces, etc. — J. T. Wilkin, *Trans. A. S. M. E.*, Vol. xxiv.

STEAM-JET BLOWER AND EXHAUSTER.

A blower and exhauster is made by L. Schutte & Co., Philadelphia, on the principle of the steam-jet ejector. The following is a table of capacities:

| Size No. | Quantity of Air per hr. in cubic feet. | Diameter of Pipes in inches. | | Size No. | Quantity of Air per hr. in cubic feet. | Diameter of Pipes in inches. | |
|----------|--|------------------------------|------------------|----------|--|------------------------------|------|
| | | Steam. | Air. | | | Steam. | Air. |
| 000 | 1,000 | 1 $\frac{1}{2}$ | 1 | 5 | 30,000 | 21 $\frac{1}{2}$ | 5 |
| 00 | 2,000 | 3 $\frac{3}{4}$ | 11 $\frac{1}{2}$ | 6 | 36,000 | 21 $\frac{1}{2}$ | 6 |
| 0 | 4,000 | 1 | 2 | 7 | 42,000 | 3 | 6 |
| 1 | 6,000 | 11 $\frac{1}{4}$ | 21 $\frac{1}{2}$ | 8 | 48,000 | 3 | 7 |
| 2 | 12,000 | 11 $\frac{1}{2}$ | 3 | 9 | 54,000 | 31 $\frac{1}{2}$ | 7 |
| 3 | 18,000 | 2 | 31 $\frac{1}{2}$ | 10 | 60,000 | 31 $\frac{1}{2}$ | 8 |
| 4 | 24,000 | 2 | 4 | | | | |

The admissible vacuum and counter-pressure, for which the apparatus is constructed, is up to a rarefaction of 20 inches of mercury, and a counter-pressure up to one-sixth of the steam-pressure.

The table of capacities is based on a steam-pressure of about 60 lbs., and a counter-pressure of about 8 lbs. With an increase of steam-pressure or decrease of counter-pressure the capacity will largely increase.

Another steam-jet blower is used for boiler-firing, ventilation, and similar purposes where a low counter-pressure or rarefaction meets the requirements.

The volumes as given in the following table of capacities are under the supposition of a steam-pressure of 45 lbs. and a counter-pressure of, say, 2 inches of water:

| Size No. | Cubic feet of Air delivered per hour. | Diam. of Steam-pipe in inches. | Diam. in inches of— | | Size No. | Cubic feet of Air delivered per hour. | Diam. of Steam-pipe in inches. | Diam. in inches of— | |
|----------|---------------------------------------|--------------------------------|---------------------|-------|----------|---------------------------------------|--------------------------------|---------------------|-------|
| | | | Inlet. | Disch | | | | Inlet. | Disch |
| 00 | 6,000 | 3/8 | 4 | 3 | 4 | 250,000 | 1 | 17 | 14 |
| 0 | 12,000 | 1/2 | 5 | 4 | 6 | 500,000 | 1 1/4 | 24 | 20 |
| 1 | 30,000 | 1/2 | 8 | 6 | 8 | 1,000,000 | 1 1/2 | 32 | 27 |
| 2 | 60,000 | 3/4 | 11 | 8 | 10 | 2,000,000 | 2 | 42 | 36 |
| 3 | 125,000 | 1 | 14 | 10 | | | | | |

The Steam-jet as a Means for Ventilation. — Between 1810 and 1850 the steam-jet was employed to a considerable extent for ventilating English collieries, and in 1852 a committee of the House of Commons reported that it was the most powerful and at the same time the cheapest method for the ventilation of mines; but experiments made shortly afterwards proved that this opinion was erroneous, and that furnace ventilation was less than half as expensive, and in consequence the jet was soon abandoned as a permanent method of ventilation.

For an account of these experiments see *Colliery Engineer*, Feb., 1890. The jet, however, is sometimes advantageously used as a substitute, for instance, in the case of a fan standing for repairs, or after an explosion, when the furnace may not be kept going, or in the case of the fan having been rendered useless.

BLOWING-ENGINES.

Corliss Horizontal Cross-compound Condensing Blowing-engines. (Philadelphia Engineering Works.)

| Indicated Horse-power. | | Revs. per min. | Cu. ft. Free Air per min. | Blast-pr sure per sq. in., lbs. | H. P. Cyl-inder, in. Diam., in. | L. P. Cyl-inder, in. Diam., in. | Blast Cyl-inder, 2 Diam., in. | Stroke of All, in. | Approx. Shipping Weight. | Approx. Shipping Weight of Vert. Eng. |
|-------------------------|-------------------------|----------------|---------------------------|---------------------------------|---------------------------------|---------------------------------|-------------------------------|--------------------|--------------------------|---------------------------------------|
| 15 Exp. 125 lbs. Steam. | 13 Exp. 100 lbs. Steam. | | | | | | | | | |
| | 1,572 | 40 | 30,400 | 15 | 44 | 78 | (2) 84 | 60 | 505,000 | 605,000 |
| | 2,280 | 60 | 45,600 | | | | | | | |
| | 1,290 | 40 | 30,400 | 12 | 42 | 72 | (2) 84 | 60 | 475,000 | 550,000 |
| | 2,060 | 60 | 45,600 | | | | | | | |
| 1,050 1,596 | | 40 | 30,400 | 10 | 32 | 60 | (2) 84 | 60 | 355,000 | 436,000 |
| | | 60 | 45,600 | | | | | | | |
| | 1,340 | 40 | 26,800 | 15 | 40 | 72 | (2) 78 | 60 | 445,000 | 545,000 |
| | 1,980 | 60 | 39,600 | | | | | | | |
| | 1,152 | 40 | 26,800 | 12 | 38 | 70 | (2) 78 | 60 | 425,000 | 491,000 |
| | 1,702 | 60 | 39,600 | | | | | | | |
| | 938 | 40 | 26,800 | 10 | 36 | 66 | (2) 78 | 60 | 415,000 | 450,000 |
| | 1,386 | 60 | 39,600 | | | | | | | |
| | 780 | 40 | 15,680 | 15 | 34 | 60 | (2) 72 | 60 | 340,000 | 430,000 |
| | 1,175 | 60 | 23,500 | | | | | | | |
| | 548 | 40 | 15,680 | 10 | 28 | 50 | (2) 72 | 60 | 270,000 | 300,000 |
| | 822 | 60 | 23,500 | | | | | | | |

Vertical engines are built of the same dimensions as above, except that the stroke is 48 in. instead of 60, and they are run at a higher number of revolutions to give the same piston-speed and the same I.H.P.

The calculations of power, capacity, etc., of blowing-engines are the same as those for air-compressors. They are built without any provision for cooling the air during compression. About 400 feet per minute is the usual piston-speed for recent forms of engines, but with positive air-valves, which have been introduced to some extent, this speed may be increased. The efficiency of the engine, that is, the ratio of the I.H.P. of the air-cylinder to that of the steam-cylinder, is usually taken at 90 per cent, the losses by friction, leakage, etc., being taken at 10 per cent.

HEATING AND VENTILATION.

Ventilation. (A. R. Wolff, *Stevens Indicator*, April, 1890.) — The popular impression that the impure air falls to the bottom of a crowded room is erroneous. There is a constant mingling of the fresh air admitted with the impure air due to the law of diffusion of gases, to difference of temperature, etc. The process of ventilation is one of dilution of the impure air by the fresh, and a room is properly ventilated in the opinion of the hygienists when the dilution is such that the carbonic acid in the air does not exceed from 6 to 8 parts by volume in 10,000. Pure country air contains about 4 parts CO₂ in 10,000, and badly-ventilated quarters as high as 80 parts.

An ordinary man exhales 0.6 of a cubic foot of CO₂ per hour. New York gas gives out 0.75 of a cubic feet of CO₂ for each cubic foot of gas burnt. An ordinary lamp gives out 1 cu. ft. of CO₂ per hour. An ordinary candle gives out 0.3 cu. ft. per hour. One ordinary gaslight equals in vitiating effect about 5½ men, an ordinary lamp 12/3 men, and an ordinary candle 1½ man.

To determine the quantity of air to be supplied to the inmates of an unlighted room, to dilute the air to a desired standard of purity, we can establish equations as follows:

Let v = cubic feet of fresh air to be supplied per hour;

r = cubic feet of CO₂ in each 10,000 cu. ft. of the entering air;

R = cubic feet of CO₂ which each 10,000 cu. ft. of the air in the room may contain for proper health conditions;

n = number of persons in the room;

0.6 = cubic feet of CO₂ exhaled by one man per hour.

Then $\frac{v \times r}{10,000} + 0.6 n$ equals cubic feet of CO₂ communicated to the room during one hour.

This value divided by v and multiplied by 10,000 gives the proportion of CO₂ in 10,000 parts of the air in the room, and this should equal R , the standard of purity desired. Therefore

$$R = \frac{10,000 \left[\frac{v \times r}{10,000} + 0.6 n \right]}{v}, \text{ or } v = \frac{6000 n}{R - r}.$$

If we place r at 4 and R at 6, $v = 6000 n \div (6 - 4) = 3000 n$, or the quantity of air to be supplied per person is 3000 cubic feet per hour.

If the original air in the room is of the purity of external air, and the cubic contents of the room is equal to 100 cu. ft. per inmate, only $3000 - 100 = 2900$ cu. ft. of fresh air from without will have to be supplied the first hour to keep the air within the standard purity of 6 parts of CO₂ in 10,000. If the cubic contents of the room equals 200 cu. ft. per inmate, only $3000 - 200 = 2800$ cu. ft. will have to be supplied the first hour to keep the air within the standard purity, and so on.

Again, if we only desire to maintain a standard of purity of 8 parts of carbonic acid in 10,000, the equation gives as the required air-supply per hour

$$v = \frac{6000}{8 - 4} n = 1500 n, \text{ or } 1500 \text{ cu. ft. of fresh air per inmate per hour.}$$

Cubic feet of air containing 4 parts of carbonic acid in 10,000 necessary per person per hour to keep the air in room at the composition of

| 6 | 7 | 8 | 9 | 10 | 15 | 20 | parts of CO ₂ in 10,000. |
|------|------|------|------|------|-----|-----|-------------------------------------|
| 3000 | 2000 | 1500 | 1200 | 1000 | 545 | 375 | cubic feet. |

If the original air in the room is of purity of external atmosphere (4 parts of carbonic acid in 10,000), the amount of air to be supplied the first hour, for given cubic spaces per inmate, to have given standards of purity not exceeded at the end of the hour, is obtained from the following table:

| Cubic Feet of Space in Room per Individual. | Proportion of Carbonic Acid in 10,000 Parts of the Air, not to be Exceeded at End of Hour. | | | | | | |
|---|--|-------|-------|-------|-------|-------|-------|
| | 6 | 7 | 8 | 9 | 10 | 15 | 20 |
| | Cubic Feet of Air, of Composition 4 Parts of Carbonic Acid in 10,000, to be Supplied the First Hour. | | | | | | |
| 100 | 2900 | 1900 | 1400 | 1100 | 900 | 445 | 275 |
| 200 | 2800 | 1800 | 1300 | 1000 | 800 | 345 | 175 |
| 300 | 2700 | 1700 | 1200 | 900 | 700 | 245 | 75 |
| 400 | 2600 | 1600 | 1100 | 800 | 600 | 145 | None |
| 500 | 2500 | 1500 | 1000 | 700 | 500 | 45 | |
| 600 | 2400 | 1400 | 900 | 600 | 400 | None | |
| 700 | 2300 | 1300 | 800 | 500 | 300 | | |
| 800 | 2200 | 1200 | 700 | 400 | 200 | | |
| 900 | 2100 | 1100 | 600 | 300 | 100 | | |
| 1000 | 2000 | 1000 | 500 | 200 | None | | |
| 1500 | 1500 | 500 | None | None | | | |
| 2000 | 1000 | None | | | | | |
| 2500 | 500 | | | | | | |

It is exceptional that systematic ventilation supplies the 3000 cubic feet per inmate per hour, which adequate health considerations demand. For large auditoriums in which the cubic space per individual is great, and in which the atmosphere is thoroughly fresh before the rooms are occupied, and the occupancy is of two or three hours' duration, the systematic air-supply may be reduced, and 2000 to 2500 cubic feet per inmate per hour is a satisfactory allowance.

In hospitals where, on account of unhealthy excretions of various kinds, the air-dilution must be largest, an air-supply of from 4000 to 6000 cubic feet per inmate per hour should be provided, and this is actually secured in some hospitals. A report dated March 15, 1882, by a commission appointed to examine the public schools of the District of Columbia, says:

"In each class-room not less than 15 square feet of floor-space should be allotted to each pupil. In each class-room the window-space should not be less than one-fourth the floor-space, and the distance of desk most remote from the window should not be more than one and a half times the height of the top of the window from the floor. The height of the class-room should never exceed 14 feet. The provisions for ventilation should be such as to provide for each person in a class-room not less than 30 cubic feet of fresh air per minute (1800 per hour), which amount must be introduced and thoroughly distributed without creating unpleasant draughts, or causing any two parts of the room to differ in temperature more than 2° Fahr., or the maximum temperature to exceed 70° Fahr." [The provision of 30 cu. ft. per minute for each person in a class-room is now (1909) required by law in several states.]

When the air enters at or near the floor, it is desirable that the velocity of inlet should not exceed 2 feet per second, which means larger sizes of register openings and flues than are usually obtainable, and much higher velocities of inlet than two feet per second are the rule in practice. The velocity of current into vent-flues can safely be as high as 6 or even 10 feet per second, without being disagreeably perceptible.

The entrance of fresh air into a room is coincident with, or dependent on, the removal of an equal amount of air from the room. The ordinary means of removal is the vertical vent-duct, rising to the top of the build-

ing. Sometimes reliance for the production of the current in this vent-duct is placed solely on the difference of temperature of the air in the room and that of the external atmosphere; sometimes a steam coil is placed within the flue near its bottom to heat the air within the duct; sometimes steam pipes (risers and returns) run up the duct performing the same functions; or steam jets within the flue, or exhaust fans, driven by steam or electric power, act directly as exhausters; sometimes the heating of the air in the flue is accomplished by gas-jets.

The draft of such a duct is caused by the difference of weight of the heated air in the duct, and of a column of equal height and cross-sectional area of the external air.

Let d = density, or weight in pounds, of a cubic foot of the external air.

Let d_1 = density, or weight in pounds, of a cubic foot of the heated air within the duct.

Let h = vertical height, in feet, of the vent-duct.

$h(d - d_1)$ = the pressure, in pounds per square foot, with which the air is forced into and out of the vent-duct.

This pressure expressed in height of a column of air of density within the vent-duct is $h(d - d_1) \div d$.

Or, if t = absolute temperature of external air, and t_1 = absolute temperature of the air in the vent-duct, then the pressure = $h(t_1 - t) \div t$.

The theoretical velocity, in feet per second, with which the air, would travel through the vent-duct under this pressure is

$$v = \sqrt{\frac{2gh(t_1 - t)}{t}} = 8.02 \sqrt{\frac{h(t_1 - t)}{t}}.$$

The actual velocity will be considerably less than this, on account of loss due to friction. This friction will vary with the form and cross-sectional area of the vent-duct and its connections, and with the degree of smoothness of its interior surface. On this account, as well as to prevent leakage of air through crevices in the wall, tin lining of vent-flues is desirable.

The loss by friction may be estimated at approximately 50%, and the actual velocity of the air as it flows through the vent-duct is

$$v = \frac{1}{2} \sqrt{2gh \frac{(t_1 - t)}{t}}, \text{ or, approximately, } v = 4 \sqrt{h \frac{(t_1 - t)}{t}}.$$

If V = velocity of air in vent-duct, in feet per minute, and the external air be at 32° Fahr., since the absolute temperature on Fahrenheit scale equals thermometric temperature plus 459.4,

$$V = 240 \sqrt{h \frac{(t_1 - t)}{491.4}},$$

from which has been computed the following table:

Quantity of Air, in Cubic Feet, Discharged per Minute through a Ventilating Duct, of which the Cross-sectional Area is One Square Foot (the External Temperature of Air being 32° Fahr.).

| Height of Vent-duct in feet. | Excess of Temperature of Air in Vent-duct above that of External Air. | | | | | | | | |
|------------------------------|---|-----|-----|-----|-----|-----|-----|------|------|
| | 5° | 10° | 15° | 20° | 25° | 30° | 50° | 100° | 150° |
| 10..... | 77 | 108 | 133 | 153 | 171 | 188 | 242 | 342 | 419 |
| 15..... | 94 | 133 | 162 | 188 | 210 | 230 | 297 | 419 | 514 |
| 20..... | 108 | 153 | 188 | 217 | 242 | 265 | 342 | 484 | 593 |
| 25..... | 121 | 171 | 210 | 242 | 271 | 297 | 383 | 541 | 663 |
| 30..... | 133 | 188 | 230 | 265 | 297 | 325 | 419 | 593 | 726 |
| 35..... | 143 | 203 | 248 | 286 | 320 | 351 | 453 | 640 | 784 |
| 40..... | 153 | 217 | 265 | 306 | 342 | 375 | 484 | 683 | 838 |
| 45..... | 162 | 230 | 282 | 325 | 363 | 398 | 514 | 723 | 889 |
| 50..... | 171 | 242 | 297 | 342 | 383 | 419 | 541 | 760 | 937 |

Multiplying the figures in preceding table by 60 gives the cubic feet of air discharged per hour per square foot of cross-section of vent-duct. Knowing the cross-sectional area of vent-ducts we can find the total discharge; or for a desired air-removal, we can proportion the cross-sectional area of vent-ducts required.

Heating and Ventilating of Large Buildings. (A. R. Wolff, *Jour. Frank. Inst.*, 1893.) — The transmission of heat from the interior to the exterior of a room or building, through the walls, ceilings, windows, etc., is calculated as follows:

S = amount of transmitting surface in square feet;

t = temperature F. inside, t_0 = temperature outside;

K = a coefficient representing, for various materials composing buildings, the loss by transmission per square foot of surface in British thermal units per hour, for each degree of difference of temperature on the two sides of the material;

Q = total heat transmission = $SK(t - t_0)$.

This quantity of heat is also the amount that must be conveyed to the room in order to make good the loss by transmission, but it does not cover the additional heat to be conveyed on account of the change of air for purposes of ventilation. (See Wolff's coefficients below, page 659.)

These coefficients are to be increased respectively as follows: 10% when the exposure is a northerly one, and winds are to be counted on as important factors; 10% when the building is heated during the daytime only, and the location of the building is not an exposed one; 30% when the building is heated during the daytime only, and the location of the building is exposed; 50% when the building is heated during the winter months intermittently, with long intervals (say days or weeks) of non-heating.

The value of the radiating-surface is about as follows: Ordinary bronzed cast-iron radiating-surfaces, in American radiators (of Bundy or similar type), located in rooms, give out about 250 heat-units per hour for each square foot of surface, with ordinary steam-pressure, say 3 to 5 lbs. per sq. in., and about 0.6 this amount with ordinary hot-water heating.

Non-painted radiating-surfaces, of the ordinary "indirect" type (Climax or pin surfaces), give out about 400 heat-units per hour for each square foot of heating-surface, with ordinary steam-pressure, say 3 to 5 lbs. per sq. in.; and about 0.6 this amount with ordinary hot-water heating.

A person gives out about 400 heat-units per hour; an ordinary gas-burner, about 4800 heat-units per hour; an incandescent electric (16 candle-power) light, about 1600 heat-units per hour.

The following example is given by Mr. Wolff to show the application of the formula and coefficients:

Lecture-room 40×60 ft., 20 ft. high, 48,000 cubic feet, to be heated to 69° F.; exposures as follows: North wall, 60×20 ft., with four windows, each 14×8 feet, outside temperature 0° F. Room beyond west wall and room overhead heated to 69° , except a double skylight in ceiling, 14×24 ft., exposed to the outside temperature of 0° . Store-room beyond east wall at 36° . Door 6×12 ft. in wall. Corridor beyond south wall heated to 59° . Two doors, 6×12 , in wall. Cellar below, temperature 36° .

If we assume that the lecture-room must be heated to 69° F. in the daytime when unoccupied, so as to be at this temperature when first persons arrive, there will be required, ventilation not being considered, and bronzed direct low-pressure steam-radiators being the heating media, about $113,550 \div 250 = 455$ sq. ft. of radiating-surface.

If we assume that there are 160 persons in the lecture-room, and we provide 2500 cubic feet of fresh air per person per hour, we will supply $160 \times 2500 = 400,000$ cubic feet of air per hour (*i.e.*, over eight changes of contents of room per hour).

To heat this air from 0° F. to 69° F. will require $400,000 \times 0.01785 \times 69 = 492,660$ thermal units per hour (0.01785 being the product of the weight of a cubic foot, 0.075, by the specific heat of air, 0.238). Accordingly there must be provided $492,660 \div 400 = 1232$ sq. ft. of indirect

surface, to heat the air required for ventilation, in zero weather. If the room were to be warmed entirely indirectly, that is, by the air supplied to room (including the heat to be conveyed to cover loss by transmission through walls, etc.), there would have to be conveyed to the fresh-air supply $492,660 + 118,443 = 611,103$ heat-units. This would imply the provision of an amount of indirect heating-surface of the "Climax" type of $611,103 \div 400 = 1527$ sq. ft., and the fresh air entering the room would have to be at a temperature of about 86° F., viz.,

$$69^{\circ} + \frac{118,413}{400,000 \times 0.01785}, \text{ or } 69 + 17 = 86^{\circ} \text{ F.}$$

The above calculations do not, however, take into account that 160 persons in the lecture-room give out $160 \times 400 = 64,000$ thermal units per hour; and that, say, 50 electric lights give out $50 \times 1600 = 80,000$ thermal units per hour; or, say, 50 gaslights, $50 \times 4800 = 240,000$ thermal units per hour. The presence of 160 people and the gaslighting would diminish considerably the amount of heat required. Practically, it appears that the heat generated by the presence of 160 people, 64,000 heat-units, and by 50 electric lights, 80,000 heat-units, a total of 144,000 heat-units, more than covers the amount of heat transmitted through walls, etc. Moreover, that if the 50 gaslights give out 240,000 thermal units per hour, the air supplied for ventilation must enter considerably below 69° Fahr., or the room will be heated to an unbearably high temperature. If 400,000 cubic feet of fresh air per hour are supplied, and 240,000 thermal units per hour generated by the gas must be abstracted, it means

that the air must, under these conditions, enter $\frac{240,000}{400,000 \times 0.01785} =$ about 34° less than 86° , or at about 52° Fahr. Furthermore, the additional vitiation due to gaslighting would necessitate a much larger supply of fresh air than when the vitiation of the atmosphere by the people alone is considered, one gaslight vitiating the air as much as five men.

The following table shows the calculation of heat transmission (some figures changed from the original):

| $t - t_0$ (Fahr. degrees). | Kind of Transmitting Surface. | Thickness of Wall in inches. | Calculation of Area of Transmitting Surface. | Square feet of Surface. | $K (t - t_0)$. | Thermal Units. |
|----------------------------|--|------------------------------|--|-------------------------|-----------------|----------------|
| 69° | Outside wall..... | 36" | $63 \times 22 - 448$ | 938 | 10 | 9,380 |
| 69 | Four windows (single)..... | | $4 \times 8 \times 14$ | 448 | 83 | 37,186 |
| 33 | Inside wall (store-room)..... | 36" | $42 \times 22 - 72$ | 852 | 4 | 3,408 |
| 33 | Door..... | | 6×12 | 72 | 19 | 1,368 |
| 10 | Inside wall (corridor)..... | 24" | $45 \times 22 - 72$ | 918 | 2 | 1,836 |
| 10 | Door..... | | 6×12 | 72 | 5 | 360 |
| 10 | Inside wall (corridor)..... | 36" | $17 \times 22 - 72$ | 302 | 1 | 302 |
| 10 | Door..... | | 6×12 | 72 | 5 | 360 |
| 69 | Roof..... | | $32 \times 42 - 336$ | 1,008 | 10 | 10,080 |
| 69 | Double skylight..... | | 14×24 | 336 | 35 | 11,760 |
| 33 | Floor..... | | 62×42 | 2,604 | 4 | 10,416 |
| | | | | | | 86,454 |
| | Supplementary allowance, north outside wall, 10%..... | | | | | 938 |
| | Supplementary allowance, north outside windows, 10% | | | | | 3,718 |
| | | | | | | 91,110 |
| | Exposed location and intermittent day or night use, 30%..... | | | | | 27,333 |
| | Total thermal units | | | | | 118,443 |

STANDARD VALUES FOR USE IN CALCULATION OF HEATING AND VENTILATING PROBLEMS.

Heating Value of Coal.

| | Volatile Matter in the Combustible, per cent. | Heating Value per lb. Combustible, B.T.U. | Average. | Moisture, in Air-dried Coal, per cent. | Ash in Air-dried Coal, per cent. |
|-------------------|---|---|----------|--|----------------------------------|
| Anthracite..... | 3 to 7.5 | 14,700 to 14,900 | 14,800 | 0.5 to 1.0 | 10. to 18. |
| Semi-anthracite. | 7.5 to 12.5 | 14,900 to 15,500 | 15,200 | 0.5 to 1.0 | 10. to 18. |
| Semi-bituminous | 12.5 to 25 | 15,500 to 16,000 | 15,750 | 0.5 to 1.0 | 5. to 10. |
| Bit. eastern..... | 25 to 40 | 14,800 to 15,000 | 15,150 | 1. to 4. | 5. to 15. |
| Bit. western..... | 35 to 50 | 13,500 to 14,800 | 14,150 | 4. to 14. | 10. to 25. |
| Lignite..... | Over 50 | 11,000 to 13,500 | 12,250 | 10. to 18. | 5. to 25. |

Average Heating Value of Air-Dried Coal.—Anthracite, 12,600; semi-anthracite, 12,950; semi-bituminous, 14,450; bituminous eastern, 13,250; bituminous western, 10,400; lignite, 9,700.

Eastern bituminous coal is that of the Appalachian coal field extending from Pennsylvania and Ohio to Alabama. Western bituminous coal is that of the great coal fields west of Ohio.

Steam Boiler Efficiency.—The maximum efficiency obtainable with anthracite in low-pressure steam boilers, water heaters or hot-air furnaces is about 80 per cent, when the thickness of the coal bed and the draft are such as to cause enough air to be supplied to effect complete combustion of the carbon to CO₂. With coals high in volatile matter the maximum efficiency is probably not over 70 per cent. Very much lower efficiencies than these figures are obtained when the air supply is either deficient or greatly in excess, or when the furnace is not adapted to burn the volatile matter in the coal. D. T. Randall, in tests made in 1908 for the U. S. Geological Survey, with house-heating boilers, obtained efficiencies ranging from 0.62 with coke, 0.61 with anthracite, and 0.58 with semi-bituminous, down to 0.39 with Illinois coal.

Available Heating Value of the Coal.—Using the figures given above as the average heating value of coal stored in a dry cellar, we have the following as the probable *maximum* values in British Thermal Units, of the heat available for furnishing steam or heating water or air, for the several efficiencies stated:

| Anthracite. | Semi-An. | Semi-Bit. | Bit. East. | Bit. West. | Lignite. |
|----------------|----------|-----------|------------|------------|----------|
| Eff'y.....0.80 | 0.77 | 0.75 | 0.70 | 0.65 | 0.60 |
| B.T.U. 10,080 | 9,933 | 10,837 | 9,275 | 6,760 | 5,820 |

For average values in practice, about 10 per cent may be deducted from these figures. (It is possible that an efficiency higher than 80% may be obtained with anthracite in some forms of air-heating furnaces in which the escaping chimney gases are cooled, by contact with the cold air inlet pipes, to comparatively low temperatures.)

The value 10,000 B.T.U. is usually taken as the figure to be used in calculation for design of heating and ventilating apparatus. For coals with lower available heating values proper reductions must be made.

Heat Transmission through Walls, Windows, etc., in B.T.U. per sq. ft. per Hour per Degree of Difference of Temperature.

BRICK WALLS.

| Thick- ness, In. | Wolff. | Hauss. | Average, B.T.U.* | Thickness, In. | Wolff. | Hauss. | Average, B.T.U.* |
|------------------------|--------|--------|---------------------|-------------------|--------|--------|---------------------|
| 4 | 0.66 | | 0.537 | 25 | | 0.18 | 0.188 |
| 4 3/4 | | 0.48 | 0.508 | 28 | 0.18 | | 0.172 |
| 8 | 0.45 | | 0.397 | 30 | | 0.16 | 0.163 |
| 10 | | 0.34 | 0.351 | 32 | 0.16 | | 0.154 |
| 12 | 0.33 | | 0.313 | 35 | | 0.13 | 0.143 |
| 15 | | 0.26 | 0.272 | 36 | 0.145 | | 0.140 |
| 16 | 0.27 | | 0.260 | 40 | 0.13 | 0.12 | 0.128 |
| 20 | 0.23 | 0.22 | 0.222 | 45 | | 0.11 | 0.116 |
| 24 | 0.20 | | 0.194 | | | | |

* The average figure for brick walls was obtained by plotting the reciprocals of Wolff's and Hauss's figures and drawing a straight line between them, representing the average heat resistances, and then taking the reciprocals of the resistances for different thicknesses. The resistance corresponds to the straight line formula $R = 0.12 + 0.165 t$, where t = thickness in inches. (Hauss's figures are from a paper by Chas. F. Hauss, of Antwerp, Belgium, in *Trans. A. S. H. V. E.*, 1904.)

SOLID SANDSTONE WALLS. (Hauss.)

| | | | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|------|------|------|
| Thickness, in. . . | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| B.T.U. | 0.45 | 0.39 | 0.35 | 0.32 | 0.29 | 0.26 | 0.24 | 0.22 | 0.20 | 0.19 |

For limestone walls, add 10 per cent.

| | Wolff. B.T.U. | Hauss. B.T.U. | | Wolff. B.T.U. | Hauss. B.T.U. |
|-------------------------|------------------|------------------|---|------------------|------------------|
| GLASS SURFACES. | | | FLOORS. | | |
| Vault light..... | 1.42 | | Joists with double floor..... | 0.10 | 0.07 |
| Single window..... | 1.20 | 1.00 | Concrete floor..... | 0.31 | |
| Double window..... | 0.56 | 0.46 | Fireproof construction, planked over. | 0.124 | |
| Single skylight..... | 1.03 | 1.06 | Wooden beam construction, planked over. | 0.083 | |
| Double skylight..... | 0.50 | 0.48 | Concrete floor on brick arch..... | | 0.22 |
| DOORS. | | | Stone floor on arches..... | | 0.20 |
| Door..... | | 0.40 | Planks laid on earth. | | 0.16 |
| 1-in. pine..... | 0.40 | | Planks laid on asphalt..... | | 0.20 |
| 2-in. pine..... | 0.28 | | Arch with air space.. | | 0.09 |
| PARTITIONS. | | | Stones laid on earth. | | 0.08 |
| Solid plaster, | | | CEILINGS. | | |
| 1 3/4 to 2 1/4 in. | | 0.60 | Joists with single floor..... | | 0.10 |
| 2 1/2 to 3 1/4 in. | | 0.48 | Arches with air space | | 0.14 |
| Fireproof..... | 0.30 | | | | |
| 2-in. pine board..... | 0.28 | | | | |

Allowances for Exposures. — Wolff adds 25% for north and west exposures, 15% for east, and 5% for south exposures, also 10% additional for reheating, and 10% to the transmission through floor and ceilings. The allowance for reheating Mr. Wolff explains as follows in a letter to the author, Mar. 10, 1905. The allowance is made on the basis that the apparatus will not be run continuously; in other words, that it will not be run at all, or only lightly, overnight. The rooms will cool off below the required temperature of 70°, and to be able to heat up quickly in the morning an allowance of 10% is made to the transmission figures to meet this condition. Hauss makes allowances as follows: 5% for rooms with unusual exposure; 10% where exposures are north, east, northeast, northwest and west; $31\frac{1}{3}\%$ where the height of ceiling is more than 13 ft.; $62\frac{2}{3}\%$ where it is more than 15 ft.; 10% where it is more than 18 ft. For rooms heated daily, but where heating is interrupted at night, add

$$A = 0.0025 [(N - 1) W_1] \div Z.$$

For rooms not heated daily, add $B = [0.1 W (8 - Z)] \div Z$.

In these formulas $W_1 =$ B.T.U. transmitted per hour by exposed surfaces; $W =$ total B.T.U. necessary, including that for ventilation or changes of air; $N =$ time from cessation of heating to time of starting fire again, hours; $Z =$ time necessary after fire is started until required room temperature is reached, hours.

Allowance for Exposure and for Leakage. — In calculations of the quantity of heat required by ordinary residences, the formula total heat

$$= (T_1 - T_0) \left(\frac{W}{4} + G + \frac{nC}{56} \right) \text{ is commonly used. } T_1 = \text{temp. of room,}$$

$T_0 =$ outside temp., $W =$ exposed wall surface less window surface, $G =$ glass surface, $C =$ cubic contents of room, $n =$ number of changes of air per hour. The factor n is usually assumed arbitrarily or guessed at; some writers take its value at 1, others 1 for the rooms, 2 for the halls, etc.; others object to the use of C as a factor, saying that the allowance for exposure and leakage should be made proportional to the exposed wall and glass surface since it is on these surfaces that the leakage occurs, and omitting the term $nC/56$ they multiply the remainder of the expression by a factor for exposure, $c = 1.1$ to 1.3, depending on the direction of the exposure. To show what different results may be obtained by the use of the two methods, the following table is calculated, applying both to six rooms of widely differing sizes. Two sides of each room, north and east, are exposed. $T_1 = 70$; $T_0 = 0$; $G = \frac{1}{5} (W + G)$.

| Room. | Size, ft. | C = cu. ft. | Total Wall, (W + G) sq. ft. | Glass, G. | Ratio, (C + W + G). | $H = \frac{70(W/4 + G)}{70(W/4 + G)}$ | 70 C/56. | 0.2 H. | 0.3 H. |
|-------|--------------|-------------|-----------------------------|-----------|---------------------|---------------------------------------|----------|--------|--------|
| A | 10 × 10 × 10 | 1,000 | 20 × 10 = 200 | 40 | 5 | 5,600 | 1,250 | 1,120 | 1,680 |
| B | 10 × 20 × 10 | 2,000 | 30 × 10 = 300 | 60 | 62/3 | 8,400 | 2,500 | 1,680 | 2,520 |
| C | 20 × 20 × 12 | 4,800 | 40 × 12 = 480 | 96 | 10 | 13,440 | 6,000 | 2,688 | 4,032 |
| D | 20 × 40 × 14 | 11,200 | 60 × 14 = 840 | 168 | 171/3 | 23,520 | 14,000 | 4,704 | 7,056 |
| E | 40 × 40 × 15 | 24,000 | 80 × 15 = 1200 | 240 | 20 | 33,600 | 30,000 | 6,720 | 10,080 |
| F | 40 × 80 × 16 | 51,200 | 120 × 16 = 1920 | 384 | 262/3 | 54,460 | 64,000 | 10,892 | 16,338 |

The figures in the column headed $H = 70 (W/4 + G)$ represent the heat transmitted through the walls, those in the column 70 C/56 are the heat required for one change of air per hour; 0.2 H is the heat corresponding to an allowance of 20% for exposure and leakage, and 0.3 H corresponds to an allowance of 30%. For the small rooms A and B the difference between 70 C/56 and 0.2 H or 0.3 H is not of great importance, but it becomes very important in the largest rooms; in room F the difference between 70 C/56 and 0.2 H is nearly equal to the total heat transmitted through the walls, indicating that the use of the cubic contents as a factor in calculations of large rooms is likely to lead to great errors. This is due to the fact that the ratio $C \div (W + G)$ varies greatly with different sizes of rooms.

With forced ventilation, the quantity of heat needed depends chiefly upon the number of persons to be provided for. Assuming 2000 cu. ft. per hour per person, heated from 0° to 70° , and 1, 2 and 4 persons per 100 sq. ft. of floor surface, the heat required for the air is as follows:

| Room..... | A | B | C | D | E | F |
|------------------------------|--------|--------|--------|--------|---------|---------|
| 1 person per 100 sq. ft. | 2,500 | 5,000 | 10,000 | 20,000 | 40,000 | 80,000 |
| 2 persons per 100 sq. ft. | 5,000 | 10,000 | 20,000 | 40,000 | 80,000 | 160,000 |
| 4 persons per 100 sq. ft. | 10,000 | 20,000 | 40,000 | 80,000 | 160,000 | 320,000 |
| Ratio of last line to H .. | 1.8 | 2.4 | 3.0 | 3.4 | 4.8 | 5.9 |

Heating by Hot-air Furnaces. — A simple formula for calculating the total heat in British Thermal Units required for heating and ventilating by any system is $H = \left[c \left(G + \frac{W}{4} \right) + \frac{nC}{56} \right] (T_1 - T_0)$. (See notation above.)

The formula is derived as follows: The heat transmitted through 1 sq. ft. of single glass window is approximately 1 B.T.U. per hour per degree of difference of temperature, and that through 1 sq. ft. of 16-in. brick wall about 0.25 B.T.U. (For more accurate calculations figures taken from the tables (p. 659) should be used.) The specific heat of air is taken at 0.238, and the weight of 1 cu. ft. air at 70° F. at 0.075 lb. per cu. ft. The product of these figures is 0.01785, and its reciprocal is 56.

For a difference $T_1 - T_0 = 70^{\circ}$, $0.01785 \times 70 = 1.2495$, we may, therefore, write the formula

$$\begin{aligned} \text{Total heat} &= 70 \left[c \left(G + \frac{W}{4} \right) \right] + 1.25 A \\ &= \text{heat conducted through walls} + \text{heat exhausted in ventilation.} \end{aligned}$$

A is the cubic feet of air (measured at 70°) supplied to and exhausted from the building. This formula neglects the heat conducted through the roof, for which a proper addition should be made.

There are two methods of heating by hot-air furnaces; one in which all the air for both heating and ventilation is taken from outdoors and exhausted from the building, and the other in which only the air for ventilation is taken from outdoors, and additional air is recirculated through the furnace from the building itself. The first method is an exceedingly wasteful one in cold weather. By the second it is possible to heat a building with no greater expenditure of fuel than is required for steam or hot-water heating.

EXAMPLE. — Required the amount of heat and the quantity of air to be circulated by the two methods named for a building which has $G = 400$, $W = 2400$, $C = 16,000$, $n = 2$, $T_1 = 70^{\circ}$, $T_0 = 0^{\circ}$, T_2 , the temperature at which the air leaves the furnace, being taken for three cases as 100° , 120° , and 140° . Assume c , the coefficient for exposure, including heat lost through roof, = 1.2. When only enough air for ventilation is taken into and exhausted from the building, the formula gives

$$70 \times 1.2 (500 + 400) + 1.25 \times 32,000 = 115,600 \text{ B.T.U.} = 75,600 \text{ for heat} + 40,000 \text{ for ventilation.}$$

Suppose all the air required for heating is taken from outdoors at 0° F., and all exhausted at 70° , the quantity, A , then, instead of being 32,000 cu. ft., has to be calculated as follows:

$$\begin{aligned} \text{Total heat} &= c \left(G + \frac{W}{4} \right) (T_1 - T_0) + A \times 0.01785 \times (T_1 - T_0) \\ &= 0.01785 A (T_2 - T_0). \end{aligned}$$

Heat supplied by furnace = heat for conduction + heat for ventilation

$$\begin{aligned} \text{From which we find} \quad A &= c \left(G + \frac{W}{4} \right) (T_1 - T_0) \div 0.01785 (T_2 - T_1) \\ &= 75,600 \div 0.01785 (T_2 - 70^{\circ}). \end{aligned}$$

| | | | |
|---|-------------|-------------|-------------|
| For the value of T_2 | $T_2 = 100$ | $T_2 = 120$ | $T_2 = 140$ |
| $A =$ cu. ft. | 141,117 | 84,706 | 60,504 |
| Heat lost by exhausting this air at 70° ... | 176,396 | 105,882 | 75,630 |
| Adding 75,600 loss by walls gives total... | 251,996 | 181,482 | 152,230 |
| Excess above 115,600 actually required for heating and ventilating, % | 118.0 | 57.0 | 31.7 |

British Thermal Units Absorbed in Heating 1 Cu. Ft. of Air, or given up in cooling it. — (The air is measured at 70° F.)

$$T_1 - T_2 =$$

| | | | | | | | | | | | | | | | |
|------|------|------|------|------|----|------|------|------|------|------|------|------|------|------|-----|
| 10° | 20 | 30 | 40 | 50 | 56 | 60 | 70 | 80 | 90 | 100 | 101 | 120 | 126 | 130 | 140 |
| 0.18 | 0.36 | 0.54 | 0.71 | 0.89 | 1. | 1.07 | 1.25 | 1.43 | 1.61 | 1.78 | 1.96 | 2.14 | 2.25 | 2.32 | 2.5 |

Area in Square Inches of Pipe required to Deliver 100 Cu. Ft. of Air per Minute, at Different Velocities. — The air is measured at the temperature of the air in the pipe.

| | | | | | | | | | |
|---------------------|-----|----|----|----|----|------|----|------|----|
| Velocity per second | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Area, sq. in. | 120 | 80 | 60 | 48 | 40 | 34.3 | 30 | 26.7 | 24 |

The quantity of air required for ventilation or heating should be figured at a standard temperature, say 70° F., but when warmer air is to be delivered into the room through pipes, the area of the pipes should be calculated on the basis of the temperature of the warm air, and not on that of the room.

EXAMPLE. — A room requires to be supplied with 1000 cu. ft. per min. at 70° F. for ventilation, but the air is also used for heating and is delivered into the room at 120° F. Required, the area of the delivery pipe, if the velocity of the heated air in the pipe is 6 ft. per second.

From the table of volumes, given on the next page, 1000 cu. ft. at 70° = 1094 cu. ft. at 120°. From the above table of areas, at 6 ft. velocity 40 sq. in. area is required for 100 cu. ft., therefore 1094 cu. ft. will require $10.94 \times 40 = 437.6$ sq. in. or about 3 sq. ft.

Carrying Capacity of Air Pipes.

| Diam. | Area in sq. in. | Area, sq. ft. | Velocity, Feet per Second. | | | | | |
|-------|-----------------|---------------|----------------------------|------|------|------|------|------|
| | | | 3 | 4 | 5 | 6 | 7 | 8 |
| | | | Cu. Ft. per Min. | | | | | |
| 5 | 19.63 | .1364 | 24.6 | 32.7 | 40.9 | 49.1 | 57.3 | 65.5 |
| 6 | 28.27 | .1963 | 35.3 | 47.1 | 58.9 | 70.7 | 82.4 | 94.2 |
| 7 | 38.48 | .2673 | 48.1 | 64.2 | 80.2 | 96.2 | 112. | 128. |
| 8 | 50.27 | .3491 | 62.8 | 83.8 | 105. | 126. | 147. | 168. |
| 9 | 63.62 | .4418 | 80.0 | 106. | 133. | 159. | 186. | 212. |
| 10 | 78.54 | .5454 | 98.2 | 131. | 164. | 196. | 229. | 262. |
| 11 | 95.03 | .6600 | 119. | 158. | 198. | 238. | 277. | 317. |
| 12 | 113.1 | .7854 | 141. | 188. | 236. | 283. | 330. | 377. |
| 13 | 132.7 | .9218 | 166. | 221. | 277. | 332. | 387. | 442. |
| 14 | 153.9 | 1.069 | 192. | 257. | 321. | 385. | 449. | 513. |
| 15 | 176.7 | 1.227 | 221. | 294. | 368. | 442. | 515. | 589. |
| 11.3 | 100. | 0.694 | 125. | 167. | 208. | 250. | 292. | 333. |
| 13.6 | 144. | 1. | 180. | 240. | 300. | 360. | 420. | 480. |

The figures in the table give the carrying capacity of pipes in cu. ft. of air at the temperature of the air flowing in the pipes. To reduce the figures to cu. ft. at a standard temperature (such as 70° F.) divide by the ratio of the volume per cu. ft. of the air in the pipe to that of the air of the standard temperature, as in the following table:

Volume of Air at Different Temperatures.
(Atmospheric pressure.)

| Fahr. Deg. | Cu. Ft. in 1 lb. | Compar- ative Volume. | Fahr. Deg. | Cu. Ft. in 1 lb. | Compar- ative Volume. | Fahr. Deg. | Cu. Ft. in 1 lb. | Compar- ative Volume. |
|---------------|---------------------|-----------------------------|---------------|---------------------|-----------------------------|---------------|---------------------|-----------------------------|
| 0 | 11.583 | 0.867 | 90 | 13.845 | 1.038 | 160 | 15.603 | 1.169 |
| 32 | 12.387 | 0.928 | 100 | 14.096 | 1.056 | 170 | 15.854 | 1.188 |
| 40 | 12.586 | 0.943 | 110 | 14.346 | 1.075 | 180 | 16.106 | 1.207 |
| 50 | 12.840 | 0.962 | 120 | 14.596 | 1.094 | 190 | 16.357 | 1.226 |
| 62 | 13.141 | 0.985 | 130 | 14.848 | 1.113 | 200 | 16.608 | 1.245 |
| 70 | 13.342 | 1.000 | 140 | 15.100 | 1.132 | 210 | 16.860 | 1.264 |
| 80 | 13.593 | 1.019 | 150 | 15.351 | 1.151 | 212 | 16.910 | 1.267 |

Sizes of Air Pipes Used in Furnace Heating. (W. G. Snow, *Eng. News*, April 12, 1900.)

| W'th. of Room Ft. | Length of Room, Ft. | | | | | | | | | | |
|----------------------------|------------------------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|
| | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| | Diameter of Pipe, Ins. | | | | | | | | | | |
| 8.... | 8, 7 | 8, 7 | 9, 8 | 9, 8 | | | | | | | |
| 10.... | 8, 7 | 9, 8 | 9, 8 | 10, 8 | 10, 8 | 10, 9 | | | | | |
| 12.... | | 9, 8 | 10, 8 | 10, 8 | 10, 9 | 11, 9 | 11, 9 | 12, 10 | | | |
| 14.... | | | 10, 8 | 10, 9 | 11, 9 | 11, 9 | 12, 10 | 12, 10 | 13, 10 | 13, 10 | |
| 16.... | | | | 11, 9 | 11, 9 | 12, 10 | 12, 10 | 13, 10 | 13, 10 | 13, 11 | 13, 11 |
| 18.... | | | | | 12, 10 | 12, 10 | 13, 11 | 13, 11 | 13, 11 | 14, 12 | 14, 12 |
| 20.... | | | | | | 13, 11 | 13, 11 | 13, 11 | 14, 12 | 14, 12 | 14, 12 |

The first figure in each column shows the size of pipe for the first floor and the second figure the size for the second floor. Temperature at register, 140°; room, 70°; outside, 0°. Rooms 8 to 16 ft. in width assumed to be 9 ft. high; 18 to 20 ft. width, 10 ft. high. When first-floor pipes are longer than 15 ft. use one size larger than that stated. For third floor, use one size smaller than for second floor. For rooms with three exposures, increase the area of pipe in proportion to the exposure.

The table was calculated on the following basis:

The loss of heat is calculated by first reducing the total exposure to equivalent glass surface. This is done by adding to the actual glass surface one-quarter the area of exposed wood and plaster or brick walls and $\frac{1}{20}$ the area of floor or ceiling. Ten per cent is added where the exposure is severe. The window area assumed is 20% of the entire exposure of the room.

Multiply the equivalent of glass surface by 85. The product will be the total loss of heat by transmission per hour.

Assuming the temperature of the entering air to be 140° and that of the room to be 70°, the air escaping at approximately the latter temperature will carry away one-half the heat brought in. The other half, corresponding to the drop in temperature from 140° to 70°, is lost by transmission. With outside temperature zero, each cubic foot of air at 140° brings into the room 2.2 heat units. Since one-half of this, or 1.1 heat units, can be utilized to offset the loss by transmission, to ascertain the volume of air per hour at 140° required to heat a given room, divide the loss of heat by transmission by 1.1. This result divided by 60 gives the number of cubic feet per minute. In calculating the table, maximum velocities of 280 and 400 ft. were used for pipes leading to the first and second floors respectively. The size of the smaller pipes was based on lower velocities, according to their size, to allow for their greater resistance and loss of temperature.

Furnace-Heating with Forced Air Supply. (*The Metal Worker*, April 8, 1905.) — Tests were made of a Kelsey furnace with the air supply furnished by a 48-in. Sturtevant disk fan driven by a 5 H.P. electric motor. A connection was made from the air intake, between the fan and the furnace, to the ash pit so that the rate of combustion could be regulated independently of the chimney-draft condition. The furnace had 4.91 sq. ft. of grate surface and 238 sq. ft. of heating surface. The volume of air was determined by anemometer readings at 24 points in a cross-section of a rectangular intake of 11.88 sq. ft. area. The principal results obtained in two tests of 8 hours each are as follows:

| | | |
|---|---------|---------|
| Av. temp. of the cold air..... | 39° | 58° |
| Per cent humidity of the cold air..... | 71 | 56 |
| Av. temp. of the warm air..... | 135° | 152° |
| Air delivered to heater, cu. ft. per hour.... | 250,896 | 249,195 |
| B.T.U. absorbed by the dry air per hour.... | 451,872 | 421,496 |
| B.T.U. absorbed by the vapor per hour.... | 2,016 | 3,102 |
| Avg. no. of pounds of coal burned per hour | 36 | 33.5 |
| B.T.U. given by the coal per hour..... | 529,200 | 492,450 |
| Per cent efficiency of the furnace..... | 85.7 | 86.2 |

Grate Surface and Rate of Burning Coal.

In steam boilers for power plants, which are constantly attended by firemen, coal is generally burned at between 10 and 30 lbs. per sq. ft. of grate per hour. In small boilers, house heaters and furnaces, which even in the coldest weather are supplied with fresh coal only once in several hours, it is necessary to burn the coal at very much slower rates. Taking a cubic foot of coal as weighing 60 lbs., in a bed 12 inches deep, and 1 sq. ft. of grate area, it would be one-half burned away in 7½ hours at a rate of burning of 4 lbs. per sq. ft. of grate per hour. This figure, 4 lbs., is commonly taken in designing grate surface for house-heating boilers and furnaces. Using this figure we have the following as the rated capacity of different areas of grate surface.

Rated Capacity of Furnaces and Boilers for House Heating.

| Diam. of Round Grate. | Area in — | | Coal- burning Capacity per Hour. | Capacity, B.T.U. per Hour. | Equiv. lbs. Steam Evap. 212° per Hour. | Equiv. lbs. Air per Hour Heated 100°. | Equiv. cu. ft. Air at 70° Heated 100°. |
|--------------------------------|-----------|---------|--|-------------------------------------|---|--|---|
| Ins. | sq. in. | sq. ft. | lbs. | (a) | (b) | (c) | (d) |
| 12 | 113.1 | .785 | 3.142 | 31,420 | 32.5 | 1,320 | 17,610 |
| 14 | 153.9 | 1.069 | 4.276 | 42,760 | 44.3 | 1,797 | 23,970 |
| 16 | 201.1 | 1.396 | 5.585 | 55,850 | 57.8 | 2,347 | 31,300 |
| 18 | 254.5 | 1.767 | 7.069 | 70,690 | 73.2 | 2,970 | 39,620 |
| 20 | 314.2 | 2.182 | 8.728 | 87,280 | 90.4 | 3,667 | 48,920 |
| 22 | 380.1 | 2.640 | 10.560 | 105,600 | 109.4 | 4,437 | 59,190 |
| 24 | 452.4 | 3.142 | 12.566 | 125,660 | 130.1 | 5,280 | 70,430 |
| 26 | 530.9 | 3.687 | 14.748 | 147,480 | 152.7 | 6,197 | 82,670 |
| 28 | 615.8 | 4.276 | 17.104 | 171,040 | 177.1 | 7,187 | 95,870 |
| 30 | 706.9 | 4.909 | 19.636 | 196,360 | 203.3 | 8,260 | 110,190 |
| 32 | 804.2 | 5.585 | 22.340 | 223,400 | 231.3 | 9,387 | 125,220 |
| 34 | 907.9 | 6.305 | 25.220 | 252,200 | 261.2 | 10,597 | 141,360 |
| 36 | 1017.9 | 7.069 | 28.276 | 282,760 | 292.8 | 11,881 | 158,490 |

Figures in column (b) = (a) ÷ 965.7.

Figures in column (c) = (a) ÷ (100 × 0.238).

Figures in column (d) = (c) × 13.34.

Latent heat of steam at 212° = 965.7 B.T.U. [new steam tables give 970.4].

Specific heat of air = 0.238.

Note that the figures in the last three columns are all based on the rate of combustion of 4 lbs. of coal per sq. ft. of grate per hour, which is taken as the standard for house heating. For heating schoolhouses and other large buildings where the furnace is fed with coal more frequently a

much higher actual capacity may be obtained from the grate surface named. A committee of the Am. Soc. H. and V. Engrs. in 1909 says:

The grate surface to be provided depends on the rate of combustion, and this in turn depends on the attendance and draft, and on the size of the boiler. Small boilers are usually adapted for intermittent attention and a slow rate of combustion. The larger the boiler, the more attention is given to it, and the more heating surface is provided per square foot of grate. The following rates of combustion are common for internally fired heating boilers:

| | | | |
|--|--------|----------|----------|
| Sq. ft. of grate | 4 to 8 | 10 to 18 | 20 to 30 |
| Lbs. coal per sq. ft. grate per hr. not over | 4 | 6 | 10 |

Capacity of 1 sq. ft. and of 100 sq. in. of Grate Surface, for Steam, Hot-water, or Furnace Heating.

(Based on burning 4 lbs. of coal per sq. ft. of grate per hour and 10,000 B.T.U. available heating value of 1 lb. of coal.)

| 1 sq. ft. grate equals | 100 sq. ins. grate equals | |
|---------------------------|------------------------------|--|
| 4 | 2.775 | lbs. of coal per hour. |
| 40,000 | 27,750 | B.T.U. per hour. |
| 41.25 | 28.61 | lbs. of steam evap. from and at 212° per hr. |
| 156.5 | 108.7 | sq. ft. of steam radiating surface = B.T.U. ÷ 255.6*. |
| 261.4 | 181.5 | sq. ft. of hot-water radiating surface = B.T.U. ÷ 153 †. |
| 22,420. | 15,570. | cu. ft. of air (measured at 70° F.) per hour heated 100°. |

* Steam temperature 212°, room temperature 70°, radiator coefficient, that is the B.T.U. transmitted per sq. ft. of surface per hour per degree of difference of temperature, 1.8.

† Water temperature 160°, room temperature 70°, radiator coefficient 1.7.

For any other rate of combustion than 4 lbs., multiply the figures in the table by that rate and divide by 4.

STEAM-HEATING.

The Rating of House-heating Boilers.

(W. Kent, *Trans. A. S. H. V. E.*, 1909.)

The rating of a steam-boiler for house-heating may be based upon one or more of several data: 1, square feet of grate-surface; 2, square feet of heating-surface; 3, coal-burning capacity; 4, steam-making capacity; 5, square feet of steam-radiating-surface, including mains, that it will supply. In establishing such a rating the following considerations should be taken into account:

1. One sq. ft. of cast-iron radiator surface will give off about 250 B.T.U. per hour under ordinary conditions of temperature of steam 212°, and temperature of room 70°.

2. One pound of good anthracite or semi-bituminous coal under the best conditions of air-supply, in a boiler properly proportioned, will transmit about 10,000 B.T.U. to the boiler.

3. In order to obtain this economical result from the coal the boilers should be driven at a rate not greatly exceeding 2 lbs. of water evaporated from and at 212° per sq. ft. of heating-surface per hour, corresponding to a heat transmission of $2 \times 970 = 1940$, or, say, approximately 2000 B.T.U. per hour per sq. ft. of heating-surface.

4. A satisfactory boiler or furnace for house-heating should not require coal to be fed oftener than once in 8 hours; this requires a rate of burning of only 3 to 5 pounds of coal per sq. ft. of grate per hour.

5. For commercial and constructive reasons, it is not convenient to establish a fixed ratio of heating- to grate-surface for all sizes of boilers. The grate-surface is limited by the available area in which it may be placed, but on a given grate more heating-surface may be piled in one form of boiler than in another, and in boilers of one general form one boiler may be built higher than another, thus obtaining a greater amount of heating-surface.

6. The rate of burning coal and the ratio of heating- to grate-surface both being variable, the coal-burning rate and the ratio may be so related to each other as to establish condition 3, viz., a rate of evaporation of 2 lbs. of water from and at 212° per sq. ft. of heating-surface per hour.

These general considerations lead to the following calculations:

1 lb. of coal, 10,000 B.T.U. utilized in the boiler, will supply $10,000 \div 250 = 40$ sq. ft. radiating-surface, and will require $10,000 \div 2000 = 5$ sq. ft. boiler heating-surface. 1 sq. ft. of boiler-surface will supply $2000 \div 250$ or $40 \div 5 = 8$ sq. ft. radiating-surface.

| | Low Boiler. | Medium. | High Boiler. |
|---|-------------|---------|----------------------------|
| 1 sq. ft. of grate should burn.... | 3 | 4 | 5 lb. coal per hour. |
| 1 sq. ft. of grate should develop. | 30,000 | 40,000 | 50,000 B.T.U. per hour. |
| 1 sq. ft. of grate will require..... | 15 | 20 | 25 sq. ft. heating-surf. |
| 1 sq. ft. of grate will supply..... | 120 | 160 | 200 sq. ft. radiating-sur. |
| Type of boiler, depending on ratio heating- \div grate-surface. | A. | B. | C. |

TABLE OF RATINGS.

| Type and No. | Sq. Ft. Grate. | Sq. Ft. Heat.- surf. | Coal Burned per Hour, lbs. | Water Evap. per Hour, lbs. | Rad.- surf., Sq. Ft. | Type and No. | Sq. Ft. Grate. | Sq. Ft. Heat.- surf. | Coal Burned per Hour, lbs. | Water Evap. per Hour, lbs. | Rad.- surf., Sq. Ft. |
|--------------|----------------|----------------------|----------------------------|----------------------------|----------------------|--------------|----------------|----------------------|----------------------------|----------------------------|----------------------|
| A 1... | 1 | 15 | 3 | 30 | 120 | B 8... | 8 | 160 | 32 | 320 | 1,280 |
| A 2... | 2 | 30 | 6 | 60 | 240 | C 6... | 6 | 150 | 30 | 300 | 1,200 |
| A 3... | 3 | 45 | 9 | 90 | 360 | C 7... | 7 | 175 | 35 | 350 | 1,400 |
| A 4... | 4 | 60 | 12 | 120 | 480 | C 8... | 8 | 200 | 40 | 400 | 1,600 |
| A 5... | 5 | 75 | 15 | 150 | 600 | C 10... | 10 | 250 | 50 | 500 | 2,000 |
| B 4... | 4 | 80 | 16 | 160 | 640 | C 12... | 12 | 300 | 60 | 600 | 2,400 |
| B 5... | 5 | 100 | 20 | 200 | 800 | C 14... | 14 | 350 | 70 | 700 | 2,800 |
| B 6... | 6 | 120 | 24 | 240 | 960 | C 16... | 16 | 400 | 80 | 800 | 3,200 |
| B 7... | 7 | 140 | 28 | 280 | 1,120 | | | | | | |

The table is based on the utilization in the boiler of 10,000 B.T.U. per pound of good coal. For poorer coal the same figures will hold good except the pounds coal burned per hour, which should be increased in the ratio of the B.T.U. of the good to that of the poor coal. Thus for coal from which 8000 B.T.U. can be utilized the coal burned per hour will be 25 per cent greater.

For comparison with the above table the following figures are taken and calculated from the catalogue of a prominent maker of cast-iron boilers:

| Height. | G Grate. | H Heating-surface. | R Radiating-surface. | $\frac{H}{G}$ | $\frac{R}{G}$ | $\frac{R}{H}$ | B.T.U. per Hour = $R \times 250$ | B.T.U. $\frac{H}{H}$ | Coal per Hour per sq. ft. Grate * |
|-----------|---------------|--------------------|----------------------|---------------|---------------|---------------|----------------------------------|----------------------|-----------------------------------|
| Low..... | { 2.1 4.7 | 45 90 | 210 600 | 21.5 19.1 | 100 128 | 4.7 6.7 | 52,500 150,000 | 1,167 1,667 | 2.5 3.2 |
| Medium.. | { 4.2 8.2 | 103 195 | 600 1,500 | 24.5 23.8 | 143 183 | 5.8 7.7 | 150,000 375,000 | 1,456 1,923 | 3.6 4.6 |
| High..... | { 6.7 14.7 | 210 420 | 1,200 3,300 | 31.3 28.6 | 179 225 | 5.7 7.9 | 300,000 825,000 | 1,476 1,964 | 4.5 5.6 |

* Equals B.T.U. per hour \div 10,000 G.

TESTING CAST-IRON HOUSE-HEATING BOILERS.

The testing of the evaporating power and the economy of small-sized boilers is more difficult than the testing of large steam-boilers for the reason that the small quantity of coal burned in a day makes it impossible to procure a uniform condition of the coal on the grate throughout the test, and large errors are apt to be made in the calculation on account of the difference of condition at the beginning and end of a test. The following is suggested as a method of test which will avoid these errors.

(a) Measure the grate-surface and weigh out an amount of coal equal to 30, 40, or 50 lbs. per sq. ft. of grate, according to the type A, B, or C, or the ratio of heating- to grate-surface.

(b) Disconnect the steam-pipe, so that the steam may be wasted at atmospheric pressure. Fill the boiler with cold water to a marked level, and take the weight of this water and its temperature.

(c) Start a brisk fire with plenty of wood, so as to cause the coal to ignite rapidly; feed the coal as needed, and gradually increase the thickness of the bed of coal as it burns brightly on top, getting the fire-pot full as the last of the coal is fired. Then burn away all the coal until it ceases to make steam, when the test may be considered as at an end.

(d) Record the temperature of the gases of combustion in the flue every half-hour.

(e) Periodically, as needed, feed cold water, which has been weighed, to bring the water level to the original mark. Record the time and the weight.

CALCULATIONS.

Total water fed to the boiler, including original cold water, pounds $\times (212^{\circ} - \text{original cold-water temperature}) = \dots\dots\dots$ B.T.U.
 Water apparently evaporated, pounds $\times 970 = \dots\dots\dots$ B.T.U.

Add correction for increased bulk of hot water:

Original water, pounds $\times \frac{(62.3 - 59.8)}{62.3} \times 970 = \dots\dots\dots$ B.T.U.
 Total..... B.T.U.

Divide by 970 to obtain equivalent water evaporation from and at 212° F.

Divide by the number of pounds of coal to obtain equivalent water per pound of coal.

The last result may be considerably less than 10 pounds on account of imperfect combustion at the beginning of the test, excessive air-supply when the coal bed is thin in the latter half of the test, and loss by radiation, but the results will be fairly comparable with results from other boilers of the same size and run under the same conditions. The records of water fed and of temperature of gases should be plotted, with time as the base, for comparison with other tests.

Proportions of House-heating Boilers. — A committee of the Am. Soc. Heating and Ventilating Engineers, reporting in 1909 on the method of rating small house-heating boilers, shows the following ratings, in square feet of radiating surface supplied by certain boilers of nearly the same nominal capacity, as given in makers' catalogues.

| Boiler..... | A. | B. | C. | D. | E. | F. |
|---|-----|------|------|------|-----|------|
| Rated capacity..... | 800 | 800 | 775 | 750 | 750 | 750 |
| Square inches of grate..... | 616 | 740 | 648 | 528 | 630 | 648 |
| Ratio of grate to 100 sq. ft. of capacity | 77 | 92.5 | 83.6 | 70.4 | 84 | 86.2 |
| Estimated rate of combustion..... | 5.1 | 4.2 | 4.65 | 5.63 | 4.4 | 4.5 |

The figures in the last line are lbs. of coal per sq. ft. of grate surface per hour, and are based on the assumptions of 10,000 B.T.U. utilized per lb. of coal and 270 B.T.U. transmitted by each sq. ft. of radiating surface per hour.

"The question of heating surface in a boiler seems to be an unknown quantity, and inquiry among the manufacturers does not produce much information on the subject."

Following is the list of sizes and ratings of the "Manhattan" sectional steam boiler. The figures for sq. ft. of grate surface and for the ratio of heating to grate surface (approx.) have been computed from the sizes given in the catalogue (1909).

| Number of Sections. | Square feet of Direct Radiation Boiler will Supply. | Size of Grate. | | Square Feet of Surface in Boiler. | Ratio of Htg. to Grate Surface. | Number of Sections. | Square feet of Direct Radiation Boiler will Supply. | Size of Grate. | | Square Feet of Surface in Boiler. | Ratio of Htg. to Grate Surface. |
|---------------------|---|----------------|---------|-----------------------------------|---------------------------------|---------------------|---|----------------|---------|-----------------------------------|---------------------------------|
| | | ins. | sq. ft. | | | | | ins. | sq. ft. | | |
| 4 | 450 | 18×19 | 2.37 | 68 | 29 | 10 | 2250 | 24×63 | 10.5 | 212 | 20 |
| 5 | 600 | 18×25 | 3.75 | 84 | 23 | 6 | 2200 | 36×36 | 9 | 256 | 28 |
| 6 | 750 | 18×31 | 3.87 | 100 | 26 | 7 | 2700 | 36×43 | 11.74 | 298 | 26 |
| 7 | 900 | 18×37 | 4.65 | 116 | 25 | 8 | 3200 | 36×50 | 13.33 | 340 | 26 |
| 8 | 1050 | 18×43 | 5.37 | 132 | 25 | 9 | 3700 | 36×57 | 14.25 | 382 | 26 |
| 5 | 1000 | 24×30 | 5 | 111 | 22 | 10 | 4200 | 36×64 | 16 | 424 | 26 |
| 6 | 1250 | 24×36 | 6 | 128 | 21 | 11 | 4700 | 36×71 | 17.5 | 466 | 27 |
| 7 | 1500 | 24×43 | 7.16 | 149 | 21 | 12 | 5200 | 36×78 | 19.5 | 508 | 26 |
| 8 | 1750 | 24×50 | 8.33 | 170 | 20 | 13 | 5700 | 36×84 | 21 | 550 | 26 |
| 9 | 2000 | 24×57 | 9.5 | 191 | 20 | 14 | 6200 | 36×90 | 22.5 | 592 | 26 |

It appears from this list that there are three sets of proportions, corresponding to the three widths of grate surface. The average ratio of heating to grate surface in the three sets is respectively 25.0, 20.7, and 25.8; the rated sq. ft. of radiating surface per sq. ft. of grate is 185, 208, and 259, and the sq. ft. of radiating surface per sq. ft. of boiler heating surface is 7.4, 10.1, and 9.8. Taking 10,000 B.T.U. utilized per lb. of coal, and 250 B.T.U. emitted per sq. ft. of radiating surface per hour, the rate of combustion required to supply the radiating surface is respectively 4.62, 5.22, and 6.40 lbs. per sq. ft. of grate per hour.

Coefficient of Heat Transmission in Direct Radiation. — The value of K , or the B.T.U. transmitted per sq. ft. of radiating surface per hour per degree of difference of temperature between the steam (or hot water) and the air in the room, is commonly taken at 1.8 in steam heating, with a temperature difference of about 142°, and 1.6 in hot-water heating, with a temperature difference averaging 80°. Its value as found by test varies with the conditions; thus the total heat transmitted is not directly proportional to the temperature difference, but increases at a faster rate; single pipes exposed on all sides transmit more heat than pipes in a group; low radiators more than high ones; radiators exposed to currents of cool air more than those in relatively quiet air; radiators with a free circulation of steam throughout more than those that are partly filled with water or air, etc. The total range of the value of K , for ordinary conditions of practice, is probably between 1.5 and 2.0 for steam-heating with a temperature difference of 140°, averaging 1.8, and between 1.2 and 1.7, averaging 1.6, for hot-water heating, with a temperature difference of 80°.

C. F. Hauss, *Trans. A. S. H. V. E.*, 1904, gives as a basis for calculation, for a room heated to 70° with steam at 11½ lbs. gauge pressure (temperature difference 146° F.) 1 sq. ft. of single column radiator gives off 300 B.T.U. per hour; 2-column, 275; 3-column, 250; 4-column, 225.

Value of K in Cast-iron Direct Radiators. (J. K. Allen, *Trans. A. S. H. V. E.*, 1908.) T_s = temp. of steam; T_1 = temp. of room.

| $T_s - T_1 =$ | 110 | 120 | 130 | 140 | 150 | 160 |
|------------------|------|-------|-------|-------|-------|-------|
| 2-col. rad. | 1.71 | 1.745 | 1.76 | 1.82 | 1.855 | 1.895 |
| 3-col. rad. | 1.65 | 1.695 | 1.745 | 1.79 | 1.835 | 1.885 |
| $T_s - T_1 =$ | 170 | 180 | 200 | 220 | 240 | 260 |
| 2-col. rad. | 1.93 | 1.965 | 2.04 | 2.11 | 2.185 | 2.265 |
| 3-col. rad. | 1.93 | 1.98 | 2.075 | 2.165 | 2.260 | 2.36 |

B.T.U. Transmitted per Hour per Sq. Ft. of Heating Surface in Indirect Radiators. (W. S. Munroe, *Eng. Rec.*, Nov. 18, 1899.)

| | | Cu. ft. of air per hour per sq. ft. of surface. | | | | | | | | |
|---------------------|--------|---|-----|-----|-----|------|------|-----|-----|------|
| | | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| | | B.T.U. per hour per sq. ft. of heating surface. | | | | | | | | |
| "Gold Pin" radiator | (a)... | 200 | 325 | 450 | 560 | 670 | 780 | 870 | 950 | 1030 |
| | (b)... | 300 | 550 | 760 | 950 | 1130 | 1300 | | | |
| "Whittier" | | (b)... | 250 | 400 | 520 | 620 | 710 | | | |
| | | B.T.U. per hr. per sq. ft. per deg. diff. of temp.* | | | | | | | | |
| Gold Pin (a)..... | | 1.3 | 2.2 | 3.0 | 3.7 | 4.5 | 5.2 | 5.8 | 6.3 | 6.9 |
| Gold Pin (b)..... | | 2.0 | 3.7 | 5.1 | 6.3 | 7.7 | 8.7 | | | |
| Whittier (b)..... | | 1.7 | 2.7 | 3.5 | 4.1 | 4.7 | | | | |

Temperature difference between steam and entering air, (a) 150; (b) 215.

* Between steam and entering air.

Short Rules for Computing Radiating-Surfaces. — In the early days of steam-heating, when little was known about "British Thermal Units," it was customary to estimate the amount of radiating-surface by dividing the cubic contents of the room to be heated by a certain factor supposed to be derived from "experience." Two of these rules are as follows:

One square foot of surface will heat from 40 to 100 cu. ft. of space to 75° in — 10° latitudes. This range is intended to meet conditions of exposed or corner rooms of buildings, and those less so, as intermediate ones of a block. As a general rule, 1 sq. ft. of surface will heat 70 cu. ft. of air in outer or front rooms and 100 cu. ft. in inner rooms. In large stores in cities, with buildings on each side, 1 to 100 is ample. The following are approximate proportions:

One square foot radiating-surface will heat:

| | In Dwellings, Schoolrooms, Offices, etc. | In Hall, Stores, Lofts, Factories, etc. | In Churches, Large Auditoriums, etc. |
|-------------------------|--|---|--|
| By direct radiation.... | 60 to 80 ft. | 75 to 100 ft. | 150 to 200 ft. |
| By indirect radiation.. | 40 to 50 ft. | 50 to 70 ft. | 100 to 140 ft. |

Isolated buildings exposed to prevailing north or west winds should have a generous addition made to the heating-surface on their exposed sides.

1 sq. ft. of boiler-surface will supply from 7 to 10 sq. ft. of radiating-surface, depending upon the size of boiler and the efficiency of its surface, as well as that of the radiating-surface. Small boilers for house use should be much larger proportionately than large plants. Each horsepower of boiler will supply from 240 to 360 ft. of 1-in. steam-pipe, or 80 to 120 sq. ft. of radiating-surface. Under ordinary conditions 1 horse-power will heat, approximately, in —

| | |
|---|--------------------------|
| Brick dwellings, in blocks, as in cities..... | 15,000 to 20,000 cu. ft. |
| Brick stores, in blocks..... | 10,000 " 15,000 " |
| Brick dwellings, exposed all round..... | 10,000 " 15,000 " |
| Brick mills, shops, factories, etc..... | 7,000 " 10,000 " |
| Wooden dwellings, exposed..... | 7,000 " 10,000 " |
| Foundries and wooden shops..... | 6,000 " 10,000 " |
| Exhibition buildings, largely glass, etc..... | 4,000 " 15,000 " |

Such "rules of thumb," as they are called, are generally supplanted by the modern "heat-unit" methods.

Carrying Capacity of Pipes in Low-Pressure Steam Heating. (W. Kent, *Trans. A. S. H. V. E.*, 1907.) — The following table is based on an assumed drop of 1 pound pressure per 1000 feet, not because that is the drop which should always be used — in fact the writer believes that in large installations a far greater drop is permissible — but because it gives a basis upon which the flow for any other drop may be calculated,

merely by multiplying the figures in the tables by the square root of the assigned drop. The formula from which the tables are calculated is the

well known one, $W = c \sqrt{\frac{w (p_1 - p_2) d^5}{L}}$, in which W = weight of steam in lbs. per minute; w = weight of steam in pounds per cubic foot, at the entering pressure, p_1 ; p_2 the pressure at the end of the pipe; d the actual diameter of standard wrought-iron pipe in inches, and L the length in feet. The coefficients c are derived from Darcy's experiments on flow of water in pipes, and are believed to be as accurate as any that have been derived from the very few recorded experiments on steam.

| | | | | | | | | | |
|-------------------------|------|-------|------|-------|-------|------|-------|------|-------|
| Nominal diam. of pipe.. | 1/2 | 3/4 | 1 | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 |
| Value of c —..... | 36.8 | 42 | 45.3 | 48 | 50 | 52.7 | 54.8 | 56.2 | 57.1 |
| Nominal diam. of pipe.. | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |
| Value of c —..... | 57.8 | 58.3 | 58.7 | 59.5 | 60.2 | 60.8 | 61.3 | 61.7 | 62.1 |

FLOW OF STEAM AT LOW PRESSURES IN POUNDS PER HOUR FOR A UNIFORM DROP AT THE RATE OF ONE POUND PER 1000 FEET
LENGTH OF STRAIGHT PIPE.

| Nominal Diam. of Pipe. | Steam Pressures, by Gauge, at Entrance of Pipe. | | | | | | | | |
|---------------------------------|---|--------|--------|--------|--------|--------|--------|--------|--------|
| | 0.3 | 1.3 | 2.3 | 8.3 | 4.3 | 5.3 | 6.3 | 8.3 | 10.3 |
| | Flow of Steam, Pounds per Hour. | | | | | | | | |
| 1/2 | 4.2 | 4.3 | 4.4 | 4.6 | 4.7 | 4.8 | 4.9 | 5.1 | 5.3 |
| 3/4 | 9.7 | 10.0 | 10.3 | 10.5 | 10.8 | 11.0 | 11.3 | 11.8 | 12.3 |
| 1 | 19.0 | 19.6 | 20.2 | 20.7 | 21.2 | 21.7 | 22.3 | 23.2 | 24.2 |
| 1 1/4 | 40.1 | 41.3 | 42.5 | 43.7 | 44.8 | 45.9 | 46.9 | 49.0 | 50.9 |
| 1 1/2 | 61.4 | 63.2 | 65.1 | 66.8 | 68.6 | 70.3 | 71.9 | 75.0 | 78.0 |
| 2 | 120.8 | 124.5 | 128.2 | 131.6 | 135.0 | 138.3 | 141.5 | 147.7 | 153.6 |
| 2 1/2 | 195.7 | 201.8 | 207.5 | 213.2 | 218.7 | 224.0 | 229.2 | 239.2 | 248.8 |
| 3 | 345.5 | 356.1 | 366.5 | 376.4 | 386.1 | 395.5 | 404.7 | 422.4 | 439.3 |
| 3 1/2 | 505.3 | 520.8 | 535.9 | 550.5 | 564.7 | 578.5 | 591.8 | 618.0 | 642.6 |
| 4 | 701.4 | 723.0 | 744.0 | 764.4 | 784.2 | 803.4 | 822.0 | 857.4 | 891.6 |
| 4 1/2 | 938.7 | 967.6 | 995.8 | 1023. | 1049. | 1075. | 1100. | 1148. | 1193. |
| 5 | 1252. | 1291. | 1328. | 1364. | 1399. | 1433. | 1467. | 1531. | 1592. |
| 6 | 2011. | 2074. | 2134. | 2192. | 2248. | 2303. | 2356. | 2459. | 2557. |
| 7 | 2936. | 3027. | 3115. | 3199. | 3281. | 3362. | 3440. | 3590. | 3733. |
| 8 | 4082. | 4208. | 4331. | 4448. | 4564. | 4674. | 4783. | 4991. | 5191. |
| 9 | 5462. | 5630. | 5794. | 5951. | 6102. | 6252. | 6396. | 6678. | 6942. |
| 10 | 7314. | 7536. | 7758. | 7968. | 8172. | 8370. | 8562. | 8940. | 9294. |
| 12 | 11550. | 11916. | 12264. | 12594. | 12918. | 13236. | 13542. | 14136. | 14700. |

For any other drop of pressure per 1000 feet length, multiply the figures in the table by the square root of that drop.

In all cases the judgment of the engineer must be used in the assumption of the drop to be allowed. For small distributing pipes it will generally be desirable to assume a drop of not more than one pound per 1000 feet to insure that each single radiator shall always have an ample supply for the worst conditions, and in that case the size of piping given in the table up to two inches may be used; but for main pipes supplying totals of more than 500 square feet, greater drops may be allowed.

Proportioning Pipes to Radiating Surface.

FIGURES USED IN CALCULATION OF RADIATING SURFACE.

 P = Pressure by gauge, lbs. per sq. in.

0. 0.3 1.3 2.3 3.3 4.3 5.3 6.3 8.3 10.3

 L = latent heat of evaporation, B.T.U. per lb.*

965.7 965.0 962.6 960.4 958.3 956.3 954.4 952.6 949.1 945.8

Temperature Fahrenheit, T_1 .

212. 213. 216.3 219.4 222.4 225.2 227.9 230.5 235.4 240.0

 $T_1 = T_2 - 70^\circ$, difference of temperature.

142. 143. 146.3 149.4 152.4 155.2 157.9 160.5 165.4 170.0

 $H_1 = T_1 \times 1.8$ = heat transmission per sq. ft. radiating surface, B.T.U. per hour.

255.6 257.4 263.3 268.9 274.3 279.2 284.2 288.9 297.7 306.0

 $H_1 \div L$ = steam condensed per sq. ft. radiating surface, lbs. per hour.

0.2647 0.267 0.274 0.280 0.286 0.292 0.298 0.303 0.314 0.324

Reciprocal of above = radiating surface per lb. of steam condensed per hour.

3.78 3.75 3.65 3.57 3.50 3.42 3.36 3.30 3.18 3.09

The last three lines of figures are based on the empirical constant 1.8 for the average British thermal units transmitted per square foot of radiating surface per hour per degree of difference of temperature. This figure is approximately correct for several forms of both cast-iron radiators and pipe coils, not over 30 inches high and not over two pipes in width.

RADIATING SURFACE SUPPLIED BY DIFFERENT SIZES OF PIPE.

On basis of steam in pipe at 0.3 and 10.3 lbs. gauge pressure, temperature of room 70° , heat transmitted per square foot radiating surface 257.4 and 306 British thermal units per hour, and drop of pressure in pipe at the rate of 1 lb. per 1000 feet length; = pounds of steam per hour in the table on the preceding page, 1st column, $\times 3.75$, and last column, $\times 3.09$.

| Size of Pipe. | Radiating Surface, Sq. Ft. | | Size of Pipe. | Radiating Surface, Sq. Ft. | | Size of Pipe. | Radiating Surface, Sq. Ft. | |
|---------------|----------------------------|----------|---------------|----------------------------|----------|---------------|----------------------------|----------|
| | 0.3 lb. | 10.3 lb. | | 0.3 lb. | 10.3 lb. | | 0.3 lb. | 10.3 lb. |
| 1/2 | 16 | 16 | 2 1/2 | 734 | 769 | 6 | 7,541 | 7,901 |
| 3/4 | 36 | 38 | 3 | 1,296 | 1,357 | 7 | 11,010 | 11,535 |
| 1 | 71 | 75 | 3 1/2 | 1,895 | 1,986 | 8 | 15,307 | 16,040 |
| 1 1/4 | 150 | 157 | 4 | 2,630 | 2,755 | 9 | 20,482 | 21,451 |
| 1 1/2 | 230 | 241 | 4 1/2 | 3,520 | 3,686 | 10 | 27,427 | 28,718 |
| 2 | 453 | 475 | 5 | 4,695 | 4,919 | 12 | 43,312 | 45,423 |

For greater drops than 1 lb. per 1000 ft. length of pipe, multiply the figures by the square root of the drop.

* The latest steam tables (1909) give somewhat higher figures, but the difference is unimportant here.

Sizes of Steam Pipes in Heating Plants.—G. W. Stanton, in *Heating and Ventilating Mag.*, April, 1908, gives tables for proportioning pipes to radiating surface, from which the following table is condensed:

| Supply Pipe. Ins. | Radiating Surface Sq. Ft. | | | | Returns. | | Drips. | | Connections. | | |
|-------------------|---------------------------|--------|--------|---------|----------|------------------|--------|---------------------------------|----------------|--|-------------------------------|
| | A | B | C | D | B | C ₁ D | A | B ₁ C ₁ D | A ₁ | A ₂ B ₁ C ₁ | B ₂ C ₂ |
| 1 | 24 | 60 | 36 | 60 | 1 | 1 | 3/4 | 3/4 | 1 1/4 | 1 | 1 |
| 1 1/4 | 60 | 100 | 72 | 120 | 1 | 1 | 3/4 | 3/4 | 1 1/2 | 1 1/4 | 1 |
| 1 1/2 | 125 | 200 | 120 | 240 | 1 1/4 | 1 1/4 | 1 | 1 | 2 | 1 1/2 | 1 1/4 |
| 2 | 250 | 400 | 280 | 480 | 1 1/2 | 1 1/2 | 1 1/4 | 1 | 2 1/2 | 2 | 1 1/2 |
| 2 1/2 | 600 | 700 | 528 | 880 | 2 | 2 | 1 1/4 | 1 1/4 | 3 | | |
| 3 | 800 | 1,000 | 900 | 1,500 | 2 | 2 1/2 | 1 1/2 | 1 1/4 | 3 1/2 | | |
| 3 1/2 | 1,000 | 1,600 | 1,320 | 2,200 | 2 1/2 | 2 1/2 | 1 1/2 | 1 1/4 | 4 | | |
| 4 | 1,600 | 2,300 | 1,920 | 3,200 | 2 1/2 | 3 | 1 1/2 | 1 1/4 | 4 1/2 | | |
| 4 1/2 | 1,900 | 3,200 | 2,760 | 4,600 | 2 1/2 | 3 | 1 1/2 | 1 1/4 | | | |
| 5 | 2,300 | 4,100 | 3,720 | 6,200 | 3 | 3 1/2 | | | | | |
| 6 | 4,100 | 6,500 | 6,000 | 10,000 | 3 | 3 1/2 | | | | | |
| 7 | 6,500 | 9,600 | 9,000 | 15,000 | 3 1/2 | 4 | | | | | |
| 8 | 9,600 | 13,600 | 12,800 | 21,600 | 4 | 4 | | | | | |
| 9 | 13,600 | | 17,800 | 30,000 | | 4 1/2 | | | | | |
| 10 | | | 23,200 | 39,000 | | 5 | | | | | |
| 12 | | | 37,000 | 62,000 | | 6 | | | | | |
| 14 | | | 54,000 | 92,000 | | 7 | | | | | |
| 16 | | | 76,000 | 130,000 | | 8 | | | | | |

Supply mains and risers are of the same size. Riser connections on the two-pipe system to be the same size as the riser.

A. For single-pipe steam-heating system 0 to 5 lb. pressure. A₁, riser connections. A₂, radiator connections.

B. Two-pipe system 0 to 5 lb. pressure; B₁, C₁, radiator connections, supply; B₂, C₂, radiator connections, return.

C, D. Two-pipe system 2 and 5 lbs. respectively, mains and risers not over 100 ft. length. For other lengths, multiply the given radiating surface by factors, as below:

| | | | | | | | | | |
|----------------|------|------|-----|------|------|------|------|------|------|
| Length, ft.... | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 |
| Factor..... | 0.71 | 0.58 | 0.5 | 0.45 | 0.41 | 0.38 | 0.35 | 0.33 | 0.32 |

Mr. Stanton says: Theoretically both supply and return mains could be much smaller, but in practice it has been found that while smaller pipes can be used if a job is properly and carefully figured and proportioned and installed, for work as ordinarily installed it is far safer to use the sizes that have been tried and proven. By using the sizes given a job will circulate throughout with 1 lb. steam pressure at the boiler.

Resistance of Fittings.—Where the pipe supplying the radiation contains a large number of fittings, or other conditions make such a refinement necessary, it is advisable to add to the actual distance of the radiation from the source of supply a distance equivalent to the resistance offered by the fittings, and by the entrance to the radiator, the value of which, expressed in feet of pipe of the same diameter as the fitting, will be found in the accompanying table. *Power*, Dec., 1907.

FEET OF PIPE TO BE ADDED FOR EACH FITTING.

| Size Pipe. | 1 | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------|---|-------|-------|----|-------|----|-------|----|-------|----|----|----|----|----|----|
| Elbows... | 3 | 4 | 5 | 7 | 8 | 10 | 12 | 13 | 15 | 17 | 20 | 23 | 27 | 30 | 33 |
| Globe V.. | 7 | 8 | 10 | 13 | 17 | 20 | 23 | 27 | 30 | 33 | 40 | 47 | 53 | 60 | 67 |
| Entrance | 5 | 6 | 8 | 10 | 12 | 15 | 18 | 20 | 23 | 25 | 30 | 35 | 40 | 45 | 50 |

Overhead Steam-pipes. (A. R. Wolff, *Stevens Indicator*, 1887.) — When the overhead system of steam-heating is employed, in which system direct radiating-pipes, usually $1\frac{1}{4}$ in. in diam., are placed in rows overhead, suspended upon horizontal racks, the pipes running horizontally, and side by side, around the whole interior of the building, from 2 to 3 ft. from the walls, and from 2 to 4 ft. from the ceiling, the amount of $1\frac{1}{4}$ -in. pipe required, according to Mr. C. J. H. Woodbury, for heating mills (for which use this system is deservedly much in vogue), is about 1 ft. in length for every 90 cu. ft. of space. Of course a great range of difference exists, due to the special character of the operating machinery in the mill, both in respect to the amount of air circulated by the machinery, and also the aid to warming the room by the friction of the journals.

Removal of Air from Radiators. Vacuum Systems. — In order that a steam radiator may work at its highest capacity it is necessary that it be neither water-bound nor air-bound. Proper drainage must therefore be provided, and also means for continuously, or frequently, removing air from the system, such as automatic air-valves on each radiator, an air-pump or an air-ejector on a chamber or receiver into which the returns are carried, or separate air-pipes connecting each radiator with a vacuum chamber. When a vacuum system is used, especially with a high vacuum, much lower temperatures than usual may be used in the radiators, which is an advantage in moderate weather.

Steam-consumption in Car-heating.

C., M. & ST. PAUL RAILWAY TESTS. (*Engineering*, June 27, 1890, p. 764.)

| Outside Temperature. | Inside Temperature. | Water of Condensation per Car per Hour. |
|----------------------|---------------------|--|
| 40 | 70 | 70 lbs. |
| 30 | 70 | 85 |
| 10 | 70 | 100 |

Heating a Greenhouse by Steam. — Wm. J. Baldwin answers a question in the *American Machinist* as below: With five pounds steam-pressure, how many square feet or inches of heating-surface is necessary to heat 100 square feet of glass on the roof, ends, and sides of a greenhouse in order to maintain a night heat of 55° to 65° , while the thermometer outside ranges at from 15° to 20° below zero; also, what boiler-surface is necessary? Which is the best for the purpose to use — 2" pipe or $1\frac{1}{4}$ " pipe?

Ans. — Reliable authorities agree that 1.25 to 1.50 cubic feet of air in an enclosed space will be cooled per minute per sq. ft. of glass as many degrees as the internal temperature of the house exceeds that of the air outside. Between $+65^{\circ}$ and -20° there will be a difference of 85° , or, say, one cubic foot of air cooled 127.5° F. for each sq. ft. of glass for the most extreme condition mentioned. Multiply this by the number of square feet of glass and by 60, and we have the number of cubic feet of air cooled 1° per hour within the building or house. Divide the number thus found by 48, and it gives the units of heat required, approximately. Divide again by 953, and it will give the number of pounds of steam that must be condensed from a pressure and temperature of five pounds above atmosphere to water at the same temperature in an hour to maintain the heat. Each square foot of surface of pipe will condense from $\frac{1}{4}$ to nearly $\frac{1}{2}$ lb. of steam per hour, according as the coils are exposed or well or poorly arranged, for which an average of $\frac{1}{3}$ lb. may be taken. According to this, it will require 3 sq. ft. of pipe surface per lb. of steam to be condensed. Proportion the heating-surface of the boiler to have about one fifth the actual radiating-surface, if you wish to keep steam over night, and proportion the grate to burn not more than six pounds of coal per sq. ft. of grate per hour. With very slow combustion, such as takes place in base-burning boilers, the grate might be proportioned for four to five pounds of coal per hour. It is cheaper to make coils of $1\frac{1}{4}$ " pipe than of 2", and there is nothing to be gained by using 2" pipe unless the coils are very long. The pipes in a greenhouse should be under or in front of the benches, with every chance for a good circulation

of air. "Header" coils are better than "return-bend" coils for this purpose.

Mr. Baldwin's rule may be given the following form: Let H = heat-units transferred per hour, T = temperature inside the greenhouse, t = temperature outside, S = sq. ft. of glass surface; then $H = 1.5 S (T - t) \times 60 \div 48 = 1.875 S (T - t)$. Mr. Wolff's coefficient K for single skylights gives $H = 1.03 S (T - t)$, and for single windows, $1.20 S (T - t)$.

Heating a Greenhouse by Hot Water. — W. M. Mackay, of the Richardson & Boynton Co., in a lecture before the Master Plumbers' Association, N. Y., 1889, says: I find that while greenhouses were formerly heated by 4-inch and 3-inch cast-iron pipe, on account of the large body of water which they contained, and the supposition that they gave better satisfaction and a more even temperature, florists of long experience who have tried 4-inch and 3-inch cast-iron pipe, and also 2-inch wrought-iron pipe for a number of years in heating their greenhouses by hot water, and who have also tried steam-heat, tell me that they get better satisfaction, greater economy, and are able to maintain a more even temperature with 2-inch wrought-iron pipe and hot water than by any other system they have used. They attribute this result principally to the fact that this size pipe contains less water and on this account the heat can be raised and lowered quicker than by any other arrangement of pipes, and a more uniform temperature maintained than by steam or any other system.

HOT-WATER HEATING.

The following notes are from the catalogue of the Nason Mfg. Co.:

There are two distinct forms or modifications of hot-water apparatus, depending upon the temperature of the water.

In the first or open-tank system the water is never above 212° temperature, and rarely above 200° . This method always gives satisfaction where the surface is sufficiently liberal, but in making it so its cost is considerably greater than that for a steam-heating apparatus.

In the second method, sometimes called (erroneously) high-pressure hot-water heating, or the closed-system apparatus, the tank is closed. If it is provided with a safety-valve set at 10 lbs. it is practically as safe as the open-tank system.

Law of Velocity of Flow. — The motive power of the circulation in a hot-water apparatus is the difference between the specific gravities of the water in the ascending and the descending pipes. This effective pressure is very small, and is equal to about one grain for each foot in height for each degree difference between the pipes; thus, with a height of 12 in "up" pipe, and a difference between the temperatures of the up and down pipes of 8° , the difference in their specific gravities is equal to 8.16 grains (0.001166 lb.) on each square inch of the section of return-pipe, and the velocity of the circulation is proportioned to these differences in temperature and height.

Main flow-pipes from the heater, from which branches may be taken, are to be preferred to the practice of taking off nearly as many pipes from the heater as there are radiators to supply.

It is not necessary that the main flow and return pipes should equal in capacity that of all their branches. The hottest water will seek the highest level, while gravity will cause an even distribution of the heated water if the surface is properly proportioned.

It is good practice to reduce the size of the vertical mains as they ascend, say at the rate of one size for each floor.

As with steam, so with hot water, the pipes must be unconfined to allow for expansion of the pipes consequent on having their temperatures increased.

An expansion tank is required to keep the apparatus filled with water, which latter expands $\frac{1}{24}$ of its bulk on being heated from 40° to 212° , and the cistern must have capacity to hold certainly this increased bulk. It is recommended that the supply cistern be placed on level with or above the highest pipes of the apparatus, in order to receive the air which collects in the mains and radiators, and capable of holding at least $\frac{1}{20}$ of the water in the entire apparatus.

Arrangement of Mains for Hot-water Heating. (W. M. Mackay, Lecture before Master Plumbers' Assoc., N. Y., 1889). — There are two different systems of mains in general use, either of which, if properly

placed, will give good satisfaction. One is the taking of a single large-flow main from the heater to supply all the radiators on the several floors, with a corresponding return main of the same size. The other is the taking of a number of 2-inch wrought-iron mains from the heater, with the same number of return mains of the same size, branching off to the several radiators or coils with $1\frac{1}{4}$ -inch or 1-inch pipe, according to the size of the radiator or coil. A 2-inch main will supply three $1\frac{1}{4}$ -inch or four 1-inch branches, and these branches should be taken from the top of the horizontal main with a nipple and elbow, except in special cases where it is found necessary to retard the flow of water to the near radiator, for the purpose of assisting the circulation in the far radiator; in this case the branch is taken from the side of the horizontal main. The flow and return mains are usually run side by side, suspended from the basement ceiling, and should have a gradual ascent from the heater to the radiators of at least 1 inch in 10 feet. It is customary, and an advantage where 2-inch mains are used, to reduce the size of the main at every point where a branch is taken off.

The single or large main system is best adapted for large buildings; but there is a limit as to size of main which it is not wise to go beyond — generally 6-inch, except in special cases.

The proper area of cold-air pipe necessary for 100 square feet of indirect radiation in hot-water heating is 75 square inches, while the hot-air pipe should have at least 100 square inches of area. There should be a damper in the cold-air pipe for the purpose of controlling the amount of air admitted to the radiator, depending on the severity of the weather.

Sizes of Pipe for Hot-water Heating. — A theoretical calculation of the required size of pipe in hot-water heating may be made in the following manner. Having given the amount of heat, in B.T.U. to be emitted by a radiator per minute, assume the temperatures of the water entering and leaving, say 160° and 140° . Dividing the B.T.U. by the difference in temperatures gives the number of pounds of water to be circulated, and this divided by the weight of water per cubic foot gives the number of cubic feet per minute. The motive force to move this water, per square inch of the area of the riser, is the difference in weight per cu. ft. of water at the two temperatures, divided by 144, and multiplied by H , the height of the riser, or for $T_1 = 160$ and $T_2 = 140$, $(61.37 - 60.98) \div 144 = 0.00271$ lb. per sq. in. for each foot of the riser. Dividing 144 by 61.37 gives 2.34, the ft. head of water corresponding to 1 lb. per sq. in., and $0.00271 \times 2.34 = 0.0066$ ft. head, or if the riser is 20 ft. high, $20 \times 0.0066 = 0.132$ ft. head, which is the motive force to move the water over the whole length of the circuit, overcoming the friction of the riser, the return pipe, the radiator and its connections. If the circuit has a resistance equal to that of a 50-ft. pipe, then $50 \div 0.132 = 380$ is the ratio of length of pipe to the head, which ratio is to be taken with the number of cubic feet to be circulated, and by means of formulæ for flow of water, such as Darcy's, or hydraulic tables, the diameter of pipe required to convey the given quantity of water with this ratio of length of pipe to head is found. This tedious calculation is made more complicated by the fact that estimates have to be made of the frictional resistance of the radiator and its connections, elbows, valves, etc., so that in practice it is almost never used, and "rules of thumb" and tables derived from experience are used instead.

On this subject a committee of the Am. Soc. Heating and Ventilating Engineers reported in 1909 as follows:

The amount of water of a certain temperature required per hour by radiation may be determined by the following formula:

$$\frac{R \times X}{20 \times 60.8 \times 60} = \text{cu. ft. of water per minute.}$$

R = square feet of radiation; X = B.T.U. given off per hour by 1 sq. ft. of radiation (150 for direct and 230 for indirect) with water at 170° . Twenty is the drop in temperature in degrees between the water entering the radiation and that leaving it; 60.8 is the weight of a cubic foot of water at 170 degrees; 60 is to reduce the result from hours to minutes.

The average sizes of mains, as used by seven prominent engineers in regular practice for 1800 square feet of radiation, are given below:

2-pipe open-tank system, 100 ft. mains, 5-in. pipe = 26.6 ft. per min.
 1-pipe open-tank system, 100 ft. mains, 6-in. pipe = 18.4 ft. per min.
 Overhead open-tank system, 100 ft. mains, 4-in. pipe = 41.8 ft. per min.
 Overhead open-tank system, 100 ft. mains, 3-in. pipe = 72.1 ft. per min.

For 1200 sq. ft. indirect radiation with separate main, 100 ft. long, direct from boiler, open system, the bottom of the radiator being 1 ft. above the top of the boiler — 5-in. pipe = 22.4 ft. per min.

CAPACITY OF MAINS 100 FT. LONG.

Expressed in the number of square feet of hot-water radiating surface they will supply, the radiators being placed in rooms at 70° F., and 20° drop assumed.

| Diameter of Pipes, Ins. | Two-Pipe up Feed Open Tank. | One-Pipe up Feed Open Tank. | Overhead Open Tank. | Overhead Closed Tank. | Two-Pipe Open Tank. |
|-------------------------|-----------------------------|-----------------------------|---------------------|-----------------------|---------------------|
| 1 1/4 | 75 | 45 | 127 | 250 | 48 |
| 1 1/2 | 107 | 65 | 181 | 335 | 69 |
| 2 | 200 | 121 | 339 | 667 | 129 |
| 2 1/2 | 314 | 190 | 533 | 1,060 | 202 |
| 3 | 540 | 328 | 916 | 1,800 | 348 |
| 3 1/2 | 780 | 474 | 1,334 | 2,600 | 502 |
| 4 | 1,060 | 645 | 1,800 | 3,350 | 684 |
| 5 | 1,860 | 1,130 | 3,150 | 6,200 | 1,200 |
| 6 | 2,960 | 1,800 | 5,000 | 9,800 | 1,910 |
| 7 | 4,280 | 2,700 | 7,200 | 13,900 | 2,760 |
| 8 | 5,850 | 3,500 | 9,900 | 19,500 | 3,778 |

The figures are for direct radiation except the last column which is for indirect, 12 in. above boiler.

CAPACITY OF RISERS.

Expressed in the number of sq. ft. of direct hot-water radiating surface they will supply, the radiators being placed in rooms at 70° F., and 20° drop assumed. The figures in the last column are for the closed-tank overhead system the others are for the open-tank system.

| Diameter of Riser, Inches. | 1st Floor. | 2d Floor. | 3d Floor. | 4th Floor. | Drop Risers, not exceeding 4 floors. |
|----------------------------|------------|-----------|-----------|------------|--------------------------------------|
| 1 | 33 | 46 | 57 | 64 | 48 |
| 1 1/4 | 71 | 104 | 124 | 142 | 112 |
| 1 1/2 | 100 | 140 | 175 | 200 | 160 |
| 2 | 187 | 262 | 325 | 375 | 300 |
| 2 1/2 | 292 | 410 | 492 | 580 | 471 |
| 3 | 500 | 755 | 875 | 1,000 | 810 |

All horizontal branches from mains to risers or from risers to radiators, more than 10 ft. long (unless within 15 ft. of the boiler), should be increased one size over that indicated for risers in the above table.

For indirect radiation, the amount of surface may be computed as follows:

Temperature of the air entering the room, 110° = T .

Average temperature of the air passing through the radiator, 55°.

Temperature of the air leaving the room, 70° = t .

Velocity of the air passing through the radiator, 240 ft. per min.

Cubic feet of air to be conveyed per hour, = $C = (H \times 55) \div (T - t)$.

H = exposure loss in B.T.U. per hour.

Heat necessary to raise this air to the entering temperature from 0° F., $T \times C + 55 = H$.

The amount of radiation is found by dividing the total heat by the emission of heat by indirect radiators per square foot per hour per degree difference in temperature. This varies with the velocity, as shown below:

| | | | | | | | | | | |
|---------------------------|------|------|------|------|------|------|------|------|------|------|
| Velocity, ft. per min.... | 174 | 246 | 300 | 342 | 378 | 400 | 428 | 450 | 474 | 492 |
| B.T.U..... | 1.70 | 2.00 | 2.22 | 2.38 | 2.52 | 2.60 | 2.67 | 2.72 | 2.76 | 2.80 |

The difference between 170 degrees (average temperature of the water in the radiator) and 55 degrees (average temperature of the air in the radiator) being 115, the emission at 240 ft. per min. is 2. per degree difference or 230 B.T.U.

Ordinarily the amount of indirect radiation required is computed by adding a percentage to the amount of direct radiation [computed by the usual rules], and an addition of 50% has been found sufficient in many cases; but in buildings where a standard of ventilation is to be maintained, the formula mentioned seems more likely to give satisfactory results. Free area between the sections of radiation to allow passage of the required volume of air at the assumed velocity must be maintained. The cold-air supply duct, on account of less frictional resistance, may ordinarily have 80% of the area between the radiator sections. The hot-air flues may safely be proportioned for the following air velocities per minute: First floor, 200 feet; second floor, 300 feet; third floor, 400 feet.

PIPE SIZES FOR HOT-WATER HEATING.

Based on 20° difference in temperature between flow and return water. (C. L. Hubbard, *The Engineer* July 1, 1902.)

| Diam. of Pipe. | 1 | 1 1/4 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 5 | 6 | 7 |
|--|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Length of Run. | Square Feet of Direct Radiating Surface. | | | | | | | | | | |
| Feet. | 30 | 60 | 100 | 200 | 350 | 550 | 850 | 1,200 | | | |
| 100 | | 50 | 75 | 150 | 250 | 400 | 600 | 850 | 1,400 | | |
| 200 | | | 50 | 125 | 200 | 300 | 450 | 700 | 1,150 | | |
| 300 | | | | 100 | 175 | 275 | 400 | 600 | 1,000 | 1,600 | |
| 400 | | | | 75 | 150 | 250 | 350 | 525 | 700 | 1,400 | |
| 500 | | | | | 125 | 225 | 325 | 475 | 850 | 1,300 | |
| 600 | | | | | | 200 | 300 | 450 | 775 | 1,200 | 1,700 |
| 700 | | | | | | 175 | 250 | 400 | 725 | 1,150 | 1,600 |
| 800 | | | | | | 150 | 225 | 350 | 650 | 1,000 | 1,500 |
| 1000 | | | | | | | | | | | |
| Square Feet of Indirect Radiation. | | | | | | | | | | | |
| 100 | 15 | 30 | 50 | 100 | 200 | 300 | 400 | 600 | 1,000 | | |
| 200 | | 20 | 30 | 70 | 120 | 200 | 300 | 400 | 700 | | |
| Square Feet of Direct Radiating Surface. | | | | | | | | | | | |
| 1st story | 30 | 60 | 100 | 200 | 350 | 550 | 850 | | | | |
| 2d " | 55 | 90 | 140 | 275 | 275 | | | | | | |
| 3d " | 65 | 110 | 165 | 375 | | | | | | | |
| 4th " | 75 | 125 | 185 | 425 | | | | | | | |
| 5th " | 85 | 140 | 210 | 500 | | | | | | | |
| 6th " | 95 | 160 | 240 | | | | | | | | |

The size of pipe required to supply any given amount of hot-water radiating surface depends upon (1) The square feet of radiation; (2) its elevation above the boiler; (3) the difference in temperature of the water in the supply and return pipes; (4) the length of the pipe connecting the radiator with the boiler.

In estimating the length of a pipe the number of bends and valves must be taken into account. It is customary to consider an elbow as equivalent to a pipe 60 diameters in length, and a return bend to 120 diameters. A globe valve may be taken about the same as an elbow.

A series of articles on The Determination of the Sizes of Pipe for Hot Water Heating, by F. E. Geisecke, is printed in *Domestic Engineering*, beginning in May, 1909.

Sizes of Flow and Return Pipes Approximately Proportioned to Surface of Direct Radiators for Gravity Hot-Water Heating.
(G. W. Stanton, *Heat. & Ventg. Mag.*, April, 1908.)

| Size of Mains. | Mains. | | Branches of Mains. | | | |
|-----------------------------------|------------------------|---------------------------------|----------------------|-----------------------|----------------------|--------------------------------|
| | In Cellar or Basement. | On One or More Floors. Average. | First Floor 10'-15'. | Second Floor 15'-25'. | Third Floor 25'-35'. | Fourth or Fifth Floor 35'-45'. |
| Square Feet of Radiating Surface. | | | | | | |
| 3/4 | | | | 40 | 45 | 50 |
| 1 | | | 50 | 75 | 80 | 85 |
| 1 1/4 | 100 | 135 | 110 | 120 | 135 | 150 |
| 1 1/2 | 135 | 220 | 180 | 195 | 210 | 230 |
| 2 | 225 | 350 | 290 | 320 | 350 | 370 |
| 2 1/2 | 320 | 460 | 400 | 490 | 525 | 550 |
| 3 | 500 | 675 | 620 | 650 | 690 | 730 |
| 3 1/2 | 650 | 850 | 820 | 870 | 920 | 970 |
| 4 | 850 | 1,100 | 1,050 | 1,120 | 1,185 | 1,250 |
| 4 1/2 | 1,050 | 1,350 | 1,325 | 1,400 | 1,485 | 1,560 |
| 5 | 1,350 | 1,700 | | | | |
| 6 | 2,900 | 3,600 | | | | |
| 7 | 3,900 | 4,800 | | | | |
| 8 | 5,000 | 6,200 | | | | |
| 9 | 6,300 | 7,700 | | | | |
| 10 | 7,900 | 9,800 | | | | |
| 11 | 9,500 | 11,800 | | | | |
| 12 | 11,400 | 14,000 | | | | |

Note.—The heights of the several floors are taken as:
1st. 10 to 15 ft.; 2d. 15 to 25 ft.
3d. 25 to 35 ft.; 4th. 35 to 45 ft.

Heating by Hot Water, with Forced Circulation.—The principal defect of gravity hot-water systems, that the motive force is only the difference in weight of two columns of water of different temperatures, is overcome by giving the water a forced circulation, either by means of a pump or by a steam ejector. For large installations a pump gives facilities for forcing the hot water to any distance required. The design of such a system is chiefly a problem in hydraulics. After determining the quantity of heat to be given out by each radiator, a certain drop in temperature is assumed, and from that the volume of water required by each radiator is calculated. The piping system then has to be designed so that it will carry the proper supply of water to each radiator without short-circuiting, and with a minimum total cost for power to force the water, for loss by radiation, and for interest, etc., on cost of plant. No short rules or formulæ have been established for designing a forced hot-water system, and each case has to be studied as an original problem to be solved by application of the laws of heat transmission and hydraulics. Forced systems using steam ejectors have come into use to some extent in Europe in small installations, and some of them are described in the Transactions of the Amer. Soc'y of Heating and Ventilating Engineers.

A system of distributing heat and power to customers by means of hot water pumped from a central station was adopted by the Boston Heating Co. in 1888. It was not commercially successful. A description of the plant is given by A. V. Abbott in *Trans. A. I. M. E.*, 1888.

THE BLOWER SYSTEM OF HEATING.

The system provides for the use of a fan or blower which takes its supply of fresh air from the outside of the building to be heated, forces it over steam coils, located either centrally or divided up into a number of independent groups, and then into the several ducts or flues leading to the various rooms. The movement of the warmed air is positive, and the delivery of the air to the various points of supply is certain and entirely independent of atmospheric conditions.

Advantages and Disadvantages of the Plenum System. (Prof. W. F. Barrett, *Brit. Inst. H. & V. Engrs.*, 1905.)—Advantages: (1) The

evenness of temperature produced; (2) the ventilation of the building is concurrent with its warming; (3) the air can be drawn from sources free from contamination and can be filtered from suspended impurities, warmed and brought to the proper hygrometric state before its introduction to the different rooms or wards; (4) the degree of temperature and of ventilation can be easily controlled in any part of the building, and (5) the removal of ugly pipes running through the rooms has a great architectural and esthetic advantage.

Disadvantages: (1) The most obvious is that no windows can be opened nor doors left open; double doors with an air lock between must also be provided if the doors are frequently opened and closed; (2) the mechanical arrangements are elaborate and the system requires to be used with intelligent care; (3) the whole elaborate system needs to be set going even if only one or two rooms in a large building require to be warmed, as often happens in the winter vacation of a college; (4) the temporary failure of the system, through the breakdown of the engines or other cause, throws the whole system into confusion, and if, as in the Royal Victoria Hospital, the windows are not made to open, imminent danger results; (5) then, also, in the case of hospital wards and asylums it is possible that the outlet ducts may become coated with disease germs, and unless periodically cleansed, a back current through a high wind or temporary failure of the system may bring a cloud of these disease germs back into the wards.

Heat Radiated from Coils in the Blower System. — The committee on Fan-blast Heating, of the A. S. H. V. E., in 1909, gives the following formula for amount of heat radiated from hot-blast coils with different velocities of air passing through the heater: $E = \text{B.T.U. per sq. ft. of surface per hour per degree of difference between the average temperature of the air and the steam temperature,} = \sqrt{4V}$, in which $V =$ velocity of the air through the free area of the coil in feet per second. A plotted curve of 20 tests of different heaters shows that the formula represents the average results, but individual tests show a wide variation from the average, thus: For velocity 1000 ft. per min., average 9 B.T.U., range 7.5 to 11; 1600 ft. per min., average 10.4, range 9.5 to 12.

The committee also gives the following formula for the rise in temperature of each two-row section of a coil:

$$R = \frac{(T_s - T_a) \times H \times E}{A \times V_m \times W \times 60 \times 0.2377}$$

In which $R =$ degrees F. rise for each two-row section; $T_s =$ temperature of steam; $T_a =$ temperature of air; $H =$ square feet of surface in two-row section; $E = \text{B.T.U. per degree difference between air and steam; } E = \sqrt{4V_s}$, in which $V_s =$ air velocity in ft. per sec.; $A =$ area through heater in sq. ft.; $V_m =$ velocity of air in ft. per min.; $W =$ weight of 1 cu. ft. of air, lbs.

The value of R is computed for each two-row section in a coil, and the results added. From a set of curves plotted from the formula the following figures are taken.

| | Number of Rows. | | | | | | |
|------------------------------------|----------------------------|----|-----|-----|-----|-----|-----|
| | 4 | 8 | 12 | 16 | 20 | 24 | 28 |
| | Temperature Rise, Degrees. | | | | | | |
| Steam, 80 lbs. $V_m = 1,200$ | 43 | 83 | 115 | 144 | 167 | 189 | 209 |
| Steam, 80 lbs. $V_m = 1,800$ | 36 | 68 | 96 | 122 | 145 | 165 | 182 |
| Steam, 5 lbs. $V_m = 1,200$ | 31 | 53 | 80 | 100 | 118 | 133 | 146 |
| Steam, 5 lbs. $V_m = 1,800$ | 25 | 48 | 68 | 86 | 101 | 115 | 128 |

A formula for the rise in temperature of air in passing through the coils of a hot-blast heater is given by E. F. Child in *The Metal Worker*, Oct. 5, 1907, as follows: $R = KDZ^m N \div \sqrt[n]{V}$, in which $R =$ rise in

temperature of the air; K = a constant depending on the kind of heating surface; D = an average of the summation of temperature differences between the air and the steam = $(T_1 - T_0) \div \log_e [(T_s - T_0) \div (T_s - T_1)]$; Z = number of sq. ft. of heating surface per sq. ft. of clear area per unit depth of heater. m = a power applicable to Z and depending on the type of heating surface; N = number of units in depth of heater; V = velocity of the air at 70° F. in ft. per min. through the clear area; n = a root applicable to V and depending on experiment.

For practical purposes and within the range of present knowledge on the subject the formula may be written $R = 0.85 DZN \div \sqrt[3]{V}$, and from this formula with $T_s = 227^\circ$ and $T_0 = 0^\circ$, with different values of T_1 , the temperature of the air leaving the coils, a set of curves is plotted, from which the figures in the following table are taken.

| Velocity, Ft. per Min. | Sq. ft. of heating surface \div sq. ft. free area through heater. | | | | | | | | | |
|---------------------------|---|----|----|----|-----|-----|-----|-----|-----|-----|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 120 |
| | Rise in Temperature, Degrees F. | | | | | | | | | |
| 500..... | 43 | 63 | 79 | 95 | 108 | 120 | 131 | 141 | 151 | 170 |
| 800..... | 38 | 55 | 70 | 84 | 97 | 108 | 118 | 128 | 138 | 157 |
| 1000..... | 36 | 52 | 66 | 79 | 92 | 102 | 112 | 121 | 130 | 147 |
| 1200..... | 34 | 49 | 63 | 75 | 87 | 98 | 108 | 117 | 125 | 140 |
| 2000..... | 29 | 42 | 55 | 66 | 76 | 86 | 95 | 104 | 112 | 127 |

Burt S. Harrison (*Htg. and Ventg. Mag.*, Oct. and Nov., 1907) gives the following formula, $R = \frac{1}{\sqrt[3]{V}} (T - t) \frac{1}{8/N + 0.24}$, in which T = temp. of steam

in coils, t = temp. of air entering coils, V = velocity of air through coils in ft. per sec., N = no. of rows of 1-in. pipe in depth of heater. Charts are given by means of which heaters may be designed for any set of conditions.

Tests of Cast-iron Heaters for Hot-blast Work. — An extensive series of tests of the Amer. Radiator Co's, "Vento" cast-iron heater is described by Theo. Weinshank in *Trans. A. S. H. V. E.*, 1908. The tests were made under the supervision of Prof. J. H. Kinealy. The principal results are given below.

TESTS OF A "VENTO" CAST-IRON HEATER.

| Velocity, ft. per Min. | Number of sections heater is deep. | | | | | | Number of sections heater is deep. | | | | | |
|---------------------------|--|-------|-------|-------|-------|-------|--|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| | Rise of temperature, K , per degree difference between temperature of steam and mean temperature of air for different velocities of air. | | | | | | Heat units transmitted per square foot of heating surface per hour per degree difference between the temperature of the steam and the mean temperature of the air. | | | | | |
| 1600..... | 0.124 | 0.253 | 0.395 | 0.527 | 0.649 | 0.761 | 11.94 | 12.17 | 12.67 | 12.67 | 12.50 | 12.20 |
| 1500..... | 0.132 | 0.261 | 0.403 | 0.535 | 0.657 | 0.769 | 11.91 | 11.76 | 12.11 | 12.06 | 11.86 | 11.56 |
| 1400..... | 0.139 | 0.268 | 0.410 | 0.542 | 0.664 | 0.776 | 11.70 | 11.28 | 11.50 | 11.41 | 11.18 | 10.89 |
| 1300..... | 0.147 | 0.276 | 0.418 | 0.550 | 0.672 | 0.784 | 11.50 | 10.79 | 10.89 | 10.75 | 10.51 | 10.22 |
| 1200..... | 0.154 | 0.283 | 0.425 | 0.557 | 0.679 | 0.791 | 11.11 | 10.21 | 10.22 | 10.05 | 9.81 | 9.52 |
| 1100..... | 0.162 | 0.291 | 0.433 | 0.565 | 0.687 | 0.799 | 10.72 | 9.63 | 9.55 | 9.34 | 9.09 | 8.82 |
| 1000..... | 0.170 | 0.299 | 0.441 | 0.573 | 0.695 | 0.807 | 10.23 | 8.99 | 8.84 | 8.61 | 8.36 | 8.10 |
| 900..... | 0.177 | 0.306 | 0.448 | 0.580 | 0.702 | 0.814 | 9.59 | 8.28 | 8.08 | 7.85 | 7.60 | 7.35 |
| 800..... | 0.185 | 0.314 | 0.456 | 0.588 | 0.710 | 0.822 | 8.90 | 7.56 | 7.31 | 7.08 | 6.48 | 6.60 |

TESTS OF A "VENTO" CAST-IRON HEATER. — *Continued.*

| Velocity, ft. per min. | Final temperature, T , of air when entering heater at 0° F. Temperature of steam in heater, 227°. | | | | | | Friction loss in inches of water due to the sections. | | | | | | | |
|---------------------------|--|------|------|-------|-------|-------|--|-------|-------|-------|-------|-------|--|--|
| | | | | | | | | | | | | | | |
| 1600..... | 26.5 | 51.0 | 74.9 | 94.7 | 111.3 | 125.2 | 0.236 | 0.288 | 0.416 | 0.543 | 0.672 | 0.800 | | |
| 1500..... | 28.1 | 52.4 | 76.3 | 95.8 | 112.4 | 126.0 | 0.207 | 0.253 | 0.366 | 0.477 | 0.590 | 0.703 | | |
| 1400..... | 29.5 | 53.8 | 77.2 | 96.7 | 113.3 | 126.8 | 0.180 | 0.220 | 0.318 | 0.415 | 0.514 | 0.613 | | |
| 1300..... | 31.1 | 55.0 | 77.6 | 97.9 | 114.3 | 127.7 | 0.156 | 0.190 | 0.274 | 0.358 | 0.443 | 0.528 | | |
| 1200..... | 32.4 | 56.4 | 79.6 | 99.0 | 115.3 | 128.7 | 0.133 | 0.162 | 0.234 | 0.306 | 0.378 | 0.450 | | |
| 1100..... | 34.0 | 57.7 | 80.5 | 100.0 | 116.2 | 129.6 | 0.111 | 0.136 | 0.197 | 0.257 | 0.318 | 0.378 | | |
| 1000..... | 35.6 | 59.1 | 82.0 | 100.1 | 117.2 | 130.5 | 0.092 | 0.112 | 0.162 | 0.212 | 0.262 | 0.312 | | |
| 900..... | 36.9 | 60.1 | 83.0 | 102.1 | 118.0 | 131.3 | 0.074 | 0.091 | 0.132 | 0.172 | 0.212 | 0.253 | | |
| 800..... | 38.5 | 61.6 | 84.3 | 103.1 | 119.0 | 132.3 | 0.059 | 0.072 | 0.104 | 0.136 | 0.167 | 0.200 | | |

Formulae. — s = no. of sections; V = velocity, ft. per min., air measured at 70°; k = rise of temp. per degree difference; t = final temperature. f = friction loss in in. of water. $t = 454k + (2 + k)$. $k = s(0.167 - 0.005s) - 0.061\left(\frac{v-800}{800}\right)$. $f = (0.8s + 0.2)(V/4000)^2$. Values of k and f when $s=2$ or more.

Factory Heating by the Fan System.

In factories where the space provided per operative is large, warm air is recirculated, sufficient air for ventilation being provided by leakage through the walls and windows. The air is commonly heated by steam coils furnished with exhaust steam from the factory engine. When the engine is not running, or when it does not supply enough exhaust steam for the purpose, steam from the boilers is admitted to the coils through a reducing valve. The following proportions are commonly used in designing. Coils, pipes 1-in., set $2\frac{1}{8}$ in. centers; free area through coils, 40% of cross area. Velocity of air through free area, 1200 to 1800 ft. per min.; number of coils in series 8 to 20; circumferential speed of fan, 4000 to 6000 ft. per min.; temperature of air leaving coils, 120° to 160° F.; velocity of air at outlet of coil stack, 3000 to 4000 ft. per min.; velocity in branch pipes, 2000 to 2800 ft., the lower velocities in the longest pipes.

In factories in which mechanical ventilation as well as heating is required, outlet flues at proper points must be provided, to avoid the necessity of opening windows, and the outflow of air in them may be assisted either by exhaust fans or by steam coils in the flues.

Cooling Air for Ventilation.

The chief difficulty in the artificial cooling of air is due to the moisture it contains, and the great quantity of heat that has to be absorbed or abstracted from the air in order to condense this moisture. The cooled and moisture-laden air also needs to be partially reheated in order to bring it to a degree of relative humidity that will make it suitable for ventilation. To cool 1 lb. of dry air from 82° to 72° requires the abstracting of 10×0.2375 B.T.U. (0.2375 being the specific heat at constant pressure). If the air at 82° is saturated, or 100% relative humidity, it contains 0.0235 lb. of water vapor, while 1 lb. at 72° contains 0.0167 lb., so that 0.0068 lb. will be condensed in cooling from vapor at 82° to water at 72°. The total heat (above 32°) in 1 lb. vapor at 82° is 1095.6 B.T.U. and that in 1 lb. of water at 72° is 40 B.T.U. The difference, $1055.6 \times 0.0068 = 7.178$ B.T.U., is the amount of heat abstracted in condensing the moisture. The B.T.U. in 1 lb. vapor at 72° is 1091.2,

and the B.T.U. abstracted in cooling the remaining vapor from 82° to 72° is $0.0167 \times (1095.6 - 1091.2) = 0.073$ B.T.U. The sum, 7.251 B.T.U., is more than three times that required to cool the dry air from 82° to 72°. Expressing these principles in formulæ we have:

Let T_1 = original and T_2 the final temperature of the air,
 a = vapor in 1 lb. saturated air at T_1 ; b = do. at T_2 ,
 H = relative humidity of the air at T_1 ; h = desired do. at T_2 ,
 U = total heat, in B.T.U., in 1 lb. vapor at T_1 ; u = do. at T_2 ,
 w = total heat in water at T_2 .

Then total heat abstracted in cooling air from T_1 to $T_2 = (aH - bh) \times (U - w) + bh(U - u) + 0.2375(T_1 - T_2)$, or $aHU - bhu - (aH - bh)w + 0.2375(T_1 - T_2)$, or $aH(U - w) - bh(u - w) + 0.2375(T_1 - T_2)$.

EXAMPLE. — Required the amount of heat to be abstracted per hour in cooling the air for an audience chamber containing 1000 persons, 1500 cu. ft. (measured at 70° F.), being supplied per person per hour, the temperature of the air before cooling being 82°, with relative humidity 80%, and after cooling 72°, with humidity 70%.

$$1000 \times 1500 = 1,500,000 \text{ cu. ft., at } 0.075 \text{ lb. per cu. ft.} \\ = 112,500 \text{ lbs.}$$

For 1 lb. $aH(U - w) - bh(u - w) + 0.2375(T_1 - T_2)$.

$$0.0235 \times 0.8 \times (1095.6 - 40) - 0.0167 \times 0.7 \times (1091.2 - 40) \\ + 2.375 = 9.932 \text{ B.T.U.} \\ 112,500 \times 9.932 = 1,061,100 \text{ B.T.U.}$$

Taking 142 B.T.U. as the latent heat of melting ice, this amount is equivalent to the heat that would melt 7472 lbs. of ice per hour.

See also paper by W. W. Macon, *Trans. A. S. H. V. E.*, 1909, and Air-cooling of the New York Stock Exchange, *Eng. Rec.*, April, 1905, and *The Metal Worker*, Aug. 5, 1905.

Capacities of Fans or Blowers for Hot-Blast or Plenum Heating.

(Computed by F. R. Still, American Blower Co., Detroit, Mich.)

| Size of Blower-Housing. | Diam. of Fan-Wheel. | Revolutions per Minute. | H. P. Required to Drive Fan. | Cu. Ft. of Air Delivered per Minute by Fan through Heater. | Cu. Ft. of Air per Hour. | Heat Units Required per Hour to Raise Air from 0° to 120°. | Velocity of Air through Coils in Ft. per Minute. | Free Area between Pipes in Sq. Ft. | Heat Units Given off per Sq. Ft. Surface per Hour. | Sq. Ft. Heating Surface Required. |
|-------------------------|---------------------|-------------------------|------------------------------|--|--------------------------|--|--|------------------------------------|--|-----------------------------------|
| 70 | 42 | 360 | 21 $\frac{1}{2}$ | 6,900 | 415,200 | 1,021,000 | 900 | 7.7 | 1760 | 580 |
| 80 | 48 | 320 | 3 | 8,500 | 510,000 | 1,255,000 | " | 9.45 | " | 714 |
| 90 | 54 | 280 | 4 | 10,500 | 630,000 | 1,550,000 | " | 11.66 | " | 880 |
| 100 | 60 | 250 | 5 | 12,500 | 750,000 | 1,845,000 | " | 13.9 | " | 1050 |
| 110 | 66 | 230 | 6 | 15,800 | 948,000 | 2,335,000 | " | 17.55 | " | 1325 |
| 120 | 72 | 210 | 8 | 19,800 | 1,118,000 | 2,900,000 | " | 22. | " | 1650 |
| 140 | 84 | 180 | 10 | 26,200 | 1,572,000 | 3,870,000 | " | 29.1 | " | 2200 |
| 160 | 96 | 160 | 12 | 33,000 | 1,980,000 | 4,870,000 | " | 36.7 | " | 2770 |
| 180 | 108 | 140 | 15 | 41,600 | 2,496,000 | 6,130,000 | " | 46.3 | " | 3490 |
| 200 | 120 | 125 | 18 | 50,000 | 3,000,000 | 7,375,000 | " | 55.5 | " | 4140 |

Capacities of Fans or Blowers for Hot-blast or Plenum Heating —
Continued.

| Size of Blower-Housing. | Lineal Feet of One-Inch Pipe Required. | Pounds of Steam Condensed per Hour to 212°. | Size Steam-Main Required. | Size Return-Main Required. | Boiler Capacity Required, H.P.; 30 Lbs. Steam per Hour = 1 H.P. | Sq. Ft. Heating Surface in Boiler at 15 Sq. Ft. per H.P. | Sq. Ft. Grate-Surface at 35 Sq. Ft. Heating Surface to Sq. Ft. Grate. | Volume Air Will Expand to by Heating from 0° to 120° Capacity per Minute. | Area of Conduit in Sq. Ft. for 900 Ft. Velocity per Minute. | Net Volume Delivered, Allowance Being Made for Friction Equal to 100 Ft. of Conduit. |
|-------------------------|--|---|---------------------------|----------------------------|---|--|---|---|---|--|
| 70 | 1,740 | 1055 | 3 1/2 | 2 | 35 | 525 | 15 | 8,700 | 9.67 | 8,200 |
| 80 | 2,142 | 1295 | 4 | 2 | 43 | 645 | 18 | 10,700 | 13.05 | 10,000 |
| 90 | 2,640 | 1600 | 4 1/2 | 2 1/2 | 53 | 795 | 23 | 13,200 | 14.72 | 12,500 |
| 100 | 3,150 | 1900 | 5 | 2 1/2 | 63 | 945 | 27 | 15,800 | 17.55 | 15,000 |
| 110 | 3,975 | 2410 | 5 1/2 | 3 | 80 | 1200 | 34 | 19,900 | 22.20 | 18,900 |
| 120 | 4,950 | 2990 | 6 | 3 | 100 | 1500 | 43 | 25,000 | 27.80 | 23,800 |
| 140 | 6,600 | 3990 | 7 | 3 1/2 | 133 | 1995 | 57 | 33,100 | 36.80 | 31,400 |
| 160 | 8,310 | 5025 | 8 | 4 | 167 | 2505 | 72 | 41,700 | 46.30 | 39,600 |
| 180 | 10,470 | 6325 | 9 | 4 1/2 | 211 | 3165 | 90 | 52,500 | 58.40 | 50,000 |
| 200 | 12,420 | 7560 | 10 | 5 | 252 | 3780 | 108 | 63,200 | 70.25 | 60,000 |

Temperature of fresh air, 0°; of air from coils, 120°; of steam, 227°; Pressure of steam, 5 lbs.

Peripheral velocity of fan-tips, 4000 ft.; number of pipes deep in coil, 24; depth of coil, 60 inches; area of coils approximately twice free area.

Relative Efficiency of Fans and Heated Chimneys for Ventilation. — W. P. Trowbridge, *Trans. A. S. M. E.*, vii. 531, gives a theoretical solution of the relative amounts of heat expended to remove a given volume of impure air by a fan and by a chimney. Assuming the total efficiency of a fan to be only 1/25, which is made up of an efficiency of 1/10 for the engine, 5/10 for the fan itself, and 8/10 for efficiency as regards friction, the fan requires an expenditure of heat to drive it of only 1/38 of the amount that would be required to produce the same ventilation by a chimney 100 ft. high. For a chimney 500 ft. high the fan will be 7.6 times more efficient.

The following figures are given by Atkinson (*Coll. Engr.*, 1889), showing the minimum depth at which a furnace would be equal to a ventilating-machine, assuming that the sources of loss are the same in each case, *i.e.*, that the loss of fuel in a furnace from the cooling in the upcast is equivalent to the power expended in overcoming the friction in the machine, and also assuming that the ventilating-machine utilizes 60 per cent of the engine-power. The coal consumption of the engine per I.H.P. is taken at 8 lbs. per hour.

Average temperature in upcast 100° F. 150° F. 200° F.
 Minimum depth for equal economy.. 960 yards. 1040 yards. 1130 yards.

PERFORMANCE OF HEATING GUARANTEE.

Heating a Building to 70° F. Inside when the Outside Temperature is Zero. — It is customary in some contracts for heating to guarantee that the apparatus will heat the interior of the building to 70° in zero weather. As it may not be practicable to obtain zero weather for the purpose of a test, it may be difficult to prove the performance of the guarantee unless an equivalent test may be made when the outside temperature is above zero, heating the building to a higher temperature than 70°. The following method was proposed by the author (*Eng. Rec.*,

Aug. 11, 1894) for determining to what temperature the rooms should be heated for various temperatures of the outside atmosphere and of the steam or hot water in the radiators.

Let S = sq. ft. of surface of the steam or hot-water radiator;

W = sq. ft. of surface of exposed walls, windows, etc.;

T_s = temp. of the steam or hot water, T_1 = temp. of inside of building or room, T_0 = temp. of outside of building or room;

a = heat-units transmitted per sq. ft. of surface of radiator per hour per degree of difference of temperature;

b = average heat-units transmitted per sq. ft. of walls per hour per degree of difference of temperature, including allowance for ventilation.

It is assumed that within the range of temperatures considered Newton's law of cooling holds good, viz., that it is proportional to the difference of temperature between the two sides of the radiating-surface.

Then $aS(T_s - T_1) = bW(T_1 - T_0)$. Let $\frac{bW}{aS} = C$; then

$$T_s - T_1 = C(T_1 - T_0); \quad T_1 = \frac{T_s + CT_0}{1 + C}; \quad C = \frac{T_s - T_1}{T_1 - T_0}.$$

If $T_1 = 70$, and $T_0 = 0$, $C = \frac{T_s - 70}{70}$.

Let $T_s = 140^\circ \quad 160^\circ \quad 180^\circ \quad 200^\circ \quad 212^\circ \quad 220^\circ \quad 250^\circ \quad 300^\circ$

Then $C = 1 \quad 1.286 \quad 1.571 \quad 1.857 \quad 2.029 \quad 2.143 \quad 2.571 \quad 3.286$

and from the formula $T_1 = (T_s + CT_0) \div (1 + C)$ we find the inside temperatures corresponding to the given values of T_s and T_0 which should be produced by an apparatus capable of heating the building to 70° in zero weather.

For $T_0 = -20 \quad -10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40^\circ \text{ F.}$

Inside Temperatures T_1 .

| | | | | | | | |
|----------------------------------|------|------|----|------|------|------|-------|
| For $T_s = 140^\circ \text{ F.}$ | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| 160 | 58.7 | 64.3 | 70 | 75.6 | 81.3 | 86.9 | 92.5 |
| 180 | 57.8 | 63.9 | 70 | 76.1 | 82.2 | 88.4 | 94.5 |
| 200 | 57.0 | 63.5 | 70 | 76.5 | 83.0 | 89.5 | 96.0 |
| 212 | 56.6 | 63.3 | 70 | 76.7 | 83.4 | 90.1 | 96.8 |
| 220 | 56.4 | 63.2 | 70 | 76.8 | 83.6 | 90.5 | 97.3 |
| 250 | 55.6 | 62.8 | 70 | 77.2 | 84.4 | 91.6 | 98.8 |
| 300 | 54.7 | 62.4 | 70 | 77.7 | 85.3 | 93.0 | 100.7 |

J. K. Allen (*Trans. A. S. H. V. E.*, 1908) develops a complex formula for the inside temperature which takes into consideration the fact that the coefficient of transmission of the radiator is not constant but increases with the temperature. With $T_s = 227$ and a two-column cast-iron radiator he finds for $T_0 = -20 \quad -10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40$

$T_1 = 58 \quad 64 \quad 70 \quad 77.5 \quad 83 \quad 90 \quad 97$

For all values of T_0 between -10 and 40 these figures are within one degree of those computed by the author's method.

ELECTRICAL HEATING.

Heating by Electricity. — If the electric currents are generated by a dynamo driven by a steam-engine, electric heating will prove very expensive, since the steam-engine wastes in the exhaust-steam and by radiation about 90% of the heat-units supplied to it. In direct steam-heating, with a good boiler and properly covered supply-pipes, we can utilize about 60% of the total heat value of the fuel. One pound of coal, with a heating value of 13,000 heat-units, would supply to the radiators about $13,000 \times 0.60 = 7800$ heat-units. In electric heating, suppose we have a first-class condensing-engine developing 1 H.P. for every 2 lbs. of coal burned per hour. This would be equivalent to 1,980,000 ft.-lbs. +

778 = 2545 heat-units, or 1272 heat-units for 1 lb. of coal. The friction of the engine and of the dynamo and the loss by electric leakage and by heat radiation from the conducting wires might reduce the heat-units delivered as electric current to the electric radiator, and there converted into heat, to 50% of this, or only 636 heat-units, or less than one twelfth of that delivered to the steam-radiators in direct steam-heating. Electric heating, therefore, will prove uneconomical unless the electric current is derived from water or wind power which would otherwise be wasted. (See Electrical Engineering.)

MINE-VENTILATION.

Friction of Air in Underground Passages.—In ventilating a mine or other underground passage the resistance to be overcome is, according to most writers on the subject, proportional to the extent of the frictional surface exposed; that is, to the product lo of the length of the gangway by its perimeter, to the density of the air in circulation, to the square of its average speed, v , and lastly to a coefficient k , whose numerical value varies according to the nature of the sides of the gangway and the irregularities of its course.

The formula for the loss of head, neglecting the variation in density as unimportant, is $p = \frac{ksv^2}{a}$, in which p = loss of pressure in pounds per square foot, s = square feet of rubbing-surface exposed to the air, v the velocity of the air in feet per minute, a the area of the passage in square feet, and k the coefficient of friction. W. Fairley, in *Colliery Engineer*, Oct. and Nov., 1893, gives the following formulæ for all the quantities involved, using the same notation as the above, with these additions: h = horse-power of ventilation; l = length of air-channel; o = perimeter of air-channel; q = quantity of air circulating in cubic feet per minute; u = units of work, in foot-pounds, applied to circulate the air; w = water-gauge in inches. Then,

$$1. \quad a = \frac{ksv^2}{p} = \frac{ksv^2q}{u} = \frac{ksv^3}{pv} = \frac{u}{pv} = \frac{q}{v}.$$

$$2. \quad h = \frac{u}{33,000} = \frac{qp}{33,000} = \frac{5.2qw}{33,000}.$$

$$3. \quad k = \frac{pa}{sv^2} = \frac{u}{sv^3} = \frac{p}{sv^2 + a} = \frac{5.2w}{sv^2 + a}.$$

$$4. \quad l = \frac{s}{o} = \frac{pa}{kv^2o}.$$

$$5. \quad o = \frac{s}{l} = \frac{pa}{kv^2l}.$$

$$6. \quad p = \frac{ksv^2}{a} = \frac{u}{q} = 5.2w = \left(\sqrt[3]{\frac{u}{ks}}\right)^2 \frac{ks}{a} = \frac{ksv^3}{q} = \frac{u}{av}.$$

$$7. \quad pa = ksv^2 = \left(\sqrt[3]{\frac{u}{ks}}\right)^2 ks = \frac{u}{v}; \quad pa^3 = ksq^2.$$

$$8. \quad q = va = \frac{u}{p} = \frac{ksv^3}{p} = \sqrt{\frac{pa}{ks}} a = \sqrt{\frac{u}{ks}} a.$$

$$9. \quad s = \frac{pa}{kv^2} = \frac{u}{kv^3} = \frac{qp}{kv^3} = \frac{vpa}{kv^3} = lo.$$

$$10. \quad u = qp = vpa = \frac{ksv^2q}{a} = ksv^3 = 5.2qw = 33,000h.$$

$$11. \quad v = \frac{u}{pa} = \frac{q}{a} = \sqrt[3]{\frac{u}{ks}} = \sqrt[3]{\frac{qp}{ks}} = \sqrt{\frac{pa}{ks}}.$$

$$12. \quad v^2 = \frac{pa}{ks} = \left(\sqrt[3]{\frac{u}{ks}}\right)^2.$$

$$13. v^3 = \frac{u}{ks} = \frac{qp}{ks} = \frac{vpa}{ks}.$$

$$14. w = \frac{p}{5.2} = \frac{ksv^2}{5.2 a}.$$

To find the quantity of air with a given horse-power and efficiency (e) of engine:

$$q = \frac{h \times 33,000 \times e}{p}.$$

The value of k , the coefficient of friction, as stated, varies according to the nature of the sides of the gangway. Widely divergent values have been given by different authorities (see *Colliery Engineer*, Nov., 1893), the most generally accepted one until recently being probably that of J. J. Atkinson, .0000000217, which is the pressure per square foot in decimals of a pound for each square foot of rubbing-surface and a velocity of one foot per minute. Mr. Fairley, in his "Theory and Practice of Ventilating Coal-mines," gives a value less than half of Atkinson's or .00000001; and recent experiments by D. Murgue show that even this value is high under most conditions. Murgue's results are given in his paper on Experimental Investigations in the Loss of Head of Air-currents in Underground Workings, *Trans. A. I. M. E.*, 1893, vol. xxiii. 63. His coefficients are given in the following table, as determined in twelve experiments:

| | | Coefficient of Loss of Head by Friction. | |
|------------------------------|---------------------------------------|--|----------------|
| | | French. | British. |
| Rock gangways. | Straight, normal section..... | .00092 | .000,000,00486 |
| | Straight, normal section..... | .00094 | .000,000,00497 |
| | Straight, large section..... | .00104 | .000,000,00549 |
| | Straight, normal section..... | .00122 | .000,000,00645 |
| Brick-lined arched gangways. | Straight, normal section..... | .00030 | .000,000,00158 |
| | Straight, normal section..... | .00036 | .000,000,00190 |
| | Continuous curve, normal section..... | .00062 | .000,000,00328 |
| | Sinuuous, intermediate section.... | .00051 | .000,000,00269 |
| Timbered gangways. | Sinuuous, small section..... | .00055 | .000,000,00291 |
| | Straight, normal section..... | .00168 | .000,000,00888 |
| | Straight, normal section..... | .00144 | .000,000,00761 |
| | Slightly sinuous, small section... | .00238 | .000,000,01257 |

The French coefficients which are given by Murgue represent the height of water-gauge in millimeters for each square meter of rubbing-surface and a velocity of one meter per second. To convert them to the British measure of pounds per square foot for each square foot of rubbing-surface and a velocity of one foot per minute they have been multiplied by the factor of conversion, .000005283. For a velocity of 1000 feet per minute, since the loss of head varies as v^2 , move the decimal point in the coefficients six places to the right.

Equivalent Orifice. — The head absorbed by the working-chambers of a mine cannot be computed *a priori*, because the openings, cross-passages, irregular-shaped gob-piles, and daily changes in the size and shape of the chambers present much too complicated a network for accurate analysis. In order to overcome this difficulty Murgue proposed in 1872 the method of *equivalent orifice*. This method consists in substituting for the mine to be considered the equivalent thin-lipped orifice, requiring the same height of head for the discharge of an equal volume of air. The area of this orifice is obtained when the head and the discharge are known, by means of the following formulæ, as given by Fairley:

Let Q = quantity of air in thousands of cubic feet per minute;

w = inches of water-gauge;

A = area in square feet of equivalent orifice.

Then

$$A = \frac{0.37 Q}{\sqrt{w}} = \frac{Q}{2.7 \sqrt{w}}; * \quad Q = \frac{A \times \sqrt{w}}{0.37}; \quad w = 0.1369 \times \left(\frac{Q}{A}\right)^2.$$

* Murgue gives $A = \frac{0.38 Q}{\sqrt{w}}$, and Norris $A = \frac{0.403 Q}{\sqrt{w}}$. See page 644, *ante*.

Motive Column or the Head of Air Due to Differences of Temperature, etc. (Fairley.)

Let M = motive column in feet;
 T = temperature of upcast;
 f = weight of one cubic foot of the flowing air;
 t = temperature of downcast;
 D = depth of downcast.

Then

$$M = D \frac{T-t}{T \times 459} \text{ or } \frac{5.2 \times w}{f}; \quad p = f \times M; \quad w = \frac{f \times M}{5.2} = \frac{p}{5.2}.$$

To find diameter of a round airway to pass the same amount of air as a square airway, the length and power remaining the same:

Let D = diameter of round airway, A = area of square airway; O = perimeter of square airway. Then $D^3 = \sqrt[5]{\frac{A^3 \times 3.1416}{0.7854^3 \times O}}$.

If two fans are employed to ventilate a mine, each of which when worked separately produces a certain quantity, which may be indicated by A and B , then the quantity of air that will pass when the two fans are worked together will be $\sqrt[3]{A^3 + B^3}$. (For mine-ventilating fans, see page 644.)

WATER.

Expansion of Water. — The following table gives the relative volumes of water at different temperatures, compared with its volume at 4° C. according to Kopp, as corrected by Porter.

| Cent. | Fahr. | Volume. | Cent. | Fahr. | Volume. | Cent. | Fahr. | Volume. |
|-------|-------|---------|-------|-------|---------|-------|-------|---------|
| 4° | 39.1° | 1.00000 | 35° | 95° | 1.00586 | 70° | 158° | 1.02241 |
| 5 | 41 | 1.00001 | 40 | 104 | 1.00767 | 75 | 167 | 1.02548 |
| 10 | 50 | 1.00025 | 45 | 113 | 1.00967 | 80 | 176 | 1.02872 |
| 15 | 59 | 1.00083 | 50 | 122 | 1.01186 | 85 | 185 | 1.03213 |
| 20 | 68 | 1.00171 | 55 | 131 | 1.01423 | 90 | 194 | 1.03570 |
| 25 | 77 | 1.00286 | 60 | 140 | 1.01678 | 95 | 203 | 1.03943 |
| 30 | 86 | 1.00425 | 65 | 149 | 1.01951 | 100 | 212 | 1.04332 |

Weight of 1 cu. ft. at 39.1° F. = 62.4245 lb. \div 1.04332 = 59.833, weight of 1 cu. ft. at 212° F.

Weight of Water at Different Temperatures. — The weight of water at maximum density, 39.1°, is generally taken at the figure given by Rankine, 62.425 lbs. per cubic foot. Some authorities give as low as 62.379. The figure 62.5 commonly given is approximate. The highest authoritative figure is 62.428. At 62° F. the figures range from 62.291 to 62.360. The figure 62.355 is generally accepted as the most accurate.

At 32° F. figures given by different writers range from 62.379 to 62.418. Hamilton Smith, Jr. (from Rosetti) gives 62.416.

Weight of Water at Temperatures above 200° F. (Landolt and Börnstein's Tables, 1905.)

| Deg. F. | Lbs. Per Cu. Ft. | Deg. F. | Lbs. Per Cu. Ft. | Deg. F. | Lbs. Per Cu. Ft. | Deg. F. | Lbs. Per Cu. Ft. | Deg. F. | Lbs. Per Cu. Ft. | Deg. F. | Lbs. Per Cu. Ft. |
|---------|------------------|---------|------------------|---------|------------------|---------|------------------|---------|------------------|---------|------------------|
| 200 | 60.12 | 270 | 58.26 | 340 | 55.94 | 410 | 53.0 | 480 | 49.7 | 550 | 45.6 |
| 210 | 59.88 | 280 | 57.96 | 350 | 55.57 | 420 | 52.6 | 490 | 49.2 | 560 | 44.9 |
| 220 | 59.63 | 290 | 57.65 | 360 | 55.18 | 430 | 52.2 | 500 | 48.7 | 570 | 44.1 |
| 230 | 59.37 | 300 | 57.33 | 370 | 54.78 | 440 | 51.7 | 510 | 48.1 | 580 | 43.3 |
| 240 | 59.11 | 310 | 57.00 | 380 | 54.36 | 450 | 51.2 | 520 | 47.6 | 590 | 42.6 |
| 250 | 58.83 | 320 | 56.66 | 390 | 53.94 | 460 | 50.7 | 530 | 47.0 | 600 | 41.8 |
| 260 | 58.55 | 330 | 56.30 | 400 | 53.5 | 470 | 50.2 | 540 | 46.3 | | |

Weight of Water per Cubic Foot, from 32° to 212° F., and heat-units per pound, reckoned above 32° F.: The figures for weight of water in following table, made by interpolating the table given by Clark as calculated from Rankine's formula, with corrections for apparent errors, was published by the author in 1884, *Trans. A. S. M. E.*, vi. 90. The figures for heat units are from Marks and Davis's Steam Tables, 1909.

| Temp., deg. F. | Weight, lbs. per cubic foot. | Heat-units. | Tempera- ture, deg. F. | Weight, lbs. per cubic foot. | Heat-units. | Tempera- ture, deg. F. | Weight, lbs. per cubic foot. | Heat-units. | Tempera- ture, deg. F. | Weight, lbs. per cubic foot. | Heat-units. |
|-------------------|------------------------------------|-------------|------------------------------|------------------------------------|-------------|------------------------------|------------------------------------|-------------|------------------------------|------------------------------------|-------------|
| 32 | 62.42 | 0. | 78 | 62.25 | 46.04 | 123 | 61.68 | 90.90 | 168 | 60.81 | 135.86 |
| 33 | 62.42 | 1.01 | 79 | 62.24 | 47.04 | 124 | 61.67 | 91.90 | 169 | 60.79 | 136.86 |
| 34 | 62.42 | 2.02 | 80 | 62.23 | 48.03 | 125 | 61.65 | 92.90 | 170 | 60.77 | 137.87 |
| 35 | 62.42 | 3.02 | 81 | 62.22 | 49.03 | 126 | 61.63 | 93.90 | 171 | 60.75 | 138.87 |
| 36 | 62.42 | 4.03 | 82 | 62.21 | 50.03 | 127 | 61.61 | 94.89 | 172 | 60.73 | 139.87 |
| 37 | 62.42 | 5.04 | 83 | 62.20 | 51.02 | 128 | 61.60 | 95.89 | 173 | 60.70 | 140.87 |
| 38 | 62.42 | 6.04 | 84 | 62.19 | 52.02 | 129 | 61.58 | 96.89 | 174 | 60.68 | 141.87 |
| 39 | 62.42 | 7.05 | 85 | 62.18 | 53.02 | 130 | 61.56 | 97.89 | 175 | 60.66 | 142.87 |
| 40 | 62.42 | 8.05 | 86 | 62.17 | 54.01 | 131 | 61.54 | 98.89 | 176 | 60.64 | 143.87 |
| 41 | 62.42 | 9.05 | 87 | 62.16 | 55.01 | 132 | 61.52 | 99.88 | 177 | 60.62 | 144.88 |
| 42 | 62.42 | 10.05 | 88 | 62.15 | 56.01 | 133 | 61.51 | 100.88 | 178 | 60.59 | 145.88 |
| 43 | 62.42 | 11.06 | 89 | 62.14 | 57.00 | 134 | 61.49 | 101.88 | 179 | 60.57 | 146.88 |
| 44 | 62.42 | 12.06 | 90 | 62.13 | 58.00 | 135 | 61.47 | 102.88 | 180 | 60.55 | 147.88 |
| 45 | 62.42 | 13.07 | 91 | 62.12 | 59.00 | 136 | 61.45 | 103.88 | 181 | 60.53 | 148.88 |
| 46 | 62.42 | 14.07 | 92 | 62.11 | 60.00 | 137 | 61.43 | 104.87 | 182 | 60.50 | 149.89 |
| 47 | 62.42 | 15.07 | 93 | 62.10 | 60.99 | 138 | 61.41 | 105.87 | 183 | 60.48 | 150.89 |
| 48 | 62.41 | 16.07 | 94 | 62.09 | 61.99 | 139 | 61.39 | 106.87 | 184 | 60.46 | 151.89 |
| 49 | 62.41 | 17.08 | 95 | 62.08 | 62.99 | 140 | 61.37 | 107.87 | 185 | 60.44 | 152.89 |
| 50 | 62.41 | 18.08 | 96 | 62.07 | 63.98 | 141 | 61.36 | 108.87 | 186 | 60.41 | 153.89 |
| 51 | 62.41 | 19.08 | 97 | 62.06 | 64.98 | 142 | 61.34 | 109.87 | 187 | 60.39 | 154.90 |
| 52 | 62.40 | 20.08 | 98 | 62.05 | 65.98 | 143 | 61.32 | 110.87 | 188 | 60.37 | 155.90 |
| 53 | 62.40 | 21.08 | 99 | 62.03 | 66.97 | 144 | 61.30 | 111.87 | 189 | 60.34 | 156.90 |
| 54 | 62.40 | 22.08 | 100 | 62.02 | 67.97 | 145 | 61.28 | 112.86 | 190 | 60.32 | 157.91 |
| 55 | 62.39 | 23.08 | 101 | 62.01 | 68.97 | 146 | 61.26 | 113.86 | 191 | 60.29 | 158.91 |
| 56 | 62.39 | 24.08 | 102 | 62.00 | 69.96 | 147 | 61.24 | 114.86 | 192 | 60.27 | 159.91 |
| 57 | 62.39 | 25.08 | 103 | 61.99 | 70.96 | 148 | 61.22 | 115.86 | 193 | 60.25 | 160.91 |
| 58 | 62.38 | 26.08 | 104 | 61.97 | 71.96 | 149 | 61.20 | 116.86 | 194 | 60.22 | 161.92 |
| 59 | 62.38 | 27.08 | 105 | 61.96 | 72.95 | 150 | 61.18 | 117.86 | 195 | 60.20 | 162.92 |
| 60 | 62.37 | 28.08 | 106 | 61.95 | 73.95 | 151 | 61.16 | 118.86 | 196 | 60.17 | 163.92 |
| 61 | 62.37 | 29.08 | 107 | 61.93 | 74.95 | 152 | 61.14 | 119.86 | 197 | 60.15 | 164.93 |
| 62 | 62.36 | 30.08 | 108 | 61.92 | 75.95 | 153 | 61.12 | 120.86 | 198 | 60.12 | 165.93 |
| 63 | 62.36 | 31.07 | 109 | 61.91 | 76.94 | 154 | 61.10 | 121.86 | 199 | 60.10 | 166.94 |
| 64 | 62.35 | 32.07 | 110 | 61.89 | 77.94 | 155 | 61.08 | 122.86 | 200 | 60.07 | 167.94 |
| 65 | 62.34 | 33.07 | 111 | 61.88 | 78.94 | 156 | 61.06 | 123.86 | 201 | 60.05 | 168.94 |
| 66 | 62.34 | 34.07 | 112 | 61.86 | 79.93 | 157 | 61.04 | 124.86 | 202 | 60.02 | 169.95 |
| 67 | 62.33 | 35.07 | 113 | 61.85 | 80.93 | 158 | 61.02 | 125.86 | 203 | 60.00 | 170.95 |
| 68 | 62.33 | 36.07 | 114 | 61.83 | 81.93 | 159 | 61.00 | 126.86 | 204 | 59.97 | 171.96 |
| 69 | 62.32 | 37.06 | 115 | 61.82 | 82.92 | 160 | 60.98 | 127.86 | 205 | 59.95 | 172.96 |
| 70 | 62.31 | 38.06 | 116 | 61.80 | 83.92 | 161 | 60.96 | 128.86 | 206 | 59.92 | 173.97 |
| 71 | 62.31 | 39.06 | 117 | 61.78 | 84.92 | 162 | 60.94 | 129.86 | 207 | 59.89 | 174.97 |
| 72 | 62.30 | 40.05 | 118 | 61.77 | 85.92 | 163 | 60.92 | 130.86 | 208 | 59.87 | 175.98 |
| 73 | 62.29 | 41.05 | 119 | 61.75 | 86.91 | 164 | 60.90 | 131.86 | 209 | 59.84 | 176.98 |
| 74 | 62.28 | 42.05 | 120 | 61.74 | 87.91 | 165 | 60.87 | 132.86 | 210 | 59.82 | 177.99 |
| 75 | 62.28 | 43.05 | 121 | 61.72 | 88.91 | 166 | 60.85 | 133.86 | 211 | 59.79 | 178.99 |
| 76 | 62.27 | 43.04 | 122 | 61.70 | 89.91 | 167 | 60.83 | 134.86 | 212 | 59.76 | 180.00 |
| 77 | 62.26 | 45.04 | | | | | | | | | |

Later authorities give figures for the weight of water which differ in the second decimal place only from those given above, as follows:

| | | | | | | |
|-----------------------|-------|-------|-------|-------|-------|-------|
| Temp. F. | 40 | 50 | 60 | 70 | 80 | 90 |
| Lbs. per cu. ft. | 62.43 | 62.42 | 62.37 | 62.30 | 62.22 | 62.11 |
| Temp. F. | 100 | 110 | 120 | 130 | 140 | 150 |
| Lbs. per cu. ft. | 62.00 | 61.86 | 61.71 | 61.55 | 61.38 | 61.18 |
| Temp. F. | 160 | 170 | 180 | 190 | 200 | 210 |
| Lbs. per cu. ft. | 61.00 | 60.80 | 60.50 | 60.36 | 60.12 | 59.88 |

Comparison of Heads of Water in Feet with Pressures in Various Units.

| | | |
|--|---|---|
| One foot of water at 39.1° Fahr. | = | 62.425 lbs. on the square foot; |
| “ “ “ | = | 0.4335 lbs. on the square inch; |
| “ “ “ | = | 0.0295 atmosphere; |
| “ “ “ | = | 0.8826 inch of mercury at 30°; |
| “ “ “ | = | 773.3 { feet of air at 32° and atmospheric pressure; |
| One lb. on the square foot, at 39.1° Fahr.. | = | 0.01602 foot of water; |
| One lb. on the square inch, at 39.1° Fahr.. | = | 2.307 feet of water; |
| One atmosphere of 29.922 in. of mercury .. | = | 33.9 feet of water; |
| One inch of mercury at 32.1° | = | 1.133 feet of water; |
| One foot of air at 32°, and 1 atmosphere.. | = | 0.001293 feet of water; |
| One foot of average sea-water | = | 1.026 foot of pure water; |
| One foot of water at 62° F. | = | 62.355 lbs. per sq. foot; |
| One foot of water at 62° F. | = | 0.43302 lb. per sq. inch; |
| One inch of water at 62° F. = 0.5774 ounce | = | 0.036085 lb. per sq. inch; |
| One lb. of water on the square inch at 62° F | = | 2.3094 feet of water. |
| One ounce of water on the square inch at 62° F. | = | 1.732 inches of water. |

Pressure in Pounds per Square Inch for Different Heads of Water.

At 62° F. 1 foot head = 0.433 lb. per square inch, $0.433 \times 144 = 62.352$ lbs. per cubic foot.

| Head, feet. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | | 0.433 | 0.866 | 1.299 | 1.732 | 2.165 | 2.598 | 3.031 | 3.464 | 3.897 |
| 10 | 4.330 | 4.763 | 5.196 | 5.629 | 6.062 | 6.495 | 6.928 | 7.361 | 7.794 | 8.227 |
| 20 | 8.660 | 9.093 | 9.526 | 9.959 | 10.392 | 10.825 | 11.258 | 11.691 | 12.124 | 12.557 |
| 30 | 12.990 | 13.423 | 13.856 | 14.289 | 14.722 | 15.155 | 15.588 | 16.021 | 16.454 | 16.887 |
| 40 | 17.320 | 17.753 | 18.186 | 18.619 | 19.052 | 19.485 | 19.918 | 20.351 | 20.784 | 21.217 |
| 50 | 21.650 | 22.083 | 22.516 | 22.949 | 23.382 | 23.815 | 24.248 | 24.681 | 25.114 | 25.547 |
| 60 | 25.980 | 26.413 | 26.846 | 27.279 | 27.712 | 28.145 | 28.578 | 29.011 | 29.444 | 29.877 |
| 70 | 30.310 | 30.743 | 31.176 | 31.609 | 32.042 | 32.475 | 32.908 | 33.341 | 33.774 | 34.207 |
| 80 | 34.640 | 35.073 | 35.506 | 35.939 | 36.372 | 36.805 | 37.238 | 37.671 | 38.104 | 38.537 |
| 90 | 38.970 | 39.403 | 39.836 | 40.269 | 40.702 | 41.135 | 41.568 | 42.001 | 42.436 | 42.867 |

Head in Feet of Water, Corresponding to Pressures in Pounds per Square Inch.

1 lb. per square inch = 2.30947 feet head, 1 atmosphere = 14.7 lbs. per sq. inch = 33.94 ft. head.

| Pressure. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | | 2.309 | 4.619 | 6.928 | 9.238 | 11.547 | 13.857 | 16.166 | 18.476 | 20.785 |
| 10 | 23.0947 | 25.404 | 27.714 | 30.023 | 32.333 | 34.642 | 36.952 | 39.261 | 41.570 | 43.880 |
| 20 | 46.1894 | 48.499 | 50.808 | 53.118 | 55.427 | 57.737 | 60.046 | 62.356 | 64.665 | 66.975 |
| 30 | 69.2841 | 71.594 | 73.903 | 76.213 | 78.522 | 80.831 | 83.141 | 85.450 | 87.760 | 90.069 |
| 40 | 92.3788 | 94.688 | 96.998 | 99.307 | 101.62 | 103.93 | 106.24 | 108.55 | 110.85 | 113.16 |
| 50 | 115.4735 | 117.78 | 120.09 | 122.40 | 124.71 | 127.02 | 129.33 | 131.64 | 133.95 | 136.26 |
| 60 | 138.5682 | 140.88 | 143.19 | 145.50 | 147.81 | 150.12 | 152.42 | 154.73 | 157.04 | 159.35 |
| 70 | 161.6629 | 163.97 | 166.28 | 168.59 | 170.90 | 173.21 | 175.52 | 177.83 | 180.14 | 182.45 |
| 80 | 184.7576 | 187.07 | 189.38 | 191.69 | 194.00 | 196.31 | 198.61 | 200.92 | 203.23 | 205.54 |
| 90 | 207.8523 | 210.16 | 212.47 | 214.78 | 217.09 | 219.40 | 221.71 | 224.02 | 226.33 | 228.64 |

Pressure of Water due to its Weight. — The pressure of still water in pounds per square inch against the sides of any pipe, channel, or vessel of any shape whatever is due solely to the "head," or height of the level surface of the water above the point at which the pressure is considered, and is equal to 0.43302 lb. per square inch for every foot of head, or 62.355 lbs. per square foot for every foot of head (at 62° F.).

The pressure per square inch is equal in all directions, downwards, upwards, or sideways, and is independent of the shape or size of the containing vessel.

The pressure against a vertical surface, as a retaining-wall, at any point is in direct ratio to the head above that point, increasing from 0 at the level surface to a maximum at the bottom. The total pressure against a vertical strip of a unit's breadth increases as the area of a right-angled triangle whose perpendicular represents the height of the strip and whose base represents the pressure on a unit of surface at the bottom; that is, it increases as the square of the depth. The sum of all the horizontal pressures is represented by the area of the triangle, and the resultant of this sum is equal to this sum exerted at a point one third of the height from the bottom. (The center of gravity of the area of a triangle is one third of its height.)

The horizontal pressure is the same if the surface is inclined instead of vertical.

(For an elaboration of these principles see Trautwine's Pocket-Book, or the chapter on Hydrostatics in any work on Physics. For dams, retaining-walls, etc., see Trautwine.)

The amount of pressure on the interior walls of a pipe has no appreciable effect upon the amount of flow.

Buoyancy. — When a body is immersed in a liquid, whether it float or sink, it is buoyed up by a force equal to the weight of the bulk of the liquid displaced by the body. The weight of a floating body is equal to the weight of the bulk of the liquid that it displaces. The upward pressure or buoyancy of the liquid may be regarded as exerted at the center of gravity of the displaced water, which is called the center of pressure or of buoyancy. A vertical line drawn through it is called the axis of buoyancy or of flotation. In a floating body at rest a line joining the center of gravity and the center of buoyancy is vertical, and is called the axis of equilibrium. When an external force causes the axis of equilibrium to lean, if a vertical line be drawn upward from the center of buoyancy to this axis, the point where it cuts the axis is called the *metacenter*. If the metacenter is above the center of gravity the distance between them is called the metacentric height, and the body is then said to be in stable equilibrium, tending to return to its original position when the external force is removed.

Boiling-point. — Water boils at 212° F. (100° C.) at mean atmospheric pressure at the sea-level, 14.696 lbs. per square inch. The temperature at which water boils at any given pressure is the same as the temperature of saturated steam at the same pressure. For boiling-point of water at other pressure than 14.696 lbs. per square inch, see table of the Properties of Saturated Steam.

The Boiling-point of Water may be Raised. — When water is entirely freed of air, which may be accomplished by freezing or boiling, the cohesion of its atoms is greatly increased, so that its temperature may be raised over 50° above the ordinary boiling-point before ebullition takes place. It was found by Faraday that when such air-free water did boil the rupture of the liquid was like an explosion. When water is surrounded by a film of oil, its boiling temperature may be raised considerably above its normal standard. This has been applied as a theoretical explanation in the instance of boiler explosions.

The freezing-point also may be lowered, if the water is perfectly quiet, to -10° C., or 18° Fahrenheit below the normal freezing-point. (Hamilton Smith, Jr., on Hydraulics, p. 13.)

Freezing-point. — Water freezes at 32° F. at the ordinary atmospheric pressure, and ice melts at the same temperature. In the melting of 1 pound of ice into water at 32° F. about 142 heat-units are absorbed, or become latent; and in freezing 1 lb. of water into ice a like quantity of heat is given out to the surrounding medium.

Sea-water freezes at 27° F. The ice is fresh. (Trautwine.)

Ice and Snow. (From Clark.) — 1 cubic foot of ice at 32° F. weighs 57.50 lbs.; 1 pound of ice at 32° F. has a volume of 0.0174 cu. ft. = 30.067 cu. in.

Relative volume of ice to water at 32° F., 1.0855, the expansion in passing into the solid state being 8.55%. Specific gravity of ice = 0.922, water at 62° F. being 1.

At high pressures the melting-point of ice is lower than 32° F., being at the rate of 0.0133° F. for each additional atmosphere of pressure.

The specific heat of ice is 0.504, that of water being 1.

1 cubic foot of fresh snow, according to humidity of atmosphere: 5 lbs. to 12 lbs. 1 cubic foot of snow moistened and compacted by rain: 15 lbs. to 50 lbs. (Trautwine.)

Specific Heat of Water. (From Davis and Marks's Steam Tables.)

| Deg. F. | Sp. Ht. | Deg. F. | Sp. Ht. | Deg. F. | Sp. Ht. | Deg. F. | Sp. Ht. | Deg. F. | Sp. Ht. | Deg. F. | Sp. Ht. |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 20 | 1.0168 | 120 | 0.9974 | 220 | 1.007 | 320 | 1.035 | 420 | 1.072 | 520 | 1.123 |
| 30 | 1.0098 | 130 | 0.9974 | 230 | 1.009 | 330 | 1.038 | 430 | 1.077 | 530 | 1.128 |
| 40 | 1.0045 | 140 | 0.9986 | 240 | 1.012 | 340 | 1.041 | 440 | 1.082 | 540 | 1.134 |
| 50 | 1.0012 | 150 | 0.9994 | 250 | 1.015 | 350 | 1.045 | 450 | 1.086 | 550 | 1.140 |
| 60 | 0.9990 | 160 | 1.0002 | 260 | 1.018 | 360 | 1.048 | 460 | 1.091 | 560 | 1.146 |
| 70 | 0.9977 | 170 | 1.0010 | 270 | 1.021 | 370 | 1.052 | 470 | 1.096 | 570 | 1.152 |
| 80 | 0.9970 | 180 | 1.0019 | 280 | 1.023 | 380 | 1.056 | 480 | 1.101 | 580 | 1.158 |
| 90 | 0.9967 | 190 | 1.0029 | 290 | 1.026 | 390 | 1.060 | 490 | 1.106 | 590 | 1.165 |
| 100 | 0.9967 | 200 | 1.0039 | 300 | 1.029 | 400 | 1.064 | 500 | 1.112 | 600 | 1.172 |
| 110 | 0.9970 | 210 | 1.0050 | 310 | 1.032 | 410 | 1.068 | 510 | 1.117 | | |

These figures are based on the mean value of the heat unit, that is, $\frac{1}{180}$ of the heat needed to raise 1 lb. of water from 32° to 212°.

Compressibility of Water. — Water is very slightly compressible. Its compressibility is from 0.000040 to 0.000051 for one atmosphere, decreasing with increase of temperature. For each foot of pressure distilled water will be diminished in volume 0.0000015 to 0.0000013. Water is so incompressible that even at a depth of a mile a cubic foot of water will weigh only about half a pound more than at the surface.

THE IMPURITIES OF WATER.

(A. E. Hunt and G. H. Clapp, *Trans. A. I. M. E.*, xvii. 338.)

Commercial analyses are made to determine concerning a given water: (1) its applicability for making steam; (2) its hardness, or the facility with which it will "form a lather" necessary for washing; or (3) its adaptation to other manufacturing purposes.

At the Buffalo meeting of the Chemical Section of the A. A. A. S. it was decided to report all water analyses in parts per thousand, hundred-thousand, and million.

To convert grains per imperial (British) gallon into parts per 100,000, divide by 0.7. To convert parts per 100,000 into grains per U. S. gallon, multiply by 0.5835. To convert grains per U. S. gallon into parts per million multiply by 17.14.

The most common commercial analysis of water is made to determine its fitness for making steam. Water containing more than 5 parts per 100,000 of free sulphuric or nitric acid is liable to cause serious corrosion, not only of the metal of the boiler itself, but of the pipes, cylinders, pistons, and valves with which the steam comes in contact.

The total residue in water used for making steam causes the interior linings of boilers to become coated, and often produces a dangerous hard

scale, which prevents the cooling action of the water from protecting the metal against burning.

Lime and magnesia bicarbonates in water lose their excess of carbonic acid on boiling, and often, especially when the water contains sulphuric acid, produce, with the other solid residues constantly being formed by the evaporation, a very hard and insoluble scale. A larger amount than 100 parts per 100,000 of total solid residue will ordinarily cause troublesome scale, and should condemn the water for use in steam-boilers, unless a better supply cannot be obtained.

The following is a tabulated form of the causes of trouble with water for steam purposes, and the proposed remedies, given by Prof. L. M. Norton.

CAUSES OF INCRUSTATION.

1. Deposition of suspended matter.
2. Deposition of deposited salts from concentration.
3. Deposition of carbonates of lime and magnesia by boiling off carbonic acid, which holds them in solution.
4. Deposition of sulphates of lime, because sulphate of lime is but slightly soluble in cold water, less soluble in hot water, insoluble above 270° F.
5. Deposition of magnesia, because magnesium salts decompose at high temperature.
6. Deposition of lime soap, iron soap, etc., formed by saponification of grease.

MEANS FOR PREVENTING INCRUSTATION.

1. Filtration.
2. Blowing off.
3. Use of internal collecting apparatus or devices for directing the circulation.
4. Heating feed-water.
5. Chemical or other treatment of water in boiler.
6. Introduction of zinc into boiler.
7. Chemical treatment of water outside of boiler.

TABULAR VIEW.

| Troublesome Substance. | Trouble. | Remedy or Palliation. |
|--|--|---|
| Sediment, mud, clay, etc. | Incrustation. | Filtration; blowing off. |
| Readily soluble salts. | " | Blowing off. |
| Bicarbonates of lime, magnesia, } iron. | " | { Heating feed. Addition of caustic soda, lime, or magnesia, etc. |
| Sulphate of lime. | " | { Addition of carb. soda, barium hydrate, etc. |
| Chloride and sulphate of mag- } nesium. | Corrosion. | { Addition of carbonate of soda, etc. |
| Carbonate of soda in large } amounts. | Priming. | { Addition of barium chlo- ride, etc. |
| Acid (in mine waters). | Corrosion. | Alkali. |
| Dissolved carbonic acid and } oxygen. | Corrosion. | { Feed milk of lime to the boiler, to form a thin in- ternal coating. |
| Grease (from condensed water). | Corrosion or } incrustation. | Different cases require dif- ferent remedies. Consult a specialist on the sub- ject. |
| Organic matter (sewage). | { Priming, corrosion, or incrustation. | |

The mineral matters causing the most troublesome boiler-scales are bicarbonates and sulphates of lime and magnesia, oxides of iron and alumina, and silica. The analyses of some of the most common and troublesome boiler-scales are given in the following table:

Analyses of Boiler-scale. (Chandler.)

| | Sulphate of Lime. | Magnesia. | Silica. | Peroxide of Iron. | Water. | Carbonate of Lime. |
|-----------------------------|-------------------|-----------|---------|-------------------|--------|--------------------|
| N. Y. C. & H. R. Ry., No. 1 | 74.07 | 9.19 | 0.65 | 0.08 | 1.14 | 14.78 |
| " " " No. 2 | 71.37 | | 1.76 | | | |
| " " " No. 3 | 62.86 | 18.95 | 2.60 | 0.92 | 1.28 | 12.62 |
| " " " No. 4 | 53.05 | | 4.79 | | | |
| " " " No. 5 | 46.83 | | 5.32 | | | |
| " " " No. 6 | 30.80 | 31.17 | 7.75 | 1.08 | 2.44 | 26.93 |
| " " " No. 7 | 4.95 | 2.61 | 2.07 | 1.03 | 0.63 | 86.25 |
| " " " No. 8 | 0.88 | 2.84 | 0.65 | 0.36 | 0.15 | 93.19 |
| " " " No. 9 | 4.81 | | 2.92 | | | |
| " " " No. 10 | 30.07 | | 8.24 | | | |

Analyses in parts per 100,000 of Water giving Bad Results in Steam-boilers. (A. E. Hunt.)

| | Bicarbonate of Lime deposited on Boiling. | Bicarbonate of Magnesia deposited on Boiling. | Total Lime. | Total Magnesia. | Sulphuric Acid. | Chlorine. | Iron. | Organic Matter. | Alumina. | Chloride of Sodium. |
|--------------------------------|---|---|-------------|-----------------|-----------------|-----------|-------|-----------------|----------|---------------------|
| Coal-mine water..... | 110 | 25 | 119 | 39 | 890 | 590 | 780 | 30 | 640 | |
| Salt-well..... | 151 | 38 | 190 | 48 | 360 | 990 | 38 | 21 | 30 | 1310 |
| Spring..... | 75 | 89 | 95 | 120 | 310 | 21 | 75 | 10 | 80 | 36 |
| Monongahela River..... | 130 | 21 | 161 | 33 | 210 | 38 | 70 | | | |
| " " "..... | 80 | 70 | 94 | 81 | 219 | 210 | 90 | | | |
| " " "..... | 32 | 82 | 61 | 104 | 28 | 190 | 38 | | | |
| Allegheny R., near Oil-works.. | 30 | 50 | 41 | 68 | 890 | 42 | 23 | | | |

Many substances have been added with the idea of causing chemical action which will prevent boiler-scale. As a general rule, these do more harm than good, for a boiler is one of the worst possible places in which to carry on chemical reaction, where it nearly always causes more or less corrosion of the metal, and is liable to cause dangerous explosions.

In cases where water containing large amounts of total solid residue is necessarily used, a heavy petroleum oil, free from tar or wax, which is not acted upon by acids or alkalis, not having sufficient wax in it to cause saponification, and which has a vaporizing-point at nearly 600° F., will give the best results in preventing boiler-scale. Its action is to form a thin greasy film over the boiler linings, protecting them largely from the action of acids in the water and greasing the sediment which is formed, thus preventing the formation of scale and keeping the solid residue from the evaporation of the water in such a plastic suspended condition that it can be easily ejected from the boiler by the process of "blowing off." If the water is not blown off sufficiently often, this sediment forms into a "putty" that will necessitate cleaning the boilers. Any boiler using bad water should be blown off every twelve hours.

Hardness of Water. — The hardness of water, or its opposite quality, indicated by the ease with which it will form a lather with soap, depends almost altogether upon the presence of compounds of lime and magnesia. Almost all soaps consist, chemically, of oleate, stearate, and palmitate of an alkaline base, usually soda and potash. The more lime and magnesia in a sample of water, the more soap a given volume of the water will decompose, so as to give insoluble oleate, palmitate, and stearate of lime and magnesia, and consequently the more soap must be added in order that the necessary quantity of soap may remain in solution to form the lather. The relative hardness of samples of water is generally expressed in terms of the number of standard soap-measures consumed by a gallon of water in yielding a permanent lather.

In Great Britain the standard soap-measure is the quantity required to precipitate one grain of carbonate of lime: in the U. S. it is the quantity required to precipitate one milligramme.

If a water charged with a bicarbonate of lime, magnesia, or iron is boiled, it will, on the excess of the carbonic acid being expelled, deposit a considerable quantity of the lime, magnesia, or iron, and consequently the water will be softer. The hardness of the water after this deposit of lime, after long boiling, is called the *permanent hardness* and the difference between it and the total hardness is called *temporary hardness*.

Lime salts in water react immediately on soap-solutions, precipitating the oleate, palmitate, or stearate of lime at once. Magnesia salts, on the contrary, require some considerable time for reaction. They are, however, more powerful hardeners; one equivalent of magnesia salts consuming as much soap as one and one-half equivalents of lime.

The presence of soda and potash salts softens rather than hardens water. Each grain of carbonate of lime per gallon of water causes an increased expenditure for soap of about 2 ounces per 100 gallons of water. (*Eng'g News*, Jan. 31, 1885.)

Low degrees of hardness (down to 200 parts of calcium carbonate (CaCO_3) per million) are usually determined by means of a standard solution of soap. To 50 c.c. of the water is added alcoholic soap solution from a burette, shaking well after each addition, until a lather is obtained which covers the entire surface of the liquid when the bottle is laid on its side and which lasts five minutes. From the number of c.c. of soap solution used, the hardness of the water may be calculated by the use of Clark's table, given below, in parts of CaCO_3 per million.

| c.c. Soap Sol. | Pts. CaCO_3 . | c.c. Soap Sol. | Pts. CaCO_3 . | c.c. Soap Sol. | Pts. CaCO_3 . | c.c. Soap Sol. | Pts. CaCO_3 . |
|----------------|------------------------|----------------|------------------------|----------------|------------------------|----------------|------------------------|
| 0.7..... | 0 | 4.0..... | 46 | 8.0..... | 103 | 12.0..... | 164 |
| 1.0..... | 5 | 5.0..... | 60 | 9.0..... | 118 | 13.0..... | 180 |
| 2.0..... | 19 | 6.0..... | 74 | 10.0..... | 133 | 14.0..... | 196 |
| 3.0..... | 32 | 7.0..... | 89 | 11.0..... | 148 | 15.0..... | 212 |

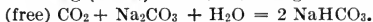
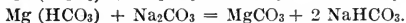
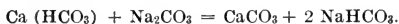
For waters which are harder than 200 parts per million, a solution of soap ten times as strong may be used, the end or determining point being reached when sufficient soap has been added to deaden the harsh sound produced on shaking the bottle containing the water. — A. H. Gill, Engine-Room Chemistry.

Purifying Feed-water for Steam-boilers. (See also Incrustation and Corrosion, p. 897.) — When the water used for steam-boilers contains a large amount of scale-forming material it is usually advisable to purify it before allowing it to enter the boiler rather than to attempt the prevention of scale by the introduction of chemicals into the boiler. Carbonates of lime and magnesia may be removed to a considerable extent by simple heating of the water in an exhaust-steam feed-water heater or, still better, by a live-steam heater. (See circular of the Hoppes Mfg. Co., Springfield, O.) When the water is very bad it is best treated

with chemicals — lime, soda-ash, caustic soda, etc. — in tanks, the precipitates being separated by settling or filtering. For a description of several systems of water purification see a series of articles on the subject by Albert A. Cary in *Eng'g Mag.*, 1897.

Mr. H. E. Smith, chemist of the Chicago, Milwaukee & St. Paul Ry. Co., in a letter to the author, June, 1902, writes as follows concerning the chemical action of soda-ash on the scale-forming substances in boiler waters:

Soda-ash acts on carbonates of lime and magnesia in boiler water in the following manner: — The carbonates are held in solution by means of the carbonic acid gas also present which probably forms bicarbonates of lime and magnesia. Any means which will expel or absorb this carbonic acid will cause the precipitation of the carbonates. One of these means is soda ash (carbonate of soda), which absorbs the gas with the formation of bicarbonate of soda. This method would not be practicable for softening cold water, but it serves in a boiler. The carbonates precipitated in this manner are in flocculent condition instead of semi-crystalline as when thrown down by heat. In practice it is desirable and sufficient to precipitate only a portion of the lime and magnesia in flocculent condition. As to equations, the following represent what occurs: —



Chemical equivalents: — 106 pounds of pure carbonate of soda — equal to about 109 pounds of commercial 58 degree soda-ash — are chemically equivalent to — i.e., react exactly with — the following weights of the substances named: Calcium sulphate, 136 lbs.; magnesium sulphate, 120 lbs.; calcium carbonate, 100 lbs.; magnesium carbonate, 84 lbs.; calcium chloride, 111 lbs.; magnesium chloride, 95 lbs.

Such numbers are simply the molecular weights of the substances reduced to a common basis with regard to the valence of the component atoms.

Important work in this line should not be undertaken by an amateur. "Recipes" have a certain field of usefulness, but will not cover the whole subject. In water purification, as in a problem of mechanical engineering, methods and apparatus must be adapted to the conditions presented. Not only must the character of the raw water be considered but also the conditions of purification and use.

Water-softening Apparatus. (From the Report of the Committee on Water Service, of the Am. Railway Eng'g and Maintenance of Way Assn., *Eng. Rec.*, April 20, 1907). — Between three and four hours is necessary for reaction and precipitation. Water taken from running streams in winter should have at least four hours' time. At least three feet of the bottom of each settling tank should be reserved for the accumulation of the precipitates.

The proper capacities for settling tanks, measured above the space reserved for sludge, can be determined as follows: a = capacity of softener in gallons per hour; b = hours required for reaction and precipitation; c = number of settling tanks (never less than two); x = number of hours required to fill the portion of settling tank above the sludge portion; y = number of hours required to transfer treated water from one settling tank to the storage tank (y should never be greater than x).

Where one pump alternates between filling and emptying settling tanks, $x = y$. Settling capacity in each tank = $2ax = ab \div (c - 1)$.

For plants where the quantity of water supplied to the softener and the capacity of the plant are equal, the settling capacity of each tank is equal to ax . The number of hours required to fill all the settling tanks should equal the number of hours required to fill, precipitate and empty one tank, as expressed by the following equation: $cx = x + b + y$.

$$\text{If } y = x, ax = ab \div (c - 2).$$

$$\text{If } y = 1/2 x, ax = ab \div (c - 1.5).$$

An article on "The Present Status of Water Softening," by G. C. Whipple, in *Cass. Mag.*, Mar., 1907, illustrates several different forms of water-purifying apparatus. A classification of degrees of hardness corresponding to parts of carbonates and sulphates of lime and magnesia per million parts of water is given as follows: Very soft, 0 to 10 parts; soft, 10 to 20; slightly hard, 25 to 50; hard, 50 to 100; very hard, 100 to 200; excessively hard, 200 to 500; mineral water, 500 or more. The same article gives the following figures showing the quantity of chemicals required for the various constituents of hard water. For each part per million of the substances mentioned it is necessary to add the stated number of pounds per million gallons of lime and soda.

| For Each Part per Million of | Pounds per Million Gallons. | |
|--|-----------------------------|-------|
| | Lime. | Soda. |
| Free CO ₂ | 10.62 | 0 |
| Free acid (calculated as H ₂ SO ₄)..... | 4.77 | 9.03 |
| Alkalinity..... | 4.67 | 0 |
| Incrustants..... | 0.00 | 8.85 |
| Magnesium..... | 19.48 | 0 |

The above figures do not take into account any impurities in the chemicals. These have to be considered in actual operation.

An illustrated description of a water-purifying plant on the Chicago & Northwestern Ry. by G. M. Davidson is found in *Eng. News*, April 2, 1903. Two precipitation tanks are used, each 30 ft. diam., 16 ft. high, or 70,000 gallons each. As some water is left with the sludge in the bottom after each emptying, their net capacity is about 60,000 gallons each. The time required for filling, precipitating, settling and transferring the clear water to supply tanks is 12 hours. Once a month the sludge is removed, and it is found to make a good whitewash. Lime and soda-ash, in predetermined quantity, as found by analysis of the water, are used as precipitants. The following table shows the effect of treatment of well water at Council Bluffs, Iowa.

| | Before Treatment. | After Treatment. |
|---|-------------------|------------------|
| Total solid matter, grains per gallon..... | 53.67 | 31.35 |
| Carbonates of lime and magnesia..... | 25.57 | 3.14 |
| Sulphates of lime and magnesia..... | 19.55 | |
| Silica and oxides of iron and aluminum..... | 1.76 | 0.40 |
| Total incrusting solids..... | 46.88 | 3.54 |
| Alkali chlorides..... | 1.21 | 1.27 |
| Alkali sulphates..... | 5.58 | 26.32 |
| Total non-incrusting solids..... | 6.79 | 27.81 |
| Pounds scale-forming matter in 1000 gals..... | 6.69 | 0.51 |

The minimum amount of scaling matter which will justify treatment cannot be stated in terms of analysis alone, but should be stated in terms of pounds incrusting matter held in solution in a day's supply. Besides the scale-forming solids, nearly all water contains more or less free carbonic acid. Sulphuric acid is also found, particularly in streams adjacent to coal mines. Serious trouble from corrosion will result from a small amount of this acid. In treating waters, the acids can be neutralized, and the incrusting matter can be reduced to at least 5 grains per gallon in most cases.

**QUANTITY OF PURE REAGENTS REQUIRED TO REMOVE ONE POUND OF
INCRUSTING OR CORROSIVE MATTER FROM THE WATER.**

| Incrusting or Corrosive Substance Held in Solution. | Amount of Reagent. (Pure.) | Foaming Mat- ter Increased. |
|---|---------------------------------------|--------------------------------|
| Sulphuric acid..... | 0.57 lb. lime plus 1.08 lbs. soda ash | 1.45 lbs. |
| Free carbonic acid..... | 1.27 lbs. lime..... | None |
| Calcium carbonate..... | 0.56 lb. lime..... | None |
| Calcium sulphate..... | 0.78 lb. soda ash..... | 1.04 lbs. |
| Calcium chloride..... | 0.96 lb. soda ash..... | 1.05 lbs. |
| Calcium nitrate..... | 0.65 lb. soda ash..... | 1.04 lbs. |
| Magnesium carbonate... | 1.33 lbs. lime..... | None |
| Magnesium sulphate.... | 0.47 lb. lime plus 0.88 lb. soda ash. | 1.18 lbs. |
| Magnesium chloride..... | 0.59 lb. lime plus 1.11 lbs. soda ash | 1.22 lbs. |
| Magnesium nitrate..... | 0.38 lb. lime plus 0.72 lb. soda ash. | 1.15 lbs. |
| Calcium carbonate..... | 1.71 lbs. barium hydrate..... | None |
| Magnesium carbonate... | 4.05 lbs. barium hydrate..... | None |
| Magnesium sulphate.... | 1.42 lbs. barium hydrate..... | None |
| *Calcium sulphate..... | 1.26 lbs. barium hydrate..... | None |

* In precipitating the calcium sulphate, there would also be precipitated 0.74 lb. of calcium carbonate or 0.31 lb. of magnesium carbonate, the 1.26 lbs. of barium hydrate performing the work of 0.41 lb. of lime and 0.78 lb. of soda-ash, or for reacting on either magnesium or calcium sulphate, 1 lb. of barium hydrate performs the work of 0.33 lb. of lime plus 0.62 lb. of soda-ash, and the lime treatment can be correspondingly reduced.

Barium hydrate has no advantage over lime as a reagent to precipitate the carbonates of lime and magnesia and should not be considered except in connection with the treating of water containing calcium sulphate.

HYDRAULICS—FLOW OF WATER.

Formulae for Discharge of Water through Orifices and Weirs.—For rectangular or circular orifices, with the head measured from center of the orifice to the surface of the still water in the feeding reservoir:

$$Q = C \sqrt{2gH} \times a. \quad (1)$$

For weirs with no allowance for increased head due to velocity of approach:

$$Q = C^{2/3} \sqrt{2gH} \times LH. \quad (2)$$

For rectangular and circular or other shaped vertical or inclined orifices; formula based on the proposition that each successive horizontal layer of water passing through the orifice has a velocity due to its respective head:

$$Q = c L^{2/3} \sqrt{2g} \times (\sqrt{H_b^3} - \sqrt{H_t^3}). \quad (3)$$

For rectangular vertical weirs:

$$Q = c^{2/3} \sqrt{2gH} \times Lh. \quad (4)$$

Q = quantity of water discharged in cubic feet per second; C = approximate coefficient for formulas (1) and (2); c = correct coefficient for (3) and (4).

Values of the coefficients c and C are given below.

$g = 32.16$; $\sqrt{2g} = 8.02$; H = head in feet measured from center of orifice to level of still water; H_b = head measured from bottom of orifice; H_t = head measured from top of orifice; $h = H$, corrected for velocity of approach, $V_a = H + 1.33 V_a^2/2g$ for weirs with no end contraction, and $H + 1.4 V_a^2/2g$ for weirs with end contraction; a = area in square feet; L = length in feet.

HYDRAULIC FORMULÆ. — FLOW OF WATER IN OPEN AND CLOSED CHANNELS.

Flow of Water in Pipes. — The quantity of water discharged through a pipe depends on the "head"; that is, the vertical distance between the level surface of still water in the chamber at the entrance end of the pipe and the level of the center of the discharge end of the pipe; also upon the length of the pipe, upon the character of its interior surface as to smoothness, and upon the number and sharpness of the bends; but it is independent of the position of the pipe, as horizontal, or inclined upwards or downwards.

The head, instead of being an actual distance between levels, may be caused by pressure, as by a pump, in which case the head is calculated as a vertical distance corresponding to the pressure, 1 lb. per sq. in. = 2.309 ft. head, or 1 ft. head = 0.433 lb. per sq. in.

The total head operating to cause flow is divided into three parts: 1. The *velocity-head*, which is the height through which a body must fall *in vacuo* to acquire the velocity with which the water flows into the pipe = $v^2 \div 2g$, in which v is the velocity in ft. per sec. and $2g = 64.32$; 2. the *entry-head*, that required to overcome the resistance to entrance to the pipe. With sharp-edged entrance the entry-head = about $\frac{1}{2}$ the velocity-head; with smooth rounded entrance the entry-head is inappreciable; 3. the *friction-head*, due to the frictional resistance to flow within the pipe.

In ordinary cases of pipes of considerable length the sum of the entry and velocity heads required scarcely exceeds 1 foot. In the case of long pipes with low heads the sum of the velocity and entry heads is generally so small that it may be neglected.

General Formula for Flow of Water in Pipes or Conduits.

Mean velocity in ft. per sec. = $c \sqrt{\text{mean hydraulic radius} \times \text{slope}}$

Do. for pipes running full = $c \sqrt{\frac{\text{diameter}}{4} \times \text{slope}},$

in which c is a coefficient determined by experiment. (See pages following.)

The mean hydraulic radius = $\frac{\text{area of wet cross-section}}{\text{wet perimeter}}$

In pipes running full, or exactly half full, and in semicircular open channels running full it is equal to $\frac{1}{4}$ diameter.

The slope = the head (or pressure expressed as a head, in feet)

\div length of pipe measured in a straight line from end to end.

In open channels the slope is the actual slope of the surface, or its fall per unit of length, or the sine of the angle of the slope with the horizon.

Chezy's Formula: $v = c \sqrt{r} \sqrt{s} = c \sqrt{rs}$; r = mean hydraulic radius, s = slope = head \div length, v = velocity in feet per second, all dimensions in feet.

Quantity of Water Discharged. — If Q = discharge in cubic feet per second and a = area of channel, $Q = av = ac \sqrt{rs}$.

$a \sqrt{r}$ is approximately proportional to the discharge. It is a maximum at 308° of the circumference, corresponding to $\frac{19}{20}$ of the diameter, and the flow of a conduit $\frac{19}{20}$ full is about 5 per cent greater than that of one completely filled.

Values of the Coefficient c . (Chiefly condensed from P. J. Flynn on Flow of Water.) — Almost all the old hydraulic formulæ for finding the

mean velocity in open and closed channels have constant coefficients, and are therefore correct for only a small range of channels. They have often been found to give incorrect results with disastrous effects. Ganguillet and Kutter thoroughly investigated the American, French, and other experiments, and they gave as the result of their labors the formula now generally known as Kutter's formula. There are so many varying conditions affecting the flow of water, that all hydraulic formulæ are only approximations to the correct result.

When the surface-slope measurement is good, Kutter's formula will give results seldom exceeding $7\frac{1}{2}\%$ error, provided the rugosity coefficient of the formula is known for the site. For small open channels Darcy's and Bazin's formulæ, and for cast-iron pipes Darcy's formulæ, are generally accepted as being approximately correct.

Table giving Fall in Feet per Mile, the Distance on Slope corresponding to a Fall of 1 Ft., and also the Values of s and \sqrt{s} for Use in the Formula $v = C\sqrt{rs}$.

$s = H \div L$ = sine of angle of slope = fall of water-surface (H), in any distance (L), divided by that distance.

| Fall in Feet per Mi. | Slope, 1 Foot in | Sine of Slope, s . | \sqrt{s} . | Fall in Feet per Mi. | Slope, 1 Foot in | Sine of Slope, s . | \sqrt{s} . |
|----------------------------|------------------------|----------------------------|--------------|----------------------------|------------------------|----------------------------|--------------|
| 0.25 | 21120 | 0.0000473 | 0.006881 | 17 | 310.6 | 0.0032197 | 0.056742 |
| .30 | 17600 | .0000568 | .007538 | 18 | 293.3 | .0034091 | .058388 |
| .40 | 13200 | .0000758 | .008704 | 19 | 277.9 | .0035985 | .059988 |
| .50 | 10560 | .0000947 | .009731 | 20 | 264 | .0037879 | .061546 |
| .60 | 8800 | .0001136 | .010660 | 22 | 240 | .0041667 | .064549 |
| .702 | 7520 | .0001330 | .011532 | 24 | 220 | .0045455 | .067419 |
| .805 | 6560 | .0001524 | .012347 | 26 | 203.1 | .0049242 | .070173 |
| .904 | 5840 | .0001712 | .013085 | 28 | 188.6 | .0053030 | .072822 |
| 1 | 5280 | .0001894 | .013762 | 30 | 176 | .0056818 | .075378 |
| 1.25 | 4224 | .0002367 | .015386 | 35.20 | 150 | .0066667 | .081650 |
| 1.5 | 3520 | .0002841 | .016854 | 40 | 132 | .0075758 | .087039 |
| 1.75 | 3017 | .0003314 | .018205 | 44 | 120 | .0083333 | .091287 |
| 2 | 2640 | .0003788 | .019463 | 48 | 110 | .0090909 | .095346 |
| 2.25 | 2347 | .0004261 | .020641 | 52.8 | 100 | .010 | .1 |
| 2.5 | 2112 | .0004735 | .021760 | 60 | 88 | .0113636 | .1066 |
| 2.75 | 1920 | .0005208 | .022822 | 66 | 80 | .0125 | .111803 |
| 3 | 1760 | .0005682 | .023837 | 70.4 | 75 | .0133333 | .115470 |
| 3.25 | 1625 | .0006154 | .024807 | 80 | 66 | .0151515 | .123091 |
| 3.5 | 1508 | .0006631 | .025751 | 88 | 60 | .0166667 | .1291 |
| 3.75 | 1408 | .0007102 | .026650 | 96 | 55 | .0181818 | .134839 |
| 4 | 1320 | .0007576 | .027524 | 105.6 | 50 | .02 | .141421 |
| 5 | 1056 | .0009470 | .030773 | 120 | 44 | .0227273 | .150756 |
| 6 | 880 | .0011364 | .03371 | 132 | 40 | .025 | .158114 |
| 7 | 754.3 | .0013257 | .036416 | 160 | 33 | .0303030 | .174077 |
| 8 | 660 | .0015152 | .038925 | 220 | 24 | .0416667 | .204124 |
| 9 | 586.6 | .0017044 | .041286 | 264 | 20 | .05 | .223607 |
| 10 | 528 | .0018939 | .043519 | 330 | 16 | .0625 | .25 |
| 11 | 443.6 | .0020833 | .045643 | 340 | 12 | .0833333 | .288675 |
| 12 | 440 | .0022727 | .047673 | 528 | 10 | .1 | .316228 |
| 13 | 406.1 | .0024621 | .04962 | 660 | 8 | .125 | .353553 |
| 14 | 377.1 | .0026515 | .051493 | 880 | 6 | .1666667 | .408248 |
| 15 | 352 | .0028409 | .0533 | 1056 | 5 | .2 | .447214 |
| 16 | 330 | .0030303 | .055048 | 1320 | 4 | .25 | .5 |

Values of \sqrt{r} for Circular Pipes, Sewers, and Conduits of Different Diameters.

r = mean hydraulic depth = $\frac{\text{area}}{\text{perimeter}} = \frac{1}{4}$ diam. for circular pipes running full or exactly half full.

| Diam., ft. in. | \sqrt{r} in Feet. | Diam., ft. in. | \sqrt{r} in Feet. | Diam., ft. in. | \sqrt{r} in Feet. | Diam., ft. in. | \sqrt{r} in Feet. |
|-------------------|------------------------|-------------------|------------------------|-------------------|------------------------|-------------------|------------------------|
| 3/8 | 0.088 | 2 | 0.707 | 4 6 | 1.061 | 9 | 1.500 |
| 1/2 | .102 | 2 1 | .722 | 4 7 | 1.070 | 9 3 | 1.521 |
| 3/4 | .125 | 2 2 | .736 | 4 8 | 1.080 | 9 6 | 1.541 |
| 1 | .144 | 2 3 | .750 | 4 9 | 1.089 | 9 9 | 1.561 |
| 1 1/4 | .161 | 2 4 | .764 | 4 10 | 1.099 | 10 | 1.581 |
| 1 1/2 | .177 | 2 5 | .777 | 4 11 | 1.109 | 10 3 | 1.601 |
| 1 3/4 | .191 | 2 6 | .790 | 5 | 1.118 | 10 6 | 1.620 |
| 2 | .204 | 2 7 | .804 | 5 1 | 1.127 | 10 9 | 1.639 |
| 2 1/2 | .228 | 2 8 | .817 | 5 2 | 1.137 | 11 | 1.658 |
| 3 | .251 | 2 9 | .829 | 5 3 | 1.146 | 11 3 | 1.677 |
| 4 | .290 | 2 10 | .842 | 5 4 | 1.155 | 11 6 | 1.696 |
| 5 | .323 | 2 11 | .854 | 5 5 | 1.164 | 11 9 | 1.714 |
| 6 | .354 | 3 | .866 | 5 6 | 1.173 | 12 | 1.732 |
| 7 | .382 | 3 1 | .878 | 5 7 | 1.181 | 12 3 | 1.750 |
| 8 | .408 | 3 2 | .890 | 5 8 | 1.190 | 12 6 | 1.768 |
| 9 | .433 | 3 3 | .901 | 5 9 | 1.199 | 12 9 | 1.785 |
| 10 | .456 | 3 4 | .913 | 5 10 | 1.208 | 13 | 1.803 |
| 11 | .479 | 3 5 | .924 | 5 11 | 1.216 | 13 3 | 1.820 |
| 1 | .500 | 3 6 | .935 | 6 | 1.225 | 13 6 | 1.837 |
| 1 1 | .520 | 3 7 | .946 | 6 3 | 1.250 | 14 | 1.871 |
| 1 2 | .540 | 3 8 | .957 | 6 6 | 1.275 | 14 6 | 1.904 |
| 1 3 | .559 | 3 9 | .968 | 6 9 | 1.299 | 15 | 1.936 |
| 1 4 | .577 | 3 10 | .979 | 7 | 1.323 | 15 6 | 1.968 |
| 1 5 | .595 | 3 11 | .990 | 7 3 | 1.346 | 16 | 2. |
| 1 6 | .612 | 4 | 1. | 7 6 | 1.369 | 16 6 | 2.031 |
| 1 7 | .629 | 4 1 | 1.010 | 7 9 | 1.392 | 17 | 2.061 |
| 1 8 | .646 | 4 2 | 1.021 | 8 | 1.414 | 17 6 | 2.091 |
| 1 9 | .661 | 4 3 | 1.031 | 8 3 | 1.436 | 18 | 2.121 |
| 1 10 | .677 | 4 4 | 1.041 | 8 6 | 1.458 | 19 | 2.180 |
| 1 11 | .692 | 4 5 | 1.051 | 8 9 | 1.479 | 20 | 2.236 |

Kutter's Formula for measures in feet is

$$v = \left\{ \frac{\frac{1.811}{n} + 41.6 + \frac{0.00281}{s}}{1 + \left(41.6 + \frac{0.00281}{s} \right) \times \frac{n}{\sqrt{r}}} \right\} \times \sqrt{rs},$$

in which v = mean velocity in feet per second; $r = \frac{a}{p}$ = hydraulic mean depth in feet = area of cross-section in square feet divided by wetted perimeter in lineal feet; s = fall of water-surface (h) in any distance (l) divided by that distance, $= \frac{h}{l}$, = sine of slope; n = the coefficient of rugosity, depending on the nature of the lining or surface of the channel. If we let the first term of the right-hand side of the equation equal c , we have Chezy's formula, $v = c \sqrt{rs} = c \times \sqrt{r} \times \sqrt{s}$.

Values of n in Kutter's Formula. — The accuracy of Kutter's formula depends, in a great measure, on the proper selection of the coefficient

of roughness n . Experience is required in order to give the right value to this coefficient, and to this end great assistance can be obtained, in making this selection, by consulting and comparing the results obtained from experiments on the flow of water already made in different channels.

In some cases it would be well to provide for the contingency of future deterioration of channel, by selecting a high value of n , as, for instance, where a dense growth of weeds is likely to occur in small channels, and also where channels are likely not to be kept in a state of good repair.

The following table, giving the value of n for different materials, is compiled from Kutter, Jackson, and Hering, and this value of n applies also in each instance to the surfaces of other materials equally rough.

VALUE OF n IN KUTTER'S FORMULA FOR DIFFERENT CHANNELS.

$n = .009$, well-planed timber, in perfect order and alignment; otherwise, perhaps $.01$ would be suitable.

$n = .010$, plaster in pure cement; planed timber; glazed, coated, or enameled stoneware and iron pipes; glazed surfaces of every sort in perfect order.

$n = .011$, plaster in cement with one-third sand, in good condition; also for iron, cement, and terra-cotta pipes, well joined, and in best order.

$n = .012$, unplanned timber, when perfectly continuous on the inside; flumes.

$n = .013$, ashlar and well-laid brickwork; ordinary metal; earthen and stoneware pipe in good condition, but not new; cement and terra-cotta pipe not well jointed nor in perfect order, plaster and planed wood in imperfect or inferior condition; and, generally, the materials mentioned with $n = .010$, when in imperfect or inferior condition.

$n = .015$, second class or rough-faced brickwork; well-dressed stonework; foul and slightly tuberculated iron; cement and terra-cotta pipes, with imperfect joints and in bad order; and canvas lining on wooden frames.

$n = .017$, brickwork, ashlar, and stoneware in an inferior condition; tuberculated iron pipes; rubble in cement or plaster in good order; fine gravel, well rammed, $\frac{1}{3}$ to $\frac{2}{3}$ inch diameter; and, generally, the materials mentioned with $n = .013$ when in bad order and condition.

$n = .020$, rubble in cement in an inferior condition; coarse rubble, rough set in a normal condition; coarse rubble set dry; ruined brickwork and masonry; coarse gravel well rammed, from 1 to $1\frac{1}{3}$ inch diameter; canals with beds and banks of very firm, regular gravel, carefully trimmed and rammed in defective places; rough rubble with bed partially covered with silt and mud; rectangular wooden troughs with battens on the inside two inches apart; trimmed earth in perfect order.

$n = .0225$, canals in earth above the average in order and regimen.

$n = .025$, canals and rivers in earth of tolerably uniform cross-section; slope and direction, in moderately good order and regimen, and free from stones and weeds.

$n = .0275$, canals and rivers in earth below the average in order and regimen.

$n = .030$, canals and rivers in earth in rather bad order and regimen, having stones and weeds occasionally, and obstructed by detritus.

$n = .035$, suitable for rivers and canals with earthen beds in bad order and regimen, and having stones and weeds in great quantities.

$n = .05$, torrents encumbered with detritus.

Kutter's formula has the advantage of being easily adapted to a change in the surface of the pipe exposed to the flow of water, by a change in the value of n . For cast-iron pipes it is usual to use $n = .013$ to provide for the future deterioration of the surface.

Reducing Kutter's formula to the form $v = c \times \sqrt{r} \times \sqrt{s}$, and taking n , the coefficient of roughness in the formula, $= .011$, $.012$, and $.013$, and $s = .001$, we have the following values of the coefficient c of different diameters of conduit.

Values of c in Formula $v = c \times \sqrt{r} \times \sqrt{s}$ for Metal Pipes and Moderately Smooth Conduits Generally.

By KUTTER'S FORMULA. ($s = .001$ or greater.)

| Diameter. | | $n = .011$ | $n = .012$ | $n = .013$ | Diameter. | | $n = .011$ | $n = .012$ | $n = .013$ |
|-----------|-----|------------|------------|------------|-----------|-------|------------|------------|------------|
| ft. | in. | $c =$ | $c =$ | $c =$ | ft. | in. | $c =$ | $c =$ | $c =$ |
| 0 | 1 | 47.1 | | | 7 | | 152.7 | 139.2 | 127.9 |
| | 2 | 61.5 | | | 8 | | 155.4 | 141.9 | 130.4 |
| | 4 | 77.4 | | | 9 | | 157.7 | 144.1 | 132.7 |
| | 6 | 87.4 | 77.5 | 69.5 | 10 | | 159.7 | 146 | 134.5 |
| 1 | | 105.7 | 94.6 | 85.3 | 11 | | 161.5 | 147.8 | 136.2 |
| 1 | 6 | 116.1 | 104.3 | 94.4 | 12 | | 163 | 149.3 | 137.7 |
| 2 | | 123.6 | 111.3 | 101.1 | 14 | | 165.8 | 152 | 140.4 |
| 3 | | 133.6 | 120.8 | 110.1 | 16 | | 168 | 154.2 | 142.1 |
| 4 | | 140.4 | 127.4 | 116.5 | 18 | | 169.9 | 156.1 | 144.4 |
| 5 | | 145.4 | 132.3 | 121.1 | 20 | | 171.6 | 157.7 | 146 |
| 6 | | 149.4 | 136.1 | 124.8 | | | | | |

For circular pipes the hydraulic mean depth r equals $1/4$ of the diameter.

According to Kutter's formula the value of c , the coefficient of discharge, is the same for all slopes greater than 1 in 1000; that is, within these limits c is constant. We further find that up to a slope of 1 in 2640 the value of c is, for all practical purposes, constant, and even up to a slope of 1 in 5000 the difference in the value of c is very little. This is exemplified in the following:

Value of c for Different Values of \sqrt{r} and s in Kutter's Formula, with $n = .013$.

| \sqrt{r} | Slope. 1 in 1000 | Slope. 1 in 2500 | Slope. 1 in 3333.3 | Slope. 1 in 5000 | Slope. 1 in 10,000 |
|------------|---------------------|---------------------|-----------------------|---------------------|-----------------------|
| 0.6 | 93.6 | 91.5 | 90.4 | 88.4 | 83.3 |
| 1 | 116.5 | 115.2 | 114.4 | 113.2 | 109.7 |
| 2 | 142.6 | 142.8 | 143.0 | 143.1 | 143.8 |

The reliability of the values of the coefficient of Kutter's formula for pipes of less than 6 in. diameter is considered doubtful. (See note under table on page 704.)

Values of c for Earthen Channels, by Kutter's Formula, for Use in Formula $v = c \sqrt{rs}$.

| | Coefficient of Roughness, $n = .0225$. | | | | | Coefficient of Roughness, $n = .035$. | | | | |
|-------------|--|------|------|------|-------|---|------|------|------|------|
| | \sqrt{r} in feet. | | | | | \sqrt{r} in feet. | | | | |
| | 0.4 | 1.0 | 1.8 | 2.5 | 4.0 | 0.4 | 1.0 | 1.8 | 2.5 | 4.0 |
| Slope, 1 in | c | c | c | c | c | c | c | c | c | c |
| 1,000 | 35.7 | 62.5 | 80.3 | 89.2 | 99.9 | 19.7 | 37.6 | 51.6 | 59.3 | 69.2 |
| 1,250 | 35.5 | 62.3 | 80.3 | 89.3 | 100.2 | 19.6 | 37.6 | 51.6 | 59.4 | 69.4 |
| 1,667 | 35.2 | 62.1 | 80.3 | 89.5 | 100.6 | 19.4 | 37.4 | 51.6 | 59.5 | 69.8 |
| 2,500 | 34.6 | 61.7 | 80.3 | 89.8 | 101.4 | 19.1 | 37.1 | 51.6 | 59.7 | 70.4 |
| 3,333 | 34. | 61.2 | 80.3 | 90.1 | 102.2 | 18.8 | 36.9 | 51.6 | 59.9 | 71.0 |
| 5,000 | 33. | 60.5 | 80.3 | 90.7 | 103.7 | 18.3 | 36.4 | 51.6 | 60.4 | 72.2 |
| 7,500 | 31.6 | 59.4 | 80.3 | 91.5 | 106.0 | 17.6 | 35.8 | 51.6 | 60.9 | 73.9 |
| 10,000 | 30.5 | 58.5 | 80.3 | 92.3 | 107.9 | 17.1 | 35.3 | 51.6 | 60.5 | 75.4 |
| 15,840 | 28.5 | 56.7 | 80.2 | 93.9 | 112.2 | 16.2 | 34.3 | 51.6 | 62.5 | 78.6 |
| 20,000 | 27.4 | 55.7 | 80.2 | 94.8 | 115.0 | 15.6 | 33.8 | 51.5 | 63.1 | 80.6 |

Darcy's Formula for clean iron pipes under pressure is

$$v = \left\{ \frac{rs}{0.00007726 + \frac{0.00000162}{r}} \right\}^{1/2}$$

Darcy's formula, as given by J. B. Francis, C. E., for old cast-iron pipe, lined with deposit and under pressure, is

$$v = \left(\frac{144 d^2 s}{0.00082 (12 d + 1)} \right)^{1/2}.$$

In which d = diameter in feet.

For Pipes Less than 5 inches in Diameter, coefficients (c) in the formula $v = c \sqrt{rs}$, from the formula of Darcy, Kutter, and Fanning.

| Diam. in inches. | Darcy, for Clean Pipes. | Kutter, for $n = .011$ $s = .001$ | Fanning, for Clean Iron Pipes. | Diam. in inches. | Darcy, for Clean Pipes. | Kutter, for $n = .011$ $s = .001$ | Fanning, for Clean Iron Pipes. |
|------------------------|-------------------------------|--|---|------------------------|-------------------------------|--|---|
| 3/8 | 59.4 | 32. | | 1 3/4 | 90.7 | 58.8 | 92.5 |
| 1/2 | 65.7 | 36.1 | | 2 | 92.9 | 61.5 | 94.8 |
| 3/4 | 74.5 | 42.6 | | 2 1/2 | 96.1 | 66. | |
| 1 | 80.4 | 47.4 | 80.4 | 3 | 98.5 | 70.1 | 96.6 |
| 1 1/4 | 84.8 | 51.9 | | 4 | 101.7 | 77.4 | 103.4 |
| 1 1/2 | 88.1 | 55.4 | 88. | 5 | 103.8 | 82.9 | |

Mr. Flynn, in giving the above table, says that the facts show that the coefficients diminish from a diameter of 5 inches to smaller diameters, and it is a safer plan to adopt coefficients varying with the diameter than a constant coefficient. No opinion is advanced as to what coefficients should be used with Kutter's formula for small diameters.

VELOCITY OF WATER IN OPEN CHANNELS.

Irrigation Canals. — The minimum mean velocity required to prevent the deposit of silt or the growth of aquatic plants is in Northern India taken at 1 1/2 feet per second. It is stated that in America a higher velocity is required for this purpose, and it varies from 2 to 3 1/2 feet per second. The maximum allowable velocity will vary with the nature of the soil of the bed. A sandy bed will be disturbed if the velocity exceeds 3 feet per second. Good loam with not too much sand will bear a velocity of 4 feet per second. The Cavour Canal in Italy, over a gravel bed, has a velocity of about 5 per second. (Flynn's "Irrigation Canals.")

Mean Surface and Bottom Velocities. — According to the formula of Bazin,

$$v = v_{\max} - 25.4 \sqrt{rs}; \quad v = v_b + 10.87 \sqrt{rs}.$$

$\therefore v_b = v - 10.87 \sqrt{rs}$, in which v = mean velocity in feet per second, v_{\max} = maximum surface velocity in feet per second, v_b = bottom velocity in feet per second, r = hydraulic mean depth in feet = area of cross-section in square feet divided by wetted perimeter in feet, s = sine of slope.

The least velocity, or that of the particles in contact with the bed, is almost as much less than the mean velocity as the greatest velocity is greater than the mean.

Rankine states that in ordinary cases the velocities may be taken as bearing to each other nearly the proportions of 3, 4, and 5. In very slow currents they are nearly as 2, 3, and 4.

Safe Bottom and Mean Velocities. — Ganguillet & Kutter give the following table of safe bottom and mean velocity in channels, calculated from the formula $v = v_b + 10.87 \sqrt{rs}$:

| Material of Channel. | Safe Bottom Velocity v_b , in feet per second. | Mean Velocity v , in feet per second. |
|------------------------------|--|---|
| Soft brown earth..... | 0.249 | 0.328 |
| Soft loam..... | 0.499 | 0.656 |
| Sand..... | 1.000 | 1.312 |
| Gravel..... | 1.998 | 2.625 |
| Pebbles..... | 2.999 | 3.938 |
| Broken stone, flint..... | 4.003 | 5.579 |
| Conglomerate, soft slate.... | 4.988 | 6.564 |
| Stratified rock..... | 6.006 | 8.204 |
| Hard rock..... | 10.009 | 13.127 |

Ganguillet & Kutter state that they are unable for want of observations to judge how far these figures are trustworthy. They consider them to be rather disproportionately small than too large, and therefore recommend them more confidently.

Water flowing at a high velocity and carrying large quantities of silt is very destructive to channels, even when constructed of the best masonry.

Resistance of Soils to Erosion by Water. — W. A. Burr, *Eng'g News*, Feb. 8, 1894, gives a diagram showing the resistance of various soils to erosion by flowing water.

Experiments show that a velocity greater than 1.1 feet per second will erode sand, while pure clay will stand a velocity of 7.35 feet per second. The greater the proportion of clay carried by any soil, the higher the permissible velocity. Mr. Burr states that experiments have shown that the line describing the power of soils to resist erosion is parabolic. From his diagram the following figures are selected as representing different classes of soils:

| | | |
|---|------|------------------|
| Pure sand resists erosion by flow of..... | 1.1 | feet per second. |
| Sandy soil, 15% clay..... | 1.2 | " " |
| Sandy loam, 40% clay..... | 1.8 | " " |
| Loamy soil, 65% clay..... | 3.0 | " " |
| Clay loam, 85% clay..... | 4.8 | " " |
| Agricultural clay, 95% clay..... | 6.2 | " " |
| Clay..... | 7.35 | " " |

Abrading and Transporting Power of Water. — Prof. J. LeConte, in his "Elements of Geology," states:

The erosive power of water, or its power of overcoming cohesion, varies as the square of the velocity of the current.

The transporting power of a current varies as the sixth power of the velocity. * * * If the velocity therefore be increased ten times, the transporting power is increased 1,000,000 times. A current running three feet per second, or about two miles per hour, will bear fragments of stone of the size of a hen's egg, or about three ounces weight. A current of ten miles an hour will bear fragments of one and a half tons, and a torrent of twenty miles an hour will carry fragments of 100 tons.

The transporting power of water must not be confounded with its erosive power. The resistance to be overcome in the one case is weight, in the other, cohesion; the latter varies as the square: the former as the sixth power of the velocity.

In many cases of removal of slightly cohering material, the resistance is a mixture of these two resistances, and the power of removing material will vary at some rate between v^2 and v^6 .

Baldwin Latham has found that in order to prevent deposits of sewage silt in small sewers or drains, such as those from 6 inches to 9 inches diameter, a mean velocity of not less than 3 feet per second should be produced. Sewers from 12 to 24 inches diameter should have a velocity

of not less than $2\frac{1}{2}$ feet per second, and in sewers of larger dimensions in no case should the velocity be less than 2 feet per second.

The specific gravity of the materials has a marked effect upon the mean velocities necessary to move them. T. E. Blackwell found that coal of a sp. gr. of 1.26 was moved by a current of from 1.25 to 1.50 ft. per second, while stones of a sp. gr. of 2.32 to 3.00 required a velocity of 2.5 to 2.75 ft. per second.

Chailly gives the following formula for finding the velocity required to move rounded stones or shingle:

$$v = 5.67 \sqrt{ag},$$

in which v = velocity of water in feet per second, a = average diameter in feet of the body to be moved, g = its specific gravity.

Geo. Y. Wisner, *Eng'g News*, Jan. 10, 1895, doubts the general accuracy of statements made by many authorities concerning the rate of flow of a current and the size of particles which different velocities will move. He says:

The scouring action of any river, for any given rate of current, must be an inverse function of the depth. The fact that some engineer has found that a given velocity of current on some stream of unknown depth will move sand or gravel has no bearing whatever on what may be expected of currents of the same velocity in streams of greater depths. In channels 3 to 5 ft. deep a mean velocity of 3 to 5 ft. per second may produce rapid scouring, while in depths of 18 ft. and upwards current velocities of 6 to 8 ft. per second often have no effect whatever on the channel bed.

Grade of Sewers. — The following empirical formula is given in Baumeister's "Cleaning and Sewerage of Cities," for the minimum grade for a sewer of clear diameter equal to d inches, and either circular or oval in section:

$$\text{Minimum grade, in per cent,} = \frac{100}{5d + 50}.$$

As the lowest limit of grades which can be flushed, 0.1 to 0.2 per cent may be assumed for sewers which are sometimes dry, while 0.3 per cent is allowable for the trunk sewers in large cities. The sewers should run dry as rarely as possible.

FLOW OF WATER — EXPERIMENTS AND TABLES.

The Flow of Water through New Cast-Iron Pipe was measured by S. Bent Russell, of the St. Louis, Mo., Water-works. The pipe was 12 inches in diameter, 1631 feet long, and laid on a uniform grade from end to end. Under an average total head of 3.36 feet the flow was 43,200 cubic feet in seven hours; under an average head of 3.37 feet the flow was the same; under an average total head of 3.41 feet the flow was 46,700 cubic feet in 8 hours and 35 minutes. Making allowance for loss of head due to entrance and to curves, it was found that the value of c in the formula $v = c \sqrt{rs}$ was from 88 to 93. (*Eng'g Record*, April 14, 1894.)

Flow of Water in a 20-inch Pipe 75,000 Feet Long. — A comparison of experimental data with calculations by different formulæ is given by Chas. B. Brush, *Trans. A. S. C. E.*, 1888. The pipe experimented with was that supplying the city of Hoboken, N. J.

RESULTS OBTAINED BY THE HACKENSACK WATER CO., FROM 1882-1887, IN PUMPING THROUGH A 20-IN. CAST-IRON MAIN 75,000 FEET LONG.

Pressure in lbs. per sq. in. at pumping-station:

| | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|
| 95 | 100 | 105 | 110 | 115 | 120 | 125 | 130 |
|----|-----|-----|-----|-----|-----|-----|-----|

Total effective head in feet:

| | | | | | | | |
|----|----|----|----|-----|-----|-----|-----|
| 55 | 66 | 77 | 89 | 100 | 112 | 123 | 135 |
|----|----|----|----|-----|-----|-----|-----|

Discharge in U. S. gallons in 24 hours, 1 = 1000:

| | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 2,848 | 3,165 | 3,354 | 3,566 | 3,804 | 3,904 | 4,116 | 4,255 |
|-------|-------|-------|-------|-------|-------|-------|-------|

Theoretical discharge by Darcy's formula:

| | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 2,743 | 3,004 | 3,244 | 3,488 | 3,699 | 3,915 | 4,102 | 4,297 |
|-------|-------|-------|-------|-------|-------|-------|-------|

Actual velocity in main in feet per second:

| | | | | | | | |
|------|------|------|------|------|------|------|------|
| 2.00 | 2.24 | 2.36 | 2.52 | 2.68 | 2.76 | 2.92 | 3.00 |
|------|------|------|------|------|------|------|------|

**Flow of Water in Circular Pipes, Sewers, etc., Flowing Full.
Based on Kutter's Formula, with $n = .013$.**

Discharge in cubic feet per second.

| Diam- eter. | Slope, or Head Divided by Length of Pipe. | | | | | | | |
|----------------|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| | 1 in 40 | 1 in 70 | 1 in 100 | 1 in 200 | 1 in 300 | 1 in 400 | 1 in 500 | 1 in 600 |
| 5 in. | 0.456 | 0.344 | 0.288 | 0.204 | 0.166 | 0.144 | 0.137 | 0.118 |
| 6 " | 0.762 | 0.576 | 0.482 | 0.341 | 0.278 | 0.241 | 0.230 | 0.197 |
| 7 " | 1.17 | 0.889 | 0.744 | 0.526 | 0.430 | 0.372 | 0.355 | 0.304 |
| 8 " | 1.70 | 1.29 | 1.08 | 0.765 | 0.624 | 0.54 | 0.516 | 0.441 |
| 9 " | 2.37 | 1.79 | 1.50 | 1.06 | 0.868 | 0.75 | 0.717 | 0.613 |
| <hr/> | | | | | | | | |
| s = | 1 in 60 | 1 in 80 | 1 in 100 | 1 in 200 | 1 in 300 | 1 in 400 | 1 in 500 | 1 in 600 |
| 10 in. | 2.59 | 2.24 | 2.01 | 1.42 | 1.16 | 1.00 | 0.90 | 0.82 |
| 11 " | 3.39 | 2.94 | 2.63 | 1.86 | 1.52 | 1.31 | 1.17 | 1.07 |
| 12 " | 4.32 | 3.74 | 3.35 | 2.37 | 1.93 | 1.67 | 1.5 | 1.37 |
| 13 " | 5.38 | 4.66 | 4.16 | 2.95 | 2.40 | 2.08 | 1.86 | 1.70 |
| 14 " | 6.60 | 5.72 | 5.15 | 3.62 | 2.95 | 2.57 | 2.29 | 2.09 |
| <hr/> | | | | | | | | |
| s = | 1 in 100 | 1 in 200 | 1 in 300 | 1 in 400 | 1 in 500 | 1 in 600 | 1 in 700 | 1 in 800 |
| 15 in. | 6.18 | 4.37 | 3.57 | 3.09 | 2.77 | 2.52 | 2.34 | 2.19 |
| 16 " | 7.38 | 5.22 | 4.26 | 3.69 | 3.30 | 3.01 | 2.79 | 2.61 |
| 18 " | 10.21 | 7.22 | 5.89 | 5.10 | 4.56 | 4.17 | 3.86 | 3.61 |
| 20 " | 13.65 | 9.65 | 7.88 | 6.82 | 6.10 | 5.57 | 5.16 | 4.83 |
| 22 " | 17.71 | 12.52 | 10.22 | 8.85 | 7.92 | 7.23 | 6.69 | 6.26 |
| <hr/> | | | | | | | | |
| s = | 1 in 200 | 1 in 400 | 1 in 600 | 1 in 800 | 1 in 1000 | 1 in 1250 | 1 in 1500 | 1 in 1800 |
| 2 ft. | 15.88 | 11.23 | 9.17 | 7.94 | 7.10 | 6.35 | 5.80 | 5.29 |
| 2 ft. 2 in. | 19.73 | 13.96 | 11.39 | 9.87 | 8.82 | 7.89 | 7.20 | 6.58 |
| 2 " 4 " | 24.15 | 17.07 | 13.94 | 12.07 | 10.80 | 9.66 | 8.82 | 8.05 |
| 2 " 6 " | 29.08 | 20.56 | 16.79 | 14.54 | 13.00 | 11.63 | 10.62 | 9.69 |
| 2 " 8 " | 34.71 | 24.54 | 20.04 | 17.35 | 15.52 | 13.88 | 12.67 | 11.57 |
| <hr/> | | | | | | | | |
| s = | 1 in 500 | 1 in 750 | 1 in 1000 | 1 in 1250 | 1 in 1500 | 1 in 1750 | 1 in 2000 | 1 in 2500 |
| 2 ft. 10 in. | 25.84 | 21.10 | 18.27 | 16.34 | 14.92 | 13.81 | 12.92 | 11.55 |
| 3 " | 30.14 | 24.61 | 21.31 | 19.06 | 17.40 | 16.11 | 15.07 | 13.48 |
| 3 " 2 in. | 34.90 | 28.50 | 24.68 | 22.07 | 20.15 | 18.66 | 17.45 | 15.61 |
| 3 " 4 " | 40.08 | 32.72 | 28.34 | 25.35 | 23.14 | 21.42 | 20.04 | 17.93 |
| 3 " 6 " | 45.66 | 37.28 | 32.28 | 28.87 | 26.36 | 24.40 | 22.83 | 20.41 |
| <hr/> | | | | | | | | |
| s = | 1 in 500 | 1 in 750 | 1 in 1000 | 1 in 1250 | 1 in 1500 | 1 in 1750 | 1 in 2000 | 1 in 2500 |
| 3 ft. 8 in. | 51.74 | 42.52 | 36.59 | 32.72 | 29.87 | 27.66 | 25.87 | 23.14 |
| 3 " 10 " | 58.36 | 47.65 | 41.27 | 36.91 | 33.69 | 31.20 | 29.18 | 26.10 |
| 4 " | 65.47 | 53.46 | 46.30 | 41.41 | 37.80 | 34.50 | 32.74 | 29.28 |
| 4 " 6 in. | 89.75 | 73.28 | 63.47 | 56.76 | 51.82 | 47.97 | 44.88 | 40.14 |
| 5 " | 118.9 | 97.09 | 84.08 | 75.21 | 68.65 | 63.56 | 59.46 | 53.18 |
| <hr/> | | | | | | | | |
| s = | 1 in 750 | 1 in 1000 | 1 in 1500 | 1 in 2000 | 1 in 2500 | 1 in 3000 | 1 in 3500 | 1 in 4000 |
| 5 ft. 6 in. | 125.2 | 108.4 | 88.54 | 76.67 | 68.58 | 62.60 | 57.96 | 54.21 |
| 6 " | 157.8 | 136.7 | 111.6 | 96.66 | 86.45 | 78.92 | 73.07 | 68.35 |
| 6 " 6 " | 195.0 | 168.8 | 137.9 | 119.4 | 106.8 | 97.49 | 90.26 | 84.43 |
| 7 " | 237.7 | 205.9 | 168.1 | 145.6 | 130.2 | 118.8 | 110.00 | 102.9 |
| 7 " 6 " | 285.3 | 247.1 | 201.7 | 174.7 | 156.3 | 142.6 | 132.1 | 123.5 |
| <hr/> | | | | | | | | |
| s = | 1 in 1500 | 1 in 2000 | 1 in 2500 | 1 in 3000 | 1 in 3500 | 1 in 4000 | 1 in 4500 | 1 in 5000 |
| 8 ft. | 239.4 | 207.3 | 195.4 | 169.3 | 156.7 | 146.6 | 138.2 | 131.1 |
| 8 " 6 in. | 281.1 | 243.5 | 217.8 | 198.8 | 184.0 | 172.2 | 162.3 | 154.0 |
| 9 " | 327.0 | 283.1 | 253.3 | 231.2 | 214.0 | 200.2 | 188.7 | 179.1 |
| 9 " 6 " | 376.9 | 326.4 | 291.9 | 266.5 | 246.7 | 230.8 | 217.6 | 206.4 |
| 10 " | 431.4 | 373.6 | 334.1 | 305.0 | 282.4 | 264.2 | 249.1 | 236.3 |

For U. S. gallons multiply the figures in the table by 7.4805.

For a given diameter the quantity of flow varies as the square root of the sine of the slope. From this principle the flow for other slopes than those given in the table may be found. Thus, what is the flow for a pipe 8 feet diameter, slope 1 in 125? From the table take $Q = 207.3$ for slope 1 in 2000. The given slope 1 in 125 is to 1 in 2000 as 16 to 1, and the square root of this ratio is 4 to 1. Therefore the flow required is $207.3 \times 4 = 829.2$ cu. ft.

Circular Pipes, Conduits, etc., Flowing Full.

Values of the factor $ac\sqrt{r}$ in the formula $Q = ac\sqrt{r} \times \sqrt{s}$ corresponding to different values of the coefficient of roughness, n . (Based on Kutter's formula.)

| Diam. ft. in. | Value of $ac\sqrt{r}$. | | | | | |
|------------------|-------------------------|--------------|--------------|--------------|--------------|--------------|
| | $n = .010$. | $n = .011$. | $n = .012$. | $n = .013$. | $n = .015$. | $n = .017$. |
| 6 | 6.906 | 6.0627 | 5.3800 | 4.8216 | 3.9604 | 3.329 |
| 9 | 21.25 | 18.742 | 16.708 | 15.029 | 12.421 | 10.50 |
| 1 | 46.93 | 41.487 | 37.149 | 33.497 | 27.803 | 23.60 |
| 1 3 | 86.05 | 76.347 | 68.44 | 61.867 | 51.600 | 43.93 |
| 1 6 | 141.2 | 125.60 | 112.79 | 102.14 | 85.496 | 72.99 |
| 1 9 | 214.1 | 190.79 | 171.66 | 155.68 | 130.58 | 111.8 |
| 2 | 307.6 | 274.50 | 247.33 | 224.63 | 188.77 | 164 |
| 2 3 | 421.9 | 377.07 | 340.10 | 309.23 | 260.47 | 223.9 |
| 2 6 | 559.6 | 500.78 | 452.07 | 411.27 | 347.28 | 299.3 |
| 2 9 | 722.4 | 647.18 | 584.90 | 532.76 | 451.23 | 388.8 |
| 3 | 911.8 | 817.50 | 739.59 | 674.09 | 570.90 | 493.3 |
| 3 3 | 1128.9 | 1013.1 | 917.41 | 836.69 | 709.56 | 613.9 |
| 3 6 | 1374.7 | 1234.4 | 1118.6 | 1021.1 | 866.91 | 750.8 |
| 3 9 | 1652.1 | 1484.2 | 1345.9 | 1229.7 | 1045 | 906 |
| 4 | 1962.8 | 1764.3 | 1600.9 | 1463.9 | 1245.3 | 1080.7 |
| 4 6 | 2682.1 | 2413.3 | 2193 | 2007 | 1711.4 | 1487.3 |
| 5 | 3543 | 3191.8 | 2903.6 | 2659 | 2272.7 | 1977 |
| 5 6 | 4557.8 | 4111.9 | 3742.7 | 3429 | 2934.8 | 2557.2 |
| 6 | 5731.5 | 5176.3 | 4713.9 | 4322 | 3702.3 | 3232.5 |
| 6 6 | 7075.2 | 6394.9 | 5825.9 | 5339 | 4588.3 | 4010 |
| 7 | 8595.1 | 7774.3 | 7087 | 6510 | 5591.6 | 4893.2 |
| 7 6 | 10296 | 9318.3 | 8501.8 | 7814 | 6717 | 5884.3 |
| 8 | 12196 | 11044 | 10083 | 9272 | 7978.3 | 6995.3 |
| 8 6 | 14298 | 12954 | 11832 | 10889 | 9377.9 | 8226.7 |
| 9 | 16604 | 15049 | 13751 | 12663 | 10917 | 9580 |
| 9 6 | 19118 | 17338 | 15847 | 14597 | 12594 | 11061 |
| 10 | 21858 | 19834 | 18134 | 16709 | 14426 | 12678 |
| 10 6 | 24823 | 22534 | 20612 | 18996 | 16412 | 14434 |
| 11 | 28020 | 25444 | 23285 | 21464 | 18555 | 16333 |
| 11 6 | 31482 | 28593 | 26179 | 24139 | 20879 | 18395 |
| 12 | 35156 | 31937 | 29254 | 26981 | 23352 | 20584 |
| 12 6 | 39104 | 35529 | 32558 | 30041 | 26012 | 22938 |
| 13 | 43307 | 39358 | 36077 | 33301 | 28850 | 25451 |
| 13 6 | 47751 | 43412 | 39802 | 36752 | 31860 | 28117 |
| 14 | 52491 | 47739 | 43773 | 40432 | 35073 | 30965 |
| 14 6 | 57496 | 52308 | 47969 | 44322 | 38454 | 33975 |
| 15 | 62748 | 57103 | 52382 | 48413 | 42040 | 37147 |
| 16 | 74191 | 67557 | 62008 | 57343 | 49823 | 44073 |
| 17 | 86769 | 79050 | 72594 | 67140 | 58387 | 51669 |
| 18 | 100617 | 91711 | 84247 | 77932 | 67839 | 60067 |
| 19 | 115769 | 105570 | 96991 | 89759 | 78201 | 69301 |
| 20 | 132133 | 120570 | 110905 | 102559 | 89423 | 79259 |

Flow of Water in Circular Pipes, Conduits, etc., Flowing under Pressure.

Based on Darcy's formulæ for the flow of water through cast-iron pipes. With comparison of results obtained by Kutter's formula, with $n = 0.013$. (Condensed from Flynn on Water Power.)

Values of a , and also the values of the factors $c\sqrt{r}$ and $ac\sqrt{r}$ for use in the formulæ $Q = av$; $v = c\sqrt{r} \times \sqrt{s}$, and $Q = ac\sqrt{r} \times \sqrt{s}$.

Q = discharge in cubic feet per second, a = area in square feet, v = velocity in feet per second, r = mean hydraulic depth, $1/4$ diam. for pipes running full, s = sine of slope.

(For values of \sqrt{s} see page 700.)

| Size of Pipe. | | Clean Cast-iron Pipes. | | Value of $ac\sqrt{r}$ by Kutter's Formula, when $n = .013$. | Old Cast-iron Pipes Lined with Deposit. | |
|-----------------------|---------------------------|-----------------------------|-------------------------------|--|---|-------------------------------|
| d =diam. in ft. in. | a =area in square feet. | For Velocity, $c\sqrt{r}$. | For Discharge, $ac\sqrt{r}$. | | For Velocity, $c\sqrt{r}$. | For Discharge, $ac\sqrt{r}$. |
| $3/8$ | .00077 | 5.251 | .00403 | | 3.532 | .00272 |
| $1/2$ | .00136 | 6.702 | .00914 | | 4.507 | .00613 |
| $3/4$ | .00307 | 9.309 | .02855 | | 6.261 | .01922 |
| 1 | .00545 | 11.61 | .06334 | | 7.811 | .04257 |
| $1 1/4$ | .00852 | 13.68 | .11659 | | 9.255 | .07885 |
| $1 1/2$ | .01227 | 15.58 | .19115 | | 10.48 | .12855 |
| $1 3/4$ | .01670 | 17.32 | .28936 | | 11.65 | .19462 |
| 2 | .02182 | 18.96 | .41357 | | 12.75 | .27824 |
| $2 1/2$ | .0341 | 21.94 | .74786 | | 14.76 | .50321 |
| 3 | .0491 | 24.63 | 1.2089 | | 16.56 | .81333 |
| 4 | .0873 | 29.37 | 2.5630 | | 19.75 | 1.7246 |
| 5 | .136 | 33.54 | 4.5610 | | 22.56 | 3.0681 |
| 6 | .196 | 37.28 | 7.3068 | 4.822 | 25.07 | 4.9147 |
| 7 | .267 | 40.65 | 10.852 | | 27.34 | 7.2995 |
| 8 | .349 | 43.75 | 15.270 | | 29.43 | 10.271 |
| 9 | .442 | 46.73 | 20.652 | 15.03 | 31.42 | 13.891 |
| 10 | .545 | 49.45 | 26.952 | | 33.26 | 18.129 |
| 11 | .660 | 52.16 | 34.428 | | 35.09 | 23.158 |
| 1 | .785 | 54.65 | 42.918 | 33.50 | 36.75 | 28.867 |
| 1 2 | 1.000 | 59.34 | 63.435 | | 39.91 | 42.668 |
| 1 4 | 1.396 | 63.67 | 88.886 | | 42.83 | 59.788 |
| 1 6 | 1.767 | 67.75 | 119.72 | 102.14 | 45.57 | 80.531 |
| 1 8 | 2.182 | 71.71 | 156.46 | | 48.34 | 105.25 |
| 1 10 | 2.640 | 75.32 | 198.83 | | 50.658 | 133.74 |
| 2 | 3.142 | 78.80 | 247.57 | 224.63 | 52.961 | 166.41 |
| 2 2 | 3.687 | 82.15 | 302.90 | | 55.258 | 203.74 |
| 2 4 | 4.276 | 85.39 | 365.14 | | 57.436 | 245.60 |
| 2 6 | 4.909 | 88.39 | 433.92 | 411.37 | 59.455 | 291.87 |
| 2 8 | 5.585 | 91.51 | 511.10 | | 61.55 | 343.8 |
| 2 10 | 6.305 | 94.40 | 595.17 | | 63.49 | 400.3 |
| 3 | 7.068 | 97.17 | 686.76 | 674.09 | 65.35 | 461.9 |
| 3 2 | 7.875 | 99.93 | 786.94 | | 67.21 | 529.3 |
| 3 4 | 8.726 | 102.6 | 895.7 | | 69 | 602 |
| 3 6 | 9.621 | 105.1 | 1011.2 | 1021.1 | 70.70 | 680.2 |
| 3 8 | 10.559 | 107.6 | 1136.5 | | 72.40 | 764.5 |
| 3 10 | 11.541 | 110.2 | 1271.4 | | 74.10 | 855.2 |
| 4 | 12.566 | 112.6 | 1414.7 | 1463.9 | 75.73 | 951.6 |
| 4 3 | 14.186 | 116.1 | 1647.6 | | 78.12 | 1108.2 |
| 4 6 | 15.904 | 119.6 | 1901.9 | 2007 | 80.43 | 1279.2 |
| 4 9 | 17.721 | 122.8 | 2176.1 | | 82.20 | 1456.8 |
| 5 | 19.635 | 126.1 | 2476.4 | 2659 | 84.83 | 1665.7 |
| 5 3 | 21.648 | 129.3 | 2799.7 | | 86.99 | 1883.2 |
| 5 6 | 23.758 | 132.4 | 3146.3 | 3429 | 89.07 | 2116.2 |
| 5 9 | 25.967 | 135.4 | 3516 | | 91.08 | 2365 |
| 6 | 28.274 | 138.4 | 3912.8 | 4322 | 93.08 | 2631.7 |
| 6 6 | 33.183 | 144.1 | 4782.1 | 5339 | 96.93 | 3216.4 |
| 7 | 38.485 | 149.6 | 5757.5 | 6510 | 100.61 | 3872.5 |

| Size of Pipe. | | Clean Cast-iron Pipes. | | Value of $ac\sqrt{r}$ by Kutter's Formula, when $n = .013$ | Old Cast-iron Pipes Lined with Deposit. | |
|-----------------------|---------------------------|-----------------------------|-------------------------------|--|---|-------------------------------|
| d =diam. in ft. in. | a =area in square feet. | For Velocity, $c\sqrt{r}$. | For Discharge, $ac\sqrt{r}$. | | For Velocity, $c\sqrt{r}$. | For Discharge, $ac\sqrt{r}$. |
| 7 6 | 44.179 | 154.9 | 6841.6 | 7814 | 104.11 | 4601.9 |
| 8 | 50.266 | 160 | 8043 | 9272 | 107.61 | 5409.9 |
| 8 6 | 56.745 | 165 | 9364.7 | 10889 | 111 | 6299.1 |
| 9 | 63.617 | 169.8 | 10804 | 12663 | 114.2 | 7267.3 |
| 9 6 | 70.882 | 174.5 | 12370 | 14597 | 117.4 | 8320.6 |
| 10 | 78.540 | 179.1 | 14066 | 16709 | 120.4 | 9460.9 |
| 10 6 | 86.590 | 183.6 | 15893 | 18996 | 123.4 | 10690 |
| 11 | 95.033 | 187.9 | 17855 | 21464 | 126.3 | 12010 |
| 11 6 | 103.869 | 192.2 | 19966 | 24139 | 129.3 | 13429 |
| 12 | 113.098 | 196.3 | 22204 | 26981 | 132 | 14935 |
| 12 6 | 122.719 | 200.4 | 24598 | 30041 | 134.8 | 16545 |
| 13 | 132.733 | 204.4 | 27134 | 33301 | 137.5 | 18252 |
| 13 6 | 143.139 | 208.3 | 29818 | 36752 | 140.1 | 20056 |
| 14 | 153.938 | 212.2 | 32664 | 40432 | 142.7 | 21971 |
| 14 6 | 165.130 | 216.0 | 35660 | 44322 | 145.2 | 23986 |
| 15 | 176.715 | 219.6 | 38807 | 48413 | 147.7 | 26103 |
| 15 6 | 188.692 | 223.3 | 42125 | 52753 | 150.1 | 28335 |
| 16 | 201.062 | 226.9 | 45621 | 57343 | 152.6 | 30686 |
| 16 6 | 213.825 | 230.4 | 49273 | 62132 | 155 | 33144 |
| 17 | 226.981 | 233.9 | 53082 | 67140 | 157.3 | 35704 |
| 17 6 | 240.529 | 237.3 | 57074 | 72409 | 159.6 | 38389 |
| 18 | 254.470 | 240.7 | 61249 | 77932 | 161.9 | 41199 |
| 19 | 283.529 | 247.4 | 70154 | 89759 | 166.4 | 47186 |
| 20 | 314.159 | 253.8 | 79736 | 102559 | 170.7 | 53633 |

Flow of Water in Pipes from $\frac{3}{8}$ Inch to 12 Inches Diameter for a Uniform Velocity of 100 Ft. per Min.

| Diam. in In. | Area Sq. Ft. | Cu. Ft. per Min. | U. S. Gallons per Min. | Diam. in In. | Area Sq. Ft. | Cu. Ft. per Min. | U. S. Gallons per Min. |
|----------------|--------------|------------------|------------------------|--------------|--------------|------------------|------------------------|
| $\frac{3}{8}$ | .00077 | 0.077 | .57 | 4 | .0873 | 8.73 | 65.28 |
| $\frac{1}{2}$ | .00136 | 0.136 | 1.02 | 5 | .136 | 13.6 | 102.00 |
| $\frac{3}{4}$ | .00307 | 0.307 | 2.30 | 6 | .196 | 19.6 | 146.88 |
| 1 | .00545 | 0.545 | 4.08 | 7 | .267 | 26.7 | 199.92 |
| $1\frac{1}{4}$ | .00852 | 0.852 | 6.38 | 8 | .349 | 34.9 | 261.12 |
| $1\frac{1}{2}$ | .01227 | 1.227 | 9.18 | 9 | .442 | 44.2 | 330.48 |
| $1\frac{3}{4}$ | .01670 | 1.670 | 12.50 | 10 | .545 | 54.5 | 408.00 |
| 2 | .02182 | 2.182 | 16.32 | 11 | .660 | 66.0 | 493.68 |
| $2\frac{1}{2}$ | .0341 | 3.41 | 25.50 | 12 | .785 | 78.5 | 587.52 |
| 3 | .0491 | 4.91 | 36.72 | | | | |

Short Formulæ. E. Sherman Gould, *Eng. News*, Sept. 6, 1900, shows that Darcy's formulæ for cast-iron pipes may be reduced to the following approximate forms, in which h is loss of head or drop of hydraulic grade line in feet per 1000, d in ft., v in ft. per sec., Q in cu. ft. per sec.

$$\begin{aligned}
 &8 \text{ in. to } 48 \text{ in. diam. } \begin{cases} \text{Rough, } Q^2 = hd^5; & v = 1.27 \sqrt{dh}. \\ \text{Smooth, } Q^2 = 2hd^5; & v = 1.80 \sqrt{dh}. \end{cases} \\
 &3 \text{ to } 6 \text{ in. diam. } \begin{cases} \text{Rough, } Q^2 = 0.785hd^5; & v = 1.13 \sqrt{dh}. \\ \text{Smooth, } Q^2 = 1.57hd^5; & v = 1.60 \sqrt{dh}. \end{cases}
 \end{aligned}$$

Flow of Water in Circular Pipes from 3/8 Inch to 12 Inches Diameter.

Based on Darcy's formula for clean cast-iron pipes. $Q = ac \sqrt{r} \sqrt{s}$.

| Value of $ac\sqrt{r}$. | Dia. in. | Slope, or Head Divided by Length of Pipe. | | | | | | | |
|----------------------------|-------------|---|---------|---------|---------|---------|----------|----------|----------|
| | | 1 in 10 | 1 in 20 | 1 in 40 | 1 in 60 | 1 in 80 | 1 in 100 | 1 in 150 | 1 in 200 |
| | | Quantity in cubic feet per second. | | | | | | | |
| .00403 | 3/8 | .00127 | .00090 | .00064 | .00052 | .00045 | .00040 | .00033 | .00028 |
| .00914 | 1/2 | .00289 | .00204 | .00145 | .00118 | .00102 | .00091 | .00075 | .00065 |
| .02855 | 3/4 | .00903 | .00638 | .00451 | .00369 | .00319 | .00286 | .00233 | .00202 |
| .06334 | 1 | .02003 | .01416 | .01001 | .00818 | .00708 | .00633 | .00517 | .00448 |
| .11659 | 1 1/4 | .03687 | .02607 | .01843 | .01505 | .01303 | .01166 | .00952 | .00824 |
| .19115 | 1 1/2 | .06044 | .04274 | .03022 | .02468 | .02137 | .01912 | .01561 | .01352 |
| .28936 | 1 3/4 | .09140 | .06470 | .04575 | .03736 | .03235 | .02894 | .02363 | .02046 |
| .41357 | 2 | .13077 | .09247 | .06539 | .05339 | .04624 | .04136 | .03377 | .02927 |
| .74786 | 2 1/2 | .23647 | .16722 | .11824 | .09655 | .08361 | .07479 | .06106 | .05288 |
| 1.2089 | 3 | .38225 | .27031 | .19113 | .15607 | .13515 | .12089 | .09871 | .08548 |
| 2.5630 | 4 | .81042 | .57309 | .40521 | .33088 | .28654 | .25630 | .20927 | .18123 |
| 4.5610 | 5 | 1.4422 | 1.0198 | .72109 | .58882 | .50992 | .45610 | .37241 | .32251 |
| 7.3068 | 6 | 2.3104 | 1.6338 | 1.1552 | .94331 | .81690 | .73068 | .59660 | .51666 |
| 10.852 | 7 | 3.4314 | 2.4265 | 1.7157 | 1.4110 | 1.2132 | 1.0852 | .88607 | .76734 |
| 15.270 | 8 | 4.8284 | 3.4143 | 2.4141 | 1.9713 | 1.7072 | 1.5270 | 1.2468 | 1.0797 |
| 20.652 | 9 | 6.5302 | 4.6178 | 3.2651 | 2.6662 | 2.3089 | 2.0652 | 1.6862 | 1.4603 |
| 26.952 | 10 | 8.5222 | 6.0265 | 4.2611 | 3.4795 | 3.0132 | 2.6952 | 2.2006 | 1.9058 |
| 34.428 | 11 | 10.886 | 7.6981 | 5.4431 | 4.4447 | 3.8491 | 3.4428 | 2.8110 | 2.4344 |
| 42.918 | 12 | 13.571 | 9.5965 | 6.7853 | 5.5407 | 4.7982 | 4.2918 | 3.5043 | 3.0347 |
| Value of \sqrt{s} | | 0.3162 | 0.2236 | 0.1581 | 0.1291 | 0.1118 | 0.1 | 0.08165 | 0.07071 |

| Value of $ac\sqrt{r}$. | Dia. in. | 1 in 250 | 1 in 300 | 1 in 350 | 1 in 400 | 1 in 450 | 1 in 500 | 1 in 550 | 1 in 600 |
|----------------------------|-------------|----------|----------|----------|----------|----------|----------|----------|----------|
| .00403 | 3/8 | .00025 | .00023 | .00022 | .00020 | .00019 | .00018 | .00017 | .00016 |
| .00914 | 1/2 | .00058 | .00053 | .00049 | .00046 | .00043 | .00041 | .00039 | .00037 |
| .02855 | 3/4 | .00181 | .00165 | .00153 | .00143 | .00134 | .00128 | .00122 | .00117 |
| .06334 | 1 | .00400 | .00366 | .00339 | .00317 | .00298 | .00283 | .00270 | .00259 |
| .11659 | 1 1/4 | .00737 | .00673 | .00623 | .00583 | .00549 | .00521 | .00497 | .00476 |
| .19115 | 1 1/2 | .01209 | .01104 | .01022 | .00956 | .00901 | .00855 | .00815 | .00780 |
| .28936 | 1 3/4 | .01830 | .01671 | .01547 | .01447 | .01363 | .01294 | .01234 | .01181 |
| .41357 | 2 | .02615 | .02388 | .02211 | .02068 | .01948 | .01849 | .01763 | .01688 |
| .74786 | 2 1/2 | .04730 | .04318 | .03997 | .03739 | .03523 | .03344 | .03189 | .03053 |
| 1.2089 | 3 | .07645 | .06980 | .06462 | .06045 | .05695 | .05406 | .05155 | .04935 |
| 2.5630 | 4 | .16208 | .14799 | .13699 | .12815 | .12074 | .11461 | .10929 | .10463 |
| 4.5610 | 5 | .28843 | .26335 | .24379 | .22805 | .21487 | .20397 | .19448 | .18620 |
| 7.3068 | 6 | .46208 | .42189 | .39055 | .36534 | .34422 | .32676 | .31156 | .29830 |
| 10.852 | 7 | .68628 | .62660 | .58005 | .54260 | .51124 | .48530 | .46273 | .44303 |
| 15.270 | 8 | .95657 | .88158 | .81617 | .76350 | .71936 | .68286 | .65111 | .62340 |
| 20.652 | 9 | 1.3060 | 1.1924 | 1.1038 | 1.0326 | .97292 | .92356 | .88060 | .84310 |
| 26.952 | 10 | 1.7044 | 1.5562 | 1.4405 | 1.3476 | 1.2697 | 1.2053 | 1.1492 | 1.1003 |
| 34.428 | 11 | 2.1772 | 1.9878 | 1.8402 | 1.7214 | 1.6219 | 1.5396 | 1.4680 | 1.4055 |
| 42.918 | 12 | 2.7141 | 2.4781 | 2.2940 | 2.1459 | 2.0219 | 1.9193 | 1.8300 | 1.7521 |
| Value of \sqrt{s} | | .06324 | .05774 | .05345 | .05 | .04711 | .04472 | .04264 | .04082 |

For U. S. gals. per sec., multiply the figures in the table by 7.4805
 " " " " min., " " " " ... 48.83
 " " " " hour, " " " " ... 26929.8
 " " " " 24 hrs., " " " " ... 646315.

For any other slope the flow is proportional to the square root of the slope; thus, flow in slope of 1 in 100 is double that in slope of 1 in 400.

Flow of Water in House-service Pipes.

Mr. E. Kuichling, C. E., furnished the following table to the Thomson Meter Co.:

| Condition of Discharge. | Pressure in Main, pounds per square inch. | Discharge, or Quantity capable of being delivered, in Cubic Feet per Minute, from the Pipe, under the conditions specified in the first column. | | | | | | | | |
|--|---|---|------|------|-------|-------|-------|--------|--------|--------|
| | | Nominal Diameters of Iron or Lead Service-pipe in Inches. | | | | | | | | |
| | | 1/2 | 5/8 | 3/4 | 1 | 1 1/2 | 2 | 3 | 4 | 6 |
| Through 35 feet of service-pipe, no back pressure. | 30 | 1.10 | 1.92 | 3.01 | 6.13 | 16.58 | 33.34 | 88.16 | 173.85 | 444.63 |
| | 40 | 1.27 | 2.22 | 3.48 | 7.08 | 19.14 | 38.50 | 101.80 | 200.75 | 513.42 |
| | 50 | 1.42 | 2.48 | 3.89 | 7.92 | 21.40 | 43.04 | 113.82 | 224.44 | 574.02 |
| | 60 | 1.56 | 2.71 | 4.26 | 8.67 | 23.44 | 47.15 | 124.68 | 245.87 | 628.81 |
| | 75 | 1.74 | 3.03 | 4.77 | 9.70 | 26.21 | 52.71 | 139.39 | 274.89 | 703.03 |
| | 100 | 2.01 | 3.50 | 5.50 | 11.20 | 30.27 | 60.87 | 160.96 | 317.41 | 811.79 |
| | 130 | 2.29 | 3.99 | 6.28 | 12.77 | 34.51 | 69.40 | 183.52 | 361.91 | 925.58 |
| Through 100 feet of service-pipe, no back pressure. | 30 | 0.66 | 1.16 | 1.84 | 3.78 | 10.40 | 21.30 | 58.19 | 118.13 | 317.23 |
| | 40 | 0.77 | 1.34 | 2.12 | 4.36 | 12.01 | 24.59 | 67.19 | 136.41 | 366.30 |
| | 50 | 0.86 | 1.50 | 2.37 | 4.88 | 13.43 | 27.50 | 75.13 | 152.51 | 409.54 |
| | 60 | 0.94 | 1.65 | 2.60 | 5.34 | 14.71 | 30.12 | 82.30 | 167.06 | 448.63 |
| | 75 | 1.05 | 1.84 | 2.91 | 5.97 | 16.45 | 33.68 | 92.01 | 186.78 | 501.58 |
| | 100 | 1.22 | 2.13 | 3.36 | 6.90 | 18.99 | 38.89 | 106.24 | 215.68 | 579.18 |
| | 130 | 1.39 | 2.42 | 3.83 | 7.86 | 21.66 | 44.34 | 121.14 | 245.91 | 660.36 |
| Through 100 feet of service-pipe, and 15 feet vertical rise. | 30 | 0.55 | 0.96 | 1.52 | 3.11 | 8.57 | 17.55 | 47.90 | 97.17 | 260.56 |
| | 40 | 0.66 | 1.15 | 1.81 | 3.72 | 10.24 | 20.95 | 57.20 | 116.01 | 311.09 |
| | 50 | 0.75 | 1.31 | 2.06 | 4.24 | 11.67 | 23.87 | 65.18 | 132.20 | 354.49 |
| | 60 | 0.83 | 1.45 | 2.29 | 4.70 | 12.94 | 26.48 | 72.28 | 146.61 | 393.13 |
| | 75 | 0.94 | 1.64 | 2.59 | 5.32 | 14.64 | 29.96 | 81.79 | 165.90 | 444.85 |
| | 100 | 1.10 | 1.92 | 3.02 | 6.21 | 17.10 | 35.00 | 95.55 | 193.82 | 519.72 |
| | 130 | 1.26 | 2.20 | 3.48 | 7.14 | 19.66 | 40.23 | 109.82 | 222.75 | 597.31 |
| Through 100 feet of service-pipe, and 30 feet vertical rise. | 30 | 0.44 | 0.77 | 1.22 | 2.50 | 6.80 | 14.11 | 38.63 | 78.54 | 211.54 |
| | 40 | 0.55 | 0.97 | 1.53 | 3.15 | 8.68 | 17.79 | 48.68 | 98.98 | 266.59 |
| | 50 | 0.65 | 1.14 | 1.79 | 3.69 | 10.16 | 20.82 | 56.98 | 115.87 | 312.08 |
| | 60 | 0.73 | 1.28 | 2.02 | 4.15 | 11.45 | 23.47 | 64.22 | 130.59 | 351.73 |
| | 75 | 0.84 | 1.47 | 2.32 | 4.77 | 13.15 | 26.95 | 73.76 | 149.99 | 403.98 |
| | 100 | 1.00 | 1.74 | 2.75 | 5.65 | 15.58 | 31.93 | 87.38 | 177.67 | 478.55 |
| | 130 | 1.15 | 2.02 | 3.19 | 6.55 | 18.07 | 37.02 | 101.33 | 206.04 | 554.96 |

In this table it is assumed that the pipe is straight and smooth inside; that the friction of the main and meter are disregarded; that the inlet from the main is of ordinary character, sharp, not flaring or rounded, and that the outlet is the full diameter of pipe. The deliveries given will be increased if, first, the pipe between the meter and the main is of larger diameter than the outlet; second, if the main is tapped, say for 1-inch pipe, but is enlarged from the tap to 1 1/4 or 1 1/2 inch; or, third, if pipe on the outlet is larger than that on the inlet side of the meter. The exact details of the conditions given are rarely met in practice; consequently the quantities of the table may be expected to be decreased, because the pipe is liable to be throttled at the joints, additional bends may interpose, or stop-cocks may be used, or the back-pressure may be increased.

Flow of Water Through Nozzles in Cubic Feet per Second. (Joshua Hendy Iron Works.)

H = head in feet at the nozzle. P = pressure due to head lbs. per sq. in. V = theoretical velocity, ft. per sec.

| Diam. Nozzle, In. | | 1 | 1 1/2 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------------------|--------|--------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| H . | P . | V . | | | | | | | | | | | | | | | |
| | | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
| 5 | 2.17 | 17.93 | 0.098 | 0.221 | 0.392 | 0.611 | 0.880 | 1.197 | 1.565 | 2.040 | 2.445 | 3.519 | 4.810 | 6.26 | 7.92 | 9.78 | 12.83 |
| 10 | 4.33 | 25.36 | 0.138 | 0.312 | 0.553 | 0.864 | 1.245 | 1.694 | 2.213 | 2.875 | 3.459 | 4.978 | 6.778 | 8.85 | 11.20 | 13.83 | 16.74 |
| 15 | 6.66 | 35.86 | 0.195 | 0.441 | 0.784 | 1.222 | 1.760 | 2.395 | 3.130 | 4.066 | 4.885 | 7.039 | 9.585 | 12.52 | 15.84 | 19.56 | 23.67 |
| 20 | 12.99 | 43.92 | 0.241 | 0.540 | 0.957 | 1.497 | 2.156 | 2.933 | 3.834 | 4.980 | 5.990 | 8.621 | 11.739 | 15.33 | 19.40 | 23.95 | 28.99 |
| 30 | 17.32 | 50.72 | 0.276 | 0.624 | 1.105 | 1.729 | 2.490 | 3.388 | 4.427 | 5.751 | 6.918 | 9.956 | 13.557 | 17.71 | 22.41 | 27.66 | 33.48 |
| 40 | 21.65 | 56.71 | 0.311 | 0.697 | 1.236 | 1.933 | 2.784 | 3.788 | 4.950 | 6.429 | 7.735 | 11.132 | 15.158 | 19.80 | 25.05 | 30.93 | 37.43 |
| 50 | 25.99 | 62.12 | 0.341 | 0.764 | 1.354 | 2.118 | 3.050 | 4.149 | 5.423 | 7.044 | 8.437 | 12.194 | 16.604 | 21.69 | 27.44 | 33.88 | 41.00 |
| 60 | 30.62 | 67.10 | 0.369 | 0.825 | 1.464 | 2.288 | 3.294 | 4.482 | 5.857 | 7.609 | 9.152 | 13.171 | 17.935 | 23.42 | 29.64 | 36.60 | 44.29 |
| 70 | 34.65 | 71.73 | 0.394 | 0.882 | 1.564 | 2.445 | 3.521 | 4.791 | 6.262 | 8.134 | 9.783 | 14.080 | 19.173 | 25.04 | 31.69 | 39.12 | 47.34 |
| 80 | 38.98 | 76.08 | 0.418 | 0.936 | 1.659 | 2.594 | 3.735 | 5.082 | 6.641 | 8.627 | 10.377 | 14.934 | 20.346 | 26.56 | 33.61 | 41.49 | 50.21 |
| 90 | 43.31 | 80.20 | 0.441 | 0.986 | 1.748 | 2.734 | 3.937 | 5.377 | 7.001 | 9.094 | 10.940 | 15.743 | 21.437 | 28.00 | 35.43 | 43.74 | 52.93 |
| 100 | 47.64 | 84.20 | 0.461 | 1.031 | 1.831 | 2.866 | 4.114 | 5.570 | 7.271 | 9.465 | 11.366 | 16.525 | 22.490 | 29.68 | 37.83 | 46.93 | 56.00 |
| 120 | 51.97 | 87.88 | 0.489 | 1.080 | 1.866 | 2.996 | 4.314 | 5.870 | 7.671 | 9.965 | 11.986 | 17.250 | 23.490 | 30.68 | 38.83 | 47.93 | 58.00 |
| 140 | 56.30 | 91.49 | 0.514 | 1.127 | 1.909 | 3.125 | 4.509 | 6.059 | 8.056 | 10.560 | 12.625 | 18.225 | 24.590 | 31.68 | 39.83 | 48.93 | 59.00 |
| 160 | 60.63 | 94.89 | 0.537 | 1.167 | 1.952 | 3.254 | 4.698 | 6.248 | 8.243 | 10.750 | 12.815 | 18.415 | 24.780 | 32.68 | 40.73 | 49.93 | 60.00 |
| 180 | 64.96 | 98.10 | 0.557 | 1.202 | 2.000 | 3.383 | 4.887 | 6.437 | 8.428 | 10.940 | 13.005 | 18.605 | 24.970 | 33.68 | 41.63 | 50.93 | 61.00 |
| 200 | 69.29 | 101.45 | 0.577 | 1.239 | 2.047 | 3.512 | 5.076 | 6.626 | 8.619 | 11.130 | 13.195 | 18.795 | 25.160 | 34.63 | 42.53 | 51.93 | 62.00 |
| 220 | 73.62 | 104.80 | 0.597 | 1.275 | 2.094 | 3.641 | 5.265 | 6.815 | 8.809 | 11.320 | 13.385 | 18.985 | 25.350 | 35.63 | 43.43 | 52.93 | 63.00 |
| 250 | 82.00 | 113.41 | 0.621 | 1.394 | 2.472 | 3.867 | 5.568 | 7.187 | 9.392 | 12.200 | 14.675 | 21.119 | 28.758 | 37.36 | 47.53 | 58.68 | 71.01 |
| 280 | 86.62 | 117.41 | 0.641 | 1.441 | 2.545 | 4.056 | 5.856 | 7.476 | 9.671 | 12.490 | 14.965 | 21.409 | 29.048 | 38.46 | 48.43 | 59.58 | 72.01 |
| 300 | 90.50 | 126.80 | 0.668 | 1.560 | 2.764 | 4.323 | 6.225 | 8.470 | 11.069 | 14.379 | 17.295 | 24.857 | 33.893 | 44.27 | 56.02 | 69.16 | 83.69 |
| 350 | 130.20 | 138.91 | 0.761 | 1.709 | 3.028 | 4.736 | 6.820 | 9.270 | 12.126 | 15.712 | 18.899 | 27.268 | 37.130 | 48.49 | 61.37 | 75.76 | 91.68 |
| 400 | 150.00 | 150.04 | 0.825 | 1.845 | 3.271 | 5.116 | 7.366 | 10.022 | 13.098 | 17.014 | 20.465 | 29.452 | 40.105 | 52.38 | 66.29 | 81.83 | 99.03 |
| 450 | 173.60 | 160.40 | 0.883 | 1.973 | 3.497 | 5.469 | 7.875 | 10.714 | 14.002 | 18.189 | 21.878 | 31.486 | 42.874 | 56.00 | 70.86 | 87.48 | 105.86 |
| 500 | 195.30 | 170.12 | 0.935 | 2.092 | 3.709 | 5.801 | 8.352 | 11.364 | 14.851 | 19.391 | 23.204 | 33.945 | 45.263 | 59.39 | 75.16 | 92.78 | 112.23 |
| 550 | 216.00 | 179.33 | 0.986 | 2.206 | 3.909 | 6.115 | 8.645 | 11.979 | 15.655 | 20.336 | 24.460 | 35.202 | 47.934 | 62.60 | 79.23 | 97.81 | 118.36 |
| 600 | 260.11 | 196.44 | 1.080 | 2.416 | 4.282 | 6.698 | 9.645 | 13.122 | 17.149 | 22.276 | 26.794 | 38.561 | 52.508 | 68.58 | 86.79 | 107.14 | 129.65 |
| 700 | 303.80 | 212.18 | 1.167 | 2.610 | 4.626 | 7.235 | 10.418 | 14.173 | 18.523 | 24.061 | 28.941 | 41.650 | 56.715 | 74.07 | 93.74 | 115.72 | 140.04 |
| 800 | 347.20 | 226.84 | 1.247 | 2.790 | 4.945 | 7.735 | 11.137 | 15.152 | 19.803 | 25.723 | 30.940 | 44.528 | 60.634 | 79.19 | 100.22 | 123.72 | 149.71 |
| 900 | 390.60 | 240.60 | 1.323 | 2.959 | 5.245 | 8.204 | 11.813 | 16.072 | 20.804 | 27.284 | 32.817 | 47.229 | 64.312 | 83.99 | 106.30 | 131.22 | 158.80 |
| 1000 | 434.00 | 253.61 | 1.394 | 3.119 | 5.529 | 8.648 | 12.452 | 16.941 | 22.141 | 28.759 | 34.592 | 49.783 | 67.789 | 88.54 | 112.04 | 138.32 | 167.38 |
| 1200 | 520.80 | 277.81 | 1.527 | 3.417 | 6.056 | 9.473 | 13.640 | 18.557 | 24.260 | 31.503 | 37.893 | 54.534 | 74.258 | 96.98 | 122.74 | 151.52 | 183.35 |
| 1400 | 607.60 | 300.10 | 1.650 | 3.691 | 6.542 | 10.223 | 14.734 | 20.046 | 26.198 | 34.031 | 40.933 | 58.909 | 80.216 | 104.76 | 132.58 | 163.67 | 198.07 |
| 1600 | 694.40 | 320.80 | 1.764 | 3.946 | 6.993 | 10.939 | 15.751 | 21.429 | 27.907 | 36.378 | 43.757 | 62.973 | 85.749 | 111.99 | 141.73 | 174.96 | 211.73 |
| 1800 | 781.20 | 340.28 | 1.872 | 4.185 | 7.418 | 11.503 | 16.707 | 22.730 | 29.706 | 38.587 | 46.414 | 66.796 | 90.956 | 118.79 | 150.34 | 185.59 | 224.58 |
| 2000 | 868.00 | 358.65 | 1.973 | 4.411 | 7.818 | 12.229 | 17.609 | 23.937 | 31.310 | 40.670 | 48.919 | 70.402 | 95.867 | 125.21 | 158.45 | 195.61 | 236.71 |

LOSS OF HEAD.

The loss of head due to friction when water, steam, air, or gas of any kind flows through a straight tube is represented by the formula

$$h = f \frac{4l}{d} \frac{v^2}{2g}; \quad \text{whence } v = \sqrt{\frac{64.4}{4f} \frac{hd}{l}},$$

in which l = the length and d = the diameter of the tube, both in feet; v = velocity in feet per second, and f is a coefficient to be determined by experiment. According to Weisbach, $f = 0.00644$, in which case

$$\sqrt{\frac{64.4}{4f}} = 50, \text{ and } v = 50 \sqrt{\frac{hd}{l}},$$

which is one of the older formulæ for flow of water (Downing's). Prof. Unwin says that the value of f is possibly too small for tubes of small bore, and he would put $f = 0.006$ to 0.01 for 4-inch tubes, and $f = 0.0084$ to 0.012 for 2-inch tubes. Another formula by Weisbach is

$$h = \left(0.0144 + \frac{0.01716}{\sqrt{v}}\right) \frac{l}{d} \frac{v^2}{2g}.$$

Rankine gives

$$f = 0.005 \left(1 + \frac{1}{12d}\right).$$

From the general equation for velocity of flow of water $v = c \sqrt{r} \sqrt{s}$ = for round pipes $c \sqrt{\frac{d}{4}} \sqrt{\frac{h}{l}}$, we have $v^2 = c^2 \frac{d}{4} \frac{h}{l}$ and $h = \frac{4lv^2}{c^2d}$, in which

c is the coefficient c of Darcy's, Bazin's, Kutter's, or other formula as found by experiment. Since this coefficient varies with the condition of the inner surface of the tube, as well as with the velocity, it is to be expected that values of the loss of head given by different writers will vary as much as those of quantity of flow.

The relation of the value of c in Chezy's formula $V = c \sqrt{rs}$ to the value of the coefficient of friction f is $c = \sqrt{2g/f}$.

| | | | | | | |
|-------------|---------|---------|---------|---------|---------|---------|
| $f = .0035$ | $.0040$ | $.0045$ | $.0050$ | $.0055$ | $.0060$ | $.0065$ |
| $c = 135.5$ | 127.8 | 119.6 | 113.4 | 108.1 | 103.5 | 99.4 |
| $f = .0070$ | $.0075$ | $.0080$ | $.0090$ | $.010$ | $.011$ | $.012$ |
| $c = 95.8$ | 92.6 | 89.7 | 84.5 | 80.2 | 76.5 | 73.2 |
| $c = 60$ | 70 | 80 | 90 | 100 | 110 | 120 |
| $f = .018$ | $.013$ | $.010$ | $.008$ | $.0064$ | $.0053$ | $.0045$ |
| | | | | | | $.0038$ |
| | | | | | | $.0033$ |
| | | | | | | $.0029$ |

Equations derived from the formulæ. (Unwin.)

Velocity, ft. per sec. $v = 4.012 \sqrt{dh/(fl)} = 1.273 Q/d^2 = c \sqrt{d/4} \times \sqrt{s}$.

Diameter, ft. $d = 0.0622 fvl/h = 1.128 \sqrt{Q/v}$.

Quantity, cu. ft. per sec. $Q = 3.149 \sqrt{hd^5/fl}$.

Head, ft. $h = 0.1008 fQ^2/d^5$.

Rough preliminary calculations may be made by the following approximate formulæ. They are least accurate for small pipes. s = slope, $= h/l$.

New and clean pipes.

Old and incrustèd pipes.

$$v = 56 \sqrt{ds}.$$

$$v = 40 \sqrt{ds}.$$

$$Q = 44 \sqrt{d^5s}.$$

$$Q = 31.4 \sqrt{d^5s}.$$

$$d = 0.22 \sqrt[5]{Q^2/s}.$$

$$d = 0.252 \sqrt[5]{Q^2/s}.$$

Flow of Water in Riveted Steel Pipes. — The laps and rivets tend to decrease the carrying capacity of the pipe. See paper on "New Formulas for Calculating the Flow of Water in Pipes and Channels," by W. E. Foss, *Jour. Assoc. Eng. Soc.*, xiii, 295. Also Clemens Herschel's book on "115 Experiments on the Carrying Capacity of Large Riveted Metal Conduits," John Wiley & Sons, 1897.

Values of the Coefficient of Friction. Unwin's "Hydraulics" gives values of f , based on Darcy's experiments, as follows: Clean and smooth pipes, $f = 0.005 (1 + \frac{1}{12} d)$. Incrusted pipes, $f = 0.01 (1 + \frac{1}{12} d)$. In 1886 Unwin examined all the more carefully made experiments on flow in pipes, including those of Darcy, classifying them according to the quality and condition of their surfaces, and showing the relation of the value of f to both diameter and velocity. The results agree fairly closely with the following values, $f = \alpha (1 + \beta/d)$.

| Kind of pipe. | Values of α for velocities in ft. per second. | | | | Values of β . |
|---------------------------------------|--|--------|--------|--------|---------------------|
| | 1-2 | 2-3 | 3-4 | 4-5 | |
| Drawn wrought iron... | .00375 | .00322 | .00297 | .00275 | 0.37 |
| Asphalted cast iron... | .00492 | .00455 | .00432 | .00415 | 0.20 |
| Clean cast iron..... | .00405 | .00395 | .00387 | .00382 | 0.28 |
| Incrusted cast iron at all velocities | $\alpha = 0.00855$ | | | | 0.26 |

From the experiments of Clemens Herschel, 1892-6, on clean steel riveted pipes, Unwin derives the following values of f for different velocities.

| Ft. per sec..... | 1 | 2 | 3 | 4 | 5 | 6 |
|------------------------|-------|-------|-------|-------|-------|-------|
| 48-in. pipe, av. of 2. | .0066 | .0060 | .0057 | .0055 | .0055 | .0055 |
| 42-in. pipe, av. of 2. | .0067 | .0058 | .0054 | .0054 | .0054 | .0054 |
| 36-in. pipe..... | .0087 | .0071 | .0060 | .0053 | .0047 | .0042 |

Unwin attributes the anomalies in this table to errors of observation. In comparing the results with those on cast-iron pipes, the roughness of the rivet heads and joints must be considered, and the resistance can only be determined by direct experiment on riveted pipes.

Two portions of the 48-in. main were tested after being four years in use, and the coefficients derived from them differ remarkably.

| Ft. per sec..... | 1 | 2 | 3 | 4 | 5 | 6 |
|------------------|-------|-------|-------|-------|-------|-------|
| Upper part..... | .0106 | .0080 | .0075 | .0073 | .0072 | .0072 |
| Lower part..... | .0068 | .0060 | .0058 | .0060 | .0060 | .0060 |

Marx, Wing, and Hopkins in 1897 and 1899 made gaugings on a 6-ft. main, part of which was of riveted steel and part of wood staves. (*Trans. A. S. C. E.*, xl, 471, and xlv, 34.) From these tests Unwin derives the following values of f .

| Ft. per sec. | 1 | 1.5 | 2 | 2.5 | 3 | 4 | 5 | 5.5 |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Steel pipe: | | | | | | | | |
| 1897... $f =$ | .0053 | .0052 | .0053 | .0055 | .0055 | .0052 | .0058 | .0058 |
| 1899... $f =$ | .0097 | .0076 | .0067 | .0063 | .0061 | .0060 | .0058 | .0058 |
| Wood staves: | | | | | | | | |
| 1897... $f =$ | .0064 | .0053 | .0048 | | .0043 | .0041 | | .0043 |
| 1899... $f =$ | .0048 | .0046 | .0045 | | .0044 | .0043 | .0043 | .0043 |

Freeman's experiments on fire hose pipes (*Trans. A. S. C. E.*, xxi, 303) give the following values of f .

| Velocity, ft. per sec..... | 4 | 6 | 10 | 15 | 20 |
|---------------------------------|-------|-------|-------|-------|-------|
| Unlined canvas..... | .0095 | .0095 | .0093 | .0088 | .0085 |
| Rough rubber-lined cotton..... | .0078 | .0078 | .0078 | .0075 | .0073 |
| Smooth rubber-lined cotton..... | .0060 | .0058 | .0055 | .0048 | .0045 |

The Resistance at the Inlet of a Pipe is equal to the frictional resistance of a straight pipe whose length is $l_0 = (1 + f_0) d \div 4 f$. Values of f_0 are: (A) for end of pipe flush with reservoir wall, 0.5; (B) pipe entering wall, straight edges, 0.56; (C) pipe entering wall, sharp edges, 1.30; (D) bell-mouthed inlet, 0.02 to 0.05. Values of l_0/d are for

| | | | | |
|-------------|-------|-------|-------|--------|
| $f = 0.005$ | A, 53 | B, 75 | C, 78 | D, 115 |
| 0.010 | 26 | 38 | 39 | 58 |

Multiplying these figures by d gives the length of straight pipe to be added to the actual length to allow for the inlet resistance. In long lengths of pipe the relative value of this length is so small that it may be neglected in practical calculations. — (Unwin.)

Loss of Head in Pipe by Friction. — Loss of head by friction in each 100 feet in length of riveted pipe when discharging the following quantities of water per minute (Pelton Water-wheel Co.).

V = velocity in feet per second; h = loss of head in feet; Q = discharge in cubic feet per minute.

| V | Inside Diameter of Pipe in Inches. | | | | | | | | | | | |
|-----|------------------------------------|-------|-------|------|-------|------|-------|------|-------|------|-------|------|
| | 7 | | 8 | | 9 | | 10 | | 11 | | 12 | |
| | h | Q | h | Q | h | Q | h | Q | h | Q | h | Q |
| 2.0 | 0.338 | 32.0 | 0.296 | 41.9 | 0.264 | 53 | 0.237 | 65.4 | 0.216 | 79.2 | 0.198 | 94.2 |
| 3.0 | 0.698 | 48.1 | 0.611 | 62.8 | 0.544 | 79.5 | 0.488 | 98.2 | 0.444 | 119 | 0.407 | 141 |
| 4.0 | 1.175 | 64.1 | 1.027 | 83.7 | 0.913 | 106 | 0.822 | 131 | 0.747 | 158 | 0.685 | 188 |
| 5.0 | 1.76 | 80.2 | 1.54 | 105 | 1.37 | 132 | 1.23 | 163 | 1.122 | 198 | 1.028 | 235 |
| 6.0 | 2.46 | 96.2 | 2.15 | 125 | 1.92 | 159 | 1.71 | 196 | 1.56 | 237 | 1.43 | 283 |
| 7.0 | 3.26 | 112.0 | 2.85 | 146 | 2.52 | 185 | 2.28 | 229 | 2.07 | 277 | 1.91 | 330 |

| V | 13 in. | | 14 in. | | 15 in. | | 16 in. | | 18 in. | | 20 in. | |
|-----|--------|-----|--------|-----|--------|-----|--------|-----|--------|-----|--------|-----|
| | h | Q | h | Q | h | Q | h | Q | h | Q | h | Q |
| | h | Q | h | Q | h | Q | h | Q | h | Q | h | Q |
| 2.0 | 0.183 | 110 | 0.169 | 128 | 0.158 | 147 | 0.147 | 167 | 0.132 | 212 | 0.119 | 262 |
| 3.0 | .375 | 166 | .349 | 192 | .325 | 221 | .306 | 251 | .271 | 318 | .245 | 393 |
| 4.0 | .632 | 221 | .587 | 256 | .548 | 294 | .513 | 335 | .456 | 424 | .410 | 523 |
| 5.0 | .949 | 276 | .881 | 321 | .822 | 368 | .770 | 419 | .685 | 530 | .617 | 654 |
| 6.0 | 1.325 | 332 | 1.229 | 385 | 1.148 | 442 | 1.076 | 502 | .957 | 636 | .861 | 785 |
| 7.0 | 1.75 | 387 | 1.63 | 449 | 1.52 | 515 | 1.43 | 586 | 1.27 | 742 | 1.143 | 916 |

| V | 22 in. | | 24 in. | | 26 in. | | 28 in. | | 30 in. | | 36 in. | |
|-----|--------|------|--------|------|--------|------|--------|------|--------|------|--------|------|
| | h | Q | h | Q | h | Q | h | Q | h | Q | h | Q |
| | h | Q | h | Q | h | Q | h | Q | h | Q | h | Q |
| 2.0 | 0.108 | 316 | 0.098 | 377 | 0.091 | 442 | 0.084 | 513 | 0.079 | 589 | 0.066 | 848 |
| 3.0 | .222 | 475 | .204 | 565 | .188 | 663 | .174 | 770 | .163 | 883 | .135 | 1273 |
| 4.0 | .373 | 633 | .342 | 754 | .315 | 885 | .293 | 1026 | .273 | 1178 | .228 | 1697 |
| 5.0 | .561 | 792 | .513 | 942 | .474 | 1106 | .440 | 1283 | .411 | 1472 | .342 | 2121 |
| 6.0 | .782 | 950 | .717 | 1131 | .662 | 1327 | .615 | 1539 | .574 | 1767 | .479 | 2545 |
| 7.0 | 1.040 | 1109 | .953 | 1319 | .879 | 1548 | .817 | 1796 | .762 | 2061 | .636 | 2868 |

This table is based on Cox's reconstruction of Weisbach's formula, using the denominator 1000 instead of 1200, to be on the safe side, allowing 20% for the loss of head due to the laps and rivet-heads in the pipe.

EXAMPLE. — Given 200 ft. head and 600 ft. of 11-inch pipe, carrying 119 cubic feet of water per minute. To find effective head: In right-hand column, under 11-inch pipe, find 119 cubic ft.; opposite this will be found the loss by friction in 100 ft. of length for this amount of water, which is 0.444. Multiply this by the number of hundred feet of pipe, which is 6, and we have 2.66 ft., which is the loss of head. Therefore the effective head is $200 - 2.66 = 197.34$.

EXPLANATION. — The loss of head by friction in a pipe depends not only upon diameter and length, but upon the quantity of water passed through it. The head or pressure is what would be indicated by a

pressure-gauge attached to the pipe near the wheel. Readings of gauge should be taken while the water is flowing from the nozzle.

To reduce heads in feet to pressure in pounds multiply by 0.433. To reduce pounds pressure to feet multiply by 2.309.

Cox's Formula. — Weisbach's formula for loss of head caused by the friction of water in pipes is as follows:

$$\text{Friction-head} = \left(0.0144 + \frac{0.01716}{\sqrt{V}}\right) \frac{L \cdot V^2}{5.367 d^5}$$

where L = length of pipe in feet;
 V = velocity of the water in feet per second;
 d = diameter of pipe in inches.

William Cox (*Amer. Mach.*, Dec. 28, 1893) gives a simpler formula which gives almost identical results:

$$H = \text{friction-head in feet} = \frac{L}{d} \frac{4V^2 + 5V - 2}{1200} \quad (1)$$

$$\frac{Hd}{L} = \frac{4V^2 + 5V - 2}{1200} \quad (2)$$

He gives a table by means of which the value of $\frac{4V^2 + 5V - 2}{1200}$ is at once obtained when V is known, and *vice versa*.

VALUES OF $\frac{4V^2 + 5V - 2}{1200}$.

| V | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1 | .00583 | .00695 | .00813 | .00938 | .01070 | .01208 | .01353 | .01505 | .01663 | .01828 |
| 2 | .02000 | .02178 | .02363 | .02555 | .02753 | .02958 | .03170 | .03388 | .03613 | .03845 |
| 3 | .04083 | .04328 | .04580 | .04838 | .05103 | .05375 | .05653 | .05938 | .06230 | .06528 |
| 4 | .06833 | .07145 | .07463 | .07788 | .08120 | .08458 | .08803 | .09155 | .09513 | .09878 |
| 5 | .10250 | .10628 | .11013 | .11405 | .11803 | .12208 | .12620 | .13038 | .13463 | .13895 |
| 6 | .14333 | .14778 | .15230 | .15688 | .16153 | .16625 | .17103 | .17588 | .18080 | .18578 |
| 7 | .19083 | .19595 | .20113 | .20638 | .21170 | .21708 | .22253 | .22805 | .23363 | .23928 |
| 8 | .24500 | .25078 | .25663 | .26255 | .26853 | .27458 | .28070 | .28688 | .29313 | .29945 |
| 9 | .30583 | .31228 | .31880 | .32538 | .33203 | .33875 | .34553 | .35238 | .35930 | .36628 |
| 10 | .37333 | .38045 | .38763 | .39488 | .40220 | .40958 | .41703 | .42455 | .43213 | .43978 |
| 11 | .44750 | .45528 | .46313 | .47105 | .47903 | .48708 | .49520 | .50338 | .51163 | .51995 |
| 12 | .52833 | .53678 | .54530 | .55388 | .56253 | .57125 | .58003 | .58888 | .59780 | .60678 |
| 13 | .61583 | .62495 | .63413 | .64338 | .65270 | .66208 | .67153 | .68105 | .69063 | .70028 |
| 14 | .71000 | .71978 | .72963 | .73955 | .74953 | .75958 | .76970 | .77988 | .79013 | .80045 |
| 15 | .81083 | .82128 | .83180 | .84238 | .85303 | .86375 | .87453 | .88538 | .89630 | .90728 |
| 16 | .91833 | .92945 | .94063 | .95188 | .96320 | .97458 | .98603 | .99755 | 1.00913 | 1.02078 |
| 17 | 1.03250 | 1.04428 | 1.05613 | 1.06805 | 1.08003 | 1.09208 | 1.10420 | 1.11638 | 1.12863 | 1.14095 |
| 18 | 1.15333 | 1.16578 | 1.17830 | 1.19088 | 1.20353 | 1.21625 | 1.22903 | 1.24188 | 1.25480 | 1.26778 |
| 19 | 1.28083 | 1.29395 | 1.30713 | 1.32038 | 1.33370 | 1.34708 | 1.36053 | 1.37405 | 1.38763 | 1.40128 |
| 20 | 1.41500 | 1.42878 | 1.44263 | 1.45655 | 1.47053 | 1.48458 | 1.49870 | 1.51288 | 1.52713 | 1.54145 |
| 21 | 1.55583 | 1.57028 | 1.58480 | 1.59938 | 1.61403 | 1.62875 | 1.64353 | 1.65838 | 1.67330 | 1.68828 |

The use of the formula and table is illustrated as follows:

Given a pipe 5 inches diameter and 1000 feet long, with 49 feet head, what will the discharge be?

If the velocity V is known in feet per second, the discharge is $0.32725 d^2 V$ cubic foot per minute.

By equation 2 we have

$$\frac{4V^2 + 5V - 2}{1200} = \frac{Hd}{L} = \frac{49 \times 5}{1000} = 0.245;$$

whence, by table, V = real velocity = 8 feet per second.

The discharge in cubic feet per minute, if V is velocity in feet per second and d diameter in inches, is $0.32725 d^2 V$, whence, discharge

$$= 0.3275 \times 25 \times 8 = 65.45 \text{ cubic feet per minute.}$$

The velocity due the head, if there were no friction, is $8.025 \sqrt{H}$ = 56.175 feet per second, and the discharge at that velocity would be

$$0.32725 \times 25 \times 56.175 = 460 \text{ cubic feet per minute.}$$

Suppose it is required to deliver this amount, 460 cubic feet, at a velocity of 2 feet per second, what diameter of pipe of the same length and under the same head will be required and what will be the loss of head by friction?

$$d = \text{diameter} = \sqrt{\frac{Q}{V \times 0.32725}} = \sqrt{\frac{460}{2 \times 0.32725}} = \sqrt{703} = 26.5 \text{ inches.}$$

Having now the diameter, the velocity, and the discharge, the friction-head is calculated by equation 1 and use of the table; thus,

$$H = \frac{L}{d} \frac{4V^2 + 5V - 2}{1200} = \frac{1000}{26.5} \times 0.02 = \frac{20}{26.5} = 0.75 \text{ foot,}$$

thus leaving $49 - 0.75$ = say 48 feet effective head applicable to power-producing purposes.

Problems of the loss of head may be solved rapidly by means of Cox's Pipe Computer, a mechanical device on the principle of the slide-rule, for sale by Keuffel & Esser, New York.

Exponential Formulæ. Williams and Hazen's Tables. — From Chezy's formula, $v = c \sqrt{rs}$, it would appear that the velocity varies as the square root of the head, or that the head varies as the square of the velocity; this is not true, however, for c is not a constant, but a variable, depending on both r and s . Hazen and Williams, as a result of a study of the best records of experiments and plotting them on logarithmic ruled paper, found an exponential formula $v = cr^{0.63} s^{0.54}$, in which the coefficient c is practically independent of the diameter and the slope, and varies only with the condition of the surface. In order to equalize the numerical value of c to that of the c in the Chezy formula, at a slope of 0.001, they added the factor $0.001^{-0.04}$ to the formula, so that the working formula of Hazen and Williams is

$$v = cr^{0.63} s^{0.54} 0.001^{-0.04}.$$

Approximate values given for c are:

140 for the very best cast-iron pipe, laid straight and when new.

130 for good, new cast-iron pipe, very smooth; good masonry aqueducts; small brass pipes.*

120 for cast-iron pipe 5 years old; riveted steel pipe, new.

110 for cast-iron pipe 10 years old; steel pipe 10 years old; brick sewers.

100 for cast-iron pipe 17 years old, rough.

90 for cast-iron pipe 26 years old, rough.

80 for cast-iron pipe 37 years old, very rough.

* 130 may also be used for straight lead, tin, and drawn copper pipes. Computations of the exponential formula are made by logarithms, or by the Hazen-Williams hydraulic slide rule. On logarithmic ruled paper values of v for different values of c , r and s may be plotted in straight lines. (See "Hydraulic Tables," by Williams and Hazen, John Wiley & Sons.)

Friction Loss in Clean Cast-Iron Pipe.

Compiled from Weston's "Friction of Water in Pipes" as computed from formulas of Henry Darcy.

Pounds loss per 1000 feet in pipe of given diameter. (Small lower figures give Velocity in Feet per Second.)

| U. S. Gals per Min. and (Cu. Ft. per Sec.) | Diameter of Pipe in Inches. | | | | | | | | | | | |
|--|-----------------------------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|
| | 3 | 4 | 5 | 6 | 8 | 10 | 12 | 14 | 16 | 20 | 24 | 30 |
| 250 (0.56) | 60 | 20 | 6.4 | 2.5 | 0.6 | 0.2 | 0.07 | 0.03 | 0.02 | 0.01 | 0.00 | |
| 500 (1.11) | 11 | 6.4 | 4.0 | 2.8 | 1.6 | 1.2 | 0.7 | 0.52 | 0.4 | 0.26 | 0.18 | |
| 750 (1.67) | 220 | 82 | 25.8 | 10.0 | 2.3 | 0.7 | 0.29 | 0.13 | 0.07 | 0.02 | 0.01 | 0.00 |
| 1,000 (2.23) | 23 | 13.0 | 8.2 | 6.0 | 3.2 | 2.4 | 1.4 | 1.04 | 0.8 | 0.51 | 0.35 | 0.23 |
| | 477 | 184 | 58.0 | 23.0 | 5.0 | 1.6 | 0.66 | 0.30 | 0.15 | 0.05 | 0.02 | |
| | 34 | 19.0 | 12.2 | 8.0 | 4.8 | 3.1 | 2.1 | 1.56 | 1.2 | 0.77 | 0.53 | |
| | | 328 | 103.0 | 40.0 | 9.0 | 2.9 | 1.20 | 0.53 | 0.27 | 0.09 | 0.03 | 0.01 |
| | | 26.0 | 16.3 | 11.0 | 6.4 | 4.1 | 2.8 | 2.08 | 1.6 | 1.0 | 0.71 | 0.45 |
| 1,250 (2.79) | | | 161.0 | 63.0 | 14.0 | 4.6 | 1.80 | 0.83 | 0.42 | 0.14 | 0.06 | |
| 1,500 (3.34) | | | 20.4 | 14.0 | 8.0 | 5.1 | 3.6 | 2.60 | 2.0 | 1.3 | 0.89 | |
| 1,750 (3.90) | | | 231.9 | 91.0 | 21.0 | 6.6 | 2.60 | 1.10 | 0.61 | 0.20 | 0.08 | 0.03 |
| 2,000 (4.46) | | | 24.5 | 17.0 | 10.0 | 6.1 | 4.3 | 3.13 | 2.4 | 1.5 | 1.06 | 0.68 |
| | | | | 123.0 | 28.0 | 9.0 | 3.60 | 1.6 | 0.83 | 0.27 | 0.11 | |
| | | | | 20.0 | 11.0 | 7.1 | 5.0 | 3.65 | 2.8 | 1.8 | 1.24 | |
| | | | | 160.0 | 37.0 | 12.0 | 4.70 | 2.14 | 1.10 | 0.35 | 0.14 | 0.05 |
| | | | | 23.0 | 13.0 | 8.2 | 5.7 | 4.17 | 3.2 | 2.0 | 1.42 | 0.91 |
| 2,500 (5.57) | Diam. of Pipe in In. | | | | 58.0 | 18.0 | 7.30 | 3.34 | 1.70 | 0.55 | 0.22 | 0.07 |
| 3,000 (6.68) | | | | | 16.0 | 10.2 | 7.1 | 5.21 | 4.0 | 2.6 | 1.80 | 1.13 |
| 4,000 (8.91) | | | | | | 26.0 | 10.00 | 4.81 | 2.40 | 0.79 | 0.32 | 0.10 |
| 5,000 (11.14) | 36 | 48 | | | | 12.0 | 8.5 | 6.25 | 4.8 | 3.1 | 2.10 | 1.40 |
| | | | | | | | | 8.55 | 4.30 | 1.40 | 0.56 | 0.18 |
| | | | | | | | | 8.34 | 6.4 | 4.1 | 2.80 | 1.80 |
| | 0.11 | 0.03 | | | | | | | 6.80 | 2.20 | 1.00 | 0.29 |
| | 1.6 | 0.89 | | | | | | | 8.0 | 5.1 | 3.60 | 2.30 |
| 6,000 (13.37) | 0.16 | 0.04 | | | | | | | | 3.20 | 1.30 | 0.41 |
| 7,000 (15.60) | 1.9 | 1.06 | | | | | | | | 6.1 | 4.30 | 2.70 |
| 8,000 (17.82) | 0.23 | 0.05 | | | | | | | | 4.30 | 1.70 | 0.56 |
| 9,000 (20.05) | 2.2 | 1.2 | | | | | | | | 7.1 | 5.00 | 3.20 |
| 10,000 (22.28) | 0.29 | 0.07 | | | | | | | | | 2.20 | 0.73 |
| | 2.5 | 1.4 | | | | | | | | | 5.70 | 3.60 |
| | 0.37 | 0.09 | | | | | | | | | 2.80 | 0.92 |
| | 2.8 | 1.6 | | | | | | | | | 6.40 | 4.10 |
| | 0.45 | 0.11 | | | | | | | | | | 1.13 |
| | 3.1 | 1.8 | | | | | | | | | | 4.50 |

| | | | | | | | | | | | | |
|--------------------|-------|-------|------|------|------|------|------|-----|-----|------|------|------|
| Vel. ft. per sec.. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Hd. due vel. ft. | 0.016 | 0.062 | 0.14 | 0.25 | 0.39 | 0.56 | 0.76 | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 |
| Vel. ft. per sec.. | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 25 | 30 | 40 | 50 |
| Hd. due vel. ft.. | 2.6 | 3.1 | 3.5 | 4.0 | 4.5 | 5.0 | 5.6 | 6.2 | 9.3 | 14.0 | 24.8 | 38.8 |

These losses are for *new, clean, straight, tar-coated, cast-iron pipes*. For pipes that have been in service a number of years the losses will be larger on account of corrosion and incrustation, and the losses in the tables should be multiplied under average conditions by the factors opposite; but they must be used with much discretion, for some waters corrode pipes much more rapidly than others.

The same figures may be used for wrought-iron pipes which are not subject to a frequent change of water.

Approximate Hydraulic Formulæ. (The Lombard Governor Co., Boston, Mass.)

Head (H) in feet. Pressure (P) in lbs. per sq. in. Diameter (D) in feet. Area (A) in sq. ft. Quantity (Q) in cubic ft. per second. Time (T) in seconds.

$$\text{Spouting velocity} = 8.02 \sqrt{H}.$$

Time (T_1) to acquire spouting velocity in a vertical pipe, or (T_2) in a pipe on an angle (θ) from horizontal:

$$T_1 = 8.02 \sqrt{H} + 32.17, \quad T_2 = 8.02 \sqrt{H} + 32.17 \sin \theta.$$

Head (H) or pressure (P) which will vent any quantity (Q) through a round orifice of any diameter (D) or area (A):

$$H = Q^2 + 14.1 D^4 = Q^2 + 23.75 A^2; \quad P = Q^2 + 34.1 D^4 = Q^2 + 55.3 A^2.$$

Quantity (Q) discharged through a round orifice of any diameter (D) or area (A) under any pressure (P) or under any head (H):

$$Q = \sqrt{P \times 55.3 \times A^2} = \sqrt{P \times 34.1 \times D^4}; \\ = \sqrt{H \times 23.75 \times A^2} = \sqrt{H \times 14.71 \times D^4}.$$

Diameter (D) or area (A) of a round orifice to vent any quantity (Q) under any head (H) or under any pressure (P):

$$D = \sqrt{Q + 3.84 \sqrt{H}} = \sqrt{Q + 5.8 \sqrt{P}}; \quad A = Q + 4.89 \sqrt{H} = Q + 7.35 \sqrt{P}.$$

Time (T) of emptying a vessel of any area (A) through an orifice of any area (a) anywhere in its side: $T = 0.416 A \sqrt{H} \div a$.

Time (T) of lowering a water level from (H) to (h) in a tank of area A through an orifice of any area (a) in its side. $T = 0.416 A (\sqrt{H} - \sqrt{h}) \div a$.

Kinetic energy (K) or foot-pounds in water in a round pipe of any diameter (D) when moving at velocity (V): $K = 0.76 \times D^2 \times L \times V$.

Area (a) of an orifice to empty a tank of any area (A) in any time (T) from any head (H): $a = T \div 0.409 A \sqrt{H}$.

Area (a) of an orifice to lower water in a tank of area (A) from head (H) to (h) in time (T): $a = T \div 0.409 \times A \times (\sqrt{H} - \sqrt{h})$.

Compound Pipes and Pipes with Branches. (Unwin.) — Loss of head in a main consisting of different diameters. (1) Constant discharge. Total loss of head $H = h_1 + h_2 + h_3 = 0.1008 f Q^2 (l_1/d_1^5 + l_2/d_2^5 + l_3/d_3^5)$.

(2) Constant velocity in the main, the discharge diminishing from section to section. $H = 0.0551 f v^{5/2} (l_1/\sqrt{Q_1} + l_2/\sqrt{Q_2} + l_3/\sqrt{Q_3})$. Equivalent main of uniform diameter. Length of equivalent main

$$l = d^5 (l_1/d_1^5 + l_2/d_2^5 + l_3/d_3^5).$$

Loss of head in a main of uniform diameter in which the discharge decreases uniformly along its length, such as a main with numerous branch pipes uniformly spaced and delivering equal quantities: $h = 0.0336 f Q^2 l/d^5$, Q being the quantity entering the pipe. The loss of head is just one-third of the loss in a pipe carrying the uniform quantity Q throughout its length.

Loss of head in a pipe that receives Q cu. ft. per sec. at the inlet, and delivers Q_x cu. ft. at x ft. from the inlet, having distributed qx cu. ft. uniformly in that distance, $h_x = 0.1008 f x (Q_x + 0.55 qx)/d^5$.

Delivery by two or more mains, in parallel. Total discharge $= Q_1 + Q_2 + Q_3 = 3.149 \sqrt{h/f} (\sqrt{d_1^5/l_1} + \sqrt{d_2^5/l_2} + \sqrt{d_3^5/l_3})$. Diameter of an equivalent main to discharge the same total quantity, $d = (\sqrt{d_1^5} + \sqrt{d_2^5} + \sqrt{d_3^5})^{2/5}$.

Long Pipe Lines. — (1) Vyrnwy to Liverpool, 68 miles; 40 million gals. (British) per day. Three lines of cast-iron pipe, 42 to 39 in. diam. One of the 42-in. lines after being laid 12 years, with a hydraulic gradient of

4.5 ft. per mile, discharged 15 million gallons per day; velocity, 2.892 ft. per sec., $f = 0.00574$.

(2) East Jersey riveted steel pipe line, Newark, N. J., 21 miles long, 48 in. diam., 50 million U. S. gals. per day; velocity about 6 ft. per sec.

(3) Perth to Coolgarlie, Western Australia, 351 miles, 30 in. steel pipe with lock-bar joints. Eight pumping stations in the line. Two tests showed delivery of 5 and 5.6 million gals. per day; hydraulic gradient, 2.25 and 2.8 ft. per mile; velocity, 1.889 and 2.115 ft. per sec.; $f = 0.00480$ and 0.00486.

Rifled Pipes for Conveying Heavy Oils. (*Eng. Rec.*, May 23, 1908.)—The oil from the California fields is a heavy, viscous fluid. Attempts to handle it in long pipe lines of the ordinary type have not been practically successful. High pumping pressures are required, resulting in large expense for pipe and for pumping equipment.

The method of pumping in the rifled-pipe line is to inject about 10 per cent of water with the oil and to give the oil and water a centrifugal motion, by means of the rifled pipe, sufficient to throw the water to the outside, where it forms a thin film of lubrication between the oil and the sides of the pipe that greatly reduces the friction. The rifled pipe delivers at ordinary temperatures eight to ten times as much oil, through a long line, as does a line of ordinary pipe under similar conditions. An 8-in. rifled pipe line 282 miles in length has been built from the Kern oil fields to Porta Costa, on tidewater near San Francisco. The pipe is rifled with six helical grooves to the circumference, these grooves making a complete turn through 360 deg. in 10 ft. of length.

Loss of Pressure Caused by Valves and Fittings —The data given below are condensed from the results of experiments by John R. Freeman for the Inspection Department of the Assoc. Fauty. Mut. Ins. Cos. The friction losses in ells and tees are approximate. Fittings of the same nominal size with the different curvatures and different smoothness as made by different manufacturers will cause materially different friction losses. The figures are the number of feet of clean, straight pipe of same size which would cause the same loss as the fitting. Grinnell dry-pipe valve, 6-in., 80 ft.; 4-in., 47 ft. Grinnell alarm check, 6-in., 100 ft.; 4-in., 47 ft. Pratt & Cady check valve, 6-in., 50 ft.; 4-in., 25 ft. 4-in. Walworth globe check valve, 6-in., 200 ft.; 4-in., 130 ft. 2½ in. to 8-in. ells, long-turn, 4 ft.; short-turn 9 ft. 3-in. to 8-in. tees, long-turn, 9 ft.; short-turn, 17 ft. One-eighth bend, 5 ft.

Effect of Bends and Curves in Pipes. — Weisbach's rule for bends: Loss of head in feet = $\left[0.131 + 1.847 \left(\frac{r}{R}\right)^{7/2}\right] \times \frac{v^2}{64.4} \times \frac{a}{180}$, in which r = internal radius of pipe in feet, R = radius of curvature of axis of pipe, v = velocity in feet per second, and a = the central angle, or angle subtended by the bend.

Hamilton Smith, Jr., in his work on Hydraulics, says: The experimental data at hand are entirely insufficient to permit a satisfactory analysis of this quite complicated subject; in fact, about the only experiments of value are those made by Bossut and Dubuat with small pipes.

Curves. — If the pipe has easy curves, say with radius not less than 5 diameters of the pipe, the flow will not be materially diminished, provided the tops of all curves are kept below the hydraulic grade-line and provision be made for escape of air from the tops of all curves. (Trautwine.)

Williams, Hubbell and Fenkel (*Trans. A. S. C. E.*, 1901) conclude from an extensive series of experiments that curves of short radius, down to about 2½ diameters, offer less resistance to the flow of water than do those of longer radius, and that earlier theories and practices regarding curve resistance are incorrect. For a 90° curve in 30 in. cast-iron pipe, 6 ft. radius, they found the loss of head 15.7% greater than that of a straight pipe of equal length; with 10 ft. radius, 17.3% greater; with 25 ft. radius, 52.7 % greater; and with 60 ft. radius, 90.2% greater.

Hydraulic Grade-line. — In a straight tube of uniform diameter throughout, running full and discharging freely into the air, the hydraulic grade-line is a straight line drawn from the discharge end to a point immediately over the entry end of the pipe and at a depth below the surface equal to the entry and velocity heads. (Trautwine.)

In a pipe leading from a reservoir, no part of its length should be above the hydraulic grade-line.

Air-bound Pipes. — A pipe is said to be air-bound when, in consequence of air being entrapped at the high points of vertical curves in the line, water will not flow out of the pipe, although the supply is higher than the outlet. The remedy is to provide cocks or valves at the high points, through which the air may be discharged. The valve may be made automatic by means of a float.

Water-hammer. — Prof. I. P. Church gives the following formula for the pressure developed by the instantaneous closing of a valve in a water pipe:

$$p = vC\gamma/g,$$

in which p is pressure in lbs. per sq. in., v velocity in inches per second, C velocity of pressure wave in inches per second, and $g = 386.4$ ins. The value of C is $\sqrt{gEE_1t/\gamma(tE_1 + 2rE)}$, in which E_1 = modulus of elasticity, 30,000,000 for steel, E = bulk modulus of water = 300,000 lbs. per sq. in. at 50° F., $\gamma = 0.03604$ = lbs. of water in 1 cu. in., t = thickness of pipe, ins., and r = internal radius of pipe, ins. Example, a 16-in. steel pipe with $1/4$ -in. walls, and $v = 60$ ins. per second, gives a velocity of the pressure wave $C = 44,285$ ins. per second and a pressure per sq. in. of 2478 lbs. If the elasticity of the pipe is not considered, the formula reduces to $p = 5.29 v$, which in the example given gives a pressure of 317.4 lbs. per sq. in.

Vertical Jets. (Molesworth.) — H = head of water, h = height of jet, d = diameter of jet, K = coefficient, varying with ratio of diameter of jet to head; then $h = KH$.

| | | | | | | | | |
|-------------------|------|-----|------|------|------|------|------|------|
| If $H = d \times$ | 300 | 600 | 1000 | 1500 | 1800 | 2800 | 3500 | 4500 |
| $K =$ | 0.96 | 0.9 | 0.85 | 0.8 | 0.7 | 0.6 | 0.5 | 0.25 |

Water Delivered through Meters. (Thomson Meter Co.) — The best modern practice limits the velocity in water-pipes to 10 lineal feet per second. Assume this as a basis of delivery, and we find, for the several sizes of pipes usually metered, the following approximate results:
Nominal diameter of pipe in inches:

| | | | | | | | | | |
|---|------|------|------|------|-------|------|------|------|-------|
| | 3/8 | 5/8 | 3/4 | 1 | 1 1/2 | 2 | 3 | 4 | 6 |
| Quantity delivered, in cubic feet per minute, due to said velocity: | 0.46 | 1.28 | 1.85 | 3.28 | 7.36 | 13.1 | 29.5 | 52.4 | 117.9 |

Prices Charged for Water in Different Cities. (National Meter Co.)
Average minimum price for 1000 gallons in 163 places. 9.4 cents.
Average maximum price for 1000 gallons in 163 places. 28 "
Extremes, 2 1/2 cents to. 100 "

FIRE-STREAMS.

Fire-Stream Tables. — The table on the following page is condensed from one contained in the pamphlet of "Fire-Stream Tables" of the Associated Factory Mutual Fire Ins. Cos., based on the experiments of John R. Freeman, *Trans. A. S. C. E.*, vol. xxi, 1889.

The pressure in the first column is that indicated by a gauge attached at the base of the play pipe and set level with the end of the nozzle. The vertical and horizontal distances, in 2d and 3d cols., are those of effective fire-streams with moderate wind. The maximum limit of a "fair stream" is about 10% greater for a vertical stream; 12% for a horizontal stream. In still air much greater distances are reached by the extreme drops. The pressures given are for the best quality of rubber-lined hose, smooth inside. The hose friction varies greatly in different kinds of hose, according to smoothness of inside surface, and pressures as much as 50% greater are required for the same delivery in long lengths of inferior rubber-lined or linen hose. The pressures at the hydrant are those while the stream is flowing, and are those required with smooth nozzles. Ring nozzles require greater pressures. With the same pressures at the base of the play pipe, the discharge of a 3/4-in. smooth nozzle is the same as that of a 7/8-in. ring nozzle; of a 7/8-in. smooth nozzle, the same as that of a 1-in. ring nozzle.

The figures for hydrant pressure in the body of the table are derived by adding to the nozzle or play-pipe pressure the friction loss in the hose, and also the friction loss of a Chapman 4-way independent gate

hydrant ranging from 0.86 lb. for 200 gals. per min. flowing to 2.31 lbs. for 600 gals.

The following notes are taken from the pamphlet referred to. The discharge as stated in Ellis's tables and in their numerous copies in trade catalogues is from 15 to 20% in error.

In the best rubber-lined hose, 2½-in. diam., the loss of head due to friction, for a discharge of 240 gallons per minute, is 14.1 lbs. per 100 ft. length; in inferior rubber-lined mill hose, 25.5 lbs., and in unlined linen hose, 33.2 lbs.

Less than a 1½-in. smooth-nozzle stream with 40 lbs. pressure at the base of the play pipe, discharging about 240 gals. per min., cannot be called a first-class stream for a factory fire. 80 lbs. per sq. in. is considered the best hydrant pressure for general use; 100 lbs. should not be exceeded, except for very high buildings, or lengths of hose over 300 ft.

Hydrant Pressures Required with Different Sizes and Lengths of Hose. (J. R. Freeman, *Trans. A. S. C. E.*, 1889.)

¾-inch smooth nozzle.

| Press. Lbs. | Fire-stream Distance. | | Gal. per Min. | Hydrant Pressure with Different Lengths of Hose to Maintain Pressure at Base of Play Pipe. | | | | | | | | |
|-------------|-----------------------|------|---------------|--|---------|---------|---------|---------|---------|---------|---------|----------|
| | Vert. | Hor. | | 50 ft. | 100 ft. | 200 ft. | 300 ft. | 400 ft. | 500 ft. | 600 ft. | 800 ft. | 1000 ft. |
| 10 | 17 | 19 | 52 | 10 | 11 | 11 | 12 | 13 | 13 | 14 | 15 | 16 |
| 20 | 33 | 29 | 73 | 21 | 22 | 23 | 24 | 25 | 26 | 28 | 30 | 32 |
| 30 | 48 | 37 | 90 | 31 | 32 | 34 | 36 | 38 | 40 | 41 | 45 | 49 |
| 40 | 60 | 44 | 104 | 42 | 43 | 46 | 48 | 50 | 53 | 55 | 60 | 65 |
| 50 | 67 | 50 | 116 | 52 | 54 | 57 | 60 | 63 | 66 | 69 | 75 | 81 |
| 60 | 72 | 54 | 127 | 63 | 65 | 68 | 72 | 76 | 79 | 83 | 90 | 97 |
| 70 | 76 | 58 | 137 | 73 | 75 | 80 | 84 | 88 | 92 | 97 | 105 | 114 |
| 80 | 79 | 62 | 147 | 84 | 86 | 91 | 96 | 101 | 106 | 111 | 120 | 130 |
| 90 | 81 | 65 | 156 | 94 | 97 | 102 | 108 | 113 | 119 | 124 | 135 | 146 |
| 100 | 83 | 68 | 164 | 105 | 108 | 114 | 120 | 126 | 132 | 138 | 150 | 163 |

7/8-inch smooth nozzle.

| | | | | | | | | | | | | |
|-----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 10 | 18 | 21 | 71 | 11 | 11 | 13 | 14 | 15 | 16 | 17 | 19 | 22 |
| 20 | 34 | 33 | 100 | 22 | 23 | 25 | 27 | 30 | 32 | 34 | 39 | 43 |
| 30 | 49 | 42 | 123 | 33 | 34 | 38 | 41 | 45 | 48 | 51 | 58 | 65 |
| 40 | 62 | 49 | 142 | 43 | 46 | 50 | 55 | 59 | 64 | 68 | 78 | 87 |
| 50 | 71 | 55 | 159 | 54 | 57 | 63 | 69 | 74 | 80 | 86 | 97 | 108 |
| 60 | 77 | 61 | 174 | 65 | 69 | 75 | 82 | 89 | 96 | 103 | 116 | 130 |
| 70 | 81 | 66 | 188 | 76 | 80 | 88 | 96 | 104 | 112 | 120 | 136 | 152 |
| 80 | 85 | 70 | 201 | 87 | 91 | 101 | 110 | 119 | 128 | 137 | 155 | 173 |
| 90 | 88 | 74 | 213 | 98 | 103 | 113 | 123 | 134 | 144 | 154 | 174 | 195 |
| 100 | 90 | 76 | 224 | 109 | 114 | 126 | 137 | 148 | 160 | 171 | 194 | 216 |

1-inch smooth nozzle.

| | | | | | | | | | | | | |
|-----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-------|------|
| 10 | 18 | 21 | 93 | 12 | 12 | 14 | 16 | 18 | 20 | 22 | 26 | 30 |
| 20 | 35 | 37 | 132 | 23 | 25 | 29 | 33 | 37 | 41 | 45 | 52 | 60 |
| 30 | 51 | 47 | 161 | 34 | 37 | 43 | 49 | 55 | 61 | 67 | 79 | 90 |
| 40 | 64 | 55 | 186 | 46 | 50 | 58 | 66 | 73 | 81 | 89 | 105 | 120 |
| 50 | 73 | 61 | 208 | 57 | 62 | 72 | 82 | 92 | 102 | 111 | 131 | 151 |
| 60 | 79 | 67 | 228 | 69 | 75 | 87 | 98 | 110 | 122 | 134 | 157 | 181 |
| 70 | 85 | 72 | 246 | 80 | 87 | 101 | 115 | 128 | 142 | 156 | 183 | 211 |
| 80 | 89 | 76 | 263 | 92 | 100 | 115 | 131 | 147 | 162 | 178 | 209 | 241 |
| 90 | 92 | 80 | 279 | 103 | 112 | 130 | 147 | 165 | 183 | 200 | 236 | |
| 100 | 96 | 83 | 295 | 115 | 125 | 144 | 164 | 183 | 203 | 223 | | |

Pump Inspection Table.

Discharge of nozzles attached to 50 ft. of 2½-in. best quality rubber-lined hose, inside smooth. (J. R. Freeman.)

| Hydrant Pressure. | Size of Smooth Nozzle. | | | | | | | | Ring Nozzle. | | |
|-------------------|------------------------|-----|-----|-----|-----|-----|-----|-----|--------------|-----|-----|
| | 1¾ | 1½ | 1⅜ | 1¼ | 1⅛ | 1 | 7/8 | ¾ | 1⅜ | 1¼ | 1⅛ |
| 10 | 193 | 163 | 146 | 127 | 107 | 87 | 68 | 51 | 118 | 101 | 84 |
| 20 | 274 | 232 | 206 | 179 | 151 | 123 | 96 | 72 | 167 | 143 | 119 |
| 30 | 335 | 283 | 251 | 219 | 184 | 150 | 118 | 88 | 205 | 175 | 145 |
| 40 | 387 | 327 | 291 | 253 | 213 | 173 | 136 | 101 | 237 | 202 | 168 |
| 50 | 432 | 366 | 325 | 283 | 238 | 194 | 152 | 113 | 264 | 226 | 188 |
| 60 | 473 | 400 | 357 | 309 | 261 | 213 | 167 | 124 | 289 | 247 | 205 |
| 70 | 510 | 432 | 385 | 334 | 281 | 230 | 180 | 134 | 313 | 267 | 222 |
| 80 | 546 | 461 | 412 | 357 | 301 | 246 | 192 | 144 | 334 | 285 | 237 |
| 90 | 579 | 490 | 437 | 379 | 319 | 261 | 204 | 152 | 355 | 303 | 252 |
| 100 | 610 | 515 | 461 | 400 | 337 | 275 | 215 | 161 | 374 | 319 | 266 |

Friction Loss in Rubber-Lined Cotton Hose with Smoothest Lining.

| Diam. of Hose. | Gallons per Minute Flowing. | | | | | | | | | Velocity, Ft. per Sec. | Velocity Head $V^2 \div 2g$. | |
|----------------|---|------|------|------|------|-------|-------|-------|-------|---------------------------|----------------------------------|------|
| | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 1000 | | Ft. | Lbs. |
| | Friction Loss, Pounds per 100 ft. Length. | | | | | | | | | | | |
| 2 | 6.836 | 27.3 | 61.5 | 109 | 171 | | | | | 5 | 0.39 | 0.17 |
| 2 1/8 | 5.170 | 20.7 | 46.5 | 82.7 | 129 | 189 | | | | 10 | 1.6 | 0.69 |
| 2 1/4 | 3.790 | 15.2 | 34.1 | 60.6 | 94.7 | 136 | 186 | | | 15 | 3.5 | 1.5 |
| 2 3/8 | 2.895 | 11.6 | 26.1 | 46.3 | 72.4 | 104 | 138 | 185 | | 20 | 6.2 | 2.7 |
| 2 1/2 | 2.240 | 9.0 | 20.2 | 35.8 | 56.0 | 80.6 | 110 | 143 | 224 | 25 | 9.7 | 4.2 |
| 2 5/8 | 1.748 | 7.0 | 15.7 | 28.0 | 43.7 | 62.9 | 85.7 | 112 | 175 | 30 | 14.0 | 6.1 |
| 2 3/4 | 1.391 | 5.6 | 12.5 | 22.3 | 34.8 | 50.1 | 68.2 | 89.0 | 139 | 35 | 19.0 | 8.2 |
| 2 7/8 | 1.097 | 4.4 | 9.9 | 17.6 | 27.4 | 39.5 | 53.8 | 70.2 | 110 | 40 | 24.8 | 10.7 |
| 3 | 0.900 | 3.6 | 8.1 | 14.4 | 22.5 | 32.4 | 44.1 | 57.6 | 90 | 45 | 31.4 | 13.6 |
| 3 1/2 | 0.416 | 1.7 | 3.7 | 6.7 | 10.4 | 15.0 | 20.4 | 26.6 | 41.6 | 50 | 38.8 | 16.7 |
| 4 | 0.214 | 0.9 | 1.9 | 3.4 | 5.4 | 7.7 | 10.5 | 13.7 | 21.4 | | | |

The above table is computed on the basis of 14 lbs. per 100 ft. length of 2½-in. hose with 250 gals. per min. flowing, as found in Freeman's tests, assuming that the loss varies as the square of the quantity, and for different diameters and the same quantity inversely as the 5th power of the diameter.

Rated Capacities of Steam Fire-engines, which is perhaps one third greater than their ordinary rate of work at fires, are substantially as follows:

| | | |
|----------|------------------------|-----------------------------|
| 3d size, | 550 gals. per min., or | 792,000 gals. per 24 hours. |
| 2d " | 700 " | 1,008,000 " |
| 1st " | 900 " | 1,296,000 " |
| 1 ext., | 1,100 " | 1,584,000 " |

THE SIPHON.

The Siphon is a bent tube of unequal branches, open at both ends, and is used to convey a liquid from a higher to a lower level, over an intermediate point higher than either. Its parallel branches being in a vertical plane and plunged into two bodies of liquid whose upper surfaces are at different levels, the fluid will stand at the same level both within and without each branch of the tube when a vent or small opening is made at the bend. If the air be withdrawn from the siphon through this vent, the water will rise in the branches by the atmospheric pressure without, and when the two columns unite and the vent is closed, the liquid will flow from the upper reservoir as long as the end of the shorter branch of the siphon is below the surface of the liquid in the reservoir.

If the water was free from air the height of the bend above the supply level might be as great as 33 feet.

If A = area of cross-section of the tube in square feet, H = the difference in level between the two reservoirs in feet, D the density of the liquid in pounds per cubic foot, then ADH measures the intensity of the force which causes the movement of the fluid, and $V = \sqrt{2gH} = 8.02 \sqrt{H}$ is the theoretical velocity, in feet per second, which is reduced by the loss of head for entry and friction, as in other cases of flow of liquids through pipes. In the case of the difference of level being greater than 33 feet, however, the velocity of the water in the shorter leg is limited to that due to a height of 33 feet, or that due to the difference between the atmospheric pressure at the entrance and the vacuum at the bend.

Long Siphons. — Prof. Joseph Torrey, in the *Amer. Machinist*, describes a long siphon which was a partial failure.

The length of the pipe was 1792 feet. The pipe was 3 inches diameter, and rose at one point 9 feet above the initial level. The final level was 20 feet below the initial level. No automatic air valve was provided. The highest point in the siphon was about one third the total distance from the pond and nearest the pond. At this point a pump was placed, whose mission was to fill the pipe when necessary. This siphon would flow for about two hours and then cease, owing to accumulation of air in the pipe. When in full operation it discharged $43\frac{1}{2}$ gallons per minute. The theoretical discharge from such a sized pipe with the specified head is $55\frac{1}{2}$ gallons per minute.

Siphon on the Water-supply of Mount Vernon, N. Y. (*Eng'g News*, May 4, 1893.) — A 12-inch siphon, 925 feet long, with a maximum lift of 22.12 feet and a 45° change in alignment, was put in use in 1892 by the New York City Suburban Water Co. At its summit the siphon crosses a supply main, which is tapped to charge the siphon. The air-chamber at the siphon is 12 inches by 16 feet long. A $\frac{1}{2}$ -inch tap and cock at the top of the chamber provide an outlet for the collected air.

It was found that the siphon with air-chamber as described would run until 125 cubic feet of air had gathered, and that this took place only half as soon with a 14-foot lift as with the full lift of 22.12 feet. The siphon will operate about 12 hours without being recharged, but more water can be gotten over by charging every six hours. It can be kept running 23 hours out of 24 with only one man in attendance. With the siphon as described above it is necessary to close the valves at each end of the siphon to recharge it. It has been found by weir measurements that the discharge of the siphon before air accumulates at the summit is practically the same as through a straight pipe.

A successful siphon is described by R. S. Hale in *Jour. Assoc. Eng. Soc.*, 1900. A 2-in. galvanized pipe had been used, and it had been necessary to open a waste-pipe and thus secure a continuous flow in order to keep the siphon in operation. The trouble seemed to be due to very small air leaks in the joints. When the 2-in. iron pipe was replaced by a 1-in. lead pipe, the siphon was entirely successful. The maximum rise of the pipe above the level of the pond was 12 ft., the discharge about 350 ft. below the level, and the length 500 ft.

MEASUREMENT OF FLOWING WATER.

Piezometer. — If a vertical or oblique tube be inserted into a pipe containing water under pressure, the water will rise in the former, and the vertical height to which it rises will be the head producing the pressure at the point where the tube is attached. Such a tube is called a piezometer or pressure measure. If the water in the piezometer falls below its proper level it shows that the pressure in the main pipe has been reduced by an obstruction between the piezometer and the reservoir. If the water rises above its proper level, it indicates that the pressure there has been increased by an obstruction beyond the piezometer.

If we imagine a pipe full of water to be provided with a number of piezometers, then a line joining the tops of the columns of water in them is the hydraulic grade-line.

Pitot Tube Gauge. — The Pitot tube is used for measuring the velocity of fluids in motion. It has been used with great success in measuring the flow of natural gas. (S. W. Robinson, Report Ohio Geol. Survey, 1890.) (See also *Van Nostrand's Mag.*, vol. xxxv.) It is simply a tube so bent that a short leg extends into the current of fluid flowing from a tube, with the plane of the entering orifice opposed at right angles to the direction of the current. The pressure caused by the impact of the current is transmitted through the tube to a pressure-gauge of any kind, such as a column of water or of mercury, or a Bourdon spring-gauge. From the pressure thus indicated and the known density and temperature of the flowing gas is obtained the head corresponding to the pressure, and from this the velocity. In a modification of the Pitot tube described by Prof. Robinson, there are two tubes inserted into the pipe conveying the gas, one of which has the plane of the orifice at right angles to the current, to receive the static pressure plus the pressure due to impact; the other has the plane of its orifice parallel to the current, so as to receive the static pressure only. These tubes are connected to the legs of a *U* tube partly filled with mercury, which then registers the difference in pressure in the two tubes, from which the velocity may be calculated. Comparative tests of Pitot tubes with gas-meters, for measurement of the flow of natural gas, have shown an agreement within 3%.

It appears from experiments made by W. M. White, described in a paper before the Louisiana Eng'g Socy., 1901, by Williams, Hubbell and Fenkel (*Trans. A. S. C. E.*, 1901), and by W. B. Gregory (*Trans. A. S. M. E.*, 1903), that in the formula for the Pitot tube, $V = c \sqrt{2gH}$, in which V is the velocity of the current in feet per second, H the head in feet of the fluid corresponding to the pressure measured by the tube, and c an experimental coefficient, $c = 1$ when the plane at the point of the tube is exactly at right angles with the direction of the current, and when the static pressure is correctly measured. The total pressure produced by a jet striking an extended plane surface at right angles to it, and escaping parallel to the plate, equals twice the product of the area of the jet into the pressure calculated from the "head due the velocity," and for this case $H = 2 \times V^2/2g$ instead of $V^2/2g$; but as found in White's experiments the maximum pressure at a point on the plate exactly opposite the jet corresponds to $h = V^2/2g$. Experiments made with four different shapes of nozzles placed under the center of a falling stream of water showed that the pressure produced was capable of sustaining a column of water almost exactly equal to the height of the source of the falling water.

Tests by J. A. Knesche (*Indust. Eng'g*, Nov., 1909), in which a Pitot tube was inserted in a 4-in. water pipe, gave C = about 0.77 for velocities of 2.5 to 8 ft. per sec., and smaller values for lower velocities. He holds that the coefficient of a tube should be determined by experiment before its readings can be considered accurate.

Maximum and Mean Velocities in Pipes. — Williams, Hubbell and Fenkel (*Trans. A. S. C. E.*, 1901) found a ratio of 0.84 between the mean and the maximum velocities of water flowing in closed circular conduits, under normal conditions, at ordinary velocities; whereby observations of velocity taken at the center under such conditions, with a properly rated Pitot tube, may be relied on to give results within 3% of correctness.

The Venturi Meter, invented by Clemens Herschel, and described in a pamphlet issued by the Builders' Iron Foundry of Providence, R.I., is named from Venturi, who first called attention, in 1796, to the relation between the velocities and pressures of fluids when flowing through converging and diverging tubes. It consists of two parts — the tube, through which the water flows, and the recorder, which registers the quantity of water that passes through the tube. The tube takes the shape of two truncated cones joined in their smallest diameters by a short throat-piece. At the up-stream end and at the throat there are pressure-chambers, at which points the pressures are taken.

The action of the tube is based on that property which causes the small section of a gently expanding frustum of a cone to receive, without material resultant loss of head, as much water at the smallest diameter as is discharged at the large end, and on that further property which causes the pressure of the water flowing through the throat to be less, by virtue of its greater velocity, than the pressure at the up-stream end of the tube, each pressure being at the same time a function of the velocity at that point and of the hydrostatic pressure which would obtain were the water motionless within the pipe.

The recorder is connected with the tube by pressure-pipes which lead to it from the chambers surrounding the up-stream end and the throat of the tube. It may be placed in any convenient position within 1000 feet of the meter. It is operated by a weight and clockwork. The difference of pressure or head at the entrance and at the throat of the meter is balanced in the recorder by the difference of level in two columns of mercury in cylindrical receivers, one within the other. The inner carries a float, the position of which is indicative of the quantity of water flowing through the tube. By its rise and fall the float varies the time of contact between an integrating drum and the counters by which the successive readings are registered.

There is no limit to the sizes of the meters nor the quantity of water that may be measured. Meters with 24-inch, 36-inch, 48-inch, and even 20-foot tubes can be readily made.

Measurement by Venturi Tubes. (*Trans. A. S. C. E.*, Nov., 1887, and Jan., 1888.) — Mr. Herschel recommends the use of a Venturi tube, inserted in the force-main of the pumping engine, for determining the quantity of water discharged. Such a tube applied to a 24-inch main has a total length of about 20 feet. At a distance of 4 feet from the end nearest the engine the inside diameter of the tube is contracted to a throat having a diameter of about 8 inches. A pressure-gauge is attached to each of two chambers, the ones surrounding and communicating with the entrance or main pipe, the other with the throat. According to experiments made upon two tubes of this kind, one 4 in. in diameter at the throat and 12 in. at the entrance, and the other about 36 in. in diameter at the throat and 9 feet at its entrance, the quantity of water which passes through the tube is very nearly the theoretical discharge through an opening having an area equal to that of the throat, and a velocity which is that due to the difference in head shown by the two gauges. Mr. Herschel states that the coefficient for these two widely-varying sizes of tubes and for a wide range of velocity through the pipe, was found to be within two per cent, either way, of 98%. In other words, the quantity of water flowing through the tube per second is expressed within two per cent by the formula $W = 0.98 \times A \times \sqrt{2gh}$, in which A is the area of the throat of the tube, h the head, in feet, corresponding to the difference in the pressure of the water entering the tube and that found at the throat, and $g = 32.16$.

Measurement of Discharge of Pumping-engines by means of Nozzles. (*Trans. A. S. M. E.*, xii, 575.) — The measurement of water by computation from its discharge through orifices, or through the nozzles of fire-hose, furnishes a means of determining the quantity of water delivered by a pumping-engine which can be applied without much difficulty. John R. Freeman, *Trans. A. S. C. E.*, Nov., 1889, describes a series of experiments covering a wide range of pressures and sizes, and the results showed that the coefficient of discharge for a smooth nozzle of ordinary good form was within one-half of one per cent, either way, of 0.977; the diameter of the nozzle being accurately calipered, and the pressures being determined by means of an accurate gauge attached to a suitable piezometer at the base of the play-pipe.

In order to use this method for determining the quantity of water discharged by a pumping-engine, it would be necessary to provide a pressure-box, to which the water would be conducted, and attach to the box as many nozzles as would be required to carry off the water. According to Mr. Freeman's estimate, four 1 1/4-inch nozzles, thus connected, with a pressure of 80 lbs. per square inch, would discharge the full capacity of a two-and-a-half-million engine. He also suggests the use of a portable apparatus with a single opening for discharge, consisting essentially of a Slamese nozzle, so-called, the water being carried to it by three or more lines of fire-hose.

To insure reliability for these measurements, it is necessary that the shut-off valve in the force-main, or the several shut-off valves, should be tight, so that all the water discharged by the engine may pass through the nozzles.

Flow through Rectangular Orifices. (Approximate. See p. 698.)

CUBIC FEET OF WATER DISCHARGED PER MINUTE THROUGH AN ORIFICE ONE INCH SQUARE, UNDER ANY HEAD OF WATER FROM 3 TO 72 INCHES.

For any other orifice multiply by its area in square inches.

Formula, $Q' = 0.624 \sqrt{h'} \times a$. Q' = cu. ft. per min.; a = area in sq. in.

| Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. | Heads in inches. | Cubic Feet Discharged per min. |
|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|------------------|--------------------------------|
| 3 | 1.12 | 13 | 2.20 | 23 | 2.90 | 33 | 3.47 | 43 | 3.95 | 53 | 4.39 | 63 | 4.78 | | |
| 4 | 1.27 | 14 | 2.28 | 24 | 2.97 | 34 | 3.52 | 44 | 4.00 | 54 | 4.42 | 64 | 4.81 | | |
| 5 | 1.40 | 15 | 2.36 | 25 | 3.03 | 35 | 3.57 | 45 | 4.05 | 55 | 4.46 | 65 | 4.85 | | |
| 6 | 1.52 | 16 | 2.43 | 26 | 3.08 | 36 | 3.62 | 46 | 4.09 | 56 | 4.52 | 66 | 4.89 | | |
| 7 | 1.64 | 17 | 2.51 | 27 | 3.14 | 37 | 3.67 | 47 | 4.12 | 57 | 4.55 | 67 | 4.92 | | |
| 8 | 1.75 | 18 | 2.58 | 28 | 3.20 | 38 | 3.72 | 48 | 4.18 | 58 | 4.58 | 68 | 4.97 | | |
| 9 | 1.84 | 19 | 2.64 | 29 | 3.25 | 39 | 3.77 | 49 | 4.21 | 59 | 4.63 | 69 | 5.00 | | |
| 10 | 1.94 | 20 | 2.71 | 30 | 3.31 | 40 | 3.81 | 50 | 4.27 | 60 | 4.65 | 70 | 5.03 | | |
| 11 | 2.03 | 21 | 2.78 | 31 | 3.36 | 41 | 3.86 | 51 | 4.30 | 61 | 4.72 | 71 | 5.07 | | |
| 12 | 2.12 | 22 | 2.84 | 32 | 3.41 | 42 | 3.91 | 52 | 4.34 | 62 | 4.74 | 72 | 5.09 | | |

Measurement of an Open Stream by Velocity and Cross-section. —

Measure the depth of the water at from 6 to 12 points across the stream at equal distances between. Add all the depths in feet together and divide by the number of measurements made; this will be the average depth of the stream, which multiplied by its width will give its area or cross-section. Multiply this by the velocity of the stream in feet per minute, and the result will be the discharge in cubic feet per minute of the stream.

The velocity of the stream can be found by laying off 100 feet of the bank and throwing a float into the middle, noting the time taken in passing over the 100 ft. Do this a number of times and take the average; then, dividing this distance by the time gives the velocity at the surface. As the top of the stream flows faster than the bottom or sides — the average velocity being about 83% of the surface velocity at the middle — it is convenient to measure a distance of 120 feet for the float and reckon it as 100.

Miner's Inch Measurements. (Pelton Water Wheel Co.)

The cut, Fig. 141, shows the form of measuring-box ordinarily used, and the following table gives the discharge in cubic feet per minute of a miner's inch of water, as measured under the various heads and different lengths and heights of apertures used in California.

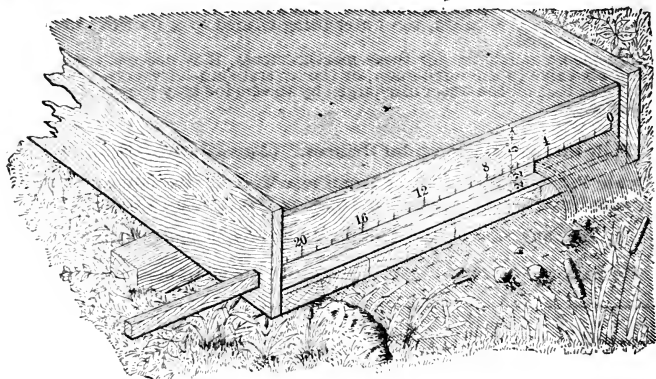


FIG. 141.

| Length of Opening in Inches. | Openings 2 Inches High. | | | Openings 4 Inches High. | | |
|--|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| | Head to Center, 5 inches. | Head to Center, 6 inches. | Head to Center, 7 inches. | Head to Center, 5 inches. | Head to Center, 6 inches. | Head to Center, 7 inches. |
| | Cu. ft. | Cu. ft. | Cu. ft. | Cu. ft. | Cu. ft. | Cu. ft. |
| 4 | 1.348 | 1.473 | 1.589 | 1.320 | 1.450 | 1.570 |
| 6 | 1.355 | 1.480 | 1.596 | 1.336 | 1.470 | 1.595 |
| 8 | 1.359 | 1.484 | 1.600 | 1.344 | 1.481 | 1.608 |
| 10 | 1.361 | 1.485 | 1.602 | 1.349 | 1.487 | 1.615 |
| 12 | 1.363 | 1.487 | 1.604 | 1.352 | 1.491 | 1.620 |
| 14 | 1.364 | 1.488 | 1.604 | 1.354 | 1.494 | 1.623 |
| 16 | 1.365 | 1.489 | 1.605 | 1.356 | 1.496 | 1.626 |
| 18 | 1.365 | 1.489 | 1.606 | 1.357 | 1.498 | 1.628 |
| 20 | 1.365 | 1.490 | 1.606 | 1.359 | 1.499 | 1.630 |
| 22 | 1.366 | 1.490 | 1.607 | 1.359 | 1.500 | 1.631 |
| 24 | 1.366 | 1.490 | 1.607 | 1.360 | 1.501 | 1.632 |
| 26 | 1.366 | 1.490 | 1.607 | 1.361 | 1.502 | 1.633 |
| 28 | 1.367 | 1.491 | 1.607 | 1.361 | 1.503 | 1.634 |
| 30 | 1.367 | 1.491 | 1.608 | 1.362 | 1.503 | 1.635 |
| 40 | 1.367 | 1.492 | 1.608 | 1.363 | 1.505 | 1.637 |
| 50 | 1.368 | 1.493 | 1.609 | 1.364 | 1.507 | 1.639 |
| 60 | 1.368 | 1.493 | 1.609 | 1.365 | 1.508 | 1.640 |
| 70 | 1.368 | 1.493 | 1.609 | 1.365 | 1.508 | 1.641 |
| 80 | 1.368 | 1.493 | 1.609 | 1.366 | 1.509 | 1.641 |
| 90 | 1.369 | 1.493 | 1.610 | 1.366 | 1.509 | 1.641 |
| 100 | 1.369 | 1.494 | 1.610 | 1.366 | 1.509 | 1.642 |

NOTE. — The apertures from which the above measurements were obtained were through material $1\frac{1}{4}$ inches thick, and the lower edge 2 inches above the bottom of the measuring-box, thus giving full contraction.

Flow of Water Over Weirs. Weir Dam Measurement. (Pelton Water Wheel Co.) — Place a board or plank in the stream, as shown in the sketch, at some point where a pond will form above. The length of the notch in the dam should be from two to four times its depth for small quantities and longer for large quantities. The edges of the notch should be beveled toward the intake side, as shown. The overfall below the notch should not be less than twice its depth. Francis says a fall below the crest equal to one-half the head is sufficient, but there must be a free access of air under the sheet.

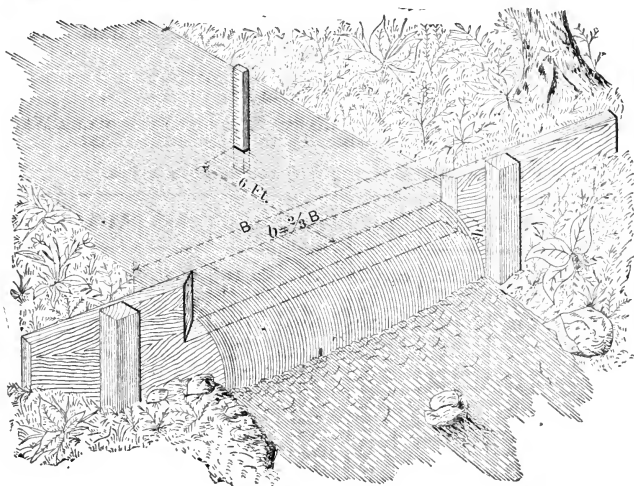


FIG. 142.

In the pond, about 6 ft. above the dam, drive a stake, and then obstruct the water until it rises precisely to the bottom of the notch and mark the stake at this level. Then complete the dam so as to cause all the water to flow through the notch, and, after time for the water to settle, mark the stake again for this new level. If preferred the stake can be driven with its top precisely level with the bottom of the notch and the depth of the water be measured with a rule after the water is flowing free, but the marks are preferable in most cases. The stake can then be withdrawn; and the distance between the marks is the theoretical depth of flow corresponding to the quantities in the weir table on the following page.

Francis's Formulæ for Weirs.

| | As given by Francis. | As modified by Smith. |
|---|--------------------------------|--|
| Weirs with both end contractions suppressed | $Q = 3.33 l h^{3/2}$ | $3.29 \left(l + \frac{h}{7} \right) h^{3/2}$ |
| Weirs with one end contraction suppressed | $Q = 3.33 (l - 0.1 h) h^{3/2}$ | $3.29 l h^{3/2}$ |
| Weirs with full contraction. | $Q = 3.33 (l - 0.2 h) h^{3/2}$ | $3.29 \left(l - \frac{h}{10} \right) h^{3/2}$ |

The greatest variation of the Francis formulæ from the values of c given by Smith amounts to $3 \frac{1}{2}\%$. The modified Francis formulæ, says Smith,

will give results sufficiently exact, when great accuracy is not required, within the limits of h , from 0.5 ft. to 2 ft., l being not less than $3h$.

Q = discharge in cubic feet per second, l = length of weir in feet, h = effective head in feet, measured from the level of the crest to the level of still water above the weir.

If Q' = discharge in cubic feet per minute, and l' and h' are taken in inches, the first of the above formulæ reduces to $Q' = 0.4 l' h'^{3/2}$. From this formula the following table is calculated. The values are sufficiently accurate for ordinary computations of water-power for weirs without end contraction, that is, for a weir the full width of the channel of approach. For weirs with full end contraction multiply the values taken from the table by the length of the weir crest in inches less 0.2 times the head in inches, to obtain the discharge.

Weir Table.

GIVING CUBIC FEET OF WATER PER MINUTE THAT WILL FLOW OVER A WEIR ONE INCH WIDE AND FROM $1/8$ TO $207/8$ INCHES DEEP.

For other widths multiply by the width in inches.

| Depth. | | $1/8$ in. | $1/4$ in. | $3/8$ in. | $1/2$ in. | $5/8$ in. | $3/4$ in. | $7/8$ in. |
|--------|---------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| in. | cu. ft. | cu. ft. | cu. ft. | cu. ft. | cu. ft. | cu. ft. | cu. ft. | cu. ft. |
| 0 | .00 | .01 | .05 | .09 | .14 | .19 | .26 | .32 |
| 1 | .40 | .47 | .55 | .64 | .73 | .82 | .92 | 1.02 |
| 2 | 1.13 | 1.23 | 1.35 | 1.46 | 1.58 | 1.70 | 1.82 | 1.95 |
| 3 | 2.07 | 2.21 | 2.34 | 2.48 | 2.61 | 2.76 | 2.90 | 3.05 |
| 4 | 3.20 | 3.35 | 3.50 | 3.66 | 3.81 | 3.97 | 4.14 | 4.30 |
| 5 | 4.47 | 4.64 | 4.81 | 4.98 | 5.15 | 5.33 | 5.51 | 5.69 |
| 6 | 5.87 | 6.06 | 6.25 | 6.44 | 6.62 | 6.82 | 7.01 | 7.21 |
| 7 | 7.40 | 7.60 | 7.80 | 8.01 | 8.21 | 8.42 | 8.63 | 8.83 |
| 8 | 9.05 | 9.26 | 9.47 | 9.69 | 9.91 | 10.13 | 10.35 | 10.57 |
| 9 | 10.80 | 11.02 | 11.25 | 11.48 | 11.71 | 11.94 | 12.17 | 12.41 |
| 10 | 12.64 | 12.88 | 13.12 | 13.36 | 13.60 | 13.85 | 14.09 | 14.34 |
| 11 | 14.59 | 14.84 | 15.09 | 15.34 | 15.59 | 15.85 | 16.11 | 16.36 |
| 12 | 16.62 | 16.88 | 17.15 | 17.41 | 17.67 | 17.94 | 18.21 | 18.47 |
| 13 | 18.74 | 19.01 | 19.29 | 19.56 | 19.84 | 20.11 | 20.39 | 20.67 |
| 14 | 20.95 | 21.23 | 21.51 | 21.80 | 22.08 | 22.37 | 22.65 | 22.94 |
| 15 | 23.23 | 23.52 | 23.82 | 24.11 | 24.40 | 24.70 | 25.00 | 25.30 |
| 16 | 25.60 | 25.90 | 26.20 | 26.50 | 26.80 | 27.11 | 27.42 | 27.72 |
| 17 | 28.03 | 28.34 | 28.65 | 28.97 | 29.28 | 29.59 | 29.91 | 30.22 |
| 18 | 30.54 | 30.86 | 31.18 | 31.50 | 31.82 | 32.15 | 32.47 | 32.80 |
| 19 | 33.12 | 33.45 | 33.78 | 34.11 | 34.44 | 34.77 | 35.10 | 35.44 |
| 20 | 35.77 | 36.11 | 36.45 | 36.78 | 37.12 | 37.46 | 37.80 | 38.15 |

When the velocity of the approaching water is less than $1/2$ foot per second, the result obtained by the table is fairly accurate. When the velocity of approach is greater than $1/2$ foot per second, a correction should be applied, see page 698.

For more accurate computations, the coefficients of flow of Hamilton Smith, Jr., or of Bazin should be used. In Smith's *Hydraulics* will be found a collection of results of experiments on orifices and weirs of various shapes made by many different authorities, together with a discussion of their several formulæ. (See also Trautwine's *Pocket Book*, Unwin's *Hydraulics*, and Church's *Mechanics of Engineering*.)

Bazin's Experiments. — M. Bazin (*Annales des Ponts et Chaussées*, Oct., 1888, translated by Marichal and Trautwine, Proc. Engrs. Club of Phila., Jan., 1890) made an extensive series of experiments with a sharp-crested weir without lateral contraction, the air being admitted freely behind the falling sheet, and found values of m varying from 0.42 to 0.50, with variations of the length of the weir from $19\frac{3}{4}$ to $78\frac{3}{4}$ in., of the height of the crest above the bottom of the channel from 0.79 to 2.46 ft.,

and of the head from 1.97 to 23.62 in. From these experiments he deduces the following formula:

$$Q = \left[0.425 + 0.21 \left(\frac{H}{P+H} \right)^2 \right] LH \sqrt{2gH},$$

In which P is the height in feet of the crest of the weir above the bottom of the channel of approach, L the length of the weir, H the head, both in feet, and Q the discharge in cu. ft. per sec. This formula, says M. Bazin, is entirely practical where errors of 2% to 3% are admissible. The following table is condensed from M. Bazin's paper:

VALUES OF THE COEFFICIENT m IN THE FORMULA $Q = mLH \sqrt{2gH}$, FOR A SHARP-CRESTED WEIR WITHOUT LATERAL CONTRACTION; THE AIR BEING ADMITTED FREELY BEHIND THE FALLING SHEET.

| Head, H . | | Height of Crest of Weir Above Bed of Channel. | | | | | | | | | |
|-------------|-------|---|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|----------------------|
| | | Feet . . . 0.65 Inches 7.87 | 0.98 11.81 | 1.31 15.75 | 1.64 19.69 | 1.97 23.62 | 2.62 31.50 | 3.28 39.38 | 4.92 59.07 | 6.56 78.76 | ∞ ∞ |
| Ft. | In. | m | m | m | m | m | m | m | m | m | m |
| 0.164 | 1.97 | 0.458 | 0.453 | 0.451 | 0.450 | 0.449 | 0.449 | 0.449 | 0.448 | 0.448 | 0.4481 |
| 0.230 | 2.76 | 0.455 | 0.448 | 0.445 | 0.443 | 0.442 | 0.441 | 0.440 | 0.440 | 0.439 | 0.4391 |
| 0.295 | 3.54 | 0.457 | 0.447 | 0.442 | 0.440 | 0.438 | 0.436 | 0.436 | 0.435 | 0.434 | 0.4340 |
| 0.394 | 4.72 | 0.462 | 0.443 | 0.442 | 0.438 | 0.436 | 0.433 | 0.432 | 0.430 | 0.430 | 0.4291 |
| 0.525 | 6.30 | 0.471 | 0.453 | 0.44 | 0.438 | 0.435 | 0.431 | 0.429 | 0.427 | 0.426 | 0.4246 |
| 0.656 | 7.87 | 0.480 | 0.459 | 0.447 | 0.440 | 0.436 | 0.431 | 0.428 | 0.425 | 0.423 | 0.4215 |
| 0.787 | 9.45 | 0.488 | 0.465 | 0.452 | 0.444 | 0.438 | 0.432 | 0.428 | 0.424 | 0.422 | 0.4194 |
| 0.919 | 11.02 | 0.496 | 0.472 | 0.457 | 0.448 | 0.441 | 0.433 | 0.429 | 0.424 | 0.422 | 0.4181 |
| 1.050 | 12.60 | | 0.478 | 0.462 | 0.452 | 0.444 | 0.436 | 0.430 | 0.424 | 0.421 | 0.4168 |
| 1.181 | 14.17 | | 0.483 | 0.457 | 0.456 | 0.448 | 0.438 | 0.432 | 0.424 | 0.421 | 0.4156 |
| 1.312 | 15.75 | | 0.489 | 0.472 | 0.459 | 0.451 | 0.440 | 0.433 | 0.424 | 0.421 | 0.4144 |
| 1.444 | 17.32 | | 0.494 | 0.476 | 0.463 | 0.454 | 0.442 | 0.435 | 0.425 | 0.421 | 0.4134 |
| 1.575 | 18.90 | | | 0.480 | 0.467 | 0.457 | 0.444 | 0.436 | 0.425 | 0.421 | 0.4122 |
| 1.706 | 20.47 | | | 0.483 | 0.470 | 0.460 | 0.446 | 0.438 | 0.426 | 0.421 | 0.4112 |
| 1.837 | 22.05 | | | 0.487 | 0.473 | 0.463 | 0.448 | 0.439 | 0.427 | 0.421 | 0.4101 |
| 1.969 | 23.62 | | | 0.490 | 0.476 | 0.466 | 0.451 | 0.441 | 0.427 | 0.421 | 0.4092 |

A comparison of the results of this formula with those of experiments, says M. Bazin, justifies us in believing that, except in the unusual case of a very low weir (which should always be avoided), the preceding table will give the coefficient m in all cases within 1%; provided, however, that the arrangements of the standard weir are exactly reproduced. It is especially important that the admission of the air behind the falling sheet be perfectly assured. If this condition is not complied with, m may vary within much wider limits. The type adopted gives the least possible variation in the coefficient.

The Cippoletti, or Trapezoidal Weir. — Cippoletti found that by using a trapezoidal weir with the sides inclined 1 horizontal to 4 vertical, with end contraction, the discharge is equal to that of a rectangular weir without end contraction (that is with the width of the weir equal to the width of the channel) and is represented by the simple formula $Q = 3.367 LH^{3/2}$. A. D. Flinn and C. W. D. Dyer (*Trans. A. S. C. E.*, 1894), in experiments with a trapezoidal weir, with values of L from 3 to 9 ft. and of H from 0.24 to 1.40 ft., found the value of the coefficient to average 3.334, the water being measured by a rectangular weir and the results being computed by Francis's formula, and 3.354 when Smith's formula was used. They conclude that Cippoletti's formula when applied to a properly constructed trapezoidal weir will give the discharge with an error due to combined inaccuracies, not greater than 1%.

WATER-POWER.

Power of a Fall of Water — Efficiency. — The gross power of a fall of water is the product of the weight of water discharged in a unit of time into the total head, i.e., the difference of vertical elevation of the upper surface of the water at the points where the fall in question begins and ends. The term "head" used in connection with water-wheels is the difference in height from the surface of the water in the wheel-pit to the surface in the pen-stock when the wheel is running.

If Q = cubic feet of water discharged per second, D = weight of a cubic foot of water = 62.36 lbs. at 60° F., H = total head in feet; then

DQH = gross power in foot-pounds per second,
and $DQH \div 550 = 0.1134 QH$ = gross horse-power.

If Q' is taken in cubic feet per minute, $H.P. = \frac{Q'H \times 62.36}{33,000} = .00189 Q'H$.

A water-wheel or motor of any kind cannot utilize the whole of the head H , since there are losses of head at both the entrance to and the exit from the wheel. There are also losses of energy due to friction of the water in its passage through the wheel. The ratio of the power developed by the wheel to the gross power of the fall is the efficiency of the wheel. For 75% efficiency, net horse-power = $0.00142 Q'H = \frac{Q'H}{706}$.

A head of water can be made use of in one or other of the following ways, viz.:

- 1st. By its weight, as in the water-balance and in the overshot-wheel.
- 2d. By its pressure, as in turbines and in the hydraulic engine, hydraulic press, crane, etc.
- 3d. By its impulse, as in the undershot-wheel, and in the Pelton wheel.
- 4th. By a combination of the above.

Horse-power of a Running Stream. — The gross horse-power is $H.P. = QH \times 62.36 \div 550 = 0.1134 QH$, in which Q is the discharge in cubic feet per second actually impinging on the float or bucket, and H = theoretical head due to the velocity of the stream = $\frac{v^2}{2g} = \frac{v^2}{64.4}$, in which v is the velocity in feet per second. If Q' be taken in cubic feet per minute, $H.P. = 0.00189 Q'H$.

Thus, if the floats of an undershot-wheel driven by a current alone be 5 feet \times 1 foot, and the velocity of stream = 210 ft. per minute, or 3½ ft. per sec., of which the theoretical head is 0.19 ft., $Q = 5$ sq. ft. \times 210 = 1050 cu. ft. per minute; $H.P. = 1050 \times 0.19 \times 0.00189 = 0.377$ H.P.

The wheels would realize only about 0.4 of this power, on account of friction and slip, or 0.151 H.P., or about 0.03 H.P. per square foot of float, which is equivalent to 33 sq. ft. of float per H.P.

Current Motors. — A current motor could only utilize the whole power of a running stream if it could take all the velocity out of the water, so that it would leave the floats or buckets with no velocity at all; or in other words, it would require the backing up of the whole volume of the stream until the actual head was equivalent to the theoretical head due to the velocity of the stream. As but a small fraction of the velocity of the stream can be taken up by a current motor, its efficiency is very small. Current motors may be used to obtain small amounts of power from large streams, but for large powers they are not practicable.

Bernoulli's Theorem. — **Energy of Water Flowing in a Tube.** —

The head due to the velocity is $\frac{v^2}{2g}$; the head due to the pressure is $\frac{f}{w}$; the head due to actual height above the datum plane is h feet. The total head is the sum of these = $\frac{v^2}{2g} + h + \frac{f}{w}$, in feet, in which v = velocity in feet per second, f = pressure in lbs. per sq. ft., w = weight of 1 cu. ft. of water =

62.36 lbs. If p = pressure in lbs. per sq. in., $\frac{f}{w} = 2.309 p$. If a constant quantity of water is flowing through a tube in a given time, the velocity varying at different points on account of changes in the diameter, the energy remains constant (loss by friction excepted) and the sum of the three heads is constant, the pressure head increasing as the velocity decreases, and *vice-versa*. This principle is known as "Bernoulli's Theorem."

In hydraulic transmission the velocity and the height above datum are usually small compared with the pressure-head. The work or energy of a given quantity of water under pressure = its volume in cubic feet \times its pressure in lbs. per sq. ft.; or if Q = quantity in cubic feet per second, and p = pressure in lbs. per square inch, $W = 144 pQ$, and the H.P. = $\frac{144 pQ}{550} = 0.2618 pQ$.

Maximum Efficiency of a Long Conduit. — A. L. Adams and R. C. Gemmell (*Eng'g News*, May 4, 1893) show by mathematical analysis that the conditions for securing the maximum amount of power through a long conduit of fixed diameter, without regard to the economy of water, is that the draught from the pipe should be such that the frictional loss in the pipe will be equal to one-third of the entire static head.

Mill-Power. — A "mill-power" is a unit used to rate a water-power for the purpose of renting it. The value of the unit is different in different localities. The following are examples (from Emerson):

Holyoke, Mass. — Each mill-power at the respective falls is declared to be the right during 16 hours in a day to draw 38 cu. ft. of water per second at the upper fall when the head there is 20 feet, or a quantity proportionate to the height at the falls. This is equal to 86.2 horse-power as a maximum.

Lowell, Mass. — The right to draw during 15 hours in the day so much water as shall give a power equal to 25 cu. ft. a second at the great fall, when the fall there is 30 feet. Equal to 85 H.P. maximum.

Lawrence, Mass. — The right to draw during 16 hours in a day so much water as shall give a power equal to 30 cu. ft. per second when the head is 25 feet. Equal to 85 H.P. maximum.

Minneapolis, Minn. — 30 cu. ft. of water per second with head of 22 feet. Equal to 74.8 H.P.

Manchester, N.H. — Divide 725 by the number of feet of fall minus 1, and the quotient will be the number of cubic feet per second in that fall. For 20 feet fall this equals 38.1 cu. ft., equal to 86.4 H.P. maximum.

Cohoes, N.Y. — "Mill-power" equivalent to the power given by 6 cu. ft. per second, when the fall is 20 feet. Equal to 13.6 H.P., maximum.

Passaic, N.J. — Mill-power: The right to draw $8\frac{1}{2}$ cu. ft. of water per sec., fall of 22 feet, equal to 21.2 horse-power. Maximum rental \$700 per year for each mill-power = \$33.00 per H.P.

The horse-power maximum above given is that due theoretically to the weight of water and the height of the fall, assuming the water-wheel to have perfect efficiency. It should be multiplied by the efficiency of the wheel, say 75% for good turbines, to obtain the H.P. delivered by the wheel.

Value of a Water-power. — In estimating the value of a water-power, especially where such value is used as testimony for a plaintiff whose water-power has been diminished or confiscated, it is a common custom for the person making such estimate to say that the value is represented by a sum of money which, when put at interest, would maintain a steam-plant of the same power in the same place.

Mr. Charles T. Main (*Trans. A. S. M. E.*, xiii. 140) points out that this system of estimating is erroneous; that the value of a power depends upon a great number of conditions, such as location, quantity of water, fall or head, uniformity of flow, conditions which fix the expense of dams, canals, foundations of buildings, freight charges for fuel, raw materials and finished product, etc. He gives an estimate of relative cost of steam and water-power for a 500 H.P. plant from which the following is condensed:

The amount of heat required per H.P. varies with different kinds of business, but in an average plain cotton-mill, the steam required for heating and slashing is equivalent to about 25% of steam exhausted from the high-pressure cylinder of a compound engine of the power required to run that mill, the steam to be taken from the receiver.

The coal consumption per H. P. per hour for a compound engine is taken at $1\frac{3}{4}$ lbs. per hour, when no steam is taken from the receiver for heating purposes. The gross consumption when 25% is taken from the receiver is about 2.06 lbs.

75% of the steam is used as in a compound engine at 1.75 lbs. = 1.31 lbs.
25% of the steam is used as in a high-pressure engine at 3.00 lbs. = .75 lb.

2.06 lbs.

The running expenses per H. P. per year are as follows:

| | |
|--|--------|
| 2.06 lbs. coal per hour = 21.115 lbs. for $10\frac{1}{4}$ hours or one day = | |
| 6503.42 lbs. for 308 days, which, at \$3.00 per long ton = | \$8.71 |
| Attendance of boilers, one man @ \$2.00, and one man @ \$1.25 = | 2.00 |
| Attendance of engine, one man @ \$3.50. | 2.16 |
| Oil, waste, and supplies. | .80 |

The cost of such a steam-plant in New England and vicinity of 500 H. P. is about \$65 per H. P. Taking the fixed expenses as 4% on engine, 5% on boilers, and 2% on other portions, repairs at 2%, interest at 5%, taxes at $1\frac{1}{2}$ % on $\frac{3}{4}$ cost, and insurance at $\frac{1}{2}$ % on exposed portion, the total average per cent is about $12\frac{1}{2}$ %, or $\$65 \times 0.12\frac{1}{2}$ =

8.13

Gross cost of power and low-pressure steam per H. P. **\$21.80**

Comparing this with water-power, Mr. Main says: "At Lawrence the cost of dam and canals was about \$650,000, or \$65 per H. P. The cost per H. P. of wheel-plant from canal to river is about \$45 per H. P. of plant, or about \$65 per H. P. used, the additional \$20 being caused by making the plant large enough to compensate for fluctuation of power due to rise and fall of river. The total cost per H. P. of developed plant is then about \$130 per H. P. Placing the depreciation on the whole plant at 2%, repairs at 1%, interest at 5%, taxes and insurance at 1%, or a total of 9%, gives:

| | |
|--|---------|
| Fixed expenses per H. P. $\$1.30 \times .09 =$ | \$11.70 |
| Running expenses per H. P. (Estimated) | 2.00 |

\$13.70

"To this has to be added the amount of steam required for heating purposes, said to be about 25% of the total amount used, but in winter months the consumption is at least $37\frac{1}{2}$ %. It is therefore necessary to have a boiler plant of about $37\frac{1}{2}$ % of the size of the one considered with the steam-plant, costing about $\$20 \times 0.375 = \7.50 per H. P. of total power used. The expense of running this boiler-plant is, per H. P. of the total plant per year:

| | |
|---|---------------|
| Fixed expenses $12\frac{1}{2}$ % on \$7.50..... | \$0.94 |
| Coal..... | 3.26 |
| Labor..... | 1.23 |
| Total..... | \$5.43 |

Making a total cost per year for water-power with the auxiliary boiler plant $\$13.70 + \$5.43 = \$19.13$ which deducted from \$21.80 makes a difference in favor of water-power of \$2.67, or for 10,000 H. P. a saving of \$26,700 per year.

"It is fair to say," says Mr. Main, "that the value of this constant power is a sum of money which when put at interest will produce the saving; or if 6% is a fair interest to receive on money thus invested the value would be $\$26,700 \div 0.06 = \$445,000$."

Mr. Main makes the following general statements as to the value of a water-power: "The value of an undeveloped variable power is usually nothing if its variation is great, unless it is to be supplemented by a steam-plant. It is of value then only when the cost per horse-power for the double-plant is less than the cost of steam-power under the same conditions as mentioned for a permanent power, and its value can be represented in the same manner as the value of a permanent power has been represented.

"The value of a developed power is as follows: If the power can be run cheaper than steam, the value is that of the power, plus the cost of plant, less depreciation. If it cannot be run as cheaply as steam, considering its cost, etc., the value of the power itself is nothing, but the value of the plant is such as could be paid for it new, which would bring the total cost of running down to the cost of steam-power, less depreciation."

Mr. Samuel Webber, *Iron Age*, Feb. and March, 1893, writes a series of articles showing the development of American turbine wheels, and incidentally criticises the statements of Mr. Main and others who have made comparisons of costs of steam and of water-power unfavorable to the latter. He says: "They have based their calculations on the cost of steam, on large compound engines of 1000 or more H. P. and 120 pounds pressure of steam in their boilers, and by careful 10-hour trials succeeded in figuring down steam to a cost of about \$20 per H. P., ignoring the well-known fact that its average cost in practical use, except near the coal mines, is from \$40 to \$50. In many instances dams, canals, and modern turbines can be all completed for a cost of \$100 per H. P.; and the interest on that, and the cost of attendance and oil, will bring water-power up to about \$10 or \$12 per annum; and with a man competent to attend the dynamo in attendance, it can probably be safely estimated at not over \$15 per H. P."

WATER-WHEELS.

Water-wheels are classified as vertical wheels (including current motors, undershot, breast, and overshot wheels), turbine wheels, and impulse wheels. Undershot and breast wheels give very low efficiency, and are now no longer built. The overshot wheel when made of large diameter (wheels as high as 72 ft. diameter have been made) and properly designed have given efficiencies of over 80%, but they have been almost entirely supplanted by turbines, on account of their cumbersomeness, high cost, leakage, and inability to work in back water.

Turbines are generally classified according to the direction in which the water flows through them, as follows:

Tangential flow: Barker's mill. Parallel flow: Jonval. Radial outward flow: Fourneyron. Radial inward flow: Thompson vortex; Francis. Inward and downward flow: Central discharge scroll wheels and earlier American type of wheels; Swain turbine. Inward, downward, and outward flow: The American type of turbine.

TURBINE WHEELS.

Proportions of Turbines. — Prof. De Volson Wood discusses at length the theory of turbines in his paper on Hydraulic Reaction Motors, *Trans. A. S. M. E.* xiv. 266. His principal deductions which have an immediate bearing upon practice are condensed in the following:

Notation.

Q = volume of water passing through the wheel per second,
 h_1 = head in the supply chamber above the entrance to the buckets,
 h_2 = head in the tail-race above the exit from the buckets,
 z_1 = fall in passing through the buckets,
 $H = h_1 + z_1 - h_2$, the effective head,
 μ_1 = coefficient of resistance along the guides,
 μ_2 = coefficient of resistance along the buckets,
 r_1 = radius of the initial rim,
 r_2 = radius of the terminal rim,
 V = velocity of the water issuing from supply chamber,
 v_1 = initial velocity of the water in the bucket in reference to the bucket,
 v_2 = terminal velocity in the bucket,
 ω = angular velocity of the wheel,
 α = terminal angle between the guide and initial rim = CAB , Fig. 143,
 γ_1 = angle between the initial element of bucket and initial rim = EAD ,
 $\gamma_2 = GFI$, the angle between the terminal rim and terminal element of the bucket,
 $a = eb$, Fig. 144 = the arc subtending one gate opening,

a_1 = the arc subtending one bucket at entrance. (In practice a_1 is larger than a .)

$a_2 = gh$, the arc subtending one bucket at exit,

$K = bf$, normal section of passage, it being assumed that the passages and buckets are very narrow.

$k_1 = bd$, initial normal section of bucket,

$k_2 = gi$, terminal normal section,

ωr_1 = velocity of initial rim,

ωr_2 = velocity of terminal rim.

$\theta = HFI$, angle between the terminal rim and actual direction of the water at exit.

Y = depth of K , y , of a_1 , and y_2 of K_2 , then

$K = Ya \sin \alpha$; $K_1 = y_1 a_1 \sin \gamma_1$; $K_2 = y_2 a_2 \sin \gamma_2$.

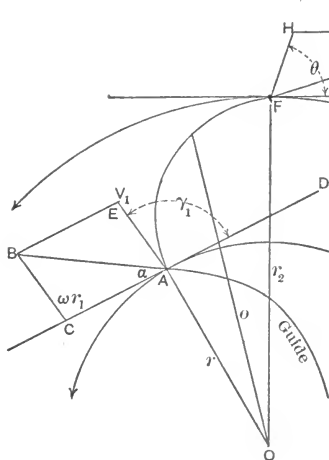


FIG. 143.

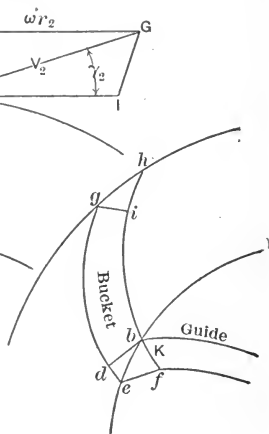


FIG. 144.

Three simple systems are recognized, $r_1 < r_2$, called outward flow; $r_1 > r_2$, called inward flow; $r_1 = r_2$, called parallel flow. The first and second may be combined with the third, making a mixed system.

Value of γ_2 (the quitting angle). — The efficiency is increased as γ_2 decreases, and is greatest for $\gamma_2 = 0$. Hence, theoretically, the terminal element of the bucket should be tangent to the quitting rim for best efficiency. This, however, for the discharge of a finite quantity of water, would require an infinite depth of bucket. In practice, therefore, this angle must have a finite value. The larger the diameter of the terminal rim the smaller may be this angle for a given depth of wheel and given quantity of water discharged. In practice γ_2 is from 10° to 20° .

In a wheel in which all the elements except γ_2 are fixed, the velocity of the wheel for best effect must increase as the quitting angle of the bucket decreases.

Values of $\alpha + \gamma_1$ must be less than 180° , but the best relation cannot be determined by analysis. However, since the water should be deflected from its course as much as possible from its entering to its leaving the wheel, the angle α for this reason should be as small as practicable.

In practice, α cannot be zero, and is made from 20° to 30° .

The value $r_1 = 1.4 r_2$ makes the width of the crown for internal flow about the same as for $r_1 = r_2 \sqrt{1/2}$ for outward flow, being approximately 0.3 of the external radius.

Values of μ_1 and μ_2 . — The frictional resistances depend upon the construction of the wheel as to smoothness of the surfaces, sharpness of the angles, regularity of the curved parts, and also upon the speed it is run. These values cannot be definitely assigned beforehand, but Weisbach gives for good conditions $\mu_1 = \mu_2 = 0.05$ to 0.10 .

They are not necessarily equal, and μ_1 may be from 0.05 to 0.075 , and μ_2 from 0.06 to 0.10 or even larger.

Values of γ_1 must be less than $180^\circ - \alpha$.

To be on the safe side, γ_1 may be 20 or 30 degrees less than $180^\circ - 2\alpha$, giving

$$\gamma_1 = 180^\circ - 2\alpha - 25 \text{ (say)} = 155^\circ - 2\alpha.$$

Then if $\alpha = 30^\circ$, $\gamma_1 = 95^\circ$. Some designers make $\gamma_1 90^\circ$; others more, and still others less, than that amount. Weisbach suggests that it be less, so that the bucket will be shorter and friction less. This reasoning appears to be correct for the inflow wheel, but not for the outflow wheel. In the Tremont turbines, described in the Lowell Hydraulic Experiments, this angle is 90° , the angle $\alpha 20^\circ$, and $\gamma_2 10^\circ$, which proportions insured a positive pressure in the wheel. Fourneyron made $\gamma_1 = 90^\circ$, and α from 30° to 33° , which values made the initial pressure in the wheel near zero.

Form of Bucket. — The form of the bucket cannot be determined analytically. From the initial and terminal directions and the volume of the water flowing through the wheel, the area of the normal sections may be found.

The normal section of the buckets will be: $K = \frac{Q}{V}$; $k_1 = \frac{Q}{v_1}$; $k_2 = \frac{Q}{v_2}$.

The depths of those sections will be:

$$Y = \frac{K}{a \sin \alpha}; \quad y_1 = \frac{k_1}{a_1 \sin \gamma_1}; \quad y_2 = \frac{k_2}{a_2 \sin \gamma_2}.$$

The changes of curvature and section must be gradual, and the general form regular, so that eddies and whirls shall not be formed. For the same reason the wheel must be run with the correct velocity to secure the best effect. In practice the buckets are made of two or three arcs of circles, mutually tangential.

The Value of ω . — So far as analysis indicates, the wheel may run at any speed; but in order that the stream shall flow smoothly from the supply chamber into the bucket, the velocity V should be properly regulated.

If $\mu_1 = \mu_2 = 0.10$, $r_2 \div r_1 = 1.40$, $\alpha = 25^\circ$, $\gamma_1 = 90^\circ$, $\gamma_2 = 12^\circ$, the velocity of the initial rim for outward flow will be for maximum efficiency 0.614 of the velocity due to the head, or $\omega r_1 = 0.614 \sqrt{2 gH}$.

The velocity due to the head would be $\sqrt{2 gH} = 1.414 \sqrt{gH}$.

For an inflow wheel for the case in which $r_1^2 = 2 r_2^2$, and the other dimensions as given above, $\omega r_1 = 0.682 \sqrt{2 gH}$.

The highest efficiency of the Tremont turbine, found experimentally, was 0.79375 , and the corresponding velocity, 0.62645 of that due to the head, and for all velocities above and below this value the efficiency was less.

In the Tremont wheel $\alpha = 20^\circ$ instead of 25° , and $\gamma_2 = 10^\circ$ instead of 12° . These would make the theoretical efficiency and velocity of the wheel somewhat greater. Experiment showed that the velocity might be considerably larger or smaller than this amount without much diminution of the efficiency.

It was found that if the velocity of the initial (or interior) rim was not less than 44% nor more than 75% of that due to the fall, the efficiency was 75% or more. This wheel was allowed to run freely without any brake except its own friction, and the velocity of the initial rim was observed to be $1.335 \sqrt{2 gH}$, half of which is $0.6675 \sqrt{2 gH}$, which is not far from the velocity giving maximum effect; that is to say, when the gate is fully raised the coefficient of effect is a maximum when the wheel is moving with about half its maximum velocity.

Number of Buckets. — Successful wheels have been made in which the distance between the buckets was as small as 0.75 of an inch, and others as much as 2.75 inches. Turbines at the Centennial Exposition had buckets from $4\frac{1}{2}$ inches to 9 inches from center to center. If too large they will not work properly. Neither should they be too deep. Horizontal parti-

tions are sometimes introduced. These secure more efficient working in case the gates are only partly opened. The form and number of buckets for commercial purposes are chiefly the result of experience.

Ratio of Radii. — Theory does not limit the dimensions of the wheel. In practice,

for outward flow, $r_2 \div r_1$ is from 1.25 to 1.50;
for inward flow, $r_2 \div r_1$ is from 0.66 to 0.80.

It appears that the inflow-wheel has a higher efficiency than the outward-flow wheel. The inflow-wheel also runs somewhat slower for best effect. The centrifugal force in the outward-flow wheel tends to force the water outward faster than it would otherwise flow; while in the inward-flow wheel it has the contrary effect, acting as it does in opposition to the velocity in the buckets.

It also appears that the efficiency of the outward-flow wheel increases slightly as the width of the crown is less and the velocity for maximum efficiency is slower; while for the inflow-wheel the efficiency slightly increases for increased width of crown, and the velocity of the outer rim at the same time also increases.

Efficiency. — The exact value or the efficiency for a particular wheel must be found by experiment.

It seems hardly possible for the effective efficiency to equal, much less exceed, 86%, and all claims of 90 or more per cent for these motors should be discarded as improbable. A turbine yielding from 75% to 80% is extremely good. Experiments with higher efficiencies have been reported.

The celebrated Tremont turbine gave 79 1/4% without the "diffuser," which might have added some 2%. A Jonval turbine (parallel flow) was reported as yielding 0.75 to 0.90, but Morin suggested corrections reducing it to 0.63 to 0.71. Weisbach gives the results of many experiments, in which the efficiency ranged from 50% to 84%. Numerous experiments give $E = 0.60$ to 0.65 . The efficiency, considering only the energy imparted to the wheel, will exceed by several per cent the efficiency of the wheel, for the latter will include the friction of the support and leakage at the joint, between the sluice and wheel, which are not included in the former; also as a plant the resistances and losses in the supply-chamber are to be still further deducted.

The Crowns. — The crowns may be plane annular disks, or conical, or curved. If the partitions forming the buckets be so thin that they may be discarded, the law of radial flow will be determined by the form of the crowns. If the crowns be plane, the radial flow (or radial component) will diminish, for the outward-flow wheel, as the distance from the axis increases — the buckets being full — for the angular space will be greater.

Prof. Wood deduces from the formulæ in his paper the tables on the next page.

It appears from these tables: 1. That the terminal angle, α , has frequently been made too large in practice for the best efficiency.

2. That the terminal angle, α , of the guide should be for the inflow less than 10° for the wheels here considered, but when the initial angle of the bucket is 90° , and the terminal angle of the guide is $5^\circ 28'$, the gain of efficiency is not 2% greater than when the latter is 25° .

3. That the initial angle of the bucket should exceed 90° for best effect for out flow-wheels.

4. That with the initial angle between 60° and 120° for best effect on inflow wheels the efficiency varies scarcely 1%.

5. In the outflow-wheel, column (9) shows that for the outflow for best effect the direction of the quitting water in reference to the earth should be nearly radial (from 76° to 97°), but for the inflow wheel the water is thrown forward in quitting. This shows that the velocity of the rim should somewhat exceed the relative final velocity backward in the bucket, as shown in columns (4) and (5).

6. In these tables the velocities given are in terms of $\sqrt{2gh}$, and the coefficients of this expression will be the part of the head which would produce that velocity if the water issued freely. There is only one case, column (5), where the coefficient exceeds unity, and the excess is so small it may be discarded; and it may be said that in a properly proportioned turbine with the conditions here given none of the velocities will equal that due to the head in the supply-chamber when running at best effect.

Outward-flow Turbine.

| $r_1 = r_2 \sqrt{1/2}$ | | $\mu_1 = \mu_2 = 0.10$ | | $\gamma_2 = 12^\circ$ | | Parallel Crowns. | | $k_1 v_1 = k_2 v_2 = K V = Q = 1$ | |
|------------------------------|--------------------|--------------------------------------|---|-------------------------------------|---|--|--------------------------------------|--|--|
| Initial Angle. γ_1 | Efficiency. E | Velocity Outer Rim. $r_1 \omega'$ | Velocity Inner Rim. $r_1 \omega' = \sqrt{1/2} r_2 \omega'$ | Relative Velocity of Exit. v_2 | Relative Velocity of Entrance. v_1 | Velocity of Exit from Supply-Chamber. V | Terminal Angle of Guide. α | Direction of quitting Water. θ | Head Equivalent of Energy in quitting Water. $\frac{w^2}{2g}$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 60° | 0.804 | $0.972 \sqrt{2gH}$ | $0.687 \sqrt{2gH}$ | $1.048 \sqrt{2gH}$ | $0.356 \sqrt{2gH}$ | $0.595 \sqrt{2gH}$ | 31° 17' | 76° | 0.051 H |
| 90° | 0.828 | $0.874 \sqrt{2gH}$ | $0.619 \sqrt{2gH}$ | $0.931 \sqrt{2gH}$ | $0.274 \sqrt{2gH}$ | $0.676 \sqrt{2gH}$ | 23° 56' | 79° | 0.039 H |
| 120° | 0.839 | $0.798 \sqrt{2gH}$ | $0.565 \sqrt{2gH}$ | $0.843 \sqrt{2gH}$ | $0.286 \sqrt{2gH}$ | $0.749 \sqrt{2gH}$ | 19° 5' | 82° | 0.031 H |
| 150° | 0.921 | $0.709 \sqrt{2gH}$ | $0.501 \sqrt{2gH}$ | $0.707 \sqrt{2gH}$ | $0.416 \sqrt{2gH}$ | $0.886 \sqrt{2gH}$ | 13° 31' | 97° | 0.022 H |

Inward-flow Turbine.

| $r_1 = \sqrt{2} r_2$ | | $\mu_1 = \mu_2 = 0.10$ | | $\gamma_2 = 12^\circ$ | | Parallel Crowns. | | $k_1 v_1 = k_2 v_2 = K V = Q = 1$ | |
|----------------------|-------|--------------------------------------|--------------------------------------|-----------------------|--------------------|--------------------|----------|-----------------------------------|------------------|
| γ_1 | E | Velocity Outer Rim. $r_1 \omega'$ | Velocity Inner Rim. $r_2 \omega'$ | v_2 | v_1 | V | α | θ | $\frac{w^2}{2g}$ |
| 60° | 0.920 | $0.709 \sqrt{2gH}$ | $0.501 \sqrt{2gH}$ | $0.476 \sqrt{2gH}$ | $0.089 \sqrt{2gH}$ | $0.672 \sqrt{2gH}$ | 7° 0' | 110° | 0.010 H |
| 90° | 0.920 | $0.688 \sqrt{2gH}$ | $0.487 \sqrt{2gH}$ | $0.470 \sqrt{2gH}$ | $0.069 \sqrt{2gH}$ | $0.691 \sqrt{2gH}$ | 5° 28' | 106° | 0.010 H |
| 120° | 0.919 | $0.668 \sqrt{2gH}$ | $0.473 \sqrt{2gH}$ | $0.456 \sqrt{2gH}$ | $0.077 \sqrt{2gH}$ | $0.709 \sqrt{2gH}$ | 4° 46' | 105° | 0.010 H |
| 150° | 0.918 | $0.634 \sqrt{2gH}$ | $0.448 \sqrt{2gH}$ | $0.429 \sqrt{2gH}$ | $0.126 \sqrt{2gH}$ | $0.743 \sqrt{2gH}$ | 3° 08' | 107° | 0.009 H |

 $k_2 \sqrt{gH}$

1.48

1.50

1.55

1.65

7. The inflow turbine presents the best conditions for construction for producing a given effect, the only apparent disadvantage being an increased first cost due to an increased depth, or an increased diameter for producing a given amount of work. The larger efficiency should, however, more than neutralize the increased first cost.

Tests of Turbines. — Emerson says that in testing turbines it is a rare thing to find two of the same size which can be made to do their best at the same speed. The best speed of one of the leading wheels is invariably wide from the tabled rate. It was found that a 54-in. Leffel wheel under 12 ft. head gave much better results at 78 revolutions per minute than at 90.

Overshot wheels have been known to give 75% efficiency, but the average performance is not over 60%.

A fair average for a good turbine wheel may be taken at 75%. In tests of 18 wheels made at the Philadelphia Water-works in 1859 and 1860, one wheel gave less than 50% efficiency, two between 50% and 60%, six between 60% and 70%, seven between 71% and 77%, two 82%, and one 87.77%. (Emerson.)

Tests of Turbine Wheels at the Centennial Exhibition, 1876. (From a paper by R. H. Thurston on The Systematic Testing of Turbine Wheels in the United States, *Trans. A. S. M. E.*, viii. 359.) — In 1876 the judges at the International Exhibition conducted a series of trials of turbines. Many of the wheels offered for tests were found to be more or less defective in fitting and workmanship. The following is a statement of the results of all turbines entered which gave an efficiency of over 75%. Seven other wheels were tested, giving results between 65% and 75%.

| Maker's Name, or Name the Wheel is Known by. | Per Cent at Full Gate or Discharge. | Per Cent at about $\frac{9}{10}$ of Full Discharge. | Per Cent at about $\frac{7}{8}$ of Full Discharge. | Per Cent at about $\frac{3}{4}$ of Full Discharge. | Per Cent at about $\frac{5}{8}$ of Full Discharge. | Per Cent at about $\frac{1}{2}$ of Full Discharge. | Per Cent at about $\frac{4}{10}$ of Full Discharge. |
|--|-------------------------------------|---|--|--|--|--|---|
| Risdon..... | 87.68 | | 86.20 | 82.41 | | 75.35 | |
| National..... | 83.79 | | | 70.79 | | | |
| Geyelin (single)..... | 83.30 | | | | | | |
| Thos. Taft..... | 82.13 | | | 70.40 | 66.35 | | 55.00 |
| Goldie & McCullough..... | 81.21 | | 71.01 | 55.90 | | | |
| Rodney Hunt Mach. Co..... | 78.70 | 71.66 | | 68.60 | 51.03 | | |
| Tyler Wheel..... | 79.59 | | 81.24 | 79.92 | 67.23 | 69.59 | |
| Geyelin (duplex)..... | 77.57 | | | | | | |
| Knowlton & Dolan..... | 77.43 | 74.25 | | | 62.75 | | |
| E. T. Cope & Sons..... | 76.94 | | 69.92 | | | | |
| Barber & Harris..... | 76.16 | 73.33 | | | 70.87 | 71.74 | |
| York Manufacturing Co..... | 75.70 | | 67.08 | 67.57 | 62.06 | | |
| W. F. Mosser & Co..... | 75.15 | 74.89 | 71.90 | 70.52 | | 66.04 | |

The limits of error of the tests, says Prof. Thurston, were very uncertain; they are undoubtedly considerable as compared with the later work done in the permanent flume at Holyoke — possibly as much as 4% or 5%.

Experiments with "draught-tubes," or "suction-tubes," which were actually "diffusers" in their effect, so far as Prof. Thurston has analyzed them, indicate the loss by friction which should be anticipated in such cases, this loss decreasing as the tube increased in size, and increasing as its diameter approached that of the wheel — the minimum diameter tried. It was sometimes found very difficult to free the tube from air completely, and next to impossible, during the interval, to control the speed with the brake. Several trials were often necessary before the power due to the full head could be obtained. The loss of power by gearing and by belting was variable with the proportions and arrangement of the gears and pulleys, length of belt, etc., but averaged not far from 30% for a single pair of bevel-

gears, uncut and dry, but smooth for such gearing, and but 10% for the same gears, well lubricated, after they had been a short time in operation. The amount of power transmitted was, however, small, and these figures are probably much higher than those representing ordinary practice. Introducing a second pair — spur-gears — the best figures were but little changed, although the difference between the case in which the larger gear was the driver, and the case in which the small wheel was the driver, was perceptible, and was in favor of the former arrangement. A single straight belt gave a loss of but 2% or 3%, a crossed belt 6% to 8%, when transmitting 14 horse-power with maximum tightness and transmitting power. A "quarter turn" wasted about 10% as a maximum, and a "quarter twist" about 5%.

Dimensions of Turbines. — For dimensions, power, etc., of standard makes of turbines consult the catalogues of different manufacturers. The wheels of different makers vary greatly in their proportions for any given capacity.

Rating and Efficiency of Turbines. — The following notes and tables are condensed from a pamphlet entitled "Turbine Water-wheel Tests and Power Tables," by R. E. Horton. Water-supply and Irrigation Paper No. 180, U. S. Geol. Survey, 1906.

Theory does not indicate the numbers of guides or buckets most desirable. If, however, they are too few, the stream will not properly follow the flow lines indicated by theory. If the buckets are too small and too numerous, the surface-friction factor will be large.

It is customary to make the number of guide chutes greater than the number of buckets, so that any object passing through the chutes will be likely to pass through the buckets also.

With most forms of gates the size of the jet is decreased as the gate is closed, the bucket area remaining unchanged, so that the wheel operates mostly by reaction at full gate and by impulse to an increasing extent as the gate is closed. Hence, the speed of maximum efficiency varies as the gate is closed. The ratio peripheral velocity ÷ velocity due head for maximum efficiency for a 36-inch Hercules turbine is given below:

| | | | | |
|------------------------------------|-------|-------|-------|-------|
| Proportional gate opening ... Full | 0.806 | 0.647 | 0.489 | 0.379 |
| Maximum efficiency..... | 85.6 | 87.1 | 86.3 | 80 |
| Periph. vel. ÷ vel. due head.. | 0.677 | 0.648 | 0.641 | 0.603 |
| | | | | 0.585 |

American turbine practice differs from European practice in that water wheels are placed on the market in standard or stock sizes, whereas in Europe, notably on the Continent, each turbine is designed for the special conditions under which it is to operate, the designs being based on mathematical theory and following chiefly the Jonval and Fourneyron types.

Having been developed by experiment after successive Holyoke tests, American stock pattern turbines probably give their best efficiencies at about the head under which those tests are made—i.e., 14 to 17 ft. The shafts, runners, and cases are so constructed as to enable stock sizes of wheels to be used under heads ranging from 6 to 60 ft. For very low heads they are perhaps unnecessarily cumbersome. For heads exceeding 60 ft. American builders commonly resort to the use of bronze buckets and "special wheels," not designed along theoretical lines, as in Europe, but representing modifications of the standard patterns.

The double Fourneyron turbine used in the first installation of the Niagara Falls Power Co. is operated under a head of about 135 ft. Two wheels are used, one being placed at the top and the other at the bottom of the globe penstock. The runner and buckets are attached to the vertical shaft. Holes are provided in the upper penstock drum to allow water under full pressure of the head to pass through and act vertically against the upper runner. In this way the vertical pressure of the great column of water is neutralized and a means is provided to counterbalance the weight of the long vertical shaft and the armature of the dynamo at its upper end. These turbines discharge 430 cu. ft. per second, make 250 rev. per min., and are rated at 5000 H.P.

A Fourneyron turbine at Trenton Falls, N. Y., operates under 265 ft. gross head and has 37 buckets, each $5\frac{1}{2}$ in. deep and $\frac{1}{8}$ inch wide at the least section. The total area of outflow at the minimum section is 165 sq. in. The wheel develops 950 H.P.

The theoretical horse-power of a given quantity of water Q , in cu. ft. per min., falling through a height H , in ft., is H.P. = $0.00189 QH$.

In practice the theoretical power is multiplied by an efficiency factor E to obtain the net power available on the turbine shaft as determinable by dynamometrical test.

Manufacturers' rating tables are usually based on efficiencies of about 80%. In selecting turbines from a maker's list the rated efficiency may be obtained by the following formula:

E = tabled efficiency. H.P. = tabled horse-power, and Q = tabled discharge (C.F.M.) for any head H . $E = \frac{33,000 \times \text{H.P.}}{62.4 \times Q \times H} = 528.8 \frac{\text{H.P.}}{Q \times H}$.

Relations of Power, Speed and Discharge.—Nearly all American turbine builders publish rating tables showing the discharge in cu. ft. per min., rev. per min., and H.P. for each size pattern under heads varying from 3 or 4 ft. to 40 ft. or more.

Examples of each size of a number of the leading types of turbines have been tested in the Holyoke flume. For such turbines the rating tables have usually been prepared directly from the tests.

Let M , R , and Q denote, respectively, the H.P., r.p.m., and discharge in cu. ft. per min. of a turbine, as expressed in the tables, for any head H in feet. The subscripts 1 and 16 added signify the power, speed, and discharge for the particular heads 1 and 16 ft., respectively.

Let P , N , and F denote coefficients of power, speed, and discharge, which represent, respectively, the H.P., r.p.m., and discharge in cu. ft. per sec. under a head of 1 ft.

The speed of a turbine or the number of rev. per min. and the discharge are proportional to the square root of the head. The H.P. varies with the product of the head and discharge, and is consequently proportional to the three-halves power of the head.

Given the values of M , R , and Q from the tables for any head H , these quantities for any other head h are:

$$M_H : M_h :: H^{3/2} : h^{3/2} ; R_H : R_h :: H^{1/2} : h^{1/2} ; Q_H : Q_h :: H^{1/2} : h^{1/2}.$$

If H and h are taken at 16 ft. and 1 ft., respectively, the values of the coefficients P , N , and F are:

$$P = M_{16}/H^{3/2} = M_{16}/64 = 0.01562 M_{16}$$

$$N = R_{16}/H^{1/2} = R_{16}/4 = 0.25 R_{16}$$

$$F = Q_{16}/60 H^{1/2} = Q_{16}/240 = 0.00417 Q_{16}.$$

P , N , and F , when derived for a given wheel, enable the power, speed, and discharge to be calculated without the aid of the tables, and for any head H , by means of the following formulas:

$$\begin{aligned} M &= M_1 H^{3/2}/H_1 = P H^{3/2} \\ R &= R_1 \sqrt{H/H_1} = N \sqrt{H} \\ Q &= Q_1 \sqrt{H/H_1} = 60 F \sqrt{H}. \end{aligned}$$

Since at a head of 1 ft., and M_1 , R_1 , and Q_1 equal P , N , and $60 F$, respectively, $H_1^{3/2}$ and $\sqrt{H_1}$ each equals 1. Calculations involving $H^{3/2}$ may be facilitated by the use of the appended table of three-halves powers. Rating tables for sizes other than those tested are computed usually on the following basis:

1. The efficiency and coefficients of gate and bucket discharge for the sizes tested are assumed to apply to the other sizes also.

2. The discharge for additional sizes is computed in proportion to the measured area of the vent or discharge orifices.

Having these data, together with the efficiency, the tables of discharge and horse-power can be prepared. The peripheral speed corresponding to maximum efficiency determined from tests of one size of turbine may be assumed to apply to the other sizes also. From this datum the revolutions per minute can be computed, the number of revolutions required to give a constant peripheral speed being inversely proportional to the diameter of the turbine.

In point of discharge, the writer's observation has been that the rating tables are usually fairly accurate. In the matter of efficiency there are undoubtedly much larger discrepancies.

TABLE OF $H^{3/2}$ FOR CALCULATING HORSE-POWER OF TURBINES.

| Head ft. | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | Head ft. | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|-------------|--------|--------|--------|--------|--------|-------------|---------|--------|--------|--------|--------|
| 0 | 0.00 | 0.09 | 0.25 | 0.46 | 0.72 | 51 | 364.21 | 366.36 | 368.50 | 370.66 | 372.82 |
| 1 | 1.00 | 1.32 | 1.66 | 2.02 | 2.42 | 52 | 374.98 | 377.14 | 379.31 | 381.48 | 383.66 |
| 2 | 2.83 | 3.26 | 3.72 | 4.19 | 4.69 | 53 | 385.85 | 388.03 | 390.22 | 392.4 | 394.61 |
| 3 | 5.20 | 5.72 | 6.27 | 6.83 | 7.41 | 54 | 396.81 | 399.02 | 401.23 | 403.45 | 405.67 |
| 4 | 8.00 | 8.61 | 9.23 | 9.87 | 10.52 | 55 | 407.89 | 410.11 | 412.35 | 414.58 | 416.82 |
| 5 | 11.18 | 11.86 | 12.55 | 13.25 | 13.97 | 56 | 419.07 | 421.31 | 423.56 | 425.81 | 428.07 |
| 6 | 14.70 | 15.44 | 16.19 | 16.96 | 17.73 | 57 | 430.34 | 432.60 | 434.87 | 437.15 | 439.43 |
| 7 | 18.52 | 19.32 | 20.13 | 20.95 | 21.78 | 58 | 441.71 | 444.00 | 446.29 | 448.58 | 450.88 |
| 8 | 22.63 | 23.48 | 24.35 | 25.22 | 26.11 | 59 | 453.09 | 455.49 | 457.80 | 460.12 | 462.43 |
| 9 | 27.00 | 27.91 | 28.82 | 29.75 | 30.68 | 60 | 464.75 | 467.08 | 469.41 | 471.75 | 474.08 |
| 10 | 31.62 | 32.55 | 33.54 | 34.51 | 35.49 | 61 | 476.42 | 478.77 | 481.12 | 483.47 | 485.82 |
| 11 | 36.48 | 37.48 | 38.49 | 39.51 | 40.53 | 62 | 488.19 | 490.55 | 492.92 | 495.29 | 497.67 |
| 12 | 41.57 | 42.61 | 43.66 | 44.73 | 45.79 | 63 | 500.04 | 502.43 | 504.82 | 507.20 | 509.60 |
| 13 | 46.87 | 47.9 | 49.05 | 50.15 | 51.26 | 64 | 512.00 | 514.40 | 516.80 | 519.22 | 521.63 |
| 14 | 52.38 | 53.51 | 54.64 | 55.79 | 56.94 | 65 | 524.04 | 526.46 | 528.89 | 531.31 | 533.75 |
| 15 | 58.09 | 59.26 | 60.43 | 61.61 | 62.80 | 66 | 536.18 | 538.62 | 541.07 | 543.51 | 545.96 |
| 16 | 64.00 | 65.20 | 66.41 | 67.63 | 68.85 | 67 | 548.42 | 550.87 | 553.33 | 555.80 | 558.27 |
| 17 | 70.09 | 71.33 | 72.58 | 73.84 | 75.10 | 68 | 560.74 | 563.22 | 565.70 | 568.18 | 570.66 |
| 18 | 76.37 | 77.64 | 78.93 | 80.22 | 81.52 | 69 | 573.16 | 575.65 | 578.14 | 580.65 | 583.15 |
| 19 | 82.82 | 84.13 | 85.45 | 86.77 | 88.10 | 70 | 585.66 | 588.17 | 590.68 | 593.20 | 595.73 |
| 20 | 89.44 | 90.79 | 92.14 | 93.50 | 94.86 | 71 | 598.25 | 600.77 | 603.32 | 605.85 | 608.39 |
| 21 | 96.23 | 97.61 | 99.00 | 100.39 | 101.79 | 72 | 610.93 | 613.45 | 616.04 | 618.59 | 621.15 |
| 22 | 103.19 | 104.60 | 106.02 | 107.44 | 108.87 | 73 | 623.71 | 626.27 | 628.84 | 631.41 | 633.99 |
| 23 | 110.30 | 111.74 | 113.19 | 114.65 | 116.11 | 74 | 636.57 | 639.15 | 641.74 | 644.33 | 646.92 |
| 24 | 117.58 | 119.05 | 120.53 | 122.01 | 123.50 | 75 | 649.52 | 652.11 | 654.72 | 657.33 | 659.94 |
| 25 | 125.00 | 126.50 | 128.01 | 129.53 | 131.05 | 76 | 662.55 | 665.17 | 667.79 | 670.41 | 673.04 |
| 26 | 132.57 | 134.11 | 135.65 | 137.19 | 138.74 | 77 | 675.67 | 678.20 | 680.94 | 683.58 | 686.23 |
| 27 | 140.30 | 141.86 | 143.43 | 145.00 | 146.58 | 78 | 688.87 | 691.52 | 694.18 | 696.84 | 699.50 |
| 28 | 148.16 | 149.75 | 151.35 | 152.95 | 154.56 | 79 | 702.16 | 704.83 | 707.50 | 710.13 | 712.85 |
| 29 | 156.17 | 157.79 | 159.41 | 161.04 | 162.68 | 80 | 715.54 | 718.22 | 720.92 | 723.60 | 726.30 |
| 30 | 164.32 | 165.96 | 167.61 | 169.27 | 170.93 | 81 | 729.00 | 731.70 | 734.40 | 737.11 | 739.82 |
| 31 | 172.60 | 174.27 | 175.95 | 177.64 | 179.33 | 82 | 742.54 | 745.26 | 747.98 | 750.70 | 753.43 |
| 32 | 181.02 | 182.72 | 184.42 | 186.13 | 187.85 | 83 | 756.16 | 758.90 | 761.63 | 764.38 | 767.12 |
| 33 | 189.57 | 191.30 | 193.03 | 194.76 | 196.51 | 84 | 769.87 | 772.62 | 775.37 | 778.13 | 780.89 |
| 34 | 198.25 | 200.00 | 201.76 | 203.52 | 205.29 | 85 | 783.66 | 786.42 | 789.21 | 791.97 | 794.75 |
| 35 | 207.06 | 208.84 | 210.62 | 212.41 | 214.20 | 86 | 797.53 | 800.31 | 803.10 | 805.89 | 808.68 |
| 36 | 216.00 | 217.80 | 219.61 | 221.42 | 223.24 | 87 | 811.43 | 814.27 | 817.03 | 819.88 | 822.70 |
| 37 | 225.06 | 226.89 | 228.72 | 230.56 | 232.40 | 88 | 825.51 | 828.32 | 831.15 | 833.97 | 836.79 |
| 38 | 234.25 | 236.10 | 237.96 | 239.82 | 241.68 | 89 | 839.62 | 842.45 | 845.29 | 848.13 | 850.96 |
| 39 | 243.56 | 245.43 | 247.31 | 249.20 | 251.09 | 90 | 853.81 | 856.66 | 859.51 | 862.37 | 865.22 |
| 40 | 252.98 | 254.88 | 256.79 | 258.70 | 260.61 | 91 | 868.08 | 870.94 | 873.81 | 876.68 | 879.55 |
| 41 | 262.53 | 264.45 | 266.33 | 268.31 | 270.25 | 92 | 882.43 | 885.30 | 888.17 | 891.07 | 893.96 |
| 42 | 272.19 | 274.14 | 276.09 | 278.05 | 280.01 | 93 | 896.86 | 899.75 | 902.6 | 905.55 | 908.45 |
| 43 | 281.97 | 283.91 | 285.91 | 287.89 | 289.88 | 94 | 911.36 | 914.27 | 917.18 | 920.10 | 923.02 |
| 44 | 291.86 | 293.8 | 295.85 | 297.85 | 299.86 | 95 | 925.94 | 928.87 | 931.79 | 934.73 | 937.66 |
| 45 | 301.87 | 303.88 | 305.90 | 307.93 | 309.95 | 96 | 940.60 | 943.54 | 946.48 | 949.43 | 952.38 |
| 46 | 311.97 | 314.02 | 316.07 | 318.11 | 320.16 | 97 | 955.33 | 958.29 | 961.25 | 964.21 | 967.17 |
| 47 | 322.22 | 324.27 | 326.34 | 328.41 | 330.48 | 98 | 970.14 | 973.11 | 976.09 | 979.07 | 982.05 |
| 48 | 332.55 | 334.63 | 336.72 | 338.81 | 340.90 | 99 | 985.03 | 988.02 | 991.01 | 994.00 | 996.99 |
| 49 | 343.00 | 345.10 | 347.21 | 349.32 | 351.43 | 100 | 1000.00 | | | | |
| 50 | 353.55 | 355.67 | 357.80 | 359.93 | 362.07 | | | | | | |

Rating Table for Turbines.

LEFFEL STANDARD (NEW TYPE). PIVOT GATE. [1900 list.]

| Diameter of Runner in Inches. | Manufacturer's Rating for a Head of 16 Ft. | | | Coefficients. | | |
|-------------------------------|--|------------------------|----------------------|---------------|-------------------|-------------|
| | H.P. (=M). | Cu. Ft. per min. (=Q). | Revs. per min. (=R). | Power (=P). | Dis-charge. (=F). | Speed (=N). |
| 10..... | 3.70 | 53 | 535 | 0.058 | 0.220 | 133.8 |
| 11 1/2..... | 4.9 | 201 | 463 | .076 | .838 | 115.8 |
| 13 1/4..... | 6.5 | 267 | 404 | .101 | 1.113 | 101.0 |
| 15 1/4..... | 8.4 | 348 | 351 | .131 | 1.451 | 87.8 |
| 17 1/2..... | 11.00 | 455 | 306 | .172 | 1.897 | 76.5 |
| 20..... | 14.9 | 602 | 268 | .232 | 2.510 | 67 |
| 23..... | 19.4 | 802 | 233 | .303 | 3.344 | 58.2 |
| 25 1/2..... | 25.25 | 1,043 | 202 | .393 | 4.339 | 50.5 |
| 30 1/2..... | 33.61 | 1,390 | 176 | .524 | 5.796 | 44 |
| 35..... | 44.3 | 1,831 | 153 | .691 | 7.635 | 38.2 |
| 40..... | 58.2 | 2,406 | 134 | .908 | 10.033 | 33.5 |
| 44..... | 67.75 | 2,800 | 122 | 1.058 | 11.676 | 30.5 |
| 48..... | 84.1 | 3,475 | 110 | 1.312 | 14.490 | 27.5 |
| 55..... | 142 | 5,858 | 96 | 2.215 | 24.428 | 24 |
| 61..... | 168 | 6,950 | 87 | 2.621 | 28.982 | 21.8 |
| 66..... | 202 | 8,340 | 80 | 3.151 | 34.778 | 20 |
| 74..... | 247 | 10,222 | 72 | 3.853 | 42.623 | 18 |

LEFFEL IMPROVED SAMSON. PIVOT GATE. [1897 and 1900 lists.]

| | | | | | | |
|---------|------|--------|-----|--------|---------|------|
| 20..... | 51.7 | 2,111 | 325 | 0.806 | 8.803 | 81.3 |
| 23..... | 68.3 | 2,792 | 283 | 1.065 | 11.643 | 70.8 |
| 26..... | 87.3 | 3,569 | 250 | 1.362 | 14.883 | 62.5 |
| 30..... | 116 | 4,751 | 217 | 1.810 | 19.812 | 54.3 |
| 35..... | 158 | 6,440 | 186 | 2.465 | 26.855 | 46.5 |
| 40..... | 207 | 8,446 | 163 | 3.229 | 35.220 | 40.8 |
| 45..... | 262 | 10,689 | 145 | 4.087 | 44.573 | 36.3 |
| 50..... | 324 | 13,196 | 130 | 5.054 | 55.027 | 32.5 |
| 56..... | 405 | 16,554 | 116 | 6.318 | 69.030 | 29.0 |
| 62..... | 497 | 20,292 | 105 | 7.753 | 84.618 | 26.3 |
| 68..... | 597 | 24,409 | 96 | 9.313 | 101.786 | 24.0 |
| 74..... | 708 | 28,906 | 88 | 11.045 | 120.538 | 22.0 |

VICTOR HIGH PRESSURE TURBINE. CYLINDER GATE. [1903 list.]

Ratings for 100 Ft. Head.

| | | | | | | |
|---------|-----|-------|-----|-------|-------|------|
| 14..... | 37 | 247 | 656 | 0.037 | 0.412 | 65.6 |
| 16..... | 50 | 332 | 574 | .050 | .553 | 57.4 |
| 18..... | 66 | 442 | 510 | .066 | .733 | 51.0 |
| 20..... | 82 | 542 | 459 | .082 | .903 | 45.9 |
| 22..... | 106 | 707 | 417 | .106 | 1.178 | 41.7 |
| 24..... | 128 | 850 | 383 | .128 | 1.417 | 38.3 |
| 26..... | 151 | 1,001 | 353 | .151 | 1.668 | 35.3 |
| 28..... | 173 | 1,147 | 328 | .173 | 1.912 | 32.8 |
| 30..... | 191 | 1,265 | 306 | .191 | 2.108 | 30.6 |
| 33..... | 228 | 1,512 | 278 | .228 | 2.520 | 27.8 |
| 36..... | 272 | 1,805 | 255 | .272 | 3.008 | 25.5 |
| 39..... | 303 | 2,005 | 235 | .303 | 3.342 | 23.5 |
| 42..... | 343 | 2,277 | 219 | .343 | 3.795 | 21.9 |
| 45..... | 387 | 2,563 | 204 | .387 | 4.272 | 20.4 |
| 48..... | 426 | 2,820 | 191 | .426 | 4.700 | 19.1 |
| 51..... | 462 | 3,063 | 180 | .462 | 5.105 | 18.0 |
| 54..... | 504 | 3,340 | 170 | .504 | 5.567 | 17.0 |
| 57..... | 544 | 3,605 | 161 | .544 | 6.008 | 16.0 |
| 60..... | 590 | 3,907 | 153 | .590 | 6.512 | 15.3 |
| 63..... | 619 | 4,100 | 146 | .619 | 6.833 | 14.6 |
| 66..... | 680 | 4,505 | 139 | .680 | 7.508 | 13.9 |
| 69..... | 742 | 4,910 | 133 | .742 | 8.183 | 13.3 |
| 72..... | 799 | 5,290 | 127 | .799 | 8.817 | 12.7 |

The discharge of turbines is nearly always expressed in cubic feet per minute. The "vent" in square inches is also used by millwrights and manufacturers, although to a decreasing extent. The vent of a turbine is the area of an orifice which would, under any given head, theoretically discharge the same quantity of water that is vented or passed through a turbine under that same head when the wheel is so loaded as to be running at maximum efficiency.

If V = vent in sq. in., Q = discharge in cu. ft. per min. under a head H , F = discharge in cu. ft. per sec. under a head of 1 foot, then $Q = 60 V/144 \sqrt{2gH} = 3.344 V\sqrt{H}$, and $V = 0.3 Q/\sqrt{H}$; also $V = 17.94 F$ and $F = 0.0557 V$.

The vent of a turbine should not be confused with the area of the outlet orifice of the buckets. The actual discharge through a turbine is commonly from 40 to 60% of the theoretical discharge of an orifice whose area equals the combined cross-sectional areas of the outlet ports measured in the narrowest section.

The high-pressure turbine is a recent design (1903), and is tabled for heads of 70 to 675 feet.

A 10,000 H.P. Turbine at Snoqualmie, Wash. (Arthur Giesler, *Eng. News*, Mar. 20, 1906.)—The fall is about 270 ft. high. The machinery is placed in an underground chamber excavated in the rock about 250 ft. below the surface, and 300 ft. up-stream from the crest of the falls. A tail-race tunnel runs to the lower reach of the river. The wheel was designed by the Platt Iron Works Co., Dayton, O., for an effective head of 260 ft. and 300 r.p.m., the latter being fixed by the limitations of dynamo design. There was no precedent for a generator approximating 10,000 H.P. running at such a speed. The turbine is a horizontal shaft machine, of the Francis type, radial inward flow with central axial discharge. The turbine proper has only one bearing, $8\frac{3}{8} \times 26$ in., the generator having three bearings. The draft tube is on the generator (front) side. The shaft-bearing, thrust-bearing and thrust-balancing devices are at the back side. The wheel is 66 in. outside diam. by 9 in. wide through the vanes. It has 34 vanes which extend a short distance beyond the end plate of the wheel on the discharge side. There are 32 guide vanes, of the swivel type, connected to a rotatable ring which is actuated by a Lombard governor. The turbine wheel or runner is an annular steel casting. It is bolted to a disk 46 in. diam., which is an enlargement of the $13\frac{1}{2}$ in. hollow nickel-steel shaft. A test for efficiency was made, in which the output was measured on the electrical side, and the input by the drop of head across the head gate. At 10,000 H.P. the efficiency shown was 84%, the figure being subject to the inaccuracy of the water measurement. The maximum capacity registered was 8250 K.W. or 11,000 H.P. With the generator and the governor disconnected, with full gates and no load, the wheel ran at 505 r.p.m.

Turbines of 13,500 H.P.—Four Francis turbines, with vertical shafts, rated at 13,500 H.P. each, have been built by Allis-Chalmers Co., for the Great Northern Power Co., Duluth, Minn. The available head is 365 ft., and the wheels run at 375 r.p.m.; discharging, at full load, about 400 cu. ft. per second, each. The runners are 62 in. diam. The penstock for each wheel is 84 in. diam., reduced gradually to 66 in. at the wheel. (Bulletin No. 1613, A.-C. Co.)

The "Fall-increaser" for Turbines.—A circular issued Nov., 1908, by Clemens Herschel, the inventor of the Venturi Meter, illustrates a device, based on the principle of the meter, for diminishing the back-water head which acts against the turbine. The surplus water, which would otherwise run to waste, is caused to flow into a tube of the Venturi shape, and the pressure in the narrow section, or throat of this tube, is less than that due to the head of the back-water into which the tube discharges. The throat is perforated with a great number of 6-in. holes, through which the discharge-water of the turbine is caused to flow, the velocity through the holes being never over 4 ft. per second. The circular says:

The fall-increaser is a form of power-house foundation construction so made that by running through it water, which would otherwise waste over the dam, the fall acting on the turbines is increased, and the output of power is kept at its maximum quantity, in spite of the back-water

which always accompanies an abundance of river flow passing down the river.

The results show that fall-increasers add about 10% to the annual output of power with no appreciable increase in operating expenses.

For half the days of the year the fall-increasers are shut down because there is not enough, or only enough, water to supply the plain turbines; but for the other half of the year the fall-increasers keep the output of power practically constant, and at the full output, where this power output would fall to half the full output or less if the fall-increasers had not been built.

An illustrated description of the fall-increaser, with results of tests, is given in the *Harvard Eng'g Journal*, June, 1908. See also U. S. Pat. No. 873,435 and *Eng. News*, June 11, 1908.

TANGENTIAL OR IMPULSE WATER-WHEELS.

The Pelton Water-wheel. — Mr. Ross E. Browne (*Eng'g News*, Feb. 20, 1892) thus outlines the principles upon which this water-wheel is constructed:

The function of a water-wheel, operated by a jet of water escaping from a nozzle, is to convert the energy of the jet, due to its velocity, into useful work. In order to utilize this energy fully the wheel-bucket, after catching the jet, must bring it to rest before discharging it, without inducing turbulence or agitation of the particles.

This cannot be fully effected, and unavoidable difficulties necessitate the loss of a portion of the energy. The principal losses occur as follows: First, in sharp or angular diversion of the jet in entering, or in its course through the bucket, causing impact, or the conversion of a portion of the energy into heat instead of useful work. Second, in the so-called frictional resistance offered to the motion of the water by the wetted surfaces of the buckets, causing also the conversion of a portion of the energy into heat instead of useful work. Third, in the velocity of the water, as it leaves the bucket, representing energy which has not been converted into work.

Hence, in seeking a high efficiency: 1. The bucket-surface at the entrance will be approximately parallel to the relative course of the jet, and the bucket should be curved in such a manner as to avoid sharp angular deflection of the stream. If, for example, a jet strikes a surface at an angle and is sharply deflected, a portion of the water is backed, the smoothness of the stream is disturbed, and there results considerable loss by impact and otherwise.

2. The path of the jet in the bucket should be short; in other words, the total wetted surface of the bucket should be small, as the loss by friction will be proportional to this.

3. The discharge end of the bucket should be as nearly tangential to the wheel periphery as compatible with the clearance of the bucket which follows; and great differences of velocity in the parts of the escaping water should be avoided. In order to bring the water to rest at the discharge end of the bucket, it is shown, mathematically, that the velocity of the bucket should be one half the velocity of the jet.

A bucket, such as shown in Fig. 145, will cause the heaping of more or less dead or turbulent water at the point indicated by dark shading. This dead water is subsequently thrown from the wheel with considerable velocity, and represents a large loss of energy. The introduction of the wedge in the Pelton bucket (see Fig. 146) is an efficient means of avoiding this loss.



FIG. 145.



FIG. 146.



FIG. 147.

A wheel of the form of the Pelton (Fig. 147) conforms closely in construction to each of these requirements. [In wheels as now made (1909)

the sharp corners shown in this bucket are eliminated. See catalogues of the Pelton Water Wheel Co., Joshua Hendy Iron Works, and Abner Doble Co., all of San Francisco.]

Considerations in the Choice of a Tangential Wheel (Joshua Hendy Iron Works.) — The horse-power that can be developed by a tangential wheel does not depend upon the size of the wheel but *solely upon the head and volume of water available*. The number of revolutions per minute that a wheel makes (running under normal conditions) depends solely upon two factors, viz., its diameter and the head of water.

The choice of the diameter of a wheel is not therefore controlled by the power required but by the speed required when working under a given head. If a wheel has no load, and is not governed, it will speed up until the periphery is revolving at approximately the same velocity as the spouting velocity of the jet, but as soon as the wheel commences to develop power by driving machinery, etc., its velocity will drop. In a properly designed wheel the velocity of the rim in lineal feet per minute, at full load, will be from 48 to 50% of the spouting velocity of the jet.

The diameter of pulley wheels on wheel shaft and countershafts of machinery should be so proportioned that the water wheel shall run at the speed given in the table.

The width, area and curvature of buckets are designed to meet conditions of volume of flow under given heads. The higher the peripheral velocity of the wheel, the greater the volume of water that the buckets can handle, and consequently the same standard wheel can handle more water, the higher the head.

Standard wheels can generally be adapted one size larger or one size smaller to meet conditions of a variation of speed or volume of flow under a given head. Wheels designed for a given horse-power can be used for smaller powers (within reasonable limits) with very little loss of efficiency, but an increase in the volume to be used requires a larger bucket. If, for the purpose of maintaining the same speed conditions, the same diameter of wheel is to be adhered to, then a special wheel must be built with either very large buckets or with two or more nozzles, or else a double or multiple unit must be adopted.

It is advised to subdivide large streams between two, three or more runners, as this insures a greater freedom from breakdown and is often cheapest in the end. Single-nozzle, multiple runner units are easier to govern than multiple-nozzle, single runner units. When two or more nozzles are used in combination on one runner, the increased volume to be dealt with is divided between the different nozzles, which are so arranged that their respective jets impinge on different buckets at different parts of the periphery. Three-nozzle and five-nozzle wheels have many disadvantages, when governing is required, and should only be adopted for handling a very large volume of water when other designs cannot be used.

Combined Heads. — When two or more water powers are available at the same site, but under different heads, it is possible to utilize them by mounting wheels of different diameters in parallel, or, when the difference of head and volume is very great, it would even be possible to arrange for a turbine for the low head and a tangential wheel for the high head, although, in the latter case, it would probably be best to mount them independently and connect to the machinery through the medium of belts and countershafts. In either case, separate pipe lines must be employed.

Reversible Wheels. — In the case of reversible wheels desired for use with hoists, cableways, etc., two wheels of proper dimensions and the same type may be mounted parallel on the same shaft, one of the wheels having the buckets and nozzles arranged to run in the opposite direction to the other. Suitable valves, levers and pipe connections can be arranged to cut the water off one wheel and turn it on to the other.

Horizontal Wheels. — For electric generating stations, when it is desired to place the wheels below the floor of the generators, where vertical direct-connected equipments are used, tangential wheels may be mounted horizontally with vertical shafts and step bearings.

Notes on Hydraulic Power Installations. (Joshua Hendy Iron Works.) — Apertures of screens must be slightly smaller than the diameter of the smallest nozzle used.

When not in use, keep the pipe full by closing the valve at the lower end. There is less liability for trouble from expansion and contraction with a full pipe line.

Equip the pipe line with air valves, approximately one for every 20 ft. of head.

When operating under high heads, when no other precaution has been taken to avoid water ram during the process of governing, it is advisable to install relief valves between the lower end of the pipe line and the gates or controllers.

When operating under even moderately high heads, if no safety device or by-pass has been installed, and a plain valve is to be used, sliding gates and butterfly valves should not be employed, but only screw gates, as the former would be too rapid in their action and might set up a dangerous water ram.

The size of the nozzle that must be used on a wheel for maximum efficiency must be such as will just keep the pipe line full. If water overflows, put on a larger nozzle. If the pipe remains partly empty, put on a smaller nozzle, as otherwise the effective head is reduced, with considerable loss of efficiency.

The nozzles may be placed either above or below the wheel, depending on the direction of rotation required.

Control of Tangential Water-Wheels. — The methods of regulating tangential water-wheels may be classified under five heads:

1. Permanently or semi-permanently altering the area of efflux of the nozzles, with water economy and without loss of efficiency.
2. Reducing the volume of flow without altering the area of efflux, with water economy but with loss of efficiency.
3. Variable alteration of the area of efflux without loss of efficiency and with water economy.
4. Deflection of the jet, so that only a portion of its energy is transmitted to the wheel, without water economy.
5. Combined regulation of 3 and 4, producing an effect whereby the energy of the jet is reduced rapidly without water ram and the area of efflux reduced slowly to effect water economy, or by a combination of 3 with some form of by-pass.

Governors. — Of the five methods of control enumerated above, the first cannot be done automatically; the other four, however, are susceptible to either hand regulation or automatic regulation by means of governors, the function of the governor being merely to automatically bring into action the particular controlling device with which the wheel has been equipped. There are two leading types of governors, the hydraulic and the mechanical. In the first, the mechanism of the water-wheel regulator is actuated by a hydraulically operated piston, the motive power being taken from a small branch pipe from the main water supply, or from an independent high-pressure oil-pumping system, the position of the piston in the cylinder and consequent relative position of the controlling mechanism being dependent upon the amount of fluid under pressure admitted to the cylinder at either end. This is controlled by a main valve, operated by a very sensitive relay valve which, in turn, is directly controlled by the centrifugal balls of the governor.

The second type, or mechanically operated governor, consists of a device for automatically controlling and directing the transmission of the requisite amount of energy taken from the wheel shaft, to operate the water-regulating mechanism. The Lombard governor is a representative of the first type, and the Lombard-Replogle governor of the second.

The close regulation that can be obtained with the latter is remarkable. Any size will go into operation and make connection at so slight a deviation as one-tenth of one per cent from normal, and in installations which have been made they will not permit of a departure of more than five to eight per cent temporarily where there is an instantaneous drop from full load to practically no load. When there is sufficient fly-wheel effect, the deviation will not be over two per cent. The adoption of fly wheels greatly facilitates many problems of governing.

Tangential Water-Wheel Table. (Joshua Hendy Iron Works.)

P = horse-power, Q = cubic feet per minute, R = revs. per min. The smaller figures in the first column give the spouting velocity of the jet in feet per minute. (The table is greatly condensed from the original; 6-in., 15-in., and 30-in. wheels are also listed. P and Q are the same, with any given head, for a 30 as for a 36-in. wheel, but R is 20% greater.)

| Head in Ft. | | 12 Inch. | 18 Inch. | 24 Inch. | 36 Inch. | 48 Inch. | 60 Inch. | 72 Inch. | 8 Feet. | 10 Feet. | 12 Feet. |
|-------------------|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|-------------|-------------|
| 20 2152 | P | .12 | .37 | .66 | 1.50 | 2.64 | 4.18 | 6.00 | 10.64 | 16.48 | 23.80 |
| | Q | 3.91 | 11.72 | 20.83 | 46.93 | 83.32 | 130.36 | 187.72 | 332.70 | 515.04 | 748.95 |
| | R | 342 | 228 | 171 | 114 | 85 | 70 | 57 | 43 | 34 | 29 |
| 30 2635 | P | .23 | .69 | 1.22 | 2.76 | 4.88 | 7.69 | 11.04 | 19.53 | 30.00 | 43.80 |
| | Q | 4.79 | 14.36 | 25.51 | 57.44 | 102.04 | 159.66 | 229.76 | 407.03 | 630.00 | 916.47 |
| | R | 418 | 279 | 209 | 139 | 104 | 83 | 69 | 52 | 41 | 35 |
| 40 3043 | P | .35 | 1.05 | 1.89 | 4.24 | 7.58 | 11.85 | 16.96 | 30.08 | 46.60 | 67.60 |
| | Q | 5.53 | 16.59 | 29.45 | 66.36 | 107.84 | 184.36 | 265.44 | 470.27 | 728.16 | 1058.86 |
| | R | 484 | 323 | 242 | 161 | 121 | 96 | 80 | 62 | 49 | 40 |
| 50 3403 | P | .49 | 1.49 | 2.65 | 5.98 | 10.60 | 16.63 | 23.93 | 42.05 | 65.00 | 94.50 |
| | Q | 6.18 | 18.54 | 32.93 | 74.17 | 131.72 | 206.13 | 296.70 | 525.90 | 814.32 | 1184.15 |
| | R | 541 | 361 | 270 | 180 | 135 | 108 | 90 | 69 | 55 | 46 |
| 60 3727 | P | .65 | 1.96 | 3.48 | 7.84 | 13.94 | 21.77 | 31.36 | 55.20 | 85.62 | 124.50 |
| | Q | 6.77 | 20.31 | 36.08 | 81.25 | 144.32 | 225.80 | 325.00 | 576.00 | 892.00 | 1297.00 |
| | R | 592 | 395 | 296 | 197 | 148 | 118 | 98 | 75 | 60 | 50 |
| 70 4026 | P | .82 | 2.47 | 4.39 | 9.88 | 17.58 | 27.51 | 39.52 | 70.00 | 107.80 | 157.50 |
| | Q | 7.31 | 21.94 | 38.97 | 87.76 | 155.88 | 243.89 | 351.04 | 624.00 | 966.24 | 1405.17 |
| | R | 640 | 427 | 320 | 213 | 160 | 130 | 106 | 81 | 64 | 54 |
| 80 4304 | P | 1.00 | 3.01 | 5.36 | 12.04 | 21.44 | 33.54 | 48.16 | 85.76 | 134.16 | 192.64 |
| | Q | 7.82 | 23.46 | 41.66 | 93.84 | 166.64 | 260.73 | 375.36 | 666.56 | 1042.92 | 1501.44 |
| | R | 684 | 456 | 342 | 228 | 171 | 137 | 114 | 87 | 69 | 58 |
| 90 4555 | P | 1.20 | 3.60 | 6.39 | 14.40 | 25.59 | 40.04 | 57.60 | 102.36 | 160.16 | 230.40 |
| | Q | 8.29 | 24.88 | 44.19 | 99.52 | 176.75 | 276.55 | 398.08 | 707.00 | 1106.20 | 1592.32 |
| | R | 726 | 484 | 363 | 242 | 181 | 145 | 121 | 93 | 73 | 62 |
| 100 4812 | P | 1.40 | 4.21 | 7.49 | 16.84 | 29.93 | 46.85 | 67.36 | 119.72 | 187.40 | 269.44 |
| | Q | 8.74 | 26.22 | 46.58 | 104.88 | 186.32 | 291.51 | 419.52 | 745.28 | 1166.04 | 1678.08 |
| | R | 765 | 510 | 382 | 255 | 191 | 152 | 127 | 96 | 77 | 64 |
| 120 5271 | P | 1.84 | 5.54 | 9.85 | 22.18 | 39.41 | 61.66 | 88.75 | 157.64 | 246.64 | 355.00 |
| | Q | 9.57 | 28.72 | 51.02 | 114.91 | 204.10 | 319.33 | 459.64 | 816.40 | 1277.32 | 1838.56 |
| | R | 838 | 559 | 419 | 279 | 209 | 167 | 139 | 105 | 83 | 70 |
| 140 5694 | P | 2.33 | 6.99 | 12.41 | 27.96 | 49.64 | 77.71 | 111.85 | 198.56 | 310.84 | 447.40 |
| | Q | 10.34 | 31.03 | 55.11 | 124.12 | 220.44 | 344.92 | 496.43 | 881.76 | 1379.68 | 1985.92 |
| | R | 906 | 604 | 453 | 302 | 226 | 181 | 151 | 114 | 90 | 75 |
| 160 6087 | P | 2.84 | 8.54 | 15.17 | 34.16 | 60.68 | 94.94 | 136.65 | 242.72 | 377.76 | 546.60 |
| | Q | 11.05 | 33.17 | 58.92 | 132.68 | 235.68 | 368.73 | 530.75 | 942.72 | 1474.92 | 2123.00 |
| | R | 969 | 646 | 484 | 323 | 242 | 193 | 161 | 121 | 97 | 81 |
| 180 6456 | P | 3.39 | 10.19 | 18.10 | 40.77 | 72.41 | 113.30 | 163.08 | 289.64 | 453.20 | 652.32 |
| | Q | 11.72 | 35.18 | 62.49 | 140.74 | 249.97 | 391.10 | 562.96 | 999.83 | 1564.40 | 2251.84 |
| | R | 1024 | 683 | 513 | 342 | 256 | 206 | 171 | 128 | 103 | 86 |
| 200 6805 | P | 3.97 | 11.93 | 21.20 | 47.75 | 84.81 | 132.70 | 191.00 | 339.24 | 530.80 | 764.00 |
| | Q | 12.36 | 37.08 | 65.87 | 148.35 | 263.49 | 412.25 | 593.40 | 1053.96 | 1649.00 | 2373.60 |
| | R | 1080 | 720 | 540 | 360 | 270 | 216 | 180 | 135 | 108 | 90 |
| 225 7215 | P | | | | 56.99 | 101.20 | 158.38 | 227.96 | 404.80 | 633.52 | 911.84 |
| | Q | | | | 157.33 | 279.44 | 437.23 | 629.32 | 1117.76 | 1748.92 | 2517.28 |
| | R | | | | 382 | 287 | 229 | 191 | 144 | 115 | 96 |
| 250 7608 | P | 5.56 | 16.68 | 29.63 | 66.74 | 118.54 | 185.47 | 266.96 | 474.16 | 741.88 | 1067.84 |
| | Q | 13.82 | 41.46 | 73.64 | 165.86 | 294.59 | 460.91 | 663.45 | 1178.36 | 1843.64 | 2653.80 |
| | R | 1209 | 806 | 605 | 403 | 302 | 241 | 202 | 151 | 121 | 101 |

Tangential Water-Wheel Table.—Continued.

| Head in Ft. | | 12 Inch. | 18 Inch. | 24 Inch. | 36 Inch. | 48 Inch. | 60 Inch. | 72 Inch. | 8 Feet. | 10 Feet. | 12 Feet. |
|----------------|---|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|-------------|-------------|
| 275 7975 | P | | | | 77.00 | 136.76 | 214.00 | 308.00 | 547.04 | 856.00 | 1232.00 |
| | Q | | | | 173.94 | 308.92 | 483.39 | 695.76 | 1235.68 | 1933.56 | 2783.04 |
| | R | | | | 423 | 317 | 253 | 211 | 159 | 127 | 106 |
| 300 8335 | P | 7.31 | 21.93 | 38.95 | 87.73 | 155.83 | 243.82 | 350.94 | 623.32 | 975.28 | 1403.76 |
| | Q | 15.13 | 45.42 | 80.67 | 181.59 | 322.71 | 504.91 | 726.76 | 1290.84 | 2019.64 | 2907.04 |
| | R | 1326 | 884 | 663 | 442 | 331 | 265 | 221 | 166 | 133 | 111 |
| 325 8672 | P | | | | 98.93 | 175.68 | 274.94 | 395.72 | 702.72 | 1099.76 | 1582.88 |
| | Q | | | | 189.10 | 335.84 | 525.50 | 756.40 | 1343.36 | 2102.00 | 3025.60 |
| | R | | | | 460 | 344 | 276 | 230 | 172 | 138 | 115 |
| 350 9002 | P | 9.21 | 27.64 | 49.09 | 110.56 | 196.38 | 307.25 | 442.27 | 785.52 | 1229.00 | 1769.08 |
| | Q | 16.35 | 49.06 | 87.14 | 196.25 | 348.57 | 545.36 | 785.00 | 1394.28 | 2181.44 | 3140.00 |
| | R | 1432 | 955 | 716 | 477 | 358 | 275 | 238 | 179 | 143 | 119 |
| 400 9624 | P | 11.25 | 33.77 | 59.98 | 135.08 | 239.94 | 375.40 | 540.35 | 959.76 | 1501.60 | 2161.40 |
| | Q | 17.48 | 52.45 | 93.16 | 209.80 | 372.64 | 583.02 | 839.20 | 1490.56 | 2332.08 | 3356.80 |
| | R | 1531 | 1021 | 765 | 510 | 382 | 305 | 255 | 101 | 153 | 128 |
| 450 10208 | P | 13.43 | 40.79 | 71.57 | 161.19 | 286.31 | 447.95 | 644.78 | 1145.24 | 1791.80 | 2579.12 |
| | Q | 18.54 | 55.63 | 98.81 | 222.52 | 395.24 | 618.38 | 890.11 | 1580.96 | 2473.52 | 3560.44 |
| | R | 1624 | 1083 | 812 | 541 | 406 | 324 | 270 | 203 | 162 | 135 |
| 500 10760 | P | 15.73 | 47.20 | 83.83 | 188.80 | 335.34 | 524.66 | 755.20 | 1341.36 | 2098.64 | 3020.80 |
| | Q | 19.54 | 58.64 | 104.15 | 234.56 | 416.62 | 651.83 | 938.25 | 1666.48 | 2607.02 | 3753.00 |
| | R | 1713 | 1142 | 856 | 571 | 428 | 342 | 285 | 214 | 171 | 143 |
| 550 11279 | P | | | | 217.82 | 386.84 | 605.31 | 871.28 | 1547.36 | 2421.24 | 3485.12 |
| | Q | | | | 246.00 | 436.92 | 683.62 | 984.00 | 1747.68 | 2734.48 | 3936.00 |
| | R | | | | 599 | 449 | 359 | 299 | 225 | 179 | 150 |
| 600 11787 | P | 24.26 | 62.04 | 110.19 | 248.16 | 440.77 | 689.63 | 992.65 | 1763.08 | 2758.52 | 3970.60 |
| | Q | 25.12 | 64.24 | 114.09 | 256.95 | 456.38 | 714.05 | 1027.80 | 1825.52 | 2856.20 | 4111.20 |
| | R | 1876 | 1251 | 938 | 625 | 469 | 375 | 312 | 235 | 188 | 156 |
| 640 12169 | P | | | | 270.97 | 484.16 | 748.80 | 1083.88 | 1936.64 | 2995.20 | 4335.52 |
| | Q | | | | 264.63 | 456.12 | 731.59 | 1058.52 | 1864.48 | 2926.36 | 4234.08 |
| | R | | | | 644 | 483 | 387 | 322 | 242 | 194 | 161 |
| 700 12731 | P | 30.57 | 78.18 | 138.86 | 312.73 | 555.46 | 869.06 | 1250.92 | 2221.84 | 3476.24 | 5003.68 |
| | Q | 27.13 | 69.38 | 123.23 | 277.54 | 492.95 | 771.25 | 1110.16 | 1971.80 | 3085.04 | 4440.64 |
| | R | 2026 | 1351 | 1013 | 675 | 506 | 405 | 337 | 253 | 203 | 169 |
| 750 13178 | P | 33.91 | 86.70 | 154.00 | 346.83 | 616.03 | 963.82 | 1387.34 | 2464.12 | 3855.28 | 5549.36 |
| | Q | 28.08 | 71.82 | 127.56 | 287.28 | 510.25 | 798.33 | 1149.13 | 2041.00 | 3193.32 | 4596.52 |
| | R | 2098 | 1309 | 1049 | 699 | 524 | 419 | 349 | 262 | 210 | 175 |
| 800 13610 | P | 37.35 | 95.52 | 169.66 | 382.09 | 678.66 | 1061.81 | 1528.36 | 2714.64 | 4247.24 | 6113.44 |
| | Q | 29.00 | 74.17 | 131.74 | 296.70 | 526.99 | 824.51 | 1186.81 | 2107.96 | 3298.04 | 4747.24 |
| | R | 2166 | 1444 | 1083 | 722 | 542 | 433 | 361 | 271 | 217 | 181 |
| 900 14436 | P | 44.57 | 113.98 | 202.45 | 455.94 | 809.82 | 1267.02 | 1823.76 | 3239.28 | 5068.08 | 7295.04 |
| | Q | 30.76 | 78.67 | 139.74 | 314.70 | 558.96 | 874.53 | 1258.81 | 2235.84 | 3498.12 | 5035.24 |
| | R | 2298 | 1532 | 1149 | 766 | 574 | 459 | 383 | 287 | 229 | 192 |
| 1000 15217 | P | 52.20 | 133.50 | 237.12 | 534.01 | 948.48 | 1483.97 | 2136.04 | 3793.92 | 5935.88 | 8544.16 |
| | Q | 32.42 | 82.93 | 147.30 | 331.72 | 589.19 | 921.83 | 1326.91 | 2356.76 | 3687.32 | 5287.64 |
| | R | 2420 | 1615 | 1210 | 807 | 605 | 484 | 403 | 303 | 242 | 202 |

The above tables are compiled on the following basis:

The head (h) is the net effective head at the nozzle. Proper allowance must be made for all losses in the pipe line.

The velocity of efflux (V) is the approximate spouting velocity of the jet in feet per minute as it issues from the nozzle = $\sqrt{2gh} \times 60 = 481.2 \sqrt{h}$.

The discharge in cubic feet per minute = $Q = V \times a$, where a equals the cross-section area of nozzle opening in sq. ft., no allowance being made for friction in the nozzle.

The weight of a cubic foot of water is taken at 39.2° Fahr. = 62.425 lbs.

The theoretical horse-power = $Q \times 62.425 \times h \div 33,000 = 0.00189 Qh$.

The horse-power in the tables is based on 85% mechanical efficiency for the wheels.

The diameter is the effective diameter at the line of the nozzle center, where the jet impinges on the center of the bucket.

The number of revolutions is based on a peripheral speed for the effective diameter, of half the velocity of efflux of the jet, and equals $V \div 2 C$, where C = the circumference (in feet) of the effective diameter.

Small wheels, up to 24-in. diam., are commonly called motors.

Amount of Water Required to Develop a Given Horse-Power, with a Given Available Effective Head.

| Effective Head in Feet. | Horse-Power Based on 85% Efficiency of the Water Wheel. | | | | | | | | | |
|-------------------------|---|-----|-----|-----|-----|-----|-----|------|------|------|
| | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| | Flow in Cubic Feet of Water per Minute Required to Develop Power. | | | | | | | | | |
| 50..... | 125 | 250 | 375 | 500 | 625 | 750 | 875 | 1000 | 1125 | 1250 |
| 60..... | 104 | 208 | 312 | 416 | 520 | 624 | 726 | 830 | 934 | 1038 |
| 70..... | 88 | 177 | 266 | 355 | 444 | 532 | 621 | 709 | 798 | 886 |
| 80..... | 77 | 155 | 232 | 311 | 388 | 466 | 544 | 622 | 699 | 876 |
| 90..... | 70 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 | 700 |
| 100..... | 63 | 125 | 186 | 248 | 312 | 372 | 435 | 498 | 558 | 622 |
| 110..... | 59 | 118 | 176 | 234 | 293 | 350 | 410 | 467 | 525 | 585 |
| 120..... | 52 | 104 | 156 | 208 | 260 | 312 | 364 | 415 | 467 | 520 |
| 130..... | 48 | 96 | 143 | 192 | 240 | 287 | 335 | 385 | 430 | 478 |
| 140..... | 45 | 89 | 133 | 178 | 222 | 266 | 310 | 355 | 400 | 443 |
| 150..... | 42 | 83 | 125 | 166 | 208 | 250 | 292 | 332 | 375 | 416 |
| 160..... | 39 | 78 | 117 | 155 | 195 | 233 | 272 | 312 | 350 | 388 |
| 170..... | 37 | 73 | 110 | 146 | 183 | 220 | 256 | 293 | 330 | 365 |
| 180..... | 35 | 69 | 104 | 138 | 172 | 207 | 242 | 276 | 310 | 345 |
| 190..... | 33 | 65 | 98 | 132 | 164 | 198 | 230 | 262 | 295 | 326 |
| 200..... | 31 | 62 | 93 | 124 | 155 | 186 | 218 | 248 | 280 | 310 |
| 210..... | 30 | 59 | 89 | 118 | 148 | 177 | 206 | 236 | 266 | 295 |
| 220..... | 28 | 57 | 85 | 113 | 141 | 169 | 198 | 225 | 255 | 283 |
| 230..... | 27 | 54 | 81 | 108 | 135 | 162 | 190 | 216 | 243 | 270 |
| 240..... | 26 | 52 | 78 | 104 | 130 | 155 | 181 | 207 | 233 | 258 |
| 250..... | 25 | 50 | 75 | 100 | 125 | 149 | 174 | 199 | 224 | 248 |
| 260..... | 24 | 48 | 72 | 96 | 120 | 144 | 167 | 191 | 215 | 238 |
| 270..... | 23 | 46 | 69 | 92 | 115 | 138 | 161 | 184 | 207 | 230 |
| 280..... | 22 | 45 | 67 | 89 | 111 | 133 | 156 | 178 | 200 | 222 |
| 290..... | 21 | 43 | 65 | 86 | 107 | 129 | 150 | 172 | 193 | 215 |
| 300..... | 20 | 42 | 62 | 83 | 104 | 124 | 145 | 166 | 187 | 208 |
| 310..... | 19 | 41 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 |
| 320..... | 19 | 40 | 59 | 78 | 97 | 117 | 136 | 156 | 175 | 194 |
| 330..... | 19 | 38 | 57 | 76 | 94 | 113 | 132 | 151 | 170 | 188 |
| 340..... | 18 | 37 | 55 | 74 | 92 | 110 | 128 | 146 | 165 | 183 |
| 350..... | 18 | 36 | 53 | 71 | 89 | 106 | 124 | 142 | 160 | 178 |
| 360..... | 18 | 35 | 52 | 69 | 86 | 102 | 121 | 138 | 155 | 172 |
| 370..... | 17 | 34 | 50 | 67 | 84 | 100 | 117 | 134 | 151 | 168 |
| 380..... | 17 | 33 | 49 | 66 | 82 | 98 | 114 | 130 | 147 | 164 |
| 390..... | 16 | 32 | 48 | 64 | 80 | 96 | 111 | 127 | 144 | 160 |
| 400..... | 16 | 31 | 47 | 63 | 77 | 94 | 105 | 124 | 140 | 156 |

Efficiency of the Doble Nozzle.—The nozzle tip is of brass, highly polished in the interior, with concave curves near the end. It contains a conical regulating needle, which is set at any desired distance from the opening to regulate the size of the opening and the diameter of the jet. A jet flowing from the nozzle has a clear, glassy appearance. Tests

by H. C. Crowell and G. C. D. Lenth, at Mass. Inst. of Tech., 1903, gave efficiencies under constant head from 96.4 to 99.3% for different settings of the needle, the coefficient of velocity being from 0.982 to 0.997. The efficiency of a jet is equal to the ratio of the velocity head in the jet to the total head at the entrance to the nozzle, and equal to the square of the coefficient of velocity. — *Bulletin of the Abner Doble Co.*, No. 6, 1904.

Tests of a 12-in. Doble Laboratory Motor (*Bulletin* No. 12, 1908, Abner Doble Co.).—The tests were made by students at the University of Missouri. The available head was 46 ft. The needle valve was opened two, four, six and eight turns in the four series of tests, and with each opening different loads were applied by a Prony brake. The results were recorded and plotted in curves showing the relation of speed, load and efficiency, and from these curves the following approximate figures are taken:

Speed, Revolutions per Minute.

| Valve open | 200 | 300 | 400 | 500 | 600 | 700 | 800 |
|---------------------------|------|------|------|------|------|------|------|
| Two turns { B.H.P. | 0.20 | 0.26 | 0.27 | 0.26 | 0.22 | 0.14 | 0.03 |
| { Effy. % | 62 | 75 | 80 | 77 | 64 | 41 | 13 |
| Four turns { B.H.P. | 0.36 | 0.45 | 0.51 | 0.50 | 0.42 | 0.30 | 0.12 |
| { Effy. % | 57 | 75 | 85 | 85 | 71 | 50 | 19 |
| Six turns { B.H.P. | 0.41 | 0.55 | 0.63 | 0.66 | 0.60 | 0.41 | 0.20 |
| { Effy. % | 48 | 64 | 73 | 76 | 74 | 66 | 51 |
| Eight turns { B.H.P. | 0.48 | 0.62 | 0.70 | 0.71 | 0.64 | 0.43 | 0.19 |
| { Effy. % | 53 | 70 | 79 | 81 | 72 | 50 | 23 |

Water-power Plants Operating under High Pressures.—The following notes are contributed by the Pelton Water Wheel Co.:

The Consolidated Virginia & Col. Mining Co., Virginia, Nev., has a 3-ft. steel-disk Pelton wheel operating under 2100 ft. fall, equal to 911 lbs. per sq. in. It runs at a peripheral velocity of 10,804 ft. per minute and has a capacity of over 100 H.P. The rigidity with which water under such a high pressure as this leaves the nozzle is shown in the fact that it is impossible to cut the stream with an axe, however heavy the blow, as it will rebound just as it would from a steel rod travelling at a high rate of speed.

The London Hydraulic Power Co. has a large number of Pelton wheels from 12 to 18 in. diameter running under pressure of about 1000 lbs. per sq. in. from a system of pressure-mains. The 18-in. wheels weighing 30 lbs. have a capacity of over 20 H.P. (See Blaine's "Hydraulic Machinery.")

Hydraulic Power-hoist of Milwaukee Mining Co., Idaho. — One cage travels up as the other descends; the maximum load of 5500 lbs. at a speed of 400 ft. per min. is carried by one of a pair of Pelton wheels (one for each cage). Wheels are started and stopped by opening and closing a small hydraulic valve at the engineer's stand which operates the larger valves by hydraulic pressure. An air-chamber takes up the shock that would otherwise occur on the pipe line under the pressure due to 850 ft. fall.

The Mannesmann Cycle Tube Works, North Adams, Mass., are using four Pelton wheels, having a fly-wheel rim, under a pump pressure of 600 lbs. per sq. in. These wheels are direct-connected to the rolls through which the ingots are passed for drawing out seamless tubing.

The Alaska Gold Mining Co., Douglass Island, Alaska, has a 22-ft. Pelton wheel on the shaft of a Riedler duplex compressor. It is used as a fly-wheel as well, weighing 25,000 lbs., and develops 500 H.P. at 75 revolutions. A valve connected to the pressure-chamber starts and stops the wheel automatically, thus maintaining the pressure in the air-receiver.

At Pachuca in Mexico five Pelton wheels having a capacity of 600 H.P. each under 800 ft. head are driving an electric transmission plant. These wheels weigh less than 500 lbs. each, showing over a horse-power per pound of metal.

Formulae for Calculating the Power of Jet Water-wheels, such as the Pelton (F. K. Blue). — HP = horse-power delivered; δ = 62.36 lbs. per cu. ft.; E = efficiency of turbine; q = quantity of water, cubic feet per minute; h = feet effective head; d = inches diameter of jet; p = pounds per square inch effective head; c = coefficient of discharge from nozzle, which may be ordinarily taken at 0.9.

$$HP = \frac{\delta E q h}{33000} = .00189 E q h = .00436 E q p = .00496 E c d^2 \sqrt{h^3} = .0174 E c d^2 \sqrt{p^3}.$$

$$q = 529.2 \frac{HP}{Eh} = 229 \frac{HP}{Ep} = 2.62 c d^2 \sqrt{h} = 3.99 c d^2 \sqrt{p}.$$

$$d^2 = 201.6 \frac{HP}{Ec \sqrt{h^3}} = 57.4 \frac{HP}{Ec \sqrt{p^3}} = 0.381 \frac{q}{c \sqrt{h}} = 0.25 \frac{q}{c \sqrt{p}}.$$

THE POWER OF OCEAN WAVES.

Albert W. Stahl, U. S. N. (*Trans. A. S. M. E.*, xiii. 438), gives the following formulæ and table, based upon a theoretical discussion of wave motion:

The total energy of one whole wave-length of a wave H feet high, L feet long, and one foot in breadth, the length being the distance between successive crests, and the height the vertical distance between the crest and the trough, is $E = 8 LH^2 \left(1 - 4.935 \frac{H^2}{L^2}\right)$ foot-pounds.

The time required for each wave to travel through a distance equal to its own length is $P = \sqrt{\frac{L}{5.123}}$ seconds, and the number of waves passing any given point in one minute is $N = \frac{60}{P} = 60 \sqrt{\frac{5.123}{L}}$. Hence the total energy of an indefinite series of such waves, expressed in horse-power per foot of breadth, is

$$\frac{E \times N}{33,000} = 0.0329 \frac{H^2 L}{\sqrt{L}} \left(1 - 4.935 \frac{H^2}{L^2}\right).$$

By substituting various values for $H \div L$, within the limits of such values actually occurring in nature, we obtain the following table of

TOTAL ENERGY OF DEEP-SEA WAVES IN TERMS OF HORSE-POWER PER FOOT OF BREADTH.

| Ratio of Length to Height of Waves. | Length of Waves in Feet. | | | | | | | |
|--|--------------------------|-------|-------|--------|--------|--------|---------|---------|
| | 25 | 50 | 75 | 100 | 150 | 200 | 300 | 400 |
| 50 | 0.04 | 0.23 | 0.64 | 1.31 | 3.62 | 7.43 | 20.46 | 42.01 |
| 40 | 0.06 | 0.36 | 1.00 | 2.05 | 5.65 | 11.59 | 31.95 | 65.58 |
| 30 | 0.12 | 0.64 | 1.77 | 3.64 | 10.02 | 20.57 | 56.70 | 116.38 |
| 20 | 0.25 | 1.44 | 3.96 | 8.13 | 21.79 | 45.98 | 120.70 | 260.08 |
| 15 | 0.42 | 2.83 | 6.97 | 14.31 | 39.43 | 80.94 | 223.06 | 457.89 |
| 10 | 0.98 | 5.53 | 15.24 | 31.29 | 86.22 | 177.00 | 487.75 | 1001.25 |
| 5 | 3.30 | 18.68 | 51.48 | 105.68 | 291.20 | 597.78 | 1647.31 | 3381.60 |

The figures are correct for trochoidal deep-sea waves only, but they give a close approximation for any nearly regular series of waves in deep water and a fair approximation for waves in shallow water.

The question of the practical utilization of the energy which exists in ocean waves divides itself into several parts:

1. The various motions of the water which may be utilized for power purposes.

2. The wave-motor proper. That is, the portion of the apparatus in direct contact with the water, and receiving and transmitting the energy thereof; together with the mechanism for transmitting this energy to the machinery for utilizing the same.

3. Regulating devices, for obtaining a uniform motion from the irregular and more or less spasmodic action of the waves, as well as for adjusting the apparatus to the state of the tide and condition of the sea.

4. Storage arrangements for insuring a continuous and uniform output of power during a calm, or when the waves are comparatively small.

The motions that may be utilized for power purposes are the following: 1. Vertical rise and fall of particles at and near the surface. 2. Horizontal to-and-fro motion of particles at and near the surface. 3. Varying slope of surface of wave. 4. Impetus of waves rolling up the beach in the form of breakers. 5. Motion of distorted verticals. All of these motions, except the last one mentioned, have at various times been proposed to be utilized for power purposes; and the last is proposed to be used in apparatus described by Mr. Stahl.

The motion of distorted verticals is thus defined: A set of particles, originally in the same vertical straight line when the water is at rest, does not remain in a vertical line during the passage of the wave; so that the line connecting a set of such particles, while vertical and straight in still water, becomes distorted, as well as displaced, during the passage of the wave, its upper portion moving farther and more rapidly than its lower portion.

Mr. Stahl's paper contains illustrations of several wave-motors designed upon various principles. His conclusion as to their practicability is as follows: "Possibly none of the methods described in this paper may ever prove commercially successful; indeed the problem may not be susceptible of a financially successful solution. My own investigations, however, so far as I have yet been able to carry them, incline me to the belief that wave-power can and will be utilized on a paying basis."

Continuous Utilization of Tidal Power. (P. DeCour, *Proc. Inst. C. E.* 1890.) — In connection with the training-walls to be constructed in the estuary of the Seine, it is proposed to construct large basins, by means of which the power available from the rise and fall of the tide could be utilized. The method proposed is to have two basins separated by a bank rising above high water, within which turbines would be placed. The upper basin would be in communication with the sea during the higher one-third of the tidal range, rising, and the lower basin during the lower one-third of the tidal range, falling. If H be the range in feet, the level in the upper basin would never fall below $\frac{2}{3}H$ measured from low water, and the level in the lower basin would never rise above $\frac{1}{3}H$. The available head varies between $0.53H$ and $0.80H$, the mean value being $\frac{2}{3}H$. If S square feet be the area of the lower basin, and the above conditions are fulfilled, a quantity $\frac{1}{3}SH$ cu. ft. of water is delivered through the turbines in the space of $9\frac{1}{4}$ hours. The mean flow is, therefore, $SH \div 99,900$ cu. ft. per sec., and, the mean fall being $\frac{2}{3}H$, the available gross horse-power is about $\frac{1}{30}S'H^2$, where S' is measured in acres. This might be increased by about one-third if a variation of level in the basins amounting to $\frac{1}{2}H$ were permitted. But to reach this end the number of turbines would have to be doubled, the mean head being reduced to $\frac{1}{2}H$, and it would be more difficult to transmit a constant power from the turbines. The turbine proposed is of an improved model designed to utilize a large flow with a moderate diameter. One has been designed to produce 300 horse-power, with a minimum head of 5 ft. 3 in. at a speed of 15 revolutions per minute, the vanes having 13 ft. internal diameter. The speed would be maintained constant by regulating sluices.

PUMPS AND PUMPING ENGINES.

Theoretical Capacity of a Pump. — Let Q' = cu. ft. per min.; G' = U. S. gals. per min. = $7.4805 Q'$; d = diam. of pump in inches; l = stroke in inches; N = number of single strokes per min.

$$\text{Capacity in cu. ft. per min.} = Q' = \frac{\pi}{4} \cdot \frac{d^2}{144} \cdot \frac{LN}{12} = 0.0004545 N d^2 l;$$

$$\text{Capacity in U. S. gals. per min. } G' = \frac{\pi}{4} \cdot \frac{N d^2 l}{231} \dots\dots\dots = 0.0034 N d^2 l;$$

$$\text{Capacity in gals. per hour} \dots\dots\dots = 0.204 N d^2 l.$$

$$\text{Diameter required for a } \left. \begin{array}{l} \text{given capacity per min.} \end{array} \right\} d = 46.9 \sqrt{\frac{Q'}{Nl}} = 17.15 \sqrt{\frac{G'}{Nl}}$$

$$\text{If } v = \text{piston speed in feet per min., } d = 13.54 \sqrt{\frac{Q'}{v}} = 4.95 \sqrt{\frac{G'}{v}}.$$

If the piston speed is 100 feet per min.:

$$Nl = 1200, \text{ and } d = 1.354 \sqrt{Q'} = 0.495 \sqrt{G'}; \quad G' = 4.08 d^2 \text{ per min.}$$

The actual capacity will be from 60% to 95% of the theoretical, according to the tightness of the piston, valves, suction-pipe, etc.

Theoretical Horse-power Required to Raise Water to a Given Height. — Horse-power =

$$\frac{\text{Volume in cu. ft. per min.} \times \text{pressure per sq. ft.}}{33,000} = \frac{\text{Weight} \times \text{height of lift}}{33,000}$$

Q' = cu. ft. per min.; G' = gals. per min.; W = wt. in lbs.; P = pressure in lbs. per sq. ft.; p = pressure in lbs. per sq. in.; H = height of lift in ft.; $W = 62.355 Q'$, $P = 144 p$, $p = 0.433 H$, $H = 2.3094 p$, $G' = 7.4805 Q'$.

$$\text{HP.} = \frac{Q'P}{33,000} = \frac{Q'H \times 144 \times 0.433}{33,000} = \frac{Q'H}{529.23} = \frac{G'H}{3958.9} = \frac{1.0104 G'H}{4000}$$

$$\text{HP.} = \frac{WH}{33,000} = \frac{Q' \times 62.355 \times 2.3094 p}{33,000} = \frac{Q'p}{229.17} = \frac{G'p}{1714.3}.$$

For the actual horse-power required an allowance must be made for the friction, slips, etc., of engine, pump, valves, and passages.

Depth of Suction. — Theoretically a perfect pump will draw water from a height of nearly 34 feet, or the height corresponding to a perfect vacuum (14.7 lbs. \times 2.309 = 33.95 feet): but since a perfect vacuum cannot be obtained on account of valve-leakage, air contained in the water, and the vapor of the water itself, the actual height is generally less than 30 feet. When the water is warm the height to which it can be lifted by suction decreases, on account of the increased pressure of the vapor. In pumping hot water, therefore, the water must flow into the pump by gravity. The following table shows the theoretical maximum depth of suction for different temperatures, leakage not considered:

| Temp. Fahr. | Absolute Pressure of Vapor, lbs. per sq. in. | Vacuum in Inches of Mercury. | Max. Depth of Suction, feet. | Temp. Fahr. | Absolute Pressure of Vapor, lbs. per sq. in. | Vacuum in Inches of Mercury. | Max. Depth of Suction, feet. |
|----------------|--|---------------------------------------|--|----------------|--|---------------------------------------|--|
| 102.1 | 1 | 27.88 | 31.6 | 182.9 | 8 | 13.63 | 15.4 |
| 126.3 | 2 | 25.85 | 29.3 | 188.3 | 9 | 11.60 | 13.1 |
| 141.6 | 3 | 23.83 | 27.0 | 193.2 | 10 | 9.56 | 10.8 |
| 153.1 | 4 | 21.78 | 24.7 | 197.8 | 11 | 7.52 | 8.5 |
| 162.3 | 5 | 19.74 | 22.3 | 202.0 | 12 | 5.49 | 6.2 |
| 170.1 | 6 | 17.70 | 20.0 | 205.9 | 13 | 3.45 | 3.9 |
| 176.9 | 7 | 15.67 | 17.7 | 209.6 | 14 | 1.41 | 1.6 |

The Deane Single Boiler-feed or Pressure Pump. — Suitable for pumping clear liquids at a pressure not exceeding 150 lbs.

| Number. | Sizes. | | | Gallons per Stroke. | Capacity per min. at Given Speed. | | Length in inches. | Width in inches. | Sizes of Pipes. | | | |
|---------|------------------|------------------|-------------------|---------------------|-----------------------------------|----------|-------------------|------------------|-----------------|----------|----------|------------|
| | Steam-cyl-inder. | Water-cyl-inder. | Length of Stroke. | | Strokes. | Gallons. | | | Steam. | Exhaust. | Suction. | Discharge. |
| 0 | 3 | 2 | 5 | .07 | 150 | 10 | 29 1/2 | 7 | 1/2 | 3/4 | 1 1/4 | 1 |
| 1 | 3 1/2 | 2 1/4 | 5 | .09 | 150 | 13 | 33 1/2 | 7 1/2 | 1/2 | 3/4 | 1 1/4 | 1 |
| 1 1/2 | 4 | 2 3/8 | 5 | .10 | 150 | 15 | 33 1/2 | 7 1/2 | 1/2 | 3/4 | 1 1/4 | 1 |
| 2 | 4 | 2 1/2 | 5 | .11 | 150 | 16 | 33 1/2 | 7 1/2 | 1/2 | 3/4 | 1 1/4 | 1 |
| 2 1/2 | 4 3/4 | 3 | 5 | .15 | 150 | 22 | 34 | 8 1/2 | 1/2 | 3/4 | 1 1/2 | 1 1/4 |
| 3 | 5 | 3 1/4 | 7 | .25 | 125 | 31 | 43 1/2 | 9 1/4 | 3/4 | 1 | 2 | 1 1/2 |
| 4 | 5 1/2 | 3 3/4 | 7 | .33 | 125 | 42 | 43 1/2 | 9 1/4 | 3/4 | 1 | 2 | 1 1/2 |
| 4 1/2 | 7 | 4 1/4 | 8 | .49 | 120 | 58 | 51 1/2 | 12 | 1 | 1 1/2 | 3 | 2 |
| 5 | 7 | 4 1/2 | 10 | .69 | 100 | 69 | 55 | 12 | 1 | 1 1/2 | 3 | 2 |
| 5 1/2 | 7 1/2 | 5 | 10 | .85 | 100 | 85 | 55 | 12 | 1 | 1 1/2 | 3 | 2 |
| 6 | 8 | 5 | 12 | 1.02 | 100 | 102 | 63 | 14 | 1 | 1 1/2 | 3 | 2 1/2 |
| 6 1/2 | 10 | 6 | 12 | 1.47 | 100 | 147 | 69 | 19 | 1 1/2 | 2 | 4 | 4 |
| 7 | 12 | 7 | 12 | 2.00 | 100 | 200 | 69 | 19 | 2 | 2 1/2 | 5 | 4 |
| 8 | 14 | 8 | 12 | 2.61 | 100 | 261 | 69 | 21 | 2 | 2 1/2 | 5 | 5 |

The Deane Single Tank or Light-service Pump. — These pumps will all stand a constant working pressure of 75 lbs. on the water-cylinders.

| Sizes. | | | Gallons per Stroke. | Capacity per min. at Given Speed. | | Length in inches. | Width in inches. | Sizes of Pipes. | | | |
|------------------|------------------|-------------------|---------------------|-----------------------------------|----------|-------------------|------------------|-----------------|----------|----------|------------|
| Steam-cyl-inder. | Water-cyl-inder. | Length of Stroke. | | Strokes. | Gallons. | | | Steam. | Exhaust. | Suction. | Discharge. |
| 4 | 4 | 5 | .27 | 130 | 35 | 33 | 9 1/2 | 1/2 | 3/4 | 2 | 1 1/2 |
| 5 | 4 | 7 | .38 | 125 | 48 | 45 1/2 | 15 | 3/4 | 1 | 3 | 2 1/2 |
| 5 1/2 | 5 1/2 | 7 | .72 | 125 | 90 | 45 1/2 | 15 | 3/4 | 1 | 3 | 2 1/2 |
| 7 1/2 | 7 1/2 | 10 | 1.91 | 110 | 210 | 58 | 17 | 1 | 1 1/2 | 5 | 4 |
| 8 | 6 | 12 | 1.46 | 100 | 146 | 67 | 20 1/2 | 1 | 1 1/2 | 4 | 4 |
| 6 | 7 | 12 | 2.00 | 100 | 200 | 66 | 17 | 3/4 | 1 | 4 | 4 |
| 8 | 7 | 12 | 2.00 | 100 | 209 | 67 | 20 1/2 | 1 | 1 1/2 | 5 | 4 |
| 8 | 8 | 12 | 2.61 | 100 | 261 | 68 | 30 | 1 | 1 1/2 | 5 | 5 |
| 10 | 8 | 12 | 2.61 | 100 | 261 | 68 1/2 | 30 | 1 1/2 | 2 | 5 | 5 |
| 8 | 10 | 12 | 4.08 | 100 | 408 | 68 | 20 1/2 | 1 | 1 1/2 | 8 | 8 |
| 10 | 10 | 12 | 4.08 | 100 | 408 | 68 1/2 | 30 | 1 1/2 | 2 | 8 | 8 |
| 12 | 10 | 12 | 4.08 | 100 | 408 | 64 | 24 | 2 | 2 1/2 | 8 | 8 |
| 10 | 12 | 12 | 5.87 | 100 | 587 | 68 1/2 | 30 | 1 1/2 | 2 | 8 | 8 |
| 12 | 12 | 12 | 5.87 | 100 | 587 | 64 | 28 1/2 | 2 | 2 1/2 | 8 | 8 |
| 10 | 12 | 18 | 8.79 | 70 | 616 | 95 | 25 | 1 1/2 | 2 | 8 | 8 |
| 12 | 12 | 18 | 8.79 | 70 | 616 | 95 | 28 1/2 | 2 | 2 1/2 | 8 | 8 |
| 12 | 14 | 18 | 12.00 | 70 | 840 | 95 | 28 1/2 | 2 | 2 1/2 | 8 | 8 |
| 14 | 16 | 18 | 15.66 | 70 | 1096 | 95 | 34 | 2 | 2 1/2 | 12 | 10 |
| 16 | 16 | 18 | 15.66 | 70 | 1096 | 95 | 34 | 2 | 2 1/2 | 12 | 10 |
| 18 | 16 | 18 | 15.66 | 70 | 1096 | 97 | 34 | 3 | 3 1/2 | 12 | 10 |
| 16 | 18 | 24 | 26.42 | 50 | 1321 | 115 | 40 | 2 | 2 1/2 | 14 | 12 |
| 18 | 18 | 24 | 26.42 | 50 | 1321 | 135 | 40 | 3 | 3 1/2 | 14 | 12 |

Amount of Water raised by a Single-acting Lift-pump. — It is common to estimate that the quantity of water raised by a single-acting bucket-valve pump per minute is equal to the number of strokes in one direction per minute, multiplied by the volume traversed by the piston in a single stroke, on the theory that the water rises in the pump only when the piston or bucket ascends; but the fact is that the column of water does not cease flowing when the bucket descends, but flows on continuously through the valve in the bucket, so that the discharge of the pump, if it is operated at a high speed, may amount to considerably more than that calculated from the displacement multiplied by the number of single strokes in one direction.

Proportioning the Steam-cylinder of a Direct-acting Pump. — Let

A = area of steam-cylinder; a = area of pump-cylinder;
 D = diameter of steam-cylinder; d = diameter of pump-cylinder;
 P = steam-pressure, lbs. per sq. in.; p = resistance per sq. in. on pumps;
 H = head = 2.309 p ; $p = 0.433 H$;

E = efficiency of the pump = $\frac{\text{work done in pump-cylinder}}{\text{work done by the steam-cylinder}}$.

$$A = \frac{ap}{EP}; a = \frac{EAP}{p}; D = d \sqrt{\frac{p}{EP}}; d = D \sqrt{\frac{EP}{p}}; P = \frac{ap}{EA}; p = \frac{EAP}{a}.$$

$$\frac{A}{a} = \frac{p}{EP} = \frac{0.433 H}{EP}; H = 2.309 EP \frac{A}{a}. \text{ If } E = 75\%, H = 1.732 P \frac{A}{a}.$$

E is commonly taken at 0.7 to 0.8 for ordinary direct-acting pumps. For the highest class of pumping-engines it may amount to 0.9. The steam-pressure P is the mean effective pressure, according to the indicator-diagram; the water-pressure p is the mean total pressure acting on the pump plunger or piston, including the suction, as could be shown by an indicator-diagram of the water-cylinder. The pressure on the pump-piston is frequently much greater than that due to the height of the lift, on account of the friction of the valves and passages, which increases rapidly with velocity of flow.

Speed of Water through Pipes and Pump-passages. — The speed of the water is commonly from 100 to 200 feet per minute. If 200 feet per minute is exceeded, the loss from friction may be considerable.

The diameter of pipe required is $4.95 \sqrt{\frac{\text{gallons per minute}}{\text{velocity in feet per minute}}}$.

For a velocity of 200 feet per minute, diam. = $0.35 \times \sqrt{\text{gallons per min.}}$

Sizes of Direct-acting Pumps. — The tables on pages 758 and 760 are selected from catalogues of manufacturers, as representing the two common types of direct-acting pump, viz., the single-cylinder and the duplex. Both types are made by most of the leading manufacturers.

Efficiency of Small Direct-acting Pumps. — Chas. E. Emery, in Reports of Judges of Philadelphia Exhibition, 1876, Group xx., says: "Experiments made with steam-pumps at the American Institute Exhibition of 1867 showed that average-sized steam-pumps do not, on the average, utilize more than 50 per cent of the indicated power in the steam-cylinders, the remainder being absorbed in the friction of the engine, but more particularly in the passage of the water through the pump. It may be safely stated that ordinary steam-pumps rarely require less than 120 pounds of steam per hour for each horse-power utilized in raising water, equivalent to a duty of only 15,000,000 foot-pounds per 100 pounds of coal. With larger steam-pumps, particularly when they are proportioned for the work to be done, the duty will be materially increased."

The Worthington Duplex Pump.

STANDARD SIZES FOR ORDINARY SERVICE.

| Diameter of Steam-cylinders. | Diameter of Water-plungers. | Length of Stroke. | Displacement in Gallons per Stroke of One Plunger. | Proper Strokes per Minute of One Plunger, varying with kind of work and pressure. | Gallons delivered per Minute by both Plungers at stated Number of Strokes. | Diameter of Plunger required in any single-cylinder pump to do the same work at same speed. | Sizes of Pipes for Short Lengths. To be increased as length increases. | | | |
|------------------------------|-----------------------------|-------------------|--|---|--|---|---|---------------|---------------|-----------------|
| | | | | | | | Steam-pipe. | Exhaust-pipe. | Suction-pipe. | Discharge-pipe. |
| 3 | 2 | 3 | .04 | 100 to 250 | 8 to 20 | 27/8 | 3/8 | 1/2 | 1 1/4 | 1 |
| 4 1/2 | 2 3/4 | 4 | .10 | 100 to 200 | 20 to 40 | 4 | 1/2 | 3/4 | 2 | 1 1/2 |
| 5 1/4 | 3 1/2 | 5 | .20 | 100 to 200 | 40 to 80 | 5 | 3/4 | 1 1/4 | 2 1/2 | 1 1/2 |
| 6 | 4 | 6 | .33 | 100 to 150 | 70 to 100 | 5 5/8 | 1 | 1 1/2 | 3 | 2 |
| 7 1/2 | 4 1/2 | 6 | .42 | 100 to 150 | 85 to 125 | 6 3/8 | 1 1/2 | 2 | 4 | 3 |
| 7 1/2 | 5 | 6 | .51 | 100 to 150 | 100 to 150 | 7 | 1 1/2 | 2 | 4 | 3 |
| 7 1/2 | 4 1/2 | 10 | .69 | 75 to 125 | 100 to 170 | 6 3/8 | 1 1/2 | 2 | 4 | 3 |
| 9 | 5 1/4 | 10 | .93 | 75 to 125 | 135 to 230 | 7 1/2 | 2 | 2 1/2 | 4 | 3 |
| 10 | 6 | 10 | 1.22 | 75 to 125 | 180 to 300 | 8 1/2 | 2 | 2 1/2 | 5 | 4 |
| 10 | 7 | 10 | 1.66 | 75 to 125 | 245 to 410 | 9 7/8 | 2 | 2 1/2 | 6 | 5 |
| 12 | 7 | 10 | 1.66 | 75 to 125 | 245 to 410 | 9 7/8 | 2 1/2 | 3 | 6 | 5 |
| 14 | 7 | 10 | 1.66 | 75 to 125 | 245 to 410 | 9 7/8 | 2 1/2 | 3 | 6 | 5 |
| 12 | 8 1/2 | 10 | 2.45 | 75 to 125 | 365 to 610 | 12 | 2 1/2 | 3 | 6 | 5 |
| 14 | 8 1/2 | 10 | 2.45 | 75 to 125 | 365 to 610 | 12 | 2 1/2 | 3 | 6 | 5 |
| 16 | 8 1/2 | 10 | 2.45 | 75 to 125 | 365 to 610 | 12 | 2 1/2 | 3 | 6 | 5 |
| 18 1/2 | 8 1/2 | 10 | 2.45 | 75 to 125 | 365 to 610 | 12 | 3 | 3 1/2 | 6 | 5 |
| 20 | 8 1/2 | 10 | 2.45 | 75 to 125 | 365 to 610 | 12 | 4 | 5 | 6 | 5 |
| 12 | 10 1/4 | 10 | 3.57 | 75 to 125 | 530 to 890 | 14 1/4 | 2 1/2 | 3 | 8 | 7 |
| 14 | 10 1/4 | 10 | 3.57 | 75 to 125 | 530 to 890 | 14 1/4 | 2 1/2 | 3 | 8 | 7 |
| 16 | 10 1/4 | 10 | 3.57 | 75 to 125 | 530 to 890 | 14 1/4 | 2 1/2 | 3 | 8 | 7 |
| 18 1/2 | 10 1/4 | 10 | 3.57 | 75 to 125 | 530 to 890 | 14 1/4 | 3 | 3 1/2 | 8 | 7 |
| 20 | 10 1/4 | 10 | 3.57 | 75 to 125 | 530 to 890 | 14 1/4 | 4 | 5 | 8 | 7 |
| 14 | 12 | 10 | 4.89 | 75 to 125 | 730 to 1220 | 17 | 2 1/2 | 3 | 10 | 8 |
| 16 | 12 | 10 | 4.89 | 75 to 125 | 730 to 1220 | 17 | 2 1/2 | 3 | 10 | 8 |
| 18 1/2 | 12 | 10 | 4.89 | 75 to 125 | 730 to 1220 | 17 | 3 | 3 1/2 | 10 | 8 |
| 20 | 12 | 10 | 4.89 | 75 to 125 | 730 to 1220 | 17 | 4 | 5 | 10 | 8 |
| 18 1/2 | 14 | 10 | 6.66 | 75 to 125 | 990 to 1660 | 19 3/4 | 3 | 3 1/2 | 12 | 10 |
| 20 | 14 | 10 | 6.66 | 75 to 125 | 990 to 1660 | 19 3/4 | 4 | 5 | 12 | 10 |
| 17 | 10 | 15 | 5.10 | 50 to 100 | 510 to 1020 | 14 | 3 | 3 1/2 | 8 | 7 |
| 20 | 12 | 15 | 7.34 | 50 to 100 | 730 to 1460 | 17 | 4 | 5 | 12 | 10 |
| 20 | 15 | 15 | 11.47 | 50 to 100 | 1145 to 2290 | 21 | | | | |
| 25 | 15 | 15 | 11.47 | 50 to 100 | 1145 to 2290 | 21 | | | | |

Speed of Piston. — A piston speed of 100 feet per minute is commonly assumed as correct in practice, but for short-stroke pumps this gives too high a speed of rotation, requiring too frequent a reversal of the valves. For long-stroke pumps, 2 feet and upward, this speed may be considerably exceeded, if valves and passages are of ample area.

Number of Strokes Required to Attain a Piston Speed from 50 to 125 Feet per Minute for Pumps Having Strokes from 3 to 18 Inches in Length.

| Speed of Piston, in feet per min. | Length of Stroke in Inches. | | | | | | | | | |
|-----------------------------------|-------------------------------|-----|-----|-----|-----|-------|-----|-----|-----|----|
| | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 | 15 | 18 |
| | Number of Strokes per Minute. | | | | | | | | | |
| 50 | 200 | 150 | 120 | 100 | 86 | 75 | 60 | 50 | 40 | 33 |
| 55 | 220 | 165 | 132 | 110 | 94 | 82.5 | 66 | 55 | 44 | 37 |
| 60 | 240 | 180 | 144 | 120 | 103 | 90 | 72 | 60 | 48 | 40 |
| 65 | 260 | 195 | 156 | 130 | 111 | 97.5 | 78 | 65 | 52 | 43 |
| 70 | 280 | 210 | 168 | 140 | 120 | 105 | 84 | 70 | 56 | 47 |
| 75 | 300 | 225 | 180 | 150 | 128 | 112.5 | 90 | 75 | 60 | 50 |
| 80 | 320 | 240 | 192 | 160 | 137 | 120 | 96 | 80 | 64 | 53 |
| 85 | 340 | 255 | 204 | 170 | 146 | 127.5 | 102 | 85 | 68 | 57 |
| 90 | 360 | 270 | 216 | 180 | 154 | 135 | 108 | 90 | 72 | 60 |
| 95 | 380 | 285 | 228 | 190 | 163 | 142.5 | 114 | 95 | 76 | 63 |
| 100 | 400 | 300 | 240 | 200 | 171 | 150 | 120 | 100 | 80 | 67 |
| 105 | 420 | 315 | 252 | 210 | 180 | 157.5 | 126 | 105 | 84 | 70 |
| 110 | 440 | 330 | 264 | 220 | 188 | 165 | 132 | 110 | 88 | 73 |
| 115 | 460 | 345 | 276 | 230 | 197 | 172.5 | 138 | 115 | 92 | 77 |
| 120 | 480 | 360 | 288 | 240 | 206 | 180 | 144 | 120 | 96 | 80 |
| 125 | 500 | 375 | 300 | 250 | 214 | 187.5 | 150 | 125 | 100 | 83 |

Piston Speed of Pumping-engines. — (John Birkinbine, *Trans. A. I. M. E.*, v. 459.) — In dealing with such a ponderous and unyielding substance as water there are many difficulties to overcome in making a pump work with a high piston speed. The attainment of moderately high speed is, however, easily accomplished. Well-proportioned pumping-engines of large capacity, provided with ample water-ways and properly constructed valves, are operated successfully against heavy pressures at a speed of 250 ft. per minute, without "thug," concussion, or injury to the apparatus, and there is no doubt that the speed can be still further increased.

Speed of Water through Valves. — If areas through valves and water passages are sufficient to give a velocity of 250 ft. per min. or less, they are ample. The water should be carefully guided and not too abruptly deflected. (F. W. Dean, *Eng. News*, Aug. 10, 1893.)

Boiler-feed Pumps. — Practice has shown that 100 ft. of piston speed per minute is the limit, if excessive wear and tear is to be avoided.

The velocity of water through the suction-pipe must not exceed 200 ft. per minute, else the resistance of the suction is too great.

The approximate size of suction-pipe, where the length does not exceed 25 ft. and there are not more than two elbows, may be found as follows:

$\frac{7}{10}$ of the diameter of the cylinder multiplied by $\frac{1}{100}$ of the piston speed in feet. For duplex pumps of small size, a pipe one size larger is usually employed. The velocity of flow in the discharge-pipe should not exceed 500 ft. per minute. The volume of discharge and length of pipe vary so greatly in different installations that where the water is to be forced more than 50 ft. the size of discharge-pipe should be calculated for the particular conditions, allowing no greater velocity than 500 ft. per minute. The size of discharge-pipe is calculated in single-cylinder pumps from 250 to 400 ft. per minute. Greater velocity is permitted in the larger pipes.

In determining the proper size of pump for a steam-boiler, allowance must be made for a supply of water sufficient for the maximum capacity of the boiler when over driven, with an additional allowance for feeding water beyond this maximum capacity when the water level in the boiler becomes low. The average run of horizontal tubular boilers will evaporate from 2 to 3 lbs. of water per sq. ft. of heating-surface per hour, but

may be driven up to 6 lbs. if the grate-surface is too large or the draught too great for economical working.

Pump-Valves. — A. F. Nagle (*Trans. A. S. M. E.*, x. 521) gives a number of designs with dimensions of double-beat or Cornish valves used in large pumping-engines, with a discussion of the theory of their proportions. Mr. Nagle says: There is one feature in which the Cornish valves are necessarily defective, namely, the lift must always be quite large, unless great power is sacrificed to reduce it. A small valve presents proportionately a larger surface of discharge with the same lift than a larger valve, so that whatever the total area of valve-seat opening, its full contents can be discharged with less lift through numerous small valves than with one large one. See also Mr. Nagle's paper on Pump Valves and Valve Areas, *Trans. A. S. M. E.*, 1909.

Henry R. Worthington was the first to use numerous small rubber valves in preference to the larger metal valves. These valves work well under all the conditions of a city pumping-engine. A volute spring is generally used to limit the rise of the valve.

In the Leavitt high-duty sewerage-engine at Boston (*Am. Machinist*, May 31, 1884), the valves are of rubber, $\frac{3}{4}$ inch thick, the opening in valve-seat being $13\frac{1}{2} \times 4\frac{1}{2}$ inches. The valves have iron face and back-plates, and form their own hinges.

The large pumping engines at the St. Louis water works have rubber valves $3\frac{1}{2}$ in. outside diam. There are seven valve cages in each of the suction and discharge diaphragms, each cage having 28 valves. The aggregate free area of 196 valves is 7.76 sq. ft., the area of one plunger being 6.26 sq. ft. The suction and discharge pipes are each 36 in. diam., = 7.07 sq. ft. area. (Bull. No. 1609, Allis-Chalmers Co. Such liberal proportions of valves are found usually only in the highest grade of large high-duty engines. In small and medium sized pumps a valve area equal to one-third the plunger area is commonly used.)

The Worthington "High-Duty" Pumping Engine dispenses with a fly-wheel, and substitutes for it a pair of oscillating hydraulic cylinders, which receive part of the energy exerted by the steam during the first half of the stroke, and give it out in the latter half. For description see catalogue of H. R. Worthington, New York. A test of a triple expansion condensing engine of this type is reported in *Eng. News*, Nov. 29, 1904. Steam cylinders 13, 21, 34 ins.; plungers 30 in., stroke 25 in. Steam pressure, 124 lbs. Total head, 79 ft.; capacity, 14,267,000 gal. in 24 hrs. Duty per million B.T.U., 102,224,000 ft.-lbs.

The d'Auria Pumping Engine substitutes for a fly-wheel a compensating cylinder in line with the plunger, with a piston which pushes water to and fro through a pipe connecting the ends of the cylinder. It is built by the Builders' Iron Foundry, Providence, R. I.

A 72,000,000-gallon Pumping Engine at the Calf Pasture Station of the Boston Main Drainage Works is described in *Eng. News*, July 6, 1905. It has three cylinders, $18\frac{1}{2}$, 33 and $52\frac{3}{4}$ ins., and two plungers, 60-in. diam.; stroke of all, 10 ft. The piston-rods of the two smaller cylinders connect to one end of a walking beam and the rod of the third cylinder to the other. Steam pressure 185 lbs. gauge; revolutions per min., 17; static head 37 to 43 ft. Suction valves 128; ports, $4 \times 16\frac{1}{4}$ in.; total port area 8576 sq. in. Delivery valves, 96; ports, $4 \times 16\frac{3}{4}$ to $20\frac{3}{4}$ in.; total port area 7215 sq. in. The valves are rectangular, rubber flaps, backed and faced with bronze and weighted with lead. They are set with their longest dimension horizontal, on ports which incline about 45° to the horizontal. At 17 r.p.m. the displacement is 72,000,000 gallons in 24 hours.

The Screw Pumping Engine of the Kinnickinick Flushing Tunnel, Milwaukee, has a capacity of 30,000 cubic feet per minute (= 323,000,000 gal. in 24 hrs.) at 55 r.p.m. The head is $3\frac{1}{2}$ ft. The wheel 12.5 ft. diam., made of six blades, revolves in a casing set in the tunnel lining. A cone, 6 ft. diam. at the base, placed concentric with the wheel on the approach side diverts the water to the blades. A casing beyond the wheel contains stationary deflector blades which reduce the swirling motion of the water (Allis-Chalmers Co., Bulletin No. 1610). The two screw pumping engines of the Chicago sewerage system have wheels $14\frac{3}{4}$ ft. diam., consisting of a hexagonal hub surmounted by six blades, and revolving in cylindrical casings 16 ft. long, allowing $\frac{1}{4}$ in. clearance at the sides. The pumps are driven by vertical triple-expansion engines with cylinders 22, 38 and 62 in. diam., and 42 in. stroke.

Finance of Pumping Engine Economy.—A critical discussion of the results obtained by the Nordberg and other high-duty engines is printed in *Eng. News*, Sept. 27, 1900. It is shown that the practical question in most cases is not how great fuel economy can be reached, but how economical an engine it will pay to install, taking into consideration interest, depreciation, repairs, cost of labor and of fuel, etc. The following table is given, showing that with low cost of fuel and labor it does not pay to put in a very high duty engine. Accuracy is not claimed for the figures; they are given only to show the method of computation that should be used, and to show the influence of different factors on the final result.

TABULAR STATEMENT OF TOTAL ANNUAL COST OF PUMPING WITH AN 800-H.P. ENGINE, AS INFLUENCED BY VARYING DUTY OF ENGINE, VARYING PRICE OF FUEL, AND VARYING TIME OF OPERATION.

| | Duty per million B.T.U. | | | | |
|-------------------------------|-------------------------|----------|----------|-----------|-----------|
| | 50. | 100. | 120. | 150. | 180. |
| First cost: | | | | | |
| Engine..... | \$24,000 | \$48,000 | \$68,000 | \$118,000 | \$148,000 |
| Engine, per H.P..... | 30.00 | 60.00 | 85.00 | 147.50 | 185.00 |
| Boilers, economizers..... | 27,000 | 13,500 | 11,250 | 9,000 | 7,500 |
| Engine and boilers..... | 51,000 | 61,500 | 79,250 | 127,000 | 155,500 |
| Int. and depreciation: | | | | | |
| On engine, at 6%..... | 1,440 | 2,880 | 4,080 | 7,080 | 8,880 |
| Boilers, 8%..... | 2,160 | 1,080 | 900 | 720 | 600 |
| Total..... | 3,600 | 3,960 | 4,980 | 7,800 | 9,480 |
| Labor per annum..... | 6,022 | 6,022 | 7,655 | 9,307 | 10,220 |
| Fuel cost: | | | | | |
| 4,000 hrs. per yr.: | | | | | |
| \$3.00 per ton..... | 17,280 | 8,640 | 7,200 | 5,760 | 4,800 |
| 4.00 per ton..... | 23,040 | 11,520 | 9,600 | 7,680 | 6,400 |
| 5.00 per ton..... | 28,800 | 14,400 | 12,400 | 9,600 | 8,000 |
| 6,000 hrs. per yr.: | | | | | |
| \$3.00 per ton..... | 25,920 | 12,960 | 10,800 | 8,640 | 7,200 |
| 4.00 per ton..... | 34,560 | 17,280 | 14,400 | 11,520 | 9,600 |
| 5.00 per ton..... | 43,200 | 21,600 | 18,600 | 14,400 | 12,000 |
| Total annual cost: | | | | | |
| 4,000 hrs. per yr.: | | | | | |
| Coal, \$3 per ton..... | 26,902 | 18,622 | 19,835 | 22,867 | 24,500 |
| 4 per ton..... | 32,662 | 21,502 | 22,235 | 24,787 | 25,100 |
| 5 per ton..... | 38,422 | 24,382 | 25,035 | 26,707 | 27,700 |
| 6,000 hrs. per yr. | | | | | |
| Coal, \$3 per ton..... | 35,522 | 22,942 | 23,435 | 25,747 | 26,900 |
| 4 per ton..... | 44,182 | 27,262 | 27,035 | 28,627 | 29,300 |
| 5 per ton..... | 52,822 | 31,582 | 31,235 | 31,507 | 31,700 |

Cost of Electric Current for Pumping 1000 Gallons per Minute 100 ft. High. (Theoretical H.P. with 100% efficiency = $100,000 \div 3958.9 = 25.259$ H.P.)

Assume cost of current = 1 cent per K.W. hour delivered to the motor; efficiency of motor = 90%; mechanical efficiency of triplex pumps = 80%; of centrifugal pumps = 72%; combined efficiency, triplex pumps, 72%; centrifugal, 64.8%. 1 K.W. = 1.34 electrical H.P. on wire.

Triplex, $1.34 \times 0.72 = 0.9648$ pump H.P.; $\times 33,000 = 31,838$ ft.-lbs. per min.

Centrifugal, $1.34 \times 0.648 = 0.86382$ pump H.P.; $\times 33,000 = 28,654$ ft.-lbs. per min.

1000 gallons 100 ft. high = 833,400 ft.-lbs. per min.

Triplex, $833,400 \div 31,838 = 26.1763$ K.W. $\times 8760$ hours per year $\times \$0.01 = \2293.04 .

Centrifugal, $833,400 \div 28,655 = 29.0840$ K.W. $\times 8760$ hours per year $\times \$0.01 = \2547.76 .

For 100% efficiency, $\$2293.04 \times 0.72 = \1650.00 . For any other efficiency, divide \$1650.00 by the efficiency. For any other cost per K.W. hour, in cents, multiply by that cost.

**Cost of Fuel per Year for Pumping 1,000 Gallons per Minute
100 Ft. High by Steam Pumps.**

| (1) | (2) | | (3) | (4) | (5) | (6) | (7) |
|--------|------------|---------|--------|--------|---------|--------|---------|
| | 100% Effy. | 90% | | | | | |
| 10. | 198. | 178.2 | 142.56 | 0.5846 | 0.42090 | 153.63 | 460.89 |
| 11.88 | 166.667 | 150. | 120. | 0.6945 | 0.50004 | 182.51 | 547.53 |
| 14. | 141.433 | 127.87 | 101.83 | 0.8184 | 0.58926 | 215.08 | 645.24 |
| 14.256 | 138.889 | 125. | 100. | 0.8334 | 0.60005 | 219.02 | 657.06 |
| 15. | 132. | 118.8 | 95.04 | 0.8769 | 0.63125 | 230.44 | 691.32 |
| 16. | 123.75 | 111.375 | 89.10 | 0.9354 | 0.67344 | 245.80 | 737.40 |
| 17.82 | 111.111 | 100. | 80. | 1.0417 | 0.75006 | 273.77 | 821.31 |
| 20. | 99. | 89.1 | 71.28 | 1.1692 | 0.84180 | 307.26 | 921.78 |
| 23.76 | 83.333 | 75. | 60. | 1.3890 | 1.00008 | 365.03 | 1095.09 |
| 30. | 66. | 59.4 | 47.52 | 1.7538 | 1.26270 | 460.89 | 1382.67 |
| 35.64 | 55.556 | 50. | 40. | 2.0835 | 1.50012 | 547.54 | 1642.62 |
| 40. | 49.5 | 44.5 | 35.64 | 2.3384 | 1.68360 | 614.52 | 1843.56 |
| 47.52 | 41.667 | 37.5 | 30. | 2.7780 | 2.00016 | 730.06 | 2190.18 |
| 50. | 39.6 | 35.64 | 28.51 | 2.9230 | 2.10450 | 768.15 | 2304.45 |
| a | b | c | d | e | f | g | h |

(1) Lbs. steam per I.H.P. per hour.

(2) Duty million ft.-lbs. per 1000 lbs. steam, *b*, 100% effy., *c*, 90%.

(3) Duty per 100 lbs. coal, 90% effy., 8 lbs. steam per lb. coal.

(4) Lbs. coal per min. for 1000 gals., 100 ft. high.

(5) Tons, 2000 lbs. in 24 hours.

(6) Tons per year, 365 days.

(7) Cost of fuel per year at \$3.00 per ton.

Factors for calculation: $b = 1980 \div a$; $c = b \times 0.9$; $d = c \times 0.8$; $e = 8334 \div 100 d$; $f = e \times 0.72$; $g = f \times 365$; $h = g \times 3$.

For any other cost of coal per ton, multiply the figures in the last column by the ratio of that cost to \$3.00.

**Cost of Pumping 1000 Gallons per Minute 100 ft. High by
Gas Engines.**

Assume a gas engine supplied by an anthracite gas producer using 1.5 lbs. of coal per brake H.P. hour, coal costing \$3.00 per ton of 2000 lbs.

Efficiency of triplex pump 80%, of centrifugal pump, 72%.

1000 gals. per min. 100 ft. high = 833,400 ft.-lbs. per min. \div 33,000 = 25.2545 H.P.

Fuel cost per brake H.P. hour 1.5 lbs. \times 300 cents \div 2000 = 0.225 cent \times 8760 hours per year = \$19.71 per H.P. \times 25.2545 = \$497.766 for 100% efficiency.

For 80% effy., \$622.21; for 72% effy., \$691.34; or the same as the cost with a steam pumping engine of 95,000,000 foot-pounds duty per 100 lbs. of coal.

Cost of Fuel for Electric Current.

Based on 10 lbs. steam per I.H.P. hour, 8 lbs. steam per lb. coal, or 1.25 lbs. coal per I.H.P. per hour. (Electric line loss not included.)

Efficiency of engine 0.90, of generator 0.90, combined effy. 0.81.

I.H.P. = 0.746 K.W., $0.746 \times 0.81 = 0.6426$ K.W. on wire for 10 lbs. steam. Reciprocal = 16.5492 lbs. steam per K.W. hour. 8 lbs. steam per lb. coal = 2.06865 lbs. coal, at \$3.00 per ton of 2,000 lbs. = 0.3103 cents per K.W. hour.

Lbs. steam per I.H.P. hr. —

12 14 16 18 20 30 40

Fuel cost, cents per K.W. hr. —

0.3724 0.4344 0.4965 0.5585 0.6206 0.9309 1.2412

CENTRIFUGAL PUMPS.

Theory of Centrifugal Pumps. — Bulletin No. 173 of the Univ. of Wisconsin, 1907, contains an investigation by C. B. Stewart of a 6-in. centrifugal pump which gave a maximum efficiency, under the best conditions of load, of only 32%, together with a discussion of the general theory of M. Combe, 1840, which has been followed by Weisbach, Rankine, and Unwin. Mr. Stewart says that the theory of the centrifugal

pump, at the times of these writers, seemed practically settled, but it was found later that the pump did not follow the theoretical laws derived, and the subject is still open for investigation. The theoretical head developed by the impeller can be stated for the condition of impending delivery, but as soon as flow begins the ordinary theory does not seem to apply. Experiment shows that the main difficulty to be overcome in order to secure high efficiency with the centrifugal pump is in providing some means of transforming the portion of the energy which exists in the kinetic form, at the outlet of the impeller, to the pressure form, or of reducing the loss of head in the pump casing to a minimum. The theoretical head for impending delivery is $V^2 \div g$, while experiment shows that the maximum actual head approaches $V^2 \div 2g$ as a limit. As the flow commences each pound of water discharged will possess the kinetic energy $V^2 \div 2g$ in addition to its pressure energy. To secure high efficiency some means must be found of utilizing this kinetic energy. The use of a free vortex or whirlpool, surrounding the impeller, and this surrounded by a suitable spiral discharge chamber, is practically accepted as one means of utilizing the energy of the velocity head. Guide vanes surrounding the impeller also provide a means of changing velocity head to pressure head, but the comparative advantage of these two means cannot be stated until more experimental data are obtained.

The catalogue of the Alberger Pump Co., 1908, contains the following:

It was not until the year 1901 that the centrifugal pump was shown to be nothing more or less than a water turbine reversed, and when designed on similar lines was capable of dealing with heads as great, and with efficiencies as good, as could be obtained with the turbines themselves. Since this date great progress has been made in both the theory and design, until now it is quite possible to build a pump for any reasonable conditions and to accurately estimate the efficiency and other characteristics to be expected during actual operation.

The mechanical power delivered to the shaft of a centrifugal pump by the prime mover is transmitted to the water by means of a series of radial vanes mounted together to form a single member called the impeller, and revolved by the shaft. The water is led to the inner ends of the impeller vanes, which gently pick it up and with a rapidly accelerating motion cause it to flow radially between them so that upon reaching the outer circumference of the impeller the water, owing to the velocity and pressure acquired, has absorbed all the power transmitted to the pump shaft. The problem to be solved in impeller design is to obtain the required velocity and pressure with the minimum loss in shock and friction. Since the energy of the water on leaving the pump is required to be mostly in the form of pressure, the next problem is to transform into pressure the kinetic energy of the water due to its velocity on leaving the impeller and furthermore to accomplish this with the least possible loss.

The next consideration in impeller design is the proportions of the vanes and the water passages, and to properly solve this problem an extensive use of intricate mathematical formulæ is necessary in addition to a wide knowledge of the practical side of the question. It is possible to obtain the same results as to capacity and head with practically an infinite number of different shapes, each of which gives a different efficiency as well as other varied characteristics. The change from velocity to pressure is accomplished by slowing down the speed of the water in an annular diffusion space extending from the impeller to the volute casing itself and so designed that there is the least loss from eddies or shock. It is necessary that this change shall take place gradually and uniformly, as otherwise most of the velocity would be consumed in producing eddies. With a proper design of the diffusion space and volute it is possible to transform practically the whole of the velocity into pressure so that the loss from this source may be very small.

It is necessary also to furnish a uniform supply of water to all parts of the inlet or suction opening of the impeller, for unless all the impeller vanes receive the same quantity of water at their inner edges, they cannot deliver an equal quantity at their outer edges, and this would seriously interfere with the continuity of the flow of water and the successful operation of the pump.

Design of a Four-stage Turbine Pump. — C. W. Clifford, in *Am. Mach.*, Oct. 17, 1907, describes the design of a four-stage pump of a capacity of 2300 gallons per minute = 5.124 cu. ft. per sec. Following

is an abstract of the method adopted. The total head was 1000 ft. Three sets of four-stage pumps were used at elevations of 16, 332 and 666 ft., the discharge of the first being the suction of the second, and so on.

The speed of the motor shaft is 850 r.p.m. This gives, for the diameter of the impeller, $d = 12 \times 60 \times 75.05 \div 850 \pi = 20.24$ in. Circumference $C = 63.6$ in; h = head for each impeller, in ft.

V = peripheral speed $= 1.015 \sqrt{2gh} = 75.05$ ft. per sec., 1.015 being an assumed coefficient. The velocity V is divided into two parts by the formula $V_1 = V - V_2$; $V_2 = 2gh \div 2V$; whence $V_1 = 38.65$ ft. per sec. This is the tangential component of the actual velocity of the water as it leaves the vane of the impeller. The radial component, or the radial velocity, was taken approximately at 8 ft. per sec.; $8 \div 38.65 = \text{tang. of } 11^\circ 42'$, the calculated angle between the vane and a tangent at the periphery. Taking this at 12° gives $\text{tang. } 12^\circ \times 38.65 = 8.215$ ft. per sec. = radial velocity V . The outflow area at the impeller then is $5.124 \times 144 \div (8.215 \times 0.85) = 105$ sq. in.; the 0.85 is an allowance for contraction of area in the impeller. The thickness of the vane measured on the periphery is approximately $1\frac{3}{4}$ in.; taking this into account the width of the impeller was made $17\frac{7}{8}$ in. [$105 \div (63.6 - 6 \times 1\frac{3}{4}) = 1.98$ in.]. The vanes were then plotted as shown in Fig. 148, keeping the distance between them nearly constant and of uniform section. Care was taken to increase the velocity as gradually as possible.

The suction velocity was 9.37 ft. per sec., the diam. of the opening being 10 in. This was increased to 11 ft. per sec. at the opening of the impeller, from which, after deducting the area of the shaft, the diameter, d , of the impeller inlet was found. Three long and three short vanes were used to reduce the shock.

The diffusive vanes, Fig. 149, were then designed, the object being to change the direction of the water to a radial one, and to reduce the velocity gradually to 2 ft. per sec. at the discharge through the ports.

Fig. 150 shows a cross-section of the pump. The pumps were thoroughly tested, and the following figures are derived from a mean curve of the results:

| | | | | | | | | | |
|----------------|-----|------|------|------|------|------|------|------|------|
| Gals. per min. | 500 | 1000 | 1500 | 2000 | 2200 | 2400 | 2500 | 3000 | 3500 |
| Efficiency, % | 30 | 51 | 68 | 78 | 79 | 78 | 76 | 61 | 31 |

Relation of the Peripheral Speed to the Head. — For constant speed the discharge of a centrifugal pump for any lift varies with the square root of the difference between the actual lift and the hydrostatic head created by the pump without discharge. If any centrifugal pump connected to a source of supply and to a discharge pipe of considerable height is put in revolution, it will be found that it is necessary to maintain a certain peripheral runner speed to hold the water 1 ft. high without discharge, and that for any other height the requisite speed will be very nearly as the square of the velocity for 1 ft.

Experiments prove that the peripheral speed in ft. per min. necessary to lift water to a given height with vanes of different forms is approximately as follows: $a, 481 \sqrt{h}$; $b, 554 \sqrt{h}$; $c, 610 \sqrt{h}$; $d, 780 \sqrt{h}$; $e, 394 \sqrt{h}$. a is a straight radial vane, b is a straight vane bent backward, c is a curved vane, its extremity making an angle of 27° with a tangent to the impeller, d is a curved vane with an angle of 18° , e is a vane curved in the reverse direction so that outer end is radial.

Applying the above formula, speed ft. per min. = coeff. $\times \sqrt{h}$, to the design of Mr. Clifford, gives $60 \times 75.05 = C \times \sqrt{85}$, whence $C = 488$. The vane angle was 12° . It is evident that the value of C depends on other things than the shape or angle of the vanes, such as smoothness of the vanes and other surfaces, shape and area of the diffusion vanes, and resistance due to eddies in the pump passages.

The coefficient varies with the shape of the vanes; this means that different speeds are necessary to hold water to the same heights with these different forms of vanes, and for any constant speed or lift there must be a form of vane more suitable than any other. It would seem at first glance that the runner which creates a given hydrostatic head with the least peripheral velocity must be the most efficient, but practically it is apparent from tests that the curvature of the vanes can be designed to suit the speed and lift without materially lowering the efficiency. (L. A. Hicks, *Eng. News*, Aug. 9, 1900.)

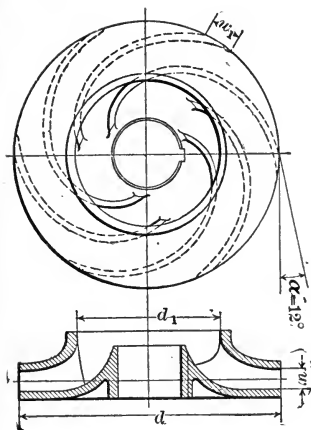


FIG. 148.



FIG. 149.

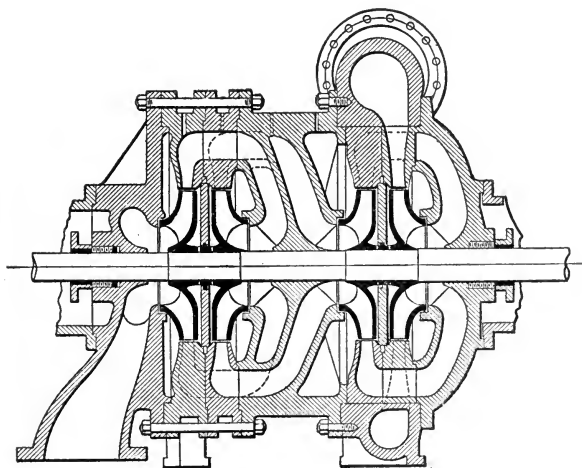


FIG. 150.

A Combination Single-stage and Two-stage Pump, for low and high heads, designed by Rateau, is described by J. B. Sperry in *Power*, July 13, 1909. It has two runners, one carried on the main driving-shaft, and the other on a hollow shaft, driven from the main shaft by a clutch. It has two discharge pipes, either one of which may be closed. When the hollow shaft is uncoupled, one runner only is used, and the pump is then a single-stage pump for low heads. When the shafts are coupled, the water passes through both runners, and may then be delivered against a high head.

Tests of De Laval Centrifugal Pumps. — The tables given below contain a condensed record of tests of three De Laval pumps made by Prof. J. E. Denton and the author in April, 1904. Two of the pumps were driven by De Laval steam turbines, and the other one by an electric motor. In the two-stage pump the small wheel was coupled direct to the high-speed shaft of the turbine, running at about 20,500 r.p.m., and the large wheel was coupled to the low-speed shaft, which is driven by the first through gears of a ratio of 1 to 10. The water delivery and the duty were computed from weir measurements, Francis's formula being used, and this was checked by calibration of the weir at different heads by a tank, the error of the formula for the weir used being less than 1%. Pitot tube measurements of the water delivered through a nozzle were also made.

One inch below the center of the nozzle was located one end of a thin half-inch brass tube, tapered so as to make an orifice of $\frac{3}{32}$ inch diameter. The other end of this tube was connected to a vertical glass tube, fastened to the wall of the testing room, graduated in inches over a height of about 30 ft. The stream of water issuing from the nozzle impinged upon the orifice of the brass tube, and thereby maintained a height of water in the glass tube. This height afforded a "Pitot Tube Basis" of measurement of the quantity of water flowing, the reliability of which was tested by the flow as determined from the weir. The Pitot tube gave the same result as the weir from the formula $Q_1 = C \times \text{Area of Nozzle} \times \sqrt{2gh}$ with a value of C varying only between 0.953 and 0.977 for the large nozzle, and between 0.942 and 0.960 for the small nozzle.

TEST OF STEAM TURBINE CENTRIFUGAL PUMP, RATED AT 1700 GALS. PER MIN., 100 FT. HEAD.

| No. of Test. | Steam Press. at the Governor Valve. Lbs. per Sq. In. | | Inches Vacuum. | Revolutions per Minute. | *Brake Horse-Power Calculated. | *Steam per Brake Horse-power. | Duty. — Millions of Foot-Lbs. per 1000 Lbs. of Steam. | Water Horse-Power. | Total Head, including Suction, Feet. | Water Pumped, Gals. per Min. | Efficiency of Pump. |
|--------------|--|--------|-------------------|-------------------------|--------------------------------|-------------------------------|---|--------------------|--------------------------------------|------------------------------|---------------------|
| | Above. | Below. | | | | | | | | | |
| 6 | 190 | 126 | 251 $\frac{1}{4}$ | 1,547 | 47.7 | 25.45 | 37.43 | 22.95 | 45.97 | 1,978 | 0.481 |
| 10 | 190 | 148 | 251 $\frac{1}{2}$ | 1,536 | 56.65 | 24.42 | 50.44 | 34.95 | 70.75 | 1,958 | 0.617 |
| 1 | 188 | 155.2 | 25 | 1,553 | 59.6 | 24.06 | 61.50 | 44.54 | 94.9 | 1,860 | 0.747 |
| 2 | 188 | 153.5 | 251 $\frac{1}{4}$ | 1,547 | 58.9 | 24.21 | 61.86 | 44.55 | 100.37 | 1,759 | 0.756 |
| 3 | 188 | 150.7 | 251 $\frac{1}{4}$ | 1,540 | 57.7 | 24.33 | 61.47 | 43.59 | 106.94 | 1,615 | 0.753 |
| 4 | 188 | 143.5 | 251 $\frac{1}{2}$ | 1,549 | 54.8 | 24.53 | 60.00 | 40.72 | 115.46 | 1,398 | 0.743 |
| 5 | 188 | 161 | 253 $\frac{3}{8}$ | 1,540 | 47.5 | 24.5 | 54.47 | 31.80 | 125.85 | 1,001 | 0.676 |
| 6A | 189.5 | 170 | 251 $\frac{1}{2}$ | 1,565 | 24.9 | | Shut-off T. | | 142.15 | | |
| †7 | 189 | 169.5 | | 1,537 | | | 45.15 | 43.85 | 95.14 | 1,826 | |
| †8 | 189 | 169 | | 1,535 | | | 45.12 | 43.82 | 99.05 | 1,753 | |
| †9 | 189 | 169.7 | | 1,538 | | | 44.62 | 42.93 | 104.42 | 1,629 | |

* The brake H.P. and the steam per B.H.P. hour were calculated by a formula derived from Prony brake tests of the turbine.

† Non-condensing.

TEST OF ELECTRIC MOTOR CENTRIFUGAL PUMP. DIAM. OF PUMP WHEEL
8⁹/₃₂ IN. RATED AT 1200 GALS. PER MIN. — 45 FT. HEAD.
2000 REVS. PER MIN.

| No. of Test. | Volts. | Amperes. | E.H.P. | * Brake Horse-Power. | Revolutions per Minute. | Cubic Feet of Water per Sec. by Weir. | Water Horse-Power. | Total Head, Feet. | Water Pumped, Gals. per Min. | Efficiency of Pump. |
|--------------|--------|----------|--------|----------------------|-------------------------|---------------------------------------|--------------------|-------------------|------------------------------|---------------------|
| 1..... | 242.5 | 55.2 | 17.94 | 15.07 | 2,006 | 3.158 | 10.25 | 28.52 | 1,417 | 0.680 |
| 2..... | 242.3 | 54.8 | 17.80 | 14.94 | 1,996 | 3.126 | 10.67 | 30.12 | 1,403 | 0.714 |
| 3..... | 242 | 59 | 19.14 | 16.22 | 1,996 | 2.885 | 11.80 | 36.1 | 1,295 | 0.728 |
| 4..... | 242 | 62.4 | 20.24 | 17.27 | 2,005 | 2.826 | 12.18 | 38.05 | 1,258 | 0.706† |
| 5..... | 241.8 | 62.9 | 20.39 | 17.41 | 2,000 | 2.525 | 13.06 | 45.66 | 1,133 | 0.750 |
| 6..... | 240.8 | 66 | 21.30 | 18.28 | 2,005 | 2.504 | 13.40 | 47.25 | 1,124 | 0.733† |
| 7..... | 241.4 | 64 | 20.71 | 17.71 | 2,003 | 2.197 | 13.12 | 52.7 | 986 | 0.742 |
| 8..... | 239.7 | 66.3 | 21.30 | 18.28 | 1,997 | 2.179 | 13.15 | 53.28 | 978 | 0.720† |
| 9..... | 240.9 | 63.2 | 20.41 | 17.43 | 2,007 | 1.735 | 11.42 | 58.10 | 779 | 0.665† |
| 10..... | 242 | 62 | 20.11 | 17.14 | 2,003 | 1.760 | 11.71 | 58.76 | 790 | 0.683 |
| 11..... | 248 | 34 | 11.30 | 8.74 | 2,040 | Shut-off | | 68.39 | | |

* Brake H.P. calculated from a formula derived from a brake test of the motor.

† Tests marked † were made with the pump suction throttled so as to make the suction equal to about 22 ft. of water column. In the other tests the suction was from 5.6 to 10.9 ft.

TEST OF STEAM TURBINE TWO-STAGE CENTRIFUGAL PUMP. RATED AT
250 GALS. PER MIN. 700 FT. HEAD. LARGE PUMP WHEEL,
2050 R.P.M.; SMALL WHEEL, 20,500 R.P.M.

| Steam Press. at the Governor Valve. Lbs. per Sq. In. | | Pressure between Pumps. Lbs. Sq. In. | Vacuum, In. | Steam Consumption, Lbs. | Revolutions per Minute. | Cu. Ft. of Water per Sec. by Weir. | Total Head. Feet. | Water Horse-Power. | Water Quantity, Gals. per Min. by Weir. | Duty, — Millions of Ft.-Lbs. per 1000 Lbs. of Steam. | Lbs. of Steam per W.H.P. per Hour. |
|--|--------|--------------------------------------|-------------|-------------------------|-------------------------|------------------------------------|-------------------|--------------------|---|--|------------------------------------|
| Above. | Below. | | | | | | | | | | |
| 186 | 120.7 | 28.1 | 25.25 | 341 | 2,104 | 0.830 | 135.76 | 12.83 | 373 | 18.63 | 106.2 |
| 175 | 138.3 | 27.5 | 24.4 | ... | 2,092 | 0.799 | 193.85 | 17.54 | 359 | ... | ... |
| 181 | 162.3 | 27.05 | 25.5 | 385 | 2,074 | 0.790 | 288 | 25.78 | 354 | 28.73 | 68.9 |
| 178 | 173.7 | 26.2 | 25.5 | 316 | 2,056 | 0.775 | 358.78 | 31.50 | 347 | 32.9 | 60.2 |
| 180 | 180.3 | 26 | 25.3 | 326 | 2,027 | 0.750 | 420.5 | 35.60 | 336 | 36.00 | 54.9 |
| 181 | 182 | 25.3 | 25.25 | 325 | 2,001 | 0.731 | 494.35 | 40.92 | 328 | 41.55 | 47.7 |
| 180 | 182 | 24.9 | 25.35 | ... | 1,962 | 0.697 | 585.06 | 46.19 | 312 | ... | ... |
| 186 | 188.3 | 25.5 | 26.3 | 331 | 2,014 | 0.664 | 632.6 | 47.58 | 299 | 47.43 | 41.77 |
| 185 | 185 | 30 | 25.3 | 331 | 2,012 | 0.558 | 756.38 | 47.81 | 251 | 47.67 | 41.5 |
| 185 | 184 | 29 | 26.5 | 325 | 2,029 | 0.544 | 781.4 | 48.15 | 244 | 48.88 | 40.50 |

A Test of a Lea-Deagan Two-Stage Pump, by Prof. J. E. Denton, is reported in *Eng. Rec.*, Sept. 29, 1906. The pump had a 10-in. suction and discharge line, and impellers 24 in. diam., each with 8 blades. The following table shows the principal results, as taken from plotted curves of the tests. The pump was designed to give equal efficiency at different speeds.

Gal. per min.

| | 400 | 800 | 1200 | 1600 | 2000 | 2400 | 2800 | 3000 | 3200 | 3400 | 3600 | 3800 |
|-------------|-----|-----|------|------|------|------|------|-------|-------|-------|-------|-------|
| Efficiency. | | | | | | | | | | | | |
| 400 r.p.m. | 42 | 61 | 69 | 75 | 77 | 77 | 70 | | | | | |
| 500 " | 39 | 56 | 65 | 71 | 75 | 77 | 77.6 | 77 | 74 | 70 | | |
| 600 " | 35 | 50 | 62 | 68 | 71 | 74 | 76 | 77 | 78 | 78 | 76 | 54 |
| Head. | | | | | | | | | | | | |
| 400 r.p.m. | 55 | 55 | 53 | 51 | 47 | 42 | 34 | | | | | |
| 500 " | 63 | 86 | 84 | 82 | 78 | 73 | 67 | 63 | 58 | 51 | | |
| 600 " | 126 | 127 | 125 | 122 | 118 | 115 | 107 | 104 | 101 | 97 | 87 | 55 |

The following results were obtained under conditions of maximum efficiency:

| | | | |
|------------|-------------|---------------------|---------------|
| 400 r.p.m. | 77.7% effy. | 2296 gals. per min. | 43.6 ft. lift |
| 500 " | 77.6 | 2794 " | 67.4 " |
| 600 " | 77.97 | 3235 " | 100.7 " |

A High-Duty Centrifugal Pump.—A 45,000,000 gal. centrifugal pump at the Deer Island sewage pumping station, Boston, Mass., was tested in 1896 and showed a duty of 95,867,476 ft.-lbs., based on coal fired to the boilers. — (Allis-Chalmers Co., Bulletin No. 1062.)

Rotary Pumps.—Pumps with two parallel geared shafts carrying vanes or impellers which mesh with each other, and other forms of positive driven apparatus, in which the water is pushed at a moderate velocity, instead of being rotated at a high velocity as in centrifugal pumps, are known as rotary pumps. They have an advantage over reciprocating pumps in being valveless, and over centrifugal pumps in working under variable heads. They are usually not economical, but when carefully designed with the impellers of the correct cycloidal shape, like those used in positive rotary blowers, they give a moderately high efficiency.

Tests of Centrifugal and Rotary Pumps. (W. B. Gregory, *Bull.* 183, U. S. Dept. of Agriculture, 1907.)—These pumps are used for irrigation and drainage in Louisiana. A few records of small pumps, giving very low efficiencies, are omitted. Oil was used as fuel in the boilers, except in the pump of the New Orleans drainage station No. 7 (figures in the last column), which was driven by a gas-engine.

| | | | | | | | | | | |
|---------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Actual lift..... | 15.5 | 16.2 | 11.2 | 30.2 | 9.5 | 28.7 | 31.7 | 6.8 | 31.6 | 13.4 |
| Disch. cu. ft. per sec... | 72.6 | 157.0 | 116.0 | 93.2 | 71.4 | 68.7 | 85.6 | 130.5 | 152.9 | 30.5 |
| Water horse-power.... | 127.5 | 287.4 | 147.1 | 318.0 | 76.5 | 222.8 | 306.8 | 98.8 | 547.9 | 46.2 |
| I.H.P..... | 155.6 | 671.2 | 229.8 | 648.0 | 137.7 | 503.9 | 452.3 | 193.6 | 657.7 | 90.6 |
| Effy., engine, gearing and pumps..... | 81.7 | 42.9 | 64.2 | 49.0 | 55.6 | 44.3 | 67.9 | 51.0 | 83.3 | 51.0 |
| Duty, per 1000 lbs. stea. | 72.1 | 34.3 | 40.7 | 33.8 | | 33.9 | 78.2 | 31.4 | 75.4 | |
| Duty, per million B.T.U. in fuel..... | 37.8 | 18.3 | 20.7 | 24.2 | 22.1 | 17.3 | 51.1 | 16.7 | 50.1 | 82.4 |
| Therm. effy. from stea. | 8.16 | 4.23 | 4.68 | 4.16 | | 4.09 | 9.70 | 3.93 | 9.61 | |
| Kind of engine, and pump..... | a, f | b, g | b, g | b, g | c, g | b, g | a, g | d, g | a, g | e, g |

a, Tandem compound condensing Corliss; *b*, Simple condensing Corliss; *c*, Simple non-condensing Corliss; *d*, Triple-expansion condensing, vertical; *e*, Three-cylinder vertical gas-engine, with gas-producer, 0.85 lb. coal per I.H.P. per hour; *f*, Rotary pump; *g*, Cycloidal rotary.

The relatively low duty per million B.T.U. is due to the low efficiency of the boilers. The test whose figures are given in the next to the last column is reported by Prof. Gregory in *Trans. A. S. M. E.*, to vol. xxviii.

DUTY TRIALS OF PUMPING-ENGINES.

A committee of the *A. S. M. E.* (*Trans.*, xii. 530) reported in 1891 on a standard method of conducting duty trials. Instead of the old unit of duty of foot-pounds of work per 100 lbs. of coal used, the committee recommend a new unit, foot-pounds of work per million heat-units furnished by the boiler. The variations in quantity of coal make the old standard unfit as a basis of duty ratings. The new unit is the precise equivalent of 100 lbs. of coal in cases where each pound of coal imparts 10,000 heat-units to the water in the boiler, or where the evaporation is $10,000 \div 965.7 = 10.355$ lbs. of water from and at 212° per pound of fuel. This evaporative result is readily obtained from all grades of Cumberland or other semi-bituminous coal used in horizontal return tubular boilers, and, in many cases, from the best grades of anthracite coal.

The committee also recommends that the work done be determined by plunger displacement, after making a test for leakage, instead of by measurement of flow by weirs or other apparatus, but advises the use of such apparatus when practicable for obtaining additional data. The following extracts are taken from the report. When important tests are to be made the complete report should be consulted.

The necessary data having been obtained, the duty of an engine, and other quantities relating to its performance, may be computed by the use of the following formulæ:

$$\begin{aligned} 1. \text{ Duty} &= \frac{\text{Foot-pounds of work done}}{\text{Total number of heat-units consumed}} \times 1,000,000 \\ &= \frac{A (P \pm p + s) \times L \times N}{H} \times 1,000,000 \text{ (foot-pounds).} \end{aligned}$$

$$2. \text{ Percentage of leakage} = \frac{C \times 144}{A \times L \times N} \times 100 \text{ (per cent).}$$

$$\begin{aligned} 3. \text{ Capacity} &= \text{number of gallons of water discharged in 24 hours} \\ &= \frac{A \times L \times N \times 7.4805 \times 24}{D \times 144} = \frac{A \times L \times N \times 1.24675}{D} \text{ (gallons).} \end{aligned}$$

$$\begin{aligned} 4. \text{ Percentage of total frictions,} \\ &= \left[\frac{\text{I.H.P.} - \frac{A (P \pm p + s) \times L \times N}{D \times 60 \times 33,000}}{\text{I.H.P.}} \right] \times 100 \\ &= \left[1 - \frac{A (P \pm p + s) \times L \times N}{A_s \times \text{M.E.P.} \times L_s \times N_s} \right] \times 100 \text{ (per cent);} \end{aligned}$$

or, in the usual case, where the length of the stroke and number of strokes of the plunger are the same as that of the steam-piston, this last formula becomes:

$$\text{Percentage of total frictions} = \left[1 - \frac{A (P \pm p + s)}{A_s \times \text{M.E.P.}} \right] \times 100 \text{ (per cent).}$$

In these formulæ the letters refer to the following quantities:

A = Area, in square inches, of pump plunger or piston, corrected for area of piston rod or rods;

P = Pressure, in pounds per square inch, indicated by the gauge on the force main; *

* E. T. Sederholm, chief engineer of Fraser & Chalmers, in a letter to the author, Feb. 20, 1900, shows that the sum $P \pm p + s$ may lead to erroneous results unless the two gauges are placed below the levels of the water in the discharge and suction air chambers respectively, and the connecting pipes to the gauges run so they will always be full of water. He prefers to connect these gauges to the air spaces of the two air chambers, running the connecting pipes so they will be full of air only, and to add to the sum of the indications of the two gauges the difference in water level of the two chambers.

- p = Pressure, in pounds per square inch, corresponding to indication of the vacuum-gauge on suction-main (or pressure-gauge, if the suction-pipe is under a head). The indication of the vacuum-gauge, in inches of mercury, may be converted into pounds by dividing it by 2.035;
- s = Pressure, in pounds per square inch, corresponding to distance between the centers of the two gauges. The computation for this pressure is made by multiplying the distance, expressed in feet, by the weight of one cubic foot of water at the temperature of the pump-well, and dividing the product by 144;
- L = Average length of stroke of pump-plunger, in feet;
- N = Total number of single strokes of pump-plunger made during the trial;
- As = Area of steam-cylinder, in square inches, corrected for area of piston-rod. The quantity $As \times \text{M.E.P.}$, in an engine having more than one cylinder, is the sum of the various quantities relating to the respective cylinders;
- L_s = Average length of stroke of steam-piston, in feet;
- N_s = Total number of single strokes of steam-piston during trial;
- M.E.P. = Average mean effective pressure, in pounds per square inch, measured from the indicator-diagrams taken from the steam-cylinder;
- I.H.P. = Indicated horse-power developed by the steam-cylinder;
- C = Total number of cubic feet of water which leaked by the pump-plunger during the trial, estimated from the results of the leakage test;
- D = Duration of trial in hours;
- H = Total number of heat-units (B.T.U.) consumed by engine = weight of water supplied to boiler by main feed-pump \times total heat of steam of boiler pressure reckoned from temperature of main feed-water + weight of water supplied by jacket-pump \times total heat of steam of boiler-pressure reckoned from temperature of jacket-water + weight of any other water supplied \times total heat of steam reckoned from its temperature of supply. The total heat of the steam is corrected for the moisture or superheat which the steam may contain. No allowance is made for water added to the feed-water, which is derived from any source except the engine or some accessory of the engine. Heat added to the water by the use of a flue-heater at the boiler is not to be deducted. Should heat be abstracted from the flue by means of a steam reheater connected with the intermediate receiver of the engine, this heat must be included in the total quantity supplied by the boiler.

Leakage Test of Pump. — The leakage of an inside plunger (the only type which requires testing) is most satisfactorily determined by making the test with the cylinder-head removed. A wide board or plank may be temporarily bolted to the lower part of the end of the cylinder, so as to hold back the water in the manner of a dam, and an opening made in the temporary head thus provided for the reception of an overflow-pipe. The plunger is blocked at some intermediate point in the stroke (or, if this position is not practicable, at the end of the stroke), and the water from the force main is admitted at full pressure behind it. The leakage escapes through the overflow-pipe, and it is collected in barrels and measured. The test should be made, if possible, with the plunger in various positions.

In the case of a pump so planned that it is difficult to remove the cylinder-head, it may be desirable to take the leakage from one of the openings which are provided for the inspection of the suction-valves, the head being allowed to remain in place.

It is assumed that there is a practical absence of valve leakage. Examination for such leakage should be made, and if it occurs, and it is found to be due to disordered valves, it should be remedied before making the plunger test. Leakage of the discharge valves will be shown by water passing down into the empty cylinder at either end when they are under pressure. Leakage of the suction-valves will be shown by the disappearance of water which covers them.

If valve leakage is found which cannot be remedied the quantity of

water thus lost should also be tested. One method is to measure the amount of water required to maintain a certain pressure in the pump cylinder when this is introduced through a pipe temporarily erected, no water being allowed to enter through the discharge valves of the pump.

Table of Data and Results. — In order that uniformity may be secured, it is suggested that the data and results, worked out in accordance with the standard method, be tabulated in the manner indicated in the following scheme:

DUTY TRIAL OF ENGINE.

DIMENSIONS.

1. Number of steam-cylinders.....
 2. Diameter of steam-cylinders..... ins.
 3. Diameter of piston-rods of steam-cylinders..... ins.
 4. Nominal stroke of steam-pistons..... ft.
 5. Number of water-plungers.....
 6. Diameter of plungers..... ins.
 7. Diameter of piston-rods of water-cylinders..... ins.
 8. Nominal stroke of plungers..... ft.
 9. Net area of steam-pistons..... sq. ins.
 10. Net area of plungers..... sq. ins.
 11. Average length of stroke of steam-pistons during trial.... ft.
 12. Average length of stroke of plungers during trial..... ft.
- (Give also complete description of plant.)

TEMPERATURES.

13. Temperature of water in pump-well..... degs.
14. Temp. of water supplied to boiler by main feed-pump degs.
15. Temp. of water supplied to boiler from other sources degs.

FEED-WATER.

16. Weight of water supplied to boiler by main feed-pump... lbs.
17. Weight of water supplied to boiler from other sources lbs.
18. Total weight of feed-water supplied from all sources lbs

PRESSURES.

19. Boiler pressure indicated by gauge..... lbs.
20. Pressure indicated by gauge on force main..... lbs.
21. Vacuum indicated by gauge on suction main..... ins.
22. Pressure corresponding to vacuum given in preceding line lbs.
23. Vertical distance between the centers of the two gauges.. ins.
24. Pressure equivalent to distance between the two gauges.. lbs.

MISCELLANEOUS DATA.

25. Duration of trial..... hrs.
26. Total number of single strokes during trial.....
27. Percentage of moisture in steam supplied to engine, or number of degrees of superheating..... % or deg.
28. Total leakage of pump during trial, determined from results of leakage test..... lbs.
29. Mean effective pressure, measured from diagrams taken from steam-cylinders..... M.E.P.

PRINCIPAL RESULTS.

30. Duty ft.-lbs.
31. Percentage of leakage..... %
32. Capacity gals.
33. Percentage of total friction..... %

ADDITIONAL RESULTS

34. Number of double strokes of steam-piston per minute....
35. Indicated horse-power developed by the various steam-cylinders..... I.H.P.
36. Feed-water consumed by the plant per hour..... lbs.
37. Feed-water consumed by the plant per indicated horse-power per hour, corrected for moisture in steam..... lbs.

38. Heat units consumed per I.H.P. per hour..... B.T.U.
 39. Heat units consumed per I.H.P. per minute B.T.U.
 40. Steam accounted for by indicator at cut-off and release in
 the various steam-cylinders lbs.
 41. Proportion which steam accounted for by indicator bears
 to the feed-water consumption.....
 42. Number of double strokes of pump per minute.....
 43. Mean effective pressure, measured from pump diagrams . M.E.P.
 44. Indicated horse-power exerted in pump-cylinders..... I.H.P.
 45. Work done (or duty) per 100 lbs. of coal ft.-lbs.

SAMPLE DIAGRAM TAKEN FROM STEAM-CYLINDERS.

(Also, if possible, full measurement of the diagrams, embracing pressures at the initial point, cut-off, release, and compression; also back pressure, and the proportions of the stroke completed at the various points noted.)

SAMPLE DIAGRAM TAKEN FROM PUMP-CYLINDERS.

These are not necessary to the main object, but it is desirable to give them.

DATA AND RESULTS OF BOILER TEST.

(In accordance with the scheme recommended by the Boiler-test Committee of the Society.)

Notable High-duty Pumping Engine Records.

| Date of test..... | (1) 1899 | (2) 1900 | (3) 1900 | (4) 1901 | (5) 1906 |
|--------------------------------|------------------|-----------------------|-------------------------------|--------------------------|--|
| Locality..... | Wildwood, Pa. | St. Louis (10). | Boston, Chest- nut Hill | Boston, Spot Pond. | St. Louis (3) Bissell's Point. |
| Capacity, mil. gal., 24 hrs... | 6 | 15 | 30 | 30 | 20 |
| Diam. of steam cylinders, in. | 19.5, 29, 49.5 | 34, 62, 92 | 30, 56, 87 | 22, 41.5, 62 | 34, 62, 94 |
| Stroke, in..... | 57.5 × 42 | × 42 | × 66 | × 60 | 72 |
| No. and diam. of plungers... | (2) 14 3/4 | (3) 29 1/2 | (3) 42 | (3) 30.5 | (3) 33 7/8 |
| Piston speed, ft. per min.... | 256 | 197 | 195 | 244 | 198 |
| Total head, ft..... | 504 | 292 | 140 | 125 | 238 |
| Steam pressure..... | 200 | 126 | 185 | 151 | 146 |
| Indicated Horse-power..... | 712 | 801 | 801 | 464 | 859 |
| Friction, %..... | 6.95 | 3.16 | 6.71 | 3.47 | 2.27 |
| Mechanical efficiency, %..... | 93.05 | 96.84 | 93.29 | 96.53 | 97.73 |
| Dry steam per I.H.P. hr.... | 12.26, 11.4 | 10.68 | 10.34 | 11.09 | |
| B.T.U. per I.H.P. per min.... | 186* | 202 | 196 | 203 | 202.8 |
| Duty, B.T.U. basis..... | 162.9* 147.5† | 158.07 | 156.8 | 156.59 | 158.85 |
| Duty per 1000 lbs. steam | 150.2* | 179.45 | 178.49 | 172.40 | 181.30 |
| Thermal efficiency, %..... | 22.81 | 21.00 | 21.63 | 20.84 | 20.92 |

* With reheaters.

† Without reheaters.

(1), (2). From *Eng. News*, Sept. 27, 1900. (3) Do. Aug. 23, 1900.
 (4) Do. Nov. 4, 1901. (5) Allis-Chalmers Co., Bulletin No. 1609. The Wildwood engine has double-acting plungers.

The coal consumption of the Chestnut Hill engine was 1.062 lbs. per I.H.P. per hour, the lowest figure on record at that date, 1901.

The Nordberg Pumping Engine at Wildwood, Pa. — *Eng. News*, May 4, 1899, Aug. 23, 1900, *Trans. A. S. M. E.*, 1899. The peculiar feature of this engine is the method used in heating the feed-water. The engine is quadruple expansion, with four cylinders and three receivers. There are five feed-water heaters in series, *a, b, c, d, e*. The water is taken from the hot-well and passed in succession through *a* which is heated by the exhaust steam on its passage to the condenser; *b* receives its heat from the fourth cylinder, and *c, d* and *e* respectively from the

third, second and first receivers. An approach is made to the requirement of the Carnot thermodynamic cycle, *i.e.*, that heat entering the system should be entered at the highest temperature; in this case the water receives the heat from the receivers at gradually increasing temperatures. The temperatures of the water leaving the several heaters were, on the test, 105°, 136°, 193°, 260°, and 311° F. The economy obtained with this engine was the highest on record at the date (1900) *viz.*, 162,948,824 ft. lbs. per million B.T.U., and it has not yet been exceeded (1909).

VACUUM PUMPS.

The Pulsometer. — In the pulsometer the water is raised by suction into the pump-chamber by the condensation of steam within it, and is then forced into the delivery-pipe by the pressure of a new quantity of steam on the surface of the water. Two chambers are used which work alternately, one raising while the other is discharging.

Test of a Pulsometer. — A test of a pulsometer is described by De Volson Wood in *Trans. A. S. M. E.*, xiii. It had a 3½-inch suction-pipe, stood 40 in. high, and weighed 695 lbs.

The steam-pipe was 1 inch in diameter. A throttle was placed about 2 feet from the pump, and pressure gauges placed on both sides of the throttle, and a mercury well and thermometer placed beyond the throttle. The wire drawing due to throttling caused superheating.

The pounds of steam used were computed from the increase of the temperature of the water in passing through the pump.

Pounds of steam \times loss of heat = lbs. of water sucked in \times increase of temp.

The loss of heat in a pound of steam is the total heat in a pound of saturated steam as found from "steam tables" for the given pressure, plus the heat of superheating, minus the temperature of the discharged water; or

$$\text{Pounds of steam} = \frac{\text{lbs. water} \times \text{increase of temp.}}{H - 0.48 t - T}$$

The results for the four tests are given in the following table:

| Data and Results. | 1 | 2 | 3 | 4 |
|---|------------|------------|------------|------------|
| Strokes per minute..... | 71 | 60 | 57 | 64 |
| Steam pressure in pipe before throttling..... | 114 | 110 | 127 | 104.3 |
| Steam pressure after throttling.. | 19 | 30 | 43.8 | 26.1 |
| Steam temp. after throttling, °F.. | 270.4 | 277 | 309.0 | 270.1 |
| Steam superheating, °F..... | 3.1 | 3.4 | 17.4 | 1.4 |
| Steam used, lbs..... | 1617 | 931 | 1518 | 1019.9 |
| Water pumped, lbs..... | 404,786 | 186,362 | 228,425 | 248,053 |
| Water temp. before entering pump | 75.15 | 80.6 | 76.3 | 70.25 |
| Water temperature, rise of..... | 4.47 | 5.5 | 7.49 | 4.55 |
| Water head by gauge on lift, ft.... | 29.90 | 54.05 | 54.05 | 29.90 |
| Water head by gauge on suction... | 12.26 | 12.26 | 19.67 | 19.67 |
| Water head by gauge, total (<i>H</i>) .. | 42.16 | 66.31 | 73.72 | 49.57 |
| Water head by measure, total (<i>h</i>) | 32.8 | 57.80 | 66.6 | 41.60 |
| Coeff. of friction of plant, <i>h/H</i> | 0.777 | 0.877 | 0.911 | 0.839 |
| Efficiency of pulsometer..... | 0.012 | 0.0155 | 0.0126 | 0.0138 |
| Eff'y of plant exclusive of boiler | 0.0093 | 0.0136 | 0.0115 | 0.0116 |
| Eff'y of plant if that of boiler be 0.7 | 0.0065 | 0.0095 | 0.0080 | 0.0081 |
| Duty, if 1 lb. evaporates 10 lbs. water..... | 10,511,400 | 13,391,000 | 11,059,000 | 12,036,300 |

Of the two tests having the highest lift (54.05 ft.), that was more efficient which had the smaller suction (12.26 ft.), and this was also the most efficient of the four tests. But, on the other hand, the other two tests having the same lift (29.9 ft.), that was the more efficient which had the greater suction (19.67), so that no law in this regard was established. The pressures used, 19, 30, 43.8, 26.1, follow the order of magnitude of

the total heads, but are not proportional thereto. No attempt was made to determine what pressure would give the best efficiency for any particular head. The pressure used was intrusted to a practical runner, and he judged that when the pump was running regularly and well, the pressure then existing was the proper one. It is peculiar that, in the first test, a pressure of 19 lbs. of steam should produce a greater number of strokes and pump over 50% more water than 26.1 lbs., the lift being the same as in the fourth experiment.

Chas. E. Emery in discussion of Prof. Wood's paper says, referring to tests made by himself and others at the Centennial Exhibition in 1876 (see Report of the Judges, Group xx.), that a vacuum-pump tested by him in 1871 gave a duty of 4.7 millions; one tested by J. F. Flagg, at the Cincinnati Exposition in 1875, gave a maximum duty of 3.25 millions. Several vacuum and small steam-pumps, compared later on the same basis, were reported to have given duties of 10 to 11 millions, the steam-pumps doing no better than the vacuum-pumps. Injectors, when used for lifting water not required to be heated, have an efficiency of 2 to 5 millions; vacuum-pumps vary generally between 3 and 10; small steam-pumps between 8 and 15; larger steam-pumps, between 15 and 30, and pumping-engines between 30 and 140 millions.

A very high record of test of a pulsometer is given in *Eng'g*, Nov. 24, 1893, p. 639, viz.: Height of suction 11.27 ft.; total height of lift, 102.6 ft.; horizontal length of delivery-pipe, 118 ft.; quantity delivered per hour, 26,188 British gallons. Weight of steam used per H. P. per hour, 92.76 lbs.; work done per pound of steam 21,345 foot-pounds, equal to a duty of 21,345,000 foot-pounds per 100 lbs. of coal, if 10 lbs. of steam were generated per pound of coal.

The Jet-pump. — This machine works by means of the tendency of a stream or jet of fluid to drive or carry contiguous particles of fluid along with it. The water-jet pump, in its present form, was invented by Prof. James Thomson, and first described in 1852. In some experiments on a small scale as to the efficiency of the jet-pump, the greatest efficiency was found to take place when the depth from which the water was drawn by the suction-pipe was about nine tenths of the height from which the water fell to form the jet; the flow up the suction-pipe being in that case about one fifth of that of the jet, and the efficiency, consequently, $\frac{9}{10} \times \frac{1}{5} = 0.18$. This is but a low efficiency; but it is probable that it may be increased by improvements in proportions of the machine. (Rankine, S. E.)

The Injector when used as a pump has a very low efficiency. (See Injectors, under Steam-boilers.)

PUMPING BY COMPRESSED AIR — THE AIR-LIFT PUMP.

Air-lift Pump. — The air-lift pump consists of a vertical water-pipe with its lower end submerged in a well, and a smaller pipe delivering air into it at the bottom. The rising column in the pipe consists of air mingled with water, the air being in bubbles of various sizes, and is therefore lighter than a column of water of the same height; consequently the water in the pipe is raised above the level of the surrounding water. This method of raising water was proposed as early as 1797, by Loescher, of Freiberg, and was mentioned by Collon in lectures in Paris in 1876, but its first practical application probably was by Werner Siemens in Berlin in 1885. Dr. J. G. Pohle experimented on the principle in California in 1886, and U. S. patents on apparatus involving it were granted to Pohle and Hill in the same year. A paper describing tests of the air-lift pump made by Randall, Browne and Behr was read before the Technical Society of the Pacific Coast in Feb., 1890.

The diameter of the pump-column was 3 in., of the air-pipe 0.9 in., and of the air-discharge nozzle $\frac{5}{8}$ in. The air-pipe had four sharp bends and a length of 35 ft. plus the depth of submersion.

The water was pumped from a closed pipe-well (55 ft. deep and 10 in. in diameter). The efficiency of the pump was based on the least work theoretically required to compress the air and deliver it to the receiver. If the efficiency of the compressor be taken at 70%, the efficiency of the pump and compressor together would be 70% of the efficiency found for the pump alone.

For a given submersion (h) and lift (H), the ratio of the two being kept within reasonable limits, (H) being not much greater than (h), the efficiency was greatest when the pressure in the receiver did not greatly exceed the head due to the submersion. The smaller the ratio $H \div h$, the higher was the efficiency.

The pump, as erected, showed the following efficiencies:

| | | | | |
|------------------|-----|-----|-----|-----|
| For $H \div h =$ | 0.5 | 1.0 | 1.5 | 2.0 |
| Efficiency = | 50% | 40% | 30% | 25% |

The fact that there are absolutely no moving parts makes the pump especially fitted for handling dirty or gritty water, sewage, mine water, and acid or alkali solutions in chemical or metallurgical works.

In Newark, N. J., pumps of this type are at work having a total capacity of 1,000,000 gallons daily, lifting water from three 8-in. artesian wells. The Newark Chemical Works use an air-lift pump to raise sulphuric acid of 1.72° gravity. The Colorado Central Consolidated Mining Co., in one of its mines at Georgetown, Colo., lifts water in one case 250 ft., using a series of lifts.

For a full account of the theory of the pump, and details of the tests above referred to, see *Eng'g News*, June 8, 1893.

Air-Lifts for Deep Oil-Wells are described by E. M. Ivens, in *Trans. A. S. M. E.* 1909, p. 341. The following are some results obtained in wells in Evangeline, La.:

| | | | | |
|--|------|------|------|------|
| Cu. ft. free air per minute, displacement of compressor..... | 650 | 442 | 702 | 536 |
| Cu. ft. oil pumped per minute..... | 4.35 | 4.87 | 13.7 | 5.54 |
| Air pressure at well, lbs. per sq. in..... | 155 | 200 | 202 | 252 |
| Pumping head, from oil level while pumping, ft. | 1155 | 1081 | 1076 | 917 |
| Submergence, from oil level to air entrance, ft. | 358 | 412 | 419 | 583 |
| Submergence \div total ft. of vertical pipe, %... | 23.6 | 27.6 | 28 | 39 |
| Pumping efficiency, %..... | 9.3 | 13.4 | 19.5 | 10.3 |

Artesian Well Pumping by Compressed Air.—H. Tipper, *Eng. News*, Jan. 16, 1908, mentions cases where 1-in. air lines supplied air for 6-in. wells, with the inside air-pipe system; the length of the pipe was 300 ft. from the well top, and another 350 ft. to the compressor. The wells pumped 75 gals. per min., using 200 cu. ft. of air, the efficiency being 61½%. Changing the pipes to 2½ in. above the well, and 2 in. in the well, and putting an air receiver near the compressor, raised the delivery to 180 gals. per min., with a little less air, and the efficiency to 23%. A large receiver capacity, a large pipe above ground, a submergence of 55%, well piping proportioned for a friction loss of not over 5%, with lifts not over 200 ft., gave the best results, 1 gal. of water being raised per cu. ft. of air. The utmost net efficiency of the air-lift is not over 25 to 30%.

Eng. News, June 18, 1908, contains an account of tests of eleven wells at Atlantic City. The Atlantic City wells were 10 in. diam., water pipes, 4 to 5¼ in., air pipes, ¾ to 1¼ in. The maximum lift of the several wells ranged from 26 to 40 ft., the submergence, 37 to 49 ft., ratio of submergence to lift, 0.9 to 1.8, submergence % of length of pipe, 53 to 64. Capacity test, 3,544,900 gals. in 24 hrs., mean lift, 26.88 ft., air pressure, 31 lbs., duty of whole plant, 19,900,000 ft. lbs. per 1000 lbs. of steam used by the compressors. Two-thirds capacity test, delivery, 2,642,900 gals., mean lift, 25.43 ft., air pressure, 26 lbs., duty, 24,207,000.

An article in *The Engineer* (Chicago), Aug. 15, 1904, gives the following formulæ and rules for the design of air-lifts of maximum efficiency. The authority is not given.

Ratio of area of air pipe to area of water pipe, 0.16.

Submerged portion = 65% of total length of pipe.

Economical range of submerision ratio, 55 to 80%.

Velocity of air in air pipe, not over 4000 ft. per min.

Volume of air to raise 1 cu. ft. of water, 3.9 to 4.5 cu. ft.

C = cu. ft. of water raised per min., A = cu. ft. of air used, L = lift above water level, D = submergence, in feet.

$A = LC \div 16.824$; $C = 8.24 AD \div L^2$.

Where L exceeds 180 ft. it will be more economical to use two or more air-lifts in series.

THE HYDRAULIC RAM.

Efficiency. — The hydraulic ram is used where a considerable flow of water with a moderate fall is available, to raise a small portion of that flow to a height exceeding that of the fall. The following are rules given by Eytelwein as the results of his experiments* (from Rankine):

Let Q be the whole supply of water in cubic feet per second, of which q is lifted to the height h above the pond, and $Q - q$ runs to waste at the depth H below the pond; L , the length of the supply-pipe, from the pond to the waste-clack; D , its diameter in feet; then

$$D = \sqrt{(1.63 Q)}; L = H + h + \frac{h}{H} \times 2 \text{ feet};$$

$$\text{Efficiency, } \frac{qh}{(Q-q)H} = 1.12 - 0.2 \sqrt{\frac{h}{H}}, \text{ when } \frac{h}{H} \text{ does not exceed } 20;$$

or

$$1 \div (1 + h/10 H) \text{ nearly, when } h/H \text{ does not exceed } 12.$$

$$\text{D'Aubuisson gives } \frac{q(H+h)}{QH} = 1.42 - 0.28 \sqrt{\frac{h}{H}}.$$

Clark, using five sixths of the values given by D'Aubuisson's formula, gives:

| | | | | | | | | | | | | |
|------------------------|----|----|----|----|----|----|----|----|----|----|----|----|
| Ratio of lift to fall. | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
| Efficiency per cent. | 72 | 61 | 52 | 44 | 37 | 31 | 25 | 19 | 14 | 9 | 4 | 0 |

The efficiency as calculated by the two formulæ given above is nearly the same for high ratios of lift, but for low ratios there is considerable difference. For example:

| | | | | |
|---|------------------|------|------|------|
| Let $Q = 100$, $H = 10$, $H + h =$ | 20 | 40 | 100 | 200 |
| Efficiency, D'Aubuisson's formula, % | 80 | 72 | 44 | 14 |
| $q = \text{effy.} \times QH \div (H + h) =$ | 40 | 18 | 4.4 | 0.7 |
| Efficiency by Rankine's formula, % | 66 $\frac{2}{3}$ | 65.9 | 41.4 | 13.4 |

D'Aubuisson's formula is that of the machine itself, on the basis that the energy put into the machine is that of the whole column of water, Q , falling through the height h and that the energy delivered is that of q raised through the whole height above the ram, $H + h$; while Rankine's efficiency is that of the whole plant, assuming that the energy put in is only that of the water that runs to waste, and that the work done is lifting the quantity q not from the level of the ram but only from that of the supply pond. D'Aubuisson's formula is the one in harmony with the usual definition of efficiency. It also is applicable (as Rankine's is not) to the case of a ram which uses the quantity Q from one source of supply to pump water of different quality from a source at the level of the ram.

An extensive mathematical investigation of the hydraulic ram, by L. F. Harza, is contained in Bulletin No. 205 of the University of Wisconsin, 1908, together with results of tests of a Rife "hydraulic engine," which appear to verify the theory. It was found both by theory and by experiment that the efficiency bears a relation to the velocity in the drive pipe. From plotted diagrams of the results the following figures (roughly approximate) are taken: Length of 2-in. drive pipe, 85.4 ft.; supply head, 8.2 ft.

| | | | | | | |
|---|---------------------------|----|----|----|----|----|
| Max. vel. in drive pipe, ft. per sec. . . . | 1.5 | 2 | 3 | 4 | 5 | 6 |
| | Efficiency of machine, %. | | | | | |
| Pumping head, ft. | 2.6 | 30 | 20 | 15 | 7 | 0 |
| | 12.3 | 60 | 60 | 45 | 33 | 18 |
| | 23.2 | 60 | 65 | 53 | 40 | 20 |
| | 43.5 | 55 | 60 | 53 | 42 | 30 |
| | 63.1 | 60 | 55 | 50 | 28 | 0 |

The author of the paper concludes that the comparison of experiment and theory has demonstrated the practicability of the logical design of a hydraulic ram for any given working conditions.

An interesting historical account, with illustrations, of the development of the hydraulic ram, with a description of Pearsall's hydraulic engine, is given by J. Richards in *Jour. Assn. Eng'g Societies*, Jan., 1898. For a description of the Rife hydraulic engine see *Eng. News*, Dec. 31, 1896.

The Columbia Steel Co., Portland, Ore., furnished the author in July, 1908, records of tests of four hydraulic rams, from which the following is condensed, the efficiency, by D'Aubuisson's formula, being calculated from the data given. L = length in ft. and D = diam. in ins. of the drive pipe, l and d , length and diameter of the discharge pipe.

| Size of Ram. | H | $h + H$ | Q^* | q^* | L | D | l | d | Effy. % |
|--------------|------|---------|-------|-------|-------|-------|------|-------|---------|
| Ins. | Ft. | Ft. | | | Ft. | Ins. | Ft. | Ins. | |
| 3..... | 4 | 28 | 35 | 3.5 | 28 | 3 | 1008 | 11/2 | 58.9 |
| 4 1/2..... | 5 | 45 | 100 | 8 | 40 | 4 1/2 | 325 | | 72.0 |
| 6..... | 12 | 36.4 | 200 | 50.5 | 60 | 4 1/2 | 945 | 21/2 | 76.6 |
| 6..... | 37.6 | 144.1 | 6.26 | 1.15 | 192.5 | 6 | 1785 | 10† | 70.4 |

* Q and q are in gallons per min.; except the last line, which is in cu. ft. per sec.

† Eleven rams discharge into one 10-in. jointed wood pipe. The loss of head in the drive pipe was 0.7 ft., and in the discharge pipe, 2.7 ft. On another test 1 cu. ft. per sec. was delivered with less than 5 cu. ft. entering the drive pipe. Taking 5 cu. ft. gives 76.6% efficiency.

A description and record of test of the Foster "impact engine" is given in *Eng'g News*, Aug. 3, 1905. Two engines are connected into one 8-in. delivery pipe. Using the same notation as before, the data of the tests of the two engines are as follows: Q , gal. per min., 582, 578; q , 232, 228; H , 36.75, 37.25; $H + h$, 84, 84; strokes per min., 130, 130; Effy. (D'Aubuisson), 91.23, 89.06%.

Prof. R. C. Carpenter (*Eng'g Mechanics*, 1894) reports the results of four tests of a ram constructed by Rumsey & Co., Seneca Falls. The supply-pipe used was 1 1/2 inches in diameter, about 50 feet long, with 3 elbows. Each run was made with a different stroke for the waste-valve, the supply and delivery head being constant; the object of the experiment was to find that stroke of clack-valve which would give the highest efficiency.

| | | | | |
|------------------------------------|-------|-------|-------|--------|
| Length of stroke, per cent | 100 | 80 | 60 | 46 |
| Number of strokes per minute | 52 | 56 | 61 | 66 |
| Supply head, feet of water | 5.67 | 5.77 | 5.58 | 5.65 |
| Delivery head, feet of water | 19.75 | 19.75 | 19.75 | 19.75 |
| Total water pumped, pounds | 297 | 296 | 301 | 297.5 |
| Total water supplied, pounds | 1615 | 1567 | 1518 | 1455.5 |
| Efficiency, per cent | 64.1 | 64.7 | 70.2 | 71.4 |

The highest efficiency realized was obtained when the clack-valve travelled 60% of its full stroke, the full travel being 15/16 in.

HYDRAULIC-PRESSURE TRANSMISSION.

Water under high pressure (700 to 2000 lbs. per sq. in. and upwards) affords a satisfactory method of transmitting power to a distance, especially for the movement of heavy loads at small velocities, as by cranes and elevators. The system consists usually of one or more pumps capable of developing the required pressure; accumulators, which are vertical cylinders with heavily-weighted plungers passing through stuffing-boxes in the upper end, by which a quantity of water may be accumulated at the pressure to which the plunger is weighted; the distributing-pipes; and the presses, cranes, or other machinery to be operated.

The earliest important use of hydraulic pressure probably was in the Bramah hydraulic press, patented in 1796. Sir. W. G. Armstrong in 1846 was one of the pioneers in the adaptation of the hydraulic system to cranes. The use of the accumulator by Armstrong led to the extended use of hydraulic machinery. Recent developments and applications of the system are largely due to Ralph Tweddell, of London, and Sir Joseph Whitworth. Sir Henry Bessemer, in his patent of May 13, 1856, No.

1292, first suggested the use of hydraulic pressure for compressing steel ingots while in the fluid state.

The Gross Amount of Energy of the water under pressure stored in the accumulator, measured in foot-pounds, is its volume in cubic feet \times its pressure in pounds per square foot. The horse-power of a given quantity steadily flowing is $H.P. = 144 pQ/550 = 0.2618 pQ$, in which Q is the quantity flowing in cubic feet per second and p the pressure in pounds per square inch.

The loss of energy due to velocity of flow in the pipe is calculated as follows (R. G. Blaine, *Eng'g*, May 22 and June 5, 1891):

According to Darcy, every pound of water loses $\lambda 4L/D$ times its kinetic energy, or energy due to its velocity, in passing along a straight pipe L feet in length and D feet diameter, where λ is a variable coefficient. For clean cast-iron pipes it may be taken as $\lambda = 0.005 \left(1 + \frac{1}{12D}\right)$, or for diameter in inches $= d$.

$d = 1/2 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 12$
 $\lambda = .015 .01 \quad .0075 \quad .00667 \quad .00625 \quad .006 \quad .00583 \quad .00571 \quad .00563 \quad .00556 \quad .0055 \quad .00542$

The loss of energy per minute is $60 \times 62.36 Q \times \frac{\lambda 4L}{D} \frac{v^2}{2g}$, and the

horse-power wasted in the pipe is $W = \frac{0.6363 \lambda L (H.P.)^3}{p^3 D^5}$, in which λ varies with the diameter as above. p = pressure at entrance in pounds per square inch. Values of 0.6363λ for different diameters of pipe in inches are:

$d = 1/2 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$
 $.00954 \quad .00636 \quad .00477 \quad .00424 \quad .00398 \quad .00382 \quad .00371 \quad .00363 \quad .00358$
 $\quad \quad \quad 9 \quad 10 \quad 12$
 $.00353 \quad .00350 \quad .00345$

Efficiency of Hydraulic Apparatus. — The useful effect of a direct hydraulic plunger or ram is usually taken at 93%. The following is given as the efficiency of a ram with chain-and-pulley multiplying gear properly proportioned and well lubricated:

| | | | | | | | | |
|-------|--------|--------|--------|--------|---------|---------|---------|---------|
| Gear | 2 to 1 | 4 to 1 | 6 to 1 | 8 to 1 | 10 to 1 | 12 to 1 | 14 to 1 | 16 to 1 |
| Eff'y | 0.80 | 0.76 | 0.72 | 0.67 | 0.63 | 0.59 | 0.51 | 0.50 |

With large sheaves, small steel pins, and wire rope for multiplying gear the efficiency has been found as high as 66% for a multiplication of 20 to 1.

Henry Adams gives the following formula for effective pressure in cranes and hoists: P = accumulator pressure in pounds per square inch; m = ratio of multiplying power; E = effective pressure in pounds per square inch, including all allowances for friction;

$$E = P (0.84 - 0.02 m).$$

J. E. Tuit (*Eng'g*, June 15, 1888) describes some experiments on the friction of hydraulic jacks from $3\frac{1}{4}$ to $13\frac{5}{8}$ -inch diameter, fitted with cupped leather packings. The friction loss varied from 5.6% to 18.8% according to the condition of the leather, the distribution of the load on the ram, etc. The friction increased considerably with eccentric loads. With hemp packing a plunger, 14-inch diameter, showed a friction loss of from 11.4% to 3.4%, the load being central, and from 15.0% to 7.6% with eccentric load, the percentage of loss decreasing in both cases with increase of load.

Thickness of Hydraulic Cylinders. — Sir W. G. Armstrong gives the following, for cast-iron cylinders, for a pressure of 1000 lbs. per sq. in.:

| | | | | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|-------|------|------|------|
| Diam. of cylinder, inches — | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
| Thickness, inches — | 0.832 | 1.146 | 1.552 | 1.875 | 2.222 | 2.578 | 3.19 | 3.69 | 4.11 |

For any other pressure multiply by the ratio of that pressure to 1000. These figures correspond nearly to the formula $t = 0.175 d + 0.48$, in which t = thickness and d = diameter in inches, up to 16 inches diameter, but for 20 inches diameter the addition 0.48 is reduced to 0.19 and at 24 inches it disappears. For formulæ for thick cylinders see page 316.

Cast iron should not be used for pressures exceeding 2000 lbs. per square inch. For higher pressures steel castings or forged steel should be used. For working pressures of 750 lbs. per square inch the test pressure should be 2500 lbs. per square inch, and for 1500 lbs. the test pressure should not be less than 3500 lbs.

Speed of Hoisting by Hydraulic Pressure. — The maximum allowable speed for warehouse cranes is 6 feet per second; for platform cranes 4 feet per second; for passenger and wagon hoists, heavy loads, 2 feet per second. The maximum speed under any circumstances should never exceed 10 feet per second.

The Speed of Water Through Valves should never be greater than 100 feet per second.

Speed of Water Through Pipes. — Experiments on water at 1600 lbs. pressure per square inch flowing into a flanging-machine ram, 20-inch diameter, through a $\frac{1}{2}$ -inch pipe contracted at one point to $\frac{1}{4}$ -inch, gave a velocity of 114 feet per second in the pipe, and 456 feet at the reduced section. Through a $\frac{1}{2}$ -inch pipe reduced to $\frac{3}{8}$ -inch at one point the velocity was 213 feet per second in the pipe and 381 feet at the reduced section. In a $\frac{1}{2}$ -inch pipe without contraction the velocity was 355 feet per second.

For many of the above notes the author is indebted to Mr. John Platt, consulting engineer, of New York.

High-pressure Hydraulic Presses in Iron-works are described by R. M. Daelen, of Germany, in *Trans. A. I. M. E.*, 1892. The following distinct arrangements used in different systems of high-pressure hydraulic work are discussed and illustrated:

1. Steam-pump, with fly-wheel and accumulator.
2. Steam-pump, without fly-wheel and with accumulator.
3. Steam-pump, without fly-wheel and without accumulator.

In these three systems the valve-motion of the working press is operated in the high-pressure column. This is avoided in the following:

4. Single-acting steam-intensifier without accumulator.
5. Steam-pump with fly-wheel, without accumulator and with pipe-circuit.
6. Steam-pump with fly-wheel, without accumulator and without pipe-circuit.

The disadvantages of accumulators are thus stated: The weighted plungers which formerly served in most cases as accumulators, cause violent shocks in the pipe-line when changes take place in the movement of the water, so that in many places, in order to avoid bursting from this cause, the pipes are made exclusively of forged and bored steel. The seats and cones of the metallic valves are cut by the water (at high speed), and in such cases only the most careful maintenance can prevent great losses of power.

Hydraulic Power in London. — The general principle involved is pumping water into mains laid in the streets, from which service-pipes are carried into the houses to work lifts or three-cylinder motors when rotary power is required. In some cases a small Pelton wheel has been tried, working under a pressure of over 700 lbs. on the square inch. Over 55 miles of hydraulic mains are at present laid (1892).

The reservoir of power consists of capacious accumulators, loaded to 800 lbs. per sq. in.

The engine-house contains six sets of triple-expansion pumping engines. Each pump will deliver 300 gallons of water per minute.

The water delivered from the main pumps passes into the accumulators. The rams are 20 inches in diameter, and have a stroke of 23 feet. They are each loaded with 110 tons of slag, contained in a wrought-iron cylindrical box suspended from a cross-head on the top of the ram. One of the accumulators is loaded a little more heavily than the other, so that they rise and fall successively; the more heavily loaded actuates a stop-valve on the main steam-pipe.

The mains in the public streets are so constructed and laid as to be perfectly trustworthy and free from leakage. Every pipe and valve used throughout the system is tested to 2500 lbs. per sq. in. before being placed on the ground and again tested to a reduced pressure in the trenches to insure the perfect tightness of the joints. The jointing material used is gutta-percha.

The average rate obtained by the company is about 3 shillings per thousand gallons. The principal use of the power is for intermittent work in cases where direct pressure can be employed, as, for instance, passenger elevators, cranes, presses, warehouse hoists, etc.

An important use of the hydraulic power is its application to the extinguishing of fire by means of Greathead's injector hydrant. By the use of these hydrants a continuous fire-engine is available.

Hydraulic Riveting-machines. — Hydraulic riveting was introduced in England by Mr. R. H. Tweddell. Fixed riveters were first used about 1868. Portable riveting-machines were introduced in 1872.

The riveting of the large steel plates in the Forth Bridge was done by small portable machines working with a pressure of 1000 lbs. per square inch. In exceptional cases 3 tons per inch were used. (*Proc. Inst. M. E.*, May, 1889.)

An application of hydraulic pressure invented by Andrew Higginson, of Liverpool, dispenses with the necessity of accumulators. It consists of a three-throw pump driven by shafting or worked by steam and depends partially upon the work accumulated in a heavy fly-wheel. The water in its passage from the pumps and back to them is in constant circulation at a very feeble pressure, requiring a minimum of power to preserve the tube of water ready for action at the desired moment, when by the use of a tap the current is stopped from going back to the pumps, and is thrown upon the piston of the tool to be set in motion. The water is now confined, and the driving-belt or steam-engine, supplemented by the momentum of the heavy fly-wheel, is employed in closing up the rivet, or bending or forging the object subjected to its operation.

Hydraulic Forging-press.

For a very complete illustrated account of the development of the hydraulic forging-press, see a paper by R. H. Tweddell in *Proc. Inst. C. E.*, vol. cxvii. 1893-4.

In the Allen forging-press the force-pump and the large or main cylinder of the press are in direct and constant communication. There are no intermediate valves of any kind, nor has the pump any check-valves, but it simply forces its cylinder full of water direct into the cylinder of the press, and receives the same water, as it were, back again on the return stroke. Thus, when both cylinders and the pipe connecting them are full, the large ram of the press rises and falls simultaneously with each stroke of the pump, keeping up a continuous oscillating motion, the ram, of course, traveling the shorter distance, owing to the larger capacity of the press cylinder. (*Journal Iron and Steel Institute*, 1891. See also illustrated article in "Modern Mechanism," page 668.)

A 2000-ton forging-press erected at the Couillet forges in Belgium is described in *Eng. and M. Jour.*, Nov. 25, 1893. The press is composed essentially of two parts — the press itself and the compressor. The compressor is formed of a vertical steam-cylinder and a hydraulic cylinder. The piston-rod of the former forms the piston of the latter. The hydraulic piston discharges the water into the press proper. The distribution is made by a cylindrical balanced valve; as soon as the pressure is released the steam-piston falls automatically under the action of gravity. During its descent the steam passes to the other face of the piston to reheat the cylinder, and finally escapes from the upper end.

When steam enters under the piston of the compressor-cylinder the piston rises, and its rod forces the water into the press proper. The pressure thus exerted on the piston of the latter is transmitted through a cross-head to the forging which is upon the anvil. To raise the cross-head two small single-acting steam-cylinders are used, their piston-rods being connected to the cross-head; steam acts only on the pistons of these cylinders from below. The admission of steam to the cylinders, which stand on top of the press frame, is regulated by the same lever which directs the motions of the compressor. The movement given to the dies is sufficient for all the ordinary purposes of forging.

A speed of 30 blows per minute has been attained. A double press on the same system, having two compressors and giving a maximum pressure of 6000 tons, has been erected in the Krupp works, at Essen.

Hydraulic Engine driving an Air-compressor and a Forging-hammer. (*Iron Age*, May 12, 1892.) — The great hammer in Terni, near Rome, is one of the largest in existence. Its falling weight amounts to 100 tons, and the foundation belonging to it consists of a block of cast iron of 1000 tons. The stroke is 16 feet $4\frac{3}{4}$ inches; the diameter of the cylinder 6 feet $3\frac{1}{2}$ inches; diameter of piston-rod $13\frac{3}{4}$ inches; total height of the hammer, 62 feet 4 inches. The power to work the hammer, as well as the two cranes of 100 and 150 tons respectively, and other auxiliary appliances belonging to it, is furnished by four air-compressors coupled together and driven directly by water-pressure engines, by means of which the air is compressed to 73.5 pounds per square inch. The cylinders of the water-pressure engines, which are provided with a bronze lining, have a $13\frac{3}{4}$ -inch bore. The stroke is $47\frac{3}{4}$ inches, with a pressure of water on the piston amounting to 264.6 pounds per square inch. The compressors are bored out to $31\frac{1}{2}$ inches diameter, and have $47\frac{3}{4}$ -inch stroke. Each of the four cylinders requires a power equal to 280 horse-power. The compressed air is delivered into huge reservoirs, where a uniform pressure is kept up by means of a suitable water-column.

The Hydraulic Forging Plant at Bethlehem, Pa., is described in a paper by R. W. Davenport, read before the Society of Naval Engineers and Marine Architects, 1893. It includes two hydraulic forging-presses complete, with engines and pumps, one of 1500 and one of 4500 tons capacity, together with two Whitworth hydraulic traveling forging-cranes and other necessary appliances for each press; and a complete fluid-compression plant, including a press of 7000 tons capacity and a 125-ton hydraulic traveling crane for serving it (the upper and lower heads of this press weighing respectively about 135 and 120 tons).

A new forging-press designed by Mr. John Fritz, for the Bethlehem Works, of 14,000 tons capacity, is run by engines and pumps of 15,000 horse-power. The plant is served by four open-hearth steel furnaces of a united capacity of 120 tons of steel per heat.

The Davy High-speed Steam-hydraulic Forging Press is described in the *Iron Age*, April 15, 1909. It is built in sizes ranging from 150 to 12,000 tons capacity. In the four-column type, in which all but the smaller sizes are built, there is a central press operated by hydraulic pressure from a steam intensifier, and two steam balance cylinders carried on top of the entablature. A single lever controls the press. The operator admits steam to the balance cylinders, lifting the cross head and the main plunger, and forcing the water from the press cylinder into the water cylinder of the intensifier. Exhausting the steam from the balance cylinders, allows the plunger to descend and rest on the forging. To and fro motions of the lever, slow or fast as the operator desires, up to 120 a minute, then are made to reduce the forging. The smaller, or single frame, type has only one balance cylinder, immediately above the press cylinder. The Davy press is made in the United States by the United Engineering & Foundry Co., Pittsburgh.

Some References on Hydraulic Transmission. — Reuleaux's "Constructor;" "Hydraulic Motors, Turbines, and Pressure-engines," G. Bodmer, London, 1889; Robinson's "Hydraulic Power and Hydraulic Machinery," London, 1888; Colyer's "Hydraulic Steam, and Hand-power Lifting and Pressing Machinery," London, 1881. See also *Engineering* (London), Aug. 1, 1884, p. 99; March 13, 1885, p. 262; May 22 and June 5, 1891, pp. 612, 665; Feb. 19, 1892, p. 25; Feb. 10, 1893, p. 170.

FUEL.

Theory of Combustion of Solid Fuel. (From Rankine, somewhat altered.) — The ingredients of every kind of fuel commonly used may be thus classed: (1) Fixed or free carbon, which is left in the form of charcoal or coke after the volatile ingredients of the fuel have been distilled away. These ingredients burn either wholly in the solid state (C to CO_2), or part in the solid state and part in the gaseous state ($CO + O = CO_2$), the latter part being first dissolved by previously formed carbon dioxide by the reaction $CO_2 + C = 2 CO$. Carbon monoxide, CO , is produced when the supply of air to the fire is insufficient.

(2) Hydrocarbons, such as olefiant gas, pitch, tar, naphtha, etc., all of which must pass into the gaseous state before being burned.

If mixed on their first issuing from amongst the burning carbon with a large quantity of hot air, these inflammable gases are completely burned with a transparent blue flame, producing carbon dioxide and steam. When mixed with cold air they are apt to be chilled and pass off unburned. When raised to a red heat, or thereabouts, before being mixed with a sufficient quantity of air for perfect combustion, they disengage carbon in fine powder, and pass to the condition partly of marsh gas, CH_4 and partly of free hydrogen; and the higher the temperature, the greater is the proportion of carbon thus disengaged.

If the disengaged carbon is cooled below the temperature of ignition before coming in contact with oxygen, it constitutes, while floating in the gas, smoke, and when deposited on solid bodies, soot.

But if the disengaged carbon is maintained at the temperature of ignition and supplied with oxygen sufficient for its combustion, it burns while floating in the inflammable gas, and forms red, yellow, or white flame. The flame from fuel is the larger the more slowly its combustion is effected. The flame itself is apt to be chilled by radiation, as into the heating surface of a steam-boiler, so that the combustion is not completed, and part of the gas and smoke pass off unburned.

(3) Oxygen or hydrogen either actually forming water, or existing in combination with the other constituents in the proportions which form water. Such quantities of oxygen and hydrogen are to be left out of account in determining the heat generated by the combustion. If the quantity of water actually or virtually present in each pound of fuel is so great as to make its latent heat of evaporation worth considering, that heat is to be deducted from the total available heat of combustion of the fuel.

(4) Nitrogen, either free or in combination with other constituents. This substance is simply inert.

(5) Sulphide of iron, which exists in coal and is detrimental, as tending to cause spontaneous combustion.

(6) Other mineral compounds of various kinds, which are also inert, and form the ash left after complete combustion of the fuel, and also the clinker or glassy material produced by fusion of the ash, which tends to choke the grate.

Oxygen and Air Required for the Combustion of Carbon, Hydrogen, etc.

| Chemical Reaction. | Lbs. O per lb. Fuel. | Lbs. N, = 3.32 O | Air per lb. = 4.32 O. | Gase- ous Prod- ucts per lb. | Heat of Combustion, B.T.U. per lb. |
|--------------------------------|----------------------------|---------------------|-----------------------------|--|---|
| C to CO_2 $C + 2O = CO_2$ | 2 1/3 | 8.85 | 11.52 | 12.52 | 14,600 |
| C to CO $C + O = CO$ | 1 1/3 | 4.43 | 5.76 | 6.76 | 4,450 |
| CO to CO_2 $CO + O = CO_2$ | 4/7 | 1.90 | 2.47 | 3.47 | 10,150 |
| H to H_2O $2H + O = H_2O$ | 8 | 26.56 | 34.56 | 35.56 | 62,000 |
| CH_4 to CO_2 } $CH_4 + 4O$ | | | | | |
| and H_2O } $= CO_2 + 2H_2O$ | 4 | 13.28 | 17.28 | 18.28 | 23,600 |
| S to SO_2 $S + 2O = SO_2$ | 1 | 3.32 | 4.32 | 5.32 | 4,050 |

For heat of combustion of various fuels see Heat, page 533.

The imperfect combustion of carbon, making carbon monoxide, produces less than one-third of the heat which is yielded by the complete combustion, making carbon dioxide.

The total heat of combustion of any compound of hydrogen and carbon is nearly the sum of the quantities of heat which the constituents would produce separately by their combustion. (Marsh-gas is an exception.)

In computing the total heat of combustion of compounds containing oxygen as well as hydrogen and carbon, the following principle is to be observed: When hydrogen and oxygen exist in a compound in the proper proportion to form water (that is, by weight one part of hydrogen to eight of oxygen), these constituents have no effect on the total heat of combustion. If hydrogen exists in a greater proportion, only the surplus of hydrogen above that which is required by the oxygen is to be taken into account.

The following is a general formula (Dulong's) for the total heat of combustion of any compound of carbon, hydrogen, and oxygen:

Let C , H , and O be the fractions of one pound of the compound, which consists respectively of carbon, hydrogen, and oxygen, the remainder being nitrogen, ash, and other impurities. Let h be the total heat of combustion of one pound of the compound in British thermal units.

Then
$$h = 14,600 C + 62,000 (H - \frac{1}{8} O).$$

Analyses of Gases of Combustion.—The following are selected from a large number of analyses of gases from locomotive boilers, to show the range of composition under different circumstances (P. H. Dudley, *Trans. A. I. M. E.*, iv. 250):

| Test. | CO ₂ | CO | O | N | |
|-------|-----------------|-------|------|------|--|
| 1 | 13.8 | 2.5 | 2.5 | 81.6 | No smoke visible. |
| 2 | 11.5 | | 6 | 82.5 | Old fire, escaping gas white, engine working hard. |
| 3 | 8.5 | | 8 | 83 | Fresh fire, much black gas, engine working hard. |
| 4 | 2.3 | | 17.2 | 80.5 | Old fire, damper closed, engine standing still. |
| 5 | 5.7 | | 14.7 | 79.6 | " " smoke white, engine working hard. |
| 6 | 8.4 | 1.2 | 8.4 | 82 | New fire, engine not working hard. |
| 7 | 12 | 1 | 4.4 | 82.6 | Smoke black, engine not working hard. |
| 8 | 3.4 | | 16.8 | 76.8 | " dark, blower on, engine standing still. |
| 9 | 6 | | 13.5 | 81.5 | " white, engine working hard. |

In analyses on the Cleveland and Pittsburgh road, in every instance when the smoke was the blackest, there was found the greatest percentage of unconsumed oxygen in the product, showing that something besides the mere presence of oxygen is required to effect the combustion of the volatile carbon of fuels. (What is needed is thorough mixture of the oxygen with the volatile gases in a hot combustion chamber.)

Temperature of the Fire. (Rankine, S. E., p. 283.)—By temperature of the fire is meant the temperature of the products of combustion at the instant that the combustion is complete. The elevation of that temperature above the temperature at which the air and the fuel are supplied to the furnace may be computed by dividing the total heat of combustion of one lb. of fuel by the weight and by the mean specific heat of the whole products of combustion, and of the air employed for their dilution under constant pressure.

Temperature of the Fire, the Fuel Containing Hydrogen and Water.—The following formula is developed in the author's "Steam-boiler Economy" on the assumptions that all the hydrogen and the water exist in the combustion chamber as superheated steam at the temperature of the fire, and that the specific heat of the gases is a constant, = 0.237. The last assumption is probably largely in error, since it is now known that the specific heat of gases increases with the temperature. (See page 537.) The formula will give approximate results, however, and is sufficiently accurate when relative figures only are desired.

Let C , H , O , and W represent respectively the percentages of carbon, hydrogen, oxygen, and water in a fuel, and f the pounds of dry gas per

pound of fuel, = $CO_2 + N$ + excess air, then the theoretical elevation of the temperature of the fire above the temperature of the atmosphere,

$$T = \frac{616 C + 2200 H - 327 O - 44 W}{f + 0.02 W + 0.18 H}$$

EXAMPLE. — Required the maximum temperature obtainable by burning moist wood of the composition C , 38; H , 5; O , 32; ash, 1; moisture 24; the dry gas being 15 lbs. per pound of wood, and the temperature of the atmosphere 62° .

$$T = \frac{616 \times 38 + 2200 \times 5 - 327 \times 32 - 44 \times 24}{15 + 0.02 \times 24 + 0.18 \times 5} = 1403, \text{ add } 62^\circ = 1465^\circ.$$

Rise of Temperature in Combustion of Gases. (*Eng'g*, March 12 and April 2, 1886.) — It is found that the temperatures obtained by experiment fall short of those obtained by calculation. Three theories have been given to account for this: 1. The cooling effect of the sides of the containing vessel; 2. The retardation of the evolution of heat caused by dissociation; 3. The increase of the specific heat of the gases at very high temperatures. The calculated temperatures are obtainable only on the condition that the gases shall combine instantaneously and simultaneously throughout their whole mass. This condition is practically impossible in experiments. The gases formed at the beginning of an explosion dilute the remaining combustible gases and tend to retard or check the combustion of the remainder.

CLASSIFICATION OF SOLID FUELS.

Gruner classifies solid fuels as follows (*Eng'g and M'g Jour.*, July, 1874).

| Name of Fuel. | Ratio $\frac{O}{H}$ or $\frac{O+N^*}{H}$. | Proportion of Coke or Charcoal yielded by the Dry Pure Fuel. |
|---|---|--|
| Pure cellulose..... | 8 | 0.28 @ 0.30 |
| Wood (cellulose and encasing matter)... | 7 | .30 @ .35 |
| Peat and fossil fuel | 6 @ 5 | .35 @ .40 |
| Lignite, or brown coal | 5 | .40 @ .50 |
| Bituminous coals | 4 @ 1 | .50 @ .90 |
| Anthracite | 1 @ 0.75 | .90 @ .92 |

* The nitrogen rarely exceeds 1 per cent of the weight of the fuel.

Progressive Change from Wood to Graphite.

(J. S. Newberry in Johnson's Cyclopaedia.)

| | Wood. | Loss. | Lignite. | Loss. | Bituminous coal. | Loss. | Anthracite. | Loss. | Graphite. |
|---------------|-------|-------|----------|-------|------------------|-------|-------------|-------|-----------|
| Carbon..... | 49.1 | 18.65 | 30.45 | 12.35 | 18.10 | 3.57 | 14.53 | 1.42 | 13.11 |
| Hydrogen..... | 6.3 | 3.25 | 3.05 | 1.85 | 1.20 | 0.93 | 0.27 | 0.14 | 0.13 |
| Oxygen..... | 44.6 | 24.40 | 20.20 | 18.13 | 2.07 | 1.32 | 0.65 | 0.65 | 0.00 |
| | 100.0 | 46.30 | 53.70 | 32.33 | 21.37 | 5.82 | 15.45 | 2.21 | 13.24 |

Classification of Coals.

It is convenient to classify the several varieties of coal according to the relative percentages of carbon and volatile matter contained in their combustible portion as determined by proximate analysis. The following is the classification given in the author's "Steam-boiler Economy":

CLASSIFICATION OF COALS.

| | Fixed Carbon. | Volatile Matter. — | Heating Value per lb. of Combustible | Relative Value of Combustible Semi-bit. = 100 |
|---------------------------|------------------|--------------------------|---|---|
| Anthracite | 97 to 92.5 | 3 to 7.5 | 14600 to 14800 | 93 |
| Semi-anthracite | 92.5 to 87.5 | 7.5 to 12.5 | 14700 to 15500 | 96 |
| Semi-bituminous | 87.5 to 75 | 12.5 to 25 | 15500 to 16000 | 100 |
| Bituminous, Eastern | 75 to 60 | 25 to 40 | 14800 to 15500 | 96 |
| Bituminous, Western | 65 to 50 | 35 to 50 | 13500 to 14800 | 90 |
| Lignite | under 50 | over 50 | 11000 to 13500 | 77 |

The anthracites, with some unimportant exceptions, are confined to three small fields in eastern Pennsylvania. The semi-anthracites are found in a few small areas in the western part of the anthracite field. The semi-bituminous coals are found on the eastern border of the great Appalachian coal field, extending from north central Pennsylvania across the southern boundary of Virginia into Tennessee, a distance of over 300 miles. They include the coals of Clearfield, Cambria, and Somerset counties, Pennsylvania, and the Cumberland, Md., the Pocahontas, Va., and the New River, W. Va., coals.

It is a peculiarity of the semi-bituminous coals that their combustible portion is of remarkably uniform composition, the volatile matter usually ranging between 18 and 22% of the combustible, and approaching in its analysis marsh gas, CH_4 , with very little oxygen. They are usually low also in moisture, ash, and sulphur, and rank among the best steaming coals in the world.

The eastern bituminous coals occupy the remainder of the Appalachian coal field, from Pennsylvania and eastern Ohio to Alabama. They are higher in volatile matter, ranging from 25 to over 40%, the higher figures in the western portion of the field. The volatile matter is of lower heating value, being higher in oxygen. The western bituminous coals are found in most of the states west of Ohio. They are higher in volatile matter and in oxygen and moisture than the bituminous coals of the Appalachian field, and usually give off a denser smoke when burned in ordinary furnaces.

The U. S. Geological Survey classifies coals into six groups, as follows: (1) anthracite; (2) semi-anthracite; (3) semi-bituminous; (4) bituminous; (5) sub-bituminous, or black lignite; and (6) lignite.

Classes 5 and 6 are described as follows:

Sub-bituminous coal is commonly known as "lignite," "lignitic coal," "black lignite," "brown coal," etc. It is generally black and shining, closely resembling bituminous coal, but it weathers more rapidly on exposure and lacks the prismatic structure of bituminous coal. Its calorific value is generally less than that of bituminous coal. The localities in which this sub-bituminous coal is found include Montana, Idaho, Washington, Oregon, California, Wyoming, Utah, Colorado, New Mexico, and Texas.

Lignite is commonly known as "lignite," "brown lignite," or "brown coal." It usually has a woody structure and is distinctly brown in color, even on a fresh fracture. It carries a higher percentage of moisture than any other class of coals, its mine samples showing from 30 to 40% of moisture. The localities in which lignite is found are chiefly North Dakota, South Dakota, Texas, Arkansas, Louisiana, Mississippi, and Alabama.

The following analyses of representative coals of the six classes are given by Prof. N. W. Lord:

Class 1 — Anthracite Culm. Penna.

Class 2 — Semi-anthracite. Arkansas.

Class 3 — Semi-bituminous. W. Va.

Class 4(a) — Bituminous coking. Connellsville, Pa.

Class 4(b) — Bituminous non-coking. Hocking Valley, Ohio.

Class 5 — Sub-bituminous. Wyoming, black lignite.

Class 6 — Lignite. Texas.

COMPOSITION OF ILLUSTRATIVE COALS — CAR-LOAD SAMPLES.

Proximate Analysis of "Air-dried" Sample.

| Class..... | 1 | 2 | 3 | 4a | 4b | 5 | 6 |
|--------------------|-------|-------|-------|-------|--------|-------|-------|
| Moisture..... | 2.08 | 1.28 | 0.65 | 0.97 | 7.55 | 8.68 | 9.88 |
| Vol. comb..... | 7.27 | 12.82 | 18.80 | 29.09 | 34.03 | 41.31 | 36.17 |
| Fixed carbon..... | 74.32 | 73.69 | 75.92 | 60.85 | 52.57 | 46.49 | 43.65 |
| Ash..... | 16.33 | 12.21 | 4.63 | 9.09 | 5.85 | 3.52 | 10.30 |
| Loss on air-drying | 3.40 | 1.10 | 1.10 | 4.20 | Undet. | 11.30 | 23.50 |

Ultimate Analysis of Coal Dried at 105° C.

| | | | | | | | |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| Hydrogen..... | 2.63 | 3.63 | 4.54 | 4.57 | 5.06 | 5.31 | 4.47 |
| Carbon..... | 76.86 | 78.32 | 86.47 | 77.10 | 75.82 | 73.31 | 64.84 |
| Oxygen..... | 2.27 | 2.25 | 2.68 | 6.67 | 10.47 | 15.72 | 16.52 |
| Nitrogen..... | 0.82 | 1.41 | 1.08 | 1.58 | 1.50 | 1.21 | 1.30 |
| Sulphur..... | 0.78 | 2.03 | 0.57 | 0.90 | 0.82 | 0.60 | 1.44 |
| Ash..... | 16.64 | 12.36 | 4.66 | 9.18 | 6.33 | 3.85 | 11.43 |

Results Calculated to an Ash and Moisture Free Basis.

| | | | | | | | |
|--------------------|-------|-------|-------|-------|-------|-------|-------|
| Volatile comb..... | 8.91 | 14.82 | 19.85 | 32.34 | 39.30 | 47.05 | 45.31 |
| Fixed carbon..... | 91.09 | 85.18 | 80.15 | 67.66 | 60.70 | 52.95 | 54.69 |

Ultimate Analysis.

| | | | | | | | |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| Hydrogen..... | 3.16 | 4.14 | 4.76 | 5.03 | 5.41 | 5.50 | 5.05 |
| Carbon..... | 92.20 | 89.36 | 90.70 | 84.89 | 80.93 | 76.35 | 73.21 |
| Oxygen..... | 2.72 | 2.57 | 2.81 | 7.34 | 11.18 | 16.28 | 18.65 |
| Nitrogen..... | 0.98 | 1.61 | 1.13 | 1.74 | 1.61 | 1.25 | 1.47 |
| Sulphur..... | 0.94 | 2.32 | 0.60 | 1.00 | 0.87 | 0.62 | 1.62 |

Calorific Value in B.T.U. per lb., by Dulong's formula.

| | | | | | | | |
|----------------|--------|--------|--------|--------|--------|--------|--------|
| Air-dried coal | 12,472 | 13,406 | 15,190 | 13,951 | 12,510 | 11,620 | 10,288 |
| Combustible | 15,286 | 15,496 | 16,037 | 15,511 | 14,446 | 13,235 | 12,889 |

Caking and Non-caking Coals. — Bituminous coals are sometimes classified as caking and non-caking coals, according to their behavior when subjected to the process of coking. The former undergo an incipient fusion or softening when heated, so that the fragments coalesce and yield a compact coke, while the latter (also called free-burning) preserve their form, producing a coke which is only serviceable when made from large pieces of coal, the smaller pieces being incoherent. The reason of this difference is not clearly understood, as non-caking coals are often of similar ultimate chemical composition to caking coals. Some coals which cannot be made into coke in a bee-hive oven are easily coked in gas-heated ovens.

Cannel Coals are coals that are higher in hydrogen than ordinary coals. They are valuable as enrichers in gas-making. The following are some ultimate analyses:

| | C. | H. | O+N. | S. | Ash. | Combustible. | | |
|--------------------------|-------|-------|------|-------|-------|--------------|-------|------|
| | | | | | | C. | H. | O+N. |
| Boghead, Scotland..... | 63.10 | 8.91 | 7.25 | 0.96 | 19.78 | 79.61 | 11.24 | 9.15 |
| Albertite, Nova Scotia.. | 82.67 | 9.14 | 8.19 | | | 82.67 | 9.14 | 8.19 |
| Tasmanite, Tasmania... | 79.34 | 10.41 | 4.93 | 5.32 | | 83.80 | 10.99 | 5.21 |

Rhode Island Graphitic Anthracite. — A peculiar variety of coal is found in the central part of Rhode Island and in Eastern Massachusetts. It resembles both graphite and anthracite coal, and has about the following composition (A. E. Hunt, *Trans. A. I. M. E.*, xvii. 678: Graphitic carbon, 78%; volatile matter, 2.60%; silica, 15.06%; phosphorus, .045%. It burns with extreme difficulty.

ANALYSIS AND HEATING VALUE OF COALS.

Coal is composed of four different things, which may be separated by proximate analysis, viz.: fixed carbon, volatile hydrocarbon, ash and moisture. In making a proximate analysis of a weighed quantity, such as a gram of coal, the moisture is first driven off by heating it to about 250° F. then the volatile matter is driven off by heating it in a closed crucible to a red heat, then the carbon is burned out of the remaining coke at a white heat, with sufficient air supplied, until nothing is left but the ash.

The fixed carbon has a constant heating value of about 14,600 B.T.U. per lb. The value of the volatile hydrocarbon depends on its composition, and that depends chiefly on the district in which the coal is mined. It may be as high as 21,000 B.T.U. per lb., or about the heating value of marsh gas, in the best semi-bituminous coals, which contain very small percentages of oxygen, or as low as 12,000 B.T.U. per lb., as in those from some of the western states, which are high in oxygen. The ash has no heating value, and the moisture has in effect less than none, for its evaporation and the superheating of the steam made from it to the temperature of the chimney gases, absorb some of the heat generated by the combustion of the fixed carbon and volatile matter.

The analysis of a coal may be reported in three different forms, as percentages of the moist coal, of the dry coal or of the combustible, as in the following table. By "combustible" is always meant the sum of the fixed carbon and volatile matter, the moisture and ash being excluded. By some writers it is called "coal dry and free from ash" and by others "pure coal."

| | Moist Coal. | Dry Coal. | Combustible. |
|----------------------|-------------|-----------|--------------|
| Moisture..... | 10 | | |
| Volatile matter..... | 30 | 33.33 | 37.50 |
| Fixed carbon..... | 50 | 55.56 | 62.50 |
| Ash..... | 10 | 11.11 | |
| | 100 | 100.00 | 100.00 |

The sulphur, commonly reported with a proximate analysis, is determined separately. In the proximate analysis part of it escapes with the volatile matter and the rest of it is found in the ash as sulphide of iron. The sulphur should be given separately in the report of the analysis.

The relation of the volatile matter and of the fixed carbon in the combustible portion of the coal enables us to judge the class to which the coal belongs, as anthracite, semi-anthracite, semi-bituminous, bituminous, or lignite. Coals containing less than 7.5 per cent volatile matter in the combustible, would be classed as anthracite, between 7.5 and 12.5 per cent as semi-anthracite, between 12.5 and 25 per cent as semi-bituminous, between 25 and 50 per cent as bituminous, and over 50 per cent as lignitic coals or lignites. In the classification of the U. S. Geological Survey the sub-bituminous coals and lignites are distinguished by their structure and color rather than by analysis.

The figures in the second column, representing the percentages in the dry coal, are useful in comparing different lots of coal of one class, and they are better for this purpose than the figures in the first column, for the moisture is a variable constituent, depending to a large extent on the weather to which the coal has been subjected since it was mined, on the amount of moisture in the atmosphere at the time when it is analyzed, and on the extent to which it may have accidentally been dried during the process of sampling.

The heating value of a coal depends on its percentage of total combustible matter, and on the heating value per pound of that combustible. The latter differs in different districts and bears a relation to the percentage of volatile matter. It is highest in the semi-bituminous coals, being nearly constant at about 15,750 B.T.U. per pound. It is between 14,500 and 15,000 B.T.U. in anthracite, and ranges from 15,500 down to

13,000 in the bituminous coals, decreasing usually as we go westward, and as the volatile matter contains an increasing percentage of oxygen. In some lignites it is as low as 10,000.

In reporting the heating value of a coal, the B.T.U. per pound of combustible should always be stated, for convenient comparison with other reports.

Proximate Analyses and Heating Values of American Coals.

The accompanying table of proximate analyses and heating values of American coals is condensed from one compiled by the author for the 1898 edition of the Babcock & Wilcox Co.'s book, "Steam." The analyses are selected from various sources, and in general are averages of many samples. The heating values per pound of combustible are either obtained from direct calorimetric determinations or calculated from ultimate analyses, except those marked (?) which are estimated from the heating values of coals of similar composition.

TABLE OF HEATING VALUE OF COALS.

| | Moisture. | Volatile Matter. | Fixed Carbon. | Ash. | Sulphur. | Heating Value per lb. Coal, B.T.U. | Volatile Matter per Cent of Combustible. | Heating Value Per lb. Combustible. | Theoretical Evaporation from and at 212° per lb. Combustible. |
|-------------------------|-----------|------------------|---------------|-------|----------|------------------------------------|--|------------------------------------|---|
| Anthracite. | | | | | | | | | |
| Northern Coal Field... | 3.42 | 4.38 | 83.27 | 8.20 | 0.73 | 13160 | 5.00 | 14900 | 15.42 |
| East Middle Field.... | 3.71 | 3.08 | 86.40 | 6.22 | 0.58 | 13420 | 3.44 | 14900 | 15.42 |
| West Middle Field.... | 3.16 | 3.72 | 81.59 | 10.65 | 0.50 | 12840 | 4.36 | 14900 | 15.42 |
| Southern Coal Field.. | 3.09 | 4.28 | 83.81 | 8.18 | 0.64 | 13220 | 4.85 | 14900 | 15.42 |
| Semi-anthracite. | | | | | | | | | |
| Loyalsock Field..... | 1.30 | 8.10 | 83.34 | 6.23 | 1.63 | 13920 | 8.86 | 15500 | 16.05 |
| Bernice Basin..... | 0.65 | 9.40 | 83.69 | 5.34 | 0.91 | 13700 | 10.98 | 15500 | 16.05 |
| Semi-bituminous. | | | | | | | | | |
| Clearfield Co., Pa..... | 0.76 | 22.52 | 71.82 | 3.99 | 0.91 | 14950 | 24.60 | 15700 | 16.25 |
| Cambria Co., Pa..... | 0.94 | 19.20 | 71.12 | 7.04 | 1.70 | 14450 | 22.71 | 15700 | 16.25 |
| Somerset Co., Pa..... | 1.58 | 16.42 | 71.51 | 8.62 | 1.87 | 14200 | 20.37 | 15800 | 16.36 |
| Cumberland, Md..... | 1.09 | 17.30 | 73.12 | 7.75 | 0.74 | 14400 | 19.79 | 15800 | 16.36 |
| Pocahontas, Va..... | 1.00 | 21.00 | 74.39 | 3.03 | 0.58 | 15070 | 22.50 | 15700 | 16.25 |
| New River, W. Va..... | 0.85 | 17.88 | 77.64 | 3.36 | 0.27 | 15220 | 18.95 | 15800 | 16.36 |
| Bituminous. | | | | | | | | | |
| Connellsville, Pa..... | 1.26 | 30.12 | 59.61 | 8.23 | 0.78 | 14050 | 34.03 | 15300 | 15.84 |
| Youghiogheny, Pa.... | 1.03 | 36.50 | 59.05 | 2.61 | 0.81 | 14450 | 38.73 | 15000 | 15.53 |
| Jefferson Co., Pa..... | 1.21 | 32.53 | 60.99 | 4.27 | 1.00 | 14370 | 35.47 | 15200 | 15.74 |
| Brier Hill, Ohio..... | 4.80 | 34.60 | 56.30 | 4.30 | | 13010 | 38.20 | 14300 | 14.80 |
| Vanderpool, Ky..... | 4.00 | 34.10 | 54.60 | 7.30 | | 12770 | 38.50 | 14400 | 14.91 |
| Muhlenberg Co., Ky.. | 4.33 | 33.65 | 55.50 | 4.95 | 1.57 | 13060 | 38.86 | 14400(?) | 14.91 |
| Scott Co., Tenn..... | 1.26 | 35.76 | 53.14 | 8.02 | 1.80 | 13700 | 34.17 | 15100(?) | 15.63 |
| Jefferson Co., Ala..... | 1.55 | 34.44 | 59.77 | 2.62 | 1.42 | 13770 | 37.63 | 14400(?) | 14.91 |
| Big Muddy, Ill..... | 7.50 | 30.70 | 53.80 | 8.00 | | 12420 | 36.30 | 14700 | 15.22 |
| Mt. Olive, Ill..... | 11.00 | 35.65 | 37.10 | 13.00 | | 10490 | 47.00 | 13800 | 14.29 |
| Streator, Ill..... | 12.00 | 33.30 | 40.70 | 14.00 | | 10580 | 45.00 | 14300 | 14.80 |
| Missouri..... | 6.44 | 37.57 | 47.94 | 8.05 | | 12230 | 43.94 | 14300(?) | 14.80 |

The heating values per pound of combustible given in the table, except those marked (?) are probably within 3% of the average actual heating values of the combustible portion of the coals of the several districts. When the percentage of moisture and ash in any given lot of coal is known

the heating value per pound of coal may be found approximately by multiplying the heating value per pound of combustible of the average coal of the district by the difference between 100% and the sum of the percentages of moisture and ash.

In 1890 the author deduced from Mahler's tests on European coals the following table of the approximate heating value of coals of different composition.

APPROXIMATE HEATING VALUES OF COALS.

| Per Cent Fixed Carbon in Coal Dry and Free from Ash. | Heating Value, B.T.U. per lb. Combustible. | Equivalent Water Evaporation from and at 212° per lb. Combustible. | Per Cent Fixed Carbon in Coal Dry and Free from Ash. | Heating Value, B.T.U. per lb. Combustible. | Equivalent Water Evaporation from and at 212° per lb. Combustible. |
|--|--|--|--|--|--|
| 100 | 14,580 | 15.09 | 68 | 15,480 | 16.03 |
| 97 | 14,940 | 15.47 | 63 | 15,120 | 15.65 |
| 94 | 15,210 | 15.75 | 60 | 14,760 | 15.28 |
| 90 | 15,480 | 16.03 | 57 | 14,220 | 14.72 |
| 87 | 15,660 | 16.21 | 55 | 13,860 | 14.35 |
| 80 | 15,840 | 16.40 | 53 | 13,320 | 13.79 |
| 72 | 15,660 | 16.21 | 51 | 12,420 | 12.86 |

The experiments of Lord and Haas on American coals (*Trans. A. I. M. E.*, 1897) practically confirm these figures for all coals in which the percentage of fixed carbon is 60% and over of the combustible, but for coals containing less than 60% fixed carbon or more than 40% volatile matter in the combustible, they are liable to an error in either direction of about 4%. It appears from these experiments that the coal of one seam in a given district has the same heating value per pound of combustible within one or two per cent, [true only of some districts] but coals of the same proximate analysis, and containing over 40% volatile matter, but mined in different districts, may vary 6 or 8% in heating value.

The coals containing from 72 to 87 per cent of fixed carbon in the combustible have practically the same heating value. This is confirmed by Lord and Haas's tests of Pocahontas coal. A study of these tests and of Mahler's indicates that the heating value of all the semi-bituminous coals, 75 to 87.5% fixed carbon, is within 1½% of 15,750 B.T.U. per pound.

The heating value of any coal may also be calculated from its ultimate analysis, with a probable error not exceeding 2%, by Dulong's formula:

$$\text{Heating value per lb.} = 146 C + 620 \left(H - \frac{O}{8} \right) + 40 S,$$

in which *C*, *H*, and *O* are respectively the percentages of carbon, hydrogen and oxygen. Its approximate accuracy is proved by both Mahler's and Lord and Haas's experiments, and any deviation of the calorimetric determination of any coals (cannel coals and lignites excepted) more than 2% from that calculated by the formula, is more likely to proceed from an error in either the calorimetric test or the analysis, than from an error in the formula.

Tests of the U. S. Geological Survey, 1904-1906. — Coals were selected at the mines in different parts of the country for the purpose of testing their relative value in developing power through a steam boiler and engine and through a gas producer and gas engine. The full account of these tests will be found in Bulletins 261, 290 and 323, and Professional Paper 48, of the U. S. Geological Survey. The following table shows approximately the range of heating values per pound of combustible, as determined by the Mahler calorimeter, and the range of percentages of fixed carbon in the combustible (total of fixed carbon and volatile

matter) in the coals from the several states. The extreme figures, 10,200 and 15,950, fairly represent the whole range of heating values of the combustible of the coals of the United States, but the figures for each state do not nearly cover the range of values in that state, and in some cases, as in Indiana and Illinois, the figures are much lower than the average heating values of the coals of the states.

| | Fixed C. %. | B.T.U. per lb. |
|-------------------------------|--------------|------------------|
| Penna. anthracite..... | 89 | 14,900 |
| West Va. semi-bituminous..... | 80 to 76.5 | 15,950 to 15,650 |
| Arkansas semi-bituminous..... | 84 to 77 | 15,250 to 15,500 |
| Penna. bituminous..... | 67 | 15,500 |
| West Va. bituminous..... | 67.5 to 55 | 15,500 to 15,000 |
| Eastern Kentucky..... | 60 | 15,000 |
| Western Kentucky..... | 55 to 50.5 | 14,400 to 13,700 |
| Alabama..... | 61.5 to 59 | 14,800 to 14,200 |
| Kansas..... | 62 to 53.5 | 14,800 to 14,100 |
| Oklahoma..... | 56 to 51 | 14,600 to 13,100 |
| Missouri..... | 50.5 to 47 | 14,300 to 12,600 |
| Illinois..... | 59 to 47.5 | 13,700 to 12,400 |
| Iowa..... | 57 to 53.5 | 13,600 to 12,700 |
| Indiana..... | 49 | 13,300 |
| New Mexico..... | 50.5 to 47 | 12,500 to 12,300 |
| Wyoming..... | 48 to 41.5 | 13,300 to 10,900 |
| Montana..... | 48.5 | 12,100 |
| Colorado..... | 46 | 11,500 |
| North Dakota..... | 48.5 to 42.5 | 10,200 to 11,400 |
| Texas..... | 44.5 to 34 | 10,900 to 11,000 |

Average Results of Lord and Haas's Tests. — ("Steam Boiler Economy," p. 104.)

| Name of Coal. | C. | H. | O. | N. | S. | Ash. | Moist. | Vol. Mat. | Fixed C. | Vol. Mat. % of Comb. | B.T.U.* |
|-------------------------------|-------|------|-------|------|------|-------|--------|-----------|----------|----------------------------|---------|
| Pocahontas, Va.. | 84.87 | 4.20 | 2.84 | 0.85 | 0.59 | 5.89 | 0.76 | 18.51 | 74.84 | 19.82 | 15766 |
| Thacker, W. Va. | 78.65 | 5.00 | 6.01 | 1.41 | 1.28 | 6.27 | 1.38 | 35.68 | 56.67 | 38.62 | 15237 |
| Pittsburg, Pa.... | 75.24 | 5.01 | 7.04 | 1.51 | 1.79 | 8.02 | 1.37 | 36.80 | 53.81 | 40.61 | 14963 |
| Middle Kittanning, Pa..... | 75.19 | 4.91 | 7.47 | 1.46 | 1.98 | 7.18 | 1.81 | 36.32 | 54.69 | 39.91 | 1480 |
| Upper Freeport, Pa. and O.... | 72.65 | 4.82 | 7.26 | 1.34 | 2.89 | 9.10 | 1.93 | 37.35 | 51.63 | 41.98 | 14755 |
| Mahoning, O.... | 71.13 | 4.56 | 7.17 | 1.23 | 1.86 | 10.90 | 3.15 | 35.00 | 50.95 | 40.72 | 14728 |
| Jackson Co., O.... | 70.72 | 4.45 | 10.82 | 1.47 | 1.13 | 3.25 | 8.17 | 35.79 | 52.78 | 40.41 | 14141 |
| Hocking Valley, O..... | 68.03 | 4.97 | 9.87 | 1.44 | 1.59 | 8.00 | 6.59 | 35.77 | 49.64 | 41.84 | 14040 |

* Per lb. of combustible, by the Mahler calorimeter. The average figures calculated from the ultimate analyses agreed within 0.5%, except in the case of the Jackson Co. coal in which the calorimetric result was 1.6% higher than that computed from the analysis.

Sizes of Anthracite Coal. — When anthracite is mined it is crushed in a "breaker," and passed over screens separating it into different sizes, which are named as follows:

Lump, passes over bars set $3\frac{1}{2}$ to 5 in. apart; steamboat, over $3\frac{1}{2}$ in. and out of screen; broken, through $3\frac{1}{2}$ in., over $2\frac{3}{4}$ in.; egg, $2\frac{3}{4}$ to 2 in.; stove, 2 to $1\frac{3}{8}$ in.; chestnut, $1\frac{3}{8}$ to $\frac{3}{4}$ in.; pea, $\frac{3}{4}$ to $\frac{1}{2}$ in.; buckwheat, $\frac{1}{2}$ to $\frac{3}{8}$ in.; rice, $\frac{3}{8}$ to $\frac{3}{16}$ in.; culm, through $\frac{3}{16}$ in.

When coal is screened into sizes for shipment the purity of the different sizes as regards ash varies greatly. Samples from one mine gave results as follows:

| Name of Coal. | Screened. | | Analyses. | |
|----------------|-----------------|--------------|---------------|-------|
| | Through Inches. | Over Inches. | Fixed Carbon. | Ash. |
| Egg..... | 2.5 | 1.75 | 88.49 | 5.66 |
| Stove..... | 1.75 | 1.25 | 83.67 | 10.17 |
| Chestnut..... | 1.25 | 0.75 | 80.72 | 12.67 |
| Pea..... | 0.75 | 0.50 | 79.05 | 14.66 |
| Buckwheat..... | 0.50 | 0.25 | 76.92 | 16.62 |

Space Occupied by Anthracite Coal. (*J. C. I. W.*, vol. iii.)—The cubic contents of 2240 lbs. of hard Lehigh coal is a little over 36 feet; an average Schuylkill white-ash, 37 to 38 feet; Shamokin, 38 to 39 feet; Lorberrry, nearly 41.

According to measurements made with Wilkesbarre anthracite coal from the Wyoming Valley, it requires 32.2 cu. ft. of lump, 33.9 cu. ft. broken, 34.5 cu. ft. egg, 34.8 cu. ft. of stove, 35.7 cu. ft. of chestnut, and 36.7 cu. ft. of pea, to make one ton of coal of 2240 lbs.; while it requires 28.8 cu. ft. of lump, 30.3 cu. ft. of broken, 30.8 cu. ft. of egg, 31.1 cu. ft. of stove, 31.9 cu. ft. of chestnut, and 32.8 cu. ft. of pea, to make one ton of 2000 lbs.

Bernice Basin, Pa., Coals.

| | Water. | Vol. H.C. | Fixed C. | Ash. | Sulphur. |
|-------------------------|--------|-----------|----------|------|----------|
| Bernice Basin, Sullivan | 0.96 | 3.56 | 82.52 | 3.27 | 0.24 |
| and Lycoming Cos.; | to | to | to | to | to |
| range of 8..... | 1.97 | 8.56 | 89.39 | 9.34 | 1.04 |

This coal is on the dividing-line between the anthracites and semi-anthracites, and is similar to the coal of the Lykens Valley district.

More recent analyses (*Trans. A. I. M. E.*, xiv. 721) give:

| | Water. | Vol. H.C. | Fixed Carb. | Ash. | Sulphur. |
|------------------------|--------|-----------|-------------|------|----------|
| Working seam..... | 0.65 | 9.40 | 83.69 | 5.34 | 0.91 |
| 60 ft. below seam..... | 3.67 | 15.42 | 71.34 | 8.97 | 0.59 |

The first is a semi-anthracite, the second a semi-bituminous.

Connellsville Coal and Coke. (*Trans. A. I. M. E.*, xiii. 332.) — The Connellsville coal-field, in the southwestern part of Pennsylvania, is a strip about 3 miles wide and 60 miles in length. The mine workings are confined to the Pittsburgh seam, which here has its best development as to size, and its quality best adapted to coke-making. It generally affords from 7 to 8 feet of coal.

The following analyses by T. T. Morrell show about its range of composition:

| | Moisture. | Vol. Mat. | Fixed C. | Ash. | Sulphur. | Phosph's. |
|------------------|-----------|-----------|----------|------|----------|-----------|
| Herold Mine..... | 1.26 | 28.83 | 60.79 | 8.44 | 0.67 | 0.013 |
| Kintz Mine..... | 0.79 | 31.91 | 56.46 | 9.52 | 1.32 | 0.02 |

In comparing the composition of coals across the Appalachian field, in the western section of Pennsylvania, it will be noted that the Connellsville variety occupies a peculiar position between the rather dry semi-bituminous coals eastward of it and the fat bituminous coals flanking it on the west.

Beneath the Connellsville or Pittsburgh coal-bed occurs an interval of from 400 to 600 feet of "barren measures," separating it from the lower productive coal-measures of Western Pennsylvania. The following tables show the great similarity in composition in the coals of these upper and lower coal-measures in the same geographical belt or basin.

Analyses from the Upper Coal-measures in a Westward Order.

| Localities. | Moisture. | Vol. Mat. | Fixed Carb. | Ash. | Sulphur. |
|------------------------|-----------|-----------|-------------|------|----------|
| Anthracite..... | 1.35 | 3.45 | 89.06 | 5.81 | 0.30 |
| Cumberland, Md..... | 0.89 | 15.52 | 74.28 | 9.29 | 0.71 |
| Salisbury, Pa..... | 1.66 | 22.35 | 68.77 | 5.96 | 1.24 |
| Connellsville, Pa..... | | 31.38 | 60.30 | 7.24 | 1.09 |
| Greensburg, Pa..... | 1.02 | 33.50 | 61.34 | 3.28 | 0.86 |
| Irwin's, Pa..... | 1.41 | 37.66 | 54.44 | 5.86 | 0.64 |

Analyses from the Lower Coal-measures in a Westward Order.

| Localities. | Moisture. | Vol. Mat. | Fixed Carb. | Ash. | Sulphur. |
|-------------------|-----------|-----------|-------------|------|----------|
| Anthracite..... | 1.35 | 3.45 | 89.06 | 5.81 | 0.30 |
| Broad Top..... | 0.77 | 18.18 | 73.34 | 6.69 | 1.02 |
| Bennington..... | 1.40 | 27.23 | 61.84 | 6.93 | 2.60 |
| Johnstown..... | 1.18 | 16.54 | 74.46 | 5.96 | 1.86 |
| Blairsville..... | 0.92 | 24.36 | 62.22 | 7.69 | 4.92 |
| Armstrong Co..... | 0.96 | 38.20 | 52.03 | 5.14 | 3.66 |

Analyses of Southern Coals.

| | Moisture. | Vol. Mat. | Fixed C. | Ash. | Sulphur. |
|---|------------------------|--------------------|--------------------------|---------------|--------------|
| VIRGINIA AND KENTUCKY. | | | | | |
| Big Stone Gap Field,* 9 analyses, range | { from 0.80 to 2.01 | 31.44 36.27 | 54.80 63.50 | 1.73 8.25 | 0.56 1.72 |
| KENTUCKY. | | | | | |
| Pulaski Co., 3 analyses, range | { from 1.26 to 1.32 | 35.15 39.44 | 60.85 52.48 | 1.23 5.52 | 0.40 1.00 |
| Muhlenberg Co., 4 analyses, range | { from 3.60 to 7.06 | 30.60 38.70 | 58.80 53.70 | 3.40 6.50 | 0.79 3.16 |
| Pike Co., Eastern Ky., 37 analyses, range | { from 1.80 to 1.60 | 26.80 41.00 | 67.60 50.37 | 3.80 7.80 | 0.97 0.03 |
| Kentucky Cannel Coals, 5 analyses, range..... | { from to | 40.20 † 66.30 † | 59.80 coke 33.70 coke | 8.81 4.80 | 0.96 1.32 |
| TENNESSEE. | | | | | |
| Scott Co., range of several †. | { from 0.70 to 1.83 | 32.33 41.29 | 46.61 61.66 | 16.94 1.11 | 3.37 0.77 |
| Roane Co., Rockwood..... | 1.75 | 26.62 | 60.11 | 11.52 | 1.49 |
| Hamilton Co., Melville..... | 2.74 | 26.50 | 67.08 | 3.68 | 0.91 |
| Marion Co., Etna..... | 0.94 | 23.72 | 63.94 | 11.40 | 1.19 |
| Sewanee Co., Tracy City.... | 1.60 | 29.30 | 61.00 | 7.80 | |
| Kelly Co., Whiteside..... | 1.30 | 21.80 | 74.20 | 2.70 | |
| GEORGIA. | | | | | |
| Dade Co..... | 1.20 | 23.05 | 60.50 | 15.16 | 0.84 |
| ALABAMA. | | | | | |
| Warren Field: | | | | | |
| Jefferson Co., Birmingham | 3.01 | 42.76 | 48.30 | 3.21 | 2.72 |
| Jefferson Co., Black Creek. | 0.12 | 26.11 | 71.64 | 2.03 | 0.10 |
| Tuscaloosa Co..... | 1.59 | 38.33 | 54.64 | 5.45 | 1.33 |
| Cahaba Field, } Helena Vein | 2.00 | 32.90 | 53.08 | 11.34 | 0.68 |
| Bibb Co..... } Coke Vein.. | 1.78 | 30.60 | 66.58 | 1.09 | 0.04 |

* This field covers about 120 square miles in Virginia, and about 30 square miles in Kentucky.

† Volatile matter including moisture.

‡ Single analyses from Morgan, Rhea, Anderson, and Roane counties fall within this range.

Analyses of Southern Coals — Continued.

| | Moisture. | Vol. Mat. | Fixed C. | Ash. | Sulphur. |
|----------------------------|-----------|-----------|----------|-------|----------|
| TEXAS. | | | | | |
| Eagle Mine..... | 3.54 | 30.84 | 50.69 | 14.93 | |
| Sabinas Field, Vein I..... | 1.91 | 20.04 | 62.71 | 15.35 | |
| “ “ “ II..... | 1.37 | 16.42 | 68.18 | 13.02 | |
| “ “ “ III..... | 0.84 | 29.35 | 50.18 | 19.63 | |
| “ “ “ IV..... | 0.45 | 21.6 | 45.75 | 29.1 | 3.15 |

Indiana Coals. (J. S. Alexander, *Trans. A. I. M. E.*, iv. 100.) — The typical block coal of the Brazil (Indiana) district differs in chemical composition but little from the coking coals of Western Pennsylvania. The physical difference, however, is quite marked; the latter has a cuboid structure made up of bituminous particles lying against each other, so that under the action of heat fusion throughout the mass readily takes place, while block coal is formed of alternate layers of rich bituminous matter and a charcoal-like substance, which is not only very slow of combustion, but so retards the transmission of heat that agglutination is prevented, and the coal burns away layer by layer, retaining its form until consumed.

An ultimate analysis of block coal from Sand Creek by E. T. Cox gave: C, 72.94; H, 4.50; O, 11.77; N, 1.79; ash, 4.50; moisture, 4.50.

Analyses of other Indiana coals are given below.

| | Moisture. | Vol. Mat. | Fixed C. | Ash. |
|----------------------|-----------|-----------|----------|------|
| Caking Coals. | | | | |
| Parke Co..... | 4.50 | 45.50 | 45.50 | 4.50 |
| Sullivan Co..... | 2.35 | 45.25 | 51.60 | 0.80 |
| Clay Co..... | 7.00 | 39.70 | 47.30 | 6.00 |
| Spencer Co..... | 3.50 | 45.00 | 46.00 | 2.50 |
| Block Coals. | | | | |
| Clay Co..... | 8.50 | 31.00 | 57.50 | 3.00 |
| Martin Co..... | 2.50 | 44.75 | 51.25 | 1.50 |
| Daviess Co..... | 5.50 | 36.00 | 53.50 | 5.00 |

Illinois Coals. The Illinois coals are generally high in moisture, volatile matter, ash and sulphur, and the volatile matter is high in oxygen; consequently the coals are low in heating value. The range of quality is a wide one. The Big Muddy coal of Jackson Co., which has a high reputation as a steam coal in the St. Louis market, has about 36% of volatile matter in the combustible, while a coal from Staunton, Macoupin Co., tested by the author in 1883 (*Trans. A. S. M. E.*, v. 266) had 68%. A boiler test with this coal gave only 6.19 lbs. of water evaporated from and at 212° per lb. of combustible, in the same boiler that had given 9.88 lbs. with Jackson, O., nut.

Prof. S. W. Parr, in Bulletin No. 3 of the Ill. State Geol. Survey, 1906, reports the analyses and calorimetric tests of 150 Illinois coals. The two having the lowest and the highest value per pound of combustible have the following analysis:

| | Air-dried Coal. | | | | | Pure Coal. | | |
|-----------|-----------------|------|-------|----------|------|------------|----------|----------------|
| | Moist. | Ash. | Vol. | Fixed C. | S. | Vol. | Fixed C. | B.T.U. per lb. |
| Lowest.. | 9.90 | 5.02 | 40.75 | 44.33 | 2.00 | 47.90 | 52.10 | 12,162 |
| Highest.. | 5.68 | 8.90 | 33.32 | 52.10 | 1.18 | 39.02 | 60.98 | 14,830 |

The poorest coal of the series had a heating value of only 8645 B.T.U. per lb., air dry; it contained 9.70 moisture and 31.18 ash, and the B.T.U. per lb. combustible was 14,623. The best coal had a heating value of 13,303 per lb.; moisture 4.20, ash 5.50, B.T.U. per lb. combustible, 14,734.

Of the 156 coals, 28 gave between 14,500 and 14,830 B.T.U. per lb. combustible; 82 between 14,000 and 14,500; 32 between 13,500 and 14,000; 6 between 13,000 and 13,500; one 12,535 and one 12,162. The average is about 14,200. The volatile matter ranged from 36.24% to 53.80% of the combustible; the sulphur from 0.62 to 4.96%; the ash from 2.32 to 31.18%, and the moisture from 3.28 to 12.74%, all calculated from the air-dried samples. The moisture in the coal as mined is not stated, but was no doubt considerably higher. The author has found over 14% moisture in a lump of Illinois coal that was apparently dry, having been exposed to air, under cover, for more than a month.

Colorado Coals. — The Colorado coals are of extremely variable composition, ranging all the way from lignite to anthracite. G. C. Hewitt (*Trans. A. I. M. E.*, xvii. 377) says: The coal seams, where unchanged by heat and flexure, carry a lignite containing from 5% to 20% of water. In the southeastern corner of the field the same have been metamorphosed so that in four miles the same seams are an anthracite, coking, and dry coal. The dry seams also present wide chemical and physical changes in short distances. A soft and loosely bedded coal has in a hundred feet become compact and hard without the intervention of a fault. A couple of hundred feet has reduced the water of combination from 12% to 5%.

Western Arkansas and Oklahoma, (formerly Indian Territory). (H. M. Chance, *Trans. A. I. M. E.*, 1890.) — The western Arkansas coals are dry semi-bituminous or semi-anthracitic coals, mostly non-coking, or with quite feeble coking properties, ranging from 14% to 16% in volatile matter, the highest percentage yet found, according to Mr. Winslow's Arkansas report, being 17.655.

In the Mitchell basin, about 10 miles west from the Arkansas line, the coal shows 19% volatile matter; the Mayberry coal, about 8 miles farther west, contains 23%; and the Bryan Mine coal, about the same distance west, shows 26%. About 30 miles farther west, the coal shows from 38% to 41½% volatile matter, which is also about the percentage in coals of the McAlester and Lehigh districts.

Western Lignites. — The ultimate analyses of some lignites from Utah, Wyoming, Oregon and Alaska are reported by R. W. Raymond in *Trans. A. I. M. E.*, vol. ii. 1873. The range of the analyses is as follows: C, 55.79 to 69.84; H, 3.26 to 5.08; O, 9.54 to 21.82; N, 0.42 to 1.93; S, 0.63 to 3.92; moisture, 3.08 to 16.52; ash, 1.68 to 9.28. The heating value in B.T.U. per lb. combustible, calculated by Dulong's formula, ranges from 10,090 to 13,970.

Analyses of Foreign Coals. (Selected from D. L. Barnes's paper on American Locomotive Practice, *Trans. A. S. C. E.*, 1893.)

| | Volatile. Matter. | Fixed Carbon. | Ash. | | Volatile. Matter. | Fixed Carbon. | Ash. |
|------------------|----------------------|------------------|------|---------------------|----------------------|------------------|-------|
| Great Britain: | | | | South America: | | | |
| South-Wales..... | 8.5 | 88.3 | 3.2 | Chili, Chiroqui.... | 24.11 | 38.98 | 36.91 |
| South-Wales..... | 6.2 | 92.3 | 1.5 | Patagonia..... | 24.35 | 62.25 | 13.4 |
| Lancashire, Eng. | 17.2 | 80.1 | 2.7 | Brazil..... | 40.5 | 57.9 | 1.6 |
| Derbyshire, " | 17.7 | 79.9 | 2.4 | Canada: | | | |
| Durham, " * | 15.05 | 86.8 | 1.1 | Nova Scotia..... | 26.8 | 60.7 | 12.5 |
| Staffordshire, " | 20.4 | 78.6 | 1.0 | Cape Breton..... | 26.9 | 67.6 | 5.5 |
| Scotland†..... | 17.1 | 63.1 | 19.8 | Australia. | | | |
| Scotland†..... | 17.5 | 80.1 | 2.4 | Lignite..... | 15.8 | 64.3 | 10.0 |
| South America: | | | | Sydney, N. S. W. | 14.98 | 82.39 | 2.04 |
| Chili..... | 21.93 | 70.55 | 7.52 | Borneo..... | 26.5 | 70.3 | 14.2 |
| | | | | Tasmania..... | 6.16 | 63.4 | 30.45 |

* Semi-bit. coking coal.

† Boghead cannel gas coal.

‡ Semi-bit. steam-coal.

An analysis of Pictou, N. S., coal, in *Trans. A. I. M. E.*, xiv. 560, is: vol., 29.63; carbon, 56.98; ash, 13.39; and one of Sydney, Cape Breton, coal is: vol., 34.07; carbon, 61.43; ash, 4.50.

Sampling Coal for Analysis.—J. P. Kimball, *Trans. A. I. M. E.*, xii. 317, says: The unsuitable sampling of a coal-seam, or the improper preparation of the sample in the laboratory, often gives rise to errors in determinations of the ash so wide in range as to vitiate the analysis for all practical purposes; every other single determination, excepting moisture, showing its relative part of the error. The determinations of sulphur and ash are especially liable to error, as they are intimately associated in the slates.

Wm. Forsyth, in his paper on The Heating Value of Western Coals (*Eng'g News*, Jan. 17, 1895), says: This trouble in getting a fairly average sample of anthracite coal has compelled the Reading R. R. Co., in getting its samples, to take as much as 300 lbs. for one sample, drawn direct from the chutes, as it stands ready for shipment.

The directions for collecting samples of coal for analysis at the C., B. & Q. laboratory are as follows:

Two samples should be taken, one marked "average," the other "select." Each sample should contain about 10 lbs., made up of lumps about the size of an orange taken from different parts of the dump or car, and so selected that they shall represent as nearly as possible, first, the average lot; second, the best coal.

An example of the difference between an "average" and a "select" sample, taken from Mr. Forsyth's paper, is the following of an Illinois coal:

| | Moisture. | Vol. Mat. | Fixed Carbon. | Ash. |
|--------------|-----------|-----------|---------------|-------|
| Average..... | 1.36 | 27.69 | 35.41 | 35.54 |
| Select..... | 1.90 | 34.70 | 48.23 | 15.17 |

The theoretical evaporative power of the former was 9.13 lbs. of water from and at 212° per lb. of coal, and that of the latter 11.44 lbs.

RELATIVE VALUE OF STEAM COALS.

The heating value of a coal may be determined, with more or less approximation to accuracy, by three different methods.

1st, by chemical analysis; 2d, by combustion in a coal calorimeter; 3d, by actual trial in a steam-boiler.

The accuracy of the first two methods depends on the precision of the method of analysis or calorimetry adopted, and upon the care and skill of the operator. The results of the third method are subject to numerous sources of variation and error, and may be taken as approximately true only for the particular conditions under which the test is made. Analysis and calorimetry give with considerable accuracy the heating value which may be obtained under the conditions of perfect combustion and complete absorption of the heat produced. A boiler test gives the actual result under conditions of more or less imperfect combustion, and of numerous and variable wastes. It may give the highest practical heating value, if the conditions of grate-bars, draft, extent of heating surface, method of firing, etc., are the best possible for the particular coal tested, and it may give results far beneath the highest if these conditions are adverse or unsuitable to the coal.

In a paper entitled Proposed Apparatus for Determining the Heating Power of Different Coals (*Trans. A. I. M. E.*, xiv. 727) the author described and illustrated an apparatus designed to test fuel on a large scale, avoiding the errors of a steam-boiler test. It consists of a fire-brick furnace enclosed in a water casing, and two cylindrical shells containing a great number of tubes, which are surrounded by cooling water and through which the gases of combustion pass while being cooled. No steam is generated in the apparatus, but water is passed through it and allowed to escape at a temperature below 200° F. The product of the weight of the water passed through the apparatus by its increase in temperature is the measure of the heating value of the fuel.

A study of M. Mahler's calorimetric tests shows that the maximum difference between the results of these tests and the calculated heating power by Dulong's law in any single case is only a little over 3%, and the results of 31 tests show that Dulong's formula gives an average of

only 47 thermal units less than the calorimetric tests, the average total heating value being over 14,000 B.T.U., a difference of less than 0.4%.*

The close agreement of the results of calorimetric tests when properly conducted, and of the heating power calculated from the ultimate chemical analysis, indicates that either the chemical or the calorimetric method may be accepted as correct enough for all practical purposes for determining the total heating power of coal. The results obtained by either method may be taken as a standard by which the results of a boiler test are to be compared, and the difference between the total heating power and the result of the boiler test is a measure of the inefficiency of the boiler under the conditions of any particular test.

The heating value that can be obtained in boiler practice from any given coal depends upon the efficiency of the boiler, and this largely upon the difficulty of thoroughly burning the volatile combustible matter in the boiler furnace.

With the best anthracite coal, in which the combustible portion is, say, 97% fixed carbon and 3% volatile matter, the highest result that can be expected in a boiler-test with all conditions favorable is 12.2 lbs. of water evaporated from and at 212° per lb. of combustible, which is 79% of 15.47 lbs., the theoretical heating-power. With the best semibituminous coals, such as Cumberland and Pocahontas, in which the fixed carbon is 80% of the total combustible, 12.5 lbs., or 76% of the theoretical 16.4 lbs., may be obtained. For Pittsburgh coal, with a fixed carbon ratio of 68%, 11 lbs., or 69% of the theoretical 16.03 lbs., is about the best practically obtainable with the best boilers when hand-fired, with ordinary furnaces. (The author has obtained 78% with an automatic stoker set in a "Dutch oven" furnace.) With some good Ohio coals, with a fixed carbon ratio of 60%, 10 lbs., or 66% of the theoretical 15.28 lbs., has been obtained, under favorable conditions, with a fire-brick arch over the furnace. With coals mined west of Ohio, with lower carbon ratios, the boiler efficiency is not apt to be as high as 60% unless a special furnace, adapted to the coal, is used.

From these figures a table of probable maximum boiler-test results with ordinary furnaces from coals of different fixed carbon ratios may be constructed as follows:

| | | | | | | |
|--|------|------|----|----|-----|-----|
| Fixed carbon ratio..... | 97 | 80 | 68 | 60 | 54 | 50 |
| Evap. from and at 212° per lb. combustible, maximum in boiler-tests: | | | | | | |
| | 12.2 | 12.5 | 11 | 10 | 8.3 | 7.0 |
| Boiler efficiency, per cent..... | 80 | 76 | 69 | 66 | 60 | 55 |
| Loss, chimney, radiation, imperfect combustion, etc: | | | | | | |
| | 20 | 24 | 31 | 34 | 40 | 45 |

The difference between the loss of 20% with anthracite and the greater losses with the other coals is chiefly due to imperfect combustion of the bituminous coals, the more highly volatile coals sending up the chimney the greater quantity of smoke and unburned hydrocarbon gases. It is a measure of the inefficiency of the boiler furnace and of the inefficiency of heating-surface caused by the deposition of soot, the latter being primarily caused by the imperfection of the ordinary furnace and its unsuitability to the proper burning of bituminous coal. If in a boiler-test with an ordinary furnace lower results are obtained than those in the above table, it is an indication of unfavorable conditions, such as bad firing, wrong proportions of boiler, defective draft, a rate of driving beyond the capacity of the furnace, or beyond the capacity of the boiler to absorb the heat produced in the furnace. It is quite possible, however, with automatic stokers and fire-brick combustion chambers to obtain an efficiency of 70% with the highly volatile western coals.

* Mahler gives Dulong's formula with Berthelot's figure for the heating value of carbon, in British thermal units,

$$\text{Heating Power} = 14,650 C + 62,025 \left(H - \frac{(O + N) - 1}{8} \right).$$

The formula commonly used in the United States is $14,600 C + 62,000 (H - \frac{1}{8} O) + 4050 S$. For a description of the Mahler calorimeter and its method of operation see the author's "Steam Boiler Economy." Prof. S. W. Parr, of the University of Illinois, has put a calorimeter on the market which gives results practically equal to those obtained with Mahler's instrument.

Purchase of Coal under Specifications. — It is customary for large users of coal to purchase it under specifications of its analysis or heating value with a penalty attached for failure to meet the specifications. The following standards for a specification were given by the author in his "Steam Boiler Economy," 1901:

Anthracite and Semi-anthracite. — The standard is a coal containing 5% volatile matter, not over 2% moisture, and not over 10% ash. A premium of 1% on the price will be given for each per cent of volatile matter above 5% up to and including 15%, and a reduction of 2% on the price will be made for each 1% of moisture and ash above the standard.

Semi-bituminous and Bituminous. — The standard is a semi-bituminous coal containing not over 20% volatile matter, 2% moisture, 6% ash. A reduction of 1% in the price will be made for each 1% of volatile matter in excess of 25%, and of 2% for each 1% of ash and moisture in excess of the standard.

For western coals in which the volatile matter differs greatly in its percentage of oxygen, the above specification based on proximate analysis may not be sufficiently accurate, and it is well to introduce either the heating value as determined by a calorimeter or the percentage of oxygen. The author has proposed the following for Illinois coal:

The standard is one containing 14,500 B.T.U. per lb. of pure coal (coal free from moisture and ash), not over 6% moisture and 10% ash in an air-dried sample. For lower heating value per lb. of pure coal, the price shall be reduced proportionately, and for every 1% increase in ash or moisture above the specified figures, 2% on the price shall be deducted.

Several departments of the U. S. government now purchase coal under specifications. See paper on the subject by D. T. Randall, Bulletin No. 339, U. S. Geological Survey, 1908.

Evaporative Power of Bituminous Coals.

(Tests with Babcock & Wilcox Boilers, *Trans. A. S. M. E.*, iv. 267.)

| Name of Coal. | Duration of Test. | Grate Surface, sq. ft. | Heating Surface, sq. ft. | Percentage of Refuse. | Coal burned per sq. ft. of Grate, pounds. | Water evaporated per sq. ft. of Heating Surface per hour, pounds. | Water per pound Coal from and at 212°, lbs. | Water per pound Combustible from and at 212°. | Rated Horse-power. | Horse-power developed. |
|--|-------------------|------------------------|--------------------------|-----------------------|---|---|---|---|--------------------|------------------------|
| 1. Welsh..... | 13 1/2 hrs | 40 | 1679 | 7.5 | 6.3 | 2.07 | 11.53 | 12.46 | 146 | 96 |
| 2. Anthracite scr's 1/5. Semi-bit. 4/5. | 10 1/4 h | 60 | 3126 | 8.8 | 17.6 | 4.32 | 11.32 | 12.42 | 272 | 448 |
| 3. Pittsb'gh fine slack | 4 hrs | 33.7 | 1679 | 12.3 | 21.9 | 4.47 | 8.12 | 9.29 | 146 | 250 |
| " 3d Pool lump | 10 " | 43.5 | 2760 | 4.8 | 27.5 | 4.76 | 10.47 | 11.00 | 240 | 419 |
| 4. Castle Shannon, nr. Pittsb'gh, 3/8 nut, 5/8 lump. | 42 1/4 h | 69.1 | 4784 | 10.5 | 27.9 | 4.13 | 10.00 | 11.17 | 416 | 570 |
| 5. Ill. "run of mine". | 6 days | | 1196 | | | 1.41 | 9.49 | | 104 | 54 |
| " Ind. block..... | 3 days | | 1196 | | | 2.95 | 9.47 | | 104 | 111 |
| 6. Jackson, O., nut.... | 8 hrs. | 48 | 3358 | 9.6 | 32.1 | 4.11 | 8.93 | 9.88 | 292 | 460 |
| " Staunton, Ill., nut.. | 8 " | 60 | 3358 | 17.7 | 25.1 | 2.27 | 5.09 | 6.19 | 292 | 246 |
| 7. Renton screenings.. | 5 h 50 m | 21.2 | 1564 | 13.8 | 31.5 | 2.95 | 6.88 | 7.98 | 136 | 151 |
| " Wellington scr'gs... | 6 h 30 m | 21.2 | 1564 | 18.3 | 27 | 2.93 | 7.89 | 9.66 | 136 | 150 |
| " Black Diam. scr'gs.. | 5 h 58 m | 21.2 | 1564 | 19.3 | 36.4 | 3.11 | 6.29 | 7.80 | 136 | 160 |
| " Seattle screenings.. | 6 h 24 m | 21.2 | 1564 | 13.4 | 31.3 | 2.91 | 6.86 | 7.92 | 136 | 150 |
| " Wellington lump.... | 6 h 19 m | 21.2 | 1564 | 13.8 | 28.2 | 3.52 | 9.02 | 10.46 | 136 | 171 |
| " Cardiff lump..... | 6 h 47 m | 21.2 | 1564 | 11.7 | 26.7 | 3.69 | 10.07 | 11.40 | 136 | 189 |
| " " | 7 h 23 m | 21.2 | 1564 | 19.1 | 25.6 | 3.35 | 9.62 | 11.89 | 136 | 174 |
| " South Paine lump.. | 6 h 35 m | 21.2 | 1564 | 13.9 | 28.9 | 3.53 | 8.95 | 10.41 | 136 | 182 |
| " Seattle lump..... | 6 h 5 m | 21.2 | 1564 | 9.5 | 34.1 | 3.57 | 7.68 | 8.49 | 136 | 184 |

Place of Test: 1. London, England; 2. Peacedale, R. I.; 3. Cincinnati; 4. Pittsburgh; 5. Chicago; 6. Springfield, O.; 7. San Francisco.

In all the above tests the furnace was supplied with a fire-brick arch for preventing the radiation of heat from the coal directly to the boiler.

Weathering of Coal. (I. P. Kimball, *Trans. A. I. M. E.*, viii. 204.) — The effect of the weathering of coal, while sometimes increasing its weight, is to diminish the carbon and disposable hydrogen and to increase the oxygen and indisposable hydrogen. Hence a reduction in the calorific value. An excess of pyrites in coal tends to produce rapid oxidation and mechanical disintegration of the mass, with development of heat, loss of coking power, and spontaneous ignition.

The only appreciable results of the weathering of anthracite are confined to the oxidation of its accessory pyrites. In coking coals, however, weathering reduces and finally destroys the coking power.

Richters found that at a temperature of 158° to 180° Fahr., three coals lost in fourteen days an average of 3.6% of calorific power. It appears from the experiments of Richters and Rader that when there is no rise of temperature of coal piled in heaps and exposed to the air for nine to twelve months, it undergoes no sensible change, but when the coal becomes heated it suffers loss of C and H by oxidation and increases in weight by the fixation of oxygen. (See also paper by R. P. Rothwell, *Trans. A. I. M. E.*, iv. 55.)

Experiments by S. W. Parr and N. D. Hamilton (Bull. No. 17 of Univ'y of Ill. Eng'g Experiment Station, 1907) on samples of about 100 lbs. each, show that no appreciable change takes place in coal submerged in water. Their conclusions are:

(a) Submerged coal does not lose appreciably in heat value.

(b) Outdoor exposure results in a loss of heating value varying from 2 to 10 per cent.

(c) Dry storage has no advantage over storage in the open except with high sulphur coals, where the disintegrating effect of sulphur in the process of oxidation facilitates the escape or oxidation of the hydrocarbons.

(d) In most cases the losses in storage appear to be practically complete at the end of five months. From the seventh to the ninth month the loss is inappreciable.

This paper contains also a historical review of the literature on weathering and on spontaneous combustion, with a summary of the opinions of various authorities.

Later experiments on storing carload lots of Illinois coals (W. F. Wheeler, *Trans. A. I. M. E.*, 1908) confirm the above conclusions, except that 4 per cent seems to be amply sufficient to cover the losses sustained by Illinois coals under regular storage-conditions, the larger losses indicated in the former series being probably due to the small size of the samples exposed. In these latter tests, the losses sustained by the submerged coal, though small in amount, are only slightly less than those indicated for the exposed coal. Screenings and 3-in. nut coal from three mines were stored outdoors, under cover and under water. The average loss in heating value at the end of one week was 0.8%, at the end of two months 1.3%, and at the end of six months 2.0%. Pillar coal exposed underground from 22 to 27 years showed less than 3% loss in heating value as compared with fresh face coal from the same mines.

An extreme case of weathering was found in coal taken from near an outcrop that had been covered with soil and forest. The coal in this case had become so changed as to appear nearly like lignite, and the analysis shows a corresponding resemblance. The dry coal analysis of the outcrop coal, as compared with fresh face coal 300 ft. from the outcrop, is as follows:

| | Ash. | Vol. Mat. | Fixed C. | Sulphur. |
|---------------------|-------|-----------|----------|----------|
| Outcrop. | 16.86 | 39.27 | 43.87 | 0.85 |
| Fresh coal. | 16.25 | 40.72 | 43.03 | 3.91 |

The moisture in the outcrop coal was 29.81% and in the fresh coal 13.86%. The heating value of the ash-, water- and sulphur-free coal from the outcrop was 11,164 B.T.U. and that of the fresh coal 14,618 B.T.U.

Pressed Fuel. (E. F. Loiseau, *Trans. A. I. M. E.*, viii, 314.)—Pressed fuel has been made from anthracite dust by mixing the dust with ten per cent of its bulk of dry pitch, which is prepared by separating from tar at a temperature of 572° F. the volatile matter it contains. The mixture is kept heated by steam to 212° , at which temperature the pitch acquires its cementing properties, and is passed between two rollers, on the periphery of which are milled out a series of semi-oval cavities. The lumps of the mixture, about the size of an egg, drop out under the rollers on an endless belt which carries them to a screen in eight minutes, which time is sufficient to cool the lumps, and they are then ready for delivery.

The enterprise of making the pressed fuel above described was not commercially successful, on account of the low price of other coal. In France, however, "*briquettes*" are regularly made of coal-dust (bituminous and semi-bituminous).

Experiments with briquets for use in locomotives have been made by the Penna. R. R. Co., with favorable results, which were reported at the convention of the Am. Ry. Mast. Mechs. Assn. (*Eng. News*, July 2, 1908). A rate of evaporation as high as 19 lbs. per sq. ft. of heating surface per hour was reached. The comparative economy of raw coal and of briquets was as follows:

| Evap. per sq. ft. heat. surf. per hr., lbs | 8 | 10 | 12 | 14 | 16 |
|--|------|------|-----|-----|-----|
| Evap. from and at } Lloydell coal.... | 9.5 | 8.8 | 8.0 | 7.3 | 6.6 |
| 212° per lb. of fuel } Briquetted coal. | 10.7 | 10.2 | 9.7 | 9.2 | 8.7 |

The fuel consumed per draw-bar horse-power with the locomotive running at 37.8 miles per hour and a cut-off of 25% was: with raw coal, 4.48 lbs.; with round briquets, 3.65 lbs.

Experiments on different binders for briquets are discussed by J. E. Mills in Bulletin No. 343 of the U. S. Geological Survey, 1908.

The experiments show that, in general, where it can be obtained, the cheapest binder will be the heavy residuum from petroleum, often known to the trade as asphalt. Four per cent of this binder being sufficient, its cost ranges from 45 to 60 cts. per ton of briquets produced. This binder is available in California, Texas, and adjacent territory.

Second in order of importance comes water-gas tar pitch. Five to six per cent usually proving sufficient, the cost of this binder ranges from 50 to 60 cts. per ton of briquets. As water-gas pitch is also derived from petroleum, it will be available in oil-producing regions.

Third in order is coal-tar pitch. This binder is very widely available. From 6.5 to 8% will usually be required, and the cost ranges from 65 to 90 cts. per ton of briquets.

Other substances are also mentioned which may possibly be used for binders, such as asphalts and tars derived from wood distillation; pitch made from producer-gas tar; and magnesia. Starch and the waste sulphite liquor from paper mills may also be used, but the briquets made with them are not waterproof.

Briquetting tests made at the St. Louis exhibition, 1904, with descriptions of the machines used are reported in Bulletin No. 261 of the U. S. Geological Survey, 1905. See also paper on Coal Briquetting in the U. S., by E. W. Parker, *Trans. A. I. M. E.*, 1907.

COKE.

Coke is the solid material left after evaporating the volatile ingredients of coal, either by means of partial combustion in furnaces called coke ovens, or by distillation in the retorts of gas-works.

Coke made in ovens is preferred to gas coke as fuel. It is of a dark gray color, with slightly metallic luster, porous, brittle, and hard.

The proportion of coke yielded by a given weight of coal is very different for different kinds of coal, ranging from 0.9 to 0.35.

Being of a porous texture, it readily attracts and retains water from the atmosphere, and sometimes, if it is kept without proper shelter, from 0.15 to 0.20 of its gross weight consists of moisture.

Analyses of Coke.

(From report of John R. Procter, Kentucky Geological Survey.)

| Where Made. | | Fixed Carbon. | Ash. | Sulphur. |
|--------------------|------------------------------|---------------|-------|----------|
| Connellsville, Pa. | (Average of 3 samples) | 88.96 | 9.74 | 0.810 |
| Chattanooga, Tenn. | " " 4 " | 80.51 | 16.34 | 1.595 |
| Birmingham, Ala. | " " 4 " | 87.29 | 10.54 | 1.195 |
| Pocahontas, Va. | " " 3 " | 92.53 | 5.74 | 0.597 |
| New River, W. Va. | " " 8 " | 92.38 | 7.21 | 0.562 |
| Big Stone Gap, Ky. | " " 7 " | 93.23 | 5.69 | 0.749 |

Experiments in Coking. CONNELLSVILLE REGION.

(John Fulton, *Amer. Mfr.*, Feb. 10, 1893.)

| No. of Test. | Time in Oven. | Coal Charged. | Ash made. | Fine Coke made. | Market Coke made. | Total Coke made. | Per cent of Yield. | | | | Per Cent Lost. |
|--------------|---------------|---------------|-----------|-----------------|-------------------|------------------|--------------------|------------|--------------|-------------|----------------|
| | | | | | | | Ash. | Fine Coke. | Market Coke. | Total Coke. | |
| 1 | h. m. | lb. | lb. | lb. | lb. | lb. | | | | | |
| 1 | 67 00 | 12,420 | 99 | 385 | 7,518 | 7,903 | 00.80 | 3.10 | 60.53 | 63.63 | 35.57 |
| 2 | 68 00 | 11,090 | 90 | 359 | 6,580 | 6,939 | 00.81 | 3.24 | 59.33 | 62.57 | 36.62 |
| 3 | 45 00 | 9,120 | 77 | 272 | 5,418 | 5,690 | 00.84 | 2.98 | 59.41 | 62.39 | 36.77 |
| 4 | 45 00 | 9,020 | 74 | 349 | 5,334 | 5,683 | 00.82 | 3.87 | 59.13 | 63.00 | 36.18 |

These results show, in a general average, that Connellsville coal carefully coked in a modern beehive oven will yield 66.17% of marketable coke, 2.30% of small coke or breeze, and 0.82% of ash.

The total average loss in volatile matter expelled from the coal in coking amounts to 30.71%.

The beehive coke oven is 12 feet in diameter and 7 feet high at crown of dome. It is used in making 48 and 72 hour coke. [The Belgian type of beehive oven is rectangular in shape.]

In making these tests the coal was weighed as it was charged into the oven; the resultant marketable coke, small coke or breeze and ashes weighed dry as they were drawn from the oven.

Coal Washing. — In making coke from coals that are high in ash and sulphur, it is advisable to crush and wash the coal before coking it. A coal-washing plant at Brookwood, Ala., has a capacity of 50 tons per hour. The average percentage of ash in the coal during ten days' run varied from 14% to 21%, in the washed coal from 4.8% to 8.1%, and in the coke from 6.1% to 10.5%. During three months the average reduction of ash was 60.9%. (*Eng. and Mining Jour.*, March 25, 1893.)

An experiment on washing Missouri No. 3 slack coal is described in Bulletin No. 3 of the Engineering Experiment Station of Iowa State College, 1905. The raw coal analyzed: moisture, 14.37; ash, 28.39; sulphur, 4.30; and the washed coal, moisture, 23.90; ash, 7.59; sulphur, 2.89. Nearly 25% of the coal was lost in the operation.

Recovery of By-products in Coke Manufacture. — In Germany considerable progress has been made in the recovery of by-products. The Hoffman-Otto oven has been most largely used, its principal feature being that it is connected with regenerators. In 1884 40 ovens on this system were running, and in 1892 the number had increased to 1209.

A Hoffman-Otto oven in Westphalia takes a charge of 6¼ tons of dry coal and converts it into coke in 48 hours. The product of an oven annually is 1025 tons in the Ruhr district, 1170 tons in Silesia, and 960 tons in the Saar district. The yield from dry coal is 75% to 77% of coke, 2.5% to 3% of tar, and 1.1% to 1.2% of sulphate of ammonia in

the Ruhr district; 65% to 70% of coke, 4% to 4.5% of tar, and 1% to 1.25% of sulphate of ammonia in the Upper Silesia region, and 68% to 72% of coke, 4% to 4.3% of tar and 1.8% to 1.9% of sulphate of ammonia in the Saar district. A group of 60 Hoffman ovens, therefore, yields annually the following:

| District. | Coke, tons. | Tar, tons. | Sulphate Ammonia, tons. |
|--------------------|----------------|---------------|-------------------------------|
| Ruhr..... | 51,300 | 1860 | 780 |
| Upper Silesia..... | 48,000 | 3000 | 840 |
| Saar..... | 40,500 | 2400 | 492 |

An oven which has been introduced lately into Germany in connection with the recovery of by-products is the Semet-Solvay, which works hotter than the Hoffman-Otto, and for this reason 73% to 77% of gas coal can be mixed with 23% to 27% of coal low in volatile matter, and yet yield a good coke. Mixtures of this kind yield a larger percentage of coke, but, on the other hand, the amount of gas is lessened, and therefore the yield of tar and ammonia is not so great.

The yield of coke by the beehive and the retort ovens respectively is given as follows in a pamphlet of the Solvay Process Co.: Connellsville coal: beehive, 66%, retort, 73%; Pocahontas: beehive, 62%, retort, 83%; Alabama: beehive, 60%, retort, 74%. (See article in *Mineral Industry*, vol. viii. 1900.)

References: F. W. Luerman, Verein Deutscher Eisenhuettenleute 1891, *Iron Age*, March 31, 1892; *Amer. Mfr.*, April 28, 1893. An excellent series of articles on the manufacture of coke, by John Fulton, of Johnstown, Pa., is published in the *Colliery Engineer*, beginning in January, 1893.

Since the above was written, great progress in the introduction of coke ovens with by-product attachments has been made in the United States, especially by the Semet-Solvay Co., Syracuse, N. Y. See paper on The Development of the Modern By-product Coke-oven, by C. G. Atwater, *Trans. A. I. M. E.*, 1902.

Generation of Steam from Waste Heat and Gases of Coke-ovens. (Erskine Ramsey, *Amer. Mfr.*, Feb. 16, 1894.) — The gases from a number of adjoining ovens of the beehive type are led into a long horizontal flue, and thence to a combustion-chamber under a battery of boilers. Two plants are in satisfactory operation at Tracy City, Tenn., and two at Pratt Mines, Ala.

A Bushel of Coal. — The weight of a bushel of coal in Indiana is 70 lbs.; in Penna., 76 lbs.; in Ala., Colo., Ga., Ill., Ohio, Tenn., and W. Va., it is 80 lbs.

A Bushel of Coke is almost uniformly 40 lbs., but in exceptional cases, when the coal is very light, 38, 36, and 33 lbs. are regarded as a bushel, in others from 42 to 50 lbs. are given as the weight of a bushel; in this case the coke would be quite heavy.

Products of the Distillation of Coal. — S. P. Sadler's Handbook of Industrial Organic Chemistry gives a diagram showing over 50 chemical products that are derived from distillation of coal. The first derivatives are coal-gas, gas-liquor, coal-tar, and coke. From the gas-liquor are derived ammonia and sulphate, chloride and carbonate of ammonia. The coal-tar is split up into oils lighter than water or crude naphtha, oils heavier than water — otherwise dead oil or tar, commonly called creosote, — and pitch. From the two former are derived a variety of chemical products.

From the coal-tar there comes an almost endless chain of known combinations. The greatest industry based upon their use is the manufacture of dyes, and the enormous extent to which this has grown can be judged from the fact that there are over 600 different coal-tar colors in use, and many more which as yet are too expensive for this purpose. Many medicinal preparations come from the series, pitch for paving

purposes, and chemicals for the photographer, the rubber manufacturers and tanners, as well as for preserving timber and cloths.

The composition of the hydrocarbons in a soft coal is uncertain and quite complex; but the ultimate analysis of the average coal shows that it approaches quite nearly to the composition of CH_4 (marsh-gas). (W. H. Biauvelt, *Trans. A. I. M. E.*, xx. 625.)

WOOD AS FUEL.

Wood, when newly felled, contains a proportion of moisture which varies very much in different kinds and in different specimens, ranging between 30% and 50%, and being on an average about 40%. After 8 or 12 months' ordinary drying in the air the proportion of moisture is from 20 to 25%. This degree of dryness, or almost perfect dryness if required, can be produced by a few days' drying in an oven supplied with air at about 240° F. When coal or coke is used as the fuel for that oven, 1 lb. of fuel suffices to expel about 3 lbs. of moisture from the wood. This is the result of experiments on a large scale by Mr. J. R. Napier. If air-dried wood were used as fuel for the oven, from 2 to 2½ lbs. of wood would probably be required to produce the same effect.

The specific gravity of different kinds of wood ranges from 0.3 to 1.2.

Perfectly dry wood contains about 50% of carbon, the remainder consisting almost entirely of oxygen and hydrogen in the proportions which form water. The coniferous family contain a small quantity of turpentine, which is a hydrocarbon. The proportion of ash in wood is from 1% to 5%. The total heat of combustion of all kinds of wood, when dry, is almost exactly the same, and is that due to the 50% of carbon.

The above is from Rankine; but according to the table by S. P. Sharpless in *Jour. C. I. W.*, iv. 36, the ash varies from 0.03% to 1.20% in American woods, and the fuel value, instead of being the same for all woods, ranges from 3667 (for white oak) to 5546 calories (for long-leaf pine) = 6600 to 9883 British thermal units for dry wood, the fuel value of 0.50 lb. carbon being 7272 B. T. U.

Heating Value of Wood. — The following table is given in several books of reference, authority and quality of coal referred to not stated.

The weight of one cord of different woods (thoroughly air-dried) is about as follows:

| | lbs. | lbs. | |
|------------------------------|------|----------|--------------------------------|
| Hickory or hard maple . . . | 4500 | equal to | 1800 coal. (Others give 2000.) |
| White oak | 3850 | " | 1540 " (" 1715.) |
| Beech, red and black oak . . | 3250 | " | 1300 " (" 1450.) |
| Poplar, chestnut, and elm . | 2350 | " | 940 " (" 1050.) |
| The average pine | 2000 | " | 800 (" 925.) |

Referring to the figures in the last column, it is said:

From the above it is safe to assume that 2¼ lbs. of dry wood are equal to 1 lb. average quality of soft coal and that the full value of the same weight of different woods is very nearly the same — that is, a pound of hickory is worth no more for fuel than a pound of pine, assuming both to be dry. It is important that the wood be dry, as each 10% of water or moisture in wood will detract about 12% from its value as fuel.

Taking an average wood of the analysis C 51%, H 6.5%, O 42.0%, ash 0.5%, perfectly dry, its fuel value per pound, according to Dulong's formula, $V = \left[14,600 C + 62,000 \left(H - \frac{O}{8} \right) \right]$, is 8221 British thermal units. If the wood, as ordinarily dried in air, contains 25% of moisture, then the heating value of a pound of such wood is three quarters of 8221 = 6165 heat-units, less the heat required to heat and evaporate the ¼ lb. of water from the atmospheric temperature, and to heat the steam made from this water to the temperature of the chimney gases, say 150 heat-units per pound to heat the water to 212°, 970 units to evaporate it at that temperature, and 100 heat-units to raise the temperature of the steam to 420° F., or 1220 in all = 305 for ¼ lb., which subtracted from the 6165, leaves 5860-heat-units as the net fuel value of the wood per pound, or about 0.4 that of a pound of carbon.

Composition of Wood.

(Analysis of Woods, by M. Eugene Chevandier.)

| Woods. | Carbon. | Hydrogen. | Oxygen. | Nitrogen. | Ash. |
|--------------|---------|-----------|---------|-----------|-------|
| Beech..... | 49.36% | 6.01% | 42.69% | 0.91% | 1.06% |
| Oak..... | 49.64 | 5.92 | 41.16 | 1.29 | 1.97 |
| Birch..... | 50.20 | 6.20 | 41.62 | 1.15 | 0.81 |
| Poplar..... | 49.37 | 6.21 | 41.60 | 0.96 | 1.86 |
| Willow..... | 49.96 | 5.96 | 39.56 | 0.96 | 3.37 |
| Average..... | 49.70% | 6.06% | 41.30% | 1.05% | 1.80% |

The following table, prepared by M. Violette, shows the proportion of water expelled from wood at gradually increasing temperatures:

| Temperature. | Water Expelled from 100 Parts of Wood. | | | |
|----------------|--|-------|--------|---------|
| | Oak. | Ash. | Elm. | Walnut. |
| 257° Fahr..... | 15.26 | 14.78 | 15.32 | 15.55 |
| 302° Fahr..... | 17.93 | 16.19 | 17.02 | 17.43 |
| 347° Fahr..... | 32.13 | 21.22 | 36.94? | 21.00 |
| 392° Fahr..... | 35.80 | 27.51 | 33.38 | 41.77? |
| 437° Fahr..... | 44.31 | 33.38 | 40.56 | 36.56 |

The wood operated upon had been kept in store during two years. When wood which has been strongly dried by means of artificial heat is left exposed to the atmosphere, it reabsorbs about as much water as it contains in its air-dried state.

A cord of wood = $4 \times 4 \times 8 = 128$ cu. ft. About 56% solid wood and 44% interstitial spaces. (Marcus Bull, Phila., 1829. *J. C. I. W.*, vol. i., p. 293.)

B. E. Fernow gives the per cent. of solid wood in a cord as determined officially in Prussia (*J. C. I. W.*, vol. iii. p. 20):

Timber cords, 74.07% = 80 cu. ft. per cord;
 Firewood cords (over 6" diam.), 69.44% = 75 cu. ft. per cord;
 "Billet" cords (over 3" diam.), 55.55% = 60 cu. ft. per cord;
 "Brush" woods less than 3" diam., 18.52%; Roots, 37.00%.

CHARCOAL.

Charcoal is made by evaporating the volatile constituents of wood and peat, either by a partial combustion of a conical heap of the material to be charred, covered with a layer of earth, or by the combustion of a separate portion of fuel in a furnace, in which are placed retorts containing the material to be charred.

According to Peclet, 100 parts by weight of wood when charred in a heap yield from 17 to 22 parts by weight of charcoal, and when charred in a retort from 28 to 30 parts.

This has reference to the ordinary condition of the wood used in charcoal-making, in which 25 parts in 100 consist of moisture. Of the remaining 75 parts the carbon amounts to one half, or 37½% of the gross weight of the wood. Hence it appears that on an average nearly half of the carbon in the wood is lost during the partial combustion in a heap, and about one quarter during the distillation in a retort.

To char 100 parts by weight of wood in a retort, 121½ parts of wood must be burned in the furnace. Hence in this process the whole expenditure of wood to produce from 28 to 30 parts of charcoal is 112½ parts;

so that if the weight of charcoal obtained is compared with the whole weight of wood expended, its amount is from 25% to 27%; and the proportion lost is on an average $11\frac{1}{2} \div 37\frac{1}{2} = 0.3$, nearly.

According to Peclet, good wood charcoal contains about 0.07 of its weight of ash. The proportion of ash in peat charcoal is very variable and is estimated on an average at about 0.18. (Rankine.)

Much information concerning charcoal may be found in the Journal of the Charcoal-iron Workers' Assn., vols. i. to vi. From this source the following notes have been taken:

Yield of Charcoal from a Cord of Wood. — From 45 to 50 bushels to the cord in the kiln, and from 30 to 35 in the meller. Prof. Egleston in *Trans. A. I. M. E.*, viii. 395, says the yield from kilns in the Lake Champlain region is often from 50 to 60 bushels for hard wood and 50 for soft wood; the average is about 50 bushels.

The apparent yield per cord depends largely upon whether the cord is a full cord of 128 cu. ft. or not.

In a four months' test of a kiln at Goodrich, Tenn., Dr. H. M. Pierce found results as follows: Dimensions of kiln — inside diameter of base, 28 ft. 8 in.; diam. at spring of arch, 26 ft. 8 in.; height of walls, 8 ft.; rise of arch, 5 ft.; capacity, 30 cords. Highest yield of charcoal per cord of wood (measured) 59.27 bushels, lowest 50.14 bushels, average 53.65 bushels.

No. of charges 12, length of each turn or period from one charging to another 11 days. (*J. C. I. W.*, vol. vi., p. 26.)

Results from Different Methods of Charcoal-making.

| Coaling Methods. | Character of Wood Used. | Yield. | | Bushels of Charcoal per Cord of Wood. | Weight in Lbs. per Bushel of Charcoal. |
|---------------------------------------|---|---------------------|---------------------|---------------------------------------|--|
| | | In Volume per cent. | In Weight per cent. | | |
| Odelstjerna's experiments | Birch dried at 230 F..... | | 35.9 | | |
| Mathieu's retorts, fuel excluded..... | { Air dry, av. good yellow pine weighing | 77.0 | 28.3 | 63.4 | 15.7 |
| Mathieu's retorts, fuel included..... | { abt. 28 lbs. per cu. ft. | 65.8 | 24.2 | 54.2 | 15.7 |
| Swedish ovens, av. results | { Good dry fir and pine, mixed. | 81.0 | 27.7 | 66.7 | 13.3 |
| Swedish ovens, av. results | { Poor wood, mixed fir and pine. | 70.0 | 25.8 | 62.0 | 13.3 |
| Swedish meilers exceptional..... | { Fir and white-pine wood, mixed. Av. 25 lbs. per cu. ft. | 72.2 | 24.7 | 59.5 | 13.3 |
| Swedish meilers, av. results | { Av. good yellow pine weighing abt. 25 lbs. per cu. ft. | 52.5 | 18.3 | 43.9 | 13.3 |
| American kilns, av. results | { | 54.7 | 22.0 | 45.0 | 17.5 |
| American meilers, av. results..... | { | 42.9 | 17.1 | 35.0 | 17.5 |

Consumption of Charcoal in Blast-furnaces per Ton of Pig Iron; average consumption according to census of 1880, 1.14 tons charcoal per ton of pig. The consumption at the best furnaces is much below this average. As low as 0.853 ton, is recorded of the Morgan furnace; Bay furnace, 0.858; Elk Rapids, 0.884. (1892.)

Absorption of Water and of Gases by Charcoal. — Svedlius, in his hand-book for charcoal-burners, prepared for the Swedish Government, says: Fresh charcoal, also reheated charcoal, contains scarcely any water, but when cool it absorbs it very rapidly, so that, after twenty-four hours, it may contain 4% to 8% of water. After the lapse of a few weeks the moisture of charcoal may not increase perceptibly, and may be estimated at 10% to 15%, or an average of 12%. A thoroughly charred piece of charcoal ought, then, to contain about 84 parts carbon, 12 parts water, 3 parts ash, and 1 part hydrogen.

M. Saussure, operating with blocks of fine boxwood charcoal, freshly burnt, found that by simply placing such blocks in contact with certain gases they absorbed them in the following proportion:

| | Volumes. | | Volumes. |
|-----------------------------------|----------|--------------------------|----------|
| Ammonia..... | 90.00 | Carbonic oxide..... | 9.42 |
| Hydrochloric acid gas..... | 85.00 | Oxygen..... | 9.25 |
| Sulphurous acid..... | 65.00 | Nitrogen..... | 6.50 |
| Sulphuretted hydrogen..... | 55.00 | Carburetted hydrogen.... | 5.00 |
| Nitrous oxide (laughing-gas)..... | 40.00 | Hydrogen..... | 1.75 |
| Carbonic acid..... | 35.00 | | |

It is this enormous absorptive power that renders of so much value a comparatively slight sprinkling of charcoal over dead animal matter, as a preventive of the escape of odors arising from decomposition.

In a box or case containing one cubic foot of charcoal may be stored without mechanical compression a little over nine cubic feet of oxygen, representing a mechanical pressure of one hundred and twenty-six pounds to the square inch. From the store thus preserved the oxygen can be drawn by a small hand-pump.

Composition of Charcoal Produced at Various Temperatures. (By M. Violette.)

| | Temperature of Carbonization. | | Carbon. | Hydrogen. | Oxygen. | Nitrogen and Loss. | Ash. |
|---|-------------------------------|------------|---------|-----------|---------|--------------------|-------|
| 1 | 150° Cent. | 302° Fahr. | 47.51 | 6.12 | 46.29 | 0.08 | 47.51 |
| 2 | 200 | 392 | 51.82 | 3.99 | 43.98 | 0.23 | 39.88 |
| 3 | 250 | 482 | 65.59 | 4.81 | 28.97 | 0.63 | 32.98 |
| 4 | 300 | 592 | 73.24 | 4.25 | 21.96 | 0.57 | 24.61 |
| 5 | 350 | 662 | 76.64 | 4.14 | 18.44 | 0.61 | 22.42 |
| 6 | 432 | 810 | 81.64 | 4.96 | 15.24 | 1.61 | 15.40 |
| 7 | 1023 | 1873 | 81.97 | 2.30 | 14.15 | 1.60 | 15.30 |

The wood experimented on was that of black alder, or alder buckthorn, which furnishes a charcoal suitable for gunpowder. It was previously dried at 150 deg. C. = 302 deg. F.

MISCELLANEOUS SOLID FUELS.

Dust Fuel — Dust Explosions. — Dust when mixed in air burns with such extreme rapidity as in some cases to cause explosions. Explosions of flour-mills have been attributed to ignition of the dust in confined passages. Experiments in England in 1876 on the effect of coal-dust in carrying flame in mines showed that in a dusty passage the flame from a blown-out shot may travel 50 yards. Prof. F. A. Abel (*Trans. A. I. M. E.*, xiii. 260) says that coal-dust in mines much promotes and extends explosions, and that it may readily be brought into operation as a fiercely burning agent which will carry flame rapidly as far as its mixture with air extends, and will operate as an explosive agent though the medium of a very small proportion of fire-damp in the air of the mine. The explosive violence of the combustion of dust is largely due to the instantaneous heating and consequent expansion of the air. (See also paper on "Coal Dust as an Explosive Agent," by Dr. R. W. Raymond, *Trans. A. I. M. E.*, 1894.) Experiments made in Germany in 1893 show that pulverized fuel may be burned without smoke, and with high economy. The fuel, instead of being introduced into the fire-box in the ordinary manner, is first reduced to a powder by pulverizers of any construction. In the place of the ordinary boiler fire-box there is a combustion chamber in the form of a closed furnace lined with fire-brick and provided with an air-injector. The nozzle throws a constant stream of fuel into the chamber, scattering it throughout the whole space of the fire-box. When this powder is once ignited, and it is very readily done by first raising the lining to a high temperature by an open fire, the combustion continues in an intense and regular manner under the action of the current of air which carries it in. (*Mfrs. Record*, April, 1893.)

Records of tests with the Wegener powdered-coal apparatus, which is now (1900) in use in Germany, are given in *Eng. News*, Sept. 16, 1897. An illustrated description is given in the author's *Steam Boiler Economy*, p. 183. Coal-dust fuel is now extensively used in the United States in rotary kilns for burning Portland cement.

Powdered fuel was used in the Crompton rotary puddling-furnace at Woolwich Arsenal, England, in 1873. (*Jour. I. & S. I.*, i. 1873, p. 91.) Numerous experiments on the use of powdered fuel for steam boilers were made in the U. S. between 1895 and 1905, but they were not commercially successful.

Peat or Turf, as usually dried in the air, contains from 25% to 30% of water, which must be allowed for in estimating its heat of combustion. This water having been evaporated, the analysis of M. Regnault gives, in 100 parts of perfectly dry peat of the best quality: C 58%, H 6%, O 31%, Ash 5%. In some examples of peat the quantity of ash is greater, amounting to 7% and sometimes to 11%.

The specific gravity of peat in its ordinary state is about 0.4 or 0.5. It can be compressed by machinery to a much greater density. (Rankine.)

Clark (*Steam-engine*, i. 61) gives as the average composition of dried Irish peat: C 59%, H 6%, O 30%, N 1.25%, Ash 4%.

Applying Dulong's formula to this analysis, we obtain for the heating value of perfectly dry peat 10,260 heat-units per pound, and for air-dried peat containing 25% of moisture, after making allowance for evaporating the water, 7391 heat-units per pound.

A paper on Peat in the U. S., by M. R. Campbell, will be found in *Mineral Resources of the U. S.* (U. S. Geol. Survey) for 1905, p. 1319.

Sawdust as Fuel. — The heating power of sawdust is naturally the same per pound as that of the wood from which it is derived, but if allowed to get wet it is more like spent tan (which see below). The conditions necessary for burning sawdust are that plenty of room should be given it in the furnace, and sufficient air supplied on the surface of the mass. The same applies to shavings, refuse lumber, etc. Sawdust is frequently burned in saw-mills, etc., by being blown into the furnace by a fan-blast.

Wet Tan Bark as Fuel. — Tan, or oak bark, after having been used in the processes of tanning, is burned as fuel. The spent tan consists of the fibrous portion of the bark. Experiments by Prof. R. H. Thurston (*Jour. Frank. Inst.*, 1874) gave with the Crockett furnace, the wet tan containing 59% of water, an evaporation from and at 212° F. of 4.24 lbs. of water per pound of the wet tan, and with the Thompson furnace an evaporation of 3.19 lbs. per pound of wet tan containing 55% of water. The Thompson furnace consisted of six fire-brick ovens, each 9 ft. × 4 ft. 4 ins., containing 234 sq. ft. of grate in all, for three boilers with a total heating surface of 2000 sq. ft., a ratio of heating to grate surface of 9 to 1. The tan was fed through holes in the top. The Crockett furnace was an ordinary fire-brick furnace, 6 × 4 ft., built in front of the boiler, instead of under it, the ratio of heating surface to grate being 14.6 to 1. The conditions of success in burning wet fuel are the surrounding of the mass so completely with heated surfaces and with burning fuel that it may be rapidly dried, and then so arranging the apparatus that thorough combustion may be secured, and that the rapidity of combustion be precisely equal to and never exceed the rapidity of desiccation. Where this rapidity of combustion is exceeded the dry portion is consumed completely, leaving an uncovered mass of fuel which refuses to take fire.

D. M. Myers (*Trans. A.S.M.E.*, 1909) describes some experiments on tan as a boiler fuel. One hundred lbs. of air dried bark fed to the mill will produce 213 lbs. of spent tan containing 65% moisture. Taking 9500 B.T.U. as the heating value per lb. of dry tan and 500° F. as the temperature of the chimney gases, the available heat in 1 lb. of wet tan is 2665 B.T.U. Based on this value as much as 71% efficiency has been obtained in a boiler test with a special furnace, or 1.93 lbs. of water evaporated from and at 212° per lb. of wet tan.

Straw as Fuel. (*Eng'g Mechanics*, Feb., 1893, p. 55.) — Experiments in Russia showed that winter-wheat straw, dried at 230° F., had the following composition: C, 46.1; H, 5.6; N, 0.42; O, 43.7; Ash, 4.1. Heating value in British thermal units: dry straw, 6290; with 6% water, 5770; with 10% water, 5448. With straws of other grains the heating value of dry straw ranged from 5590 for buckwheat to 6750 for flax.

Clark (S. E., vol. 1, p. 62) gives the mean composition of wheat and barley straw as C, 36; H, 5; O, 38; N, 0.50; Ash, 4.75; water, 15.75, the two straws varying less than 1%. The heating value of straw of this composition, according to Dulong's formula, and deducting the heat lost in evaporating the water, is 5155 heat units. Clark erroneously gives it as 8144 heat units.

Bagasse as Fuel in Sugar Manufacture. — Bagasse is the name given to refuse sugar-cane, after the juice has been extracted. Prof. L. A. Becuel, in a paper read before the Louisiana Sugar Chemists' Association, in 1892, says: "With tropical cane containing 12.5% woody fibre, a juice containing 16.13% solids, and 83.87% water, bagasse of, say, 66% and 72% mill extraction would have the following percentage composition:

| | | | |
|--------------|------------------|------------------------|------------|
| 66% bagasse: | Woody Fibre, 37; | Combustible Salts, 10; | Water, 53. |
| 72% bagasse: | " 45; | " 9; | " 46. |

"Assuming that the woody fibre contains 51% carbon, the sugar and other combustible matters an average of 42.1%, and that 12,906 units of heat are generated for every pound of carbon consumed, the 66% bagasse is capable of generating 297,834 heat-units per 100 lbs. as against 345,200, or a difference of 47,366 units in favor of the 72% bagasse.

"Assuming the temperature of the waste gases to be 450° F., that of the surrounding atmosphere and water in the bagasse at 86° F., and the quantity of air necessary for the combustion of one pound of carbon at 24 lbs., the lost heat will be as follows: In the waste gases, heating air from 86° to 450° F., and in vaporizing the moisture, etc., the 66% bagasse will require 112,546 heat units, and 116,150 for the 72% bagasse.

"Subtracting these quantities from the above, we find that the 66% bagasse will produce 185,288 available heat-units per 100 lbs., or nearly 24% less than the 72% bagasse, which gives 229,050 units. Accordingly, one ton of cane of 2000 lbs. at 66% mill extraction will produce 680 lbs. bagasse, equal to 1,259,958 available heat-units, while the same cane at 72% extraction will produce 560 lbs. bagasse, equal to 1,282,680 units.

"A similar calculation for the case of Louisiana cane containing 10% woody fibre, and 16% total solids in the juice, assuming 75% mill extraction, shows that bagasse from one ton of cane contains 1,573,956 heat-units, from which 561,465 have to be deducted.

"This would make such bagasse worth on an average nearly 92 lbs. coal per ton of cane ground. Under fairly good conditions, 1 lb. coal will evaporate 7½ lbs. water, while the best boiler plants evaporate 10 lbs. Therefore the bagasse from 1 ton of cane at 75% mill extraction should evaporate from 689 lbs. to 919 lbs. of water. The juice extracted from such cane would under these conditions contain 1260 lbs. of water. If we assume that the water added during the process of manufacture is 10% (by weight) of the juice made, the total water handled is 1410 lbs. From the juice represented in this case, the commercial massecuite would be about 15% of the weight of the original mill juice, or, say, 225 lbs. Said mill juice 1500 lbs., plus 10%, equals 1650 lbs. liquor handled; and 1650 lbs., minus 225 lbs., equals 1425 lbs., the quantity of water to be evaporated during the process of manufacture. To effect a 7½-lb. evaporation requires 190 lbs. of coal, and 142½ lbs. for a 10-lb. evaporation.

"To reduce 1650 lbs. of juice to syrup of, say, 27° Baumé, requires the evaporation of 1170 lbs. of water, leaving 480 lbs. of syrup. If this work be accomplished in the open air, it will require about 156 lbs. of coal at 7½ lbs. boiler evaporation, and 117 at 10 lbs. evaporation.

"With a double effect the fuel required would be from 59 to 78 lbs., and with a triple effect, from 36 to 52 lbs.

"To reduce the above 480 lbs. of syrup to the consistency of commercial massecuite means the further evaporation of 255 lbs. of water, requiring the expenditure of 34 lbs. coal at 7½ lbs. boiler evaporation, and 25½ lbs. with a 10-lb. evaporation. Hence, to manufacture one ton of cane into sugar and molasses, it will take from 145 to 190 lbs. additional coal to do the work by the open evaporator process; from 85 to 112 lbs. with a double effect, and only 7½ lbs. evaporation in the boilers, while with 10 lbs. boiler evaporation the bagasse alone is capable of furnishing 8% more heat than is actually required to do the work. With triple-effect evaporation depending on the excellence of the boiler plant, the 1425 lbs. of water to be evaporated from the juice will require between

62 and 86 lbs. of coal. These values show that from 6 to 30 lbs. of coal can be spared from the value of the bagasse to run engines, grind cane, etc.

"It accordingly appears," says Prof. Becuel, "that with the best boiler plants, those taking up all the available heat generated, by using this heat economically the bagasse can be made to supply all the fuel required by our sugar-houses."

E. W. Kerr, in Bulletin No. 117 of the Louisiana Agricultural Experiment Station, Baton Rouge, La., gives the results of a study of many different forms of bagasse furnaces. An equivalent evaporation of 21¼ lbs. of steam from and at 212° was obtained from 1 lb. of wet bagasse of a net calorific value of 3256 B.T.U. This net value is that calculated from the analysis by Dulong's formula, minus the heat required to evaporate the moisture and to heat the vapor to the temperature of the escaping chimney gases, 594° F. The approximate composition of bagasse of 75% extraction is given as 51% free moisture, and 28% of water combined with 21% of carbon in the fibre and sugar. For the best results the bagasse should be burned at a high rate of combustion, at least 100 lbs. per sq. ft. of grate per hour. Not more than 1.5 lbs. of bagasse per sq. ft. of heating surface per hour should be burned under ordinary conditions, and not less than 1.5 boiler horse-power should be provided per ton of coal per 24 hours.

LIQUID FUEL.

Products of the Distillation of Crude Petroleum.

Crude American petroleum of sp. gr. 0.800 may be split up by fractional distillation as follows ("Robinson's Gas and Petroleum Engines"):

| Temp. of Distillation Fahr.: | Distillate. | Per-cent-ages. | Specific Gravity. | Flashing Point. Deg. F. |
|------------------------------|--------------------------------|----------------|-------------------|-------------------------|
| 113° | Rhigolene. } | traces. | .590 to .625 | |
| 113 to 140° | Chymogene. } | | | |
| 140 to 158° | Gasoline (petroleum spirit) .. | 1.5 | .636 to .657 | |
| 158 to 248° | Benzine, naphtha C, benzolene | 10. | .680 to .700 | 14 |
| 248° | { Benzine, naphtha B. | 2.5 | .714 to .718 | |
| to | { Benzine, naphtha A. | 2. | .725 to .737 | 32 |
| 347° | { Polishing oils. | | | |
| 338° and } upwards. } | Kerosene (lamp-oil)..... | 50. | .802 to .820 | 100 to 122 |
| 482° | Lubricating oil..... | 15. | .850 to .915 | 230 |
| | Paraffine wax..... | 2. | | |
| | Residue and Loss..... | 16. | | |

Lima Petroleum, produced at Lima, Ohio, is of a dark green color, very fluid, and marks 48° Baumé at 15° C. (sp. gr., 0.792).

The distillation in fifty parts, each part representing 2% by volume, gave the following results:

| Per cent. | Sp. Gr. | Per cent. | Sp. Gr. | Per cent. | Sp. Gr. | Per cent. | Sp. Gr. | Per cent. | Sp. Gr. | Per cent. | Sp. Gr. |
|-----------|---------|-----------|---------|-----------|---------|-----------|---------|-----------|---------|-----------|---------|
| .2 | 0.680 | 18 | 0.720 | 34 | 0.764 | 50 | 0.802 | 68 | 0.820 | 88 | 0.815 |
| 4 | .683 | 20 | .728 | 36 | .768 | 52 | | 70 | .825 | 90 | .815 |
| 6 | .685 | 22 | .730 | 38 | .772 | 54 | .806 | 72 | .830 | | |
| 8 | .690 | 24 | .735 | 40 | .778 | 58 | | 73 | .830 | 92 | |
| 10 | .694 | 26 | .740 | 42 | .782 | 60 | .800 | 76 | .810 | to | |
| 12 | .698 | 28 | .742 | 44 | .788 | 62 | .804 | 78 | .820 | 100 | |
| 14 | .700 | 30 | .746 | 46 | .792 | 64 | .808 | 82 | .818 | | |
| 16 | .706 | 32 | .760 | 48 | .800 | 66 | .812 | 86 | .816 | | |

RETURNS.

16 per cent naphtha, 70° Baumé.

68 per cent burning oil.

6 per cent paraffine oil.

10 per cent residuum.

The distillation started at 23° C., this being due to the large amount of naphtha present, and when 60% was reached, at a temperature of 310° C., the hydrocarbons remaining in the retort were dissociated, then gases

escaped, lighter distillates were obtained, and, as usual in such cases, the temperature decreased from 310° C. down gradually to 200° C., until 75% of oil was obtained, and from this point the temperature remained constant until the end of the distillation. Therefore these hydrocarbons in *statu moriendi* absorbed much heat. (*Jour. Am. Chem. Soc.*)

There is not a good agreement between the character of the materials designated gasoline, kerosene, etc., and the temperature of distillation and densities employed in different places. The following table shows one set of values that is probably as good as any.

| Name. | Boiling Point. | Specific Gravity. | Density at 59° F. |
|----------------------|----------------|-------------------|-------------------|
| | ° F. | | ° Baumé. |
| Petroleum ether..... | 104-158 | 0.650-0.660 | 85-80 |
| Gasoline..... | 158-176 | .660- .670 | 80-78 |
| Naphtha C..... | 176-212 | .670- .707 | 78-68 |
| Naphtha B..... | 212-248 | .707- .722 | 68-64 |
| Naphtha A..... | 248-302 | .722- .737 | 64-60 |
| Kerosene..... | 302-572 | .753- .864 | 56-32 |

Gasoline is different from a simple substance with a fixed boiling point, and therefore theoretical calculations on the heat of combustion, air necessary, and conditions for vaporizing or carbureting air are of little value. (C. E. Lucke.)

Value of Petroleum as Fuel. — Thos. Urquhart, of Russia (*Proc. Inst. M. E.*, Jan., 1889), gives the following table of the theoretical evaporative power of petroleum in comparison with that of coal, as determined by Messrs. Favre and Silbermann:

| Fuel. | Specific Gravity at 32° F., Water = 1.000 | Chem. Comp. | | | Heating power, British Thermal Units. | Theoret. Evap., lbs. Water per lb. Fuel, from and at 212° F. |
|--|---|-------------|------|-----|---------------------------------------|--|
| | | C. | H. | O. | | |
| Penna. heavy crude oil. | 0.886 | 84.9 | 13.7 | 1.4 | 20,736 | 21.48 |
| Caucasian light crude oil.. | 0.884 | 86.3 | 13.6 | 0.1 | 22,027 | 22.79 |
| Caucasian heavy crude oil. | 0.938 | 86.6 | 12.3 | 1.1 | 20,138 | 20.85 |
| Petroleum refuse..... | 0.928 | 87.1 | 11.7 | 1.2 | 19,832 | 20.53 |
| Good English Coal, Mean of 98 Samples..... | 1.380 | 80.0 | 5.0 | 8.0 | 14,112 | 14.61 |

In experiments on Russian railways with petroleum as fuel Mr. Urquhart obtained an actual efficiency equal to 82% of the theoretical heating-value. The petroleum is fed to the furnace by means of a spray-injector driven by steam. An induced current of air is carried in around the injector-nozzle, and additional air is supplied at the bottom of the furnace.

Beaumont, Texas, oil analyzed as follows (*Eng. News*, Jan. 30, 1902): C, 84.60; H, 10.90; S, 1.63; O, 2.87. Sp. gr.; 0.92; flash point, 142° F.; burning point, 181° F.; heating value per lb., by oxygen calorimeter, 19,060 B.T.U. A test of a horizontal tubular boiler with this oil, by J. E. Denton gave an efficiency of 78.5%. As high as 82% has been reported for California oil.

Bakersfield, Cal., oil: Sp. gr. 16° Baumé; Moisture, 1%; Sulphur, 0.5%. B.T.U. per lb., 18,500.

Redondo, Cal., oil, six lots: Moisture, 1.82 to 2.70%; Sulphur, 2.17 to 2.60%; B.T.U. per lb., 17,717 to 17,966. Kilowatt-hours generated per barrel (33½ lbs.) of oil in a 5000 K.W. plant, using water-tube boilers, and reciprocating engines and generators having a combined efficiency of 90.2 to 94.75% (boiler economy and steam-rate of engine not stated). 2000 K.W. load, 237.3; 3000 K.W., 256.7; 5000 K.W., 253.4; variable load, 24 hours, 243.8. (C. R. Weymouth, *Trans. A. S. M. E.*, 1908.)

The following table showing the relative values of petroleum and coal was given by the author in *Power*, Sept., 1902. It is based on the following assumed data: B.T.U. per lb. of oil 20,000; sp. gr., 0.885; = 7.37 lbs. per gal.; 1 barrel = 41 gals. = 310 lbs.

| Coal, B.T.U. per lb. | 1 lb. coal = lbs. oil. | 1 barrel oil = lbs. coal. | 1 ton coal = barrels oil. |
|-------------------------|---------------------------|------------------------------|------------------------------|
| 10,000 | 2. | 620 | 3.23 |
| 11,000 | 1.818 | 564 | 3.55 |
| 12,000 | 1.667 | 517 | 3.87 |
| 13,000 | 1.538 | 477 | 4.19 |
| 14,000 | 1.429 | 443 | 4.52 |
| 15,000 | 1.333 | 413 | 4.84 |

From this table we see that if coal of a heating value of only 10,000 B.T.U. per lb. costs \$3.23 per ton, and coal of 14,000 B.T.U. per lb. at \$4.52 per ton, then the price of oil will have to be as low as \$1 a barrel to compete with coal; or, if the poorer coal is \$6.26 and the better coal \$9.04 per ton, then oil will be the cheaper fuel if it is below \$2 per barrel.

Fuel Oil Burners.—A great variety of burners are on the market, most of them based on the principle of using a small jet of steam at the boiler pressure to inject the oil into the furnace, in the shape of finely divided spray, and at the same time to draw in the air supply and mix it intimately with the oil. So far as economy of oil is concerned these burners are all of about equal value, but their successful operation depends on the construction of the furnace. This should have a large combustion chamber, entirely surrounded with fire brick, and the jet should be so directed that it will strike a fire-brick surface and rebound before touching the heating surface of the boiler. Burners using air at high pressure, 40 lbs. per sq. in., without steam, have been used with advantage. Lower pressures have been found not sufficient to atomize the oil.

When boilers are forced, with a combustion chamber too small to allow the oil spray to be completely burned in it before passing to the boiler surface, dense clouds of smoke result, with deposit of lampblack or soot.

Oil vs. Coal as Fuel. (*Iron Age*, Nov. 2, 1893.)—Test by the Twin City Rapid Transit Company of Minneapolis and St. Paul. This test showed that with the ordinary Lima oil weighing 6.6 pounds per gallon, and costing 2¼ cents per gallon, and coal that gave an evaporation of 7½ lbs. of water per pound of coal, the two fuels were equally economical when the price of coal was \$3.85 per ton of 2000 lbs. With the same coal at \$2.00 per ton, the coal was 37% more economical, and with the coal at \$4.85 per ton, the coal was 20% more expensive than the oil. These results include the difference in the cost of handling the coal, ashes, and oil.

In 1892 there were reported to the Engineers' Club of Philadelphia some comparative figures, from tests undertaken to ascertain the relative value of coal, petroleum, and gas.

| | Lbs. Water, from and at 212° F. |
|--|------------------------------------|
| 1 lb. anthracite coal evaporated | 9.70 |
| 1 lb. bituminous coal | 10.14 |
| 1 lb. fuel oil, 36° gravity | 16.48 |
| 1 cubic foot gas, 20 C. P. | 1.28 |

The gas used was that obtained in the distillation of petroleum, having about the same fuel-value as natural or coal-gas of equal candle-power.

Taking the efficiency of bituminous coal as a basis, the calorific energy of petroleum is more than 60% greater than that of coal; whereas, theoretically, petroleum exceeds coal only about 45% — the one containing 14,500 heat-units, and the other 21,000.

Crude Petroleum vs. Indiana Block Coal for Steam-raising at the South Chicago Steel Works. (E. C. Potter, *Trans. A. I. M. E.*, xvii, 807.)—With coal, 14 tubular boilers 16 ft. × 5 ft. required 25 men to operate them; with fuel oil, 6 men were required, a saving of 19 men at \$2 per day, or \$38 per day.

For one week's work 2731 barrels of oil were used, against 848 tons of coal required for the same work, showing 3.22 barrels of oil to be equivalent to 1 ton of coal. With oil at 60 cents per barrel and coal at \$2.15 per ton, the relative cost of oil to coal is as \$1.93 to \$2.15. No evaporation tests were made.

Petroleum as a Metallurgical Fuel. — C. E. Felton (*Trans. A. I. M. E.*, xvii, 809) reports a series of trials with oil as fuel in steel-heating and open-hearth steel-furnaces, and in raising steam, with results as follows: 1. In a run of six weeks the consumption of oil, partly refined (the paraffine and some of the naphtha being removed), in heating 14-inch ingots in Siemens furnaces was about 6½ gallons per ton of blooms. 2. In melting in a 30-ton open-hearth furnace 48 gallons of oil were used per ton of ingots. 3. In a six weeks' trial with Lima oil from 47 to 54 gallons of oil were required per ton of ingots. 4. In a six months' trial with Siemens heating-furnaces the consumption of Lima oil was 6 gallons per ton of ingots. Under the most favorable circumstances, charging hot ingots and running full capacity, 4½ to 5 gallons per ton were required. 5. In raising steam in two 100-H.P. tubular boilers, the feed-water being supplied at 160° F., the average evaporation was about 12 pounds of water per pound of oil, the best 12 hours' work being 16 pounds.

In all of the trials the oil was vaporized in the Archer producer, an apparatus for mixing the oil and superheated steam, and heating the mixture to a high temperature. From 0.5 lb. to 0.75 lb. of pea-coal was used per gallon of oil in the producer itself.

ALCOHOL AS FUEL.

Denatured alcohol is a grain or ethyl alcohol mixed with a denaturant in order to make it unfit for beverage or medicinal purposes. Under acts of Congress of June 7, 1906 and March 2, 1907, denatured alcohol became exempt from internal revenue taxation, when used in the industries.

The Government formulas for completely denatured alcohol are:

1. To every 100 gal. of ethyl or grain alcohol (of not less than 180% proof) there shall be added 10 gal. of approved methyl or wood alcohol and ½ gal. of approved benzine. (180% proof = 90% alcohol, 10% water, by volume.)

2. To every 100 gal. of ethyl alcohol (of not less than 180% proof) there shall be added 2 gal. of approved methyl alcohol and ½ gal. of approved pyridin (a petroleum product) bases.

Methyl alcohol, benzine and pyridin used as denaturants must conform to specifications of the Internal Revenue Department.

The alcohol which it is proposed to manufacture under the present law is ethyl alcohol, C_2H_5OH . This material is seldom, if ever, obtained pure, it being generally diluted with water and containing other alcohols when used for engines.

SPECIFIC GRAVITY OF ETHYL ALCOHOL AT 60° F. COMPARED WITH WATER AT 60°. (Smithsonian Tables.)

| Sp. Gr. | Per cent Alcohol. | | Sp. Gr. | Per cent Alcohol. | | Sp. Gr. | Per cent Alcohol. | |
|---------|-------------------|------|---------|-------------------|------|---------|-------------------|------|
| | Weight. | Vol. | | Weight. | Vol. | | Weight. | Vol. |
| 0.834 | 85.8 | 90.0 | 0.826 | 88.9 | 92.3 | 0.818 | 91.9 | 94.5 |
| .832 | 86.6 | 90.6 | .824 | 89.6 | 92.9 | .816 | 92.6 | 95.0 |
| .830 | 87.4 | 91.2 | .822 | 90.4 | 93.4 | .814 | 93.3 | 95.5 |
| .828 | 88.1 | 91.8 | .820 | 91.1 | 94.0 | .812 | 94.0 | 96.0 |

The heat of combustion of ethyl alcohol, 94% by volume, as determined by the calorimeter, is 11,900 B.T.U. per lb. — a little more than half that of gasoline (Lucke). Favre and Silbermann obtained 12,913 B.T.U. for absolute alcohol.

The products of complete combustion of alcohol are H_2O and CO_2 . Under certain conditions, with an insufficient supply of air, acetic acid is

formed, which causes rusting of the parts of an alcohol engine. This may be prevented by addition to the alcohol of benzol or acetylene.

With any good small stationary engine as small a consumption as 0.70 lb. of gasoline, or 1.16 lb. of alcohol per brake H.P. hour may reasonably be expected under favorable conditions (Lucke).

References. — H. Diederichs, *Int'l. Marine Eng'g.*, July, 1906; *Machy.*, Aug., 1906. C. E. Lucke and S. M. Woodward, *Farmer's Bulletin*, No. 277, U. S. Dept. of Agriculture, 1907. *Eng. Rec.*, Nov. 2, 1907. T. L. White, *Eng. Mag.*, Sept., 1908.

VAPOR PRESSURE OF SATURATION FOR VARIOUS LIQUIDS, IN MILLIMETERS OF MERCURY.

(To convert into pounds per sq. in., multiply by 0.01934; to convert into inches of mercury, multiply by 0.03937.)

| Temperature. | | Pure Ethyl Alcohol. | Pure Methyl Alcohol. | Water. | Gasoline. | Temperature. | | Pure Ethyl Alcohol. | Pure Methyl Alcohol. | Water. | Gasoline. |
|--------------|------|---------------------|----------------------|--------|-----------|--------------|------|---------------------|----------------------|--------|-----------|
| ° C. | ° F. | | | | | ° C. | ° F. | | | | |
| 0 | 32 | 12 | 30 | 5 | 99 | 35 | 95 | 103 | 204 | 42 | 301 |
| 5 | 41 | 17 | 40 | 7 | 115 | 40 | 104 | 134 | 259 | 55 | 360 |
| 10 | 50 | 24 | 54 | 9 | 133 | 45 | 113 | 172 | 327 | 71 | 422 |
| 15 | 59 | 32 | 71 | 13 | 154 | 50 | 122 | 220 | 409 | 92 | 493 |
| 20 | 68 | 44 | 94 | 17 | 179 | 55 | 131 | 279 | 508 | 117 | 561 |
| 25 | 77 | 59 | 123 | 24 | 210 | 60 | 140 | 350 | 624 | 149 | 648 |
| 30 | 86 | 78 | 159 | 32 | 251 | 65 | 149 | 437 | 761 | 187 | 739 |

VAPOR TENSION OF ALCOHOL AND WATER, AND DEGREE OF SATURATION OF AIR WITH THESE VAPORS.

| Temp. degs. F. | Vapor Tension, Inches Mercury. | | 1 Pound of Air Contains in Saturated Condition, in Pounds. | | | |
|-------------------|--------------------------------|--------------|--|--------------|------------------|--------------|
| | | | At 28.95 Inches. | | At 26.05 Inches. | |
| | Alcohol Vapor. | Water Vapor. | Alcohol Vapor. | Water Vapor. | Alcohol Vapor. | Water Vapor. |
| 50 | 0.950 | 0.359 | 0.055 | 0.008 | 0.061 | 0.009 |
| 59 | 1.283 | 0.500 | 0.075 | 0.011 | 0.084 | 0.013 |
| 68 | 1.733 | 0.687 | 0.104 | 0.016 | 0.117 | 0.018 |
| 77 | 2.325 | 0.925 | 0.144 | 0.022 | 0.162 | 0.025 |
| 86 | 3.090 | 1.240 | 0.200 | 0.031 | 0.227 | 0.036 |
| 104 | 5.270 | 2.162 | 0.390 | 0.063 | 0.450 | 0.072 |
| 122 | 8.660 | 3.620 | 0.827 | 0.135 | 1.002 | 0.164 |

FUEL GAS.

The following notes are extracted from a paper by W. J. Taylor on "The Energy of Fuel" (*Trans. A. I. M. E.*, xviii. 205):

Carbon Gas. — In the old Siemens producer, practically all the heat of primary combustion — that is, the burning of solid carbon to carbon monoxide, or about 30% of the total carbon energy — was lost, as little or no steam was used in the producer, and nearly all the sensible heat of the gas was dissipated in its passage from the producer to the furnace, which was usually placed at a considerable distance.

Modern practice has improved on this plan, by introducing steam with the air blown into the producer, and by utilizing the sensible heat of the gas in the combustion-furnace. It ought to be possible to oxidize

one out of every four lbs. of carbon with oxygen derived from water-vapor. The thermic reactions in this operation are as follows:

| | Heat-units. |
|---|--------------|
| 4 lbs. C burned to CO (3 lbs. gasified with air and 1 lb. with water) develop..... | 17,600 |
| 1.5 lbs. of water (which furnish 1.33 lbs. of oxygen to combine with 1 lb. of carbon) absorb by dissociation..... | 10,333 |
| The gas, consisting of 9.333 lbs. CO, 0.167 lb. H, and 13.39 lbs. N, heated 600°, absorbs..... | 3,748 |
| Leaving for radiation and loss..... | 3,519 |
| | <hr/> 17,600 |

The steam which is blown into a producer with the air is almost all condensed into finely-divided water before entering the fuel, and consequently is considered as water in these calculations.

The 1.5 lbs. of water liberates 0.167 lb. of hydrogen, which is delivered to the gas, and yields in combustion the same heat that it absorbs in the producer by dissociation. According to this calculation, therefore, 60% of the heat of primary combustion is theoretically recovered by the dissociation of steam, and, even if all the sensible heat of the gas be counted, with radiation and other minor items, as loss, yet the gas must carry $4 \times 14,500 - (3748 + 3519) = 50,733$ heat-units, or 87% of the calorific energy of the carbon. This estimate shows a loss in conversion of 13%, without crediting the gas with its sensible heat, or charging it with the heat required for generating the necessary steam, or taking into account the loss due to oxidizing some of the carbon to CO₂. In good producer-practice the proportion of CO₂ in the gas represents from 4% to 7% of the C burned to CO₂, but the extra heat of this combustion should be largely recovered in the dissociation of more water-vapor, and therefore does not represent as much loss as it would indicate. As a conveyor of energy, this gas has the advantage of carrying 4.46 lbs. less nitrogen than would be present if the fourth pound of coal had been gasified with air; and in practical working the use of steam reduces the amount of clinkering in the producer.

Anthracite Gas. — In anthracite coal there is a volatile combustible varying in quantity from 1.5% to over 7%. The amount of energy derived from the coal is shown in the following theoretical gasification made with coal of assumed composition: Carbon, 85%; vol. HC, 5%; ash, 10%; 80 lbs. carbon assumed to be burned to CO; 5 lbs. carbon burned to CO₂; three fourths of the necessary oxygen derived from air, and one fourth from water.

| Process. | Pounds. | Products. Cubic Feet. | Anal. by Vol. |
|---|--------------|--------------------------|---------------|
| 80 lbs. C burned to CO | 186.66 | 2529.24 | 33.4 |
| 5 lbs. C burned to CO ₂ | 18.33 | 157.64 | 2.0 |
| 5 lbs. vol. HC (distilled) | 5.00 | 116.60 | 1.6 |
| 120 lbs. oxygen are required, of which 30 lbs. from H ₂ O liberate H | 3.75 | 712.50 | 9.4 |
| 90 lbs. from air are associated with N | 301.05 | 4064.17 | 53.6 |
| | <hr/> 514.79 | <hr/> 7580.15 | <hr/> 100.0 |

Energy in the above gas obtained from 100 lbs. anthracite:

| | | |
|---------------------------------------|-----------------|-------------|
| 186.66 lbs. CO | 807,304 | heat-units. |
| 5.00 " CH ₄ | 117,500 | " |
| 3.75 " H | 232,500 | " |
| | <hr/> 1,157,304 | " |
| Total energy in gas per lb. | 2,248 | " |
| Total energy in 100 lbs. of coal | 1,349,500 | " |
| Efficiency of the conversion | 86%. | |

The sum of CO and H exceeds the results obtained in practice. The sensible heat of the gas will probably account for this discrepancy and, therefore, it is safe to assume the possibility of delivering at least 82% of the energy of the anthracite.

Bituminous Gas. — A theoretical gasification of 100 lbs. of coal, containing 55% of carbon and 32% of volatile combustible (which is above the average of Pittsburgh coal), is made in the following table. It is assumed that 50 lbs. of C are burned to CO and 5 lbs. to CO₂; one fourth of the O is derived from steam and three fourths from air; the heat value of the volatile combustible is taken at 20,000 heat-units to the pound. In computing volumetric proportions all the volatile hydrocarbons, fixed as well as condensing, are classed as marsh-gas, since it is only by some such tentative assumption that even an approximate idea of the volumetric composition can be formed. The energy, however, is calculated from weight:

| Process. | Products. | | |
|---|-----------|-------------|---------------|
| | Pounds. | Cubic Feet. | Anal. by Vol. |
| 50 lbs. C burned to CO | 116.66 | 1580.7 | 27.8 |
| 5 lbs. C burned to CO ₂ | 18.33 | 157.6 | 2.7 |
| 32 lbs. vol. HC (distilled) | 32.00 | 746.2 | 13.2 |
| 80 lbs. O are required, of which 20 lbs., derived from H ₂ O, liberate H | 2.5 | 475.0 | 8.3 |
| 60 lbs. O, derived from air, are associated with N | 200.70 | 2709.4 | 47.8 |
| | 370.19 | 5668.9 | 99.8 |
| Energy in 116.66 lbs. CO | | 504,554 | heat-units. |
| “ “ 32.00 lbs. vol. HC | | 640,000 | “ |
| “ “ 2.50 lbs. H | | 155,000 | “ |
| | | 1,299,554 | “ |
| Energy in coal | | 1,437,500 | “ |
| Per cent of energy delivered in gas | | | 90.0 |
| Heat-units in 1 lb. of gas | | | 3,484 |

Water-gas. — Water-gas is made in an intermittent process, by blowing up the fuel-bed of the producer to a high state of incandescence (and in some cases utilizing the resulting gas, which is a lean producer-gas), then shutting off the air and forcing steam through the fuel, which dissociates the water into its elements of oxygen and hydrogen, the former combining with the carbon of the coal, and the latter being liberated.

This gas can never play a very important part in the industrial field, owing to the large loss of energy entailed in its production, yet there are places and special purposes where it is desirable, even at a great excess in cost per unit of heat over producer-gas; for instance, in small high-temperature furnaces, where much regeneration is impracticable, or where the “blow-up” gas can be used for other purposes instead of being wasted.

The reactions and energy required in the production of 1000 feet of water-gas, composed, theoretically, of equal volumes of CO and H, are as follows:

| | |
|----------------------------------|------------|
| 500 cubic feet of H weigh | 2.635 lbs. |
| 500 cubic feet of CO weigh | 36.89 “ |

Total weight of 1000 cubic feet

Now, as CO is composed of 12 parts C to 16 of O, the weight of C in 36.89 lbs. is 15.81 lbs. and of O 21.08 lbs. When this oxygen is derived from water it liberates, as above, 2,635 lbs. of hydrogen. The heat developed and absorbed in these reactions (roughly, as we will not take into account the energy required to elevate the coal from the temperature of the atmosphere to, say, 1800°) is as follows:

| | |
|---|-------------|
| | Heat-units. |
| 2.635 lbs. H. absorb in dissociation from water $2.635 \times 62,000$ | = 163,370 |
| 15.81 lbs. C burned to CO develops 15.81×4400 | = 69,564 |
| Excess of heat-absorption over heat-development | = 93,806 |

If this excess could be made up from C burnt to CO₂ without loss by radiation, we would only have to burn an additional 4.83 lbs. C to supply this heat, and we could then make 1000 feet of water-gas from 20.64 lbs.

of carbon (equal 24 lbs. of 85% coal). This would be the perfection of gas-making, as the gas would contain really the same energy as the coal; but instead, we require in practice more than double this amount of coal and do not deliver more than 50% of the energy of the fuel in the gas, because the supporting heat is obtained in an indirect way and with imperfect combustion. Besides this, it is not often that the sum of CO and H exceed 90%, the balance being CO₂ and N. But water-gas should be made with much less loss of energy by burning the "blow-up" (producer) gas in brick regenerators, the stored-up heat of which can be returned to the producer by the air used in blowing-up.

The following table shows what may be considered average volumetric analyses, and the weight and energy of 1000 cubic feet, of the four types of gases used for heating and illuminating purposes:

| | Natural Gas. | Coal-gas. | Water-gas. | Producer-gas. | |
|-------------------------------------|--------------|-----------|------------|---------------|---------|
| | | | | Anthra. | Bitu. |
| CO..... | 0.50 | 6.0 | 45.0 | 27.0 | 27.0 |
| H..... | 2.18 | 46.0 | 45.0 | 12.0 | 12.0 |
| CH ₄ | 92.6 | 40.0 | 2.0 | 1.2 | 2.5 |
| C ₂ H ₄ | 0.31 | 4.0 | | | 0.4 |
| CO ₂ | 0.26 | 0.5 | 4.0 | 2.5 | 2.5 |
| N..... | 3.61 | 1.5 | 2.0 | 57.0 | 56.2 |
| O..... | 0.34 | 0.5 | 0.5 | 0.3 | 0.3 |
| Vapor..... | | 1.5 | 1.5 | | |
| Pounds in 1000 cubic feet..... | 45.6 | 32.0 | 45.6 | 65.6 | 65.9 |
| Heat-units in 1000 cubic feet.... | 1,100,000 | 735,000 | 322,000 | 137,455 | 156,917 |

Natural Gas in Ohio and Indiana.

(Eng. and M. J., April 21, 1894.)

| | Fos-toria, O. | Find-lay, O. | St. Mary's, O. | Muncie, Ind. | Ander-son, Ind. | Koko-mo, Ind. | Mar-ion, Ind. |
|--------------------|------------------|-----------------|-------------------|-----------------|--------------------|------------------|------------------|
| Hydrogen..... | 1.89 | 1.64 | 1.94 | 2.35 | 1.86 | 1.42 | 1.20 |
| Marsh-gas..... | 92.84 | 93.35 | 93.85 | 92.67 | 93.07 | 94.16 | 93.57 |
| Olefiant gas..... | .20 | .35 | .20 | .25 | .47 | .30 | .15 |
| Carbon monoxide.. | .55 | .41 | .44 | .45 | .73 | .55 | .60 |
| Carbon dioxide.... | .20 | .25 | .23 | .25 | .26 | .29 | .30 |
| Oxygen..... | .35 | .39 | .35 | .35 | .42 | .30 | .55 |
| Nitrogen..... | 3.82 | 3.41 | 2.98 | 3.53 | 3.02 | 2.80 | 3.42 |
| Hydrogen sulphide | .15 | .20 | .21 | .15 | .15 | .18 | .20 |

Natural Gas as a Fuel for Boilers. — J. M. Whitham (*Trans. A. S. M. E.*, 1905) reports the results of several tests of water-tube boilers with natural gas. The following is a condensed statement of the results:

| Kind of Boiler..... | Cook Vertical. | | Heine. | | | Cahall Vert. | |
|------------------------------|----------------|-------|--------|-------|-------|--------------|---------|
| Rated H.P. of boilers..... | 1500 | 1500 | 200 | 200 | 200 | 300 | 300 |
| H.P. developed..... | 1642 | 1507 | 155 | 218 | 258 | 340 | 260 |
| Temperature at chimney | 521 | 494 | 386 | 450 | 465 | 406 | 374 |
| Gas pressure at burners, oz. | 6.9 | 6.4 | ... | ... | ... | 4.8 | 7 to 30 |
| Cu. ft. of gas per boiler.. | | | | | | | |
| H.P.-hour..... | 44.9* | 41.0* | 46.0† | 40.7† | 38.3† | 42.3 | 34 |
| Boiler efficiency, %..... | 72.7 | ... | 65.8 | ... | 74.9 | ... | ... |

* Reduced to 4 oz. pressure and 62° F.

† Reduced to atmos. press. and 32° F.

Six tests by Daniel Ashworth on 2-flue horizontal boilers gave cu. ft. of gas per boiler H.P. hour, 58.0; 59.7; 67.0; 63.0; 74.0; 47.0.

On the first Cook boiler test, the chimney gas, analyzed by the Orsat apparatus, showed 7.8 CO₂; 8.05 O; 0.0 CO; 84.15 N. This shows an excessive air supply.

White versus Blue Flame. — Tests were made with the air supply throttled at the burners, so as to produce a white flame, and also unthrottled, producing a blue flame with the following results:

| Pressure of gas at burners, oz..... | 4 | | 6 | | 8 | |
|---|-------|------|-------|------|-------|------|
| Kind of flame..... | White | Blue | White | Blue | White | Blue |
| Boiler H.P. made per 250-H.P. boiler | 247 | 213 | 297 | 271 | 255 | 227 |
| Cu. ft. of gas (at 4 oz. and 60° F.) per H.P. hour..... | 41 | 41 | 41.6 | 37.9 | 40 | 43.1 |
| Chimney temperature..... | 436 | 503 | 478 | 511 | 502 | 508 |

Average of 6 tests — White, 266 H.P., 43.6 cu. ft.; Blue, 237 H.P., 43.8 cu. ft., showing that the economy is the same with each flame, but the capacity is greatest with the white flame. Mr. Whitham's principal conclusions from these tests are as follows:

- (1) There is but little advantage possessed by one burner over another.
- (2) As good economy is made with a blue as with a white or straw flame, and no better.
- (3) Greater capacity may be made with a straw-white than with a blue flame.
- (4) An efficiency as high as from 72 to 75 per cent in the use of gas is seldom obtained under the most expert conditions.
- (5) Fuel costs are the same under the best conditions with natural gas at 10 cents per 1000 cu. ft. and semi-bituminous coal at \$2.87 per ton of 2240 lbs.
- (6) Considering the saving of labor with natural gas, as compared with hand-firing of coal, in a plant of 1500 H.P., and coal at \$2 per ton of 2240 lbs., gas should sell for about 10 cents per 1000 cu. ft.

ANALYSES OF NATURAL GAS.

| | | | | |
|---|-------|-------|-------|------|
| Illuminants..... | 0.45 | 0.15 | 0.50 | 1.6 |
| Carbonic oxide..... | 0.00 | 0.00 | 0.15 | 1.8 |
| Hydrogen..... | 0.20 | 0.30 | 0.25 | 0.3 |
| Marsh gas..... | 81.05 | 83.20 | 83.40 | 81.9 |
| Ethane..... | 17.60 | 15.55 | 15.40 | 13.2 |
| Carbonic acid..... | 0.00 | 0.20 | 0.00 | 0.0 |
| Oxygen..... | 0.15 | 0.10 | 0.00 | 0.4 |
| Nitrogen..... | 0.55 | 0.50 | 0.30 | 0.8 |
| B.T.U. per cu. ft. at 60° F. and 14.7 lbs. barometer..... | 1030 | 1020 | 1026 | 1098 |

The first three analyses are of the gas from nine wells in Lewis Co., W. Va.; the last is from a mixture from fields in three states supplying Pittsburg, Pa., used in the tests of the Cook boiler.

Producer-gas from One Ton of Coal.

(W. H. Blauvelt, *Trans. A. I. M. E.*, xviii, 614.)

| Analysis by Vol. | Per Cent. | Cubic Feet. | Lbs. | Equal to — |
|-------------------------------------|-----------|-------------|---------|-----------------------------------|
| CO..... | 25.3 | 33,213.84 | 2451.20 | 1050.51 lbs. C + 1400.7 lbs. O. |
| H..... | 9.2 | 12,077.76 | 63.56 | " H. |
| CH ₄ | 3.1 | 4,069.68 | 174.66 | " CH ₄ . |
| C ₂ H ₄ | 0.8 | 1,050.24 | 77.78 | " C ₂ H ₄ . |
| CO ₂ | 3.4 | 4,453.52 | 519.02 | " C + 377.44 lbs. O. |
| N (by difference) | 58.2 | 76,404.96 | 5659.63 | " Air. |
| | 100.0 | 131,280.00 | 8945.85 | |

Calculated upon this basis, the 131,280 ft. of gas from the ton of coal contained 20,311,162 B.T.U., or 155 B.T.U. per cubic ft., or 2270 B. T.U. per lb.

The composition of the coal from which this gas was made was as follows: Water, 1.26%; volatile matter, 36.22%; fixed carbon, 57.98%; sulphur, 0.70%; ash, 3.78%. One ton contains 1159.6 lbs. carbon and 724.4 lbs. volatile combustible, the energy of which is 31,302,200 B.T.U. Hence, in the processes of gasification and purification there was a loss of 35.2% of the energy of the coal.

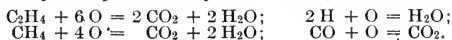
The composition of the hydrocarbons in a soft coal is uncertain and quite complex; but the ultimate analysis of the average coal shows that it approaches quite nearly to the composition of CH_4 (marsh-gas).

Mr. Blauvelt emphasizes the following points as highly important in soft-coal producer-practice:

First. That a large percentage of the energy of the coal is lost when the gas is made in the ordinary low producer and cooled to the temperature of the air before being used. To prevent these sources of loss, the producer should be placed so as to lose as little as possible of the sensible heat of the gas, and prevent condensation of the hydrocarbon vapors. A high fuel-bed should be carried, keeping the producer cool on top, thereby preventing the breaking-down of the hydrocarbons and the deposit of soot, as well as keeping the carbonic acid low.

Second. That a producer should be blown with as much steam mixed with the air as will maintain incandescence. This reduces the percentage of nitrogen and increases the hydrogen, thereby greatly enriching the gas. The temperature of the producer is kept down, diminishing the loss of heat by radiation through the walls, and in a large measure preventing clinkers.

The Combustion of Producer-gas. (H. H. Campbell, *Trans. A. I. M. E.*, xix, 128.) — The combustion of the components of ordinary producer-gas may be represented by the following formulæ:



AVERAGE COMPOSITION BY VOLUME OF PRODUCER-GAS: A, MADE WITH OPEN GRATES, NO STEAM IN BLAST; B, OPEN GRATES, STEAM-JET IN BLAST. 10 SAMPLES OF EACH.

| | CO_2 . | O. | C_2H_4 . | CO. | H. | CH_4 . | N. |
|----------------|-----------------|------|--------------------------|-------|------|-----------------|-------|
| A min..... | 3.6 | 0.4 | 0.2 | 20.0 | 5.3 | 3.0 | 58.7 |
| A max..... | 5.6 | 0.4 | 0.4 | 24.8 | 8.5 | 5.2 | 64.4 |
| A average..... | 4.84 | 0.4 | 0.34 | 22.1 | 6.8 | 3.74 | 61.78 |
| B min..... | 4.6 | 0.4 | 0.2 | 20.8 | 6.9 | 2.2 | 57.2 |
| B max..... | 6.0 | 0.8 | 0.4 | 24.0 | 9.8 | 3.4 | 62.0 |
| B average..... | 5.3 | 0.54 | 0.36 | 22.74 | 8.37 | 2.56 | 60.13 |

The coal used contained carbon 82%, hydrogen 4.7%.

The following are analyses of products of combustion:

| | CO_2 . | O. | CO. | CH_4 . | H. | N. |
|--------------|-----------------|-----|--------|-----------------|--------|------|
| Minimum..... | 15.2 | 0.2 | trace. | trace. | trace. | 80.1 |
| Maximum..... | 17.2 | 1.6 | 2.0 | 0.6 | 2.0 | 83.6 |
| Average..... | 16.3 | 0.8 | 0.4 | 0.1 | 0.2 | 82.2 |

Proportions of Gas Producers and Scrubbers. (F. C. Tryon, *Power*, Dec. 1, 1908.) — Small inside diameter means excessive draft through the fire. If a fire is forced, as will be necessary with too small an inside diameter, the results will be clinkers and blow-holes or chimneys through the fire bed, with excess CO_2 and weak gas; clinkers fused to the lining, and burning out of grates. If sufficient steam is used to keep down the excessive heat, the result is likely to be too much hydrogen in the gas, with the attendant engine troubles.

The lining should never be less than 9 in. thick even in the smaller sizes, and a 100-H.P., or larger, producer should have at least 12 in. of generator lining. The lining next to the fire bed should be of the best quality of refractory material. A good lining consists of a course of soft common bricks put in edgewise next to the steel shell of the generator, laid in Portland cement; then a good firebrick 6 in. thick laid inside to fit the circle, the bricks being dipped as laid in a fine grouting of ground firebrick.

If we take $1\frac{1}{4}$ lbs. of coal per H.P.-hour as a fair average and 10 lbs. of

coal per hour per square foot of internal fuel-bed cross-section, with 9 in. of refractory lining up to 100 H.P. and at least 12 in. of lining on larger sizes, the generator will give good gas without forcing and without excessive heat in the zone of complete combustion. A 200-H.P. producer on this basis consumes 250 lbs. of coal at full load, and at 10 lbs. per sq. ft. internal area 25 sq. ft. will be necessary. With a 12-in. lining the outside diameter will be 92 in.

Practice has shown that the depth of the fuel bed should never be less than the inside diameter up to 6 ft.; above this size the depth can be adjusted as experience indicates the best working results. Assuming for a 200-H.P. producer 18 in. for the ashpit below the grate, 12 in. for the thickness of the grate and the ashes to protect it, 68 in. depth of fuel bed, 24 in. above the fuel to the gas outlet, the height will be 10 ft. 4 in. to the top of the generator; above this the coal-feeding hopper, say 32 in. high, is mounted; this makes the height over all 13 ft.

The wet scrubber of a gas producer should be of ample size to cool the gas to atmospheric temperature and wash out most of the impurities. A good rule is to make its diameter three-fourths that of the inside diameter of the generator and the height one and one-half times the height of the generator shell. For a 100-H.P. producer, 4 ft. inside diam., the wet scrubber should be 3 ft. inside diam., and if the generator shell is 8 ft. 6 in. high, the scrubber should be 12 ft. 9 in. high. When filled with the proper amount of baffling and scrubbing material (coke is commonly used), the scrubber will have space for about 30 cu. ft. of gas. A 100-H.P. gas engine using 12,000 B.T.U. per H.P.-hour will use 160 cu. ft. of 125-B.T.U. gas per minute. The wet scrubber will therefore be emptied $5\frac{1}{3}$ times every minute, and would require about $8\frac{1}{3}$ gallons of water per minute; if the diameter of the scrubber were reduced one-third the volume of water necessary to cool and scrub the gas would have to be doubled. Gas must be cooled below 90° F. to enable it to give up the impurities it carries in suspension, and even lower than this to condense its moisture.

A separate dry scrubber with two compartments should always be provided and the piping between the two scrubbers so arranged that the gas can be turned into either part of the dry scrubber at will. The dry scrubber should be equal in area to the inside of the generator, and the depth of each part should be sufficient to accommodate at least 2 cu. ft. of scrubbing material and give 1 cu. ft. of space next to the outlet. Oil-soaked excelsior is a good scrubbing material and should be packed as closely as possible.

Taking as the standard the dimensions above stated for the different parts of a producer-gas plant, a list of dimensions for different horse-power capacities would be about as in the following table.

DIMENSIONS OF GAS PRODUCERS AND SCRUBBERS.

| H.P. | Producers. | | | Wet Scrubbers. | | Dry Scrubbers. | | |
|------|--------------|---------------|---------|----------------|---------|----------------|-------|---------|
| | Inside Diam. | Outside Diam. | Height. | Diam. | Height. | | Diam. | Height. |
| | in. | in. | ft. in. | in. | ft. in. | | in. | ft. in. |
| 25 | 24 | 42 | 6 6 | 18 | 9 9 | Single... | 24 | 3 0 |
| 35 | 28 | 46 | 6 10 | 21 | 10 3 | ...do.... | 28 | 3 0 |
| 50 | 34 | 52 | 7 4 | 26 | 11 0 | Double... | 34 | 6 0 |
| 60 | 37 | 55 | 7 7 | 28 | 11 5 | ...do.... | 37 | 6 0 |
| 75 | 42 | 60 | 8 0 | 32 | 12 0 | ...do.... | 42 | 6 0 |
| 100 | 48 | 72 | 8 6 | 36 | 12 9 | ...do.... | 48 | 7 0 |
| 125 | 54 | 78 | 9 6 | 41 | 14 3 | ...do.... | 52 | 7 0 |
| 150 | 58 | 82 | 9 10 | 44 | 14 9 | ...do.... | 58 | 7 6 |
| 175 | 63 | 87 | 10 3 | 48 | 15 5 | ...do.... | 63 | 7 6 |
| 200 | 68 | 92 | 10 8 | 51 | 16 0 | ...do.... | 68 | 7 6 |

The inside diameter of the producers corresponds to the formula
 $H.P. = 6.25d^2$.

Gas Producer Practice. — The following notes on gas producers are condensed from the catalogue of the Morgan Construction Co.

The Morgan Continuous Gas Producer is made in the following sizes:

| | | | | |
|--|----|----|------|-----|
| Diam. inside of lining, ft..... | 6 | 8 | 10 | 12 |
| Area of gas-making surface, sq. ft..... | 28 | 50 | 78.5 | 113 |
| 24-hour capacity with good coal, tons..... | 4 | 7 | 10 | 15 |
| Diam. of outlet, in. | 20 | 27 | 33 | 40 |

The best coal to buy for a producer in any locality is that which by analysis or calorimeter test shows the most heat units for a dollar. It rarely pays to buy gas coal unless it can be had at a moderate cost over the ordinary steam bituminous grade. For very high temperature melting operations a fairly high percentage of volatile matter is necessary to give a luminous flame and intensify the radiation from the roof of the furnace. Freely burning gas coals are the most easily gasified, and the capacity of the producer to handle these coals is twice as great as when a slaty, dirty coal, high in ash and sulphur, is used. It is usually best to use "run-of-mine" coal, crushed at the mine to pass a 4-in. ring. It never pays to use slack coal, for it cuts down the capacity by choking the blast, which has to be run at high pressure to get through the fire, overheating the gas and lowering the efficiency of the producer.

There is always a certain amount of CO₂ formed, even in the best practice; in fact, it is inevitable, and if kept within proper limits does not constitute a net loss of efficiency, especially with very short gas flues, because the energy of the fuel so burned is represented in the sensible heat or temperature of the gas, and results in delivering a hot gas to the furnace. The best result is at about 4% CO₂, a gas temperature between 1100° and 1200° F., and flues less than 100 ft. long.

The amount of steam required to blow a gas producer is from 33% to 40% of the weight of the fuel gasified. If 30-lbs. of steam is called a standard horse-power, we have therefore to provide about 1 H.P. of steam for every 80 lbs. of coal gasified per hour or for every ton of coal gasified in 24 hours.

In the original Siemens air-blown producer about 70% of the whole gas was inert and 30% combustible. Then with the advent of steam-blown producers the dilution was reduced to about 60%, with 40% combustible. Now, under the system of automatic feed, uniform conditions, perfect distribution and adjustment of the steam blast here presented, we are able to reduce the nitrogen to 50% and sometimes less.

In the best practice the volume of gas from the producer is now reduced to about 60 cu. ft. per pound of coal, of which 30 cu. ft. are nitrogen. These volumes are measured at 60° F.

The temperature of the gas leaving the producer under best modern conditions is about 1200° F. It can be run cooler than this, but not much, except at a sacrifice of both quantity and quality. At this temperature, the sensible heat carried by the gas is 1200×0.35 (average specific heat) = 420 B.T.U. per pound. As one pound of good gas is about 16 cu. ft. and carries about $16 \times 180 = 2880$ heat units at normal temperature, we see that the sensible heat carried away represents about one-seventh, or over 14% of the combustive energy, which is much too large a percentage to lose whenever it can be utilized by using the gas at the temperature at which it is made.

Capacity of Producers. — The capacity of a gas producer is a varying quantity, dependent upon the construction of the producer and upon the quality of the coal supplied to it. The point is, not to push the producer so hard as to burn up the gas within it; also to avoid blowing dust through into the flues. These two limitations in a well-constructed automatically fed gas producer occur at about the same rate of gasification, namely, at about 10 lbs. per sq. ft. of surface per hour with bituminous coal carrying 10% of ash and $1\frac{1}{2}$ % of sulphur. With gas coal, having high volatile percentage and low ash, this rate can be safely increased to 12 lbs. and in some cases to 15 lbs. per sq. ft. At 10 lbs. per sq. ft., the capacity of a gas producer 8 ft. internal diameter is 500 lbs. per hour, which with gas coals may be increased to a maximum of about 700 lbs. It frequently happens that the cheapest coal available is of such quality that neither of these figures can be reached, and the gasification per sq. ft. has to be cut down to 6 or 7 lbs. per hour to get the best results.

Flues. — It is necessary to provide large flue capacity and to carry the full area right up to the furnace ports, which latter may be slightly reduced to give the gas a forward impetus. Generally speaking, the net area of a flue should not be less than $\frac{1}{16}$ of the area of the gas-making surface in the producers supplying it. Or it may be stated thus: — The carrying capacity of a hot gas flue is equivalent to 200 lbs. of coal per hour per sq. ft. of section.

Loss of Energy in a Gas Producer. — The total loss from all sources in the gasification of fuel in a gas producer under fairly good conditions, when the gas is used cold or when its sensible heat is not utilized, ranges between 20% and 25%, which under very bad conditions may be increased to 50%. The loss under favorable conditions, using the gas hot, is reduced to as low as 10%, which also includes the heat of the steam used in blowing.

Test of a Morgan Producer. — The following is the record of a test made in Chicago by Robert W. Hunt & Co. The coal used was Illinois "New Kentucky" run-of-mine of the following analysis: —

Fixed carbon, 50.87; volatile matter, 37.32; moisture, 5.08; ash (1.12 sulphur), 6.73. The average of all the gas analyses by volume is as follows:

CO, 24.5; H, 17.8; CH₄ and C₂H₄, 6.8; total combustibles, 49.1%; CO₂, 3.7; O, 0.4; N, 46.8; total non-combustibles, 50.9%.

Average depth of fuel bed, 3 ft. 4 in. Average pressure of steam on blower, 4.7 lbs. per sq. in. Analysis of ash: combustible, 4.66%; non-combustible, 95.34%. Percentage of fuel lost in the ash, $4.66 \times 6.73 \div 100 = 0.3\%$.

High Temperature Required for Production of CO. — In an ordinary coal fire, with an excess of air CO₂ is produced, with a high temperature. When the thickness of the coal bed is increased so as to choke the air supply CO is produced, with a decreased temperature. It appears, however, that if the temperature is greatly lowered, CO₂ instead of CO will be produced notwithstanding the diminished air supply. Herr Ernst (*Eng'g*, April 4, 1893) holds that the oxidation of C begins at 752° F., and that CO₂ is then formed as the main product, with only a small amount of CO, whether the air be admitted in large or in small quantities. When the rate of combustion is increased and the temperature rises to 1292° F. the chief product is CO₂ even when the exhaust gases contain 20% by volume of CO₂, which is practically the maximum limit, proving that all the oxygen has been consumed. Above 1292° F. the proportion of CO rapidly increases until 1823° F. is reached, when CO is exclusively produced.

Experiments reported by J. K. Clement and H. A. Grine in Bulletin No. 393 of the U. S. Geological Survey, 1909, show that with the rate of flow of gas and the depth of fuel bed which obtain in a gas producer a temperature of 1100° C. (2012° F.) or more is required for the formation of 90% CO gas from CO₂ and charcoal, and 1300° (2372° F.) for the same percentage from CO₂ and coke, and from CO₂ and anthracite coal. With a temperature 100° C. (180° F.) lower than these the resultant gas will contain about 50% CO. It follows that the temperature of the fuel bed of the gas producer must be at least 1300° C. in order to yield the highest possible percentage of CO.

The Mond Gas Producer is described by H. A. Humphrey in *Proc. Inst. C. E.*, vol. cxxix, 1897. The producer, which is combined with a by-product recovery plant, uses cheap bituminous fuel and recovers from it 90 lbs. of sulphate of ammonia per ton, and yields a gas suitable for gas engines and all classes of furnace work. The producer is worked at a much lower temperature than usual, due to the large quantity of superheated steam introduced with the air, amounting to more than twice the weight of the fuel. The gas containing the ammonia is passed through an absorbing apparatus, and treated so that 70% of the original nitrogen of the fuel is recovered. The result of a test showed that for every ton of fuel about 2.5 tons of steam and 3 tons of air are blown through the grate, the mixture being at a temperature of about 480° F. The greater part of this steam passes through the producer undecomposed, its heat being used in a regenerator to furnish fresh steam for the producer. More than 0.5 ton of steam is decomposed in passing through the hot fuel, and nearly 4.5 tons of gas are produced from a ton of coal, equal to about 160,000 cu. ft. at ordinary atmospheric temperature. The gas has a calorific power of 81% of that of the original fuel. Mr. Humphrey gives the following table showing the relative value of different gases.

| Volume per cent. | Mond Producer Gas from Bituminous Fuel. | Siemens Producer Gas. | Dowson Producer Gas from Anthracite. | Leicauchez Producer Gas from Anthracite. | Solvay Coke-Oven Gas. | Coal-Gas (Illuminating). | Pittsburgh Natural Gas. |
|--|---|-----------------------|--------------------------------------|--|-----------------------|--------------------------|-------------------------|
| Hydrogen (H)..... | 24.8 | 8.6 | 18.73 | 20.0 | 56.9 | 48.0 | 22.0 |
| Marsh gas (CH ₄)..... | 2.3 | 2.4 | 0.31 | | 22.6 | 39.5 | 67.0 |
| C _n H _{2n} gases..... | nil | nil | 0.31 | 4.0(?) | 3.0 | 3.8 | 6.0 |
| Carbonic oxide (CO)..... | 13.2 | 24.4 | 25.07 | 21.0 | 8.7 | 7.5 | 0.6 |
| Nitrogen (N)..... | 46.8 | 59.4 | 48.98 | 49.5 | 5.8 | 0.5 | 3.0 |
| Carbonic acid (CO ₂)..... | 12.9 | 5.2 | 6.57 | 5.0 | 3.0 | nil | 0.6 |
| Total volume..... | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Total combustible gases..... | 40.3 | 35.4 | 44.42 | 45.0 | 91.2 | 98.8 | 95.6 |
| Theoretical. | | | | | | | |
| Air required for combustion.... | 112.4 | 101.4 | 113.2 | 154.0 | 410.0 | 581.0 | 806.0 |
| Calorific value per cu. ft., in lb. ° C. units..... | 85.9 | 74.7 | 88.9 | 115.3 | 284.0 | 381.0 | 495.8 |
| Do., B.T.U. per cu. ft..... | 154.6 | 134.5 | 160.0 | 207.5 | 511.2 | 658.8 | 892.4 |
| Do., per litre, gram ° C. units ... | 1,374 | 1,195 | 1,432 | 1,845 | 4,544 | 6,096 | 7,932 |

NOTE. — Where the volume per cent does not add up to 100 the slight difference is due to the presence of oxygen.

The following is the analysis of gas made in a Mond producer at the works of the Solvay Process Co. in Detroit, Mich. (*Mineral Industry*, vol. viii, 1900): CO₂, 14.1; O, 0.3; N, 42.9; H, 25.9; CH₄, 4.1; CO, 12.7. Combustible, 42.7%. Calories per litre, 1540, = 173 B.T.U. per cu. ft.

Relative Efficiencies of Different Coals in Gas Producer and Engine Tests. — The following is a condensed statement of the principal results obtained in the gas-producer tests of the U. S. Geological Survey at St. Louis in 1904. (R. H. Fernald, *Trans. A. S. M. E.*, 1905.)

| Sample. | B.t.u. per lb. combustible. | Pounds per electrical H.P. hour at switchboard. | | | Sample. | B.t.u. per lb. combustible. | Pounds per electrical H.P. hour at switchboard. | | |
|-----------------|-----------------------------|---|-----------|--------------|---------------|-----------------------------|---|-----------|--------------|
| | | Coal as fired. | Dry coal. | Combustible. | | | Coal as fired. | Dry coal. | Combustible. |
| Ala. No. 2.... | 14820 | 1.71 | 1.64 | 1.53 | Ky. No. 3... | 14650 | 2.05 | 1.91 | 1.72 |
| Colo. No. 3.... | 13210 | 2.14 | 1.71 | 1.58 | Mo. No. 2... | 14280 | 1.94 | 1.71 | 1.43 |
| Ill. No. 3..... | 14560 | 1.93 | 1.79 | 1.60 | Mont. No. 1 | 13580 | 2.54 | 2.25 | 1.98 |
| Ill. No. 4..... | 14344 | 2.01 | 1.76 | 1.57 | N. Dak. No. 2 | 12600 | 3.80 | 2.29 | 2.05 |
| Ind. No. 1..... | 14720 | 2.17 | 1.93 | 1.71 | Texas No. 1 | 12945 | 3.34 | 2.22 | 1.88 |
| Ind. No. 2.... | 14500 | 1.68 | 1.55 | 1.39 | Texas No. 2 | 12450 | 2.58 | 1.71 | 1.52 |
| Okla. No. 1.... | 14800 | 1.92 | 1.83 | 1.66 | W. Va. No. 1 | 15350 | 1.60 | 1.57 | 1.48 |
| Okla. No. 4.... | 13890 | 1.57 | 1.43 | 1.17 | W. Va. No. 4 | 15600 | 1.32 | 1.29 | 1.17 |
| Iowa No. 2.... | 13950 | 2.07 | 1.73 | 1.30 | W. Va. No. 7 | 15800 | 1.53 | 1.50 | 1.40 |
| Kan. No. 5.... | 15200 | 1.69 | 1.62 | 1.43 | Wyo. No. 2 | 13820 | 2.28 | 2.07 | 1.60 |

The gas was made in a Taylor pressure producer rated at 250 H.P. Its inside diam. was 7 ft., area of fuel bed 38.5 sq. ft., height of casing 15 ft.; rotative ash table; centrifugal tar extractor. The engine was a 3-cylinder

vertical Westinghouse, 19 in. diam., 22 in. stroke, 200 r.p.m., rated at 235 B.H.P. Comparing the results of the W. Va. No. 7 coal, the best on the list, with the North Dakota coal, the one which gave the poorest results, the heat values per lb. combustible of the coals are as 1 to 0.808; reciprocal, 1 to 1.24; the lbs. combustible per E. H. P. hour as 1 to 1.75, and lbs. coal as fired per E. H. P. hour as 1 to 2.88. The relative thermal efficiencies of the engine with the two coals are as 2.05 to 1.17, or as 1 to 0.578. The analyses by volume of the dry gas obtained from the two coals was:

| | CO ₂ | O | CO | H | CH ₄ | N | Total combustible. |
|-------------|-----------------|------|-------|-------|-----------------|-------|-----------------------|
| N. Dak..... | 10.16 | 0.24 | 15.82 | 11.16 | 3.74 | 58.88 | 40.06 |
| W. Va..... | 8.69 | 0.23 | 20.90 | 14.33 | 4.85 | 51.02 | 30.72 |

The dry-gas analysis shows the North Dakota gas to be by far the best; its much lower result in the engine test is due to the smaller quantity of gas produced per lb. of coal, which was 22.7 cu. ft. per lb. of coal as fired, as compared with 70.6 cu. ft. for the W. Va. coal, measured at 62° F. and 14.7 lb. absolute pressure.

Use of Steam in Producers and in Boiler-furnaces. (R. W. Raymond, *Trans. A. I. M. E.*, xx, 635.) — No possible use of steam can cause a gain of heat. If steam be introduced into a bed of incandescent carbon it is decomposed into hydrogen and oxygen.

The heat absorbed by the reduction of one pound of steam to hydrogen is much greater in amount than the heat generated by the union of the oxygen thus set free with carbon, forming either carbonic oxide or carbonic acid. Consequently, the effect of steam alone upon a bed of incandescent fuel is to chill it. In every water-gas apparatus, designed to produce by means of the decomposition of steam a fuel-gas relatively free from nitrogen, the loss of heat in the producer must be compensated by some reheating device.

This loss may be recovered if the hydrogen of the steam is subsequently burned, to form steam again. Such a combustion of the hydrogen is contemplated, in the case of fuel-gas, as secured in the subsequent use of that gas. Assuming the oxidation of H to be complete, the use of steam will cause neither gain nor loss of heat, but a simple transference, the heat absorbed by steam decomposition being restored by hydrogen combustion. In practice, it may be doubted whether this restoration is ever complete. But it is certain that an excess of steam would defeat the reaction altogether, and that there must be a certain proportion of steam, which permits the realization of important advantages, without too great a net loss in heat.

The advantage to be secured (in boiler furnaces using small sizes of anthracite) consists principally in the transfer of heat from the lower side of the fire, where it is not wanted, to the upper side, where it is wanted. The decomposition of the steam below cools the fuel and the grate-bars, whereas a blast of air alone would produce, at that point, intense combustion (forming at first CO₂), to the injury of the grate, the fusion of part of the fuel, etc.

Gas Analyses by Volume and by Weight. — To convert an analysis of a mixed gas by volume into analysis by weight: Multiply the percentage of each constituent gas by its relative density, viz: CO₂ by 11, O by 8, CO and N each by 7, and divide each product by the sum of the products. Conversely, to convert analysis by weight into analysis by volume, divide the percentage by weight of each gas by its relative density, and divide each quotient by the sum of the quotients.

Gas-fuel for Small Furnaces. — E. P. Reichhelm (*Am. Mach.*, Jan. 10, 1895) discusses the use of gaseous fuel for forge fires, for drop-forging, in annealing-ovens and furnaces for melting brass and copper, for case-hardening, muffle-furnaces, and kilns. Under ordinary conditions, in such furnaces he estimates that the loss by draught, radiation, and the heating of space not occupied by work is, with coal, 80%, with petroleum 70%, and with gas above the grade of producer-gas 25%. He gives the following table of comparative cost of fuels, as used in these furnaces:

| Kind of Gas. | No. of Heat-units in 1000 cu. ft. used. | No. of Heat-units in Furnaces after Deducting 25 % Loss. | Average Cost per 1000 Ft. | Cost of 1,000-Heat-units Obtained in Furnaces. |
|---|---|--|---------------------------|--|
| Natural gas..... | 1,000,000 | 750,000 | | |
| Coal-gas, 20 candle-power..... | 675,000 | 506,250 | \$1.25 | \$2.46 |
| Carburetted water-gas..... | 646,000 | 484,500 | 1.00 | 2.06 |
| Gasolene gas, 20 candle-power..... | 690,000 | 517,500 | .90 | 1.73 |
| Water-gas from coke..... | 313,000 | 234,750 | .40 | 1.70 |
| Water-gas from bituminous coal..... | 377,000 | 282,750 | .45 | 1.59 |
| Water-gas and producer-gas mixed..... | 185,000 | 138,750 | .20 | 1.44 |
| Producer-gas..... | 150,000 | 112,500 | .15 | 1.33 |
| Naphtha-gas, fuel 2½ gals. per 1000 ft..... | 306,365 | 229,774 | .15 | .65 |
| Coal, \$4 per ton, per 1,000,000 heat-units utilized..... | | | | .73 |
| Crude petroleum, 3 cts. per gal., per 1,000,000 heat-units..... | | | | .73 |

Mr. Reichhelm gives the following figures from practice in melting brass with coal and with naphtha converted into gas: 1800 lbs. of metal require 1080 lbs. of coal, at \$4.65 per ton, equal to \$2.51, or, say, 15 cents per 100 lbs. Mr. T.'s report: 2500 lbs. of metal require 47 gals. of naphtha, at 6 cents per gal., equal to \$2.82, or, say, 11¼ cents per 100 lbs.

Blast-Furnace Gas.—The waste-gases from iron blast furnaces were formerly utilized only for heating the blast in the hot-blast ovens and for raising steam for the blowing-engine pumps, hoists and other auxiliary apparatus. Since the introduction of gas engines for blowing and other purposes it has been found that there is a great amount of surplus gas available for other uses, so that a large power plant for furnishing electric current to outside consumers may easily be run by it. H. Freyn, in a paper presented before the Western Society of Engineers (*Eng. Rec.*, Jan. 13, 1906), makes an elaborate calculation for the design of such a plant in connection with two blast furnaces of a capacity of 400 tons of pig iron each per day. Some of his figures are as follows: The two furnaces would supply 4,350,000 cu. ft. of gas per hour, of 90 B.T.U. average heat value per cu. ft. The hot-blast stoves would require 30% of this, or 1,305,000 cu. ft.; the gas-blowing engines 720,000 cu. ft.; pumps, hoists and lighting machinery, 120,000 cu. ft.; gas-cleaning machinery, 120,000 cu. ft.; losses in piping, 48,000 cu. ft.; leaving available for outside uses, in round numbers, 2,000,000 cu. ft. per hour. At the rate of 100 cu. ft. of gas per brake H.P. hour this would supply engines of 20,000 H.P., but assuming that on account of irregular working of the furnaces only half this amount would be available for part of the time, a 10,000-H.P. plant could be run with the surplus gas of the two furnaces. Taking into account the cost of the plant, figured at \$61.60 per B.H.P., interest, depreciation, labor, etc., the annual cost of producing one B.H.P., 24 hours a day, is \$17.88, no value being placed on the blast-furnace gas, and 1 K.W. hour would cost 0.295 cent, which is far below the lowest figure ever reached with a steam-engine power plant.

Blast-furnace gas is composed of nitrogen, carbon dioxide and carbon monoxide, the latter being the combustible constituent. An analysis reported in *Trans. A.I.M.E.*, xvii, 50, is, by volume, CO₂, 7.08; CO, 27.80; O, 0.10; N, 65.02. The relative proportions of CO₂ and CO vary considerably with the conditions of the furnace.

ACETYLENE AND CALCIUM CARBIDE.

Acetylene, C₂H₂, contains 12 parts C and 1 part H, or 92.3% C, 7.7% H. It is described as follows in a paper on Calcium Carbide and Acetylene by J. B. Morehead (*Am. Gas Light Jour.*, July 10, 1905):

Acetylene is a colorless and tasteless gas. When pure it has a sweet, ethereal odor, but in the commercial form it carries small percentages of phosphoreted and sulphureted hydrogen which give it a pungent odor. One cu. ft. requires 11.91 cu. ft. of air for its complete combustion. Its

specific gravity is 0.92, air being 1. It is the nearest approach to gaseous carbon, and it possesses a higher candle power than any other known substance, or 240 candles for 5 cu. ft. It is soluble in its own volume of water, and in varying proportions in ether, alcohol, turpentine and acetone. It liquefies under a pressure of 700 lbs. per sq. in. at 70° F. The pressure necessary for liquefaction varies directly with the temperature up to 98°, which is its critical temperature, beyond which it is impossible to liquefy the gas at any pressure.

When calcium carbide is brought into contact with water, the calcium robs the water of its oxygen and forms lime and thus frees the hydrogen, which combines with the carbon of the carbide to form acetylene. Sixty-four lbs. of calcium carbide combine with 36 lbs. of water and produce 26 lbs. of acetylene and 17 lbs. of pure, slacked lime. [The chemical reaction is $\text{CaC}_2 + 2\text{H}_2\text{O} = \text{C}_2\text{H}_2 + \text{Ca(OH)}_2$.]

Chemically pure calcium carbide will yield at 70° F. and 30 in. mercury, 5.83 cu. ft. acetylene per pound of carbide. Commercially pure carbide is guaranteed to yield 5 cu. ft. of acetylene per pound, and usually exceeds the guarantee by a few per cent. The reaction between calcium carbide and water, and the subsequent slacking of the calcium oxide produced, give rise to considerable heat. This heat from one pound of chemically pure calcium carbide amounts to sufficient to raise the temperature of 4.1 lbs. of water from the freezing to the boiling point.

There are two types of generators; one in which a varying quantity of water is dropped on to the carbide, the other in which the carbide is dropped into a large excess of water. Owing to the large amount of heat generated by the reaction, and the susceptibility of the acetylene to heat, the first, or dry type, is confined to lamps and to small machines.

Acetylene contains 1685 B.T.U. per cubic foot as compared with 1000 for natural gas and 600 for coal or water gas. At the present state of development of the acetylene industry and the calcium carbide manufacture, this gas will not compete with coal gas or water gas, or with electricity as supplied in our cities. Acetylene may be stored under pressure for railway and other portable lighting, and it may be absorbed in acetone and used for the same purpose.

Calcium carbide was discovered on May 4, 1892, at the plant of the Willson Aluminum Co., in North Carolina. It is a crystalline body, hard, brittle and varying in color from almost black to brick red. Its specific gravity is 2.26. A cubic foot of crushed carbide weighs 138 lbs., and in weight, color and most of its physical characteristics is about like granite. If broken hot, the fracture shows a handsome, bluish purple iridescence and the crystals are apt to be quite large.

Calcium carbide, CaC_2 , contains 62.5% Ca and 37.5% C. It is insoluble in most acids and in all alkalies, it is non-inflammable, infusible, non-explosive, unaffected by jars, concussions or time, and, except for the property of giving off acetylene when brought in contact with water, it is an inert and stable body. It is made by the reduction in an electric arc furnace of a mixture of finely pulverized and intimately mixed calcium oxide or quicklime and carbon in the shape of coke. [$3\text{C} + \text{CaO} = \text{CaC}_2 + \text{CO}$.] The temperature is calculated to be from 5000 to 8000° F. The furnaces employ from 250 to 350 electric H.P. each and produce about one ton a day. The output is crushed to different sizes and it is sold for \$70 per ton at the works.

The entire use for calcium carbide is for the production of acetylene. [Wohler, in 1862, obtained calcium carbide by heating an alloy of calcium and zinc together with carbon to a very high temperature.]

Acetylene Generators and Burners. — Lewes classifies acetylene generators under four types: (1) Those in which water drips or flows slowly on a mass of carbide; (2) those in which water rises, coming in contact with a mass of carbide; (3) those in which water rises, coming in contact with successive layers of carbide; (4) those in which the carbide is dropped or plunged into an excess of water. He shows that the first two classes are dangerous; that some generators of the third class are good, but that those of the fourth are the best.

Of the various burners used for acetylene, those of the Naphey type are among the most satisfactory. Two tubes leading from the base of the burner are so adjusted as to cause two jets of flame to impinge upon each other at some little distance from the nozzles, and mutually to splay each other out into a flat flame. The tips of the nozzles, usually of steatite, are

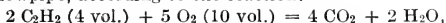
formed on the principle of the Bunsen burner, insuring a thorough mixture of the acetylene with enough air to give the best illumination. (H. C. Biddle, *Cal. Jour. of Tech.*, 1907.)

Acetylene gas is an endothermic compound. In its formation heat is absorbed, and there resides in the acetylene molecule the power of spontaneously decomposing and liberating this heat if it is subjected to a temperature or pressure beyond the capacity of its unstable nature to withstand. (Thos. L. White, *Eng. Mag.*, Sept., 1908.) Mr. White recommends the use of acetylene for carbureting the alcohol used in alcohol motors for automobiles.

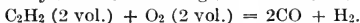
The Acetylene Blowpipe. — (*Machy.*, July, 1907.) — The acetylene is produced in a generator and stored in a tank at a pressure of 2.2 to 3 lbs. per sq. in. The oxygen is compressed in a tank at about 150 lbs. pressure. The acetylene is conveyed to the burner through a 1-in. pipe with one 3/8-in. branch leading to each blowpipe connection. The oxygen is conveyed through 3/8-in. pipe with 1/4-in. branches. The blowpipe is of brass, made on the injector principle. As acetylene is so rich in carbon — containing 92.3 % — it is possible, when mixed with air in a Bunsen burner, to obtain 3100° F., and when combined with oxygen, 6300° F., which is the hottest flame known as a product of combustion, and nearly equals the electric arc. This is about 1200° higher than the oxy-hydrogen blowpipe flame.

In lighting the blowpipe, the acetylene is first turned on full; then the oxygen is added until the flame is only a single cone. At the apex of this cone is a temperature of 6300° F. In welding, this point is held from 1/8 to 1/4 in. distant from the metal to be welded. Too much acetylene produces two cones and a white color; an excess of oxygen is indicated by a violet tint.

Theoretically, 2 1/2 volumes of oxygen are required for complete combustion of 1 volume of acetylene. Practically, however, with the blowpipe the best welding results are obtained with 1.7 volumes of oxygen to 1 volume of acetylene. The acetylene is, therefore, not completely burned with the blowpipe, according to the reaction:

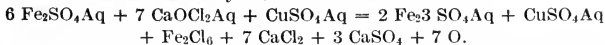


but it is incompletely burned according to the reaction:



Making Oxygen for the Blowpipe. — The distinctive feature which has done the most to make the acetylene welding process of wide commercial value is the introduction of a means for producing oxygen. By combining a chemical product, known as "epurite," with water, pure oxygen is easily obtained. Epurite is composed of chloride of lime, sulphate of copper and sulphate of iron. The sulphate of copper is pulverized and mixed dry with the chloride of lime. In making oxygen, 50 lbs. of this dry mixture are dissolved in warm water. To this solution is added a solution of about 7 lbs. of sulphate of iron dissolved in one gallon of water.

The oxygen-generating apparatus consists of two lead-lined chambers with a scrubber and settling chamber between. One generator is filled with lukewarm water to which one chemical charge is added. While this solution is being stirred with an agitator a solution of iron sulphate is added which acts as a catalyzer. The reaction is:



The oxygen, liberated, passes through a scrubber and a water-sealed trap into a gasometer; from which it is compressed to 10 atmospheres, with an air compressor, into a pressure storage tank.

The Theory and Practice of Oxy-Acetylene Welding is described in an illustrated article by J. F. Springer in *Indust. Eng'g.*, Oct., 1909.

IGNITION TEMPERATURE OF GASES.

Mayer and Münch (*Berichte der deutscher Gesellschaft*, xxvi, 2241) give the following:

| | | |
|-------------------------------------|---------|----------|
| Marsh gas, C_2H_4 , | 667° C. | 1233° F. |
| Ethane, C_2H_6 , | 616 | 1141 |
| Propane, C_3H_8 , | 547 | 1017 |
| Acetylene, C_2H_2 , | 580 | 1076 |
| Propylene, C_3H_6 , | 504 | 939 |

ILLUMINATING-GAS.

Coal-gas is made by distilling bituminous coal in retorts. The retort is usually a long horizontal semi-cylindrical or α shaped chamber, holding from 160 to 300 lbs. of coal. The retorts are set in "benches" of from 3 to 9, heated by one fire, which is generally of coke. The vapors distilled from the coal are converted into a fixed gas by passing through the retort, which is heated almost to whiteness.

The gas passes out of the retort through an "ascension-pipe" into a long horizontal pipe called the hydraulic main, where it deposits a portion of the tar it contains; thence it goes into a condenser, a series of iron tubes surrounded by cold water, where it is freed from condensable vapors, as ammonia-water, then into a washer, where it is exposed to jets of water, and into a scrubber, a large chamber partially filled with trays made of wood or iron, containing coke, fragments of brick or paving-stones, which are wet with a spray of water. By the washer and scrubber the gas is freed from the last portion of tar and ammonia and from some of the sulphur compounds. The gas is then finally purified from sulphur compounds by passing it through lime or oxide of iron. The gas is drawn from the hydraulic main and forced through the washer, scrubber, etc., by an exhaustor or gas pump.

The kind of coal used is generally caking bituminous, but as usually this coal is deficient in gases of high illuminating power, there is added to it a portion of cannel coal or other enricher.

The following table, abridged from one in Johnson's Cyclopaedia, shows the analysis, candle-power, etc., of some gas-coals and enrichers:

| Gas-coals, etc. | Vol. Matter. | Fixed Carb. | Ash. | Gas per ton of 2240 lbs. in cu. ft. | Cand.-power of Gas. | Coke per ton of 2240 lbs. | | Gas purified by 1 bush. of lime, in cu. ft. |
|-----------------------|--------------|-------------|-------|-------------------------------------|---------------------|---------------------------|-------|---|
| | | | | | | lbs. | bush. | |
| Pittsburgh, Pa..... | 36.76 | 51.93 | 7.07 | | | | | |
| Westmoreland, Pa..... | 36.00 | 58.00 | 6.00 | 10,642 | 16.62 | 1544 | 40 | 6420 |
| Sterling, O..... | 37.50 | 56.90 | 5.60 | 10,528 | 18.81 | 1480 | 36 | 3993 |
| Despard, W. Va..... | 40.00 | 53.30 | 6.70 | 10,765 | 20.41 | 1540 | 36 | 2494 |
| Darlington, O..... | 43.00 | 40.00 | 17.00 | 9,800 | 34.98 | 1320 | 32 | 2806 |
| Petonia, W. Va..... | 46.00 | 41.00 | 13.00 | 13,200 | 42.79 | 1380 | 32 | 4510 |
| Grahamite, W. Va..... | 53.50 | 44.50 | 2.00 | 15,000 | 28.70 | 1056 | 44 | |

The products of the distillation of 100 lbs. of average gas-coal are about as follows. They vary according to the quality of coal and the temperature of distillation.

Coke, 64 to 65 lbs.; tar, 6.5 to 7.5 lbs.; ammonia liquor, 10 to 12 lbs.; purified gas, 15 to 12 lbs.; impurities and loss, 4.5% to 3.5%.

The composition of the gas by volume ranges about as follows: Hydrogen, 38% to 48%; carbonic oxide, 2% to 14%; marsh-gas (Methane, CH_4), 43% to 31%; heavy hydrocarbons (C_nH_{2n} , ethylene, propylene, benzole vapor, etc.), 7.5% to 4.5%; nitrogen, 1% to 3%.

In the burning of the gas the nitrogen is inert; the hydrogen and carbonic oxide give heat but no light. The luminosity of the flame is due to the decomposition by heat of the heavy hydrocarbons into lighter hydrocarbons and carbon, the latter being separated in a state of extreme subdivision. By the heat of the flame this separated carbon is heated to intense whiteness, and the illuminating effect of the flame is due to the light of incandescence of the particles of carbon.

The attainment of the highest degree of luminosity of the flame depends upon the proper adjustment of the proportion of the heavy hydro-

carbons (with due regard to their individual character) to the nature of the diluent mixed therewith.

Investigations of Percy F. Frankland show that mixtures of ethylene and hydrogen cease to have any luminous effect when the proportion of ethylene does not exceed 10% of the whole. Mixtures of ethylene and carbonic oxide cease to have any luminous effect when the proportion of the former does not exceed 20%, while all mixtures of ethylene and marsh-gas have more or less luminous effect. The luminosity of a mixture of 10% ethylene and 90% marsh-gas being equal to about 18 candles, and that of one of 20% ethylene and 80% marsh-gas about 25 candles. The illuminating effect of marsh-gas alone, when burned in an argand burner, is by no means inconsiderable.

For further description, see the treatises on gas by King, Richards, and Hughes; also Appleton's Cyc. Mech., vol. i. p. 900.

Water-gas. — Water-gas is obtained by passing steam through a bed of coal, coke, or charcoal heated to redness or beyond. The steam is decomposed, its hydrogen being liberated and its oxygen burning the carbon of the fuel, producing carbonic-oxide gas. The chemical reaction is, $C + H_2O = CO + 2H$, or $2C + 2H_2O = C + CO_2 + 4H$, followed by a splitting up of the CO_2 , making $2CO + 4H$. By weight the normal gas $CO + 2H$ is composed of $C + O + H = 28$ parts CO and 2 parts H ,

or 93.33% CO and 6.67% H ; by volume it is composed of equal parts of carbonic oxide and hydrogen. Water-gas produced as above described has great heating-power, but no illuminating-power. It may, however, be used for lighting by causing it to heat to whiteness some solid substance, as is done in the Welsbach incandescent light.

An illuminating-gas is made from water-gas by adding to it hydrocarbon gases or vapors, which are usually obtained from petroleum or some of its products. A history of the development of modern illuminating water-gas processes, together with a description of the most recent forms of apparatus, is given by Alex. C. Humphreys, in a paper on "Water-gas in the United States," read before the Mechanical Section of the British Association for Advancement of Science, in 1889. After describing many earlier patents, he states that success in the manufacture of water-gas may be said to date from 1874, when the process of T. S. C. Lowe was introduced. All the later most successful processes are the modifications of Lowe's, the essential features of which were "an apparatus consisting of a generator and superheater internally fired; the superheater being heated by the secondary combustion from the generator, the heat so stored up in the loose brick of the superheater being used, in the second part of the process, in the fixing or rendering permanent of the hydrocarbon gases; the second part of the process consisting in the passing of steam through the generator fire, and the admission of oil or hydrocarbon at some point between the fire of the generator and the loose filling of the superheater."

The water-gas process thus has two periods: first the "blow," during which air is blown through the bed coal in the generator, and the partially burned gaseous products are completely burned in the superheater, giving up a great portion of their heat to the fire-brick work contained in it, and then pass out to a chimney; second, the "run" during which the air blast is stopped, the opening to the chimney closed, and steam is blown through the incandescent bed of fuel. The resulting water-gas passing into the carburetting chamber in the base of the superheater is there charged with hydrocarbon vapors, or spray (such as naphtha and other distillates or crude oil), and passes through the superheater, where the hydrocarbon vapors become converted into fixed illuminating gases. From the superheater the combined gases are passed, as in the coal-gas process, through washers, scrubbers, etc., to the gas-holder. In this case, however, there is no ammonia to be removed.

The specific gravity of water-gas increases with the increase of the heavy hydrocarbons which give illuminating power. The following figures, taken from different authorities, are given by F. H. Shelton in a paper on "Water-gas," read before the Ohio Gas Light Association, in 1894:

| | | | | | | | | | |
|--------------------|------|------|------|--------|------|------|------|------|----------------|
| Candle-power.... | 19.5 | 20. | 22.5 | 24. | 25.4 | 26.3 | 28.3 | 29.6 | .30 to 31.9 |
| Sp. gr. (Air=1) .. | .571 | .630 | .589 | .60 to | .67 | .64 | .602 | .70 | .65 .65 to .71 |

Analyses of Water-gas and Coal-gas Compared.

The following analyses are taken from a report of Dr. Gideon E. Moore on the Granger Water-gas, 1885:

| | Composition by Vol. | | | Composition by Weight. | | |
|---------------------------|---------------------|--------|-------------------------------|------------------------|---------|-----------|
| | Water-gas. | | Coal-gas. Heidel- berg. | Water-gas. | | Coal-gas. |
| | Wor- cester. | Lake. | | Wor- cester. | Lake. | |
| Nitrogen..... | 2.64 | 3.85 | 2.15 | 0.04402 | 0.06175 | 0.04559 |
| Carbonic acid..... | 0.14 | 0.30 | 3.01 | 0.00365 | 0.00753 | 0.09992 |
| Oxygen..... | 0.06 | 0.01 | 0.65 | 0.00114 | 0.00018 | 0.01569 |
| Ethylene..... | 11.29 | 12.80 | 2.55 | 0.18759 | 0.20454 | 0.05389 |
| Propylene..... | 0.00 | 0.00 | 1.21 | | | 0.03834 |
| Benzole vapor..... | 1.53 | 2.63 | 1.33 | 0.07077 | 0.11700 | 0.07825 |
| Carbonic oxide..... | 28.26 | 23.58 | 8.88 | 0.46934 | 0.37664 | 0.18758 |
| Marsh-gas..... | 18.88 | 20.95 | 34.02 | 0.17928 | 0.19133 | 0.41087 |
| Hydrogen..... | 37.20 | 35.88 | 46.20 | 0.04421 | 0.04103 | 0.06987 |
| | 100.00 | 100.00 | 100.00 | 1.00000 | 1.00000 | 1.00000 |
| Density: Theory..... | 0.5825 | 0.6057 | 0.4580 | | | |
| Practice..... | 0.5915 | 0.6018 | | | | |
| B.T.U. from 1 cu. ft.: | | | | | | |
| Water liquid..... | 650.1 | 688.7 | 642.0 | | | |
| " vapor..... | 597.0 | 646.6 | 577.0 | | | |
| Flame-temperature, °F ... | 5311.2 | 5281.1 | 5202.9 | | | |
| Average candle-power..... | 22.06 | 26.31 | | | | |

The heating-values (B.T.U.) of the gases are calculated from the analysis by weight, by using the multipliers given below (computed from results of J. Thomsen), and multiplying the result by the weight of 1 cu. ft. of the gas at 62° F., and atmospheric pressure.

The flame-temperatures (theoretical) are calculated on the assumption of complete combustion of the gases in air, without excess of air.

The candle-power was determined by photometric tests, using a pressure of 1/2-in. water-column, a candle consumption of 120 grains of spermaceti per hour, and a meter rate of 5 cu. ft. per hour, the result being corrected for a temperature at 62° F. and a barometric pressure of 30 in. It appears that the candle-power may be regulated at the pleasure of the person in charge of the apparatus, the range of candle-power being from 20 to 29 candles, according to the manipulation employed.

Calorific Equivalents of Constituents of Illuminating-gas.

| | Heat-units from 1 lb. | | | Heat-units from 1 lb. | |
|--------------------|-----------------------|-----------------|------------------|-----------------------|-----------------|
| | Water Liquid. | Water Vapor. | | Water Liquid. | Water Vapor. |
| Ethylene..... | 21,524.4 | 20,134.8 | Carbonic oxide . | 4,395.6 | 4,395.6 |
| Propylene..... | 21,222.0 | 19,834.2 | Marsh-gas..... | 24,021.0 | 21,592.8 |
| Benzole vapor..... | 18,954.0 | 17,847.0 | Hydrogen..... | 61,524.0 | 51,804.0 |

Efficiency of a Water-gas Plant. — The practical efficiency of an illuminating water-gas setting is discussed in a paper by A. G. Glasgow (*Proc. Am. Gaslight Assn.*, 1890) from which the following is abridged:

The results refer to 1000 cu. ft. of unpurified carburetted gas, reduced to 60° F. The total anthracite charged per 1000 cu. ft. of gas was 33.4 lbs.,

ash and unconsumed coal removed 9.9 lbs., leaving total combustible consumed 23.5 lbs., which is taken to have a fuel-value of 14,500 B.T.U. per pound, or a total of 340,750 heat-units.

| | | Com- position by Vol. | Weight per 100 cu. ft. | Com- position by W'ht. | Specific Heat. |
|---|--------------------------------------|--------------------------------|---------------------------------|---------------------------------|-------------------|
| I. Carburetted Water-gas.. | CO ₂ + H ₂ S. | 3.8 | .465842 | .09647 | .02088 |
| | C _n H _{2n} | 14.6 | 1.139968 | .23607 | .08720 |
| | CO..... | 28.0 | 2.1868 | .45285 | .11226 |
| | CH ₄ | 17.0 | .75854 | .15710 | .09314 |
| | H..... | 35.6 | .1991464 | .04124 | .14041 |
| | N..... | 1.0 | .078596 | .01627 | .00397 |
| | | 100.0 | 4.8288924 | 1.00000 | .45786 |
| II. Uncarburetted gas..... | CO ₂ | 3.5 | .429065 | .1019 | .02205 |
| | CO..... | 43.4 | 3.389540 | .8051 | .19958 |
| | H..... | 51.8 | .289821 | .0688 | .23424 |
| | N..... | 1.3 | .102175 | .0242 | .00591 |
| | | 100.0 | 4.210601 | 1.0000 | .46178 |
| III. Blast products escap- ing from superheater. | CO ₂ | 17.4 | 2.133066 | .2464 | .05342 |
| | O..... | 3.2 | .2856096 | .0329 | .00718 |
| | N..... | 79.4 | 6.2405224 | .7207 | .17585 |
| | | 100.0 | 8.6591980 | 1.0000 | .23645 |
| IV. Generator blast-gases.. | CO ₂ | 9.7 | 1.189123 | .1436 | .031075 |
| | CO..... | 17.8 | 1.390180 | .1680 | .041647 |
| | N..... | 72.5 | 5.698210 | .6884 | .167970 |
| | | 100.0 | 8.277513 | 1.0000 | .240692 |

The heat-energy absorbed by the apparatus is $23.5 \times 14,500 = 340,750$ heat-units = *A*. Its disposition is as follows:

B, the energy of the CO produced;

C, the energy absorbed in the decomposition of the steam;

D, the difference between the sensible heat of the escaping illuminating-gases and that of the entering oil;

E, the heat carried off by the escaping blast products;

F, the heat lost by radiation from the shells;

G, the heat carried away from the shells by convection (air-currents);

H, the heat rendered latent in the gasification of the oil;

I, the sensible heat in the ash and unconsumed coal recovered from the generator.

The heat equation is $A = B + C + D + E + F + G + H + I$; *A* being known. A comparison of the CO in Tables I and II show that $\frac{280}{434}$,

or 64.5% of the volume of carburetted gas, is pure water-gas, distributed thus: CO₂, 2.3%; CO, 28.0%; H, 33.4%; N, 0.8%; = 64.5%. 1 lb. of CO at 60° F. = 13,531 cu. ft. CO per 1000 cu. ft. of gas = $280 \div 13,531 = 20.694$ lbs. Energy of the CO = $20.694 \times 4395.6 = 91,043$ heat-units = *B*. 1 lb. of H at 60° F. = 189.2 cu. ft. H per M of gas = $334 \div 189.2 = 1.7653$ lbs. Energy of the H per lb. (according to Thomsen, considering the steam generated by its combustion to be condensed to water at 75° F.) = 61,524 B.T.U. In Mr. Glasgow's experiments the steam entered the generator at 331° F.; the heat required to raise the product of combustion of 1 lb. of H, viz., 8.98 lbs. H₂O, from water at 75° to steam at 331° must therefore be deducted from Thomsen's figure, or $61,524 - (8.98 \times 1140.2) = 51,285$ B.T.U. per lb. of H. Energy of the H, then, is $1.7653 \times 51,285 = 90,533$ heat-units = *C*. The heat

lost due to the sensible heat in the illuminating-gases, their temperature being 1450° F., and that of the entering oil 235° F., is 48.29 (weight) \times .45786 (sp. heat) \times 1215 (rise of temperature) = 26,864 heat-units = *D*.
(The specific heat of the entering oil is approximately that of the issuing gas.)

The heat carried off in 1000 cu. ft. of the escaping blast products is 86.592 (weight) \times .23645 (sp. heat) \times 1474° (rise of temp.) = 30,180 heat-units: the temperature of the escaping blast gases being 1550° F., and that of the entering air 76° F. But the amount of the blast gases, by registration of an anemometer, checked by a calculation from the analyses of the blast gases, was 2457 cubic feet for every 1000 cubic feet of carburetted gas made. Hence the heat carried off per M. of carburetted gas is 30,180 \times 2.457 = 74,152 heat-units = *E*.

Experiments made by a radiometer covering four square feet of the shell of the apparatus gave figures for the amount of heat lost by radiation = 12,454 heat-units = *F*, and by convection = 15,696 heat-units = *G*.

The heat rendered latent by the gasification of the oil was found by taking the difference between all the heat fed into the carburetter and superheater and the total heat dissipated therefrom to be 12,841 heat-units = *H*. The sensible heat in the ash and unconsumed coal is 9.9 lbs. \times 1500° \times .25 (sp. ht.) = 3712 heat-units = *I*.

The sum of all the items *B* + *C* + *D* + *E* + *F* + *G* + *H* + *I* = 327,295 heat-units, which subtracted from the heat-energy of the combustible consumed, 340,750 heat-units, leaves 13,455 heat-units, or 4 per cent unaccounted for.

Of the total heat-energy of the coal consumed, or 340,750 heat-units, the energy wasted is the sum of items *D*, *E*, *F*, *G*, and *I*, amounting to 132,878 heat-units, or 39 per cent; the remainder, or 207,872 heat-units, or 61 per cent, being utilized. The efficiency of the apparatus as a heat machine is therefore 61 per cent.

Five gallons, or 35 lbs. of crude petroleum, were fed into the carburetter per 1000 cu. ft. of gas made; deducting 5 lbs. of tar recovered, leaves 30 lbs. \times 20,000 = 600,000 heat-units as the net heating-value of the petroleum used. Adding this to the heating-value of the coal, 340,750 B.T.U., gives 940,750 heat-units, of which there is found as heat-energy in the carburetted gas, as in the table below, 764,050 heat-units, or 81 per cent, which is the commercial efficiency of the apparatus, i.e., the ratio of the energy contained in the finished product to the total energy of the coal and oil consumed.

| The heating-power per M. cu. ft. of the carburetted gas is | | The heating-power per M. of the uncarburetted gas is | |
|---|--|---|--|
| CO ₂ | 38.0 | CO ₂ | 35.0 |
| C ₃ H ₆ * | 146.0 \times .117220 \times 21222.0 = 363200 | CO | 434.0 \times .078100 \times 4395.6 = 148991 |
| CO | 280.0 \times .078100 \times 4395.6 = 96120 | H | 518.0 \times .005594 \times 61524.0 = 178277 |
| CH ₄ | 170.0 \times .044620 \times 24021.0 = 182210 | N | 13.0 |
| H | 356.0 \times .005594 \times 61524.0 = 122520 | | |
| N | 10.0 | | |
| | 1000.0 | | 1000.0 |
| | 764050 | | 327268 |

The candle-power of the gas is 31, or 6.2 candle-power per gallon of oil used. The calculated specific gravity is .6355, air being 1.

For description of the operation of a modern carburetted water-gas plant, see paper by J. Stelfox, *Eng'g*, July 20, 1894, p. 89.

Space Required for a Water-gas Plant. — Mr. Shelton, taking 15 modern plants of the form requiring the most floor-space, figures the average floor-space required per 1000 cubic feet of daily capacity as follows:

| Water-gas Plants of Capacity in 24 hours of | Require an Area of Floor-space for each 1000 cu. ft. of about |
|--|--|
| 100,000 cubic feet | 4 square feet. |
| 200,000 " " | 3.5 " " |
| 400,000 " " | 2.75 " " |
| 600,000 " " | 2 to 2.5 sq. ft. |
| 7 to 10 million cubic feet | 1.25 to 1.5 sq. ft. |

* The heating-value of the illuminants C_nH_{2n} is assumed to equal that of C₃H₆.

These figures include scrubbing and condensing rooms, but not boiler and engine rooms. In coal-gas plants of the most modern and compact forms one with 16 benches of 9 retorts each, with a capacity of 1,500,000 cubic feet per 24 hours, will require 4.8 sq. ft. of space per 1000 cu. ft. of gas, and one of 6 benches of 6 retorts each, with 300,000 cu. ft. capacity per 24 hours, will require 6 sq. ft. of space per 1000 cu. ft. The storage-room required for the gas-making materials is: for coal-gas, 1 cubic foot of room for every 232 cubic feet of gas made; for water-gas made from coke, 1 cubic foot of room for every 373 cu. ft. of gas made; and for water-gas made from anthracite, 1 cu. ft. of room for every 645 cu. ft. of gas made.

The comparison is still more in favor of water-gas if the case is considered of a water-gas plant added as an auxiliary to an existing coal-gas plant; for, instead of requiring further space for storage of coke, part of that already required for storage of coke produced and not at once sold can be cut off, by reason of the water-gas plant creating a constant demand for more or less of the coke so produced.

Mr. Shelton gives a calculation showing that a water-gas of 0.625 sp. gr. would require gas-mains eight per cent greater in diameter than the same quantity coal-gas of 0.425 sp. gr. if the same pressure is maintained at the holder. The same quantity may be carried in pipes of the same diameter if the pressure is increased in proportion to the specific gravity. With the same pressure the increase of candle-power about balances the decrease of flow. With five feet of coal-gas, giving, say, eighteen candle-power, 1 cubic foot equals 3.6 candle-power; with water-gas of 23 candle-power, 1 cubic foot equals 4.6 candle-power, and 4 cubic feet gives 18.4 candle-power, or more than is given by 5 cubic feet of coal-gas. Water-gas may be made from oven-coke or gas-house coke as well as from anthracite coal. A water-gas plant may be conveniently run in connection with a coal-gas plant, the surplus retort coke of the latter being used as the fuel of the former.

In coal-gas making it is impracticable to enrich the gas to over twenty candle-power without causing too great a tendency to smoke, but water-gas of as high as thirty candle-power is quite common. A mixture of coal-gas and water-gas of a higher C.P. than 20 can be advantageously distributed.

Fuel-value of Illuminating-gas. — E. G. Love (*School of Mines Quarterly*, January, 1892) describes F. W. Hartley's calorimeter for determining the calorific power of gases, and gives results obtained in tests of the carbureted water-gas made by the municipal branch of the Consolidated Co. of New York. The tests were made from time to time during the past two years, and the figures give the heat-units per cubic foot at 60° F. and 30 inches pressure: 715, 692, 725, 732, 691, 738, 735, 703, 734, 730, 731, 727. Average, 721 heat-units. Similar tests of mixtures of coal- and water-gases made by other branches of the same company give 694, 715, 684, 692, 727, 665, 695, and 686 heat-units per foot, or an average of 694.7. The average of all these tests was 710.5 heat-units, and this we may fairly take as representing the calorific power of the illuminating gas of New York. One thousand feet of this gas, costing \$1.25, would therefore yield 710,500 heat-units, which would be equivalent to 568,400 heat-units for \$1.00.

The common coal-gas of London, with an illuminating power of 16 to 17 candles, has a calorific power of about 668 units per foot, and costs from 60 to 70 cents per thousand.

The product obtained by decomposing steam by incandescent carbon, as effected in the Motay process, consists of about 40% of CO, and a little over 50% of H.

This mixture would have a heating-power of about 300 units per cubic foot, and if sold at 50 cents per 1000 cubic feet would furnish 600,000 units for \$1.00, as compared with 568,400 units for \$1.00 from illuminating gas at \$1.25 per 1000 cubic feet. This illuminating-gas if sold at \$1.15 per thousand would therefore be a more economical heating agent than the fuel-gas mentioned, at 50 cents per thousand, and be much more advantageous than the latter, in that one main, service, and meter could be used to furnish gas for both lighting and heating.

A large number of fuel-gases tested by Mr. Love gave from 184 to 470 heat-units per foot, with an average of 309 units.

Taking the cost of heat from illuminating-gas at the lowest figure given

by Mr. Love, viz., \$1.00 for 600,000 heat-units, it is a very expensive fuel, equal to coal at \$40 per ton of 2000 lbs., the coal having a calorific power of only 12,000 heat-units per pound, or about 83% of that of pure carbon: 600,000: (12,000 × 2000) :: \$1 : \$40.

FLOW OF GAS IN PIPES.

The rate of flow of gases of different densities, the diameter of pipes required, etc., are given in King's Treatise on Coal Gas, vol. ii, 374, as follows:

$$\left. \begin{array}{l} \text{If } d = \text{diameter of pipe in inches,} \\ Q = \text{quantity of gas in cu. ft. per} \\ \quad \text{hour,} \\ l = \text{length of pipe in yards,} \\ h = \text{pressure in inches of water,} \\ s = \text{specific gravity of gas, air} \\ \quad \text{being 1,} \end{array} \right\} \begin{array}{l} d = \sqrt[5]{\frac{Q^2 sl}{(1350)^2 h}}, \\ h = \frac{Q^2 sl}{(1350)^2 d^5}, \\ Q = 1350 d^2 \sqrt{\frac{dh}{sl}} = 1350 \sqrt{\frac{d^5 h}{sl}}. \end{array}$$

$$\text{Molesworth gives } Q = 1000 \sqrt{\frac{d^5 h}{sl}}.$$

$$\text{J. P. Gill, } \textit{Am. Gas-light Jour.}, 1894, \text{ gives } Q = 1291 \sqrt{\frac{d^5 h}{s(l+d)}}.$$

This formula is said to be based on experimental data, and to make allowance for obstructions by tar, water, and other bodies tending to check the flow of gas through the pipe.

King's formula translated into the form of the common formula for the flow of compressed air or steam in pipes, $Q = c \sqrt{(p_1 - p_2) d^5 / wL}$, in which $Q = \text{cu. ft. per min.}$, $p_1 - p_2 = \text{difference in pressure in lbs. per sq. in.}$; $w = \text{density in lbs. per cu. ft.}$, $L = \text{length in ft.}$, $d = \text{diam. in ins.}$, gives 56.6 for the value of the coefficient c , which is nearly the same as that commonly used (60) in calculations of the flow of air in pipes. For values of c based on Darcy's experiments on flow of water in pipes see Flow of Steam.

An experiment made by Mr. Clegg, in London, with a 4-in. pipe, 6 miles long, pressure 3 in. of water, specific gravity of gas 0.398, gave a discharge into the atmosphere of 852 cu. ft. per hour, after a correction of 33 cu. ft. was made for leakage.

Substituting this value, 852 cu. ft., for Q in the formula $Q = C \sqrt{d^5 h + sl}$, we find C , the coefficient, = 997, which corresponds nearly with the formula given by Molesworth.

Wm. Cox (*Am. Mach.*, Mar. 20, 1902) gives the following formula for flow of gas in long pipes.

$$Q = 3000 \sqrt{\frac{d^5 \times (p_1^2 - p_2^2)}{l}} = 41.3 \sqrt{\frac{d^5 \times (p_1^2 - p_2^2)}{L}}.$$

$Q = \text{discharge in cu. ft. per hour at atmospheric pressure; } d = \text{diam. of pipe in ins.; } p_1 = \text{initial and } p_2 = \text{terminal absolute pressure, lbs. per sq. in.; } l = \text{length of pipe in feet, } L = \text{length in miles. For } p_1^2 - p_2^2 \text{ may be substituted } (p_1 + p_2) (T_1 - T_2). \text{ The specific gravity of the gas is assumed to be 0.65, air being 1. For fluids of any other sp. gr., } s, \text{ multiply the coefficients 3000 or 41.3 by } \sqrt{0.65/s}. \text{ For air, } s = 1, \text{ the coefficients become 2419 and 33.3. J. E. Johnson Jr.'s formula for air, page 596, translated into the same notation as Mr. Cox's, makes the coefficients 2449 and 33.5.}$

Services for Lamps. (Molesworth.)

| Lamps. | Ft. from Main. | Require Pipe-bore. | Lamps. | Ft. from Main. | Require Pipe-bore. |
|---------|----------------|--------------------|---------|----------------|--------------------|
| 2..... | 40 | 3/8 in. | 15..... | 130 | 1 in. |
| 4..... | 40 | 1/2 in. | 20..... | 150 | 1 1/4 in. |
| 6..... | 50 | 5/8 in. | 25..... | 180 | 1 1/2 in. |
| 10..... | 100 | 3/4 in. | 30..... | 200 | 1 3/4 in. |

(In cold climates no service less than 3/4 in. should be used.)

Maximum Supply of Gas through Pipes in cu. ft. per Hour,
Specific Gravity being taken at 0.45, calculated from the
Formula $Q = 1000 \sqrt{d^5 h \div st.}$ (Molesworth.)

LENGTH OF PIPE = 10 YARDS.

| Diameter of Pipe in Inches. | Pressure by the Water-gauge in Inches. | | | | | | | | | |
|-----------------------------------|--|------|------|------|------|------|------|------|------|------|
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| 3/8 | 13 | 18 | 22 | 26 | 29 | 31 | 34 | 36 | 38 | 41 |
| 1/2 | 26 | 37 | 46 | 53 | 59 | 64 | 70 | 74 | 79 | 83 |
| 3/4 | 73 | 103 | 126 | 145 | 162 | 187 | 192 | 205 | 218 | 230 |
| 1 | 149 | 211 | 258 | 298 | 333 | 365 | 394 | 422 | 447 | 471 |
| 1 1/4 | 260 | 368 | 451 | 521 | 582 | 638 | 689 | 737 | 781 | 823 |
| 1 1/2 | 411 | 581 | 711 | 821 | 918 | 1006 | 1082 | 1162 | 1232 | 1299 |
| 2 | 843 | 1192 | 1460 | 1686 | 1886 | 2066 | 2231 | 2385 | 2530 | 2667 |

LENGTH OF PIPE = 100 YARDS.

| | Pressure by the Water-gauge in Inches. | | | | | | | | | | |
|-------|--|------|------|------|------|------|------|------|------|------|------|
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 | 2 | 2.5 |
| 1/2 | 8 | 12 | 14 | 17 | 19 | 23 | 26 | 29 | 32 | 36 | 42 |
| 3/4 | 23 | 32 | 42 | 46 | 51 | 63 | 73 | 81 | 89 | 103 | 115 |
| 1 | 47 | 67 | 82 | 94 | 105 | 129 | 149 | 167 | 183 | 211 | 236 |
| 1 1/4 | 82 | 116 | 143 | 165 | 184 | 225 | 260 | 291 | 319 | 368 | 412 |
| 1 1/2 | 130 | 184 | 225 | 260 | 290 | 356 | 411 | 459 | 503 | 581 | 649 |
| 2 | 267 | 377 | 462 | 533 | 596 | 730 | 843 | 943 | 1033 | 1193 | 1333 |
| 2 1/2 | 466 | 659 | 807 | 932 | 1042 | 1276 | 1473 | 1647 | 1804 | 2083 | 2329 |
| 3 | 735 | 1039 | 1270 | 1470 | 1643 | 2012 | 2323 | 2598 | 2846 | 3286 | 3674 |
| 3 1/2 | 1080 | 1528 | 1871 | 2161 | 2416 | 2958 | 3416 | 3820 | 4184 | 4831 | 5402 |
| 4 | 1508 | 2133 | 2613 | 3017 | 3373 | 4131 | 4770 | 5333 | 5842 | 6746 | 7542 |

LENGTH OF PIPE = 1000 YARDS.

| | Pressure by the Water-gauge in Inches. | | | | | | |
|-------|--|------|------|------|------|------|------|
| | 0.5 | 0.75 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 1 | 33 | 41 | 47 | 58 | 67 | 75 | 82 |
| 1 1/2 | 92 | 113 | 130 | 159 | 184 | 205 | 226 |
| 2 | 189 | 231 | 267 | 327 | 377 | 422 | 462 |
| 2 1/2 | 329 | 403 | 466 | 571 | 659 | 737 | 807 |
| 3 | 520 | 636 | 735 | 900 | 1039 | 1162 | 1273 |
| 4 | 1067 | 1306 | 1508 | 1847 | 2133 | 2385 | 2613 |
| 5 | 1863 | 2282 | 2635 | 3227 | 3727 | 4167 | 4564 |
| 6 | 2939 | 3600 | 4157 | 5091 | 5879 | 6573 | 7200 |

LENGTH OF PIPE = 5000 YARDS.

| Diameter of Pipe in Inches. | Pressure by the Water-gauge in Inches. | | | | |
|-----------------------------------|--|-------|-------|-------|-------|
| | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 2 | 119 | 146 | 169 | 189 | 207 |
| 3 | 329 | 402 | 465 | 520 | 569 |
| 4 | 675 | 826 | 955 | 1067 | 1168 |
| 5 | 1179 | 1443 | 1667 | 1863 | 2041 |
| 6 | 1859 | 2277 | 2629 | 2939 | 3220 |
| 7 | 2733 | 3347 | 3865 | 4321 | 4734 |
| 8 | 3816 | 4674 | 5397 | 6034 | 6610 |
| 9 | 5123 | 6274 | 7245 | 8100 | 8873 |
| 10 | 6667 | 8165 | 9428 | 10541 | 11547 |
| 12 | 10516 | 12880 | 14872 | 16628 | 18215 |

Mr. A. C. Humphreys says his experience goes to show that these tables give too small a flow, but it is difficult to accurately check the tables, on account of the extra friction introduced by rough pipes, bends, etc. For bends, one rule is to allow $\frac{1}{42}$ of an inch pressure for each right-angle bend.

Where there is apt to be trouble from frost it is well to use no service of less diameter than $\frac{3}{4}$ in., no matter how short it may be. In extremely cold climates this is now often increased to 1 in., even for a single lamp. The best practice in the U. S. now condemns any service less than $\frac{3}{4}$ in.

STEAM.

The Temperature of Steam in contact with water depends upon the pressure under which it is generated. At the ordinary atmospheric pressure (14.7 lbs. per sq. in.) its temperature is 212° F. As the pressure is increased, as by the steam being generated in a closed vessel, its temperature, and that of the water in its presence, increases.

Saturated Steam is steam of the temperature due to its pressure — not superheated.

Superheated Steam is steam heated to a temperature above that due to its pressure.

Dry Steam is steam which contains no moisture. It may be either saturated or superheated.

Wet Steam is steam containing intermingled moisture, mist, or spray. It has the same temperature as dry saturated steam of the same pressure.

Water introduced into the presence of superheated steam will flash into vapor until the temperature of the steam is reduced to that due its pressure. Water in the presence of saturated steam has the same temperature as the steam. Should cold water be introduced, lowering the temperature of the whole mass, some of the steam will be condensed, reducing the pressure and temperature of the remainder, until equilibrium is established.

Total Heat of Saturated Steam (above 32° F.). — According to Marks and Davis, the formula for total heat of steam, based on researches by Henning, Knoblauch, Linde and Klebe, is $H = 1150.3 + 0.3745(t + 212^{\circ}) - 0.000550(t - 212)^2$, in which H is the total heat in B.T.U. above water at 32° F. and t is the temperature Fahrenheit.

Latent Heat of Steam. — The latent heat, or heat of vaporization, is obtained by subtracting from the total heat at any given temperature the heat of the liquid, or total heat above 32° in water of the same temperature.

The total heat in steam (above 32°) includes three elements:

1st. The heat required to raise the temperature of the water to the temperature of the steam.

2d. The heat required to evaporate the water at that temperature, called internal latent heat.

3d. The latent heat of volume, or the external work done by the steam in making room for itself against the pressure of the superincumbent atmosphere (or surrounding steam if inclosed in a vessel).

The sum of the last two elements is called the latent heat of steam.

Heat required to Generate 1 lb. of Steam from water at 32° F. Heat-units.

| | |
|--|-------|
| Sensible heat, to raise the water from 32° to 212° = ... | 180.0 |
| Latent heat, 1, of the formation of steam at 212° = ... | 897.6 |
| 2, of expansion against the atmospheric pressure, 2116.4 lbs. per sq. ft. × 26.79 cu. ft. = 55,786 foot-pounds ÷ 778 = | 72.8 |
| | 970.4 |

Total heat above 32° F. 1150.4

The Heat-Unit, or British Thermal Unit. — The old definition of the heat-unit (Rankine), viz., the quantity of heat required to raise the temperature of 1 lb. of water 1° F., at or near its temperature of maximum density (39.1° F.), is now (1909) no longer used. Peabody defines it as the heat required to raise a pound of water from 62° to 63° F., and Marks and Davis as $\frac{1}{180}$ of the heat required to raise 1 lb. of water from 32° to 212° F. By Peabody's definition the heat required to raise 1 lb. of water from 32° to 212° is 180.3 instead of 180 units, and the heat of vaporization at 212° 969.7 instead of 970.4 units.

Specific Heat of Saturated Steam. — When a unit weight of saturated steam is increased in temperature and in pressure, the volume decreasing so as to just keep it saturated, the specific heat is negative, and decreases as temperature increases. (See Wood, *Therm.*, p. 147; Peabody, *Therm.*, p. 93.)

Absolute Zero. — The value of the absolute zero has been variously given as from 459.2 to 460.66 degrees below the Fahrenheit zero. Marks and Davis, comparing the results of Berthelot (1903), Buckingham, 1907, and Ross-Innes, 1908, give as the most probable value — 459°.64 F. The value — 460° is close enough for all engineering calculations.

The Mechanical Equivalent of Heat. — The value generally accepted, based on Rowland's experiments, is 778 ft.-lbs. Marks and Davis give the value 777.52 standard ft. lbs., based on later experiments, and on the value of $g = 980.665$ cm. per sec.², = 32.174 ft. per sec.², fixed by international agreement (1901). These values of the absolute zero and of the mechanical equivalent of heat have been used by Marks and Davis in the computation of their steam tables. In refined investigations involving the value of the mechanical equivalent of heat the value of g for the latitude in which the experiments are made must be considered.

Pressure of Saturated Steam. — Holborn and Henning, *Zeit. des Ver. deutscher Ingenieure*, Feb. 20, 1909, report results of measurements of the pressures of saturated steam at temperatures ranging from 50° to 200° C. (112° to 392° F.). Their values agree closely with those obtained in 1905 by Knoblauch, Linde and Klebe. From a table in the article giving pressures for each degree from 0° to 200° C., the following values have been transformed into English measurements (*Eng. Digest* April, 1909).

| Deg. F. | Lbs. per sq. in. | Deg. F. | Lbs. per sq. in. | Deg. F. | Lbs. per sq. in. |
|---------|------------------|---------|------------------|---------|------------------|
| 32 | 0.0885 | 150 | 3.715 | 300 | 66.972 |
| 68 | 0.3386 | 200 | 11.527 | 350 | 134.508 |
| 100 | 0.9462 | 250 | 29.819 | 400 | 248.856 |

Volume of Saturated Steam. — The values of specific volume of saturated steam are computed by Clapyron's equation (Marks and Davis's Tables) which gives results remarkably close to those found in the experiments of Knoblauch, Linde and Klebe.

Volume of Superheated Steam. — Linde's equation (1905),

$$pv = 0.5962 T - p (1 + 0.0014 p) \left(\frac{150,300,000}{T^3} - 0.0833 \right),$$

in which p is in lbs. per sq. ft., v is in cu. ft. and $T = t + 459.6$ is the absolute temperature on the Fahrenheit scale, has been used in the computation of Marks and Davis's tables.

The Specific Density of Gaseous Steam, that is, steam considerably superheated, is 0.622, that of air being 1. That is to say, the weight of a cubic foot of gaseous steam is about five-eighths of that of a cubic foot of air, of the same pressure and temperature.

The density or weight of a cubic foot of gaseous steam is expressible by the same formula as that of air, except that the multiplier or coefficient is less in proportion to the less specific density. Thus,

$$D = \frac{2.7074 p \times .622}{t + 461} = \frac{1.684 p}{t + 461},$$

in which D is the weight of a cubic foot, p the total pressure per square inch and t the temperature Fahrenheit. (Clark's Steam-engine.)

H. M. Prevost Murphy (*Eng. News*, June 18, 1908) shows that the specific density is not a constant, but varies with the temperature, and that the correct value is $0.6113 + \frac{0.092 t}{850 - t}$.

Properties of Superheated Steam.—See the table on page 843, condensed from Marks and Davis's tables.

Specific Heat of Superheated Steam.—Mean specific heats from the temperature of saturation to various temperatures at several pressures English and metric units.—Knoblauch and Jakob (from Peabody's Tables).

| Kg. per sq. cm. | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|------------------|------|-------|------|------|-------|-------|-------|-------|-------|-------|-------|
| Lbs. per sq. in. | 14.2 | 28.4 | 56.9 | 85.3 | 113.3 | 142.2 | 170.6 | 199.1 | 227.5 | 256.0 | 284.4 |
| Temp. sat., °C. | 99 | 120 | 143 | 158 | 169 | 179 | 187 | 194 | 200 | 206 | 211 |
| Temp. sat., °F. | 210 | 248 | 289 | 316 | 336 | 350 | 368 | 381 | 392 | 403 | 412 |
| ° F. | ° C. | | | | | | | | | | |
| 212 | 100 | 0.463 | | | | | | | | | |
| 302 | 150 | .462 | .478 | .515 | | | | | | | |
| 392 | 200 | .462 | .475 | .502 | .530 | .560 | .597 | .635 | .677 | | |
| 482 | 250 | .463 | .474 | .495 | .514 | .512 | .552 | .570 | .588 | .609 | .635 |
| 572 | 300 | .464 | .475 | .492 | .505 | .517 | .530 | .541 | .550 | .561 | .572 |
| 662 | 350 | .468 | .477 | .492 | .503 | .512 | .522 | .529 | .536 | .543 | .550 |
| 752 | 400 | .473 | .481 | .494 | .504 | .512 | .520 | .526 | .531 | .537 | .542 |

The Rationalization of Regnault's Experiments on Steam.—(J. McFarlane Gray, *Proc. Inst. M. E.*, July, 1889.)—The formulæ constructed by Regnault are strictly empirical, and were based entirely on his experiments. They are therefore not valid beyond the range of temperatures and pressures observed.

Mr. Gray has made a most elaborate calculation, based not on experiments but on fundamental principles of thermodynamics, from which he deduces formulæ for the pressure and total heat of steam, and presents tables calculated therefrom which show substantial agreement with Regnault's figures. He gives the following examples of steam-pressures calculated for temperatures beyond the range of Regnault's experiments.

| Temperature. | | Pounds per sq. in. | Temperature. | | Pounds in sq. in. |
|--------------|-------|--------------------|--------------|-------|-------------------|
| C. | Fahr. | | C. | Fahr. | |
| 230 | 446 | 406.9 | 340 | 644 | 2156.2 |
| 240 | 464 | 488.9 | 360 | 680 | 2742.5 |
| 250 | 482 | 579.9 | 380 | 716 | 3448.1 |
| 260 | 500 | 691.6 | 400 | 752 | 4300.2 |
| 280 | 536 | 940.0 | 415 | 779 | 5017.1 |
| 300 | 572 | 1261.8 | 427 | 800.6 | 5659.9 |
| 320 | 608 | 1661.9 | | | |

These pressures are higher than those obtained by Regnault's formula, which gives for 415° C. only 4067.1 lbs. per square inch.

Properties of Saturated Steam.

(Condensed from Marks and Davis's Steam Tables and Diagrams, 1909, by permission of the publishers, Longmans, Green & Co.)

| Vacuum, Inches of Mercury. | Absolute Pressure, Lbs. per Sq. In. | Temperature, Fahrenheit. | Total Heat above 32° F. | | Latent Heat, L $= H - h$ Heat-Units. | Volume, Cu. Ft. in 1 Lb. of Steam. | Weight of 1 Cu. Ft. Steam, Lb. | Entropy of the Water. | Entropy of Evaporation. |
|----------------------------|-------------------------------------|--------------------------|------------------------------|------------------------------|---|------------------------------------|--------------------------------|-----------------------|-------------------------|
| | | | In the Water h Heat-Units. | In the Steam H Heat-Units. | | | | | |
| 29.74 | 0.0886 | 32 | 0.00 | 1073.4 | 1073.4 | 3294 | 0.000304 | 0.0000 | 2.1832 |
| 29.67 | 0.1217 | 40 | 8.05 | 1076.9 | 1068.9 | 2438 | 0.000410 | 0.0162 | 2.1394 |
| 29.56 | 0.1780 | 50 | 18.08 | 1081.4 | 1063.3 | 1702 | 0.000587 | 0.0361 | 2.0865 |
| 29.40 | 0.2562 | 60 | 28.08 | 1085.9 | 1057.8 | 1208 | 0.000828 | 0.0555 | 2.0358 |
| 29.18 | 0.3626 | 70 | 38.06 | 1090.3 | 1052.3 | 871 | 0.001148 | 0.0745 | 1.9868 |
| 29.09 | 0.505 | 80 | 48.03 | 1094.8 | 1046.7 | 636.8 | 0.001570 | 0.0932 | 1.9398 |
| 28.50 | 0.696 | 90 | 58.00 | 1099.2 | 1041.2 | 469.3 | 0.002131 | 0.1114 | 1.8944 |
| 28.00 | 0.946 | 100 | 67.97 | 1103.6 | 1035.6 | 350.8 | 0.002851 | 0.1295 | 1.8505 |
| 27.88 | 1 | 101.83 | 69.8 | 1104.4 | 1034.6 | 333.0 | 0.00300 | 0.1327 | 1.8427 |
| 25.85 | 2 | 126.15 | 94.0 | 1115.0 | 1021.0 | 173.5 | 0.00576 | 0.1749 | 1.7431 |
| 23.81 | 3 | 141.52 | 109.4 | 1121.6 | 1012.3 | 118.5 | 0.00845 | 0.2008 | 1.6840 |
| 21.78 | 4 | 153.01 | 120.9 | 1126.5 | 1005.7 | 90.5 | 0.01107 | 0.2198 | 1.6416 |
| 19.74 | 5 | 162.28 | 130.1 | 1130.5 | 1000.3 | 73.33 | 0.01364 | 0.2348 | 1.6084 |
| 17.70 | 6 | 170.06 | 137.9 | 1133.7 | 995.8 | 61.89 | 0.01616 | 0.2471 | 1.5814 |
| 15.67 | 7 | 176.85 | 144.7 | 1136.5 | 991.8 | 53.56 | 0.01867 | 0.2579 | 1.5582 |
| 13.63 | 8 | 182.86 | 150.8 | 1139.0 | 988.2 | 47.27 | 0.02115 | 0.2673 | 1.5380 |
| 11.60 | 9 | 188.27 | 156.2 | 1141.1 | 985.0 | 42.36 | 0.02361 | 0.2756 | 1.5202 |
| 9.56 | 10 | 193.22 | 161.1 | 1143.1 | 982.0 | 38.38 | 0.02606 | 0.2832 | 1.5042 |
| 7.52 | 11 | 197.75 | 165.7 | 1144.9 | 979.2 | 35.10 | 0.02849 | 0.2902 | 1.4895 |
| 5.49 | 12 | 201.96 | 169.9 | 1146.5 | 976.6 | 32.36 | 0.03090 | 0.2967 | 1.4760 |
| 3.45 | 13 | 205.87 | 173.8 | 1148.0 | 974.2 | 30.03 | 0.03330 | 0.3025 | 1.4639 |
| 1.42 | 14 | 209.55 | 177.5 | 1149.4 | 971.9 | 28.02 | 0.03569 | 0.3081 | 1.4523 |
| lbs. gage. | 14.70 | 212 | 180.0 | 1150.4 | 970.4 | 26.79 | 0.03732 | 0.3118 | 1.4447 |
| 0.3 | 15 | 213.0 | 181.0 | 1150.7 | 969.7 | 26.27 | 0.03806 | 0.3133 | 1.4416 |
| 1.3 | 16 | 216.3 | 184.4 | 1152.0 | 967.6 | 24.79 | 0.04042 | 0.3183 | 1.4311 |
| 2.3 | 17 | 219.4 | 187.5 | 1153.1 | 965.6 | 23.38 | 0.04277 | 0.3229 | 1.4215 |
| 3.3 | 18 | 222.4 | 190.5 | 1154.2 | 963.7 | 22.16 | 0.04512 | 0.3273 | 1.4127 |
| 4.3 | 19 | 225.2 | 193.4 | 1155.2 | 961.8 | 21.07 | 0.04746 | 0.3315 | 1.4045 |
| 5.3 | 20 | 228.0 | 196.1 | 1156.2 | 960.0 | 20.08 | 0.04980 | 0.3355 | 1.3965 |
| 6.3 | 21 | 230.6 | 198.8 | 1157.1 | 958.3 | 19.18 | 0.05213 | 0.3393 | 1.3887 |
| 7.3 | 22 | 233.1 | 201.3 | 1158.0 | 956.7 | 18.37 | 0.05445 | 0.3430 | 1.3811 |
| 8.3 | 23 | 235.5 | 203.8 | 1158.8 | 955.1 | 17.62 | 0.05676 | 0.3465 | 1.3739 |
| 9.3 | 24 | 237.8 | 206.1 | 1159.6 | 953.5 | 16.93 | 0.05907 | 0.3499 | 1.3670 |
| 10.3 | 25 | 240.1 | 208.4 | 1160.4 | 952.0 | 16.30 | 0.0614 | 0.3532 | 1.3604 |
| 11.3 | 26 | 242.2 | 210.6 | 1161.2 | 950.6 | 15.72 | 0.0636 | 0.3564 | 1.3542 |
| 12.3 | 27 | 244.4 | 212.7 | 1161.9 | 949.2 | 15.18 | 0.0659 | 0.3594 | 1.3483 |
| 13.3 | 28 | 246.4 | 214.8 | 1162.6 | 947.8 | 14.67 | 0.0682 | 0.3623 | 1.3425 |
| 14.3 | 29 | 248.4 | 216.8 | 1163.2 | 946.4 | 14.19 | 0.0705 | 0.3652 | 1.3367 |
| 15.3 | 30 | 250.3 | 218.8 | 1163.9 | 945.1 | 13.74 | 0.0728 | 0.3680 | 1.3311 |
| 16.3 | 31 | 252.2 | 220.7 | 1164.5 | 943.8 | 13.32 | 0.0751 | 0.3707 | 1.3257 |
| 17.3 | 32 | 254.1 | 222.6 | 1165.1 | 942.5 | 12.93 | 0.0773 | 0.3733 | 1.3205 |
| 18.3 | 33 | 255.8 | 224.4 | 1165.7 | 941.3 | 12.57 | 0.0795 | 0.3759 | 1.3155 |
| 19.3 | 34 | 257.6 | 226.2 | 1166.3 | 940.1 | 12.22 | 0.0818 | 0.3784 | 1.3107 |
| 20.3 | 35 | 259.3 | 227.9 | 1166.8 | 938.9 | 11.89 | 0.0841 | 0.3808 | 1.3060 |
| 21.3 | 36 | 261.0 | 229.6 | 1167.3 | 937.7 | 11.58 | 0.0863 | 0.3832 | 1.3014 |
| 22.3 | 37 | 262.6 | 231.3 | 1167.8 | 936.6 | 11.29 | 0.0886 | 0.3855 | 1.2969 |
| 23.3 | 38 | 264.2 | 232.9 | 1168.4 | 935.5 | 11.01 | 0.0908 | 0.3877 | 1.2925 |
| 24.3 | 39 | 265.8 | 234.5 | 1168.9 | 934.4 | 10.74 | 0.0931 | 0.3899 | 1.2882 |
| 25.3 | 40 | 267.3 | 236.1 | 1169.4 | 933.3 | 10.49 | 0.0953 | 0.3920 | 1.2841 |
| 26.3 | 41 | 268.7 | 237.6 | 1169.8 | 932.2 | 10.25 | 0.0976 | 0.3941 | 1.2800 |

Properties of Saturated Steam. (Continued.)

| Gauge Pressure, Lbs. per Sq. In. | Absolute Pressure, Lbs. per Sq. In. | Temperature, Fahrenheit. | Total Heat above 32° F. | | Latent Heat, L $= H - h$ Heat- Units. | Volume, Cu. Ft. in 1 Lb. of Steam. | Weight of 1 Cu. Ft. Steam, Lb. | Entropy of the Water. | Entropy of Evap- oration. |
|-------------------------------------|--|-----------------------------|------------------------------------|------------------------------------|---|--|-----------------------------------|--------------------------|------------------------------|
| | | | In the Water h Heat-Units. | In the Steam H Heat-Units. | | | | | |
| 27.3 | 42 | 270.2 | 239.1 | 1170.3 | 931.2 | 10.02 | 0.0998 | 0.3962 | 1.2759 |
| 28.3 | 43 | 271.7 | 240.5 | 1170.7 | 930.2 | 9.80 | 0.1020 | 0.3982 | 1.2720 |
| 29.3 | 44 | 273.1 | 242.0 | 1171.2 | 929.2 | 9.59 | 0.1043 | 0.4002 | 1.2681 |
| 30.3 | 45 | 274.5 | 243.4 | 1171.6 | 928.2 | 9.39 | 0.1065 | 0.4021 | 1.2644 |
| 31.3 | 46 | 275.8 | 244.8 | 1172.0 | 927.2 | 9.20 | 0.1087 | 0.4040 | 1.2607 |
| 32.3 | 47 | 277.2 | 246.1 | 1172.4 | 926.3 | 9.02 | 0.1109 | 0.4059 | 1.2571 |
| 33.3 | 48 | 278.5 | 247.5 | 1172.8 | 925.3 | 8.84 | 0.1131 | 0.4077 | 1.2536 |
| 34.3 | 49 | 279.8 | 248.8 | 1173.2 | 924.4 | 8.67 | 0.1153 | 0.4095 | 1.2502 |
| 35.3 | 50 | 281.0 | 250.1 | 1173.6 | 923.5 | 8.51 | 0.1175 | 0.4113 | 1.2468 |
| 36.3 | 51 | 282.3 | 251.4 | 1174.0 | 922.6 | 8.35 | 0.1197 | 0.4130 | 1.2432 |
| 37.3 | 52 | 283.5 | 252.6 | 1174.3 | 921.7 | 8.20 | 0.1219 | 0.4147 | 1.2405 |
| 38.3 | 53 | 284.7 | 253.9 | 1174.7 | 920.8 | 8.05 | 0.1241 | 0.4164 | 1.2370 |
| 39.3 | 54 | 285.9 | 255.1 | 1175.0 | 919.9 | 7.91 | 0.1263 | 0.4180 | 1.2339 |
| 40.3 | 55 | 287.1 | 256.3 | 1175.4 | 919.0 | 7.78 | 0.1285 | 0.4196 | 1.2309 |
| 41.3 | 56 | 288.2 | 257.5 | 1175.7 | 918.2 | 7.65 | 0.1307 | 0.4212 | 1.2278 |
| 42.3 | 57 | 289.4 | 258.7 | 1176.0 | 917.4 | 7.52 | 0.1329 | 0.4227 | 1.2248 |
| 43.3 | 58 | 290.5 | 259.8 | 1176.4 | 916.5 | 7.40 | 0.1350 | 0.4242 | 1.2218 |
| 44.3 | 59 | 291.6 | 261.0 | 1176.7 | 915.7 | 7.28 | 0.1372 | 0.4257 | 1.2189 |
| 45.3 | 60 | 292.7 | 262.1 | 1177.0 | 914.9 | 7.17 | 0.1394 | 0.4272 | 1.2160 |
| 46.3 | 61 | 293.8 | 263.2 | 1177.3 | 914.1 | 7.06 | 0.1416 | 0.4287 | 1.2132 |
| 47.3 | 62 | 294.9 | 264.3 | 1177.6 | 913.3 | 6.95 | 0.1438 | 0.4302 | 1.2104 |
| 48.3 | 63 | 295.9 | 265.4 | 1177.9 | 912.5 | 6.85 | 0.1460 | 0.4316 | 1.2077 |
| 49.3 | 64 | 297.0 | 266.4 | 1178.2 | 911.8 | 6.75 | 0.1482 | 0.4330 | 1.2050 |
| 50.3 | 65 | 298.0 | 267.5 | 1178.5 | 911.0 | 6.65 | 0.1503 | 0.4344 | 1.2024 |
| 51.3 | 66 | 299.0 | 268.5 | 1178.8 | 910.2 | 6.56 | 0.1525 | 0.4358 | 1.1998 |
| 52.3 | 67 | 300.0 | 269.6 | 1179.0 | 909.5 | 6.47 | 0.1547 | 0.4371 | 1.1972 |
| 53.3 | 68 | 301.0 | 270.6 | 1179.3 | 908.7 | 6.38 | 0.1569 | 0.4385 | 1.1946 |
| 54.3 | 69 | 302.0 | 271.6 | 1179.6 | 908.0 | 6.29 | 0.1590 | 0.4398 | 1.1921 |
| 55.3 | 70 | 302.9 | 272.6 | 1179.8 | 907.2 | 6.20 | 0.1612 | 0.4411 | 1.1896 |
| 56.3 | 71 | 303.9 | 273.6 | 1180.1 | 906.5 | 6.12 | 0.1634 | 0.4424 | 1.1872 |
| 57.3 | 72 | 304.8 | 274.5 | 1180.4 | 905.8 | 6.04 | 0.1656 | 0.4437 | 1.1848 |
| 58.3 | 73 | 305.8 | 275.5 | 1180.6 | 905.1 | 5.96 | 0.1678 | 0.4449 | 1.1825 |
| 59.3 | 74 | 306.7 | 276.5 | 1180.9 | 904.4 | 5.89 | 0.1699 | 0.4462 | 1.1801 |
| 60.3 | 75 | 307.6 | 277.4 | 1181.1 | 903.7 | 5.81 | 0.1721 | 0.4474 | 1.1778 |
| 61.3 | 76 | 308.5 | 278.3 | 1181.4 | 903.0 | 5.74 | 0.1743 | 0.4487 | 1.1755 |
| 62.3 | 77 | 309.4 | 279.3 | 1181.6 | 902.3 | 5.67 | 0.1764 | 0.4499 | 1.1730 |
| 63.3 | 78 | 310.3 | 280.2 | 1181.8 | 901.7 | 5.60 | 0.1786 | 0.4511 | 1.1712 |
| 64.3 | 79 | 311.2 | 281.1 | 1182.1 | 901.0 | 5.54 | 0.1808 | 0.4523 | 1.1687 |
| 65.3 | 80 | 312.0 | 282.0 | 1182.3 | 900.3 | 5.47 | 0.1829 | 0.4535 | 1.1665 |
| 66.3 | 81 | 312.9 | 282.9 | 1182.5 | 899.7 | 5.41 | 0.1851 | 0.4546 | 1.1644 |
| 67.3 | 82 | 313.8 | 283.8 | 1182.8 | 899.0 | 5.34 | 0.1873 | 0.4557 | 1.1623 |
| 68.3 | 83 | 314.6 | 284.6 | 1183.0 | 898.4 | 5.28 | 0.1894 | 0.4568 | 1.1602 |
| 69.3 | 84 | 315.4 | 285.5 | 1183.2 | 897.7 | 5.22 | 0.1915 | 0.4579 | 1.1581 |
| 70.3 | 85 | 316.3 | 286.3 | 1183.4 | 897.1 | 5.16 | 0.1937 | 0.4590 | 1.1561 |
| 71.3 | 86 | 317.1 | 287.2 | 1183.6 | 896.4 | 5.10 | 0.1959 | 0.4601 | 1.1540 |
| 72.3 | 87 | 317.9 | 288.0 | 1183.8 | 895.8 | 5.05 | 0.1980 | 0.4612 | 1.1520 |
| 73.3 | 88 | 318.7 | 288.9 | 1184.0 | 895.2 | 5.00 | 0.2001 | 0.4623 | 1.1500 |
| 74.3 | 89 | 319.5 | 289.7 | 1184.2 | 894.6 | 4.94 | 0.2023 | 0.4633 | 1.1481 |
| 75.3 | 90 | 320.3 | 290.5 | 1184.4 | 893.9 | 4.89 | 0.2044 | 0.4644 | 1.1461 |
| 76.3 | 91 | 321.1 | 291.3 | 1184.6 | 893.3 | 4.84 | 0.2065 | 0.4654 | 1.1442 |
| 77.3 | 92 | 321.8 | 292.1 | 1184.8 | 892.7 | 4.79 | 0.2087 | 0.4664 | 1.1423 |
| 78.3 | 93 | 322.6 | 292.9 | 1185.0 | 892.1 | 4.74 | 0.2109 | 0.4674 | 1.1404 |
| 79.3 | 94 | 323.4 | 293.7 | 1185.2 | 891.5 | 4.69 | 0.2130 | 0.4684 | 1.1385 |
| 80.3 | 95 | 324.1 | 294.5 | 1185.4 | 890.9 | 4.65 | 0.2151 | 0.4694 | 1.1367 |

Properties of Saturated Steam. (Continued.)

| Gauge Pressure, Lbs. per Sq. In. | Absolute Pressure, Lbs. per Sq. In. | Temperature, Fahrenheit. | Total Heat above 32° F. | | Latent Heat, L $= H - h$ Heat- Units. | Volume, Cu. Ft. in 1 Lb. of Steam. | Weight of 1 Cu. Ft. Steam, Lb. | Entropy of the Water. | Entropy of Evap- oration. |
|-------------------------------------|--|-----------------------------|------------------------------------|------------------------------------|---|--|-----------------------------------|--------------------------|------------------------------|
| | | | In the Water h Heat-Units. | In the Steam H Heat-Units. | | | | | |
| 81.3 | 96 | 324.9 | 295.3 | 1185.6 | 890.3 | 4.60 | 0.2172 | 0.4704 | 1.1348 |
| 82.3 | 97 | 325.6 | 296.1 | 1185.8 | 889.7 | 4.56 | 0.2193 | 0.4714 | 1.1330 |
| 83.3 | 98 | 326.4 | 296.8 | 1186.0 | 889.2 | 4.51 | 0.2215 | 0.4724 | 1.1312 |
| 84.3 | 99 | 327.1 | 297.6 | 1186.2 | 888.6 | 4.47 | 0.2237 | 0.4733 | 1.1295 |
| 85.3 | 100 | 327.8 | 298.3 | 1186.3 | 888.0 | 4.429 | 0.2258 | 0.4743 | 1.1277 |
| 87.3 | 102 | 329.3 | 299.8 | 1186.7 | 886.9 | 4.347 | 0.2300 | 0.4762 | 1.1242 |
| 89.3 | 104 | 330.7 | 301.3 | 1187.0 | 885.8 | 4.268 | 0.2343 | 0.4780 | 1.1208 |
| 91.3 | 106 | 332.0 | 302.7 | 1187.4 | 884.7 | 4.192 | 0.2336 | 0.4798 | 1.1174 |
| 93.3 | 108 | 333.4 | 304.1 | 1187.7 | 883.6 | 4.118 | 0.2429 | 0.4816 | 1.1141 |
| 95.3 | 110 | 334.8 | 305.5 | 1188.0 | 882.5 | 4.047 | 0.2472 | 0.4834 | 1.1108 |
| 97.3 | 112 | 336.1 | 306.9 | 1188.4 | 881.4 | 3.978 | 0.2514 | 0.4852 | 1.1076 |
| 99.3 | 114 | 337.4 | 308.3 | 1188.7 | 880.4 | 3.912 | 0.2556 | 0.4869 | 1.1045 |
| 101.3 | 116 | 338.7 | 309.6 | 1189.0 | 879.3 | 3.848 | 0.2599 | 0.4886 | 1.1014 |
| 103.3 | 118 | 340.0 | 311.0 | 1189.3 | 878.3 | 3.786 | 0.2641 | 0.4903 | 1.0984 |
| 105.3 | 120 | 341.3 | 312.3 | 1189.6 | 877.2 | 3.726 | 0.2683 | 0.4919 | 1.0954 |
| 107.3 | 122 | 342.5 | 313.6 | 1189.8 | 876.2 | 3.668 | 0.2726 | 0.4935 | 1.0924 |
| 109.3 | 124 | 343.8 | 314.9 | 1190.1 | 875.2 | 3.611 | 0.2769 | 0.4951 | 1.0895 |
| 111.3 | 126 | 345.0 | 316.2 | 1190.4 | 874.2 | 3.556 | 0.2812 | 0.4967 | 1.0865 |
| 113.3 | 128 | 346.2 | 317.4 | 1190.7 | 873.3 | 3.504 | 0.2854 | 0.4962 | 1.0837 |
| 115.3 | 130 | 347.4 | 318.6 | 1191.0 | 872.3 | 3.452 | 0.2897 | 0.4998 | 1.0809 |
| 117.3 | 132 | 348.5 | 319.9 | 1191.2 | 871.3 | 3.402 | 0.2939 | 0.5013 | 1.0782 |
| 119.3 | 134 | 349.7 | 321.1 | 1191.5 | 870.4 | 3.354 | 0.2981 | 0.5028 | 1.0755 |
| 121.3 | 136 | 350.8 | 322.3 | 1191.7 | 869.4 | 3.308 | 0.3023 | 0.5043 | 1.0728 |
| 123.3 | 138 | 352.0 | 323.4 | 1192.0 | 868.5 | 3.263 | 0.3065 | 0.5057 | 1.0702 |
| 125.3 | 140 | 353.1 | 324.6 | 1192.2 | 867.6 | 3.219 | 0.3107 | 0.5072 | 1.0675 |
| 127.3 | 142 | 354.2 | 325.8 | 1192.5 | 866.7 | 3.175 | 0.3150 | 0.5086 | 1.0649 |
| 129.3 | 144 | 355.3 | 326.9 | 1192.7 | 865.8 | 3.133 | 0.3192 | 0.5100 | 1.0624 |
| 131.3 | 146 | 356.3 | 328.0 | 1192.9 | 864.9 | 3.092 | 0.3234 | 0.5114 | 1.0599 |
| 133.3 | 148 | 357.4 | 329.1 | 1193.2 | 864.0 | 3.052 | 0.3276 | 0.5128 | 1.0574 |
| 135.3 | 150 | 358.5 | 330.2 | 1193.4 | 863.2 | 3.012 | 0.3320 | 0.5142 | 1.0550 |
| 137.3 | 152 | 359.5 | 331.4 | 1193.6 | 862.3 | 2.974 | 0.3362 | 0.5155 | 1.0525 |
| 139.3 | 154 | 360.5 | 332.4 | 1193.8 | 861.4 | 2.938 | 0.3404 | 0.5169 | 1.0501 |
| 141.3 | 156 | 361.6 | 333.5 | 1194.1 | 860.6 | 2.902 | 0.3446 | 0.5182 | 1.0477 |
| 143.3 | 158 | 362.6 | 334.6 | 1194.3 | 859.7 | 2.868 | 0.3488 | 0.5195 | 1.0454 |
| 145.3 | 160 | 363.6 | 335.6 | 1194.5 | 858.8 | 2.834 | 0.3529 | 0.5208 | 1.0431 |
| 147.3 | 162 | 364.6 | 336.7 | 1194.7 | 858.0 | 2.801 | 0.3570 | 0.5220 | 1.0409 |
| 149.3 | 164 | 365.6 | 337.7 | 1194.9 | 857.2 | 2.769 | 0.3612 | 0.5233 | 1.0387 |
| 151.3 | 166 | 366.5 | 338.7 | 1195.1 | 856.4 | 2.737 | 0.3654 | 0.5245 | 1.0365 |
| 153.3 | 168 | 367.5 | 339.7 | 1195.3 | 855.5 | 2.706 | 0.3696 | 0.5257 | 1.0343 |
| 155.3 | 170 | 368.5 | 340.7 | 1195.4 | 854.7 | 2.675 | 0.3738 | 0.5269 | 1.0321 |
| 157.3 | 172 | 369.4 | 341.7 | 1195.6 | 853.9 | 2.645 | 0.3780 | 0.5281 | 1.0300 |
| 159.3 | 174 | 370.4 | 342.7 | 1195.8 | 853.1 | 2.616 | 0.3822 | 0.5293 | 1.0278 |
| 161.3 | 176 | 371.3 | 343.7 | 1196.0 | 852.3 | 2.588 | 0.3864 | 0.5305 | 1.0257 |
| 163.3 | 178 | 372.2 | 344.7 | 1196.2 | 851.5 | 2.560 | 0.3906 | 0.5317 | 1.0235 |
| 165.3 | 180 | 373.1 | 345.6 | 1196.4 | 850.8 | 2.533 | 0.3948 | 0.5328 | 1.0215 |
| 167.3 | 182 | 374.0 | 346.6 | 1196.6 | 850.0 | 2.507 | 0.3989 | 0.5339 | 1.0195 |
| 169.3 | 184 | 374.9 | 347.6 | 1196.8 | 849.2 | 2.481 | 0.4031 | 0.5351 | 1.0174 |
| 171.3 | 186 | 375.8 | 348.5 | 1196.9 | 848.4 | 2.455 | 0.4073 | 0.5362 | 1.0154 |
| 173.3 | 188 | 376.7 | 349.4 | 1197.1 | 847.7 | 2.430 | 0.4115 | 0.5373 | 1.0134 |
| 175.3 | 190 | 377.6 | 350.4 | 1197.3 | 846.9 | 2.406 | 0.4157 | 0.5384 | 1.0114 |
| 177.3 | 192 | 378.5 | 351.3 | 1197.4 | 846.1 | 2.381 | 0.4199 | 0.5395 | 1.0095 |
| 179.3 | 194 | 379.3 | 352.2 | 1197.6 | 845.4 | 2.358 | 0.4241 | 0.5405 | 1.0076 |
| 181.3 | 196 | 380.2 | 353.1 | 1197.8 | 844.7 | 2.335 | 0.4283 | 0.5416 | 1.0056 |
| 183.3 | 198 | 381.0 | 354.0 | 1197.9 | 843.9 | 2.312 | 0.4325 | 0.5426 | 1.0038 |

Properties of Saturated Steam. (Continued.)

| Gauge Pressure, Lbs. per Sq. In. | Absolute Pressure, Lbs. per Sq. In. | Temperature, Fahrenheit. | Total Heat above 32° F. | | Latent Heat, L $= H - h$ Heat- Units. | Volume, Cu. Ft. in 1 Lb. of Steam. | Weight of 1 Cu. Ft. Steam, Lb. | Entropy of the Water. | Entropy of Evap- oration. |
|-------------------------------------|--|-----------------------------|------------------------------------|------------------------------------|---|--|-----------------------------------|--------------------------|------------------------------|
| | | | In the Water h Heat-Units. | In the Steam H Heat-Units. | | | | | |
| 185.3 | 200 | 381.9 | 354.9 | 1198.1 | 843.2 | 2.290 | 0.437 | 0.5437 | 1.0019 |
| 190.3 | 205 | 384.0 | 357.1 | 1198.5 | 841.4 | 2.237 | 0.447 | 0.5463 | 0.9973 |
| 195.3 | 210 | 386.0 | 359.2 | 1198.8 | 839.6 | 2.187 | 0.457 | 0.5488 | 0.9928 |
| 200.3 | 215 | 388.0 | 361.4 | 1199.2 | 837.9 | 2.138 | 0.468 | 0.5513 | 0.9885 |
| 205.3 | 220 | 389.9 | 363.4 | 1199.6 | 836.2 | 2.091 | 0.478 | 0.5538 | 0.9841 |
| 210.3 | 225 | 391.9 | 365.5 | 1199.9 | 834.4 | 2.046 | 0.489 | 0.5562 | 0.9799 |
| 215.3 | 230 | 393.8 | 367.5 | 1200.2 | 832.8 | 2.004 | 0.499 | 0.5586 | 0.9758 |
| 220.3 | 235 | 395.6 | 369.4 | 1200.6 | 831.1 | 1.964 | 0.509 | 0.5610 | 0.9717 |
| 225.3 | 240 | 397.4 | 371.4 | 1200.9 | 829.5 | 1.924 | 0.520 | 0.5633 | 0.9676 |
| 230.3 | 245 | 399.3 | 373.3 | 1201.2 | 827.9 | 1.887 | 0.530 | 0.5655 | 0.9638 |
| 235.3 | 250 | 401.1 | 375.2 | 1201.5 | 826.3 | 1.850 | 0.541 | 0.5676 | 0.9600 |
| 245.3 | 260 | 404.5 | 378.9 | 1202.1 | 823.1 | 1.782 | 0.561 | 0.5719 | 0.9525 |
| 255.3 | 270 | 407.9 | 382.5 | 1202.6 | 820.1 | 1.718 | 0.582 | 0.5760 | 0.9454 |
| 265.3 | 280 | 411.2 | 386.0 | 1203.1 | 817.1 | 1.658 | 0.603 | 0.5800 | 0.9385 |
| 275.3 | 290 | 414.4 | 389.4 | 1203.6 | 814.2 | 1.602 | 0.624 | 0.5840 | 0.9316 |
| 285.3 | 300 | 417.5 | 392.7 | 1204.1 | 811.3 | 1.551 | 0.645 | 0.5878 | 0.9251 |
| 295.3 | 310 | 420.5 | 395.9 | 1204.5 | 808.5 | 1.502 | 0.666 | 0.5915 | 0.9187 |
| 305.3 | 320 | 423.4 | 399.1 | 1204.9 | 805.8 | 1.455 | 0.687 | 0.5951 | 0.9125 |
| 315.3 | 330 | 426.3 | 402.2 | 1205.3 | 803.1 | 1.413 | 0.708 | 0.5986 | 0.9065 |
| 325.3 | 340 | 429.1 | 405.3 | 1205.7 | 800.4 | 1.372 | 0.729 | 0.6020 | 0.9005 |
| 335.3 | 350 | 431.9 | 408.2 | 1206.1 | 797.8 | 1.334 | 0.750 | 0.6053 | 0.8949 |
| 345.3 | 360 | 434.6 | 411.2 | 1206.4 | 795.3 | 1.298 | 0.770 | 0.6085 | 0.8894 |
| 355.3 | 370 | 437.2 | 414.0 | 1206.8 | 792.8 | 1.264 | 0.791 | 0.6116 | 0.8840 |
| 365.3 | 380 | 439.8 | 416.8 | 1207.1 | 790.3 | 1.231 | 0.812 | 0.6147 | 0.8788 |
| 375.3 | 390 | 442.3 | 419.5 | 1207.4 | 787.9 | 1.200 | 0.833 | 0.6178 | 0.8737 |
| 385.3 | 400 | 444.8 | 422 | 1208 | 786 | 1.17 | 0.86 | 0.621 | 0.868 |
| 435.3 | 450 | 456.5 | 435 | 1209 | 774 | 1.04 | 0.96 | 0.635 | 0.844 |
| 485.3 | 500 | 467.3 | 448 | 1210 | 762 | 0.93 | 1.08 | 0.648 | 0.822 |
| 535.3 | 550 | 477.3 | 459 | 1210 | 751 | 0.83 | 1.20 | 0.659 | 0.801 |
| 585.3 | 600 | 486.6 | 469 | 1210 | 741 | 0.76 | 1.32 | 0.670 | 0.783 |

Available Energy in Expanding Steam. — Rankine Cycle. (J. B. Stanwood, *Power*, June 9, 1908.) — A simple formula for finding, with the aid of the steam and entropy tables, the available energy per pound of steam in B.T.U. when it is expanded adiabatically from a higher to a lower pressure is:

$$U = H - H_1 + T(N_1 - N).$$

U = available B.T.U. in 1 lb. of expanding steam; H and H_1 total heat in 1 lb. steam at the two pressures; T = absolute temperature at the lower pressure; $N - N_1$, difference of entropy of 1 lb. of steam at the two pressures.

EXAMPLE. — Required the available B.T.U. in 1 lb. steam expanded from 100 lbs. to 14.7 lbs. absolute. $H = 1186.3$; $H_1 = 1150.4$; $T = 672$; $N = 1.602$; $N_1 = 1.756$. $35.9 + 103.5 = 138.4$.

Efficiency of the Cycle. — Let the steam be made from feed-water at 212°. Heat required = $1186.3 - 180 = 1006.3$; efficiency = $138.4 \div 1006.3 = 0.1375$.

Rankine Cycle. — This efficiency is that of the Rankine cycle, which assumes that the steam is expanded adiabatically to the lowest pressure and temperature, and that the feed-water from which the steam is made is introduced into the system at the same low temperature.

Carnot Cycle. — The Carnot ideal cycle, which assumes that all the heat entering the system enters at the highest temperature, and in which the efficiency is $(T_1 - T_2) \div T_1$, gives $(327.8 - 212) \div (327.8 + 460) = 0.1470$ and the available energy in B.T.U. = $0.1470 \times 1006.3 = 147.9$ B.T.U.

Properties of Superheated Steam.

(Condensed from Marks and Davis's Steam Tables and Diagrams.)

 v = specific volume in cu. ft. per lb., h = total heat, from water at 32° F. in B.T.U. per lb., n = entropy, from water at 32°.

| Press. Abs. Lbs. per Sq. In. | Temp. Sat. Steam. | Degrees of Superheat. | | | | | | | | | |
|------------------------------------|----------------------|-----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 0 | 20 | 50 | 100 | 150 | 200 | 250 | 300 | 400 | 500 |
| 20 | 228.0 | v 20.08 | 20.73 | 21.69 | 23.25 | 24.80 | 26.33 | 27.85 | 29.37 | 32.39 | 35.40 |
| | | h 1156.2 | 1165.7 | 1179.9 | 1203.5 | 1227.1 | 1250.6 | 1274.1 | 1297.6 | 1344.8 | 1392.2 |
| | | n 1.7320 | 1.7456 | 1.7652 | 1.7961 | 1.8251 | 1.8524 | 1.8781 | 1.9026 | 1.9479 | 1.9893 |
| 40 | 267.3 | v 10.49 | 10.83 | 11.33 | 12.13 | 12.93 | 13.70 | 14.48 | 15.25 | 16.78 | 18.30 |
| | | h 1169.4 | 1179.3 | 1194.0 | 1218.4 | 1242.4 | 1266.4 | 1290.3 | 1314.1 | 1361.6 | 1409.3 |
| | | n 1.6761 | 1.6895 | 1.7089 | 1.7392 | 1.7674 | 1.7940 | 1.8189 | 1.8427 | 1.8867 | 1.9271 |
| 60 | 292.7 | v 7.17 | 7.40 | 7.75 | 8.30 | 8.84 | 9.36 | 9.89 | 10.41 | 11.43 | 12.45 |
| | | h 1177.0 | 1187.3 | 1202.6 | 1227.6 | 1252.1 | 1276.4 | 1300.4 | 1324.3 | 1372.2 | 1420.0 |
| | | n 1.6432 | 1.6568 | 1.6761 | 1.7062 | 1.7342 | 1.7603 | 1.7849 | 1.8081 | 1.8511 | 1.8908 |
| 80 | 312.0 | v 5.47 | 5.65 | 5.92 | 6.34 | 6.75 | 7.17 | 7.56 | 7.95 | 8.72 | 9.49 |
| | | h 1182.3 | 1193.0 | 1208.8 | 1234.3 | 1259.0 | 1283.6 | 1307.8 | 1331.9 | 1379.8 | 1427.9 |
| | | n 1.6200 | 1.6338 | 1.6532 | 1.6833 | 1.7110 | 1.7368 | 1.7612 | 1.7840 | 1.8265 | 1.8658 |
| 100 | 327.8 | v 4.43 | 4.58 | 4.79 | 5.14 | 5.47 | 5.80 | 6.12 | 6.44 | 7.07 | 7.69 |
| | | h 1186.3 | 1197.5 | 1213.8 | 1239.7 | 1264.7 | 1289.4 | 1313.6 | 1337.8 | 1385.9 | 1434.1 |
| | | n 1.6020 | 1.6160 | 1.6358 | 1.6658 | 1.6933 | 1.7188 | 1.7428 | 1.7656 | 1.8079 | 1.8468 |
| 120 | 341.3 | v 3.73 | 3.85 | 4.04 | 4.33 | 4.62 | 4.89 | 5.17 | 5.44 | 5.96 | 6.48 |
| | | h 1189.6 | 1201.1 | 1217.9 | 1244.1 | 1269.3 | 1294.1 | 1318.4 | 1342.7 | 1391.0 | 1439.4 |
| | | n 1.5873 | 1.6016 | 1.6216 | 1.6517 | 1.6789 | 1.7041 | 1.7280 | 1.7505 | 1.7924 | 1.8311 |
| 140 | 353.1 | v 3.22 | 3.32 | 3.49 | 3.75 | 4.00 | 4.24 | 4.48 | 4.71 | 5.16 | 5.61 |
| | | h 1192.2 | 1204.3 | 1221.4 | 1248.0 | 1273.3 | 1298.2 | 1322.6 | 1346.9 | 1395.4 | 1443.8 |
| | | n 1.5747 | 1.5894 | 1.6096 | 1.6395 | 1.6666 | 1.6916 | 1.7152 | 1.7376 | 1.7792 | 1.8177 |
| 160 | 363.6 | v 2.83 | 2.93 | 3.07 | 3.30 | 3.53 | 3.74 | 3.95 | 4.15 | 4.56 | 4.95 |
| | | h 1194.5 | 1207.0 | 1224.5 | 1251.3 | 1276.8 | 1301.7 | 1326.2 | 1350.6 | 1399.3 | 1447.9 |
| | | n 1.5639 | 1.5789 | 1.5993 | 1.6292 | 1.6561 | 1.6810 | 1.7043 | 1.7266 | 1.7680 | 1.8063 |
| 180 | 373.1 | v 2.53 | 2.62 | 2.75 | 2.96 | 3.16 | 3.35 | 3.54 | 3.72 | 4.09 | 4.44 |
| | | h 1196.4 | 1209.4 | 1227.2 | 1254.3 | 1279.9 | 1304.8 | 1329.5 | 1353.9 | 1402.7 | 1451.4 |
| | | n 1.5543 | 1.5697 | 1.5904 | 1.6201 | 1.6468 | 1.6716 | 1.6948 | 1.7169 | 1.7581 | 1.7962 |
| 200 | 381.9 | v 2.29 | 2.37 | 2.49 | 2.68 | 2.86 | 3.04 | 3.21 | 3.38 | 3.71 | 4.03 |
| | | h 1198.1 | 1211.6 | 1229.8 | 1257.1 | 1282.6 | 1307.7 | 1332.4 | 1357.0 | 1405.9 | 1454.7 |
| | | n 1.5456 | 1.5614 | 1.5823 | 1.6120 | 1.6385 | 1.6632 | 1.6862 | 1.7082 | 1.7493 | 1.7872 |
| 220 | 389.9 | v 2.09 | 2.16 | 2.28 | 2.45 | 2.62 | 2.78 | 2.94 | 3.10 | 3.40 | 3.69 |
| | | h 1199.6 | 1213.6 | 1232.2 | 1259.6 | 1285.2 | 1310.3 | 1335.1 | 1359.8 | 1408.8 | 1457.7 |
| | | n 1.5379 | 1.5541 | 1.5753 | 1.6049 | 1.6312 | 1.6558 | 1.6787 | 1.7005 | 1.7415 | 1.7792 |
| 240 | 397.4 | v 1.92 | 1.99 | 2.09 | 2.26 | 2.42 | 2.57 | 2.71 | 2.85 | 3.13 | 3.40 |
| | | h 1200.9 | 1215.4 | 1234.3 | 1261.9 | 1287.6 | 1312.8 | 1337.6 | 1362.3 | 1411.5 | 1460.5 |
| | | n 1.5309 | 1.5476 | 1.5690 | 1.5985 | 1.6246 | 1.6492 | 1.6720 | 1.6937 | 1.7344 | 1.7721 |
| 260 | 404.5 | v 1.78 | 1.84 | 1.94 | 2.10 | 2.24 | 2.39 | 2.52 | 2.65 | 2.91 | 3.16 |
| | | h 1202.1 | 1217.1 | 1236.4 | 1264.1 | 1289.9 | 1315.1 | 1340.0 | 1.64.7 | 1414.0 | 1463.2 |
| | | n 1.5244 | 1.5416 | 1.5631 | 1.5926 | 1.6186 | 1.6430 | 1.6658 | 1.6874 | 1.7280 | 1.7655 |
| 280 | 411.2 | v 1.66 | 1.72 | 1.81 | 1.95 | 2.09 | 2.22 | 2.35 | 2.48 | 2.72 | 2.95 |
| | | h 1203.1 | 1218.7 | 1238.4 | 1266.2 | 1291.9 | 1317.2 | 1342.2 | 1367.0 | 1416.4 | 1465.7 |
| | | n 1.5185 | 1.5362 | 1.5580 | 1.5873 | 1.6133 | 1.6375 | 1.6603 | 1.6818 | 1.7223 | 1.7597 |
| 300 | 417.5 | v 1.55 | 1.60 | 1.69 | 1.83 | 1.96 | 2.09 | 2.21 | 2.33 | 2.55 | 2.77 |
| | | h 1204.1 | 1220.2 | 1240.3 | 1268.2 | 1294.0 | 1319.3 | 1344.3 | 1369.2 | 1418.6 | 1468.0 |
| | | n 1.5129 | 1.5310 | 1.5530 | 1.5824 | 1.6082 | 1.6323 | 1.6550 | 1.6765 | 1.7168 | 1.7541 |
| 350 | 431.9 | v 1.33 | 1.38 | 1.46 | 1.58 | 1.70 | 1.81 | 1.92 | 2.02 | 2.22 | 2.41 |
| | | h 1206.1 | 1223.9 | 1244.6 | 1272.7 | 1298.7 | 1324.1 | 1349.3 | 1374.3 | 1424.0 | 1473.7 |
| | | n 1.5002 | 1.5199 | 1.5423 | 1.5715 | 1.5971 | 1.6210 | 1.6436 | 1.6650 | 1.7052 | 1.7422 |
| 400 | 444.8 | v 1.17 | 1.21 | 1.28 | 1.40 | 1.50 | 1.60 | 1.70 | 1.79 | 1.97 | 2.14 |
| | | h 1207.7 | 1227.2 | 1248.6 | 1276.9 | 1303.0 | 1328.6 | 1353.9 | 1379.1 | 1429.0 | 1478.9 |
| | | n 1.4894 | 1.5107 | 1.5336 | 1.5625 | 1.5880 | 1.6117 | 1.6342 | 1.6554 | 1.6955 | 1.7323 |
| 450 | 456.5 | v 1.04 | 1.08 | 1.14 | 1.25 | 1.35 | 1.44 | 1.53 | 1.61 | 1.77 | 1.93 |
| | | h 1209 | 1231 | 1252 | 1281 | 1307 | 1333 | 1358 | 1383 | 1434 | 1484 |
| | | n 1.479 | 1.502 | 1.526 | 1.554 | 1.580 | 1.603 | 1.626 | 1.647 | 1.687 | 1.723 |
| 500 | 467.3 | v 0.93 | 0.97 | 1.03 | 1.13 | 1.22 | 1.31 | 1.39 | 1.47 | 1.62 | 1.76 |
| | | h 1210 | 1233 | 1256 | 1285 | 1311 | 1337 | 1362 | 1388 | 1438 | 1489 |
| | | n 1.470 | 1.496 | 1.519 | 1.548 | 1.573 | 1.597 | 1.619 | 1.640 | 1.679 | 1.715 |

FLOW OF STEAM.

Flow of Steam through a Nozzle. (From Clark on the Steam-engine.) — The flow of steam of a greater pressure into an atmosphere of a less pressure increases as the difference of pressure is increased; until the external pressure becomes only 58% of the absolute pressure in the boiler. The flow of steam is neither increased nor diminished by the fall of the external pressure below 58%, or about $\frac{4}{7}$ of the inside pressure, even to the extent of a perfect vacuum. In flowing through a nozzle of the best form, the steam expands to the external pressure, and to the volume due to this pressure, so long as it is not less than 58% of the internal pressure. For an external pressure of 58%, and for lower percentages, the ratio of expansion is 1 to 1.624.

When steam of varying initial pressures is discharged into the atmosphere — the atmospheric pressure being not more than 58% of the initial pressure — the velocity of outflow at constant density, that is, supposing the initial density to be maintained, is given by the formula $V = 3.5953 \sqrt{h}$.
 V = velocity in feet per second, as for steam of the initial density;
 h = the height in feet of a column of steam of the given initial pressure, the weight of which is equal to the pressure on the unit of base.

The lowest initial pressure to which the formula applies, when the steam is discharged into the atmosphere at 14.7 lbs. per sq. in., is $(14.7 \times 100/58) = 25.37$ lbs. per sq. in.

From the contents of the table below it appears that the velocity of outflow into the atmosphere, of steam above 25 lbs. per sq. in. absolute pressure, increases very slowly with the pressure, because the density, and the weight to be moved, increase with the pressure. An average of 900 ft. per sec. may, for approximate calculations, be taken for the velocity of outflow as for constant density, that is, taking the volume of the steam at the initial volume. For a fuller discussion of this subject see "Steam Turbines," page 1065.

Outflow of Steam into the Atmosphere. — External pressure per square inch, 14.7 lbs. absolute. Ratio of expansion in nozzle, 1.624.

| Absolute Initial Pressure per square inch. | Velocity of Outflow as at Constant Density. | Actual Velocity of Outflow Expanded. | Discharge per square inch of Orifice per min. | Horse-power per sq. in. of Orifice if H.P. = 30 lbs. per hour. | Absolute Initial Pressure per square inch. | Velocity of Outflow as at Constant Density. | Actual Velocity of Outflow Expanded. | Discharge per square inch of Orifice per minute. | Horse-power per sq. in. of Orifice if H.P. = 30 lbs. per hour. |
|--|---|--------------------------------------|---|--|--|---|--------------------------------------|--|--|
| lbs. | feet p.sec. | feet per sec. | lbs. | H.P. | lbs. | feet p.sec. | feet per sec. | lbs. | H.P. |
| 25.37 | 863 | 1401 | 22.81 | 45.6 | 90 | 895 | 1454 | 77.94 | 155.9 |
| 30 | 867 | 1408 | 26.84 | 53.7 | 100 | 898 | 1459 | 86.34 | 172.7 |
| 40 | 874 | 1419 | 35.18 | 70.4 | 115 | 902 | 1466 | 98.76 | 197.5 |
| 50 | 880 | 1429 | 44.06 | 88.1 | 135 | 906 | 1472 | 115.61 | 231.2 |
| 60 | 885 | 1437 | 52.59 | 105.2 | 155 | 910 | 1478 | 132.21 | 264.4 |
| 70 | 889 | 1444 | 61.07 | 122.1 | 165 | 912 | 1481 | 140.46 | 280.9 |
| 75 | 891 | 1447 | 65.30 | 130.6 | 215 | 919 | 1493 | 181.58 | 363.2 |

Rateau's Formula. — A. Rateau, in 1895-6, made experiments with converging nozzles 0.41, 0.59 and 0.95 in. diam., on steam of pressures from 1.4 to 170 lbs. per sq. in. In his paper read at the Intl. Eng'g. Congress at Glasgow (*Eng. Rec.*, Oct. 16, 1901) he gives the following formula, applicable when the final pressure, absolute, is less than 58% of the initial. Pounds per hour per sq. in. area of orifice = $3.6 P (16.3 - 0.96 \log P)$. P = absolute pressure, lbs. per sq. in.

Napier's Approximate Rule. — Flow in pounds per second = absolute pressure \times area in square inches $\div 70$. This rule gives results

which closely correspond with those in the above table, and with results computed by Rateau's formula, as shown below.

| | | | | | | | | |
|--|-------|-------|-------|-------|-------|--------|--------|--------|
| Abs. press., lbs. per sq. in. | 25.37 | 40 | 60 | 75 | 100 | 135 | 165 | 215 |
| Discharge per min., by table, lbs. | 22.81 | 35.18 | 52.59 | 65.30 | 86.34 | 115.61 | 140.46 | 181.58 |
| By Rateau's for- mula. | 22.76 | 35.43 | 52.49 | 65.25 | 86.28 | 115.47 | 140.28 | 181.39 |
| By Napier's rule | 21.74 | 34.29 | 51.43 | 64.29 | 85.71 | 115.71 | 141.43 | 184.29 |

Flow of Steam in Pipes. — A formula formerly used for velocity of flow of steam in pipes is the same as Downing's for the flow of water in smooth cast-iron pipes, viz., $V = 50 \sqrt{HD/L}$, in which V = velocity in feet per second, L = length and D = diameter of pipe in feet, H = height in feet of a column of steam, of the pressure of the steam at the entrance, which would produce a pressure equal to the difference of pressures at the two ends of the pipe. (For derivation of the coefficient 50, see Briggs on "Warming Buildings by Steam," *Proc. Inst. C. E.*, 1882.)

If Q = quantity in cubic feet per minute, d = diameter in inches, L and H being in feet, the formula reduces to

$$Q = 4.7233 \sqrt{\frac{H}{L}} d^5. \quad H = 0.0448 \frac{Q^2 L}{d^5}, \quad d = 0.5374 \sqrt{\frac{Q^2 L}{H}}.$$

These formulæ are applicable to air and other gases as well as steam. They are not as accurate as later formulæ (see below) in which the coefficients vary with the diameter of the pipe. G. H. Babcock, in "Steam," gives the formula

$$W = 87 \sqrt{\frac{w(p_1 - p_2) d^5}{L \left(1 + \frac{3.6}{d}\right)}}$$

W = weight of steam flowing, in lbs. per minute, w = density in lbs. per cu. ft. of the steam at the entrance to the pipe, p_1 = pressure in lbs. per sq. in. at the entrance, p_2 = pressure at the exit, d = diam. in inches, L = length in feet. This formula is apparently derived from Unwin's formula for flow of fluids in *Ency. Brit.*, vol. xii, pp. 508, 516. Putting the formula in the form $W = c \sqrt{w(p_1 - p_2) d^5/L}$, in which c will vary with the diameter of the pipe, we have,

| | | | | | | | |
|-------------------------|------|------|------|----|------|------|------|
| For diameter, inches... | 1 | 2 | 3 | 4 | 6 | 9 | 12 |
| Value of c | 40.7 | 52.1 | 58.8 | 63 | 68.8 | 73.7 | 79.3 |

One of the most widely accepted formulæ for flow of water is Darcy's,

$V = c \sqrt{\frac{HD}{L^4}}$, in which c has values ranging from 65 for a 1/2-inch pipe up to 111.5 for 24-inch. Using Darcy's coefficients, and modifying his formula to make it apply to steam, to the form

$$Q = c \sqrt{\frac{(p_1 - p_2) d^5}{wL}}, \quad \text{or} \quad W = c \sqrt{\frac{w(p_1 - p_2) d^5}{L}},$$

we obtain,

| | | | | | | | | |
|-----------------------------|------|------|------|------|------|------|------|------|
| For diameter, inches .. 1/2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Value of c | 36.8 | 45.3 | 52.7 | 56.1 | 57.8 | 58.4 | 59.5 | 60.1 |
| For diameter, inches .. 9 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| Value of c | 61.2 | 61.8 | 62.1 | 62.3 | 62.6 | 62.7 | 62.9 | 63.2 |

In the absence of direct experiments these coefficients are probably as accurate as any that may be derived from formulæ for flow of water.

$$\text{Loss of pressure in lbs. per sq. in.} = p_1 - p_2 = \frac{Q^2 w L}{c^2 d^5} = \frac{W^2 L}{c^2 w d^5}.$$

For a comparison of different formulæ for flow of steam see a paper by G. F. Gebhardt, in *Power*, June, 1907.

Table of Flow of Steam in Pipes of Different Diameters and Different Drops in Pressure. (E. C. Sickles, *Trans. A. S. M. E.*, xx 354.) — The drop is calculated from the formula $p_1 - p_2 = 0.000131$

$\left(1 + \frac{3.6}{d}\right) \frac{W^2 L}{w d^5}$, or $W = 87.54 \sqrt{\frac{w (p_1 - p_2) d^5}{L (1 + 3.6/d)}}$. p_1 and p_2 , initial and final pressures, lbs. per sq. in., d = diam. in ins., W = flow in pounds per minute, w = density of steam in lbs. per cu. ft., L = length of pipe in feet. The table is calculated on the basis of $L = 1000$ ft. For any other length the drop is proportional to the length $\div 1000$.

EXAMPLE IN USE OF THE TABLE. — Required the size of pipe to carry 2500 lbs. per min. of steam of 150 lbs. absolute pressure. In the first table we find figures above 2500 lbs. per min. as follows: 2667, 13-in. pipe, line 2; 2736, 14-in. pipe, line 4; 2527, 15-in. pipe, line 8; 2638, 16-in. pipe, line 10; 2623, 18-in. pipe, line 14. In the table on the next page, under 150 lbs., we find the corresponding drops per 1000 ft. as follows: line 2, 9.60 lbs.; line 4, 6.83 lbs.; line 8, 4.10 lbs.; line 10, 3.19 lbs.; line 14, 1.72 lbs.

STEAM DISCHARGE IN POUNDS PER MINUTE.

Corresponding to Drop in Pressure in table on the next page, for Pipe Diameters in Inches in Top Line.

| Line No. | 24 | 22 | 20 | 18 | 16 | 15 | 14 | 13 | 12 | 11 | 10 |
|----------|-------|-------|------|------|------|------|------|------|------|------|------|
| 1 | 14000 | 11188 | 8772 | 6678 | 4923 | 4163 | 3481 | 2871 | 2328 | 1853 | 1443 |
| 2 | 13000 | 10392 | 8144 | 6203 | 4573 | 3867 | 3233 | 2667 | 2165 | 1721 | 1341 |
| 3 | 12000 | 9593 | 7517 | 5724 | 4220 | 3569 | 2983 | 2461 | 1996 | 1589 | 1237 |
| 4 | 11000 | 8804 | 6891 | 5247 | 3868 | 3271 | 2736 | 2256 | 1830 | 1456 | 1134 |
| 5 | 10000 | 7992 | 6265 | 4770 | 3517 | 2974 | 2486 | 2051 | 1663 | 1324 | 1031 |
| 6 | 9500 | 7705 | 5947 | 4532 | 3341 | 2825 | 2362 | 1940 | 1580 | 1258 | 979 |
| 7 | 9000 | 7205 | 5638 | 4293 | 3165 | 2676 | 2237 | 1846 | 1497 | 1192 | 928 |
| 8 | 8500 | 6905 | 5321 | 4054 | 2989 | 2527 | 2113 | 1743 | 1414 | 1125 | 876 |
| 9 | 8000 | 6506 | 5012 | 3816 | 2814 | 2379 | 1989 | 1640 | 1331 | 1059 | 825 |
| 10 | 7500 | 6106 | 4695 | 3577 | 2638 | 2230 | 1865 | 1538 | 1248 | 993 | 873 |
| 11 | 7000 | 5707 | 4385 | 3339 | 2462 | 2082 | 1740 | 1435 | 1164 | 927 | 722 |
| 12 | 6500 | 5307 | 4069 | 3100 | 2286 | 1933 | 1616 | 1333 | 1081 | 860 | 670 |
| 13 | 6000 | 4908 | 3758 | 2862 | 2110 | 1784 | 1492 | 1230 | 998 | 794 | 619 |
| 14 | 5500 | 4508 | 3443 | 2623 | 1934 | 1635 | 1368 | 1128 | 915 | 728 | 567 |
| 15 | 5000 | 4108 | 3132 | 2385 | 1758 | 1487 | 1243 | 1025 | 832 | 662 | 516 |

STEAM DISCHARGE FOR PIPE DIAMETERS IN INCHES, *Continued.*

| Line No. | 9 | 8 | 7 | 6 | 5 | 4 | 3 1/2 | 3 | 2 1/2 | 2 | 1 1/2 | 1 |
|----------|------|-----|-----|-----|------|-------|-------|------|-------|------|-------|------|
| 1 | 1093 | 799 | 560 | 371 | 227 | 123 | 71.6 | 55.9 | 28.8 | 18.1 | 6.81 | 2.52 |
| 2 | 1015 | 742 | 521 | 344 | 210 | 114.6 | 68.6 | 51.9 | 27.6 | 16.8 | 6.52 | 2.34 |
| 3 | 937 | 685 | 481 | 318 | 194 | 106.0 | 65.6 | 47.9 | 26.4 | 15.5 | 6.24 | 2.16 |
| 4 | 859 | 628 | 441 | 292 | 178 | 97.0 | 62.7 | 43.9 | 25.2 | 14.2 | 5.95 | 1.98 |
| 5 | 781 | 571 | 401 | 265 | 162 | 88.2 | 59.7 | 39.9 | 24.0 | 12.9 | 5.67 | 1.80 |
| 6 | 742 | 542 | 381 | 252 | 154 | 83.8 | 56.5 | 37.9 | 22.8 | 12.3 | 5.29 | 1.71 |
| 7 | 703 | 514 | 361 | 239 | 146 | 79.4 | 53.5 | 35.9 | 21.6 | 11.6 | 5.00 | 1.62 |
| 8 | 664 | 485 | 341 | 226 | 138 | 75.0 | 50.5 | 33.9 | 20.4 | 10.9 | 4.72 | 1.53 |
| 9 | 625 | 457 | 321 | 212 | 130 | 70.6 | 47.6 | 31.9 | 19.2 | 10.3 | 4.43 | 1.44 |
| 10 | 586 | 428 | 301 | 199 | 122 | 66.2 | 44.5 | 23.9 | 18.0 | 9.68 | 4.15 | 1.35 |
| 11 | 547 | 400 | 281 | 186 | 113 | 61.7 | 41.6 | 27.9 | 16.8 | 9.03 | 3.86 | 1.26 |
| 12 | 508 | 371 | 261 | 172 | 105 | 57.3 | 38.6 | 25.9 | 15.6 | 8.38 | 3.68 | 1.17 |
| 13 | 469 | 343 | 241 | 159 | 97.2 | 52.9 | 35.6 | 23.9 | 14.4 | 7.74 | 3.40 | 1.08 |
| 14 | 430 | 314 | 221 | 146 | 89.1 | 48.5 | 32.6 | 21.9 | 13.2 | 7.10 | 3.11 | 0.99 |
| 15 | 390 | 286 | 200 | 132 | 81.0 | 44.1 | 29.6 | 20.0 | 12.0 | 6.45 | 2.83 | 0.90 |

DROP IN PRESSURE IN POUNDS PER SQ. IN., PER 1000 FT. LENGTH.
Corresponding to Discharge in above Table.

| Density * Pressure † | 0.208 90 | 0.230 100 | 0.273 120 | 0.295 130 | 0.316 140 | 0.338 150 | 0.401 180 | 0.442 200 | 0.485 220 | 0.548 250 |
|-------------------------|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Line, 1 | 18.10 | 16.4 | 13.8 | 12.8 | 11.9 | 11.1 | 9.39 | 8.50 | 7.76 | 6.87 |
| 2 | 15.60 | 14.1 | 11.9 | 11.0 | 10.3 | 9.60 | 8.09 | 7.33 | 6.69 | 5.92 |
| 3 | 13.3 | 12.0 | 10.1 | 9.38 | 8.75 | 8.18 | 6.90 | 6.24 | 5.70 | 5.05 |
| 4 | 11.1 | 10.0 | 8.46 | 7.83 | 7.31 | 6.83 | 5.76 | 5.21 | 4.76 | 4.21 |
| 5 | 9.25 | 8.36 | 7.5 | 6.52 | 6.09 | 5.69 | 4.80 | 4.34 | 3.97 | 3.51 |
| 6 | 8.33 | 7.53 | 6.35 | 5.87 | 5.48 | 5.13 | 4.32 | 3.91 | 3.57 | 3.16 |
| 7 | 7.48 | 6.76 | 5.70 | 5.27 | 4.92 | 4.60 | 3.88 | 3.51 | 3.21 | 2.84 |
| 8 | 6.67 | 6.03 | 5.08 | 4.70 | 4.39 | 4.10 | 3.46 | 3.13 | 2.86 | 2.53 |
| 9 | 5.91 | 5.35 | 4.50 | 4.17 | 3.89 | 3.64 | 3.07 | 2.78 | 2.53 | 2.24 |
| 10 | 5.19 | 4.69 | 3.95 | 3.65 | 3.42 | 3.19 | 2.69 | 2.44 | 2.23 | 1.97 |
| 11 | 4.52 | 4.09 | 3.44 | 3.19 | 2.98 | 2.78 | 2.34 | 2.12 | 1.94 | 1.72 |
| 12 | 3.90 | 3.53 | 2.97 | 2.75 | 2.57 | 2.40 | 2.02 | 1.83 | 1.67 | 1.48 |
| 13 | 3.32 | 3.00 | 2.53 | 2.34 | 2.19 | 2.04 | 1.72 | 1.56 | 1.42 | 1.26 |
| 14 | 2.79 | 2.52 | 2.13 | 1.97 | 1.84 | 1.72 | 1.45 | 1.31 | 1.20 | 1.06 |
| 15 | 2.31 | 2.09 | 1.76 | 1.63 | 1.52 | 1.42 | 1.20 | 1.08 | 0.991 | 0.877 |

* Density in lbs. per cu. ft.

† Pressure, absolute, lbs. per sq. in.

For Flow of Steam at low pressures, see Heating and Ventilation, page 670.

Carrying Capacity of Extra Heavy Steam Pipes. (Power Speciality Co.)

| Nominal size of pipe, in. | Actual in- side area in sq. in. | 200 lbs. | 150 lbs. | 100 lbs. | 50 lbs. | Nominal size of pipe, in. | Actual in- side area in sq. in. | 200 lbs. | 150 lbs. | 100 lbs. | 50 lbs. |
|---------------------------------|---------------------------------------|-------------|-------------|-------------|------------|---------------------------------|---------------------------------------|-------------|-------------|-------------|------------|
| Pounds of steam per hour. | | | | | | Pounds of steam per hour. | | | | | |
| 1 | 0.71 | 1210 | 872 | 618 | 362 | 6 | 25.93 | 40800 | 31600 | 22600 | 13210 |
| 1 1/4 | 1.27 | 2000 | 1555 | 1105 | 646 | 7 | 34.47 | 54600 | 42250 | 30000 | 17600 |
| 1 1/2 | 1.75 | 2750 | 2140 | 1525 | 894 | 8 | 44.18 | 69500 | 54000 | 38400 | 22450 |
| 2 | 2.93 | 4610 | 3590 | 2550 | 1525 | 9 | 58.42 | 92000 | 71500 | 50800 | 29800 |
| 2 1/2 | 4.20 | 6610 | 5150 | 3660 | 2140 | 10 | 74.66 | 117300 | 91500 | 65000 | 38100 |
| 3 | 6.56 | 10300 | 8050 | 5720 | 3450 | 11 | 90.76 | 142800 | 111500 | 79200 | 46300 |
| 3 1/2 | 8.85 | 13900 | 10820 | 7720 | 4520 | 12 | 108.4 | 170500 | 133000 | 94750 | 55400 |
| 4 | 11.44 | 18000 | 14000 | 10000 | 5850 | 14 | 153.94 | 242000 | 188200 | 133900 | 78600 |
| 4 1/2 | 14.18 | 22300 | 17350 | 12320 | 7230 | 16 | 176.71 | 277500 | 216200 | 153800 | 90500 |
| 5 | 18.19 | 28610 | 22250 | 15800 | 9300 | 18 | 226.98 | 357000 | 278000 | 197500 | 115700 |

The pounds per hour in the above table are figured for the velocities given below:

Steam superheated degrees F... 0 50 100 150 200 250
Velocity, ft. per min. 8000 8500 8950 9450 9900 10450

Flow of Steam in Long Pipes. Ledoux's Formula. — In the flow of steam or other gases in long pipes, the volume and the velocity are increased as the drop in pressure increases. Taking this into account a correct formula for flow would be an exponential one. Ledoux gives

$$d = 0.699 \sqrt{\frac{W^2 L}{p_1^{1.94} - p_2^{1.94}}}$$
, his notation being reduced to English measures. (*Annales des Mines*, 1892; *Trans. A. S. M. E.*, xx., 365; *Power*, June, 1907.) See Johnson's formula for flow of air, page 596.

Resistance to Flow by Bends, Valves, etc. (From Briggs on Warming Buildings by Steam.) — The resistance at the entrance to a tube when no special bell-mouth is given consists of two parts. The head $v^2 \div 2g$ is expended in giving the velocity of flow; and the head $0.595 v^2 \div 2g$ in overcoming the resistance of the mouth of the tube. Hence the whole loss of head at the entrance is $1.595 v^2 \div 2g$. This resistance is equal to the resistance of a straight tube of a length equal to about 60 times its diameter.

The loss at each sharp right-angled elbow is the same as in flowing through a length of straight tube equal to about 40 times its diameter. For a globe steam stop-valve the resistance is taken to be $1\frac{1}{2}$ times that of the right-angled elbow.

Sizes of Steam-pipes for Stationary Engines. — An old common rule is that steam-pipes supplying engines should be of such size that the mean velocity of steam in them does not exceed 6000 feet per minute, in order that the loss of pressure due to friction may not be excessive. The velocity is calculated on the assumption that the cylinder is filled at each stroke. In modern practice with large engines and high pressures, this rule gives unnecessarily large and costly pipes. For such engines the allowable drop in steam pressure should be assumed and the diameter calculated by means of the formulæ given above.

An article in *Power*, May, 1893, on proper area of supply-pipes for engines gives a table showing the practice of leading builders. To facilitate comparison, all the engines have been rated in horse-power at 40 pounds mean effective pressure. The table contains all the varieties of simple engines, from the slide-valve to the Corliss, and it appears that there is no general difference in the sizes of pipe used in the different types.

The averages selected from this table are as follows:

DIAMETERS OF CYLINDERS CORRESPONDING TO VARIOUS SIZES OF STEAM-PIPES BASED ON PISTON-SPEED OF ENGINE OF 600 FT. PER MINUTE, AND ALLOWABLE MEAN VELOCITY OF STEAM IN PIPE OF 4000, 6000, AND 8000 FT. PER MIN. (STEAM ASSUMED TO BE ADMITTED DURING FULL STROKE.)

| | | | | | | | | |
|---------------------------------|-----|-----------------|------|-----------------|------|-----------------|------|------|
| Diam. of pipe, inches | 2 | 2 $\frac{1}{2}$ | 3 | 3 $\frac{1}{2}$ | 4 | 4 $\frac{1}{2}$ | 5 | 6 |
| Vel. 4000 | 5.2 | 6.5 | 7.7 | 9.0 | 10.3 | 11.6 | 12.9 | 15.5 |
| Vel. 6000 | 6.3 | 7.9 | 9.5 | 11.1 | 12.6 | 14.2 | 15.8 | 19.0 |
| Vel. 8000 | 7.3 | 9.1 | 10.9 | 12.8 | 14.6 | 16.4 | 18.3 | 21.9 |
| Horse-power, approx. | 20 | 31 | 45 | 62 | 80 | 100 | 125 | 180 |

| | | | | | | | | |
|------------------------------|------|------|------|------|------|------|------|------|
| Diam. of pipes, inches . . . | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Vel. 4000 | 18.1 | 20.7 | 23.2 | 25.8 | 28.4 | 31.0 | 33.6 | 36.1 |
| Vel. 6000 | 22.1 | 25.3 | 28.5 | 31.6 | 34.8 | 37.9 | 41.1 | 44.3 |
| Vel. 8000 | 25.6 | 29.2 | 32.9 | 36.5 | 40.2 | 43.8 | 47.5 | 51.1 |
| Horse-power, approx | 245 | 320 | 406 | 500 | 606 | 718 | 845 | 981 |

Formula. Area of pipe = $\frac{\text{Area of cylinder} \times \text{piston-speed}}{\text{mean velocity of steam in pipe}}$.

For piston-speed of 600 ft. per min. and velocity in pipe of 4000, 6000, and 8000 ft. per min., area of pipe = respectively 0.15, 0.10, and 0.075 \times area of cylinder. Diam. of pipe = respectively 0.3873, 0.3162, and 0.2739 \times diam. of cylinder. Reciprocals of these figures are 2.582, 3.162, and 3.651.

The first line in the above table may be used for proportioning exhaust pipes, in which a velocity not exceeding 4000 ft. per minute is advisable. The last line, approx. H.P. of engine, is based on the velocity of 6000 ft. per min. in the pipe, using the corresponding diameter of piston, and taking $\text{H.P.} = \frac{1}{2} (\text{diam. of piston in inches})^2$.

Sizes of Steam-pipes for Marine Engines. — In marine-engine practice the steam-pipes are generally not as large as in stationary practice for the same sizes of cylinder. Seaton gives the following rules:

Main Steam-pipes should be of such size that the mean velocity of flow does not exceed 8000 ft. per min.

In large engines, 1000 to 2000 H.P., cutting off at less than half stroke, the steam-pipe may be designed for a mean velocity of 9000 ft., and 10,000 ft. for still larger engines.

In small engines and engines cutting off later than half stroke, a velocity of less than 8000 ft. per minute is desirable.

Taking 8100 ft. per min. as the mean velocity, S speed of piston in feet per min., and D the diameter of the cylinder,

$$\text{Diam. of main steam-pipe} = \sqrt{D^2 S \div 8100} = D \sqrt{S \div 90}.$$

Stop and Throttle Valves should have a greater area of passages than the area of the main steam-pipe, on account of the friction through the circuitous passages. The shape of the passages should be designed so as to avoid abrupt changes of direction and of velocity of flow as far as possible.

Area of Steam Ports and Passages =

$$\frac{\text{Area of piston} \times \text{speed of piston in ft. per min.}}{6000} = \frac{(\text{Diam.})^2 \times \text{speed}}{7639}.$$

Opening of Port to Steam. — To avoid wire-drawing during admission the area of opening to steam should be such that the mean velocity of flow does not exceed 10,000 ft. per min. To avoid excessive clearance the width of port should be as short as possible, the necessary area being obtained by length (measured at right angles to the line of travel of the valve). In practice this length is usually 0.6 to 0.8 of the diameter of the cylinder, but in long-stroke engines it may equal or even exceed the diameter.

Exhaust Passages and Pipes. — The area should be such that the mean velocity of the steam should not exceed 6000 ft. per min., and the area should be greater if the length of the exhaust-pipe is comparatively long. The area of passages from cylinders to receivers should be such that the velocity will not exceed 5000 ft. per min.

The following table is computed on the basis of a mean velocity of flow of 8000 ft. per min. for the main steam-pipe, 10,000 for opening to steam, and 6000 for exhaust. A = area of piston, D its diameter.

STEAM AND EXHAUST OPENINGS.

| Piston-speed, ft. per min. | Diam. of Steam-pipe $\div D$. | Area of Steam-pipe $\div A$. | Diam. of Exhaust $\div D$. | Area of Exhaust $\div A$. | Opening to Steam $\div A$. |
|----------------------------|--------------------------------|-------------------------------|-----------------------------|----------------------------|-----------------------------|
| 300 | 0.194 | 0.0375 | 0.223 | 0.0500 | 0.03 |
| 400 | 0.224 | 0.0500 | 0.258 | 0.0667 | 0.04 |
| 500 | 0.250 | 0.0625 | 0.288 | 0.0833 | 0.05 |
| 600 | 0.274 | 0.0750 | 0.316 | 0.1000 | 0.06 |
| 700 | 0.296 | 0.0875 | 0.341 | 0.1167 | 0.07 |
| 800 | 0.316 | 0.1000 | 0.365 | 0.1333 | 0.08 |
| 900 | 0.335 | 0.1125 | 0.387 | 0.1500 | 0.09 |
| 1000 | 0.353 | 0.1250 | 0.400 | 0.1667 | 0.10 |

Proportioning Steam-Pipes for Minimum Total Loss by Radiation and Friction. — For a given size of pipe and quantity of steam to be carried the loss of pressure due to friction is calculated by formulæ given above, or taken from the tables. The work of friction, being converted into heat, tends to dry or superheat the steam, but its influence is usually so small that it may be neglected. The loss of heat by radiation tends to destroy the superheat and condense some of the steam into water. For well-covered steam-pipes this loss may be estimated at about 0.3 lb. per sq. ft. of external surface of the pipe per hour per degree of difference of temperature between that of the steam and that of the surrounding atmosphere (see Steam-pipe Coverings, p. 558).

A practical problem in power-plant design is to find the diameter of pipe to carry a given quantity of steam with a minimum total loss of available energy due to both radiation and friction, considering also the money loss due to interest and depreciation on the value of the pipe and covering as erected. Each case requires a separate arithmetical computation, no formula yet being constructed to fit the general case. An approximate method of solution, neglecting the slight gain of heat by

the steam from the work of friction, and assuming that the water condensed by radiation of heat is removed by a separator and lost, is as follows: Calculate the amount of steam required by the engine, in pounds per minute. From a steam pipe formula or table find the several drops of pressure, in lbs. per sq. in., in pipes of different assumed diameters, for the given quantity of steam and the given length of pipe. Compute from a theoretical indicator diagram of steam expanding in the engine the loss of available work done by 1 lb. of steam, due to the several drops already found, and the corresponding fraction of 1 lb. of steam that will have to be supplied to make up for this loss of work. State this loss as equivalent to so many pounds of steam per 1000 lbs. of steam carried. Calculate the loss in lbs. of steam condensed by radiation in the pipes of the different diameters, per 1000 lbs. carried. Add the two losses together for each assumed size of pipe, and by inspection find which pipe gives the lowest total loss. The money loss due to cost and depreciation may also be figured approximately in the same unit of lbs. of steam lost per 1000 lbs. carried by taking the cost of the covered pipe, assuming a rate of interest and depreciation, finding the annual loss in cents, then from the calculated value of steam, which depends on the cost of fuel, find the equivalent quantity of steam which represents this money loss, and the equivalent lbs. of steam per 1000 lbs. carried. This is to be added to the sum of the losses due to friction and radiation, and it will be found to modify somewhat the conclusion as to the diameter of pipe and the drop which corresponds to a minimum total loss.

Instead of determining the loss of available work per pound of steam from theoretical indicator diagrams, it may be computed approximately on the assumption, based on the known characteristics of the engine, that its efficiency is a certain fraction of that of an engine working between the same limits of temperature on the ideal Carnot cycle, as shown in the table below, and from the efficiency thus found, compared with the efficiency at the given initial pressure less the drop, the loss of work may be calculated.

AVAILABLE MAXIMUM THERMAL EFFICIENCY OF STEAM EXPANDED BETWEEN THE GIVEN PRESSURES AND 1 LB. ABSOLUTE, BASED ON THE CARNOT CYCLE. $E = (T_1 - T_2) \div T_1$.

| Initial Pressure less than Maxi- mum. | Maximum Initial Absolute Pressures. | | | | | | | | |
|---|-------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 |
| | Maximum Thermal Efficiency. | | | | | | | | |
| lbs. | | | | | | | | | |
| 0..... | 0.287 | 0.302 | 0.314 | 0.324 | 0.333 | 0.341 | 0.348 | 0.354 | 0.360 |
| 2..... | .286 | .301 | .313 | .323 | .332 | .340 | .347 | .354 | .359 |
| 5..... | .284 | .299 | .312 | .322 | .331 | .339 | .346 | .353 | .359 |
| 10..... | .280 | .296 | .309 | .320 | .329 | .337 | .345 | .352 | .358 |
| 20..... | .272 | .290 | .304 | .316 | .326 | .335 | .342 | .349 | .356 |

This table shows that if the initial steam pressure is lowered from 100 lbs. to 80 lbs., the efficiency of the Carnot cycle is reduced from 0.287 to 0.272, or over 5%, but if steam of 300 lbs. is lowered to 280 lbs. the efficiency is reduced only from 0.360 to 0.356 or 1.1%. With high-pressure steam, therefore, much greater loss of pressure by friction of steam pipes, valves and ports is allowable than with steam of low pressure.

Theoretically the loss of efficiency due to drop in pressure on account of friction of pipes should be less than that indicated in the above table, since the work of friction tends to superheat the steam, but practically most, if not all, of the superheating is lost by radiation.

By a method of calculation somewhat similar to that above outlined, the following figures were found, in a certain case, of the cost per day of the transmission of 50,000 lbs. of steam per hour a distance of 1000 feet, with 100 lbs. initial pressure.

| Diameter of Pipe. | 6 in. | 7 in. | 8 in. | 10 in. | 12 in. |
|------------------------------------|--------|--------|--------|--------|--------|
| 1. Interest, etc., 12% per annum.. | \$0.39 | \$0.46 | \$0.53 | \$0.66 | \$0.84 |
| 2. Condensation..... | 1.51 | 1.76 | 2.01 | 2.51 | 3.02 |
| 3. Friction | 0.86 | 0.38 | 0.19 | 0.06 | 0.02 |
| Total per day..... | \$2.76 | \$2.60 | \$2.73 | \$3.23 | \$3.88 |

STEAM PIPES.

Bursting-tests of Copper Steam-pipes. (From Report of Chief Engineer Melville, U. S. N., for 1892.) — Some tests were made at the New York Navy Yard which show the unreliability of brazed seams in copper pipes. Each pipe was 8 in. diameter inside and 3 ft. 1 5/8 in. long. Both ends were closed by ribbed heads and the pipe was subjected to a hot-water pressure, the temperature being maintained constant at 371° F. Three of the pipes were made of No. 4 sheet copper (Stubs gauge) and the fourth was made of No. 3 sheet.

The following were the results, in lbs. per sq. in., of bursting-pressure:

| | | | | | |
|----------------------------|------|------|------|------|------|
| Pipe number | 1 | 2 | 3 | 4 | 4' |
| Actual bursting-strength.. | 835 | 785 | 950 | 1225 | 1275 |
| Calculated " " | 1336 | 1336 | 1569 | 1568 | 1568 |
| Difference..... | 501 | 551 | 619 | 343 | 293 |

The tests of specimens cut from the ruptured pipes show the injurious action of heat upon copper sheets; and that, while a white heat does not change the character of the metal, a heat of only slightly greater degree causes it to lose the fibrous nature that it has acquired in rolling, and a serious reduction in its tensile strength and ductility results.

A Failure of a Brazed Copper Steam-pipe on the British steamer *Prodano* was investigated by Prof. J. O. Arnold. He found that the brazing was originally sound, but that it had deteriorated by oxidation of the zinc in the brazing alloy by electrolysis, which was due to the presence of fatty acids produced by decomposition of the oil used in the engines. A full account of the investigation is given in *The Engineer*, April 15, 1898.

Reinforcing Steam-pipes. (*Eng.*, Aug. 11, 1893.) — In the Italian Navy copper pipes above 8 in. diam. are reinforced by wrapping them with a close spiral of copper or Delta-metal wire. Two or three independent spirals are used for safety in case one wire breaks. They are wound at a tension of about 1 1/2 tons per sq. in.

Materials for Pipes and Valves for Superheated Steam. (M. W. Kellogg, *Trans. A. S. M. E.*, 1907.) — The latest practice is to do away with fittings entirely on high-pressure steam lines and put what are known as "nozzles" on the piping itself. This is accomplished by welding wrought-steel pipe on the side of another section, so as to accomplish the same result as a fitting. In this way rolled or cast steel flanges and a Rockwood or welded joint can be used. This method has three distinct advantages: 1. The quality of the metal used. 2. The lightening of the entire work. 3. The doing away with a great many joints.

As a general average, at least 50% of the joints can be left out; sometimes the proportion runs up as high as 70%.

Above 575° F. the limit of elasticity in cast iron is reached with a pressure varying from 140 to 175 pounds. Under such conditions the material is strained and does not resume its former shape, eventually showing surface cracks which increase until the pipe breaks.

It would seem that iron castings are unsuitable for both fittings and valves to be used in any superheated steam work. The only adaptable metal seems to be cast steel. Tests by Bach on this metal show that at 572° F. the reduction in breaking strength amounts only to 1.1% and at 752° F. to about 8%.

The effect of temperature on nickel is similar to that on cast steel and in consequence this material is very suitable for use in connection with

highly superheated steam. Bach recommends that bronze alloys be done away with for use on steam lines above a temperature of about 390° F.

The old-fashioned screwed joint, no matter how well made, is not suitable for superheated steam work.

In making up a joint, the face of all flanges or pipe where a joint is made should be given a fine tool finish and a plane surface, and a gasket should be used. The best results have been obtained with a corrugated soft Swedish steel gasket with "Smooth-on" applied, and with the McKim gasket, which is of copper or bronze surrounding asbestos. On superheated steam lines a corrugated copper gasket will in time pit out in some part of the flange nearly through the entire gasket.

Specifications for pipes and fittings for superheated steam service were published by Crane Co., Chicago, in the *Valve World*, 1907.

Riveted Steel Steam-pipes have been used for high pressures. See paper on A Method of Manufacture of Large Steam-pipes, by Chas. H. Manning, *Trans. A. S. M. E.*, vol. xv.

Valves in Steam-pipes. — Should a globe-valve on a steam-pipe have the steam-pressure on top or underneath the valve is a disputed question. With the steam-pressure on top, the stuffing-box around the valve-stem cannot be repacked without shutting off steam from the whole line of pipe; on the other hand, if the steam-pressure is on the bottom of the valve it all has to be sustained by the screw-thread on the valve-stem, and there is danger of stripping the thread.

A correspondent of the *American Machinist*, 1892, says that it is a very uncommon thing in the ordinary globe-valve to have the thread give out, but by water-hammer and merciless screwing the seat will be crushed down quite frequently. Therefore with plants where only one boiler is used he advises placing the valve with the boiler-pressure underneath it. On plants where several boilers are connected to one main steam-pipe he would reverse the position of the valve, then when one of the valves needs repacking the valve can be closed and the pressure in the boiler whose pipe it controls can be reduced to atmospheric by lifting the safety-valve. The repacking can then be done without interfering with the operation of the other boilers of the plant.

He proposes also the following other rules for locating valves: Place valves with the stems horizontal to avoid the formation of a water-pocket. Never put the junction-valve close to the boiler if the main pipe is above the boiler, but put it on the highest point of the junction-pipe. If the other plan is followed, the pipe fills with water whenever this boiler is stopped and the others are running, and breakage of the pipe may cause serious results. Never let a junction-pipe run into the bottom of the main pipe, but into the side or top. Always use an angle-valve where convenient, as there is more room in them. Never use a gate valve under high pressure unless a by-pass is used with it. Never open a blow-off valve on a boiler a little and then shut it; it is sure to catch the sediment and ruin the valve; throw it well open before closing. Never use a globe-valve on an indicator-pipe. For water, always use gate or angle valves or stop-cocks to obtain a clear passage. Buy if possible valves with renewable disks. Lastly, never let a man go inside a boiler to work, especially if he is to hammer on it, unless you break the joint between the boiler and the valve and put a plate of steel between the flanges.

The "Steam-Loop" is a system of piping by which water of condensation in steam-pipes is automatically returned to the boiler. In its simplest form it consists of three pipes, which are called the riser, the horizontal, and the drop-leg. When the steam-loop is used for returning to the boiler the water of condensation and entrainment from the steam-pipe through which the steam flows to the cylinder of an engine, the riser is generally attached to a separator; this riser empties at a suitable height into the horizontal, and from thence the water of condensation is led into the drop-leg, which is connected to the boiler, into which the water of condensation is fed as soon as the hydrostatic pressure in the drop-leg in connection with the steam-pressure in the pipes is sufficient to overcome the boiler-pressure. The action of the device depends on the following principles: Difference of pressure may be balanced by a water-column; vapors or liquids tend to flow to the point of lowest pressure; rate of flow depends on difference of pressure and mass; decrease of static pressure in a steam-pipe or chamber is proportional to rate of conden-

sation; in a steam-current water will be carried or swept along rapidly by friction. (Illustrated in *Modern Mechanism*, p. 807. Patented by J. H. Blessing, Feb. 13, 1872, Dec. 28, 1883.) Mr. Blessing thus describes the operation of the loop in *Eng. Review*, Sept., 1907.

The heating system is so arranged that the water of condensation from the radiators gravitates towards some low point and thence is led into the top of a receiver. After this is done it is found that owing to friction caused by the velocity of the steam passing through the different pipes and condensation due to radiation, the steam pressure in the small drip receiver is much less than that in the boiler. This difference will determine the height, or the length of the loop, that must be employed so that the water will gravitate through it into the boiler; that is to say, if there is 10 lbs. difference in pressure, the descending leg of the loop should extend about 30 feet above the water-level in the boiler, since a column of water 2.3 ft. is equal to 1 lb. pressure, and a difference in pressure of 10 lbs. would require a column 23 ft. high. If we make the loop 30 feet high we shall have an additional length of 7 ft. with which to overcome friction. The water, after it reaches the top of the loop, composed of a larger section of pipe, will flow into the boiler through the descending leg with a velocity due to the extra 7 ft. added to the discharging leg.

Loss from an Uncovered Steam-pipe. (Bjorling on Pumping-engines.) — The amount of loss by condensation in a steam-pipe carried down a deep mine-shaft has been ascertained by actual practice at the Clay Cross Colliery, near Chesterfield, where there is a pipe $7\frac{1}{2}$ in. internal diam., 1100 ft. long. The loss of steam by condensation was ascertained by direct measurement of the water deposited in a receiver, and was found to be equivalent to about 1 lb. of coal per I.H.P. per hour for every 100 ft. of steam-pipe; but there is no doubt that if the pipes had been in the up-cast shaft, and well covered with a good non-conducting material, the loss would have been less. (For Steam-pipe Coverings, see p. 558, *ante*.)

Condensation in an Underground Pipe Line. (W. W. Christie, *Eng. Rec.*, 1904.) — A length of 300 ft. of 4-in. pipe, enclosed in a box of $1\frac{1}{4}$ -in. planks, 10 ins. square inside, and packed with mineral wool, was laid in a trench, the upper end being 1 ft. and the lower end 5 ft. below the surface. With 80 lbs. gauge pressure in the pipe the condensation was equivalent to 0.275 B.T.U. per minute per sq. ft. of pipe surface when the outside temperature was 31° F., and 0.222 per min. when the temperature was 62° F.

Steam Receivers on Pipe Lines. (W. Andrews, *Steam Eng'g*, Dec. 10, 1902.) — In the four large power houses in New York City, with an ultimate capacity of 60,000 to 100,000 H.P. each, the largest steam mains are not over 20 ins. in diameter. Some of the best plants have pipes which run from the header to the engine two sizes smaller than that called for by the engine builders. These pipes before reaching the engine are carried into a steel receiver, which acts also as a separator. This receiver has a cubical capacity of three times that of the high-pressure cylinder and is placed as close as possible to the cylinder. The pipe from the receiver to the cylinder is of the full size called for by the engine builder. The objects of this arrangement are: First, to have a full supply of steam to the throttle; second, to provide a cushion near the engine on which the cut-off in the steam chest may be spent, thereby preventing vibrations from being transmitted through the piping system; and third, to produce a steady and rapid flow of steam in one direction only, by having a small pipe leading into the receiver. The steam flows rapidly enough to make good the loss caused during the first quarter of the stroke. Plants fitted up in this way are successfully running where the drop in steam pressure is not greater than 4 lbs., although the engines are 500 ft. away from the boilers.

Equation of Pipes. — For determining the number of small sized pipes that are equal in carrying capacity to one of greater size the table given under *Flow of Air*, page 597, is commonly used. It is based on the equation $N = \sqrt{d^5 + d_1^5}$, in which N is the number of smaller pipes of diameter d_1 equal in capacity to one pipe of diameter d . A more accurate equation, based on Unwin's formula for flow of fluids, is $N = \frac{d^3 \sqrt{d_1 + 3.6}}{d_1 \sqrt{d + 3.6}}$; (d and d_1 in inches). For $d = 2 d_1$, the first formula gives

$N = 5.7$, and the second $N = 6.15$, an unimportant difference, but for $d = 8 d_1$, the first gives $N = 181$ and the second $N = 274$, a considerable difference. (G. F. Gebhardt, *Power*, June, 1907).

Identification of Power House Piping by Different Colors. (W. H. Bryan, *Trans. A. S. M. E.*, 1908.) — In large power plants the multiplicity of pipe lines carrying different fluids causes confusion and may lead to danger by an operator opening a wrong valve. It has therefore become customary to paint the different lines of different colors. The paper gives several tables showing color schemes that have been adopted in different plants. The following scheme, adopted at the New York Edison Co.'s Waterside Station, is selected as an example.

| Pipe Lines. | Colors of Pipe. | Bands, Couplings, Valves, etc. |
|---|-----------------|--------------------------------|
| Steam, high pressure to engines, boiler cross-overs, leaders and headers..... | Black | Brass |
| All other steam lines..... | Buff | Black |
| Steam, exhaust..... | Orange | Red |
| Steam, drips including traps..... | Orange | Black |
| Steam trap discharge..... | Green | Black |
| Blow-offs, drips from water columns and low-pressure drips..... | Slate | Red |
| Drains from crank pits..... | Dark Brown | Blue |
| Cold water to primary heaters and jacket pumps..... | Blue | Red |
| Feed-water, pumps to boilers..... | Maroon | Same |
| Hot-water mains, primary heaters to pumps, and cooling-water returns.... | Green | Red |
| Air pump discharge to hot well..... | Slate | Black |
| Cooling water, pumps to engines..... | Blue | Black |
| Fire lines..... | Vermilion | Same |
| Cylinder oil, high pressure..... | Brown | Black |
| Cylinder oil, low pressure..... | Brown | Green |
| Engine oil..... | Brown | Red |
| Pneumatic system..... | Black | Same |

THE STEAM-BOILER.

The Horse-power of a Steam-boiler. — The term horse-power has two meanings in engineering: *First*, an absolute unit or measure of the rate of work, that is, of the work done in a certain definite period of time, by a source of energy, as a steam-boiler, a waterfall, a current of air or water, or by a prime mover, as a steam-engine, a water-wheel, or a wind-mill. The value of this unit, whenever it can be expressed in foot-pounds of energy, as in the case of steam-engines, water-wheels, and waterfalls, is 33,000 foot-pounds per minute. In the case of boilers, where the work done, the conversion of water into steam, cannot be expressed in foot-pounds of available energy, the usual value given to the term horse-power is the evaporation of 30 lbs. of water of a temperature of 100° F. into steam at 70 lbs. pressure above the atmosphere. Both of these units are arbitrary: the first, 33,000 foot-pounds per minute, first adopted by James Watt, being considered equivalent to the power exerted by a good London draught-horse, and the 30 lbs. of water evaporated per hour being considered to be the steam requirement per indicated horse-power of an average engine.

The second definition of the term horse-power is an approximate measure of the size, capacity, value, or "rating" of a boiler, engine, water-wheel, or other source or conveyer of energy, by which measure it may be described, bought and sold, advertised, etc. No definite value can be given to this measure, which varies largely with local custom or individual opinion of makers and users of machinery. The nearest approach to uniformity which can be arrived at in the term "horse-power," used in this sense, is to say that a boiler, engine, water-wheel, or other machine, "rated" at a

certain horse-power, should be capable of steadily developing that horse-power for a long period of time under ordinary conditions of use and practice, leaving to local custom, to the judgment of the buyer and seller, to written contracts of purchase and sale, or to legal decisions upon such contracts, the interpretation of what is meant by the term "ordinary conditions of use and practice." (*Trans. A. S. M. E.*, vol. vii, p. 226.)

The Committee of Judges of the Centennial Exhibition, 1876, in reporting the trials of competing boilers at that exhibition adopted the unit, 30 lbs. of water evaporated into dry steam per hour from feed-water at 100° F., and under a pressure of 70 lbs. per square inch above the atmosphere, these conditions being considered by them to represent fairly average practice.

The A. S. M. E. Committee on Boiler Tests, 1884, accepted the same unit, and defined it as equivalent to 34.5 lbs. evaporated per hour from a feed-water temperature of 212° into steam at the same temperature. The committee of 1899 adopted this definition, 34.5 lbs. per hour, from and at 212°, as the unit of commercial horse-power. Using the figures for total heat of steam given in Marks and Davis's steam tables (1909), 341½ lbs. from and at 212°, is equivalent to 33,479 B.T.U. per hour, or to an evaporation of 30.018 lbs. from 100° feed-water temperature into steam at 70 lbs. pressure.

The Committee of 1899 says: A boiler rated at any stated capacity should develop that capacity when using the best coal ordinarily sold in the market where the boiler is located, when fired by an ordinary fireman, without forcing the fires, while exhibiting good economy; and further, the boiler should develop at least one-third more than the stated capacity when using the same fuel and operated by the same fireman, the full draught being employed and the fires being crowded; the available draught at the damper, unless otherwise understood, being not less than ½ inch water column.

Unit of Evaporation. (Abbreviation, U. E.) — It is the custom to reduce results of boiler-tests to the common standard of the equivalent evaporation from and at the boiling-point at atmospheric pressure, or "from and at 212° F." This unit of evaporation, or one pound of water evaporated from and at 212°, is equivalent to 970.4 British thermal units. 1 B.T.U. = the mean quantity of heat required to raise 1 lb. of water 1° F. between 32° and 212°.

Measures for Comparing the Duty of Boilers. — The measure of the efficiency of a boiler is the number of pounds of water evaporated per pound of combustible (coal less moisture and ash), the evaporation being reduced to the standard of "from and at 212°."

The measure of the capacity of a boiler is the amount of "boiler horse-power" developed, a horse-power being defined as the evaporation of 34½ lbs. per hour from and at 212°.

The measure of relative rapidity of steaming of boilers is the number of pounds of water evaporated from and at 212° per hour per square foot of water-heating surface.

The measure of relative rapidity of combustion of fuel in boiler-furnaces is the number of pounds of coal burned per hour per square foot of grate-surface.

STEAM-BOILER PROPORTIONS.

Proportions of Grate and Heating Surface required for a given Horse-power. — The term horse-power here means capacity to evaporate 34.5 lbs. of water from and at 212° F.

Average proportions for maximum economy for land boilers fired with good anthracite coal:

| | |
|---|--------------|
| Heating surface per horse-power..... | 11.5 sq. ft. |
| Grate surface per horse-power..... | 1/3 " |
| Ratio of heating to grate surface..... | 34.5 " |
| Water evap'd from and at 212° per sq. ft. H.S. per hr. | 3 lbs. |
| Combustible burned per H.P. per hour..... | 3 " |
| Coal with 1/8 refuse, lbs. per H.P. per hour..... | 3.6 " |
| Combustible burned per sq. ft. grate per hour..... | 9 " |
| Coal with 1/8 refuse, lbs. per sq. ft. grate per hour.... | 10.8 " |
| Water evap'd from and at 212° per lb. combustible... | 11.5 " |
| Water evap'd from and at 212° per lb. coal (1/8 refuse) | 9.6 " |

Heating-surface. — For maximum economy with any kind of fuel a boiler should be proportioned so that at least one square foot of heating-surface should be given for every 3 lbs. of water to be evaporated from and at 212° F. per hour. Still more liberal proportions are required if a portion of the heating-surface has its efficiency reduced by: 1. Tendency of the heated gases to short-circuit, that is, to select passages of least resistance and flow through them with high velocity, to the neglect of other passages. 2. Deposition of soot from smoky fuel. 3. Incrustation. If the heating-surfaces are clean, and the heated gases pass over it uniformly, little if any increase in economy can be obtained by increasing the heating-surface beyond the proportion of 1 sq. ft. to every 3 lbs. of water to be evaporated, and with all conditions favorable but little decrease of economy will take place if the proportion is 1 sq. ft. to every 4 lbs. evaporated; but in order to provide for driving of the boiler beyond its rated capacity, and for possible decrease of efficiency due to the causes above named, it is better to adopt 1 sq. ft. to 3 lbs. evaporation per hour as the minimum standard proportion.

Where economy may be sacrificed to capacity, as where fuel is very cheap, it is customary to proportion the heating-surface much less liberally. The following table shows approximately the relative results that may be expected with different rates of evaporation, with anthracite coal.

| Lbs. water evapor'd from and at 212° per sq. ft. heating-surface per hour: | | | | | | | | | | |
|--|------|------|------|------|------|------|------|------|------|------|
| 2 | 2.5 | 3 | 3.5 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Sq. ft. heating-surface required per horse-power: | | | | | | | | | | |
| 17.3 | 13.8 | 11.5 | 9.8 | 8.6 | 6.8 | 5.8 | 4.9 | 4.3 | 3.8 | 3.5 |
| Ratio of heating to grate surface if $\frac{1}{3}$ sq. ft. of G.S. is required per H.P.: | | | | | | | | | | |
| 52 | 41.4 | 34.5 | 29.4 | 25.8 | 20.4 | 17.4 | 13.7 | 12.9 | 11.4 | 10.5 |
| Probable relative economy: | | | | | | | | | | |
| 100 | 100 | 100 | 95 | 90 | 85 | 80 | 75 | 70 | 65 | 60 |
| Probable temperature of chimney gases, degrees F.: | | | | | | | | | | |
| 450 | 450 | 450 | 518 | 585 | 652 | 720 | 787 | 855 | 922 | 990 |

The relative economy will vary not only with the amount of heating-surface per horse-power, but with the efficiency of that heating-surface as regards its capacity for transfer of heat from the heated gases to the water, which will depend on its freedom from soot and incrustation, and upon the circulation of the water and the heated gases.

With bituminous coal the efficiency will largely depend upon the thoroughness with which the combustion is effected in the furnace.

The efficiency with any kind of fuel will greatly depend upon the amount of air supplied to the furnace in excess of that required to support combustion. With strong draught and thin fires this excess may be very great, causing a serious loss of economy. This subject is further discussed below.

Measurement of Heating-surface. — The usual rule is to consider as heating-surface all the surfaces that are surrounded by water on one side and by flame or heated gases on the other, using the external instead of the internal diameter of tubes, for greater convenience in calculation, the external diameter of boiler-tubes usually being made in even inches or half inches. This method, however, is inaccurate, for the true heating-surface of a tube is the side exposed to the hot gases, the inner surface in a fire-tube boiler and the outer surface in a water-tube boiler. The resistance to the passage of heat from the hot gases on one side of a tube or plate to the water on the other consists almost entirely of the resistance to the passage of the heat from the gases into the metal, the resistance of the metal itself and that of the wetted surface being practically nothing. See paper by C. W. Baker, *Trans. A. S. M. E.*, vol. xix.

RULE for finding the heating-surface of vertical tubular boilers: Multiply the circumference of the fire-box (in inches) by its height above the grate; multiply the combined circumference of all the tubes by their length, and to these two products add the area of the lower tube-sheet; from this sum subtract the area of all the tubes, and divide by 144: the quotient is the number of square feet of heating-surface.

RULE for finding the heating-surface of horizontal tubular boilers: Take the dimensions in inches. Multiply two-thirds of the circumference of the shell by its length; multiply the sum of the circumferences of all the tubes

by their common length; to the sum of these products add two thirds of the area of both tube-sheets; from this sum subtract twice the combined area of all the tubes; divide the remainder by 144 to obtain the result in square feet.

RULE for finding the square feet of heating-surface in tubes: Multiply the number of tubes by the diameter of a tube in inches, by its length in feet, and by 0.2618.

Horse-power, Builder's Rating. Heating-surface per Horse-power. — It is a general practice among builders to furnish about 10 square feet of heating-surface per horse-power, but as the practice is not uniform, bids and contracts should always specify the amount of heating-surface to be furnished. Not less than one-third square foot of grate-surface should be furnished per horse-power with ordinary chimney draught, not exceeding 0.3 in. of water column at the damper, for anthracite coal, and for poor varieties of soft coal high in ash, with ordinary furnaces. A smaller ratio of grate surface may be allowed for high grade soft coal and for forced draught.

Horse-power of Marine and Locomotive Boilers. — The term horse-power is not generally used in connection with boilers in marine practice, or with locomotives. The boilers are designed to suit the engines, and are rated by extent of grate and heating-surface only.

Grate-surface. — The amount of grate-surface required per horse-power, and the proper ratio of heating-surface to grate-surface are extremely variable, depending chiefly upon the character of the coal and upon the rate of draught. With good coal, low in ash, approximately equal results may be obtained with large grate-surface and light draught and with small grate-surface and strong draught, the total amount of coal burned per hour being the same in both cases. With good bituminous coal, like Pittsburgh, low in ash, the best results apparently are obtained with strong draught and high rates of combustion, provided the grate-surfaces are cut down so that the total coal burned per hour is not too great for the capacity of the heating-surface to absorb the heat produced.

With coals high in ash, especially if the ash is easily fusible, tending to choke the grates, large grate-surface and a slow rate of combustion are required, unless means, such as shaking grates, are provided to get rid of the ash as fast as it is made.

The amount of grate-surface required per horse-power under various conditions may be estimated from the following table:

| | Lbs. Water from and at 212° per lb., Coal. | Lbs. Coal per H.P. per hour. | Pounds of Coal burned per square foot of Grate per hour. | | | | | | | | |
|--------------------------|--|------------------------------|--|------|-----|-----|-----|-----|-----|-----|-----|
| | | | 8 | 10 | 12 | 15 | 20 | 25 | 30 | 35 | 40 |
| | | | Sq. Ft. Grate per H.P. | | | | | | | | |
| Good coal and boiler, | 10 | 3.45 | .43 | .35 | .28 | .23 | .17 | .14 | .11 | .10 | .09 |
| | 9 | 3.83 | .48 | .38 | .32 | .25 | .19 | .15 | .13 | .11 | .10 |
| | 8.61 | 4. | .50 | .40 | .33 | .26 | .20 | .16 | .13 | .12 | .10 |
| Fair coal or boiler, | 8 | 4.31 | .54 | .43 | .36 | .29 | .22 | .17 | .14 | .13 | .11 |
| | 7 | 4.93 | .62 | .49 | .41 | .33 | .24 | .20 | .17 | .14 | .12 |
| | 6.9 | 5. | .63 | .50 | .42 | .34 | .25 | .20 | .17 | .15 | .13 |
| Poor coal or boiler, | 6 | 5.75 | .72 | .58 | .48 | .38 | .29 | .23 | .19 | .17 | .14 |
| | 5 | 6.9 | .86 | .69 | .58 | .46 | .35 | .28 | .23 | .22 | .17 |
| Lignite and poor boiler, | 3.45 | 10. | 1.25 | 1.00 | .83 | .67 | .50 | .40 | .33 | .29 | .25 |

In designing a boiler for a given set of conditions, the grate-surface should be made as liberal as possible, say sufficient for a rate of combustion of 10 lbs. per square foot of grate for anthracite, and 15 lbs. per square foot for bituminous coal, and in practice a portion of the grate-surface may be bricked over if it is found that the draught, fuel, or other conditions render it advisable.

Proportions of Areas of Flues and other Gas-passages. — Rules are usually given making the area of gas-passages bear a certain ratio to the area of the grate-surface; thus a common rule for horizontal tubular boilers is to make the area over the bridge wall $\frac{1}{7}$ of the grate-surface, the flue area $\frac{1}{8}$, and the chimney area $\frac{1}{9}$.

For average conditions with anthracite coal and moderate draught, say a rate of combustion of 12 lbs. coal per square foot of grate per hour, and a ratio of heating to grate surface of 30 to 1, this rule is as good as any, but it is evident that if the draught were increased so as to cause a rate of combustion of 24 lbs., requiring the grate-surface to be cut down to a ratio of 60 to 1, the areas of gas-passages should not be reduced in proportion. The amount of coal burned per hour being the same under the changed conditions, and there being no reason why the gases should travel at a higher velocity, the actual areas of the passages should remain as before, but the ratio of the area to the grate-surface would in that case be doubled.

Mr. Barrus states that the highest efficiency with anthracite coal is obtained when the tube area is $\frac{1}{9}$ to $\frac{1}{10}$ of the grate-surface, and with bituminous coal when it is $\frac{1}{8}$ to $\frac{1}{7}$, for the conditions of medium rates of combustion, such as 10 to 12 lbs. per square foot of grate per hour, and 12 square feet of heating-surface allowed to the horse-power.

The tube area should be made large enough not to choke the draught and so lessen the capacity of the boiler; if made too large the gases are apt to select the passages of least resistance and escape from them at a high velocity and high temperature.

This condition is very commonly found in horizontal tubular boilers where the gases go chiefly through the upper rows of tubes; sometimes also in vertical tubular boilers, where the gases are apt to pass most rapidly through the tubes nearest to the center. It may to some extent be remedied by placing retarders in those tubes in which the gases travel the quickest.

Air-passages through Grate-bars. — The usual practice is, air-opening = 30% to 50% of area of the grate; the larger the better, to avoid stoppage of the air-supply by clinker; but with coal free from clinker much smaller air-space may be used without detriment. See paper by F. A. Scheffler, *Trans. A. S. M. E.*, vol. xv, p. 503.

PERFORMANCE OF BOILERS.

The performance of a steam-boiler comprises both its capacity for generating steam and its economy of fuel. Capacity depends upon size, both of grate-surface and of heating-surface, upon the kind of coal burned, upon the draught, and also upon the economy. Economy of fuel depends upon the completeness with which the coal is burned in the furnace, on the proper regulation of the air-supply to the amount of coal burned, and upon the thoroughness with which the boiler absorbs the heat generated in the furnace. The absorption of heat depends on the extent of heating-surface in relation to the amount of coal burned or of water evaporated, upon the arrangement of the gas-passages, and upon the cleanness of the surfaces. The capacity of a boiler may increase with increase of economy when this is due to more thorough combustion of the coal or to better regulation of the air-supply, or it may increase at the expense of economy when the increased capacity is due to overdriving, causing an increased loss of heat in the chimney gases. The relation of capacity to economy is therefore a complex one, depending on many variable conditions.

A formula expressing the relation between capacity, rate of driving, or evaporation per square foot of heating-surface, to the economy, or evaporation per pound of combustible is given on page 865.

Selecting the highest results obtained at different rates of driving with anthracite coal in the Centennial tests (see p. 867), and the highest results with anthracite reported by Mr. Barrus in his book on Boiler Tests, the author has plotted two curves showing the maximum results which may be expected with anthracite coal, the first under exceptional conditions such as obtained in the Centennial tests, and the second under the best conditions of ordinary practice. (*Trans. A. S. M. E.*, xviii, 354.) From these curves the following figures are obtained.

Lbs. water evaporated from and at 212° per sq. ft. heating-surface per hour:

| | 1.6 | 1.7 | 2 | 2.6 | 3 | 3.5 | 4 | 4.5 | 5 | 6 | 7 | 8 |
|--|-----|-----|---|-----|---|-----|---|-----|---|---|---|---|
|--|-----|-----|---|-----|---|-----|---|-----|---|---|---|---|

Lbs. water evaporated from and at 212° per lb. combustible:

| | | | | | | | | | | | | |
|-------------------|------|------|-------|------|-------|------|-------|------|------|-------|-----|-----|
| Centennial. | 11.8 | 11.9 | 12.0 | 12.1 | 12.05 | 12 | 11.85 | 11.7 | 11.5 | 10.85 | 9.8 | 8.5 |
| Barrus. . . | 11.4 | 11.5 | 11.55 | 11.6 | 11.6 | 11.5 | 11.2 | 10.9 | 10.6 | 9.9 | 9.2 | 8.5 |
| Avg. Cent'l . . . | 12.0 | 11.6 | 11.2 | 10.8 | 10.4 | 10.0 | 9.6 | 8.8 | 8.0 | 7.2 | | |

The figures in the last line are taken from a straight line drawn as nearly as possible through the average of the plotting of all the Centennial tests. The poorest results are far below these figures. It is evident that no formula can be constructed that will express the relation of economy to rate of driving as well as do the three lines of figures given above.

For semi-bituminous and bituminous coals the relation of economy to the rate of driving no doubt follows the same general law that it does with anthracite, i.e., that beyond a rate of evaporation of 3 or 4 lbs. per sq. ft. of heating-surface per hour there is a decrease of economy, but the figures obtained in different tests will show a wider range between maximum and average results on account of the fact that it is more difficult with bituminous than with anthracite coal to secure complete combustion in the furnace.

The amount of the decrease in economy due to driving at rates exceeding 4 lbs. of water evaporated per square foot of heating-surface per hour differs greatly with different boilers, and with the same boiler it may differ with different settings and with different coal. The arrangement and size of the gas-passages seem to have an important effect upon the relation of economy to rate of driving.

A comparison of results obtained from different types of boilers leads to the general conclusion that the economy with which different types of boilers operate depends much more upon their proportions and the conditions under which they work, than upon their type; and, moreover, that when the proportions are correct, and when the conditions are favorable, the various types of boilers give substantially the same economic result.

Conditions of Fuel Economy in Steam-boilers.—1. That the boiler has sufficient heating surface to absorb from 75 to 80% of all the heat generated by the fuel. 2. That this surface is so placed, and the gas passages so controlled by baffles, that the hot gases are forced to pass uniformly over the surface, not being short-circuited. 3. That the furnace is of such a kind, and operated in such a manner, that the fuel is completely burned in it, and that no unburned gases reach the heating surface of the boiler. 4. That the fuel is burned with the minimum supply of air required to insure complete combustion, thereby avoiding the carrying of an excessive quantity of heated air out of the chimney.

There are two indices of high economy. 1. High temperature, approaching 3000° F. in the furnace, combined with low temperature, below 600° F., in the flue. 2. Analysis of the flue gases showing between 5 and 8% of free oxygen. Unfortunately neither of these indices is available to the ordinary fireman; he cannot distinguish by the eye any temperature above 2000°, and he cannot know whether or not an excessive amount of oxygen is passing through the fuel. The ordinary haphazard way of firing therefore gives an average of about 10% lower economy than can be obtained when the firing is controlled, as it is in many large plants, by recording furnace pyrometers, or by continuous gas analysis, or by both. Low CO₂ in the flue gases may indicate either excessive air supply in the furnace, or leaks of air into the setting, or deficient air supply with the presence of CO, and therefore imperfect combustion. The latter, if excessive, is indicated by low furnace temperature. The analysis for CO₂ should be made both of the gas sampled just beyond the furnace and of the gas sampled at the flue. Diminished CO₂ in the latter indicates air-leakage.

Less than 5% of free oxygen in the gases is usually accompanied with CO, and it therefore indicates imperfect combustion from deficient air supply. More than 8% means excessive air supply and corresponding waste of heat.

Air Leakage or infiltration of air through the firebrick setting is a common cause of poor economy. It may be detected by analysis as above

stated, and should be prevented by stopping all visible cracks in the brick-work, and by covering it with a coating impervious to air.

Autographic CO₂ Recorders are used in many large boiler plants for the continuous recording of the percentage of carbon dioxide in the gases. When the percentage of CO₂ is between 12 and 16, it indicates good furnace conditions, when below 12 the reverse.

Efficiency of a Steam-boiler. — The efficiency of a boiler is the percentage of the total heat generated by the combustion of the fuel which is utilized in heating the water and in raising steam. With anthracite coal the heating-value of the combustible portion is very nearly 14,800 B.T.U. per lb., equal to an evaporation from and at 212° of 14,800 ÷ 970 = 15.26 lbs. of water. A boiler which when tested with anthracite coal shows an evaporation of 12 lbs. of water per lb. of combustible, has an efficiency of $12 \div 15.26 = 78.6\%$, a figure which is approximated, but scarcely ever quite reached, in the best practice. With bituminous coal it is necessary to have a determination of its heating-power made by a coal calorimeter before the efficiency of the boiler using it can be determined, but a close estimate may be made from the chemical analysis of the coal. (See Coal.)

The difference between the efficiency obtained by test and 100% is the sum of the numerous wastes of heat, the chief of which is the necessary loss due to the temperature of the chimney-gases. If we have an analysis and a calorimetric determination of the heating-power of the coal (properly sampled), and an average analysis of the chimney-gases, the amounts of the several losses may be determined with approximate accuracy by the method described below.

Data given:

| 1. ANALYSIS OF THE COAL. | | 2. ANALYSIS OF THE DRY CHIMNEY-GASES, BY WEIGHT. | | | |
|-----------------------------|--------|--|-------|-------|-------|
| Cumberland Semi-bituminous. | | | C. | O. | N. |
| Carbon | 80.55 | CO ₂ = 13.6 = | 3.71 | 9.89 | |
| Hydrogen | 4.50 | CO = 0.2 = | 0.09 | 0.11 | |
| Oxygen | 2.70 | O = 11.2 = | | 11.20 | |
| Nitrogen | 1.08 | N = 75.0 = | | | 75.00 |
| Moisture | 2.92 | | | | |
| Ash | 8.25 | | | | |
| | 100.00 | 100.0 | 3.80 | 21.20 | 75.00 |

Heating-value of the coal by Dulong's formula, 14,243 heat-units.

The gases being collected over water, the moisture in them is not determined.

3. Ash and refuse as determined by boiler-test, 10.25, or 2% more than that found by analysis, the difference representing carbon in the ashes obtained in the boiler-test.

4. Temperature of external atmosphere, 60° F.

5. Relative humidity of air, 60%, corresponding (see air tables) to 0.007 lb. of vapor in each lb. of air.

6. Temperature of chimney-gases, 560° F.

Calculated results:

The carbon in the chimney-gases being 3.8% of their weight, the total weight of dry gases per lb. of carbon burned is $100 \div 3.8 = 26.32$ lbs. Since the carbon burned is $80.55 - 2 = 78.55\%$ of the weight of the coal, the weight of the dry gases per lb. of coal is $26.32 \times 78.55 \div 100 = 20.67$ lbs.

Each pound of coal furnishes to the dry chimney-gases 0.7855 lb. C, 0.0108 N, and $(2.70 - \frac{4.50}{8}) \div 100 = 0.0214$ lb. O; a total of 0.8177, say 0.82 lb. This subtracted from 20.67 lbs. leaves 19.85 lbs. as the quantity of dry air (not including moisture) which enters the furnace per pound of coal, not counting the air required to burn the available hydrogen, that is, the hydrogen minus one-eighth of the oxygen chemically combined in the coal. Each lb. of coal burned contained 0.045 lb. H, which requires $0.045 \times 8 = 0.36$ lb. O for its combustion. Of this, 0.027 lb. is furnished by the coal itself, leaving 0.333 lb. to come from the air. The quantity of air needed to supply this oxygen (air containing 23% by weight of oxygen) is $0.333 \div 0.23 = 1.45$ lb., which added to the 19.85 lbs. already

found gives 21.30 lbs. as the quantity of dry air supplied to the furnace per lb. of coal burned.

The air carried in as vapor is 0.0071 lb. for each lb. of dry air, or $21.3 \times 0.0071 = 0.15$ lb. for each lb. of coal. Each lb. of coal contained 0.029 lb. of moisture, which was evaporated and carried into the chimney-gases. The 0.045 lb. of H per lb. of coal when burned formed $0.045 \times 9 = 0.405$ lb. of H_2O .

From the analysis of the chimney-gas it appears that $0.09 \div 3.80 = 2.37\%$ of the carbon in the coal was burned to CO instead of to CO_2 .

We now have the data for calculating the various losses of heat, as follows, for each pound of coal burned:

| | Heat-units. | Per cent of Heat-value of the Coal. |
|---|-------------|-------------------------------------|
| 20.67 lbs. dry gas $\times (560^\circ - 60^\circ) \times$ sp. heat $0.24 =$ | 2480.4 | 17.41 |
| 0.15 lb. vapor in air $\times (560^\circ - 60^\circ) \times$ sp. ht. $0.48 =$ | 36.0 | 0.25 |
| 0.029 lb. moist. in coal heated from 60° to $212^\circ =$ | 4.4 | 0.03 |
| 0.029 lb. evap. from and at 212° : $0.029 \times 966 =$ | 28.0 | 0.20 |
| 0.029 lb. steam (heated 212° to 560°) $\times 348 \times 0.48 =$ | 4.8 | 0.03 |
| 0.405 lb. H_2O from H in coal $\times (152 + 966 + 348 \times 0.48) =$ | 520.4 | 3.65 |
| 0.0237 lb. C burned to CO; loss by incomplete combustion, $0.0237 \times (14544 - 4451) =$ | 239.2 | 1.68 |
| 0.02 lb. coal lost in ashes: $0.02 \times 14544 =$ | 290.9 | 2.04 |
| Radiation and unaccounted for, by difference = | 624.0 | 4.38 |
| | 4228.1 | 29.69 |
| Utilized in making steam, equivalent evaporation 10.37 lbs. from and at 212° per lb. of coal = | 10,014.9 | 70.31 |
| | 14,243.0 | 100.00 |

The heat lost by radiation from the boiler and furnace is not easily determined directly, especially if the boiler is enclosed in brickwork, or is protected by non-conducting covering. It is customary to estimate the heat lost by radiation by difference, that is, to charge radiation with all the heat lost which is not otherwise accounted for.

One method of determining the loss by radiation is to block off a portion of the grate-surface and build a small fire on the remainder, and drive this fire with just enough draught to keep up the steam-pressure and supply the heat lost by radiation without allowing any steam to be discharged, weighing the coal consumed for this purpose during a test of several hours' duration.

Estimates of radiation by difference are apt to be greatly in error, as in this difference are accumulated all the errors of the analyses of the coal and of the gases. An average value of the heat lost by radiation from a boiler set in brickwork is about 4 per cent. When several boilers are in a battery and enclosed in a boiler-house the loss by radiation may be very much less, since much of the heat radiated from the boiler is returned to it in the air supplied to the furnace, which is taken from the boiler-room.

An important source of error in making a "heat balance" such as the one above given, especially when highly bituminous coal is used, may be due to the non-combustion of part of the hydrocarbon gases distilled from the coal immediately after firing, when the temperature of the furnace may be reduced below the point of ignition of the gases. Each pound of hydrogen which escapes burning is equivalent to a loss of heat in the furnace of 62,000 heat-units. Another source of error, especially with bituminous slack coal high in moisture, is due to the formation of water-gas, $CO + H$, by the decomposition of the water, and the consequent absorption of heat, this water-gas escaping unburned on account of the choking of the air supply when fine fresh coal is supplied to the fire.

In analyzing the chimney-gases by the usual method the percentages of the constituent gases are obtained by volume instead of by weight. To reduce percentages by volume to percentages by weight, multiply the percentage by volume of each gas by its specific gravity as compared with air, and divide each product by the sum of the products.

Instead of using the percentages by weight of the gases, the percentage

by volume may be used directly to find the weight of gas per pound of carbon by the formula given below.

If O, CO, CO₂, and N represent the percentages by volume of oxygen, carbonic oxide, carbonic acid, and nitrogen, respectively, in the gases of combustion:

$$\left. \begin{array}{l} \text{Lbs. of air required to burn} \\ \text{one pound of carbon} \end{array} \right\} = \frac{3.032 N}{\text{CO}_2 + \text{CO}}.$$

$$\text{Ratio of total air to the theoretical requirement} = \frac{N}{N - 3.782 O}.$$

$$\left. \begin{array}{l} \text{Lbs. of air per pound} \\ \text{of coal} \end{array} \right\} = \left\{ \begin{array}{l} \text{Lbs. of air per pound} \\ \text{of carbon} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Per cent of carbon} \\ \text{in coal} \end{array} \right\}$$

$$\text{Lbs. dry gas produced per pound of carbon} = \frac{11 \text{ CO}_2 + 8 \text{ O} + 7 (\text{CO} + \text{N})}{3 (\text{CO}_2 + \text{CO})}.$$

Relation of Boiler Efficiency to the Rate of Driving, Air Supply, etc.—In the author's *Steam Boiler Economy* (p. 205) a formula is developed showing the efficiency that may be expected, when the combustion of the coal is complete, under different conditions. The formula is

$$\frac{E_a}{E_p} = \frac{K - tcf}{K (1 + RS/W)} - \frac{970}{K} \frac{ac^2f^2}{(K - tcf)} \frac{W}{S}.$$

K = heating value per lb. of combustible; E_a = actual evaporation from and at 212° per lb. of combustible; E_p = possible evaporation = $K + 970$; t = elevation of the temperature of the water in the boiler above the atmospheric temperature; c = specific heat of the chimney gases, taken at 0.24; f = weight of flue gases per lb. of combustible; S = square feet of heating surface; W = pounds of water evaporated per hour; W/S = rate of driving; R = radiation loss, in units of evaporation per sq. ft. of heating-surface per hour; a is a coefficient found by experiment; it may be called a coefficient of inefficiency of the boiler, and it depends on and increases with the resistance to the passage of heat through the metal, soot or scale on the metal, imperfect combustion, short-circuiting, air leakage, or any other defective condition, not expressed in terms in the formula, which may tend to lower the efficiency. Its value is between 200 and 400 when records of tests show high efficiency, and above 400 for lower efficiencies.

The coefficient a is a criterion of performance of a boiler when all the other terms of the formula are known as the results of a test. By transposition its value is

$$a = \left[\frac{K - tcf}{970 (1 + RS/W)} - E_a \right] \div \frac{c^2f^2}{(K - tcf)} \frac{W}{S}.$$

On the diagram below (Fig. 148), with abscissas representing rates of driving and ordinates representing efficiencies are plotted curves showing the relation of the efficiency to rate of driving for values of $a = 100$ to 400 and values of f from 20 to 35, together with a broken line showing the maximum efficiencies obtained by six boilers at the Centennial Exhibition, and other lines showing the poor results obtained from five other boilers. The curves are also based on the following values, $K = 14,800$; $c = 0.24$; $t = 300$ (except one curve, $t = 250$); $R = 0.1$.

An inspection of the curves shows the following. 1. The maximum Centennial results all lie below the curve $f = 20$, $a = 200$, by 2 to 4%, but they follow the general direction of the curve. This curve may therefore be taken as representing the maximum possible boiler performance with anthracite coal, as the results obtained in 1876 have never been exceeded with anthracite.

2. With $f = 20$ and $a = 200$ the efficiency for maximum performance, according to the curve, is a little less than 82% at 2 lbs. evaporation per sq. ft. of heating-surface per hour, but it decreases very slowly at higher rates, so that it is 80% at 3½ lbs., and 76% at 5¾ lbs.

With $a = 200$ and f greater than 20, the efficiency has a lower maximum, reaches the maximum at a lower rate of driving, and falls off rapidly as the rate increases, the more rapidly the higher the value of f . Excessive air supply is thus shown to be a most potent cause of low economy.

3. An increase in the value of a from 200 to 400 with $f = 20$ is much less detrimental to efficiency than an increase in f from 20 to 30.

In the diagram, Fig. 151, are plotted, together with the curve for $f = 20$, $a = 200$, $t = 300$, and $K = 15,750$, marked $R = 0.1$, a straight line, $R = 0$, showing the theoretical maximum efficiency when there is no loss by

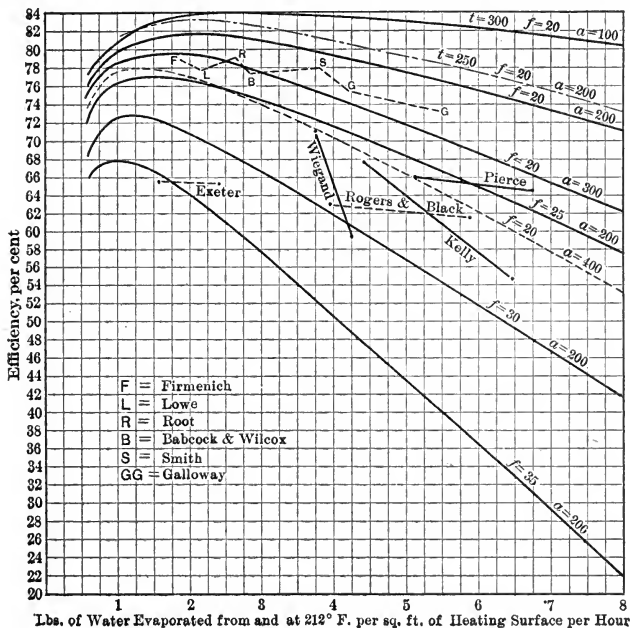


FIG. 151.

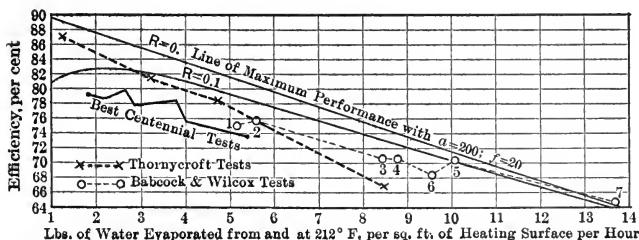


FIG. 152.

radiation, and the plottings of the results of two series of tests, one of a Thornycroft boiler, with W/S from 1.24 to 8.5, and the other of a Babcock & Wilcox marine boiler with W/S from 5.18 to 13.67, together with the

maximum Centennial tests. The calculated value of a in all these tests except one ranged from 191 to 454, the highest values being those showing the largest departure from the curve $R = 0.1$. The one exception is the Thornycroft test showing over 86% efficiency; this gives a value of $a = 57$, which indicates an error in the test, as such a low value is far below the lowest recorded in any other test.

TESTS OF STEAM-BOILERS.

Boiler-tests at the Centennial Exhibition, Philadelphia, 1876. — (See Reports and Awards Group XX, International Exhibition, Phila., 1876; also, Clark on the Steam-engine, vol. i, page 253.)

Competitive tests were made of fourteen boilers, using good anthracite coal, one boiler, the Galloway, being tested with both anthracite and semi-bituminous coal. Two tests were made with each boiler: one called the capacity trial, to determine the economy and capacity at a rapid rate of driving; and the other called the economy trial, to determine the economy when driven at a rate supposed to be near that of maximum economy and rated capacity. The following table gives the principal results obtained in the economy trial, together with the capacity and economy figures of the capacity trial for comparison.

| Name of Boiler. | Economy Tests. | | | | | | | | Capacity Tests. | |
|---------------------|---|---|--------------------------|---|--|------------------------|--------------------|------------------------|-----------------|---|
| | Ratio Water-heating Surface to Grate-surface. | Coal burned per sq. ft. Grate per hour. | Per cent Ash and Refuse. | Water evap. from 100° to 70 lbs. p. s. ft. H.S. per hr. | Water evap. from and at 212° p. lb. comb'le cor. for Quality of Steam. | Temperature in Uptake. | Moisture in Steam. | Superheating of Steam. | Horse-power. | Water evap. from and at 212° per lb. Combustible. |
| | lbs. | %. | lbs. | lbs. | deg | % | deg | H.P. | H.P. | lbs. |
| Root..... | 34.6 | 9.1 | 10.4 | 2.25 | 12.094 | 393 | 41.4 | 119.8 | 148.6 | 10.441 |
| Firmenich..... | 64.3 | 12.0 | 10.4 | 1.68 | 11.988 | 415 | 32.6 | 57.8 | 68.4 | 11.064 |
| Lowe..... | 30.6 | 6.8 | 11.3 | 1.87 | 11.923 | 333 | 9.4 | 47.0 | 69.3 | 11.163 |
| Smith..... | 45.8 | 12.1 | 11.1 | 2.42 | 11.906 | 411 | 1.3 | 99.8 | 125.0 | 11.925 |
| Babcock & Wilcox. | 37.7 | 10.0 | 11.0 | 2.43 | 11.822 | 296 | 2.7 | 135.6 | 186.6 | 10.330 |
| Galloway..... | 23.7 | 9.6 | 11.1 | 3.63 | 11.583 | 303 | 1.4 | 103.3 | 133.8 | 11.216 |
| Do. semi-bit. coal. | 23.7 | 7.9 | 8.8 | 3.20 | 12.125 | 325 | 0.3 | 50.9 | 125.1 | 11.609 |
| Andrews..... | 15.6 | 8.0 | 10.3 | 2.32 | 11.039 | 420 | 71.7 | 42.6 | 58.7 | 9.745 |
| Harrison..... | 27.3 | 12.4 | 8.5 | 2.75 | 10.930 | 517 | 0.9 | 82.4 | 108.4 | 9.889 |
| Wiegand..... | 30.7 | 12.3 | 9.5 | 3.30 | 10.834 | 524 | 20.5 | 147.5 | 162.8 | 9.145 |
| Anderson..... | 17.5 | 9.7 | 9.3 | 2.64 | 10.618 | 417 | 15.7 | 98.0 | 132.8 | 9.568 |
| Kelly..... | 20.9 | 10.8 | 9.0 | 3.82 | 10.312 | | 5.6 | 81.0 | 99.9 | 8.397 |
| Exeter..... | 33.5 | 9.3 | 11.4 | 1.38 | 10.041 | 430 | 4.2 | 72.1 | 108.0 | 9.974 |
| Pierce..... | 14.0 | 8.0 | 11.0 | 4.44 | 10.021 | 374 | 5.2 | 51.7 | 67.8 | 9.865 |
| Rogers & Black.... | 19.0 | 8.6 | 9.9 | 3.43 | 9.613 | 572 | 2.1 | 45.7 | 67.2 | 9.429 |
| Averages..... | | | | 2.77 | 11.123 | | | 85.0 | 110.8 | 10.251 |

The comparison of the economy and capacity trials shows that an average increase in capacity of 30 per cent was attended by a decrease in economy of 8 per cent, but the relation of economy to rate of driving varied greatly in the different boilers. In the Kelly boiler an increase in capacity of 22 per cent was attended by a decrease in economy of over 18 per cent, while the Smith boiler with an increase of 25 per cent in capacity showed a slight increase in economy.

One of the most important lessons gained from the above tests is that there is no necessary relation between the type of a boiler and economy.

Of the five boilers that gave the best results, the total range of variation between the highest and lowest of the five being only 2.3%, three were water-tube boilers, one was a horizontal tubular boiler, and the fifth was a combination of the two types. The next boiler on the list, the Gallo-way, was an internally fired boiler, all of the others being externally fired.

Some High Rates of Evaporation. — *Eng'g*, May 9, 1884, p. 415.

| | Locomotive. | | Torpedo-boat. | |
|--|-------------|--------|---------------|--------|
| Water evap. per sq. ft. H.S. per hour | 12.57 | 13.73 | 12.54 | 20.74 |
| Water evap. per lb. fuel from and at 212° | 8.22 | 8.94 | 8.37 | 7.04 |
| Thermal units transf'd per sq. ft. of H.S. | 12,142 | 13,263 | 12,113 | 20,034 |
| Efficiency | 0.586 | 0.637 | 0.542 | 0.468 |

It is doubtful if these figures were corrected for priming.

Economy Effected by Heating the Air Supplied to Boiler-furnaces.

— An extensive series of experiments was made by J. C. Hoadley (*Trans. A. S. M. E.*, vi, 676) on a "Warm-blast Apparatus," for utilizing the heat of the waste gases in heating the air supplied to the furnace. The apparatus, as applied to an ordinary horizontal tubular boiler 60 in. diameter, 21 ft. long, with 65 3½-in. tubes, consisted of 240 2-in. tubes, 18 ft. long, through which the hot gases passed while the air circulated around them. The net saving of fuel effected by the warm blast was from 10.7% to 15.5% of the fuel used with cold blast. The comparative temperatures averaged as follows, in degrees F.:

| | Cold-blast Boiler. | Warm-blast Boiler. | Difference. |
|-------------------------------------|-----------------------|-----------------------|-------------|
| In heat of fire | 2493 | 2793 | 300 |
| At bridge wall | 1340 | 1600 | 260 |
| In smoke box | 373 | 375 | 2 |
| Air admitted to furnace | 32 | 332 | 300 |
| Steam and water in boiler | 300 | 300 | 0 |
| Gases escaping to chimney | 373 | 162 | 211 |
| External air | 32 | 32 | 0 |

With anthracite coal the evaporation from and at 212° per lb. combustible was, for the cold-blast boiler, days 10.85 lbs., days and nights 10.51; and for the warm-blast boiler, days 11.83, days and nights 11.03.

Maximum Boiler Efficiency with Cumberland Coal. — About 12.5 lbs. of water per lb. combustible from and at 212° is about the highest evaporation that can be obtained from the best steam fuels in the United States, such as Cumberland, Pocahontas, and Clearfield. In exceptional cases 13 lbs. has been reached, and one test is on record (F. W. Dean, *Eng'g News*, Feb. 1, 1894) giving 13.23 lbs. The boiler was internally fired, of the Belpaire type, 82 inches diameter, 31 feet long, with 160 3-inch tubes 12½ feet long. Heating-surface, 1998 square feet; grate-surface, 45 square feet, reduced during the test to 30½ square feet. Double furnace, with fire-brick arches and a long combustion-chamber. Feed-water heater in smoke-box. The following are the principal results:

| | 1st Test. | 2d Test. |
|---|-----------|----------|
| Dry coal burned per sq. ft. of grate per hour, lbs. | 8.85 | 18.06 |
| Water evap. per sq. ft. of heating-surface per hour, lbs. | 1.63 | 3.00 |
| Water evap. from and at 212° per lb. combustible, including feed-water heater | 13.17 | 13.23 |
| Water evaporated, excluding feed-water heater | 12.88 | 12.90 |
| Temperature of gases after leaving heater, F. | 360° | 469° |

BOILERS USING WASTE GASES.

Water-tube Boilers using Blast-furnace Gases. — D. S. Jacobus (*Trans. A. I. M. E.*, xvii, 50) reports a test of a water-tube boiler using blast-furnace gas as fuel. The heating-surface was 2535 sq. ft. It developed 328 H.P., or 5.01 lbs. of water from and at 212° per sq. ft. of heating-surface per hour. Some of the principal data obtained were as follows: Calorific value of 1 lb. of the gas, 1413 B.T.U., including the effect

of its initial temperature, which was 650° F. Amount of air used to burn 1 lb. of the gas = 0.9 lb. Chimney draught, 11 $\frac{1}{3}$ in. of water. Area of gas inlet, 300 sq. in.; of air inlet, 100 sq. in. Temperature of the chimney gases, 775° F. Efficiency of the boiler calculated from the temperatures and analyses of the gases at exit and entrance, 61%. The average analyses were as follows, hydrocarbons being included in the nitrogen:

| | By Weight. | | By Volume. | |
|----------------------------|--------------|----------|--------------|----------|
| | At Entrance. | At Exit. | At Entrance. | At Exit. |
| CO ₂ | 10.69 | 26.37 | 7.08 | 18.64 |
| O..... | .11 | 3.05 | 0.10 | 2.96 |
| CO..... | 26.71 | 1.78 | 27.89 | 1.98 |
| Nitrogen..... | 62.48 | 68.80 | 65.02 | 76.42 |
| C in CO ₂ | 2.92 | 7.19 | | |
| C in CO..... | 11.45 | 0.76 | | |
| Total C..... | 14.37 | 7.95 | | |

Steam-boilers Fired with Waste Gases from Puddling and Heating-Furnaces. — The *Iron Age*, April 6, 1893, contains a report of a number of tests of steam-boilers utilizing the waste heat from puddling and heating-furnaces in rolling-mills. The following principal data are selected: in Nos. 1, 2, and 4 the boiler is a Babcock & Wilcox water-tube boiler, and in No. 3 it is a plain cylinder boiler, 42 in. diam. and 26 ft. long. No. 4 boiler was connected with a heating-furnace, the others with puddling furnaces.

| | No. 1. | No. 2. | No. 3. | No. 4. |
|---|--------|--------|--------|--------|
| Heating-surface, sq. ft. | 1026 | 1196 | 143 | 1380 |
| Grate-surface, sq. ft. | 19.9 | 13.6 | 13.6 | 16.7 |
| Ratio H.S. to G.S. | 52 | 87.2 | 10.5 | 82.8 |
| Water evap. per hour, lbs. | 3358 | 2159 | 1812 | 3055 |
| Water evap. per sq. ft. H.S. per hr., lbs. | 3.3 | 1.8 | 12.7 | 2.2 |
| Water evap. per lb. coal from and at 212° ... | 5.9 | 6.24 | 3.76 | 6.34 |
| Water evap. per lb. comb. from and at 212° | | 7.20 | 4.31 | 8.34 |

In No. 2, 1.38 lbs. of iron were puddled per lb. of coal.

In No. 3, 1.14 lbs. of iron were puddled per lb. of coal.

No. 3 shows that an insufficient amount of heating-surface was provided for the amount of waste heat available.

RULES FOR CONDUCTING BOILER-TESTS.

Code of 1899.

(Reported by the Committee on Boiler Trials, Am. Soc. M. E.)*

I. *Determine at the outset* the specific object of the proposed trial, whether it be to ascertain the capacity of the boiler, its efficiency as a steam-generator, its efficiency and its defects under usual working conditions, the economy of some particular kind of fuel, or the effect of changes of design, proportion, or operation; and prepare for the trial accordingly.

II. *Examine the boiler*, both outside and inside; ascertain the dimensions of grates, heating surfaces, and all important parts; and make a full record, describing the same, and illustrating special features by sketches.

III. *Notice the general condition* of the boiler and its equipment, and record such facts in relation thereto as bear upon the objects in view.

* The code is here slightly abridged. The complete report of the Committee may be obtained in pamphlet form from the Secretary of the American Society of Mechanical Engineers, 29 West 39th St., New York.

If the object of the trial is to ascertain the maximum economy or capacity of the boiler as a steam-generator, the boiler and all its appurtenances should be put in first-class condition. Clean the heating surface inside and outside, remove clinkers from the grates and from the sides of the furnace. Remove all dust, soot, and ashes from the chambers, smoke-connections and flues. Close air-leaks in the masonry and poorly fitted cleaning-doors. See that the damper will open wide and close tight. Test for air-leaks by firing a few shovels of smoky fuel and immediately closing the damper, observing the escape of smoke through the crevices; or by passing the flame of a candle over cracks in the brickwork.

IV. *Determine the character of the coal to be used.* For tests of the efficiency or capacity of the boiler for comparison with other boilers the coal should, if possible, be of some kind which is commercially regarded as a standard. For New England and that portion of the country east of the Allegheny Mountains, good anthracite egg coal, containing not over 10 per cent of ash, and semi-bituminous Clearfield (Pa.), Cumberland (Md.), and Pocahontas (Va.) coals are thus regarded. West of the Allegheny Mountains, Pocahontas (Va.) and New River (W. Va.) semi-bituminous, and Youghiogheny or Pittsburg bituminous coals are recognized as standards.*

For tests made to determine the performance of a boiler with a particular kind of coal, such as may be specified in a contract for the sale of a boiler, the coal used should not be higher in ash and in moisture than that specified, since increase in ash and moisture above a stated amount is apt to cause a falling off of both capacity and economy in greater proportion than the proportion of such increase.

V. *Establish the correctness of all apparatus used in the test for weighing and measuring.* These are:

1. Scales for weighing coal, ashes, and water.
2. Tanks or water-meters for measuring water. Water-meters, as a rule, should only be used as a check on other measurements. For accurate work the water should be weighed or measured in a tank.
3. Thermometers and pyrometers for taking temperatures of air, steam, feed-water, waste gases, etc.
4. Pressure-gauges, draught-gauges, etc.

VI. *See that the boiler is thoroughly heated before the trial to its usual working temperature.* If the boiler is new and of a form provided with a brick setting, it should be in regular use at least a week before the trial, so as to dry and heat the walls. If it has been laid off and become cold, it should be worked before the trial until the walls are well heated.

VII. *The boiler and connections should be proved to be free from leaks before beginning a test, and all water connections, including blow and extra feed-pipes, should be disconnected, stopped with blank flanges, or bled through special openings beyond the valves, except the particular pipe through which water is to be fed to the boiler during the trial.* During the test the blow-off and feed pipes should remain exposed to view.

If an injector is used, it should receive steam directly through a felted pipe from the boiler being tested.†

If the water is metered after it passes the injector, its temperature should be taken at the point where it leaves the injector. If the quantity is determined before it goes to the injector, the temperature should be determined on the suction side of the injector, and if no change of

* These coals are selected because they are about the only coals which possess the essentials of excellence of quality, adaptability to various kinds of furnaces, grates, boilers, and methods of firing, and wide distribution and general accessibility in the markets.

† In feeding a boiler undergoing test with an injector taking steam from another boiler, or from the main steam-pipe from several boilers, the evaporative results may be modified by a difference in the quality of the steam from such source compared with that supplied by the boiler being tested, and in some cases the connection to the injector may act as a drip for the main steam-pipe. If it is known that the steam from the main pipe is of the same pressure and quality as that furnished by the boiler undergoing the test, the steam may be taken from such main pipe.

temperature occurs other than that due to the injector, the temperature thus determined is properly that of the feed-water. When the temperature changes between the injector and the boiler, as by the use of a heater or by radiation, the temperature at which the water enters and leaves the injector and that at which it enters the boiler should all be taken. In that case the weight to be used is that of the water leaving the injector, computed from the heat units if not directly measured; and the temperature, that of the water entering the boiler.

Let w = weight of water entering the injector;
 x = weight of steam entering the injector;
 h_1 = heat-units per pound of water entering injector;
 h_2 = heat-units per pound of steam entering injector;
 h_3 = heat-units per pound of water leaving injector.

Then $w + x$ = weight of water leaving injector;

$$x = w \frac{h_2 - h_1}{h_2 - h_3}.$$

See that the steam-main is so arranged that water of condensation cannot run back into the boiler.

VIII. *Duration of the Test.* — For tests made to ascertain either the maximum economy or the maximum capacity of a boiler, irrespective of the particular class of service for which it is regularly used, the duration should be at least ten hours of continuous running. If the rate of combustion exceeds 25 pounds of coal per square foot of grate-surface per hour, it may be stopped when a total of 250 pounds of coal has been burned per square foot of grate.

IX. *Starting and Stopping a Test.* — The conditions of the boiler and furnace in all respects should be, as nearly as possible, the same at the end as at the beginning of the test. The steam-pressure should be the same; the water-level the same; the fire upon the grates should be the same in quantity and condition; and the walls, flues, etc., should be of the same temperature. Two methods of obtaining the desired equality of conditions of the fire may be used, viz., those which were called in the Code of 1885 "the standard method" and "the alternate method," the latter being employed where it is inconvenient to make use of the standard method.*

X. *Standard Method of Starting and Stopping a Test.* — Steam being raised to the working pressure, remove rapidly all the fire from the grate, close the damper, clean the ash-pit, and as quickly as possible start a new fire with weighed wood and coal, noting the time and the water-level, while the water is in a quiescent state, just before lighting the fire.†

At the end of the test remove the whole fire, which has been burned low, clean the grates and ash-pit, and note the water-level when the water is in a quiescent state, and record the time of hauling the fire. The water-level should be as nearly as possible the same as at the beginning of the test. If it is not the same, a correction should be made by computation, and not by operating the pump after the test is completed.

XI. *Alternate Method of Starting and Stopping a Test.* — The boiler being thoroughly heated by a preliminary run, the fires are to be burned low and well cleaned. Note the amount of coal left on the grate as nearly as it can be estimated; note the pressure of steam and the water-level. Note the time, and record it as the starting-time. Fresh coal which has

* The Committee concludes that it is best to retain the designations "standard" and "alternate," since they have become widely known and established in the minds of engineers and in the reprints in the Code of 1885. Many engineers prefer the "alternate" to the "standard" method on account of its being less liable to error due to cooling of the boiler at the beginning and end of a test.

† The gauge-glass should not be blown out within an hour before the water-level is taken at the beginning and end of a test, otherwise an error in the reading of the water-level may be caused by a change in the temperature and density of the water in the pipe leading from the bottom of the glass into the boiler.

been weighed should now be fired. The ash-pits should be thoroughly cleaned at once after starting. Before the end of the test the fires should be burned low, just as before the start, and the fires cleaned in such a manner as to leave a bed of coal on the grates of the same depth, and in the same condition, as at the start. When this stage is reached, note the time and record it as the stopping-time. The water-level and steam-pressures should previously be brought as nearly as possible to the same point as at the start. If the water-level is not the same as at the start, a correction should be made by computation, and not by operating the pump after the test is completed.

XII. Uniformity of Conditions. — In all trials made to ascertain maximum economy or capacity the conditions should be maintained uniformly constant. Arrangements should be made to dispose of the steam so that the rate of evaporation may be kept the same from beginning to end.

XIII. Keeping the Records. — Take note of every event connected with the progress of the trial, however unimportant it may appear. Record the time of every occurrence and the time of taking every weight and every observation.

The coal should be weighed and delivered to the fireman in equal proportions, each sufficient for not more than one hour's run, and a fresh portion should not be delivered until the previous one has all been fired. The time required to consume each portion should be noted, the time being recorded at the instant of firing the last of each portion. It is desirable that at the same time the amount of water fed into the boiler should be accurately noted and recorded, including the height of the water in the boiler, and the average pressure of steam and temperature of feed during the time. By thus recording the amount of water evaporated by successive portions of coal, the test may be divided into several periods if desired, and the degree of uniformity of combustion, evaporation, and economy analyzed for each period. In addition to these records of the coal and the feed-water, half-hourly observations should be made of the temperature of the feed-water, of the flue-gases, of the external air in the boiler-room, of the temperature of the furnace when a furnace-pyrometer is used, also of the pressure of steam, and of the readings of the instruments for determining the moisture in the steam. A log should be kept on properly prepared blanks containing columns for record of the various observations.

XIV. Quality of Steam. — The percentage of moisture in the steam should be determined by the use of either a throttling or a separating steam-calorimeter. The sampling-nozzle should be placed in the vertical steam-pipe rising from the boiler. It should be made of $\frac{1}{2}$ -inch pipe, and should extend across the diameter of the steam-pipe to within half an inch of the opposite side, being closed at the end and perforated with not less than twenty $\frac{1}{8}$ -inch holes equally distributed along and around its cylindrical surface, but none of these holes should be nearer than $\frac{1}{2}$ inch to the inner side of the steam-pipe. The calorimeter and the pipe leading to it should be well covered with felting. Whenever the indications of the throttling or separating calorimeter show that the percentage of moisture is irregular, or occasionally in excess of three per cent, the results should be checked by a steam-separator placed in the steam-pipe as close to the boiler as convenient, with a calorimeter in the steam-pipe just beyond the outlet from the separator. The drip from the separator should be caught and weighed, and the percentage of moisture computed therefrom added to that shown by the calorimeter.

Superheating should be determined by means of a thermometer placed in a mercury-well inserted in the steam-pipe. The degree of superheating should be taken as the difference between the reading of the thermometer for superheated steam and the readings of the same thermometer for saturated steam at the same pressure as determined by a special experiment, and not by reference to steam-tables.

XV. Sampling the Coal and Determining its Moisture. — As each barrow-load or fresh portion of coal is taken from the coal-pile, a representative shovelful is selected from it and placed in a barrel or box in a cool place and kept until the end of the trial. The samples are then mixed and broken into pieces not exceeding one inch in diameter, and reduced by the process of repeated quartering and crushing until a final sample

weighing about five pounds is obtained, and the size of the larger pieces is such that they will pass through a sieve with $\frac{1}{4}$ -inch meshes. From this sample two one-quart, air-tight glass preserving-jars, or other air-tight vessels which will prevent the escape of moisture from the sample, are to be promptly filled, and these samples are to be kept for subsequent determinations of moisture and of heating value and for chemical analyses. During the process of quartering, when the sample has been reduced to about 100 pounds, a quarter to a half of it may be taken for an approximate determination of moisture. This may be made by placing it in a shallow iron pan, not over three inches deep, carefully weighing it, and setting the pan in the hottest place that can be found on the brickwork of the boiler-setting or flues, keeping it there for at least 12 hours, and then weighing it. The determination of moisture thus made is believed to be approximately accurate for anthracite and semi-bituminous coals, and also for Pittsburg or Youghiogeny coal; but it cannot be relied upon for coals mined west of Pittsburg, or for other coals containing inherent moisture. For these latter coals it is important that a more accurate method be adopted. The method recommended by the Committee for all accurate tests, whatever the character of the coal, is described as follows:

Take one of the samples contained in the glass jars, and subject it to a thorough air-drying, by spreading it in a thin layer and exposing it for several hours to the atmosphere of a warm room, weighing it before and after, thereby determining the quantity of surface moisture it contains. Then crush the whole of it by running it through an ordinary coffee-mill adjusted so as to produce somewhat coarse grains (less than $\frac{1}{16}$ inch), thoroughly mix the crushed sample, select from it a portion of from 10 to 50 grams, weigh it in a balance which will easily show a variation as small as 1 part in 1000, and dry it in an air- or sand-bath at a temperature between 240 and 280 degrees Fahr. for one hour. Weigh it and record the loss, then heat and weigh it again repeatedly, at intervals of an hour or less, until the minimum weight has been reached and the weight begins to increase by oxidation of a portion of the coal. The difference between the original and the minimum weight is taken as the moisture in the air-dried coal. This moisture test should preferably be made on duplicate samples, and the results should agree within 0.3 to 0.4 of one per cent, the mean of the two determinations being taken as the correct result. The sum of the percentage of moisture thus found and the percentage of surface moisture previously determined is the total moisture.

XVI. Treatment of Ashes and Refuse. — The ashes and refuse are to be weighed in a dry state. If it is found desirable to show the principal characteristics of the ash, a sample should be subjected to a proximate analysis and the actual amount of incombustible material determined. For elaborate trials a complete analysis of the ash and refuse should be made.

XVII. Calorific Tests and Analysis of Coal. — The quality of the fuel should be determined either by heat test or by analysis, or by both.

The rational method of determining the total heat of combustion is to burn the sample of coal in an atmosphere of oxygen gas, the coal to be sampled as directed in Article XV of this code.

The chemical analysis of the coal should be made only by an expert chemist. The total heat of combustion computed from the results of the ultimate analysis may be obtained by the use of Dulong's formula (with constants modified by recent determinations), viz.,

$$14,600 C + 62,000 \left(H - \frac{O}{8} \right) + 4000 S,$$

in which C, H, O, and S refer to the proportions of carbon, hydrogen, oxygen, and sulphur respectively, as determined by the ultimate analysis.*

It is desirable that a proximate analysis should be made, thereby determining the relative proportions of volatile matter and fixed carbon. These proportions furnish an indication of the leading characteristics of the fuel, and serve to fix the class to which it belongs.

* Favre and Silbermann give 14,544 B.T.U. per pound carbon; Berthelot, 14,647 B.T.U. Favre and Silbermann give 62,032 B.T.U. per pound hydrogen; Thomsen, 61,816 B.T.U.

XVIII. Analysis of Flue-gases. — The analysis of the flue-gases is an especially valuable method of determining the relative value of different methods of firing or of different kinds of furnaces. In making these analyses great care should be taken to procure average samples, since the composition is apt to vary at different points of the flue. The composition is also apt to vary from minute to minute, and for this reason the drawings of gas should last a considerable period of time. Where complete determinations are desired, the analyses should be intrusted to an expert chemist. For approximate determinations the Orsat or the Hempel apparatus may be used by the engineer.

For the continuous indication of the amount of carbonic acid present in the flue-gases an instrument may be employed which shows the weight of CO₂ in the sample of gas passing through it.

XIX. Smoke Observations. — It is desirable to have a uniform system of determining and recording the quantity of smoke produced where bituminous coal is used. The system commonly employed is to express the degree of smokiness by means of percentages dependent upon the judgment of the observer. The actual measurement of a sample of soot and smoke by some form of meter is to be preferred.

XX. Miscellaneous. — In tests for purposes of scientific research, in which the determination of all the variables entering into the test is desired, certain observations should be made which are in general unnecessary for ordinary tests. As these determinations are rarely undertaken, it is not deemed advisable to give directions for making them.

XXI. Calculations of Efficiency. — Two methods of defining and calculating the efficiency of a boiler are recommended. They are:

1. Efficiency of the boiler = $\frac{\text{Heat absorbed per lb. combustible}}{\text{Calorific value of 1 lb. combustible}}$.
2. Efficiency of the boiler and grate = $\frac{\text{Heat absorbed per lb. coal}}{\text{Calorific value of 1 lb. coal}}$.

The first of these is sometimes called the efficiency based on combustible, and the second the efficiency based on coal. The first is recommended as a standard of comparison for all tests, and this is the one which is understood to be referred to when the word "efficiency" alone is used without qualification. The second, however, should be included in a report of a test, together with the first, whenever the object of the test is to determine the efficiency of the boiler and furnace together with the grate (or mechanical stoker), or to compare different furnaces, grates, fuels, or methods of firing.

The heat absorbed per pound of combustible (or per pound coal) is to be calculated by multiplying the equivalent evaporation from and at 212 degrees per pound combustible (or coal) by 965.7.

XXII. The Heat Balance. — An approximate "heat balance" may be included in the report of a test when analyses of the fuel and of the chimney-gases have been made. It should be reported in the following form: [see next page.]

XXIII. Report of the Trial. — The data and results should be reported in the manner given in either one of the two following tables [only the "Short Form" of table is given here], omitting lines where the tests have not been made as elaborately as provided for in such tables. Additional lines may be added for data relating to the specific object of the test. The Short Form of Report, Table No. 2, is recommended for commercial tests and as a convenient form of abridging the longer form for publication when saving of space is desirable. For elaborate trials it is recommended that the full log of the trial be shown graphically, by means of a chart.

HEAT BALANCE, OR DISTRIBUTION OF THE HEATING VALUE OF THE COMBUSTIBLE.

Total Heat Value of 1 lb of Combustible.....B.T.U.

| | B.T.U. | Per Cent. |
|--|--------|-----------|
| 1. Heat absorbed by the boiler = evaporation from and at 212 degrees per pound of combustible $\times 965.7$ *..... | | |
| 2. Loss due to moisture in coal = per cent of moisture referred to combustible $\div 100 \times [(212 - t) + 966 + 0.48 (T - 212)]$ (t = temperature of air in the boiler-room, T = that of the flue-gases)..... | | |
| 3. Loss due to moisture formed by the burning of hydrogen = per cent of hydrogen to combustible $\div 100 \times 9 \times [(212 - t) + 966 + 0.48 (T - 212)]$ | | |
| 4. † Loss due to heat carried away in the dry chimney-gases = weight of gas per pound of combustible $\times 0.24 \times (T - t)$ | | |
| 5. ‡ Loss due to incomplete combustion of carbon $= \frac{\text{CO}}{\text{CO}_2 + \text{CO}} \times \frac{\text{per cent C in combustible}}{100} \times 10,150$ | | |
| 6. Loss due to unconsumed hydrogen and hydrocarbons, to heating the moisture in the air, to radiation, and unaccounted for (Some of these losses may be separately itemized if data are obtained from which they may be calculated)..... | | |
| Totals..... | | 100.00 |

* [The figure 965.7 (or 966) is taken from the old steam tables. If Peabody's new table (1909) is used it should be changed to 969.7, or if Marks & Davis's table is used, to 970.4.]

† The weight of gas per pound of carbon burned may be calculated from the gas analyses as follows:

Dry gas per pound carbon = $\frac{11 \text{ CO}_2 + 8 \text{ O} + 7 (\text{CO} + \text{N})}{3 (\text{CO}_2 + \text{CO})}$, in which CO_2 ,

CO , O , and N are the percentages by volume of the several gases. As the sampling and analyses of the gases in the present state of the art are liable to considerable errors, the result of this calculation is usually only an approximate one. The heat balance itself is also only approximate for this reason, as well as for the fact that it is not possible to determine accurately the percentage of unburned hydrogen or hydrocarbons in the flue-gases.

The weight of dry gas per pound of combustible is found by multiplying the dry gas per pound of carbon by the percentage of carbon in the combustible, and dividing by 100.

‡ CO_2 and CO are respectively the percentage by volume of carbonic acid and carbonic oxide in the flue-gases. The quantity 10,150 = number of heat-units generated by burning to carbonic acid one pound of carbon contained in carbonic oxide.

TABLE NO. 2.

DATA AND RESULTS OF EVAPORATIVE TEST.

Arranged in accordance with the Short Form advised by the Boiler Test Committee of the American Society of Mechanical Engineers.
Code of 1899.

Made by on boiler, at
to determine.....
Kind of fuel
Kind of furnace

| | |
|---|----------|
| Method of starting and stopping the test ("standard" or "alternate," Arts. X and XI, Code)..... | sq. ft. |
| Grate surface..... | " |
| Water-heating surface..... | " |
| Superheating surface..... | " |
| TOTAL QUANTITIES. | |
| 1. Date of trial..... | hours |
| 2. Duration of trial..... | lbs. |
| 3. Weight of coal as fired *..... | per cent |
| 4. Percentage of moisture in coal †..... | lbs. |
| 5. Total weight of dry coal consumed..... | " |
| 6. Total ash and refuse..... | per cent |
| 7. Percentage of ash and refuse in dry coal..... | lbs. |
| 8. Total weight of water fed to the boiler ‡..... | " |
| 9. Water actually evaporated, corrected for moisture or superheat in steam..... | " |
| 9a. Factor of evaporation §..... | " |
| 10. Equivalent water evaporated into dry steam from and at 212 degrees. (Item 9 × Item 9a.)..... | " |
| HOURLY QUANTITIES. | |
| 11. Dry coal consumed per hour..... | " |
| 12. Dry coal per square foot of grate surface per hour.... | " |
| 13. Water evaporated per hour corrected for quality of steam..... | " |
| 14. Equivalent evaporation per hour from and at 212 degrees | " |
| 15. Equivalent evaporation per hour from and at 212 degrees per square foot of water-heating surface | " |

* Including equivalent of wood used in lighting the fire, not including unburned coal withdrawn from furnace at times of cleaning and at end of test. One pound of wood is taken to be equal to 0.4 pound of coal, or, in case greater accuracy is desired, as having a heat value equivalent to the evaporation of 6 pounds of water from and at 212 degrees per pound. ($6 \times 965.7 = 5794$ B.T.U.) The term "as fired" means in its actual condition, including moisture.

† This is the total moisture in the coal as found by drying it artificially, as described in Art. XV of Code.

‡ Corrected for inequality of water-level and of steam-pressure at beginning and end of test.

§ Factor of evaporation = $\frac{H - h}{965.7}$, in which H and h are respectively the total heat in steam of the average observed pressure, and in water of the average observed temperature of the feed.

|| The symbol "U.E.," meaning "units of evaporation," may be conveniently substituted for the expression "Equivalent water evaporated into dry steam from and at 212 degrees," its definition being given in a foot-note.

TABLE NO. 2 — *Continued.*
DATA AND RESULTS OF EVAPORATIVE TEST.

| AVERAGE PRESSURES, TEMPERATURES, ETC. | |
|--|---------------|
| 16. Steam pressure by gauge..... | lbs. p.sq.in. |
| 17. Temperature of feed-water entering boiler..... | deg. |
| 18. Temperature of escaping gases from boiler..... | |
| 19. Force of draught between damper and boiler..... | ins.of water |
| 20. Percentage of moisture in steam, or number of degrees of superheating..... | % or deg. |
| HORSE-POWER. | |
| 21. Horse-power developed. (Item 14 ÷ 34½)*..... | H.P. |
| 22. Builders' rated horse-power..... | |
| 23. Percentage of builders' rated horse-power developed..... | per cent |
| ECONOMIC RESULTS. | |
| 24. Water apparently evaporated under actual conditions per pound of coal as fired. (Item 8 ÷ Item 3.) | lbs. |
| 25. Equivalent evaporation from and at 212 degrees per pound of coal as fired. (Item 10 ÷ Item 3.)..... | " |
| 26. Equivalent evaporation from and at 212 degrees per pound of dry coal. (Item 10 ÷ Item 5.)..... | " |
| 27. Equivalent evaporation from and at 212 degrees per pound of combustible. [Item 10 ÷ (Item 5 - Item 6).]..... | " |
| (If Items 25, 26, and 27 are not corrected for quality of steam, the fact should be stated.) | |
| EFFICIENCY. | |
| 28. Calorific value of the dry coal per pound..... | B.T.U. |
| 29. Calorific value of the combustible per pound..... | " |
| 30. Efficiency of boiler (based on combustible) †..... | per cent |
| 31. Efficiency of boiler, including grate (based on dry coal)..... | " |
| COST OF EVAPORATION. | |
| 32. Cost of coal per ton of — lbs. delivered in boiler-room..... | \$ |
| 33. Cost of coal required for evaporating 1000 pounds of water from and at 212 degrees..... | \$ |

* Held to be the equivalent of 30 lbs. of water evaporated from 100 degrees Fahr. into dry steam at 70 lbs. gauge-pressure.

† In all cases where the word "combustible" is used, it means the coal without moisture and ash, but including all other constituents. It is the same as what is called in Europe "coal dry and free from ash."

|| See foot-note on the preceding page.

FACTORS OF EVAPORATION.

The figures in the table on the next four pages are calculated from the formula $F = (H - h) \div 970.4$, in which H is the total heat above 32° of 1 lb. of steam of the observed pressure, h the total heat above 32° of the feed water, and 970.4 the heat of vaporization, or latent heat, of steam at 212° F. The values of these total heats and of the latent heat are those given in Marks and Davis's steam tables.

The factors are given for every 3° of feed water temperature between 32° and 212°, and for every 5 or 10 lbs. steam pressure within the ordinary working limits of pressure. Intermediate values correct to the third decimal place may easily be found by interpolation.

| Feed water. | Factors of Evaporation. | | | | | | | | | |
|-------------|-------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 1.0003 | 1.0103 | 1.0169 | 1.0218 | 1.0258 | 1.0290 | 1.0316 | 1.0340 | 1.0361 | 1.0370 |
| 212° F. | 1.0003 | 1.0103 | 1.0169 | 1.0218 | 1.0258 | 1.0290 | 1.0316 | 1.0340 | 1.0361 | 1.0370 |
| 209 | 34 | 34 | 1.0200 | 50 | 89 | 1.0321 | 47 | 71 | 92 | 1.0401 |
| 206 | 65 | 65 | 31 | 81 | 1.0320 | 52 | 79 | 1.0402 | 1.0423 | 32 |
| 203 | 96 | 96 | 62 | 1.0312 | 51 | 83 | 1.0410 | 33 | 54 | 63 |
| 200 | 1.0127 | 1.0227 | 93 | 43 | 82 | 1.0414 | 41 | 64 | 85 | 94 |
| 197 | 58 | 58 | 1.0324 | 74 | 1.0413 | 45 | 72 | 95 | 1.0516 | 1.0525 |
| 194 | 89 | 89 | 55 | 1.0405 | 44 | 76 | 1.0503 | 1.0526 | 47 | 56 |
| 191 | 1.0220 | 1.0320 | 86 | 36 | 75 | 1.0507 | 34 | 57 | 78 | 87 |
| 188 | 51 | 51 | 1.0417 | 67 | 1.0506 | 38 | 65 | 88 | 1.0609 | 1.0618 |
| 185 | 82 | 82 | 48 | 98 | 37 | 69 | 96 | 1.0619 | 40 | 49 |
| 182 | 1.0313 | 1.0413 | 79 | 1.0529 | 68 | 1.0600 | 1.0627 | 50 | 71 | 80 |
| 179 | 44 | 44 | 1.0510 | 60 | 99 | 31 | 58 | 81 | 1.0702 | 1.0711 |
| 176 | 75 | 75 | 41 | 91 | 1.0630 | 62 | 89 | 1.0712 | 33 | 42 |
| 173 | 1.0406 | 1.0506 | 72 | 1.0622 | 61 | 93 | 1.0720 | 43 | 64 | 73 |
| 170 | 37 | 37 | 1.0603 | 53 | 92 | 1.0724 | 51 | 74 | 95 | 1.0804 |
| 167 | 68 | 68 | 34 | 84 | 1.0723 | 55 | 82 | 1.0805 | 1.0826 | 35 |
| 164 | 99 | 99 | 65 | 1.0715 | 54 | 86 | 1.0812 | 36 | 57 | 66 |
| 161 | 1.0530 | 1.0630 | 96 | 45 | 85 | 1.0817 | 43 | 67 | 88 | 97 |
| 158 | 61 | 61 | 1.0727 | 76 | 1.0816 | 47 | 74 | 98 | 1.0919 | 1.0928 |
| 155 | 92 | 92 | 58 | 1.0807 | 46 | 78 | 1.0905 | 1.0929 | 50 | 59 |
| 152 | 1.0623 | 1.0723 | 89 | 38 | 77 | 1.0909 | 36 | 60 | 80 | 90 |
| 149 | 54 | 54 | 1.0820 | 69 | 1.0908 | 40 | 67 | 91 | 1.1011 | 1.1021 |
| 146 | 85 | 85 | 51 | 1.0900 | 39 | 71 | 98 | 1.1022 | 42 | 52 |
| 143 | 1.0715 | 1.0815 | 81 | 31 | 70 | 1.1002 | 1.1029 | 52 | 73 | 82 |
| 140 | 46 | 46 | 1.0912 | 62 | 1.1001 | 33 | 60 | 83 | 1.1104 | 1.1113 |
| 137 | 77 | 77 | 43 | 93 | 32 | 64 | 91 | 1.1114 | 35 | 44 |
| 134 | 1.0808 | 1.0908 | 74 | 1.1023 | 63 | 95 | 1.1121 | 45 | 66 | 75 |
| 131 | 39 | 39 | 1.1005 | 54 | 93 | 1.1125 | 52 | 76 | 97 | 1.1206 |
| 128 | 70 | 70 | 36 | 85 | 1.1124 | 56 | 83 | 1.1207 | 1.1227 | 37 |
| 125 | 1.0901 | 1.1001 | 67 | 1.1116 | 55 | 87 | 1.1214 | 38 | 58 | 68 |
| 122 | 31 | 31 | 97 | 47 | 86 | 1.1218 | 45 | 69 | 89 | 98 |
| 119 | 62 | 62 | 1.1128 | 78 | 1.1217 | 49 | 76 | 99 | 1.1320 | 1.1329 |
| 116 | 93 | 93 | 59 | 1.1209 | 48 | 80 | 1.1306 | 1.1330 | 51 | 60 |
| 113 | 1.1024 | 1.1124 | 90 | 39 | 79 | 1.1310 | 37 | 61 | 82 | 91 |
| 110 | 55 | 55 | 1.1221 | 70 | 1.1309 | 41 | 68 | 92 | 1.1412 | 1.1422 |
| 107 | 86 | 86 | 52 | 1.1301 | 40 | 72 | 99 | 1.1423 | 43 | 53 |
| 104 | 1.1116 | 1.1216 | 82 | 32 | 71 | 1.1403 | 1.1430 | 53 | 74 | 83 |
| 101 | 47 | 47 | 1.1313 | 63 | 1.1402 | 34 | 61 | 84 | 1.1505 | 1.1514 |
| 98 | 78 | 78 | 44 | 93 | 33 | 65 | 91 | 1.1515 | 36 | 45 |
| 95 | 1.1209 | 1.1309 | 75 | 1.1424 | 63 | 95 | 1.1522 | 46 | 66 | 76 |
| 92 | 40 | 40 | 1.1406 | 55 | 94 | 1.1526 | 53 | 77 | 97 | 1.1607 |
| 89 | 71 | 71 | 37 | 86 | 1.1525 | 57 | 84 | 1.1608 | 1.1628 | 37 |
| 86 | 1.1301 | 1.1401 | 67 | 1.1518 | 56 | 88 | 1.1615 | 38 | 59 | 68 |
| 83 | 32 | 32 | 98 | 48 | 87 | 1.1619 | 46 | 69 | 90 | 99 |
| 80 | 63 | 63 | 1.1529 | 78 | 1.1618 | 50 | 76 | 1.1700 | 1.1721 | 1.1730 |
| 77 | 94 | 94 | 60 | 1.1609 | 48 | 80 | 1.1707 | 31 | 51 | 61 |
| 74 | 1.1425 | 1.1525 | 91 | 40 | 79 | 1.1711 | 38 | 62 | 82 | 92 |
| 71 | 55 | 55 | 1.1621 | 71 | 1.1710 | 42 | 69 | 92 | 1.1813 | 1.1822 |
| 68 | 86 | 86 | 52 | 1.1702 | 41 | 73 | 1.1800 | 1.1823 | 44 | 53 |
| 65 | 1.1517 | 1.1617 | 83 | 33 | 72 | 1.1804 | 30 | 54 | 75 | 84 |
| 62 | 48 | 48 | 1.1714 | 63 | 1.1803 | 35 | 61 | 85 | 1.1906 | 1.1915 |
| 59 | 79 | 79 | 45 | 94 | 33 | 65 | 92 | 1.1916 | 37 | 46 |
| 56 | 1.1610 | 1.1710 | 76 | 1.1825 | 64 | 96 | 1.1923 | 47 | 67 | 77 |
| 53 | 41 | 41 | 1.1807 | 56 | 95 | 1.1927 | 54 | 78 | 98 | 1.2008 |
| 50 | 72 | 72 | 38 | 87 | 1.1926 | 58 | 85 | 1.2009 | 1.2029 | 39 |
| 47 | 1.1703 | 1.1803 | 69 | 1.1918 | 57 | 89 | 1.2016 | 40 | 60 | 70 |
| 44 | 34 | 34 | 1.1900 | 49 | 88 | 1.2020 | 47 | 71 | 91 | 1.2101 |
| 41 | 65 | 65 | 31 | 80 | 1.2019 | 51 | 78 | 1.2102 | 1.2122 | 32 |
| 38 | 96 | 96 | 62 | 1.2011 | 50 | 82 | 1.2109 | 33 | 53 | 63 |
| 35 | 1.1827 | 1.1927 | 93 | 42 | 81 | 1.2113 | 40 | 64 | 84 | 94 |
| 32 | 58 | 58 | 1.2024 | 73 | 1.2113 | 44 | 71 | 95 | 1.2216 | 1.2225 |

| Gauge press. 90.3 Abs. press. 105. | Lbs. | | | | | | | | | |
|---------------------------------------|-------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | 95.3 110. | 100.3 115. | 105.3 120. | 110.3 125. | 115.3 130. | 120.3 135. | 125.3 140. | 130.3 145. | 135.3 150. | 140.3 155. |
| Feed water. | Factors of Evaporation. | | | | | | | | | |
| 212° F. | 1.0379 | 1.0387 | 1.0396 | 1.0404 | 1.0411 | 1.0418 | 1.0425 | 1.0431 | 1.0437 | 1.0443 |
| 209 | 1.0410 | 1.0419 | 1.0427 | 35 | 42 | 49 | 56 | 62 | 68 | 74 |
| 206 | 41 | 50 | 58 | 66 | 73 | 81 | 87 | 93 | 99 | 1.0505 |
| 203 | 72 | 81 | 89 | 97 | 1.0504 | 1.0512 | 1.0518 | 1.0524 | 1.0530 | 36 |
| 200 | 1.0504 | 1.0512 | 1.0520 | 1.0528 | 35 | 43 | 49 | 55 | 61 | 67 |
| 197 | 35 | 43 | 51 | 59 | 66 | 74 | 80 | 85 | 92 | 98 |
| 194 | 66 | 74 | 82 | 90 | 97 | 1.0605 | 1.0611 | 1.0617 | 1.0623 | 1.0629 |
| 191 | 97 | 1.0605 | 1.0613 | 1.0621 | 1.0629 | 36 | 42 | 48 | 54 | 60 |
| 188 | 1.0628 | 36 | 44 | 52 | 60 | 67 | 73 | 79 | 85 | 91 |
| 185 | 59 | 67 | 75 | 83 | 91 | 98 | 1.0704 | 1.0710 | 1.0716 | 1.0722 |
| 182 | 90 | 98 | 1.0706 | 1.0714 | 1.0721 | 1.0729 | 35 | 41 | 47 | 53 |
| 179 | 1.0721 | 1.0729 | 37 | 45 | 52 | 60 | 66 | 72 | 78 | 84 |
| 176 | 52 | 60 | 68 | 76 | 83 | 91 | 97 | 1.0803 | 1.0809 | 1.0815 |
| 173 | 82 | 91 | 99 | 1.0807 | 1.0814 | 1.0822 | 1.0828 | 34 | 40 | 45 |
| 170 | 1.0813 | 1.0822 | 1.0830 | 38 | 45 | 53 | 59 | 65 | 71 | 77 |
| 167 | 44 | 53 | 61 | 69 | 76 | 84 | 90 | 96 | 1.0902 | 1.0908 |
| 164 | 75 | 84 | 92 | 1.0900 | 1.0907 | 1.0914 | 1.0921 | 1.0927 | 33 | 39 |
| 161 | 1.0906 | 1.0914 | 1.0923 | 31 | 38 | 45 | 52 | 58 | 64 | 70 |
| 158 | 37 | 45 | 54 | 62 | 69 | 76 | 82 | 89 | 95 | 1.1001 |
| 155 | 68 | 76 | 85 | 93 | 1.1000 | 1.1007 | 1.1013 | 1.1020 | 1.1026 | 32 |
| 152 | 99 | 1.1007 | 1.1015 | 1.1024 | 31 | 38 | 44 | 51 | 57 | 63 |
| 149 | 1.1030 | 38 | 46 | 55 | 62 | 69 | 75 | 81 | 88 | 94 |
| 146 | 61 | 69 | 77 | 86 | 93 | 1.1100 | 1.1105 | 1.1112 | 1.1119 | 1.1125 |
| 143 | 92 | 1.1100 | 1.1108 | 1.1116 | 1.1124 | 31 | 37 | 43 | 49 | 56 |
| 140 | 1.1123 | 31 | 39 | 47 | 54 | 62 | 68 | 74 | 80 | 86 |
| 137 | 53 | 62 | 70 | 78 | 85 | 93 | 99 | 1.1205 | 1.1211 | 1.1217 |
| 134 | 84 | 93 | 1.1201 | 1.1209 | 1.1216 | 1.1223 | 1.1230 | 36 | 42 | 48 |
| 131 | 1.1215 | 1.1223 | 32 | 40 | 47 | 54 | 60 | 67 | 73 | 79 |
| 128 | 46 | 54 | 62 | 71 | 78 | 85 | 91 | 98 | 1.1304 | 1.1310 |
| 125 | 77 | 85 | 93 | 1.1302 | 1.1309 | 1.1316 | 1.1322 | 1.1328 | 35 | 41 |
| 122 | 1.1308 | 1.1316 | 1.1324 | 32 | 40 | 47 | 53 | 59 | 65 | 71 |
| 119 | 39 | 47 | 55 | 63 | 70 | 78 | 84 | 90 | 96 | 1.1402 |
| 116 | 69 | 78 | 86 | 94 | 1.1401 | 1.1408 | 1.1415 | 1.1421 | 1.1427 | 33 |
| 113 | 1.1400 | 1.1408 | 1.1417 | 1.1425 | 32 | 39 | 45 | 52 | 58 | 64 |
| 110 | 31 | 39 | 47 | 56 | 63 | 70 | 76 | 82 | 89 | 95 |
| 107 | 62 | 70 | 78 | 87 | 94 | 1.1501 | 1.1507 | 1.1513 | 1.1519 | 1.1525 |
| 104 | 92 | 1.1501 | 1.1509 | 1.1517 | 1.1525 | 32 | 38 | 44 | 50 | 56 |
| 101 | 1.1523 | 32 | 40 | 48 | 55 | 63 | 69 | 75 | 81 | 87 |
| 98 | 54 | 62 | 71 | 79 | 86 | 93 | 1.1600 | 1.1605 | 1.1612 | 1.1618 |
| 95 | 85 | 93 | 1.1602 | 1.1610 | 1.1617 | 1.1624 | 30 | 37 | 43 | 49 |
| 92 | 1.1616 | 1.1624 | 32 | 41 | 48 | 55 | 61 | 67 | 74 | 80 |
| 89 | 47 | 55 | 63 | 71 | 79 | 86 | 92 | 98 | 1.1704 | 1.1711 |
| 86 | 78 | 86 | 94 | 1.1702 | 1.1710 | 1.1717 | 1.1723 | 1.1729 | 35 | 41 |
| 83 | 1.1708 | 1.1717 | 1.1725 | 33 | 40 | 48 | 54 | 60 | 66 | 72 |
| 80 | 39 | 47 | 56 | 64 | 71 | 78 | 85 | 91 | 97 | 1.1803 |
| 77 | 70 | 78 | 86 | 95 | 1.1802 | 1.1809 | 1.1815 | 1.1822 | 1.1828 | 34 |
| 74 | 1.1801 | 1.1809 | 1.1817 | 1.1826 | 33 | 40 | 45 | 52 | 59 | 65 |
| 71 | 32 | 40 | 48 | 56 | 64 | 71 | 77 | 83 | 89 | 96 |
| 68 | 62 | 71 | 79 | 87 | 94 | 1.1902 | 1.1908 | 1.1914 | 1.1920 | 1.1926 |
| 65 | 93 | 1.1902 | 1.1910 | 1.1918 | 1.1925 | 33 | 39 | 45 | 51 | 57 |
| 62 | 1.1924 | 32 | 41 | 49 | 56 | 63 | 70 | 76 | 82 | 88 |
| 59 | 55 | 63 | 72 | 80 | 87 | 94 | 1.2000 | 1.2007 | 1.2013 | 1.2019 |
| 56 | 86 | 94 | 1.2002 | 1.2011 | 1.2018 | 1.2025 | 31 | 38 | 44 | 50 |
| 53 | 1.2017 | 1.2025 | 33 | 42 | 49 | 56 | 62 | 68 | 75 | 81 |
| 50 | 48 | 56 | 64 | 73 | 80 | 87 | 93 | 99 | 1.2105 | 1.2112 |
| 47 | 79 | 87 | 95 | 1.2104 | 1.2111 | 1.2118 | 1.2124 | 1.2130 | 37 | 43 |
| 44 | 1.2110 | 1.2118 | 1.2126 | 35 | 42 | 49 | 55 | 61 | 68 | 74 |
| 41 | 41 | 49 | 57 | 66 | 73 | 80 | 86 | 92 | 99 | 1.2205 |
| 38 | 72 | 80 | 88 | 97 | 1.2204 | 1.2211 | 1.2217 | 1.2223 | 1.2230 | 36 |
| 35 | 1.2203 | 1.2211 | 1.2219 | 1.2228 | 35 | 42 | 48 | 55 | 61 | 67 |
| 32 | 34 | 42 | 51 | 59 | 66 | 73 | 79 | 86 | 92 | 98 |

| | Lbs. augepress. 145.3 | 150.3 | 155.3 | 160.3 | 165.3 | 170.3 | 175.3 | 180.3 | 185.3 | 190.3 | 195.3 |
|----------------|--------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | Lbs. press. 160. | 165. | 170. | 175. | 180. | 185. | 190. | 195. | 200. | 205. | 210 |
| Feed water. | Factors of Evaporation. | | | | | | | | | | |
| 212° F. | 1.0454 | 1.0460 | 1.0464 | 1.0469 | 1.0474 | 1.0478 | 1.0483 | 1.0487 | 1.0492 | 1.0496 | 1.0499 |
| 209 | 86 | 91 | 95 | 1.0500 | 1.0505 | 1.0509 | 1.0514 | 1.0519 | 1.0523 | 1.0527 | 1.0530 |
| 206 | 1.0517 | 1.0522 | 1.0526 | 31 | 36 | 40 | 45 | 50 | 54 | 58 | 61 |
| 203 | 48 | 53 | 57 | 62 | 67 | 71 | 77 | 81 | 85 | 89 | 92 |
| 200 | 79 | 84 | 88 | 93 | 98 | 1.0602 | 1.0608 | 1.0612 | 1.0616 | 1.0620 | 1.0623 |
| 197 | 1.0610 | 1.0615 | 1.0619 | 1.0624 | 1.0629 | 33 | 39 | 43 | 47 | 51 | 54 |
| 194 | 41 | 46 | 50 | 55 | 60 | 64 | 70 | 74 | 78 | 82 | 85 |
| 191 | 72 | 77 | 81 | 86 | 91 | 95 | 1.0701 | 1.0705 | 1.0709 | 1.0713 | 1.0716 |
| 188 | 1.0703 | 1.0708 | 1.0712 | 1.0717 | 1.0722 | 1.0727 | 32 | 36 | 40 | 44 | 47 |
| 185 | 34 | 39 | 43 | 48 | 53 | 58 | 63 | 67 | 71 | 75 | 78 |
| 182 | 65 | 70 | 74 | 79 | 84 | 88 | 94 | 98 | 1.0802 | 1.0806 | 1.0809 |
| 179 | 96 | 1.0801 | 1.0805 | 1.0810 | 1.0815 | 1.0819 | 1.0825 | 1.0829 | 33 | 37 | 40 |
| 176 | 1.0827 | 32 | 36 | 41 | 46 | 50 | 56 | 60 | 64 | 68 | 71 |
| 173 | 58 | 63 | 67 | 72 | 77 | 81 | 87 | 91 | 95 | 99 | 1.0902 |
| 170 | 89 | 94 | 98 | 1.0903 | 1.0908 | 1.0912 | 1.0917 | 1.0922 | 1.0926 | 1.0930 | 33 |
| 167 | 1.0920 | 1.0925 | 1.0929 | 34 | 39 | 43 | 48 | 53 | 57 | 61 | 64 |
| 164 | 51 | 56 | 60 | 65 | 70 | 74 | 79 | 84 | 88 | 92 | 95 |
| 161 | 81 | 87 | 91 | 96 | 1.1001 | 1.1005 | 1.1010 | 1.1014 | 1.1019 | 1.1023 | 1.1026 |
| 158 | 1.1012 | 1.1018 | 1.1022 | 1.1027 | 32 | 36 | 41 | 45 | 49 | 54 | 57 |
| 155 | 43 | 48 | 53 | 58 | 63 | 67 | 72 | 76 | 80 | 85 | 88 |
| 152 | 74 | 79 | 83 | 89 | 94 | 98 | 1.1103 | 1.1107 | 1.1111 | 1.1115 | 1.1119 |
| 149 | 1.1105 | 1.1110 | 1.1114 | 1.1120 | 1.1125 | 1.1129 | 34 | 38 | 42 | 46 | 49 |
| 146 | 36 | 41 | 45 | 50 | 55 | 60 | 65 | 69 | 73 | 77 | 80 |
| 143 | 67 | 72 | 76 | 81 | 86 | 91 | 96 | 1.1200 | 1.1204 | 1.1208 | 1.1211 |
| 140 | 98 | 1.1203 | 1.1207 | 1.1212 | 1.1217 | 1.1221 | 1.1227 | 31 | 35 | 39 | 42 |
| 137 | 1.1229 | 34 | 38 | 43 | 48 | 52 | 58 | 62 | 66 | 70 | 73 |
| 134 | 59 | 65 | 69 | 74 | 79 | 83 | 88 | 92 | 97 | 1.1301 | 1.1304 |
| 131 | 90 | 95 | 1.1300 | 1.1305 | 1.1310 | 1.1314 | 1.1319 | 1.1323 | 1.1327 | 32 | 35 |
| 128 | 1.1321 | 1.1326 | 30 | 36 | 41 | 45 | 50 | 54 | 58 | 62 | 66 |
| 125 | 52 | 57 | 61 | 66 | 72 | 76 | 81 | 85 | 89 | 93 | 96 |
| 122 | 83 | 88 | 92 | 97 | 1.1402 | 1.1407 | 1.1412 | 1.1416 | 1.1420 | 1.1424 | 1.1427 |
| 119 | 1.1414 | 1.1419 | 1.1423 | 1.1428 | 33 | 37 | 43 | 47 | 51 | 55 | 58 |
| 116 | 45 | 50 | 54 | 59 | 64 | 68 | 73 | 78 | 82 | 86 | 89 |
| 113 | 75 | 81 | 85 | 90 | 95 | 99 | 1.1504 | 1.1508 | 1.1512 | 1.0515 | 1.1520 |
| 110 | 1.1506 | 1.1511 | 1.1515 | 1.1521 | 1.1526 | 1.1530 | 35 | 39 | 43 | 47 | 50 |
| 107 | 37 | 42 | 46 | 51 | 57 | 61 | 66 | 70 | 74 | 78 | 81 |
| 104 | 68 | 73 | 77 | 82 | 87 | 92 | 97 | 1.1601 | 1.1605 | 1.1609 | 1.1612 |
| 101 | 99 | 1.1604 | 1.1608 | 1.1613 | 1.1618 | 1.1622 | 1.1627 | 32 | 36 | 40 | 43 |
| 98 | 1.1629 | 35 | 39 | 44 | 49 | 53 | 58 | 62 | 67 | 71 | 74 |
| 95 | 60 | 65 | 70 | 75 | 80 | 84 | 89 | 93 | 97 | 1.1701 | 1.1705 |
| 92 | 91 | 96 | 1.1700 | 1.1705 | 1.1711 | 1.1715 | 1.1720 | 1.1724 | 1.1728 | 32 | 35 |
| 89 | 1.1722 | 1.1727 | 31 | 35 | 42 | 46 | 51 | 55 | 59 | 63 | 66 |
| 86 | 53 | 58 | 62 | 67 | 72 | 76 | 82 | 86 | 90 | 94 | 97 |
| 83 | 84 | 89 | 93 | 98 | 1.1803 | 1.1807 | 1.1812 | 1.1817 | 1.1821 | 1.1825 | 1.1828 |
| 80 | 1.1814 | 1.1820 | 1.1824 | 1.1829 | 34 | 38 | 43 | 47 | 52 | 56 | 59 |
| 77 | 45 | 50 | 54 | 60 | 65 | 69 | 74 | 78 | 82 | 86 | 90 |
| 74 | 76 | 81 | 85 | 90 | 96 | 1.1900 | 1.1905 | 1.1909 | 1.1913 | 1.1917 | 1.1920 |
| 71 | 1.1907 | 1.1912 | 1.1916 | 1.1921 | 1.1926 | 31 | 36 | 40 | 44 | 48 | 51 |
| 68 | 38 | 43 | 47 | 52 | 57 | 61 | 67 | 71 | 75 | 79 | 82 |
| 65 | 69 | 74 | 78 | 83 | 88 | 92 | 97 | 1.2002 | 1.2006 | 1.2010 | 1.2013 |
| 62 | 99 | 1.2005 | 1.2009 | 1.2014 | 1.2019 | 1.2023 | 1.2028 | 32 | 36 | 41 | 44 |
| 59 | 1.2030 | 35 | 40 | 45 | 50 | 54 | 59 | 63 | 67 | 72 | 75 |
| 56 | 61 | 66 | 70 | 76 | 81 | 85 | 90 | 94 | 98 | 1.2102 | 1.2106 |
| 53 | 92 | 97 | 1.2101 | 1.2107 | 1.2112 | 1.2116 | 1.2121 | 1.2125 | 1.2129 | 33 | 36 |
| 50 | 1.2123 | 1.2128 | 32 | 37 | 43 | 47 | 52 | 56 | 60 | 64 | 67 |
| 47 | 54 | 59 | 63 | 68 | 74 | 78 | 83 | 87 | 91 | 95 | 98 |
| 44 | 85 | 90 | 94 | 1.2200 | 1.2205 | 1.2209 | 1.2214 | 1.2218 | 1.2222 | 1.2226 | 1.2229 |
| 41 | 1.2216 | 1.2221 | 1.2225 | 31 | 36 | 40 | 45 | 49 | 53 | 57 | 60 |
| 38 | 47 | 52 | 56 | 62 | 67 | 71 | 76 | 80 | 84 | 88 | 91 |
| 35 | 78 | 83 | 88 | 93 | 98 | 1.2302 | 1.2307 | 1.2311 | 1.2315 | 1.2320 | 1.2323 |
| 32 | 1.2309 | 1.2315 | 1.2319 | 1.2324 | 1.2329 | 33 | 38 | 42 | 46 | 51 | 54 |

| | Lbs. | | | | | | | | | | |
|--------------|-------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Gauge press. | 200.3 | 205.3 | 210.3 | 215.3 | 220.3 | 225.3 | 230.3 | 235.3 | 240.3 | 245.3 | 250.3 |
| Abs. press. | 215. | 220. | 225. | 230. | 235. | 240. | 245. | 250. | 255. | 260. | 265. |
| Feed water. | Factors of Evaporation. | | | | | | | | | | |
| 212° F. | 1.0503 | 1.0507 | 1.0510 | 1.0513 | 1.0517 | 1.0520 | 1.0523 | 1.0527 | 1.0529 | 1.0533 | 1.0535 |
| 209 | 34 | 38 | 41 | 44 | 48 | 52 | 55 | 58 | 60 | 64 | 66 |
| 206 | 65 | 69 | 72 | 75 | 79 | 83 | 86 | 89 | 91 | 95 | 97 |
| 203 | 96 | 1.0600 | 1.0603 | 1.0606 | 1.0611 | 1.0614 | 1.0617 | 1.0620 | 1.0622 | 1.0626 | 1.0629 |
| 200 | 1.0627 | 31 | 34 | 37 | 42 | 45 | 48 | 51 | 53 | 57 | 60 |
| 197 | 58 | 62 | 65 | 68 | 73 | 76 | 79 | 82 | 84 | 88 | 91 |
| 194 | 89 | 93 | 96 | 1.0700 | 1.0704 | 1.0707 | 1.0710 | 1.0713 | 1.0715 | 1.0719 | 1.0722 |
| 191 | 1.0720 | 1.0724 | 1.0727 | 31 | 35 | 38 | 41 | 44 | 46 | 50 | 53 |
| 188 | 51 | 55 | 58 | 62 | 66 | 69 | 72 | 75 | 78 | 81 | 84 |
| 185 | 82 | 86 | 89 | 93 | 97 | 1.0800 | 1.0803 | 1.0806 | 1.0809 | 1.0812 | 1.0815 |
| 182 | 1.0813 | 1.0817 | 1.0820 | 1.0823 | 1.0828 | 31 | 34 | 37 | 39 | 43 | 46 |
| 179 | 44 | 48 | 51 | 54 | 59 | 62 | 65 | 68 | 70 | 74 | 77 |
| 176 | 75 | 79 | 82 | 86 | 90 | 93 | 96 | 99 | 1.0901 | 1.0905 | 1.0908 |
| 173 | 1.0905 | 1.0910 | 1.0913 | 1.0916 | 1.0921 | 1.0924 | 1.0927 | 1.0930 | 32 | 36 | 39 |
| 170 | 37 | 41 | 44 | 47 | 51 | 55 | 58 | 61 | 63 | 67 | 69 |
| 167 | 68 | 72 | 75 | 78 | 82 | 86 | 89 | 92 | 94 | 98 | 1.1001 |
| 164 | 99 | 1.1003 | 1.1006 | 1.1009 | 1.1013 | 1.1016 | 1.1019 | 1.1023 | 1.1025 | 1.1029 | 31 |
| 161 | 1.1030 | 34 | 37 | 40 | 44 | 47 | 50 | 54 | 56 | 60 | 62 |
| 158 | 61 | 65 | 68 | 71 | 75 | 78 | 81 | 85 | 87 | 91 | 93 |
| 155 | 92 | 96 | 99 | 1.1102 | 1.1106 | 1.1109 | 1.1112 | 1.1115 | 1.1118 | 1.1122 | 1.1124 |
| 152 | 1.1123 | 1.1127 | 1.1130 | 33 | 37 | 40 | 43 | 45 | 49 | 53 | 55 |
| 149 | 54 | 58 | 61 | 64 | 68 | 71 | 74 | 77 | 80 | 83 | 86 |
| 146 | 84 | 89 | 92 | 95 | 99 | 1.1202 | 1.1205 | 1.1208 | 1.1211 | 1.1214 | 1.1217 |
| 143 | 1.1215 | 1.1219 | 1.1223 | 1.1226 | 1.1230 | 33 | 36 | 39 | 42 | 45 | 48 |
| 140 | 46 | 50 | 53 | 56 | 61 | 64 | 67 | 70 | 72 | 76 | 79 |
| 137 | 77 | 81 | 84 | 87 | 92 | 95 | 98 | 1.1301 | 1.1303 | 1.1307 | 1.1310 |
| 134 | 1.1308 | 1.1312 | 1.1315 | 1.1318 | 1.1322 | 1.1326 | 1.1329 | 32 | 34 | 38 | 40 |
| 131 | 39 | 43 | 46 | 49 | 53 | 56 | 59 | 62 | 65 | 69 | 71 |
| 128 | 70 | 74 | 77 | 80 | 84 | 87 | 90 | 93 | 96 | 1.1400 | 1.1402 |
| 125 | 1.1400 | 1.1405 | 1.1408 | 1.1411 | 1.1415 | 1.1418 | 1.1421 | 1.1424 | 1.1427 | 30 | 33 |
| 122 | 31 | 35 | 39 | 42 | 46 | 49 | 52 | 55 | 58 | 61 | 64 |
| 119 | 62 | 66 | 69 | 72 | 77 | 80 | 83 | 86 | 88 | 92 | 95 |
| 116 | 93 | 97 | 1.1500 | 1.1503 | 1.1507 | 1.1511 | 1.1514 | 1.1517 | 1.1519 | 1.1523 | 1.1525 |
| 113 | 1.1524 | 1.1528 | 31 | 34 | 38 | 41 | 44 | 48 | 50 | 54 | 56 |
| 110 | 55 | 59 | 62 | 65 | 69 | 72 | 75 | 78 | 81 | 85 | 87 |
| 107 | 85 | 90 | 93 | 96 | 1.1600 | 1.1603 | 1.1606 | 1.1609 | 1.1612 | 1.1615 | 1.1618 |
| 104 | 1.1616 | 1.1620 | 1.1624 | 1.1627 | 31 | 34 | 37 | 40 | 43 | 46 | 49 |
| 101 | 47 | 51 | 54 | 57 | 61 | 65 | 68 | 71 | 73 | 77 | 80 |
| 98 | 78 | 82 | 85 | 88 | 92 | 95 | 98 | 1.1702 | 1.1704 | 1.1708 | 1.1710 |
| 95 | 1.1709 | 1.1713 | 1.1716 | 1.1719 | 1.1723 | 1.1726 | 1.1729 | 32 | 35 | 39 | 41 |
| 92 | 39 | 44 | 47 | 50 | 54 | 57 | 60 | 63 | 66 | 69 | 72 |
| 89 | 70 | 75 | 78 | 81 | 85 | 88 | 91 | 94 | 97 | 1.1800 | 1.1803 |
| 86 | 1.1801 | 1.1805 | 1.1808 | 1.1812 | 1.1816 | 1.1819 | 1.1822 | 1.1825 | 1.1827 | 31 | 34 |
| 83 | 32 | 36 | 39 | 42 | 46 | 50 | 53 | 56 | 58 | 62 | 64 |
| 80 | 63 | 67 | 70 | 73 | 77 | 80 | 83 | 87 | 89 | 93 | 95 |
| 77 | 94 | 98 | 1.1901 | 1.1904 | 1.1908 | 1.1911 | 1.1914 | 1.1917 | 1.1920 | 1.1924 | 1.1926 |
| 74 | 1.1924 | 1.1929 | 32 | 35 | 39 | 42 | 45 | 48 | 51 | 54 | 57 |
| 71 | 55 | 59 | 63 | 66 | 70 | 73 | 76 | 79 | 82 | 85 | 88 |
| 68 | 86 | 90 | 93 | 96 | 1.2001 | 1.2004 | 1.2007 | 1.2010 | 1.2012 | 1.2016 | 1.2019 |
| 65 | 1.2017 | 1.2021 | 1.2024 | 1.2027 | 31 | 35 | 38 | 41 | 43 | 47 | 49 |
| 62 | 48 | 52 | 55 | 58 | 62 | 65 | 68 | 72 | 74 | 78 | 80 |
| 59 | 79 | 83 | 86 | 89 | 93 | 95 | 99 | 1.2102 | 1.2105 | 1.2109 | 1.2111 |
| 56 | 1.2110 | 1.2114 | 1.2117 | 1.2120 | 1.2124 | 1.2127 | 1.2130 | 33 | 36 | 40 | 42 |
| 53 | 41 | 45 | 48 | 51 | 55 | 58 | 61 | 64 | 67 | 70 | 73 |
| 50 | 71 | 76 | 79 | 82 | 86 | 89 | 92 | 95 | 98 | 1.2201 | 1.2204 |
| 47 | 1.2202 | 1.2207 | 1.2210 | 1.2213 | 1.2217 | 1.2220 | 1.2223 | 1.2226 | 1.2229 | 32 | 35 |
| 44 | 34 | 38 | 41 | 44 | 48 | 51 | 54 | 57 | 60 | 63 | 66 |
| 41 | 65 | 69 | 72 | 75 | 79 | 82 | 85 | 88 | 91 | 94 | 97 |
| 38 | 96 | 1.2300 | 1.2303 | 1.2306 | 1.2310 | 1.2313 | 1.2316 | 1.2319 | 1.2322 | 1.2325 | 1.2328 |
| 35 | 1.2327 | 31 | 34 | 37 | 41 | 44 | 47 | 50 | 53 | 57 | 59 |
| 32 | 58 | 62 | 65 | 68 | 72 | 75 | 78 | 82 | 84 | 88 | 90 |

STRENGTH OF STEAM-BOILERS. VARIOUS RULES FOR CONSTRUCTION.*

There is a great lack of uniformity in the rules prescribed by different writers and by legislation governing the construction of steam-boilers. In the United States, boilers for merchant vessels must be constructed according to the rules and regulations prescribed by the Board of Supervising Inspectors of Steam Vessels; in the U. S. Navy, according to rules of the Navy Department, and in some cases according to special acts of Congress. On land, in some places, as in Philadelphia, the construction of boilers is governed by local laws; but generally there are no laws upon the subject, and boilers are constructed according to the idea of individual engineers and boiler-makers. In Europe the construction is generally regulated by stringent inspection laws. The rules of the U. S. Supervising Inspectors of Steam-vessels, the British Lloyd's and Board of Trade, the French Bureau Veritas, and the German Lloyd's are ably reviewed in a paper by Nelson Foley, M. Inst. Naval Architects, etc., read at the Chicago Engineering Congress, 1893, Division of Marine and Naval Engineering. From this paper the following notes are taken, chiefly with reference to the U. S. and British rules:

(*Abbreviations.* — T. S., for tensile strength; el., elongation; contr., contraction of area.)

Hydraulic Tests. — *Board of Trade, Lloyd's, and Bureau Veritas.* — Twice the working pressure.

United States Statutes. — One and a half times the working pressure.

Mr. Foley proposes that the proof pressure should be $1\frac{1}{2}$ times the working pressure + one atmosphere.

Established Nominal Factors of Safety. — *Board of Trade.* — 4.5 for a boiler of moderate length and of the best construction and workmanship.

Lloyd's. — Not very apparent, but appears to lie between 4 and 5.

United States Statutes. — Indefinite, because the strength of the joint is not considered, except by the broad distinction between single and double riveting.

Bureau Veritas: 4.4.

German Lloyd's: 5 to 4.65, according to the thickness of the plates.

Material for Riveting. — *Board of Trade.* — Tensile strength of rivet bars between 26 and 30 tons, el. in 10 in. not less than 25%, and contr. of area not less than 50%. (Tons of 2240 lbs.)

Lloyd's. — T. S., 26 to 30 tons; el. not less than 20% in 8 in. The material must stand bending to a curve, the inner radius of which is not greater than $1\frac{1}{2}$ times the thickness of the plate, after having been uniformly heated to a low cherry-red, and quenched in water at 82° F.

United States Statutes. — No special provision.

Rules Connected with Riveting. — *Board of Trade.* — The shearing resistance of the rivet steel to be taken at 23 tons per square inch, 5 to be used for the factor of safety independently of any addition to this factor for the plating. Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The diameter must not be less than the thickness of the plate and the pitch never greater than $8\frac{1}{2}$ ". The thickness of double butt-straps (each) not to be less than $\frac{5}{8}$ the thickness of the plate; single butt-straps not less than $\frac{9}{8}$.

Distance from center of rivet to edge of plate = diam. of rivet $\times 1\frac{1}{2}$.

Distance between rows of rivets

= $2 \times$ diam. of rivet or = $[(\text{diam.} \times 4) + 1] \div 2$, if chain, and

= $\frac{\sqrt{[(\text{pitch} \times 11) + (\text{diam.} \times 4)] \times (\text{pitch} + \text{diam.} \times 4)}}{10}$ if zigzag.

Diagonal pitch = $(\text{pitch} \times 6 + \text{diam.} \times 4) \div 10$.

Lloyd's. — Rivets in double shear to have only 1.75 times the single section taken in the calculation instead of 2. The shearing strength of rivet steel to be taken at 85% of the T. S. of the material of shell plates, In any case where the strength of the longitudinal joint is satisfactorily

* For specifications for steel for boilers, see p. 483. For riveted joints, see page 401.

shown by experiment to be greater than given by the formula, the actual strength may be taken in the calculation.

United States Statutes. — No rules. [The rules in 1909 give formulas equivalent to those of the British Board of Trade and tables taken from T. W. Traill's "Boilers, Marine and Land."]

Material for Cylindrical Shells Subject to Internal Pressure. — *Board of Trade.* — T. S. between 27 and 32 tons. In the normal condition, el. not less than 18% in 10 in., but should be about 25%; if annealed, not less than 20%. Strips 2 in. wide should stand bending until the sides are parallel at a distance from each other of not more than three times the plate's thickness.

Lloyd's. — T. S. between the limits of 26 and 30 tons per square inch. El. not less than 20% in 8 in. Test strips heated to a low cherry-red and plunged into water at 82° F. must stand bending to a curve, the inner radius of which is not greater than 1½ times the plate's thickness.

U. S. Statutes. — Plates ½ in. thick and under shall show a contr. of not less than 50%; when over ½ in. and up to ¾ in., not less than 45%; when over ¾ in., not less than 40%.

Mr. Foley's comments: The Board of Trade rules seem to indicate a steel of too high T. S. when a lower and more ductile one can be got: the lower tensile limit should be reduced, and the bending test might with advantage be made after tempering, and made to a smaller radius. Lloyd's rule for quality seems more satisfactory, but the temper test is not severe. The United States Statutes are not sufficiently stringent to insure an entirely satisfactory material.

Mr. Foley suggests a material which would meet the following: 25 tons lower limit in tension; 25% in 8 in. minimum elongation; radius for bending test after tempering = the plate's thickness.

Shell-plate Formulæ. — *Board of Trade:* $P = \frac{T \times B \times t \times 2}{D \times F}$.

D = diameter of boiler in inches;

P = working-pressure in lbs. per square inch;

t = thickness in inches;

B = percentage of strength of joint compared to solid plate;

T = tensile strength allowed for the material in lbs. per square inch;

F = a factor of safety, being 4.5, with certain additions depending on method of construction.

Lloyd's: $P = \frac{C \times (t - 2) \times B}{D}$

t = thickness of plate in sixteenths; B and D as before; C = a constant depending on the kind of joint.

When longitudinal seams have double butt-straps, $C = 20$. When longitudinal seams have double butt-straps of unequal width, only covering on one side the reduced section of plate at the outer line of rivets, $C = 19.5$.

When the longitudinal seams are lap-jointed, $C = 18.5$.

U. S. Statutes. — Using same notation as for Board of Trade,

$P = \frac{t \times 2 \times T}{D \times 6}$ for single-riveting; add 20% for double-riveting;

where T is the lowest T.S. stamped on any plate.

Mr. Foley criticises the rule of the United States Statutes as follows: The rule ignores the riveting, except that it distinguishes between single and double, giving the latter 20% advantage; the circumferential riveting or class of seam is altogether ignored. The rule takes no account of workmanship or method adopted of constructing the joints. The factor, one sixth, simply covers the actual nominal factor of safety as well as the loss of strength at the joint, no matter what its percentage; we may therefore dismiss it as unsatisfactory.

Rules for Flat Plates. — *Board of Trade:* $P = \frac{C(t + 1)^2}{S - 6}$.

P = working-pressure in lbs. per square inch;

S = surface supported in square inches;

t = thickness in sixteenths of an inch;

C = a constant as per following table:

$C = 125$ for plates not exposed to heat or flame, the stays fitted with nuts and washers, the latter at least three times the diameter of the stay and $\frac{2}{3}$ the thickness of the plate;

$C = 187.5$ for the same condition, but the washers $\frac{2}{3}$ the pitch of stays in diameter, and thickness not less than plate;

$C = 200$ for the same condition, but doubling plates in place of washers, the width of which is $\frac{2}{3}$ the pitch and thickness the same as the plate;

$C = 112.5$ for the same condition, but the stays with nuts only;

$C = 75$ when exposed to impact of heat or flame and steam in contact with the plates, and the stays fitted with nuts and washers three times the diameter of the stay and $\frac{2}{3}$ the plate's thickness;

$C = 67.5$ for the same condition, but stays fitted with nuts only;

$C = 100$ when exposed to heat or flame, and water in contact with the plates, and stays screwed into the plates and fitted with nuts;

$C = 66$ for the same condition, but stays with riveted heads.

U. S. Statutes. — Using same notation as for Board of Trade.

$P = \frac{C \times t^2}{p^2}$, where p = greatest pitch in inches, P and t as above;

$C = 112$ to 200 according to various specified conditions. [Rules of 1909.]

Certain experiments were carried out by the Board of Trade which showed that the resistance to bulging does not vary as the square of the plate's thickness. There seems also good reason to believe that it is not inversely as the square of the greatest pitch. Bearing in mind, says Mr. Foley, that mathematicians have signally failed to give us true theoretical foundations for calculating the resistance of bodies subject to the simplest forms of stresses, we therefore cannot expect much from their assistance in the matter of flat plates.

The Board of Trade rules for flat surfaces, being based on actual experiment, are especially worthy of respect; sound judgment appears also to have been used in framing them.

Furnace Formulæ. — BOARD OF TRADE. — *Long Furnaces.* —

$P = \frac{C \times t^2}{(L + 1) \times D}$, but not where L is shorter than $(11.5 t - 1)$, at which length the rule for short furnaces comes into play.

P = working-pressure in pounds per square inch; t = thickness in inches;

D = outside diameter in inches; L = length of furnace in feet up to 10 ft.;

C = a constant, as per following table, for drilled holes:

$C = 99,000$ for welded or butt-jointed with single straps, double-riveted;

$C = 88,000$ for butts with single straps, single-riveted;

$C = 99,000$ for butts with double straps, single-riveted.

Provided always that the pressure so found does not exceed that given by the following formulæ, which apply also to short furnaces:

$P = \frac{C \times t}{D}$ for all the patent furnaces named;

$P = \frac{C \times t}{3 \times D} \left(5 - \frac{L \times 12}{67.5 \times t} \right)$ when with Adamson rings.

$C = 8,800$ for plain furnaces;

$C = 14,000$ for Fox; minimum thickness $\frac{5}{16}$ in., greatest $\frac{5}{8}$ in.; plain part not to exceed 6 in. in length;

$C = 13,500$ for Morison; minimum thickness $\frac{5}{16}$ in., greatest $\frac{5}{8}$ in.; plain part not to exceed 6 in. in length;

$C = 14,000$ for Purves-Brown; limits of thickness $\frac{7}{16}$ in. and $\frac{5}{8}$ in., plain part 9 in. in length;

$C = 8,800$ for Adamson rings; radius of flange next fire $1\frac{1}{2}$ in.

U. S. STATUTES. — *Long Furnaces.* — Same notation.

$P = \frac{89,600 \times t^2}{L \times D}$, but L not to exceed 8 ft. [New rules are given in 1909; see page 884.]

Mr. Foley comments on the rules for long furnaces as follows: The Board of Trade general formula, where the length is a factor, has a very limited range indeed, viz., 10 ft. as the extreme length, and 135 thicknesses

— 12 in., as the short limit. The original formula, $P = \frac{C \times t^2}{L \times D}$, is that of Sir W. Fairbairn, and was, I believe, never intended by him to apply to short furnaces. On the very face of it, it is apparent, on the other hand, that if it is true for moderately long furnaces, it cannot be so for very long ones. We are therefore driven to the conclusion that any formula which includes simple L as a factor must be founded on a wrong basis.

With Mr. Traill's form of the formula, namely, substituting $(L + 1)$ for L , the results appear sufficiently satisfactory for practical purposes, and indeed, as far as can be judged, tally with the results obtained from experiment as nearly as could be expected. The experiments to which I refer were six in number, and of great variety of length to diameter; the actual factors of safety ranged from 4.4 to 6.2, the mean being 4.78, or practically 5. It seems to me, therefore, that, within the limits prescribed, the Board of Trade formula may be accepted as suitable for our requirements.

Material for Stays. — The qualities of material prescribed are as follows:

Board of Trade. — The tensile strength to lie between the limits of 27 and 32 tons per sq. in., and to have an elongation of not less than 20% in 10 in. Steel stays which have been welded or worked in the fire should not be used. [Tons of 2240 lbs.]

Lloyd's. — 26 to 30 ton steel, with elongation not less than 20% in 8 in.

U. S. Statutes. — The only condition is that the reduction of area must not be less than 40% if the test bar is over $\frac{3}{4}$ in. diameter.

Loads allowed on Stays. — *Board of Trade.* — 9000 lbs. per square inch is allowed on the net section, provided the tensile strength ranges from 27 to 32 tons. Steel stays are not to be welded or worked in the fire.

Lloyd's. — For screwed and other stays, not exceeding $1\frac{1}{2}$ in. diameter effective, 8000 lbs. per square inch is allowed; for stays above $1\frac{1}{2}$ in., 9000 lbs. No stays are to be welded.

U. S. Statutes. — Braces and stays shall not be subjected to a greater stress than 6000 lbs. per sq. in. [As high as 9000 lbs. is allowed in some cases in the rules of 1909.]

[Rankine, S. E., p. 459, says: "The iron of the stays ought not to be exposed to a greater working tension than 3000 lbs. on the square inch, in order to provide against their being weakened by corrosion. This amounts to making the factor of safety for the working pressure about 20." It is evident, however, that an allowance in the factor of safety for corrosion may reasonably be decreased with increase of diameter. W.K.]

A discussion of various rules and formulæ for stay bolts, braces and flat surfaces will be found in a paper by R. S. Hale, *Trans. A. S. M. E.*, 1904.

Girders. — *Board of Trade.* $P = \frac{C \times d^2 \times t}{(W - p) D \times L}$. P = working pressure in lbs. per sq. in.; W = width of flame-box; L = length of girder; p = pitch of bolts; D = distance between girders from center to center; d = depth of girder; t = thickness of sum of same; C = a constant = 6600 for 1 bolt, 9900 for 2 or 3 bolts, and 11,220 for 4 bolts. All dimensions in inches.

Lloyd's. — The same formula and constants, except that C = 11,000 for 4 or 5 bolts, 11,550 for 6 or 7, and 11,880 for 8 or more.

U. S. Statutes. — [The rules in 1909 are the same as Lloyd's.]

Tube-Plates. — *Board of Trade.* $P = \frac{t(D - d) \times 20,000}{W \times D}$. D = least horizontal distance between centers of tubes in inches; d = inside diameter of ordinary tubes; t = thickness of tube-plate in inches; W = extreme width of combustion-box in inches from front tube-plate to back of fire-box, or distance between combustion-box tube-plates when the boiler is double-ended and the box common to both ends.

The crushing stress on tube-plates caused by the pressure on the flame-box top is to be limited to 10,000 lbs. per square inch.

Material for Tubes. — Mr. Foley proposes the following: If iron, the quality to be such as to give at least 22 tons per square inch as the minimum tensile strength, with an elongation of not less than 15% in 8 ins. If steel, the elongation to be not less than 26% in ins. for the material before being rolled into strips; and after tempering, the test bar to stand completely closing together. Provided the steel welds well, there does not seem to be any object in providing tensile limits. The ends should be annealed after manufacture, and stay-tube ends should be annealed before screwing.

Holding-power of Boiler-tubes. (See also page 342.) — In Messrs. Yarrow's experiments on iron and steel tubes of 2 in. to 21 $\frac{1}{4}$ in. diameter the first 5 tubes gave way on an average of 23,740 lbs., which would appear to be about $\frac{2}{3}$ the ultimate strength of the tubes themselves. In all these cases the hole through the tube-plate was parallel with a sharp edge to it, and a ferrule was driven into the tube.

Tests of the next 5 tubes were made under the same conditions as the first 5, with the exception that in this case the ferrule was omitted, the tubes being simply expanded into the plates. The mean pull required was 15,270 lbs., or considerably less than half the ultimate strength of the tubes.

Effect of beading the tubes, the holes through the plate being parallel and ferrules omitted. The mean of the first 3, which are tubes of the same kind, gives 26,876 lbs. as their holding-power, under these conditions, as compared with 23,740 lbs. for the tubes fitted with ferrules only. This high figure is, however, mainly due to an exceptional case where the holding-power is greater than the average strength of the tubes themselves.

It is disadvantageous to cone the hole through the tube-plate unless its sharp edge is removed, as the results are much worse than those obtained with parallel holes, the mean pull being but 16,031 lbs., the experiments being made with tubes expanded and ferruled but not beaded over.

In experiments on tubes expanded into tapered holes, beaded over and fitted with ferrules, the net result is that the holding-power is, for the size experimented on, about $\frac{3}{4}$ of the tensile strength of the tube, the mean pull being 28,797 lbs. With tubes expanded into tapered holes and simply beaded over, better results were obtained than with ferrules; in these cases, however, the sharp edge of the hole was rounded off, which appears in general to have a good effect.

In one particular the experiments are incomplete, as it is impossible to reproduce on a machine the racking the tubes get by the expansion of a boiler as it is heated up and cooled down again, and it is quite possible, therefore, that the fastening giving the best results on the testing-machine may not prove so efficient in practice.

N.B. — It should be noted that the experiments were all made under the cold condition, so that reference should be made with caution, the circumstances in practice being very different, especially when there is scale on the tube-plates, or when the tube-plates are thick and subject to intense heat.

Iron versus Steel Boiler-tubes. (Foley.) — Mr. Blechynden prefers iron tubes to those of steel, but how far he would go in attributing the leaky-tube defect to the use of steel tubes we are not aware. It appears, however, that the results of his experiments would warrant him in going a considerable distance in this direction. The test consisted of heating and cooling two tubes, one of wrought iron and the other of steel. Both tubes were 2 $\frac{3}{4}$ in. in diameter and 0.16 in. thickness of metal. The tubes were put in the same furnace, made red-hot, and then dipped in water. The length was gauged at a temperature of 46° F.

This operation was twice repeated, with results as follows:

| | Steel. | Iron. |
|--|------------|------------|
| Original length | 55.495 in. | 55.495 in. |
| Heated to 186° F.; increase | 0.052 in. | 0.048 in. |
| Coefficient of expansion per degree F. | .0000067 | .0000062 |
| Heated red-hot and dipped in water; decrease | .007 in. | .003 in. |
| Second heating and cooling, decrease | .031 in. | .004 in. |
| Third heating and cooling, decrease | .017 in. | .006 in. |
| Total contraction | .055 in. | .013 in. |

Mr. A. C. Kirk writes: That overheating of tube ends is the cause of the leakage of the tubes in boilers is proved by the fact that the ferrules at present used by the Admiralty prevent it. These act by shielding the tube ends from the action of the flame, and consequently reducing evaporation, and so allowing free access of the water to keep them cool.

Although many causes contribute, there seems no doubt that thick tube-plates must bear a share of causing the mischief.

Rules for Construction of Boilers in Merchant Vessels in the United States.

(Extracts from General Rules and Regulations of the Board of Supervising Inspectors, Steamboat Inspection Service (as amended Jan., 1909).)

Tensile Strength of Plate. — From each plate as rolled there shall be taken two test pieces, one for tensile test and one for bending test. The piece for tensile test shall be taken from the side of the plate at about one-third of its length from the top of the plate, and the piece for bending test shall be taken transversely from the top of the plate near the center.

All the pieces shall be prepared so that the skin shall not be removed, the edges only planed or shaped.

In no case shall test pieces be prepared by annealing or reduced in size by hammering.

Tensile-test pieces shall be at least 16 ins. in length, from $1\frac{1}{2}$ to $3\frac{1}{2}$ ins. in width at the ends, which ends shall join by an easy fillet, a straight part in the center of at least 9 ins. in length and $1\frac{1}{2}$ ins. in width, . . . marked with light prick punch marks at distances 1 inch apart, spaced so as to give 8 inches in length.

Only steel plates manufactured by what is known as the basic or acid open-hearth processes will be allowed to be used in the construction or repairs of boilers for marine purposes.

No plate made by the acid process shall contain more than 0.06% of phosphorus and 0.04% of sulphur, and no plate made by the basic process shall contain more than 0.04% of phosphorus and 0.04% of sulphur.

For steel plates the sample must show, when tested, a tensile strength not lower than 50,000 lbs. and not higher than 75,000 lbs. per sq. in. of section, and no such plate shall be stamped with a higher tensile strength than 70,000 lbs.: Provided, however, that for steel plates exceeding a thickness of 0.3125 in. intended for use in externally fired boilers, the sample must show, when tested, a tensile strength not lower than 54,000 lbs. and not higher than 67,000 lbs. per sq. in. of section, and no plate exceeding a thickness of 0.3125 in. intended for use in externally fired boilers shall be stamped with a higher tensile strength than 62,000 lbs. Such sample must also show an elongation of at least 25% in a length of 2 ins. for thickness up to $\frac{1}{4}$ in., inclusive; in a length of 4 ins. for over $\frac{1}{4}$ to $\frac{7}{16}$ in., inclusive; in a length of 6 ins. for all plates over $\frac{7}{16}$ in. The sample must also show a reduction of sectional area as follows:

At least 50% for thickness up to $\frac{1}{2}$ in., inclusive; 45% for thickness over $\frac{1}{2}$ to $\frac{3}{4}$ in., inclusive, and 40% for thickness over $\frac{3}{4}$ in.

Quenching and bending test. — Quenching and bending test pieces shall be at least 12 ins. in length and from 1 to $3\frac{1}{2}$ ins. in width. The side where sheared or planed must not be rounded, but the edges may have the sharpness taken off with a fine file. The test piece shall be heated to a cherry red (as seen in a dark place) and then plunged into water at a temperature of about 82° F. Thus prepared, the sample shall be bent to a curve, the inner radius of which is not greater than $1\frac{1}{2}$ times the thickness of the sample, without cracks or flaws. The ends must be parallel after bending.

Cylindrical Shells. — The working steam pressure allowable on cylindrical shells of boilers constructed of plates inspected as required by these rules, when single riveted, shall not produce a strain to exceed one-sixth of the tensile strength of the iron or steel plates of which such boilers are constructed; but where the longitudinal laps of the cylindrical parts of such boilers are double riveted, and the rivet holes for such boilers have been fairly drilled, an addition of 20 per cent to the working pressure provided for single riveting will be allowed.

The pressure for any dimension of boilers must be ascertained by the following rule, viz.:

Multiply one-sixth of the lowest tensile strength found stamped on the plates in the cylindrical shell by the thickness — expressed in inches or part of an inch — and divide by the radius or half diameter — also expressed in inches — and the result will be the pressure allowable per square inch of surface for single riveting, to which add 20% for double riveting, when all the rivet holes in the shell of such boiler have been "fairly drilled" and no part of such holes has been punched. The pressure allowed shall be based on the plate whose tensile strength multiplied by its thickness gives the lowest product.

Cylindrical Shells of Water-tube or Coil Boilers. — The working pressure allowable, when such shells have a row or rows of pipes or tubes inserted therein, shall be determined by the formula:

$$P = (D - d) \times T \times S \div (D \times R),$$

where P = working pressure allowable in pounds; D = distance in inches between the tube or pipe centers in a line from head to head; d = diameter of hole in inches; T = thickness of plate in inches; S = one-sixth of the tensile strength of the plate; R = radius of shell in inches.

Convex Heads. — Plates used as heads, when new and made to practically true circles, shall be allowed a steam pressure in accordance with the formula: $P = T \times S \div R$, where P = steam pressure allowable in lbs. per sq. in.; T = thickness of plate in ins.; S = one-sixth of the tensile strength; R = one-half of the radius to which the head is bumped.

Add 20% when the head is double riveted to the shell and the holes are fairly drilled.

Bumped heads may contain a manhole opening flanged inwardly, when such flange is turned to a depth of three times the thickness of material in the head.

Concave Heads. — For concave heads the pressure allowable will be 0.6 times the pressure allowable for convex heads.

Flat Heads. — Where flat heads do not exceed 20 ins. diameter they may be used without being stayed, and the steam pressure allowable shall be determined by the formula: $P = C \times T^2 \div A$, where P = steam pressure allowable in pounds; T = thickness of material in sixteenths of an inch; A = one-half the area of head in inches; C = 112 for plates $7/16$ in. and under; C = 120 for plates over $7/16$ in. Provided, the flanges are made to an inside radius of at least $1\frac{1}{2}$ inches.

Flat Surfaces. — The maximum stress allowable on flat plates supported by stays shall be determined by the following formula:

All stayed surfaces formed to a curve the radius of which is over 21 ins. excepting surfaces otherwise provided for, shall be deemed flat surfaces.

$$\text{Working pressure} = C \times T^2 \div P^2,$$

where T = thickness of plates in 16ths of an inch; P = greatest pitch of stays in ins.; C = 112 for screw stays with riveted heads, plates $7/16$ thick and under; C = 120 for screw stays with riveted heads, plates above $7/16$ in. thick; C = 120 for screw stays with nuts, plates $7/16$ in. thick and under; C = 125 for screw stays with nuts, plates above $7/16$ in. thick and under $9/16$ in.; C = 135 for screw stays with nuts, plates $9/16$ in. thick and above; C = 175 for stays with double nuts having one nut on the inside and one nut on the outside of plate, without washers or doubling plates; C = 160 for stays fitted with washers or doubling strips which have a thickness of at least 0.5 of the thickness of the plate and a diameter of at least 0.5 of the greatest pitch of the stay, riveted to the outside of the plates, and stays having one nut inside of the plate, and one nut outside of the washer or doubling strip. For T take 72% of the combined thickness of the plate and washer or plate and doubling strip. C = 200 for stays fitted with doubling strips which have a thickness equal to at least 0.5 of the thickness of the plate reinforced, and covering the full area braced (up to the curvature of the flange, if any), riveted to either the inside or outside of the plate, and stays having one nut outside and one inside of the plates. Washers or doubling plates to be substantially riveted. For T take 72% of the combined thickness of the two plates. C = 200 for stays with plates

stiffened with tees or angle bars having a thickness of at least $\frac{2}{3}$ the thickness of plate and depth of webs at least $\frac{1}{4}$ of the greatest pitch of the stays, and substantially riveted on the inside of the plates, and stays having one nut inside, bearing on washers fitted to the edges of the webs that are at right angles to the plate. For T take 72% of the combined thickness of web and plate.

No such flat plates or surfaces shall be unsupported a greater distance than 18 inches.

Stays. — The maximum stress in pounds allowable per square inch of cross-sectional area for stays used in the construction of marine boilers, when they are accurately fitted and properly secured, shall be ascertained by the following formula:

$P = A \times C \div a$, where P = working pressure in lbs. per sq. in.; A = least cross-sectional area of stay in inches; a = area of surface supported by one stay, in inches; C = 9000 for tested steel stays exceeding $2\frac{1}{2}$ ins. diam.; C = 8000 for tested steel stays $1\frac{1}{4}$ ins. and not exceeding $2\frac{1}{2}$ ins. diam., when such stays are not forged or welded. The ends, however, may be upset to a sufficient diameter to allow for the depth of the thread. The diameter shall be taken at the bottom of the thread, provided it is the least diameter of the stay. All such stays after being upset shall be thoroughly annealed. C = 8000 for a tested Huston or similar type of brace, the cross-sectional area of which exceeds 5 sq. ins.; C = 7000 for such tested braces when the cross-sectional area is not less than 1.227 and not more than 5 sq. ins., provided such braces are prepared at one heat from a solid piece of plate without welds; C = 6000 for all stays not otherwise provided for.

Flues subjected to External Pressure only. — Plain lap-welded steel flues 7 to 13 ins. diameter. D = outside diam., ins.; T = thickness, ins.; P = working pressure, lbs. per sq. in.; F = factor of safety.

$T = \frac{[(F \times P) + 1386] D}{86670}$. This formula is applicable to lengths greater than six diameters of flue, to working pressures greater than 100 lbs. per sq. in., and to temperatures less than 650° F.

Riveted flues, made in sections riveted together, 6 to 9 ins. diam., maximum length of sections 60 ins.; over 9 and not over 13 ins. diam., maximum length 42 ins.; $P = 8100 \times T \div D$.

Riveted or lap-welded flues, over 13 and not over 28 ins. diam., lengths not to exceed $3\frac{1}{2}$ times the diam.:

$$P = \frac{51.5}{D} [(18.75 \times T) - (L \times 1.03)].$$

(L = length of flue in inches; T = thickness in 16ths of an inch.)

Furnaces. — The tensile strength of steel used in the construction of corrugated or ribbed furnaces shall not exceed 67,000, and be not less than 54,000 lbs.; and in all other furnaces the minimum tensile strength shall not be less than 58,000, and the maximum not more than 67,000 lbs. The minimum elongation in 8 inches shall be 20%.

All corrugated furnaces having plain parts at the ends not exceeding 9 inches in length (except flues especially provided for), when new, and made to practically true circles, shall be allowed a steam pressure in accordance with the formula: $P = C \times T \div D$.

P = pressure in lbs. per sq. in., T = thickness in inches, C = a constant, as below.

| | |
|---|---|
| Leeds suspension bulb furnace | $C = 17,000$, T not less than $\frac{5}{16}$ in. |
| Morison corrugated type | $C = 15,600$, T not less than $\frac{5}{16}$ in. |
| Fox corrugated type | $C = 14,000$, T not less than $\frac{5}{16}$ in. |
| Purves type, rib projections | $C = 14,000$, T not less than $\frac{7}{16}$ in. |
| Brown corrugated type | $C = 14,000$, T not less than $\frac{5}{16}$ in. |
| Type having sections 18 ins. long | $C = 10,000$, T not less than $\frac{7}{16}$ in. |

Limiting dimensions from center to center of the corrugations or projecting ribs, and of their depth, are given for each furnace.

Tubes. — Lap-welded tubes are allowed a working pressure of 225 lbs. per sq. in., if of the thicknesses given below, "provided they are deemed safe by the inspectors."

1 and $1\frac{1}{4}$ ins. diam., 0.072 in. thick; $1\frac{1}{2}$ ins., 0.083; $1\frac{3}{4}$, 2 and $2\frac{1}{4}$ ins., 0.095; $2\frac{1}{2}$, $2\frac{3}{4}$ and 3 ins., 0.109; $3\frac{1}{4}$, $3\frac{1}{2}$ and $3\frac{3}{4}$ ins., 0.120; 4 and $4\frac{1}{2}$ ins., 0.134; 5 ins., 0.148; 6 ins., 0.165.

Safe Working Pressure in Cylindrical Shells. — The author desires to express his condemnation of the rule of the U. S. Statutes, as giving too low a factor of safety. (See also criticism by Mr. Foley, page 880, *ante*.)

If P_b = bursting-pressure, t = thickness, T = tensile strength, c = coefficient of strength of riveted joint, that is, ratio of strength of the joint to that of the solid plate, d = diameter, $P_b = 2tTc \div d$, or if c be taken for double-riveting at 0.7, then $P_b = 1.4tT \div d$.

By the U. S. rule the allowable pressure $P_a = \frac{1/6tT}{1/2d} \times 1.20 = \frac{0.4tT}{d}$; whence $P_b = 3.5P_a$; that is, the factor of safety is only 3.5, provided the "tensile strength found stamped in the plate" is the real tensile strength of the material.

The author's formula for safe working-pressure of externally fired boilers with longitudinal seams double-riveted, is $P = \frac{14,000t}{d}$; $t = \frac{Pd}{14,000}$; P = gauge-pressure in lbs. per sq. in.; t = thickness and d = diam. in inches.

This is derived from the formula $P = \frac{2tTc}{fd}$, taking c at 0.7 and $f = 5$ for steel of 50,000 lbs. T.S., or 6 for 60,000 lbs. T.S.; the factor of safety being increased in the ratio of the T.S., since with the higher T.S. there is greater danger of cracking at the rivet-holes from the effect of punching and riveting and of expansion and contraction caused by variations of temperature. For external shells of internally fired boilers, these shells not being exposed to the fire, with rivet-holes drilled or reamed after punching, a lower factor of safety and steel of a higher T.S. may be allowable.

If the T.S. is 60,000, a working pressure $P = 16,000t \div d$ would give a factor of safety of 5.25.

The following table gives safe working pressures for different diameters of shell and thicknesses of plate calculated from the author's formula.

Safe Working Pressures in Cylindrical Shells of Boilers, Tanks, Pipes, etc., in Pounds per Square Inch.

Longitudinal seams double-riveted.

(Calculated from formula $P = 14,000 \times \text{thickness} \div \text{diameter}$.)

| Thickness in 16ths of an Inch. | Diameter in Inches. | | | | | | | | | | |
|--------------------------------------|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 24 | 30 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 |
| 1 | 36.5 | 29.2 | 24.3 | 23.0 | 21.9 | 20.8 | 19.9 | 19.0 | 18.2 | 17.5 | 16.8 |
| 2 | 72.9 | 58.3 | 48.6 | 46.1 | 43.8 | 41.7 | 39.8 | 38.0 | 36.5 | 35.0 | 33.7 |
| 3 | 109.4 | 87.5 | 72.9 | 69.1 | 65.6 | 62.5 | 59.7 | 57.1 | 54.7 | 52.5 | 50.5 |
| 4 | 145.8 | 116.7 | 97.2 | 92.1 | 87.5 | 83.3 | 79.5 | 76.1 | 72.9 | 70.0 | 67.3 |
| 5 | 182.3 | 145.8 | 121.5 | 115.1 | 109.4 | 104.2 | 99.4 | 95.1 | 91.1 | 87.5 | 84.1 |
| 6 | 218.7 | 175.0 | 145.8 | 138.2 | 131.3 | 125.0 | 119.3 | 114.1 | 109.4 | 105.0 | 101.0 |
| 7 | 255.2 | 204.1 | 170.1 | 161.2 | 153.1 | 145.9 | 139.2 | 133.2 | 127.6 | 122.5 | 117.8 |
| 8 | 291.7 | 233.3 | 194.4 | 184.2 | 175.0 | 166.7 | 159.1 | 152.2 | 145.8 | 140.0 | 134.6 |
| 9 | 328.1 | 262.5 | 218.8 | 207.2 | 196.9 | 187.5 | 179.0 | 171.2 | 164.1 | 157.5 | 151.4 |
| 10 | 364.6 | 291.7 | 243.1 | 230.3 | 218.8 | 208.3 | 198.9 | 190.2 | 182.3 | 175.0 | 168.3 |
| 11 | 401.0 | 320.8 | 267.4 | 253.3 | 240.6 | 229.2 | 218.7 | 209.2 | 200.5 | 192.5 | 185.1 |
| 12 | 437.5 | 350.0 | 291.7 | 276.3 | 262.5 | 250.0 | 238.6 | 228.3 | 218.7 | 210.0 | 201.9 |
| 13 | 473.9 | 379.2 | 316.0 | 299.3 | 284.4 | 270.9 | 258.5 | 247.3 | 237.0 | 227.5 | 218.8 |
| 14 | 510.4 | 408.3 | 340.3 | 322.4 | 306.3 | 291.7 | 278.4 | 266.3 | 255.2 | 245.0 | 235.6 |
| 15 | 546.9 | 437.5 | 364.6 | 345.4 | 328.1 | 312.5 | 298.3 | 285.3 | 273.4 | 266.5 | 252.4 |
| 16 | 583.3 | 466.7 | 388.9 | 368.4 | 350.0 | 333.3 | 318.2 | 304.4 | 291.7 | 280.0 | 269.2 |

Safe Working Pressures in Cylindrical Shells — *Continued.*

| Thickness in 16ths of an Inch. | Diameter in Inches. | | | | | | | | | | | |
|--------------------------------------|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 54 | 60 | 66 | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 |
| 1 | 16.2 | 14.6 | 13.3 | 12.2 | 11.2 | 10.4 | 9.7 | 9.1 | 8.6 | 8.1 | 7.7 | 7.3 |
| 2 | 32.4 | 29.2 | 26.5 | 24.3 | 22.4 | 20.8 | 19.4 | 18.2 | 17.2 | 16.2 | 15.4 | 14.6 |
| 3 | 48.6 | 43.7 | 39.8 | 36.5 | 33.7 | 31.3 | 29.2 | 27.3 | 25.7 | 24.3 | 23.0 | 21.9 |
| 4 | 64.8 | 58.3 | 53.0 | 48.6 | 44.9 | 41.7 | 38.9 | 36.5 | 34.3 | 32.4 | 30.7 | 29.2 |
| 5 | 81.0 | 72.9 | 66.3 | 60.8 | 56.1 | 52.1 | 48.6 | 45.6 | 42.9 | 40.5 | 38.4 | 36.5 |
| 6 | 97.2 | 87.5 | 79.5 | 72.9 | 67.3 | 62.5 | 58.3 | 54.7 | 51.5 | 48.6 | 46.1 | 43.8 |
| 7 | 113.4 | 102.1 | 92.8 | 85.1 | 78.5 | 72.9 | 68.1 | 63.8 | 60.0 | 56.7 | 53.7 | 51.0 |
| 8 | 129.6 | 116.7 | 106.1 | 97.2 | 89.7 | 83.3 | 77.8 | 72.9 | 68.6 | 64.8 | 61.4 | 58.3 |
| 9 | 145.8 | 131.2 | 119.3 | 109.4 | 101.0 | 93.8 | 87.5 | 82.0 | 77.2 | 72.9 | 69.1 | 65.6 |
| 10 | 162.0 | 145.8 | 132.6 | 121.5 | 112.3 | 104.2 | 97.2 | 91.1 | 85.8 | 81.0 | 76.8 | 72.9 |
| 11 | 178.2 | 160.4 | 145.8 | 133.7 | 123.4 | 114.6 | 106.9 | 100.3 | 94.4 | 89.1 | 84.4 | 80.2 |
| 12 | 194.4 | 175.0 | 159.1 | 145.8 | 134.6 | 125.0 | 116.7 | 109.4 | 102.9 | 97.2 | 92.1 | 87.5 |
| 13 | 210.7 | 189.6 | 172.4 | 158.0 | 145.8 | 135.4 | 126.4 | 118.5 | 111.5 | 105.3 | 99.8 | 94.8 |
| 14 | 226.9 | 204.2 | 185.6 | 170.1 | 157.1 | 145.8 | 136.1 | 127.6 | 120.1 | 113.4 | 107.5 | 102.1 |
| 15 | 243.1 | 218.7 | 198.9 | 182.3 | 168.3 | 156.3 | 145.8 | 136.7 | 128.7 | 121.5 | 115.1 | 109.4 |
| 16 | 259.3 | 233.3 | 212.1 | 194.4 | 179.5 | 166.7 | 155.6 | 145.8 | 137.3 | 129.6 | 122.8 | 116.7 |

Flat Stayed Surfaces in Steam-boilers. — Clark, in his treatise on the Steam-engine, also in his Pocket-book, gives the following formula: $p = 407ts \div d$, in which p is the internal pressure in pounds per square inch that will strain the plates to their elastic limit, t is the thickness of the plate in inches, d is the distance between two rows of stay-bolts in the clear, and s is the tensile stress in the plate, in tons of 2240 lbs., per square inch, at the elastic limit. Substituting values of s for iron, steel, and copper, 12, 14, and 8 tons respectively, we have the following:

FORMULÆ FOR ULTIMATE ELASTIC STRENGTH OF FLAT STAYED SURFACES.

| | Iron. | Steel. | Copper. |
|-------------------------|-------------------------------|-------------------------------|-------------------------------|
| Pressure..... | $p = 5000 \frac{t}{d}$ | $p = 5700 \frac{t}{d}$ | $p = 3300 \frac{t}{d}$ |
| Thickness of plate..... | $t = \frac{p \times d}{5000}$ | $t = \frac{p \times d}{5700}$ | $t = \frac{p \times d}{3300}$ |
| Pitch of bolts..... | $d = \frac{5000t}{p}$ | $d = \frac{5700t}{p}$ | $d = \frac{3300t}{p}$ |

For Diameter of the Stay-bolts, Clark gives $d' = 0.0024 \sqrt{\frac{PP'p}{s}}$,

in which d' = diameter of screwed bolt at bottom of thread, P = longitudinal and P' transverse pitch of stay-bolts between centers, p = internal pressure in lbs. per sq. in. that will strain the plate to its elastic limit, s = elastic strength of the stay-bolts, in lbs. per sq. in. Taking s = 12, 14, and 8 tons, respectively, for iron, steel, and copper, we have

For iron, $d' = 0.00069 \sqrt{PP'p}$, or if $P = P'$, $d' = 0.00069 P \sqrt{p}$.

For steel, $d' = 0.00064 \sqrt{PP'p}$, or if $P = P'$, $d' = 0.00064 P \sqrt{p}$;

For copper, $d' = 0.00084 \sqrt{PP'p}$, or if $P = P'$, $d' = 0.00084 P \sqrt{p}$.

In using formulæ for stays a large factor of safety should be taken to allow for reduction of size by corrosion. Thurston's Manual of Steam-boilers, p. 144, recommends that the factor be as large as 15 or 20. The Hartford Steam Boiler Insp. & Ins. Co. recommends not less than 10.

Strength of Stays. — A. F. Yarrow (*Engr.*, March 20, 1891) gives the following results of experiments to ascertain the strength of water-space stays:

| Description. | Length between Plates. | Diameter of Stay over Threads. | Ulti- mate Stress. |
|---|------------------------------|--|--------------------------|
| | | | lbs. |
| Hollow stays screwed into plates and hole expanded } | 4.75 in. | 1 in. (hole $7\frac{1}{16}$ in. and $5\frac{1}{16}$ in.) | 25,457 |
| | 4.64 in. | 1 in. (hole $9\frac{1}{16}$ in. and $7\frac{1}{16}$ in.) | 20,992 |
| Solid stays screwed into plates and riveted over. } | 4.80 in. | $7\frac{1}{8}$ in. | 22,008 |
| | 4.80 in. | $7\frac{1}{8}$ in. | 22,070 |

The above are taken as a fair average of numerous tests.

Fusible plugs. — Fusible plugs should be put in that portion of the heating-surface which first becomes exposed from lack of water. The rules of the U. S. Supervising Inspectors specify Banca tin for the purpose. Its melting-point is about 445° F. The rule says: Every boiler, other than boilers of the water-tube type, shall have at least one fusible plug made of a bronze casing filled with good Banca tin from end to end. Fusible plugs, except as otherwise provided for, shall have an external diameter of not less than $\frac{3}{4}$ in. pipe tap, and the Banca tin shall be at least $\frac{1}{2}$ in. in diameter at the smallest end and shall have a larger diameter at the center or at the opposite end of the plug; smaller plugs are allowed for pressures above 150 lbs., also for upright boilers. Cylinder-boilers with flues shall have one plug inserted in one flue of each boiler; and also one plug inserted in the shell of each boiler from the inside, immediately below the fire line and not less than 4 ft. from the front end. Other shell boilers shall have one plug inserted in the crown of the back connection. Upright tubular boilers shall have a fusible plug inserted in one of the tubes at a point at least 2 in. below the lowest gauge-cock, but in boilers having a cone top it shall be inserted in the upper tube sheet. All tubes are to be inserted so that the small end of the tin shall be exposed to the fire.

Steam-domes. — Steam-domes or drums were formerly almost universally used on horizontal boilers, but their use is now generally discontinued, as they are considered a useless appendage to a steam-boiler, and unless properly designed and constructed are an element of weakness.

Height of Furnace. — Recent practice in the United States makes the height of furnace much greater than it was formerly. With large sizes of anthracite there is no serious objection to having the furnace as low as 18 in., measured from the surface of the grate to the nearest portion of the heating surface of the boiler, but with coal containing much volatile matter and moisture a much greater distance is desirable. With very volatile coals the distance may be as great as 5 ft. or even 10 ft. Rankine (*S. E.*, p. 457) says: The clear height of the "crown" or roof of the furnace above the grate-bars is seldom less than about 18 in., and often considerably more. In the fire-boxes of locomotives it is on an average about 4 ft. The height of 18 in. is suitable where the crown of the furnace is a brick arch. Where the crown of the furnace, on the other hand, forms part of the heating-surface of the boiler, a greater height is desirable in every case in which it can be obtained: for the temperature of the boiler-plates, being much lower than that of the flame, tends to check the combustion of the inflammable gases which rise from the fuel. As a general principle a high furnace is favorable to complete combustion.

IMPROVED METHODS OF FEEDING COAL.

Mechanical Stokers. (William R. Roney, *Trans. A. S. M. E.*, vol. xii.) — Mechanical stokers have been used in England to a limited extent since 1785. In that year one was patented by James Watt. (See D. K. Clark's *Treatise on the Steam-engine*.)

After 1840 many styles of mechanical stokers were patented in England, but nearly all were variations and modifications of the two forms of stokers patented by John Jukes in 1841, and by E. Henderson in 1843.

The Jukes stoker consisted of longitudinal fire-bars, connected by links, so as to form an endless chain. The small coal was delivered from a hopper on the front of the boiler, on to the grate, which slowly moving

from front to rear, gradually advanced the fuel into the furnace and discharged the ash and clinker at the back.

The Henderson stoker consists primarily of two horizontal fans revolving on vertical spindles, which scatter the coal over the fire.

The first American stoker was the Murphy stoker, brought out in 1878. It consists of two coal magazines placed in the side walls of the boiler furnace, and extending back from the boiler front 6 or 7 feet. In the bottom of these magazines are rectangular iron boxes, which are moved from side to side by means of a rack and pinion, and serve to push the coal upon the grates, which incline at an angle of about 35° from the inner edge of the coal magazines, forming a V-shaped receptacle for the burning coal. The grates are composed of narrow parallel bars, so arranged that each alternate bar lifts about an inch at the lower end, while at the bottom of the V, and filling the space between the ends of the grate-bars, is placed a cast-iron toothed bar, arranged to be turned by a crank. The purpose of this bar is to grind the clinker coming in contact with it. Over this V-shaped receptacle is sprung a fire-brick arch.

In the Roney mechanical stoker the fuel to be burned is dumped into a hopper on the boiler front. Set in the lower part of the hopper is a "pusher" which, by a vibratory motion, gradually forces the fuel over the "dead-plate" and on the grate. The grate-bars in their normal condition form a series of steps. Each bar is capable of a rocking motion through an adjustable angle. All the grate-bars are coupled together by a "rocker-bar." A variable back-and-forth motion being given to the "rocker-bar," through a connecting-rod, the grate-bars rock in unison, now forming a series of steps, and now approximating to an inclined plane, with the grates partly overlapping, like shingles on a roof. When the grate-bars rock forward the fire will tend to work down in a body. But before the coal can move too far the bars rock back to the stepped position, checking the downward motion. The rocking motion is slow, being from 7 to 10 strokes per minute, according to the kind of coal. This alternate starting and checking motion is continuous, and finally lands the cinder and ash on the dumping-grate below.

The Hawley Down-draught Furnace. — A foot or more above the ordinary grate there is carried a second grate composed of a series of water-tubes, opening at both ends into steel drums or headers, through which water is circulated. The coal is fed on this upper grate, and as it is partially consumed falls through it upon the lower grate, where the combustion is completed in the ordinary manner. The draught through the coal on the upper grate is downward through the coal and the grate. The volatile gases are therefore carried down through the bed of coal, where they are thoroughly heated, and are burned in the space beneath, where they meet the excess of hot air drawn through the fire on the lower grate. In tests in Chicago, from 30 to 45 lbs. of coal were burned per square foot of grate upon this system, with good economical results. (See catalogue of the Hawley Down-draught Furnace Co., Chicago.)

The Chain Grate Stoker, made by Jukes in 1841, is now (1909) widely used in the United States. It is made by the Babcock & Wilcox Co. and others.

Under-feed Stokers. — Results similar to those that may be obtained with downward draught are obtained by feeding the coal at the bottom of the bed, pushing upward the coal already on the bed which has had its volatile matter distilled from it. The volatile matter of the freshly fired coal then has to pass through a body of ignited coke, where it meets a supply of hot air. (See circular of The Underfeed Stoker Co., Chicago.)

The Taylor Gravity Stoker, made by the Amer. Ship Windlass Co., Providence, R. I., is a combination of an underfeed stoker containing two horizontal rows of pushers with an inclined or step grate through which air is blown by a fan.

SMOKE PREVENTION.

The following article was contributed by the author to a "Report on Smoke Abatement," presented by a Committee to the Syracuse Chamber of Commerce, published by the Chamber in 1907.

Smoke may be made in two ways: (1) By direct distillation of tarry condensable vapors from coal without burning; (2) By the partial burning or splitting up of hydrocarbon gases, the hydrogen burning and the

carbon being left unburned as smoke or soot. These causes usually act conjointly.

The direct cause of smoke is that the gases distilled from the coal are not completely burned in the furnace before coming in contact with the surface of the boiler, which chills them below the temperature of ignition.

The amount and quality of smoke discharged from a chimney may vary all the way from a dense cloud of jet-black smoke, which may be carried by a light wind for a distance of a mile or more before it is finally dispersed into the atmosphere, to a thin cloud, which becomes invisible a few feet from the chimney. Often the same chimney will for a few minutes immediately after firing give off a dense black cloud and then a few minutes later the smoke will have entirely disappeared.

The quantity and density of smoke depend upon many variable causes. Anthracite coal produces no smoke under any conditions of furnace. Semi-bituminous, containing 12.5 to 25% of volatile matter in the combustible part of the coal, will give off more or less smoke, depending on the conditions under which it is burned, and bituminous coal, containing from 25 to 50% of volatile matter, will give off great quantities of smoke with all of the usual old-style furnaces, even with skillful firing, and this smoke can only be prevented by the use of special devices, together with proper methods of firing the fuel and of admission of air.

Practically the whole theory of smoke production and prevention may be illustrated by the flame of an ordinary gas burner or gas stove. When the gas is turned down very low every particle of gas, as it emerges from the burner, is brought in contact with a sufficient supply of hot air to effect its complete and instantaneous combustion, with a pale blue or almost invisible flame. Turn on the gas a little more and a white flame appears. The gas is imperfectly burned in the center of the flame. Particles of carbon have been separated which are heated to a white heat. If a cold plate is brought in contact with the white flame, these carbon particles are deposited as soot. Turn on the gas still higher, and it burns with a dull, smoky flame, although it is surrounded with an unlimited quantity of air. Now, carry this smoky flame into a hot fire-brick or porcelain chamber, where it is brought in contact with very hot air, and it will be made smokeless by the complete burning of the particles.

We thus see: (1) That smoke may be prevented from forming if each particle of gas, as it is made by distillation from coal, is immediately mixed thoroughly with hot air, and (2) That even if smoke is formed by the absence of conditions for preventing it, it may afterwards be burned if it is thoroughly mixed with air at a sufficiently high temperature. It is easy to burn smoke when it is made in small quantities, but when made in great volumes it is difficult to get the hot air mixed with it unless special apparatus is used. In boiler firing the formation of smoke must be prevented, as the conditions do not usually permit of its being burned. The essential conditions for preventing smoke in boiler fires may be enumerated as follows:

1. The gases must be distilled from the coal at a uniform rate.
2. The gases, when distilled, must be brought into intimate mixture with sufficient hot air to burn them completely.
3. The mixing should be done in a fire-brick chamber.
4. The gases should not be allowed to touch the comparatively cold surfaces of the boiler until they are completely burned. This means that the gases shall have sufficient space and time in which to burn before they are allowed to come in contact with the boiler surface.

Every one of these four conditions is violated in the ordinary method of burning coal under a steam boiler. (1) The coal is fired intermittently and often in large quantities at a time, and the distillation proceeds at so rapid a rate that enough air cannot be introduced into the furnace to burn the gas. (2) The piling of fresh coal on the grate in itself chokes the air supply. (3) The roof of the furnace is the cold shell, or tubes, of the boiler, instead of a fire-brick arch, as it should be, and the furnace is not of a sufficient size to allow the gases time and space in which to be thoroughly mixed with the air supply.

In order to obtain the conditions for preventing smoke it is necessary: (1) That the coal be delivered into the furnace in small quantities at a time. (2) That the draught be sufficient to carry enough air into the furnace to burn the gases as fast as they are distilled. (3) That the air

itself be thoroughly heated either by passing through a bed of white-hot coke or by passing through channels in hot brickwork, or by contact with hot fire-brick surfaces. (4) That the gas and the air be brought into the most complete and intimate mixture, so that each particle of carbon in the gas meets, before it escapes from the furnace, its necessary supply of air. (5) That the flame produced by the burning shall be completely extinguished by the burning of every particle of the carbon into invisible carbon dioxide.

If a white flame touches the surface of a boiler, it is apt to deposit soot and to produce smoke. A white flame itself is the visible evidence of incomplete combustion.

The first remedy for smoke is to obtain anthracite coal. If this is not commercially practicable, then obtain, if possible, coal with the smallest amount of volatile matter. Coal of from 15 to 25% of volatile matter makes much less smoke than coals containing higher percentages. Provide a proper furnace for burning coal. Any furnace is a proper furnace which secures the conditions named in the preceding paragraphs. Next, compel the firemen to follow instructions concerning the method of firing.

It is impossible with coal containing over 30% of volatile matter and with a water-tube boiler, with tubes set close to the grate and vertical gas passages, as in an anthracite setting, to prevent smoke even by the most skillful firing. This style of setting for a water-tube boiler should be absolutely condemned. A Dutch oven setting, or a longitudinal setting with fire-brick baffle walls, is highly recommended as a smoke-preventing furnace, but with such a furnace it is necessary to use considerable skill in firing.

Mechanical mixing of the gases and the air by steam jets is sometimes successful in preventing smoke, but it is not a universal preventive, especially when the coal is very high in volatile matter, when the firing is done unskillfully, or when the boiler is being driven beyond its normal capacity. It is essential to have sufficient draught to burn the coal properly and this draught may be obtained either from a chimney or a fan. There is no especial merit in forced draught, except that it enables a larger quantity of coal to be burned and the boiler to be driven harder in case of emergency, and usually the harder the boiler is driven, the more difficult it is to suppress smoke.

Down-draught furnaces and mechanical stokers of many different kinds are successfully used for smoke prevention, and when properly designed and installed and handled skillfully, and usually at a rate not beyond that for which they are designed, prevent all smoke. If these appliances are found giving smoke, it is always due either to overdriving or to unskillful handling. It is necessary, however, that the design of these stokers be suited to the quality of the coal and the quantity to be burned, and great care should be taken to provide a sufficient size of furnace with a fire-brick roof and means of introducing air to make them completely successful.

Burning Illinois Coal without Smoke. (L. P. Breckenridge, Bulletin No. 15 of the Univ. of Ill. Eng'g Experiment Station, 1907.) — Any fuel may be burned economically and without smoke if it is mixed with the proper amount of air at a proper temperature. The boiler plant of the University of Illinois consists of nine units aggregating 2000 H.P. Over 200 separate tests have been made. The following is a condensed statement of the results in regard to smoke prevention.

Boilers Nos. 1 and 2. Babcock & Wilcox. Chain-grate stoker. Usual vertical baffling. Can be run without smoke at from 50 to 120% of rated capacity.

No. 3. Stirling boiler. Chain-grate stoker. Usual baffling and combustion arches. Can be run without smoke at capacities of 50 to 140%.

No. 4. National water-tube. Chain-grate stoker. Vertical baffling. No smoke at capacities of 50 to 120%. With the Murphy furnace it was smokeless except when cleaning fires.

No. 5. Babcock & Wilcox. Roney stoker. Vertical baffling. Nearly smokeless (maximum No. 2 on a chart in which 5 represents black smoke) up to 100% of rating, but cannot be run above 100% without objectionable smoke.

No. 6. Babcock & Wilcox. Roney stoker. Horizontal tile-roof baffling. Can be run without smoke at capacities of 50 to 100% of rating.

Nos. 7 and 8. Stirling, equipped with Stirling bar-grate stoker. Usual baffling and combustion arches. Can be run without smoke at 50 to 140% of rating.

No. 9. Heine boiler. Chain-grate stoker. Combustion arch and tile-roof furnace. Can be run without smoke at capacities of 50 to 140%. It is almost impossible to make smoke with this setting under any condition of operation. As much as 46 lbs. of coal per sq. ft. of grate surface has been burned without smoke.

Conditions of Smoke Prevention. — Bulletin No. 373 of the U. S. Geological Survey, 1909 (188 pages), contains a report of an extensive research by D. T. Randall and J. T. Weeks on The Smokeless Combustion of Coal in Boiler Plants. A brief summary of the conclusions reached is as follows:

Smoke prevention is both possible and economical. There are many types of furnaces and stokers that are operated smokelessly.

Stokers or furnaces must be set so that combustion will be complete before the gases strike the heating surfaces of the boiler. When partly burned gases at a temperature of say 2500° F. strike the tubes of a boiler at say 350° F., combustion may be entirely arrested.

The most economical hand-fired plants are those that approach most nearly to the continuous feed of the mechanical stoker. The fireman is so variable a factor that the ultimate solution of the problem depends on the mechanical stoker — in other words, the personal element must be eliminated.

A well designed and operated furnace will burn many coals without smoke up to a certain number of pounds per hour, the rate varying with different coals. If more than this amount is burned, the efficiency will decrease and smoke will be made, owing to the lack of furnace capacity to supply air and mix gases.

High volatile matter in the coal gives low efficiency, and vice versa. When the furnace was forced the efficiency decreased.

With a hand-fired furnace the best results were obtained when firing was done most frequently, with the smallest charge.

Small sizes of coal burned with less smoke than large sizes, but developed lower capacities.

Peat, lignite, and sub-bituminous coal burned readily in the tile-roofed furnace and developed the rated capacity, with practically no smoke.

Coals which smoked badly gave efficiencies three to five per cent lower than the coals burning with little smoke.

Briquets were found to be an excellent form for using slack coal in a hand-fired plant.

In the average hand-fired furnace washed coal burns with lower efficiency and makes more smoke than raw coal. Moreover, washed coal offers a means of running at high capacity, with good efficiency, in a well-designed furnace.

Forced draught did not burn coal any more efficiently than natural draught. It supplied enough air for high rates of combustion, but as the capacity of the boiler increased, the efficiency decreased and the percentage of black smoke increased.

Fire-brick furnaces of sufficient length and a continuous, or nearly continuous, supply of coal and air to the fire make it possible to burn most coals efficiently and without smoke.

Coals containing a large percentage of tar and heavy hydrocarbons are difficult to burn without smoke and require special furnaces and more than ordinary care in firing.

FORCED COMBUSTION IN STEAM-BOILERS.

For the purpose of increasing the amount of steam that can be generated by a boiler of a given size, forced draught is of great importance. It is universally used in the locomotive, the draught being obtained by a steam-jet in the smoke-stack. It is now largely used in ocean steamers, especially in ships of war, and to a small extent in stationary boilers. Economy of fuel is generally not attained by its use, its advantages being confined to the securing of increased capacity from a boiler of a given bulk, weight, or cost.

There are three different modes of using the fan for promoting combustion: 1, blowing direct into a closed ash-pit; 2, exhausting the gases by the suction of the fan; 3, forcing air into an air-tight boiler-room or stoke-hold. Each of these three methods has its advantages and disadvantages.

In the use of the closed ash-pit the blast-pressure frequently forces the gases of combustion from the joint around the furnace doors in so great a quantity as to affect both the efficiency of the boiler and the health of the firemen.

The chief defect of the second plan is the great size of the fan required to produce the necessary exhaustion, on account of the higher exit temperature enlarging the volume of the waste gases.

The third method, that of forcing cold air by the fan into an air-tight boiler-room — the closed stoke-hold system — though it overcame the difficulties in working belonging to the two forms first tried, has serious defects of its own, as it cannot be worked, even with modern high-class boiler-construction, much, if at all, above the power of a good chimney draught, in most boilers, without damaging them. (J. Howden, Proc. Eng'g Congress at Chicago, in 1893.)

In 1880 Mr. Howden designed an arrangement intended to overcome the defects of both the closed ash-pit and the closed stoke-hold systems.

An air-tight chamber is placed on the front end of the boiler and surrounding the furnaces. This reservoir, which projects from 8 to 10 inches from the end of the boiler, receives the air under pressure, which is passed by valves into the ash-pits and over the fires in proportions suited to the kind of fuel and the rate of combustion. The air used above the fires is admitted to a space between the outer and inner furnace-doors, the inner having perforations and an air-distributing box through which the air passes under pressure. By means of the balance of pressure above and below the fires all tendency of the fire to blow out at the door is removed.

A feature of the system is the combination of the heating of the air of combustion by the waste gases with the controlled and regulated admission of air to the furnaces. This arrangement is effected most conveniently by passing the hot fire-gases after they leave the boiler through stacks of vertical tubes enclosed in the uptake, their lower ends being immediately above the smoke-box doors. Installations on Howden's system have been arranged for a rate of combustion to give an average of from 18 to 22 I.H.P. per square foot of fire-grate with fire-bars from 5 to 5½ ft. in length. It is believed that with suitable arrangement of proportions even 30 I.H.P. per square foot can be obtained.

For an account of uses of exhaust-fans for increasing draught, see paper by W. R. Roney, *Trans. A. S. M. E.*, vol. xv.

FUEL ECONOMIZERS.

Economizers for boiler plants are usually made of vertical cast-iron tubes contained in a long rectangular chamber of brickwork. The feed-water enters the bank of tubes at one end, while the hot gases enter the chamber at the other end and travel in the opposite direction to the water. The tubes are made of cast iron because it is more non-corrosive than wrought iron or steel when exposed to gases of combustion at low temperatures. An automatic scraping device is usually provided for the purpose of removing dust from the outer surface of the tubes.

The amount of saving of fuel that may be made by an economizer varies greatly according to the conditions of operation. With a given quantity of chimney gases to be passed through it, its economy will be greater

(1) the higher the temperature of these gases; (2) the lower the temperature of the water fed into it; and (3) the greater the amount of its heating surface. From (1) it is seen that an economizer will save more fuel if added to a boiler that is overdriven than if added to one driven at a nominal rate. From (2) it appears that less saving can be expected from an economizer in a power plant in which the feed-water is heated by exhaust steam from auxiliary engines than when the feed-water entering it is taken directly from the condenser hot-well. The amount of heating surface that should be used in any given case depends not only on the saving of fuel that may be made, but also on the cost of coal, and on the annual costs of maintenance, including interest, depreciation, etc.

The following table shows the theoretical results possibly attainable from economizers under the conditions specified. It is assumed that the coal has a heating value of 15,000 B.T.U. per lb. of combustible; that it is completely burned in the furnace at a temperature of 2500° F.; that the boiler gives efficiencies ranging from 60 to 75% according to the rate of driving; and that sufficient economizer surface is provided to reduce the temperature of the gases in all cases to 300° F. Assuming the specific heat of the gases to be constant, and neglecting the loss of heat by radiation, the temperature of the gases leaving the boiler and entering the economizer is directly proportional to $(100 - \% \text{ of boiler efficiency})$, and the combined efficiency of boiler and economizer is $(2500 - 300) \div 2500 = 88\%$, which corresponds to an evaporation of $(15,000 \div 970) \times 0.88 = 13.608$ lbs. from and at 212° per lb. of combustible; or assuming the feed-water enters the economizer at 100° F. and the boiler makes steam of 150 lbs. absolute pressure, to an evaporation of 11.729 lbs. under these conditions. Dividing this figure into the number of heat units utilized by the economizer per lb. of combustible gives the heat units added to the water, from which, by reference to a steam table, the temperature may be found. With these data we obtain the results given in the table below.

| Boiler Efficiency, %. | 60 | 65 | 70 | 75 |
|--|-------|--------|--------|--------|
| B.T.U. absorbed by boiler per lb. combustible..... | 9000 | 9750 | 10500 | 11250 |
| B.T.U. in chimney gases leaving boiler..... | 6030 | 5250 | 4500 | 3750 |
| Estimated temp. of gases leaving boiler..... | 1000° | 875° | 750° | 625° |
| Estimated temp. of gases leaving economizer..... | 300° | 300° | 300° | 300° |
| B.T.U. saved by economizer..... | 4200 | 3450 | 2700 | 1950 |
| Efficiency gained by economizer, %..... | 28 | 23 | 18 | 13 |
| Equivalent water evap. per lb. comb. in boiler..... | 9.278 | 10.051 | 10.824 | 11.598 |
| B.T.U. saved by econ. equivalent to evap. of lbs.... | 4.330 | 3.557 | 2.884 | 2.010 |
| Temp. of water leaving economizer..... | 448° | 389° | 327° | 265° |
| Efficiency of the economizer, %..... | 70 | 65.7 | 60 | 52 |

Amount of Heating Surface. — The Fuel Economizer Co. says: We have found in practice that by allowing 4 sq. ft. of heating surface per boiler H.P. (34½ lbs. evap. from and at 212° = 1 H.P.) we are able to raise the feed-water 60° F. for every 100° reduction in the temperature, the gases entering the economizer at 450° to 600°. With gases at 600° to 700° we have allowed a heating surface of 4½ to 5 sq. ft. per H.P., and for every 100° reduction in temperature of the gases we have obtained about 65° rise in temperature of the water; the feed-water entering at 60 to 120°. With 5000 sq. ft. of boiler-heating surface (plain cylinder boilers) developing 1000 H.P. we should recommend 5 sq. ft. of economizer surface per boiler H.P. developed, or an economizer of about 500 tubes, and it should heat the feed-water about 300°.

Heat Transmission in Economizers. (Carl S. Dow, *Indust. Eng'g*, April, 1909.) — The rate of heat transmission (*C*) per sq. ft. per hour per degree of difference between the average temperatures of the gases and the water passing through the economizer varies with the mean temperature of the gas about as follows: Gas, 600°, *C* = 3.25; gas 500°, *C* = 3; gas 400°, *C* = 2.75; gas 300°, *C* = 2.25.

Calculation of the Saving made by an Economizer. — The usual method of calculating the saving of fuel by an economizer when the boiler and the economizer are tested together as a unit is by the formula $(H_1 - h) \div (H_2 - h)$, in which h is the total heat above 32° of 1 lb. of water entering, H_1 the total heat of 1 lb. of water leaving the economizer, and H_2 the total heat above 32° of 1 lb. of steam at the boiler pressure. If $h = 100$, $H_1 = 210$, $H_2 = 1200$, then the saving according to the formula is $(210 - 100) \div 1100 = 10\%$. This is correct if the saving is defined as the ratio of the heat absorbed by the economizer to the total heat absorbed by the boiler and economizer together, but it is not correct if the saving is defined as the saving of fuel made by running the combined unit as compared with running the boiler alone making the same quantity of steam from feed-water at the low temperature, so as to cause the boiler to furnish $H_2 - h$ heat units per lb. instead of $H_2 - H_1$. In this case the boiler is called on to do more work, and in doing it it may be overdriven and work with lower efficiency.

In a test made by F. G. Gasche, in Kansas City in 1897, using Missouri coal analyzing moisture 7.58; volatile matter, 36.69; fixed carbon, 35.02; ash, 15.69; sulphur, 5.12, he obtained an evaporation of 5.17 lbs. from and at 212° per lb. of coal with the boiler alone, and when the boiler and economizer were tested together the equivalent evaporation credited to the boiler was 5.55, to the economizer 0.72, and to the combined unit 6.27, the saving by the combined unit as compared with the boiler alone being $(6.27 - 5.17) \div 6.27 = 17.5\%$, while the saving of heat shown by the economizer in the combined test is only $(6.27 - 5.55) \div 6.27 = 11.5\%$, or as calculated by Mr. Gasche from the formula $(H_1 - h) \div (H_2 - h)$, $(172.1 - 39.3) \div (1181.8 \div 39.3) = 11.6\%$.

The maximum saving of fuel which may be made by the use of an economizer when attached to boilers that are working with reasonable economy is about 15%. Take the case of a condensing engine using steam of 125 lbs. gauge pressure, and with a hot-well or feed-water temperature of 100° F. The economizer may be expected under the best conditions to raise this temperature about 170° , or to 270° . Then $h = 68$, $H_1 = 239$, $H_2 = 1190$. $(H_1 - h) \div (H_2 - h) = 171 \div 15.24\%$.

If the boilers are not working with fair economy on account of being overdriven, then the saving made by the addition of an economizer may be much greater.

Test of a Large Economizer. (R. D. Tomlinson, *Power*, Feb., 1904.) — Two tests were made of one of the sixteen Green economizers at the 74th St. Station of the Rapid Transit Railway, New York City. Four 520-H.P. B. & W. boilers were connected to the economizer. It had 512 tubes, 10 ft. long, $4\frac{9}{16}$ in. external diam.; total heating surface 6760 sq. ft., or 3.25 sq. ft. per rated H.P. of the boilers. Draught area through econ., 3 sq. in. per H.P. The stack for each 16 boilers and four economizers was 280 ft. high, 17 ft. internal diam. The first test was made with the boilers driven at 94% of rating, the second at 113%. The results are given below, the figures of the second test being in parentheses.

Water entering econ. 96° (93.5°); leaving 200° (203.8°); rise 104 (110.3).

Gases entering econ. 548° (603°); leaving 295 (325); drop 253 (278).

Steam, gauge pressure, 166 (165). Total B.T.U. per lb. from feed temp. 1132 (1134).

Saving of heat by economizer, %, 9.17 (9.73).

Reduction of draught in passing through econ., in. of water, 0.16 (0.23).

Results from Seven Tests of Sturtevant Economizers (Catalogue of B. F. Sturtevant Co.)

| Plants Tested. | Gases Entering. Deg. F. | Gases Leaving. Deg. F. | Water Entering. Deg. F. | Water Leaving. Deg. F. | Increase in Temperature. |
|----------------|-------------------------|------------------------|-------------------------|------------------------|--------------------------|
| 1 | 650 | 275 | 180 | 340 | 160 |
| 2 | 575 | 290 | 160 | 320 | 160 |
| 3 | 470 | 230 | 130 | 260 | 130 |
| 4 | 500 | 240 | 110 | 230 | 120 |
| 5 | 460 | 200 | 90 | 230 | 140 |
| 6 | 440 | 220 | 120 | 236 | 116 |
| 7 | 525 | 225 | 180 | 320 | 140 |

THERMAL STORAGE.

In Druitt Halpin's steam storage system (*Industries and Iron*, Mar. 22, 1895) he employs only sufficient boilers to supply the mean demand, and storage tanks sufficient to supply the maximum demand. These latter not being subjected to the fire suffer but little deterioration. The boilers working continuously at their most economical rate have their excess of energy during light load stored up in the water of the tank, from which it may be drawn at will during heavy load. He proposes that the boilers and tanks shall work under a pressure of 265 lbs. per square inch when fully charged, which corresponds to a temperature of 406° F., and that the engines be worked at 130 lbs. per square inch, which corresponds to 347° F. The total available heat stored when the reservoirs are charged is that due to a range of 59°. The falling in temperature of 141¼ lbs. of water from 407° to 347° will yield 1 lb. of steam. To allow for radiation of loss and imperfect working, this may be taken at 16 lbs. of water per pound of steam. The steam consumption per effective H.P. may be taken at 18 lbs. per hour in condensing and 25 lbs. per hour in non-condensing engines. The storage-room per effective H.P. by this method would, therefore, be $(16 \times 18) \div 62.5 = 4.06$ cu. ft. for condensing and $(16 \times 25) \div 62.5 = 6.4$ cu. ft. for non-condensing engines.

Gas storage, assuming that illuminating gas is used, would require about 20 cu. ft. of storage room per effective H.P. hour stored, and if ordinary fuel gas were stored it would require about four times this capacity. In water storage 317 cu. ft. would be required at an elevation of 100 ft. to store one H.P. hour, so that of the three methods of storing energy the thermal method is by far the most economical of space.

In the steam storage method the boiler is completely filled with water and the storage tank nearly so. The two are in free communication by means of pipes, and a constant circulation of water is maintained between the two, but the steam for the engines is taken only from the top of the storage tank through a reducing valve.

In the feed storage system, the excess of energy during light load is stored in the tank as before, but the boilers are not completely filled. In this system the steam is taken exclusively from the boilers, the superheated water of the storage tanks being used during heavy load as feed-water to the boilers.

A third method is a combination of these two. In the "combined" feed and steam storage system the pressure in boiler and storage tank is equalized by connecting the steam spaces in both by pipe, and the steam for the engines is, therefore, taken from both. In other words they work in parallel.

INCRUSTATION AND CORROSION.

Incrustation or Scale. — Incrustation (as distinguished from mere sediments due to dirty water, which are easily blown out, or gathered up, by means of sediment-collectors) is due to the presence of salts in the feed-water (carbonates and sulphates of lime and magnesia for the most part), which are precipitated when the water is heated, and form hard deposits upon the boiler-plates. (See *Impurities in Water*, p. 691, *ante*.)

Where the quantity of these salts is not very large (12 grains per gallon, say) scale preventives may be found effective. The chemical preventives either form with the salts other salts soluble in hot water; or precipitate them in the form of soft mud, which does not adhere to the plates, and can be washed out from time to time. The selection of the chemical must depend upon the composition of the water, and it should be introduced regularly with the feed.

EXAMPLES. — Sulphate-of-lime scale prevented by carbonate of soda: The sulphate of soda produced is soluble in water; and the carbonate of lime falls down in grains, does not adhere to the plates, and may therefore be blown out or gathered into sediment-collectors. The chemical reaction is:

Sulphate of lime + Carbonate of soda = Sulphate of soda + Carbonate of lime

$$\text{CaSO}_4 \quad \text{Na}_2\text{CO}_3 \quad \text{Na}_2\text{SO}_4 \quad \text{CaCO}_3$$

Where the quantity of salts is large, scale preventives are not of much use. Some other source of supply must be sought, or the bad water

purified before it is allowed to enter the boilers. The damage done to boilers by unsuitable water is enormous.

Pure water may be obtained by collecting rain, or condensing steam by means of surface condensers. The water thus obtained should be mixed with a little bad water, or treated with a little alkali, as undiluted, pure water corrodes iron; or, after each periodic cleaning, the bad water may be used for a day or two to put a skin upon the plates.

Carbonate of lime and magnesia may be precipitated either by heating the water or by mixing milk of lime (Porter-Clark process) with it, the water being then filtered.

Corrosion may be produced by the use of pure water, or by the presence of acids in the water, caused perhaps in the engine-cylinder by the action of high-pressure steam upon the grease, resulting in the production of fatty acids. Acid water may be neutralized by the addition of lime.

Amount of Sediment which may collect in a 100-H.P. steam-boiler, evaporating 3000 lbs. of water per hour, the water containing different amounts of impurity in solution, provided that no water is blown off:

Grains of solid impurities per U. S. gallon:

| 5 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---------------------------------------|-------|-------|-------|-------|-------|--------|-------|-------|-------|-------|
| Equivalent parts per 100,000: | | | | | | | | | | |
| 8.57 | 17.14 | 34.28 | 51.42 | 68.56 | 85.71 | 102.85 | 120 | 137.1 | 154.3 | 171.4 |
| Sediment deposited in 1 hour, pounds: | | | | | | | | | | |
| 0.257 | 0.514 | 1.028 | 1.542 | 2.056 | 2.571 | 3.085 | 3.6 | 4.11 | 4.63 | 5.14 |
| In one day of 10 hours, pounds: | | | | | | | | | | |
| 2.57 | 5.14 | 10.28 | 15.42 | 20.56 | 25.71 | 30.85 | 36.0 | 41.1 | 46.3 | 51.4 |
| In one week of 6 days, pounds: | | | | | | | | | | |
| 15.43 | 30.85 | 61.7 | 92.55 | 123.4 | 154.3 | 185.1 | 216.0 | 246.8 | 277.6 | 308.5 |

If a 100-H.P. boiler has 1200 sq. ft. heating-surface, one week's running without blowing off, with water containing 100 grains of solid matter per gallon in solution, would make a scale nearly 0.02 in. thick, if evenly deposited all over the heating-surface, assuming the scale to have a sp. gr. of 2.5 = 156 lbs. per cu. ft.: $0.02 \times 1200 \times 156 \times \frac{1}{12} = 312$ lbs.

Boiler-scale Compounds. — The Bavarian Steam-boiler Inspection Assn. in 1885 reported as follows:

Generally the unusual substances in water can be retained in soluble form or precipitated as mud by adding caustic soda or lime. This is especially desirable when the boilers have small interior spaces.

It is necessary to have a chemical analysis of the water in order to fully determine the kind and quantity of the preparation to be used for the above purpose.

All secret compounds for removing boiler-scale should be avoided. (A list of 27 such compounds manufactured and sold by German firms is then given which have been analyzed by the association.)

Such secret preparations are either nonsensical or fraudulent, or contain either one of the two substances recommended by the association for removing scale, generally soda, which is colored to conceal its presence, and sometimes adulterated with useless or even injurious matter.

These additions as well as giving the compound some strange, fanciful name, are meant simply to deceive the boiler owner and conceal from him the fact that he is buying colored soda or similar substances, for which he is paying an exorbitant price.

Effect of Scale on Boiler Efficiency. — The following statement, or a similar one, has been published and republished for 40 years or more by makers of "boiler compounds," feed-water heaters and water-purifying apparatus, but the author has not been able to trace it to its original source:*

"It has been estimated that scale $\frac{1}{50}$ of an inch thick requires the burning of 5 per cent of additional fuel; scale $\frac{1}{25}$ of an inch thick

* A committee of the Am. Ry. Mast. Mechs. Assn. in 1872 quoted from a paper by Dr. Jos. G. Rodgers before the Am. Assn. for Adv. of Science (date not stated): "It has been demonstrated [how and by whom not stated] that a scale $\frac{1}{16}$ in. thick requires the expenditure of 15% more fuel. As the scale thickens the ratio increases; thus when it is $\frac{1}{4}$ in. thick, 60% more is required."

requires 10 per cent more fuel; $\frac{1}{16}$ of an inch of scale requires 15 per cent additional fuel; $\frac{1}{8}$ of an inch, 30 per cent., and $\frac{1}{4}$ of an inch, 66 per cent."

The absurdity of the last statement may be shown by a simple calculation. Suppose a clean boiler is giving 75% efficiency with a furnace temperature of 2400° F. above the atmospheric temperature. Neglecting the radiation and assuming a constant specific heat for the gases, the temperature of the chimney gases will be 600°. A certain amount of fuel and air supply will furnish 100 lbs. of gas. In the boiler with $\frac{1}{4}$ in. scale 66% more fuel will make 66 lbs. more gas. As the extra fuel does no work in evaporating water, its heat must all go into the chimney gas. We have then in the chimney gases

| | |
|------------------------------|----------------|
| 100 lbs. at 600° F., product | 60,000 |
| 66 lbs. at 2400° F., product | <u>158,400</u> |
| | 218,400 |

which divided by 166 gives 1370° above atmosphere as the temperature of the chimney gas, or more than enough to make the flue connection and damper red hot. (Makers of boiler compounds, etc., please copy.)

Another writer says: "Scale of $\frac{1}{16}$ inch thickness will reduce boiler efficiency $\frac{1}{8}$, and the reduction of efficiency increases as the square of the thickness of the scale."

This is still more absurd, for according to it if $\frac{1}{16}$ in. scale reduces the efficiency $\frac{1}{8}$, then $\frac{3}{16}$ in. will reduce it $\frac{9}{8}$, or to below zero.

From a series of tests of locomotive tubes covered with different thicknesses of scale up to $\frac{1}{8}$ in. Prof. E. C. Schmidt (Bull. No. 11 Univ. of Ill. Experiment Station, 1907) draws the following conclusions:

1. Considering scale of ordinary thickness, say varying up to $\frac{1}{8}$ inch, the loss in heat transmission due to scale may vary in individual cases from insignificant amounts to as much as 10 or 12 per cent.
2. The loss increases somewhat with the thickness of the scale.
3. The mechanical structure of the scale is of as much or more importance than the thickness in producing this loss.
4. Chemical composition, except in so far as it affects the structure of the scale, has no direct influence on its heat-transmitting qualities.

In 1896 the author made a test of a water-tube boiler at Aurora, Ill., which had a coating of scale about $\frac{1}{4}$ in. thick throughout its whole heating surface, and obtained practically the same evaporation as in another test, a few days later, after the boiler had been cleaned. This is only one case, but the result is not unreasonable when it is known that the scale was very soft and porous, and was easily removed from the tubes by scraping.

Prof. R. C. Carpenter (*Am. Electrician*, Aug., 1900) says: So far as I am able to determine by tests, a lime scale, even of great thickness, has no appreciable effect on the efficiency of a boiler, as in a test which was conducted by myself the results were practically as good when the boiler was thickly covered with lime scale as when perfectly clean. . . . Observations and experiments have shown that any scale porous to water has little or no detrimental effect on economy of the boiler. There is, I think, good philosophy for this statement; the heating capacity is affected principally by the rapidity with which the heated gases will surrender heat, as the water and the metal have capacities for absorbing heat more than a hundred times faster than the air will surrender heat.

A thin film of grease, being impermeable to water, keeps the latter from contact with the metal and generally produces disastrous results. It is much more harmful than a very thick scale of carbonate of lime.

Kerosene and other Petroleum Oils; Foaming. — Kerosene has been recommended as a scale preventive. See paper by L. F. Lyne (*Trans. A. S. M. E.*, ix. 247). The *Am. Mach.*, May 22, 1890, says: Kerosene used in moderate quantities will not make the boiler foam; it is recommended and used for loosening the scale and for preventing the formation of scale. The presence of oil in combination with other impurities increases the tendency of many boilers to foam, as the oil with the impurities impedes the free escape of steam from the water surface. The use of common oil not only tends to cause foaming, but is dangerous otherwise. The grease appears to combine with the impurities of the water, and when the boiler is at rest this compound sinks to the plates

and clings to them in a loose, spongy mass, preventing the water from coming in contact with the plates, and thereby producing overheating, which may lead to an explosion. Foaming may also be caused by forcing the fire, or by taking the steam from a point over the furnace or where the ebullition is violent; the greasy and dirty state of new boilers is another good cause for foaming. Kerosene should be used at first in small quantities, the effect carefully noted, and the quantity increased if necessary for obtaining the desired results.

R. C. Carpenter (*Trans. A. S. M. E.*, vol. xi) says: The boilers of the State Agricultural College at Lansing, Mich., were badly incrustated with a hard scale. It was fully $\frac{3}{8}$ in. thick in many places. The first application of the oil was made while the boilers were being but little used, by inserting a gallon of oil, filling with water, heating to the boiling-point and allowing the water to stand in the boiler two or three weeks before removal. By this method fully one-half the scale was removed during the warm season and before the boilers were needed for heavy firing. The oil was then added in small quantities when the boiler was in actual use. For boilers 4 ft. in diam. and 12 ft. long the best results were obtained by the use of 2 qts. for each boiler per week, and for each boiler 5 ft. in diam. 3 qts. per week. The water used in the boilers has the following analysis: CaCO_3 , 206 parts in a million; MgCO_3 , 78 parts; Fe_2CO_3 , 22 parts; traces of sulphates and chlorides of potash and soda. Total solids, 325 parts in 1,000,000.

Petroleum Oils heavier than kerosene have been used with good results. Crude oil should never be used. The more volatile oils it contains make explosive gases, and its tarry constituents are apt to form a spongy incrustation.

Removal of Hard Scale. — When boilers are coated with a hard scale difficult to remove the addition of $\frac{1}{4}$ lb. caustic soda per horse-power, and steaming for some hours, according to the thickness of the scale, just before cleaning, will greatly facilitate that operation, rendering the scale soft and loose. This should be done, if possible, when the boilers are not otherwise in use. (*Steam.*)

Corrosion in Marine Boilers. (*Proc. Inst. M. E.*, Aug., 1884.) — The investigations of the Committee on Boilers served to show that the internal corrosion of boilers is greatly due to the combined action of air and sea-water when under steam, and when not under steam to the combined action of air and moisture upon the unprotected surfaces of the metal. There are other deleterious influences at work, such as the corrosive action of fatty acids, the galvanic action of copper and brass, and the inequalities of temperature; these latter, however, are considered to be of minor importance.

Of the several methods recommended for protecting the internal surfaces of boilers, the three found most effectual are: First, the formation of a thin layer of hard scale, deposited by working the boiler with sea-water; second, the coating of the surfaces with a thin wash of Portland cement, particularly wherever there are signs of decay; third, the use of zinc slabs suspended in the water and steam spaces.

As to general treatment for the preservation of boilers when laid up in the reserve, either of the two following methods is adopted. First, the boilers are dried as much as possible by airing-stoves, after which 2 to 3 cwt. of quicklime is placed on trays at the bottom of the boiler and on the tubes. The boiler is then closed and made as air-tight as possible. Inspection is made every six months, when if the lime be found slacked it is renewed. Second, the boilers are filled with sea or fresh water, having added soda to it in the proportion of 1 lb. to every 100 or 120 lbs. of water. The sufficiency of the saturation can be tested by introducing a piece of clean new iron and leaving it in the boiler for ten or twelve hours; if it shows signs of rusting, more soda should be added. It is essential that the boilers be entirely filled, to the complete exclusion of air.

Mineral oil has for many years been exclusively used for internal lubrication of engines, with the view of avoiding the effects of fatty acid, as this oil does not readily decompose and possesses no acid properties.

Of all the preservative methods adopted in the British service, the use of zinc properly distributed and fixed has been found the most effectual

In saving the iron and steel surfaces from corrosion, and also in neutralizing by its own deterioration the hurtful influences met with in water as ordinarily supplied to boilers. The zinc slabs now used in the navy boilers are 12 in. long, 6 ins. wide, and $\frac{1}{2}$ in. thick; this size being found convenient for general application. The amount of zinc used in new boilers at present is one slab of the above size for every 20 I.H.P., or about 1 sq. ft. of zinc surface to 2 sq. ft. of grate surface. Rolled zinc is found the most suitable for the purpose. Especial care must be taken to insure perfect metallic contact between the slabs and the stays or plates to which they are attached. The slabs should be placed in such positions that all the surfaces in the boiler are protected. Each slab should be periodically examined to see that its connection remains perfect, and to renew any that may have decayed; this examination is usually made at intervals not exceeding three months. Under ordinary circumstances of working these zinc slabs may be expected to last in fit condition from 60 to 90 days, immersed in hot sea-water; but in new boilers they at first decay more rapidly. The slabs are generally secured by means of iron straps 2 in. \times $\frac{3}{8}$ in., and long enough to reach the nearest stay, to which the strap is attached by screw-bolts.

To promote the proper care of boilers when not in use the following order has been issued to the French Navy by the Government: On board all ships in the reserve, as well as those which are laid up, the boilers will be completely filled with fresh water. In the case of large boilers with large tubes there will be added to the water a certain amount of milk of lime, or a solution of soda. In the case of tubulous boilers with small tubes milk of lime or soda may be added, but the solution will not be so strong as in the case of the larger tube, so as to avoid any danger of contracting the effective area by deposit from the solution; but the strength of the solution will be just sufficient to neutralize any acidity of the water. (*Iron Age*, Nov. 2, 1893.)

Use of Zinc. — Zinc is often used in boilers to prevent the corrosive action of water on the metal. The action appears to be an electrical one, the iron being one pole of the battery and the zinc being the other. The hydrogen goes to the iron shell and escapes as a gas into the steam. The oxygen goes to the zinc.

On account of this action it is generally believed that zinc will always prevent corrosion, and that it cannot be harmful to the boiler or tank. Some experiences go to disprove this belief, and in numerous cases zinc has not only been of no use, but has even been harmful. In one case a tubular boiler had been troubled with a deposit of scale consisting chiefly of organic matter and lime, and zinc was tried as a preventive. The beneficial action of the zinc was so obvious that its continued use was advised, with frequent opening of the boiler and cleaning out of detached scale until all the old scale should be removed and the boiler become clean. Eight or ten months later the water-supply was changed, it being now obtained from another stream supposed to be free from lime and to contain only organic matter. Two or three months after its introduction the tubes and shell were found to be coated with an obstinate adhesive scale, composed of zinc oxide and the organic matter or sediment of the water used. The deposit had become so heavy in places as to cause overheating and bulging of the plates over the fire. (*The Locomotive*.)

Effect of Deposit on the Fire-surface of Flues. (Rankine.) — An external crust of a carbonaceous kind is often deposited from the flame and smoke of the furnaces in the flues and tubes, and if allowed to accumulate seriously impairs the economy of fuel. It is removed from time to time by means of scrapers and wire brushes. The accumulation of this crust is the probable cause of the fact that in some steamships the consumption of coal per I.H.P. per hour goes on gradually increasing until it reaches one and a half times its original amount, and sometimes more.

Dangerous Steam-boilers discovered by Inspection. — The Hartford Steam-boiler Inspection and Insurance Co. reports that its inspectors during 1908 examined 317,537 boilers, inspected 124,990 boilers, both internally and externally, subjected 10,449 to hydrostatic pressure, and found 572 unsafe for further use. The whole number of defects reported was 151,359, of which 15,578 were considered dangerous. A summary is given below. (*The Locomotive*, Jan., 1909.)

SUMMARY, BY DEFECTS, FOR THE YEAR 1893.

| Nature of Defects. | Whole No. | Dan-gerous. | Nature of Defects. | Whole No. | Dan-gerous. |
|--------------------------------|-----------|-------------|-------------------------------|-----------|-------------|
| Deposit of sediment..... | 18,879 | 1,242 | Defective tubes..... | 8,026 | 2,136 |
| Incrustation and scale..... | 37,924 | 1,193 | Tubes too light..... | 1,636 | 432 |
| Internal grooving..... | 2,649 | 249 | Leakage at joints..... | 4,845 | 392 |
| Internal corrosion..... | 13,053 | 555 | Water-gauges defective..... | 2,411 | 585 |
| External corrosion..... | 9,400 | 698 | Blow-offs defective..... | 3,818 | 1,125 |
| Def'tive braces and stays..... | 1,993 | 503 | Deficiency of water..... | 391 | 147 |
| Settings defective..... | 5,341 | 642 | Safety-valves overloaded..... | 1,216 | 379 |
| Furnaces out of shape..... | 6,981 | 380 | Safety-valves defective..... | 1,068 | 359 |
| Fractured plates..... | 3,119 | 482 | Pressure-gauges def'tive..... | 7,120 | 531 |
| Burned plates..... | 4,605 | 440 | Without pressure-gauges..... | 322 | 322 |
| Laminated plates..... | 666 | 44 | Unclassified defects..... | 7 | 3 |
| Defective riveting..... | 3,395 | 713 | | | |
| Defective heads..... | 1,565 | 223 | | | |
| Leakage around tubes..... | 10,929 | 2,103 | Total..... | 151,359 | 15,878 |

The above-named company publishes annually a summary like the above, and also a classified list of boiler-explosions, compiled chiefly from newspaper reports, showing that from 200 to 300 explosions take place in the United States every year, killing from 200 to 300 persons, and injuring from 300 to 450. The lists are not pretended to be complete, and may include only a fraction of the actual number of explosions.

Steam-boilers as Magazines of Explosive Energy. — Prof. R. H. Thurston (*Trans. A. S. M. E.*, vol. vi), in a paper with the above title, presents calculations showing the stored energy in the hot water and steam of various boilers. Concerning the plain tubular boiler of the form and dimensions adopted as a standard by the Hartford Steam-boiler Insurance Co., he says: It is 60 ins. in diameter, containing 66 3-in. tubes, and is 15 ft. long. It has 850 sq. ft. of heating and 30 sq. ft. of grate surface is rated at 60 H.P., but is oftener driven up to 75; weighs 9500 lbs., and contains nearly its own weight of water, but only 21 lbs. of steam when under a pressure of 75 lbs. per sq. in., which is below its safe allowance. It stores 52,000,000 foot-pounds of energy, of which but 4% is in the steam, and this is enough to drive the boiler just about one mile into the air, with an initial velocity of nearly 600 ft. per second.

SAFETY-VALVES.

Calculation of Weight, etc., for Lever Safety-valves.

Let W = weight of ball at end of lever; w = weight of lever itself; V = weight of valve and spindle, all in pounds; L = distance between fulcrum and center of ball; l = distance between fulcrum and center of valve; g = distance between fulcrum and center of gravity of lever all in inches; A = area of valve, in sq. ins.; P = pressure of steam, in lbs. per sq. in., at which valve will open.

Then $PA \times l = W \times L + w \times g + V \times l$;
whence $P = (WL + wg + Vl) \div Al$; $W = (PA l - wg - Vl) \div L$; $L = (PA l - wg - Vl) \div W$.

EXAMPLE. — Diameter of valve, 4 ins.; distance from fulcrum to center of ball, 36 ins.; to center of valve, 4 ins.; to center of gravity of lever, 15½ ins.; weight of valve and spindle, 3 lbs.; weight of lever, 7 lbs.; required the weight of ball to make the blowing-off pressure 80 lbs. per sq. in.; area of 4-in. valve = 12.566 sq. ins. Then

$$W = \frac{PA l - wg - Vl}{L} = \frac{80 \times 12.566 \times 4 - 7 \times 15\frac{1}{2} - 3 \times 4}{36} = 108.4 \text{ lbs.}$$

By the rules of the U. S. Supervising Inspectors of Steam Vessels the use of lever safety-valves is prohibited on all boilers built for steam vessels after June 30, 1906.

Rules for Area of Safety-valves.

(Rule of U. S. Supervising Inspectors of Steam-vessels (as amended 1909).)

The areas of all safety-valves on boilers contracted for or the construction of which commenced on or after June 1, 1904, shall be determined in accordance with the following formula: $a = 0.2074 \times W/P$, where a = area of safety-valve, in sq. in., per sq. ft. of grate surface; W = pounds of water evaporated per sq. ft. of grate surface per hour; P = absolute pressure per sq. in. = working gauge pressure + 15.

The value of a multiplied by the square feet of grate surface gives the area of safety valve or valves required. When this calculation results in an odd size of safety-valve use the next larger standard size.

EXAMPLE. — Boiler-pressure = 215 lbs. gauge, = 230 absolute, = P . Grate surface = 110 sq. ft. Water evaporated per pound coal = 10 lbs. Coal burned per sq. ft. grate per hour = 30 lbs. Evaporation per sq. ft. grate per hour = 300 lbs. = W . $a = 0.2074 \times 300 \div 230 = 0.270$. Therefore area of safety-valve = $110 \times 0.270 = 29.7$ sq. ins., which is too large for one valve. Use two, 14.85 sq. ins. each. Diameter = $4\frac{3}{8}$ ins. Each spring-loaded valve shall be supplied with a lever that will raise the valve from its seat a distance of not less than that equal to one-eighth of the diameter of the valve opening.

The valves shall be so arranged that each boiler shall have at least one separate safety-valve, unless the arrangement is such as to preclude the possibility of shutting off the communication of any boiler with the safety valve or valves employed.

Two safety-valves may be allowed on any boiler, provided their combined area is equal to that required by rule for one valve. Whenever the area of a safety-valve, as found by the rule, will be greater than that corresponding to 6 inches in diameter, two or more safety-valves, whose combined area shall be equal at least to the area required, must be used.

The seats of all safety-valves shall have an angle of inclination of 45 degrees to the center lines of their axes.

Comparison of Various Rules for Area of Lever Safety-valves. (Condensed from an article by the author in *American Machinist*, May 24, 1894, with some alterations.) — Assume the case of a boiler rated at 100 horse-power; 40 sq. ft. grate; 1200 sq. ft. heating-surface; using 400 lbs. of coal per hour, or 10 lbs. per sq. ft. of grate per hour, and evaporating 3600 lbs. of water, or 3 lbs. per sq. ft. of heating-surface per hour; steam-pressure by gauge, 100 lbs. What size of safety-valve, of the lever type, should be required?

A compilation of various rules for finding the area of the safety-valve disk, from *The Locomotive* of July, 1892, is given in abridged form below, together with the area calculated by each rule for the above example.

| | Disk Area in sq. in. |
|--|-------------------------|
| U. S. Supervisors, heating-surface in sq. ft. \div 25 (old rule) | 48 |
| English Board of Trade, grate-surface in sq. ft. \div 2 | 20 |
| Molesworth, four-fifths of grate-surface in sq. ft. | 32 |
| Thurston, 4 times coal burned per hour \times (gauge pressure + 10) ... | 14.5 |
| Thurston, $2.5 \times$ heating-surface \div gauge pressure + 10 | 27.3 |
| Rankine, $0.006 \times$ water evaporated per hour | 21.6 |
| Committee of U. S. Supervisors, $0.005 \times$ water evaporated per hr. .. | 18 |

Suppose that, other data remaining the same, the draught were increased so as to burn $131\frac{1}{3}$ lbs. coal per sq. ft. of grate per hour, and the grate-surface cut down to 30 sq. ft. to correspond, making the coal burned per hour 400 lbs., and the water evaporated 3600 lbs., the same as before; then the English Board of Trade rule and Molesworth's rule would give an area of disk of only 15 and 24 sq. in., respectively, showing the absurdity of making the area of grate the basis of the calculation of disk area.

Other rules give for the area of safety-valve of the same 100-horse-power boiler results ranging all the way from 5.25 to 57.6 sq. ins.

All of the rules quoted give the area of the disk of the valve as the thing to be ascertained, and it is this area which is supposed to bear some direct ratio to the grate-surface, to the heating-surface, to the

water evaporated, etc. It is difficult to see why this area has been considered even approximately proportional to these quantities, for with small lifts the area of actual opening bears a direct ratio, not to the area of disk, but to the circumference.

Thus for various diameters of valve:

| Diameter, ins..... | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----------------------------------|-------|------|------|-------|-------|-------|-------|
| Area, sq. ins..... | 0.785 | 3.14 | 7.07 | 12.57 | 19.64 | 28.27 | 38.48 |
| Circumference..... | 3.14 | 6.28 | 9.42 | 12.57 | 15.71 | 18.85 | 21.99 |
| Circum. \times lift of 0.1 in. | 0.31 | 0.63 | 0.94 | 1.26 | 1.57 | 1.89 | 2.20 |
| Ratio to area..... | 0.4 | 0.2 | 0.13 | 0.1 | 0.08 | 0.067 | 0.057 |

A correct rule for size of safety-valves should make the product of the diameter and the lift proportional to the weight of steam to be discharged.

A method for calculating the size of safety-valve is given in *The Locomotive*, July, 1892, based on the assumption that the actual opening should be sufficient to discharge all the steam generated by the boiler. Napier's rule for flow of steam is taken, viz., flow through aperture of one sq. in. in lbs. per second = absolute pressure \div 70, or in lbs. per hour = $51.43 \times$ absolute pressure.

If the angle of the seat is 45° , the area of opening in sq. in. = circumference of the disk \times the lift \times 0.71, 0.71 being the cosine of 45° ; or diameter of disk \times lift \times 2.23.

Spring-loaded Safety Valves.

Spring-loaded safety valves to be used on U. S. merchant vessels must conform to the rules prescribed by the Board of Supervising Inspectors, and on vessels for the U. S. Navy to specifications made by the Bureau of Steam Engineering, U. S. N. Valves to be used on stationary boilers must conform in many cases to the special laws made by various states. Few of these rules are on a logical basis, in that they take no account of the lift of the valve, and it is quite clear that the rate of steam discharge through a safety-valve depends upon the area of opening, which varies with the circumference of the valve and the lift. Experiments made by the Consolidated Safety Valve Co. showed that valves made by the different manufacturers and employing various combinations of springs with different designs of valve lips and huddling chambers give widely different lifts. Lifts at popping point of different makes of safety-valves, at 200 lbs. pressure, are as follows:

4-in. stationary valves, in., 0.031, 0.056, 0.064, 0.082, 0.094, 0.094, 0.137.
Av. 0.079 in.

3 1/2-in. locomotive valves, in., 0.040, 0.051, 0.065, 0.072, 0.076, 0.140 ins.
Av. 0.074 in.

United States Supervising Inspectors' Rule (adopted in 1904). $A = 0.2074 W/P$. A = area of safety valve in sq. in. per sq. ft. of grate surface; W = lbs. of water evaporated per sq. ft. of grate surface per hour; P = boiler pressure, absolute, lbs. per sq. in. This rule assumes a lift of $1/32$ of the nominal diameter, and 75% of the flow calculated by Napier's rule. This 75% corresponds nearly to the cosine of 45° , or 0.707.

Massachusetts Rule of 1909. $A = 770 W/P$, in which W = lbs. evaporated per sq. ft. of grate per second; A and P as above. This is the same as the U. S. rule with a 3.2% larger constant.

Philadelphia Rule. — $A = 22.5 G \div (P + 8.62)$. A = total area of valve or valves, sq. in.; G = grate area, sq. ft.; P = boiler pressure (gauge). This rule came from France in 1868. It was recommended to the city of Philadelphia by a committee of the Franklin Institute, although the committee "had not found the reasoning upon which the rule had been based."

Philip G. Darling (*Trans. A. S. M. E.*, 1909) commenting on the above rules says: The principal defect of these rules is that they assume that valves of the same nominal size have the same capacity, and they rate them the same without distinction, in spite of the fact that in actual practice some have but one-third of the capacity of others. There are other defects, such as varying the assumed lift as the valve diameter, while in

reality with a given design the lifts are more nearly the same in the different sizes, not varying nearly as rapidly as the diameters. And further than this, the actual lifts assumed for the larger valves are nearly double the actual average obtained in practice. The direct conclusion is that existing rules and statutes are not safe to follow. Some of these rules in use were formulated before, and have not been modified since, spring safety-valves were invented, and at a time when 120 lbs. was considered high pressure. None of these rules take account of the different lifts which exist in the different makes of valves of the same nominal size, and they thus rate exactly alike valves which actually vary in lift and relieving capacity over 300%. It would therefore seem the duty of all who are responsible for steam installation and operation to no longer leave the determination of safety-valve size and selection to such statutes as may happen to exist in their territory, but to investigate for themselves.

Formulae for Spring-loaded Safety-Valves. — Let L = lift of valve in.; D = diam. in.; E = discharge, lbs. per hour; P = abs. pressure; A = area of opening; θ = angle of seat with horizontal. By Napier's formula $E = AP \times 3600 \div 70 = 51.43 AP$. $A = \pi DL \cos \theta$ (approximately). If $\theta = 45^\circ$, $\cos \theta = 0.707$, whence $E = 114.2 LDP$. Experiments with six different valves, 3, $3\frac{1}{2}$ and 4 in. stationary, and $1\frac{1}{2}$, 3 and $3\frac{1}{2}$ in. locomotive, gave an average flow equal to 92.5% of that calculated by the above formula, which is therefore modified by Mr. Darling to the forms $E = 105 LDP$, and $D = 0.0095 E \div LP$ (1)

To obtain formulae for safety valves in terms of the heating-surface of the boiler Mr. Darling takes for stationary boilers an average evaporation of $3\frac{1}{2}$ lbs. per sq. ft. of heating-surface per hour, with an overload capacity of 100%; for marine boilers, water-tube or Scotch, an overload or maximum evaporation of 10 lbs. per sq. ft. of heating-surface per hour. If H = total boiler heating-surface in sq. ft., these assumptions give for stationary boilers $D = 0.068 H \div LP$, (2) and for marine boilers $D = 0.095 H \div LP$ (3). For locomotive boilers the proper constant in the formula was deduced from numerous experiments to be 0.055 (4).

For flat valves the constants in the last four formulae are: (1) 0.0067; (2) 0.065; (3) 0.090; (4) 0.052.

The following table is calculated from Napier's formula, on the assumption of a lift of 0.1 in. and a 45° valve-seat. For any other lift than 0.1 in., the discharge is proportional to the lift. The figures should be multiplied by a coefficient expressing the relation of the discharge of actual valves to the discharge through a plain round orifice (Napier's). In the Consolidated Safety Valve Co.'s experiments the average value of this coefficient was found to be 0.925.

STEAM DISCHARGED IN LBS. PER HOUR BY A VALVE LIFTING 0.10 IN.

| Gauge Pressure. | Valve diameters, inches. | | | | | | | | | | |
|-----------------|--------------------------|----------------|------|----------------|-------|----------------|-------|----------------|-------|----------------|-------|
| | 1 | $1\frac{1}{2}$ | 2 | $2\frac{1}{2}$ | 3 | $3\frac{1}{2}$ | 4 | $4\frac{1}{2}$ | 5 | $5\frac{1}{2}$ | 6 |
| 25 | 460 | 690 | 920 | 1150 | 1380 | 1610 | 1840 | 2080 | 2300 | 2540 | 2770 |
| 50 | 750 | 1130 | 1500 | 1880 | 2250 | 2630 | 3000 | 3380 | 3760 | 4130 | 4500 |
| 75 | 1040 | 1560 | 2080 | 2600 | 3120 | 3640 | 4160 | 4680 | 5200 | 5720 | 6240 |
| 100 | 1330 | 2000 | 2660 | 3330 | 4000 | 4660 | 5320 | 6000 | 6650 | 7320 | 8000 |
| 125 | 1620 | 2440 | 3250 | 4060 | 4860 | 5670 | 6480 | 7300 | 8100 | 8920 | 9730 |
| 150 | 1910 | 2870 | 3830 | 4790 | 5740 | 6700 | 7650 | 8610 | 9560 | 10520 | 11470 |
| 175 | 2200 | 3300 | 4400 | 5500 | 6600 | 7700 | 8800 | 9900 | 11000 | 12100 | 13200 |
| 200 | 2500 | 3740 | 5000 | 6240 | 7480 | 8730 | 9970 | 11200 | 12460 | 13700 | 14950 |
| 225 | 2780 | 4180 | 5570 | 6960 | 8340 | 9730 | 11120 | 12500 | 13900 | 15300 | 16700 |
| 250 | 3070 | 4610 | 6140 | 7680 | 9200 | 10740 | 12300 | 13800 | 15360 | 16900 | 18450 |
| 275 | 3360 | 5050 | 6720 | 8400 | 10100 | 11760 | 13450 | 15150 | 16800 | 18500 | 20200 |
| 300 | 3650 | 5480 | 7310 | 9150 | 10960 | 12800 | 14600 | 16470 | 18300 | 20100 | 22000 |

Unequal expansion of safety-valve parts under steam temperatures tends to cause leakage, and as this temperature effect becomes more serious in the large sizes the manufacturers do not recommend the use of valves larger than $4\frac{1}{2}$ ins. If greater relieving capacity be required it is the best practice to use duplex valves or additional single valves.

RELIEVING CAPACITIES, CONSOLIDATED POP SAFETY VALVES, STATIONARY TYPE. (Pounds of Steam per hour.)

| Size Valve, In. | Gauge Pressures. (lbs. per sq. in.) | | | | | | | | | | | | |
|--------------------|-------------------------------------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 | 260 | 280 | 300 |
| 2 | 1890 | 2400 | 2900 | 3400 | 3900 | 4410 | 4910 | 5420 | 5920 | 6430 | 6930 | 7430 | 7940 |
| 2½ | 2360 | 3000 | 3620 | 4250 | 4880 | 5500 | 6140 | 6760 | 7400 | 8030 | 8650 | 9300 | 9900 |
| 3 | 3070 | 3890 | 4700 | 5530 | 6350 | 7170 | 8000 | 8800 | 9620 | 10400 | 11200 | 12100 | 12900 |
| 3½ | 3850 | 4880 | 5910 | 6950 | 7960 | 9020 | 10000 | 11100 | 12100 | 13100 | 14200 | 15200 | 16300 |
| 4 | 4410 | 5580 | 6770 | 7950 | 9120 | 10300 | 11500 | 12600 | 13800 | 15000 | 16200 | 17300 | 18500 |
| 4½ | 5310 | 6730 | 8150 | 9570 | 11000 | 12400 | 13800 | 15200 | 16700 | 18100 | 19500 | 20900 | 22400 |
| 5 | 6300 | 7970 | 9650 | 11330 | 13000 | 14700 | 16400 | 18100 | 19700 | 21400 | 23100 | 24800 | 26500 |

For an extended discussion on safety-valves, see *Trans. A. S. M. E.*, 1909.

THE INJECTOR.

Equation of the Injector.

Let S be the number of pounds of steam used;

W the number of pounds of water lifted and forced into the boiler;
 h the height in feet of a column of water, equivalent to the absolute pressure in the boiler;

h_0 the height in feet the water is lifted to the injector;

t_1 the temperature of the water before it enters the injector;

t_2 the temperature of the water after leaving the injector;

H the total heat above 32° F. in one pound of steam in the boiler, in heat-units;

L the work in friction and the equivalent lost work due to radiation and lost heat;

778 the mechanical equivalent of heat.

Then

$$S[H - (t_2 - 32^\circ)] = W(t_2 - t_1) + \frac{(W + S)h + Wh_0 + L}{778}.$$

An equivalent formula, neglecting $Wh_0 + L$ as small, is

$$S = \left[W(t_2 - t_1) + \frac{W + S}{d} \cdot p \cdot \frac{144}{778} \right] \frac{1}{H - (t_2 - 32^\circ)},$$

$$\text{or } S = \frac{W[(t_2 - t_1)d + 0.1851 p]}{[H - (t_2 - 32^\circ)]d - 0.1851 p},$$

in which d = weight of 1 cu. ft. of water at temperature t_2 ; p = absolute pressure of steam, lbs. per sq. in.

The rule for finding the proper sectional area for the narrowest part of the nozzles is given as follows by Rankine, S. E., p. 477:

$$\text{Area in square inches} = \frac{\text{cubic feet per hour gross feed-water}}{800 \sqrt{\text{pressure in atmospheres}}}.$$

An important condition which must be fulfilled in order that the injector will work is that the supply of water must be sufficient to condense

the steam. As the temperature of the supply or feed-water is higher, the amount of water required for condensing purposes will be greater.

The table below gives the calculated value of the maximum ratio of water to the steam, and the values obtained on actual trial, also the highest admissible temperature of the feed-water as shown by theory and the highest actually found by trial with several injectors.

| Gauge- pres- sure, pounds per sq. in. | Maximum Ratio Water to Steam. | | | | Gauge- pres- sure, pounds per sq. in. | Maximum Temperature of Feed-Water. | | | | | | | | |
|--|----------------------------------|-------------------------|------|-------|--|---------------------------------------|-----------------------------|-----------------------------|------|------|----|----|--|------|
| | Calculated from Theory. | Actual Ex- periment. | | | | Theoreti- cal. | Experimental Results. | | | | | | | |
| | | H. | P. | M. | | | Temp. discharge 180°. | Temp. discharge 212°. | H. | P. | M. | S. | | |
| | | | | | | | | | | | | | | |
| 10 | 36.5 | 30.9 | | | 10 | | | | | | | | | 132° |
| 20 | 25.6 | 22.5 | 19.9 | 21.5 | 20 | 142° | 173° | 135° | 120° | 130° | | | | 134 |
| 30 | 20.9 | 19.0 | 17.2 | 19.0 | 30 | 132 | 162 | | | | | | | 134 |
| 40 | 17.87 | 15.8 | 15.0 | 15.86 | 40 | 126 | 156 | 140 | 113 | 125 | | | | 132 |
| 50 | 16.2 | 13.3 | 14.0 | 13.3 | 50 | 120 | 150 | | | | | | | 131 |
| 60 | 14.7 | 11.2 | 11.2 | 12.6 | 60 | 114 | 143 | | 115 | 123 | | | | 130 |
| 70 | 13.7 | 12.3 | 11.7 | 12.9 | 70 | 109 | 139 | 141* | | 123 | | | | 130 |
| 80 | 12.9 | 11.4 | 11.2 | | 80 | 105 | 134 | 141* | 118 | 122 | | | | 131 |
| 90 | 12.1 | | | | 90 | 99 | 129 | | | | | | | 132* |
| 100 | 11.5 | | | | 100 | 95 | 125 | | | | | | | 132* |
| | | | | | 120 | 87 | 117 | | | | | | | 134* |
| | | | | | 150 | 77 | 107 | | | | | | | 121* |

* Temperature of delivery above 212°. Waste-valve closed.

H, Hancock inspirator; P, Park injector; M, Metropolitan injector; S, Sellers 1876 injector.

Efficiency of the Injector.—Experiments at Cornell University, described by Prof. R. C. Carpenter, in *Cassier's Magazine*, Feb., 1892, show that the injector, when considered merely as a pump, has an exceedingly low efficiency, the duty ranging from 161,000 to 2,752,000 under different circumstances of steam and delivery pressure. Small direct-acting pumps, such as are used for feeding boilers, show a duty of from 4 to 8 million ft.-lbs., and the best pumping-engines from 100 to 140 million. When used for feeding water into a boiler, however, the injector has a thermal efficiency of 100%, less the trifling loss due to radiation, since all the heat rejected passes into the water which is carried into the boiler.

The loss of work in the injector due to friction reappears as heat which is carried into the boiler, and the heat which is converted into useful work in the injector appears in the boiler as stored-up energy.

Although the injector thus has a perfect efficiency as a boiler-feeder, it is not the most economical means for feeding a boiler, since it can draw only cold or moderately warm water, while a pump can feed water which has been heated by exhaust steam which would otherwise be wasted.

Performance of Injectors.—In *Am. Mach.*, April 13, 1893, are a number of letters from different manufacturers of injectors in reply to the question: "What is the best performance of the injector in raising or lifting water to any height?" Some of the replies are tabulated below.

W. Sellers & Co. — 25.51 lbs. water delivered to boiler per lb. of steam; temperature of water, 64°; steam pressure, 65 lbs.

Schaeffer & Budenberg — 1 gal. water delivered to boiler for 0.4 to 0.8 lb. steam.

Injector will lift by suction water of

| | | | |
|----------------------------------|---------------|----------------|-----------------|
| 140° F. | 136° to 133° | 122° to 118° | 113° to 107° |
| If boiler pres. is 30 to 60 lbs. | 60 to 90 lbs. | 90 to 120 lbs. | 120 to 150 lbs. |

If the water is not over 80° F., the injector will force against a pressure 75 lbs. higher than that of the steam.

Hancock Inspirator Co.:

| | | | | |
|-------------------------------------|-------|--------|--------|--------|
| Lift in feet..... | 22 | 22 | 22 | 11 |
| Boiler pressure, absolute, lbs.... | 75.8 | 54.1 | 95.5 | 75.4 |
| Temperature of suction... .. | 34.9° | 35.4° | 47.3° | 53.2 |
| Temperature of delivery..... | 134° | 117.4° | 173.7° | 131.1° |
| Water fed per lb. of steam, lbs.... | 11.02 | 13.67 | 8.18 | 13.3 |

The theory of the injector is discussed in Wood's, Peabody's, and Rontgen's treatises on Thermodynamics. See also "Theory and Practice of the Injector," by Strickland L. Kneass, New York, 1910.

Boiler-feeding Pumps. — Since the direct-acting pump, commonly used for feeding boilers, has a very low efficiency, or less than one-tenth that of a good engine, it is generally better to use a pump driven by belt from the main engine or driving shaft. The mechanical work needed to feed a boiler may be estimated as follows: If the combination of boiler and engine is such that half a cubic foot, say 32 lbs. of water, is needed per horse-power, and the boiler-pressure is 100 lbs. per sq. in., then the work of feeding the quantity of water is 100 lbs. \times 144 sq. in. \times 1/2 ft.-lb. per hour = 120 ft.-lbs. per min. = 120/33,000 = .0036 H.P., or less than 4/10 of 1% of the power exerted by the engine. If a direct-acting pump, which discharges its exhaust steam into the atmosphere, is used for feeding, and it has only 1/10 the efficiency of the main engine, then the steam used by the pump will be equal to nearly 4% of that generated by the boiler.

The low efficiency of boiler-feeding pumps, and of other small auxiliary steam-driven machinery, is, however, of no importance if all the exhaust steam from these pumps is utilized in heating the feed-water.

The following table by Prof. D. S. Jacobus gives the relative efficiency of steam and power pumps and injector, with and without heater, as used upon a boiler with 80 lbs. gauge-pressure, the pump having a duty of 10,000,000 ft.-lbs. per 100 lbs. of coal when no heater is used; the injector heating the water from 60° to 150° F.

| | |
|---|-------|
| Direct-acting pump feeding water at 60°, without a heater..... | 1.000 |
| Injector feeding water at 150°, without a heater..... | 0.985 |
| Injector feeding water through a heater in which it is heated from 150° to 200°..... | 0.938 |
| Direct-acting pump feeding water through a heater, in which it is heated from 60° to 200°..... | 0.879 |
| Geared pump, run from the engine, feeding water through a heater, in which it is heated from 60° to 200°..... | 0.868 |

Gravity Boiler-feeders. — If a closed tank be placed above the level of the water in a boiler and the tank be filled or partly filled with water, then on shutting off the supply to the tank, admitting steam from the boiler to the upper part of the tank, so as to equalize the steam-pressure in the boiler and in the tank, and opening a valve in a pipe leading from the tank to the boiler, the water will run into the boiler. An apparatus of this kind may be made to work with practically perfect efficiency as a boiler-feeder, as an injector does, when the feed-supply is at ordinary atmospheric temperature, since after the tank is emptied of water and the valves in the pipes connecting it with the boiler are closed the condensation of the steam remaining in the tank will create a vacuum which will lift a fresh supply of water into the tank. The only loss of energy in the cycle of operations is the radiation from the tank and pipes, which may be made very small by proper covering.

When the feed-water supply is hot, such as the return water from a heating system, the gravity apparatus may be made to work by having two receivers, one at a low level, which receives the returns or other feed-supply, and the other at a point above the boilers. A partial vacuum being created in the upper tank, steam-pressure is applied above the water in the lower tank by which it is elevated into the upper. The operation of such a machine may be made automatic by suitable arrangement of valves.

FEED-WATER HEATERS.

Percentage of Saving for Each Degree of Increase in Temperature of Feed-water Heated by Waste Steam.

| Initial Temp. of Feed. | Steam Pressure in Boiler, lbs. per sq. in. above Atmosphere. | | | | | | | | | | | Initial Temp. |
|------------------------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------------|
| | 0 | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | |
| 32° | .0872 | .0861 | .0855 | .0851 | .0847 | .0844 | .0841 | .0839 | .0837 | .0835 | .0833 | 32° |
| 40 | .0878 | .0867 | .0861 | .0856 | .0853 | .0850 | .0847 | .0845 | .0843 | .0841 | .0839 | 40 |
| 50 | .0886 | .0875 | .0868 | .0864 | .0860 | .0857 | .0854 | .0852 | .0850 | .0848 | .0846 | 50 |
| 60 | .0894 | .0883 | .0876 | .0872 | .0867 | .0864 | .0862 | .0859 | .0856 | .0855 | .0853 | 60 |
| 70 | .0902 | .0890 | .0884 | .0879 | .0875 | .0872 | .0869 | .0867 | .0864 | .0862 | .0860 | 70 |
| 80 | .0910 | .0898 | .0891 | .0887 | .0883 | .0879 | .0877 | .0874 | .0872 | .0870 | .0868 | 80 |
| 90 | .0919 | .0907 | .0900 | .0895 | .0888 | .0887 | .0884 | .0883 | .0879 | .0877 | .0875 | 90 |
| 100 | .0927 | .0915 | .0908 | .0903 | .0899 | .0895 | .0892 | .0890 | .0887 | .0885 | .0883 | 100 |
| 110 | .0936 | .0923 | .0916 | .0911 | .0907 | .0903 | .0900 | .0898 | .0895 | .0893 | .0891 | 110 |
| 120 | .0945 | .0932 | .0925 | .0919 | .0915 | .0911 | .0908 | .0906 | .0903 | .0901 | .0899 | 120 |
| 130 | .0954 | .0941 | .0934 | .0928 | .0924 | .0920 | .0917 | .0914 | .0912 | .0909 | .0907 | 130 |
| 140 | .0963 | .0950 | .0943 | .0937 | .0932 | .0929 | .0925 | .0923 | .0920 | .0918 | .0916 | 140 |
| 150 | .0973 | .0959 | .0951 | .0946 | .0941 | .0937 | .0934 | .0931 | .0929 | .0926 | .0924 | 150 |
| 160 | .0982 | .0968 | .0961 | .0955 | .0950 | .0946 | .0943 | .0940 | .0937 | .0935 | .0933 | 160 |
| 170 | .0992 | .0978 | .0970 | .0964 | .0959 | .0955 | .0952 | .0949 | .0946 | .0944 | .0941 | 170 |
| 180 | .1002 | .0988 | .0981 | .0973 | .0969 | .0965 | .0961 | .0958 | .0955 | .0953 | .0951 | 180 |
| 190 | .1012 | .0998 | .0989 | .0983 | .0978 | .0974 | .0971 | .0968 | .0964 | .0962 | .0960 | 190 |
| 200 | .1022 | .1008 | .0999 | .0993 | .0988 | .0984 | .0980 | .0977 | .0974 | .0972 | .0969 | 200 |
| 210 | .1033 | .1018 | .1009 | .1003 | .0998 | .0994 | .0990 | .0987 | .0984 | .0981 | .0979 | 210 |
| 220 | | .1029 | .1019 | .1013 | .1008 | .1004 | .1000 | .0997 | .0994 | .0991 | .0989 | 220 |
| 230 | | .1039 | .1031 | .1024 | .1018 | .1012 | .1010 | .1007 | .1003 | .1001 | .0999 | 230 |
| 240 | | .1050 | .1041 | .1034 | .1029 | .1024 | .1020 | .1017 | .1014 | .1011 | .1009 | 240 |
| 250 | | .1062 | .1052 | .1045 | .1040 | .1035 | .1031 | .1027 | .1025 | .1022 | .1019 | 250 |

An approximate rule for the conditions of ordinary practice is that a saving of 1% is made by each increase of 11° in the temperature of the feed-water. This corresponds to 0.0909% per degree.

The calculation of saving is made as follows: Boiler-pressure, 100 lbs. gauge; total heat in steam above 32° = 1185 B.T.U. Feed-water, original temperature 60°, final temperature 209° F. Increase in heat-units, 150. Heat-units above 32° in feed-water of original temperature = 28. Heat-units in steam above that in cold feed-water, 1185 - 28 = 1157. Saving by the feed-water heater = 150/1157 = 12.96%. The same result is obtained by the use of the table. Increase in temperature 150° × tabular figure 0.0864 = 12.96%. Let total heat of 1 lb. of steam at the boiler-pressure = H ; total heat of 1 lb. of feed-water before entering the heater = h_1 , and after passing through the heater = h_2 ; then the saving made by the heater is $\frac{h_2 - h_1}{H - h_1}$.

Strains Caused by Cold Feed-water. — A calculation is made in *The Locomotive* of March, 1893, of the possible strains caused in the section of the shell of a boiler by cooling it by the injection of cold feed-water. Assuming the plate to be cooled 200° F., and the coefficient of expansion of steel to be 0.0000067 per degree, a strip 10 in. long would contract 0.013 in., if it were free to contract. To resist this contraction, assuming that the strip is firmly held at the ends and that the modulus of elasticity is 29,000,000, would require a force of 37,700 lbs. per sq. in. Of course this amount of strain cannot actually take place, since the strip is not firmly held at the ends, but is allowed to contract to some extent by the elasticity of the surrounding metal. But, says *The Locomotive*, we may feel pretty confident that in the case considered a longitudinal strain of somewhere in the neighborhood of 8000 or 10,000 lbs. per sq. in. may be produced by the feed-water striking directly upon the plates; and this, in addition to the normal strain produced by the steam-pressure, is quite enough to tax the girth-seams beyond their elastic limit, if the

feed-pipe discharges anywhere near them. Hence it is not surprising that the girth-seams develop leaks and cracks in 99 cases out of every 100 in which the feed discharges directly upon the fire-sheets.

Capacity of Feed-water Heaters. (W. R. Billings, *Eng. Rec.*, Feb., 1898.) — Closed feed-water heaters are seldom provided with sufficient surface to raise the feed temperature to more than 200°. The rate of heat transmission may be measured by the number of British thermal units which pass through a square foot of tubular surface in one hour for each degree of difference in temperature between the water and the steam. One set of experiments gave results as below:

| | | | | |
|---|---|------------|-----------|--|
| Difference between final temperatures of water and steam | { | 5° F..... | 67 B.T.U. | } Transmitted in one hour by each sq. ft. of surface for each degree of average difference in temper- atures. |
| | | 6° "..... | 79 " | |
| | | 8° "..... | 89 " | |
| | | 11° "..... | 114 " | |
| | | 15° "..... | 129 " | |
| | | 18° "..... | 139 " | |

Even with the rate of transmission as low as 67 B.T.U. the water was still 5° from the temperature of the steam. At what rate would the heat have been transmitted if the water could have been brought to within 2° of the temperature of the steam, or to 210° when the steam is at 212°?

For commercial purposes feed-water heaters are given a H.P. rating which allows about one-third of a square foot of surface per H.P. — a boiler H.P. being 30 lbs. of water per hour. If the figures given in the table above are accepted as substantially correct, a heater which is to raise 3000 lbs. of water per hour from 60° to 207°, using exhaust steam at 212° as a heating medium, should have nearly 84 sq. ft. of heating surface or nearly a square foot of surface per H.P. That feed-water heaters do not carry this amount of heating surface is well known.

Calculation of Surface of Heaters and Condensers. — (H. L. Hepburn, *Power*, April, 1902.) Let W = lbs. of water per hour; A = area of surface in sq. ft.; T_s = temperature of the steam; I = initial temperature of the water; F = final temperature of the water; S = lbs. of steam per hour; H = B.T.U. above 32° F. in 1 lb. of steam; N = B.T.U. in 1 lb. of condensed steam; U = B.T.U. transmitted per sq. ft. per hour per degree of mean difference of temperature between the steam and the water.

$$\text{Then } AU = W \log_e \frac{T_s - I}{T_s - F}, \text{ for heaters.}$$

$$AU = S \frac{H - N}{F - I} \times \log_e \frac{T_s - I}{T_s - F}, \text{ for condensers.}$$

The value of U varies widely according to the condition of the surface, whether clean or coated with grease or scale, and also with the velocity of the water over the surfaces. Values of 300 to 350 have been obtained in experiments with corrugated copper tubes, but ordinary heaters give much lower values. From the experiments of Loring and Emery on the U. S. S. *Dallas*, Mr. Hepburn finds $U = 192$. Using this value he finds the number of square feet of heating surface required per 1000 lbs. of feed-water per hour to be as follows, the temperature of the entering water being 60° F.

| Steam Temperature, 212°. | | | | Steam 25 in. Vacuum. | | | |
|--------------------------|-------|-----|----------|----------------------|------|-----|----------|
| F | S | F | S | F | S | F | S |
| 194 | 11.11 | 204 | 15.34 | 90 | 2.38 | 115 | 6.78 |
| 196 | 11.73 | 206 | 16.85 | 95 | 3.03 | 120 | 8.60 |
| 198 | 12.44 | 208 | 18.93 | 100 | 3.76 | 125 | 11.15 |
| 200 | 13.20 | 210 | 22.52 | 105 | 4.62 | 130 | 16.25 |
| 202 | 14.17 | 212 | Infinite | 110 | 5.65 | 133 | Infinite |

F = final temperature of feed-water. S = sq. ft. of surface. From this table it is seen that if 30 lbs. of water per hour is taken to equal 1 H.P.

and a feed-water heater is made with $\frac{1}{3}$ sq. ft. per H.P., it may be expected to heat the feed-water from 60° to something less than 194° , or if made with $\frac{1}{2}$ sq. ft. per H.P. it may heat the water to 204° F.

For a further discussion of this subject, see Heat, pages 561 to 565.

Proportions of Open Type Feed-water Heaters. — C. L. Hubbard (*Practical Engineer*, Jan. 1, 1909) gives the following:

Exhaust heaters should be proportioned according to the quality of the water to be used, the size being increased with the amount of mud or scale-producing properties which the water contains regardless of the quantity of water to be heated. The general proportions of an open heater will depend somewhat upon the arrangement of the trays or pans, but an approximation of the size of shell for a cylindrical heater is as follows: $A = H \div aL$; $L = H \div aA$; in which A = sectional area of shell in sq. ft.; L = length of shell in linear ft.; H = total weight of water to be heated per hour divided by the weight of steam used per horse-power per hour by the engine; a = 2.15 for very muddy water, 6.0 for slightly muddy water, and 8.0 for clear water.

The pan or tray surface varies according to the quality of the water, both as regards the amount of mud and the scale-making ingredients. The surface in square feet for each 1000 lbs. of water heated per hour may be taken as follows, for the vertical and horizontal types respectively:

| | |
|-----------------------------|-------------|
| Very bad water..... | 8.5 and 9.1 |
| Medium muddy water..... | 6 and 6.5 |
| Clear and little scale..... | 2 and 2.2 |

The space between the pans is made not less than 0.1 the width for rectangular and 0.25 the diameter for round pans. Under ordinary circumstances it is not customary to use more than six pans in a tier, in order to obtain a low velocity over each pan. The size of the storage or settling chamber in the horizontal type varies from 0.25 to 0.4 of the volume of the shell, depending on the quality of the water; 0.33 is about the average. In the case of vertical heaters, this varies from 0.4 to 0.6 of the volume of the shell. Filters occupy from 10 to 15% of the volume of the shell in the horizontal type and from 15 to 20% in the vertical.

Open versus Closed Feed-water Heaters. (W. E. Harrington, *St. Rwy. Jour.*, July 22, 1905.) — There still exists some difference of opinion as to the relative desirability of open or closed type of feed-water heater, but the degree of perfection which the open heater has attained has eliminated formerly objectionable features. The chief objection which attended the early use of the open heater, namely, that the oil from the exhaust steam was carried into the boiler, did much to discourage its more general adoption. This objection does not hold good against the better designs of open heaters now on the market. There are thousands of installations in which the open heater is now being used where no difficulty is experienced from the contamination of the feed-water by oil. The perfection of oil separators for use in the exhaust steam connection to the heater has rendered this possible.

STEAM SEPARATORS.

If moist steam flowing at a high velocity in a pipe has its direction suddenly changed, the particles of water are by their momentum projected in their original direction against the bend in the pipe or wall of the chamber in which the change of direction takes place. By making proper provision for drawing off the water thus separated the steam may be dried to a greater or less extent.

For long steam-pipes a large drum should be provided near the engine for trapping the water condensed in the pipe. A drum 3 feet diameter, 15 feet high, has given good results in separating the water of condensation of a steam-pipe 10 inches diameter and 800 feet long.

Efficiency of Steam Separators. — Prof. R. C. Carpenter, in 1891, made a series of tests of six steam separators, furnishing them with steam containing different percentages of moisture, and testing the quality of steam before entering and after passing the separator. A condensed table of the principal results is given below.

| Make of Separator. | Test with Steam of about 10% of Moisture. | | | Tests with Varying Moisture. | | |
|--------------------|---|-------------------------|-----------------------|------------------------------|-------------------------|-------------------|
| | Quality of Steam before. | Quality of Steam after. | Efficiency, per cent. | Quality of Steam before. | Quality of Steam after. | Av'ge Efficiency. |
| B | 87.0% | 98.8% | 90.8 | 66.1 to 97.5% | 97.8 to 99% | 87.6 |
| A | 90.1 | 98.0 | 80.0 | 51.9 " 98 | 97.9 " 99.1 | 76.4 |
| D | 89.6 | 95.8 | 59.6 | 72.2 " 96.1 | 95.5 " 98.2 | 71.7 |
| C | 90.6 | 93.7 | 33.0 | 67.1 " 96.8 | 93.7 " 98.4 | 63.4 |
| E | 88.4 | 90.2 | 15.5 | 68.6 " 98.1 | 79.3 " 98.5 | 36.9 |
| F | 88.9 | 92.1 | 28.8 | 70.4 " 97.7 | 84.1 " 97.9 | 28.4 |

Conclusions from the tests were: 1. That no relation existed between the volume of the several separators and their efficiency. 2. No marked decrease in pressure was shown by any of the separators, the most being 1.7 lbs. in E. 3. Although changed direction, reduced velocity, and perhaps centrifugal force are necessary for good separation, still some means must be provided to lead the water out of the current of the steam. The high efficiency obtained from B and A was largely due to this feature. In B the interior surfaces are corrugated and thus catch the water thrown out of the steam and readily lead it to the bottom. In A, as so on as the water falls or is precipitated from the steam, it comes in contact with the perforated diaphragm through which it runs into the space below, where it is not subjected to the action of the steam. Experiments made by Prof. Carpenter on a "Stratton" separator in 1894 showed that the moisture in the steam leaving the separator was less than 1% when that in the steam supplied ranged from 6% to 21%.

Experiments by Prof. G. F. Gebhardt (*Power*, May 11, 1909) on six separators of different makes led to the following conclusions: (1) The efficiency of separation decreases as the velocity of the steam increases. (2) The efficiency increases as the percentage of moisture in the entering steam increases. (3) The drop in pressure increases rapidly with the increase in velocity. The six separators are described as follows:

U: 2-in. vertical; no baffles; current reversed once.

V: 4-in. horizontal with single baffle plate of the fluted type; current reversed once.

W: 4-in. vertical with two baffle plates of the smooth type; current reversed once.

X: 3-in. horizontal; several fluted baffle plates; no reversal of current.

Y: 6-in. vertical; centrifugal type; current reversed once.

Z: 3-in. horizontal; current reversed twice; steam impinges on horizontal fluted baffle during reversal.

The efficiency is defined as the ratio of the water removed from the steam by the separator to the water injected into the dry steam for the purpose of the test. With steam at 100 lbs. pressure containing 10% water, the efficiencies, taken approximately from plotted curves, were as follows:

| | U | V | W | X | Y | Z |
|--------------------------|----|----|----|----|----|----|
| At 2000 ft. per min..... | 64 | 69 | 86 | 88 | 79 | 66 |
| At 3000 ft. per min..... | 37 | 45 | 80 | 60 | 61 | 48 |

DETERMINATION OF THE MOISTURE IN STEAM—STEAM CALORIMETERS.

In all boiler-tests it is important to ascertain the quality of the steam, i.e., 1st, whether the steam is "saturated" or contains the quantity of heat due to the pressure according to standard experiments; 2d, whether the quantity of heat is deficient, so that the steam is wet; and 3d, whether the heat is in excess and the steam superheated. The best method of ascertaining the quality of the steam is undoubtedly that employed by a committee which tested the boilers at the American Institute Exhibition of 1871-2, of which Prof. Thurston was chairman, i.e., condensing all the water evaporated by the boiler by means of a surface condenser, weighing

the condensing water, and taking its temperature as it enters and as it leaves the condenser; but this plan cannot always be adopted.

A substitute for this method is the barrel calorimeter, which with careful operation and fairly accurate instruments may generally be relied on to give results within two per cent of accuracy (that is, a sample of steam which gives the apparent result of 2% of moisture may contain anywhere between 0 and 4%). This calorimeter is described as follows: A sample of the steam is taken by inserting a perforated 1/2-inch pipe into and through the main pipe near the boiler, and led by a hose, thoroughly felted, to a barrel, holding preferably 400 lbs. of water, which is set upon a platform scale and provided with a cock or valve for allowing the water to flow to waste, and with a small propeller for stirring the water.

To operate the calorimeter the barrel is filled with water, the weight and temperature ascertained, steam blown through the hose outside the barrel until the pipe is thoroughly warmed, when the hose is suddenly thrust into the water, and the propeller operated until the temperature of the water is increased to the desired point, say about 110° usually. The hose is then withdrawn quickly, the temperature noted, and the weight again taken.

An error of 1/10 of a pound in weighing the condensed steam, or an error of 1/2 degree in the temperature, will cause an error of over 1% in the calculated percentage of moisture. See *Trans. A. S. M. E.*, vi, 293.

The calculation of the percentage of moisture is made as below:

$$Q = \frac{1}{H - T} \left[\frac{W}{w} (h_1 - h) - (T - h_1) \right]$$

Q = quality of the steam, dry saturated steam being unity.

H = total heat of 1 lb. of steam at the observed pressure.

T = total heat of 1 lb. of water at the temperature of steam of the observed pressure.

h = total heat of 1 lb. of condensing water, original.

h_1 = total heat of 1 lb. of condensing water, final.

W = weight of condensing water, corrected for water-equivalent of the apparatus.

w = weight of the steam condensed.

Percentage of moisture = $1 - Q$.

If Q is greater than unity, the steam is superheated, and the degrees of superheating = $2.0833 (H - T) (Q - 1)$.

Difficulty of Obtaining a Correct Sample. — Experiments by Prof. D. S. Jacobus (*Trans. A. S. M. E.*, xvi, 1017), show that it is practically impossible to obtain a true average sample of the steam flowing in a pipe. For accurate determinations all the steam made by the boiler should be passed through a separator, the water separated should be weighed and a calorimeter test made of the steam just after it has passed the separator.

Coil Calorimeters. — Instead of the open barrel in which the steam is condensed, a coil acting as a surface-condenser may be used, which is placed in the barrel, the water in coil and barrel being weighed separately. For a description of an apparatus of this kind designed by the author, which he has found to give results with a probable error not exceeding 1/2 per cent of moisture, see *Trans. A. S. M. E.*, vi, 294. This calorimeter may be used continuously, if desired, instead of intermittently. In this case a continuous flow of condensing water into and out of the barrel must be established, and the temperature of inflow and outflow and of the condensed steam read at short intervals of time.

Throttling Calorimeter. — For percentages of moisture not exceeding 3 per cent the throttling calorimeter is most useful and convenient and remarkably accurate. In this instrument the steam which reaches it in a 1/2-inch pipe is throttled by an orifice 1/16 inch diameter, opening into a chamber which has an outlet to the atmosphere. The steam in this chamber has its pressure reduced nearly or quite to the pressure of the atmosphere, but the total heat in the steam before throttling causes the steam in the chamber to be superheated more or less according to whether the steam before throttling was dry or contained moisture. The only observations required are those of the temperature and pressure of the steam on each side of the orifice.

The author's formula for reducing the observations of the throttling calorimeter is as follows (Experiments on Throttling Calorimeters, *Am. Mach.*, Aug. 4, 1892): $w = 100 \times \frac{H - h - K(T - t)}{L}$, in which w = percentage of moisture in the steam; H = total heat, and L = latent heat of steam in the main pipe; h = total heat due the pressure in the discharge side of the calorimeter, = 1146.6 at atmospheric pressure; K = specific heat of superheated steam; T = temperature of the throttled and superheated steam in the calorimeter; t = temperature due to the pressure in the calorimeter, = 212° at atmospheric pressure.

Taking K at 0.48 and the pressure in the discharge side of the calorimeter as atmospheric pressure, the formula becomes

$$w = 100 \times \frac{H - 1146.6 - 0.48(T - 212^\circ)}{L}$$

From this formula the following table is calculated:

MOISTURE IN STEAM — DETERMINATIONS BY THROTTLING CALORIMETER

| Degree of Super-heating $T - 212^\circ$. | Gauge-pressures. | | | | | | | | | | | |
|--|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 5 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 75 | 80 | 85 | 90 |
| | Per Cent of Moisture in Steam. | | | | | | | | | | | |
| 0° | 0.51 | 0.90 | 1.54 | 2.06 | 2.50 | 2.90 | 3.24 | 3.56 | 3.71 | 3.86 | 3.99 | 4.13 |
| 10° | 0.01 | 0.39 | 1.02 | 1.54 | 1.97 | 2.36 | 2.71 | 3.02 | 3.17 | 3.32 | 3.45 | 3.58 |
| 20° | | | 0.51 | 1.02 | 1.45 | 1.83 | 2.17 | 2.48 | 2.63 | 2.77 | 2.90 | 3.03 |
| 30° | | | 0.00 | 0.50 | 0.92 | 1.30 | 1.64 | 1.94 | 2.09 | 2.23 | 2.35 | 2.49 |
| 40° | | | | | 0.39 | 0.77 | 1.10 | 1.40 | 1.55 | 1.69 | 1.80 | 1.94 |
| 50° | | | | | | 0.24 | 0.57 | 0.87 | 1.01 | 1.15 | 1.26 | 1.40 |
| 60° | | | | | | | 0.03 | 0.33 | 0.47 | 0.60 | 0.72 | 0.85 |
| 70° | | | | | | | | | | 0.06 | 0.17 | 0.31 |
| Dif. p. deg. | .0503 | .0507 | .0515 | .0521 | .0526 | .0531 | .0535 | .0539 | .0541 | .0542 | .0544 | .0546 |

| Degree of Super-heating $T - 212^\circ$. | Gauge-pressures. | | | | | | | | | | | |
|--|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 | 250 |
| | Per Cent of Moisture in Steam. | | | | | | | | | | | |
| 0° | 4.39 | 4.63 | 4.85 | 5.08 | 5.29 | 5.49 | 5.68 | 5.87 | 6.05 | 6.22 | 6.39 | 7.16 |
| 10° | 3.84 | 4.08 | 4.29 | 4.52 | 4.73 | 4.93 | 5.12 | 5.30 | 5.48 | 5.65 | 5.82 | 6.58 |
| 20° | 3.29 | 3.52 | 3.74 | 3.96 | 4.17 | 4.37 | 4.56 | 4.74 | 4.91 | 5.08 | 5.25 | 6.00 |
| 30° | 2.74 | 2.97 | 3.18 | 3.41 | 3.61 | 3.80 | 3.99 | 4.17 | 4.34 | 4.51 | 4.67 | 5.41 |
| 40° | 2.19 | 2.42 | 2.63 | 2.85 | 3.05 | 3.24 | 3.43 | 3.61 | 3.78 | 3.94 | 4.10 | 4.83 |
| 50° | 1.64 | 1.87 | 2.08 | 2.29 | 2.49 | 2.68 | 2.87 | 3.04 | 3.21 | 3.37 | 3.53 | 4.25 |
| 60° | 1.09 | 1.32 | 1.52 | 1.74 | 1.93 | 2.12 | 2.30 | 2.48 | 2.64 | 2.80 | 2.96 | 3.67 |
| 70° | 0.55 | 0.77 | 0.97 | 1.18 | 1.38 | 1.56 | 1.74 | 1.91 | 2.07 | 2.23 | 2.38 | 3.09 |
| 80° | 0.00 | 0.22 | 0.42 | 0.63 | 0.82 | 1.00 | 1.18 | 1.34 | 1.50 | 1.66 | 1.81 | 2.51 |
| 90° | | | | 0.07 | 0.26 | 0.44 | 0.61 | 0.78 | 0.94 | 1.09 | 1.24 | 1.93 |
| 100° | | | | | | | 0.05 | 0.21 | 0.37 | 0.52 | 0.67 | 1.34 |
| 110° | | | | | | | | | | | 0.10 | 0.76 |
| Dif. p. deg. | .0549 | .0551 | .0554 | .0556 | .0559 | .0561 | .0564 | .0566 | .0568 | .0570 | .0572 | .0581 |

Separating Calorimeters. — For percentages of moisture beyond the range of the throttling calorimeter the separating calorimeter is used,

which is simply a steam separator on a small scale. An improved form of this calorimeter is described by Prof. Carpenter in *Power*, Feb., 1893.

For fuller information on various kinds of calorimeters, see papers by Prof. Peabody, Prof. Carpenter, and Mr. Barrus in *Trans. A. S. M. E.*, vols. x, xi, xii, 1889 to 1891; Appendix to Report of Com. on Boiler Tests, *A. S. M. E.*, vol. vi, 1884; Circular of Schaeffer & Budenberg, N. Y., "Calorimeters, Throttling and Separating."

Identification of Dry Steam by Appearance of a Jet. — Prof. Denton (*Trans. A. S. M. E.*, vol. x) found that jets of steam show unmistakable change of appearance to the eye when steam varies less than 1% from the condition of saturation in the direction of either wetness or of superheating.

If a jet of steam flow from a boiler into the atmosphere under circumstances such that very little loss of heat occurs through radiation, etc., and the jet be transparent close to the orifice, or be even a grayish-white color, the steam may be assumed to be so nearly dry that no portable condensing calorimeter will be capable of measuring the amount of water in the steam. If the jet be strongly white, the amount of water may be roughly judged up to about 2%, but beyond this only a calorimeter can determine the exact amount of moisture.

A common brass pet-cock may be used as an orifice, but it should, if possible, be set into the steam-drum of the boiler and never be placed further away from the latter than 4 feet, and then only when the intermediate reservoir or pipe is well covered.

Usual Amount of Moisture in Steam Escaping from a Boiler. — In the common forms of horizontal tubular land boilers and water-tube boilers with ample horizontal drums, and supplied with water free from substances likely to cause foaming, the moisture in the steam does not generally exceed 2% unless the boiler is overdriven or the water-level is carried too high.

CHIMNEYS.

Chimney Draught Theory. — The commonly accepted theory of chimney draught, based on Peclet's and Rankine's hypotheses (Rankine, *S. E.*), is discussed by Prof. De Volson Wood, *Trans. A. S. M. E.*, vol. xi.

Peclet represented the law of draught by the formula

$$h = \frac{u^2}{2g} \left(1 + G + \frac{fl}{m} \right)$$

in which h is the "head," defined as such a height of hot gases as, if added to the column of gases in the chimney, would produce the same pressure at the furnace as a column of outside air, of the same area of base, and a height equal to that of the chimney;

u is the required velocity of gases in the chimney;

G a constant to represent the resistance to the passage of air through the coal;

l the length of the flues and chimney;

m the mean hydraulic depth or the area of a cross-section divided by the perimeter;

f a constant depending upon the nature of the surfaces over which the gases pass, whether smooth, or sooty and rough.

Rankine's formula (*Steam Engine*, p. 288), derived by giving certain values to the constants (so-called) in Peclet's formula, is

$$h = \frac{\frac{\tau_0}{\tau_2} (0.0807)}{\frac{\tau_0}{\tau_1} (0.084)} H - H = \left(0.96 \frac{\tau_1}{\tau_2} - 1 \right) H;$$

in which H = the height of the chimney in feet;

τ_0 = 493° F., absolute (temperature of melting ice);

τ_1 = absolute temperature of the gases in the chimney;

τ_2 = absolute temperature of the external air,

Prof. Wood derives from this a still more complex formula which gives the height of chimney required for burning a given quantity of coal per second, and from it he calculates the following table, showing the height of chimney required to burn respectively 24, 20, and 16 lbs. of coal per square foot of grate per hour, for the several temperatures of the chimney gases given.

| Outside Air. τ_2 . | Chimney Gas. | | Coal per sq. ft. of grate per hour, lbs. | | |
|----------------------------|-----------------------|----------------|--|-------|------|
| | τ_1 Absolute. | Temp. Fahr. | 24 | 20 | 16 |
| | | | Height H , feet. | | |
| 520° | 700 | 239 | 250.9 | 157.6 | 67.8 |
| absolute or | 800 | 339 | 172.4 | 115.8 | 55.7 |
| 59° F. | 1000 | 539 | 149.1 | 100.0 | 48.7 |
| | 1100 | 639 | 148.8 | 98.9 | 48.2 |
| | 1200 | 739 | 152.0 | 100.9 | 49.1 |
| | 1400 | 939 | 159.9 | 105.7 | 51.2 |
| | 1600 | 1139 | 168.8 | 111.0 | 53.5 |
| | 2000 | 1539 | 206.5 | 132.2 | 63.0 |

Rankine's formula gives a maximum draught when $\tau = 2\frac{1}{12}\tau_2$, or 622° F., when the outside temperature is 60°. Prof. Wood says: "This result is not a fixed value, but departures from theory in practice do not affect the result largely. There is, then, in a properly constructed chimney properly working, a temperature giving a maximum draught,* and that temperature is not far from the value given by Rankine, although in special cases it may be 50° or 75° more or less."

All attempts to base a practical formula for chimneys upon the theoretical formula of Peclet and Rankine have failed on account of the impossibility of assigning correct values to the so-called "constants" G and f . (See *Trans. A. S. M. E.*, xi, 984.)

Force or Intensity of Draught. — The force of the draught is equal to the difference between the weight of the column of hot gases inside of the chimney and the weight of a column of the external air of the same height. It is measured by a draught-gauge, usually a U-tube partly filled with water, one leg connected by a pipe to the interior of the flue, and the other open to the external air.

If D is the density of the air outside, d the density of the hot gas inside, in lbs. per cubic foot, h the height of the chimney in feet, and 0.192 the factor for converting pressure in lbs. per sq. ft. into inches of water column, then the formula for the force of draught expressed in inches of water is,

$$F = 0.192 h (D - d).$$

The density varies with the absolute temperature (see Rankine).

$$d = \frac{\tau_0}{\tau_1} 0.084; D = 0.0807 \frac{\tau_0}{\tau_2},$$

where τ_0 is the absolute temperature at 32° F., = 493, τ_1 the absolute temperature of the chimney gases and τ_2 that of the external air. Substituting these values the formula for force of draught becomes

$$F = 0.192 h \left(\frac{39.79}{\tau_2} - \frac{41.41}{\tau_1} \right) = h \left(\frac{7.64}{\tau_2} - \frac{7.95}{\tau_1} \right).$$

* Much confusion to students of the theory of chimneys has resulted from their understanding the words maximum draught to mean maximum intensity or pressure of draught, as measured by a draught-gauge. It here means maximum quantity or weight of gases passed up the chimney. The maximum intensity is found only with maximum temperature, but after the temperature reaches about 622° F. the density of the gas decreases more rapidly than its velocity increases, so that the weight is a maximum about 622° F., as shown by Rankine. — W. K.

To find the maximum intensity of draught for any given chimney, the heated column being 600° F., and the external air 60°, multiply the height above grate in feet by 0.0073, and the product is the draught in inches of water.

Height of Water Column Due to Unbalanced Pressure in Chimney 100 Feet High. (*The Locomotive*, 1884.)

| Temp. in the Chimney. | Temperature of the External Air — Barometer, 14.7 lbs. per sq. in. | | | | | | | | | | |
|-----------------------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° | 100° |
| 200 | 0.453 | 0.419 | 0.384 | 0.353 | 0.321 | 0.292 | 0.263 | 0.234 | 0.209 | 0.182 | 0.157 |
| 220 | .488 | .453 | .419 | .388 | .355 | .326 | .298 | .269 | .244 | .217 | .192 |
| 240 | .520 | .488 | .451 | .421 | .388 | .359 | .330 | .301 | .276 | .250 | .225 |
| 260 | .555 | .528 | .484 | .453 | .420 | .392 | .363 | .334 | .309 | .282 | .257 |
| 280 | .584 | .549 | .515 | .482 | .451 | .422 | .394 | .365 | .340 | .313 | .288 |
| 300 | .611 | .576 | .541 | .511 | .478 | .449 | .420 | .392 | .367 | .340 | .315 |
| 320 | .637 | .603 | .568 | .538 | .505 | .476 | .447 | .419 | .394 | .367 | .342 |
| 340 | .662 | .638 | .593 | .563 | .530 | .501 | .472 | .443 | .419 | .392 | .367 |
| 360 | .687 | .653 | .618 | .588 | .555 | .526 | .497 | .468 | .444 | .417 | .392 |
| 380 | .710 | .676 | .641 | .611 | .578 | .549 | .520 | .492 | .467 | .440 | .415 |
| 400 | .732 | .697 | .662 | .632 | .598 | .570 | .541 | .513 | .488 | .461 | .436 |
| 420 | .753 | .718 | .684 | .653 | .620 | .591 | .563 | .534 | .509 | .482 | .457 |
| 440 | .774 | .739 | .705 | .674 | .641 | .612 | .584 | .555 | .530 | .503 | .478 |
| 460 | .793 | .758 | .724 | .694 | .660 | .632 | .603 | .574 | .549 | .522 | .497 |
| 480 | .810 | .776 | .741 | .710 | .678 | .649 | .620 | .591 | .566 | .540 | .515 |
| 500 | .829 | .791 | .760 | .730 | .697 | .669 | .639 | .610 | .586 | .559 | .534 |

For any other height of chimney than 100 ft. the height of water column is found by simple proportion, the height of water column being directly proportioned to the height of chimney.

The calculations have been made for a chimney 100 ft. high, with various temperatures outside and inside of the flue, and on the supposition that the temperature of the chimney is uniform from top to bottom. This is the basis on which all calculations respecting the draught-power of chimneys have been made by Rankine and other writers, but it is very far from the truth in most cases. The difference will be shown by comparing the reading of the draught-gauge with the table given. In one case a chimney 122 ft. high showed a temperature at the base of 320°, and at the top of 230°.

Box, in his "Treatise on Heat," gives the following table:

DRAUGHT POWERS OF CHIMNEYS, ETC., WITH THE INTERNAL AIR AT 552° AND THE EXTERNAL AIR AT 62°, AND WITH THE DAMPER NEARLY CLOSED.

| Height of Chimney in feet. | Draught Power in ins. of water. | Theoretical Velocity in feet per second. | | Height of Chimney in feet. | Draught Power in ins. of water. | Theoretical Velocity in feet per second. | |
|----------------------------|---------------------------------|--|------------------|----------------------------|---------------------------------|--|------------------|
| | | Cold Air Entering. | Hot Air at Exit. | | | Cold Air Entering. | Hot Air at Exit. |
| 10 | 0.073 | 17.8 | 35.6 | 80 | 0.585 | 50.6 | 101.2 |
| 20 | 0.146 | 25.3 | 50.6 | 90 | 0.657 | 53.7 | 107.4 |
| 30 | 0.219 | 31.0 | 62.0 | 100 | 0.730 | 56.5 | 113.0 |
| 40 | 0.292 | 35.7 | 71.4 | 120 | 0.876 | 62.0 | 124.0 |
| 50 | 0.365 | 40.0 | 80.0 | 150 | 1.095 | 69.3 | 138.6 |
| 60 | 0.438 | 43.8 | 87.6 | 175 | 1.277 | 74.3 | 149.6 |
| 70 | 0.511 | 47.3 | 94.6 | 200 | 1.460 | 80.0 | 160.0 |

Rate of Combustion Due to Height of Chimney. — Trowbridge's "Heat and Heat Engines" gives the following figures for the heights of chimney for producing certain rates of combustion per sq. ft. of grate. They may be approximately true for anthracite in moderate and large sizes, but greater heights than are given in the table are needed to secure the given rates of combustion with small sizes of anthracite, and for bituminous coal smaller heights will suffice if the coal is reasonably free from ash — 5% or less.

| Height, feet. | Lbs. of Coal per Sq. Ft. of Grate. | Height, feet. | Lbs. of Coal per Sq. Ft. of Grate. | Height, feet. | Lbs. of Coal per Sq. Ft. of Grate. | Height, feet. | Lbs. of Coal per Sq. Ft. of Grate. |
|---------------|------------------------------------|---------------|------------------------------------|---------------|------------------------------------|---------------|------------------------------------|
| 20 | 7.5 | 45 | 12.4 | 70 | 15.8 | 95 | 18.5 |
| 25 | 8.5 | 50 | 13.1 | 75 | 16.4 | 100 | 19.0 |
| 30 | 9.5 | 55 | 13.8 | 80 | 16.9 | 105 | 19.5 |
| 35 | 10.5 | 60 | 14.5 | 85 | 17.4 | 110 | 20.0 |
| 40 | 11.6 | 65 | 15.1 | 90 | 18.0 | | |

W. D. Ennis (*Eng. Mag.*, Nov., 1907), gives the following as the force of draught required for burning No. 1 buckwheat coal:

| | | | | |
|---|-----|------|-----|-----|
| Draught, in. of water | 0.3 | 0.45 | 0.7 | 1.0 |
| Lbs. coal per sq. ft. grate per hour..... | 10 | 15 | 20 | 25 |

Thurston's rule for rate of combustion effected by a given height of chimney (*Trans. A. S. M. E.*, xi, 991) is: Subtract 1 from twice the square root of the height, and the result is the rate of combustion in pounds per square foot of grate per hour, for anthracite. Or rate = $2\sqrt{h} - 1$, in which h is the height in feet. This rule gives the following:

| | | | | | | | | | | | |
|-------------------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|
| $h =$ | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 125 | 150 | 175 | 200 |
| $2\sqrt{h} - 1 =$ | 13.14 | 14.49 | 15.73 | 16.89 | 17.97 | 19 | 19.97 | 21.36 | 23.49 | 25.45 | 27.28 |

The results agree closely with Trowbridge's table given above. In practice the high rates of combustion for high chimneys given by the formula are not generally obtained, for the reason that with high chimneys there are usually long horizontal flues, serving many boilers, and the friction and the interference of currents from the several boilers are apt to cause the intensity of draught in the branch flues leading to each boiler to be much less than that at the base of the chimney. The draught of each boiler is also usually restricted by a damper and by bends in the gas-passages. In a battery of several boilers connected to a chimney 150 ft. high, the author found a draught of $\frac{3}{4}$ -inch water-column at the boiler nearest the chimney, and only $\frac{1}{4}$ -inch at the boiler farthest away. The first boiler was wasting fuel from too high temperature of the chimney-gases, 900°, having too large a grate-surface for the draught, and the last boiler was working below its rated capacity and with poor economy, on account of insufficient draught.

The effect of changing the length of the flue leading into a chimney 60 ft. high and 2 ft. 9 in. square is given in the following table, from Box on "Heat":

| Length of Flue in feet. | Horse-power. | Length of Flue in feet. | Horse-power. |
|-------------------------|--------------|-------------------------|--------------|
| 50 | 107.6 | 800 | 56.1 |
| 100 | 100.0 | 1,000 | 51.4 |
| 200 | 85.3 | 1,500 | 43.3 |
| 400 | 70.8 | 2,000 | 38.2 |
| 600 | 62.5 | 3,000 | 31.7 |

The temperature of the gases in this chimney was assumed to be 552° F., and that of the atmosphere 62°.

High Chimneys not Necessary. — Chimneys above 150 ft. in height are very costly, and their increased cost is rarely justified by increased efficiency. In recent practice it has become somewhat common to build two or more smaller chimneys instead of one large one. A notable example is the Spreckels Sugar Refinery in Philadelphia, where three separate chimneys are used for one boiler-plant of 7500 H.P. The three chimneys are said to have cost several thousand dollars less than a single chimney of their combined capacity would have cost. Very tall chimneys have been characterized by one writer as "monuments to the folly of their builders."

Heights of Chimney required for Different Fuels. — The minimum height necessary varies with the fuel, wood requiring the least, then good bituminous coal, and fine sizes of anthracite the greatest. It also varies with the character of the boiler — the smaller and more circuitous the gas-passages the higher the stack required; also with the number of boilers, a single boiler requiring less height than several that discharge into a horizontal flue. No general rule can be given.

C. L. Hubbard (*Am. Electrician*, Mar., 1904) says: The following heights have been found to give good results in plants of moderate size, and to produce sufficient draught to force the boilers from 20 to 30 per cent above their rating:

With free-burning bituminous coal, 75 feet; with anthracite of medium and large size, 100 feet; with slow-burning bituminous coal, 120 feet; with anthracite pea coal, 130 feet; with anthracite buckwheat coal, 150 feet. For plants of 700 or 800 horse-power and over, the chimney should not be less than 150 feet high regardless of the kind of coal to be used.

SIZE OF CHIMNEYS.

The formula given below, and the table calculated therefrom for chimneys up to 96 in. diameter and 200 ft. high, were first published by the author in 1884 (*Trans. A. S. M. E.*, vi, 81). They have met with much approval since that date by engineers who have used them, and have been frequently published in boiler-makers' catalogues and elsewhere. The table is now extended to cover chimneys up to 12 ft. diameter and 300 ft. high. The sizes corresponding to the given commercial horse-powers are believed to be ample for all cases in which the draught areas through the boiler-flues and connections are sufficient, say not less than 20% greater than the area of the chimney, and in which the draught between the boilers and chimney is not checked by long horizontal passages and right-angled bends.

Note that the figures in the table correspond to a coal consumption of 5 lbs. of coal per horse-power per hour. This liberal allowance is made to cover the contingencies of poor coal being used, and of the boilers being driven beyond their rated capacity. In large plants, with economical boilers and engines, good fuel and other favorable conditions, which will reduce the maximum rate of coal consumption at any one time to less than 5 lbs. per H.P. per hour, the figures in the table may be multiplied by the ratio of 5 to the maximum expected coal consumption per H.P. per hour. Thus, with conditions which make the maximum coal consumption only 2.5 lbs. per hour, the chimney 300 ft. high \times 12 ft. diameter should be sufficient for $6155 \times 2 = 12,310$ horse-power. The formula is based on the following data:

1. The draught power of the chimney varies as the square root of the height.

2. The retarding of the ascending gases by friction may be considered as equivalent to a diminution of the area of the chimney, or to a lining of the chimney by a layer of gas which has no velocity. The thickness of this lining is assumed to be 2 inches for all chimneys, or the diminution of area equal to the perimeter \times 2 inches (neglecting the overlapping of the corners of the lining). Let D = diameter in feet, A = area, and E = effective area in square feet:

$$\text{For square chimneys, } E = D^2 - \frac{8D}{12} = A - \frac{2}{3}\sqrt{A}.$$

$$\text{For round chimneys, } E = \frac{\pi}{4} \left(D^2 - \frac{8D}{12} \right) = A - 0.591\sqrt{A}.$$

For simplifying calculations, the coefficient of \sqrt{A} may be taken as 0.6 for both square and round chimneys, and the formula becomes

$$E = A - 0.6 \sqrt{A}.$$

3. The power varies directly as this effective area E .

4. A chimney should be proportioned so as to be capable of giving sufficient draught to cause the boiler to develop much more than its rated power, in case of emergencies, or to cause the combustion of 5 lbs. of fuel per rated horse-power of boiler per hour.

5. The power of the chimney varying directly as the effective area, E , and as the square root of the height, H , the formula for horse-power of boiler for a given size of chimney will take the form $H.P. = CE\sqrt{H}$, in which C is a constant, the average value of which, obtained by plotting the results obtained from numerous examples in practice, the author finds to be 3.33.

The formula for horse-power then is

$$H.P. = 3.33 E \sqrt{H}, \text{ or } H.P. = 3.33 (A - 0.6 \sqrt{A}) \sqrt{H}.$$

If the horse-power of boiler is given, to find the size of chimney, the height being assumed,

$$E = 0.3 H.P. \div \sqrt{H}; = A - 0.6 \sqrt{A}.$$

For round chimneys, diameter of chimney = diam. of $E + 4"$.

For square chimneys, side of chimney = $\sqrt{E} + 4"$.

If effective area E is taken in square feet, the diameter in inches is $d = 13.54 \sqrt{E} + 4"$, and the side of a square chimney in inches is $s = 12 \sqrt{E} + 4"$.

If horse-power is given and area assumed, the height $H = \left(\frac{0.3 H.P.}{E} \right)^2$.

An approximate formula for chimneys above 1000 H.P. is $H.P. = 2.5 D^2 \sqrt{H}$. This gives the H.P. somewhat greater than the figures in the table.

In proportioning chimneys the height should first be assumed, with due consideration of the heights of surrounding buildings or hills near to the proposed chimney, the length of horizontal flues, the character of coal to be used, etc.; then the diameter required for the assumed height and horse-power is calculated by the formula or taken from the table.

For Height of Chimneys see pages 918 and 919. No formula for height can be given which will be satisfactory for different classes of coal, kinds and amounts of ash, styles of grate-bars, etc. A formula in "Ingenieurs Taschenbuch," translated into English measures, is $h = 0.216 R^2 + 6 d$. h = height in ft; R = lbs. coal burned per sq. ft. of grate per hour; d = diam. in ft. This formula gives an insufficient height for small sizes of anthracite, and a height greater than is necessary for free-burning bituminous coal low in ash.

The Protection of Tall Chimney-shafts from Lightning. — C. Molyneux and J. M. Wood (*Industries*, March 28, 1890) recommend for tall chimneys the use of a coronal or heavy band at the top of the chimney, with copper points 1 ft. in height at intervals of 2 ft. throughout the circumference. The points should be gilded to prevent oxidation. The most approved form of conductor is a copper tape about $\frac{3}{4}$ in. by $\frac{1}{8}$ in. thick, weighing 6 ozs. per ft. If iron is used it should weigh not less than $2\frac{1}{4}$ lbs. per ft. There must be no insulation, and the copper tape should be fastened to the chimney with holdfasts of the same material, to prevent voltaic action. An allowance for expansion and contraction should be made, say 1 in. in 40 ft. Slight bends in the tape, not too abrupt, answer the purpose. For an earth terminal a plate of metal at least 3 ft. sq. and $\frac{1}{16}$ in. thick should be buried as deep as possible in a damp spot. The plate should be of the same metal as the conductor, to which it should be soldered. The best earth terminal is water, and when a deep well or other large body of water is at hand, the conductor should be carried down into it. Right-angled bends in the conductor should be avoided. No bend in it should be over 30° .

Size of Chimneys for Steam-boilers.

Formula, H.P. = $3.33 (A - 0.6 \sqrt{A}) \sqrt{H}$. (Assuming 1 H.P. = 5 lbs. of coal burned per hour.)

| Diam. inches. | Area A. sq. ft. | Effective Area. $E = A - \frac{1}{0.6 \sqrt{A}}$ sq. ft. | Height of Chimney. | | | | | | | | | | | | | Equivalent Square Chimney. Side of Square $\sqrt{E + 4 \text{ ins.}}$ | |
|------------------|--------------------|---|-----------------------------------|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|---------|---------|--|---------|
| | | | Commercial Horse-power of Boiler. | | | | | | | | | | | | | | |
| | | | 50 ft. | 60 ft. | 70 ft. | 80 ft. | 90 ft. | 100 ft. | 110 ft. | 125 ft. | 150 ft. | 175 ft. | 200 ft. | 225 ft. | 250 ft. | | 300 ft. |
| 18 | 1.77 | 0.97 | 23 | 25 | 27 | 29 | | | | | | | | | | | 16 |
| 21 | 2.41 | 1.47 | 35 | 38 | 41 | 44 | | | | | | | | | | | 19 |
| 24 | 3.14 | 2.08 | 49 | 54 | 58 | 62 | 66 | | | | | | | | | | 22 |
| 27 | 3.98 | 2.78 | 65 | 72 | 78 | 83 | 88 | | | | | | | | | | 24 |
| 30 | 4.91 | 3.58 | 84 | 92 | 100 | 107 | 113 | 119 | | | | | | | | | 27 |
| 33 | 5.94 | 4.48 | | 115 | 125 | 133 | 141 | 149 | 156 | | | | | | | | 30 |
| 36 | 7.07 | 5.47 | | 141 | 152 | 163 | 173 | 182 | 191 | 204 | | | | | | | 32 |
| 39 | 8.30 | 6.57 | | | 183 | 196 | 208 | 219 | 229 | 245 | 268 | | | | | | 35 |
| 42 | 9.62 | 7.76 | | | 216 | 231 | 245 | 258 | 271 | 289 | 316 | 342 | | | | | 38 |
| 48 | 12.57 | 10.44 | | | | 311 | 330 | 348 | 365 | 389 | 426 | 460 | 492 | | | | 43 |
| 54 | 15.90 | 13.51 | | | | | 427 | 449 | 472 | 503 | 551 | 595 | 636 | 675 | | | 48 |
| 60 | 19.64 | 16.98 | | | | | 536 | 565 | 593 | 632 | 692 | 748 | 800 | 848 | 894 | | 54 |
| 66 | 23.76 | 20.83 | | | | | | 694 | 728 | 776 | 849 | 918 | 981 | 1040 | 1097 | 1201 | 59 |
| 72 | 28.27 | 25.08 | | | | | | 835 | 876 | 934 | 1023 | 1105 | 1181 | 1253 | 1320 | 1417 | 64 |
| 78 | 33.18 | 29.73 | | | | | | | 1038 | 1107 | 1212 | 1310 | 1400 | 1485 | 1565 | 1715 | 70 |
| 84 | 38.48 | 34.76 | | | | | | | 1214 | 1294 | 1418 | 1531 | 1637 | 1736 | 1830 | 2005 | 75 |
| 90 | 44.18 | 40.19 | | | | | | | | 1496 | 1639 | 1770 | 1893 | 2008 | 2116 | 2318 | 80 |
| 96 | 50.27 | 46.01 | | | | | | | | 1712 | 1876 | 2027 | 2167 | 2298 | 2423 | 2654 | 86 |
| 102 | 56.75 | 52.23 | | | | | | | | 1944 | 2130 | 2300 | 2459 | 2609 | 2750 | 3012 | 91 |
| 108 | 63.62 | 58.83 | | | | | | | | 2090 | 2399 | 2592 | 2771 | 2939 | 3098 | 3393 | 96 |
| 114 | 70.88 | 65.83 | | | | | | | | | 2685 | 2900 | 3100 | 3288 | 3466 | 3797 | 101 |
| 120 | 78.54 | 73.22 | | | | | | | | | 2986 | 3226 | 3448 | 3657 | 3855 | 4223 | 107 |
| 132 | 95.03 | 89.18 | | | | | | | | | 3637 | 3929 | 4200 | 4453 | 4696 | 5144 | 117 |
| 144 | 113.10 | 106.72 | | | | | | | | | 4352 | 4701 | 5026 | 5331 | 5618 | 6155 | 128 |

For pounds of coal burned per hour for any given size of chimney, multiply the figures in the table by 5.

Some Tall Brick Chimneys (1895).

| | Height. | Internal Diam. | Outside Diameter. | | Capacity by the Author's Formula. | |
|---|---------|----------------|-------------------|-------|-----------------------------------|-----------------------|
| | | | Base. | Top. | H. P. | Pounds Coal per Hour. |
| 1. Hallsbrückner Hütte, Saxony..... | 460 | 15.7' | 33' | 16' | 13,221 | 66,105 |
| 2. Townsend's, Glasgow..... | 454 | | 32 | | | |
| 3. Tennant's, Glasgow..... | 435 | 13' 6" | 40 | | 9,795 | 48,975 |
| 4. Dobson & Barlow, Bolton, Eng..... | 367 1/2 | 13' 2" | 33' 10" | | 8,245 | 41,225 |
| 5. Fall River Iron Co., Boston..... | 350 | 11 | 30 | 21 | 5,558 | 27,790 |
| 6. Clark Thread Co., Newark, N. J..... | 335 | 11 | 28' 6" | 14 | 5,435 | 27,175 |
| 7. Merrimac Mills, Lowell, Mass..... | 282' 9" | 12 | | | 5,980 | 29,900 |
| 8. Washington Mills, Lawrence, Mass..... | 250 | 10 | | | 3,839 | 19,195 |
| 9. Amoskeag Mills, Manchester, N. H..... | 250 | 10 | | | 3,839 | 19,195 |
| 10. Narragansett E. L. Co., Providence, R. I..... | 238 | 14 | | | 7,515 | 37,575 |
| 11. Lower Pacific Mills, Lawrence, Mass..... | 214 | 8 | | | 2,248 | 11,240 |
| 12. Passaic Print Works, Passaic, N. J..... | 200 | 9 | | | 2,771 | 13,855 |
| 13. Edison Station Brooklyn, Two each..... | 150 | 50" x 120" | | each | 1,541 | 7,705 |

NOTES ON THE ABOVE CHIMNEYS. — 1. This chimney is situated near Freiberg, at an elevation of 219 ft. above that of the foundry works, so that its total height above the sea will be 711 3/4 ft. The furnace-gases are conveyed across river to the chimney on a bridge, through a pipe 3227 ft. long. It is built of brick, and cost about \$40,000. — *Mfr. & Blwr.*

2. Owing to the fact that it was struck by lightning, and somewhat damaged, as a precautionary measure a copper extension subsequently was added to it, making its entire height 488 feet.

1, 2, 3, and 4 were built of these great heights to remove deleterious gases from the neighborhood, as well as for draught for boilers.

5. The structure rests on a solid granite foundation, 55 x 30 feet, and 16 feet deep. In its construction there were used 1,700,000 bricks, 2000 tons of stone, 2000 barrels of mortar, 1000 loads of sand, 1000 barrels of Portland cement, and the estimated cost is \$40,000. It is arranged for two flues, 9 feet 6 inches by 6 feet, connecting with 40 boilers, which are to be run in connection with four triple-expansion engines of 1350 horsepower each.

6. It has a uniform batter of 2.85 ins. to every 10 ft. Designed for 21 boilers of 200 H.P. each. It is surmounted by a cast-iron coping which weighs six tons, and is composed of 32 sections bolted together by inside flanges so as to present a smooth exterior. The foundation is 40 ft. square and 5 ft. deep. Two qualities of brick were used: the outer portions were of the first quality North River, and the backing up was of good quality New Jersey brick. Every twenty feet in vertical measurement an iron ring, 4 ins. wide and 3/4 to 1/2 in. thick, placed edge-wise, was built into the walls about 8 ins. from the outer circle. As the chimney starts from the base it is double. The outer wall is 5 ft. 2 ins. in thickness, and inside of this is a second wall 20 ins. thick and spaced

off about 20 ins. from main wall. From the interior surface of the main wall eight buttresses are carried, nearly touching this inner or main flue wall in order to keep it in line should it tend to sag. The interior wall, starting with the thickness described, is gradually reduced until a height of about 90 ft. is reached, when it is diminished to 8 inches. At 165 ft. it ceases, and the rest of the chimney is without lining. The total weight of the chimney and foundation is 5000 tons. It was completed in September, 1888.

7. Connected to 12 boilers, with 1200 sq. ft. of grate. Draught $1\frac{9}{16}$ ins.
8. Connected to 8 boilers, 6 ft. 8 in. diam. \times 18 ft. Grate 448 sq. ft.
9. Connected to 64 Manning vertical boilers, total grate surface 1810 sq. ft. Designed to burn 18,000 lbs. anthracite per hour.
10. Designed for 12,000 H.P. of engines; (compound condensing).
11. Grate-surface 434 square feet; H.P. of boilers about 2500.
13. Eight boilers (water-tube) each 450 H.P.; 12 engines, each 300 H.P. For the first 60 feet the exterior wall is 28 ins. thick, then 24 ins. for 20 ft., 20 ins. for 30 ft., 16 ins. for 20 ft., and 12 ins. for 20 ft. The interior wall is 9 ins. thick of fire-brick for 50 ft., and then 8 ins. thick of red brick for the next 30 ft. Illustrated in *Iron Age*, Jan. 2, 1890.

A number of the above chimneys are illustrated in *Power*, Dec., 1890.

More Recent Brick Chimneys (1909). — Heller & Merz Co., Newark, N. J. 350 ft. high, inside diam., 8 ft. Outside diam., top 9 ft. $10\frac{1}{4}$ in., bottom 27 ft. $6\frac{1}{2}$ in. Outside taper 5.2 in 100. Outer shell $7\frac{1}{8}$ in. at the top, 38 in. at the bottom. Custodis radial brick laid in mortar of 1 cement, 2 lime, 5 sand. The changes in thickness are made by 2-in. offsets on the inside every 20 ft. Iron band $3\frac{1}{2} \times \frac{5}{16}$ in., three courses below the top. Lined with 4 in. of special brick to resist acids. The lining is sectional, being carried on corbels projecting from the shell every 20 ft. An air space of 2 ins. is left between the lining and the shell. The lining bricks are laid in a mortar made of silicate of soda and white asbestos wool, tempered to the consistency of fire-clay mortar. This mortar is acid-proof, and its binding power, which is considerable in comparison to that of fire-clay mortar, is unaffected by temperatures up to 2000° F. (*Eng. News*, Feb. 15, 1906.) Supported on 324 piles driven 69 ft. to solid rock, and covering an area 45 ft. square. Total cost \$32,000. The standard Custodis radial brick is $4\frac{1}{2}$ in. thick and $6\frac{1}{2}$ in. wide; radial lengths are 4, $5\frac{1}{2}$, $7\frac{1}{8}$, $8\frac{5}{8}$ and $10\frac{5}{8}$ ins. The smallest size has six vertical perforations, 1 in. square, and the largest fifteen.

Eastman Kodak Co., Rochester, N. Y. Height 366 ft.; internal diam. at top 9 ft. 10 ins., at bottom 20 ft. 10 ins.; outside diam., top 11 ft., bottom 27 ft. 10 ins. Radial brick, with 4-in. acid-resisting brick lining.

Some notable tall chimneys built by the Alphonse Custodis Chimney Construction Co. are: Dolgeville, N. Y., 6×175 ft.; Camden, N. J., 7×210 ft.; Newark, N. J., 8×350 ft.; Rochester, N. Y., 9×366 ft.; Constable Hook, N. J., 10×365 ft.; Providence, R. I., 16×308 ft.; Garfield, Utah, 30×300 ft.; Great Falls, Mont., 50×506 ft.

The Largest Chimney in the World, in 1908, is that of the Montana smelter, at Great Falls, Mont. Height 506 ft. Internal diam. at top 50 ft. Built of Custodis radial brick. Designed to remove 4,000,000 cu. ft. of gases per minute at an average temperature of 600° F. Erected on top of a hill 500 ft. above the city, and 246 ft. above the floor of the furnaces, which are about 2000 ft. distant. Designed for a wind pressure of $33\frac{1}{3}$ lbs. per sq. ft. of projected area; bearing pressure limited to 21 lbs. per sq. ft. at any section. Foundation: 111 ft. max. diam., $22\frac{1}{2}$ ft. deep; bearing pressure on bottom (shale rock) 4.83 tons per sq. ft.; octagonal outside, 103 ft. across at bottom, 81 ft. at top, with inner circular opening 47 ft. diam. at bottom, 64 ft. at top; made of 1 cement, 3 sand, 5 crushed slag. Four flue openings in the base, each 15 ft. wide, 36 ft. high. The stack proper consists of an octagonal base, 46 ft. in height, which has a taper of 8%, and above this a circular barrel, the first 180 ft. above the base having a taper of 7%, the next 100 ft. of 4%, and the remaining 180 ft. to the cap 2%.

The chimney wall varies from 66 in. at the base to $18\frac{1}{8}$ in. at the top by uniform decrements of 2 in. per section, excepting at the section immediately above the top of the base, where the thickness decreases from 60 in. to 54 in. The outside diameters of the stack are 781.2 ft. at the base, 53 ft. 9 in. at the base of the cap; the inside diameters range from $66\frac{1}{2}$ ft.

at the foundation line to 50 ft. at the top. The chimney is lined with 4-in. acid-proof brick, laid in sections carried on corbels from the main shell.

A description of the methods of design and of erection of the Great Falls chimney is given in *Eng. Rec.*, Nov. 28, 1908.

Stability of Chimneys. — Chimneys must be designed to resist the maximum force of the wind in the locality in which they are built. A general rule for diameter of base of brick chimneys, approved by many years of practice in England and the United States, is to make the diameter of the base one-tenth of the height. If the chimney is square or rectangular, make the diameter of the inscribed circle of the base one-tenth of the height. The "batter" or taper of a chimney should be from $\frac{1}{16}$ to $\frac{1}{4}$ inch to the foot on each side. The brickwork should be one brick (8 or 9 inches) thick for the first 25 feet from the top, increasing $\frac{1}{2}$ brick (4 or $4\frac{1}{2}$ inches) for each 25 feet from the top downwards. If the inside diameter exceeds 5 feet, the top length should be $1\frac{1}{2}$ bricks; and if under 3 feet, it may be $\frac{1}{2}$ brick for ten feet.

(From *The Locomotive*, 1884 and 1886.) For chimneys of four feet in diameter and one hundred feet high, and upwards, the best form is circular with a straight batter on the outside.

Chimneys of any considerable height are not built up of uniform thickness from top to bottom, nor with a uniformly varying thickness of wall, but the wall, heaviest of course at the base, is reduced by a series of steps.

Where practicable the load on a chimney foundation should not exceed two tons per square foot in compact sand, gravel, or loam. Where a solid rock-bottom is available for foundation, the load may be greatly increased. If the rock is sloping, all unsound portions should be removed, and the face dressed to a series of horizontal steps, so that there shall be no tendency to slide after the structure is finished.

All boiler-chimneys of any considerable size should consist of an outer stack of sufficient strength to give stability to the structure, and an inner stack or core independent of the outer one. This core is by many engineers extended up to a height of but 50 or 60 feet from the base of the chimney, but the better practice is to run it up the whole height of the chimney; it may be stopped off, say, a couple of feet below the top, and the outer shell contracted to the area of the core, but the better way is to run it up to about 8 or 12 inches of the top and *not* contract the outer shell. But under no circumstances should the core at its upper end be built into or connected with the outer stack. This has been done in several instances by bricklayers, and the result has been the expansion of the inner core which lifted the top of the outer stack squarely up and cracked the brickwork.

For a height of 100 feet we would make the outer shell in three steps, the first 20 feet high, 16 inches thick, the second 30 feet high, 12 inches thick, the third 50 feet high and 8 inches thick. These are the minimum thicknesses admissible for chimneys of this height, and the batter should be not less than 1 in 36 to give stability. The core should also be built in three steps, each of which may be about one-third the height of the chimney, the lowest 12 inches, the middle 8 inches, and the upper step 4 inches thick. This will insure a good sound core. The top of a chimney may be protected by a cast-iron cap; or perhaps a cheaper and equally good plan is to lay the ornamental part in some good cement, and plaster the top with the same material.

C. L. Hubbard (*Am. Electrician*, Mar., 1904) says: The following approximate method may be used for determining the thickness of walls. If the inside diameter at the top is less than 3 ft. the walls may be 4 ins. thick for the first 10 ft., and increased 4 ins. for each 25 ft. downward. If the inside diameter is more than 3 ft. and less than 5 ft., begin with a wall 8 ins. thick, increasing 4 ins. for each 25 ft. downward. If the diameter is over 5 ft., begin with a 12-in. wall, increasing below the first 10 ft. as before. The lining or core may be 4 ins. thick for the first 20 ft. from the top, 8 ins. for the next 30 ft., 12 ins. for the next 40 ft., 16 ins. for the next 50 ft., and 20 ins. for the next 50 ft. Using this method for an outer wall 200 ft. high and assuming a cubic foot of brickwork to weigh 130 lbs., it gives a maximum pressure of 8.2 tons per sq. ft. of section at the base; while a lining 190 ft. high would have a maximum pressure of 8.6 tons per sq. ft. The safe load for brickwork may be taken at from

8 to 10 tons per sq. ft., although the strength of best pressed brick will run much higher.

James B. Francis, in a report to the Lawrence Mfg. Co. in 1873 (*Eng. News*, Aug. 28, 1880), concerning the probable effects of wind on that company's chimney as then constructed, says:

The stability of the chimney to resist the force of the wind depends mainly on the weight of its outer shell, and the width of its base. The cohesion of the mortar may add considerably to its strength; but it is too uncertain to be relied upon. The inner shell will add a little to the stability, but it may be cracked by the heat, and its beneficial effect, if any, is too uncertain to be taken into account.

The effect of the joint action of the vertical pressure due to the weight of the chimney, and the horizontal pressure due to the force of the wind is to shift the center of pressure at the base of the chimney, from the axis toward one side, the extent of the shifting depending on the relative magnitude of the two forces. If the center of pressure is brought too near the side of the chimney, it will crush the brickwork on that side, and the chimney will fall. A line drawn through the center of pressure, perpendicular to the direction of the wind, must leave an area of brickwork between it and the side of the chimney, sufficient to support half the weight of the chimney; the other half of the weight being supported by the brickwork on the windward side of the line.

Different experimenters on the strength of brickwork give very different results. Kirkaldy found the weights which caused several kinds of bricks, laid in hydraulic lime mortar and in Roman and Portland cements, to fail slightly, to vary from 19 to 60 tons (of 2000 lbs.) per sq. ft. If we take in this case 25 tons per sq. ft. as the weight that would cause it to begin to fail, we shall not err greatly.

Rankine, in a paper printed in the transactions of the Institution of Engineers, in Scotland, for 1867-68, says: "It had previously been ascertained by observation of the success and failure of actual chimneys, and especially of those which respectively stood and fell during the violent storms of 1856, that, in order that a round chimney may be sufficiently stable, its weight should be such that a pressure of wind, of about 55 lbs. per sq. ft. of a plane surface, directly facing the wind, or $27\frac{1}{2}$ lbs. per sq. ft. of the plane projection of a cylindrical surface, . . . shall not cause the resultant pressure at any bed-joint to deviate from the axis of the chimney by more than one-quarter of the outside diameter at that joint."

Steel Chimneys are largely used, especially for tall chimneys of iron-works, from 150 to 300 feet in height. The advantages claimed are: greater strength and safety; smaller space required; smaller cost, by 30 to 50 per cent, as compared with brick chimneys; avoidance of infiltration of air and consequent checking of the draught, common in brick chimneys. They are usually made cylindrical in shape, with a wide curved flare for 10 to 25 feet at the bottom. A heavy cast-iron base-plate is provided, to which the chimney is riveted, and the plate is secured to a massive foundation by holding-down bolts. No guys are used.

Design of Self-supporting Steel Chimneys. — John D. Adams (*Eng. News*, July 20, 1905) gives a very full discussion of the design of steel chimneys, from which the following is adapted. The bell-shaped bottom of the chimney is assumed to occupy one-seventh of the total height, and the point of maximum strain is taken to be at the top of this bell portion. Let D = diam. in inches, H = height in feet, T = thickness in inches, S = safe tensile stress, lbs. per sq. in. The general formula for moment of resistance of a hollow cylinder is $M = \frac{1}{32} \pi (D^4 - D_1^4) S/D$. When the thickness is a small fraction of the diameter this becomes approximately $M = 0.7854 D^2 TS$.

With steel plate of 60,000 lbs. tensile strength, riveting of 0.6 efficiency, and a factor of safety of 4, we have $S = 9000$ pounds per sq. in., and the safe moment of resistance = $7070 D^2 T$.

The effect of the wind upon a cylinder is equal to the wind pressure multiplied by one-half the diametral plane, and taking the maximum wind pressure at 50 lbs. per sq. ft., we get

$$\text{Total wind pressure} = 50 \times \frac{1}{12} D \times \frac{1}{2} \times \frac{6}{7} H = 25 DH/14.$$

The distance of the center of pressure above the top of the bell portion, $= \frac{3}{7} H$, multiplied by the total wind pressure, gives us the bending moment due to the wind,

$$\text{inch pounds, } 25 DH/14 \times \frac{3}{7} H \times 12 = 9.184 DH^2.$$

Equating the bending and the resisting moment we have $T = 0.0013 H^2/D$.

With this formula the maximum thickness of plates was calculated for different sizes of chimneys, as given in the table below.

In the above formula, no attention has been paid to the weight of the steel in the stack above the bell portion, which weight has a tendency to decrease the tension on the windward side and increase the compression on the leeward side of the stack. A column of steel 150 ft. high would exert a pressure of approximately 500 lbs. per sq. in., which, with steel of 60,000 lbs. tensile strength, is less than 1% of the ultimate strength, and may safely be neglected.

From the table it appears that a chimney 12×120 ft. requires, as far as fracture by bending of a tubular section is concerned, a thickness of but little over $\frac{1}{8}$ in. In designing a stack of such extreme proportions as 12×120 ft., there are other factors besides bending to take into consideration that ordinarily could be neglected. For instance, such a stack should be provided with stiffening angles, or else made heavier, to guard against lateral flattening. Ordinarily, however, the strength of the chimney determined as a tubular section will be the prime factor in determining the maximum thickness of plates.

THICKNESS OF BASE-RING PLATES OF SELF-SUPPORTING STEEL STACKS.

For normal wind pressure of 50 lbs. per sq. ft. on half the diametral plane.

Diameter of Stack in feet.

| Hgt. ft. | 3.5 | 4 | 5 | 6 | 7 | 8 | 8.5 | 9 | 9.5 | 10 | 11 | 12 |
|-------------|-------|------|------|------|------|------|------|------|------|------|------|------|
| 70 | 0.152 | .133 | .106 | | | | | | | | | |
| 80 | 0.198 | .182 | .139 | .116 | .099 | | | | | | | |
| 90 | 0.224 | .219 | .175 | .146 | .125 | .111 | | | | | | |
| 100 | 0.310 | .271 | .217 | .181 | .155 | .135 | .127 | .120 | | | | |
| 110 | 0.375 | .328 | .262 | .218 | .187 | .164 | .154 | .146 | .138 | .131 | .119 | |
| 120 | 0.445 | .390 | .312 | .260 | .223 | .195 | .183 | .173 | .164 | .156 | .142 | .130 |
| 130 | 0.523 | .458 | .366 | .305 | .262 | .228 | .215 | .203 | .193 | .183 | .166 | .153 |
| 140 | 0.607 | .531 | .425 | .354 | .303 | .265 | .250 | .236 | .223 | .212 | .193 | .180 |
| 150 | 0.696 | .609 | .487 | .406 | .348 | .305 | .286 | .271 | .257 | .244 | .222 | .203 |
| 160 | | .693 | .555 | .462 | .396 | .346 | .326 | .308 | .292 | .277 | .252 | .231 |
| 170 | | | .626 | .522 | .447 | .391 | .368 | .348 | .330 | .313 | .285 | .261 |
| 180 | | | .702 | .585 | .501 | .439 | .413 | .390 | .370 | .351 | .319 | .293 |
| 190 | | | | .652 | .559 | .489 | .460 | .434 | .411 | .391 | .356 | .326 |
| 200 | | | | | .620 | .542 | .510 | .481 | .456 | .433 | .394 | .361 |
| 210 | | | | | .682 | .596 | .562 | .531 | .503 | .478 | .434 | .398 |
| 220 | | | | | | .655 | .617 | .582 | .552 | .524 | .476 | .437 |
| 230 | | | | | | .717 | .674 | .637 | .603 | .573 | .521 | .477 |
| 240 | | | | | | | .734 | .693 | .657 | .624 | .567 | .520 |
| 250 | | | | | | | | .752 | .713 | .677 | .615 | .564 |

Foundation. — Neglecting the increase of wind area due to the flare at the base of the chimney, which has but a very small turning effect, if all dimensions be taken in feet, we have

Total wind pressure $= \frac{1}{2} D \times H \times 50 = 25 DH$; lever-arm $= \frac{1}{2} H$; hence, turning moment $= 12.5 DH^2$.

Let d = diameter and h = height of foundation. For average conditions $h = 0.4 d$, then volume of foundation $= 0.7854 d^2 h$, and for concrete at 150 lbs. per cu. ft., weight of foundation $= W = 0.7854 d^2 h \times 150 = 47.124 d^2$.

The stability of the foundation or the tendency to resist overturning is equal to the weight of the foundation multiplied by its radius or $\frac{1}{2} Wd = 23.562 d^3$. Applying a factor of safety of $2\frac{1}{2}$, which is indicated by

current practice, gives safe stability = $9.425 d^4$. Equating this to the overturning moment we obtain $d = 1.07 \sqrt[4]{DH^2}$, in which all dimensions are in feet.

Anchor-bolts. — The holding power of the bolts depends on three factors: the number of bolts, the diameter of the bolt circle, and the diameter of the bolts. The number of bolts is largely conventional and may be selected so as not to necessitate bolts of too large a diameter. The diameter of the bolt circle is also more or less arbitrary. The bolts will be stretched and therefore strained, in proportion to their distance from the axis of turning, assuming, as we must, that the cast-iron ring at the base of the chimney is rigid. The leverage at which any bolt acts is also directly proportional to its distance from the axis of turning. Therefore, since the effectiveness of any one bolt, as regards overturning, depends upon the strain in that bolt, multiplied by its leverage, it is evident that the effectiveness of any bolt varies as the square of its distance from the axis of turning. If we lay out, say, 12 or 24 bolts equidistant on a circle and add all the squares of these distances, we will find that we may consider the total as though the bolts were all placed at a distance of $\frac{2}{3}$ the diameter of the bolt circle from the axis of turning, which is the tangent to the bolt circle.

Let b = diameter of bolt in inches, n = number of bolts, diameter of bolt circle = $\frac{2}{3}d$. Take safe working stress at 8000 pounds per sq. inch. Then resistance to overturning = $0.7854 b^2 \times 8000 \times \frac{2}{3}d \times \frac{3}{8} \times N = 6283 b^2 Nd/4$. Equating this to the turning moment, $12.5 DH^2$, gives $b = 0.0257 \frac{H}{N} \sqrt{D/d}$ for 12 bolts, $0.0222 \frac{H}{N} \sqrt{D/d}$ for 18 bolts, and $0.0182 \frac{H}{N} \sqrt{D/d}$ for 24 bolts.

The Babcock & Wilcox Co.'s book "Steam" illustrates a steel chimney at the works of the Maryland Steel Co., Sparrow's Point, Md. It is 225 ft. in height above the base, with internal brick lining 13' 9" uniform inside diameter. The shell is 25 ft. diam. at the base, tapering in a curve to 17 ft. 25 ft. above the base, thence tapering almost imperceptibly to 14' 8" at the top. The upper 40 feet is of $\frac{1}{4}$ -inch plates, the next four sections of 40 ft. each are respectively $\frac{9}{32}$, $\frac{5}{16}$, $\frac{11}{32}$, and $\frac{3}{8}$ inch.

Reinforced Concrete Chimneys began extensively to come into use in the United States in 1901. Some hundreds of them are now (1909) in use. The following description of the method of construction of these chimneys is condensed from a circular of the Weber Chimney Co., Chicago.

The foundation is comparatively light and made of concrete, consisting of 1 cement, 3 sand, and 5 gravel or macadam. The steel reinforcement consists of two networks usually made of T steel of small size. The bars for the lower network are placed diagonally and the bars for the second network (about 4 to 6 ins. above the first one) run parallel to the sides. The vertical bars, forming the reinforcement of the chimney itself, also go down into the foundation and a number of these bars are bent in order to secure an anchorage for the chimney.

The chimney shaft consists of two parts, the lower double shell and the single shell above, which are united at the offset. The inside shell is usually 4 ins. thick, while the thickness of the outer shell depends on the height and varies from 6 to 12 ins. The single shell is from 4 to 10 ins. thick. The height of the double shell depends upon the purpose of the chimney, nature and heat of the gases, etc.

Between the two shells in the lower part there is a circular air space 4 ins. in width. An expansion joint is provided where the two shells unite.

The concrete above the ground level consists of one part Portland cement and three parts of sand. No gravel or macadam is used.

The bending forces caused by wind pressure are taken up by the vertical steel reinforcement. The resistance of the concrete itself against tension is not considered in calculation.

The vertical T bars are from $1 \times 1 \times \frac{1}{8}$ to $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{2}$ in., the weight and number depending upon the dimensions of the chimney. The bars are from 16 to 30 ft. long and overlap not less than 24 ins. They are placed at regular intervals of 18 ins. and encircled by steel rings bent to the desired circle. The work of erection is done from the inside of the chimney: no outside scaffolding is needed.

The following is a list of some of the tallest concrete chimneys that have been built of their respective diameters: Butte, Mont., 350×18 ft.; Seattle,

Wash., 278×17 ft.; Portland, Ore., 230×12 ft.; Lawrence, Mass., 250×11 ft.; Cincinnati, Ohio, 200×10 ft.; Worcester, Mass., 220×9 ft.; Atlanta, Ga., 225×8 ft.; Chicago, 175×7 ft.; Rockville, Conn., 175×6 ft.; Seymour, Ind., 150×5 ft.; Iola, Kans., 143×4 ft.; St. Louis, Mo., 130×3 ft. 4 in.; Dayton, Ohio, 94×3 ft.

Sizes of Foundations for Steel Chimneys.

(Selected from circular of Phila. Engineering Works.)

HALF-LINED CHIMNEYS.

| Diameter, clear, feet..... | 3 | 4 | 5 | 6 | 7 | 9 | 11 |
|----------------------------|-------|-------|-------|--------|-------|-------|-------|
| Height, feet..... | 100 | 100 | 150 | 150 | 150 | 150 | 150 |
| Least diam. foundation.. | 15'9" | 16'4" | 20'4" | 21'10" | 22'7" | 23'8" | 24'8" |
| Least depth foundation.. | 6' | 6' | 9' | 8' | 9' | 10' | 10' |
| Height, feet..... | 125 | 200 | 200 | 250 | 275 | 300 | |
| Least diam. foundation.. | 18'5" | 23'8" | 25' | 29'8" | 33'6" | 26' | |
| Least depth foundation . | 7' | 10' | 10' | 12' | 12' | 14' | |

Weight of Sheet-iron Smoke-stacks per Foot.

(Porter Mfg. Co.)

| Diam. inches. | Thick- ness. W. G. | Weight per ft. | Diam. inches. | Thick- ness. W. G. | Weight per ft. | Diam. inches. | Thick- ness. W. G. | Weight per ft. |
|------------------|--------------------------|-------------------|------------------|--------------------------|-------------------|------------------|--------------------------|-------------------|
| 10 | No. 16 | 7.20 | 26 | No. 16 | 17.50 | 20 | No. 14 | 18.33 |
| 12 | " | 8.66 | 28 | " | 18.75 | 22 | " | 20.00 |
| 14 | " | 9.58 | 30 | " | 20.00 | 24 | " | 21.66 |
| 16 | " | 11.68 | 10 | No. 14 | 9.40 | 26 | " | 23.33 |
| 20 | " | 13.75 | 12 | " | 11.11 | 28 | " | 25.00 |
| 22 | " | 15.00 | 14 | " | 13.69 | 30 | " | 26.66 |
| 24 | " | 16.25 | 16 | " | 15.00 | | | |

Sheet-iron Chimneys. (Columbus Machine Co.)

| Diameter Chimney, inches. | Length Chimney, feet. | Thick- ness Iron, B. W. G. | Weight lbs. | Diameter Chimney, inches. | Length Chimney, feet. | Thick- ness Iron, B. W. G. | Weight lbs. |
|---------------------------------|-----------------------------|-------------------------------------|----------------|---------------------------------|-----------------------------|-------------------------------------|----------------|
| 10 | 20 | No. 16 | 160 | 30 | 40 | No. 15 | 960 |
| 15 | 20 | " 16 | 240 | 32 | 40 | " 15 | 1020 |
| 20 | 20 | " 16 | 320 | 34 | 40 | " 14 | 1170 |
| 22 | 20 | " 16 | 350 | 36 | 40 | " 14 | 1240 |
| 24 | 40 | " 16 | 760 | 38 | 40 | " 12 | 1800 |
| 26 | 40 | " 16 | 826 | 40 | 40 | " 12 | 1890 |
| 28 | 40 | " 15 | 900 | | | | |

THE STEAM-ENGINE.

Expansion of Steam. Isothermal and Adiabatic. — According to Mariotte's law, the volume of a perfect gas, the temperature being kept constant, varies inversely as its pressure, or $p \propto 1/v$; $pv = \text{a constant}$. The curve constructed from this formula is called the *isothermal* curve, or curve of equal temperatures, and is a common or rectangular hyperbola. The expansion of steam in an engine is not isothermal, since the temperature decreases with increase of volume, but its expansion curve approximates the curve of $pv = \text{a constant}$. The relation of the pressure and volume of saturated steam, as deduced from Regnault's experiments, and as given in steam tables, is approximately, according to Rankine (S. E., p. 403), for pressures not exceeding 120 lbs., $p \propto 1/v^{1\frac{1}{2}}$, or $p \propto v^{-1\frac{1}{2}}$ or $pv^{1\frac{1}{2}} = pv^{1.0625} = \text{a constant}$. Zeuner has found that the exponent 1.0646 gives a closer approximation.

When steam expands in a closed cylinder, as in an engine, according to Rankine (S. E., p. 385), the approximate law of the expansion is $p \propto 1/v^{1\frac{10}{9}}$, or $p \propto v^{-1\frac{10}{9}}$, or $pv^{1\frac{10}{9}} = \text{a constant}$. The curve constructed from this formula is called the *adiabatic* curve, or curve of no transmission of heat.

Peabody (Therm., p. 112) says: "It is probable that this equation was obtained by comparing the expansion lines on a large number of indicator-diagrams. . . . There does not appear to be any good reason for using an exponential equation in this connection, . . . and the action of a lagged steam-engine cylinder is far from being adiabatic. . . . For general purposes the hyperbola is the best curve for comparison with the expansion curve of an indicator-card. . . ." Wolff and Denton, *Trans. A. S. M. E.*, ii, 175, say: "From a number of cards examined from a variety of steam-engines in current use, we find that the actual expansion line varies between the 10/9 adiabatic curve and the Mariotte curve."

Prof. Thurston (*Trans. A. S. M. E.*, ii, 203) says he doubts if the exponent ever becomes the same in any two engines, or even in the same engine at different times of the day and under varying conditions of the day.

Expansion of Steam according to Mariotte's Law and to the Adiabatic Law. (*Trans. A. S. M. E.*, ii, 156.) — Mariotte's law $pv =$

$p_1 v_1$; values calculated from formula $\frac{P_m}{p_1} = \frac{1}{R} (1 + \text{hyp log } R)$, in which $R = v_2 \div v_1$, p_1 = absolute initial pressure, P_m = absolute mean pressure, v_1 = initial volume of steam in cylinder at pressure p_1 , v_2 = final volume of steam at final pressure. Adiabatic law: $pv^{1\frac{10}{9}} = p_1 v_1^{1\frac{10}{9}}$; values calculated from formula $\frac{P_m}{p_1} = 10 R^{-1} - 9 R^{-1\frac{10}{9}}$.

| Ratio of Ex- pansion <i>R.</i> | Ratio of Mean to Initial Pressure. | | Ratio of Ex- pansion <i>R.</i> | Ratio of Mean to Initial Pressure. | | Ratio of Ex- pansion <i>R.</i> | Ratio of Mean to Initial Pressure. | |
|---|--|--------|---|--|--------|---|--|--------|
| | Mar. | Adiab. | | Mar. | Adiab. | | Mar. | Adiab. |
| 1.00 | 1.000 | 1.000 | 3.7 | 0.624 | 0.600 | 6. | 0.465 | 0.438 |
| 1.25 | .978 | .976 | 3.8 | .614 | .590 | 6.25 | .453 | .425 |
| 1.50 | .937 | .931 | 3.9 | .605 | .580 | 6.5 | .442 | .413 |
| 1.75 | .891 | .881 | 4. | .597 | .571 | 6.75 | .431 | .403 |
| 2. | .847 | .834 | 4.1 | .588 | .562 | 7. | .421 | .393 |
| 2.2 | .813 | .798 | 4.2 | .580 | .554 | 7.25 | .411 | .383 |
| 2.4 | .781 | .765 | 4.3 | .572 | .546 | 7.5 | .402 | .374 |
| 2.5 | .766 | .748 | 4.4 | .564 | .538 | 7.75 | .393 | .365 |
| 2.6 | .752 | .733 | 4.5 | .556 | .530 | 8. | .385 | .357 |
| 2.8 | .725 | .704 | 4.6 | .549 | .523 | 8.25 | .377 | .349 |
| 3. | .700 | .678 | 4.7 | .542 | .516 | 8.5 | .369 | .342 |
| 3.1 | .688 | .666 | 4.8 | .535 | .509 | 8.75 | .362 | .335 |
| 3.2 | .676 | .654 | 4.9 | .528 | .502 | 9. | .355 | .328 |
| 3.3 | .665 | .642 | 5.0 | .522 | .495 | 9.25 | .349 | .321 |
| 3.4 | .654 | .630 | 5.25 | .506 | .479 | 9.5 | .342 | .315 |
| 3.5 | .644 | .620 | 5.5 | .492 | .464 | 9.75 | .336 | .309 |
| 3.6 | .634 | .610 | 5.75 | .478 | .450 | 10. | .330 | .303 |

Mean Pressure of Expanded Steam.— For calculations of engines it is generally assumed that steam expands according to Mariotte's law, the curve of the expansion line being a hyperbola. The mean pressure, measured above vacuum, is then obtained from the formula

$$P_m = p_1 \frac{1 + \text{hyp log } R}{R}, \text{ or } P_m = P_t (1 + \text{hyp log } R),$$

in which P_m is the absolute mean pressure, p_1 the absolute initial pressure taken as uniform up to the point of cut-off, P_t the terminal pressure, and R the ratio of expansion. If l = length of stroke to the cut-off, L = total stroke.

$$P_m = \frac{p_1 l + p_l \text{ hyp log } \frac{L}{l}}{L}; \text{ and if } R = \frac{L}{l}, P_m = p_1 \frac{1 + \text{hyp log } R}{R}.$$

Mean and Terminal Absolute Pressures.— **Mariotte's Law.**— The values in the following table are based on Mariotte's law, except those in the last column, which give the mean pressure of superheated steam, which, according to Rankine, expands in a cylinder according to the law $p \propto v^{-\frac{1}{2}}$. These latter values are calculated from the formula

$$\frac{P_m}{p_1} = \frac{17 - 16 R^{-\frac{1}{2}}}{R}. \quad R^{-\frac{1}{2}} \text{ may be found by extracting the square root of } \frac{1}{R} \text{ four times.}$$

From the mean absolute pressures given deduct the mean back pressure (absolute) to obtain the mean effective pressure.

| Rate of Expansion. | Cut-off. | Ratio of Mean to Initial Pressure. | Ratio of Mean to Terminal Pressure. | Ratio of Terminal to Mean Pressure. | Ratio of Initial to Mean Pressure. | Ratio of Mean to Initial Dry Steam. |
|--------------------|----------|------------------------------------|-------------------------------------|-------------------------------------|------------------------------------|-------------------------------------|
| 30 | 0.033 | 0.1467 | 4.40 | 0.227 | 6.82 | 0.136 |
| 28 | 0.036 | 0.1547 | 4.33 | 0.231 | 6.46 | |
| 26 | 0.038 | 0.1638 | 4.26 | 0.235 | 6.11 | |
| 24 | 0.042 | 0.1741 | 4.18 | 0.239 | 5.75 | |
| 22 | 0.045 | 0.1860 | 4.09 | 0.244 | 5.38 | |
| 20 | 0.050 | 0.1998 | 4.00 | 0.250 | 5.00 | 0.186 |
| 18 | 0.055 | 0.2161 | 3.89 | 0.256 | 4.63 | |
| 16 | 0.062 | 0.2358 | 3.77 | 0.265 | 4.24 | |
| 15 | 0.066 | 0.2472 | 3.71 | 0.269 | 4.05 | |
| 14 | 0.071 | 0.2599 | 3.64 | 0.275 | 3.85 | |
| 13.33 | 0.075 | 0.2690 | 3.59 | 0.279 | 3.72 | 0.254 |
| 13 | 0.077 | 0.2742 | 3.56 | 0.280 | 3.65 | |
| 12 | 0.083 | 0.2904 | 3.48 | 0.287 | 3.44 | |
| 11 | 0.091 | 0.3089 | 3.40 | 0.294 | 3.24 | |
| 10 | 0.100 | 0.3303 | 3.30 | 0.303 | 3.03 | 0.314 |
| 9 | 0.111 | 0.3552 | 3.20 | 0.312 | 2.81 | |
| 8 | 0.125 | 0.3849 | 3.08 | 0.321 | 2.60 | 0.370 |
| 7 | 0.143 | 0.4210 | 2.95 | 0.339 | 2.37 | |
| 6.66 | 0.150 | 0.4347 | 2.90 | 0.345 | 2.30 | 0.417 |
| 6.00 | 0.166 | 0.4653 | 2.79 | 0.360 | 2.15 | |
| 5.71 | 0.175 | 0.4807 | 2.74 | 0.364 | 2.08 | |
| 5.00 | 0.200 | 0.5218 | 2.61 | 0.383 | 1.92 | 0.506 |
| 4.44 | 0.225 | 0.5608 | 2.50 | 0.400 | 1.78 | |
| 4.00 | 0.250 | 0.5965 | 2.39 | 0.419 | 1.68 | 0.582 |
| 3.63 | 0.275 | 0.6308 | 2.29 | 0.437 | 1.58 | |
| 3.33 | 0.300 | 0.6615 | 2.20 | 0.454 | 1.51 | 0.68 |
| 3.00 | 0.333 | 0.6995 | 2.10 | 0.476 | 1.43 | |
| 2.86 | 0.350 | 0.7171 | 2.05 | 0.488 | 1.39 | 0.707 |
| 2.66 | 0.375 | 0.7440 | 1.98 | 0.505 | 1.34 | |
| 2.50 | 0.400 | 0.7664 | 1.91 | 0.523 | 1.31 | 0.756 |
| 2.22 | 0.450 | 0.8095 | 1.80 | 0.556 | 1.24 | 0.800 |
| 2.00 | 0.500 | 0.8465 | 1.69 | 0.591 | 1.18 | 0.840 |
| 1.82 | 0.550 | 0.8786 | 1.60 | 0.626 | 1.14 | 0.874 |
| 1.66 | 0.600 | 0.9066 | 1.51 | 0.662 | 1.10 | 0.900 |
| 1.60 | 0.625 | 0.9187 | 1.47 | 0.680 | 1.09 | |
| 1.54 | 0.650 | 0.9292 | 1.43 | 0.699 | 1.07 | 0.926 |
| 1.48 | 0.675 | 0.9405 | 1.39 | 0.718 | 1.06 | |

Calculation of Mean Effective Pressure, Clearance and Compression Considered.—In the above tables no account is taken of clearance, which in actual steam-engines modifies the ratio of expansion and the mean pressure; nor of compression and back-pressure, which diminish the mean effective pressure. In the following calculation these elements are considered.

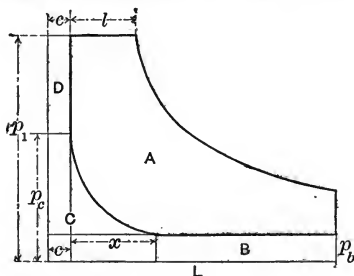


FIG. 153.

L = length of stroke, l = length before cut-off, x = length of compression part of stroke, c = clearance, p_1 = initial pressure, p_b = back pressure, p_c = pressure of clearance steam at end of compression. All pressures are absolute, that is, measured from a perfect vacuum.

$$\text{Area of ABCD} = p_1(l+c) \left(1 + \text{hyp log } \frac{L+c}{l+c}\right);$$

$$B = p_b(L-x);$$

$$C = p_c c \left(1 + \text{hyp log } \frac{x+c}{c}\right) = p_b(x+c) \left(1 + \text{hyp log } \frac{x+c}{c}\right);$$

$$D = (p_1 - p_c)c = p_1c - p_b(x+c).$$

$$\text{Area of A} = \text{ABCD} - (B + C + D)$$

$$= p_1(l+c) \left(1 + \text{hyp log } \frac{L+c}{l+c}\right)$$

$$- \left[p_b(L-x) + p_b(x+c) \left(1 + \text{hyp log } \frac{x+c}{c}\right) + p_1c - p_b(x+c) \right]$$

$$= p_1(l+c) \left(1 + \text{hyp log } \frac{L+c}{l+c}\right)$$

$$- p_b \left[(L-x) + (x+c) \text{hyp log } \frac{x+c}{c} \right] - p_1c.$$

$$\text{Mean effective pressure} = \frac{\text{area of A}}{L}.$$

EXAMPLE.—Let $L=1$, $l=0.25$, $x=0.25$, $c=0.1$, $p_1=60$ lbs., $p_b=2$ lbs.

$$\text{Area A} = 60(0.25 + 0.1) \left(1 + \text{hyp log } \frac{1.1}{0.35}\right)$$

$$- 2 \left[(1 - 0.25) + 0.35 \text{hyp log } \frac{0.35}{0.1} \right] - 60 \times 0.1.$$

$$= 21(1 + 1.145) - 2[0.75 + 35 \times 1.253] - 6$$

$$= 45.045 - 2.377 - 6 = 36.668 = \text{mean effective pressure.}$$

The actual indicator-diagram generally shows a mean pressure considerably less than that due to the initial pressure and the rate of expansion. The causes of loss of pressure are: 1. Friction in the stop-valves and steam-pipes. 2. Friction or wire-drawing of the steam during admission and cut-off, due chiefly to defective valve-gear and contracted steam-passages. 3. Liquefaction during expansion. 4. Exhausting before the engine has completed its stroke. 5. Compression due to early closure of exhaust. 6. Friction in the exhaust-ports, passages, and pipes.

Re-evaporation during expansion of the steam condensed during admission, and valve-leakage after cut-off, tend to elevate the expansion line of the diagram and increase the mean pressure.

If the theoretical mean pressure be calculated from the initial pressure and the rate of expansion on the supposition that the expansion curve follows Mariotte's law, $pv = a$ constant, and the necessary corrections are made for clearance and compression, the expected mean pressure in practice may be found by multiplying the calculated results by the factor (commonly called the "diagram factor") in the following table, according to Scaton.

| Particulars of Engine. | Factor. |
|--|-------------|
| Expansive engine, special valve-gear, or with a separate cut-off valve, cylinder jacketed..... | 0.94 |
| Expansive engine having large ports, etc., and good ordinary valves, cylinders jacketed..... | 0.9 to 0.92 |
| Expansive engines with the ordinary valves and gear as in general practice, and unjacketed..... | 0.8 to 0.85 |
| Compound engines, with expansion valve to h.p. cylinder; cylinders jacketed, and with large ports, etc..... | 0.9 to 0.92 |
| Compound engines, with ordinary slide-valves, cylinders jacketed, and good ports, etc..... | 0.8 to 0.85 |
| Compound engines as in general practice in the merchant service, with early cut-off in both cylinders, without jackets and expansion-valves..... | 0.7 to 0.8 |
| Fast-running engines of the type and design usually fitted in war-ships..... | 0.6 to 0.8 |

If no correction be made for clearance and compression, and the engine is in accordance with general modern practice, the theoretical mean pressure may be multiplied by 0.96, and the product by the proper factor in the table, to obtain the expected mean pressure.

Given the Initial Pressure and the Average Pressure, to Find the Ratio of Expansion and the Period of Admission.

P = initial absolute pressure in lbs. per sq. in.;
 p = average total pressure during stroke in lbs. per sq. in.;
 L = length of stroke in inches;
 l = period of admission measured from beginning of stroke;
 c = clearance in inches;

$$R = \text{actual ratio of expansion} = \frac{L+c}{l+c} \quad \dots \quad (1)$$

$$p = \frac{P(1 + \text{hyp log } R)}{R}$$

To find average pressure p , taking account of clearance,

$$p = \frac{P(l+c) + P(l+c) \text{ hyp log } R - Pc}{L} \quad \dots \quad (2)$$

whence

$$pL + Pc = P(l+c)(1 + \text{hyp log } R);$$

$$\text{hyp log } R = \frac{pL + Pc}{Pl + Pc} - 1 = \frac{\frac{p}{P}L + c}{l + c} - 1 \quad \dots \quad (3)$$

Given p and P , to find R and l (by trial and error). — There being two unknown quantities R and l , assume one of them, viz., the period of admission l , substitute it in equation (3) and solve for R . Substitute this value of R in the formula (1), or $l = \frac{L+c}{R} - c$, obtained from formula (1), and find l . If the result is greater than the assumed value of l , then the assumed value of the period of admission is too long; if less, the assumed value is too short. Assume a new value of l , substitute it in formula (3) as before, and continue by this method of trial and error till the required values of R and l are obtained.

EXAMPLE. — $P=70$, $p=42.78$, $L=60$ in., $c=3$ in., to find l . Assume $l=21$ in.

$$\text{hyp log } R = \frac{\frac{p}{P} L + c}{l + c} - 1 = \frac{\frac{42.78}{70} \times 60 + 3}{21 + 3} - 1 = 1.653 - 1 = 0.653;$$

$$\text{hyp log } R = 0.653, \text{ whence } R = 1.92.$$

$$l = \frac{L + c}{R} - c = \frac{63}{1.92} - 3 = 29.8,$$

which is greater than the assumed value, 21 inches.

Now assume $l=15$ inches:

$$\text{hyp log } R = \frac{\frac{42.78}{70} \times 60 + 3}{15 + 3} - 1 = 1.204, \text{ whence } R = 3.5;$$

$$l = \frac{L + c}{R} - c = \frac{63}{3.5} - 3 = 18 - 3 = 15 \text{ inches, the value assumed.}$$

Therefore $R=3.5$, and $l=15$ inches.

Period of Admission Required for a Given Actual Ratio of Expansion:

$$l = \frac{L + c}{R} - c, \text{ in inches (4)}$$

$$\text{In percentage of stroke, } l = \frac{100 + \text{p. ct. clearance}}{R} - \text{p. ct. clearance . . . (5)}$$

$$\text{Terminal pressure} = \frac{P(l + c)}{L + c} = \frac{P}{R} \text{ (6)}$$

Pressure at any other Point of the Expansion. — Let L_1 = length of stroke up to the given point.

$$\text{Pressure at the given point} = \frac{P(l + c)}{L_1 + c} \text{ (7)}$$

Mechanical Energy of Steam Expanded Adiabatically to Various Pressures. — The figures in the following table are taken from a chart constructed by R. M. Neilson in *Power*, Mar. 16, 1909. The pressures are absolute, lbs per sq. in.

| Initial Press. | | | | | | | | | | | | |
|----------------|---|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 15 | 20 | 25 | 40 | 60 | 80 | 100 | 120 | 140 | 170 | 200 | 250 |
| Final Press. | Mechanical Energy, Thousands of Foot-Pounds per Lb. of Steam. | | | | | | | | | | | |
| 15 | 0 | 17 | 29.5 | 55.5 | 77.5 | 94.5 | 107 | 116.5 | 121 | 136.5 | 146 | 160 |
| 12 | 12 | 29 | 41 | 66.5 | 88 | 104 | 116 | 126 | 135 | 145 | 154.5 | 168.5 |
| 10 | 22 | 39 | 50.5 | 75.5 | 97 | 113 | 125 | 135.5 | 144 | 154 | 163.5 | 176 |
| 8 | 34 | 50 | 62 | 86.5 | 109 | 124 | 136 | 147 | 155 | 165.5 | 174.5 | 186 |
| 6 | 49 | 64 | 76 | 101 | 123 | 138 | 150 | 160 | 168.5 | 179.5 | 188 | 199 |
| 4 | 68 | 85 | 95.5 | 120 | 142 | 157 | 168 | 177.5 | 186 | 196 | 204.5 | 216 |
| 2 | 100 | 116 | 128 | 151 | 171 | 186.5 | 197.5 | 207 | 215 | 224 | 232.5 | 244 |
| 1 | 131 | 147 | 157.5 | 181.5 | 200.5 | 215 | 225 | 234.5 | 243 | 250.5 | 260.5 | 270.5 |

Measures for Comparing the Duty of Engines. — Capacity is measured in horse-powers, expressed by the initials, H.P.: 1 H.P.=33,000 ft.-lbs. per minute, =550 ft.-lbs. per second, = 1,980,000 ft.-lbs. per hour. 1 ft.-lb. = a pressure of 1 lb. exerted through a space of 1 ft.

Economy is measured, 1, in pounds of coal per horse-power per hour; 2, in pounds of steam per horse-power per hour. The second of these measures is the more accurate and scientific, since the engine uses steam and not coal, and it is independent of the economy of the boiler.

In gas-engine tests the common measure is the number of cubic feet of gas (measured at atmospheric pressure) per horse-power, but as all gas is not of the same quality, it is necessary for comparison of tests to give the analysis of the gas. When the gas for one engine is made in one gas-producer, then the number of pounds of coal used in the producer per hour per horse-power of the engine is a measure of economy. Since different coals vary in heating value, a more accurate measure is the number of heat units required per horse-power per hour.

Economy, or duty of an engine, is also measured in the number of foot-pounds of work done per pound of fuel. As 1 horse-power is equal to 1,980,000 ft.-lbs. of work in an hour, a duty of 1 lb. of coal per H.P. per hour would be equal to 1,980,000 ft.-lbs. per lb. of fuel; 2 lbs. per H.P. per hour equals 990,000 ft.-lbs. per lb. of fuel, etc.

The duty of pumping-engines is expressed by the number of foot-pounds of work done per 100 lbs. of coal, per 1000 lbs. of steam, or per million heat units.

When the duty of a pumping-engine is given, in ft.-lbs. per 100 lbs. of coal, the equivalent number of pounds of fuel consumed per horse-power per hour is found by dividing 198 by the number of millions of foot-pounds of duty. Thus a pumping-engine giving a duty of 99 millions is equivalent to $198/99 = 2$ lbs. of fuel per horse-power per hour.

Efficiency Measured in Thermal Units per Minute. — The efficiency of an engine is sometimes expressed in terms of the number of thermal units used by the engine per minute for each indicated horse-power, instead of by the number of pounds of steam used per hour.

The heat chargeable to an engine per pound of steam is the difference between the total heat in a pound of steam at the boiler-pressure and that in a pound of the feed-water entering the boiler. In the case of condensing engines, suppose we have a temperature in the hot-well of 100° F., corresponding to a vacuum of 28 in. of mercury; we may feed the water into the boiler at that temperature. In the case of a non-condensing engine, by using a portion of the exhaust steam in a good feed-water heater, at a pressure a trifle above the atmosphere (due to the resistance of the exhaust passages through the heater), we may obtain feed-water at 212°. One pound of steam used by the engine then would be equivalent to thermal units as follows:

| | | | | | | | |
|--------------------------------|--------|--------|--------|--------|--------|--------|--------|
| Gauge pressure..... | 50 | 75 | 100 | 125 | 150 | 175 | 200 |
| Absolute pressure.... | 65 | 90 | 115 | 140 | 165 | 190 | 215 |
| Total heat in steam above 32°: | | | | | | | |
| | 1178.5 | 1184.4 | 1188.8 | 1192.2 | 1195.0 | 1197.3 | 1199.2 |

Subtracting 68 and 180 heat-units, respectively, the heat above 32° in feed-water of 100° and 212° F., we have —

Heat given by boiler per pound of steam:

| | | | | | | | |
|-------------------|--------|--------|--------|--------|--------|--------|--------|
| Feed at 100°..... | 1110.5 | 1116.4 | 1120.8 | 1124.2 | 1127.0 | 1129.3 | 1131.2 |
| Feed at 212°..... | 998.5 | 1004.4 | 1008.8 | 1012.2 | 1015.0 | 1017.3 | 1019.2 |

Thermal units per minute used by an engine for each pound of steam used per indicated horse-power per hour:

| | | | | | | | |
|-------------------|-------|-------|-------|-------|-------|-------|-------|
| Feed at 100°..... | 18.51 | 18.61 | 18.68 | 18.74 | 18.78 | 18.82 | 18.85 |
| Feed at 212°..... | 16.64 | 16.76 | 16.78 | 16.87 | 16.92 | 16.96 | 16.99 |

EXAMPLES. — A triple-expansion engine, condensing, with steam at 175 lbs. gauge, and vacuum 28 in., uses 13 lbs. of water per I.H.P. per hour, and a high-speed non-condensing engine, with steam at 100 lbs. gauge, uses 30 lbs. How many thermal units per minute does each consume?

Ans. — $13 \times 18.82 = 244.7$, and $30 \times 16.78 = 503.4$ thermal units per minute.

A perfect engine converting all the heat-energy of the steam into work would require $33,000 \text{ ft.-lbs.} \div 778 = 42.4164$ thermal units per minute per indicated horse-power. This figure, 42.4164, therefore, divided by the number of thermal units per minute per I.H.P. consumed by an engine, gives its efficiency as compared with an ideally perfect engine. In the examples above, 42.4164 divided by 244.3 and by 503.4 gives 17.33% and 8.42% efficiency, respectively.

ACTUAL EXPANSIONS
With Different Clearances and Cut-offs.
 Computed by A. F. Nagle.

| Cut-off. | Per Cent of Clearance. | | | | | | | | | | |
|----------|------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| .01 | 100.00 | 50.5 | 34.0 | 25.75 | 20.8 | 17.5 | 15.14 | 13.38 | 12.00 | 10.9 | 10 |
| .02 | 50.00 | 33.67 | 25.50 | 20.60 | 17.33 | 15.00 | 13.25 | 11.89 | 10.80 | 9.91 | 9.17 |
| .03 | 33.33 | 25.25 | 20.40 | 17.16 | 14.86 | 13.12 | 11.78 | 10.70 | 9.82 | 9.08 | 8.46 |
| .04 | 25.00 | 20.20 | 17.00 | 14.71 | 13.00 | 11.66 | 10.60 | 9.73 | 9.00 | 8.39 | 7.86 |
| .05 | 20.00 | 16.83 | 14.57 | 12.87 | 11.55 | 10.50 | 9.64 | 8.92 | 8.31 | 7.79 | 7.33 |
| .06 | 16.67 | 14.43 | 12.75 | 11.44 | 10.40 | 9.55 | 8.83 | 8.23 | 7.71 | 7.27 | 6.88 |
| .07 | 14.28 | 12.62 | 11.33 | 10.30 | 9.46 | 8.75 | 8.15 | 7.64 | 7.20 | 6.81 | 6.47 |
| .08 | 12.50 | 11.22 | 10.2 | 9.36 | 8.67 | 8.08 | 7.57 | 7.13 | 6.75 | 6.41 | 6.11 |
| .09 | 11.11 | 10.10 | 9.27 | 8.58 | 8.00 | 7.50 | 7.07 | 6.69 | 6.35 | 6.06 | 5.79 |
| .10 | 10.00 | 9.18 | 8.50 | 7.92 | 7.43 | 7.00 | 6.62 | 6.30 | 6.00 | 5.74 | 5.50 |
| .11 | 9.09 | 8.42 | 7.84 | 7.36 | 6.93 | 6.56 | 6.24 | 5.94 | 5.68 | 5.45 | 5.24 |
| .12 | 8.33 | 7.78 | 7.29 | 6.86 | 6.50 | 6.18 | 5.89 | 5.63 | 5.40 | 5.19 | 5.00 |
| .14 | 7.14 | 6.73 | 6.37 | 6.06 | 5.78 | 5.53 | 5.30 | 5.10 | 4.91 | 4.74 | 4.58 |
| .16 | 6.25 | 5.94 | 5.67 | 5.42 | 5.20 | 5.00 | 4.82 | 4.65 | 4.50 | 4.36 | 4.23 |
| .20 | 5.00 | 4.81 | 4.64 | 4.48 | 4.33 | 4.20 | 4.08 | 3.96 | 3.86 | 3.76 | 3.67 |
| .25 | 4.00 | 3.88 | 3.77 | 3.68 | 3.58 | 3.50 | 3.42 | 3.34 | 3.27 | 3.21 | 3.14 |
| .30 | 3.33 | 3.26 | 3.19 | 3.12 | 3.06 | 3.00 | 2.94 | 2.90 | 2.84 | 2.80 | 2.75 |
| .40 | 2.50 | 2.46 | 2.43 | 2.40 | 2.36 | 2.33 | 2.30 | 2.28 | 2.25 | 2.22 | 2.20 |
| .50 | 2.00 | 1.98 | 1.96 | 1.94 | 1.92 | 1.90 | 1.89 | 1.88 | 1.86 | 1.85 | 1.83 |
| .60 | 1.67 | 1.66 | 1.65 | 1.64 | 1.63 | 1.615 | 1.606 | 1.597 | 1.588 | 1.580 | 1.571 |
| .70 | 1.43 | 1.42 | 1.42 | 1.41 | 1.41 | 1.400 | 1.395 | 1.390 | 1.385 | 1.380 | 1.375 |
| .80 | 1.25 | 1.25 | 1.244 | 1.241 | 1.238 | 1.235 | 1.233 | 1.230 | 1.227 | 1.224 | 1.222 |
| .90 | 1.111 | 1.11 | 1.109 | 1.108 | 1.106 | 1.105 | 1.104 | 1.103 | 1.102 | 1.101 | 1.100 |
| 1.00 | 1.00 | 1.00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Relative Efficiency of 1 lb. of Steam with and without Clearance;
 back pressure and compression not considered.

$$\text{Mean total pressure} = p = \frac{P(l+c) + P(l+c) \text{ hyp log } R - Pc}{L}$$

Let $P = 1$; $L = 100$; $l = 25$; $c = 7$.

$$p = \frac{32 + 32 \text{ hyp log } \frac{107}{32} - 7}{100} = \frac{32 + 32 \times 1.209 - 7}{100} = 0.637.$$

If the clearance be added to the stroke, so that clearance becomes zero, the same quantity of steam being used, admission l being then $= l + c = 32$, and stroke $L + c = 107$,

$$p_1 = \frac{32 + 32 \text{ hyp log } \frac{107}{32} - 0}{107} = \frac{32 + 32 \times 1.209}{107} = 0.707.$$

That is, if the clearance be reduced to 0, the amount of the clearance 7 being added to both the admission and the stroke, the same quantity of steam will do more work than when the clearance is 7 in the ratio 707:637, or 11% more.

Back Pressure Considered. — If back pressure $= 0.10$ of P , this amount has to be subtracted from p and p_1 giving $p = 0.537$, $p_1 = 0.607$, the work of a given quantity of steam used without clearance being greater than when clearance is 7 per cent in the ratio of 607:537, or 13% more.

Effect of Compression. — By early closure of the exhaust, so that a portion of the exhaust-steam is compressed into the clearance-space, much of the loss due to clearance may be avoided. If expansion is continued down to the back pressure, if the back pressure is uniform throughout the exhaust-stroke, and if compression begins at such point that the

exhaust-steam remaining in the cylinder is compressed to the initial pressure at the end of the back stroke, then the work of compression of the exhaust-steam equals the work done during expansion by the clearance-steam. The clearance-space being filled by the exhaust-steam thus compressed, no new steam is required to fill the clearance-space for the next forward stroke, and the work and efficiency of the steam used in the cylinder are just the same as if there were no clearance and no compression. When, however, there is a drop in pressure from the final pressure of the expansion, or the terminal pressure, to the exhaust or back pressure (the usual case), the work of compression to the initial pressure is greater than the work done by the expansion of the clearance-steam, so that a loss of efficiency results. In this case a greater efficiency can be attained by inclosing for compression a less quantity of steam than that needed to fill the clearance-space with steam of the initial pressure. (See Clark, S. E., p. 399, *et seq.*; also F. H. Ball, *Trans. A. S. M. E.*, xiv, 1067.) It is shown by Clark that a somewhat greater efficiency is thus attained whether or not the pressure of the steam be carried down by expansion to the back exhaust-pressure.

Cylinder-condensation may have considerable effect upon the best point of compression, but it has not yet (1893) been determined by experiment. (*Trans. A. S. M. E.*, xiv, 1078.)

Clearance in Low- and High-speed Engines. (Harris Tabor, *Am. Mach.*, Sept. 17, 1891.) — The construction of the high-speed engine is such, with its relatively short stroke, that the clearance must be much larger than in the releasing-valve type. The short-stroke engine is, of necessity, an engine with large clearance, which is aggravated when variable compression is a feature. Conversely, the engine with releasing-valve gear is, from necessity, an engine of slow rotative speed, where great power is obtainable from long stroke, and small clearance is a feature in its construction. In one case the clearance will vary from 8% to 12% of the piston-displacement, and in the other from 2% to 3%. In the case of an engine with a clearance equaling 10% of the piston-displacement the waste room becomes enormous when considered in connection with an early cut-off. The system of compounding reduces the waste due to clearance in proportion as the steam is expanded to a lower pressure. The farther expansion is carried through a train of cylinders the greater will be the reduction of waste due to clearance. This is shown from the fact that the high-speed engine, expanding steam much less than the Corliss, will show a greater gain when changed from simple to compound than its rival under similar conditions.

Cylinder-condensation. — Rankine, S. E., p. 421, says: Conduction of heat to and from the metal of the cylinder, or to and from liquid water contained in the cylinder, has the effect of lowering the pressure at the beginning and raising it at the end of the stroke, the lowering effect being on the whole greater than the raising effect. In some experiments the quantity of steam wasted through alternate liquefaction and evaporation in the cylinder has been found to be greater than the quantity which performed the work.

Percentage of Loss by Cylinder-condensation, taken at Cut-off.

(From circular of the Ashcroft Mfg. Co. on the Tabor Indicator, 1889.)

| Percentage of Stroke completed at Cut-off. | Per cent of Feed-water accounted for by the Indicator. | | | Per cent of Feed-water due to Cylinder-condensation. | | |
|--|--|-----------------------------|-------------------------------------|--|-----------------------------|-------------------------------------|
| | Simple Engines. | Compound Engines, h.p. cyl. | Triple-expansion Engines, h.p. cyl. | Simple Engines. | Compound Engines, h.p. cyl. | Triple-expansion Engines, h.p. cyl. |
| 5 | 58 | | | 42 | | |
| 10 | 66 | 74 | | 34 | 26 | |
| 15 | 71 | 76 | 78 | 29 | 24 | 22 |
| 20 | 74 | 78 | 80 | 26 | 22 | 20 |
| 30 | 78 | 82 | 84 | 22 | 18 | 16 |
| 40 | 82 | 85 | 87 | 18 | 15 | 13 |
| 50 | 86 | 88 | 90 | 14 | 12 | 10 |

Theoretical Compared with Actual Water-consumption, Single-cylinder Automatic Cut-off Engines. (From the catalogue of the Buckeye Engine Co.) — The following table has been prepared on the basis of the pressures that result in practice with a constant boiler-pressure of 80 lbs. and different points of cut-off, with Buckeye engines and others with similar clearance. Fractions are omitted, except in the percentage column, as the degree of accuracy their use would seem to imply is not attained or aimed at.

| Cut-off Part of Stroke. | Mean Effective Pressure, lbs. per sq. in. | Total Terminal Pressure, lbs. per sq. in. | Indicated Rate, lbs. Water per I.H.P. per hour. | Assumed. | | Product of Cols. 1 and 6. |
|-------------------------|---|---|---|-------------|---------|---------------------------|
| | | | | Act'l Rate. | % Loss. | |
| 0.10 | 18 | 11 | 20 | 32 | 58 | 5.8 |
| 0.15 | 27 | 15 | 19 | 27 | 41 | 6.15 |
| 0.20 | 35 | 20 | 19 | 25 | 31.5 | 6.3 |
| 0.25 | 42 | 25 | 20 | 25 | 25 | 6.25 |
| 0.30 | 48 | 30 | 20 | 24 | 21.8 | 6.54 |
| 0.35 | 53 | 35 | 21 | 25 | 19 | 6.65 |
| 0.40 | 57 | 38 | 22 | 26 | 16.7 | 6.68 |
| 0.45 | 61 | 43 | 23 | 27 | 15 | 6.75 |
| 0.50 | 64 | 48 | 24 | 27 | 13.6 | 6.8 |

It will be seen that while the best indicated economy is when the cut-off is about at 0.15 or 0.20 of the stroke, giving about 30 lbs. M.E.P., and a terminal 3 or 4 lbs. above atmosphere, when we come to add the percentages due to a constant amount of unindicated loss, as per sixth column, the most economical point of cut-off is found to be about 0.30 of the stroke, giving 48 lbs. M.E.P. and 30 lbs. terminal pressure. This showing agrees substantially with modern experience under automatic cut-off regulation.

The last column shows that the actual amount of cylinder condensation is nearly a constant quantity, increasing only from 5.8% of the cylinder volume at 0.10 cut-off to 6.8% at 0.50 cut-off.

Experiments on Cylinder-condensation. — Experiments by Major Thos. English (*Eng'g*, Oct. 7, 1887, p. 386) with an engine 10 × 14 in., jacketed in the sides but not on the ends, indicate that the net initial condensation (or excess of condensation over re-evaporation) by the clearance surface varies directly as the initial density of the steam, and inversely as the square root of the number of revolutions per unit of time. The mean results gave for the net initial condensation by clearance-space per sq. ft. of surface at one rev. per second 6.06 thermal units in the engine when run non-condensing and 5.75 units when condensing.

G. R. Bodmer (*Eng'g*, March 4, 1892, p. 299) says: Within the ordinary limits of expansion desirable in one cylinder the expansion ratio has practically no influence on the amount of condensation per stroke, which for simple engines can be expressed by the following formula for the weight of water condensed [per minute, probably; the original does not

state]: $W = C \frac{S(T-t)}{L \sqrt{N^2}}$, where T denotes the mean admission temper-

ature, t the mean exhaust temperature, S clearance-surface (square feet), N the number of revolutions per second, L latent heat of steam at the mean admission temperature, and C a constant for any given type of engine.

Mr. Bodmer found from experimental data that for high-pressure non-jacketed engines C = about 0.11, for condensing non-jacketed engines 0.085 to 0.11, for condensing jacketed engines 0.085 to 0.053. The figures for jacketed engines apply to those jacketed in the usual way, and not at the ends.

C varies for different engines of the same class, but is practically constant for any given engine. For simple high-pressure non-jacketed engines it was found to range from 0.1 to 0.112.

Applying Mr. Bodmer's formula to the case of a Corliss non-jacketed

non-condensing engine, 4-ft. stroke, 24 in. diam., 60 revs. per min., initial pressure 90 lbs. gauge, exhaust pressure 2 lbs., we have $T - t = 112^\circ$, $N = 1$, $L = 880$, $S = 7$ sq. ft.; and, taking $C = 0.112$ and $W =$ lbs. water condensed per minute, $W = \frac{0.112 \times 112 \times 7}{1 \times 880} = 0.09$ lb. per minute, or 5.4 lbs. per hour. If the steam used per I.H.P. per hour according to the diagram is 20 lbs., the actual water consumption is 25.4 lbs., corresponding to a cylinder condensation of 27%.

INDICATOR-DIAGRAM OF A SINGLE-CYLINDER ENGINE.

Definitions. — *The Atmospheric Line, AB*, is a line drawn by the pencil of the indicator when the connections with the engine are closed and both sides of the piston are open to the atmosphere.

The Vacuum Line, OX, is a reference line usually drawn about 14.7 pounds by scale below the atmospheric line.

The Clearance Line, OY, is a reference line drawn at a distance from the end of the diagram equal to the same per cent of its length as the clearance and waste room is of the piston-displacement.

The Line of Boiler-pressure, JK, is drawn parallel to the

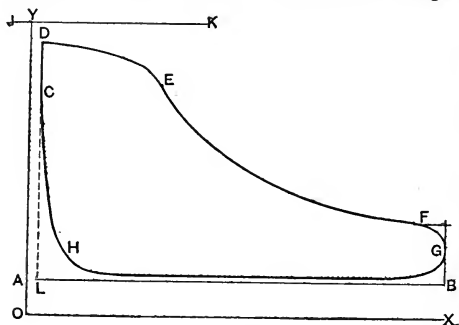


FIG. 154.

atmospheric line, and at a distance from it by scale equal to the boiler-pressure shown by the gauge.

The Admission Line, CD, shows the rise of pressure due to the admission of steam to the cylinder by opening the steam-valve.

The Steam Line, DE, is drawn when the steam-valve is open and steam is being admitted to the cylinder.

The Point of Cut-off, E, is the point where the admission of steam is stopped by the closing of the valve. It is often difficult to determine the exact point at which the cut-off takes place. It is usually located where the outline of the diagram changes its curvature from convex to concave.

The Expansion Curve, EF, shows the fall in pressure as the steam in the cylinder expands doing work.

The Point of Release, F, shows when the exhaust-valve opens.

The Exhaust Line, FG, represents the change in pressure that takes place when the exhaust-valve opens.

The Back-pressure Line, GH, shows the pressure against which the piston acts during its return stroke.

The Point of Exhaust Closure, H, is the point where the exhaust-valve closes. It cannot be located definitely, as the change in pressure is at first due to the gradual closing of the valve.

The Compression Curve, HC, shows the rise in pressure due to the compression of the steam remaining in the cylinder after the exhaust-valve has closed.

The Mean Height of the Diagram equals its area divided by its length.

The Mean Effective Pressure is the mean net pressure urging the piston forward = the mean height \times the scale of the indicator-spring.

To find the Mean Effective Pressure from the Diagram. — Divide the length, LB , into a number, say 10, equal parts, setting off half a part at L , half a part at B , and nine other parts between; erect ordinates perpendicular to the atmospheric line at the points of division of LB , cutting the diagram; add together the lengths of these ordinates intercepted

between the upper and lower lines of the diagram and divide by their number. This gives the mean height, which multiplied by the scale of the indicator-spring gives the M.E.P. Or find the area by a planimeter, or other means (see Mensuration, p. 57), and divide by the length *LB* to obtain the mean height.

The Initial Pressure is the pressure acting on the piston at the beginning of the stroke.

The Terminal Pressure is the pressure above the line of perfect vacuum that would exist at the end of the stroke if the steam had not been released earlier. It is found by continuing the expansion-curve to the end of the diagram.

A single indicator card shows the pressure exerted by the steam at each instant on one side of the piston; a card taken simultaneously from the opposite end of the engine shows the pressure exerted on the other side. By superposing these cards the pressure or tension on the piston rod may be determined. The pressure or pull on the crank pin at any instant is the pressure or tension in the rod modified by the angle of the connecting rod and by the effect of the inertia of the reciprocating parts. For discussion of this subject see Klein's "High-speed Steam Engine," also papers by S. A. Moss, *Trans. A. S. M. E.*, 1904, and by F. W. Hollmann, in *Power*, April 6, 1909.

Errors of Indicators. — The most common error is that of the spring, which may vary from its normal rating; the error may be determined by proper testing apparatus and allowed for. But after making this correction, even with the best work, the results are liable to variable errors which may amount to 2 or 3 per cent. See Barrus, *Trans. A. S. M. E.*, v, 310; Denton, *Trans. A. S. M. E.*, xi, 329; David Smith, U. S. N., *Proc. Eng'g Congress*, 1893, Marine Division.

Other errors of indicator diagrams are those due to inaccuracy of the straight-line motion of the indicator, to the incorrect design or position of the "rig" or reducing motion, to long pipes between the indicator and the engine, to throttling of these pipes, to friction or lost motion in the indicator mechanism, and to drum-motion distortion. For discussion of the last named see *Power*, April, 1909. For methods of testing indicators, see paper by D. S. Jacobus, *Trans. A. S. M. E.*, 1898.

Indicator "Rigs," or Reducing-motions; Interpretation of Diagrams for Errors of Steam-distribution, etc. For these see circulars of manufacturers of Indicators; also works on the Indicator.

Pendulum Indicator Rig. — *Power* (Feb., 1893) gives a graphical representation of the errors in indicator-diagrams, caused by the use of incorrect forms of the pendulum rigging. It is shown that the "brumbo" pulley on the pendulum, to which the cord is attached, does not generally give as good a reduction as a simple pin attachment. When the end of the pendulum is slotted, working in a pin on the crosshead, the error is apt to be considerable at both ends of the card. With a vertical slot in a plate fixed to the crosshead, and a pin on the pendulum working in this slot, the reduction is perfect, when the cord is attached to a pin on the pendulum, a slight error being introduced if the brumbo pulley is used. With the connection between the pendulum and the crosshead made by means of a horizontal link, the reduction is nearly perfect, if the construction is such that the connecting link vibrates equally above and below the horizontal, and the cord is attached by a pin. If the link is horizontal at mid-stroke a serious error is introduced, which is magnified if a brumbo pulley also is used. The adjoining figures show the two forms recommended.

The Manograph, for indicating engines of very high speed, invented by Prof. Hospitalier, is described by Howard Greene in *Power*, June, 1907. It is made by Carpentier, of Paris. A small mirror is tilted upward and downward by a diaphragm which responds to the pressure variations in the cylinder, and the same mirror is rocked from side to side by a reducing mechanism which is geared to the engine and reproduces the reciprocations

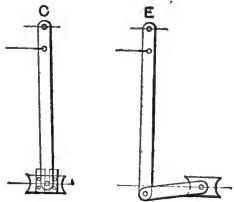


FIG. 155.

of the engine piston on a smaller scale. A beam of light is reflected by the mirror to the ground-glass screen, and this beam, by the oscillations of the mirror, is made to traverse a path corresponding to that of the pencil point of an ordinary indicator. The diagram, therefore, is made continuously but varies with varying conditions in the cylinder.

A plate-holder carrying a photographic dry plate can be substituted for the ground-glass screen, and the diagram photographed, the exposure required varying from half a second to three seconds. By the use of special diaphragms and springs the effects of low pressures and vacuums can be magnified, and thus the instrument can be made to show with remarkable clearness the action of the valves of a gas engine on the suction and exhaust strokes.

The Lea Continuous Recorder, for recording the steam consumption of an engine, is described by W. H. Booth in *Power*, Aug. 31, 1909. It comprises a tank into which flows the condensed steam from a condenser, a triangular notch through which the water flows from the tank, and a mechanical device through which the variations in the level of the water in the tank are translated into the motion of a pencil, which motion is made proportionate to the quantity flowing, and is recorded on paper moved by clockwork.

INDICATED HORSE-POWER OF ENGINES, SINGLE-CYLINDER.

$$\text{Indicated Horse-power, I.H.P.} = \frac{P L a n}{33,000},$$

in which P = mean effective pressure in lbs. per sq. in.; L = length of stroke in feet; a = area of piston in square inches. For accuracy, one half of the sectional area of the piston-rod must be subtracted from the area of the piston if the rod passes through one head, or the whole area of the rod if it passes through both heads; n = No. of single strokes per min. = $2 \times$ No. of revolutions of a double-acting engine.

$$\text{I.H.P.} = \frac{P a S}{33,000}, \text{ in which } S = \text{piston speed in feet per minute.}$$

$$\text{I.H.P.} = \frac{P L d^2 n}{42,017} = \frac{P d^2 S}{42,017} = 0.0000238 P L d^2 n = 0.0000238 P d^2 S,$$

in which d = diam. of cyl. in inches. (The figures 238 are exact, since $7854 \div 33 = 238$ exactly.) If product of piston-speed \times mean effective pressure = 42,017, then the horse-power would equal the square of the diameter in inches.

Handy Rule for Estimating the Horse-power of a Single-cylinder Engine. — Square the diameter and divide by 2. This is correct whenever the product of the mean effective pressure and the piston-speed = $\frac{1}{2}$ of 42,017, or, say, 21,000, viz., when M.E.P. = 30 and $S = 700$; when M.E.P. = 35 and $S = 600$; when M.E.P. = 38.2 and $S = 550$; and when M.E.P. = 42 and $S = 500$. These conditions correspond to those of ordinary practice with both Corliss engines and shaft-governor high-speed engines.

Given Horse-power, Mean Effective Pressure, and Piston-speed, to find Size of Cylinder. —

$$\text{Area} = \frac{33,000 \times \text{I.H.P.}}{P L n}. \quad \text{Diameter} = 205 \sqrt{\frac{\text{I.H.P.}}{P S}}.$$

Brake Horse-power is the actual horse-power of the engine as measured at the fly-wheel by a friction-brake or dynamometer. It is the indicated horse-power minus the friction of the engine.

Electrical Horse-power is the power in an electric current, usually measured in kilowatts, translated into horse-power. 1 H.P. = 33,000 ft. lbs. per min.; 1 K.W. = 1.3405 H.P.; 1 H.P. = 0.746 kilowatts, or 746 watts.

EXAMPLE. — A 100-H.P. engine, with a friction loss of 10% at rated load, drives a generator whose efficiency is 90%, furnishing current to a motor of 90% effy., through a line whose loss is 5%. I.H.P. = 100; B.H.P. = 90; E.H.P. at generator 81, at end of line 76.95. H.P. delivered by motor 69.26.

Table for Roughly Approximating the Horse-power of a Compound Engine from the Diameter of its Low-pressure Cylinder. —

The indicated horse-power of an engine being $\frac{Psd^2}{42,017}$, in which P = mean effective pressure per sq. in., s = piston-speed in ft. per min., and d = diam. of cylinder in inches; if s = 600 ft. per min., which is approximately the speed of modern stationary engines, and P = 35 lbs., which is an approximately average figure for the M.E.P. of single-cylinder engines, and of compound engines referred to the low-pressure cylinder, then I.H.P. = $\frac{1}{2}d^2$; hence the rough-and-ready rule for horse-power given above: Square the diameter in inches and divide by 2. This applies to triple and quadruple expansion engines as well as to single cylinder and compound. For most economical loading, the M.E.P. referred to the low-pressure cylinder of compound engines is usually not greater than that of simple engines; for the greater economy is obtained by a greater number of expansions of steam of higher pressures, and the greater the number of expansions for a given initial pressure the lower the mean effective pressure. The following table gives approximately the figures of mean total and effective pressures for the different types of engines, together with the factor by which the square of the diameter is to be multiplied to obtain the horse-power at most economical loading, for a piston-speed of 600 ft. per minute.

| Type of Engine. | Initial Absolute Steam-Pressure. | Number of Expansions. | Terminal Absolute Press., lbs. | Ratio Mean Total to Initial Pressure. | Mean Total Pressure, lbs. | Total Back Pressure, Mean, lbs. | Mean Effective Pressure, lbs. | Piston-speed, ft. per min. | Horse-power = diam. ² × |
|---------------------|----------------------------------|-----------------------|--------------------------------|---------------------------------------|---------------------------|---------------------------------|-------------------------------|----------------------------|------------------------------------|
| Non-condensing. | | | | | | | | | |
| Single Cylinder.. | 100 | 5. | 20 | 0.522 | 52.2 | 15.5 | 36.7 | 600 | 0.524 |
| Compound..... | 120 | 7.5 | 16 | .402 | 48.2 | 15.5 | 32.7 | " | .467 |
| Triple..... | 160 | 10. | 16 | .330 | 52.8 | 15.5 | 37.3 | " | .533 |
| Quadruple..... | 200 | 12.5 | 16 | .282 | 56.4 | 15.5 | 40.9 | " | .584 |
| Condensing Engines. | | | | | | | | | |
| Single Cylinder.. | 100 | 10. | 10 | 0.330 | 33.0 | 2 | 31.0 | 600 | 0.443 |
| Compound..... | 120 | 15. | 8 | .247 | 29.6 | 2 | 27.6 | " | .390 |
| Triple..... | 160 | 20. | 8 | .200 | 32.0 | 2 | 30.0 | " | .429 |
| Quadruple..... | 200 | 25. | 8 | .169 | 33.8 | 2 | 31.8 | " | .454 |

For any other piston-speed than 600 ft. per min., multiply the figures in the last column by the ratio of the piston-speed to 600 ft.

Horse-power Constant of a given Engine for a Fixed Speed = product of its area of piston in square inches, length of stroke in feet and number of single strokes per minute divided by 33,000, or $\frac{Lan}{33,000}$ = C . The product of the mean effective pressure as found by the diagram and this constant is the indicated horse-power.

Horse-power Constant of any Engine of a given Diameter of Cylinder, whatever the length of stroke, = area of piston ÷ 33,000 = square of the diameter of piston in inches × 0.0000238. A table of constants derived from this formula is given on page 943.

The constant multiplied by the piston-speed in feet per minute and by the M.E.P. gives the I.H.P.

Table of Engine Constants for Use in Figuring Horse-power. — "Horse-power constant" for cylinders from 1 inch to 60 inches in diameter, advancing by 8ths, for one foot of piston-speed per minute and one pound of M.E.P. Find the diameter of the cylinder in the column at the side. If the diameter contains no fraction the constant will be found in the column headed Even Inches. If the diameter is not in even inches, follow the line horizontally to the column corresponding to the required fraction. The constants multiplied by the piston-speed and by the M.E.P. give the horse-power.

Engine Constants, Constant \times Piston Speed \times M.E.P. = H.P.

| Diam. of Cylinder. | Even Inches. | + 1/8 | + 1/4 | + 3/8 | + 1/2 | + 5/8 | + 3/4 | + 7/8 |
|-----------------------|-----------------|---------|---------|---------|---------|---------|---------|---------|
| 1 | 0000238 | 0000301 | 0000372 | 0000450 | 0000535 | 0000628 | 0000729 | 0000837 |
| 2 | 0000952 | 0001074 | 0001205 | 0001342 | 0001487 | 0001640 | 0001800 | 0001967 |
| 3 | 0002142 | 0002324 | 0002514 | 0002711 | 0002915 | 0003127 | 0003347 | 0003574 |
| 4 | 0003808 | 0004050 | 0004299 | 0004554 | 0004819 | 0005091 | 0005370 | 0005656 |
| 5 | 0005950 | 0006251 | 0006560 | 0006876 | 0007199 | 0007530 | 0007869 | 0008215 |
| 6 | 0008568 | 0008929 | 0009297 | 0009672 | 0010055 | 0010445 | 0010844 | 0011249 |
| 7 | 0011662 | 0012082 | 0012510 | 0012944 | 0013387 | 0013837 | 0014295 | 0014759 |
| 8 | 0015232 | 0015711 | 0016198 | 0016693 | 0017195 | 0017705 | 0018222 | 0018746 |
| 9 | 0019278 | 0019817 | 0020363 | 0020916 | 0021479 | 0022048 | 0022625 | 0023209 |
| 10 | 0023800 | 0024398 | 0025004 | 0025618 | 0026239 | 0026867 | 0027502 | 0028147 |
| 11 | 0028798 | 0029456 | 0030121 | 0030794 | 0031475 | 0032163 | 0032859 | 0033561 |
| 12 | 0034272 | 0034990 | 0035714 | 0036447 | 0037187 | 0037934 | 0038690 | 0039452 |
| 13 | 0040222 | 0040999 | 0041783 | 0042576 | 0043375 | 0044182 | 0044997 | 0045819 |
| 14 | 0046648 | 0047484 | 0048328 | 0049181 | 0050039 | 0050906 | 0051780 | 0052661 |
| 15 | 0053550 | 0054446 | 0055349 | 0056261 | 0057179 | 0058105 | 0059039 | 0059979 |
| 16 | 0060928 | 0061884 | 0062847 | 0063817 | 0064795 | 0065780 | 0066774 | 0067774 |
| 17 | 0068782 | 0069797 | 0070819 | 0071850 | 0072887 | 0073932 | 0074985 | 0076044 |
| 18 | 0077112 | 0078187 | 0079268 | 0080360 | 0081452 | 0082560 | 0083672 | 0084791 |
| 19 | 0085918 | 0087052 | 0088193 | 0089343 | 0090499 | 0091663 | 0092835 | 0094013 |
| 20 | 0095200 | 0096393 | 0097594 | 0098803 | 0100019 | 0101243 | 0102474 | 0103712 |
| 21 | 0104958 | 0106211 | 0107472 | 0108739 | 0110015 | 0111299 | 0112589 | 0113886 |
| 22 | 0115192 | 0116505 | 0117825 | 0119152 | 0120487 | 0121830 | 0123179 | 0124537 |
| 23 | 0125902 | 0127274 | 0128654 | 0130040 | 0131435 | 0132837 | 0134247 | 0135664 |
| 24 | 0137088 | 0138519 | 0139959 | 0141405 | 0142859 | 0144321 | 0145789 | 0147266 |
| 25 | 0148750 | 0150241 | 0151739 | 0153246 | 0154759 | 0156280 | 0157809 | 0159345 |
| 26 | 0160888 | 0162439 | 0163997 | 0165563 | 0167135 | 0168716 | 0170304 | 0171899 |
| 27 | 0173502 | 0175112 | 0176729 | 0178355 | 0179988 | 0181627 | 0183275 | 0184929 |
| 28 | 0186592 | 0188262 | 0189939 | 0191624 | 0193316 | 0195015 | 0196722 | 0198436 |
| 29 | 0200158 | 0201887 | 0203634 | 0205368 | 0207119 | 0208879 | 0210645 | 0212418 |
| 30 | 0214200 | 0215988 | 0217785 | 0219588 | 0221399 | 0223218 | 0225044 | 0226877 |
| 31 | 0228718 | 0230566 | 0232422 | 0234285 | 0236155 | 0238033 | 0239919 | 0241812 |
| 32 | 0243712 | 0245619 | 0247535 | 0249457 | 0251387 | 0253325 | 0255269 | 0257222 |
| 33 | 0259182 | 0261149 | 0263124 | 0265106 | 0267095 | 0269092 | 0271097 | 0273109 |
| 34 | 0275128 | 0277155 | 0279189 | 0281231 | 0283279 | 0285336 | 0287399 | 0289471 |
| 35 | 0291550 | 0293636 | 0295729 | 0297831 | 0299939 | 0302056 | 0304179 | 0306309 |
| 36 | 0308448 | 0310594 | 0312747 | 0314908 | 0317075 | 0319251 | 0321434 | 0323624 |
| 37 | 0325822 | 0328027 | 0330239 | 0332460 | 0334687 | 0336922 | 0339165 | 0341415 |
| 38 | 0343672 | 0345937 | 0348209 | 0350489 | 0352775 | 0355070 | 0357372 | 0359681 |
| 39 | 0361998 | 0364322 | 0366654 | 0368993 | 0371339 | 0373694 | 0376055 | 0378424 |
| 40 | 0380800 | 0383184 | 0385575 | 0387973 | 0390379 | 0392793 | 0395214 | 0397642 |
| 41 | 0400078 | 0402521 | 0404972 | 0407430 | 0409895 | 0412368 | 0414849 | 0417337 |
| 42 | 0419832 | 0422335 | 0424845 | 0427362 | 0429887 | 0432420 | 0434959 | 0437507 |
| 43 | 0440062 | 0442624 | 0445194 | 0447771 | 0450355 | 0452947 | 0455547 | 0458154 |
| 44 | 0460768 | 0463389 | 0466019 | 0468655 | 0471299 | 0473951 | 0476609 | 0479276 |
| 45 | 0481950 | 0484631 | 0487320 | 0490016 | 0492719 | 0495430 | 0498149 | 0500875 |
| 46 | 0503608 | 0506349 | 0509097 | 0511853 | 0514615 | 0517386 | 0520164 | 0522949 |
| 47 | 0525742 | 0528542 | 0531349 | 0534165 | 0536988 | 0539818 | 0542655 | 0545499 |
| 48 | 0548352 | 0551212 | 0554079 | 0556953 | 0559835 | 0562725 | 0565622 | 0568526 |
| 49 | 0571438 | 0574357 | 0577284 | 0580218 | 0583159 | 0586109 | 0589065 | 0592029 |
| 50 | 0595000 | 0597979 | 0600965 | 0603959 | 0606959 | 0609969 | 0612984 | 0616007 |
| 51 | 0619038 | 0622076 | 0625122 | 0628175 | 0631235 | 0634304 | 0637379 | 0640462 |
| 52 | 0643552 | 0646649 | 0649753 | 0652867 | 0655987 | 0659115 | 0662250 | 0665392 |
| 53 | 0668542 | 0671699 | 0674864 | 0678036 | 0681215 | 0684402 | 0687597 | 0690799 |
| 54 | 0694008 | 0697225 | 0700449 | 0703681 | 0706923 | 0710166 | 0713419 | 0716681 |
| 55 | 0719950 | 0724226 | 0728510 | 0732801 | 0737099 | 0741406 | 0745719 | 0750039 |
| 56 | 0746368 | 0749704 | 0753047 | 0756398 | 0759755 | 0763120 | 0766494 | 0769874 |
| 57 | 0773262 | 0776657 | 0780060 | 0783476 | 0786887 | 0790312 | 0793745 | 0797185 |
| 58 | 0800632 | 0804087 | 0807549 | 0811019 | 0814495 | 0817980 | 0821472 | 0824971 |
| 59 | 0828478 | 0831992 | 0835514 | 0839043 | 0842579 | 0846123 | 0849675 | 0853234 |
| 60 | 0856800 | 0860374 | 0863955 | 0867543 | 0871139 | 0874743 | 0878354 | 0881973 |

Horse-power per Pound Mean Effective Pressure.Formula, Area in sq. in. \times piston-speed \div 33,000.

| Diam of Cylinder, inches. | Speed of Piston in feet per minute. | | | | | | | | |
|---------------------------------|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| 4 | .0381 | .0762 | .1142 | .1523 | .1904 | .2285 | .2666 | .3046 | .3427 |
| 4 1/2 | .0482 | .0964 | .1446 | .1928 | .2410 | .2892 | .3374 | .3856 | .4338 |
| 5 | .0595 | .1190 | .1785 | .2380 | .2975 | .3570 | .4165 | .4760 | .5355 |
| 5 1/2 | .0720 | .1440 | .2160 | .2880 | .3600 | .4320 | .5040 | .5760 | .6480 |
| 6 | .0857 | .1714 | .2570 | .3427 | .4284 | .5141 | .5998 | .6854 | .7711 |
| 6 1/2 | .1006 | .2011 | .3017 | .4022 | .5028 | .6033 | .7039 | .8044 | .9050 |
| 7 | .1166 | .2332 | .3499 | .4665 | .5831 | .6997 | .8163 | .9330 | 1.0496 |
| 7 1/2 | .1339 | .2678 | .4016 | .5355 | .6694 | .8033 | .9371 | 1.0710 | 1.2049 |
| 8 | .1523 | .3046 | .4570 | .6093 | .7616 | .9139 | 1.0662 | 1.2186 | 1.3709 |
| 8 1/2 | .1720 | .3439 | .5159 | .6878 | .8598 | 1.0317 | 1.2037 | 1.3756 | 1.5476 |
| 9 | .1928 | .3856 | .5783 | .7711 | .9639 | 1.1567 | 1.3495 | 1.5422 | 1.7350 |
| 9 1/2 | .2148 | .4296 | .6444 | .8592 | 1.0740 | 1.2888 | 1.5036 | 1.7184 | 1.9532 |
| 10 | .2380 | .4760 | .7140 | .9520 | 1.1900 | 1.4280 | 1.6660 | 1.9040 | 2.1420 |
| 11 | .2880 | .5760 | .8639 | 1.1519 | 1.4399 | 1.7279 | 2.0159 | 2.3038 | 2.5818 |
| 12 | .3427 | .6854 | 1.0282 | 1.3709 | 1.7136 | 2.0563 | 2.3990 | 2.7418 | 3.0845 |
| 13 | .4022 | .8044 | 1.2067 | 1.6089 | 2.0111 | 2.4133 | 2.8155 | 3.2178 | 3.6200 |
| 14 | .4665 | .9330 | 1.3994 | 1.8659 | 2.3324 | 2.7989 | 3.2654 | 3.7318 | 4.1983 |
| 15 | .5355 | 1.0710 | 1.6065 | 2.1420 | 2.6775 | 3.2130 | 3.7485 | 4.2840 | 4.8195 |
| 16 | .6093 | 1.2186 | 1.8278 | 2.4371 | 3.0464 | 3.6557 | 4.2650 | 4.8742 | 5.4835 |
| 17 | .6878 | 1.3756 | 2.0635 | 2.7513 | 3.4391 | 4.1269 | 4.8147 | 5.5026 | 6.1904 |
| 18 | .7711 | 1.5422 | 2.3134 | 3.0845 | 3.8556 | 4.6267 | 5.3978 | 6.1690 | 6.9401 |
| 19 | .8592 | 1.7184 | 2.5775 | 3.4367 | 4.2959 | 5.1551 | 6.0143 | 6.8734 | 7.7326 |
| 20 | .9520 | 1.9040 | 2.8560 | 3.8080 | 4.7600 | 5.7120 | 6.6640 | 7.6160 | 8.5680 |
| 21 | 1.0496 | 2.0992 | 3.1488 | 4.1983 | 5.2479 | 6.2975 | 7.3471 | 8.3966 | 9.4462 |
| 22 | 1.1519 | 2.3038 | 3.4558 | 4.6077 | 5.7596 | 6.9115 | 8.0634 | 9.2154 | 10.367 |
| 23 | 1.2590 | 2.5180 | 3.7771 | 5.0361 | 6.2951 | 7.5541 | 8.8131 | 10.072 | 11.331 |
| 24 | 1.3709 | 2.7418 | 4.1126 | 5.4835 | 6.8544 | 8.2253 | 9.5962 | 10.967 | 12.338 |
| 25 | 1.4875 | 2.9750 | 4.4625 | 5.9500 | 7.4375 | 8.9250 | 10.413 | 11.900 | 13.388 |
| 26 | 1.6089 | 3.2178 | 4.8266 | 6.4355 | 8.0444 | 9.6534 | 11.262 | 12.871 | 14.480 |
| 27 | 1.7350 | 3.4700 | 5.2051 | 6.9401 | 8.6751 | 10.410 | 12.145 | 13.880 | 15.615 |
| 28 | 1.8659 | 3.7318 | 5.5978 | 7.4637 | 9.3296 | 11.196 | 13.061 | 14.927 | 16.793 |
| 29 | 2.0016 | 4.0032 | 6.0047 | 8.0063 | 10.008 | 12.009 | 14.011 | 16.013 | 18.014 |
| 30 | 2.1420 | 4.2840 | 6.4260 | 8.5680 | 10.710 | 12.852 | 14.994 | 17.136 | 19.278 |
| 31 | 2.2872 | 4.5744 | 6.8615 | 9.1487 | 11.436 | 13.723 | 16.010 | 18.297 | 20.585 |
| 32 | 2.4371 | 4.8742 | 7.3114 | 9.7485 | 12.186 | 14.623 | 17.060 | 19.497 | 21.934 |
| 33 | 2.5918 | 5.1836 | 7.7755 | 10.367 | 12.959 | 15.551 | 18.143 | 20.735 | 23.326 |
| 34 | 2.7513 | 5.5026 | 8.2538 | 11.005 | 13.756 | 16.508 | 19.259 | 22.010 | 24.762 |
| 35 | 2.9155 | 5.8310 | 8.7465 | 11.662 | 14.578 | 17.493 | 20.409 | 23.324 | 26.240 |
| 36 | 3.0845 | 6.1690 | 9.2534 | 12.338 | 15.422 | 18.507 | 21.591 | 24.676 | 27.760 |
| 37 | 3.2582 | 6.5164 | 9.7747 | 13.033 | 16.291 | 19.549 | 22.808 | 26.066 | 29.324 |
| 38 | 3.4367 | 6.8734 | 10.310 | 13.747 | 17.184 | 20.620 | 24.057 | 27.494 | 30.930 |
| 39 | 3.6200 | 7.2400 | 10.860 | 14.480 | 18.100 | 21.720 | 25.340 | 28.960 | 32.580 |
| 40 | 3.8080 | 7.6160 | 11.424 | 15.232 | 19.040 | 22.848 | 26.656 | 30.464 | 34.272 |
| 41 | 4.0008 | 8.0016 | 12.002 | 16.003 | 20.004 | 24.005 | 28.005 | 32.006 | 36.007 |
| 42 | 4.1983 | 8.3866 | 12.585 | 16.783 | 20.982 | 25.180 | 29.378 | 33.577 | 37.775 |
| 43 | 4.4006 | 8.8012 | 13.202 | 17.602 | 22.003 | 26.404 | 30.804 | 35.205 | 39.606 |
| 44 | 4.6077 | 9.2154 | 13.823 | 18.431 | 23.038 | 27.646 | 32.254 | 36.861 | 41.469 |
| 45 | 4.8195 | 9.6390 | 14.459 | 19.278 | 24.098 | 28.917 | 33.737 | 38.556 | 43.376 |
| 46 | 5.0361 | 10.072 | 15.108 | 20.144 | 25.180 | 30.216 | 35.253 | 40.289 | 45.325 |
| 47 | 5.2574 | 10.515 | 15.772 | 21.030 | 26.287 | 31.545 | 36.802 | 42.059 | 47.317 |
| 48 | 5.4835 | 10.967 | 16.451 | 21.934 | 27.418 | 32.901 | 38.385 | 43.868 | 49.352 |
| 49 | 5.7144 | 11.429 | 17.143 | 22.858 | 28.572 | 34.286 | 40.001 | 45.715 | 51.429 |
| 50 | 5.9500 | 11.900 | 17.850 | 23.800 | 29.750 | 35.700 | 41.650 | 47.600 | 53.550 |
| 51 | 6.1904 | 12.381 | 18.571 | 24.762 | 30.952 | 37.142 | 43.333 | 49.523 | 55.713 |
| 52 | 6.4355 | 12.871 | 19.307 | 25.742 | 32.178 | 38.613 | 45.049 | 51.484 | 57.920 |
| 53 | 6.6854 | 13.371 | 20.056 | 26.742 | 33.427 | 40.113 | 46.798 | 53.483 | 60.169 |
| 54 | 6.9401 | 13.880 | 20.820 | 27.760 | 34.700 | 41.640 | 48.581 | 55.521 | 62.461 |
| 55 | 7.1995 | 14.399 | 21.599 | 28.798 | 35.998 | 43.197 | 50.397 | 57.596 | 64.796 |
| 56 | 7.4637 | 14.927 | 22.391 | 29.855 | 37.318 | 44.782 | 52.246 | 59.709 | 67.173 |
| 57 | 7.7326 | 15.465 | 23.198 | 30.930 | 38.663 | 46.396 | 54.128 | 61.861 | 69.597 |
| 58 | 8.0063 | 16.013 | 24.019 | 32.025 | 40.032 | 48.038 | 56.044 | 64.051 | 72.054 |
| 59 | 8.2848 | 16.570 | 24.854 | 33.139 | 41.424 | 49.709 | 57.993 | 66.278 | 74.563 |
| 60 | 8.5680 | 17.136 | 25.704 | 34.272 | 42.840 | 51.408 | 59.976 | 68.544 | 77.112 |

Nominal Horse-power.—The term "nominal horse-power" originated in the time of Watt, and was used to express approximately the power of an engine as calculated from its diameter, estimating the mean pressure in the cylinder at 7 lbs. above the atmosphere. It has long been obsolete.

Horse-power Constant of a given Engine for Varying Speeds = product of its area of piston and length of stroke divided by 33,000. This multiplied by the mean effective pressure and by the number of single strokes per minute is the indicated horse-power.

To draw the Clearance-line on the Indicator-diagram, the actual clearance not being known. — The clearance-line may be obtained approximately by drawing a straight line, *cbad*, across the compression

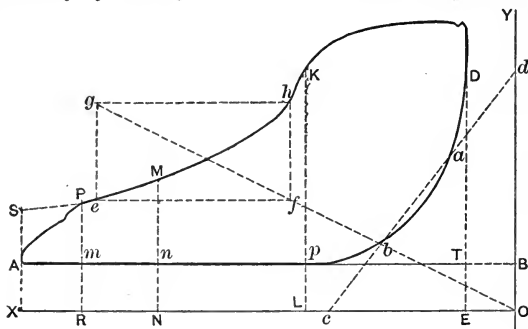


FIG. 156.

curve, first having drawn *OX* parallel to the atmospheric line and 14.7 lbs. below. Measure from *a* the distance *ad*, equal to *cb*, and draw *YO* perpendicular to *OX* through *d*; then will *TB* divided by *AT* be the percentage of clearance. The clearance may also be found from the expansion-line by constructing a rectangle *efhg*, and drawing a diagonal *gf* to intersect the line *XO*. This will give the point *O*, and by erecting a perpendicular to *XO* we obtain a clearance-line *OY*.

Both these methods for finding the clearance require that the expansion and compression curves be hyperbolas. Prof. Carpenter (*Power*, Sept., 1893) says that with good diagrams the methods are usually very accurate, and give results which check substantially.

The Buckeye Engine Co., however, says that, as the results obtained are seldom correct, being sometimes too little, but more frequently too much, and as the indications from the two curves seldom agree, the operation has little practical value, though when a clearly defined and apparently undistorted compression curve exists of sufficient extent to admit of the application of the process, it may be relied on to give much more correct results than the expansion curve.

To draw the Hyperbolic Curve on the Indicator-diagram. — Select

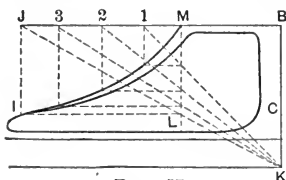


FIG. 157.

any point *I* in the actual curve, and from this point draw a line perpendicular to the line *JB*, meeting the latter in the point *J*. The line *JB* may be the line of boiler-pressure, but this is not material; it may be drawn at any convenient height near the top of the diagram and parallel to the atmospheric line. From *J* draw a diagonal to *K*, the latter point being the intersection of the vacuum and clearance lines; from *I* draw *IL* parallel with the atmospheric line. From *L*, the point of intersection of the diagonal *JK* and the horizontal line *IL*, draw the verti-

Intersection of the diagonal *JK* and the horizontal line *IL*, draw the verti-

cal line LM . The point M is the theoretical point of cut-off, and LM the cut-off line. Fix upon any number of points 1, 2, 3, etc., on the line JB , and from these points draw diagonals to K . From the intersection of these diagonals with LM draw horizontal lines, and from 1, 2, 3, etc., vertical lines. Where these lines meet will be points in the hyperbolic curve.

Theoretical Water-consumption calculated from the Indicator-card.—The following method is given by Prof. Carpenter (*Power*, Sept., 1893): p = mean effective pressure, l = length of stroke in feet, a = area of piston in square inches, $a \div 144$ = area in square feet, c = percentage of clearance to the stroke, b = percentage of stroke at point where water rate is to be computed, n = number of strokes per minute, $60n$ = number per hour, w = weight of a cubic foot of steam having a pressure as shown by the diagram corresponding to that at the point where water rate is required, w' = that corresponding to pressure at end of compression.

$$\text{Number of cubic feet per stroke} = l \left(\frac{b+c}{100} \right) \frac{a}{144}.$$

$$\text{Corresponding weight of steam per stroke in lbs.} = l \left(\frac{b+c}{100} \right) \frac{a}{144} w.$$

$$\text{Volume of clearance} = \frac{lca}{14,400}.$$

$$\text{Weight of steam in clearance} = \frac{lcaw'}{14,400}.$$

$$\text{Total weight of steam per stroke} = l \left(\frac{b+c}{100} \right) \frac{wa}{144} - \frac{lcaw'}{14,400} = \frac{la}{14,400} [(b+c)w - cw'].$$

$$\text{Total weight of steam from diagram per hour} = \frac{60nla}{14,400} [(b+c)w - cw'].$$

The indicated horse-power is $p l a n \div 33,000$. Hence the steam-consumption per hour per indicated horse-power is

$$\frac{\frac{60nla}{14,400} [(b+c)w - cw']}{\frac{p l a n}{33,000}} = \frac{137.50}{p} [(b+c)w - cw'].$$

Changing the formula to a rule, we have: To find the water rate from the indicator diagram at any point in the stroke.

RULE.—To the percentage of the entire stroke which has been completed by the piston at the point under consideration add the percentage of clearance. Multiply this result by the weight of a cubic foot of steam, having a pressure of that at the required point. Subtract from this the product of percentage of clearance multiplied by weight of a cubic foot of steam having a pressure equal to that at the end of the compression. Multiply this result by 137.50 divided by the mean effective pressure.*

NOTE.—This method applies only to points in the expansion curve or between cut-off and release.

The beneficial effect of compression in reducing the water-consumption of an engine is clearly shown by the formula. If the compression is carried to such a point that it produces a pressure equal to that at the point under consideration, the weight of steam per cubic foot is equal, and $w = w'$. In this case the effect of clearance entirely disappears, and the formula becomes $\frac{137.5}{p} (bw)$.

In case of no compression, w' becomes zero, and the water-rate =

$$\frac{137.5}{p} [(b+c)w].$$

* For compound or triple-expansion engines read: divided by the equivalent mean effective pressure, on the supposition that all work is done in one cylinder.

Prof. Denton (*Trans. A. S. M. E.*, xiv, 1363) gives the following table of the theoretical water-consumption for a perfect Mariotte expansion with steam at 150 lbs. above atmosphere, and 2 lbs. absolute back pressure:

| Ratio of Expansion, r . | M.E.P., lbs. per sq. in. | Lbs. of Water per hour per horse-power, W . |
|---------------------------|--------------------------|---|
| 10 | 52.4 | 9.68 |
| 15 | 38.7 | 8.74 |
| 20 | 30.9 | 8.20 |
| 25 | 25.9 | 7.84 |
| 30 | 22.2 | 7.63 |
| 35 | 19.5 | 7.45 |

The difference between the theoretical water-consumption found by the formula and the actual consumption as found by test represents "water not accounted for by the indicator," due to cylinder condensation, leakage through ports, radiation, etc.

Leakage of Steam. — Leakage of steam, except in rare instances, has so little effect upon the lines of the diagram that it can scarcely be detected. The only satisfactory way to determine the tightness of an engine is to take it when not in motion, apply a full boiler-pressure to the valve, placed in a closed position, and to the piston as well, which is blocked for the purpose at some point away from the end of the stroke, and see by the eye whether leakage occurs. The indicator-cocks provide means for bringing into view steam which leaks through the steam-valves, and in most cases that which leaks by the piston, and an opening made in the exhaust-pipe or observations at the atmospheric escape-pipe, are generally sufficient to determine the fact with regard to the exhaust-valves.

The steam accounted for by the indicator should be computed for both the cut-off and the release points of the diagram. If the expansion-line departs much from the hyperbolic curve a very different result is shown at one point from that shown at the other. In such cases the extent of the loss occasioned by cylinder condensation and leakage is indicated in a much more truthful manner at the cut-off than at the release. (Tabor Indicator Circular.)

COMPOUND ENGINES.

Compound, Triple- and Quadruple-expansion Engines. — A compound engine is one having two or more cylinders, and in which the steam after doing work in the first or high-pressure cylinder completes its expansion in the other cylinder or cylinders.

The term "compound" is commonly restricted, however, to engines in which the expansion takes place in two stages only — high and low pressure, the terms triple-expansion and quadruple-expansion engines being used when the expansion takes place respectively in three and four stages. The number of cylinders may be greater than the number of stages of expansion, for constructive reasons; thus in the compound or two-stage expansion engine the low-pressure stage may be effected in two cylinders so as to obtain the advantages of nearly equal sizes of cylinders and of three cranks at angles of 120° . In triple-expansion engines there are frequently two low-pressure cylinders, one of them being placed tandem with the high-pressure, and the other with the intermediate cylinder, as in mill engines with two cranks at 90° . In the triple-expansion engines of the steamers *Campania* and *Lucania*, with three cranks at 120° , there are five cylinders, two high, one intermediate, and two low, the high-pressure cylinders being tandem with the low.

Advantages of Compounding. — The advantages secured by dividing the expansion into two or more stages are twofold: 1. Reduction of wastes of steam by cylinder-condensation, clearance, and leakage; 2. Dividing the pressures on the cranks, shafts, etc., in large engines so as to avoid excessive pressures and consequent friction. The diminished

loss by cylinder-condensation is effected by decreasing the range of temperature of the metal surfaces of the cylinders, or the difference of temperature of the steam at admission and exhaust. When high-pressure steam is admitted into a single-cylinder engine a large portion is condensed by the comparatively cold metal surfaces; at the end of the stroke and during the exhaust the water is re-evaporated, but the steam so formed escapes into the atmosphere or into the condenser, doing no work; while if it is taken into a second cylinder, as in a compound engine, it does work. The steam lost in the first cylinder by leakage and clearance also does work in the second cylinder. Also, if there is a second cylinder, the temperature of the steam exhausted from the first cylinder is higher than if there is only one cylinder, and the metal surfaces therefore are not cooled to the same degree. The difference in temperatures and in pressures corresponding to the work of steam of 150 lbs. gauge-pressure expanded 20 times, in one, two, and three cylinders, is shown in the following table, by W. H. Weightman, *Am. Mach.*, July 28, 1892:

| | Single Cyl- inder. | Compound Cylinders. | | Triple-expansion Cylinders. | | |
|---|--------------------------|------------------------|--------|--------------------------------|--------|--------|
| Diameter of cylinders, in. . | 60 | 33 | 61 | 28 | 46 | 61 |
| Area ratios..... | | 1 | 3.416 | 1 | 2.70 | 4.740 |
| Expansions..... | 20 | 5 | 4 | 2.714 | 2.714 | 2.714 |
| Initial steam-pressures — absolute — pounds..... | 165 | 165 | 33 | 165 | 60.8 | 22.4 |
| Mean pressures, pounds.. | 32.96 | 86.11 | 19.68 | 121.44 | 44.75 | 16.49 |
| Mean effective pressures, pounds..... | 28.96 | 53.11 | 15.68 | 60.64 | 22.35 | 12.49 |
| Steam temperatures into cylinders..... | 366° | 366° | 259.9° | 366° | 293.5° | 234.1° |
| Steam temperatures out of the cylinders..... | 184.2° | 259.9° | 184.2° | 293.5° | 234.1° | 184.2° |
| Difference in temperatures | 181.8 | 106.1 | 75.7 | 72.5 | 59.4 | 49.9 |

“Woolf” and Receiver Types of Compound Engines.—The compound steam-engine, consisting of two cylinders, is reducible to two forms, 1, in which the steam from the h.p. cylinder is exhausted direct into the l.p. cylinder, as in the Woolf engine; and 2, in which the steam from the h.p. cylinder is exhausted into an intermediate reservoir, whence the steam is supplied to, and expanded in, the l.p. cylinder, as in the “receiver-engine.”

If the steam be cut off in the first cylinder before the end of the stroke, the total ratio of expansion is the product of the two ratios of expansion: that is, the product of the ratio of expansion in the first cylinder, into the ratio of the volume of the second to that of the first cylinder.

Thus, let the areas of the first and second cylinders be as 1 to $3\frac{1}{2}$, the strokes being equal, and let the steam be cut off in the first at $\frac{1}{2}$ stroke; then

Expansion in the 1st cylinder 1 to 2
Expansion in the 2d cylinder..... 1 to $3\frac{1}{2}$

Total or combined expansion, the product of the two ratios 1 to 7

Woolf Engine, without Clearance — Ideal Diagrams.—The diagrams of pressure of an ideal Woolf engine are shown in Fig. 158, as they would be described by the indicator, according to the arrows. In these diagrams *pq* is the atmospheric line, *mn* the vacuum line, *cd* the admission line, *dg* the hyperbolic curve of expansion in the first cylinder, and *gh* the consecutive expansion-line of back pressure for the return-stroke of the first piston, and of positive pressure for the steam-stroke of the second piston. At the point *h*, at the end of the stroke of the second piston, the steam is exhausted into the condenser, and the pressure falls to the level of perfect vacuum, *mn*.

The diagram of the second cylinder, below gh , is characterized by the absence of any specific period of admission; the whole of the steam-line gh being expansional, generated by the expansion of the initial body of steam contained in the first cylinder into the second.

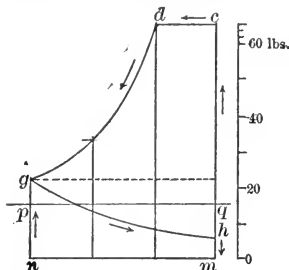


FIG. 158. — WOOLF ENGINE, IDEAL INDICATOR-DIAGRAMS.

same shaft. The receiver takes the steam exhausted from the first cylinder and supplies it to the second, in which the steam is cut off and then expanded to the end of the stroke. On the assumption that the initial pressure in the second cylinder is equal to the final pressure in the first, and of course equal to the pressure in the receiver, the volume cut off in the second cylinder must be equal to the volume of the first cylinder, for the second cylinder must admit as much steam at each stroke as is discharged from the first cylinder.

In Fig. 159, cd is the line of admission and hg the exhaust-line for the first cylinder; and dg is the expansion-curve and pq the atmospheric line.

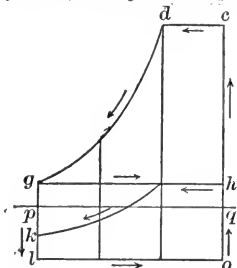


FIG. 159. — RECEIVER-ENGINE, IDEAL INDICATOR-DIAGRAM.

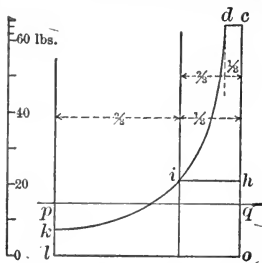


FIG. 160. — RECEIVER ENGINE, IDEAL DIAGRAMS REDUCED AND COMBINED.

In the region below the exhaust-line of the first cylinder, between it and the line of perfect vacuum, ol , the diagram of the second cylinder is formed; hi , the second line of admission, coincides with the exhaust-line hg of the first cylinder, showing in the ideal diagram no intermediate fall of pressure, and ik is the expansion-curve. The arrows indicate the order in which the diagrams are formed.

In the action of the receiver-engine, the expansive working of the steam, though clearly divided into two consecutive stages, is, as in the Woolf engine, essentially continuous from the point of cut-off in the first cylinder to the end of the stroke of the second cylinder, where it is delivered to the condenser; and the first and second diagrams may be placed together and combined to form a continuous diagram. For this purpose take the second diagram as the basis of the combined diagram, namely, $hiklo$, Fig. 160. The period of admission, hi , is one-third of the stroke, and as the ratios of the cylinders areas 1 to 3, hi is also the propor-

tional length of the first diagram as applied to the second. Produce oh upwards, and set off oc equal to the total height of the first diagram above the vacuum-line; and, upon the shortened base hi , and the height hc , complete the first diagram with the steam-line cd and the expansion line di .

It is shown by Clark (S. E., p. 432 *et seq.*) in a series of arithmetical calculations, that the receiver-engine is an elastic system of compound engine, in which considerable latitude is afforded for adapting the pressure in the receiver to the demands of the second cylinder, without considerably diminishing the effective work of the engine. In the Woolf engine, on the contrary, it is of much importance that the intermediate volume of space between the first and second cylinders, which is the cause of an intermediate fall of pressure, should be reduced to the lowest practicable amount.

Supposing that there is no loss of steam in passing through the engine, by cooling and condensation, it is obvious that whatever steam passes through the first cylinder must also find its way through the second cylinder. By varying, therefore, in the receiver-engine, the period of admission in the second cylinder, and thus also the volume of steam admitted for each stroke, the steam will be measured into it at a higher pressure and of a less bulk, or at a lower pressure and of a greater bulk; the pressure and density naturally adjusting themselves to the volume that the steam from the receiver is permitted to occupy in the second cylinder. With a sufficiently restricted admission, the pressure in the receiver may be maintained at the pressure of the steam as exhausted from the first cylinder. On the contrary, with a wider admission, the pressure in the receiver may fall or "drop" to three-fourths or even one-half of the pressure of the exhaust steam from the first cylinder.

(For a more complete discussion of the action of steam in the Woolf and receiver engines, see Clark on the Steam-engine.)

Combined Diagrams of Compound Engines.—The only way of making a correct combined diagram from the indicator-diagrams of

the several cylinders in a compound engine is to set off all the diagrams on the same horizontal scale of volumes, adding the clearances to the cylinder capacities proper. When this is attended to, the successive diagrams fall exactly into their right places relatively to one another, and would compare properly with any theoretical expansion-curve, (Prof. A. B. W. Kennedy, *Proc. Inst. M. E.*, Oct., 1886.)

This method of combining diagrams is commonly adopted, but there are objections to its accuracy, since the whole quantity of steam consumed in the first cylinder at the end of the stroke is not carried forward to the second, but a part of it is retained in the first cylinder for compression.

For a method of combining diagrams in which compression is taken account of, see discussions by Thomas Mudd and others, in *Proc. Inst. M. E.*, Feb., 1887, p. 48. The usual method of combining diagrams is also criticised by Frank H. Ball as inaccurate and misleading (*Am. Mach.*, April 12, 1894; *Trans. A. S. M. E.*, xiv, 1405, and xv, 403).

Figure 161 shows a combined diagram of a quadruple-expansion engine, drawn according to the usual method, that is, the diagrams are first reduced in length to relative scales that correspond with the relative

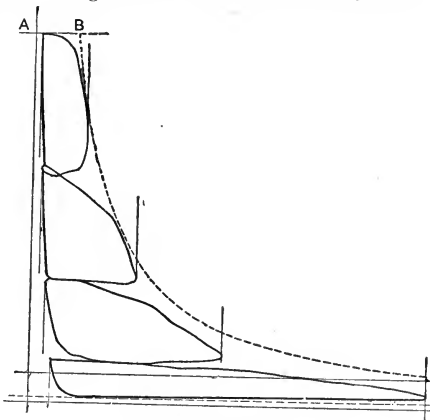


FIG. 161.

piston-displacement of the three cylinders. Then the diagrams are placed at such distances from the clearance-line of the proposed combined diagram as to represent correctly the clearance in each cylinder.

Proportions of Cylinders in Compound Engines. — Authorities differ as to the proportions by volume of the high and low pressure cylinders v and V . Thus Grashof gives $V \div v = 0.85 \sqrt{r}$; Hrabak, $0.90 \sqrt{r}$; Werner, \sqrt{r} ; and Rankine, $\sqrt{r^2}$, r being the ratio of expansion. Busley makes the ratio dependent on the boiler-pressure thus:

| | | | | |
|----------------------|----|----|-----|-----|
| Lbs. per sq. in..... | 60 | 90 | 105 | 120 |
| $V \div v$ | =3 | 4 | 4.5 | 5 |

(See Seaton's Manual, p. 95, etc., for analytical method; Sennett, p. 496, etc.; Clark's Steam-engine, p. 445, etc.; Clark's Rules, Tables, Data, p. 849, etc.)

Mr. J. McFarlane Gray states that he finds the mean effective pressure in the compound engine reduced to the low-pressure cylinder to be approximately the square root of 6 times the boiler-pressure.

Ratio of Cylinder Capacity in Compound Marine Engines. (Seaton.) — The low-pressure cylinder is the measure of the power of a compound engine, for so long as the initial steam-pressure and rate of expansion are the same, it signifies very little, so far as total power only is concerned, whether the ratio between the low and high pressure cylinders is 3 or 4; but as the power developed should be nearly equally divided between the two cylinders, in order to get a good and steady working engine, there is a necessity for exercising a considerable amount of discretion in fixing on the ratio.

In choosing a particular ratio the objects are to divide the power evenly and to avoid as much as possible "drop" and high initial strain. [Some writers advocate drop in the high-pressure cylinder making it smaller than is the usual practice and making the cylinder ratio as high as 6 or 7.]

If increased economy is to be obtained by increased boiler-pressures the rate of expansion should vary with the initial pressure, so that the pressure at which the steam enters the condenser should remain constant. In this case, with the ratio of cylinders constant, the cut-off in the high-pressure cylinder will vary inversely as the initial pressure.

Let R be the ratio of the cylinders; r the rate of expansion; p_1 the initial pressure: then cut-off in high-pressure cylinder = $R \div r$; r varies with p_1 , so that the terminal pressure p_n is constant, and consequently $r = p_1 \div p_n$; therefore, cut-off in high-pressure cylinder = $R \times p_n \div p_1$.

Ratios of Cylinders as Found in Marine Practice. — The rate of expansion may be taken at one-tenth of the boiler-pressure (or about one-twelfth the absolute pressure), to work economically at full speed. Therefore, when the diameter of the low-pressure cylinder does not exceed 100 inches, and the boiler-pressure 70 lbs., the ratio of the low-pressure to the high-pressure cylinder should be 3.5; for a boiler-pressure of 80 lbs., 3.75; for 90 lbs., 4.0; for 100 lbs., 4.5. If these proportions are adhered to, there will be no need of an expansion-valve to either cylinder. If, however, to avoid "drop," the ratio be reduced, an expansion-valve should be fitted to the high-pressure cylinder.

Where economy of steam is not of first importance, but rather a large power, the ratio of cylinder capacities may with advantage be decreased, so that with a boiler-pressure of 100 lbs. it may be 3.75 to 4.

In tandem engines there is no necessity to divide the work equally. The ratio is generally 4, but when the steam-pressure exceeds 90 lbs. absolute 4.5 is better, and for 100 lbs. 5.0.

When the power requires that the l.p. cylinder shall be more than 100 in. diameter, it should be divided in two cylinders. In this case the ratio of the combined capacity of the two l.p. cylinders to that of the h.p. may be 3.0 for 85 lbs. absolute, 3.4 for 95 lbs., 3.7 for 105 lbs., and 4.0 for 115 lbs.

Receiver Space in Compound Engines should be from 1 to 1.5 times the capacity of the high-pressure cylinder, when the cranks are at an angle of from 90° to 120° . When the cranks are at 180° or nearly this, the space may be very much reduced. In the case of triple-compound engines, with cranks at 120° , and the intermediate cylinder leading the high-pressure, a very small receiver will do. The pressure in the receiver should never exceed half the boiler-pressure. (Seaton.)

Formula for Calculating the Expansion and the Work of Steam in Compound Engines.

(Condensed from Clark on the "Steam-engine.")

- a = area of the first cylinder in square inches;
 a' = area of the second cylinder in square inches;
 r = ratio of the capacity of the second cylinder to that of the first;
 L = length of stroke in feet, supposed to be the same for both cylinders;
 l = period of admission to the first cylinder in feet, excluding clearance;
 c = clearance at each end of the cylinders, in parts of the stroke, in ft.;
 L' = length of the stroke plus the clearance, in feet;
 l' = period of admission plus the clearance, in feet;
 s = length of a given part of the stroke of the second cylinder, in feet;
 P = total initial pressure in the first cylinder, in lbs. per square inch, supposed to be uniform during admission;
 P' = total pressure at the end of the given part of the stroke s ;
 p = average total pressure for the whole stroke;
 R = nominal ratio of expansion in the first cylinder, or $L + l$;
 R' = actual ratio of expansion in the first cylinder, or $L' + l'$;
 R'' = actual combined ratio of expansion, in the first and second cylinders together;
 n = ratio of the final pressure in the first cylinder to any intermediate fall of pressure between the first and second cylinders;
 N = ratio of the volume of the intermediate space in the Woolf engine, reckoned up to, and including the clearance of, the second piston, to the capacity of the first cylinder plus its clearance. The value of N is correctly expressed by the actual ratio of the volumes as stated, on the assumption that the intermediate space is a vacuum when it receives the exhaust-steam from the first cylinder. In point of fact, there is a residuum of unexhausted steam in the intermediate space, at low pressure, and the value of N is thereby practically reduced below the ratio here stated.

$$N = \frac{n}{n-1} - 1.$$

w = whole net work in one stroke, in foot-pounds.

Ratio of expansion in the second cylinder:

$$\text{In the Woolf engine, } \frac{\left(r \frac{L}{L'}\right) + N}{1 + N};$$

$$\text{In the receiver-engine, } \frac{(n-1)r}{n}.$$

Total actual ratio of expansion = product of the ratios of the three consecutive expansions, in the first cylinder, in the intermediate space, and in the second cylinder,

$$\text{In the Woolf engine, } R' \left(r \frac{L}{L'} + N \right);$$

$$\text{In the receiver-engine, } r \frac{L'}{l'}, \text{ or } rR'.$$

$$\text{Combined ratio of expansion behind the pistons} = \frac{n-1}{n} rR' = R''.$$

Work done in the two cylinders for one stroke, with a given cut-off and a given combined actual ratio of expansion:

$$\text{Woolf engine, } w = aP [l' (1 + \text{hyp log } R'') - c];$$

$$\text{Receiver engine, } w = aP \left[l' (1 + \text{hyp log } R'') - c \left(1 + \frac{r-1}{R'} \right) \right],$$

when there is no intermediate fall of pressure.

When there is an intermediate fall, when the pressure falls to $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$ of the final pressure in the 1st cylinder, the reduction of work is 0.2%, 1.0%, 4.6% of that when there is no fall.

Total work in the two cylinders of a receiver-engine, for one stroke for any intermediate fall of pressure,

$$w = aP \left[\frac{n+1}{n} + \text{hyp log } R'' \right] - c \left(1 + \frac{(n-1)(r-1)}{nR'} \right).$$

EXAMPLE. — Let $a = 1$ sq. in., $P = 63$ lbs., $\nu = 2.42$ ft., $n = 4$, $R'' = 5.969$, $c = 0.42$ ft., $r = 3$, $R' = 2.653$;

$$w = 1 \times 63 \left[2.42 \left(\frac{5}{4} \text{ hyp log } 5.969 \right) - .42 \left(1 + \frac{3 \times 2}{4 \times 2.653} \right) \right] = 421.55 \text{ ft.-lbs.}$$

Calculation of Diameters of Cylinders of a compound condensing engine of 2000 H.P. at a speed of 700 feet per minute, with 100 lbs. boiler-pressure.

100 lbs. gauge-pressure = 115 absolute, less drop of 5 lbs. between boiler and cylinder = 110 lbs. initial absolute pressure. Assuming terminal pressure in l.p. cylinder = 6 lbs., the total expansion of steam in both cylinders = $110 \div 6 = 18.33$. Hyp log 18.33 = 2.909. Back pressure in l.p. cylinder, 3 lbs. absolute.

The following formulæ are used in the calculation of each cylinder:

$$(1) \text{ Area of cylinder} = \frac{\text{H.P.} \times 33,000}{\text{M.E.P.} \times \text{piston-speed}}$$

(2) Mean effective pressure = mean total pressure — back pressure.

(3) Mean total pressure = terminal pressure $\times (1 + \text{hyp log } R)$.

(4) Absolute initial pressure = absolute terminal pressure \times ratio of expansion.

First calculate the area of the low-pressure cylinder as if all the work were done in that cylinder.

From (3), mean total pressure = $6 \times (1 + \text{hyp log } 18.33) = 23.454$ lbs.

From (2), mean effective pressure = $23.454 - 3 = 20.454$ lbs.

From (1), area of cylinder = $\frac{2000 \times 33,000}{20.454 \times 700} = 4610$ sq. ins. = 76.6 ins. diam.

If half the work, or 1000 H.P., is done in the l.p. cylinder the M.E.P. will be half that found above, or 10.227 lbs., and the mean total pressure $10.227 + 3 = 13.227$ lbs.

From (3), $1 + \text{hyp log } R = 13.227 \div 6 = 2.2045$.

Hyp log $R = 1.2045$, whence R in l.p. cyl. = 3.335.

From (4), $3.335 \times 6 = 20.01$ lbs. initial pressure in l.p. cyl. and terminal pressure in h.p. cyl., assuming no drop between cylinders.

$110 \div 20.01 = 5.497$, R in h.p. cyl.

From (3), mean total pres. in h.p. cyl. = $20.01 \times (1 + \text{hyp log } 5.497) = 54.11$.

From (2), $54.11 - 20.01 = 34.10$, M.E.P. in h.p. cyl.

From (1), area of h.p. cyl. = $\frac{1000 \times 33,000}{34.1 \times 700} = 1382$ sq. ins. = 42 ins. diam.

Cylinder ratio = $4610 \div 1382 = 3.336$.

The area of the h.p. cylinder may be found more directly by dividing the area of the l.p. cyl. by the ratio of expansion in that cylinder. $4610 \div 3.335 = 1382$ sq. ins.

In the above calculation no account is taken of clearance, of compression, of drop between cylinders, nor of area of piston-rods. It also assumes that the diagram in each cylinder is the full theoretical diagram, with a horizontal steam-line and a hyperbolic expansion line, with no allowance for rounding of the corners. To make allowance for these, the mean effective pressure in each cylinder must be multiplied by a diagram factor, or the ratio of the area of an actual diagram of the class of engine considered, with the given initial and terminal pressures, to the area of the theoretical diagram. Such diagram factors will range from 0.6 to 0.94, as in the table on p. 932.

Best Ratios of Cylinders. — The question what is the best ratio of areas of the two cylinders of a compound engine is still (1901) a disputed

one, but there appears to be an increasing tendency in favor of large ratios, even as great as 7 or 8 to 1, with considerable terminal drop in the high-pressure cylinder. A discussion of the subject, together with a description of a new method of drawing theoretical diagrams of multiple-expansion engines, taking into consideration drop, clearance, and compression will be found in a paper by Bert C. Ball, in *Trans. A. S. M. E.* xxi, 1002.

TRIPLE-EXPANSION ENGINES.

Proportions of Cylinders. — H. H. Supplee, *Mechanics*, Nov., 1887, gives the following method of proportioning cylinders of triple-expansion engines:

As in the case of compound engines the diameter of the low-pressure cylinder is first determined, being made large enough to furnish the entire power required at the mean pressure due to the initial pressure and expansion ratio given; and then this cylinder is given only pressure enough to perform one-third of the work, and the other cylinders are proportioned so as to divide the other two-thirds between them.

Let us suppose that an initial pressure of 150 lbs. is used and that 900 H.P. is to be developed at a piston-speed of 800 ft. per min., and that an expansion ratio of 16 is to be reached with an absolute back-pressure of 2 lbs.

The theoretical M.E.P. with an absolute initial pressure of $150 + 14.7 = 164.7$ lbs. initial at 16 expansions is

$$\frac{P(1 + \text{hyp log } 16)}{16} = 164.7 \times \frac{3.7726}{16} = 38.83,$$

less 2 lbs. back pressure, $= 38.83 - 2 = 36.83$.

In practice only about 0.7 of this pressure is actually attained, so that $36.83 \times 0.7 = 25.781$ lbs. is the M.E.P. upon which the engine is to be proportioned.

To obtain 900 H.P. we must have $33,000 \times 900 = 29,700,000$ foot-pounds, and this divided by the mean pressure (25.78) and by the speed in feet (800) will give 1440 sq. in. as the area of the l.p. cylinder, about equivalent to 43 in. diam.

Now as one-third of the work is to be done in the l.p. cylinder, the M.E.P. in it will be $25.78 \div 3 = 8.59$ lbs.

The cut-off in the high-pressure cylinder is generally arranged to cut off at 0.6 of the stroke, and so the ratio of the h.p. to the l.p. cylinder is equal to $16 \times 0.6 = 9.6$, and the h.p. cylinder will be $1440 \div 9.6 = 150$ sq. in. area, or about 14 in. diameter, and the M.E.P. in the h.p. cylinder is equal to $9.6 \times 8.59 = 82.46$ lbs.

If the intermediate cylinder is made a mean size between the other two, its size would be determined by dividing the area of the l.p. cylinder by the square root of the ratio between the low and the high; but in practice this is found to give a result too large to equalize the stresses, so that instead the area of the int. cylinder is found by dividing the area of the l.p. piston by 1.1 times the square root of the ratio of l.p. to h.p. cylinder, which in this case is $1440 \div (1.1 \sqrt{9.6}) = 422.5$ sq. in., or a little more than 23 in. diam.

The choice of expansion ratio is governed by the initial pressure, and is generally chosen so that the terminal pressure in the l.p. cylinder shall be about 10 lbs. absolute.

Formulae for Proportioning Cylinder Areas of Triple-Expansion Engines. — The following formulæ are based on the method of first finding the cylinder areas that would be required if an ideal hyperbolic diagram were obtainable from each cylinder, with no clearance, compression, wire-drawing, drop by free expansion in receivers, or loss by cylinder condensation, assuming equal work to be done in each cylinder, and then dividing the areas thus found by a suitable diagram factor, such as those given on page 932, expressing the ratio which the area of an actual diagram, obtained in practice from an engine of the type under consideration, bears to the ideal or theoretical diagram. It will vary in different classes of engine and in different cylinders of the same engine, usual

values ranging from 0.6 to 0.9. When any one of the three stages of expansion takes place in two cylinders, the combined area of these cylinders equals the area found by the formulæ.

NOTATION.

- p_1 = initial pressure in the high-pressure cylinder.
 p_t = terminal pressure in the low-pressure cylinder.
 p_b = back pressure in the low-pressure cylinder.
 p_2 = term. press. in h.p. cyl. and initial press. in intermediate cyl.
 p_3 = term. press. in int. cyl. and initial press. in l.p. cyl.
 R_1, R_2, R_3 , ratio of exp. in h.p. int. and l.p. cyls.
 R = total ratio of exp. = $R_1 \times R_2 \times R_3$.
 P = mean effec. press. of the combined ideal diagram, referred to the l.p. cyl.
 P_1, P_2, P_3 = M.E.P. in the h.p., int., and l.p. cyls.
 HP = horse-power of the engine = $PLA_3N \div 33,000$.
 L = length of stroke in feet; N = number of single strokes per min.
 A_1, A_2, A_3 , areas (sq. ins.) of h.p. int. and l.p. cyls. (ideal).
 W = work done in one cylinder per foot of stroke.
 r_2 = ratio of A_2 to A_1 ; r_3 = ratio of A_3 to A_1 .
 F_1, F_2, F_3 , diagram factors of h.p. int. and l.p. cyl.
 a_1, a_2, a_3 , areas (actual) of h.p. int. and l.p. cyl.

Formulæ.

- (1) $R = p_1 \div p_t$.
- (2) $P = p_t (1 + \text{hyp log } R) - p_b$.
- (3) $P_3 = \frac{1}{3} P$.
- (4) $\text{Hyp log } R_3 = (P_3 - p_t + p_b) \div p_t$.
- (5) $R_1 R_2 = R \div R_3$; $R_1 = R_2 = \sqrt{R_1 R_2}$.
- (6) $p_3 = p_t \times R_3$.
- (7) $p_2 = p_3 \times R_2$.
- (8) $p_1 = p_2 \times R_1$.
- (9) $P_2 = p_3 (\text{hyp log } R_2) = P_3 R_3$.
- (10) $P_1 = p_2 (\text{hyp log } R_1) = P_2 R_2$.
- (11) $W = 11,000 HP \div LN$.
- (12) $A_1 = W \div P_1$; $A_2 = W \div P_2$; $A_3 = W \div P_3$.
- (13) $r_2 = A_2 \div A_1 = P_1 \div P_2 = R_1$ or R_2 ; $r_3 = A_3 \div A_1 = P_1 \div P_3$.
- (14) $a_1 = A_1 \div F_1$; $a_2 = A_2 \div F_2$; $a_3 = A_3 \div F_3$.

From these formulæ the figures in the following tables have been calculated:

THEORETICAL MEAN EFFECTIVE PRESSURES, CYLINDER RATIOS, ETC., OF TRIPLE EXPANSION ENGINES.

Back pressure 3 lbs. Terminal pressure, 8 lbs. (absolute).

| p_1 . | R . | P . | P_3 . | R_3 . | R_1, R_2 , or r_2 . | p_3 . | p_2 . | P_2 . | P_1 . | r_3 . |
|---------|-------|-------|---------|---------|----------------------------|---------|---------|---------|---------|---------|
| 120 | 15 | 26.66 | 8.89 | 1.626 | 3.037 | 13.01 | 39.51 | 14.45 | 43.89 | 4.939 |
| 140 | 17.5 | 27.90 | 9.30 | 1.712 | 3.197 | 13.70 | 43.79 | 15.92 | 50.89 | 5.472 |
| 160 | 20 | 28.97 | 9.66 | 1.790 | 3.343 | 14.32 | 47.86 | 17.29 | 57.76 | 5.980 |
| 180 | 22.5 | 29.91 | 9.97 | 1.861 | 3.477 | 14.89 | 51.77 | 18.55 | 64.52 | 6.471 |
| 200 | 25 | 30.75 | 10.25 | 1.928 | 3.601 | 15.42 | 55.54 | 19.76 | 71.16 | 6.942 |
| 220 | 27.5 | 31.51 | 10.50 | 1.990 | 3.718 | 15.91 | 59.16 | 20.90 | 77.69 | 7.397 |
| 240 | 30 | 32.21 | 10.74 | 2.049 | 3.826 | 16.39 | 62.72 | 22.00 | 84.16 | 7.839 |

THEORETICAL MEAN EFFECTIVE PRESSURES, CYLINDER RATIOS, ETC.,
OF TRIPLE EXPANSION ENGINES.

Back pressure, 3 lbs. Terminal pressure, 10 lbs. (absolute).

| p_1 . | R . | P . | P_3 . | R_3 . | $R_1, R_2,$ or r_2 . | p_3 . | p_2 . | P_2 . | P_1 . | r_3 . |
|---------|-------|-------|---------|---------|---------------------------|---------|---------|---------|---------|---------|
| 120 | 12 | 31.85 | 10.62 | 1.436 | 2.890 | 14.36 | 41.50 | 15.24 | 44.04 | 4.148 |
| 140 | 14 | 33.39 | 11.13 | 1.511 | 3.044 | 15.11 | 45.99 | 16.82 | 51.20 | 4.600 |
| 160 | 16 | 34.73 | 11.58 | 1.580 | 3.182 | 15.80 | 50.28 | 18.29 | 58.20 | 5.027 |
| 180 | 18 | 35.90 | 11.97 | 1.643 | 3.310 | 16.43 | 54.38 | 19.66 | 65.09 | 5.439 |
| 200 | 20 | 36.96 | 12.32 | 1.702 | 3.428 | 17.02 | 58.34 | 20.97 | 71.88 | 5.834 |
| 220 | 22 | 37.91 | 12.64 | 1.757 | 3.538 | 17.57 | 62.15 | 22.20 | 78.54 | 6.215 |
| 240 | 24 | 38.78 | 12.93 | 1.809 | 3.642 | 18.09 | 65.88 | 23.38 | 85.15 | 6.587 |

Given the required H.P. of an engine, its speed and length of stroke, and the assumed diagram factors F_1, F_2, F_3 for the three cylinders, the areas of the cylinders may be found by using formulæ (11), (12), and (14), and the values of P_1, P_2 , and P_3 in the above table.

A Common Rule for Proportioning the Cylinders of multiple-expansion engines is: for two-cylinder compound engines, the cylinder ratio is the square root of the number of expansions, and for triple-expansion engines the ratios of the high to the intermediate and of the intermediate to the low are each equal to the cube root of the number of expansions, the ratio of the high to the low being the product of the two ratios, that is, the square of the cube root of the number of expansions. Applying this rule to the pressures above given, assuming a terminal pressure (absolute) of 10 lbs. and 8 lbs. respectively, we have, for triple-expansion engines:

| Boiler-pressure (Absolute). | Terminal Pressure, 10 lbs. | | Terminal Pressure, 8 lbs. | |
|--------------------------------|----------------------------|----------------------------|---------------------------|----------------------------|
| | No. of Ex- pansions. | Cylinder Ratios, areas. | No. of Ex- pansions. | Cylinder Ratios, areas. |
| 130 | 13 | 1 to 2.35 to 5.53 | 16 1/4 | 1 to 2.53 to 6.42 |
| 140 | 14 | 1 to 2.41 to 5.81 | 17 1/2 | 1 to 2.60 to 6.74 |
| 150 | 15 | 1 to 2.47 to 6.08 | 18 3/4 | 1 to 2.66 to 7.06 |
| 160 | 16 | 1 to 2.52 to 6.35 | 20 | 1 to 2.71 to 7.37 |

The ratio of the diameters is the square root of the ratios of the areas, and the ratio of the diameters of the first and third cylinders is the same as the ratio of the areas of first and second.

Seaton, in his *Marine Engineering*, says: When the pressure of steam employed exceeds 115 lbs. absolute, it is advisable to employ three cylinders, through each of which the steam expands in turn. The ratio of the low-pressure to high-pressure cylinder in this system should be 5, when the steam-pressure is 125 lbs. absolute; when 135 lbs., 5.4; when 145 lbs., 5.8; when 155 lbs., 6.2; when 165 lbs., 6.6. The ratio of low-pressure to intermediate cylinder should be about one-half that between low-pressure and high-pressure, as given above. That is, if the ratio of l.p. to h.p. is 6, that of l.p. to int. should be about 3, and consequently that of int. to h.p. about 2. In practice the ratio of int. to h.p. is nearly 2.25, so that the diameter of the int. cylinder is 1.5 that of the h.p. The introduction of the triple-compound engine has admitted of ships being propelled at higher rates of speed than formerly obtained without exceeding the consumption of fuel of similar ships fitted with ordinary compound engines; in such cases the higher power to obtain the speed has been developed by decreasing the rate of expansion, the low-pressure cylinder being only 6 times the capacity of the high-pressure, with a working pressure of 170 lbs. absolute. It is now a very general practice to make the diameter of the low-pressure cylinder equal to the sum of the diameters of the h.p. and int. cylinders; hence

Diameter of int. cylinder = 1.5 diameter of h.p. cylinder;
Diameter of l.p. cylinder = 2.5 diameter of h.p. cylinder.

In this case the ratio of l.p. to h.p. is 6.25; the ratio of int. to h.p. is 2.25; and ratio of l.p. to int. is 2.78.

Ratios of Cylinders for Different Classes of Engines. (*Proc. Inst. M. E.*, Feb., 1887, p. 36.)—As to the best ratios for the cylinders in a triple engine there seems to be great difference of opinion. Considerable latitude, however, is due to the requirements of the case, inasmuch as it would not be expected that the same ratio would be suitable for an economical land engine, where the space occupied and the weight were of minor importance, as in a war-ship, where the conditions were reversed. In the land engine, for example, a theoretical terminal pressure of about 7 lbs. above absolute vacuum would probably be aimed at, which would give a ratio of capacity of high pressure to low pressure of 1 to $8\frac{1}{2}$ or 1 to 9; whilst in a war-ship a terminal pressure would be required of 12 to 13 lbs. which would need a ratio of capacity of 1 to 5; yet in both these instances the cylinders were correctly proportioned and suitable to the requirements of the case. It is obviously unwise, therefore, to introduce any hard-and-fast rule.

Types of Three-stage Expansion Engines.—1. Three cranks at 120 deg. 2. Two cranks with 1st and 2d cylinders tandem. 3. Two cranks with 1st and 3d cylinders tandem. The most common type is the first, with cylinders arranged in the sequence high, intermediate, low.

Sequence of Cranks.—Mr. Wyllie (*Proc. Inst. M. E.*, 1887) favors the sequence high, low, intermediate, while Mr. Mudd favors high, intermediate, low. The former sequence, high, low, intermediate, gave an approximately horizontal exhaust-line, and thus minimizes the range of temperature and the initial load; the latter sequence high, intermediate, low, increased the range and also the load.

Mr. Morrison, in discussing the question of sequence of cranks, presented a diagram showing that with the cranks arranged in the sequence high, low, intermediate, the mean compression into the receiver was $19\frac{1}{2}$ per cent of the stroke; with the sequence high, intermediate, low, it was 57 per cent.

In the former case the compression was just what was required to keep the receiver-pressure practically uniform; in the latter case the compression caused a variation in the receiver-pressure to the extent sometimes of $22\frac{1}{2}$ lbs.

Velocity of Steam through Passages in Compound Engines. (*Proc. Inst. M. E.*, Feb., 1887.)—In the *SS. Para*, taking the area of the cylinder multiplied by the piston-speed in feet per second and dividing by the area of the port the velocity of the initial steam through the high-pressure cylinder port would be about 100 feet per second; the exhaust would be about 90. In the intermediate cylinder the initial steam had a velocity of about 180, and the exhaust of 120. In the low-pressure cylinder, the initial steam entered through the port with a velocity of 250, and in the exhaust-port the velocity was about 140 feet per second.

A Double-tandem Triple-expansion Engine, built by Watts, Campbell & Co., Newark, N. J., is described in *Am. Mach.*, April 26, 1894. It is two three-cylinder tandem engines coupled to one shaft, cranks at 90° , cylinders 21, 32 and 48 by 60 in. stroke, 65 revolutions per minute, rated H.P. 2000; fly-wheel 28 ft. diameter, 12 ft. face, weight 174,000 lbs.; main shaft 22 in. diameter at the swell; main journals 19×38 in.; crank-pins $9\frac{1}{2} \times 10$ in.; distance between center lines of two engines 24 ft. $7\frac{1}{2}$ in.; Corliss valves, with separate eccentrics for the exhaust-valves of the l.p. cylinder.

QUADRUPLE-EXPANSION ENGINES.

H. H. Suplee (*Trans. A. S. M. E.*, x, 583) states that a study of 14 different quadruple-expansion engines, nearly all intended to be operated at a pressure of 180 lbs. per sq. in., gave average cylinder ratios of 1 to 2, to 3.78, to 7.70, or nearly in the proportions 1, 2, 4, 8.

If we take the ratio of areas of any two adjoining cylinders as the fourth root of the number of expansions, the ratio of the 1st to the 4th will be the cube of the fourth root. On this basis the ratios of areas for different pressures and rates of expansion will be as follows:

| Gauge-pressures. | Absolute Pressures. | Terminal Pressures. | Ratio of Expansion. | Ratios of Areas of Cylinders. |
|------------------|---------------------|---------------------|---------------------|-------------------------------|
| 160 | 175 | { 12 | 14.6 | 1 : 1.95 : 3.81 : 7.43 |
| | | { 10 | 17.5 | 1 : 2.05 : 4.18 : 8.55 |
| | | { 8 | 21.9 | 1 : 2.16 : 4.68 : 10.12 |
| 180 | 195 | { 12 | 16.2 | 1 : 2.01 : 4.02 : 8.07 |
| | | { 10 | 19.5 | 1 : 2.10 : 4.42 : 9.28 |
| | | { 8 | 24.4 | 1 : 2.22 : 4.94 : 10.98 |
| 200 | 215 | { 12 | 17.9 | 1 : 2.06 : 4.23 : 8.70 |
| | | { 10 | 21.5 | 1 : 2.15 : 4.64 : 9.98 |
| | | { 8 | 26.9 | 1 : 2.28 : 5.19 : 11.81 |
| 220 | 235 | { 12 | 19.6 | 1 : 2.10 : 4.43 : 9.31 |
| | | { 10 | 23.5 | 1 : 2.20 : 4.85 : 10.67 |
| | | { 8 | 29.4 | 1 : 2.33 : 5.42 : 12.62 |

Seaton says: When the pressure of steam employed exceeds 190 lbs. absolute, four cylinders should be employed, with the steam expanding through each successively; and the ratio of l.p. to h.p. should be at least 7.5, and if economy of fuel is of prime consideration it should be 8; then the ratio of first intermediate to h.p. should be 1.8, that of second intermediate to first int. 2, and that of l.p. to second int. 2.2.

In a paper read before the North East Coast Institution of Engineers and Shipbuilders, 1890, William Russell Cummins advocates the use of a four-cylinder engine with four cranks as being more suitable for high speeds than the three-cylinder three-crank engine. The cylinder ratios, he claims, should be designed so as to obtain equal initial loads in each cylinder. The ratios determined for the triple engine are 1, 2.04, 6.54, and for the quadruple, 1, 2.08, 4.46, 10.47. He advocates long stroke, high piston-speed, 100 revolutions per minute, and 250 lbs. boiler-pressure, unjacketed cylinders, and separate steam and exhaust valves.

ECONOMIC PERFORMANCE OF STEAM-ENGINES.

Economy of Expansive Working under Various Conditions, Single Cylinder.

(Abridged from Clark on the Steam Engine.)

1. SINGLE CYLINDERS WITH SUPERHEATED STEAM, NON-CONDENSING.—Inside cylinder locomotive, cylinders and steam-pipes enveloped by the hot gases in the smoke-box. Net boiler pressure 100 lbs.; net maximum pressure in cylinders 80 lbs. per sq. in.

| | | | | | | | | | |
|---|------|------|------|------|------|------|------|------|------|
| Cut-off, per cent. | 20 | 25 | 30 | 35 | 40 | 50 | 60 | 70 | 80 |
| Actual ratio of expansion. | 3.91 | 3.31 | 2.87 | 2.53 | 2.26 | 1.86 | 1.59 | 1.39 | 1.23 |
| Water per I.H.P. per hour, lbs. | 18.5 | 19.4 | 20 | 21.2 | 22.2 | 24.5 | 27 | 30 | 33 |

2. SINGLE CYLINDERS WITH SUPERHEATED STEAM, CONDENSING.—The best results obtained by Hirn, with a cylinder $23\frac{3}{4} \times 67$ in. and steam superheated 150° F., expansion ratio $3\frac{3}{4}$ to $4\frac{1}{2}$, total maximum pressure in cylinder 63 to 69 lbs., were 15.63 and 15.69 lbs. of water per I.H.P. per hour.

3. SINGLE CYLINDERS OF SMALL SIZE, 8 or 9 IN. DIAM., JACKETED, NON-CONDENSING.—The best results are obtained at a cut-off of 20 per cent, with 75 lbs. maximum pressure in the cylinder; about 25 lbs. of water per I.H.P. per hour.

4. SINGLE CYLINDERS, NOT STEAM-JACKETED, CONDENSING.—The best result is from a Corliss-Wheelock engine 18×48 in.; cut-off, 12.5%; actual expansion ratio, 6.95; maximum absolute pressure in cylinder, 104 lbs.; steam per I.H.P. hour, 19.58 lbs. Other engines, with lower steam pressures, gave a steam consumption as high as 26.7 lbs.

Feed-water Consumption of Different Types of Engines.—The following tables are taken from the circular of the *Tabor Indicator* (Ashcroft Mfg. Co., 1889). In the first of the two columns under Feed-water required, in the tables for simple engines, the figures are obtained by

computation from nearly perfect indicator diagrams, with allowance for cylinder condensation according to the table on page 936, but without allowance for leakage, with back-pressure in the non-condensing table taken at 16 lbs. above zero, and in the condensing table at 3 lbs. above zero. The compression curve is supposed to be hyperbolic, and commences at 0.91 of the return-stroke, with a clearance of 3% of the piston-displacement.

Table No. 2 gives the feed-water consumption for jacketed compound-condensing engines of the best class. The water condensed in the jackets is included in the quantities given. The ratio of areas of the two cylinders is as 1 to 4 for 120 lbs. pressure: the clearance of each cylinder is 3% and the cut-off in the two cylinders occurs at the same point of stroke. The initial pressure in the l.p. cylinder is 1 lb. per sq. in. below the back-pressure of the h.p. cylinder. The average back-pressure of the whole stroke in the l.p. cylinder is 4.5 lbs. for 10% cut-off; 4.75 lbs. for 20% cut-off; and 5 lbs. for 30% cut-off. The steam accounted for by the indicator at cut-off in the h.p. cylinder (allowing a small amount for leakage) is 0.74 at 10% cut-off, 0.78 at 20%, and 0.82 at 30% cut-off. The loss by condensation between the cylinders is such that the steam accounted for at cut-off in the l.p. cylinder, expressed in proportion of that shown at release in the h.p. cylinder, is 0.85 at 10% cut-off, 0.87 at 20% cut-off, and 0.89 at 30% cut-off.

TABLE No. 1.

FEED-WATER CONSUMPTION, SIMPLE ENGINES.

NON-CONDENSING ENGINES.

CONDENSING ENGINES.

| Per cent Cut-off. | Initial Pressure above Atmosphere, lbs. | Mean Effective Pressure, lbs. | Feed-water Required per I.H.P. per Hour. | | Per cent Cut-off. | Initial Pressure above Atmosphere, lbs. | Mean effective Pressure, lbs. | Feed-water Required per I.H.P. per Hour. | |
|-------------------|---|-------------------------------|---|--|-------------------|---|-------------------------------|---|--|
| | | | Corresponding to Diagrams with no Leakage, lbs. | Corresponding to Actual Results Attained in Practice, assuming Slight Leakage. | | | | Corresponding to Diagrams with no Leakage, lbs. | Corresponding to Actual Results Attained in Practice, assuming Slight Leakage. |
| 10 { | 80 | 16.07 | 27.61 | 29.88 | 10 { | 80 | 29.72 | 17.30 | 18.89 |
| | 90 | 19.76 | 25.43 | 27.43 | | 90 | 33.41 | 17.15 | 18.70 |
| | 100 | 23.45 | 23.90 | 25.73 | | 100 | 37.10 | 17.02 | 18.56 |
| 20 { | 80 | 32.02 | 21.04 | 25.68 | 15 { | 80 | 38.28 | 17.60 | 19.09 |
| | 90 | 37.47 | 23.00 | 24.57 | | 90 | 42.92 | 17.45 | 18.91 |
| | 100 | 42.92 | 22.25 | 23.77 | | 100 | 47.56 | 17.32 | 18.74 |
| 30 { | 80 | 43.97 | 24.71 | 26.29 | 20 { | 80 | 45.63 | 18.27 | 19.69 |
| | 90 | 50.73 | 23.91 | 25.38 | | 90 | 51.08 | 18.14 | 19.51 |
| | 100 | 57.49 | 23.27 | 24.68 | | 100 | 56.53 | 18.02 | 19.36 |
| 40 { | 80 | 53.25 | 25.76 | 27.17 | 30 { | 80 | 57.57 | 19.91 | 21.25 |
| | 90 | 61.01 | 25.03 | 26.35 | | 90 | 64.32 | 19.78 | 21.06 |
| | 100 | 68.76 | 24.47 | 25.73 | | 100 | 71.08 | 19.67 | 20.93 |
| 50 { | 80 | 60.44 | 26.99 | 28.38 | 40 { | 80 | 66.85 | 21.36 | 22.56 |
| | 90 | 68.96 | 26.32 | 27.62 | | 90 | 74.60 | 21.24 | 22.41 |
| | 100 | 77.48 | 25.78 | 26.99 | | 100 | 82.36 | 21.13 | 22.24 |

The data upon which table No. 3 is calculated are not given, but the feed-water consumption is somewhat lower than has yet been reached (1894), the lowest steam consumption of a triple-expansion engine yet recorded being 11.7 lbs.

TABLE No. 2.

FEED-WATER CONSUMPTION FOR COMPOUND CONDENSING ENGINES.

| Cut-off, per cent. | Initial Pressure above Atmosphere. | | Mean Effective Press. | | Feed-water Required per I.H.P. per Hour, lbs. |
|-----------------------|---------------------------------------|--------------------|-----------------------|--------------------|--|
| | H.P. Cyl., lbs. | L.P. Cyl., lbs. | H.P. Cyl., lbs. | L.P. Cyl., lbs. | |
| 10 | 80 | 4.0 | 11.67 | 2.65 | 16.92 |
| | 100 | 7.3 | 15.33 | 3.87 | 15.00 |
| | 120 | 11.0 | 18.54 | 5.23 | 13.86 |
| 20 | 80 | 4.3 | 26.73 | 5.48 | 14.60 |
| | 100 | 8.1 | 33.13 | 7.56 | 13.67 |
| | 120 | 12.1 | 39.29 | 9.74 | 13.09 |
| 30 | 80 | 4.6 | 37.61 | 7.48 | 14.99 |
| | 100 | 8.5 | 46.41 | 10.10 | 14.21 |
| | 120 | 11.7 | 56.00 | 12.26 | 13.87 |

TABLE No. 3.

FEED-WATER CONSUMPTION FOR TRIPLE-EXPANSION CONDENSING ENGINES.

| Cut-off, per cent. | Initial Pressure above Atmosphere. | | | Mean Effective Pressure. | | | Feed-water Required per I.H.P. per Hour, lbs. |
|-----------------------|---------------------------------------|------------------|--------------------|--------------------------|------------------|--------------------|---|
| | H.P. Cyl., lb. | I. Cyl., lbs. | L.P. Cyl., lbs. | H.P. Cyl., lbs. | I. Cyl., lbs. | L.P. Cyl., lbs. | |
| 30 | 120 | 37.8 | 1.3 | 38.5 | 17.1 | 6.5 | 12.05 |
| | 140 | 43.8 | 2.8 | 46.5 | 18.6 | 7.1 | 11.4 |
| | 160 | 49.3 | 3.8 | 55.0 | 20.0 | 8.0 | 10.75 |
| 40 | 120 | 38.8 | 2.8 | 51.5 | 22.8 | 8.6 | 11.65 |
| | 140 | 45.8 | 3.9 | 59.5 | 23.7 | 9.1 | 11.4 |
| | 160 | 51.3 | 5.3 | 70.0 | 25.5 | 10.0 | 10.85 |
| 50 | 120 | 39.8 | 3.7 | 60.5 | 26.7 | 10.1 | 12.2 |
| | 140 | 46.8 | 4.8 | 70.5 | 28.0 | 10.8 | 11.6 |
| | 160 | 52.8 | 6.3 | 82.5 | 30.0 | 11.8 | 11.15 |

Sizes and Calculated Performances of Vertical High-speed Engines. — The following tables are taken from an old circular, describing the engines made by the Lake Erie Engineering Works, Buffalo, N. Y. The engines are fair representatives of the type largely used for driving dynamos directly without belts. The tables were calculated by E. F. Williams, designer of the engines. They are here somewhat abridged to save space.

Simple Engines — Non-condensing.

| Diam. of Cyl- inder, ins. | Stroke, ins. | Revs. per Minute. | H.P. when cutting off at 1/5 stroke. | | | H.P. when cutting off at 1/4 stroke. | | | H.P. when cutting off at 1/3 stroke | | | Dimen- sions of Wheels. dia. face | | Steam-pipe, ins. | Exhaust- pipe. |
|-------------------------------------|--------------|----------------------|--|------------|------------|--|------------|------------|---|------------|------------|--|-------|---------------------|-------------------|
| | | | 70 lbs. | 80 lbs. | 90 lbs. | 70 lbs. | 80 lbs. | 90 lbs. | 70 lbs. | 80 lbs. | 90 lbs. | Ft. | In. | | |
| 7 1/2 | 10 | 370 | 20 | 25 | 30 | 26 | 31 | 36 | 32 | 37 | 43 | 4 | 4 | 21 1/2 | 3 |
| 8 1/2 | 12 | 318 | 27 | 32 | 39 | 34 | 41 | 47 | 41 | 48 | 56 | 4 1/2 | 5 | 23 3/4 | 3 1/2 |
| 10 1/2 | 14 | 277 | 41 | 49 | 60 | 52 | 62 | 71 | 63 | 74 | 85 | 5 9/16 | 6 1/2 | 31 1/2 | 4 |
| 12 | 16 | 246 | 53 | 64 | 77 | 67 | 81 | 93 | 82 | 96 | 111 | 6 8/8 | 9 | 4 | 4 1/2 |
| 13 1/2 | 18 | 222 | 66 | 80 | 96 | 84 | 100 | 116 | 102 | 120 | 138 | 7 1/2 | 11 | 4 | 5 |
| 16 | 20 | 181 | 95 | 115 | 138 | 120 | 144 | 166 | 146 | 172 | 198 | 8 4/8 | 15 | 4 1/2 | 6 |
| 18 | 24 | 158 | 119 | 144 | 173 | 151 | 181 | 208 | 183 | 215 | 248 | 10 | 19 | 5 | 7 |
| 22 | 28 | 138 | 179 | 216 | 261 | 227 | 272 | 313 | 276 | 324 | 373 | 11 8/8 | 28 | 6 | 8 |
| 24 1/2 | 32 | 120 | 221 | 267 | 322 | 281 | 336 | 386 | 340 | 400 | 460 | 13 4/8 | 34 | 7 | 9 |
| 27 | 34 | 112 | 269 | 325 | 392 | 342 | 409 | 470 | 414 | 487 | 560 | 14 2/8 | 41 | 8 | 10 |
| M.E.P., lbs. ... | | | 24 | 29 | 35 | 30.5 | 36.5 | 42 | 37 | 43.5 | 50 | Note. — The nominal-power rating of the en- gines is at 80 lbs. gauge pressure, steam cut-off at 1/4 stroke. | | | |
| Ratio of exp.... | | | 5 | | | 4 | | | 3 | | | | | | |
| Term'l press. (about), lbs. . | | | 17.9 | 20 | 22.3 | 22.4 | 25 | 27.6 | 29.8 | 33.3 | 36.8 | | | | |
| Cyl. cond'n, %. | | | 26 | 26 | 26 | 24 | 24 | 24 | 21 | 21 | 21 | | | | |
| Steam per I.H.P. hour, lbs. | | | 32.9 | 30 | 27.4 | 31.2 | 29.0 | 27.9 | 32 | 31.4 | 30 | | | | |

Compound Engines — Non-condensing — High-pressure Cylinder and Receiver Jacketed.

| Diam. Cylinder, inches. | | | Stroke, inches. | Revolutions per Minute. | H.P., cutting off at 1/4 Stroke in h.p. Cylinder. | | | | H.P., cutting off at 1/3 Stroke in h.p. Cylinder. | | | | H.P., cutting off at 1/2 Stroke in h.p. Cylinder. | | | |
|---|--------|--------|-----------------|-------------------------|---|---------|------------------|----------|---|---------|------------------|----------|---|---------|------------------|----------|
| | | | | | Cyls. 3 1/3 : 1. | | Cyls. 4 1/2 : 1. | | Cyls. 3 1/3 : 1. | | Cyls. 4 1/2 : 1. | | Cyls. 3 1/3 : 1. | | Cyls. 4 1/2 : 1. | |
| | | | | | 80 lbs. | 90 lbs. | 130 lbs. | 150 lbs. | 80 lbs. | 90 lbs. | 130 lbs. | 150 lbs. | 80 lbs. | 90 lbs. | 130 lbs. | 150 lbs. |
| 5 3/4 | 6 1/2 | 12 | 10 | 370 | 7 | 15 | 19 | 32 | 23 | 31 | 35 | 46 | 44 | 55 | 64 | 79 |
| 6 3/8 | 7 1/2 | 13 1/2 | 12 | 318 | 9 | 19 | 24 | 40 | 29 | 39 | 45 | 59 | 56 | 70 | 81 | 101 |
| 7 3/4 | 9 | 16 1/2 | 14 | 277 | 14 | 28 | 36 | 60 | 43 | 58 | 67 | 87 | 83 | 104 | 121 | 159 |
| 9 | 10 1/2 | 19 | 16 | 246 | 18 | 37 | 47 | 78 | 57 | 76 | 87 | 114 | 109 | 136 | 158 | 196 |
| 10 1/2 | 12 | 22 1/2 | 18 | 222 | 26 | 53 | 68 | 112 | 81 | 109 | 125 | 164 | 156 | 195 | 226 | 281 |
| 12 | 13 1/2 | 25 | 20 | 185 | 32 | 65 | 84 | 139 | 100 | 135 | 154 | 202 | 192 | 241 | 279 | 346 |
| 13 1/2 | 15 1/2 | 28 1/2 | 24 | 158 | 43 | 88 | 112 | 186 | 135 | 181 | 206 | 271 | 258 | 323 | 374 | 464 |
| 16 | 18 1/2 | 33 1/2 | 28 | 138 | 57 | 118 | 151 | 249 | 180 | 242 | 277 | 363 | 346 | 433 | 502 | 623 |
| 18 | 20 1/2 | 38 | 32 | 120 | 74 | 152 | 194 | 321 | 232 | 312 | 357 | 468 | 446 | 558 | 647 | 803 |
| 20 | 22 1/2 | 43 | 34 | 112 | 94 | 194 | 249 | 412 | 297 | 400 | 457 | 601 | 572 | 715 | 829 | 1036 |
| 24 1/2 | 28 1/2 | 52 | 42 | 93 | 138 | 285 | 365 | 603 | 436 | 587 | 670 | 880 | 838 | 1048 | 1215 | 1506 |
| 28 1/2 | 33 | 60 | 48 | 80 | 180 | 374 | 477 | 789 | 570 | 767 | 877 | 1151 | 1096 | 1370 | 1589 | 1973 |
| Mean eff. pressure, lbs.. | | | | | 3.3 | 6.8 | 8.7 | 14.4 | 10.4 | 14.0 | 16 | 21 | 20 | 25 | 29 | 36 |
| Ratio of expansion | | | | | 13 1/2 | | 18 1/4 | | 10 1/4 | | 13 3/4 | | 6 3/4 | | 9 1/4 | |
| Cyl. condensation, %.. | | | | | 14 | 14 | 16 | 16 | 12 | 12 | 13 | 13 | 10 | 10 | 11 | 11 |
| Ter. pres. (abt.), lbs... | | | | | 7.3 | 7.7 | 7.9 | 9 | 9.2 | 10.4 | 10.5 | 12 | 14 | 15.5 | 14.6 | 17.6 |
| Loss from expanding below atmosphere, % | | | | | 34 | 15 | 17 | 3 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| St. per I.H.P. hour, lbs. | | | | | 55 | 42 | 47 | 29 | 33.3 | 27.7 | 28.7 | 25.4 | 30 | 26.2 | 21 | 20 |

Compound Engines — Condensing — Steam-jacketed.

| Diam. Cylinder, inches. | | | Stroke, inches. | Revolutions per Minute. | H.P. when cutting off at 1/4 Stroke in h.p. Cylinder. | | | | H.P. when cutting off at 1/3 Stroke in h.p. Cylinder. | | | | H.P. when cutting off at 1/2 Stroke in h.p. Cylinder. | | | |
|----------------------------|--------|--------|-----------------|-------------------------|---|----------|---------------|----------|---|----------|---------------|----------|---|----------|---------------|----------|
| H.P. | H.P. | L.P. | | | Ratio, 3 1/3 : 1. | | Ratio, 4 : 1. | | Ratio, 3 1/3 : 1. | | Ratio, 4 : 1. | | Ratio, 3 1/3 : 1. | | Ratio, 4 : 1. | |
| | | | | | 80 lbs. | 110 lbs. | 115 lbs. | 125 lbs. | 80 lbs. | 110 lbs. | 115 lbs. | 125 lbs. | 80 lbs. | 110 lbs. | 115 lbs. | 125 lbs. |
| 6 | 6 1/2 | 12 | 10 | 370 | 44 | 59 | 53 | 62 | 55 | 70 | 68 | 75 | 70 | 97 | 95 | 106 |
| 6 1/2 | 7 1/2 | 13 1/2 | 12 | 318 | 56 | 76 | 67 | 78 | 70 | 90 | 87 | 95 | 90 | 123 | 120 | 134 |
| 8 1/4 | 9 | 16 1/2 | 14 | 277 | 83 | 112 | 100 | 116 | 104 | 133 | 129 | 141 | 133 | 183 | 179 | 200 |
| 9 1/2 | 10 1/2 | 19 | 16 | 246 | 109 | 147 | 131 | 152 | 136 | 174 | 169 | 185 | 174 | 239 | 234 | 261 |
| 11 | 12 | 22 1/2 | 18 | 222 | 156 | 210 | 187 | 218 | 195 | 250 | 242 | 265 | 250 | 343 | 335 | 374 |
| 12 1/2 | 13 1/2 | 25 | 20 | 185 | 192 | 260 | 231 | 269 | 241 | 308 | 298 | 327 | 308 | 423 | 414 | 462 |
| 14 | 15 1/2 | 28 1/2 | 24 | 158 | 258 | 348 | 310 | 361 | 323 | 413 | 400 | 439 | 413 | 568 | 555 | 619 |
| 17 | 18 1/2 | 33 1/2 | 28 | 138 | 346 | 467 | 415 | 484 | 433 | 554 | 536 | 588 | 554 | 761 | 744 | 830 |
| 19 | 20 1/2 | 38 | 32 | 120 | 446 | 602 | 535 | 624 | 558 | 714 | 691 | 758 | 714 | 981 | 959 | 1070 |
| 21 | 22 1/2 | 43 | 34 | 112 | 572 | 772 | 686 | 801 | 715 | 915 | 887 | 972 | 915 | 1258 | 1230 | 1373 |
| 26 | 28 1/2 | 52 | 42 | 93 | 838 | 1131 | 1006 | 1174 | 1048 | 1341 | 1299 | 1425 | 1341 | 1844 | 1801 | 2012 |
| 30 | 33 | 60 | 48 | 80 | 1096 | 1480 | 1316 | 1534 | 1370 | 1757 | 1699 | 1863 | 1757 | 2411 | 2356 | 2632 |
| Mean eff. press., lbs..... | | | | | 20 | 27 | 24 | 28 | 25 | 32 | 31 | 34 | 32 | 44 | 43 | 48 |
| Ratio of expansion | | | | | 13 1/2 | | 16 1/4 | | 10 | | 12 1/4 | | 6 3/4 | | 8 1/4 | |
| Cyl. condensation, %... | | | | | 18 | 18 | 20 | 20 | 15 | 15 | 18 | 18 | 12 | 12 | 14 | 14 |
| St. per I.H.P. hour, lbs. | | | | | 17.3 | 16.6 | 16.6 | 15.2 | 17.0 | 16.4 | 16.3 | 15.8 | 17.5 | 17.0 | 16.8 | 16.0 |

Triple-expansion Engines, Non-condensing — Receiver only Jacketed.

| Diameter Cylinders, inches. | | | Stroke, inches. | Revolutions per Minute. | Horse-power when cutting off at 42% of Stroke in First Cylinder. | | Horse-power when cutting off at 50% of Stroke in First Cylinder. | | Horse-power when cutting off at 67% of Stroke in First Cylinder. | |
|--------------------------------|--------|--------|-----------------|-------------------------|--|----------|--|----------|--|----------|
| H.P. | I.P. | L.P. | | | 180 lbs. | 200 lbs. | 180 lbs. | 200 lbs. | 180 lbs. | 200 lbs. |
| 43/4 | 7 1/2 | 12 | 10 | 370 | 55 | 64 | 70 | 84 | 95 | 108 |
| 5 1/2 | 8 1/2 | 13 1/2 | 12 | 318 | 70 | 81 | 90 | 106 | 120 | 137 |
| 6 1/2 | 10 1/2 | 16 1/2 | 14 | 277 | 104 | 121 | 133 | 158 | 179 | 204 |
| 7 1/2 | 12 | 19 | 16 | 246 | 136 | 158 | 174 | 207 | 234 | 267 |
| 9 | 14 1/2 | 22 1/2 | 18 | 222 | 195 | 226 | 250 | 296 | 335 | 382 |
| 10 | 16 | 25 | 20 | 185 | 241 | 279 | 308 | 366 | 414 | 471 |
| 11 1/2 | 18 | 28 1/2 | 24 | 158 | 323 | 374 | 413 | 490 | 555 | 632 |
| 13 | 22 | 33 1/2 | 28 | 138 | 433 | 502 | 554 | 657 | 744 | 848 |
| 15 | 24 1/2 | 38 | 32 | 120 | 558 | 647 | 714 | 847 | 959 | 1093 |
| 17 | 27 | 43 | 34 | 112 | 715 | 829 | 915 | 1089 | 1230 | 1401 |
| 20 | 33 | 52 | 42 | 93 | 1048 | 1215 | 1341 | 1592 | 1801 | 2053 |
| 23 1/2 | 38 | 60 | 48 | 80 | 1370 | 1589 | 1754 | 2082 | 2356 | 2685 |
| Mean eff. press., lbs..... | | | | | 25 | 29 | 32 | 38 | 43 | 49 |
| No. of expansions..... | | | | | 16 | | 13 | | 10 | |
| Cyl. condensation, %... | | | | | 14 | | 12 | | 10 | |
| Steam p. I.H.P.p.hr., lbs. | | | | | 20.76 | 19.36 | 19.25 | 17.00 | 17.89 | 17.20 |
| Lbs. coal at 8 lb. evap., lbs. | | | | | 2.59 | 2.39 | 2.40 | 2.12 | 2.23 | 2.15 |

Triple-expansion Engines — Condensing — Steam-jacketed.

| Diameter Cylinders, inches. | | | Stroke, inches. | Revolutions per Minute. | Horse- power when cutting off at 1/4 Stroke in First Cyl. | | | Horse- power when cutting off at 1/3 Stroke in First Cyl. | | | Horse- power when cutting off at 1/2 Stroke in First Cyl. | | | Horse- power when cutting off at 3/4 Stroke in First Cyl. | | |
|-----------------------------------|-------|-------|--------------------|----------------------------|---|-------------|-------------|---|-------------|-------------|---|-------------|-------------|---|-------------|-------------|
| H.P. | I.P. | L.P. | | | 120 lbs. | 140 lbs. | 160 lbs. | 120 lbs. | 140 lbs. | 160 lbs. | 120 lbs. | 140 lbs. | 160 lbs. | 120 lbs. | 140 lbs. | 160 lbs. |
| 43/4 | 71/2 | 12 | 10 | 370 | 35 | 42 | 48 | 44 | 53 | 59 | 57 | 72 | 84 | 81 | 97 | 110 |
| 51/2 | 81/2 | 131/2 | 12 | 318 | 45 | 53 | 62 | 56 | 67 | 76 | 73 | 92 | 107 | 104 | 123 | 140 |
| 61/2 | 101/2 | 161/2 | 14 | 277 | 67 | 79 | 92 | 83 | 100 | 112 | 108 | 137 | 159 | 154 | 183 | 208 |
| 71/2 | 12 | 19 | 16 | 246 | 87 | 103 | 120 | 109 | 131 | 147 | 141 | 160 | 208 | 201 | 239 | 272 |
| 9 | 141/2 | 221/2 | 18 | 222 | 125 | 148 | 172 | 156 | 187 | 211 | 203 | 257 | 299 | 289 | 343 | 390 |
| 10 | 16 | 25 | 20 | 185 | 154 | 183 | 212 | 192 | 231 | 260 | 250 | 317 | 368 | 356 | 423 | 481 |
| 111/2 | 18 | 281/2 | 24 | 158 | 206 | 245 | 284 | 258 | 310 | 348 | 335 | 426 | 494 | 477 | 568 | 645 |
| 13 | 22 | 331/2 | 28 | 138 | 277 | 329 | 381 | 346 | 415 | 467 | 450 | 571 | 663 | 640 | 761 | 865 |
| 15 | 241/2 | 38 | 32 | 120 | 357 | 424 | 491 | 446 | 535 | 602 | 580 | 736 | 854 | 825 | 981 | 1115 |
| 17 | 27 | 43 | 34 | 112 | 458 | 543 | 629 | 572 | 686 | 772 | 744 | 944 | 1095 | 1058 | 1258 | 1430 |
| 20 | 33 | 52 | 42 | 93 | 670 | 796 | 922 | 838 | 1006 | 1131 | 1089 | 1383 | 1605 | 1551 | 1844 | 2096 |
| 231/2 | 38 | 60 | 48 | 80 | 877 | 1041 | 1206 | 1096 | 1316 | 1480 | 1424 | 1808 | 2099 | 2028 | 2411 | 2740 |
| Mean eff. press., lbs.... | | | | | 16 | 19 | 22 | 20 | 24 | 27 | 26 | 33 | 38.3 | 37 | 44 | 50 |
| No. of expansions..... | | | | | 26.8 | | | 20.1 | | | 13.4 | | | 8.9 | | |
| Cyl. condensation, %... | | | | | 19 | 19 | 19 | 16 | 16 | 16 | 12 | 12 | 12 | 8 | 8 | 8 |
| St. p. I.H.P. p. hr., lbs.. | | | | | 14.7 | 13.9 | 13.3 | 14.3 | 13.9 | 13.2 | 14.3 | 13.6 | 13.0 | 15.7 | 14.9 | 14.7 |
| Coal at 8 lbs. evap., lbs. | | | | | 1.8 | 1.73 | 1.66 | 1.78 | 1.7 | 1.65 | 1.78 | 1.70 | 1.62 | 1.96 | 1.86 | 1.72 |

The Willans Law. Total Steam Consumption at Different Loads.

— Mr. Willans found with his engine that when the total steam consumption at different loads was plotted as ordinates, the loads being abscissas, the result would be a straight inclined line cutting the axis of ordinates at some distance above the origin of coördinates, this distance representing the steam consumption due to cylinder condensation at zero load. This statement applies generally to throttling engines, and is known as the Willans law. It applies also approximately to automatic cut-off engines of the Corliss, and probably of other types, up to the most economical load. In Mr. Barrus's book there is a record of six tests of a 16 × 42-in. Corliss twin-cylinder non-condensing engine, which gave results as follows:

| | | | | | | | |
|------------------------------|-------|-------|-------|-------|-------|-------|-------|
| I.H.P..... | 37 | 100 | 146 | 222 | 250* | 287 | 342 |
| Feed-water per I.H.P. hour. | 73.63 | 38.28 | 31.47 | 25.83 | 25.0* | 25.39 | 25.91 |
| Total feed-water per hour... | 2724 | 3825 | 4595 | 5734 | 6250 | 7287 | 8861 |

* Interpolated from the plotted curve.

The first five figures in the last line plot in a straight line whose equation is $y = 2122 + 16.55$ H.P., and a straight line through the plotted position of the last two figures has the equation $y = 28.62$ H.P. — 927. These two lines cross at 253 H.P., which is the most economical load, the water rate being 24.96 lbs. and the total feed 6314 lbs. The figure 2122 represents the constant loss due to cylinder condensation, which is just over one-third of the total feed-water at the most economical load.

In Geo. H. Barrus's book on "Engine Tests" there is a diagram of condensation and leakage in tight or fairly tight simple engines using saturated steam. The average curve drawn through the several observations shows the condensation and leakage to be about as follows for different percentages of cut-off:

| | | | | | | | | |
|---|-----|-----|----|-----|-----|-----|-----|-----|
| Cut-off, % of stroke = l | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 42 |
| Condens. and leakage, % = p ... | 60 | 43 | 35 | 29 | 24 | 20 | 17 | 15 |
| $c = l \times p \div (100 - p)$ = | 7.5 | 7.5 | 8 | 8.2 | 7.9 | 7.5 | 7.2 | 7.4 |

The figures in the last line represent the condensation and leakage as a percentage of the volume of the stroke of the piston. that is, in the same

terms as the first line, instead of as a percentage of the total steam supplied, in which terms the figures of the second line are expressed. They indicate that the amount of cylinder condensation is nearly a constant quantity for a given engine with a given steam pressure and speed, whatever may be the point of cut-off.

Economy of Engines under Varying Loads. (From Prof. W. C. Unwin's lecture before the Society of Arts, London, 1892.) — The general result of numerous trials with large engines was that with a constant load an indicated horse-power should be obtained with a consumption of $1\frac{1}{2}$ lbs. of coal per I.H.P. for a condensing engine, and $1\frac{3}{4}$ lbs. for a non-condensing engine, corresponding to about $1\frac{3}{4}$ lbs. to $2\frac{1}{8}$ lbs. per effective H.P.

In electric-lighting stations the engines work under a very fluctuating load, and the results are far more unfavorable. An excellent Willans non-condensing engine, which on full-load trials worked with under 2 lbs. per effective H.P. hour, in the ordinary daily working of the station used $7\frac{1}{2}$ lbs. in 1886, which was reduced to 4.3 lbs. in 1890 and 3.8 lbs. in 1891. Probably in very few cases were the engines at electric-light stations working under a consumption of $4\frac{1}{2}$ lbs. per effective H.P. hour. In the case of small isolated motors working with a fluctuating load, still more extravagant results were obtained.

At electric-lighting stations the load factor, viz., the ratio of the average load to the maximum, is extremely small, and the engines worked under very unfavorable conditions, which largely accounted for the excessive fuel consumption at these stations.

In steam-engines the fuel consumption has generally been reckoned on the indicated horse-power. At full-power trials this was satisfactory enough, as the internal friction is then usually a small fraction of the total.

Experiment has, however, shown that the internal friction is nearly constant, and hence, when the engine is lightly loaded, its mechanical efficiency is greatly reduced. At full load small engines have a mechanical efficiency of 0.8 to 0.85, and large engines might reach at least 0.9, but if the internal friction remained constant this efficiency would be much reduced at low powers. Thus, if an engine working at 100 I.H.P. had an efficiency of 0.85, then when the I.H.P. fell to 50 the effective H.P. would be 35 H.P. and the efficiency only 0.7. Similarly, at 25 H.P. the effective H.P. would be 10 and the efficiency 0.4.

Experiments on a Corliss engine at Creusot gave the following results:

| | | | | | |
|--|------|------|------|------|-------|
| Effective power at full load | 1.0 | 0.75 | 0.50 | 0.25 | 0.125 |
| Condensing, mechanical efficiency.... | 0.82 | 0.79 | 0.74 | 0.63 | 0.48 |
| Non-condensing, mechanical efficiency. | 0.86 | 0.83 | 0.78 | 0.67 | 0.52 |

Steam Consumption of Engines of Various Sizes. — W. C. Unwin (*Cassier's Magazine*, 1894) gives a table showing results of 49 tests of engines of different types. In non-condensing simple engines, the steam consumption ranged from 65 lbs. per hour in a 5-horse-power engine to 22 lbs. in a 134-H.P. Harris-Corliss engine. In non-condensing compound engines, the only type tested was the Willans, which ranged from 27 lbs. in a 10-H.P. slow-speed engine, 122 ft. per minute, with steam-pressure of 84 lbs., to 19.2 lbs. in a 40-H.P. engine, 401 ft. per minute, with steam-pressure 165 lbs. A Willans triple-expansion non-condensing engine, 39 H.P., 172 lbs. pressure, and 400 ft. piston speed per minute, gave a consumption of 18.5 lbs. In condensing engines, nine tests of simple engines gave results ranging only from 18.4 to 22 lbs. In compound-condensing engines over 100 H.P., in 13 tests the range is from 13.9 to 20 lbs. In three triple-expansion engines the figures are 11.7, 12.2, and 12.45 lbs., the lowest being a Sulzer engine of 360 H.P. In marine compound engines, the Fusiya and Colchester, tested by Prof. Kennedy, gave steam consumption of 21.2 and 21.7 lbs.; and the Meteor and Tartar triple-expansion engines gave 15.0 and 19.8 lbs.

Taking the most favorable results which can be regarded as not exceptional it appears that in test trials, with constant and full load, the expenditure of steam and coal is about as follows:

| Kind of Engine. | lbs. Per I.H.P. hour. | | Per Effective H.P. hr. | |
|---------------------|-----------------------|--------|------------------------|--------|
| | Coal, | Steam, | Coal, | Steam, |
| Non-condensing..... | 1.80 | 16.5 | 2.00 | 18.0 |
| Condensing..... | 1.50 | 13.5 | 1.75 | 15.8 |

These may be regarded as minimum values, rarely surpassed by the most efficient machinery, and only reached with very good machinery in the favorable conditions of a test trial.

Small Engines and Engines with Fluctuating Loads are usually very wasteful of fuel. The following figures, illustrating their low economy, are given by Prof. Unwin, *Cassier's Magazine*, 1894. Small engines in workshops in Birmingham, Eng.

| | | | | | | | |
|---|------|-------|-------|-------|-------|-------|-------|
| Probable I.H.P. at full load..... | 12 | 45 | 60 | 45 | 75 | 60 | 60 |
| Average I.H.P. during observation..... | 2.96 | 7.37 | 8.2 | 8.6 | 23.64 | 19.08 | 20.08 |
| Coal per I.H.P. per hour during observation, lbs. | 36.0 | 21.25 | 22.61 | 18.13 | 11.68 | 9.53 | 8.50 |

It is largely to replace such engines as the above that power will be distributed from central stations.

Tests at Royal Agricultural Society's show at Plymouth, Eng. *Engineering*, June 27, 1890.

| Rated H.P. | Compound or Simple. | Diam. of Cylinders. | | Stroke, ins. | Max. Steam-pressure. | Per Brake H.P. per hour. | | Water per lb. Coal. |
|------------|---------------------|---------------------|------|--------------|----------------------|--------------------------|-----------|---------------------|
| | | b.p. | l.p. | | | Coal. | Water. | |
| 5 | simple | 7 | | 10 | 75 | 12.12 | 78.1 lbs. | 6.1 lb. |
| 3 | compound | 3 | 6 | 6 | 110 | 4.87 | 42.03 " | 8.72 " |
| 2 | simple | 4 1/2 | | 7 1/2 | 75 | 11.77 | 89.9 " | 7.64 " |

Steam-consumption of Engines at Various Speeds. (Profs. Denton and Jacobus, *Trans. A. S. M. E.*, x, 722.) — 17 × 30 in. engine, non-condensing, fixed cut-off, Meyer valve. (From plotted diagrams.)

| | | | | | | | | | | | |
|---------------------|----|----|----|------|------|------|------|------|------|------|------|
| Revs. per min... | 8 | 12 | 16 | 20 | 24 | 32 | 40 | 48 | 56 | 72 | 88 |
| 1/8 cut-off, lbs... | 39 | 35 | 32 | 30 | 29.3 | 29 | 28.7 | 28.5 | 28.3 | 28 | 27.7 |
| 1/4 cut-off, lbs... | 39 | 34 | 31 | 29.5 | 29 | 28.4 | 28 | 27.5 | 27.1 | 26.3 | 25.6 |
| 1/2 cut-off, lbs... | 39 | 36 | 34 | 33 | 32 | 30.8 | 29.8 | 29.2 | 28.8 | 28.7 | |

Steam-consumption of same engine; fixed speed, 60 revs. per minute. Varying cut-off compared with throttling-engine for same horse-power and boiler-pressures:

| | | | | | | | | | | |
|---------------------------------|-----|------|------|------|------|------|------|------|------|-----|
| Cut-off, fraction of stroke.... | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| Steam, 90 lbs... | 29 | 27.5 | 27 | 27 | 27.2 | 27.8 | 28.5 | ... | ... | ... |
| Steam, 60 lbs... | 39 | 34.2 | 32.2 | 31.5 | 31.4 | 31.6 | 32.2 | 34.1 | 36.5 | 39 |

Throttling-engine, 7/8 cut-off, for corresponding horse-powers.

| | | | | | | | | | | |
|------------------|-----|------|------|------|------|-----|-----|-----|-----|-----|
| Steam, 90 lbs... | 42 | 37 | 33.8 | 31.5 | 29.8 | ... | ... | ... | ... | ... |
| Steam, 60 lbs... | ... | 50.1 | 49 | 46.8 | 44.6 | 41 | ... | ... | ... | ... |

Some of the principal conclusions from this series of tests are as follows:

1. There is a distinct gain in economy of steam as the speed increases for 1/2, 1/8, and 1/4 cut-off at 90 lbs. pressure. The loss in economy for about 1/4 cut-off is at the rate of 1/12 lb. of water per I.H.P. per hour for each decrease of a revolution per minute from 86 to 26 revolutions, and at the rate of 5/8 lb. of water below 26 revolutions. Also, at all speeds the 1/4 cut-off is more economical than either the 1/2 or 1/8 cut-off.

2. At 90 lbs. boiler-pressure and above 1/3 cut-off, to produce a given H.P. requires about 20% less steam than to cut off at 7/8 stroke and regulate by the throttle.

3. For the same conditions with 60 lbs. boiler-pressure, to obtain, by throttling, the same mean effective pressure at 7/8 cut-off that is obtained by cutting off about 1/3, requires about 30% more steam than for the latter condition.

Capacity and Economy of Steam Fire Engines. (*Eng. News*, Mar. 28, 1895.) — The tests were made by Dexter Brackett for the Board of Fire Commissioners. Boston. Mass.

| No. of engine. | Boiler heating Surface. | Coal per sq. ft. of grate, per hour. | Water evap. per lb. coal, from and at 212°. | Av. steam pressure. | Av. water pressure. | Duty, ft.-lbs. per 100 lbs. of coal. | Av. water pumped per min. |
|----------------|-------------------------|--------------------------------------|---|---------------------|---------------------|--------------------------------------|---------------------------|
| 1..... | 101.0 | lbs. 191.0 | lbs. 2.26 | lbs. 90.2 | lbs. 143.2 | 7,619,800 | galls. 549 |
| 1..... | | 184.0 | | 92.3 | 124.0 | 9,632,700 | 499 |
| 2..... | 85.0 | 191.0 | 2.66 | 78.4 | 123.3 | 5,900,000 | 535 |
| 3..... | 74.0 | 141.6 | 3.57 | 75.7 | 113.8 | 5,882,000 | 482 |
| 4..... | 86.5 | 138.4 | 2.88 | 71.5 | 136.4 | 8,112,900 | 459 |
| 5..... | 86.0 | 163.7 | | 102.7 | 121.2 | 8,736,300 | 449 |
| 5..... | | 103.3 | 5.87 | 72.1 | 119.6 | 14,026,000 | 545 |
| 6..... | 86.0 | 181.6 | 3.45 | 92.7 | 143.0 | 9,678,400 | 536 |
| 7..... | 112.0 | 117.3 | 4.94 | 68.8 | 119.2 | 10,201,600 | 596 |
| 8..... | 140.5 | 172.1 | 3.51 | 101.3 | 112.8 | 7,758,300 | 910 |
| 9..... | 174.0 | 142.5 | 4.49 | 76.5 | 111.5 | 7,187,400 | 482 |
| 10..... | 225.0 | 91.1 | 4.22 | 59.0 | 102.1 | 6,482,100 | 419 |
| 10..... | | 151.4 | 4.10 | 87.8 | 126.8 | 7,993,400 | 564 |
| 11..... | 229.0 | 148.4 | 3.76 | 74.7 | 128.1 | 7,265,000 | 572 |

Nos. 1, 2, 3 and 4, Amoskeag engines; Nos. 5, 6, 7 and 8, Clapp & Jones; Nos. 9, 10, 11, Silsby. The engines all show an exceedingly high rate of combustion, and correspondingly low boiler efficiency and pump duty.

Economy Tests of High-speed Engines. (F. W. Dean and A. C. Wood, *Jour. A. S. M. E.*, June, 1908.) — Some of these engines had been in service for a long time, and therefore their valves may not have been in the best condition. The results may be taken as fairly representing the economy of average engines of the type, under usual working conditions. The engines were all non-condensing. The 16 × 15-in. engine was vertical, the others horizontal. They were all direct-connected to generators.

| No. of Test. | 1 | 2 | 3 | 4 |
|-------------------------|----------------|--------------|--------------|--------------|
| Size of engine, ins.... | 15 × 14 | 16 × 15 | 14 × 12 | 16 × 14 |
| Hours in service..... | 15,216 | 20,000 | 28,644 | 719 |
| Revs. per min..... | 240 | 240 | 300 | 270 |
| Valves..... | 1 flat | 1 flat | 1 flat | 4 flat |
| Generator, K.W..... | 100 | 2-50 | 2-40 | 125 |
| Steam per I.H.P.-hr.. | 37.2, † 36.2 * | 36.7, † 35.8 | 31.7, † 32.0 | 37.5, * 36.7 |
| Steam per K.W.-hr.. | 60.2, 58.4 | 61.0 59.7 | 57.1, 57.4 | 54.9, 54.7 |

| No. of Test. | 5 | 6 | 7 |
|-------------------------|------------------------|--------------|----------------------------------|
| Size of engine, ins.... | 18 × 18 | 15 × 16 | 12 × 18 |
| Hours in service..... | 32,000 | 5,600 | 10,800 |
| Revs. per min..... | 220 | 250 | 190 |
| Valves..... | 1 piston | 1 piston | { 2 flat inlet 2 Corliss exh. |
| Generator, K.W..... | 150 | 100 | 75 |
| Steam per I.H.P.-hr.. | 39.8, † 34.7, * 29.5 † | 36.3, * 33.6 | 44.0, † 36.7, 34.1 \$ |
| Steam per K.W.-hr.... | 61.8, 51.8, 43.4 | 55.2, 49.4 | 79.3, 60.5, 53.7 |

* $\frac{3}{4}$ load; † $\frac{1}{2}$ load; ‡ $1\frac{1}{4}$ load; § $1\frac{1}{2}$ load; the others full load.

Some of the conclusions of the authors from the results of these tests are as follows:

The performances of the perfectly balanced flat valve engines are so relatively poor as to disqualify them, unless this type of valve can be made with some mechanism by which wear will not increase leakage. The four valve engines, which were built to be more economical than single-valve

engines, have utterly failed in their object. The duplication of valves used in both four-valve engines simply increased the opportunity for leakage. The most economical result was obtained from a piston valve engine, No. 5, heavily loaded. With the lighter loads that are comparable the flat valve engine, No. 3, surpassed No. 5 in economy. The flat valve engines give a flatter load curve than the piston valve engines. Comparing the results of the flat valve engines, the most economical results were obtained from engine No. 3, which had a valve which automatically takes up wear, and if it does not cut, must maintain itself tight for long periods.

From the results we are justified in thinking that most high-speed engines rapidly deteriorate in economy. On the contrary, slower running Corliss or gridiron valve engines improve in economy for some time and then maintain the economy for many years. It is difficult to see that the speed is the cause of this, and it must depend on the nature of the valve.

The steam consumption of small single-valve high-speed engines non-condensing, is not often less than 30 lbs. per I.H.P. per hour. The Watertown engines, 10 × 12 tested by J. W. Hill for the Philadelphia Dept. of Public Works in 1904, gave respectively 30.67 and 29.70 lbs. at full load, 61.8 and 63.9 I.H.P., and 28.87 and 29.54 lbs. at approximately half-load, 37.63 and 36.36 I.H.P.

High Piston-speed in Engines. (*Proc. Inst. M. E.*, July, 1883, p. 321.) — The torpedo boat is an excellent example of the advance towards high speeds, and shows what can be accomplished by studying lightness and strength in combination. In running at 22½ knots an hour, an engine with cylinders of 16 in. stroke will make 480 revolutions per minute, which gives 1280 ft. per minute for piston-speed; and it is remarked that engines running at that high rate work much more smoothly than at lower speeds, and that the difficulty of lubrication diminishes as the speed increases.

A High-speed Corliss Engine. — A Corliss engine, 20 × 42 in., has been running a wire-rod mill at the Trenton Iron Co.'s works since 1877, at 160 revolutions or 1120 ft. piston-speed per minute (*Trans. A. S. M. E.*, ii, 72). A piston-speed of 1200 ft. per min. has been realized in locomotive practice.

The Limitation of Engine-speed. (Chas. T. Porter, in a paper on the Limitation of Engine-speed, *Trans. A. S. M. E.*, xiv, 806.) — The practical limitation to high rotative speed in stationary reciprocating steam-engines is not found in the danger of heating or of excessive wear, nor, as is generally believed, in the centrifugal force of the fly-wheel, nor in the tendency to knock in the centers, nor in vibration. He gives two objections to very high speeds: First, that "engines ought not to be run as fast as they can be;" second, the large amount of waste room in the port, which is required for proper steam distribution. In the important respect of economy of steam, the high-speed engine has thus far proved a failure. Large gain was looked for from high speed, because the loss by condensation on a given surface would be divided into a greater weight of steam, but this expectation has not been realized. For this unsatisfactory result we have to lay the blame chiefly on the excessive amount of waste room. The ordinary method of expressing the amount of waste room in the percentage added by it to the total piston displacement, is a misleading one. It should be expressed as the percentage which it adds to the length of steam admission. For example, if the steam is cut off at 1/5 of the stroke, 8% added by the waste room to the total piston displacement means 40% added to the volume of steam admitted. Engines of four, five and six feet stroke may properly be run at from 700 to 800 ft. of piston travel per minute, but for ordinary sizes, says Mr. Porter, 600 ft. per minute should be the limit.

British High-speed Engines. (John Davidson, *Power*, Feb. 9, 1909.) — The following figures show the general practice of leading builders:

| I.H.P. | 50 | 100 | 200 | 500 | 750 | 1000 | 1500 | 2000 |
|----------------------------|---------|---------|-----|---------|-----|------|------|---------|
| Revs. per min. | | | | | | | | |
| | 600-700 | 550-600 | 500 | 350-375 | 325 | 250 | 200 | 160-180 |
| Piston speed, ft. per min. | | | | | | | | |
| | 600 | 650 | 675 | 750 | 775 | 800 | 900 | 1000 |

Rapid strides have been made during the last few years, despite the

competition of the steam turbine. The single-acting type (Brotherhood, Willans and others) has been superseded by double-acting engines with forced lubrication. There is less wear in a high-speed than in a low-speed engine. A 500-H.P. 3-crank engine after running 7 years, 12 hours per day and 300 days per year, showed the greatest wear to be as follows: crank pins, 0.003 in.; main bearings, 0.003 in.; eccentric sheaves, 0.015 in.; crosshead pins, 0.005 in. All pins, where possible, are of steel, case-hardened. High-speed engines have at least as high economy and efficiency as any other type of engine manufactured. A triple-expansion mill engine, with steam at 175 lbs., vacuum 26 ins., superheat 100° F., gave results as shown below, [figures taken from curves in the original].

| | | | | | | | | | | |
|------------------------------|------|-------|------|------|------|-------|-------|-------|------|------|
| Fraction of full load..... | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| Lbs. steam per I.H.P. hour.. | 12.7 | 11.85 | 11.4 | 11.1 | 10.9 | 10.8 | 10.75 | 10.75 | 10.8 | 11.0 |
| Lbs. steam per B.H.P. hour.. | 16.0 | 14.8 | 13.7 | 12.9 | 12.4 | 12.05 | 11.85 | 11.8 | 11.8 | 11.8 |

Owing to the forced lubrication and throttle-governing, the economical performance at light loads is relatively much better than in slow-speed engines. The piston valves render the use of superheat practicable. At 200° superheat the saving in steam consumption of a triple-expansion engine is 26%. [A curve of the relation of superheat to saving shows that the percentage of saving is almost uniformly 1.4% for each additional 10° from 0° to 160° of superheat.]

The method of governing small high-speed engines is by means of a plain centrifugal governor fixed to the crank shaft and acting directly on a throttle. Several makers use a governor which at light loads acts by throttling, and at heavy loads by altering the expansion in the high-pressure cylinder. The crank-shaft governor used in America has been found impracticable for high speeds, except perhaps for small engines.

Advantage of High Initial and Low Back Pressure. — The theoretical advantage due to the use of low back pressures or high vacua is shown by the following table, in which the efficiencies are those of the Carnot cycle, $E = (T_1 - T_2) \div T_1$. With 100 lbs. absolute initial pressure the efficiency is increased from 0.270 to 0.353, or 30.7%, by raising the vacuum from 27.02 to 29.56 ins. of mercury, and with 200 lbs. it is increased from 0.317 to 0.394, or 24.3%, with the same change in the vacuum.

| Abs. Initial Pressure. | | | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 |
|------------------------|-------------------------------|---------------------|----------------------|------|------|------|------|------|------|------|------|
| Temp. ° F. | Vacuum, In. of Mercury. | Lbs. per Sq. In. | Carnot Efficiencies. | | | | | | | | |
| 115 | 27.02 | 1.47 | 0.270 | .285 | .298 | .308 | .317 | .325 | .332 | .339 | .345 |
| 108 | 27.48 | 1.20 | 0.279 | .293 | .306 | .316 | .325 | .333 | .341 | .347 | .353 |
| 100 | 28.00 | 0.95 | 0.289 | .303 | .316 | .325 | .335 | .343 | .350 | .356 | .362 |
| 90 | 28.50 | 0.70 | 0.302 | .316 | .328 | .338 | .347 | .355 | .361 | .368 | .373 |
| 70 | 29.18 | 0.74 | 0.327 | .341 | .353 | .362 | .371 | .378 | .385 | .391 | .396 |
| 50 | 29.56 | 0.36 | 0.353 | .366 | .377 | .386 | .394 | .402 | .408 | .414 | .419 |

The same table shows the advantage of high initial pressure. Thus with a vacuum 27.02 ins. the efficiency is increased from 0.270 to 0.317, or 17.4%, by raising the initial absolute pressure from 100 to 200 lbs., and with a vacuum of 28.5 ins. the efficiency is increased from 0.302 to 0.347, or 14.9%, by the same rise of pressure. In practice the efficiencies given in the table for the given pressures and temperatures cannot be reached on account of imperfections of the steam-engine, and the fact that the engine does not work on the ideal Carnot cycle. The relative advantages, however, are probably proportional to those indicated by the table, provided the expansion is divided into two or more stages at pressures above

100 lbs. The possibility of obtaining very high vacua is limited by the temperature of the condensing water available and by the imperfections of the air pump. The use of high initial pressures is limited by the safe working pressure of the boiler and engine.

Comparison of the Economy of Compound and Single-cylinder Corliss Condensing Engines, each expanding about Sixteen Times. (D. S. Jacobus, *Trans., A. S. M. E.*, xii, 943.)

The engines used in obtaining comparative results are located at Stations I and II of the Pawtucket Water Co.

The tests show that the compound engine is about 30% more economical than the single-cylinder engine. The dimensions of the two engines are as follows: Single 20×48 ins.; compound 15 and $30\frac{1}{8} \times 30$ ins. The steam used per I.H.P. hour was: single 20.35 lbs., compound 13.73 lbs.

Both of the engines are steam-jacketed, practically on the barrels only, with steam at full boiler-pressure, viz., single 106.3 lbs., compound 127.5 lbs.

The steam-pressure in the case of the compound engine is 127 lbs., or 21 lbs. higher than for the single engine. If the steam-pressure be raised this amount in the case of the single engine, and the indicator-cards be increased accordingly, the consumption for the single-cylinder engine would be 19.97 lbs. per hour per horse-power.

Two-cylinder vs. Three-cylinder Compound Engine. — A Wheelock triple-expansion engine, built for the Merrick Thread Co., Holyoke, Mass., is constructed so that the intermediate cylinder may be cut out of the circuit and the high-pressure and low-pressure cylinders run as a two-cylinder compound, using the same conditions of initial steam-pressure and load. The diameters of the cylinders are 12, 16, and $24\frac{13}{32}$ ins., the stroke of the first two being 36 ins. and that of the low-pressure cylinder 48 ins. The results of a test reported by S. M. Green and G. I. Rockwood, *Trans. A. S. M. E.*, vol. xiii, 647, are as follows: In lbs. of dry steam used per I.H.P. per hour, 12 and $24\frac{13}{32}$ in. cylinders only used, two tests 13.06 and 12.76 lbs., average 12.91. All three cylinders used, two tests 12.67 and 12.90 lbs., average 12.79. The difference is only 1%, and would indicate that more than two cylinders are unnecessary in a compound engine, but it is pointed out by Prof. Jacobus, that the conditions of the test were especially favorable for the two-cylinder engine, and not relatively so favorable for the three cylinders. The steam-pressure was 142 lbs. and the number of expansions about 25. (See also discussion on the Rockwood type of engine, *Trans. A. S. M. E.*, vol. xvi.)

Economy of a Compound Engine. (D. S. Jacobus, *Trans. A. S. M. E.*, 1903.) — A Rice & Sargent engine, 20 and 40×42 ins., was tested with steam about 149 lbs., vacuum 27.3 to 28.8 ins. or 0.82 to 1.16 lbs. absolute, r.p.m. 120 to 122, with results as follows:

| | | | | | | |
|-------------------------------|-------|-------|-------|-------|-------|-------|
| I.H.P. | 1004 | 853 | 820 | 627 | 491 | 340 |
| Water per I.H.P. per hr. | 12.75 | 12.33 | 12.55 | 12.10 | 13.92 | 14.58 |
| B.T.U. per I.H.P. per min. . | 231.8 | 226.3 | 229.9 | 222.7 | 256.8 | 267.7 |

The Lentz Compound Engine is described in *The Engineer* (London), July 10, 1908. It is the latest development of the reciprocating engine with four double-seated poppet valves to each cylinder, each valve operated by a separate eccentric mounted on a lay-shaft driven by bevel-gearing from the main shaft. The throw of the high-pressure steam eccentrics is varied by slide-blocks which are caused to slide along the lay-shaft by the action of a centrifugal inertia governor, which is also mounted on the lay-shaft. No elastic packing is used in the engine, the piston-rod stuffing box being fitted with ground cast-iron rings, and the valve stems being provided with grooves and ground to fit long bushings to 0.001 in. Two tests of a Lentz engine built in England, $14\frac{1}{2}$ and $24\frac{3}{4}$ by $27\frac{1}{2}$ in., gave results as follows:

Saturated steam, 170 lbs., vacuum 26 in., I.H.P. 366, steam per I.H.P. per hour 12.3 lbs. Steam 170 lbs. superheated 150° F., vac. 26 in., I.H.P. 366, steam per I.H.P. per hour, 10.4 lbs. Revs. per min. in both cases, 167. Piston speed 767 ft. per min. Engines are built for speeds up to 900 ft. per min., and up to 350 r.p.m.

The Lentz engine is built in the United States by the Erie City Iron Works.

Steam Consumption of Sulzer Compound and Triple-expansion Engines with Superheated Steam.

The figures in the table below were furnished to the author (Aug., 1902) by Sulzer Bros., Winterthur, Switzerland. They are the results of official tests by Prof. Schröter of Munich, Prof. Weber of Zurich, and other eminent engineers.

COMPOUND ENGINES.

| Normal Power, I.H.P. | Dimensions of Cylinders, Inches. | Revolutions per Minute. | Initial Pressure, Pounds. | Temp. of Steam, Deg. F. | Vacuum, Inches. | I.H.P. | Steam Cons. per I.H.P. Hour, Pounds. |
|-------------------------|--|----------------------------|--|--|--|--|--|
| 1500 to 1800 | 30.5 and 49.2 × 59.1 | 85 | 130 132 122 | 356 428 482 | 26.4 26.4 26.6 | 850 842 1719 | 13.30 12.05 12.42 |
| 800 to 1000 | 24 and 40.4 × 51.2 | 83 | 136 134 135 135 132 134 | 357 356 356 547 533 545 | 28 28 27.6 28 27.8 27.2 | 481 750 1078 515 788 1100 | 13.00 13.10 14.10 11.32 11.52 11.88 |
| 950 to 1150 | 26 and 42.3 × 51.2 do., non-cond'g | 86 | 130 129 132 136 | 358 358 496 527 | 28.2 28 28.3 | 1076 1316 1071 1021 | 14.10 14.50 11.73 15.37 |
| 400 to 500 | 17.7 and 30.5 × 35.4 | 110 | 135 135 | 577 554 | 26.4 26.4 | 519 347 | 10.80* 10.35* |
| 1000 to 1200 | 26.9 and 47.2 × 66.9 | 65 | 127 127 128 | 655 664 572 | 27.2 27.2 27.1 | 788 797 788 | 9.91* 9.68* 10.70* |

TRIPLE-EXPANSION ENGINES.

| | | | | | | | |
|------|-------------------------|------|------------|------------|--------------|--------------|---------------|
| 3000 | 32 1/4, 47 1/4, 58 × 59 | 85 | 188 190 | 606 397 | 28 27 1/4 | 2860 2880 | 8.97 11.28 |
| 3000 | 34, 49, 61 × 51 | 83.5 | 189 196 | 613 381 | 27 26 1/4 | 2908 3040 | 9.41 11.57 |

* With intermediate superheating. Temperature of steam at entrance to l.p. cylinder, 307 to 349° F.

Steam Consumption of Different Types of Engines.

Tests of a Ridgway 4-valve non-condensing engine, 19 × 18 in., at 200 r.p.m. and 100 lbs. pressure, are reported in *Power*, June, 1909, as follows:

| | | | | | |
|----------------------------|------|------|------|------|------|
| Load..... | 1/4 | 1/2 | 3/4 | Full | 11/4 |
| Steam per I.H.P. hour..... | 30.7 | 24.4 | 23.2 | 23.8 | 25.4 |

The best result obtained at 130 lbs. pressure was 21.6 lbs., at 115 lbs. pressure 22.6 lbs., and at 85 lbs. pressure 24.3 lbs. Maintained economy

In this type of engine is dependent upon reduction of unnecessary over-travel, properly fitted valves, valves which do not span a wide arc, close approach of the movement of the valves to that of a Corliss engine, and good materials.

The probable steam consumption of condensing engines of different types with different pressures of steam is given in a set of curves by R. H. Thurston and L. L. Brinsmade, *Trans. A. S. M. E.*, 1897, from which curves the following approximate figures are derived.

| | Steam pressure, absolute, lbs. per sq. in. | | | | | | | |
|-----------------|--|-------|-------|-------|-------|-------|-------|------|
| | 400 | 300 | 250 | 200 | 150 | 100 | 75 | 50 |
| Ideal Engine | | | | | | | | |
| (Rankine cycle) | 6.95 | 7.5 | 7.9 | 8.45 | 9.20 | 10.50 | 11.40 | 12.9 |
| Quadruple Exp. | | | | | | | | |
| Wastes 20% | 8.75 | 9.15 | 9.75 | 10.50 | 11.60 | 13.0 | 14.0 | 15.6 |
| Triple Exp. | | | | | | | | |
| Wastes 25% | 9.25 | 9.95 | 10.50 | 11.15 | 12.30 | 14.0 | 15.1 | 16.7 |
| Compound. | | | | | | | | |
| Wastes 33% | 10.50 | 11.25 | 11.80 | 12.70 | 13.90 | 15.6 | 16.9 | 18.9 |
| Simple Engine. | | | | | | | | |
| Wastes 50% | 14.00 | 15.00 | 15.80 | 16.80 | 18.40 | 20.4 | 22.7 | 25.2 |

The same authors give the records of tests of a three-cylinder engine at Cornell University, cylinders 9, 16 and 24 ins., 36-in. stroke, first as a triple-expansion engine; second, with the intermediate cylinder omitted, making a compound engine with a cylinder ratio of 7 to 1 and third, omitting the third cylinder, making a compound engine with a ratio of a little over 3 to 1. The boiler pressure in the first case was 119 lbs., in the second 115, and in the third 117 lbs. Charts are given showing the steam consumption per I.H.P. and per B.H.P. at different loads, from which the following figures are taken.

Indicated Horse-Power..... 40 60 80 100 110 120 130

Steam consumption per I.H.P. per hour.

| | | | | | | | |
|-------------------|------|------|------|------|------|------|------|
| Triple Exp..... | 19.1 | 16.7 | 15.3 | 14.2 | 13.7 | 13.8 | 14.4 |
| Comp. 7 to 1..... | 19.6 | 18.2 | 17.0 | 16.3 | 16. | 15.8 | 15.8 |
| Comp. 3 to 1..... | 19.7 | 18.4 | 18.1 | 18.5 | | | |

Steam consumption per B.H.P. hour.

| | | | | | | | |
|-------------------|------|------|------|------|------|------|------|
| Triple Exp..... | 30.5 | 23.0 | 19.6 | 17.1 | 16.2 | 16.2 | 16.7 |
| Comp. 7 to 1..... | 26.2 | 21.7 | 19.3 | 18.7 | 18.5 | 18.4 | 18.5 |
| Comp. 3 to 1..... | 23.4 | 20.6 | 20. | 20. | | | |

The most economical performance was as follows:

| | Triple | Comp. 7 to 1 | Comp. 3 to 1 |
|----------------------------|--------|--------------|--------------|
| Indicated Horse-Power..... | 112.7 | 130.0 | 67.7 |
| Steam per I.H.P. hour..... | 13.68 | 15.8 | 18.03 |

A test of a 7500-H.P. engine, at the 59th St. Station of the Interborough Rapid Transit Co., New York, is reported in *Power*, Feb., 1906. It is a double cross compound engine, with horizontal h.p. and vertical l.p. cylinders. With steam at 175 lbs. gauge and vacuum 25.02 ins., 75 r.p.m. it developed 7365 I.H.P., 5079 K.W. at switchboard. Friction and electrical losses 417.3 K.W. Dry steam per K.W. hour 17.34 lbs.; per I.H.P. hour, 11.96 lbs.

A test of a Fleming 4-valve engine, 15 and 40.5 in. diam., 27-in. stroke, positive-driven Corliss valves, fly-wheel governor, is reported by B. T. Allen in *Trans. A. S. M. E.*, 1903. The following results were obtained. The speed was above 150 r.p.m. and the vacuum 26 in.

| | | | | | |
|----------------------------------|-------|-------|-------|-----------|-------|
| Fraction of full load about..... | 1/6 | 5/8 | 7/10 | Full load | 1.1 |
| Horse-power..... | 87.1 | 321.5 | 348.3 | 501.6 | 553.5 |
| Steam per I.H.P. hour..... | 14.42 | 13.59 | 12.33 | 12.66 | 12.7 |

Relative Economy of Compound Non-condensing Engines under Variable Loads. — F. M. Rites, in a paper on the Steam Distribution in a Form of Single-acting Engine (*Trans. A. S. M. E.*, xiii, 537), discusses an engine designed to meet the following problem: Given an extreme range of conditions as to load or steam-pressure, either or both, to fluctuate together or apart, violently or with easy gradations, to construct an engine whose economical performance should be as good as though the engine were specially designed for a momentary condition — the adjustment to be complete and automatic. In the ordinary non-condensing compound engine with light loads the high-pressure cylinder is frequently forced to supply all the power and in addition drag along with it the low-pressure piston, whose cylinder indicates negative work. Mr. Rites shows the peculiar value of a receiver of predetermined volume which acts as a clearance chamber for compression in the high-pressure cylinder. The Westinghouse compound single-acting engine is designed upon this principle. The following results of tests of one of these engines rated at 175 H.P. for most economical load are given:

WATER RATES UNDER VARYING LOADS, LBS. PER H.P. PER HOUR.

| | | | | | | | |
|---------------------|------|------|------|------|------|------|------|
| Horse-power..... | 210 | 170 | 140 | 115 | 100 | 80 | 50 |
| Non-condensing..... | 22.6 | 21.9 | 22.2 | 22.2 | 22.4 | 24.6 | 28.8 |
| Condensing..... | 18.4 | 18.1 | 18.2 | 18.2 | 18.3 | 18.3 | 20.4 |

Efficiency of Non-condensing Compound Engines. (W. Lee Church, *Am. Mach.*, Nov. 19, 1891.) — The compound engine, non-condensing, at its best performance will exhaust from the low-pressure cylinder at a pressure 2 to 6 pounds above atmosphere. Such an engine will be limited in its economy to a very short range of power, for the reason that its valve-motion will not permit of any great increase beyond its rated power, and any material decrease below its rated power at once brings the expansion curve in the low-pressure cylinder below atmosphere. In other words, decrease of load tells upon the compound engine somewhat sooner, and much more severely, than upon the non-compound engine. The loss commences the moment the expansion line crosses a line parallel to the atmospheric line, and at a distance above it representing the mean effective pressure necessary to carry the frictional load of the engine. When expansion falls to this point the low-pressure cylinder becomes an air-pump over more or less of its stroke, the power to drive which must come from the high-pressure cylinder alone. Under the light loads common in many industries the low-pressure cylinder is thus a positive resistance for the greater portion of its stroke. A careful study of this problem revealed the functions of a fixed intermediate clearance, always in communication with the high-pressure cylinder, and having a volume bearing the same ratio to that of the high-pressure cylinder that the high-pressure cylinder bears to the low-pressure. Engines laid down on these lines have fully confirmed the judgment of the designers. The effect of this constant clearance is to supply sufficient steam to the low-pressure cylinder under light loads to hold its expansion curve up to atmosphere, and at the same time leave a sufficient clearance volume in the high-pressure cylinder to permit of governing the engine on its compression under light loads.

Tests of two non-condensing Corliss engines by G. H. Barrus are reported in *Power*, April 27, 1909. The engines were built by Rice & Sargent. One is a simple engine 22 × 30, and the other a tandem compound 22 and 36 × 36 ins. Both engines are jacketed in both heads, and the compound engine has a reheating receiver with 0.6 sq. ft. of brass pipes per rated H.P. (600). The guarantees were: compound engine, not to exceed 19 lbs. of steam per I.H.P. per hour, with 130 lbs. steam pressure and 1 lb. back pressure in the exhaust pipe, and the simple engine not to exceed 23 lbs. The friction load, engine run with the brushes off the generator and the field not excited, was not to exceed 4½ H.P. in either engine. The results were: compound engine, 99.2 r.p.m.; 608.3 H.P.; 18.33 lbs. steam per I.H.P. per hour; friction load 3.8% of 600 H.P.; simple engine, 98.5 r.p.m.; 306.2 I.H.P.; 20.98 lbs. per I.H.P. per hour; friction 3.6% of 300 H.P.

A single-cylinder engine 12 × 12 ins., made by the Buffalo Forge Co., was tested by Profs. Reeve and Allen. *El. World*, May 23, 1903. Some of the results were:

I.H.P. 16.39 37.20 56.00 69.00 74.10 81.4 89.3 125.9* 86.42†
 Water-rate 52.3 35.3 33.3 31.9 30.6 34.6 33.1 27.6 27.5

* Steam pressure 125 lbs. gauge, all the other tests 80 lbs. † Condensing, other tests all non-condensing.

Effect of Water contained in Steam on the Efficiency of the Steam-engine. (From a lecture by Walter C. Kerr, before the Franklin Institute, 1891.) — Standard writers make little mention of the effect of entrained moisture on the expansive properties of steam, but by common consent rather than any demonstration they seem to agree that moisture produces an ill effect simply proportional to the percentage amount of its presence. That is, 5% moisture will increase the water rate of an engine 5%.

Experiments reported in 1893 by R. C. Carpenter and L. S. Marks, *Trans. A. S. M. E.*, xv, in which water in varying quantity was introduced into the steam-pipe, causing the quality of the steam to range from 99% to 58% dry, showed that throughout the range of qualities used the consumption of dry steam per indicated horse-power per hour remains practically constant, and indicated that the water was an inert quantity, doing neither good nor harm.

Influence of Vacuum and Superheat on Steam Consumption. (*Eng. Digest*, Mar., 1909.) — Herr Roginsky ("Die Turbine") discusses the economies effected by the use of superheat and high vacuums.

In a certain triple-expansion engine, working under good average conditions, there was found a saving of approximately 6% for each 10% increase in vacuum beyond 50%.

The Batulli-Tumlirz formula for superheated steam is: $p(v + a) = RT$, in which p = steam pressure in kgs. per sq. meter, v = cubic meters in 1 kg. of superheated steam at pressure p , $a = 0.0084$, $R = 46.7$, and T = absolute temperature in deg. C.

Using this expression, it is found that, neglecting the fuel used for superheating, for each 10° C. of superheat at pressures ranging from 100 to 185 lbs. per sq. in. there is an average increase of volume of 2.8%. The work done by the expansion of superheated steam, as shown by diagrams, is about 1.6% less for 10° of superheating, so that the net saving for each 10° of superheat is $2.8 - 1.6 = 1.2\%$, approx. (0.66% for each 10° F.).

Rateau's formula for the steam consumption (K) per H.P.-hr. of an ideal steam turbine, in which the steam expands from pressure p_1 to p_2 , is

$$K = 0.85 (6.95 - 0.92 \log p_2) / (\log p_1 - \log p_2),$$

K being in kilograms and p_1 and p_2 in kgs. per sq. meter. From this formula the following table is calculated, the values being transformed into British units.

| p_1 Lbs. per sq. in. | Lbs. Steam at 50% Vacuum. | Reduction of Steam Consumption (%) by using a Vacuum of | | | | |
|------------------------------|---------------------------------|--|------|------|------|------|
| | | 60% | 70% | 80% | 90% | 95% |
| 184.9 | 11.11 | 5. | 11.1 | 13.1 | 27.8 | 34.6 |
| 156.5 | 11.75 | 5.8 | 11.8 | 19.3 | 28.8 | 36.4 |
| 128 | 12.57 | 6.6 | 12.9 | 20.5 | 29.8 | 38.5 |
| 99.6 | 13.84 | 7.6 | 14.4 | 22. | 33.3 | 40.6 |

From the entropy diagram it is seen that in expanding from pressures in excess of 100 lbs. per sq. in. down to 1.42 lbs. absolute, approximately 1% more work is performed for every 10° F. of superheat. The effect of increasing the degree of vacuum is summed up in the following table:

| Increasing the Vacuum from | Decreases Steam Consumption | |
|----------------------------------|------------------------------|-----------------------|
| | in Reciprocating Engines. | in Steam Turbines. |
| 50% to 60% | 5.8% | 6.2% |
| 50% to 70% | 11.6% | 12.6% |
| 50% to 80% | 17.3% | 20.0% |
| 50% to 90% | 23.1% | 30.1% |
| 50% to 95% | 26.0% | 37.4% |

In the last case (from 50% to 95%) the decrease in steam consumption is 44% greater for a steam turbine than for a reciprocating engine.

The following results of tests of a compound engine using superheated steam are reported in *Power*, Aug., 1905. The cylinders were 21 and 36 × 36 ins. The steam pressure was about 117 lbs. gauge. R.p.m. 100, vacuum 26.5 ins.

| Test No. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---------|-------|-------|-------|-------|-------|
| Indicated H.P. | 481 | 461 | 347 | 145 | 333 | 258 |
| Superheat of steam entering h.p. cyl. | 253° F. | 242° | 221° | 202° | 232° | 210° |
| B.T.U. supplied per I.H.P. per min. | 198.2 | 201.7 | 197.6 | 192.1 | 194.0 | 194.0 |
| B.T.U. theoretically required. Rankine cycle. | 142.4 | 142.5 | 130.2 | 128.0 | 126.0 | 128.5 |
| Efficiency ratio | 0.72 | 0.71 | 0.66 | 0.67 | 0.65 | 0.66 |
| Thermal efficiency % | 21.39 | 21.02 | 21.46 | 22.07 | 21.86 | 21.86 |
| Lbs. steam per I.H.P. hour | 9.098 | 9.267 | 8.886 | 8.585 | 8.682 | 8.742 |

The Practical Application of Superheated Steam is discussed in a paper by G. A. Hutchinson in *Trans. A. S. M. E.*, 1901. Many different forms of superheater are illustrated.

Some results of tests on a 3000-H.P., four-cylinder, vertical, triple-expansion Sulzer engine, using steam from Schmidt independently fired superheaters, are as follows. (*Eng. Rec.*, Oct. 13, 1900.)

| Tests Using Steam. | Highly Superheated. | | | Moderately Superheated | Saturated. | |
|---|---------------------|-------|-------|---------------------------|------------|-------|
| | | | | | | |
| Initial pressure in h.p. cyl. (absolute), lbs. | 187.3 | 195.5 | 188.4 | 190.3 | 194.6 | 195.9 |
| Temp. of steam in valve chest, deg. F. | 582 | 585 | 614 | 531 | 381 | 381 |
| Total I.H.P. | 2,900 | 2,779 | 2,868 | 2,850 | 2,951 | 2,999 |
| Lbs. steam per I.H.P. hour | 9.64 | 9.67 | 9.56 | 10.29 | 11.77 | 11.75 |
| Watt hours per lb. of coal. | 477 | 482 | 479 | 447 | 438 | 435 |

The saving due to the use of highly superheated steam is (482-438) ÷ 482 = 9.1%.

Tests of a 4000-H.P. double-compound engine (Van den Kerchove, of Brussels) with superheated steam are reported in *Power*, Dec. 29, 1908. The cylinders are 34 1/4 and 60 ins., stroke 5 ft. Ratio of areas 2.97. The following are the principal results, the first figures given being for the full-load test, and the second (in parentheses) for the half-load test. Steam

pressure at drier, 136.5 lbs. (137.9). R.p.m. 84.3 (84.06). Temp. of steam entering engine 519° F. (498), leaving l.p. cyl. 121.5° (121.5). Vacuum in condenser, ins., 27.5 (27). I.H.P. 3776 (2019). Steam per I.H.P. hour, lbs., 9.62 (9.60).

The saving due to the use of superheated steam is reported in numerous tests as being all the way from less than 10% to more than 40%. The greater saving is usually found with engines that are the most inefficient with saturated steam, such as single-cylinder engines with light loads, in which the cylinder condensation is excessive.

R. P. Bolton (*Eng. Mag.*, May, 1907) states that tests of superheated steam in locomotives, by the Prussian Railway authorities in 1904, with 50°, 104° and 158° F. superheat, showed a saving of water respectively of 2.5, 10 and 16%, and a saving of coal of 2, 7 and 12%. Mr. Bolton's paper concludes with a long list of references on the subject of superheated steam. A paper by J. R. Bibbins in *Elec. Jour.*, March, 1906, gives a series of charts showing the saving made by different degrees of superheating in different types of engines, including steam turbines.

For description of the Foster superheater, see catalogue of the Power Specialty Co., New York.

The Wolf (French) semi-portable compound engine of 40 H.P. with superheater and reheater, the engine being mounted on the boiler, is reported by R. E. Mathot, *Power*, July, 1906, to have given a steam consumption as low as 9.9 lbs. per I.H.P. hour, and 10.98 lbs. per B.H.P. hour. The steam pressure in the boiler was 172.6 lbs., and was superheated initially to 657° F., and reheated to 361° before entering the l.p. cylinder. This is a remarkable record for a small engine.

A test of a Rice & Sargent cross-compound horizontal engine 16 and 28 X 42 ins., with superheated steam, is reported by D. S. Jacobus in *Trans. A. S. M. E.*, 1904. The steam pressure at the throttle was 140 lbs. gauge, the superheating was 350 to 400°, and the vacuum 25 to 26 ins., r.p.m. 102. In three tests with superheated and one with saturated steam the results were:

| | | | | |
|--|-------|-------|-------|-------|
| I.H.P. developed | 474.5 | 420.4 | 276.8 | 406.7 |
| Water consumption per I.H.P. hour..... | 9.76 | 9.56 | 9.70 | 13.84 |
| Coal consumption per I.H.P. hour..... | 1.265 | 1.257 | 1.288 | 1.497 |
| B.T.U. per min. per I.H.P..... | 205.0 | 203.7 | 208.8 | 248.2 |
| Temp. of steam entering h.p. cyl..... | 634 | 659 | 672 | |
| Temp. of steam leaving h.p. cyl..... | 346 | 331 | 288 | 262 |
| Temp. of steam entering l.p. cyl..... | 408 | 396 | 354 | 269 |
| Temp. of steam leaving l.p. cyl..... | 135 | 141 | 117 | |

Performance of a Quadruple Engine.—O. P. Hood (*Trans. A. S. M. E.*, 1906) describes a test of a high-duty air compressor, with four steam cylinders, 14.5, 22, 38 and 54 in. diam., 48-in. stroke. The clearances were respectively 6, 5.7, 4.4 and 3.5%. R.p.m. 57. Steam pressure, gauge, near throttle, 242.8 lbs., in 1st. receiver 120.7 lbs., in 2d, 30.8 lbs., in 3d, vac., - 1.24 ins. Moisture in steam near throttle, 5.74%. Steam in No. 1 receiver, dry; in No. 2, 17° superheat; in No. 3, 9° superheat. The engine has poppet valves on the h.p. cylinder and Corliss valves on the other cylinders. The feed-water heaters are four in number, in series, on the Nordberg system; No. 1 receives its steam from the exhaust of No. 4 cylinder; No. 2 from the jacket of No. 4 cyl.; No. 3 from the jackets of No. 3 cylinder and No. 3 reheater; No. 4 from the jacket of No. 2 cylinder. The reheaters are supplied with steam from the boilers. The temperatures of steam and water were as follows: Temperatures of steam: Fed to No. 1 engine, 403°; leaving receivers, No. 1, 351°; No. 2, 291°; No. 3, 216°. Exhaust entering preheater, 114°. Temperature corresponding to condenser pressure, 109.6°. Temperatures of water: Fed to preheater, 93°; fed to heaters, No. 1, 114°; No. 2, 173°; No. 3, 202°; No. 4, 269°; leaving heater No. 4 as boiler feed, 334°. Mr. Hood gives a diagram showing graphically the transfer of heat through the several parts of the apparatus, from which the following is taken. The figures are in B.T.U. transferred per minute,

| | Received from Boiler or Receiv'rs. | Received from Jackets. | Convert- ed into Work. | Delivered to Heater. | Delivered to Jackets. |
|-------------------------|---|------------------------------|------------------------------|----------------------------|-----------------------------|
| No. 1 Cylinder..... | 194,183 | 862 | 7,697 | | |
| No. 1 Receiver..... | 187,348 | 6,624 | | 17,100 | 2,000 |
| No. 2 Cylinder..... | 174,872 | 2,000 | 10,899 | | |
| No. 2 Receiver..... | 165,973 | 8,060 | | 12,800 | 1,150 |
| No. 3 Cylinder..... | 160,083 | 1,150 | 11,695 | | |
| No. 3 Receiver..... | 149,538 | 5,185 | | 5,100 | 940 |
| No. 4 Cylinder..... | 148,683 | 940 | 11,688 | 9,100 | |
| Preheater..... | 128,835 | | | 2,350 | |
| Del'd to Condenser..... | 125,885 | | | 5,600 | |
| Disch'gd from "..... | 120,285 | | | | |

The principal results of the test are as follows:

| Cylinder..... | 1 | 2 | 3 | 4 |
|---|--------|--------|--------|--------|
| I.H.P. developed in steam cylinders..... | 181.47 | 256.96 | 275.71 | 275.56 |
| I.H.P. used in the cylinders..... | 220.04 | 222.12 | 226.20 | 214.84 |
| Total indicated horse-power, steam cylinders..... | | | | 989.7 |
| Total horse-power used in air cylinders..... | | | | 883.2 |
| Total indicated horse-power of auxiliaries..... | | | | 11.0 |
| Horse-power representing friction of the machine..... | | | | 95.5 |
| Per cent of friction..... | | | | 9.65% |
| Mechanical efficiency engine and compressor..... | | | | 90.35% |

Heat consumed by engine per hour per I.H.P., 10,157 B.T.U.; per B.H.P., 11,382 B.T.U. Equivalent standard coal consumption per hour assuming 10,000 B.T.U. imparted to the boiler per pound coal, per I.H.P., 1.016 lbs.; per B.H.P., 1.138 lbs. Dry steam per hour per I.H.P., 11.23 lbs.; per B.H.P., 12.58 lbs. Heat units consumed per minute, per I.H.P., 169.29 B.T.U.; per B.H.P., 189.70 B.T.U.

| | |
|---|-------------|
| Efficiency of Carnot cycle between the temperature of incoming steam and that corresponding to pressure in the condenser..... | 34.0 % |
| Actual heat efficiency attained by this engine..... | 25.05 % |
| Relative efficiency compared with Carnot cycle..... | 73.69 % |
| Relative efficiency compared with Rankine cycle..... | 88.2 % |
| Duty, ft.-lbs. per million B.T.U. supplied..... | 194,930,000 |

This engine establishes a new low record for the heat consumed per hour per I.H.P., being 9% lower than that used by the Wildwood pumping engine reported in 1900. (See Pumping Engines.)

The Use of Reheaters in the receivers of multiple-expansion engines is discussed by R. H. Thurston in *Trans. A. S. M. E.*, xxi, 893. He shows that such receivers improve the economy of an engine very little unless they are also superheaters; in which case marked economy may be effected by the reduction of cylinder condensation. The larger the amount of cylinder condensation and the greater the losses, exterior and interior, the greater the effect of any given amount of superheating. The same statement will hold of the use of reheaters: the more wasteful the engine without them and the more effectively they superheat, the larger the gain by their use. A reheater should be given such area of heating surface as will insure at least moderate superheating.

Influence of the Steam-jacket. — Tests of numerous engines with and without steam-jackets show an exceeding diversity of results, ranging all the way from 30% saving down to zero, or even in some cases showing an actual loss. The opinions of engineers at this date (1894) is also as diverse as the results, but there is a tendency towards a general belief that the jacket is not as valuable an appendage to an engine as was formerly supposed. An extensive *résumé* of facts and opinions on the steam-jacket is given by Prof. Thurston in *Trans. A. S. M. E.*, xiv, 462. See

also *Trans. A. S. M. E.*, xiv, 873 and 1340; xiii, 176; xii, 426 and 1340; and *Jour. F. I.*, April, 1891, p. 276. The following are a few statements selected from these papers.

The results of tests reported by the research committee on steam-jackets appointed by the British Institution of Mechanical Engineers in 1886, indicate an increased efficiency due to the use of the steam-jacket of from 1% to over 30%, according to varying circumstances.

Sennett asserts that "it has been abundantly proved that steam-jackets are not only advisable but absolutely necessary, in order that high rates of expansion may be efficiently carried out and the greatest possible economy of heat attained."

Isherwood finds the gain by its use, under the conditions of ordinary practice, as a general average, to be about 20% on small and 8% or 9% on large engines, varying through intermediate values with intermediate sizes, it being understood that the jacket has an effective circulation, and that both heads and sides are jacketed.

Professor Unwin considers that "in all cases and on all cylinders the jacket is useful; provided, of course, ordinary, not superheated, steam is used; but the advantages may diminish to an amount not worth the interest on extra cost."

Professor Cotterill says: Experience shows that a steam-jacket is advantageous, but the amount to be gained will vary according to circumstances. In many cases it may be that the advantage is small. Great caution is necessary in drawing conclusions from any special set of experiments on the influence of jacketing.

Mr. E. D. Leavitt has expressed the opinion that, in his practice, steam-jackets produce an increase of efficiency of from 15% to 20%.

In the Pawtucket pumping-engine, 15 and 30 1/8 X 30 in., 50 revs. per min., steam-pressure 125 lbs. gauge, cut-off 1/4 in h.p. and 1/3 in l.p. cylinder, the barrels only jacketed, the saving by the jackets was from 1% to 4%.

The superintendent of the Holly Mfg. Co. (compound pumping-engines) says: "In regard to the benefits derived from steam-jackets on our steam-cylinders, I am somewhat of a skeptic. From data taken on our own engines and tests made I am yet to be convinced that there is any practical value in the steam-jacket."

Professor Schröoter from his work on the triple-expansion engines at Augsburg, and from the results of his tests of the jacket efficiency on a small engine of the Sulzer type in his own laboratory, concludes: (1) The value of the jacket may vary within very wide limits, or even become negative. (2) The shorter the cut-off the greater the gain by the use of a jacket. (3) The use of higher pressure in the jacket than in the cylinder produces an advantage. The greater this difference the better. (4) The high-pressure cylinder may be left unjacketed without great loss, but the other should always be jacketed.

The test of the Laketon triple-expansion pumping-engine showed a gain of 8.3% by the use of the jackets, but Prof. Denton points out (*Trans. A. S. M. E.*, xiv, 1412) that all but 1.9% of the gain was ascribable to the greater range of expansion used with the jackets.

Test of a Compound Condensing Engine with and without Jackets at different Loads. (R. C. Carpenter, *Trans. A. S. M. E.*, xiv, 428.) — Cylinders 9 and 16 in. X 14 in. stroke; 112 lbs. boiler-pressure; rated capacity 100 H.P.; 265 revs. per min. Vacuum, 23 in. From the results of several tests curves are plotted, from which the following principal figures are taken.

| Indicated H.P. | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 125 |
|----------------------------|------|------|------|------|------|------|------|------|------|------|------|
| Steam per I.H.P. per hr. | | | | | | | | | | | |
| With jackets, lbs. | 22.6 | 21.4 | 20.3 | 19.6 | 19 | 18.7 | 18.6 | 18.9 | 19.5 | 20.4 | 21.0 |
| Without jackets, lbs. | | | | 22 | 20.5 | 19.6 | 19.2 | 19.1 | 19.3 | 20.1 | |
| Saving by jacket, % | | | | 10.9 | 7.3 | 4.6 | 3.1 | 1.0 | -1.0 | -1.5 | |

This table gives a clue to the great variation in the apparent saving due to the steam-jacket as reported by different experimenters. With this

particular engine it appears that when running at its most economical rate of 100 H.P., without jackets, very little saving is made by use of the jackets. When running light the jacket makes a considerable saving, but when overloaded it is a detriment.

At the load which corresponds to the most economical rate, with no steam in jackets, or 100 H.P., the use of the jacket makes a saving of only 1%; but at a load of 60 H.P. the saving by use of the jacket is about 11%, and the shape of the curve indicates that the relative advantage of the jacket would be still greater at lighter loads than 60 H.P.

The Best Economy of the Piston Steam Engine at the Advent of the Steam Turbine is the subject of a paper by J. E. Denton at the International Congress of Arts and Sciences, St. Louis, 1904. (*Power*, Oct. 26, 1905.) Prof. Denton says:

During the last two years the following records have been established:

(1) With an 850-H.P. Rice & Sargent compound Corliss engine, running at 120 r.p.m., having a 4 to 1 cylinder ratio, clearances of 4% and 7%, live jackets on cylinder heads and live steam in reheater, Prof. Jacobus found for 600 H.P. of load, with 150 lbs. saturated steam, 23.6 ins. vacuum, and 33 expansions, 12.1 lbs. of water per I.H.P., with a cylinder-condensation loss of 22%, and a jacket consumption of 10.7% of the total steam consumption.

(2) With a 250-H.P. Belgian poppet-valve compound engine, 126 r.p.m., with 2.97 to 1 cylinder ratio, clearances of 4%, steam-chest jackets on barrels and head, and no reheater, Prof. Schröter, of Munich, found with 117 H.P. of load, 130 lbs. saturated steam, 27.6 ins. of vacuum, and 32 expansions, 11.98 lbs. of water per H.P. per hour, with a cylinder-condensation loss of 23.5%, and a jacket consumption of 7% of the total steam consumption in the high cylinder jacket and 7% in the low jacket.

(3) With the Westinghouse twin compound combined poppet-valve and Corliss-valve engine, at the New York Edison plant, running 76 r.p.m., with 5.8 to 1 cylinder ratio, clearances of 10.5% and 4%, without jackets or reheater, Messrs. Andrew, Whitham and Wells found for the full load of 5400 H.P., 185 lbs. steam pressure, 27.3 ins. vacuum, and 29 expansions, 11.93 lbs. of water per I.H.P. per hour, with an initial condensation of about 32%.

These facts show that the minimum water consumption of the compound engine of the present date, using saturated steam, is not dependent upon any particular cylinder ratio and clearance nor upon any system of jacketing, but that the essential condition is the use of a ratio of expansion of about 30, above which the cylinder-condensation loss is liable to prevail over the influence of the law of expansion. The conclusion appears warranted, therefore, that if this ratio of expansion is secured with any of the current cylinder and clearance ratios, and with any existing system of jackets and reheaters, or without them, a water consumption of 12.4 lbs. per horse-power is possible, and that a variation of 0.4 lb. below or above this figure may occur by the accidental favorable, or unfavorable, jacket and cylinder-wall expenses which are beyond the exact control of the designer.

Compound Piston Engine Economy vs. that of Steam Turbine. — In order to compare the economy of the piston engine with that of the steam turbine, we must use the water consumption per brake horse-power, since no indicator card is possible from the turbine; and furthermore, we must use the average water consumption for the range of loads to which engines are subject in practice.

In all of the public turbine tests to date, with one exception the output was measured through the electric power of a dynamo whose efficiency is not given for the range of loading employed, so that the average brake horse-power is not known. This exception is the Dean and Main test of a 600-H.P. Westinghouse-Parsons turbine using saturated steam at 150 lbs. pressure, and a 28-in. vacuum. We may compare the results of this test with that of the 850-H.P. Rice & Sargent and of the 250-H.P. Belgian engine, by assuming that the power absorbed by friction in these engines is 3% of the indicated load plus the power shown by friction cards taken with the engine unloaded. The latter showed 5% of the rated power in the R. & S. engine and 8% in the Belgian engine. The results are:

| Per cent of full load..... | 41 | 75 | 100 | 125 | Avg. 85% |
|------------------------------|---------------------------------|-------|-------|-------|----------|
| | Lbs. Water per Brake H.P. Hour. | | | | |
| 600-H.P. Turbine..... | 13.62 | 13.91 | 14.48 | 16.05 | 14.51 |
| 800-H.P. Comp. Engine..... | 13.78 | 13.44 | 13.66 | 17.36 | 14.56 |
| 250 H.P. Belgian Engine..... | 15.10 | 14.15 | 13.99 | 15.31 | 14.64 |

These figures show practical equality in economy of the types of engines. The full report of the Van den Kerchove Belgian engine is given in *Power*, June, 1903.

For large-sized units Prof. Denton compares the Elberfeld test of a Parsons turbine at the full load of 1500 electric H.P., allowing 5% for attached air pump, 95% for generator efficiency, with the 5400-H.P. Westinghouse compound engine at the New York Edison station, whose friction at full load was found to be 4%. The turbine with 150 lbs. steam and 28 ins. vacuum required 13.08 lbs. of saturated steam per B.H.P. hour, a gain of 4% over the 600-H.P. turbine. The engine with 18.5 lbs. boiler pressure gave 12.5 lbs. per B.H.P. hour. Crediting the turbine with the possible influence of the difference in size and steam pressure, there is again practical equality in economy between it and the piston engine.

Triple-expansion Pumping Engines. — The triple-expansion engine has failed to supplant the compound for electric light and mill service, because the gain in fuel economy due to its use was not sufficient to overcome its higher first cost, depreciation, etc. It is, however, almost universally used in marine practice, and also in large-sized pumping engines. Prof. Denton says: Pumping engines in the United States have been developed in the triple-expansion fly-wheel type to a degree of economy superior to that afforded by any compound mill or electric engine, and, for saturated steam, superior to that of the pumping engines of any other country. This is because their slow speed permits of greater benefit from jackets and reheaters and of less losses from wire-drawing and back pressure. These causes, together with the greater subdivision of the range of expansion, have resulted in records made between 1894 and 1900 of 11.22, 11.26 and 11.05 lbs. of saturated steam per I.H.P., with 175 lbs. steam pressure and from 25 to 33 expansions, in the cases of the Leavitt, Snow and Allis pumping engines, respectively, the corresponding heat consumption being by different dispositions of the jacket drainage, 204, 208 and 212 thermal units per I.H.P. minute; while later the Allis pump, with 85 lbs. steam pressure, has lowered the record to 10.33 lbs. of saturated steam per I.H.P., with 196 B.T.U. per H.P. minute.

Gain from Superheating. — In the Belgian compound engine above described, with steam at 130 lbs., vacuum 27.6 ins., the average consumption of saturated steam, between 45 and 125% of load, was 12.45 lbs. per I.H.P. hour, or 225 B.T.U. per I.H.P. minute. With steam superheated 224° F. the average consumption for the same loads was 10.09 lbs. per I.H.P. hour, computed to be equivalent to 209 B.T.U. per H.P. minute, a gain due to superheating of 7%. With steam superheated 307° and the load about 80% of rating the water consumption was 8.99 lbs. per I.H.P. hour, equivalent to 192 B.T.U. per H.P. minute. The same load with saturated steam requires 221 B.T.U., showing a gain due to superheating of 13%.

The best performance reported for superheated steam used in the turbine is that of Brown & Boveri Parsons Frankfort 4000-H.P. machine, which, with 183 lbs. gauge pressure and 190° F. superheat, afforded 10.28 lbs. per B.H.P. hour, assuming a generator efficiency of 0.95. Reckoning from the feed temperature of its vacuum of 27.5 ins., the heat consumption is 214 B.T.U. per H.P. minute.

The heat consumption of the 250-H.P. Belgian compound engine per B.H.P. hour at the highest superheating of 307° F. is 220 B.T.U. The turbine, therefore, probably holds the record for brake horse-power economy over the piston engine for superheated steam by a margin of about 3%, although had the compound engine been of the same horse-power as the turbine, so that its friction load would be only 8% of its power instead of the 13% here allowed, it would have excelled the turbine in brake horse-power economy by a margin of about 2.5%.

The Sulphur-dioxide Addendum. — If the expansion in piston engines

could continue until the pressure of 1 pound was attained before exhaust occurred, considerable more work could be obtained from the steam. This cannot be done, for two reasons: first, because the low cylinder would have to be about five times greater in volume, which is commercially impracticable; and, second, because the velocity of exit through the largest exhaust ports possible is so great that the frictional resistance of the steam makes the back pressure from 1 to 3 pounds higher than the condenser pressure in the best engines of ordinary piston speed.

All the work due to this extra expansion can be obtained by exhausting the steam at 6 lbs. pressure against a nest of tubes containing sulphur dioxide which is thereby boiled to a vapor at about 170 lbs. pressure.

Professor Josse, of Berlin, has perfected this sulphur-dioxide system of improvement, and reliable tests have shown that if cooling water of 65° is available, and to the extent of about twice the quantity usually employed for condensing steam under 28 ins. of vacuum, a sulphur-dioxide cylinder of about half the size of the high-pressure cylinder of a compound engine will do sufficient work to improve the best economy of such engines at least 15%. The steam turbine expands its steam to the pressure of its exhaust chamber, and as unlimited escape ports can be provided from this chamber to a condenser, it follows that the turbine can practically expand its steam to the pressure of the condenser. Therefore a steam turbine attached to a piston engine to operate with the latter's exhaust should effect the same saving as the sulphur-dioxide cylinder.

Standard Dimensions of Direct-connected Generator Sets. From a report by a committee of the A. S. M. E., 1901.

| | | | | | | | |
|--------------------------------------|-------|-------|-------|-------|-------|-----|-----|
| Capacity of unit, K.W..... | 25 | 35 | 50 | 75 | 100 | 150 | 200 |
| Revolutions per minute..... | 310 | 300 | 290 | 275 | 260 | 225 | 200 |
| Armature bore, center crank engines. | 4 | 4 | 4 1/2 | 5 1/2 | 6 | 7 | 8 |
| Armature bore, side-crank engines... | 4 1/2 | 5 1/2 | 6 1/2 | 7 1/2 | 8 1/2 | 10 | 11 |

The diameter of the engine shaft at the armature fit is 0.001 in. greater than the bore, for bores up to and including 6 ins., and 0.002 in. greater for bores 6 1/2 ins. and larger.

Dimensions of Some Parts of Large Engines in Electric Plants. — The *Electrical World*, Sept. 27, 1902, gives a table of dimensions of the engines in the five large power stations in New York City at that date. The following figures are selected from the table.

| Name of station..... | Metro- politan. | Manhat- tan. | Kings- bridge. | Rapid Transit. | Edison. |
|--|--------------------------|---------------------------------------|--------------------------|---------------------------------------|------------------|
| Type of engine..... | Vert. Cross- Comp. | Double, 2 hor. 2 vert. Cyls. | Vert. Cross- Comp. | Double, 2 hor. 2 vert. Cyls. | 3 Cyl. Vert. |
| Rated H.P. | 4500 | 8000 | 4500 | 8900 | 5200 |
| Cylinders, all 60-in. stroke, in..... | 46, 86 | 44, 88 | 46, 86 | 42, 86 | 43 1/2, 2-75 1/2 |
| Piston rods, diam. in., | 9, 10 | 8 | 9, 10 | 8, 10 | 9 |
| Crank pins | 14 × 14 | 18 × 18 | 14 × 14 | 20 × 18 | 22 & 16 × 14 |
| Wrist pins | 14 × 14 | 12 × 12 | 14 × 14 | 12 × 12 | 14 × 14 |
| Shaft length..... | 27 ft. 4 in. | 25 ft. 3 in. | 27 ft. | 25 ft. 3 in. | 35 ft. |
| max. diam..... | 37 in. | 37 in. | 39 in. | 37 in. | 29 3/8 in. |
| bearings | 34 × 60 | 34 × 60 | 34 × 60 | 34 × 60 | 26 × 60 |

The shafts are hollow, with a 16-in. hole, except the Edison which has 10 in. The speed of all the engines is 75 r.p.m., or 750 ft. per min. The crank pins of the Manhattan and Rapid Transit engines each are attached to two connecting rods, side by side, hor. and vert., each rod having a bearing 9 in. long on the pin. The crank pins of the Edison engine are 16 in. diam. for the side-cranks, and 22 in. for the center-crank.

Some Large Rolling-Mill Engines.

| No. | Cylinders. | R.P.M. | Type. | Press. lbs. | Fly-wheel. | | Location. | Builders. |
|-----|------------------------|--------|-------------------|----------------|------------|---------|---|-----------------------------|
| | | | | | Diam. | Wt. | | |
| 1 | 44 & 82 × 60 | 65 | Cross-C. | 140 | 24 | 150,000 | Republic I. & S. Co., Youngstown, Ohio. | Filer & Stowell. |
| 2 | 46 & 80 × 60 | 80 | Tandem. | 150 | 24 | 110,000 | Carnegie S. Co., Donora, Pa. | Wisconsin Eng. Co. |
| 3 | 52 & 90 × 60 | | Tandem. | | 25 | 250,000 | Carnegie S. Co., Youngstown, Ohio. | Wm. Tod Co. |
| 4 | 2 each 42 & 70 × 54 | | Double Tandem. | 150 | none | | Carnegie S. Co., S. Sharon, Pa. | Allis Chalmers Co. |
| 5 | 2 each 44 & 70 × 60 | 60 | Double Tandem | 150 | none | | { Carnegie S. Co., Duquesne, Pa. Jones & Laughlin Steel Co., Allequippa, Pa. } | Mackintosh, Hemp-hill & Co. |

Some details: Main bearings, No. 1, 25 × 43½ in.; No. 2, 30 × 52 in.; No. 3, 30 × 60 in. Shaft diam. at wheel pit, No. 1, 26 in.; No. 3, 36 in. Crank pins, No. 1, h.p. 14 × 14; l.p., 14 × 23 in.; No. 2, 18 × 18 in. Crosshead pins, No. 1, 12 × 14; No. 2, 16 × 20 in. No. 4 is a reversing engine, with the Marshall gear. No. 5 is a reversing engine with piston valves below the cylinders.

Counterbalancing Engines. — Prof. Unwin gives the formula for counterbalancing vertical engines: $W_1 = W_2 r/p$, (1)
in which W_1 denotes the weight of the balance weight and p the radius to its center of gravity, W_2 the weight of the crank-pin and half the weight of the connecting-rod, and r the length of the crank. For horizontal engines:

$$W_1 = 2/3 (W_2 + W_3) r/p \text{ to } 3/4 (W_2 + W_3) r/p, (2)$$

in which W_3 denotes the weight of the piston, piston-rod, cross-head, and the other half of the weight of the connecting-rod.

The *American Machinist*, commenting on these formulæ, says: For horizontal engines formula (2) is often used; formula (1) will give a counterbalance too light for vertical engines. We should use formula (2) for computing the counterbalance for both horizontal and vertical engines, excepting locomotives, in which the counterbalance should be heavier.

For an account of experiments on counterbalancing large engines, with a method of recording vibrations, see paper by D. S. Jacobus, *Trans. A. S. M. E.*, 1905.

Preventing Vibrations of Engines. — Many suggestions have been made for remedying the vibration and noise attendant on the working of the big engines which are employed to run dynamos. A plan which has given great satisfaction is to build hair-felt into the foundations of the engine. An electric company has had a 90-horse-power engine removed from its foundations, which were then taken up to the depth of 4 feet. A layer of felt 5 inches thick was then placed on the foundations and run up 2 feet on all sides, and on the top of this the brickwork was built up. — *Safety Valve*.

Steam-engine Foundations Embedded in Air. — In the sugar-refinery of Claus Spreckels, at Philadelphia, Pa., the engines are distributed practically all over the buildings, a large proportion of them being on upper floors. Some are bolted to iron beams or girders, and are con-

sequently innocent of all foundation. Some of these engines ran noiselessly and satisfactorily, while others produced more or less vibration and rattle. To correct the latter the engineers suspended foundations from the bottoms of the engines, so that, in looking at them from the lower floors, they were literally hanging in the air. — *Iron Age*, Mar. 13, 1890.

COMMERCIAL ECONOMY. — COSTS OF POWER.

The Cost of Steam Power is an exceedingly variable quantity. The principal items to be considered in estimating total annual cost are: load factor; hours run per year; percentage of full load at different hours of the day; cost and quality of fuel; boiler efficiency and steam consumption of engines at different loads; cost of water and other supplies; cost of labor, first cost of plant, depreciation, repairs, interest, insurance and taxes.

In figuring depreciation not only should the probable life of the several parts of the plant, such as buildings, boilers, engines, condensers, etc., be considered, but also the possibility of part of the plant, or the whole of it, depreciating rapidly in value on account of obsolescence of the machinery or of changes in the conditions of the business.

When all of the heat in the exhaust steam from engines and pumps, including water of condensation, is used for heating purposes the fuel cost of steam-engine power may be practically nothing, since the exhaust contains all of the heat in the steam delivered to the engine except from 5 to 10 per cent which is converted into work, and a trifling amount lost by radiation.

Most Economical Point of Cut-off in Steam-engines. (See paper by Wolff and Denton, *Trans. A. S. M. E.*, vol. ii, p. 147-281; also, Ratio of Expansion at Maximum Efficiency, R. H. Thurston, vol. ii, p. 128.) — The problem of the best ratio of expansion is not one of economy of consumption of fuel and economy of cost of boiler alone. The question of interest on cost of engine, depreciation of value of engine, repairs of engine, etc., enters as well; for as we increase the rate of expansion, and thus, within certain limits fixed by the back-pressure and condensation of steam, decrease the amount of fuel required and cost of boiler per unit of work, we have to increase the dimensions of the cylinder and the size of the engine, to attain the required power. We thus increase the cost of the engine, etc., as we increase the rate of expansion, while at the same time we decrease the fuel consumption, the cost of boiler, etc. So that there is in every engine some point of cut-off, determinable by calculation and graphical construction, which will secure the greatest efficiency for a given expenditure of money, taking into consideration the cost of fuel, wages of engineer and firemen, interest on cost, depreciation of value, repairs to and insurance of boiler and engine, and oil, waste, etc., used for engine. In case of freight-carrying vessels, the value of the room occupied by fuel should be considered in estimating the cost of fuel.

Type of Engine to be used where Exhaust-steam is needed for Heating. — In many factories more or less of the steam exhausted from the engines is utilized for boiling, drying, heating, etc. Where all the exhaust-steam is so used the question of economical use of steam in the engine itself is eliminated, and the high-pressure simple engine is entirely suitable. Where only part of the exhaust-steam is used, and the quantity so used varies at different times, the question of adopting a simple, a condensing, or a compound engine becomes more complex. This problem is treated by C. T. Main in *Trans. A. S. M. E.*, vol. x, p. 48. He shows that the ratios of the volumes of the cylinders in compound engines should vary according to the amount of exhaust-steam that can be used for heating. A case is given in which three different pressures of steam are required or could be used, as in a worsted dye-house: the high or boiler pressure for the engine, an intermediate pressure for crabbing, and low-pressure for boiling, drying, etc. If it did not make too much complication of parts in the engine, the boiler-pressure might be used in the high-pressure cylinder, exhausting into a receiver from which steam could be taken for running small engines and crabbing, the steam remaining in the receiver passing into the intermediate cylinder and expanded there to from 5 to 10 lbs. above the atmosphere and exhausted into a second receiver. From this receiver is drawn the low-pressure steam needed for drying, boiling, warming mills, etc., the steam remaining in the receiver passing into the condensing cylinder.

Cost of Steam-power. (Chas. T. Main, *Trans. A. S. M. E.*, x., 48.)—Estimated costs in New England in 1888, per horse-power, based on engines of 1000 H.P.

| | Compound Engine. | Condens- ing Engine. | Non-con- densing Engine. |
|--|---------------------|-------------------------|--------------------------------|
| 1. Cost engine and piping, complete.... | \$25.00 | \$20.00 | \$17.50 |
| 2. Engine-house..... | 8.00 | 7.50 | 7.50 |
| 3. Engine foundations..... | 7.00 | 5.50 | 4.50 |
| 4. Total engine plant..... | 40.00 | 33.00 | 29.50 |
| 5. Depreciation, 4% on total cost..... | 1.60 | 1.32 | 1.18 |
| 6. Repairs, 2% on total cost..... | 0.80 | 0.66 | 0.59 |
| 7. Interest, 5% on total cost..... | 2.00 | 1.65 | 1.475 |
| 8. Taxation, 1.5% on $\frac{3}{4}$ cost..... | 0.45 | 0.371 | 0.332 |
| 9. Insurance on engine and house..... | 0.163 | 0.138 | 0.125 |
| 10. Total of lines 5, 6, 7, 8, 9..... | 5.015 | 4.139 | 3.702 |
| 11. Cost boilers, feed-pumps, etc..... | 9.33 | 13.33 | 16.00 |
| 12. Boiler-house..... | 2.92 | 4.17 | 5.00 |
| 13. Chimney and flues..... | 6.11 | 7.30 | 8.00 |
| 14. Total boiler-plant..... | 18.36 | 24.80 | 29.00 |
| 15. Depreciation, 5% on total cost..... | 0.918 | 1.240 | 1.450 |
| 16. Repairs, 2% on total cost..... | 0.367 | 0.496 | 0.580 |
| 17. Interest, 5% on total cost..... | 0.918 | 1.240 | 1.450 |
| 18. Taxation, 1.5% on $\frac{3}{4}$ cost..... | 0.207 | 0.279 | 0.326 |
| 19. Insurance, 0.5% on total cost..... | 0.092 | 0.124 | 0.145 |
| 20. Total of lines 15 to 19..... | 2.502 | 3.379 | 3.951 |
| 21. Coal used per I.H.P. per hour, lbs. ... | 1.75 | 2.50 | 3.00 |
| 22. Cost of coal per I.H.P. per day of 10 $\frac{1}{4}$ hours at \$5.00 per ton of 2240 lbs.... | cts. 4.00 | cts. 5.72 | cts. 6.86 |
| 23. Attendance of engine per day..... | 0.60 | 0.40 | 0.35 |
| 24. Attendance of boilers per day..... | 0.53 | 0.75 | 0.90 |
| 25. Oil, waste, and supplies, per day..... | 0.25 | 0.22 | 0.20 |
| 26. Total daily expense..... | 5.38 | 7.09 | 8.31 |
| 27. Yearly running expense, 308 days, per I.H.P..... | \$16.570 | \$21.837 | \$25.595 |
| 28. Total yearly expense, lines 10, 20, and 27..... | 24.087 | 29.355 | 33.248 |
| 29. Total yearly expense per I.H.P. for power if 50% of exhaust-steam is used for heating..... | 12.597 | 14.907 | 16.663 |
| 30. Total if all exhaust-steam is used for heating..... | 8.624 | 7.916 | 7.700 |

When exhaust-steam or a part of the receiver-steam is used for heating, or if part of the steam in a condensing engine is diverted from the condenser, and used for other purposes than power, the value of such steam should be deducted from the cost of the total amount of steam generated in order to arrive at the cost properly chargeable to power. The figures in lines 29 and 30 are based on an assumption made by Mr. Main of losses of heat amounting to 25% between the boiler and the exhaust-pipe, an allowance which is probably too large.

See also two papers by Chas. E. Emery on "Cost of Steam Power," *Trans. A. S. M. E.*, vol. xii, Nov., 1883, and *Trans. A. I. E. E.*, vol. x, Mar., 1893.

Decourcey May (*Trans. A. S. M. E.*, 1894) gives the following estimates

of the annual cost of power with different types of engine. He figures interest and depreciation each at 5%, insurance at 1%, and taxes at 1 1/2% of the cost of the plant. No cost of water is charged.

| Cost of coal per 2240 lbs. | \$2 | 3 | 4 | 5 | \$2 | 3 | 4 | 5 |
|--|-----------------------|-----|-----|-----|---------------------------|----|----|----|
| Cost of 1 I.H.P. per year. | 365 days of 24 hours. | | | | 308 days of 10 1/4 hours. | | | |
| Triple-expansion pumping, 20 revs..... | 48 | 55 | 61 | 67 | 31 | 33 | 35 | 37 |
| Triple-expansion without pumps, 50 revs..... | 27 | 33 | 39 | 45 | 16 | 18 | 20 | 22 |
| Compound mill, best engine | 29 | 36 | 44 | 51 | 17 | 19 | 21 | 24 |
| Compound mill, average... | 39 | 46 | 52 | 58 | 22 | 25 | 28 | 30 |
| Compound elec. light, av... | 122 | 139 | 157 | 174 | 78 | 84 | 90 | 96 |
| Compound trolley..... | 48 | 58 | 68 | 79 | 29 | 32 | 36 | 39 |
| Triple-expansion trolley... | 45 | 54 | 64 | 74 | 26 | 29 | 33 | 36 |
| Condensing mill..... | 44 | 52 | 61 | 69 | 25 | 29 | 33 | 38 |
| Non-cond., 50 to 200 H.P... | 70 | 76 | 81 | 88 | 49 | 53 | 57 | 62 |

Cost of Coal for Steam-power. — The following table shows the amount and the cost of coal per day and per year for various horse-powers, from 1 to 1000, based on the assumption of 4 lbs. of coal being used per hour per horse-power. It is useful, among other things, in estimating the saving that may be made in fuel by substituting more economical boilers and engines for those already in use. Thus with coal at \$3.00 per ton of 2000 lbs., a saving of \$9000 per year in fuel may be made by replacing a steam plant of 1000 H.P., requiring 4 lbs. of coal per hour per horse-power, with one requiring only 2 lbs.

| Horse-power. | Coal Consumption, at 4 lbs. per H.P. hour; 10 hours a day; 300 days per Year. | | | | | \$2 per Short Ton. | | \$3 per Short Ton. | | \$4 per Short Ton. | |
|--------------|---|-------------|-------------|----------------|-------------|--------------------------|---------------------|--------------------------|---------------------|--------------------------|---------------------|
| | Lbs. | Long Tons. | | Short Tons. | | | | | | | |
| | | Per Day. | Per Day. | Per Year. | Per Day. | Per Yr. | Cost in Dollars. | | Cost in Dollars. | | Cost in Dollars. |
| | Day. | | | | | | Yr. | Day. | Yr. | Day. | Yr. |
| 1 | 40 | 0.0179 | 53.57 | 0.02 | 6 | 0.04 | 12 | 0.06 | 18 | 0.08 | 24 |
| 10 | 400 | 0.1786 | 53.57 | 0.20 | 60 | 0.40 | 120 | 0.60 | 180 | 0.80 | 240 |
| 25 | 1,000 | 0.4464 | 133.92 | 0.50 | 150 | 1.00 | 300 | 1.50 | 450 | 2.00 | 600 |
| 50 | 2,000 | 0.8928 | 267.85 | 1.00 | 300 | 2.00 | 600 | 3.00 | 900 | 4.00 | 1,200 |
| 75 | 3,000 | 1.3393 | 401.78 | 1.50 | 450 | 3.00 | 900 | 4.50 | 1,350 | 6.00 | 1,800 |
| 100 | 4,000 | 1.7857 | 535.71 | 2.00 | 600 | 4.00 | 1,200 | 6.00 | 1,800 | 8.00 | 2,400 |
| 150 | 6,000 | 2.6785 | 803.56 | 3.00 | 900 | 6.00 | 1,800 | 9.00 | 2,700 | 12.00 | 3,600 |
| 200 | 8,000 | 3.5714 | 1,071.42 | 4.00 | 1,200 | 8.00 | 2,400 | 12.00 | 3,600 | 16.00 | 4,800 |
| 250 | 10,000 | 4.4642 | 1,339.27 | 5.00 | 1,500 | 10.00 | 3,000 | 15.00 | 4,500 | 20.00 | 6,000 |
| 300 | 12,000 | 5.3571 | 1,607.13 | 6.00 | 1,800 | 12.00 | 3,600 | 18.00 | 5,400 | 24.00 | 7,200 |
| 350 | 14,000 | 6.2500 | 1,874.98 | 7.00 | 2,100 | 14.00 | 4,200 | 21.00 | 6,200 | 28.00 | 8,400 |
| 400 | 16,000 | 7.1428 | 2,142.84 | 8.00 | 2,400 | 16.00 | 4,800 | 24.00 | 7,200 | 32.00 | 9,600 |
| 450 | 18,000 | 8.0356 | 2,410.69 | 9.00 | 2,700 | 18.00 | 5,400 | 27.00 | 8,100 | 36.00 | 10,800 |
| 500 | 20,000 | 8.9285 | 2,678.55 | 10.00 | 3,000 | 20.00 | 6,000 | 30.00 | 9,000 | 40.00 | 12,000 |
| 600 | 24,000 | 10.7142 | 3,214.26 | 12.00 | 3,600 | 24.00 | 7,200 | 36.00 | 10,800 | 48.00 | 14,400 |
| 700 | 28,000 | 12.4999 | 3,749.97 | 14.00 | 4,200 | 28.00 | 8,400 | 42.00 | 11,600 | 56.00 | 16,800 |
| 800 | 32,000 | 14.2856 | 4,285.68 | 16.00 | 4,800 | 32.00 | 9,600 | 48.00 | 12,400 | 64.00 | 19,200 |
| 900 | 36,000 | 16.0713 | 4,821.39 | 18.00 | 5,400 | 36.00 | 10,800 | 54.00 | 14,200 | 72.00 | 21,600 |
| 1000 | 40,000 | 17.8570 | 5,357.10 | 20.00 | 6,000 | 40.00 | 12,000 | 60.00 | 18,000 | 80.00 | 24,000 |

It is usual to consider that a factory working 10 hours a day requires $10\frac{1}{2}$ hours coal consumption on account of the coal used in banking or in starting the fires, and that there are 306 working days in the year. For these conditions multiply the costs given in the table by 1.071. For 24 hours a day 365 days in the year, multiply them by 2.68. For other rates of coal consumption than 4 lbs. per H.P. hour, the figures are to be modified proportionately.

Relative Cost of Different Sizes of Steam-engines.
(From catalogue of the Buckeye Engine Co., Part III.)

| | | | | | | | | | | | | | | |
|-------------------|----|--------|-----|-----|--------|--------|-----|--------|------|------|------|--------|-----|-----|
| Horse-power.... | 50 | 75 | 100 | 125 | 150 | 200 | 250 | 300 | 350 | 400 | 500 | 600 | 700 | 800 |
| Cost per H.P., \$ | 20 | 17 1/2 | 16 | 15 | 14 1/2 | 13 1/2 | 13 | 12 3/4 | 12.5 | 12.6 | 12.8 | 13 1/4 | 14 | 15 |

Relative Commercial Economy of Best Modern Types of Compound and Triple-expansion Engines. (J. E. Denton, *American Machinist*, Dec. 17, 1891.) — The following table and deductions show the relative commercial economy of the compound and triple types for the best stationary practice in steam plants of 500 indicated horse-power. The table is based on the tests of Prof. Schröter, of Munich, of engines built at Augsburg, and those of Geo. H. Barrus on the best plants of America, and of detailed estimates of cost obtained from several first-class builders.

| | | | | |
|--|---|--|-------|-------|
| Trip motion, or Corliss engines of the twin-compound-receiver condensing type, expanding 16 times. Boiler pressure 120 lbs. | { | Lbs. water per hour per H.P., by measurement. | 13.6 | 14.0 |
| | | Lbs. coal per hour per H.P., assuming 8.5 lbs. actual evaporation. | 1.60 | 1.65 |
| Trip motion, or Corliss engines of the triple-expansion four-cylinder-receiver condensing type, expanding 22 times. Boiler pressure 150 lbs. | { | Lbs. water per hour per H.P., by measurement. | 12.56 | 12.80 |
| | | Lbs. coal per hour per H.P., assuming 8.5 lbs. actual evaporation. | 1.48 | 1.50 |

The figures in the first column represent the best recorded performance (1891), and those in the second column the probable reliable performance.

The following table shows the total annual cost of operation, with coal at \$4.00 per ton, the plant running 300 days in the year, for 10 hours and for 24 hours per day.

| Hours running per day..... | 10 | 24 |
|---|----------|----------|
| | Per H.P. | Per H.P. |
| Expense for coal. Compound plant..... | \$9.90 | \$28.50 |
| Expense for coal. Triple plant..... | 9.00 | 25.92 |
| Annual saving of triple plant in fuel..... | 0.90 | 2.60 |
| Annual interest at 5% on \$4.50..... | \$0.23 | \$0.23 |
| Annual depreciation at 5% on \$4.50..... | 0.23 | 0.23 |
| Annual extra cost of oil, 1 gallon per 24-hour day, at \$0.50, or 15% of extra fuel cost..... | 0.15 | 0.36 |
| Annual extra cost of repairs at 3% on \$4.50 per 24 hours..... | 0.06 | 0.14 |
| | \$0.67 | \$0.96 |
| Annual saving per H.P..... | \$0.23 | \$1.64 |

Increased cost of triple-expansion plant per horse-power, including boilers, chimney, heaters, foundations, piping and erection \$4.50

Taking the total cost of plants at \$36.50 and \$41 per horse-power respectively, the figures in the table imply that for coal at \$4 per ton a

triple expansion 500 H.P. plant costs \$20,500, and saves about \$114 per year in 10-hour service, or \$826 in 24-hour service, over a compound plant, thereby saving its extra cost in 10-hour service in about $19\frac{3}{4}$ years, or in 24-hour service in about $23\frac{1}{4}$ years.

Power Plant Economics. (H. G. Stott, *Trans. A. I. E. E.*, 1906.) — The following table gives an analysis of the heat losses found in a year's operation of one of the most efficient plants in existence.

AVERAGE LOSSES IN THE CONVERSION OF 1 LB. OF COAL INTO ELECTRICITY.

| | B.T.U. | % | B.T.U. | % |
|---|--------|-------|--------|------|
| 1. B.T.U. per lb. of coal supplied | 14,150 | 100.0 | | |
| 2. Loss in ashes | | | 340 | 2.4 |
| 3. Loss to stack | | | 3,212 | 22.7 |
| 4. Loss in boiler radiation and air leakage | | | 1,131 | 8.0 |
| 5. Returned by feed-water heater | 441 | 3.1 | | |
| 6. Returned by economizer | 960 | 6.8 | | |
| 7. Loss in pipe radiation | | | 28 | 0.2 |
| 8. Delivered to circulator | | | 223 | 1.6 |
| 9. Delivered to feed pump | | | 203 | 1.4 |
| 10. Loss in leakage and high-pressure drips | | | 152 | 1.1 |
| 11. Delivered to small auxiliaries | | | 51 | 0.4 |
| 12. Heating | | | 31 | 0.2 |
| 13. Loss in engine friction | | | 111 | 0.8 |
| 14. Electrical losses | | | 36 | 0.3 |
| 15. Engine radiation losses | | | 28 | 0.2 |
| 16. Rejected to condenser | | | 8,524 | 60.1 |
| 17. To house auxiliaries | | | 29 | 0.2 |
| | 15,551 | 109.9 | 14,099 | 99.6 |
| | 14,099 | 99.6 | | |
| Delivered to bus bar | 1,452 | 10.3 | | |

The following notes concerning power-plant economy are condensed from Mr. Stott's paper.

Item 1. B.T.U. per lb. of coal. The coal is bought and paid for on the basis of the B.T.U. found by a bomb calorimeter.

Item 3. The chimney loss is very large, due to admitting too much air to the combustion chamber. This loss can be reduced about half by the use of a CO₂ recorder and proper management of the fire.

Item 4. This loss is largely due to infiltration of air into the brick setting. It can be saved by having an air-tight sheet-iron casing enclosing a magnesia lining outside of the brickwork.

Item 5. All auxiliaries should be driven by steam, so that their exhaust may be utilized in the feed-water heater.

Item 6. In all cases where the load factor exceeds 25% the investment in economizers will be justified.

Item 7. The pipes are covered with two layers of covering, each about 1.5 in. thick.

Item 10. The high-pressure drips can be returned to the boiler, so practically all the loss under this heading is recoverable.

Item 13. Recent tests of a 7500-H.P. reciprocating engine show a mechanical efficiency of 93.65%, or an engine friction of 6.35%. The engine is lubricated by the flushing system.

Item 16. The maximum theoretical efficiency of an engine working between 175 lbs. gauge and 28 ins. vacuum is

$$(T_1 - T_2) \div T_1 = (837 - 560) \div 837 = 33\%.$$

The actual best efficiency of this engine is 17 lbs. per K.W.-hour = 16.7% thermal efficiency; dividing by 0.98, the generator efficiency, gives the net thermodynamic efficiency of the engine, = 17%. The difference between the theoretical and the actual efficiency is $33 - 17 = 16\%$, of which 6.35% is due to engine friction, and the balance, 9.65%, is due to cylinder con-

densation, incomplete expansion, and radiation. [Some of this difference is due to the fact that the engine does not work on the Carnot cycle, in which the heat is all received at the highest temperature, and part of this loss might be saved by the Nordberg feed-water heating system. There may also be a slight loss from leakage. W.K.J. Superheated steam, to such an extent as to insure dry steam at the point of cut-off in the low-pressure cylinder, might save 5 or 6%.

The present type of power plant using reciprocating engines can be improved in efficiency as follows: Reduction of stack losses, 12%; boiler radiation and leakage, 5%; by superheating, 6%; resulting in a net increase of thermal efficiency of the entire plant of 4.14% and bringing the total from 10.3 to 14.44%.

The Steam Turbine. — The best results from the steam turbine up to date show that its economy on dry saturated steam is practically equal to that of the reciprocating engine, and that 200° superheat reduces its steam consumption 13.5%. The shape of the economy curve is much flatter [from 3300 to 8000 K.W. the range of steam consumption is between 14.6 and 15.0 lbs. per K.W.-hour], so that the all-day efficiency would be considerably better than that of the reciprocating engine, and the cost would be about 33% less for the combined steam motor and electric generator.

High-pressure Reciprocating Engine with Low-pressure Turbine. — The reciprocating engine is more efficient than the turbine in the higher pressures, while the turbine can expand to lower pressures and utilize the gain of full expansion. The combination of the two would therefore be more efficient than a turbine alone.

The Gas Engine. — The best result up to date obtained from gas producers and gas engines is about as follows: Loss in producer and auxiliaries, 20%; in jacket water, 19%; in exhaust gases, 30%; in engine friction, 6.5%; in electric generator, 0.5%. Total losses, 76%. Converted into electric energy, 24%. Only one important objection can be raised to this motor, that its range of economical load is practically limited to between 50% and full load. This lack of overload capacity is probably a fatal defect for the ordinary railway power plant acting under a violently fluctuating load, unless protected by a large storage-battery.

At light loads the economy of gas and liquid fuel engines fell off even more rapidly than in steam-engines. The engine friction was large and nearly constant, and in some cases the combustion was also less perfect at light loads. At the Dresden Central Station the gas-engines were kept working at nearly their full power by the use of storage-batteries. The results of some experiments are given below:

| Brake-load, per cent of full Power. | Gas-engine, cu. ft. of Gas per Brake H.P. per hour. | Petroleum Eng., Lbs. of Oil per B.H.P. per hr. | Petroleum Eng., Lbs. of Oil per B.H.P. per hr. |
|---|---|--|--|
| 100 | 22.2 | 0.96 | 0.88 |
| 75 | 23.8 | 1.11 | 0.99 |
| 59 | 28.0 | 1.44 | 1.20 |
| 20 | 40.8 | 2.38 | 1.82 |
| 12½ | 66.3 | 4.25 | 3.07 |

Combination of Gas Engines and Turbines. — A steam turbine unit can be designed to take care of 100% overload for a few seconds. If a plant were designed with 50% of its normal capacity in gas engines and 50% in steam turbines, any fluctuations in load likely to arise in practice could be taken care of. By utilizing the waste heat of the gas engine in economizers and superheaters there can be saved approximately 37% of this waste heat, to make steam for the turbines. The average total thermal efficiency of such a combination plant would be 24.5%. This combination offers the possibility of producing the kilowatt-hour for less than one-half its present cost.

The following table shows the distribution of estimated relative maintenance and operation costs of five different types of plant, the total cost of current with the reciprocating engine plant being taken at 100.

| | Recip- rocating Engines. | Steam Turbines | Recip- rocating Engines and Steam Turbines. | Gas- Engine Plant. | Gas Engines and Steam Turbines. |
|--|--------------------------------|-------------------|--|--------------------------|---|
| MAINTENANCE. | | | | | |
| 1. Engine room mechan- ical..... | 2.57 | 0.51 | 1.54 | 2.57 | 1.54 |
| 2. Boiler room or pro- ducer room..... | 4.61 | 4.30 | 3.52 | 1.15 | 1.95 |
| 3. Coal- and ash-hand- ling apparatus... | 0.58 | 0.54 | 0.44 | 0.29 | 0.29 |
| 4. Electrical apparatus | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 |
| OPERATION. | | | | | |
| 5. Coal- and ash-hand- ling labor..... | 2.26 | 2.11 | 1.74 | 1.13 | 1.13 |
| 6. Removal of ashes... | 1.06 | 0.94 | 0.80 | 0.53 | 0.53 |
| 7. Dock rental..... | 0.74 | 0.74 | 0.74 | 0.74 | 0.74 |
| 8. Boiler-room labor... | 7.15 | 6.68 | 5.46 | 1.79 | 3.03 |
| 9. Boiler-room oil, waste, etc. | 0.17 | 0.17 | 0.17 | 0.17 | 0.17 |
| 10. Coal..... | 61.30 | 57.30 | 46.87 | 26.31 | 25.77 |
| 11. Water..... | 7.14 | 0.71 | 5.46 | 3.57 | 2.14 |
| 12. Engine-room me- chanical labor..... | 6.71 | 1.35 | 4.03 | 6.71 | 4.03 |
| 13. Lubrication..... | 1.77 | 0.35 | 1.01 | 1.77 | 1.06 |
| 14. Waste, etc..... | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| 15. Electrical labor..... | 2.52 | 2.52 | 2.52 | 2.52 | 2.52 |
| Relative cost of mainte- nance and operation .. | 100.00 | 79.64 | 75.72 | 50.67 | 46.32 |
| Relative investment in per cent..... | 100.00 | 82.50 | 77.00 | 100.00 | 91.20 |

Storing Heat in Hot Water. —(See also p. 897.) There is no satisfactory method for equalizing the load on the engines and boilers in electric-light stations. Storage-batteries have been used, but they are expensive in first cost, repairs, and attention. Mr. Halpin, of London, proposes to store heat during the day in specially constructed reservoirs. As the water in the boilers is raised to 250 lbs. pressure, it is conducted to cylindrical reservoirs resembling English horizontal boilers, and stored there for use when wanted. In this way a comparatively small boiler-plant can be used for heating the water to 250 lbs. pressure all through the twenty-four hours of the day, and the stored water may be drawn on at any time, according to the magnitude of the demand. The steam-engines are to be worked by the steam generated by the release of pressure from this water, and the valves are to be arranged in such a way that the steam shall work at 130 lbs. pressure. A reservoir 8 ft. in diameter and 30 ft. long, containing 84,000 lbs. of heated water at 250 lbs. pressure, would supply 5250 lbs. of steam at 130 lbs. pressure. As the steam consumption of a condensing electric-light engine is about 18 lbs. per horse-power hour, such a reservoir would supply 286 effective horse-power hours. In 1878, in France, this method of storing steam was used on a tramway. M. Francq, the engineer, designed a smokeless locomotive to work by steam-power supplied by a reservoir containing 400 gallons of water at 220 lbs. pressure. The reservoir was charged with steam from a stationary boiler at one end of the tramway.

An installation of the Rateau low-pressure turbine and regenerator system at the rolling mill of the International Harvester Co., in Chicago, is described in *Power*, June, 1907. The regenerator is a cylindrical shell 11½ ft. diam., 30 ft. long, containing six large elliptical tubes perforated with many ¾-in. holes through which exhaust steam from a reversing

blooming-mill engine enters the water contained in the shell. A large steam pipe leads from the shell to the turbine. A series of tests of the combination was made, giving results as follows: The 42 X 60 in. blooming mill engine developed 820 I.H.P. on the average, with a water rate of 64 lbs. per I.H.P. hour. It delivered its exhaust, averaging a little above atmospheric pressure, to the regenerator, at an irregular rate corresponding to the varying work of the rolling-mill engine. The regenerator furnished steam to the turbine, which in four different tests developed 444, 544, 727 and 869 brake H.P. at the turbine shaft, with a steam consumption of 47.7, 37.1, 30.7 and 33.7 lbs. of steam per B.H.P. hour at the turbine. Had the turbine been of sufficient capacity to use all the exhaust of the mill engine, 1510 H.P. might have been delivered at the switchboard, which added to the 820 of the mill engine would make 2330 H.P. for 52,400 lbs. of steam, or a steam rate of 22.5 lbs. per H.P. hour for the combination.

UTILIZING THE SUN'S HEAT AS A SOURCE OF POWER.

John Ericsson, 1868-1875, experimented on "solar engines," in which reflecting surfaces concentrated the sun's rays at a central point causing them to boil water. A large motor of this type was built at Pasadena, Cal., in 1898. The rays were concentrated upon a water heater through which ether or sulphur dioxide was pumped in pipes, and utilized in a vapor engine. The apparatus was commercially unsuccessful on account of variable weather conditions. *Eng. News*, May 13, 1909, describes the solar heat systems of F. Shuman and of H. E. Willsie and John Boyle, Jr.

In the Shuman invention a tract of land is rolled level, forming a shallow trough. This is lined with asphaltum pitch and covered with about 3 ins. of water. Over the water about $\frac{1}{16}$ in. of paraffine is flowed, leaving between this and a glass cover about 6 ins. of dead air space. It is estimated that a power plant of this type to cover a heat-absorption area of 160,000 sq. ft., or nearly four acres, would develop about 1000 H.P. Provision is made for storing hot water in excess of the requirements of a low-pressure turbine during the day, to be utilized for running the turbine during the period when there is no absorption of heat. The heated water is run from the heat absorber to the storage tank, thence to the turbine, through a condenser and back to the heat absorber. The water enters the thermally insulated storage tank, or the turbine, at about 202° F. With a vacuum of 28 ins. in the condenser, the boiling-point of the water is reduced to 102°, and as it enters the turbine nearly 10% explodes into steam. Mr. Shuman estimates that a 1000-H.P. plant built upon his plan would cost about \$40,000.

The Willsie and Boyle plant also utilizes the indirect system of absorbing solar heat and storing the hot water in tanks. This hot water circulates in a boiler containing some volatile liquid, and the vapor generated is used to operate the engine, is condensed, and returned to the boiler to be used again. Mr. Willsie compares the cost per H.P.-hour in a 400-H.P. steam-electric and solar-electric power plant, and finds that the steam plant would have to obtain its coal for \$0.66 a ton to compete with the sun power plant in districts favorable to the latter.

RULES FOR CONDUCTING STEAM-ENGINE TESTS.

A committee of the Am. Soc. M. E. in 1902 made a report on Engine Tests, which is printed in the Transactions for that year, and also in a pamphlet of 78 pages. A greatly condensed abstract only can be given here. Engineers making tests of engines should have the complete report.

In the introduction to the report the Committee says:

The heat consumption of a steam-engine plant is ascertained by measuring the quantity of steam consumed by the plant, calculating the total heat of the entire quantity, and crediting this total with that portion of the heat rejected by the plant which is utilized and returned to the boiler. The term "engine plant" as here used should include the entire equipment of the steam plant which is concerned in the production of the power, embracing the main cylinder or cylinders; the jackets and reheaters; the air, circulating, and boiler-feed pumps, if steam driven; and any other

steam-driven mechanism or auxiliaries necessary to the working of the engine. It is obligatory to thus charge the engine with the steam used by necessary auxiliaries in determining the plant economy, for the reason that it is itself finally benefited, or should be so benefited, by the heat which they return; it being generally agreed that exhaust steam from such auxiliaries should be passed through a feed-water heater, and the heat thereby carried back to the boiler and saved.

In that large class of steam engines which are required to run at a certain limited and constant speed, there should be a considerable reserve of capacity beyond the rated power. It is our recommendation that when a steam engine is operating at its rated power at a given pressure there should be a sufficient reserve to allow a drop of at least 15 per cent in the gauge pressure without sensible reduction in the working speed of the engine, and to allow an overload at the stated pressure amounting to at least 25 per cent.

RULES FOR CONDUCTING STEAM-ENGINE TESTS. CODE OF 1902.

I. *Object of Test.* — Ascertain at the outset the specific object of the test, whether it be to determine the fulfillment of a contract guarantee, to ascertain the highest economy obtainable, to find the working economy and defects under conditions as they exist, to ascertain the performance under special conditions, to determine the effect of changes in the conditions, or to find the performance of the entire boiler and engine plant, and prepare for the test accordingly.

II. *General Condition of the Plant.* — Examine the engine and the entire plant concerned in the test; note its general condition and any points of design, construction, or operation which bear on the objects in view. Make a special examination of the valves and pistons for leakage by applying the working pressures with the engine at rest, and observe the quantity of steam, if any, blowing through per hour.

III. *Dimensions, etc.* — Measure or check the dimensions of the cylinders when they are hot. If they are much worn, the average diameter should be determined. Measure also the clearance. If the clearance cannot be measured directly, it can be determined approximately from the working drawings of the cylinder.

IV. *Coal.* — When the trial involves the complete plant, embracing boilers as well as engine, determine the character of coal to be used. The class, name of the mine, size, moisture, and quality of the coal should be stated in the report. It is desirable, for purposes of comparison, that the coal should be of some recognized standard quality for the locality where the plant is situated.

V. *Calibration of Instruments.* — All instruments and apparatus should be calibrated and their reliability and accuracy verified by comparison with recognized standards.

VI. *Leakages of Steam, Water, etc.* — In all tests except those of a complete plant made under conditions as they exist, the boiler and its connections, both steam and feed, as also the steam piping leading to the engine and its connections, should, so far as possible, be made tight. All connections should, so far as possible, be visible and be blanked off, and where this cannot be done, satisfactory assurance should be obtained that there is no leakage either in or out.

VII. *Duration of Test.* — The duration of a test should depend largely upon its character and the objects in view. The standard heat test of an engine, and, likewise, a test for the simple determination of the feed-water consumption, should be continued for at least five hours, unless the class of service precludes a continuous run of so long duration. It is desirable to prolong the test the number of hours stated to obtain a number of consecutive hourly records as a guide in analyzing the reliability of the whole.

The commercial test of a complete plant, embracing boilers as well as engine, should continue at least one full day of twenty-four hours, whether the engine is in motion during the entire time or not. A continuous coal test of a boiler and engine should be of at least ten hours' duration, or the nearest multiple of the interval between times of cleaning fires.

VIII. *Starting and Stopping a Test.* — (a) *Standard Heat Test and Feed-Water Test of Engine:* The engine having been brought to the normal

condition of running, and operated a sufficient length of time to be thoroughly heated in all its parts, and the measuring apparatus having been adjusted and set to work, the height of water in the gauge glasses of the boilers is observed, the depth of water in the reservoir from which the feed water is supplied is noted, the exact time of day is observed, and the test held to commence. Thereafter the measurements determined upon for the test are begun and carried forward until its close. When the time for the close of the test arrives, the water should, if possible, be brought to the same height in the glasses and to the same depth in the feed-water reservoir as at the beginning, delaying the conclusion of the test if necessary to bring about this similarity of conditions. If differences occur, the proper corrections must be made.

(b) *Complete Engine and Boiler Test:* For a continuous running test of combined engine or engines, and boiler or boilers, the same directions apply for beginning and ending the feed-water measurements as those just referred to. The time of beginning and ending such a test should be the regular time of cleaning the fires, and the exact time of beginning and ending should be the time when the fires are fully cleaned, just preparatory to putting on fresh coal.

For a commercial test of a combined engine and boiler, whether the engine runs continuously for the full twenty-four hours of the day, or only a portion of the time, the fires in the boilers being banked during the time when the engine is not in motion, the beginning and ending of the test should occur at the regular time of cleaning the fires, the method followed being that already given. In cases where the engine is not in continuous motion, as, for example, in textile mills, where the working time is ten or eleven hours out of the twenty-four, and the fires are cleaned and banked at the close of the day's work, the best time for starting and stopping a test is the time just before banking, when the fires are well burned down and the thickness and condition can be most satisfactorily judged.

IX. *Measurement of Heat Units Consumed by the Engine.* — The measurement of the heat consumption requires the measurement of each supply of feed water to the boiler — that is, the water supplied by the main feed pump, that supplied by auxiliary pumps, such as jacket water, water from separators, drips, etc., and water supplied by gravity or other means; also the determination of the temperature of the water supplied from each source, together with the pressure and quality of the steam. The temperatures at the various points should be those applying to the working conditions.

The heat to be determined is that used by the entire engine equipment, embracing the main cylinders and all auxiliary cylinders and mechanism concerned in the operation of the engine, including the air pump, circulating pump, and feed pumps, also the jacket and reheater when these are used.

The steam pressure and the quality of the steam are to be taken at some point conveniently near the throttle valve. The quantity of steam used by the calorimeter must be determined and properly allowed for.

X. *Measurement of Feed Water or Steam Consumption of Engine, etc.* — The method of determining the steam consumption applicable to all plants is to measure all the feed water supplied to the boilers, and deduct therefrom the water discharged by separators and drips, as also the water and steam which escapes on account of leakage of the boiler and its pipe connections and leakage of the steam main and branches connecting the boiler and the engine. In plants where the engine exhausts into a surface condenser the steam consumption can be measured by determining the quantity of water discharged by the air pump, corrected for any leakage of the condenser, and adding thereto the steam used by jackets, reheaters, and auxiliaries as determined independently.

The corrections or deductions to be made for leakage above referred to should be applied only to the standard heat-unit test and tests for determining simply the steam or feed-water consumption, and not to coal tests of combined engine and boiler equipment. In the latter, no corrections should be made except for leakage of valves connecting to other engines and boilers, or for steam used for purposes other than the operation of the plant under test. Losses of heat due to imperfections of the plant should be charged to the plant, and only such losses as are concerned in the working of the engine alone should be charged to the engine.

XI. Measurement of Steam used by Auxiliaries. — It is highly desirable that the quantity of steam used by the auxiliaries, and in many cases that used by each auxiliary, should be determined exactly, so that the net consumption of the main engine cylinders may be ascertained and a complete analysis made of the entire work of the engine plant.

XII. Coal Measurement. — The coal consumption should be determined for the entire time of the test. If the engine runs but a part of the time, and during the remaining portion the fires are banked, the measurement of coal should include that used for banking.

XIII. Indicated Horse-power. — The indicated horse-power should be determined from the average mean effective pressure of diagrams taken at intervals of twenty minutes, and at more frequent intervals if the nature of the test makes this necessary, for each end of each cylinder. With variable loads, such as those of engines driving generators for electric railroad work, and of rubber-grinding and rolling-mill engines, the diagrams cannot be taken too often.

The most satisfactory driving rig for indicating seems to be some form of well-made pantograph, with driving cord of fine annealed wire leading to the indicator. The reducing motion, whatever it may be, and the connections to the indicator, should be so perfect as to produce diagrams of equal lengths when the same indicator is attached to either end of the cylinder, and produce a proportionate reduction of the motion of the piston at every point of the stroke, as proved by test.

The use of a three-way cock and a single indicator connected to the two ends of the cylinder is not advised, except in cases where it is impracticable to use an indicator close to each end. If a three-way cock is used, the error produced should be determined and allowed for.

XIV. Testing Indicator Springs. — To make a perfectly satisfactory comparison of indicator springs with standards, the calibration should be made, if this were practical, under the same conditions as those pertaining to their ordinary use.

XV. Brake Horse-power. — This term applies to the power delivered from the flywheel shaft of the engine. It is the power absorbed by a friction brake applied to the rim of the wheel, or to the shaft. A form of brake is preferred that is self-adjusting to a certain extent, so that it will, of itself, tend to maintain a constant resistance at the rim of the wheel. One of the simplest brakes for comparatively small engines, which may be made to embody this principle, consists of a cotton or hemp rope, or a number of ropes, encircling the wheel, arranged with weighing scales, or other means for showing the strain. An ordinary band brake may also be constructed so as to embody the principle. The wheel should be provided with interior flanges for holding water used for keeping the rim cool.

XVI. Quality of Steam. — When ordinary saturated steam is used, its quality should be obtained by the use of a throttling calorimeter attached to the main steam pipe near the throttle valve. When the steam is superheated, the amount of superheating should be found by the use of a thermometer placed in a thermometer-well filled with mercury, inserted in the pipe. The sampling pipe for the calorimeter should, if possible, be attached to a section of the main pipe having a vertical direction, with the steam preferably passing upward, and the sampling nozzle should be made of a half-inch pipe, having at least 20 $\frac{1}{8}$ -in. holes in its perforated surface.

XVII. Speed. — There are several reliable methods of ascertaining the speed, or the number of revolutions of the engine crank-shaft per minute. The most reliable method is the use of a continuous recording engine register or counter, taking the total reading each time that the general test data are recorded, and computing the revolutions per minute corresponding to the difference in the readings of the instrument. When the speed is above 250 revolutions per minute, it is almost impossible to make a satisfactory counting of the revolutions without the use of some form of mechanical counter.

XVIII. Recording the Data. — Take note of every event connected with the progress of the trial whether it seems at the time to be important or unimportant. Record the time of every event, and time of taking every weight, and every observation. Observe the pressures, temperatures, water heights, speeds, etc., every twenty or thirty minutes when the con-

ditions are practically uniform, and at much more frequent intervals if the conditions vary.

XIX. *Uniformity of Conditions.* — In a test having for an object the determination of the maximum economy obtainable from an engine, or where it is desired to ascertain with special accuracy the effect of predetermined conditions of operation, it is important that all the conditions under which the engine is operated should be maintained uniformly constant.

XX. *Analysis of Indicator Diagrams.* — (a) *Steam Accounted for by the Indicator:* The simplest method of computing the steam accounted for by the indicator is the use of the formula,

$$M = \frac{13750}{\text{M.E.P.}} [(C + E) \times Wc - (H + E) \times Wh],$$

which gives the weight in pounds per indicated horse-power per hour. In this formula the symbol "M.E.P." refers to the mean effective pressure. In multiple-expansion engines, this is the combined mean effective pressure referred to the cylinder in question. C is the proportion of the stroke completed at points on the expansion line of the diagram near the actual cut-off or release; H the proportion of compression; and E the proportion of clearance; all of which are determined from the indicator diagram. Wc is the weight of one cubic foot of steam at the cut-off or release pressure; and Wh the weight of one cubic foot of steam at the compression pressure; these weights being taken from steam tables.

Should the point in the compression curve be at the same height as the point in the expansion curve, then $Wc = Wh$, and the formula becomes

$$(13,750 \div \text{M.E.P.}) \times (C - H) \times Wc,$$

in which $(C - H)$ represents the distance between the two points divided by the length of the diagram.

When the load and all other conditions are substantially uniform, it is unnecessary to work up the steam accounted for by the indicator from all the diagrams taken. Five or more sample diagrams may be selected and the computations based on the samples instead of on the whole.

(b) *Sample Indicator Diagrams:* In order that the report of a test may afford complete information regarding the conditions of the test, sample indicator diagrams should be selected from those taken and copies appended to the tables of results. In cases where the engine is of the multiple-expansion type these sample diagrams may also be arranged in the form of a "combined" diagram.

(c) *The Point of Cut-off:* The term "cut-off" as applied to steam engines, although somewhat indefinite, is usually considered to be at an earlier point in the stroke than the beginning of the real expansion line. That the cut-off point may be defined in exact terms for commercial purposes as used in steam-engine specifications and contracts, the Committee recommends that, unless otherwise specified, the *commercial cut-off*, which seems to be an appropriate expression for this term, be ascertained as follows: Through a point showing the maximum pressure during admission, draw a line parallel to the atmospheric line. Through the point on the expansion line near the actual cut-off, referred to in Section XX (a), draw a hyperbolic curve. The point where these two lines intersect is to be considered the *commercial cut-off* point. The percentage is then found by dividing the length of the diagram measured to this point, by the total length of the diagram, and multiplying the result by 100.

The *commercial cut-off*, as thus determined, is situated at an earlier point of the stroke than the actual cut-off used in computing the "steam accounted for" by the indicator and referred to in Section XX (a).

(d) *Ratio of Expansion:* The "commercial" ratio of expansion is the quotient obtained by dividing the volume corresponding to the piston displacement, including clearance, by the volume of the steam at the commercial cut-off, including clearance. In a multiple-expansion engine the volumes are those pertaining to the low-pressure cylinder and high-pressure cylinder, respectively.

The "ideal" ratio of expansion is the quotient obtained by dividing the volume of the piston displacement by the volume of the steam at the

cut-off (the latter being referred to the throttle-valve pressure), less the volume equivalent to that retained at compression. In a multiple-expansion engine, the volumes to be used are those pertaining to the low-pressure cylinder and high-pressure cylinder, respectively.

(e) *Diagram Factor*: The diagram factor is the proportion borne by the actual mean effective pressure measured from the indicator diagram to that of a diagram in which the various operations of admission, expansion, release and compression are carried on under assumed conditions. The factor recommended refers to an ideal diagram which represents the maximum power obtainable from the steam accounted for by the indicator diagrams at the point of cut-off, assuming first that the engine has no clearance; second, that there are no losses through wire-drawing the steam during either the admission or the release; third, that the expansion line is a hyperbolic curve; and fourth, that the initial pressure is that of the boiler and the back pressure that of the atmosphere for a non-condensing engine, and of the condenser for a condensing engine.

In cases where there is a considerable loss of pressure between the boiler and the engine, as where steam is transmitted from a central plant to a number of consumers, the pressure of the steam in the supply main should be used in place of the boiler pressure in constructing the diagrams.

XXI. *Standards of Economy and Efficiency.* — The hourly consumption of heat, determined by employing the actual temperature of the feed water to the boiler, as pointed out in Article IX of the Code, divided by the indicated and brake horse-power, that is, the number of heat units consumed per indicated and per brake horse-power per hour, are the standards of engine efficiency recommended by the Committee. The consumption *per hour* is chosen rather than the consumption per minute, so as to conform with the designation of time applied to the more familiar units of coal and water measurement, which have heretofore been used. The British standard, where the temperature of the feed water is taken as that corresponding to the temperature of the back-pressure steam, allowance being made for any drips from jackets or reheaters, is also included in the tables.

It is useful in this connection to express the efficiency in its more scientific form, or what is called the "thermal efficiency ratio." The thermal efficiency ratio is the proportion which the heat equivalent of the power developed bears to the total amount of heat actually consumed, as determined by test. The heat converted into work represented by one horse-power is 1,980,000 foot-pounds per hour, and this divided by 778 equals 2545 British thermal units. Consequently, the thermal efficiency ratio is expressed by the fraction

$$2545 \div \text{B.T.U. per H.P. per hour.}$$

XXII. *Heat Analysis.* — For certain scientific investigations, it is useful to make a heat analysis of the diagram, to show the interchange of heat from steam to cylinder walls, etc., which is going on within the cylinder. This is unnecessary for commercial tests.

XXIII. *Temperature-Entropy Diagram.* — The study of the heat analysis is facilitated by the use of the temperature-entropy diagram in which areas represent quantities of heat, the coördinates being the absolute temperature and entropy.

XXIV. *Ratio of Economy of an Engine to that of an Ideal Engine.* — The ideal engine recommended for obtaining this ratio is that which was adopted by the Committee appointed by the Civil Engineers, of London, to consider and report a standard thermal efficiency for steam engines. This engine is one which follows the Rankine cycle, where steam at a constant pressure is admitted into the cylinder with no clearance, and after the point of cut-off, is expanded adiabatically to the back pressure. In obtaining the economy of this engine the feed water is assumed to be returned to the boiler at the exhaust temperature.

The ratio of the economy of an engine to that of the ideal engine is obtained by dividing the heat consumption per indicated horse-power per minute for the ideal engine by that of the actual engine.

XXV. *Miscellaneous.* — In the case of tests of combined engines and boiler plants, where the full data of the boiler performance are to be determined, reference should be made to the directions given by the Boiler Test Committee of the Society, Code of 1899. (See Vol. XXI, p. 34.)

In testing steam pumping engines and locomotives in accordance with the standard methods of conducting such tests, recommended by the committees of the Society, reference should be made to the reports of those committees in the *Transactions*, Volume XII, p. 530, and in Volume XIV, p. 1312.

XXVI. *Report of Test.* — The data and results of the test should be reported in the manner and in the order outlined in one of the following tables, the first of which gives a summary of all the data and results as applied not only to the standard heat-unit test, but also to tests of combined engine and boiler for determining all questions of performance, whatever the class of service; the second refers to a short form of report giving the necessary data and results for the standard heat test; and the third to a short form of report for a feed-water test.

It is recommended that any report be supplemented by a chart in which the data of the test are graphically presented. [Of the three forms of report mentioned above, the second is given below.]

DATA AND RESULTS OF STANDARD HEAT TEST OF STEAM ENGINE.

Arranged according to the Short Form advised by the Engine Test Committee of the American Society of Mechanical Engineers. Code of 1902.

1. Made by..... of.....
on engine located at.....
to determine.....
 2. Date of trial.....
 3. Type and class of engine; also of condenser.....
 4. Dimensions of main engine.....

| | | | |
|--|----------|---------|---------|
| (a) Diameter of cylinder..... in. | 1st Cyl. | 2d Cyl. | 3d Cyl. |
| (b) Stroke of piston..... in. | | | |
| (c) Diameter of piston rod..... in. | | | |
| (d) Average clearance..... p.c. | | | |
| (e) Ratio of volume of cylinder to high-pressure cylinder..... | | | |
| (f) Horse-power constant for one pound mean effective pressure and one revolution per minute | | | |
 5. Dimensions and type of auxiliaries.....
- Total Quantities, Time, etc.*
6. Duration of test..... hours
 7. Total water fed to boilers from main source of supply..... lbs.
 8. Total water fed from auxiliary supplies:.....

| | |
|----------|---|
| (a)..... | " |
| (b)..... | " |
| (c)..... | " |
 9. Total water fed to boilers from all sources.....
 10. Moisture in steam or superheating near throttle..... p. c. or deg.
 11. Factor of correction for quality of steam.....
 12. Total dry steam consumed for all purposes..... lbs.

Hourly Quantities.

13. Water fed from main source of supply..... lbs.
14. Water fed from auxiliary supplies:.....

| | |
|----------|---|
| (a)..... | " |
| (b)..... | " |
| (c)..... | " |
15. Total water fed to boilers per hour.....
16. Total dry steam consumed per hour.....
17. Loss of steam and water per hour due to drips from main steam pipes and to leakage of plant.....
18. Net dry steam consumed per hour by engine and auxiliaries.....

Pressures and Temperatures (Corrected).

- | | |
|---|------------------|
| 19. Pressure in steam pipe near throttle by gauge..... | lbs. per sq. in. |
| 20. Barometric pressure of atmosphere in ins. of mercury..... | ins. |
| 21. Pressure in receivers by gauge..... | lbs. per sq. in. |
| 22. Vacuum in condenser in inches of mercury..... | ins. |
| 23. Pressure in jackets and reheaters by gauge..... | lbs. per sq. in. |
| 24. Temperature of main supply of feed water..... | deg. Fahr. |
| 25. Temperature of auxiliary supplies of feed water: | |
| (a)..... | " |
| (b)..... | " |
| (c)..... | " |
| 26. Ideal feed-water temperature corresponding to pressure of steam in the exhaust pipe, allowance being made for heat derived from jacket or reheater drips. | " |

Data Relating to Heat Measurement.

- | | |
|---|--------|
| 27. Heat units per pound of feed water, main supply.... | B.T.U. |
| 28. Heat units per pound of feed water, auxiliary supplies: | |
| (a)..... | " |
| (b)..... | " |
| (c)..... | " |
| 29. Heat units consumed per hour, main supply..... | " |
| 30. Heat units consumed per hour, auxiliary supplies: | |
| (a)..... | " |
| (b)..... | " |
| (c)..... | " |
| 31. Total heat units consumed per hour for all purposes. | " |
| 32. Loss of heat per hour due to leakage of plant, drips, etc..... | " |
| 33. Net heat units consumed per hour: | |
| (a) By engine alone..... | " |
| (b) By auxiliaries..... | " |
| 34. Heat units consumed per hour by engine alone, reckoned from temperature given in line 26..... | " |

Indicator Diagrams.

- | | |
|--|------------|
| 35. Commercial cut-off in per cent of stroke..... | [Separate |
| 36. Initial pressure, lbs. per sq. in. above atmosphere ... | Columns |
| 37. Back pressure at mid-stroke, above or below atmosphere, in lbs. per sq. in. | for each |
| 38. Mean effective pressure in lbs. per sq. in. | Cylinder.] |
| 39. Equivalent M.E.P. in lbs. per sq. in.: | |
| (a) Referred to first cylinder..... | |
| (b) Referred to second cylinder..... | |
| (c) Referred to third cylinder..... | |
| 40. Pressure above zero in lbs. per sq. in.: | |
| (a) Near cut-off..... | |
| (b) Near release..... | |
| (c) Near beginning of compression..... | |
| Percentage of stroke at points where pressures are measured: | |
| (a) Near cut-off | |
| (b) Near release..... | |
| (c) Near beginning of compression..... | |
| 41. Steam accounted for by indicator in (pounds per I.H.P. per hour: (a) Near cut-off; (b) Near release. | |
| 42. Ratio of expansion: (a) Commercial; (b) Ideal..... | |

Speed.

- | | |
|---------------------------------|------|
| 43. Revolutions per minute..... | rev. |
|---------------------------------|------|

Power.

- | | |
|---|------|
| 44. Indicated horse-power developed by main-engine cylinders: | |
| First cylinder..... | H.P. |
| Second cylinder..... | " |
| Third cylinder..... | " |
| Total..... | " |
| 45. Brake horse-power developed by engine..... | " |

*Standard Efficiency and other Results.**

46. Heat units consumed by engine and auxiliaries per hour:
 (a) per indicated horse-power..... B.T.U.
 (b) per brake horse-power..... " "
47. Equivalent standard coal in lbs. per hour:
 (a) per indicated horse-power..... lbs.
 (b) per brake horse-power..... " "
48. Heat units consumed by main engine per hour corresponding to ideal maximum temperature of feed water given in line 26:
 (a) per indicated horse-power..... B.T.U.
 (b) per brake horse-power..... " "
49. Dry steam consumed per indicated horse-power per hour:
 (a) Main cylinders including jackets lbs.
 (b) Auxiliary cylinders..... "
 (c) Engine and auxiliaries..... " "
50. Dry steam consumed per brake horse-power per hour:
 (a) Main cylinders including jackets..... "
 (b) Auxiliary cylinders..... "
 (c) Engine and auxiliaries..... " "
51. Percentage of steam used by main-engine cylinders accounted for by indicator diagrams, near cut-off of high-pressure cylinder..... per cent.

Additional Data.

Add any additional data bearing on the particular objects of the test or relating to the special class of service for which the engine is used. Also give copies of indicator diagrams nearest the mean, and the corresponding scales.

DIMENSIONS OF PARTS OF ENGINES.

The treatment of this subject by the leading authorities on the steam-engine is very unsatisfactory, being a confused mass of rules and formulæ based partly upon theory and partly upon practice. The practice of builders shows an exceeding diversity of opinion as to correct dimensions. The treatment given below is chiefly the result of a study of the works of Rankine, Seaton, Unwin, Thurston, Marks, and Whitham, and is largely a condensation of a series of articles by the author published in the *American Machinist*, in 1894, with many alterations and much additional matter. In order to make a comparison of many of the formulæ they have been applied to the assumed cases of six engines of different sizes, and in some cases this comparison has led to the construction of new formulæ.

[NOTE, 1909. Since the first edition of this book was published, in 1895, no satisfactory treatise on this entire subject has appeared, and therefore the matter on pages 997 to 1020 has been left, in the revision for the 8th edition, in practically its original shape. Two notable papers on the subject, however, have appeared: 1, Current Practice in Engine Proportions, by Prof. John H. Barr, 1897, and 2, Current Practice in Steam-engine Design, by Ole N. Trooien, 1909. Both of these are abstracted on pages 1021 and 1022.]

Cylinder. (Whitham.)—Length of bore = stroke + breadth of piston-ring - $\frac{1}{8}$ to $\frac{1}{2}$ in.; length between heads = stroke + thickness of piston + sum of clearances at both ends; thickness of piston = breadth of ring + thickness of flange on one side to carry the ring + thickness of follower-plate.

Thickness of flange or follower..... $\frac{3}{8}$ to $\frac{1}{2}$ in. $\frac{3}{4}$ in. 1 in.
 For cylinder of diameter..... 8 to 10 in. 36 in. 60 to 100 in.

Clearance of Piston. (Seaton.)—The clearance allowed varies with the size of the engine from $\frac{1}{8}$ to $\frac{3}{8}$ in. for roughness of castings and $\frac{1}{16}$ to $\frac{1}{8}$ in. for each working joint. Naval and other very fast-running engines

* The horse-power referred to above items 46-50 is that of the main engine, exclusive of auxiliaries.

have a larger allowance. In a vertical direct-acting engine the parts which wear so as to bring the piston nearer the bottom are three, viz., the shaft journals, the crank-pin brasses, and piston-rod gudgeon-brasses.

Thickness of Cylinder. (Thurston.) — For engines of the older types and under moderate steam-pressures, some builders have for many years restricted the stress to about 2550 lbs. per sq. in.

$$t = ap_1D + b \quad \dots \dots \dots (1)$$

is a common proportion; t , D , and b being thickness, diam., and a constant added quantity varying from 0 to $\frac{1}{2}$, all in inches; p_1 is the initial unbalanced steam-pressure, lbs. per sq. in. In this expression b is made larger for horizontal than for vertical cylinders, as, for example, in large engines 0.5 in the one case and 0.2 in the other, the one requiring rebor-ing more than the other. The constant a is from 0.0004 to 0.0005; the first value for vertical cylinders, or short strokes; the second for horizontal engines, or for long strokes.

Thickness of Cylinder and its Connections for Marine Engines. (Seaton.) — D = the diam. of the cylinder in inches; p = load on the safety-valves in lbs. per sq. in.; f , a constant multiplier, = thickness of barrel + 0.25 in.

Thickness of metal of cylinder barrel or liner, not to be less than $p \times D + 3000$ when of cast iron * $\dots \dots \dots (2)$

Thickness of cylinder-barrel = $p \times D \div 5000 + 0.6$ in. $\dots \dots \dots (3)$

Thickness of liner = $1.1 \times f$ $\dots \dots \dots (4)$

Thickness of liner when of steel = $p \times D \div 6000 + 0.5$ in.

Thickness of metal of steam-ports = $0.6 \times f$.

Thickness of metal valve-box sides = $0.65 \times f$.

| | |
|--|--|
| Thickness of metal of valve-box covers | = $0.7 \times f$. |
| " " cylinder bottom | = $1.1 \times f$, if single thickness. |
| " " " " | = $0.65 \times f$, if double " |
| " " covers | = $1.0 \times f$, if single " |
| " " " " | = $0.6 \times f$, if double " |
| " cylinder flange | = $1.4 \times f$. |
| " " cover-flange | = $1.3 \times f$. |
| " " valve-box flange | = $1.0 \times f$. |
| " " door-flange | = $0.9 \times f$. |
| " " face over ports | = $1.2 \times f$. |
| " " " " | = $1.0 \times f$, when there is a false-face. |
| " " false-face | = $0.8 \times f$, when cast iron. |
| " " " " | = $0.6 \times f$, when steel or bronze. |

Whitham gives the following from different authorities:

$$\text{Van Buren: } \begin{cases} t = 0.0001 \frac{Dp}{\sqrt{D}} + 0.15 \sqrt{D}; & \dots \dots (5) \\ t = 0.03 \sqrt{Dp} & \dots \dots \dots (6) \end{cases}$$

$$\text{Tredgold: } t = (D + 2.5) p \div 1900. \quad \dots \dots \dots (7)$$

$$\text{Weisbach: } t = 0.8 + 0.00033 pD \quad \dots \dots \dots (8)$$

$$\text{Seaton: } t = 0.5 + 0.0004 pD \quad \dots \dots \dots (9)$$

$$\text{Haswell: } \begin{cases} t = 0.0004 pD + \frac{1}{8} \text{ (vertical); } & \dots \dots (10) \\ t = 0.0005 pD + \frac{1}{8} \text{ (horizontal) } & \dots \dots (11) \end{cases}$$

Whitham recommends (6) where provision is made for the rebor-ing, and where ample strength and rigidity are secured, for horizontal or vertical cylinders of large or small diameter; (9) for large cylinders using steam under 100 lbs. gauge-pressure, and

$$t = 0.003 D \sqrt{p} \text{ for small cylinders } \dots \dots \dots (12)$$

The following table gives the calculated thickness of cylinders of engines of 10, 30, and 50 in. diam., assuming p the maximum unbalanced pressure on the piston = 100 lbs. per sq. in. As the same engines will be used for calculations of other dimensions, other particulars concerning them are here given for reference.

* When made of exceedingly good material, at least twice melted, the thickness may be 0.8 of that given by the above rules.

DIMENSIONS, ETC., OF ENGINES.

| Engine, No. | 1 and 2. | 3 and 4. | 5 and 6. |
|--|-------------|------------|-----------|
| Indicated horse-power.....I.H.P. | 50 | 450 | 1250 |
| Diam. of cyl., in..... <i>D</i> | 10 | 30 | 50 |
| Stroke, feet..... <i>L</i> | 1 ... 2 | 21½ ... 5 | 4 ... 8 |
| Revs. per min..... <i>r</i> | 250 ... 125 | 130 ... 65 | 90 ... 45 |
| Piston speed, ft. per min..... <i>S</i> | 500 | 650 | 700 |
| Area of piston, sq. in..... <i>a</i> | 78.54 | 706.86 | 1963.5 |
| Mean effective pressure.....M.E.P. | 42 | 32.3 | 30 |
| Max. total unbalanced pressure..... <i>P</i> | 7854 | 70,686 | 196,350 |
| Max. total pressure per sq. in..... <i>p</i> | 100 | 100 | 100 |

The thickness of the cylinders of these engines, according to the first eleven formulæ above quoted, ranges for engines 1 and 2 from 0.33 to 1.13 ins., for 3 and 4 from 0.99 to 2.00 ins., and for 5 and 6 from 1.56 to 3.00 ins. The averages of the 11 are, for 1 and 2, 0.76 in.; for 3 and 4, 1.48 ins.; for 5 and 6, 2.26 ins.

The average corresponds nearly to the formula $t = 0.00037 Dp + 0.4$ in. A convenient approximation is $t = 0.0004 Dp + 0.3$ in., which gives for

| | | | | | | |
|-------------------|------|------|------|------|------|----------|
| Diameters | 10 | 20 | 30 | 40 | 50 | 60 in. |
| Thicknesses | 0.70 | 1.10 | 1.50 | 1.90 | 2.30 | 2.70 in. |

The last formula corresponds to a tensile strength of cast iron of 12,500 lbs., with a factor of safety of 10 and an allowance of 0.3 in. for reborring.

Cylinder-heads. — Thurston says: Cylinder-heads may be given a thickness, at the edges and in the flanges, exceeding somewhat that of the cylinder. An excess of not less than 25% is usual. It may be thinner in the middle. Where made, as is usual in large engines, of two disks with intermediate radiating, connecting ribs or webs, that section which is safe against shearing is probably ample. An examination of the designs of experienced builders, by Professor Thurston, gave

$$t = Dp + 3000 + \frac{1}{4} \text{ inch,} \quad \dots \dots \dots (1)$$

D being the diameter of that circle in which the thickness is taken.

$$\text{Thurston also gives} \quad t = 0.005 D \sqrt{p} + 0.25 \quad \dots \dots \dots (2)$$

$$\text{Marks gives} \quad t = 0.003 \sqrt{p} \quad \dots \dots \dots (3)$$

He also says a good practical rule for pressures under 100 lbs. per sq. in. is to make the thickness of the cylinder-heads $1\frac{1}{4}$ times that of the walls; and applying this factor to his formula for thickness of walls, or $0.00028 pD$, we have

$$t = 0.00035 pD \quad \dots \dots \dots (4)$$

Whitham quotes from Seaton,

$$t = (pD + 500) + 2000, \text{ which is equal to } 0.0005 pD + 0.25 \text{ inch} \quad \dots \dots (5)$$

Seaton's formula for cylinder bottoms, quoted above, is

$$t = 0.1 f, \text{ in which } f = 0.0002 pD + 0.85 \text{ in., or } t = 0.00022 pD + 0.93 \quad \dots (6)$$

Applying the above formulæ to the engines of 10, 30, and 50 inches diameter, with maximum unbalanced steam-pressure of 100 lbs. per sq. in., we have

For cylinder 10-in. diam., 0.35 to 1.15 in.; for 30-in. diam., 0.90 to 1.75 in.; for 50-in. diam., 1.50 to 2.75 in. The averages are respectively 0.65, 1.38 and 2.10 in.

The average is expressed by the formula $t = 0.00036 Dp + 0.31$ inch.

Web-stiffened Cylinder-covers. — Seaton objects to webs for stiffening cast-iron cylinder-covers as a source of danger. The strain on the web is one of tension, and if there should be a nick or defect in the outer edge of the web the sudden application of strain is apt to start a crack. He recommends that high-pressure cylinders over 24 in. and

low-pressure cylinders over 40 in. diam. should have their covers cast hollow, with two thicknesses of metal. The depth of the cover at the middle should be about $\frac{1}{4}$ the diam. of the piston for pressures of 80 lbs. and upwards, and that of the low-pressure cylinder-cover of a compound engine equal to that of the high-pressure cylinder. Another rule is to make the depth at the middle not less than 1.3 times the diameter of the piston-rod. In the British Navy the cylinder-covers are made of steel castings, $\frac{3}{4}$ to $1\frac{1}{4}$ in. thick, generally cast without webs, stiffness being obtained by their form, which is often a series of corrugations.

Cylinder-head Bolts. — Diameter of bolt-circle for cylinder-head = diameter of cylinder + $2 \times$ thickness of cylinder + $2 \times$ diameter of bolts. The bolts should not be more than 6 inches apart (Whitham).

Marks gives for number of bolts $b = 0.7854 D^2 p \div 5000 c$, in which c = area of a single bolt, p = boiler-pressure in lbs. per sq. in.; 5000 lbs. is taken as the safe strain per sq. in. on the nominal area of the bolt.

Seaton says: Cylinder-cover studs and bolts, when made of steel, should be of such a size that the strain in them does not exceed 5000 lbs. per sq. in. When of less than $\frac{7}{8}$ inch diameter it should not exceed 4500 lbs. per sq. in. When of iron the strain should be 20% less.

Thurston says: Cylinder flanges are made a little thicker than the cylinder, and usually of equal thickness with the flanges of the heads. Cylinder-bolts should be so closely spaced as not to allow springing of the flanges and leakage, say, 4 to 5 times the thickness of the flanges. Their diameter should be proportioned for a maximum stress of not over 4000 to 5000 lbs. per square inch.

If D = diameter of cylinder, p = maximum steam-pressure, b = number of bolts, s = size or diameter of each bolt, and 5000 lbs. be allowed per sq. in. of actual area at the root of the thread, $0.7854 D^2 p = 3927 bs^2$; whence $bs^2 = 0.0002 D^2 p$;

$$b = 0.0002 \frac{D^2 p}{s^2}; s = 0.01414 D \sqrt{\frac{p}{b}}. \quad \text{For the three engines we have:}$$

| | | | |
|--|------|-----|------|
| Diameter of cylinder, inches..... | 10 | 30 | 50 |
| Diameter of bolt-circle, approx..... | 13 | 35 | 57.5 |
| Circumference of circle, approx..... | 40.8 | 110 | 180 |
| Minimum no. of bolts, circ. $\div 6$ | 7 | 18 | 30 |

$$\text{Diam. of bolts, } s = 0.01414 D \sqrt{\frac{p}{b}} \dots\dots \frac{3}{4} \text{ in.} \quad 1.00 \quad 1.29$$

The diameter of bolt for the 10-inch cylinder is 0.54 in. by the formula, but $\frac{3}{4}$ inch is as small as should be taken, on account of possible over-strain by the wrench in screwing up the nut.

The Piston. Details of Construction of Ordinary Pistons. (Seaton.) — Let D be the diameter of the piston in inches, p the effective pressure per square inch on it, x a constant multiplier, found as follows:

$$x = (D \div 50) \times \sqrt{p} + 1.$$

| | | |
|--|-----------------------|-------------------------------|
| The thickness of front of piston near the boss | = 0.2 | $\times x$. |
| “ “ “ “ rim | = 0.17 | $\times x$. |
| “ back “ | = 0.18 | $\times x$. |
| “ boss around the rod | = 0.3 | $\times x$. |
| “ flange inside packing-ring | = 0.23 | $\times x$. |
| “ “ at edge | = 0.25 | $\times x$. |
| “ packing-ring | = 0.15 | $\times x$. |
| “ junk-ring at edge | = 0.23 | $\times x$. |
| “ “ inside packing-ring | = 0.21 | $\times x$. |
| “ “ at bolt-holes | = 0.35 | $\times x$. |
| “ metal around piston edge | = 0.25 | $\times x$. |
| The breadth of packing-ring | = 0.63 | $\times x$. |
| “ depth of piston at center | = 1.4 | $\times x$. |
| “ lap of junk-ring on the piston | = 0.45 | $\times x$. |
| “ space between piston body and packing-ring | = 0.3 | $\times x$. |
| “ diameter of junk-ring bolts | = 0.1 | $\times x + 0.25 \text{ in.}$ |
| “ pitch of junk-ring bolts | = 10 | diameters. |
| “ number of webs in the piston | = (D + 20) \div 12. | |
| “ thickness of webs in the piston | = 0.18 | $\times x$. |

Marks gives the approximate rule: Thickness of piston-head = $\sqrt[3]{lD}$, in which l = length of stroke, and D = diameter of cylinder in inches. Whitham says: in a horizontal engine the rings support the piston, or at least a part of it, under ordinary conditions. The pressure due to the weight of the piston upon an area equal to 0.7 the diameter of the cylinder \times breadth of ring-face, should never exceed 200 lbs. per sq. in. He also gives a formula much used in this country: Breadth of ring-face = $0.15 \times$ diameter of cylinder.

| For our engines we have diameter = | 10 | 30 | 50 |
|---|---------------------------|------|-------|
| | Thickness of piston-head. | | |
| Marks, $\sqrt[3]{lD}$; long stroke..... | 3.31 | 5.48 | 7.00 |
| Marks, $\sqrt[3]{lD}$; short stroke..... | 3.94 | 6.51 | 8.32 |
| Seaton, depth at center = $1.4x$ | 4.20 | 9.80 | 15.40 |
| Seaton, breadth of ring = $0.63x$ | 1.89 | 4.41 | 6.93 |
| Whitham, breadth of ring = $0.15D$ | 1.50 | 4.50 | 7.50 |

Diameter of Piston Packing-rings. — These are generally turned, before they are cut, about $\frac{1}{4}$ inch diameter larger than the cylinder, for cylinders up to 20 inches diameter, and then enough is cut out of the rings to spring them to the diameter of the cylinder. For larger cylinders the rings are turned proportionately larger. Seaton recommends an excess of 1% of the diameter of the cylinder.

A theoretical paper on Piston Packing Rings of Modern Steam Engines by O. C. Reymann will be found in *Jour. Frank. Inst.*, Aug., 1897.

Cross-section of the Rings. — The thickness is commonly made $\frac{1}{30}$ of the diam. of cyl. + $\frac{1}{8}$ inch, and the width = thickness + $\frac{1}{8}$ inch. For an eccentric ring the mean thickness may be the same as for a ring of uniform thickness, and the minimum thickness = $\frac{2}{3}$ the maximum.

A circular issued by J. H. Dunbar, manufacturer of packing-rings, Youngstown, Ohio, says: Unless otherwise ordered, the thickness of rings will be made equal to $0.03 \times$ their diameter. This thickness has been found to be satisfactory in practice. It admits of the ring being made about $\frac{3}{16}$ in. to the foot larger than the cylinder, and has, when new, a tension of about two pounds per inch of circumference, which is ample to prevent leakage if the surface of the ring and cylinder are smooth.

As regards the width of rings, authorities "scatter" from very narrow to very wide, the latter being fully ten times the former. For instance, Unwin gives $W = 0.014d + 0.08$. Whitham's formula is $W = 0.15d$. In both formulæ W is the width of the ring in inches, and d the diameter of the cylinder in inches. Unwin's formula makes the width of a 20 in. ring $W = 20 \times 0.014 + 0.08 = 0.36$ in., while Whitham's is $20 \times 0.15 = 3$ in. for the same diameter of ring. There is much less difference in the practice of engine-builders in this respect, but there is still room for a standard width of ring. It is believed that for cylinders over 16 in. diameter $\frac{3}{4}$ in. is a popular and practical width, and $\frac{1}{2}$ in. for cylinders of that size and under.

E. R. McGahey, *Machy.*, Feb., 1906, gives the following tables for sizes of piston rings for cylinders 6 to 20 in., diameter. A = (outside diam. of ring — bore of cylinder); B = thickness (radial) of equal section ring, or least thickness of eccentric ring; C = width of ring (axial); D = amount cut out or lap; E = greatest thickness of eccentric ring.

EQUAL SECTION RINGS.

| Diam. | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|-------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|---------------|-----------------|-----------------|
| A | $\frac{5}{32}$ | $\frac{5}{32}$ | $\frac{3}{16}$ | $\frac{3}{16}$ | $\frac{7}{32}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{9}{32}$ | $\frac{9}{32}$ | $\frac{5}{16}$ | $\frac{11}{32}$ | $\frac{11}{32}$ | $\frac{3}{8}$ | $\frac{13}{32}$ | $\frac{13}{32}$ |
| B | $\frac{1}{4}$ | $\frac{9}{32}$ | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{13}{32}$ | $\frac{7}{16}$ | $\frac{15}{32}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{19}{32}$ | $\frac{5}{8}$ | $\frac{11}{16}$ | $\frac{7}{4}$ | $\frac{3}{4}$ | $\frac{17}{16}$ |
| C | $\frac{5}{16}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{7}{16}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{9}{16}$ | $\frac{9}{16}$ | $\frac{11}{16}$ | $\frac{11}{16}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{13}{16}$ | $\frac{12}{16}$ |
| D | $\frac{3}{64}$ | $\frac{39}{64}$ | $\frac{21}{32}$ | $\frac{23}{32}$ | $\frac{25}{32}$ | $\frac{27}{32}$ | $\frac{7}{8}$ | $\frac{15}{16}$ | 1 | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{9}{32}$ | $\frac{11}{32}$ |

ECCENTRIC RINGS.

| Diam. | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|--------|---------|
| A | 5/32 | 5/32 | 3/16 | 3/16 | 7/32 | 1/4 | 1/4 | 9/32 | 9/32 | 5/16 | 11/32 | 11/32 | 3/8 | 13/32 | 13/32 |
| B | 3/16 | 7/32 | 1/4 | 9/32 | 9/32 | 5/16 | 11/32 | 3/8 | 13/32 | 7/16 | 15/32 | 15/32 | 1/2 | 17/32 | 9/16 |
| C | 5/16 | 3/8 | 3/8 | 7/16 | 7/16 | 1/2 | 1/2 | 9/16 | 9/16 | 11/16 | 11/16 | 3/4 | 3/4 | 13/16 | 13/16 |
| D | 35/64 | 39/64 | 21/32 | 23/32 | 25/32 | 27/32 | 7/8 | 15/16 | 1 | 11/16 | 1 1/8 | 1 3/16 | 1 1/4 | 1 9/32 | 1 11/32 |
| E | 9/32 | 5/16 | 11/32 | 3/8 | 13/32 | 7/16 | 15/32 | 1/2 | 9/16 | 5/8 | 11/16 | 11/16 | 3/4 | 13/16 | 7/8 |

Fit of Piston-rod into Piston. (Seaton.) — The most convenient and reliable practice is to turn the piston-rod end with a shoulder of 1/16 inch for small engines, and 1/8 inch for large ones, make the taper 3 in. to the foot until the section of the rod is three-fourths of that of the body, then turn the remaining part parallel; the rod should then fit into the piston so as to leave 1/8 in. between it and the shoulder for large pistons and 1/16 in. for small. The shoulder prevents the rod from splitting the piston, and allows of the rod being turned true after long wear without encroaching on the taper.

The piston is secured to the rod by a nut, and the size of the rod should be such that the strain on the section at the bottom of the thread does not exceed 5500 lbs. per sq. in. for iron, 7000 lbs. for steel. The depth of this nut need not exceed the diameter which would be found by allowing these strains. The nut should be locked to prevent its working loose.

Diameter of Piston-rods. — Unwin gives

$$d'' = bD \sqrt{p}, \quad \dots \dots \dots (1)$$

in which D is the cylinder diameter in inches, p is the maximum unbalanced pressure in lbs. per sq. in., and the constant $b = 0.0167$ for iron, and $b = 0.0144$ for steel. Thurston, from an examination of a considerable number of rods in use, gives

$$d'' = \sqrt[4]{\frac{D^2 p L^2}{a}} + \frac{D}{80}, \text{ nearly } \dots \dots \dots (2)$$

(L in feet, D and d in inches), in which $a = 10,000$ and upward in the various types of engines, the marine screw engines or ordinary fast engines on shore are given the lowest values, while "low-speed engines" being less liable to accident from shock are given $a = 15,000$, often.

Connections of the piston-rod to the piston and to the cross-head should have a factor of safety of at least 8 or 10. Marks gives

$$d'' = 0.0179 D \sqrt{p}, \text{ for iron; for steel } d'' = 0.0105 D \sqrt{p}; \quad \dots \dots (3)$$

$$\text{and } d'' = 0.03901 \sqrt[4]{D^2 l^2 p}, \text{ for iron; for steel } d'' = 0.03525 \sqrt[4]{D^2 l^2 p}, \quad \dots \dots (4)$$

in which l is the length of stroke, all dimensions in inches. Deduce the diameter of piston-rod by (3), and if this diameter is less than 1/12 l , then use (4).

$$\text{Seaton gives: Diameter of piston-rod} = \frac{\text{Diameter of cylinder}}{F} \sqrt{p}.$$

The following are the values of F :

| | | |
|---|-------|----------|
| Naval engines, direct-acting | | $F = 60$ |
| return connecting-rod, 2 rods | | $F = 80$ |
| Mercantile ordinary stroke, direct-acting | | $F = 50$ |
| long " " " " " " | | $F = 48$ |
| very long " " " " " " | | $F = 45$ |
| medium stroke, oscillating | | $F = 45$ |

NOTE. — Long and very long, as compared with the stroke usual for the power of engine or size of cylinder.

In considering an expansive engine, p , the effective pressure, should be taken as the absolute working pressure, or 15 lbs. above that to which the boiler safety-valve is loaded; for a compound engine the value of p for the high-pressure piston should be taken as the absolute pressure, less 15 lbs., or the same as the load on the safety-valve; for the medium-pressure the load may be taken as that due to half the absolute boiler-pressure; and for the low-pressure cylinder the pressure to which the escape-valve is loaded + 15 lbs., or the maximum absolute pressure which can be got in the receiver, or about 25 lbs. It is an advantage to make all the rods of a compound engine alike, and this is now the rule.

Applying the above formulæ to the engines of 10, 30, and 50 in. diameter, both short and long stroke, we have:

Diameter of Piston-rods.

| Diameter of Cylinder, inches..... | 10 | | 30 | | 50 | |
|---|--------|-------|--------|-------|--------|-------|
| Stroke, inches..... | 12 | 24 | 30 | 60 | 48 | 96 |
| Unwin, iron, $0.0167 D \sqrt{p}$ | 1.67 | 1.67 | 5.01 | 5.01 | 8.35 | 8.35 |
| Unwin, steel, $0.0144 D \sqrt{p}$ | 1.44 | 1.44 | 4.32 | 4.32 | 7.20 | 7.20 |
| Thurston $\sqrt[4]{\frac{D^2 p L^2}{10,000}} + \frac{D}{80}$ (L in feet).... | 1.13 | | 3.12 | | 5.10 | |
| Thurston, same with $a = 15,000$ | | 1.40 | | 3.88 | | 6.35 |
| Marks, iron, $0.0179 D \sqrt{p}$ | 1.79 | | 5.37 | 5.37 | 8.95 | 8.95 |
| Marks, iron, $0.03901 \sqrt[4]{D^2 L^2 p}$ | 1.35 | 1.91 | 3.70 | 5.13 | 6.04 | 8.54 |
| Marks, steel, $0.0105 D \sqrt{p}$ | (1.05) | | (3.15) | | (5.25) | |
| Marks, steel, $0.03525 \sqrt[4]{D^2 L^2 p}$ | 1.22 | 1.73 | 3.34 | 4.72 | 5.46 | 7.72 |
| Seaton, naval engines, $\frac{D}{60} \sqrt{p}$ | 1.67 | | 5.01 | | 8.35 | |
| Seaton, land engine, $\frac{D}{45} \sqrt{p}$ | | 2.22 | | 6.67 | | 11.11 |
| Average of four for iron..... | 1.49 | 1.82 | 4.30 | 5.26 | 7.11 | 8.74 |

The figures in parentheses opposite Marks's third formula would be rejected since they are less than $\frac{1}{8}$ of the stroke, and the figures derived by his fourth formula would be taken instead. The figure 1.79 opposite his first formula would be rejected for the engine of 24-inch stroke.

An empirical formula which gives results approximating the above averages is $d'' = 0.0145 \sqrt{Dlp}$ for short stroke and $0.013 \sqrt{Dlp}$ for long stroke engines.

The calculated results for this formula, for the six engines, are, respectively, 1.58, 2.02, 4.35, 5.52, 7.10, 9.01.

Piston-rod Guides. — The thrust on the guide, when the connecting-rod is at its maximum angle with the line of the piston-rod, is found from the formula: Thrust = total load on piston \times tangent of maximum angle of connecting rod = $p \tan \theta$. This angle, θ , is the angle whose sine = half stroke of piston \div length of connecting-rod.

| | | | |
|--|---------|---------|--------|
| Ratio of length of connecting-rod to stroke.... | 2 | 2½ | 3 |
| Maximum angle of connecting-rod with line of piston-rod..... | 14° 29' | 11° 33' | 9° 36' |
| Tangent of the angle..... | 0.258 | 0.204 | 0.169 |
| Secant of the angle..... | 1.0327 | 1.0206 | 1.014 |

Seaton says: The area of the guide-block or slipper surface on which the thrust is taken should in no case be less than will admit of a pressure of 400 lbs., on the square inch; and for good working those surfaces which take the thrust when going ahead should be sufficiently large to prevent

the maximum pressure exceeding 100 lbs. per sq. in. When the surfaces are kept well lubricated this allowance may be exceeded.

Thurston says: The rubbing surfaces of guides are so proportioned that if V be their relative velocity in feet per minute, and p be the intensity of pressure on the guide in lbs. per sq. in., $pV < 60,000$ and $pV > 40,000$.

The lower is the safer limit; but for marine and stationary engines it is allowable to take $p = 60,000 \div V$. According to Rankine, for loco-

motives, $p = \frac{44,800}{V + 20}$, where p is the pressure in lbs. per sq. in. and V the velocity of rubbing in feet per minute. This includes the sum of all pressures forcing the two rubbing surfaces together.

Some British builders of portable engines restrict the pressure between the guides and cross-heads to less than 40, sometimes 35 lbs. per square inch.

For a mean velocity of 600 feet per minute, Prof. Thurston's formulas give, $p < 100$, $p > 66.7$; Rankine's gives $p = 72.2$ lbs. per sq. in.

Whitham gives,

$$A = \text{area of slides in square inches} = \frac{P}{p_0 \sqrt{n^2 - 1}} = \frac{0.7854 d^2 p_1}{p_0 \sqrt{n^2 - 1}}$$

in which P = total unbalanced pressure, p_1 = pressure per square inch on piston, d = diameter of cylinder, p_0 = pressure allowable per square inch on slides, and n = length of connecting-rod \div length of crank. This is equivalent to the formula, $A = P \tan \theta \div p_0$. For $n = 5$, $p_1 = 100$ and $p_0 = 80$, $A = 0.2004 d^2$. For the three engines 10, 30 and 50 in. diam., this would give for area of slides, $A = 20, 180$ and 500 sq. in., respectively. Whitham says: The normal pressure on the slide may be as high as 500 lbs. per sq. in., but this is when there is good lubrication and freedom from dust. Stationary and marine engines are usually designed to carry 100 lbs. per sq. in., and the area in this case is reduced from 50% to 60% by grooves. In locomotive engines the pressure ranges from 40 to 50 lbs. per sq. in. of slide, on account of the inaccessibility of the slide, dirt, cinder, etc.

There is perfect agreement among the authorities as to the formula for area of the slides, $A = P \tan \theta \div p_0$; but the value given to p_0 , the allowable pressure per square inch, ranges all the way from 35 lbs. to 500 lbs.

The Connecting-rod. *Ratio of length of connecting-rod to length of stroke.* — Experience has led generally to the ratio of 2 or $2\frac{1}{2}$ to 1, the latter giving a long and easy-working rod, the former a rather short, but yet a manageable one (Thurston). Whitham gives the ratio of from 2 to $4\frac{1}{2}$, and Marks from 2 to 4.

Dimensions of the Connecting-rod. — The calculation of the diameter of a connecting-rod on a theoretical basis, considering it as a strut subject to both compressive and bending stresses, and also to stress due to its inertia, in high-speed engines, is quite complicated. See Whitham, *Steam-engine Design*, p. 217; Thurston, *Manual of S. E.*, p. 100. Empirical formulas are as follows: For circular rods, largest at the middle, D = diam. of cylinder, l = length of connecting-rod in inches, p = maximum steam-pressure, lbs. per sq. in.

- (1) Whitham, diam. at middle, $d'' = 0.0272 \sqrt{Dl \sqrt{p}}$.
- (2) Whitham, diam. at necks, $d'' = 1.0$ to $1.1 \times$ diam. of piston-rod.
- (3) Sennett, diam. at middle, $d'' = D \sqrt{p} \div 55$.
- (4) Sennett, diam. at necks, $d'' = D \sqrt{p} \div 60$.
- (5) Marks, diam., $d'' = 0.0179 D \sqrt{p}$, if diam. is greater than $\frac{1}{24}$ length.
- (6) Marks, diam., $d'' = 0.02758 \sqrt{Dl \sqrt{p}}$, if diam. found by (5) is less than $\frac{1}{24}$ length.
- (7) Thurston, diam., at middle, $d'' = a \sqrt{DL \sqrt{p}} + C$, D in inches, L in feet, $a = 0.15$ and $C = \frac{1}{2}$ inch for fast engines, $a = 0.08$ and $C = \frac{3}{4}$ inch for moderate speed.

(8) Seaton says: The rod may be considered as a strut free at both ends, and, calculating its diameter accordingly,

$$\text{diameter at middle} = \sqrt{R(1 + 4ar^2)} \div 48.5,$$

where R = the total load on piston P multiplied by the secant of the maximum angle of obliquity of the connecting-rod.

For wrought iron and mild steel a is taken at $1/3000$. The following are the values of r in practice:

| | | |
|-----------------|----------------------------|--------------------------|
| Naval engines — | Direct-acting | $r = 9$ to 11 ; |
| “ “ | Return connecting-rod | $r = 10$ to 13 , old; |
| “ “ | Return connecting-rod | $r = 8$ to 9 , modern; |
| “ “ | Trunk | $r = 11.5$ to 13 . |
| Mercantile “ | Direct-acting, ordinary | $r = 12$. |
| Mercantile “ | Direct-acting, long stroke | $r = 13$ to 16 . |

(9) The following empirical formula is given by Seaton as agreeing closely with good modern practice:

Diameter of connecting-rod at middle = $\sqrt{lK} \div 4$, l = length of rod in inches, and $K = 0.03 \sqrt{\text{effective load on piston in pounds}}$.

The diam. at the ends may be 0.875 of the diam. at the middle.

Seaton's empirical formula when translated into terms of D and p is the same as the second one by Marks, viz., $d'' = 0.02758 \sqrt{Dl \sqrt{p}}$. Whitham's (1) is also practically the same.

(10) Taking Seaton's more complex formula, with length of connecting-rod = $2.5 \times$ length of stroke, and $r = 12$ and 16 , respectively, it reduces to: Diam. at middle = $0.02294 \sqrt{P}$ and $0.02411 \sqrt{P}$ for short and long stroke engines, respectively.

Applying the above formulas to the engines of our list, we have

Diameter of Connecting-rods.

| Diameter of Cylinder, inches..... | 10 | | 30 | | 50 | |
|---|-------|-------|-------|-------|-------|-------|
| Stroke, inches..... | 12 | 24 | 30 | 60 | 48 | 96 |
| Length of connecting-rod l | 30 | 60 | 75 | 150 | 120 | 240 |
| (3) $d'' = \frac{D}{55} \sqrt{p} = 0.0182 D \sqrt{p}$ | 1.82 | 1.82 | 5.46 | 5.46 | 9.09 | 9.09 |
| (5) $d'' = 0.0179 D \sqrt{p}$ | 1.79 | | 5.37 | | 8.95 | |
| (6) $d'' = 0.02758 \sqrt{Dl \sqrt{p}}$ | | 2.14 | | 5.85 | | 9.51 |
| (7) $d'' = 0.15 \sqrt{DL \sqrt{p} + 1/2}$ | 2.87 | | 7.00 | | 11.11 | |
| (7) $d'' = 0.08 \sqrt{DL \sqrt{p} + 3/4}$ | | 2.54 | | 5.65 | | 8.75 |
| (9) $d'' = 0.03 \sqrt{P}$ | 2.67 | 2.67 | 7.97 | 7.97 | 13.29 | 13.29 |
| (10) $d'' = 0.02294 \sqrt{P}$; $0.02411 \sqrt{P}$ | 2.03 | 2.14 | 6.09 | 6.41 | 10.16 | 10.68 |
| Average..... | 2.24 | 2.26 | 6.38 | 6.27 | 10.52 | 10.26 |

Formulae 5 and 6 (Marks), and also formula 10 (Seaton), give the larger diameters for the long-stroke engine; formulae 7 give the larger diameters for the short-stroke engines. The average figures show but little difference in diameter between long- and short-stroke engines; this is what might be expected, for while the connecting-rod, considered simply as a column, would require an increase of diameter for an increase of length, the load remaining the same, yet in an engine generally the shorter the connecting-rod the greater the number of revolutions, and consequently the greater the strains due to inertia. The influences tending to increase the diameter therefore tend to balance each other, and to render the diameter to some extent independent of the length. The average figures correspond nearly to the simple formula $d'' = 0.021 D \sqrt{p}$. The diameters of rod for the three diameters of engine by this formula are, respectively, 2.10, 6.30, and 10.50 in. Since the total pressure on the piston $P = 0.7854 D^2 p$, the formula is equivalent to $d'' = 0.0237 \sqrt{P}$.

Connecting-rod Ends. — For a connecting-rod end of the marine type, where the end is secured with two bolts, each bolt should be proportioned for a safe tensile strength equal to two-thirds the maximum pull or thrust in the connecting-rod.

The cap is to be proportioned as a beam loaded with the maximum pull of the connecting-rod, and supported at both ends. The calculation should be made for rigidity as well as strength, allowing a maximum deflection of $\frac{1}{100}$ inch. For a strap-and-key connecting-rod end the strap is designed for tensile strength, considering that two-thirds of the pull on the connecting-rod may come on one arm. At the point where the metal is slotted for the key and gib, the straps must be thickened to make the cross-section equal to that of the remainder of the strap. Between the end of the strap and the slot the strap is liable to fail in double shear, and sufficient metal must be provided at the end to prevent such failure.

The breadth of the key is generally one-fourth of the width of the strap, and the length, parallel to the strap, should be such that the cross-section will have a shearing strength equal to the tensile strength of the section of the strap. The taper of the key is generally about $\frac{5}{8}$ inch to the foot.

Tapered Connecting-rods. — In modern high-speed engines it is customary to make the connecting-rods of rectangular instead of circular section, the sides being parallel, and the depth increasing regularly from the crosshead end to the crank-pin end. According to Grashof, the bending action on the rod due to its inertia is greatest at $\frac{6}{10}$ the length from the crosshead end, and, according to this theory, that is the point at which the section should be greatest, although in practice the section is made greatest at the crank-pin end.

Professor Thurston furnishes the author with the following rule for tapered connecting-rod of rectangular section: Take the section as computed by the formula $d'' = 0.1 \sqrt{DL \sqrt{p} + 3/4}$ for a circular section,

and for a rod $\frac{4}{3}$ the actual length, placing the computed section at $\frac{2}{3}$ the length from the small end, and carrying the taper straight through this fixed section to the large end. This brings the computed section at the surge point and makes it heavier than the rod for which a tapered form is not required.

Taking the above formula, multiplying L by $\frac{4}{3}$, and changing it to l

in inches, it becomes $d = \frac{1}{30} \sqrt{Dl \sqrt{p} + 3/4}$ in. Taking a rectangular section of the same area as the round section whose diameter is d , and making the depth of the section $h =$ twice the thickness t , we have

$0.7854 d^2 = ht = 2 t^2$, whence $t = 0.627, d = 0.0209 \sqrt{Dl \sqrt{p} + 0.47}$ in., which is the formula for the thickness or distance between the parallel sides of the rod. Making the depth at the crosshead end $= 1.5 t$, and at $\frac{2}{3}$ the length $= 2 t$, the equivalent depth at the crank end is $2.25 t$. Applying the formula to the short-stroke engines of our examples, we have

| | | | |
|---|------|------|-------|
| Diameter of cylinder, inches..... | 10 | 30 | 50 |
| Stroke, inches..... | 12 | 30 | 48 |
| Length of connecting-rod..... | 30 | 75 | 120 |
| Thickness, $t = 0.0209 \sqrt{Dl \sqrt{p} + 0.47} =$ | 1.61 | 3.60 | 5.59 |
| Depth at crosshead end, $1.5 t =$ | 2.42 | 5.41 | 8.39 |
| Depth at crank end, $2\frac{1}{4} t =$ | 3.62 | 8.11 | 12.58 |

The thicknesses t , found by the formula $t = 0.0209 \sqrt{Dl \sqrt{p} + 0.47}$, agree closely with the more simple formula $t = 0.01 D \sqrt{p} + 0.60$ in., the thicknesses calculated by this formula being respectively 1.6, 3.6, and 5.6 inches.

The Crank-pin. — A crank-pin should be designed (1) to avoid heating, (2) for strength, (3) for rigidity. The heating of a crank-pin depends on the pressure on its rubbing surface, and on the coefficient of friction, which latter varies greatly according to the effectiveness of

the lubrication. It also depends upon the facility with which the heat produced may be carried away: thus it appears that locomotive crank-pins may be prevented to some degree from overheating by the cooling action of the air through which they pass at a high speed.

Marks gives $l = 0.0000247 fpND^2 = 1.038 f (\text{I.H.P.}) \div L$. . . (1)

Whitham gives $l = 0.9075 f. (\text{I.H.P.}) \div L$ (2)

in which l = length of crank-pin journal in inches, f = coefficient of friction, which may be taken at 0.03 to 0.05 for perfect lubrication, and 0.08 to 0.10 for imperfect; p = mean pressure in the cylinder in pounds per square inch; D = diameter of cylinder in inches; N = number of single strokes per minute; I.H.P. = indicated horse-power; L = length of stroke in feet. These formulæ are independent of the diameter of the pin, and Marks states as a general law, within reasonable limits as to pressure and speed of rubbing, the longer a bearing is made, for a given pressure and number of revolutions, the cooler it will work; and its diameter has no effect upon its heating. Both of the above formulæ are deduced empirically from dimensions of crank-pins of existing marine engines. Marks says that about one-fourth the length required for crank-pins of propeller engines will serve for the pins of side-wheel engines, and one-tenth for locomotive engines, making the formula for locomotive crank-pins $l = 0.00000247 fpND^2$, or if $p = 150$, $f = 0.06$, and $N = 600$, $l = 0.013D^2$.

Whitham recommends for pressure per square inch of projected area, for naval engines 500 pounds, for merchant engines 400 pounds, for paddle-wheel engines 800 to 900 pounds.

Thurston says the pressure should, in the steam-engine, never exceed 500 or 600 pounds per square inch for wrought-iron pins, or about twice that figure for steel. He gives the formula for length of a steel pin, in inches,

$$l = PR \div 600,000, \quad (3)$$

in which P and R are the mean total load on the pin in pounds, and the number of revolutions per minute. For locomotives, the divisor may be taken as 500,000. Where iron is used this figure should be reduced to 300,000 and 250,000 for the two cases taken. Pins so proportioned, if well made and well lubricated, may always be depended upon to run cool; if not well formed, perfectly cylindrical, well finished, and kept well oiled, no crank-pin can be relied upon. It is assumed above that good bronze or white-metal bearings are used.

Thurston also says: The size of crank-pins required to prevent heating of the journals may be determined with a fair degree of precision by either of the formulæ given below:

$$l = P(V + 20) \div 44,800 d \text{ (Rankine, 1865); } . . . (4)$$

$$l = PV \div 60,000 d \text{ (Thurston, 1862); } . . . (5)$$

$$l = PN - 350,000 \text{ (Van Buren, 1866). } . . . (6)$$

The first two formulæ give what are considered by their authors fair working proportions, and the last gives minimum length for iron pins. (V = velocity of rubbing surface in feet per minute.)

Formula (1) was obtained by observing locomotive practice in which great liability exists of annoyance by dust, and great risk occurs from inaccessibility while running, and (2) by observation of crank-pins of naval screw-engines. The first formula is therefore not well suited for marine practice.

Steel can usually be worked at nearly double the pressure admissible with iron running at similar speed.

Since the length of the crank-pin will be directly as the power expended upon it and inversely as the pressure, we may take it as

$$l = a (\text{I.H.P.}) \div L, \quad (7)$$

In which a is a constant, and L the stroke of piston, in feet. The values of the constant, as obtained by Mr. Skeel, are about as follows: $a = 0.04$ where water can be constantly used; $a = 0.045$ where water is not generally used; $a = 0.05$ where water is seldom used; $a = 0.06$ where water is never needed. Unwin gives

$$l = a (\text{I.H.P.}) + r, \quad (8)$$

In which r = crank radius in inches, $a = 0.3$ to $a = 0.4$ for iron and for marine engines, and $a = 0.066$ to $a = 0.1$ for the case of the best steel and for locomotive work, where it is often necessary to shorten up outside pins as much as possible.

J. B. Stanwood (*Eng'g*, June 12, 1891), in a table of dimensions of parts of American Corliss engines from 10 to 30 inches diameter of cylinder, gives sizes of crank-pins which approximate closely to the formula

$$l = 0.275 D'' + 0.5 \text{ in.}; \quad d = 0.25 D'' \quad \dots \quad (9)$$

By calculating lengths of iron crank-pins for the engines 10, 30, and 50 inches diameter, long and short stroke, by the several formulæ above given, it is found that there is a great difference in the results, so that one formula in certain cases gives a length three times as great as another. Nos. (4), (5), and (6) give lengths much greater than the others. Marks (1), Whitham (2), Thurston (7), $l = 0.06 \text{ I.H.P.} \div L$, and Unwin (8), $l = 0.4 \text{ I.H.P.} \div r$, give results which agree more closely.

The calculated lengths of iron crank-pins for the several cases by formulæ (1), (2), (7), and (8) are as follows:

Length of Crank-pins.

| Diameter of cylinder..... D | 10 | 10 | 30 | 30 | 50 | 50 |
|--|-------|-------|--------|--------|---------|---------|
| Stroke..... L (ft.) | 1 | 2 | 21/2 | 5 | 4 | 8 |
| Revolutions per minute..... R | 250 | 125 | 130 | 65 | 90 | 45 |
| Horse-power.....I.H.P. | 50 | 50 | 450 | 450 | 1,250 | 1,250 |
| Maximum pressure.....lbs. | 7,854 | 7,854 | 70,686 | 70,686 | 196,350 | 196,350 |
| Mean pressure per cent of max..... | 42 | 42 | 32.3 | 32.3 | 30 | 30 |
| Mean pressure..... P | 3,299 | 3,299 | 22,832 | 22,832 | 58,905 | 58,905 |
| Length of crank-pin: | | | | | | |
| (1) Whitham, $l = 0.9075 \times .05 \text{ I.H.P.} \div L$ | 2.18 | 1.09 | 8.17 | 4.08 | 14.18 | 7.09 |
| (2) Marks, $l = 1.038 \times .05 \text{ I.H.P.} \div L$ | 2.59 | 1.30 | 9.34 | 4.67 | 16.22 | 8.11 |
| (7) Thurston, $l = 0.06 \text{ I.H.P.} \div L$ | 3.00 | 1.50 | 10.80 | 5.40 | 18.75 | 9.38 |
| (8) Unwin, $l = 0.4 \text{ I.H.P.} \div r$ | 3.33 | 1.67 | 12.0 | 6.0 | 20.83 | 10.42 |
| (8) Unwin, $l = 0.3 \text{ I.H.P.} \div r$ | 2.50 | 1.25 | 9.0 | 4.5 | 15.62 | 7.81 |
| Average..... | 2.72 | 1.36 | 9.86 | 4.93 | 17.12 | 8.56 |

| | | | | | | |
|--|------|------|------|------|------|------|
| (8) Unwin, best steel, $l = 0.1 \text{ I.H.P.} \div r$ | 1.83 | 0.42 | 3.0 | 1.5 | 5.21 | 2.61 |
| (3) Thurston, steel, $l = PR \div 600,000$ | 1.37 | 0.69 | 4.95 | 2.47 | 8.84 | 4.42 |

The calculated lengths for the long-stroke engines are too low to prevent excessive pressures. See "Pressures on the Crank-pins," below.

The Strength of the Crank-pin is determined substantially as is that of the crank. In overhung cranks the load is usually assumed as carried at its extremity, and, equating its moment with that of the resistance of the pin,

$$\frac{1}{2} Pl = \frac{1}{32} \pi d^3, \text{ and } d = \sqrt[3]{\frac{5.1 Pl}{t}},$$

In which d = diameter of pin in inches, P = maximum load on the piston, t = the maximum allowable stress on a square inch of the metal. For iron it may be taken at 9000 lbs. For steel the diameters found by this formula may be reduced 10%. (Thurston.)

Unwin gives the same formula in another form, viz.:

$$d = \sqrt[3]{\frac{5.1}{t}} \sqrt[3]{Pl} = \sqrt{\frac{5.1}{t}} \sqrt{P} \frac{l}{d},$$

the last form to be used when the ratio of length to diameter is assumed.

For wrought iron, $t = 6000$ to 9000 lbs. per sq. in.,

$$\sqrt[3]{5.1/t} = 0.0947 \text{ to } 0.0827; \quad \sqrt{5.1/t} = 0.0291 \text{ to } 0.0238.$$

For steel, $t = 9000$ to 13,000 lbs. per sq. in.,

$$\sqrt[3]{5.1/t} = 0.0827 \text{ to } 0.0723; \quad \sqrt{5.1/t} = 0.0238 \text{ to } 0.0194.$$

Whitham gives $d = 0.0827 \sqrt[3]{Pl} = 2.1058 \sqrt[3]{l \times \text{I.H.P.} + LR}$ for strength, and $d = 0.0405 \sqrt[4]{Pl^3}$ for rigidity, and recommends that the diameter be calculated by both formulæ, and the largest result taken. The first is the same as Unwin's formula, with t taken at 9000 lbs. per sq. in. The second is based upon an arbitrary assumption of a deflection of $\frac{1}{300}$ in. at the center of pressure (one-third of the length from the free end).

Marks, calculating the diameter for rigidity, gives

$$d = 0.066 \sqrt[4]{pl^3D^2} = 0.945 \sqrt[4]{(\text{H.P.})l^3 + LN};$$

p = maximum steam-pressure in pounds per square inch, D = diameter of cylinder in inches, L = length of stroke in feet, N = number of single strokes per minute. He says there is no need of an investigation of the strength of a crank-pin, as the condition of rigidity gives a great excess of strength.

Marks's formula is based upon the assumption that the whole load may be concentrated at the outer end, and cause a deflection of 0.01 in. at that point. It is serviceable, he says, for steel and for wrought iron alike.

Using the average lengths of the crank-pins already found, we have the following for our six engines:

Diameter of Crank-pins.

| Diameter of cylinder..... | 10 | 10 | 30 | 30 | 50 | 50 |
|---|------|------|------|------|-------|------|
| Stroke, ft..... | 1 | 2 | 2½ | 5 | 4 | 8 |
| Length of crank-pin..... | 2.72 | 1.36 | 9.86 | 4.93 | 17.12 | 8.56 |
| Unwin, $d = \sqrt[3]{\frac{5.1 Pl}{t}}$ | 2.29 | 1.82 | 7.34 | 5.82 | 12.40 | 9.84 |
| Marks, $d = 0.066 \sqrt[4]{pl^3D^2}$ | 1.39 | 0.85 | 6.44 | 3.78 | 12.41 | 7.39 |

Pressures on the Crank-pins. — If we take the mean pressure upon the crank-pin = mean pressure on piston, neglecting the effect of the varying angle of the connecting-rod, we have the following, using the average lengths already found, and the diameters according to Unwin and Marks:

| Engine No..... | 1 | 2 | 3 | 4 | 5 | 6 |
|--------------------------------------|-------|-------|--------|--------|--------|--------|
| Diameter of cylinder, inches..... | 10 | 10 | 30 | 30 | 50 | 50 |
| Stroke, feet..... | 1 | 2 | 2½ | 5 | 4 | 8 |
| Mean pressure on pin, pounds..... | 3,299 | 3,299 | 22,832 | 22,832 | 58,905 | 58,905 |
| Projected area of pin, Unwin..... | 6.23 | 2.36 | 72.4 | 28.7 | 212.3 | 84.2 |
| Projected area of pin, Marks..... | 3.78 | 1.16 | 63.5 | 18.6 | 212.5 | 63.3 |
| Pressure per square inch, Unwin..... | 530 | 1,398 | 315 | 796 | 277 | 700 |
| Pressure per square inch, Marks..... | 873 | 2,845 | 360 | 1,228 | 277 | 930 |

The results show that the application of the formulæ for length and diameter of crank-pins give quite low pressures per square inch of projected area for the short-stroke high-speed engines of the larger sizes, but too high pressures for all the other engines. It is therefore evident that after calculating the dimensions of a crank-pin according to the formulæ given the results should be modified, if necessary, to bring the pressure per square inch down to a reasonable figure.

In order to bring the pressures down to 500 pounds per square inch, we divide the mean pressures by 500 to obtain the projected area, or product of length by diameter. Making $l = 1.5 d$ for engines Nos. 1, 2, 4, and 6, the revised table for the six engines is as follows:

| | | | | | | |
|-------------------------------|------|------|------|------|-------|-------|
| Engine No..... | 1 | 2 | 3 | 4 | 5 | 6 |
| Length of crank-pin, inches.. | 3.15 | 3.15 | 9.86 | 8.37 | 17.12 | 13.30 |
| Diameter of crank-pin..... | 2.10 | 2.10 | 7.34 | 5.58 | 12.40 | 8.87 |

Crosshead-pin or Wrist-pin. — Whitham says the bearing surface for the wrist-pin is found by the formula for crank-pin design. Seaton says the diameter at the middle must, of course, be sufficient to withstand the bending action, and generally from this cause ample surface is provided for good working; but in any case the area, calculated by multiplying the diameter of the journal by its length, should be such that the pressure does not exceed 1200 lbs. per sq. in., taking the maximum load on the piston as the total pressure on it.

For small engines with the gudgeon shrunk into the jaws of the connecting-rod, and working in brasses fitted into a recess in the piston-rod end and secured by a wrought-iron cap and two bolts, Seaton gives:

Diameter of gudgeon = $1.25 \times$ diam. of piston-rod,

Length of gudgeon = $1.4 \times$ diam. of piston-rod.

If the pressure on the section, as calculated by multiplying length by diameter, exceeds 1200 lbs. per sq. in., this length should be increased.

J. B. Stanwood, in his "Ready Reference" book, gives for length of crosshead-pin 0.25 to 0.3 diam. of piston, and diam. = 0.18 to 0.2 diam. of piston. Since he gives for diam. of piston-rod 0.14 to 0.17 diam. of piston, his dimensions for diameter and length of crosshead-pin are about 1.25 and 1.8 diam. of piston-rod respectively. Taking the maximum allowable pressure at 1200 lbs. per sq. in. and making the length of the crosshead-pin = $\frac{4}{3}$ of its diameter, we have $d = \sqrt{P \div 40}$, $l = \sqrt{P} + 30$, in which P = maximum total load on piston in lbs., d = diam., and l = length of pin in inches. For the engines of our example we have:

| | | | |
|---|----------|----------|------------|
| Diameter of piston, inches..... | 10 | 30 | 50 |
| Maximum load on piston, lbs..... | 7854 | 70,686 | 196,350 |
| Diameter of crosshead-pin, inches.... | 2.22 | 6.65 | 11.08 |
| Length of crosshead-pin, inches..... | 2.96 | 8.86 | 14.77 |
| Stanwood's rule gives diameter, ins.... | 1.8 to 2 | 5.4 to 6 | 9.0 to 10 |
| Stanwood's rule gives length, inches.... | 2.5 to 3 | 7.5 to 9 | 12.5 to 15 |
| Stanwood's largest dimensions give pressure per sq. in., lbs..... | 1309 | 1329 | 1309 |

Which pressures are greater than the maximum allowed by Seaton.

The Crank-arm. — The crank-arm is to be treated as a lever, so that if a is the thickness in a direction parallel to the shaft-axis and b its breadth at a section x inches from the crank-pin center, then, bending moment M at that section = Px , P being the thrust of the connecting-rod, and f the safe strain per square inch,

$$Px = \frac{fab^2}{6} \text{ and } \frac{a \times b^2}{6} = \frac{T}{f}, \text{ or } a = \frac{6T}{b^2 \times f}; b = \sqrt{\frac{6T}{fa}}.$$

If a crank-arm were constructed so that b varied as \sqrt{x} (as given by the above rule) it would be of such a curved form as to be inconvenient to manufacture, and consequently it is customary in practice to find the maximum value of b and draw tangent lines to the curve at the points; these lines are generally, for the same reason, tangential to the boss of the crank-arm at the shaft.

The shearing strain is the same throughout the crank-arm; and, consequently, is large compared with the bending strain close to the crank-pin; and so it is not sufficient to provide there only for bending strains. The section at this point should be such that, in addition to what is given by the calculation from the bending moment, there is an extra square inch for every 8000 lbs. of thrust on the connecting-rod (Seaton).

The length of the boss h into which the shaft is fitted is from 0.75 to 1.0 of the diameter of the shaft D , and its thickness e must be calculated from the twisting strain PL . (L = length of crank.)

For different values of length of boss h , the following values of thickness of boss e are given by Seaton:

| | |
|----------------|-------------------------------------|
| When $h = D$, | then $e = 0.35 D$; if steel, 0.3. |
| $h = 0.9 D$, | then $e = 0.38 D$; if steel, 0.32. |
| $h = 0.8 D$, | then $e = 0.40 D$; if steel, 0.33. |
| $h = 0.7 D$, | then $e = 0.41 D$; if steel, 0.34. |

The crank-eye or boss into which the pin is fitted should bear the same relation to the pin that the boss does to the shaft.

The diameter of the shaft-end onto which the crank is fitted should be $1.1 \times$ diameter of shaft.

Thurston says: The empirical proportions adopted by builders will commonly be found to fall well within the calculated safe margin. These proportions are, from the practice of successful designers, about as follows:

For the wrought-iron crank, the hub is 1.75 to 1.8 times the least diameter of that part of the shaft carrying full load; the eye is 2.0 to 2.25 the diameter of the inserted portion of the pin, and their depths are, for the hub, 1.0 to 1.2 the diameter of shaft, and for the eye, 1.25 to 1.5 the diameter of pin. The web is made 0.7 to 0.75 the width of adjacent hub or eye, and is given a depth of 0.5 to 0.6 that of adjacent hub or eye.

For the cast-iron crank the hub and eye are a little larger, ranging in diameter respectively from 1.8 to 2 and from 2 to 2.2 times the diameters of shaft and pin. The flanges are made at either end of nearly the full depth of hub or eye. Cast iron has, however, fallen very generally into disuse.

The crank-shaft is usually enlarged at the seat of the crank to about 1.1 its diameter at the journal. The size should be nicely adjusted to allow for the shrinkage or forcing on of the crank. A difference of diameter of 0.2% will usually suffice; and a common rule of practice gives an allowance of but one-half of this, or 0.1%.

The formulæ given by different writers for crank-arms practically agree, since they all consider the crank as a beam loaded at one end and fixed at the other. The relation of breadth to thickness may vary according to the taste of the designer. Calculated dimensions for our six engines are as follows:

Dimensions of Crank-arms.

| | | | | | | |
|---|--------|--------|---------|-----------|-----------|-----------|
| Diam. of cylinder, ins..... | 10 | 10 | 30 | 30 | 50 | 50 |
| Stroke S , ins..... | 12 | 24 | 30 | 60 | 48 | 96 |
| Max. pressure on pin P (approx.), lbs..... | 7854 | 7854 | 70,686 | 70,686 | 196,350 | 196,350 |
| Diam. crank-pin d | 2.10 | 2.10 | 7.34 | 5.58 | 12.40 | 8.87 |
| Dia. shaft, $a \sqrt[3]{\frac{\text{I.H.P.}}{R}}, D$ ($a = 4.69, 5.09$ and 5.22).... | 2.74 | 3.46 | 7.70 | 9.70 | 12.55 | 15.82 |
| Length of boss, $0.8 D$ | 2.19 | 2.77 | 6.16 | 7.76 | 10.04 | 12.65 |
| Thickness of boss, $0.4 D$ | 1.10 | 1.39 | 3.08 | 3.88 | 5.02 | 6.32 |
| Diam. of boss, $1.8 D$ | 4.93 | 6.23 | 13.86 | 17.46 | 22.59 | 28.47 |
| Length crank-pin eye, $0.8 d$ | 1.76 | 1.76 | 5.87 | 4.46 | 9.92 | 7.10 |
| Thickness of crank-pin eye. $0.4 d$ | 0.88 | 0.88 | 2.94 | 2.23 | 4.46 | 3.55 |
| Max. mom. T at distance $\frac{1}{2}S - \frac{1}{2}D$ from center of pin, inch-lbs..... | 37,149 | 80,661 | 788,149 | 1,848,439 | 3,479,322 | 7,871,671 |
| Thickness of crank-arm $a =$ $0.75 D$ | 2.05 | 2.60 | 5.78 | 7.28 | 9.41 | 11.87 |
| Greatest breadth, $b = \sqrt{6 T \div 9000 a}$ | 3.48 | 4.55 | 9.54 | 13.0 | 15.7 | 21.0 |
| Min. mom. T_0 at distance d from center of pin = Pd . | 16,493 | 16,493 | 528,835 | 394,428 | 2,434,740 | 1,741,625 |
| Least breadth, $b_1 = \sqrt{6 T_0 \div 9000 a}$ | 2.32 | 2.06 | 7.81 | 6.01 | 13.13 | 9.89 |

The Shaft. — Twisting Resistance. — From the general formula

for torsion, we have: $T = \frac{\pi}{16} d^3 S = 0.19635 d^3 S$, whence $d = \sqrt[3]{\frac{5.1 T}{S}}$,

in which T = torsional moment in inch-pounds, d = diameter in inches, and S = the shearing resistance of the material in pounds per square inch.

If a constant force P were applied to the crank-pin tangentially to its path, the work done per minute would be

$$P \times L \times 2\pi \div 12 \times R = 33,000 \times \text{I.H.P.},$$

in which L = length of crank in inches, and R = revs. per min., and the mean twisting moment $T = \text{I.H.P.} \div R \times 63,025$. Therefore

$$d = \sqrt[3]{5.1 T \div S} = \sqrt[3]{321,427 \text{ I.H.P.} \div RS}.$$

This may take the form

$$d = \sqrt[3]{\text{I.H.P.} \times F/R}, \text{ or } d = a \sqrt[3]{\text{I.H.P.} \div R},$$

in which F and a are factors that depend on the strength of the material and on the factor of safety. Taking S at 45,000 pounds per square inch for wrought iron, and at 60,000 for steel, we have, for simple twisting by a uniform tangential force,

| | | | | | | | | | |
|------------------|---|------|------|------|------|-----------|------|------|------|
| Factor of safety | = | 5 | 6 | 8 | 10 | 5 | 6 | 8 | 10 |
| Iron..... F | = | 35.7 | 42.8 | 57.1 | 71.4 | $a = 3.3$ | 3.5 | 3.85 | 4.15 |
| Steel.... F | = | 26.8 | 32.1 | 42.8 | 53.5 | $a = 3.0$ | 3.18 | 3.5 | 3.77 |

Unwin, taking for safe working strength of wrought iron 9000 lbs., steel 13,500 lbs., and cast iron 4500 lbs., gives $a = 3.294$ for wrought iron, 2.877 for steel, and 4.15 for cast iron. Thurston, for crank-axes of wrought iron, gives $a = 4.15$ or more.

Seaton says: For wrought iron, f , the safe strain per square inch, should not exceed 9000 lbs., and when the shafts are more than 10 inches diameter, 8000 lbs. Steel, when made from the ingot and of good materials, will admit of a stress of 12,000 lbs. for small shafts, and 10,000 lbs. for those above 10 inches diameter.

The difference in the allowance between large and small shafts is to compensate for the defective material observable in the heart of large shafting, owing to the hammering failing to affect it.

The formula $d = a \sqrt[3]{\text{I.H.P.} \div R}$ assumes the tangential force to be uniform and that it is the only acting force. For engines, in which the tangential force varies with the angle between the crank and the connecting-rod, and with the variation in steam-pressure in the cylinder, and also is influenced by the inertia of the reciprocating parts, and in which also the shaft may be subjected to bending as well as torsion, the factor a must be increased, to provide for the maximum tangential force and for bending.

Seaton gives the following table showing the relation between the maximum and mean twisting moments of engines working under various conditions, the momentum of the moving parts being neglected, which is allowable:

| Description of Engine. | Steam Cut-off at | Max. Twist Divided by Mean Twist. Moment. | Cube Root of the Ratio. |
|--|---------------------|---|----------------------------------|
| Single-crank expansive..... | 0.2 | 2.625 | 1.38 |
| “ “ | 0.4 | 2.125 | 1.29 |
| “ “ | 0.6 | 1.835 | 1.22 |
| “ “ | 0.8 | 1.698 | 1.20 |
| Two-cylinder expansive, cranks at 90°..... | 0.2 | 1.616 | 1.17 |
| “ “ “ | 0.3 | 1.415 | 1.12 |
| “ “ “ | 0.4 | 1.298 | 1.09 |
| “ “ “ | 0.5 | 1.256 | 1.08 |
| “ “ “ | 0.6 | 1.270 | 1.08 |
| “ “ “ | 0.7 | 1.329 | 1.10 |
| “ “ “ | 0.8 | 1.357 | 1.11 |
| Three-cylinder compound, cranks 120°..... | h.p. 0.5, l.p. 0.66 | 1.40 | 1.12 |
| Three-cylinder compound, l.p. cranks opposite one another, and h.p. midway } | “ “ | 1.26 | 1.08 |

Seaton also gives the following rules for ordinary practice for ordinary two-cylinder marine engines:

Diameter of the tunnel-shafts = $\sqrt[3]{\text{I.H.P.} \times F/R}$, or a $\sqrt[3]{\text{I.H.P.} \div R}$.

Compound engines, cranks at right angles:

Boiler pressure 70 lbs., rate of expansion 6 to 7, $F = 70$, $a = 4.12$.

Boiler pressure 80 lbs., rate of expansion 7 to 8, $F = 72$, $a = 4.16$.

Boiler pressure 90 lbs., rate of expansion 8 to 9, $F = 75$, $a = 4.22$.

Triple compound, three cranks at 120 degrees:

Boiler pressure 150 lbs., rate of expansion 10 to 12, $F = 62$, $a = 3.96$.

Boiler pressure 160 lbs., rate of expansion 11 to 13, $F = 64$, $a = 4$.

Boiler pressure 170 lbs., rate of expansion 12 to 15, $F = 67$, $a = 4.06$.

Expansive engines, cranks at right angles, and the rate of expansion 5, boiler-pressure 60 lbs., $F = 90$, $a = 4.48$.

Single-crank compound engines, pressure 80 lbs., $F = 96$, $a = 4.58$.

For the engines we are considering it will be a very liberal allowance for ratio of maximum to mean twisting moment if we take it as equal to the ratio of the maximum to the mean pressure on the piston. The factor a , then, in the formula for diameter of the shaft will be multiplied by the cube

root of this ratio, or $\sqrt[3]{\frac{100}{42}} = 1.34$, $\sqrt[3]{\frac{100}{32.3}} = 1.45$, and $\sqrt[3]{\frac{100}{30}} = 1.49$

for the 10, 30, and 50-in. engines, respectively. Taking $a = 3.5$, which corresponds to a shearing strength of 60,000 and a factor of safety of 8 for steel, or to 45,000 and a factor of 6 for iron, we have for the new coefficient a_1 in the formula $d_1 = a_1 \sqrt[3]{\text{I.H.P.} \div R}$, the values 4.69, 5.08, and 5.22 from which we obtain the diameters of shafts of the six engines as follows:

| Engine No. | 1 | 2 | 3 | 4 | 5 | 6 |
|----------------------|------|------|------|------|-------|-------|
| Diam. of cyl. | 10 | 10 | 30 | 30 | 50 | 50 |
| Horse-power, I.H.P. | 50 | 50 | 450 | 450 | 1250 | 1250 |
| Revs. per min., R | 250 | 125 | 130 | 65 | 90 | 45 |
| Diam. of shaft $d =$ | 2.74 | 3.46 | 7.67 | 9.70 | 12.55 | 15.82 |

These diameters are calculated for twisting only. When the shaft is also subjected to bending strain the calculation must be modified as below:

Resistance to Bending. — The strength of a circular-section shaft to resist bending is one-half of that to resist twisting. If B is the bending moment in inch-lbs., and d the diameter of the shaft in inches,

$$B = \frac{\pi d^3}{32} \times f; \text{ and } d = \sqrt[3]{\frac{B}{f} \times 10.2};$$

f is the safe strain per square inch of the material of which the shaft is composed, and its value may be taken as given above for twisting (Seaton).

Equivalent Twisting Moment. — When a shaft is subject to both twisting and bending simultaneously, the combined strain on any section of it may be measured by calculating what is called the *equivalent twisting moment*; that is, the two strains are so combined as to be treated as a twisting strain only of the same magnitude and the size of shaft calculated accordingly. Rankine gave the following solution of the combined action of the two strains.

If T = the twisting moment, and B = the bending moment on a section of a shaft, then the equivalent twisting moment $T_1 = B + \sqrt{B^2 + T^2}$.

Seaton says: Crank-shafts are subject always to twisting, bending, and shearing strains; the latter are so small compared with the former that they are usually neglected directly, but allowed for indirectly by means of the factor f .

The two principal strains vary throughout the revolution, and the maximum equivalent twisting moment can only be obtained accurately by a series of calculations of bending and twisting moments taken at fixed intervals, and from them constructing a curve of strains.

Considering the engines of our examples to have overhung cranks, the maximum bending moment resulting from the thrust of the connecting-

rod on the crank-pin will take place when the engine is passing its centers (neglecting the effect of the inertia of the reciprocating parts), and it will be the product of the total pressure on the piston by the distance between two parallel lines passing through the centers of the crank-pin and of the shaft bearing, at right angles to their axes; which distance is equal to $\frac{1}{2}$ length of crank-pin bearing + length of hub + $\frac{1}{2}$ length of shaft-bearing + any clearance that may be allowed between the crank and the two bearings. For our six engines we may take this distance as equal to $\frac{1}{2}$ length of crank-pin + thickness of crank-arm + $1.5 \times$ the diameter of the shaft as already found by the calculation for twisting. The calculation of diameter is then as below:

| Engine No. | 1 | 2 | 3 | 4 | 5 | 6 |
|--|---------|---------|-----------|-----------|------------|------------|
| Diam. of cyl., in.... | 10 | 10 | 30 | 30 | 50 | 50 |
| Horse-power..... | 50 | 50 | 450 | 450 | 1250 | 1250 |
| Revs. per min..... | 250 | 125 | 130 | 65 | 90 | 45 |
| Max. press. on pis, P | 7,854 | 7,854 | 70,686 | 70,686 | 196,350 | 196,350 |
| Leverage, * L in.... | 6.32 | 7.94 | 22.20 | 26.00 | 36.80 | 42.25 |
| Bd.mo. $PL = B$ in.-lb | 49,637 | 62,361 | 1,569,222 | 1,837,836 | 7,225,680 | 8,295,788 |
| Twist. mom. T | 47,124 | 94,248 | 1,060,290 | 2,120,580 | 4,712,400 | 9,424,800 |
| Equiv. twist mom. | | | | | | |
| $T_1 = B + \sqrt{B^2 + T^2}$ (approx.)..... | 118,000 | 175,000 | 3,463,000 | 4,647,000 | 15,840,000 | 20,850,000 |

* Leverage = distance between centers of crank-pin and shaft bearing = $\frac{1}{2}l + 2.25 d$.

Having already found the diameters, on the assumption that the shafts were subjected to a twisting moment T only, we may find the diameter for resisting combined bending and twisting by multiplying the diameters already found by the cube roots of the ratio $T_1 \div T$, or

Giving corrected diameters $d_1 =$

| | | | | | |
|------|------|-------|-------|-------|-------|
| 1.40 | 1.27 | 1.46 | 1.34 | 1.64 | 1.36 |
| 3.84 | 4.39 | 11.35 | 12.99 | 20.58 | 21.52 |

By plotting these results, using the diameters of the cylinders for abscissas and diameters of the shafts for ordinates, we find that for the long-stroke engines the results lie almost in a straight line expressed by the formula, diameter of shaft = $0.43 \times$ diameter of cylinder; for the short-stroke engines the line is slightly curved, but does not diverge far from a straight line whose equation is, diameter of shaft = 0.4 diameter of cylinder. Using these two formulas, the diameters of the shafts will be 4.0, 4.3, 12.0, 12.9, 20.0, 21.5.

J. B. Stanwood, in *Engineering*, June 12, 1891, gives dimensions of shafts of Corliss engines in American practice for cylinders 10 to 30 in. diameter. The diameters range from $4\frac{15}{16}$ to $14\frac{15}{16}$, following precisely the equation, diameter of shaft = $\frac{1}{2}$ diameter of cylinder - $\frac{1}{16}$ in.

Fly-wheel Shafts. — Thus far we have considered the shaft as resisting the force of torsion and the bending moment produced by the pressure on the crank-pin. In the case of fly-wheel engines the shaft on the opposite side of the bearing from the crank-pin has to be designed with reference to the bending moment caused by the weight of the fly-wheel, the weight of the shaft itself, and the strain of the belt. For engines in which there is an outboard bearing, the weight of fly-wheel and shaft being supported by two bearings, the point of the shaft at which the bending moment is a maximum may be taken as the point midway between the two bearings or at the middle of the fly-wheel hub, and the amount of the moment is the product of the weight supported by one of the bearings into the distance from the center of that bearing to the middle point of the shaft. The shaft is thus to be treated as a beam supported at the ends and loaded in the middle. In the case of an overhung fly-wheel, the shaft having only one bearing, the point of maximum moment should be taken as the middle of the bearing, and its amount is very nearly the product of half the weight of the fly-wheel and the shaft

into the distance of the middle of its hub from the middle of the bearing. The bending moment should be calculated and combined with the twisting moment as above shown, to obtain the equivalent twisting moment, and the diameter necessary at the point of maximum moment calculated therefrom.

In the case of our six engines we assume that the weights of the fly-wheels, together with the shaft, are double the weight of fly-wheel rim obtained from the formula $W = 785,400 \frac{d^2 s}{R^2 D^2}$ (given under Fly-wheels);

that the shaft is supported by an outboard bearing, the distance between the two bearings being 2½, 5, and 10 feet for the 10-in., 30-in., and 50-in. engines, respectively. The diameters of the fly-wheels are taken such that their rim velocity will be a little less than 6000 feet per minute.

| Engine No. | 1 | 2 | 3 | 4 | 5 | 6 |
|------------------------------------|------|------|---------|---------|-----------|-----------|
| Diam. of cyl., inches | 10 | 10 | 30 | 30 | 50 | 50 |
| Diam. of fly-wheel, ft. | 7.5 | 15 | 14.5 | 29 | 21 | 42 |
| Revs. per min. | 250 | 125 | 130 | 65 | 90 | 45 |
| Half wt. fly-wheel and shaft, lbs. | 268 | 536 | 5,968 | 11,936 | 26,384 | 52,769 |
| Lever arm for maximum moment, in. | 15 | 15 | 30 | 30 | 60 | 60 |
| Maximum bending moment, in.-lbs. | 4020 | 8040 | 179,040 | 358,080 | 1,583,070 | 3,166,140 |

As these are very much less than the bending moments calculated from the pressures on the crank-pin, the diameters already found are sufficient for the diameter of the shaft at the fly-wheel hub.

In the case of engines with heavy band fly-wheels and with long fly-wheel shafts it is of the utmost importance to calculate the diameter of the shaft with reference to the bending moment due to the weight of the fly-wheel and the shaft.

B. H. Coffey (*Power*, October, 1892) gives the formula for combined bending and twisting resistance, $T_1 = 0.196 d^2 S$, in which $T_1 = B + \sqrt{B^2 + T^2}$; T being the maximum, not the mean twisting moment; and finds empirical working values for $0.196 S$ as below. He says: Four points should be considered in determining this value: First, the nature of the material; second, the manner of applying the loads, with shock or otherwise; third, the ratio of the bending moment to the torsional moment — the bending moment in a revolving shaft produces reversed strains in the material, which tend to rupture it; fourth, the size of the section. Inch for inch, large sections are weaker than small ones. He puts the dividing line between large and small sections at 10 in. diameter, and gives the following safe values of $S \times 0.196$ for steel, wrought iron, and cast iron, for these conditions.

VALUE OF $S \times 0.196$.

| Ratio. | Heavy Shafts with Shock. | | | Light Shafts with Shock. Heavy Shafts No Shock. | | | Light Shafts No Shock. | | |
|------------------------|--------------------------|-------------|------------|--|-------------|------------|------------------------|-------------|------------|
| | Steel. | Wro't Iron. | Cast Iron. | Steel. | Wro't Iron. | Cast Iron. | Steel. | Wro't Iron. | Cast Iron. |
| B to T . | | | | | | | | | |
| 3 to 10 or less | 1045 | 880 | 440 | 1566 | 1320 | 660 | 2090 | 1760 | 880 |
| 3 to 5 or less | 941 | 785 | 393 | 1410 | 1179 | 589 | 1882 | 1570 | 785 |
| 1 to 1 or less | 855 | 715 | 358 | 1281 | 1074 | 537 | 1710 | 1430 | 715 |
| B greater than T . | 784 | 655 | 328 | 1176 | 984 | 492 | 1568 | 1310 | 655 |

Mr. Coffey gives as an example of improper dimensions the fly-wheel shaft of a 1500 H.P. engine at Willimantic, Conn., which broke while the engine was running at 425 H.P. The shaft was 17 ft. 5 in. long between

centers of bearings, 18 in. diam. for 8 ft. in the middle, and 15 in. diam. for the remainder, including the bearings. It broke at the base of the fillet connecting the two large diameters, or $56\frac{1}{2}$ in. from the center of the bearing. He calculates the mean torsional moment to be 446,654 inch-pounds, and the maximum at twice the mean; and the total weight on one bearing at 87,530 lbs., which, multiplied by $56\frac{1}{2}$ in., gives 4,945,445 in.-lbs. bending moment at the fillet. Applying the formula $T_1 = B + \sqrt{B^2 + T^2}$, gives for equivalent twisting moment 9,971,045 in.-lbs. Substituting this value in the formula $T_1 = 0.196 Sa^3$ gives for S the shearing strain 15,070 lbs. per sq. in., or if the metal had a shearing strength of 45,000 lbs., a factor of safety of only 3. Mr. Coffey considers that 6000 lbs. is all that should be allowed for S under these circumstances. This would give $d = 20.35$ in. If we take from Mr. Coffey's table a value of $0.196 S = 1100$, we obtain $d^3 = 9000$ nearly, or $d = 20.8$ in. instead of 15 in., the actual diameter.

Length of Shaft-bearings. — There is as great a difference of opinion among writers, and as great a variation in practice concerning length of journal-bearings, as there is concerning crank-pins. The length of a journal being determined from considerations of its heating, the observations concerning heating of crank-pins apply also to shaft-bearings, and the formulæ for length of crank-pins to avoid heating may also be used, using for the total load upon the bearing the resultant of all the pressures brought upon it, by the pressure on the crank, by the weight of the fly-wheel, and by the pull of the belt. After determining this pressure, however, we must resort to empirical values for the so-called constants of the formulæ, really variables, which depend on the power of the bearing to carry away heat, and upon the quantity of heat generated, which latter depends on the pressure, on the number of square feet of rubbing surface passed over in a minute, and upon the coefficient of friction. This coefficient is an exceedingly variable quantity, ranging from 0.01 or less with perfectly polished journals, having end-play, and lubricated by a pad or oil-bath, to 0.10 or more with ordinary oil-cup lubrication.

For shafts resisting torsion only, Marks gives for length of bearing $l = 0.0000247 fpND^2$, in which f is the coefficient of friction, p the mean pressure in pounds per square inch on the piston, N the number of single strokes per minute, and D the diameter of the piston. For shafts under the combined stress due to pressure on the crank-pin, weight of fly-wheel, etc., he gives the following: Let Q = reaction at bearing due to weight, S = stress due steam pressure on piston, and R_1 = the resultant force; for horizontal engines, $R_1 = \sqrt{Q^2 + S^2}$, for vertical engines $R_1 = Q + S$, when the pressure on the crank is in the same direction as the pressure of the shaft on its bearings, and $R_1 = Q - S$ when the steam pressure tends to lift the shaft from its bearings. Using empirical values for the work of friction per square inch of projected area, taken from dimensions of crank-pins in marine vessels, he finds the formula for length of shaft-journals $l = 0.0000325 fR_1N$, and recommends that to cover the defects of workmanship, neglect of oiling, and the introduction of dust, f be taken at 0.16 or even greater. He says that 500 lbs. per sq. in. of projected area may be allowed for steel or wrought-iron shafts in brass bearings with good results if a less pressure is not attainable without inconvenience. Marks says that the use of empirical rules that do not take account of the number of turns per minute has resulted in bearings much too long for slow-speed engines and too short for high-speed engines.

Whitham gives the same formula, with the coefficient 0.00002575.

Thurston says that the maximum allowable mean intensity of pressure may be, for all cases, computed by his formula for journals, $l = PV + 60,000 d$, or by Rankine's, $l = P(V + 20) \div 44,800 d$, in which P is the mean total pressure in pounds, V the velocity of rubbing surface in feet per minute, and d the diameter of the shaft in inches. It must be borne in mind, he says, that the friction work on the main bearing next the crank is the sum of that due the action of the piston on the pin and that due that portion of the weight of wheel and shaft and of pull of the belt which is carried there. The outboard bearing carries practically only the latter two parts of the total. The crank-shaft journals will be made longer on one side, and perhaps shorter on the other, than that of the crank-pin, in proportion to the work falling upon each, i.e., to their

respective products of mean total pressure, speed of rubbing surfaces, and coefficients of friction.

Unwin says: Journals running at 150 revolutions per minute are often only one diameter long. Fan shafts running 150 revolutions per minute have journals six or eight diameters long. The ordinary empirical mode of proportioning the length of journals is to make the length proportional to the diameter, and to make the ratio of length to diameter increase with the speed. For wrought-iron journals:

Revs. per min. = 50 100 150 200 250 500 1000 $l/d = 0.004 R + 1$.
Length \div diam. = 1.2 1.4 1.6 1.8 2.0 3.0 5.0.

Cast-iron journals may have $l+d = 9/10$, and steel journals $l+d = 1\frac{1}{4}$, of the above values.

Unwin gives the following, calculated from the formula $l = 0.4 \text{ H.P.} \div r$, in which r is the crank radius in inches, and H.P. the horse-power transmitted to the crank-pin.

THEORETICAL JOURNAL LENGTH IN INCHES.

| Load on Journal in pounds. | Revolutions of Journal per Minute. | | | | | |
|----------------------------|------------------------------------|-----|-----|-------|-------|-------|
| | 50 | 100 | 200 | 300 | 500 | 1000 |
| 1,000 | 0.2 | 0.4 | 0.8 | 1.2 | 2. | 4. |
| 2,000 | 0.4 | 0.8 | 1.6 | 2.4 | 4. | 8. |
| 4,000 | 0.8 | 1.6 | 3.2 | 4.8 | 8. | 16. |
| 5,000 | 1.0 | 2. | 4. | 6. | 10. | 20. |
| 10,000 | 2. | 4. | 8. | 12. | 20. | 40. |
| 15,000 | 3. | 6. | 12. | 18. | 30. | |
| 20,000 | 4. | 8. | 16. | 24. | 40. | |
| 30,000 | 6. | 12. | 24. | 36. | | |
| 40,000 | 8. | 16. | 32. | | | |
| 50,000 | 10. | 20. | 40. | | | |

Applying these different formulæ to our six engines, we have:

| Engine No..... | 1 | 2 | 3 | 4 | 5 | 6 |
|---|-------|-------|--------|--------|--------|--------|
| Diam. cyl..... | 10 | 10 | 30 | 30 | 50 | 50 |
| Horse-power..... | 50 | 50 | 450 | 450 | 1,250 | 1,250 |
| Revs. per min..... | 250 | 125 | 130 | 65 | 90 | 45 |
| Mean pressure on crank-pin = S | 3,299 | 3,299 | 23,185 | 23,185 | 58,905 | 58,905 |
| Half wt. of fly-wheel and shaft = Q | 268 | 536 | 5,968 | 11,936 | 26,470 | 52,940 |
| Resultant pressure on bearing $\sqrt{Q^2 + S^2} = R_1$ | 3,310 | 3,335 | 23,924 | 26,194 | 64,580 | 79,200 |
| Diam. of shaft journal..... | 3.84 | 4.39 | 11.35 | 12.99 | 20.58 | 21.52 |
| Length of shaft journal: | | | | | | |
| Marks, $l = 0.0000325 f R_1 N (f = 0.10)$ | 5.38 | 2.71 | 20.87 | 11.07 | 37.78 | 23.17 |
| Whitham, $l = 0.0000515 f R_1 R (f = 0.10)$ | 4.27 | 2.15 | 16.53 | 8.77 | 29.95 | 18.35 |
| Thurston, $l = P V \div (60,000 d)$ | 3.61 | 1.82 | 14.00 | 7.43 | 25.36 | 15.55 |
| Rankine, $l = P (V + 20) \div (44,800 d)$ | 5.22 | 2.78 | 21.70 | 10.85 | 35.16 | 22.47 |
| Unwin, $l = (0.004 R + 1) d$ | 7.68 | 6.59 | 17.25 | 16.36 | 27.99 | 25.39 |
| Unwin, $l = 0.4 \text{ H.P.} \div r$ | 3.33 | 1.60 | 12.00 | 6.00 | 20.83 | 10.42 |
| Average..... | 4.92 | 2.99 | 17.05 | 10.00 | 29.54 | 19.22 |

If we divide the mean resultant pressure on the bearing by the projected area, that is, by the product of the diameter and length of the journal, using the greatest and smallest lengths out of the seven lengths

for each journal given above, we obtain the pressure per square inch upon the bearing, as follows:

| Engine No..... | 1 | 2 | 3 | 4 | 5 | 6 |
|---|-------|-----|-------|-----|-------|-----|
| Press. per sq. in., shortest journal..... | 259 | 455 | 176 | 336 | 151 | 353 |
| Longest journal..... | 112 | 115 | 97 | 123 | 83 | 145 |
| Average journal..... | 175 | 254 | 124 | 202 | 106 | 191 |
| Journal of length = diam..... | | 173 | | 155 | | 175 |

Many of the formulæ give for the long-stroke engines a length of journal less than the diameter, but such short journals are rarely used in practice. The last line in the above table has been calculated on the supposition that the journals of the long-stroke engines are made of a length equal to the diameter.

In the dimensions of Corliss engines given by J. B. Stanwood (*Eng.*, June 12, 1891), the lengths of the journals for engines of diam. of cyl. 10 to 20 in. are the same as the diam. of the cylinder, and a little more than twice the diam. of the journal. For engines above 20 in. diam. of cyl. the ratio of length to diam. is decreased so that an engine of 30 in. diam. has a journal 26 in. long, its diameter being $14\frac{15}{16}$ in. These lengths of journal are greater than those given by any of the formulæ above quoted.

There thus appears to be a hopeless confusion in the various formulæ for length of shaft journals, but this is no more than is to be expected from the variation in the coefficient of friction, and in the heat-conducting power of journals in actual use, the coefficient varying from 0.10 (or even 0.16 as given by Marks) down to 0.01, according to the condition of the bearing surfaces and the efficiency of lubrication. Thurston's

formula, $l = \frac{PV}{60,000 d}$, reduces to the form $l = 0.000004363 PR$, in which

P = mean total load on journal, and R = revolutions per minute. This is of the same form as Marks's and Whitham's formulæ, in which, if f , the coefficient of friction, be taken at 0.10, the coefficients of PR are, respectively, 0.0000065 and 0.00000515. Taking the mean of these three formulæ, we have $l = 0.0000053 PR$, if $f = 0.10$ or $l = 0.000053 fPR$ for any other value of f . The author believes this to be as safe a formula as any for length of journals, with the limitation that if it brings a result of length of journal less than the diameter, then the length should be made equal to the diameter. Whenever, with $f = 0.10$ it gives a length which is inconvenient or impossible of construction on account of limited space, then provision should be made to reduce the value of the coefficient of friction below 0.10 by means of forced lubrication, end play, etc., and to carry away the heat, as by water-cooled journal-boxes. The value of P should be taken as the resultant of the mean pressure on the crank, and the load brought on the bearing by the weight of the shaft, fly-wheel, etc., as calculated by the formula already given, viz., $R_1 = \sqrt{Q^2 + S^2}$ for horizontal engines, and $R_1 = Q + S$ for vertical engines.

For our six engines the formula $l = 0.0000053 PR$ gives, with the limitation for the long-stroke engines that the length shall not be less than the diameter, the following:

| | | | | | | |
|--|------|------|-------|-------|-------|-------|
| Engine No..... | 1 | 2 | 3 | 4 | 5 | 6 |
| Length of journal..... | 4.39 | 4.39 | 16.48 | 12.99 | 30.80 | 21.52 |
| Pressure per square inch on journal..... | 196 | 173 | 128 | 155 | 102 | 171 |

Crank-shafts with Center-crank and Double-crank Arms. — In center-crank engines, one of the crank-arms, and its adjoining journal, called the after journal, usually transmit the power of the engine to the work to be done, and the journal resists both twisting and bending moments, while the other journal is subjected to bending moment only. For the after crank-journal the diameter should be calculated the same as for an overhung crank, using the formula for combined bending and

twisting moment, $T_1 = B + \sqrt{B^2 + T^2}$, in which T_1 is the equivalent twisting moment, B the bending moment, and T the twisting moment.

This value of T_1 is to be used in the formula diameter = $\sqrt[3]{5.1 T/S}$. The bending moment is taken as the maximum load on piston multiplied by one-fourth of the length of the crank-shaft between middle points of the two journal bearings, if the center is midway between the bearings, or by one-half the distance measured parallel to the shaft from the middle of the crank-pin to the middle of the after bearing. This supposes the crank-shaft to be a beam loaded at its middle and supported at the ends, but Whitham would make the bending moment only one-half of this, considering the shaft to be a beam secured or fixed at the ends, with a point of contraflexure one-fourth of the length from the end. The first supposition is the safer, but since the bending moment will in any case be much less than the twisting moment, the resulting diameter will be but little greater than if Whitham's supposition is used. For the forward journal, which is subjected to bending moment only, diameter of shaft = $\sqrt[3]{10.2 B/S}$, in which B is the maximum bending moment and S the safe shearing strength of the metal per square inch.

For our six engines, assuming them to be center-crank engines, and considering the crank-shaft to be a beam supported at the ends and loaded in the middle, and assuming lengths between centers of shaft bearings as given below, we have:

| Engine No. | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---------|---------|-----------|-----------|-----------|------------|
| Length of shaft, assumed, in., L ... | 20 | 24 | 48 | 60 | 76 | 96 |
| Max. press. on crank-pin, P | 7,854 | 7,854 | 70,686 | 70,686 | 196,350 | 196,350 |
| Max. bending moment, $B = 1/4 PL$... | 39,270 | 49,637 | 848,232 | 1,060,290 | 3,729,750 | 4,712,400 |
| Twisting mom., T ... | 47,124 | 94,248 | 1,060,290 | 2,120,580 | 4,712,400 | 9,424,800 |
| Equiv. twist. mom. $B + \sqrt{B^2 + T^2}$... | 101,000 | 156,000 | 2,208,000 | 3,430,000 | 9,740,000 | 15,240,000 |
| Diam. of after jour. | | | | | | |
| $d = \sqrt[3]{\frac{5.1 T_1}{8000}}$... | 3.98 | 4.60 | 11.15 | 13.00 | 18.25 | 21.20 |
| Diam. of forw. jour., | | | | | | |
| $d_1 = \sqrt[3]{\frac{10.2 B}{8000}}$ | 3.68 | 3.99 | 10.28 | 11.16 | 16.82 | 18.18 |

The lengths of the journals would be calculated in the same manner as in the case of overhung cranks, by the formula $l = 0.000053 fPR$, in which P is the resultant of the mean pressure due to pressure of steam on the piston, and the load of the fly-wheel, shaft, etc., on each of the two bearings. Unless the pressures are equally divided between the two bearings, the calculated lengths of the two will be different; but it is usually customary to make them both of the same length, and in no case to make the length less than the diameter. The diameters also are usually made alike for the two journals, using the largest diameter found by calculation.

The crank-pin for a center crank should be of the same length as for an overhung crank, since the length is determined from considerations of heating, and not of strength. The diameter also will usually be the same, since it is made great enough to make the pressure per square inch on the projected area (product of length by diameter) small enough to allow of free lubrication, and the diameter so calculated will be greater than is required for strength.

Crank-shaft with Two Cranks coupled at 90°.—If the whole power of the engine is transmitted through the after journal of the after

crank-shaft, the greatest twisting moment is equal to 1.414 times the maximum twisting moment due to the pressure on one of the crank-pins. If T = the maximum twisting moment produced by the steam-pressure on one of the pistons, then T_1 , the maximum twisting moment on the after part of the crank-shaft, and on the line-shaft produced, when each crank makes an angle of 45° with the center line of the engine, is $1.414 T$. Substituting this value in the formula for diameter to resist simple

torsion, viz., $d = \sqrt[3]{5.1 T \div S}$, we have $d = \sqrt[3]{5.1 \times 1.414 T \div S}$, or $d = 1.932 \sqrt[3]{T/S}$, in which T is the maximum twisting moment produced by one of the pistons, d = diameter in inches, and S = safe working shearing strength of the material. For the forward journal of the after crank, and the after journal of the forward crank, the torsional moment is that due to the pressure of steam on the forward piston only, and for the forward journal of the forward crank, if none of the power of the engine is transmitted through it, the torsional moment is zero, and its diameter is to be calculated for bending moment only.

For Combined Torsion and Flexure. — Let B_1 = bending moment on either journal of the forward crank due to maximum pressure on forward piston, B_2 = bending moment on either journal of the after crank due to maximum pressure on after piston, T_1 = maximum twisting moment on after journal of forward crank, and T_2 = maximum twisting moment on after journal of after crank, due to pressure on the after piston.

Then equivalent twisting moment on after journal of forward crank = $B_1 + \sqrt{B_1^2 + T_1^2}$.

On forward journal of after crank = $B_2 + \sqrt{B_2^2 + T_1^2}$.

On after journal of after crank = $B_2 + \sqrt{B_2^2 + (T_1 + T_2)^2}$.

These values of equivalent twisting moment are to be used in the formula for diameter of journals $d = \sqrt[3]{5.1 T/S}$. For the forward journal of the forward crank-shaft $d = \sqrt[3]{10.2 B_1/S}$.

It is customary to make the two journals of the forward crank of one diameter, viz., that calculated for the after journal.

For a Three-cylinder Engine with cranks at 120° , the greatest twisting moment on the after part of the shaft, if the maximum pressures on the three pistons are equal, is equal to twice the maximum pressure on any one piston, and it takes place when two of the cranks make angles of 30° with the center line, the third crank being at right angles to it. (For demonstration, see Whitham's "Steam-engine Design," p. 252.) For combined torsion and flexure the same method as above given for two crank engines is adopted for the first two cranks; and for the third, or after crank, if all the power of the three cylinders is transmitted through it, we have the equivalent twisting moment on the forward

journal = $B_3 + \sqrt{B_3^2 + (T_1 + T_2)^2}$, and on the after journal = $B_3 + \sqrt{B_3^2 + (T_1 + T_2 + T_3)^2}$, B_3 and T_3 being respectively the bending and twisting moments due to the pressure on the third piston.

Crank-shafts for Triple-expansion Marine Engines, according to an article in *The Engineer*, April 25, 1890, should be made larger than the formulæ would call for, in order to provide for the stresses due to the racing of the propeller in a sea-way, which can scarcely be calculated. A kind of unwritten law has sprung up for fixing the size of a crank-shaft, according to which the diameter of the shaft is made about $0.45 D$, where D is the diameter of the high-pressure cylinder. This is for solid shafts. When the speeds are high, as in war-ships, and the stroke short, the formula becomes $0.4 D$, even for hollow shafts.

The Valve-stem or Valve-rod. — The valve-rod should be designed to move the valve under the most unfavorable conditions, which are when the stem acts by thrusting, as a long column, when the valve is unbalanced (a balanced valve may become unbalanced by the joint leaking) and when it is imperfectly lubricated. The load on the valve is the product of the area into the greatest unbalanced pressure upon it per square inch, and the coefficient of friction may be as high as 20%. The product of this coefficient and the load is the force necessary to move the valve, which

same time under the most unfavorable circumstances; it will then have the stiffness requisite for good working.

Assuming the work done in reversing the link-motion, W , to be only that due to overcoming the friction of the valves themselves through their whole travel, then, if T be the travel of valves in inches, for a compound engine

$$W = \frac{T}{12} \left(\frac{l \times b \times p}{5} \right) + \frac{T}{12} \left(\frac{l_1 \times b_1 \times p_1}{5} \right);$$

l_1 , b_1 , and p_1 being length, breadth, and maximum steam-pressure on valve of the second cylinder; and for an expansive engine

$$W = 2 \times \frac{T}{12} \left(\frac{l \times b \times p}{5} \right); \text{ or } \frac{T}{30} (l \times b \times p).$$

To provide for the friction of link-motion, eccentrics, and other gear, and for abnormal conditions of the same, take the work at one and a half times the above amount.

To find the strain at any part of the gear having motion when reversing, divide the work so found by the space moved through by that part in feet; the quotient is the strain in pounds; and the size may be found from the ordinary rules of construction for any of the parts of the gear. (Seaton.)

Current Practice in Engine Proportions, 1897. (Compare pages 996 to 1020.) — A paper with this title by Prof. John H. Barr, in *Trans. A. S. M. E.*, xviii, 737, gives the results of an examination of the proportions of parts of a great number of single-cylinder engines made by different builders. The engines classed as low speed (L. S.) are Corliss or other long-stroke engines usually making not more than 100 or 125 revs. per min. Those classed as high speed (H. S.) have a stroke generally of 1 to 1½ diameters and a speed of 200 to 300 revs. per min. The results are expressed in formulas of rational form with empirical coefficients, and are here abridged as follows (dimensions in inches):

Thickness of Shell, L. S. only. — $t = CD + B$; D = diam. of piston in in.; $B = 0.3$ in.; C varies from 0.04 to 0.06, mean = 0.05.

Flanges and Cylinder-heads. — 1 to 1.5 × thickness of shell, mean 1.2.

Cylinder-head Studs. — No studs less than ¾ in. nor greater than 1⅜ in. diam. Least number, 8, for 10 in. diam. Average number = 0.7 D . Average diam. = $D/40 + 1/2$ in.

Ports and Pipes. — a = area of port (or pipe) in sq. in.; A = area of piston, sq. in.; V = mean piston-speed, ft. per min.; $a = AV/C$, in which C = mean velocity of steam through the port or pipe in ft. per min.

Ports, H. S. (same ports for steam as for exhaust). — $C = 4500$ to 6500, mean 5500. For ordinary piston-speed of 600 ft. per min. $a = KA$; $K = 0.09$ to 0.13, mean 0.11.

Steam-ports, L. S. — $C = 5000$ to 9000, mean 6800; $K = 0.08$ to 0.10, mean 0.09.

Exhaust-ports, L. S. — $C = 4000$ to 7000, mean 5500; $K = 0.10$ to 0.125, mean 0.11.

Steam-pipes, H. S. — $C = 5800$ to 7000, mean 6500. If d = diam. of pipe and D = diam. of piston, $d = 0.29 D$ to 0.32 D , mean 0.30 D .

Steam-pipes, L. S. — $C = 5000$ to 8000, mean 6000; $d = 0.27$ to 0.35 D ; mean 0.32 D .

Exhaust-pipes, H. S. — $C = 2500$ to 5500, mean 4400; $d = 0.33$ to 0.50 D , mean 0.37 D .

Exhaust-pipes, L. S. — $C = 2800$ to 4700, mean 3800; $d = 0.35$ to 0.45 D , mean 0.40 D .

Face of Pistons. — F = face; D = diameter. $F = CD$. H. S.: $C = 0.30$ to 0.60, mean 0.46. L. S.: $C = 0.25$ to 0.45, mean 0.32.

Piston-rods. — d = diam. of rod; D = diam. of piston; L = stroke, in.; $d = C \sqrt{DL}$. H. S.: $C = 0.12$ to 0.175, mean 0.145. L. S.: $C = 0.10$ to 0.13, mean 0.11.

Connecting-rods. — H. S. (generally 6 cranks long, rectangular section): b = breadth; h = height of section; L_1 = length of connecting-rod; D = diam. of piston; $b = C \sqrt{DL_1}$; $C = 0.045$ to 0.07, mean 0.057; $h = Kb$; $K = 2.2$ to 4, mean 2.7. L. S. (generally 5 cranks long, circular sections only): $C = 0.082$ to 0.105, mean 0.092.

Cross-head Slides. — Maximum pressure in lbs. per sq. in. of shoe, due to the vertical component of the force on the connecting-rod. H. S.: 10.5 to 38, mean 27. L. S.: 29 to 58, mean 40.

Cross-head Pins. — l = length; d = diam.; projected area = $a = dl = CA$; A = area of piston; $l = Kd$. H. S.: $C = 0.06$ to 0.11 , mean 0.08 ; $K = 1$ to 2 , mean 1.25 . L. S.: $C = 0.054$ to 0.10 , mean 0.07 ; $K = 1$ to 1.5 , mean 1.3 .

Crank-pin. — H.P. = horse-power of engine; L = length of stroke; l = length of pin; $l = C \times \text{H.P.} / L + B$; d = diam. of pin; A = area of piston; $dl = KA$. H. S.: $C = 0.13$ to 0.46 , mean 0.30 ; $B = 2.5$ in.; $K = 0.17$ to 0.44 , mean 0.24 . L. S.: $C = 0.4$ to 0.8 , mean 0.6 ; $B = 2$ in.; $K = 0.065$ to 0.115 , mean 0.09 .

Crank-shaft Main Journal. — $d = C \sqrt[3]{\text{H.P.} \div N}$; d = diam.; l = length; N = revs. per min.; projected area = MA ; A = area of piston. H. S.: $C = 6.5$ to 8.5 , mean 7.3 ; $l = Kd$; $K = 2$ to 3 , mean 2.2 ; $M = 0.37$ to 0.70 , mean 0.46 . L. S.: $C = 6$ to 8 , mean 6.8 ; $K = 1.7$ to 2.1 , mean 1.9 ; $M = 0.46$ to 0.64 , mean 0.56 .

Piston-speed. — H. S.: 530 to 660, mean 600; L. S.: 500 to 850, mean 600.

Weight of Reciprocating Parts (piston, piston-rod, cross-head, and one-half of connecting-rod). — $W = CD^2 \div LN^2$; D = diam. of piston; L = length of stroke, in.; N = revs. per min. H. S. only: $C = 1,200,000$ to $2,300,000$, mean $1,860,000$.

Belt-surface per I.H.P. — $S = C \times \text{H.P.} + B$; S = product of width of belt in feet by velocity of belt in ft. per min. H. S.: $C = 21$ to 40 , mean 28 ; $B = 1800$. L. S.: $S = C \times \text{H.P.}$, $C = 30$ to 42 , mean $= 35$.

Fly-wheel (H. S. only). — Weight of rim in lbs.: $W = C \times \text{H.P.} \div D_1^2 N^3$; D_1 = diam. of wheel in in.; $C = 65 \times 10^{10}$ to 2×10^{12} mean $= 12 \times 10^{11}$, or $1,200,000,000,000$.

Weight of Engine per I.H.P. in lbs., including fly-wheel. — $W = C \times \text{H.P.}$. H. S.: $C = 100$ to 135 , mean 115 . L. S.: $C = 135$ to 240 , mean 175 .

Current Practice in Steam-engine Design, 1909. (Ole N. Troöien, *Bull. Univ'y of Wis.*, No. 252; *Am. Mach.*, April 22, 1909.) — Practice in proportioning standard steam-engine parts has settled down to certain definite values, which have by long usage been found to give satisfactory results. These values can readily be expressed in formulas showing the relation between the more important factors entering the problem of design.

These formulas may be considered as partly rational and partly empirical; rational in the sense that the variables enter in the same manner as in a strict analysis, and empirical in the sense that the constants, instead of being obtained from assumed working strength, bearing pressures, etc., are derived from actual practice and include elements whose values are not accurately known but which have been found safe and economical.

The following symbols of notation are used in the formulas given:

D = diameter of piston. A = area of piston. L = length of stroke. p = unit steam pressure, taken as 125 lbs. per sq. in. above exhaust as a standard pressure. H.P. = rated horse-power. N = revs. per min. C and K , constants, and d = diam. and l = length of unit under consideration. All dimensions in inches.

The commercial point of cut-off is taken at $1/4$ of the stroke. H. S., high-speed engines. L. S., low-speed, or long-stroke engines.

Piston Rod. — $d = C \sqrt{DL}$. H. S.: $C = 0.15$ (min., 0.125 ; max., 0.187); L. S.: $C = 0.114$ (min., 0.1 ; max., 0.156).

Cylinder. — Thickness of wall in ins. = $CD + 0.28$. $C = 0.054$ (min., 0.035 ; max., 0.072). Clearance volume 5 to 11% for H. S. engines, and from 2 to 5% for Corliss engines.

Stud Bolts. — Number = $0.72 D$ for H. S. ($0.65 D$ for Corliss.) Diam. in ins. = $0.04 D + 0.375$.

Ratio (C) of Stroke to Cylinder Diameter (L/D). — For $N > 200$, $C = 1.07$ (min., 0.82 ; max., 1.55); for $N = 110$ to 200 , $C = 1.36$ (min., 1.03 ; max., 1.88); for $N < 110$ (Corliss engines), $C = (L - 8) / D = 1.63$ (min., 1.15 ; max., 2.4).

Piston. — Width of face in ins. = $CD + 1$. Mean value of $C = 0.32$

for H. S. (0.26 for Corliss). Thickness of shell = thickness of cylinder wall $\times 0.6$ (0.7 for Corliss).

Piston Speeds. — H. S., 605 ft. per min. (min. 320; max., 920); Corliss, 592 ft. per min. (min., 400; max., 800).

Cross-head. — Area of shoes in sq. ins. = $0.53 A$ (min., 0.37; max., 0.72).

Cross-head Pin. — Diameter = $0.25 D$ (min., 0.17; max., 0.28). Length for H. S. = diam. $\times 1.25$ (min., 1; max., 1.5); for Corliss = diam. $\times 1.43$ (min., 1; max., 1.9).

Connecting-rods. — Breadth for H. S. = $0.073 \sqrt{L_c D}$ (min., 0.55; max., 0.094). Height = breadth $\times 2.28$ (min., 1.85; max., 3). For L. S., diam. of circular rod = $0.092 \sqrt{L_c D}$ (min., 0.081; max., 0.104). L_c = length center to center of bearings.

Crank-pin. — Diam. for H. S. center-crank engines = $0.4 D$ (min., 0.28; max., 0.526). Diam. for side-crank Corliss = $0.27 D$ (min., 0.21; max., 0.32). Length for H. S. = diam. $\times 0.87$ (min., 0.66; max., 1.25). Length for Corliss = diam. $\times 1.14$ (min., 1; max., 1.3).

Main Journals of Crank-shaft. — For H. S. center-crank engines, diam. = $6.6 \sqrt[3]{H.P./N}$ (min., 5.4; max., 8.2). For Corliss, diameter = $7.2 \sqrt[3]{(H.P./N) - 0.3}$ (min., 6.4; max., 8).

Fly-wheels. — Total weight in pounds for H. S. up to 175 H.P. = $1,300,000,000,000 H.P./D_1^2 N^3$, where D_1 = diam. of wheel in ins. (min., 660,000,000,000; max., 2,800,000,000,000). For larger H. S. engines, weight = $(C \times H.P./D_1^2 N^3) + 1000$, where $C = 720,000,000,000$ (min., 330,000,000,000; max., 1,140,000,000,000). For Corliss engines, weight = $(C \times H.P./D_1^2 N^3) - K$, where $C = 890,000,000,000$ (min., 625,000,000,000; max., 1,330,000,000,000), and $K = 4000$ (min., 2,800; max., 6000). Diam. in ins. = $4.4 \times$ length of stroke.

Belt Surface per I.H.P. — Square feet of belt surface per minute (S) for H. S. = $H.P. \times 26.5$ (min., 10; max., 55). For Corliss engines, $S = 1000 + (21 \times H.P.)$ (min., 18.2; max., 35).

Velocity of Wheel Rim. — For H. S. 70 ft. per sec. (min., 48; max., 70); for Corliss, 68 ft. per sec. (min., 40; max., 68).

Weight of Reciprocating Parts (Piston + piston rod + crosshead + $\frac{1}{2}$ connecting-rod). — Weight in lbs. $W = (D^2/LN^2) \times 2,000,000$ (min., 1,370,000; max., 3,400,000). Balance weight opposite crank-pin = $0.75 W$.

Weight of engine per I.H.P. — Lbs. per I.H.P. for belt-connected H. S. engines = $H.P. \times 82$ (min., 52; max., 120). Do., for Corliss = $H.P. \times 132$ (min., 102; max., 164).

Shafts and Bearings of Engines. (James Christie, *Proc. Engrs. Club of Phila.*, 1898.) — The dimensions are determined by two independent considerations: 1. Sufficient size to prevent excessive deflection or torsional yield. 2. To provide sufficient wearing surface; to prevent excessive wear of journals. Usually, when the first condition is preserved, the other is provided for. When the bearings are flexible, — and excessive deflection within the limit of ordinary safety affects nothing external to the bearings, — considerable deflection can be tolerated. When bearings are rigid, or deflection may derange external mechanism, — for example, an overhung crank, — then the deflection must be more restricted. The effect of deflection is to concentrate pressure on the ends of journals, rendering the apparent bearing surface inefficient.

In direct-driven electric generators a deflection of 0.01 in. per foot of length has caused much trouble from hot bearings. I have proportioned such shafts so that the deflection will not exceed one-half this extent.

In some shafts, especially those having an oscillating movement, torsional elasticity is a prime consideration, and the limits can be known only by experience. Reuleaux says: "Limit the torsional yield to 0.1 degree per foot of length." This in some cases can be readily tolerated; in others, it has proved excessive. I have adopted the following as a general guide: Permissible twist per foot of length = 0.10 degree for easy service, without severe fluctuation of load; 0.075 degree for fluctuating loads suddenly applied; 0.050 degree for loads suddenly reversed.

Sufficiency of wearing surface and the limitation of pressure per unit

of surface are determined by several conditions: 1. Speed of movement. 2. Character of material. 3. Permissible wear of journals or bearings. 4. Constancy of pressure in one direction. 5. Alternation of the direction of pressure.

Taking the product of pressure per sq. in. of surface in lbs., and speed of movement in ft. per min., we obtain a quantity, which we can term the permissible foot-pounds per minute for each sq. in. of wearing surface. This product varies in good practice under various conditions from 50,000 to 500,000 ft.-lbs. per min. For instance, good practice, in later years, has largely increased the area of crosshead slide surfaces. For crossheads having maximum speed of 1000 feet per minute, the pressure per inch of wearing surface should not exceed 50 pounds, giving 50,000 ft.-lbs. per min.; whereas crank-pins of the requisite grade of steel, with good living metal in the boxes and efficient lubrication, will endure 200,000 ft.-lbs. per min. satisfactorily, and more than double this when speeds are very high and the pressure intermittent. On main shafts, with pressures constant in one direction, it is advisable not to exceed 50,000 ft.-lbs. per min. for heavily loaded shafts at low velocity. This may be increased to 100,000 for lighter loads and higher velocities. It can be inferred, therefore, that the product of speed and pressure cannot be used, in any comprehensive way, as a rational basis for proportioning wearing surfaces. The pressure per unit of surface must be reduced as the speed is increased, but not in a constant ratio. A good example of journals severely tested are the recent 110,000-pound freight cars, which bear a pressure of 400 lbs. per sq. in. of journal bearing, and at a speed of ten miles per hour make about 60,000 foot-pounds per minute.

Calculating the Dimensions of Bearings. (F. E. Cardullo, *Mach'y*, Feb., 1907.) — The durability of the lubricating film is affected in great measure by the character of the load that the bearing carries. When the load is unvarying in amount and direction, as in the case of a shaft carrying a heavy bandwheel, the film is easily ruptured. In those cases where the pressure is variable in amount and direction, as in railway journals and crank-pins, the film is much more durable. When the journal only rotates through a small arc, as with the wrist-pin of a steam-engine, the circumstances are most favorable. It has been found that when all other circumstances are exactly similar, a car journal will stand about twice the unit pressure that a fly-wheel journal will. A crank-pin, since the load completely reverses every revolution, will stand three times, and a wrist-pin will stand four times the unit pressure that the fly-wheel journal will.

The amount of pressure that commercial oils will endure at low speeds without breaking down varies from 500 to 1000 lbs. per sq. in., where the load is steady. It is not safe, however, to load a bearing to this extent, since it is only under favorable circumstances that the film will stand this pressure without rupturing. On this account, journal bearings should not be required to stand more than two-thirds of this pressure at slow speeds, and the pressure should be reduced when the speed increases. The approximate unit pressure which a bearing will endure without seizing is $p = PK + (DN + K)$ (1). p = allowable pressure in lbs. per sq. in. of projected area, D = diam. of the bearing in ins., N = revs. per min., and P and K depend upon the kind of oil, manner of lubrication, etc.

P is the maximum safe unit pressure for the given circumstances, at a very slow speed. In ordinary cases, its value is 200 for collar thrust bearings, 400 for shaft bearings, 800 for car journals, 1200 for crank-pins, and 1600 for wrist-pins. In exceptional circumstances, these values may be increased by as much as 50%, but only when the workmanship is of the best, the care the most skillful, the bearing readily accessible, and the oil of the best quality, and unusually viscous. In the great units of the Subway power plant in New York, the value of P for the crank-pins is 2000.

The factor K depends upon the method of oiling, the rapidity of cooling, and the care which the journal is likely to get. It will have about the following values: Ordinary work, drop-feed lubrication, 700; first-class care, drop-feed lubrication, 1000; force-feed lubrication or ring-oiling, 1200 to 1500; extreme limit for perfect lubrication and air-cooled bearings, 2000. The value 2000 is seldom used, except in locomotive

work where the rapid circulation of the air cools the journals. Higher values than this may only be used in the case of water-cooled bearings.

In case the bearing is some form of a sliding shoe, the quantity $240 V$ should be substituted for the quantity DN , V being the velocity of rubbing in feet per second. There are a few cases where a unit pressure sufficient to break down the oil film is allowable, such as the pins of punching and shearing machines, pivots of swing bridges, etc.

In general, the diameter of a shaft or pin is fixed from considerations of strength or stiffness. Having obtained the proper diameter, we must next make the bearing long enough so that the unit pressure shall not exceed the required value. This length may be found by means of the equation:

$$L = (W \div PK) \times (N + K/D), \quad (2)$$

where L is the length of the bearing in ins., W the load upon it in lbs., and P , K , N , and D are as before.

A bearing may give poor satisfaction because it is too long, as well as because it is too short. Almost every bearing is in the condition of a loaded beam, and therefore it has some deflection.

Shafts and crank-pins must not be made so long that they will allow the load to concentrate at any point. A good rule for the length is to make the ratio of length to diameter about equal to $\frac{1}{8}\sqrt{N}$. This quantity may be diminished by from 10 to 20% in the case of crank-pins and increased in the same proportion in the case of shaft bearings, but it is not wise to depart too far from it. In the case of an engine making 100 r.p.m., the bearings would be by this rule from $1\frac{1}{4}$ to $1\frac{1}{2}$ diams. in length. In the case of a motor running at 1000 r.p.m., the bearings would be about 4 diams. long.

The diameter of a shaft or pin must be such that it will be strong and stiff enough to do its work properly. In order to design it for strength and stiffness, it is first necessary to know its length. This may be assumed tentatively from the equation

$$L = 20 W \sqrt{N} \div PK. \quad (3)$$

The diameter may then be found by any of the standard equations for the strength of shafts or pins given in the different works on machine design. [See *The Strength of the Crank-pin*, page 1007.] The length is then recomputed from formula No. 2, taking this new value if it does not differ materially from the one first assumed. If it does, and especially if it is greater than the assumed length, take the mean value of the assumed and computed lengths, and try again.

EXAMPLE. — We will take the case of the crank-pin of an engine with a 20-in. cylinder, running at 80 r.p.m., and having a maximum unbalanced steam pressure of 100 lbs. per sq. in. The total steam load on the piston is 31,400 pounds. P is taken at 1200, and K as 1000. We will therefore obtain for our trial length:

$$L = (20 \times 31,400 \times \sqrt{80}) \div (1200 \times 1000) = 4.7, \text{ or say } 4\frac{3}{4} \text{ ins.}$$

In order that the deflection of the pin shall not be sufficient to destroy the lubricating film we have

$$D = 0.09 \sqrt[4]{WL^3},$$

which limits the deflection to 0.003 in. This gives $D = 3.85$ or say $3\frac{7}{8}$ ins. With this diameter, formula No. 2 gives $L = 8.9$, say 9 ins.

The mean of this value and the one obtained before is about 7 ins. Substituting this in the equation for the diameter, we get $5\frac{1}{4}$ ins. Substituting this new diameter in equation No. 2 we have $L = 7.05$, say 7 ins.

Probably most good designers would prefer to take about half an inch off the length of this pin, and add it to the diameter, making it $5\frac{3}{4} \times 6\frac{1}{2}$ inches, and this will bring the ratio of the length to the diameter nearer to $\frac{1}{8}\sqrt{N}$.

Engine-frames or Bed-plates.—No definite rules for the design of engine-frames have been given by authors of works on the steam-engine. The proportions are left to the designer who uses "rule of thumb" or copies from existing engines. F. A. Halsey (*Am. Mach.*,

Feb. 14, 1895) has made a comparison of proportions of the frames of horizontal Corliss engines of several builders. The method of comparison is to compute from the measurements the number of square inches in the smallest cross-section of the frame, that is, immediately behind the pillow block, also to compute the total maximum pressure upon the piston, and to divide the latter quantity by the former. The result gives the number of pounds pressure upon the piston allowed for each square inch of metal in the frame. He finds that the number of lbs. per sq. in. of smallest section of frame ranges from 217 for a 10×30 in. engine up to 575 for a 28×48 in. A 30×60 in. engine shows 350 lbs., and a 32-in. engine which has been running for many years shows 667 lbs. Generally the strains increase with the size of the engine, and more cross-section of metal is allowed with relatively long strokes than with short ones.

From the above Mr. Halsey formulates the general rule that in engines of moderate speed, and having strokes up to $1\frac{1}{2}$ times the diameter of the cylinder, the load per square inch of smallest section should be for a 10-in. engine 300 lbs., which figure should be increased for larger bores up to 500 lbs. for a 30-in. cylinder of the same relative stroke. For high speeds or for longer strokes the load per square inch should be reduced.

FLY-WHEELS.

The function of a fly-wheel is to store up and to restore the periodical fluctuations of energy given to or taken from an engine or machine, and thus to keep approximately constant the velocity of rotation. Rankine calls the quantity $\frac{\Delta E}{2 E_0}$ the coefficient of fluctuation of speed or of unsteadiness, in which E_0 is the mean actual energy, and ΔE the excess of energy received or of work performed, above the mean, during a given interval. The ratio of the periodical excess or deficiency of energy ΔE to the whole energy exerted in one period or revolution General Morin found to be from $\frac{1}{6}$ to $\frac{1}{4}$ for single-cylinder engines using expansion; the shorter the cut-off the higher the value. For a pair of engines with cranks coupled at 90° the value of the ratio is about $\frac{1}{4}$, and for three engines with cranks at 120° , $\frac{1}{12}$ of its value for single-cylinder engines. For tools working at intervals, such as punching, slotting and plate-cutting machines, coining-presses, etc., ΔE is nearly equal to the whole work performed at each operation.

A fly-wheel reduces the coefficient $\frac{\Delta E}{2 E_0}$ to a certain fixed amount, being about $\frac{1}{32}$ for ordinary machinery; and $\frac{1}{50}$ or $\frac{1}{60}$ for machinery for fine purposes.

If m be the reciprocal of the intended value of the coefficient of fluctuation of speed, ΔE the fluctuation of energy, I the moment of inertia of the fly-wheel alone, and a_0 its mean angular velocity, $I = \frac{mg\Delta E}{a_0^2}$. As the rim of a fly-wheel is usually heavy in comparison with the arms, I may be taken to equal Wr^2 , in which W = weight of rim in pounds, and r the radius of the wheel; then $W = \frac{mg\Delta E}{a_0^2 r^2} = \frac{mg\Delta E}{v^2}$, if v be the velocity of the rim in feet per second. The usual mean radius of the fly-wheel in steam-engines is from three to five times the length of the crank. The ordinary values of the product mg , the unit of time being the second, lie between 1000 and 2000 feet. (Abridged from Rankine, S. E., p. 62.)

Thurston gives for engines with automatic valve-gear $W = 250,000 \frac{ASp}{R^2 D^2}$, in which A = area of piston in square inches, S = stroke in feet, p = mean steam-pressure in lbs. per sq. in., R = revolutions per minute, D = outside diameter of wheel in feet. Thurston also gives for ordinary forms of non-condensing engine with a ratio of expansion between 3 and 5, $W = \frac{aAS}{R^2 D^2}$, in which a ranges from 10,000,000 to 15,000,000, averaging 12,000,000. For gas-engines, in which the charge is fired with every revolution, the *American Machinist* gives this latter formula, with a

doubled, or 24,000,000. Presumably, if the charge is fired every other revolution, a should be again doubled.

Rankine ("Useful Rules and Tables," p. 247) gives $W = 475,000 \frac{ASp}{VD^2R^2}$, in which V is the variation of speed per cent of the mean speed.

Thurston's first rule above given corresponds with this if we take $V = 1.9$.

Hartnell (*Proc. Inst. M. E.*, 1882, 427) says: The value of V , or the variation permissible in portable engines, should not exceed 3% with an ordinary load, and 4% when heavily loaded. In fixed engines, for ordinary purposes, $V = 2\frac{1}{2}$ to 3%. For good governing or special purposes, such as cotton-spinning, the variation should not exceed $1\frac{1}{2}$ to 2%.

F. M. Rites (*Trans. A. S. M. E.*, xiv, 100) develops a new formula for weight of rim, viz., $W = \frac{C \times \text{I.H.P.}}{R^3D^2}$, and weight of rim per horse-power

$= \frac{C}{R^3D^2}$, in which C varies from 10,000,000,000 to 20,000,000,000; also using the latter value of C , he obtains for the energy of the fly-wheel $\frac{Mv^2}{2} = \frac{W}{64.4} \frac{(3.14)^2 D^2 R^2}{3600} = \frac{C \times \text{H.P.} (3.14)^2 D^2 R^2}{R^3 D^2 \times 64.4 \times 3600} = \frac{850,000 \text{ H.P.}}{R}$. Fly-wheel energy per H.P. = 850,000 $\div R$.

The limit of variation of speed with such a weight of wheel from excess of power per fraction of revolution is less than 0.0023.

The value of the constant C given by Mr. Rites was derived from practice of the Westinghouse single-acting engines used for electric-lighting. For double-acting engines in ordinary service a value of $C = 5,000,000,000$ would probably be ample.

From these formulæ it appears that the weight of the fly-wheel for a given horse-power should vary inversely with the cube of the revolutions and the square of the diameter.

J. B. Stanwood (*Eng'g*, June 12, 1891) says: Whenever 480 feet is the lowest piston-speed probable for an engine of a certain size, the fly-wheel weight for that speed approximates closely to the formula

$$W = 700,000 d^2 s \div D^2 R^2.$$

W = weight in pounds, d = diameter of cylinder in inches, s = stroke in inches, D = diameter of wheel in feet, R = revolutions per minute, corresponding to 480 feet piston-speed.

In a Ready Reference Book published by Mr. Stanwood, Cincinnati, 1892, he gives the same formula, with coefficients as follows: For slide-valve engines, ordinary duty, 350,000; same, electric lighting, 700,000; for automatic high-speed engines, 1,000,000; for Corliss engines, ordinary duty 700,000, electric lighting 1,000,000.

Thurston's formula above given, $W = aAS \div R^2D^2$ with $a = 12,000,000$, when reduced to terms of d and s in inches, becomes $W = 785,400 d^2 s \div R^2D^2$.

If we reduce it to terms of horse-power, we have $\text{I.H.P.} = 2ASPR \div 33,000$, in which P = mean effective pressure. Taking this at 40 lbs., we obtain $W = 5,000,000,000 \text{ I.H.P.} \div R^3D^2$. If mean effective pressure = 30 lbs., then $W = 6,666,000,000 \text{ I.H.P.} \div R^3D^2$.

Emil Theiss (*Am. Mach.*, Sept. 7 and 14, 1893) gives the following values of d , the coefficient of steadiness, which is the reciprocal of what Rankine calls the coefficient of fluctuation:

For engines operating —

| | |
|---|-----------------|
| Hammering and crushing machinery..... | $d = 5$ |
| Pumping and shearing machinery..... | $d = 20$ to 30 |
| Weaving and paper-making machinery | $d = 40$ |
| Milling machinery | $d = 50$ |
| Spinning machinery..... | $d = 50$ to 100 |
| Ordinary driving-engines (mounted on bed-plate), belt transmission..... | $d = 35$ |
| Gear-wheel transmission..... | $d = 50$ |

Mr. Theiss's formula for weight of fly-wheel in pounds is $W = i \times \frac{d \times \text{I.H.P.}}{V^2 \times n}$,

where d is the coefficient of steadiness, V the mean velocity of the fly-wheel rim in feet per second, n the number of revolutions per minute,

i = a coefficient obtained by graphical solution, the values of which for different conditions are given in the following table. In the lines under "cut-off," p means "compression to initial pressure," and O "no compression."

VALUES OF i . SINGLE-CYLINDER NON-CONDENSING ENGINES.

| Piston-speed, ft. per min. | Cut-off, 1/6. | | Cut-off, 1/4. | | Cut-off, 1/3. | | Cut-off, 1/2. | |
|----------------------------|---------------|---------|---------------|---------|---------------|---------|---------------|---------|
| | Comp. p | O | Comp. p | O | Comp. p | O | Comp. p | O |
| 200 | 272,690 | 218,580 | 242,010 | 209,170 | 220,760 | 201,920 | 193,340 | 182,840 |
| 400 | 240,810 | 187,430 | 208,200 | 179,460 | 188,510 | 170,040 | 174,630 | 167,860 |
| 600 | 194,670 | 145,400 | 168,590 | 136,460 | 165,210 | 146,610 | | |
| 800 | 158,200 | 108,690 | 162,070 | 135,260 | | | | |

SINGLE-CYLINDER CONDENSING ENGINES.

| Piston-speed, ft. per min. | Cut-off, 1/8. | | Cut-off, 1/6. | | Cut-off, 1/4. | | Cut-off, 1/3. | | Cut-off, 1/2. | |
|----------------------------|---------------|---------|---------------|---------|---------------|---------|---------------|---------|---------------|---------|
| | Comp. p | O | Comp. p | O | Comp. p | O | Comp. p | O | Comp. p | O |
| 200 | 265,560 | 176,560 | 234,160 | 173,660 | 204,210 | 167,140 | 189,600 | 161,830 | 172,690 | 156,990 |
| 400 | 194,550 | 117,870 | 174,380 | 118,350 | 164,720 | 133,080 | 174,630 | 151,680 | | |
| 600 | 148,780 | 140,090 | | | | | | | | |

TWO-CYLINDER ENGINES, CRANKS AT 90°.

| Piston-speed, ft. per min. | Cut-off, 1/6. | | Cut-off, 1/4. | | Cut-off, 1/3. | | Cut-off, 1/2. | |
|----------------------------|---------------|------------------|---------------|------------------|---------------|------------------|---------------|------------------|
| | Comp. p | O | Comp. p | O | Comp. p | O | Comp. p | O |
| 200 | 71,980 | } Mean 60,140 | 59,420 | } Mean 54,340 | 49,272 | } Mean 50,000 | 37,920 | } Mean 36,950 |
| 400 | 70,160 | | 57,000 | | 49,150 | | 35,000 | |
| 600 | 70,040 | | 57,480 | | 49,220 | | | |
| 800 | 70,040 | | 60,140 | | | | | |

THREE-CYLINDER ENGINES, CRANKS AT 120°.

| Piston-speed, ft. per min. | Cut-off, 1/6. | | Cut-off, 1/4. | | Cut-off, 1/3. | | Cut-off, 1/2. | |
|----------------------------|---------------|--------|---------------|--------|---------------|--------|---------------|--------|
| | Comp. p | O | Comp. p | O | Comp. p | O | Comp. p | O |
| 200 | 33,810 | 32,240 | 33,810 | 35,500 | 34,540 | 33,450 | 35,260 | 32,370 |
| 800 | 30,190 | 31,570 | 35,140 | 33,810 | 36,470 | 32,850 | 33,810 | 32,370 |

As a mean value of i for these engines we may use 33,810.

Weight of Fly-wheels for Alternating-current Units. (J. Begtrup, *Am. Mach.*, July 10, 1902.) —

$$WD^2 + W_1D_1^2 = \frac{14,000,000 HU}{N^3 V},$$

in which W = weight of rim of fly-wheel in pounds, D = mean diameter of rim in feet, W_1 = weight of armature in pounds, D_1 = mean diameter of armature in feet, H = rated horse-power of engine, U = a factor of steadiness, N = number of revolutions per minute, V = maximum instantaneous displacement in degrees, not to exceed 5 degrees divided by the number of poles on the generator, according to the rule of the General Electric Company.

For simple horizontal engines, length of connecting-rod = 5 cranks, $U = 90$; (ditto, no account being taken of angularity of connecting-rod, $U = 64$); cross-compound horizontal engines, connecting-rod = 5 cranks, $U = 51$; ditto, vertical engines, heavy reciprocating parts, unbalanced, $U = 78$; vertical compound engines, cranks 180 degrees apart, reciprocating parts balanced, $U = 60$.

The small periodical variation in velocity (not angular displacement) can be determined from the following formula:

$$F = \frac{387,700,000 HZ}{N^3 (WD^2 + W_1D_1^2)},$$

in which H = rated horse-power, Z = a factor of steadiness, N = revs. per min., D = mean diameter of fly-wheel rim in feet, W = weight of fly-wheel rim in pounds, D_1 = mean diameter of armature or field in feet, W_1 = weight of armature, F = variation in per cent of mean speed.

For simple engines and tandem compounds, $Z = 16$; for horizontal cross-compounds, $Z = 8.5$; for vertical cross-compounds, heavy reciprocating parts, $Z = 12.5$; for vertical compounds, cranks opposite, weights balanced, $Z = 14$. F represents here the entire variation, between extremes — not variation from mean speed. It generally varies from 0.25% of mean speed to 0.75% — evidently a negligible quantity.

A mathematical treatment of this subject will be found in a paper by J. L. Astrom, in *Trans. A. S. M. E.*, 1901.

Centrifugal Force in Fly-wheels. — Let W = weight of rim in pounds; R = mean radius of rim in feet; r = revolutions per minute, $g = 32.16$; v = velocity of rim in feet per second = $\frac{2\pi Rr}{60}$.

$$\text{Centrifugal force of whole rim} = F = \frac{Wv^2}{gR} = \frac{4W\pi^2 Rr^2}{3600g} = 0.000341 WRr^2.$$

The resultant, acting at right angles to a diameter, of half of this force, tends to disrupt one half of the wheel from the other half, and is resisted by the section of the rim at each end of the diameter. The resultant of

half the radial forces taken at right angles to the diameter is $1 + \frac{1}{2}\pi = \frac{2}{\pi}$

of the sum of these forces; hence the total force F is to be divided by $2 \times 2 \times 1.5708 = 6.2832$ to obtain the tensile strain on the cross-section of the rim, or, total strain on the cross-section = $S = 0.00005427 WRr^2$. The weight W_1 of a rim of cast iron 1 inch square in section is $2\pi R \times 3.125 = 19.635 R$ pounds, whence strain per square inch of sectional area of rim = $S_1 = 0.0010656 R^2r^2 = 0.0002664 D^2r^2 = 0.0000270 V^2$, in which D = diameter of wheel in feet, and V is velocity of rim in feet per minute. $S_1 = 0.0972 v^2$, if v is taken in feet per second.

For wrought iron:

$$S_1 = 0.0011366 R^2r^2 = 0.0002842 D^2r^2 = 0.0000288 V^2.$$

For steel:

$$S_1 = 0.0011593 R^2r^2 = 0.0002901 D^2r^2 = 0.0000294 V^2.$$

For wood:

$$S_1 = 0.0000888 R^2r^2 = 0.0000222 D^2r^2 = 0.00000225 V^2.$$

The specific gravity of the wood being taken at 0.6 = 37.5 lbs. per cu. ft., or $\frac{1}{12}$ the weight of cast iron.

EXAMPLE. — Required the strain per square inch in the rim of a cast-iron wheel 30 ft. diameter, 60 revolutions per minute:

Answer. — $15^2 \times 60^2 \times 0.0010656 = 863.1$ lbs.

Required the strain per square inch in a cast-iron wheel-rim running a mile a minute. *Answer.* — $0.000027 \times 5280^2 = 752.7$ lbs.

In cast-iron fly-wheel rims, on account of their thickness, there is difficulty in securing soundness, and a tensile strength of 10,000 lbs. per sq. in. is as much as can be assumed with safety. Using a factor of

safety of 10 gives a maximum allowable strain in the rim of 1000 lbs. per sq. in., which corresponds to a rim velocity of 6085 ft. per minute.

For any given material, as cast iron, the strength to resist centrifugal force depends only on the velocity of the rim, and not upon its bulk or weight.

Chas. E. Emery (*Cass. Maj.*, 1892) says: It does not appear that fly-wheels of customary construction should be unsafe at the comparatively low speeds now in common use if proper materials are used in construction. The cause of rupture of fly-wheels that have failed is usually either the "running away" of the engine, such as may be caused by the breaking or slackness of a governor-belt, or incorrect design or defective materials of the fly-wheel.

Chas. T. Porter (*Trans. A. S. M. E.*, xiv, 808) states that no case of the bursting of a fly-wheel with a solid rim in a high-speed engine is known. He attributes the bursting of wheels built in segments to insufficient strength of the flanges and bolts by which the segments are held together. [The author, however, since the above was written, saw a solid rim fly-wheel of a high-speed engine which had burst, the cause being a large shrinkage hole at the junction between one of the arms and the rim. The wheel was about 6 ft. diam. Fortunately no one was injured by the accident.] (See also Thurston, "Manual of the Steam-engine," Part II, page 413.)

Diameters of Fly-wheels for Various Speeds. — If 6000 feet per minute be the maximum velocity of rim allowable, then $6000 = \pi R D$, in which R = revolutions per minute, and D = diameter of wheel in feet, whence $D = 6000 \div \pi R = 1910 \div R$.

W. H. Boehm, Supt. of the Fly-wheel Dept. of the Fidelity and Casualty Co. (*Eng. News*, Oct. 2, 1902), says: For a given material there is a definite speed at which disruption will occur, regardless of the amount of material used. This mathematical truth is expressed by the formula:

$$V = 1.6 \sqrt{S/W},$$

in which V is the velocity of the rim of the wheel in feet per second at which disruption will occur, W the weight of a cubic inch of the material used, and S the tensile strength of 1 square inch of the material.

For cast-iron wheels made in one piece, assuming 20,000 lbs. per sq. in. as the strength of small test bars, and 10,000 lbs. per sq. in. in large castings, and applying a factor of safety of 10, $V = 1.6 \sqrt{1000/0.26} = 100$ ft. per second for the safe speed. For cast steel of 60,000 lbs. per sq. in., $V = 1.6 \sqrt{6000 \div 0.28} = 233$ ft. per second. This is for wheels made in one piece. If the wheel is well made in halves, or sections, the efficiency of the rim joint must be taken into consideration. For belt wheels with flanged and bolted rim joints located between the arms, the joints average only one-fifth the strength of the rim, and no such joint can be designed having a strength greater than one-fourth the strength of the rim. If the rim is thick enough to allow the joint to be reinforced by steel links shrunk on, as in heavy balance wheels, one-third the strength of the rim may be secured in the joint; but this construction can not be applied to belt wheels having thin rims.

For hard maple, having a tensile strength of 10,500 lbs. per sq. in., and weighing 0.0283 lb. per cu. in., we have, using a factor of safety of 20, and remembering that the strength is reduced one-half because the wheel is built up of segments, $V = 1.6 \sqrt{262.5 \div 0.0283} = 154$ ft. per second. The stress in a wheel varies as the square of the speed, and the factor of safety on speed is the square root of the factor of safety on strength.

Mr. Boehm gives the following table of safe revolutions per minute of cast-iron wheels of different diameters. The flange joint is taken at 0.25 of the strength of a wheel with no joint, the pad joint, that is a wheel made in six segments, with bolted flanges or pads on the arms, = 0.50, and the link joint = 0.60 of the strength of a solid rim.

SAFE REVOLUTIONS PER MINUTE OF CAST-IRON FLY-WHEELS.

| | No joint. | Flange joint. | Pad joint. | Link joint. | | No joint. | Flange joint. | Pad joint. | Link joint. |
|--------------|-----------|---------------|------------|-------------|--------------|-----------|---------------|------------|-------------|
| Diam. in Ft. | R.P.M. | R.P.M. | R.P.M. | R.P.M. | Diam. in Ft. | R.P.M. | R.P.M. | R.P.M. | R.P.M. |
| 1 | 1910 | 955 | 1350 | 1480 | 16 | 120 | 60 | 84 | 92 |
| 2 | 955 | 478 | 675 | 740 | 17 | 112 | 56 | 79 | 87 |
| 3 | 637 | 318 | 450 | 493 | 18 | 106 | 53 | 75 | 82 |
| 4 | 478 | 239 | 338 | 370 | 19 | 100 | 50 | 71 | 78 |
| 5 | 382 | 191 | 270 | 296 | 20 | 95 | 48 | 68 | 74 |
| 6 | 318 | 159 | 225 | 247 | 21 | 91 | 46 | 65 | 70 |
| 7 | 273 | 136 | 193 | 212 | 22 | 87 | 44 | 62 | 67 |
| 8 | 239 | 119 | 169 | 185 | 23 | 84 | 42 | 59 | 64 |
| 9 | 212 | 106 | 150 | 164 | 24 | 80 | 40 | 56 | 62 |
| 10 | 191 | 96 | 135 | 148 | 25 | 76 | 38 | 54 | 59 |
| 11 | 174 | 87 | 123 | 135 | 26 | 74 | 37 | 52 | 57 |
| 12 | 159 | 80 | 113 | 124 | 27 | 71 | 35 | 50 | 55 |
| 13 | 147 | 73 | 104 | 114 | 28 | 68 | 34 | 48 | 53 |
| 14 | 136 | 68 | 96 | 106 | 29 | 66 | 33 | 47 | 51 |
| 15 | 128 | 64 | 90 | 99 | 30 | 64 | 32 | 45 | 49 |

The table is figured for a margin of safety on speed of approximately 3, which is equivalent to a margin on stress developed, or factor of safety in the usual sense, of 9. (*Am. Mach.*, Nov. 17, 1904.)

Strains in the Rims of Fly-band Wheels Produced by Centrifugal Force. (James B. Stanwood, *Trans. A. S. M. E.*, xiv, 251.)—Mr. Stanwood mentions one case of a fly-band wheel where the periphery velocity on a 17 ft. 9 in. wheel is over 7500 ft. per minute.

In band-saw mills the blade of the saw is operated successfully over wheels 8 and 9 ft. in diameter, at a periphery velocity of 9000 to 10,000 ft. per minute. These wheels are of cast iron throughout, of heavy thickness, with a large number of arms.

In shingle-machines and chipping-machines where cast-iron disks from 2 to 5 ft. in diameter are employed, with knives inserted radially, the speed is frequently 10,000 to 11,000 ft. per minute at the periphery.

If the rim of a fly-wheel alone be considered, the tensile strain in pounds per square inch of the rim section is $T = V^2/10$ nearly, in which V = velocity in feet per second; but this strain is modified by the resistance of the arms, which prevent the uniform circumferential expansion of the rim, and induce a bending as well as a tensile strain. Mr. Stanwood discusses the strains in band-wheels due to transverse bending of a section of the rim between a pair of arms.

When the arms are few in number, and of large cross-section, the rim will be strained transversely to a greater degree than with a greater number of lighter arms. To illustrate the necessary rim thicknesses for various rim velocities, pulley diameters, number of arms, etc., the following table is given, based upon the formula

$$t = 0.475 d \div N^2 \left(\frac{F}{V^2} - \frac{1}{10} \right),$$

in which t = thickness of rim in inches, d = diameter of pulley in inches, N = number of arms, V = velocity of rim in feet per second, and F = the greatest strain in pounds per square inch to which any fiber is subjected. The value of F is taken at 6000 lbs. per sq. in.

THICKNESS OF RIMS IN SOLID WHEELS.

| Diameter of Pulley in inches. | Velocity of Rim in feet per second. | Velocity of Rim in feet per minute. | No. of Arms. | Thickness in inches. |
|-------------------------------|-------------------------------------|-------------------------------------|--------------|----------------------|
| 24 | 50 | 3,000 | 6 | $\frac{2}{10}$ |
| 24 | 88 | 5,280 | 6 | $\frac{15}{32}$ |
| 48 | 88 | 5,280 | 6 | $\frac{15}{16}$ |
| 108 | 184 | 11,040 | 16 | $2\frac{1}{2}$ |
| 108 | 184 | 11,040 | 36 | $\frac{1}{2}$ |

If the limit of rim velocity for all wheels be assumed to be 88 ft. per second, equal to 1 mile per minute, $F = 6000$ lbs., the formula becomes

$$t = 0.475 d \div 0.67 N^2 = 0.7 d \div N^2.$$

When wheels are made in halves or in sections, the bending strain may be such as to make t greater than that given above. Thus, when the joint comes half way between the arms, the bending action is similar to a beam supported simply at the ends, uniformly loaded, and t is 50% greater. Then the formula becomes $t = 0.712 d \div N^2 \left(\frac{F}{V^2} - \frac{1}{10} \right)$, or for a fixed maximum rim velocity of 88 ft. per second and $F = 6000$ lbs., $t = 1.05 d \div N^2$. In segmental wheels it is preferable to have the joints opposite the arms. Wheels in halves, if very thin rims are to be employed, should have double arms along the line of separation.

Attention should be given to the proportions of large receiving and tightening pulleys. The thickness of rim for a 48-in. wheel (shown in table) with a rim velocity of 88 ft. per second, is $1\frac{1}{6}$ in. Many wrecks have been caused by the failure of receiving or tightening pulleys whose rims have been too thin. Fly-wheels calculated for a given coefficient of steadiness are frequently lighter than the minimum safe weight. This is true especially of large wheels. A rough guide to the minimum weight of wheels can be deduced from our formulæ. The arms, hub, lugs, etc., usually form from one-quarter to one-third the entire weight of the wheel. If b represents the face of a wheel in inches, the weight of the rim (considered as a simple annular ring) will be $w = 0.82 d b$ lbs. If the limit of speed is 88 ft. per second, then for solid wheels $t = 0.7 d \div N^2$. For sectional wheels (joint between arms) $t = 1.05 d \div N^2$. Weight of rim for solid wheels, $w = 0.57 d^2 b \div N^2$, in pounds. Weight of rim in sectional wheels with joints between arms, $w = 0.86 d^2 b \div N^2$, in pounds. Total weight of wheel: for solid wheel, $W = 0.76 d^2 b \div N^2$ to $0.86 d^2 b \div N^2$, in pounds. For segmental wheels with joint between arms, $W = 1.05 d^2 b \div N^2$ to $1.3 d^2 b \div N^2$, in pounds.

(This subject is further discussed by Mr. Stanwood, in vol. xv, and by Prof. Gaetano Lanza, in vol. xvi, *Trans. A. S. M. E.*)

Arms of Fly-wheels and Pulleys. — Professor Torrey (*Am. Mach.*, July 30, 1891) gives the following formula for arms of elliptical cross-section of cast-iron wheels:

W = load in pounds acting on one arm; S = strain on belt in pounds per inch of width, taken at 56 for single and 112 for double belts; v = width of belt in inches; n = number of arms; L = length of arm in feet; b = breadth of arm at hub; d = depth of arm at hub, both in inches: $W = Sv \div n$; $b = WL \div 30 d^2$. The breadth of the arm is its least dimension = minor axis of the ellipse, and the depth the major axis. This formula is based on a factor of safety of 10.

In using the formula, first assume some depth for the arm, and calculate the required breadth to go with it. If it gives too round an arm, assume the depth a little greater, and repeat the calculation. A second trial will almost always give a good section.

The size of the arms at the hub having been calculated, they may be somewhat reduced at the rim end. The actual amount cannot be calculated, as there are too many unknown quantities. However, the depth

and breadth can be reduced about one-third at the rim without danger, and this will give a well-shaped arm.

Pulleys are often cast in halves, and bolted together. When this is done the greatest care should be taken to provide sufficient metal in the bolts. This is apt to be the very weakest point in such pulleys. The combined area of the bolts at each joint should be about 28/100 the cross-section of the pulley at that point. (Torrey.)

Unwin gives $d = 0.6337 \sqrt[3]{BD/n}$ for single belts;

$d = 0.798 \sqrt[3]{BD/n}$ for double belts;

D being the diameter of the pulley, and B the breadth of the rim, both in inches. These formulae are based on an elliptical section of arm in which $b = 0.4 d$ or $d = 2.5 b$ on a width of belt = $\frac{4}{5}$ the width of the pulley rim, a maximum driving force transmitted by the belt of 56 lbs. per inch of width for a single belt and 112 lbs. for a double belt, and a safe working stress of cast iron of 2250 lbs. per square inch.

If in Torrey's formula we make $b = 0.4 d$, it reduces to

$$b = \sqrt[3]{\frac{WL}{187.5}}; d = \sqrt[3]{\frac{WL}{12}}.$$

EXAMPLE. — Given a pulley 10 feet diameter; 8 arms, each 4 feet long; face, 36 inches wide; belt, 30 inches: required the breadth and depth of the arm at the hub. According to Unwin,

$d = 0.6337 \sqrt[3]{BD/n} = 0.6337 \sqrt[3]{36 \times 120/8} = 5.16$ for single belt, $b = 2.06$;

$d = 0.798 \sqrt[3]{BD/n} = 0.798 \sqrt[3]{36 \times 120/8} = 6.50$ for double belt, $b = 2.60$.

According to Torrey, if we take the formula $b = WL \div 30 d^2$ and assume $d = 5$ and 6.5 inches, respectively, for single and double belts, we obtain $b = 1.08$ and 1.33 , respectively, or practically only one-half of the breadth according to Unwin, and, since transverse strength is proportional to breadth, an arm only one-half as strong.

Torrey's formula is said to be based on a factor of safety of 10, but this factor can be only apparent and not real, since the assumption that the strain on each arm is equal to the strain on the belt divided by the number of arms, is, to say the least, inaccurate. It would be more nearly correct to say that the strain of the belt is divided among half the number of arms. Unwin makes the same assumption in developing his formula, but says it is only in a rough sense true, and that a large factor of safety must be allowed. He therefore takes the low figure of 2250 lbs. per square inch for the safe working strength of cast iron. Unwin says that his equations agree well with practice.

A Wooden-rim Fly-wheel, built in 1891 for a pair of Corliss engines at the Amoskeag Mfg. Co.'s mill, Manchester, N.H., is described by C. H. Manning in *Trans. A. S. M. E.*, xiii, 618. It is 30 ft. diam. and 108 in. face. The rim is 12 inches thick, and is built up of 44 courses of ash plank, 2, 3, and 4 inches thick, reduced about $\frac{1}{2}$ inch in dressing, set edgewise, so as to break joints, and glued and bolted together. There are two hubs and two sets of arms, 12 in each, all of cast iron. The weights are as follows:

| | | |
|---|----------|------|
| Weight (calculated) of ash rim..... | 31,855 | lbs. |
| Weight of 24 arms (foundry 45,020)..... | 40,349 | " |
| Weight of 2 hubs (foundry 35,030)..... | 31,394± | " |
| Counter-weights in 6 arms..... | 664 | " |
| Total, excluding bolts and screws..... | 104,262± | " |

The wheel was tested at 76 revs. per min., being a surface speed of nearly 7200 feet per minute.

Wooden Fly-wheel of the Willimantic Linen Co. (Illustrated in *Power*, March, 1893.) — Rim 28 ft. diam., 110 in. face. The rim is carried upon three sets of arms, one under the center of each belt, with 12 arms in each set.

The material of the rim is ordinary whitewood, $\frac{7}{8}$ in. in thickness, cut into segments not exceeding 4 feet in length, and either 5 or 8 inches in

width. These were assembled by building a complete circle 13 inches in width, first with the 8-inch inside and the 5-inch outside, and then beside it another circle with the widths reversed, so as to break joints. Each piece as it was added was brushed over with glue and nailed with three-inch wire nails to the pieces already in position. The nails pass through three and into the fourth thickness. At the end of each arm four 14-inch bolts secure the rim, the ends being covered by wooden plugs glued and driven into the face of the wheel.

Wire-wound Fly-wheels for Extreme Speeds. (*Eng'g News*, August 2, 1890.) — The power required to produce the Mannesmann tubes is very large, varying from 2000 to 10,000 H.P., according to the dimensions of the tube. Since this power is needed for only a short time (it takes only 30 to 45 seconds to convert a bar 10 to 12 ft. long and 4 in. in diameter into a tube), and then some time elapses before the next bar is ready, an engine of 1200 H.P. provided with a large fly-wheel for storing the energy will supply power enough for one set of rolls. These fly-wheels are so large and run at such great speeds that the ordinary method of constructing them cannot be followed. A wheel at the Mannesmann Works, made in Komotau, Hungary, in the usual manner, broke at a tangential velocity of 125 ft. per second. The fly-wheels designed to hold at more than double this speed consist of a cast-iron hub to which two steel disks, 20 ft. in diameter, are bolted; around the circumference of the wheel thus formed 70 tons of No. 5 wire are wound under a tension of 50 lbs. In the Mannesmann Works at Landore, Wales, such a wheel makes 240 revolutions a minute, corresponding to a tangential velocity of 15,080 ft. or 2.85 miles per minute.

THE SLIDE-VALVE.

Definitions. — *Travel* = total distance moved by the valve.

Throw of the Eccentric = eccentricity of the eccentric = distance from the center of the shaft to the center of the eccentric disk = $\frac{1}{2}$ the travel of the valve.

Lap of the valve, also called outside lap or steam-lap = distance the outer or steam edge of the valve extends beyond or laps over the steam edge of the port when the valve is in its central position.

Inside lap, or *exhaust-lap* = distance the inner or exhaust edge of the valve extends beyond or laps over the exhaust edge of the port when the valve is in its central position. The inside lap is sometimes made zero, or even negative, in which latter case the distance between the edge of the valve and the edge of the port is sometimes called exhaust clearance, or inside clearance.

Lead of the valve = the distance the steam-port is opened when the engine is on its center and the piston is at the beginning of the stroke.

Lead-angle = the angle between the position of the crank when the valve begins to be opened and its position when the piston is at the beginning of the stroke.

The valve is said to have lead when the steam-port opens before the piston begins its stroke. If the piston begins its stroke before the admission of steam begins, the valve is said to have negative lead, and its amount is the lap of the edge of the valve over the edge of the port at the instant when the piston stroke begins.

Lap-angle = the angle through which the eccentric must be rotated to cause the steam edge to travel from its central position the distance of the lap.

Angular advance of the eccentric = lap-angle + lead-angle.

Linear advance = lap + lead.

Effect of Lap, Lead, etc., upon the Steam Distribution. — Given valve-travel $2\frac{3}{4}$ in., lap $\frac{3}{4}$ in., lead $\frac{1}{16}$ in., exhaust-lap $\frac{1}{8}$ in., required crank position for admission, cut-off, release and compression, and greatest port-opening. (Halsey on Slide-valve Gears.) Draw a circle of diameter fh = travel of valve. From O the center set off Oa = lap and ab = lead, erect perpendiculars Oe , ac , bd ; then ec is the lap-angle and cd the lead-angle, measured as arcs. Set off fg = cd , the lead-angle; then Og is the position of the crank for steam admission. Set off $2ec + cd$ from h to i ; then Oi is the crank-angle for cut-off, and $fk + fh$ is the fraction of stroke completed at cut-off. Set off Ol = exhaust-

lap and draw lm ; em is the exhaust-lap angle. Set off $hn = ec + cd - em$, and On is the position of crank at release. Set off $fp = ec + cd + em$, and Op is the position of crank for compression, $fo + fh$ is the fraction of stroke completed at release, and $hq + hf$ is the fraction of the return stroke completed when compression begins; Oh , the throw of the eccentric, minus Oa the lap, equals ah the maximum port-opening.

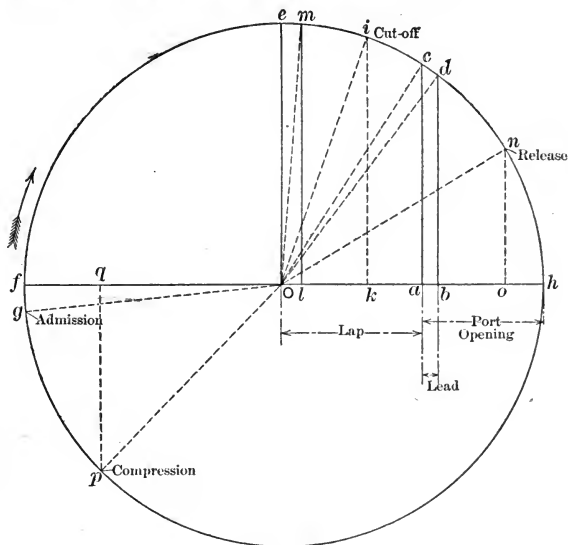


FIG. 162.

If a valve has neither lap nor lead, the line joining the center of the eccentric disk and the center of the shaft being at right angles to the line of the crank, the engine would follow full stroke, admission of steam beginning at the beginning of the stroke and ending at the end of the stroke.

Adding lap to the valve enables us to cut off steam before the end of the stroke. The eccentric being advanced on the shaft an amount equal to the lap-angle enables steam to be admitted at the beginning of the stroke, as before lap was added, and advancing it a further amount equal to the lead-angle causes steam to be admitted before the beginning of the stroke.

Having given lap to the valve, and having advanced the eccentric on the shaft from its central position at right angles to the crank, through the angular advance = lap-angle + lead-angle, the four events, admission, cut-off, release or exhaust-opening, and compression or exhaust-closure, take place as follows: Admission, when the crank lacks the lead-angle of having reached the center; cut-off, when the crank lacks two lap-angles and one lead-angle of having reached the center. During the admission of steam the crank turns through a semicircle less twice the lap-angle. The greatest port-opening is equal to half the travel of the valve less the lap. Therefore for a given port-opening the travel of the valve must be increased if the lap is increased. When exhaust-lap is added to the valve it delays the opening of the exhaust and hastens its closing by an angle of rotation equal to the exhaust-lap angle, which is the angle through which the eccentric rotates from its middle position

are given; and (2) for obtaining the points of admission, cut-off, release, and compression, etc., when the travel, the laps, and the lead of the valve are given. In working out these two problems, the connecting-rod is supposed to be of infinite length.

Determination of the Lap, Lead, etc., of a Slide-valve for Given Steam Distribution. — Given the points of admission, cut-off, and release, to find the point of compression, the lap, the lead, the exhaust lap, the angular advance, and the port-openings at different fractions of the stroke.

Draw a straight line AA' , Fig. 164, to represent on any scale the travel of the valve, and on it draw a circle, with the center O , to represent the path of the center of the eccentric. The line and the circle will also represent on a different scale the length of stroke of the piston and the path of the crank-pin. On the circle, which is called the *crank circle*, mark B ,

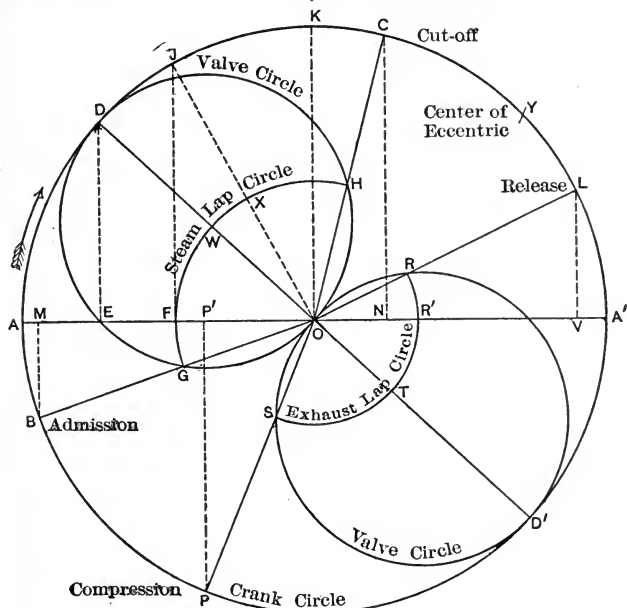


FIG. 164.—Zeuner's Valve Diagram.

the position of the crank-pin when *admission* of steam begins, the direction of motion of the crank being shown by the arrow; C , the position of the crank-pin at *cut-off*; and L , its position at *release*. From these points draw perpendiculars BM , CN , and LV , to the line AA' ; M , N , and V will then represent the positions of the piston at admission, cut-off, and release respectively, the admission taking place, as shown, before the piston reaches the end of the stroke in the direction OA , and release taking place before the end of the stroke in the direction OA' .

Bisect the arc BC at D , and draw the diameter DOD' . On DO draw the circle $DHOGE$, called the *valve circle*. Draw OB , cutting the valve circle at G ; and OC , cutting it at H . Then $OG = OH$ is the *lap* of the valve, measured on the scale in which OA is the half-travel of the valve. With OG as radius draw the arc GFH , called the *steam-lap circle*, or, for short, the *lap circle*.

Mark the point E , at which the valve circle cuts the line OA . The distance FE represents the *lead* of the valve, and $BG = AF$ is the *maximum port-opening*. A perpendicular drawn from OA at E will cut the valve circle and the crank circle at D , since the triangle DEO is a right-angled triangle drawn in the semicircle $DEGO$.

Erect the perpendicular FJ , then angle $DOJ = AOB$ is the *lead-angle* and JOK is the *lap-angle*, OK being a perpendicular to AA' drawn from O . DOK is the sum of the lap and lead angles, that is, the *angular advance*, by which the eccentric must be set beyond 90° ahead of the crank. Set off $KY = KD$; then Y is the position of the *center of the eccentric* when the crank is in the position OA .

To find the point of compression, set off $D'P = D'L$; then P is the *point of compression*.

Draw OP and OL . On OD' draw the valve circle $ORD'S$, cutting OL at R and OP at S . With OR as a radius draw the arc of the *exhaust-lap circle*, RTS ; $OR = OS$ is the *exhaust lap*.

The *port-opening* at any part of the stroke, or corresponding position of the crank, is represented by the radial distances, as EF , DW , and JX , intercepted between the lap and the valve circles on radii drawn from O . Thus, on the radius OB , the port-opening is zero when steam admission is about to begin; on the radius OA , when the crank is on the dead center the opening is EF , or equal to the lead of the valve; on the radius DO , midway between the point of admission and the point of cut-off, the opening is a maximum $DW = AF = BG$; on the radius OC it is zero again when steam has just been cut off.

In like manner the *exhaust opening* is represented by the radial distances intercepted between the exhaust-lap circle, $RR'TS$, and the valve circle, $ORD'S$. On the radius OL it is zero when release begins; on OD' it is TD' , a maximum; and on OP it is zero again when compression begins.

Determination of the Steam Distribution, etc., for a Given Valve. — Given the valve travel, the lap, the lead, and the exhaust lap, to find the maximum port-opening, the angular advance, and the points of admission, cut-off, release, and compression.

This problem is the reverse of the preceding. Draw AOA' to represent the valve travel on a certain scale, O being the middle point, and on this line on the same scale set off $OF =$ the lap, $FE =$ the lead, and $OR' =$ the exhaust lap. AF then will be the *maximum port-opening*. Draw the perpendiculars OK and ED . DOK is the *angular advance*.

Draw the diameter DOD' , and on DO and $D'O$ draw the two valve circles. From O , the center, with a radius OF , the lap, draw the arc of the steam-lap circle cutting the valve circle in G and H . Through G draw OB , and through H draw OC ; B then is the *point of admission*, and C the *point of cut-off*. With OR , the exhaust lap, as a radius, draw the arc of the exhaust-lap circle, RTS , cutting the valve circle in R and S . Through R draw OL , and through S draw OP . Then L is the *point of release* and P the *point of compression*. Draw the perpendiculars BM , CN , LV , and PP' , to find M , N , V , and P' , the respective positions on the stroke of the piston when *admission*, *cut-off*, *release*, and *compression* take place.

Practical Application of Zeuner's Diagram. — In problems solved by means of the Zeuner diagram, the results obtained on the drawings are *relative dimensions* or the *ratios* of the several dimensions to a given dimension the scale of which is known, such as the valve travel, the maximum port-opening, or the length of stroke. In problems similar to the first problem given above, the known dimensions are usually the length of stroke, the maximum port-opening, AF , which is calculated from data of the dimensions of cylinder, the piston speed, and the allowable velocity of steam through the port. The length of the stroke being represented on a certain scale by AA' , the points of admission, cut-off, release, and compression, in fractions of the stroke, are measured respectively by $A'M$, AN , AV , and $A'P$ on the same scale. The actual dimension of the maximum port-opening is represented on a different scale by AF , therefore the actual dimensions of the lap, lead, and exhaust lap are measured respectively by OF , FE , and OR' on the same scale as AF ; or, in other words, the lap, lead, and exhaust lap are respectively the ratios $\frac{OF}{AF}$, $\frac{FE}{AF}$, and $\frac{OR'}{AF}$, each multiplied by the maximum port-opening.

In problems similar to the second problem, the actual dimensions of the lap, the lead, the exhaust lap, and the valve travel are all known, and are laid down on the same scale on the line AA' , representing the valve travel; and the maximum port-opening is found by the solution of the problem to be AF , measured on the same scale; or the maximum port-opening = $\frac{1}{2}$ valve travel minus the lap. Also in this problem AA' represents the known length of stroke on a certain scale, and the points of admission, cut-off, release, and compression, in fractions of the stroke, are represented by the ratios which $A'M$, AN , AV , and $A'P$, respectively, bear to AA' .

Port-opening. — The area of port-opening is usually made such that the velocity of the steam in passing through it should not exceed 6000 ft. per min. The ratio of port area to piston area will vary with the piston-speed as follows:

| | | | | | | | | | | | | |
|-------------------------------------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|
| For speed of piston, | { | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | 1000 | 1200 |
| ft. per min. | | | | | | | | | | | | |
| Port area = piston area \times | { | | | | | | | | | | | |

For a velocity of 6000 ft. per min.,

$$\text{Port area} = \text{sq. of diam. of cyl.} \times \text{piston speed} \div 7639.$$

The length of the port-opening may be equal to or something less than the diameter of the cylinder, and the width = area of port-opening \div its length.

The bridge between steam and exhaust ports should be wide enough to prevent a leak of steam into the exhaust due to overtravel of the valve.

The width of exhaust port = width of steam port + $\frac{1}{2}$ travel of valve + inside lap - width of bridge.

Lead. (From Peabody's Valve-gears.) — The lead, or the amount that the valve is open when the engine is on a dead point, varies, with the type and size of the engine, from a very small amount, or even nothing, up to $\frac{3}{8}$ of an inch or more. Stationary-engines running at slow speed may have from $\frac{1}{64}$ to $\frac{1}{16}$ inch lead. The effect of compression is to fill the waste space at the end of the cylinder with steam; consequently, engines having much compression need less lead. Locomotive-engines having the valves controlled by the ordinary form of Stephenson link-motion may have a small lead when running slowly and with a long cut-off, but when at speed with a short cut-off the lead is at least $\frac{1}{4}$ inch; and locomotives that have valve-gear which gives constant lead commonly have $\frac{1}{4}$ inch lead. The lead-angle is the angle the crank makes with the line of dead points at admission. It may vary from 0° to 8° .

Inside Lead. — Weisbach (vol. ii, p. 296) says: Experiment shows that the earlier opening of the exhaust ports is especially of advantage, and in the best engines the lead of the valve upon the side of the exhaust, or the inside lead, is $\frac{1}{25}$ to $\frac{1}{15}$; i.e., the slide-valve at the lowest or highest position of the piston has made an opening whose height is $\frac{1}{25}$ to $\frac{1}{15}$ of the whole throw of the slide-valve. The outside lead of the slide-valve or the lead on the steam side, on the other hand, is much smaller, and is often only $\frac{1}{100}$ of the whole throw of the valve.

Effect of Changing Outside Lap, Inside Lap, Travel and Angular Advance. (Thurston.)

| | Admission. | Expansion. | Exhaust. | Compression. |
|------------|----------------------------------|------------------------------------|--------------------------------|---------------------------------|
| Incr. O.L. | is later, ceases sooner | occurs earlier, continues longer | is unchanged | begins at same point |
| Incr. I.L. | unchanged | begins as before, continues longer | occurs later, ceases earlier | begins sooner, continues longer |
| Incr. T. | begins sooner, continues longer | begins later, ceases sooner | begins later, ceases later | begins later, ends sooner |
| Incr. A.A. | begins earlier, period unaltered | begins sooner, per. the same | begins earlier, per. unchanged | begins earlier, per. the same |

Zeuner gives the following relations (Weisbach-Dubois, vol. ii, p. 307):

If S = travel of valve, p = maximum port opening;

L = steam-lap, l = exhaust-lap;

R = ratio of steam-lap to half travel = $\frac{L}{0.5 S}$, $L = \frac{R}{2} \times S$;

r = ratio of exhaust-lap to half travel = $\frac{l}{0.5 S}$, $l = \frac{r}{2} \times S$;

$S = 2p + 2L = 2p + R \times S$; $S = \frac{2p}{1-R}$.

If α = angle BOC between positions of crank at admission and at cut-off, and β = angle LOP between positions of crank at release and at compression, then $R = \frac{1}{2} \frac{\sin(180^\circ - \alpha)}{\sin \frac{1}{2} \alpha}$; $r = \frac{1}{2} \frac{\sin(180^\circ - \beta)}{\sin \frac{1}{2} \beta}$.

Crank-angles for Connecting-rods of Different Lengths.

FORWARD AND RETURN STROKES.

| Fraction of Stroke from Commencement. | Ratio of Length of Connecting-rod to Length of Stroke. | | | | | | | | | | | | Infinite |
|---|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------------------|
| | 2 | | 2 1/2 | | 3 | | 3 1/2 | | 4 | | 5 | | |
| | For. | Ret. | For. | Ret. | For. | Ret. | For. | Ret. | For. | Ret. | For. | Ret. | For. or Ret. |
| .01 | 10.3 | 13.2 | 10.5 | 12.8 | 10.6 | 12.6 | 10.7 | 12.4 | 10.8 | 12.3 | 10.9 | 12.1 | 11.5 |
| .02 | 14.6 | 18.7 | 14.9 | 18.1 | 15.1 | 17.8 | 15.2 | 17.5 | 15.3 | 17.4 | 15.5 | 17.1 | 16.3 |
| .03 | 17.9 | 22.9 | 18.2 | 22.2 | 18.5 | 21.8 | 18.7 | 21.5 | 18.8 | 21.3 | 19.0 | 21.0 | 19.9 |
| .04 | 20.7 | 26.5 | 21.1 | 25.7 | 21.4 | 25.2 | 21.6 | 24.9 | 21.8 | 24.6 | 22.0 | 24.3 | 23.1 |
| .05 | 23.2 | 29.6 | 23.6 | 28.7 | 24.0 | 28.2 | 24.2 | 27.8 | 24.4 | 27.5 | 24.7 | 27.2 | 25.8 |
| .10 | 33.1 | 41.9 | 33.8 | 40.8 | 34.3 | 40.1 | 34.6 | 39.6 | 34.9 | 39.2 | 35.2 | 38.7 | 36.9 |
| .15 | 41 | 51.5 | 41.9 | 50.2 | 42.4 | 49.3 | 42.9 | 48.7 | 43.2 | 48.3 | 43.6 | 47.7 | 45.6 |
| .20 | 48 | 59.6 | 48.9 | 58.2 | 49.6 | 57.3 | 50.1 | 56.6 | 50.4 | 56.2 | 50.9 | 55.5 | 53.1 |
| .25 | 54.3 | 66.9 | 55.4 | 65.4 | 56.1 | 64.4 | 56.6 | 63.7 | 57.0 | 63.3 | 57.6 | 62.6 | 60.0 |
| .30 | 60.3 | 73.5 | 61.5 | 72.0 | 62.2 | 71.0 | 62.8 | 70.3 | 63.3 | 69.8 | 63.9 | 69.1 | 66.4 |
| .35 | 66.1 | 79.8 | 67.3 | 78.3 | 68.1 | 77.3 | 68.8 | 76.6 | 69.2 | 76.1 | 69.9 | 75.3 | 72.5 |
| .40 | 71.7 | 85.8 | 73.0 | 84.3 | 73.9 | 83.3 | 74.5 | 82.6 | 75.0 | 82.0 | 75.7 | 81.3 | 78.5 |
| .45 | 77.2 | 91.5 | 78.6 | 90.1 | 79.6 | 89.1 | 80.2 | 88.4 | 80.7 | 87.9 | 81.4 | 87.1 | 84.3 |
| .50 | 82.8 | 97.2 | 84.3 | 95.7 | 85.2 | 94.8 | 85.9 | 94.1 | 86.4 | 93.6 | 87.1 | 92.9 | 90.0 |
| .55 | 88.5 | 102.8 | 89.9 | 101.4 | 90.9 | 100.4 | 91.6 | 99.8 | 92.1 | 99.3 | 92.9 | 98.6 | 95.7 |
| .60 | 94.2 | 108.3 | 95.7 | 107.0 | 96.7 | 106.1 | 97.4 | 105.5 | 98.0 | 105.0 | 98.7 | 104.3 | 101.5 |
| .65 | 100.2 | 113.9 | 101.7 | 112.7 | 102.7 | 111.9 | 103.4 | 111.2 | 103.9 | 110.8 | 104.7 | 110.1 | 107.5 |
| .70 | 106.5 | 119.7 | 108.0 | 118.5 | 109.0 | 117.8 | 109.7 | 117.2 | 110.2 | 116.7 | 110.9 | 116.1 | 113.6 |
| .75 | 113.1 | 125.7 | 114.6 | 124.6 | 115.6 | 123.9 | 116.3 | 123.4 | 116.7 | 123.0 | 117.4 | 122.4 | 120.0 |
| .80 | 120.4 | 132 | 121.8 | 131.1 | 122.7 | 130.4 | 123.4 | 129.9 | 123.8 | 129.6 | 124.5 | 129.1 | 126.9 |
| .85 | 128.5 | 139 | 129.8 | 138.1 | 130.7 | 137.6 | 131.3 | 137.1 | 131.7 | 136.8 | 132.3 | 136.4 | 134.4 |
| .90 | 138.1 | 146.9 | 139.2 | 146.2 | 139.9 | 145.7 | 140.4 | 145.4 | 140.8 | 145.1 | 141.3 | 144.8 | 143.1 |
| .95 | 150.4 | 156.8 | 151.3 | 156.4 | 151.8 | 156.0 | 152.2 | 155.8 | 152.5 | 155.6 | 152.8 | 155.3 | 154.2 |
| .96 | 153.5 | 159.3 | 154.3 | 158.9 | 154.8 | 158.6 | 155.1 | 158.4 | 155.4 | 158.2 | 155.7 | 158.0 | 156.9 |
| .97 | 157.1 | 162.1 | 157.8 | 161.8 | 158.2 | 161.5 | 158.5 | 161.3 | 158.7 | 161.2 | 159.0 | 161.0 | 160.1 |
| .98 | 161.3 | 165.4 | 161.9 | 165.1 | 162.2 | 164.9 | 162.5 | 164.8 | 162.6 | 164.7 | 162.9 | 164.5 | 163.7 |
| .99 | 166.8 | 169.7 | 167.2 | 169.5 | 167.4 | 169.4 | 167.6 | 169.3 | 167.7 | 169.2 | 167.9 | 169.1 | 168.5 |
| 1.00 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 |

Ratio of Lap and of Port-opening to Valve-travel. — The table on page 1041, giving the ratio of lap to travel of valve and ratio of travel to port-opening, is abridged from one given by Buel in Weisbach-Dubois,

vol. ii. It is calculated from the above formulæ. Intermediate values may be found by the formulæ, or with sufficient accuracy by interpolation from the figures in the table. By the table on page 1040 the crank-angle may be found, that is, the angle between its position when the engine is on the center and its position at cut-off, release, or compression, when these are known in fractions of the stroke. To illustrate the use of the tables the following example is given by Buel: width of port = 2.2 in.; width of port-opening = width of port + 0.3 in.; overtravel = 2.5 in.; length of connecting-rod = $2\frac{1}{2}$ times stroke; cut-off = 0.75 of stroke; release = 0.95 of stroke; lead-angle, 10° . From the first table we find crank-angle = 114.6° ; add lead-angle, making 124.6° . From the second table, for angle between admission and cut-off, 125° , we have ratio of travel to port-opening = 3.72, or for $124.6^\circ = 3.74$, which, multiplied by port-opening 2.5, gives 9.45 in. travel. The ratio of lap to travel, by the table, is 0.2324, or $9.45 \times 0.2324 = 2.2$ in. lap. For exhaust-lap, we have for release at 0.95, crank-angle = 151.3° ; add lead-angle $10^\circ = 161.3^\circ$. From the second table, by interpolation, ratio of lap to travel = 0.0811, and $0.0811 \times 9.45 = 0.77$ in., the exhaust-lap.

Lap-angle = $\frac{1}{2}(180^\circ - \text{lead-angle} - \text{crank-angle at cut-off})$;

$$= \frac{1}{2}(180^\circ - 10 - 114.6) = 27.7^\circ.$$

Angular advance = lap-angle + lead-angle = $27.7 + 10 = 37.7^\circ$.

Exhaust lap-angle = crank-angle at release + lap-angle + lead-angle - 180°
 $= 151.3 + 27.7 + 10 - 180^\circ = 9^\circ$.

Crank-angle at compression measured on return stroke } $= 180^\circ - \text{lap-angle} - \text{lead-angle} - \text{exhaust lap-angle}$
 $= 180 - 27.7 - 10 - 9 = 133.3^\circ$; corresponding, by

table, to a piston position of 0.81 of the return stroke; or

Crank-angle at compression = $180^\circ - (\text{angle at release} - \text{angle at cut-off})$
 $+ \text{lead-angle}$
 $= 180 - (151.3 - 114.6) + 10 = 133.3^\circ$.

The positions determined above for cut-off and release are for the forward stroke of the piston. On the return stroke the cut-off will take place at the same angle, 114.6° , corresponding by table to 66.6% of the return stroke, instead of 75%. By a slight adjustment of the angular advance and the length of the eccentric-rod the cut-off can be equalized. The width of the bridge should be at least $2.5 + 0.25 - 2.2 = 0.55$ in.

Lap and Travel of Valve.

| Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression. | Ratio of Lap to Travel of Valve. | Ratio of Travel of Valve to Width of Port-opening. | Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression. | Ratio of Lap to Travel of Valve. | Ratio of Travel of Valve to Width of Port-opening. | Angle between Positions of Crank at Points of Admission and Cut-off, or Release and Compression. | Ratio of Lap to Travel of Valve. | Ratio of Travel of Valve to Width of Port-opening. |
|--|----------------------------------|--|--|----------------------------------|--|--|----------------------------------|--|
| 30° | 0.4830 | 58.70 | 85° | 0.3686 | 7.61 | 135° | 0.1913 | 3.24 |
| 35 | .4769 | 43.22 | 90 | .3536 | 6.83 | 140 | .1710 | 3.04 |
| 40 | .4699 | 33.17 | 95 | .3378 | 6.17 | 145 | .1504 | 2.86 |
| 45 | .4619 | 26.27 | 100 | .3214 | 5.60 | 150 | .1294 | 2.70 |
| 50 | .4532 | 21.34 | 105 | .3044 | 5.11 | 155 | .1082 | 2.55 |
| 55 | .4435 | 17.70 | 110 | .2868 | 4.69 | 160 | .0868 | 2.42 |
| 60 | .4330 | 14.93 | 115 | .2687 | 4.32 | 165 | .0653 | 2.30 |
| 65 | .4217 | 12.77 | 120 | .2500 | 4.00 | 170 | .0436 | 2.19 |
| 70 | .4096 | 11.06 | 125 | .2309 | 3.72 | 175 | .0218 | 2.09 |
| 75 | .3967 | 9.68 | 130 | .2113 | 3.46 | 180 | .0000 | 2.00 |
| 80 | .3830 | 8.55 | | | | | | |

Relative Motions of Crosshead and Crank. — L = length of connecting-rod, R = length of crank, θ = angle of crank with center line of engine, D = displacement of crosshead from the beginning of its stroke, V = velocity of crank-pin, V_1 = velocity of piston.

For $R=1$, $D = \text{ver sin } \theta \pm (L - \sqrt{L^2 - \sin^2 \theta})$,

$$V_1 = V \sin \theta \left(1 \pm \frac{\cos \theta}{\sqrt{L^2 - \sin^2 \theta}} \right).$$

From these formulæ Mr. A. F. Nagle computes the following:

PISTON DISPLACEMENT AND PISTON VELOCITY FOR EACH 10° OF MOTION OF CRANK. Length of crank = 1. Length of connecting-rod = 5. Piston velocity V_1 for vel. of crank-pin = 1.

| Angle of Cr'nk | Displacement. | | Velocity. | | Angle of Cr'nk | Displacement. | | Velocity. | |
|----------------|---------------|-------|-----------|-------|----------------|---------------|-------|-----------|-------|
| | For-ward. | Back. | For-ward. | Back. | | For-ward. | Back. | For-ward. | Back. |
| 10° | 0.018 | 0.012 | 0.207 | | 60° | 0.576 | 0.424 | 0.954 | 0.778 |
| 20° | 0.072 | 0.048 | 0.406 | | 70° | 0.747 | 0.569 | 1.005 | 0.875 |
| 30° | 0.159 | 0.109 | 0.587 | | 80° | 0.924 | 0.728 | 1.019 | 0.950 |
| 40° | 0.276 | 0.192 | 0.742 | | 84° | 1.000 | | 1.011 | |
| 50° | 0.416 | 0.298 | 0.865 | | 90° | 1.101 | 0.899 | 1.000 | 1.000 |

PERIODS OF ADMISSION, OR CUT-OFF, FOR VARIOUS LAPS AND TRAVELS OF SLIDE-VALVES.

The two following tables are from Clark on the Steam-engine. In the first table are given the periods of admission corresponding to travels of valve of from 12 in. to 2 in., and laps of from 2 in. to 3/8 in., with 1/4 in. and 1/8 in. of lead. With greater leads than those tabulated, the steam would be cut off earlier than as shown in the table.

The influence of a lead of 5/16 in. for travels of from 15/8 in. to 6 in., and laps of from 1/2 in. to 1 1/2 in., as calculated for in the second table, is exhibited by comparison of the periods of admission in the table, for the same lap and travel. The greater lead shortens the period of admission, and increases the range for expansive working.

Periods of Admission, or Points of Cut-off, for Given Travels and Laps of Slide-valves.

| Travel of Valve. | Lead. | Periods of Admission, or Points of Cut-off, for the following Laps of Valves in inches. | | | | | | | | | |
|------------------|-------|---|-------|-------|-------|-------|-------|-----|-----|-----|-----|
| | | 2 | 1 3/4 | 1 1/2 | 1 1/4 | 1 | 7/8 | 3/4 | 5/8 | 1/2 | 3/8 |
| in. | in. | % | % | % | % | % | % | % | % | % | % |
| 12 | 1/4 | 88 | 90 | 93 | 95 | 96 | 97 | 98 | 98 | 99 | 99 |
| 10 | 1/4 | 82 | 87 | 89 | 92 | 95 | 96 | 97 | 98 | 98 | 99 |
| 8 | 1/4 | 72 | 78 | 84 | 88 | 92 | 94 | 95 | 96 | 98 | 98 |
| 6 | 1/4 | 50 | 62 | 71 | 79 | 86 | 89 | 91 | 94 | 96 | 97 |
| 5 1/2 | 1/8 | 43 | 56 | 68 | 77 | 85 | 88 | 91 | 94 | 96 | 97 |
| 5 | 1/8 | 32 | 47 | 61 | 72 | 82 | 86 | 89 | 92 | 95 | 97 |
| 4 1/2 | 1/8 | 14 | 35 | 51 | 66 | 78 | 83 | 87 | 90 | 94 | 96 |
| 4 | 1/8 | | 17 | 39 | 57 | 72 | 78 | 83 | 88 | 92 | 95 |
| 3 1/2 | 1/8 | | | 20 | 44 | 63 | 71 | 79 | 84 | 90 | 94 |
| 3 | 1/8 | | | | 23 | 50 | 61 | 71 | 79 | 86 | 91 |
| 2 1/2 | 1/8 | | | | | 27 | 43 | 57 | 70 | 80 | 88 |
| 2 | 1/8 | | | | | | | 33 | 52 | 70 | 81 |

Periods of Admission, or Points of Cut-off, for given Travels and Laps of Slide-valves.

Constant lead, $\frac{5}{16}$.

| Travel. | Lap. | | | | | | | | |
|----------------|---------------|---------------|---------------|---------------|-------|----------------|----------------|----------------|----------------|
| Inches. | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 | $1\frac{1}{8}$ | $1\frac{1}{4}$ | $1\frac{3}{8}$ | $1\frac{1}{2}$ |
| $1\frac{5}{8}$ | 19 | | | | | | | | |
| $1\frac{3}{4}$ | 39 | | | | | | | | |
| $1\frac{7}{8}$ | 47 | 17 | | | | | | | |
| 2 | 55 | 34 | | | | | | | |
| $2\frac{1}{8}$ | 61 | 42 | 14 | | | | | | |
| $2\frac{1}{4}$ | 65 | 50 | 30 | | | | | | |
| $2\frac{3}{8}$ | 68 | 55 | 38 | 13 | | | | | |
| $2\frac{1}{2}$ | 71 | 59 | 45 | 27 | | | | | |
| $2\frac{5}{8}$ | 74 | 63 | 49 | 36 | 12 | | | | |
| $2\frac{3}{4}$ | 76 | 67 | 56 | 43 | 26 | | | | |
| $2\frac{7}{8}$ | 78 | 70 | 59 | 47 | 32 | 11 | | | |
| 3 | 80 | 73 | 62 | 50 | 38 | 23 | | | |
| $3\frac{1}{8}$ | 81 | 74 | 65 | 55 | 44 | 30 | 10 | | |
| $3\frac{1}{4}$ | 83 | 76 | 68 | 59 | 48 | 34 | 22 | | |
| $3\frac{3}{8}$ | 84 | 78 | 71 | 62 | 51 | 40 | 29 | 9 | |
| $3\frac{1}{2}$ | 85 | 80 | 73 | 64 | 53 | 45 | 34 | 20 | |
| $3\frac{5}{8}$ | 86 | 81 | 75 | 66 | 57 | 49 | 38 | 26 | 9 |
| $3\frac{3}{4}$ | 87 | 82 | 76 | 68 | 60 | 52 | 42 | 32 | 19 |
| $3\frac{7}{8}$ | 87 | 83 | 78 | 70 | 63 | 55 | 46 | 36 | 25 |
| 4 | 88 | 84 | 79 | 72 | 66 | 58 | 49 | 40 | 29 |
| $4\frac{1}{4}$ | 89 | 86 | 81 | 76 | 70 | 63 | 56 | 47 | 37 |
| $4\frac{1}{2}$ | 90 | 87 | 83 | 79 | 73 | 67 | 61 | 54 | 45 |
| $4\frac{3}{4}$ | 92 | 89 | 85 | 81 | 76 | 70 | 65 | 58 | 51 |
| 5 | 93 | 90 | 87 | 83 | 78 | 73 | 67 | 62 | 56 |
| $5\frac{1}{2}$ | 94 | 92 | 89 | 86 | 82 | 78 | 73 | 68 | 63 |
| 6 | 95 | 93 | 91 | 88 | 85 | 82 | 78 | 74 | 69 |

Piston-valve. — The piston-valve is a modified form of the slide-valve. The lap, lead, etc., are calculated in the same manner as for the common slide-valve. The diameter of valve and amount of port-opening are calculated on the basis that the most contracted portion of the steam-passage between the valve and the cylinder should have an area such that the velocity of steam through it will not exceed 6000 ft. per minute. The area of the opening around the circumference of the valve should be about double the area of the steam-passage, since that portion of the opening that is opposite from the steam-passage is of little effect.

Setting the Valves of an Engine. — The principles discussed above are applicable not only to the designing of valves, but also to adjustment of valves that have been improperly set; but the final adjustment of the eccentric and of the length of the rod depends upon the amount of lost motion, temperature, etc.; and can be effected only after trial. After the valve has been set as accurately as possible when cold, the lead and lap for the forward and return strokes being equalized, indicator diagrams should be taken and the length of the eccentric-rod adjusted, if necessary, to correct slight irregularities.

To Put an Engine on its Center. — Place the engine in a position where the piston will have nearly completed its outward stroke, and opposite some point on the crosshead, such as a corner, make a mark upon the guide. Against the rim of the pulley or crank-disk place a pointer and mark a line with it on the pulley. Then turn the engine over the center until the crosshead is again in the same position on its inward stroke. This will bring the crank as much below the center as it was above it before. With the pointer in the same position as before make a second mark on the pulley rim. Divide the distance between the marks in two and mark the middle point. Turn the engine until the pointer is opposite this middle point, and it will then be on its center. To avoid

the error that may arise from the looseness of crank-pin and wrist-pin bearings, the engine should be turned a little above the center and then be brought up to it, so that the crank-pin will press against the same brass that it does when the first two marks are made.

Link-motion. — Link-motions, of which the Stephenson link is the most commonly used, are designed for two purposes: first, for reversing the motion of the engine, and second, for varying the point of cut-off by varying the travel of the valve. The Stephenson link-motion is a combination of two eccentrics, called forward and back eccentrics, with a link connecting the extremities of the eccentric-rods; so that by varying the position of the link the valve-rod may be put in direct connection with either eccentric, or may be given a movement controlled in part by one and in part by the other eccentric. When the link is moved by the reversing lever into a position such that the block to which the valve-rod is attached is at either end of the link, the valve receives its maximum travel, and when the link is in mid-gear the travel is the least and cut-off takes place early in the stroke.

In the ordinary shifting-link with open rods, that is, not crossed, the lead of the valve increases as the link is moved from full to mid-gear, that is, as the period of steam admission is shortened. The variation of lead is equalized for the front and back strokes by curving the link to the radius of the eccentric-rods concavely to the axles. With crossed eccentric-rods the lead decreases as the link is moved from full to mid-gear. In a valve-motion with stationary link the lead is constant. (For illustration see Clark's Steam-engine, vol. ii, p. 22.)

The linear advance of each eccentric is equal to that of the valve in full gear, that is, to lap + lead of the valve, when the eccentric-rods are attached to the link in such position as to cause the half-travel of the valve to equal the eccentricity of the eccentric.

The angle between the two eccentric radii, that is, between lines drawn from the center of the eccentric disks to the center of the shaft, equals 180° less twice the angular advance.

Buel, in Appleton's Cyclopaedia of Mechanics, vol. ii, p. 316, discusses the Stephenson link as follows: "The Stephenson link does not give a perfectly correct distribution of steam: the lead varies for different points of cut-off. The period of admission and the beginning of exhaust are not alike for both ends of the cylinder, and the forward motion varies from the backward.

"The correctness of the distribution of steam by Stephenson's link-motion depends upon conditions which, as much as the circumstances will permit, ought to be fulfilled, namely: 1. The link should be curved in the arc of a circle whose radius is equal to the length of the eccentric-rod. 2. The eccentric-rods ought to be long: the longer they are in proportion to the eccentricity the more symmetrical will the travel of the valve be on both sides of the center of motion. 3. The link ought to be short. Each of its points describes a curve in a vertical plane, whose ordinates grow larger the farther the considered point is from the center of the link: and as the horizontal motion only is transmitted to the valve, vertical oscillation will cause irregularities. 4. The link-hanger ought to be long. The longer it is the nearer will be the arc in which the link swings to a straight line, and thus the less its vertical oscillation. If the link is suspended in its center, the curves that are described by points equidistant on both sides from the center are not alike, and hence results the variation between the forward and backward gears. If the link is suspended at its lower end, its lower half will have less vertical oscillation and the upper half more. 5. The center from which the link-hanger swings changes its position as the link is lowered or raised, and also causes irregularities. To reduce them to the smallest amount the arm of the lifting-shaft should be made as long as the eccentric-rod, and the center of the lifting-shaft should be placed at the height corresponding to the central position of the center on which the link-hanger swings."

All these conditions can never be fulfilled in practice, and the variations in the lead and the period of admission can be somewhat regulated in an artificial way, but for one gear only. This is accomplished by giving different lead to the two eccentrics, which difference will be smaller the longer the eccentric-rods are and the shorter the link, and by suspending

the link not exactly on its center line but at a certain distance from it, giving what is called "the offset."

For application of the Zeuner diagram to link-motion, see Holmes on the Steam-engine, p. 290. See also Clark's Railway Machinery (1855), Clark's Steam-engine, Zeuner's and Auchincloss's Treatises on Slide-valve Gears, and Halsey's Locomotive Link Motion. (See page 1095.)

The following rules are given by the *American Machinist* for laying out a link for an upright slide-valve engine. By the term radius of link is meant the radius of the link-arc, *ab*, Fig. 165, drawn through the center of the slot; this radius is generally made equal to the distance from the

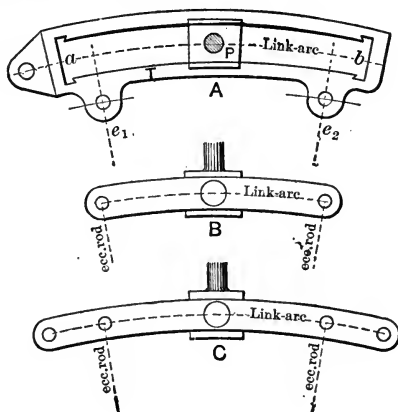


Fig. 165.

center of shaft to center of the link-block pin *P* when the latter stands midway of its travel. The distance between the centers of the eccentric-rod pins *e*₁ *e*₂ should not be less than 2½ times, and, when space will permit, three times the throw of the eccentric. By the throw we mean twice the eccentricity of the eccentric. The slot link is generally suspended from the end next to the forward eccentric at a point in the link-arc prolonged. This will give comparatively a small amount of slip to the link-block when the link is in forward gear; but this slip will be increased when the link is in backward gear. This increase of slip is, however, considered of little importance, because marine engines, as a rule, work but very little in the backward gear. When it is necessary that the motion shall be as efficient in backward gear as in forward gear, then the link should be suspended from a point midway between the two eccentric-rod pins; in marine engine practice this point is generally located on the link-arc; for equal cut-offs it is better to move the point of suspension a small amount towards the eccentrics.

For obtaining the dimensions of the link in inches: Let *L* denote the length of the valve, *B* the breadth, *p* the absolute steam-pressure per sq. in., and *R* a factor of computation used as below; then $R = 0.01 \sqrt{L \times B \times p}$

| | |
|--|--------------------------------------|
| Breadth of the link..... | = $R \times 1.6$ |
| Thickness <i>T</i> of the bar..... | = $R \times 0.8$ |
| Length of sliding-block..... | = $R \times 2.5$ |
| Diameter of eccentric-rod pins..... | = $(R \times 0.7) + \frac{1}{4}$ in. |
| Diameter of suspension-rod pin..... | = $(R \times 0.6) + \frac{1}{4}$ in. |
| Diameter of suspension-rod pin when overhung... | = $(R \times 0.8) + \frac{1}{4}$ in. |
| Diameter of block-pin when overhung..... | = $R \times \frac{1}{4}$ |
| Diameter of block-pin when secured at both ends. | = $(R \times 0.8) + \frac{1}{4}$ in. |

The length of the link, that is, the distance from *a* to *b*, measured on a straight line joining the ends of the link-arc in the slot, should be such as to allow the center of the link-block pin *P* to be placed in a line with the eccentric-rod pins, leaving sufficient room for the slip of the block. Another type of link frequently used in marine engines is the double-bar link, and this type is again divided into two classes: one class embraces those links which have the eccentric-rod ends as well as the valve-spindle end between the bars, as shown at *B* (with these links the travel of the valve is less than the throw of the eccentric); the other class embraces those links, shown at *C*, for which the eccentric-rods are made with fork-ends, so as to connect to studs on the outside of the bars, allowing the block to slide to the end of the link, so that the centers of the eccentric-rod ends and the block-pin are in line when in full gear, making the travel of the valve equal to the throw of the eccentric. The dimensions of these links when the distance between the eccentric-rod pins is $2\frac{1}{2}$ to $2\frac{3}{4}$ times the throw of eccentrics can be found as follows:

| | |
|--|---|
| Depth of bars..... | $= (R \times 1.25) + \frac{1}{2} \text{ in.}$ |
| Thickness of bars..... | $= (R \times 0.5) + \frac{1}{4} \text{ in.}$ |
| Diameter of center of sliding-block..... | $= R \times 1.3$ |

When the distance between the eccentric-rod pins is equal to 3 or 4 times the throw of the eccentrics, then

| | |
|------------------------|---|
| Depth of bars..... | $= (R \times 1.25) + \frac{3}{4} \text{ in.}$ |
| Thickness of bars..... | $= (R \times 0.5) + \frac{1}{4} \text{ in.}$ |

All the other dimensions may be found by the first table. These are empirical rules, and the results may have to be slightly changed to suit given conditions. In marine engines the eccentric-rod ends for all classes of links have adjustable brasses. In locomotives the slot-link is usually employed, and in these the pin-holes have case-hardened bushes driven into the pin-holes, and have no adjustable brasses in the ends of the eccentric-rods. The link in *B* is generally suspended by one of the eccentric-rod pins; and the link in *C* is suspended by one of the pins in the end of the link, or by one of the eccentric-rod pins. (See note on Locomotive Link Motion, p. 1095.)

The Walschaert Valve-gear. Fig. 166. — This gear, which was invented in Belgium, has for many years been used on locomotives in Europe, and it has now (1909) come largely into use in the United States. The return crank *Q*, which takes the place of an eccentric, through the rod *B* oscillates the link on the fixed pin *F*. The block *D* is raised and

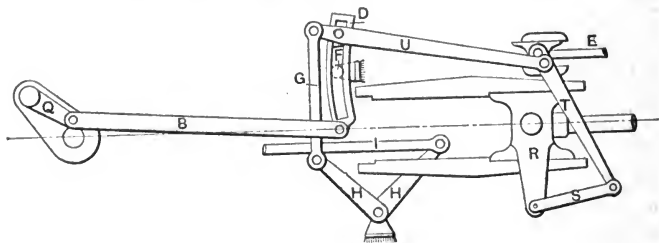


FIG. 166.— The Walschaert Valve-gear.

lowered in the link by the reversing rod *I*, operating through the bell-crank levers *H*, *H* and the supporting rod *G*. When the block is in its lowest position the radius rod *U* has a motion corresponding in direction to that of the rod *B*; when the block is at its upper position *U* moves in an opposite direction to *B*. The valve-rod *E* is moved by the combined action of *U* and a lever *T* whose lower end is connected through the rod *S* to the crosshead *R*. Constant lead is secured by this gear.

Other Forms of Valve-gear, as the Joy, Marshall, Hackworth, Bremme, Walschaert, Corliss, etc., are described in Clark's *Steam-engine*, vol. ii. *Power*, May 11, 1909, illustrates the Stephenson, Gooch, Allen, Polenceau, Marshall, Joy, Waldegg, Walschaert, Fink, and Baker-Pilliod gears. The design of the Reynolds-Corliss valve-gear is discussed by A. H. Eldridge in *Power*, Sept., 1893. See also Henthorn on the Corliss Engine. Rules for laying down the center lines of the Joy valve-gear are given in *American Machinist*, Nov. 13, 1890. For Joy's "Fluid-pressure Reversing-valve," see *Eng'g*, May 25, 1894.

GOVERNORS.

Pendulum or Fly-ball Governor. — The inclination of the arms of a revolving pendulum to a vertical axis is such that the height of the point of suspension h above the horizontal plane in which the center of gravity of the balls revolves (assuming the weight of the rods to be small compared with the weight of the balls) bears to the radius r of the circle described by the centers of the balls the ratio

$$\frac{h}{r} = \frac{\text{weight}}{\text{centrifugal force}} = \frac{w}{\frac{wv^2}{gr}} = \frac{gr}{v^2},$$

which ratio is independent of the weight of the balls, v being the velocity of the centers of the balls in feet per second.

If T = number of revolutions of the balls in 1 second, $v = 2\pi rT = ar$, in which a = the angular velocity, or $2\pi T$, and

$$h = \frac{gr^2}{v^2} = \frac{g}{4\pi^2 T^2}, \text{ or } h = \frac{0.8146}{T^2} \text{ feet} = \frac{9.775}{T^2} \text{ inches,}$$

$g = 32.16$. If N = revs. per minute, $h = 35,190 \div N^2$.

| | | | | | |
|---------------------------------|-------|-------|-------|-------|-------|
| For revolutions per minute.... | 40 | 45 | 50 | 60 | 75 |
| The height in inches will be... | 21.99 | 17.38 | 14.08 | 9.775 | 6.256 |

Number of turns per minute required to cause the arms to take a given angle with the vertical axis: Let l = length of the arm in inches from the center of suspension to the center of gyration, and α the required angle; then

$$N = \sqrt{\frac{35190}{l \cos \alpha}} = 187.6 \sqrt{\frac{1}{l \cos \alpha}} = 187.6 \sqrt{\frac{l}{h}}.$$

The simple governor is not isochronous; that is, it does not revolve at a uniform speed in all positions, the speed changing as the angle of the arms changes. To remedy this defect loaded governors, such as Porter's, are used. From the balls of a common governor whose collective weight is A let there be hung by a pair of links of lengths equal to the pendulum arms a load B capable of sliding on the spindle, having its center of gravity in the axis of rotation. Then the centrifugal force is that due to A alone, and the effect of gravity is that due to $A + 2B$; consequently the altitude for a given speed is increased in the ratio $(A + 2B) : A$, as compared with that of a simple revolving pendulum, and a given absolute variation in altitude produces a smaller proportionate variation in speed than in the common governor. (Rankine, S. E., p. 551.)

For the weighted governor let l = the length of the arm from the point of suspension to the center of gravity of the ball, and let the length of the suspending-link l_1 = the length of the portion of the arm from the point of suspension of the arm to the point of attachment of the link; G = the weight of one ball, Q = half the weight of the sliding weight, h = the height of the governor from the point of suspension to the plane of revolution of the balls, a = the angular velocity = $2\pi T$, T being the number of

revolutions per second; then $a = \sqrt{\frac{32.16}{h} \left(1 + \frac{2l_1}{l} \frac{Q}{G}\right)}$; $h = \frac{32.16}{a^2} \left(1 + \frac{2l_1}{l} \frac{Q}{G}\right)$

in feet, or $h = \frac{35190}{N^2} \left(1 + \frac{2l_1}{l} \frac{Q}{G}\right)$ in inches, N being the number of revolutions per minute.

J. H. Barr gives $h = \left(\frac{187.7}{N}\right)^2 \frac{B + 2W}{B}$, in which B is the combined weight of the two balls and W the central weight.

For various forms of governor see App. Cyl. Mech., vol. ii, 61, and Clark's Steam-engine, vol. ii, p. 65.

To Change the Speed of an Engine Having a Fly-ball Governor. — A slight difference in the speed of a governor changes the position of its weights from that required for full load to that required for no load. It is evident therefore that, whatever the speed of the engine, the normal speed of the governor must be that for which the governor was designed; i.e., the speed of the governor must be kept the same. To change the speed of the engine the problem is to so adjust the pulleys which drive the governor that the engine at its new speed shall drive it just as fast as it was driven at its original speed. In order to increase the engine-speed we must decrease the pulley upon the shaft of the engine, i.e., the driver, or increase that on the governor, i.e., the driven, in the proportion that the speed of the engine is to be increased.

Fly-wheel or Shaft-governors. — At the Centennial Exhibition in 1876 there were shown a few steam-engines in which the governors were contained in the fly-wheel or band-wheel, the fly-balls or weights revolving around the shaft in a vertical plane with the wheel and shifting the eccentric so as automatically to vary the travel of the valve and the point of cut-off. This form of governor has since come into extensive use, especially for high-speed engines. In its usual form two weights are carried on arms the ends of which are pivoted to two points on the pulley near its circumference, 180° apart. Links connect these arms to the eccentric. The eccentric is not rigidly keyed to the shaft but is free to move transversely across it for a certain distance, having an oblong hole which allows of this movement. Centrifugal force causes the weights to fly towards the circumference of the wheel and to pull the eccentric into a position of minimum eccentricity. This force is resisted by a spring attached to each arm which tends to pull the weights towards the shaft and shift the eccentric to the position of maximum eccentricity. The travel of the valve is thus varied, so that it tends to cut off earlier in the stroke as the engine increases its speed. Many modifications of this general form are in use. In the Buckeye and the McIntosh & Seymour engines the governor shifts the eccentric around on the shaft so as to vary the angular advance. In the Sweet "Straight-line" engine and in some others a single weight and a single spring are used. For discussions of this form of governor see Hartnell, *Proc. Inst. M. E.*, 1882, p. 408; *Trans. A. S. M. E.*, ix, 300; xi, 1081; xiv, 92; xv, 929; Modern Mechanism, p. 399; Whitham's Constructive Steam Engineering; J. Begtrup, *Am. Mach.*, Oct. 19 and Dec. 14, 1893, Jan. 18 and March 1, 1894.

More recent references are: J. Richardson, *Proc. Inst. M. E.*, 1895 (includes electrical regulation of steam-engines); A. K. Mansfield, *Trans. A. S. M. E.*, 1894; F. H. Ball, *Trans. A. S. M. E.*, 1896; R. C. Carpenter, *Power*, May and June, 1898; Thos. Hall, *El. World*, June 4, 1898; F. M. Rites, *Power*, July, 1902; E. R. Briggs, *Am. Mach.*, Dec. 17, 1903.

The Rites Inertia Governor, which is the most common form of the shaft governor at this date (1909), has a long bar, usually made heavy at the ends, like a dumb-bell, instead of the usual weights. This is carried on an arm of the fly-wheel by a pin located at some distance from the center line of the bar, and also at some distance from its middle point. To pins located at two other points are attached the valve-rod and the spring. The bar acts both by inertia and by centrifugal force. When the wheel increases its speed the inertia of the bar tends to make it fall behind, and thus to change the relative position of the fly-wheel arm and the bar, and to change the travel of the valve. A small book on "Shaft Governors" (Hill Pub. Co., 1908) describes and illustrates this and many other forms of shaft governors, and gives practical directions for adjusting them.

Calculation of Springs for Shaft-governors. (Wilson Hartnell, *Proc. Inst. M. E.*, Aug., 1882.) — The springs for shaft-governors may be conveniently calculated as follows, dimensions being in inches:

Let W = weight of the balls or weights, in pounds;

r_1 and r_2 = the maximum and minimum radial distances of the center of the balls or of the centers of gravity of the weights;

l_1 and l_2 = the leverages, i.e., the perpendicular distances from the center of the weight-pin to a line in the direction of the centrifugal force drawn through the center of gravity of the weights or balls at radii r_1 and r_2 ;

m_1 and m_2 = the corresponding leverages of the springs;

C_1 and C_2 = the centrifugal forces, for 100 revolutions per minute, at radii r_1 and r_2 ;

P_1 and P_2 = the corresponding pressures on the spring;

(It is convenient to calculate these and note them down for reference.)

C_3 and C_4 = maximum and minimum centrifugal forces;

S = mean speed (revolutions per minute);

S_1 and S_2 = the maximum and minimum number of revolutions per minute;

P_3 and P_4 = the pressures on the spring at the limiting number of revolutions (S_1 and S_2);

$P_4 - P_3 = D$ = the difference of the maximum and minimum pressures on the springs;

V = the percentage of variation from the mean speed, or the sensitiveness;

t = the travel of the spring;

u = the initial extension of the spring;

v = the stiffness in pounds per inch;

w = the maximum extension = $u + t$.

The mean speed and sensitiveness desired are supposed to be given. Then

$$S_1 = S - \frac{SV}{100};$$

$$S_2 = S + \frac{SV}{100};$$

$$C_1 = 0.28 \times r_1 \times W;$$

$$C_2 = 0.28 \times r_2 \times W;$$

$$P_1 = C_1 \times \frac{l_1}{m_1};$$

$$P_2 = C_2 \times \frac{l_2}{m_2};$$

$$P_3 = P_1 \times \left(\frac{S_1}{100}\right)^2;$$

$$P_4 = P_2 \times \left(\frac{S_2}{100}\right)^2;$$

$$v = \frac{D}{t}, u = \frac{P_3}{v}, w = \frac{P_4}{v}.$$

It is usual to give the spring-maker the values of P_4 and of v or w . To ensure proper space being provided, the dimensions of the spring should be calculated by the formulæ for strength and extension of springs, and the least length of the spring as compressed be determined.

$$\text{The governor-power} = \frac{P_3 + P_4}{2} \times \frac{t}{12}.$$

With a straight centripetal line, the governor-power

$$= \frac{C_3 + C_4}{2} \times \left(\frac{r_2 - r_1}{12}\right).$$

For a preliminary determination of the governor-power it may be taken as equal to this in all cases, although it is evident that with a curved centripetal line it will be slightly less. The difference D must be constant for the same spring, however great or little its initial compression. Let the spring be screwed up until its minimum pressure is P_5 . Then to find the speed $P_6 = P_5 + D$,

$$S_5 = 100 \sqrt{\frac{P_5}{P_1}}, \quad S_6 = 100 \sqrt{\frac{P_6}{P_2}}.$$

The speed at which the governor would be isochronous would be

$$100 \sqrt{\frac{D}{P_2 - P_1}}.$$

Suppose the pressure on the spring with a speed of 100 revolutions, at the maximum and minimum radii, was 200 lbs. and 100 lbs., respectively,

then the pressure of the spring to suit a variation from 95 to 105 revolutions will be $100 \times \left(\frac{95}{100}\right)^2 = 90.2$ and $200 \times \left(\frac{105}{100}\right)^2 = 220.5$. That is, the increase of resistance from the minimum to the maximum radius must be $220 - 90 = 130$ lbs.

The extreme speeds due to such a spring, screwed up to different pressures, are shown in the following table:

| | | | | | | |
|---|-----|-----|-----|-----|-----|-----|
| Revolutions per minute, balls shut..... | 80 | 90 | 95 | 100 | 110 | 120 |
| Pressure on springs, balls shut..... | 64 | 81 | 90 | 100 | 121 | 144 |
| Increase of pressure when balls open fully..... | 130 | 130 | 130 | 130 | 130 | 130 |
| Pressure on springs, balls open fully..... | 194 | 211 | 220 | 230 | 251 | 274 |
| Revolutions per minute, balls open fully..... | 98 | 102 | 105 | 107 | 112 | 117 |
| Variation, per cent of mean speed..... | 10 | 6 | 5 | 3 | 1 | -1 |

The speed at which the governor would become isochronous is 114.

Any spring will give the right variation at some speed; hence in experimenting with a governor the correct spring may be found from any wrong one by a very simple calculation. Thus, if a governor with a spring whose stiffness is 50 lbs. per inch acts best when the engine runs at 95, 90 being its proper speed, then $50 \times \left(\frac{90}{95}\right)^2 = 45$ lbs. is the stiffness of spring required.

To determine the speed at which the governor acts best, the spring may be screwed up until the governor begins to "hunt" and then be slackened until it is as sensitive as is compatible with steadiness.

CONDENSERS, AIR-PUMPS, CIRCULATING-PUMPS, ETC.

The Jet Condenser. — In practice the temperature in the hot-well varies from 110° to 120° , and occasionally as much as 130° is maintained. To find the quantity of injection-water per pound of steam to be condensed: Let T_1 = temperature of steam at the exhaust pressure; T_0 = temperature of the cooling-water; T_2 = temperature of the water after condensation, or of the hot-well; Q = pounds of the cooling-water per lb. of steam condensed; then

$$Q = \frac{1114^\circ + 0.3 T_1 - T_2}{T_2 - T_0}.$$

Another formula is: $Q = \frac{WH}{R}$, in which W is the weight of steam condensed, H the units of heat given up by 1 lb. of steam in condensing, and R the rise in temperature of the cooling-water. This is applicable both to jet and to surface condensers.

Quantity of Cooling-water. — The quantity depends chiefly upon its initial temperature, which in Atlantic practice may vary from 40° in the winter of temperate zone to 80° in subtropical seas. To raise the temperature to 100° in the condenser will require three times as many thermal units in the former case as in the latter, and therefore only one-third as much cooling-water will be required in the former case as in the latter. It is usual to provide pumping power sufficient to supply 40 times the weight of steam for general traders, and as much as 50 times for ships stationed in subtropical seas, when the engines are compound. If the circulating pump is double-acting, its capacity may be $\frac{1}{53}$ in the former and $\frac{1}{42}$ in the latter case of the capacity of the low-pressure cylinder. (Seaton.)

The following table, condensed from one given by W. V. Terry in *Power*, Nov. 30, 1909, shows the amount of circulating water required under different conditions of vacuum, temperature of water entering the condenser, and drop. The "drop" is the difference between the temperature of steam due to a given vacuum and the temperature of the water leaving the condenser.

POUNDS OF CIRCULATING WATER PER POUND OF STEAM CONDENSED.

| Vacuum. Ins. | Drop. Deg. F. | Injection Water Temperature, Deg. F. | | | | | | | | | |
|-----------------|---------------------|--------------------------------------|------|------|------|------|------|------|------|------|------|
| | | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
| 29.0 | 6 | 37.5 | 45.7 | 58.3 | 80.8 | | | | | | |
| | 12 | 47.8 | 61.8 | 87.5 | | | | | | | |
| | 18 | 65.7 | 95.5 | | | | | | | | |
| 28.5 | 6 | 25.6 | 29.2 | 33.9 | 40.3 | 50.0 | 65.7 | 95.5 | | | |
| | 12 | 30.0 | 35.0 | 42.0 | 52.5 | 70.0 | | | | | |
| | 18 | 36.2 | 43.8 | 55.3 | 75.0 | | | | | | |
| 28.0 | 6 | 21.5 | 23.9 | 26.9 | 30.9 | 36.3 | 43.8 | 55.3 | 75.0 | | |
| | 12 | 24.4 | 27.7 | 31.8 | 37.5 | 45.7 | 58.3 | 80.8 | | | |
| | 18 | 28.4 | 32.8 | 38.9 | 47.8 | 61.8 | 87.5 | | | | |
| 27.0 | 6 | 16.4 | 17.8 | 19.5 | 21.5 | 23.9 | 27.0 | 30.9 | 36.2 | 43.8 | 55.3 |
| | 12 | 18.1 | 19.8 | 21.9 | 24.4 | 27.7 | 31.8 | 37.5 | 45.7 | 58.3 | 80.8 |
| | 18 | 20.2 | 22.4 | 25.0 | 28.4 | 32.8 | 38.9 | 47.8 | 61.8 | 87.5 | |
| 26.0 | 6 | 14.0 | 15.0 | 16.2 | 17.5 | 19.1 | 21.0 | 23.4 | 26.3 | 30.0 | 35.0 |
| | 12 | 15.2 | 16.4 | 17.8 | 19.5 | 21.5 | 23.9 | 26.9 | 30.9 | 36.3 | 43.8 |
| | 18 | 16.8 | 18.1 | 19.8 | 21.9 | 24.4 | 27.7 | 31.8 | 37.5 | 45.7 | 58.3 |

Ejector Condensers. — For ejector or injector condensers (Bulkley's, Schutte's, etc.) the calculations for quantity of condensing-water is the same as for jet condensers.

The Barometric Condenser consists of a vertical cylindrical chamber mounted on top of a discharge pipe whose length is 34 ft. above the level of the hot well. The exhaust steam and the condensing water meet in the upper chamber, the water being delivered in such a manner as to expose a large surface to the steam. The external atmosphere maintains a column of water in the tube, as a column of mercury is maintained in a barometer, and no air pump is needed. The Bulkley condenser is the original form of the type. In some modern forms a small air pump draws from the chamber the residue of air which is not drawn out by the descending column of water, discharging it into the column below the chamber.

The Surface Condenser — Cooling Surface. — In practice, with the compound engine, brass condenser-tubes, 18 B.W.G. thick, 13 lbs. of steam per sq. ft. per hour, with the cooling-water at an initial temperature of 60°, is considered very fair work when the temperature of the feed-water is to be maintained at 120°. It has been found that the surface in the condenser may be half the heating surface of the boiler, and under some circumstances considerably less than this. In general practice the following holds good when the temperature of sea-water is about 60°:

| | | | | | | | |
|------------------------------------|----|------|------|------|------|------|------|
| Terminal pres., lbs., abs. | 30 | 20 | 15 | 12½ | 10 | 8 | 6 |
| Sq. ft. per I.H.P. | 3 | 2.50 | 2.25 | 2.00 | 1.80 | 1.60 | 1.50 |

For ships whose station is in the tropics the allowance should be increased by 20%, and for ships which occasionally visit the tropics 10% increase will give satisfactory results. If a ship is constantly employed in cold climates 10% less suffices. (Seaton, Marine Engineering.)

Whitham (Steam-engine Design, p. 283, also *Trans. A. S. M. E.*, ix, 431)

gives the following: $S = \frac{WL}{ck(T_1 - t)}$, in which S = condensing-surface in sq. ft.; T_1 = temperature Fahr. of steam of the pressure indicated by the vacuum-gauge; t = mean temperature of the circulating water, or the arithmetical mean of the initial and final temperatures; L = latent heat of saturated steam at temperature T_1 ; k = perfect conductivity of 1 sq. ft. of the metal used for the condensing-surface for a range of 1° F. (or 550 B.T.U. per hour for brass, according to Isherwood's experiments); c = fraction denoting the efficiency of the condensing-surface; W =

pounds of steam condensed per hour. From experiments by Loring and Emery, on U.S.S. Dallas, c is found to be 0.323, and $ck = 180$; making the equation $S = \frac{WL}{180(T_1 - t)}$.

Whitham recommends this formula for designing engines having independent circulating-pumps. When the pump is worked by the main engine the value of S should be increased about 10%.

Taking T_1 at 135° F., and $L = 1020$, corresponding to 25 in. vacuum, and t for summer temperatures at 75°, we have: $S = \frac{1020W}{180(135 - 75)} = \frac{17W}{180}$.

Much higher results than those quoted by Whitham are obtained from modern forms of condensers. The literature on the subject of condensers from 1900 to 1909 has been quite voluminous, and much difference of opinion as to rules of proportioning condensers is shown.

Coefficient of Heat Transference in Condensers. (Prof. E. Josse of Berlin. Condensed from an abstract in *Power*, Feb. 2, 1909. See also Transmission of Heat from Steam to Water, pages 561 to 563.)

The coefficient U , the number of heat units transferred per hour through 1 sq. ft. of metallic condenser wall when the temperature of the steam is 1° F. higher than that of the water, can be deduced from the formula

$$1/U = 1/A_1 + d/L + 1/A_2,$$

in which $1/A_1$ is the resistance to transmission from steam to metal, $1/A_2$ the resistance to transmission from metal to water, and d/L the resistance to transmission of heat through the metal, d being the usual thickness of condenser tubes (1 m.m. or 0.0393 in.). For this thickness the value of L is fairly well known and may be given as 18,430 for brass, 6,500 for copper, 11,270 for iron, 5740 for zinc, 11,050 for tin and 2660 for aluminum. The middle term d/L would have the value of 1/18,430 and be of comparatively little importance.

The term $1/A_2$ is the most important and has been investigated with the aid of two concentric tubes, water being sent both through the inner tube and the annular jacket. The values of various experimenters differ greatly. Ser gives the approximate formula

$$A - 2 = 510 \sqrt{V},$$

where V is the velocity of water through the tubes in ft. per sec. This velocity is far more important than the material of the condenser tubes and their thickness, and also of greater consequence than the velocity of the steam, about which, or, rather, the term $1/A_1$, there is even less agreement. Prof. Josse adopts the figure 3900. The velocity of the steam has its influence, but the whole term does not count for much. For water flowing at the rate of 1.64 ft. per sec. Josse's formula would be:

$$1/U = 1/3900 + 1/18,430 + 1/653 = 1/445,$$

and $U = 445$.

If A_1 be increased to twice its value U would rise only to 475, and if the tube thickness be doubled U would hardly be affected. An increase, however, in the rate of flow of water from 1.64 to 5 feet per second would raise U to 625. As an increase of the steam flow is undesirable the best plan is to accelerate the flow of the circulating water, and by introducing the baffle strips or retarders into his condenser tubes, in order to break the water currents up into vortices, Josse raised the value of U at a velocity of 3.28 feet per second from 614 to 922.

Opinions differ concerning the increase of U with greater differences of temperature. According to some the heat transferred should increase proportionately to the difference; according to Weiss and others, proportionally to the square of the temperature differences. Josse's investigations were conducted by placing thermo couples in different portions of the condenser tubes. If the heat transferred increases as a linear function of the difference, then the rise of the temperature in the cooling water should follow an exponential law, and it was found to be so.

Curves showing the relation of the extent of surface to the temperatures of steam and water show an agreement with the formula

$$\text{Surface} = S = \frac{Q}{U} \log_e \frac{t_s - t_e}{t_s - t},$$

where t_s is the saturation temperature and t_e the temperature of the cooling-water at entrance, t being the discharge temperature.

Air Leakage. — Air passes into the condenser with the exhaust steam, the temperature of the air being that of the steam; the pressure of the mixture will be the sum of the partial steam pressure and of the partial air pressure. The air must be withdrawn by the air-pump. If the withdrawal takes place at the temperature corresponding to the condenser pressure the partial steam pressure would be equal to the condenser pressure, and the pump would have to deal with an enormous air volume. The air temperature should, therefore, be lowered, at the spot where the air is withdrawn, below the saturation temperature of the condenser pressure.

In steam turbines it is more easy to keep air out than in reciprocating engines. Experiments with a 300-kw. Parsons turbine show that not more than $1\frac{1}{2}$ lb. of air was delivered per hour when 6600 lbs. of steam was used per hour.

Condenser Pumps. — The air and condensed water may either be removed separately, by a so-called dry-air pump, or both together, by a wet-air pump. As dry-air pumps have to deal with high compression ratios, with high vacua and single-stage pumps, the clearances must be small. When the clearance amounts to 5% the vacuum cannot be maintained at more than 95%, and the clearance must be reduced, or other expedients adopted. Three are mentioned: (1) the air-pump may be built in two stages; (2) the pump may be fitted with an equalizing pipe so that the two sides of the piston are connected near the end of each stroke; the volumetric efficiency is raised by this expedient, but considerably more power is absorbed to accomplish the result; (3) with the wet-air pump the clearance space is made to receive the condensed water, which will fill at least part of it.

Contraflow and Ordinary Flow. — Prof. Josse questions the distinction between contraflow and ordinary flow. For the greater portion of the condenser there is a rise of temperature only on the water side; the temperature of the steam side remains that of the saturated steam, and the term "contraflow" should, strictly speaking, only be applied if there is a temperature fall in the one direction and a corresponding temperature rise in the opposite direction. As far as the condensation is concerned, it is immaterial in which direction the water flows. The contraflow principle is, however, correct and necessary for the smaller portion of the condenser in which the condensed liquid is cooled together with the air; for the air must be withdrawn from the coldest spot. It seems inadvisable to attempt to direct the flow of the steam on the contraflow principle, as that would obstruct the steam flow and create a pressure difference between different portions of the condenser which would be injurious to the maintenance of high vacua.

The Power Used for Condensing Apparatus varies from about $1\frac{1}{2}$ to 5% of the indicated power of the main engine, depending on the efficiency of the apparatus, on the degree of vacuum obtained, the temperature of the cooling-water, the load on the engine, etc. J. R. Bibbins (*Power*, Feb., 1905) gives the records of test of a 300-kw. plant from which the following figures are taken. Cooling-water per lb. of steam 32 to 37 lbs. Vacuum 27.3 to 27.8 ins. Temp. cooling-water 73. Hot-well 102 to 105.

| Indicated H.P..... | 151 | 220 | 238 | 260 | 291 | 294 | 457 | 589 |
|--------------------------|------|------|------|------|------|------|------|------|
| % of total power used... | 4.69 | 3.51 | 3.22 | 3.22 | 3.08 | 2.97 | 2.80 | 2.47 |
| % for air cylinder..... | 1.63 | 1.36 | 1.27 | 1.21 | 1.19 | 1.09 | 0.95 | 0.85 |
| % for water pump..... | 3.07 | 2.14 | 1.95 | 2.00 | 1.90 | 1.89 | 1.85 | 1.52 |

Vacuum, Ins. of Mercury, and Absolute Pressures. — The vacuum as shown by a mercury column is not a direct measure of pressure, but only of the difference between the atmospheric pressure and the absolute pressure in the vacuum chamber. Since the atmospheric pressure varies with the altitude and also with atmospheric conditions, it is necessary when accuracy is desired to give the reading of the barometer as well as

that of the vacuum gauge, or preferably to give the absolute pressure in lbs. per sq. in. above a perfect vacuum.

Temperatures, Pressures and Volumes of Saturated Air. (D. B. Morison, on The Influence of Air on Vacuum in Surface Condensers, *Eng'g*, April 17, 1908.)

VOLUME OF 1 LB. OF AIR WITH ACCOMPANYING VAPOR.

| Temp. T° F. | Pressure at T° F. | Vacuum, ins. of Mercury, and lbs. absolute. | | | | | | | | | | | | | |
|----------------|----------------------|---|-----|-------------------|-----|-------------------|-------|--------------------|-------|----------------------|-------|----------------------|-------|--------------------|-------|
| | | 24 in., 2.947. | | 26 in., 1.962. | | 27 in., 1.474. | | 28 in., 0.9823. | | 28.5 in., 0.7368. | | 28.8 in., 0.5894. | | 29 in., 0.4912. | |
| | | P | V | P | V | P | V | P | V | P | V | P | V | P | V |
| 50° | 0.17 | 2.78 | 68 | 1.79 | 105 | 1.30 | 147 | 0.81 | 233 | 0.57 | 336 | 0.42 | 450 | 0.32 | 592 |
| 60° | 0.25 | 2.70 | 71 | 1.71 | 113 | 1.22 | 158 | 0.73 | 263 | 0.49 | 393 | 0.34 | 566 | 0.24 | 800 |
| 70° | 0.35 | 2.59 | 75 | 1.60 | 124 | 1.11 | 178 | 0.62 | 315 | 0.38 | 520 | 0.23 | 852 | 0.13 | 1536 |
| 80° | 0.50 | 2.45 | 81 | 1.46 | 137 | 0.97 | 204 | 0.48 | 420 | 0.24 | 832 | 0.09 | (d) | | |
| 90° | 0.69 | 2.26 | 90 | 1.27 | 163 | 0.78 | 260 | 0.29 | 700 | 0.05 | (c) | | | | |
| 100° | 0.94 | 2.01 | 103 | 1.02 | 203 | 0.53 | 390 | 0.042 | (b) | | | | | | |
| 110° | 1.25 | 1.69 | 125 | 0.70 | 304 | 0.21 | (a) | | | | | | | | |
| 120° | 1.68 | 1.27 | 170 | 0.28 | 770 | | | | | | | | | | |

P = partial pressure of air, lbs. per sq. in. V = volume of 1 lb. of air with accompanying vapor, cu. ft. (a) over 1000; (b) nearly 5000; (c) about 4000; (d) over 2000.

TEMPERATURES AND PRESSURES OF SATURATED AIR.

| Vacuum, Ins. with Barom. at 30 in. | Proportions of Air and Steam by Weight. | | | | |
|--|---|-------------------------|------------------------|-------------------------|----------------------|
| | Saturated Steam. | Air, 0.25. Steam, 1. | Air, 0.5. Steam, 1. | Air, 0.75. Steam, 1. | Air, 1. Steam, 1. |
| 29 | 79.5° F. | 75 | 71 | 67.5 | 64.5 |
| 28 | 101.5 | 96.5 | 92.4 | 88.8 | 85.3 |
| 27 | 115 | 110 | 105.6 | 101.7 | 98.6 |
| 26 | 126 | 120.2 | 115.5 | 111.5 | 108.3 |
| 25 | 134 | 128.4 | 123.5 | 119.2 | 116.2 |
| 24 | 141 | 135.2 | 130.3 | 125.8 | 122.3 |

From this table it is seen that a temperature of 126° F. corresponds to a 24-in. vacuum if the steam in the condenser has 75% of its weight of air mingled with it, and to a 26-in. vacuum if it is free from air.

One cubic foot of air measured at 60° F. and atmospheric pressure becomes 10 cu. ft. at 27 in. and 30 cu. ft. at 29 in. vacuum at the same temperature; 10.9 cu. ft. at 105° and 27 in.; 30.5 cu. ft. at 70° F. and 29 in. The same cu. ft. of air saturated with water vapor at 70° F. and 29 in. becomes 124.3 cu. ft., or 44.9 cu. ft. at 105° and 27 in. vacuum. The temperatures 105° and 70° are about 10% below the temperatures of saturated steam at 27 in. and 29 in. respectively.

Condenser Tubes are generally made of solid-drawn brass tubes, and tested both by hydraulic pressure and steam. They are usually made of a composition of 68% of best selected copper and 32% of best Silesian spelter. The Admiralty, however, always specify the tubes to be made of 70% of best selected copper and to have 1% of tin in the composition, and test the tubes to a pressure of 300 lbs. per sq. in. (Seaton.)

The diameter of the condenser tubes varies from 1/2 in. in small condensers, when they are very short, to 1 in. in very large condensers and long tubes. In the mercantile marine the tubes are, as a rule, 3/4 in. diam. externally, and 18 B.W.G. thick (0.049 inch); and 16 B.W.G. (0.065), under some exceptional circumstances. In the British Navy the tubes are also, as a rule, 3/4 in. diam., and 18 to 19 B.W.G., tinned on both sides; when the condenser is brass the tubes are not required to be tinned. Some of the smaller engines have tubes 5/8 in. diam., and 19

B.W.G. The smaller the tubes, the larger is the surface which can be got in a certain space. (Seaton.)

In the merchant service the almost universal practice is to circulate the water through the tubes.

Whitham says the velocity of flow through the tubes should not be less than 400 nor more than 700 ft. per min.

Bimetallic Condenser Tubes. (E. K. Davis, *Eng. News*, Sept. 2, 1909.) — Condenser tubes are usually made of a brass containing about 40% zinc. When this alloy is found to be short-lived, due to the presence of corrosive substances in the cooling-water, recourse is had to bronze tubing of "admiralty mixture" (87% copper, 8% tin, 5% zinc) or to pure copper. Sometimes also the tubes for further protection are tinned on the inside or on both sides.

A condenser tube should not split, should be comparatively free from localized corrosion or pit holes, and should not become brittle under the combined action of steam and cooling-water.

A bimetallic tube, composed of a copper envelope over an aluminum lining (or vice versa) is unlikely to split, owing to its being composed of two layers of metal. It is slow to corrode with the aluminum surface exposed to the cooling-water, and there is no tendency shown toward becoming brittle. Aluminum, being electro-positive to copper, protects it from corrosion in somewhat the same way that even porous galvanizing protects iron. No corrosion of the copper will take place until the aluminum has been entirely eaten away for a considerable distance around the perforation, thus leaving a sound tube for a much longer time than is the case when brass or copper is used alone. The usual proportions of metal are, 0.022 in. thickness of copper and 0.043 in. of aluminum, making a total of 0.065 in., or No. 16 Stubs gauge.

Tube-plates are usually made of brass. Rolled-brass tube-plates should be from 1.1 to 1.5 times the diameter of tubes in thickness, depending on the method of packing. When the packings go completely through the plates, the latter thickness, but when only partly through, the former, is sufficient. Hence, for $\frac{3}{4}$ -in. tubes the plates are usually $\frac{7}{8}$ to 1 in. thick with glands and tape-packings, and 1 to $1\frac{1}{4}$ ins. thick with wooden ferrules. The tube-plates should be secured to their seatings by brass studs and nuts, or brass screw-bolts; in fact there must be no wrought iron of any kind inside a condenser. When the tube-plates are of large area it is advisable to stay them by brass rods, to prevent them from collapsing.

Spacing of Tubes, etc. — The holes for ferrules, glands, or india-rubber are usually $\frac{1}{4}$ inch larger in diameter than the tubes; but when absolutely necessary the wood ferrules may be only $\frac{3}{32}$ inch thick.

The pitch of tubes when packed with wood ferrules is usually $\frac{1}{4}$ inch more than the diameter of the ferrule-hole. For example, the tubes are generally arranged zigzag, and the number which may be fitted into a square foot of plate is as follows:

| Pitch of Tubes. in. | No. in a sq. ft. | Pitch of Tubes. in. | No. in a sq. ft. | Pitch of Tubes. in. | No. in a sq. ft. |
|------------------------|------------------|------------------------|------------------|------------------------|------------------|
| 1 | 172 | $1\frac{5}{32}$ | 128 | $1\frac{1}{4}$ | 110 |
| $1\frac{1}{16}$ | 150 | $1\frac{3}{16}$ | 121 | $1\frac{9}{32}$ | 106 |
| $1\frac{1}{8}$ | 137 | $1\frac{7}{32}$ | 116 | $1\frac{5}{16}$ | 99 |

Air-pump. — The air-pump in all condensers abstracts the water condensed and the air originally contained in the water when it entered the boiler. In the case of jet-condensers it also pumps out the water of condensation and the air which it contained. The size of the pump is calculated from these conditions, making allowance for efficiency of the pump.

In surface condensation allowance must be made for the water occasionally admitted to the boilers to make up for waste, and the air contained in it, also for slight leaks in the joints and glands, so that the air-pump is made about half as large as for jet-condensation.

Seaton says: The efficiency of a single-acting air-pump is generally taken at 0.5 and that of a double-acting pump at 0.35. When the tem-

perature of the sea is 60° , and that of the (jet) condenser is 120° , Q being the volume of the cooling-water and q the volume of the condensed water in cubic feet, and n the number of strokes per minute,

The volume of the single-acting pump = $2.74 (Q + q) \div n$.

The volume of the double-acting pump = $4 (Q + q) \div n$.

W. H. Booth, in his "Treatise on Condensing Plant," says the volume to be generated by an air-pump bucket should not be less than 0.75 cu. ft. per pound of steam dealt with by the condensing plant. Mr. R. W. Allen has made tests with a little air-pump capacity as 0.5 cu. ft. and he gives 0.6 cu. ft. as a minimum. An Edwards pump with three 14-in. barrels, 12 in. stroke, single-acting, 150 r.p.m., is rated at 45,000 lbs. of steam per hour from a surface condenser, which is equivalent to 0.66 cu. ft. per pound of feed-water.

In the Edwards pump, the base of the pump and the bottom of the piston are conical in shape. The water from the condenser flows by gravity into the space below the piston, which descending projects it through ports into the space in the barrel above the piston, whence on the ascending stroke of the piston it is discharged through the outlet valves. There are no bucket or foot-valves, and the pump may be run at much higher speeds than older forms of pump. (See Catalogue of the Wheeler Condenser and Engineering Co.)

The Area through Valve-seats and past the valves should not be less than will admit the full quantity of water for condensation at a velocity not exceeding 400 ft. per minute. In practice the area is generally in excess of this. (Seaton.)

Area through foot-valves = $D^2 \times S \div 1000$ square inches.

Area through head-valves = $D^2 \times S \div 800$ square inches.

Diameter of discharge-pipe = $D \times \sqrt{S} \div 35$ inches.

D = diam. of air-pump in inches, S = its speed in ft. per min.

James Tribe (*Am. Mach.*, Oct. 8, 1891) gives the following rule for air-pumps used with jet-condensers: Volume of single-acting air-pump driven by main engine = volume of low-pressure cylinder in cubic feet, multiplied by 3.5 and divided by the number of cubic feet contained in one pound of exhaust steam of the given density. For a double-acting air-pump the same rule will apply, but the volume of steam for each stroke of the pump will be but one-half. Should the pump be driven independently of the engine, then the relative speed must be considered. Volume of jet-condenser = volume of air-pump $\times 4$. Area of injection valve = vol. of air-pump in cubic inches $\div 520$.

The Work done by an Air-pump, per stroke, is a maximum theoretically, when the vacuum is between 21 and 22 ins. of mercury. Assuming adiabatic compression, the mean effective pressure per stroke

is $P = 3.46 p_1 \left[\left(\frac{p_2}{p_1} \right)^{0.29} - 1 \right]$, where p = absolute pressure of the vacuum

and p_2 the terminal, or atmospheric, pressure, = 14.7 lbs. per sq. in. The horse-power required to compress and deliver 1 cu. ft. of air per minute, measured at the lower pressure, is, neglecting friction, $P \times 144 \div 33,000$.

The following table is calculated from these formulæ (R. R. Pratt, *Power*, Sept. 7, 1909).

| Vac. in Ins. of Mer- cury. | Abs. Press., Ins. of Mer- cury. | $\frac{p_2}{p_1}$ | Theo- retic. M.E.P. | Theo- retic. H.P. | Vac. in Ins. of Mer- cury. | Abs. Press., Ins. of Mer- cury. | $\frac{p_2}{p_1}$ | Theo- retic. M.E.P. | Theo- retic. H.P. |
|-------------------------------------|---|-------------------|---------------------------|-------------------------|-------------------------------------|---|-------------------|---------------------------|-------------------------|
| 29 | 1 | 30.00 | 2.86 | 0.0124 | 18 | 12 | 2.50 | 6.21 | 0.0271 |
| 28 | 2 | 15.00 | 4.05 | 0.0177 | 16 | 14 | 2.14 | 5.89 | 0.0256 |
| 27 | 3 | 10.00 | 4.83 | 0.0211 | 14 | 16 | 1.87 | 5.42 | 0.0236 |
| 26 | 4 | 7.50 | 5.40 | 0.0235 | 12 | 18 | 1.67 | 4.88 | 0.0212 |
| 25 | 5 | 6.00 | 5.78 | 0.0252 | 10 | 20 | 1.50 | 4.23 | 0.0184 |
| 24 | 6 | 5.00 | 6.05 | 0.0264 | 8 | 22 | 1.36 | 3.52 | 0.0153 |
| 23 | 7 | 4.28 | 6.23 | 0.0271 | 6 | 24 | 1.25 | 2.73 | 0.0119 |
| 22 | 8 | 3.75 | 6.33 | 0.0276 | 4 | 26 | 1.15 | 1.88 | 0.0082 |
| 21 | 9 | 3.33 | 6.37 | 0.0278 | 2 | 28 | 1.07 | 0.96 | 0.0042 |
| 20 | 10 | 3.00 | 6.36 | 0.0277 | 1 | 29 | 1.03 | 0.49 | 0.0021 |

Circulating-pump. — Let Q be the quantity of cooling-water in cubic feet, n the number of strokes per minute, and S the length of stroke in feet.

Capacity of circulating-pump = $Q \div n$ cubic feet.

Diameter of circulating-pump = $13.55 \sqrt{Q \div nS}$ inches.

The clear area through the valve-seats and past the valves should be such that the mean velocity of flow does not exceed 450 feet per minute. The flow through the pipes should not exceed 500 ft. per min. in small pipes and 600 in large pipes. (Seaton.)

For Centrifugal Circulating-pumps, the velocity of flow in the inlet and outlet pipes should not exceed 400 ft. per min. The diameter of the fan-wheel is from $2\frac{1}{2}$ to 3 times the diam. of the pipe, and the speed at its periphery 450 to 500 ft. per min.

The Leblanc Condenser (made by the Westinghouse Machine Co.) accomplishes the separate removal of water and air by means of a pair of relatively small turbine-type rotors on a common shaft in a single casing, which is integral with or attached directly to the lower portion of the condensing chamber. The condensing chamber itself is but little more than an enlargement of the exhaust pipe. The injection water is projected downwards through a spray nozzle, and the combined injection water and condensed steam flow downward to a centrifugal discharge pump under a head of 2 or 3 ft., which insures the filling of the pump. The space above the water level in the condensing chamber is occupied by water vapor plus the air which entered with the injection water and with the exhaust steam, and this space communicates with the air-pump through a relatively small pipe.

The air-pump differs from pumps of the ejector type in that the vanes in traversing the discharge nozzle at high speed constitute a series of pistons, each one of which forces ahead of it a small pocket of air, the high velocity of which effectually prevents its return to the condenser. A small quantity of water is supplied to the suction side of the air-pump to assist in the performance of its functions. The power required for the pumps is said to approximate 2 to 3 per cent of the power generated by the main engine.

Feed-pumps for Marine Engines. — With surface-condensing engines the amount of water to be fed by the pump is the amount condensed from the main engine plus what may be needed to supply auxiliary engines and to supply leakage and waste. Since an accident may happen to the surface-condenser, requiring the use of jet-condensation, the pumps of engines fitted with surface-condensers must be sufficiently large to do duty under such circumstances. With jet-condensers and boilers using salt water the dense salt water in the boiler must be blown off at intervals to keep the density so low that deposits of salt will not be formed. Sea-water contains about $\frac{1}{32}$ of its weight of solid matter in solution. The boiler of a surface-condensing engine may be worked with safety when the quantity of salt is four times that in sea-water. If Q = net quantity of feed-water required in a given time to make up for what is used as steam, n = number of times the saltiness of the water in the boiler is to that of sea-water, then the gross feed-water = $nQ \div (n - 1)$. In order to be capable of filling the boiler rapidly each feed-pump is made of a capacity equal to twice the gross feed-water. Two feed-pumps should be supplied, so that one may be kept in reserve to be used while the other is out of repair. If Q be the quantity of net feed-water in cubic feet, l the length of stroke of feed-pump in feet, and n the number of strokes per minute,

Diameter of each feed-pump plunger in inches = $\sqrt{550 Q \div nl}$.

If W be the net feed-water in pounds,

Diameter of each feed-pump plunger in inches = $\sqrt{8.9 W \div nl}$.

An Evaporative Surface Condenser built at the Virginia Agricultural College is described by James H. Fitts (*Trans. A. S. M. E.*, xiv, 690). It consists of two rectangular end chambers connected by a series of horizontal rows of tubes, each row of tubes immersed in a pan of water. Through the spaces between the surface of the water in each pan and the bottom of the pan above air is drawn by means of an exhaust-fan. At the top of one of the end chambers is an inlet for steam, and a horizontal

diaphragm about midway causes the steam to traverse the upper half of the tubes and back through the lower. An outlet at the bottom leads to the air-pump. The passage of air over the water surfaces removes the vapor as it rises and thus hastens evaporation. The heat necessary to produce evaporation is obtained from the steam in the tubes, causing the steam to condense. It was designed to condense 800 lbs. steam per hour and give a vacuum of 22 in., with a terminal pressure in the cylinder of 20 lbs. absolute. Results of tests show that the cooling-water required is practically equal in amount to the steam used by the engine. And since the consumption of steam is reduced by the application of a condenser, its use will actually reduce the total quantity of water required.

The Continuous Use of Condensing-water is described in a series of articles in *Power*, Aug.-Dec., 1892. It finds its application in situations where water for condensing purposes is expensive or difficult to obtain.

The different methods described include cooling pans on the roof; fountains and other spray pipes in ponds, fine spray discharged at an elevation above a pond; trickling the water discharged from the hot-well over parallel narrow metal tanks contained in a large wooden structure, while a fan blower drives a current of air against the films of water falling from the tanks, etc. These methods are suitable for small powers, but for large powers they are cumbersome and require too much space, and are practically supplanted by cooling towers.

The Increase of Power that may be obtained by adding a condenser giving a vacuum of 26 inches of mercury to a non-condensing engine may

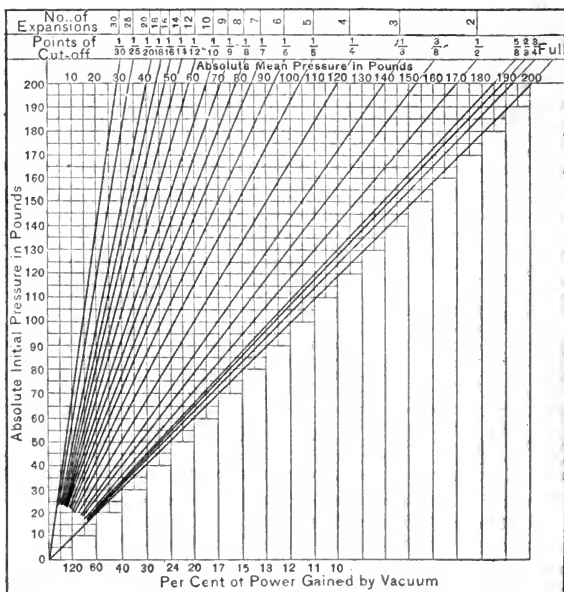


Fig. 166.

be approximated by considering it to be equivalent to a net gain of 12 lbs. mean effective pressure per sq. in. of piston area. If A = area of piston

in sq. ins., S = piston speed in ft. per min., then $12 AS \div 33,000 = AS \div 2750 = \text{H.P. made available by the vacuum}$. If the vacuum = 13.2 lbs. per sq. in. = 27.9 in. of mercury, then $\text{H.P.} = AS \div 2500$.

The saving of steam for a given horse-power will be represented approximately by the shortening of the cut-off when the engine is run with the condenser. Clearance should be included in the calculation. To the mean effective pressure non-condensing, with a given actual cut-off, clearance considered, add 3 lbs. to obtain the approximate mean *total* pressure, condensing. From tables of expansion of steam find what actual cut-off will give this mean total pressure. The difference between this and the original actual cut-off, divided by the latter and by 100, will give the percentage of saving.

The diagram on page 1058 (from catalogue of H. R. Worthington) shows the percentage of power that may be gained by attaching a condenser to a non-condensing engine, assuming that the vacuum is 12 lbs. per sq. in. The diagram also shows the mean pressure in the cylinder for a given initial pressure and cut-off, clearance and compression not considered.

The pressures given in the diagram are absolute pressures above a vacuum.

To find the mean effective pressure produced in an engine cylinder with 90 lbs. gauge (= 105 lbs. absolute) pressure, cut-off at $\frac{1}{4}$ stroke: find 105 in the left-hand or initial-pressure column, follow the horizontal line to the right until it intersects the oblique line that corresponds to the $\frac{1}{4}$ cut-off, and read the mean total pressure from the row of figures directly above the point of intersection, which in this case is 63 lbs. From this subtract the mean absolute back pressure (say 3 lbs. for a condensing engine and 15 lbs. for a non-condensing engine exhausting into the atmosphere) to obtain the mean effective pressure, which in this case, for a non-condensing engine, gives 48 lbs. To find the gain of power by the use of a condenser with this engine, read on the lower scale the figures that correspond in position to 48 lbs. in the upper row, in this case 25%. As the diagram does not take into consideration clearance or compression, the results are only approximate.

Advantage of High Vacuum in Reciprocating Engines. (R. D. Tomlinson, *Power*, Feb. 23, 1909.) — Among the transatlantic liners, the best ships with reciprocating engines are carrying from 26 to 28 and more inches of vacuum. Where the results are looked into, the engineers are required to keep the vacuum system tight and carry all the vacuum they can get, and while it is true that greater benefits can be derived from high vacua in a steam turbine than in a reciprocating engine, it is also true that, where primary heaters are not used, the higher the vacuum carried the greater is the justifiable economy which can be obtained from the plant.

The Interborough Rapid Transit Company, New York City, changed the motor-driven air-pump and jet-condenser for a barometric type of condenser and increased the vacuum on each of the 8000-H.P. Allis-Chalmers horizontal vertical engines at the 74th Street station from 26 to 28 ins., thereby increasing the power on each of the eight units approximately 275 H.P., and the economy of the station was increased nearly in the same ratio. This change was made about seven years ago and the plant is still operating with 28 ins. of vacuum, measured with mercury columns connected to the exhaust pipe at a point just below the exhaust nozzle of the low-pressure cylinders.

A careful test made on the 59th Street station showed a decrease in steam consumption of 8% when the vacuum was raised from 25 to 28 ins. These engines drive 5000-kw. generators.

The Choice of a Condenser. — Condensers may be divided into two general classes:

First. — Jet condensers, including barometric condensers, siphon condensers, ejector condensers, etc., in which the cooling-water mingles with the steam to be condensed.

Second. — Surface condensers, in which the cooling-water is separated from the steam, the cooling-water circulating on one side of this surface and the steam coming into contact with the other.

In the jet-condenser the steam, as soon as condensed, becomes mixed with the cooling-water, and if the latter should be unsuitable for boiler-feed because of scale-forming impurities, acids, salt, etc., the pure distilled

water represented by the condensed steam is wasted, and, if it were necessary to purchase other water for boiler-feeding, this might represent a considerable waste of money. On the other hand, if the cooling-water is suitable for boiler-feeding, or if a fresh supply of good water is easily obtainable, the jet-condenser, because of its simplicity and low cost, is unexcelled.

Surface condensers are recommended where the cooling-water is unfitted for boiler-feed and where no suitable and cheap supply of pure boiler-feed is available.

Where a natural supply of cooling-water, as from a well, spring, lake or river, is not available, a water-cooling tower can be installed and the same cooling-water used over and over again. (Wheeler Condenser and Eng. Co.)

Owing to their great cost as compared with jet-condensers, surface condensers should not be used except where absolutely necessary, i.e., where lack of feed-water for the boiler warrants the extra cost. Of course there are cases, such as at sea, where surface condensers are indispensable. On land, suitable feed-water can always be obtained at some expense, and that cost capitalized makes it a simple arithmetical problem to determine the extra investment permissible in order to be able to return condensed steam as feed-water to the boiler. Unfortunately there is another point which greatly complicates the matter, and one which makes it impossible to give exact figures, viz., the corrosion and deterioration of the condenser tubes themselves, the exact cause of which is not often understood. With clean, fresh water, free from acid, the tubes of a condenser last indefinitely, but where the cooling-water contains sulphur, as in drainage from coal mines, or sea-water contaminated by sewage, such as harbor water, the deterioration is exceedingly rapid.

A better vacuum may possibly be obtained from a surface condenser where there is plenty of cooling-water easily handled. The better vacuum is due to the fact that the air-pump will have much less air to handle inasmuch as the air carried in suspension by the cooling-water does not have to be extracted as in the case of jet-condensers. Water in open rivers, the ocean, etc., is said to carry in suspension 5% by volume of air. It may be said that except for leakages, which should not exist, the air-pump will have no work to do at all inasmuch as the water will have no opportunity to become aerated. On the other hand, if the cooling-water is limited, these advantages are offset by the fact that a surface condenser cannot heat the cooling-water so near to the temperature of the exhaust steam as can a jet-condenser. (F. Hodgkinson, *El. Jour.*, Aug., 1909.)

A barometric condenser used in connection with a 15,000-k.w. steam-engine-turbine unit at the 59th St. station of the Rapid Transit Co., New York, contains approximately 25,000 sq. ft. of cooling surface arranged in the double two-pass system of water circulation, with a 30-in. centrifugal circulating pump having a maximum capacity of 30,000 gal. per hour. The dry vacuum pump is of the single-stage type, 12- and 29-in. \times 24-in., with Corliss valves on the air cylinder. The condensing plant is capable of maintaining a vacuum within 1.1 in. of the barometer when condensing 150,000 lb. of steam per hour when supplied with circulating water at 70° F. — (H. G. Stott, *Jour. A.S.M.E.*, Mar., 1910.)

Cooling Towers are usually made in the shape of large cylinders of sheet steel, filled with narrow boards or lath arranged in geometrical forms, or hollow tile, or wire network, so arranged that while the water, which is sprayed over them at the top, trickles down through the spaces it is met by an ascending air column. The air is furnished either by disk fans at the bottom or is drawn in by natural draught. In the latter case the tower is made very high, say 60 to 100 ft., so as to act like a chimney. When used in connection with steam condensers, the water produced by the condensation of the exhaust steam is sufficient to compensate for the evaporation in the tower, and none need be supplied to the system. There is, on the contrary, a slight overflow, which carries with it the oil from the engine cylinders, and tends to clean the system of oil that would otherwise accumulate in the hot-well.

The cooling of water in a pond, spray, or tower goes on in three ways — first, by radiation, which is practically negligible; second, by conduction or absorption of heat by the air, which may vary from one-fifth to one-third the entire effect; and, lastly, by evaporation. The latter is the

chief effect. Under certain conditions the water in a cooling tower can actually be cooled below the temperature of the atmosphere, as water is cooled by exposing it in porous vessels to the winds of hot and dry climates.

The evaporation of 1 lb. of water absorbs about 1000 heat units. The rapidity of evaporation is determined, first, by the temperature of the water, and, second, by the vapor tension in the air in immediate contact with the water. In ordinary air the vapor present is generally in a condition corresponding to superheated steam, that is, the air is not saturated. If saturated air be brought into contact with colder water, the cooling of the vapor will cause some of it to be precipitated out of the air; on the other hand, if saturated air be brought into contact with warmer water, some of the latter will pass into the form of vapor. This is what occurs in the cooling tower, so that the latter is in a large measure independent of climatic conditions; for even if the air be saturated, the rise in temperature of the atmospheric air from contact with the hot water in the cooling tower will greatly increase the water-carrying capacity of the air, enabling a large amount of heat to be absorbed through the evaporation of the water. The two things to be sought after in cooling-tower design are, therefore, first, to present a large surface of water to the air, and, second, to provide for bringing constantly into contact with this surface the largest possible volume of new air at the least possible expenditure of energy. (Wheeler Condenser and Engineering Co.)

The great advantage of the cooling tower lies in the fact that large surfaces of water can be presented to the air while the latter is kept in rapid motion.

Tests of a Cooling Tower and Condenser are reported by J. H. Vail in *Trans. A. S. M. E.*, 1898. The tower was of the Barnard type, with two chambers, each 12 ft. 3 in. \times 18 ft. \times 29 ft. 6 in. high, containing galvanized-wire mats. Four fans supplied a strong draught to the two chambers. The rated capacity of each section was to cool the circulating water needed to condense 12,500 lbs. of steam, from 132° to 80° F., when the atmosphere does not exceed 75° F. nor the humidity 85%. The following is a record of some observations.

| Date, 1898. | Jan. 31. | Feb. | June 20. | July | Aug. 26. | Nov. 4. | Aug. 2. | |
|--------------------------------|-------------|------|-------------|--------|-------------|------------|---------|------|
| | | | | | | | Max. | Min. |
| Temperature atmosphere..... | 30° | 36° | 78° | 96° | 85° | 59° | 103 | 83 |
| Temp. condenser discharge..... | 110° | 110° | 120° | 130° | 118° | 129° | 128 | 106 |
| Temp. water from tower..... | 65° | 84° | 84° | 93° | 88° | 92° | 98 | 91 |
| Heat extracted by tower..... | 45° | 26° | 36° | 37° | 30° | 37° | 32 | 21 |
| Speed of fans, r.p.m..... | 36 | 0 | 145 | 162 | 150 | 148 | 160 | 140 |
| Vacuum, inches..... | 25 1/2 | 26 | 25 | 24 1/2 | 25 1/2 | 25 | 26 | 26 |

The quantity of steam condensed or of water circulated is not stated, but in the two tests on Aug. 2 the H.P. developed was 900 I.H.P. in the first and 400 in the second, the engine being a tandem compound, Corliss type, 20 and 36 \times 42 in., 120 r.p.m.

J. R. Bibbins (*Trans. A. S. M. E.*, 1909) gives a large amount of information on the construction and performance of different styles of cooling towers. He suggests a type of combined fan and natural draft tower suited to most efficient running on peak as well as light loads.

Evaporators and Distillers are used with marine engines for the purpose of providing fresh water for the boilers or for drinking purposes.

Weir's Evaporator consists of a small horizontal boiler, contrived so as to be easily taken to pieces and cleaned. The water in it is evaporated by the steam from the main boilers passing through a set of tubes placed in its bottom. The steam generated in this boiler is admitted to the low-pressure valve-chest, so that there is no loss of energy, and the water condensed in it is returned to the main boilers.

In *Weir's Feed-heater* the feed-water before entering the boiler is heated up very nearly to boiling-point by means of the waste water and steam from the low-pressure valve-chest of a compound engine.

ROTARY STEAM-ENGINES — STEAM TURBINES.

Rotary Steam-engines, other than steam turbines, have been invented by the thousands, but not one has attained a commercial success, as regards economy of steam. For all ordinary uses the possible advantages, such as saving of space, to be gained by a rotary engine are overbalanced by its waste of steam. Rotary engines are in use, however, for special purposes, such as steam fire-engines and steam feeds for sawmills, in which steam economy is not a matter of importance.

Impulse and Reaction Turbines.— A steam turbine of the simplest form is a wheel similar to a water wheel, which is moved by a jet of steam impinging at high velocity on its blades. Such a wheel was designed by Branca, an Italian, in 1629. The De Laval steam turbine, which is similar in many respects to a Pelton water wheel, is of this class. It is known as an impulse turbine. In a book written by Hero, of Alexandria, about 150 B.C., there is shown a revolving hollow metal ball, into which steam enters through a trunnion from a boiler beneath, and escapes tangentially from the outer rim through two arms which are bent backwards, so that the steam by its reaction causes the ball to rotate in an opposite direction to that of the escaping jets. This wheel is the prototype of a reaction turbine. In most modern steam turbines both the impulse and reaction principles are used, jets of steam striking blades or buckets inserted in the rim of a wheel, so as to give it a forward impulse, and escaping from it in a reverse direction so as to react upon it. The name impulse wheel, however, is now generally given to wheels like the De Laval, in which the pressure on the two sides of a wheel containing the blades is the same, and the name reaction wheel to one in which the steam decreases in pressure in passing through the blades. The Parsons turbine is of this class.

The De Laval Turbine.— The distinguishing features of this turbine are the diverging nozzles, in which the steam expands down to the atmospheric pressure in non-condensing, and to the vacuum pressure in condensing wheels; a single forged steel disk carrying the blades on its periphery; a slender, flexible shaft on which the wheel is mounted and which rotates about its center of gravity; and a set of reducing gears, usually 10 to 1 reduction, to change the very high speed of the turbine to a moderate speed for driving machinery. Following are the sizes and speeds of some De Laval turbines:

| | | | | |
|---------------------------------|--------|--------|--------|--------|
| Horse-power..... | 5 | 30 | 100 | 300 |
| Revolutions per minute..... | 30,000 | 20,000 | 13,000 | 10,000 |
| Diam. to center of blades, ins. | 3.94 | 8.86 | 19.68 | 29.92 |

The number and size of nozzles vary with the size of the turbine. The nozzles are provided with valves, so that for light loads some of them may be closed, and a relatively high efficiency is obtained at light loads. The taper of the nozzles differs for condensing and non-condensing turbines. Some turbines are provided with two sets of nozzles, one for condensing and the other for non-condensing operation.

The Zolley or Rateau Turbine.— The Zolley or Rateau turbines are developments of the De Laval and consist of a number of De Laval elements in series, each succeeding element utilizing the exhaust steam from the preceding. The steam is partly expanded in the first row of nozzles, strikes the first row of buckets and leaves them with practically zero velocity. It is then further expanded through the second row of nozzles, strikes a second row of moving buckets and again leaves them with zero velocity. This process is repeated until the steam is completely expanded.

The Parsons Turbine.— In the Parsons, or reaction type of turbine, there are a large number of rows of blades, mounted on a rotor or revolving drum. Between each pair of rows there is a row of stationary blades attached to the casing, which take the place of nozzles. A set of stationary blades and the following set of moving blades constitute what is known as a stage. The steam expands and loses pressure in both sets. The speed of rotation, the peripheral speed of the blades and the velocity of the steam through the blades are very much lower than in the De Laval turbine. The rotor, or drum, on which the moving blades are carried, is usually made in three sections of different diameters, the smallest at the high-pressure end, where steam is admitted, and the largest at the

exhaust end. In each section the radial length of the blades and also their width increase from one end to the other, to correspond with the increased volume of steam. The Parsons turbine is built in the United States by the Westinghouse Machine Co. and by the Allis-Chalmers Co.

The Westinghouse Double-flow Turbine.—For sizes above 5000 K.W. a turbine is built in which the impulse and reaction types are combined. It has a set of non-expanding nozzles, an impulse wheel with two velocity stages (that is two wheels with a set of stationary non-expanding blades between), one intermediate section and two low-pressure sections with Parsons blading. After steam has passed through the impulse wheel and the intermediate section it is divided into two parts, one going to the right and the other to the left hand low-pressure section. There is an exhaust pipe at each end. In this turbine, the end thrust, which has to be balanced in reaction turbines of the usual type, is almost entirely avoided. Other advantages are the reduction in size and weight, due to higher permissible speed; blades and casing are not exposed to high temperatures; reduction of size of exhaust pipes and of length of shaft; avoidance of large balance pistons.

The Curtis Turbine, made by the General Electric Company, is an impulse wheel of several stages. Steam is expanded in nozzles and enters a set of three or more blades, at least one of which is stationary. The blades are all non-expanding, and the pressure is practically the same on both sides of any row of blades. In smaller sizes of turbines, only one set of stationary and movable blades is used, but in large sizes there are from two to five sets, each forming a pressure stage, separated by diaphragms containing additional sets of nozzles. The smaller sizes have horizontal shafts, but the larger ones have vertical shafts supported on a step bearing supplied with oil or water under a pressure sufficient to support the whole weight of the shaft and its attached rotating disks. Curtis turbines are made in sizes from 15 K.W. at 3600 to 4000 revs. per minute up to 9000 K.W. at 750 revs. per minute.

Mechanical Theory of the Steam Turbine.—In the impulse turbine of the De Laval type, with a single disk containing blades at its rim, steam at high pressure enters the smaller end or throat of a tapering nozzle, and, as it passes through the nozzle, is expanded adiabatically down to the pressure in the casing of the turbine, that is to the pressure of the atmosphere, in a non-condensing turbine, or to the pressure of the vacuum, if the turbine is connected to a condenser. The steam thus expanded has its volume and its velocity enormously increased, its pressure energy being converted into energy of velocity. It then strikes tangentially the concave surfaces of the curved blades, and thus drives the wheel forward. In passing through the blades it has its direction reversed, and the reaction of the escaping jet also helps to drive the wheel forward. If it were possible for the direction of the jet to be completely reversed, or through an arc of 180° , and the velocity of the blade in the direction of the entering jet was one-half the velocity of the jet, then all the kinetic energy due to the velocity of the jet would be converted into work on the blade, and the velocity of the jet with reference to the earth would be zero. This complete reversal, however, is impossible, since room has to be allowed between the blades for the passage of the steam, and the blades, therefore, are curved through an arc considerably less than 180° , and the jet on leaving the wheel still has some kinetic energy, which is lost. The velocity of the entering steam jet also is so great that it is not practicable to give the wheel rim a velocity equal to one-half that of the jet, since that would be beyond a safe speed. The speed of the wheel being less than half that of the entering jet, also causes the jet to leave the wheel with some of its energy unutilized. The mechanical efficiency of the wheel, neglecting radiation, friction, and other internal losses, is expressed by the fraction $(E_1 - E_2) \div E_1$, in which E_1 is the kinetic energy of the steam jet impinging on the wheel and E_2 that of the steam as it leaves the blades.

In multiple-stage impulse turbines, the high velocity of the wheel is reduced by causing the steam to pass through two or more rows of blades, which rows are separated by a row of stationary curved blades which direct the steam from the outlet of one row to the inlet of the next. The passages through all the blades, both movable and secondary, are parallel, or non-expanding, so that the steam does not change its

pressure in passing through them. The wheel with two rows of movable blades running at half the velocity of a single-stage turbine, or one with three rows at one-third the velocity, causes the same total reduction in velocity as the single-stage wheel; and a greater reduction in the velocity of the wheel can be obtained by increasing the number of rows. It is, therefore, possible by having a sufficient number of rows of blades, or velocity stages, to run a wheel at comparatively slow speed and yet have the steam escape from the last set of blades at a lower absolute velocity than is possible with a single-stage turbine. In the reaction turbine the reduction of the pressure and its conversion into kinetic energy, or energy of velocity, takes place in the blades, which are made of such shape as to allow the steam to expand while passing through them.

The stationary blades also allow of expansion in volume, thus taking the place of nozzles.

In all turbines, whether of the impulse, reaction, or combination type, the object is to take in steam at high pressure and to discharge it into the atmosphere, or into the condenser, at the lowest pressure and largest volume possible, and with the lowest possible absolute velocity, or velocity with reference to the earth, consistent with getting the steam away from the wheel, and to do this with the least loss of energy in the wheel due to friction of the steam through the passages, to shock due to incorrect shape, or position of the blades, to windage or frictional resistance of the steam in contact with the rotating wheel, or other causes. The minimizing of these several losses is a problem of extreme difficulty which is being solved by costly experiments.

Heat Theory of the Steam Turbine.

— The steam turbine may also be considered as a heat engine, the object of which is to take a pound of steam containing a certain quantity of heat, H_1 , transform as great a part of this heat as possible into work, and

discharge the remaining part, H_2 , into the condenser. The thermal efficiency of the operation is $(H_1 - H_2) \div H_1$, and the theoretical limit of this efficiency is $(T_1 - T_2) \div T_2$, in which T_1 is the initial and T_2 the final absolute temperature.

Referring to temperature entropy diagram, Fig. 167, the total heat above 32° F. of 1 lb. of steam at the temperature T_1 is represented by the area $OACDG$ and its entropy is ϕ_1 . Expanding adiabatically to T_2 part of its heat energy is converted into work, represented by the area $BCDF$, while $OABFG$ represents the heat discharged into the condenser. The total heat of 1 lb. of dry saturated steam at T_2 is greater than this by the area $EFGH$, the fraction $FE \div BE$ representing moisture in the 1 lb. of wet steam discharged. If H_1 = heat units in 1 lb. of dry steam at the state-point D , and H_2 = heat units in 1 lb. of dry steam at the state-point E , at the temperature T_2 , then the energy converted into work = $BCDF = H_1 - H_2 + (\phi_2 - \phi_1) T_2$. This quantity is called the available energy E_a , of 1 lb. of steam between the temperatures T_1 and T_2 .

If the steam is initially wet, as represented by the state-point d and entropy ϕ_x , then the work done in adiabatic expansion is $BCdFB$, which is equal to $E_a = H_1 - H_2 + (\phi_2 - \phi_1) T_2 - (\phi_1 - \phi_x)(T_1 - T_2)$. The quantity $\phi_1 - \phi_x = (L/T_1)(1 - x)$, in which L = latent heat of evaporation at the temperature T_1 , and x = the moisture in 1 lb. of steam. The values of H_1 , H_2 , ϕ_1 , ϕ_2 , etc., for different temperatures, may be taken from steam tables or diagrams.

If the steam is initially superheated to the temperature T_s , as represented by the state-point j , the entropy being ϕ_3 , then the total heat at j is $H_1 + C(T_s - T_1)$, in which C is the mean specific heat of superheated steam between T_1 and T_s . The increase of entropy above ϕ_1

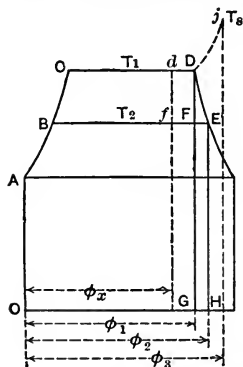


FIG. 167.

is $\phi_3 - \phi_1 = C \log_e (T_3/T_1)$. The energy converted into work is $E_a = H_1 - H_2 + (\phi_2 - \phi_1) T_2 + [1/2 (T_3 + T_1) - T_2] (\phi_3 - \phi_1)$.

Velocity of Steam in Nozzles. — Having obtained the total available energy in steam expanding adiabatically between two temperatures, as shown above, the maximum possible flow into a vacuum is obtained from the common formula, Energy, in foot-pounds, $= 1/2 W/g \times V^2$, in which W is the weight (in this case 1 lb.). V is the velocity in feet per second, and $g = 32.2$. As the energy E_a is in heat units, it is multiplied by 778 to convert it into foot-pounds, and we have

$$V = \sqrt{778 \times 2 g E_a} = 223.8 \sqrt{E_a}.$$

This is the theoretically maximum possible velocity. It cannot be obtained in a short nozzle or orifice, but is approximated in the long expanding nozzles used in turbines. In the throat or narrow section of an orifice, the velocity and the weight of steam flowing per second may be found by Napier's or Rateau's formula, see page 847, or from Grashof's formula as given by Moyer, $F = A_0 P_1^{0.97} \div 60$, or $A_0 = 60 F \div P_1^{0.97}$, in which A_0 is the area of the smallest section of the nozzle, sq. in., F is the flow of steam (initially dry saturated) in lbs. per sec., and P is the absolute pressure, lbs. per sq. in. This formula is applicable in all cases where the final pressure P_2 does not exceed 58% of the initial pressure. For wet steam the formula becomes $F = A_0 P_1^{0.97} \div 60 \sqrt{x}$, $A_0 = 60 F \sqrt{x} \div P_1^{0.97}$, in which x is the dryness quality of the inflowing steam, $1 - x$ being the moisture.

For superheated steam $F = A_0 P_1^{0.97} (1 + 0.00065 D) \div 60$; $A_0 = 60 F \div P_1^{0.97} (1 + 0.00065 D)$, D being the superheat in degrees F.

When the final pressure P_2 is greater than 0.58 P_1 , a coefficient is to be applied to F in the above formulae, the value of which is most conveniently taken from a curve given by Rateau. The values of this coefficient, c , for different ratios of P_1/P_2 , are approximately as follows:

| | | | | | | | | | | | |
|------------------|------|-------|-------|-------|-------|-------|-------|------|------|------|------|
| $P_2 \div P_1 =$ | 0.58 | 0.60 | 0.62 | 0.64 | 0.66 | 0.68 | 0.70 | 0.72 | 0.74 | 0.76 | 0.78 |
| $c =$ | 1.00 | 0.995 | 0.985 | 0.975 | 0.965 | 0.955 | 0.945 | 0.93 | 0.91 | 0.88 | 0.85 |
| $P_2 \div P_1 =$ | 0.80 | 0.82 | 0.84 | 0.86 | 0.88 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 1.00 |
| $c =$ | 0.82 | 0.79 | 0.76 | 0.72 | 0.675 | 0.625 | 0.57 | 0.51 | 0.42 | 0.30 | 0.00 |

The quality of steam after adiabatic expansion, x_2 , is found from the formula

$$x_2 = (x_1 L_1 / T_1 + \theta_1 - \theta_2) T_2 / L_2, \quad (8)$$

in which θ_1 and θ_2 are the entropies of the liquid, L_1 and L_2 the latent heats of evaporation, and x_1 and x_2 the dryness quality, at the initial and final conditions respectively. Curves of steam quality are plotted in an entropy-total heat chart given in Moyer's "Steam Turbines" and also in Marks and Davis's "Steam Tables and Diagrams."

The area of the smallest section or throat of the nozzle being found, the area of any section beyond the throat is inversely proportional to the velocity and directly proportional to the specific volume and to the dryness, or $A_1/A_0 = V_0/V_1 \times v_1/v_0 \times x_1/x_0$, in which A is in the area in sq. ins., V the velocity in ft. per sec., v the volume of 1 lb. of steam in cu. ft., and x the dryness fraction, the subscript 0 referring to the smallest section and the subscript 1 to any other section. The ratio A_1/A_0 for the largest cross section of a properly designed nozzle is nearly proportional to the ratio of the initial to the final pressure. Moyer gives it as $A_1/A_0 = 0.172 P_1/P_2 + 0.70$, and for P_1/P_2 greater than 25, $A_1/A_0 = 0.175 (P_1/P_2)^{0.91} + 0.70$.

In practice expanding nozzles are usually made so that an axial section shows the inner walls in straight lines. The transverse section is usually either a circle or a square with rounded corners. The divergence of the walls is about 6 degrees from the axis for the non-condensing and as much as 12 degrees for condensing turbines for low vacuums. Moyer gives an empirical formula for the length between the throat and

the mouth, $L = \sqrt{15 A_0}$ inches. The De Laval turbine uses a much longer nozzle for mechanical reasons. The entrance to the nozzle above the throat should be well rounded. The efficiency of a well-made nozzle with smooth surfaces as measured by the velocity is about 96 to 97%, corresponding to an energy efficiency of 92 to 94%.

Speed of the Blades.—If V_b = peripheral velocity of the blade, V_1 = absolute velocity of the steam entering the blades and α the nozzle angle, or angle of the nozzle to the plane of the wheel, then (in impulse turbines with equal entrance and exit angles of the blade with the plane of the wheel) for maximum theoretical efficiency of the blade, $V_b = \frac{1}{2} V_1 \cos \alpha$. The nozzle angle is usually about 20° , $\cos \alpha = 0.940$, and the efficiency of a single row of blades is $(0.94 - V_b/V_1) \pm V_b/V_1$.

For $V_1 = 3000$ ft. per sec., the efficiency for different blade speeds is about as follows:

| | | | | | | | | | | |
|--------------|-----|-----|-----|-----|------|------|------|------|------|------|
| $V_b =$ | 200 | 400 | 600 | 800 | 1000 | 1200 | 1400 | 1600 | 1800 | 2000 |
| Efficiency % | 23 | 44 | 60 | 72 | 81 | 87 | 89 | 87 | 80 | 71 |

The highest efficiency is obtained when $V_b = \text{about } \frac{1}{2} V_1$. It is difficult, for mechanical reasons, to use speeds much greater than 500 ft. per sec., therefore the highest efficiencies are often sacrificed in commercial machines. The blade speeds used in practice vary from 500 to 1200 ft. per sec. For an impulse wheel with more than one row of moving

blades in a single pressure stage, efficiency = $\frac{4 NV_b}{V_1} \left(\cos \alpha - \frac{NV_b}{V_1} \right)$.

Referring to Fig. 168, if V_1 is the absolute direction and velocity of the entering jet, V_b the direction and velocity of the blade, the resultant,

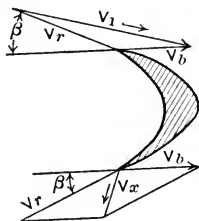


FIG. 168.

V_r is the velocity and direction of the jet relatively to the blade, and the edge of the blade is made tangent to this direction. Also V_x , the resultant of V_b and V_r at the other edge of the blade, is the absolute velocity and direction of the steam escaping from the wheel. If β is the angle between V_r and V_b , the maximum energy is abstracted from the steam when the angle between V_x and $V_b = 90 - \frac{1}{2} \beta$, and the efficiency is $\cos \beta \div \cos^2 \frac{1}{2} \beta$.

For details of design of blades, and of turbines in general, see Moyer, Foster, Thomas, Stodola and other works on Steam Turbines, also Peabody's "Thermodynamics." Calculations of stages, nozzles, etc., are much facilitated by the use of Peabody's "Steam Tables" and Marks and Davis's "Steam Tables and Diagrams."

Comparison of Commercial Impulse and Reaction Turbines. (Moyer.)

- | IMPULSE. | REACTION. |
|--|--|
| 1. Few stages. | 1. Many stages. |
| 2. Expansion in nozzles. | 2. No nozzles. |
| 3. Large drop in pressure in a stage. | 3. Small drop in pressure in a stage. |
| 4. Initial steam velocities 1000 to 4000 ft. per sec. | 4. All steam velocities low, 300 to 600 ft. per sec. |
| 5. Blade velocities 400 to 1200 ft. per sec. | 5. Blade velocities 150 to 400 ft. per sec. |
| 6. Best efficiency when the blade velocity is nearly half the initial velocity of steam. | 6. Best efficiency when the blade velocity is nearly equal to the highest velocity of the steam. |

Loss due to Windage (or friction of a turbine wheel rotating in steam). — Moyer gives for the friction of a plain disk without blades, F_w , and of one row of blades without the disk, F_b , in horse-power:

$$F_w = 0.08 d^2 (u/100)^{2.8} w \div (1 + 0.00065 D)^2,$$

$$F_b = 0.3 d u^{1.2} (u/100)^{2.8} w \div (1 + 0.00065 D)^2,$$

in which d = diam. of disk to inner edge of blade, in feet; u = peripheral velocity of disk, in ft. per sec.; w = density of dry saturated steam at the pressure surrounding the disk, in lbs. per cu. ft., and D = superheat in degrees F. The sum of F_w and F_b is the friction of the disk and blades. For moist steam the term $1 + 0.00065 D$ is to be omitted, and the expression multiplied by a coefficient c , whose value is approximately as follows:

Per cent moisture in steam 2 4 6 8 10 12 16 20 24
Coefficient c ... 1.01 1.05 1.10 1.16 1.25 1.37 1.65 2.00 2.44
At high rotative speeds the rotation loss of a non-condensing turbine with wheels revolving in steam at atmospheric pressure is quite large, and in small turbines it may be as much as 20% of the total output. The loss decreases rapidly with increasing vacuum. In a turbine with more than one stage part of the friction loss of rotation is converted into heat which in the next stage is converted into kinetic energy, thus partly compensating for the loss.

Efficiency of the Machine. — The maximum possible thermodynamic efficiency of a steam turbine, as of any other steam engine, is expressed by the ratio which the available energy between two temperatures bears to the total heat, measured above absolute zero, of the steam at the higher temperature. In the temperature-entropy diagram Fig. 167 it is represented by the ratio of the area $BCDF$ to $OACDG$. Of this available energy, from 50 to 75 and possibly 80 per cent is obtainable at the shaft of turbines of different sizes and designs. As with steam engines, the highest mechanical and thermal efficiencies are reached only with large sizes and the most expensive designs. The several losses which tend to reduce the efficiency of turbines below the theoretical maximum are: 1, residual velocity, or the kinetic energy due to the velocity of the steam escaping from the turbine; 2, friction and imperfect expansion in the nozzles; 3, windage, or friction due to rotation of the wheel in steam; 4, friction of the steam traveling through the blades; 5, shocks, impacts, eddies, etc., due to imperfect shape or roughness of blades; 6, leakage around the ends of the blades or through clearance spaces; 7, shaft friction; 8, radiation. The sum of all these losses amounts to about 25% of the available energy in the largest and best designs and to 50% or more in small sizes or poor designs.

Steam Consumption of Turbines. — The steam consumption of any steam turbine is so greatly influenced by the conditions of pressure, moisture or superheat, and vacuum, that it is necessary to know the effect of these conditions on any turbines whose performances are to be compared with each other or with a given standard. Manufacturers usually furnish with their guarantees of performance under standard conditions of pressure, superheat and vacuum, a statement or set of curves showing the amount that the steam consumption per K.W.-hour will be increased or diminished by stated variations from these standard conditions. When a test of steam consumption is made under any conditions varying from the standard, the results should be corrected in order to compare them with other tests. Moyer gives the following example of applying corrections to a pair of tests made in 1907, to reduce them both to a steam pressure of 179 lbs. gauge, 28.5 ins. vacuum, and 100° F. superheat.

| | 7500-K.W. Westing- house- Parsons. | Correc- tions, per cent. | 9000-K.W. Curtis. | Correc- tions, per cent. |
|--|---|--------------------------------|----------------------|--------------------------------|
| Average steam pressure..... | 177.5 | -0.15 | 179 | 0 |
| Average vacuum, ins., referred to 30-in. barometer..... | 27.3 | -3.36 | 29.55 | +12.39 |
| Average superheat, deg. F..... | 95.7 | -0.29 | 116 | +1.28 |
| Average load on generator, K.W. | 9830.5 | | 8070 | |
| Steam cons., lbs. per K.W.-hr.. | 15.15 | | 13.0 | |
| Net correction, per cent..... | | -3.80 | | +13.67 |
| Corr. st. cons., lbs. per K.W.-hr. | 14.57 | | 14.77 | |

For the 7500-K.W. turbine, the following corrections given by the manufacturer were used: pressure, 0.1% for each pound; vacuum, 2.8% for each inch; superheat, 7% for each 100° F. For the 9000-K.W. turbine, the following corrections were used: superheat, 8% for 100° F.; vacuum, 8% for each inch.

The results as corrected show that the two turbines would give practically the same economy if tested under uniform conditions. The results

are equivalent respectively to 9.58 and 9.72 lbs. per I.H.P.-hour, assuming 97% generator efficiency and 91% mechanical efficiency of a steam engine.

The proper correction for moisture in a steam turbine test is stated to be a little more than twice the percentage of moisture. There is a large increase in the disk and blade rotation losses when wet steam is used.

The gain in economy per inch of vacuum at different vacuums is given as follows in *Mech. Engr.*, Feb. 24, 1906.

| Inches of Vacuum. | 28 | 27 | 26 | 25 |
|---|------|------|------|------|
| Curtis, per cent gain per inch of vacuum.. | 5.1 | 4.8 | 4.6 | 4.2 |
| Parsons, per cent gain per inch of vacuum | 5.0 | 4.0 | 3.5 | 3.0 |
| Westinghouse-Parsons, per cent gain per inch of vacuum..... | 3.14 | 3.05 | 2.95 | 2.87 |
| Theoretical per cent gain per inch of vac.. | 5.2 | 4.4 | 3.7 | 3.0 |

The following results of tests of turbines of different makes are selected from a series of tables in Moyer's "Steam Turbines".

| Rated K.W. | Output K.W. | Gauge Press. | Super- heat, deg. F. | Vacuum, ins. | Lbs. per K.W.- hour. | Rated K.W. | Output B.H.P. | Gauge Press. | Super- heat, deg. F. | Vacuum, ins. | Lbs. per B.H.P.- hour. |
|---------------|----------------|-----------------|----------------------------|-----------------|----------------------------|---------------|------------------|-----------------|----------------------------|-----------------|------------------------------|
| 2000 C. | 555 | 155 | 204 | 28.5 | 18.09 | 300 W.-P. | 233 | 145 | 4.1 | 28.0 | 15.99 |
| | 1067 | 170 | 120 | 28.4 | 16.31 | | 461 | 145 | 4.8 | 28.0 | 13.99 |
| | 2024 | 166 | 207 | 28.5 | 15.02 | | 688 | 140 | 7.0 | 27.2 | 15.73 |
| 9000 C. | 5374 | 182 | 133 | 29.4 | 13.15 | 500 W.-P. | 383 | 153 | 2 | 28.2 | 14.15 |
| | 8070 | 179 | 116 | 29.4 | 13.00 | | 756 | 149 | 1 | 27.8 | 13.28 |
| | 10186 | 176 | 147 | 29.5 | 12.90 | | 1122 | 149 | 5 | 26.5 | 14.32 |
| 1500 P. | 13900 | 198 | 140 | 29.3 | 13.60 | 1000 W.-P. | 386 | 148 | 3 | 0.8 | 24.94 |
| | 530 | 145 | 110 | 28.9 | 21.58 | | 767 | 147 | 3 | 0.8 | 22.10 |
| | 1071 | 131 | 124 | 28.3 | 18.24 | | 1144 | 126 | 11 | 0.8 | 24.36 |
| 300 P. | 1585 | 128 | 125 | 27.5 | 17.60 | 3000 W.-P. | 752 | 151 | 0 | 27.5 | 14.77 |
| | 303 | 158 | 0 | 26.6 | 23.15 | | 1503 | 147 | 0 | 27.0 | 13.61 |
| | 297 | 161 | 0 | 0 | 34.20 | | 2253 | 145 | 0 | 25.2 | 15.29 |
| 1000 R. | 194 | 171 | 47 | 27.7 | 31.97 | 300 D | 2295 | 152 | 102 | 26.2 | 12.36 |
| | 425 | 144 | 21 | 27.6 | 24.91 | | 4410 | 144 | 87 | 26.2 | 11.85 |
| | 871 | 166 | 11 | 23.6 | 24.61 | | 196 | 198 | 16 | 27.4 | 15.62 |
| | 1024 | 164 | 10 | 25.0 | 21.98 | | 298 | 197 | 64 | 27.4 | 14.35 |
| | | | | | | | 352 | 199 | 84 | 27.2 | 13.94 |

C., Curtis; P., Parsons; W.-P., Westinghouse-Parsons; R., Rateau; D., De Laval. Note that the figures of steam consumption in the first half of the table are in lbs. per K.W.-hour; in second half, in lbs. per Brake H.P.-hour.

A test of a Westinghouse double-flow turbine at the Williamsburg power station, Brooklyn N. Y., gave the following results (*Eng. News*, Dec. 30, 1909): Speed, 750 r.p.m.; Steam pressure at throttle, 203.4 lbs.; Superheat, 80.1° F.; Vacuum, 28.6 ins.; Load, 13,384 K.W.; Steam per K.W.-hour, 14.4 lbs.; Efficiency of generator, 98%; Windage, 2.0%; Equivalent B.H.P., 18,620; Steam per B.H.P.-hour, 10.3 lbs.

The Largest Steam Turbine, 1909. (*Eng. News*, Dec. 30.) — A Westinghouse combination double-flow turbine is about to be tested which is capable of developing 22,000 H.P. with 1.75 lbs. steam pressure and 28 ins. vacuum, and it is estimated that the steam consumption will be about 10 lbs. per B.H.P.-hour. The principal dimensions are: length over all, 19 ft. 8 ins.; height, 9 ft.; width, 9 ft.; weight, 110,000 lbs.; weight per H.P. developed, 5 lbs.; speed, 1800 r.p.m.

Steam Consumption of Small Steam Turbines. — Small turbines, from 5 to 200 H.P., are extensively used for purposes where high speed of rotation is not an objection, such as for driving electric generators, centrifugal fans, etc., and where economy of fuel is not as important as saving of space, convenience of operation, etc. The steam consumption of these turbines varies as greatly as does that of small high-speed steam-engines, according to the design, speed, etc. A paper by Geo. A. Orrok in *Trans. A. S. M. E.*, 1909, discusses the details of several makes of machines. From a curve presented by R. H. Rice in discussion of this paper the following figures are taken showing the steam consumption in lbs. per B.H.P.-hour of different makes of impulse turbines.

| Type. | Sturtevant. | Terry. | Bliss. | Bliss. | Kerr. | Curtis. | Curtis. |
|--------------------------------------|-------------|--------|--------|--------|-------|---------|---------|
| Rated H.P..... | 20 | 50 | 100 | 200 | 150 | 50 | 200 |
| Water rate at $\frac{1}{2}$ load.... | 72 | 59 | 58 | 55 | 52 | 44 | 32 |
| $\frac{3}{4}$ load.... | 65 | 49 | 48 | 47 | 44 | 36 | 30 |
| Full load.... | 61 | 46 | 43 | 42 | 41 | 33 | 29 |
| $1\frac{1}{4}$ load.... | 58 | 44 | 40 | 39 | 39 | 31 | 28 |

Dry steam, 150 lbs. pressure; atmospheric exhaust.

Mr. Orrok shows that the steam consumption of these turbines largely depends on their peripheral speed. From a set of curves plotted with speed as the base it appears that the steam consumption per B.H.P.-hour ranges about as follows:

| | | | | | |
|---------------------------------------|----------|----------|----------|----------|----------|
| Peripheral speed, ft. per min..... | 5,000 | 10,000 | 15,000 | 20,000 | 25,000 |
| Steam per B.H.P.-hour | 45 to 70 | 38 to 60 | 31 to 52 | 29 to 45 | 29 to 40 |

Low-Pressure Steam Turbines.—Turbines designed to utilize the exhaust steam from reciprocating engines are used to some extent. For steam at or below atmospheric pressure the turbine has a great advantage over reciprocating engines in its ability to expand the steam down to the vacuum pressure, while a reciprocating condensing engine generally does not expand below 8 or 10 lbs. absolute pressure. In order to expand to lower pressures the low-pressure cylinder would have to be inordinately large, and therefore costly, and the increased loss from cylinder condensation and radiation would more than counterbalance the gain due to greater expansion.

Mr. Parsons (*Proc. Inst. Nav. Arch.*, 1908) gives the following figures showing that the theoretical economy of the combination of a reciprocating engine and an exhaust steam turbine is about the same whether the turbine receives its steam at atmospheric pressure or at 7 lbs. absolute, the initial steam pressure in the engine being 200 lbs. absolute and the vacuum 28 ins.

| | | | |
|---|-----|-----|-----|
| Back pressure of engines, lbs. abs. | 16 | 13½ | 8 |
| Initial pressure, turbine, lbs. abs. | 15 | 12½ | 7 |
| Theoretical B.T.U. { in engine..... | 178 | 189 | 218 |
| utilized per lb. of steam { in turbine..... | 142 | 131 | 100 |
| { total..... | 320 | 320 | 318 |

The following figures, by the General Electric Co., show the percentage over the output of a condensing reciprocating engine that may be made by installing a low-pressure turbine between the engine and the condenser, the vacuum being 28½ ins.

| | | | | | | | |
|--|------|------|------|------|------|------|----|
| Inches vacuum at admission valve..... | 0 | 4 | 8 | 12 | 16 | 20 | 24 |
| Per cent. of work gained ... | 26.1 | 26.5 | 26.8 | 26.3 | 25.3 | 23.6 | 20 |

It appears that a well-designed reciprocating compound engine working down to about atmospheric pressure is a more efficient machine than a turbine with the same terminal pressure, and that between the atmosphere and the condenser pressure the turbine is far more economical; therefore a combination of an engine and a turbine can be designed which will give higher economy than either an engine or a turbine working through the whole range of pressure.

When engines are run intermittently, such as rolling-mill and hoisting engines, their exhaust steam may be made to run low-pressure turbines by passing it first into a heat accumulator, or thermal storage system, where it gives up its heat to water, the latter furnishing steam continuously to the turbines. (See Thermal Storage, pages 897 and 987.)

The following results of tests of a Westinghouse low-pressure turbine are reported by Francis Hodgkinson.

| | | | | | | | | | |
|---------------|------|------|------|------|------|------|------|------|------|
| Steam press., | | | | | | | | | |
| lb. abs.... | 17.4 | 12.4 | 11.8 | 7.7 | 5.2 | 11.6 | 8.7 | 6.1 | 4.5 |
| Vacuum, ins. | 26.0 | 26.0 | 27.0 | 27.0 | 27.0 | 27.8 | 28.0 | 27.9 | 28.0 |
| Brake H.P.. | 920 | 472 | 592 | 321 | 102 | 586 | 458 | 234 | 114 |
| Steam per | | | | | | | | | |
| B.H.P.-hr., | | | | | | | | | |
| lbs..... | 27.9 | 37.1 | 29.9 | 37.3 | 64.4 | 28.0 | 30.4 | 38.6 | 54.8 |

Tests of a 1000-K.W. low-pressure double-flow Westinghouse turbine are reported to have given results as follows. (Approximate figures, from a curve.)

| | | | | | | | | | |
|-------------------------|--------------|-----|------|-----|------|------|------|------|------|
| Load, Brake H.P..... | 200 | 400 | 600 | 800 | 1000 | 1200 | 1500 | 2000 | |
| Pressure at inlet, lbs. | | | | | | | | | |
| abs..... | 4.1 | 5.1 | 6.1 | 7.2 | 8.3 | 9.4 | 11.0 | 13.5 | |
| Steam per | | | | | | | | | |
| B.H.P.- | | | | | | | | | |
| hour, lbs. } | 27½ in. vac. | 75 | 47.5 | 38 | 33 | 30 | 28 | 26.5 | 24.5 |
| | 28 in. vac. | 62 | 42 | 33 | 29 | 27 | 25.5 | 24.5 | 22.5 |

The total steam consumption per hour followed the Willans law, being directly proportional to the power after adding a constant for 0 load, viz.: for 27½-in. vacuum the total steam consumption per hour was 12,000 lbs. + 18 × H.P., and for 28-in. vacuum, 9000 lbs. + 18 × H.P. (approx.).

The guaranteed steam consumption of a 7000-K.W. Rateau-Smoot low-pressure turbine generator is given in a curve by R. C. Smoot (*Power*, June 22, 1909), from which the following figures are taken. The admission pressure is taken at 16 lbs. absolute and the vacuum 28½ ins.

| | | | | | | | |
|-----------------------------|------|------|------|------|------|------|------|
| K.W. output..... | 1500 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 |
| Steam per K.W.-hr., lb.... | 40 | 37 | 32.5 | 29.5 | 27.6 | 26.2 | 25.7 |
| Over-all efficiency, %..... | 43 | 47 | 54 | 60 | 65 | 68 | 70 |

The performance of a combined plant of several reciprocating 2000-K.W. engines and a 7000-K.W. low-pressure turbine is estimated as follows, the engines expanding the steam from 215 to 16 lbs. absolute, and the turbines from 16 lbs. to 0.75 lb., the vacuum being 28.5 ins. with the barometer at 30 ins.

| | Engine. | Turbine. |
|---|-----------|----------|
| Theoretical steam per K.W.-hour, lbs..... | 18 | 17.8 |
| Steam per K.W.-hr. at switchboard, lbs..... | 27.7 | 26.6 |
| Combined efficiency of engine and dynamo, per cent.... | 65 | 67 |
| Steam per K.W.-hour for combined plant = $1 \div (1/27.7 + 1/26.6) =$ | 13.6 lbs. | |

The combined efficiency is 66%, representing the ratio of the energy at the switchboard to the available energy of the steam delivered to the engine and expanded down to the condenser pressure, after allowing for all losses in engine, turbine, and dynamo.

Very little difference is made in the plant efficiency if the intermediate pressure is taken anywhere from 3 or 4 lbs. below atmosphere to 15 or 20 lbs. above.

M. B. Carroll (*Gen. Elec. Rev.*, 1909) gives an estimate of the steam consumption of a combined unit of a 1000-K.W. engine and a low-pressure turbine. The engine, non-condensing, will develop 1000 H.P., with 32,000 lbs. of steam per hour. Allowing 8% for moisture in the exhaust, 29,440 lbs. of dry steam will be available for the turbine, which at 33 lbs. per K.W.-hour will develop 893 K.W., making a total output of 1893 K.W. for 32,000 lbs. steam, or 16.9 lbs. per K.W.-hour. The engine alone as a condensing engine will develop 1320 K.W. at 24.2 lbs. per K.W.-hour. The combined unit therefore develops 573 K.W., or 43.5% more than the condensing engine using the same amount of steam. The maximum capacity of the engine, non-condensing, is 1265 K.W., and condensing, 1470 K.W., and of the combined unit 2500 K.W.

Tests of a 15,000 K.W. Steam-Engine-Turbine Unit are reported by H. G. Stott and R. J. S. Pigott in *Jour. A.S.M.E.*, Mar., 1910. The steam-engine is one of the 7500 K.W. Manhattan type engines at the 59th St. station of the Rapid Transit Co., New York, with two 42-in. horizontal h.p. and two 86-in. vertical l.p. cylinders, and the turbine, also 7500 K.W., is of the vertical three-stage impulse type. The principal results are summarized as follows: An increase of 100% in the maximum capacity and 146% in the economical capacity of the plant; a saving of about 85% of the condensed steam for return to the boilers [it was previously wasted]; an average improvement in economy of 13% over the best high-pressure turbine results, and of 2.5% (between 7500 and 15,000 K.W.) over the results obtained by the engine alone; an average thermal efficiency between 6500 and 15,500 K.W. of 20.6%. [This efficiency is not quite equal to that reached by triple-expansion pumping engines. See page 774.]

Reduction Gear for Steam Turbines.—Double spiral reduction gears, usually of a ratio of 1 to 10, are used with the DeLaval turbine to obtain a velocity of rotation suitable for dynamos, centrifugal pumps, etc. G. W. Melville and J. H. McAlpine have designed a similar gear, with the pinion carried in a floating frame supported at a single point between the bearings to equalize the strain on the gear teeth, for reducing the speed of large horizontal turbines to suitable speeds for marine propellers. A 6000 H.P. gear with reduction from 1500 to 300 r.p.m. has been tested, giving an efficiency of 98.5% (*Eng'g*, Sept. 17; *Eng. News*, Oct. 21 and Dec. 30, 1909).

NAPHTHA ENGINES. — HOT-AIR ENGINES.

Naphtha engines are in use to some extent in small yachts and launches. The naphtha is vaporized in a boiler, and the vapor is used expansively in the engine cylinder, as steam is used; it is then condensed and returned to the boiler. A portion of the naphtha vapor is used for fuel under the boiler. According to the circular of the builders, the Gas Engine and Power Co. of New York, a 2-H.P. engine requires from 3 to 4 quarts of naphtha per hour, and a 4-H.P. engine from 4 to 6 quarts. The chief advantages of the naphtha-engine and boiler for launches are the saving of weight and the quickness of operation. A 2-H.P. engine weighs 200 lbs., a 4-H.P. 300 lbs. It takes only about two minutes to get under headway. (*Modern Mechanism*, p. 270.)

Hot-air (or Caloric) Engines.—Hot-air engines are used to some extent, but their bulk is enormous compared with their effective power. For an account of the largest hot-air engine ever built (a total failure) see Church's Life of Ericsson. For theoretical investigation, see Rankin's Steam-engine and Roentgen's Thermodynamics. For description of constructions, see Appleton's Cyc. of Mechanics and Modern Mechanism, and Babcock on Substitutes for Steam, *Trans. A. S. M. E.*, vii, p. 693.

Test of a Hot-air Engine (Robinson).—A vertical double-cylinder (Caloric Engine Co.'s) 12 nominal H.P. engine gave 20.19 I.H.P. in the working cylinder and 11.38 I.H.P. in the pump, leaving 8.81 net I.H.P.; while the effective brake H.P. was 5.9, giving a mechanical efficiency of 67%. Consumption of coke, 3.7 lbs. per brake H.P. per hour. Mean pressure on pistons 15.37 lbs. per square inch, and in pumps 15.9 lbs., the area of working cylinders being twice that of the pumps. The hot air supplied was about 1160° F. and that rejected at end of stroke about 890° F.

INTERNAL-COMBUSTION ENGINES.

References.—For theory of the internal-combustion engine, see paper by Dugald Clerk, *Proc. Inst. C. E.*, 1882, vol. lxi; and Van Nostrand's Science Series, No. 62. See also Wood's Thermodynamics. Standard works on gas-engines are "A Text-book on Gas, Air, and Oil Engines," by Bryan Donkin; "The Gas and Oil Engine," by Dugald Clerk; "Internal Combustion Engines," by Carpenter and Diederichs; "Gas Engine Design," by C. E. Lucke; "Gas and Petroleum Engines," by W. Robinson; "The Modern Gas Engine and the Gas Producer," by A. M. Levin. For practical operation of gas and oil engines, see "The Gas Engine," by F. R. Jones, and "The Gas Engine Handbook," by E. W. Roberts.

For descriptions of large gas-engines using blast furnace gas see papers in *Proc. Iron and Steel Inst.*, 1906, and *Trans. A. I. M. E.*, 1906. Many papers on gas-engines are in *Trans. A.S.M.E.*, 1905 to 1909.

An Internal-combustion Engine is an engine in which combustible gas, vapor, or oil is burned in a cylinder, generating a high temperature and high pressure in the gases of combustion, which expand behind a piston, driving it forward. (Rotary gas-engines or gas turbines, are still, 1910, in the experimental stage.)

Four-cycle and Two-cycle Gas-Engines.—In the ordinary type of single-cylinder gas-engine (for example the Otto) known as a four-cycle engine, one ignition of gas takes place in one end of the cylinder every two revolutions of the fly-wheel, or every two double strokes. The following sequence of operations takes place during four consecutive strokes: (a) inspiration of a mixture of gas and air during an entire stroke; (b) compression during the second (return) stroke; (c) ignition at or near the dead-point, and expansion during the third stroke; (d) expulsion of the burned gas during the fourth (return) stroke. Beau de Rochas in 1862 laid down the law that there are four conditions necessary to realize the best results from the elastic force of gas: (1) The cylinders should have the greatest capacity with the smallest circumferential surface; (2) the speed should be as high as possible; (3) the cut-off should be as early as possible; (4) the initial pressure should be as high as possible.

(Strictly speaking four-cycle should be called four-stroke-cycle, but the term four-cycle is generally used in the trade.)

The two great sources of waste in gas-engines are: 1. The high temperature of the rejected products of combustion; 2. Loss of heat through the cylinder walls to the water-jacket. As the temperature of the water-jacket is increased the efficiency of the engine becomes higher.

Fig. 169 is an indicator diagram of a four-cycle gas-engine. *AB*, the lower line, shows the admission of the mixture, at a pressure slightly

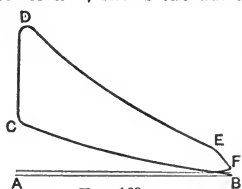


FIG. 169.

below the atmosphere on account of the resistance of the inlet valve, *BC* is the compression into the clearance space, ignition taking place at *C* and combustion with increase of pressure continuing from *C* to *D*. The gradual termination of the combustion is shown by the rounded corner at *D*. *DE* is the expansion line, *EF* the line of pressure drop as the exhaust valve opens, and *FA* the line of expulsion of the burned gases, the pressure being slightly above the atmosphere on account of the resistance of the exhaust valve.

In a two-cycle single-acting engine an explosion takes place with every revolution, or with each forward stroke of the piston. Referring to the diagram Fig. 169 and beginning at *E*, when the exhaust port begins to open to allow the burned gases to escape, the pressure drops rapidly to *F*. Before the end of the stroke is reached an inlet port opens, admitting a mixture of gas and air from a reservoir in which it has been compressed. This mixture being under pressure assists in driving the burned gases out through the exhaust port. The inlet port and the exhaust port close early in the return stroke, and during the remainder of the stroke *BC* the mixture, which may include some of the burned gas, is compressed and the ignition takes place at *C*, as in the four-cycle engine.

In one form of the two-cycle engine only compressed air is admitted while the exhaust port is open, the fuel gas being admitted under pressure after the exhaust port is closed. By this means a greater proportion of the burned gases are swept out of the cylinder. This operation is known as "scavenging."

Theoretical Pressures and Temperatures in Gas-Engines.—Referring to Fig. 169, let P_s be the absolute pressure at *B*, the end of the suction stroke, P_c the pressure at *C*, the end of the compression stroke; P_x the maximum pressure at *D*, when the gases of combustion are at their highest temperature; P_e the pressure at *E*, when the exhaust valve begins to open. For the hypothetical case of a cylinder with walls incapable of absorbing or conducting heat, and of perfect and instantaneous combustion

or explosion of the fuel, an ideal diagram might be constructed which would have the following characteristics. In a four-cycle engine receiving a charge of air and gas at atmospheric pressure and temperature, the pressure at B , or P_s , would be 14.7 lbs. per sq. in. absolute, and the temperature say 62°F. , or $522^\circ \text{absolute}$. The pressure at C , or P_c , would depend on the ratio $V_1 \div V_2$, V_1 being the original volume of the mixture in the cylinder before compression, or the piston displacement plus the volume of the clearance space, and V_2 the volume after compression, or the clearance volume, and its value would be $P_c = P_s (V_1/V_2)^n$. The absolute temperature at the end of compression would be $T_c = 522 \times (V_1/V_2)^{n-1}$, or it may be found from the formula $P_s V_s \div T_s = P_c V_c \div T_c$, the subscripts s and c referring respectively to conditions at the beginning and end of compression. The compression would be adiabatic; and the value of the exponent n would be about the value for air, or 1.406. The work done in compressing the mixture would be calculated by the formula for compressed air (see page 607). The theoretical rise of temperature at the end of the explosion, T_x , above the temperature at the end of the compression T_c may be found from the formula $(T_x - T_c) C_v = H$, in which H is the amount of heat in British thermal units generated by the combustion of the fuel in 1 lb. of the mixture, and C_v the mean specific heat, at constant volume, of the gases of combustion between the temperatures T_x and T_c . Having obtained the temperature, the correspond-

ing pressure P_x may be found from the formula $P_x = P_c \times (T_x/T_c)^{\frac{n}{n-1}}$. In like manner the pressure and temperature at the end of expansion, P_e and T_e , and the work done during expansion, may be calculated by the formula for adiabatic expansion of air.

The ideal diagram of the adiabatic compression of air, instantaneous heating, and adiabatic expansion, differs greatly from the actual diagram of a gas-engine, and the pressures, temperatures, and amount of work done are different from those obtained by the method described above. In the first place the mixture at the beginning of the compression stroke is usually below atmospheric pressure, on account of the resistance of the inlet valve, in a four-cycle engine, but may be above atmospheric pressure in a two-cycle engine, in which the mixture is delivered from a receiver under pressure. Then the temperature is much higher than that of the atmosphere, since it is heated by the walls of the cylinder as it enters. The compression is not adiabatic, since heat is received from the walls during the first part of the stroke. If the clearance space is small and the pressure and temperature at the end of compression therefore high, the gas may give up some heat to the walls during the latter part of the stroke. The explosion is not instantaneous, and during its continuance heat is absorbed by the cylinder walls, and therefore neither the temperature nor the pressure found by calculation will be actually reached. Poole states that the rise in temperature produced by combustion is from 0.4 to 0.7 of what it would be with instantaneous combustion and no heat loss to the cylinder walls. Finally the expansion is not adiabatic, as the gases of combustion, at least during the first part of the expanding stroke, are giving up heat to the cylinder.

Calculation of the Power of Gas-Engines.—If the mean effective pressure in a gas-engine cylinder be obtained from an indicator diagram, its power is found by the usual formula for steam-engines, $\text{H.P.} = PLAN \div 33,000$, in which P is the mean effective pressure in lbs. per sq. in., L the length of stroke in feet, A the area of the piston in square inches, and N the number of explosion strokes per minute.

For purposes of design, however, the mean effective pressure either has to be assumed from a knowledge of that found in other engines of the same type and working under the same conditions as those of the design, or it may be calculated from the ideal air diagram and modified by the use of a coefficient or diagram factor depending on the kind of fuel used and the compression pressure. Lucke gives the following

factors for four-cycle engines by which the mean effective pressure of a theoretical air diagram is to be multiplied to obtain the actual M.E.P. for the several conditions named.

| Kind of Fuel and Method of Use. | Compression. Gauge Pressure. | Factor. Per Cent. |
|--|---------------------------------|----------------------|
| Kerosene, when previously vaporized | Lb. 45-75 | 30-40 |
| Kerosene, injected on a hot bulb, may be as low as | | 20 |
| Casoline, used in carburetor requiring a vacuum | | 25-40 |
| Gasoline, with but little initial vacuum | 80-130 | 50-30 |
| Producer gas | 100-160 | 56-40 |
| Coal gas | Av. 80 | Av. 45 |
| Blast-furnace gas | 130-180 | 48-30 |
| Natural gas | 90-140 | 52-40 |

Factors for two-cycle engines are about 0.8 those for four-cycle engines.

Pressures and Temperatures at end of Compression and at Release.—The following tables, greatly condensed from very full tables given by C. P. Poole, show approximately the pressures and temperatures that may be realized in practice under different conditions. Poole says that the value of n , the exponent in the formula for compression, ranges from 1.2 to 1.38, these being extreme cases; the values most commonly obtained are from 1.28 to 1.35. The tables for compression pressures and temperatures are based on $n = 1.3$ and 1.4, on compression ratios or V_1/V_2 from 3 to 8, on absolute pressures in the cylinder before compression from 13 to 16 lbs., and on absolute temperatures before compression of 620° to 780° (160° to 320° F.). The release pressures and temperatures are based on values of n of 1.29 and 1.32, absolute pressures at the end of the explosion from 240 to 360 lbs. per sq. in., and absolute temperatures at the end of the explosion of 1800° to 3000° F.

COMPRESSION PRESSURES.

| Compression Ratio r_c | $n = 1.3.$ | | | | | Compression Ratio r_c | $n = 1.34.$ | | | | |
|----------------------------|------------|-------|-------|-------|-------|----------------------------|-------------|-------|-------|-------|-------|
| | $P_s = 13$ | 13.5 | 14 | 15 | 16 | | $P_s = 13$ | 13.5 | 14 | 15 | 16 |
| 3.00 | 54.2 | 56.3 | 58.4 | 62.6 | 66.7 | 3.00 | 56.7 | 58.9 | 61.0 | 65.4 | 69.7 |
| 4.00 | 78.8 | 81.9 | 84.9 | 90.9 | 97.0 | 4.00 | 83.3 | 86.5 | 89.7 | 96.1 | 102.5 |
| 5.00 | 105.4 | 109.4 | 113.5 | 121.6 | 129.7 | 5.00 | 112.3 | 116.7 | 121.0 | 129.6 | 138.3 |
| 6.00 | 133.5 | 138.7 | 143.8 | 154.1 | 164.3 | 6.00 | 143.4 | 148.9 | 154.5 | 165.5 | 176.5 |
| 7.00 | 163.2 | 169.4 | 175.7 | 188.3 | 200.8 | 7.00 | 176.3 | 183.1 | 189.9 | 203.5 | 217.0 |
| 8.00 | 194.0 | 201.5 | 209.0 | 223.9 | 238.7 | 8.00 | 210.9 | 219.0 | 227.1 | 243.4 | 259.6 |

COMPRESSION TEMPERATURES.

| Compression Ratio r_c | $n = 1.3.$ | | | | | Compression Ratio r_c | $n = 1.34.$ | | | | |
|----------------------------|-------------------|------|------|------|------|----------------------------|-------------------|------|------|------|------|
| | $T_s = 620^\circ$ | 660° | 700° | 740° | 780° | | $T_s = 620^\circ$ | 660° | 700° | 740° | 780° |
| 3.00 | 862 | 918 | 973 | 1029 | 1084 | 3.00 | 901 | 959 | 1017 | 1075 | 1132 |
| 4.00 | 940 | 1000 | 1061 | 1122 | 1182 | 4.00 | 993 | 1057 | 1122 | 1186 | 1250 |
| 5.00 | 1005 | 1070 | 1134 | 1199 | 1264 | 5.00 | 1072 | 1141 | 1210 | 1279 | 1348 |
| 6.00 | 1061 | 1130 | 1198 | 1267 | 1335 | 6.00 | 1140 | 1214 | 1287 | 1361 | 1434 |
| 7.00 | 1112 | 1183 | 1255 | 1327 | 1398 | 7.00 | 1201 | 1279 | 1357 | 1434 | 1512 |
| 8.00 | 1157 | 1232 | 1306 | 1381 | 1456 | 8.00 | 1257 | 1338 | 1420 | 1501 | 1582 |

ABSOLUTE PRESSURES PER SQUARE INCH AT RELEASE.

Corresponding to Explosion Pressures commonly obtained.

NOTE:—The expansion ratios in the left-hand column are based on the volume behind the piston when the exhaust valve begins to open.

| Expansion Ratio r_e . | $n_e = 1.29.$ | | | | | Expansion Ratio r_e . | $n_e = 1.32.$ | | | | |
|----------------------------|----------------|------|------|------|------|----------------------------|----------------|------|------|------|------|
| | Value of P_x | | | | | | Value of P_x | | | | |
| | 240 | 270 | 300 | 330 | 360 | | 240 | 270 | 300 | 330 | 360 |
| 3.00 | 58.2 | 65.4 | 72.7 | 80.0 | 87.2 | 3.00 | 56.3 | 63.3 | 70.4 | 77.4 | 84.4 |
| 4.00 | 40.1 | 45.2 | 50.2 | 55.2 | 60.2 | 4.00 | 38.5 | 43.3 | 48.1 | 52.9 | 57.8 |
| 5.00 | 30.1 | 33.9 | 37.6 | 41.4 | 45.1 | 5.00 | 28.7 | 32.3 | 35.8 | 39.4 | 43.0 |
| 6.00 | 23.8 | 26.8 | 29.7 | 32.7 | 35.7 | 6.00 | 22.5 | 25.4 | 28.2 | 31.0 | 33.8 |
| 7.00 | 19.5 | 21.9 | 24.4 | 26.8 | 29.2 | 7.00 | 18.4 | 20.7 | 23.0 | 25.3 | 27.6 |
| 8.00 | 16.4 | 18.5 | 20.5 | 22.6 | 24.6 | 8.00 | 15.4 | 17.3 | 19.3 | 21.2 | 23.1 |

ABSOLUTE TEMPERATURES AT RELEASE.

Corresponding to Explosion Temperatures commonly obtained.

| Expansion Ratio r_e | $n_e=1.29.$ | | | | | Expansion Ratio r_e | $n_e=1.32.$ | | | | |
|--------------------------|----------------|------|------|------|------|--------------------------|----------------|------|------|------|------|
| | Value of T_x | | | | | | Value of T_x | | | | |
| | 1800 | 2100 | 2400 | 2700 | 3000 | | 1800 | 2100 | 2400 | 2700 | 3000 |
| 3.00 | 1309 | 1527 | 1745 | 1963 | 2182 | 3.00 | 1266 | 1478 | 1689 | 1900 | 2111 |
| 4.00 | 1204 | 1405 | 1606 | 1806 | 2007 | 4.00 | 1155 | 1348 | 1540 | 1733 | 1925 |
| 5.00 | 1129 | 1317 | 1505 | 1693 | 1881 | 5.00 | 1075 | 1255 | 1434 | 1613 | 1792 |
| 6.00 | 1070 | 1249 | 1427 | 1606 | 1784 | 6.00 | 1015 | 1184 | 1353 | 1522 | 1691 |
| 7.00 | 1024 | 1194 | 1365 | 1536 | 1706 | 7.00 | 966 | 1127 | 1288 | 1449 | 1610 |
| 8.00 | 985 | 1149 | 1313 | 1477 | 1641 | 8.00 | 925 | 1079 | 1234 | 1388 | 1542 |

Pressures and Temperatures after Combustion.—According to Poole, the maximum temperature after combustion may be as high as 3000° absolute, F., and the maximum pressure as high as 400 lbs. per sq. in. absolute; these are high figures, however, the more usual figures being about 2300° and 250 lbs. Poole gives the following figures for the average rise in pressure, above the pressure at the end of compression, produced by combustion of different fuels, with different ratios of compression.

AVERAGE PRESSURE RISE IN LBS. PER SQ. IN. PRODUCED BY COMBUSTION.

| Comp. Ratio. | Illum. Gas 650 B.T.U.* | Gasoline. | Kerosene. | Comp. Ratio. | Natural Gas 1000 B.T.U.* | Comp. Ratio. | Producer Gas 150 B.T.U.* | Comp. Ratio. | Blast-Furnace Gas 100 B.T.U.* |
|--------------|---------------------------|-----------|-----------|--------------|-----------------------------|--------------|-----------------------------|--------------|-------------------------------------|
| 4.0 | 146 | 195 | 168 | 5.0 | 192 | 6.0 | 225 | 7.0 | 211 |
| 4.2 | 156 | 208 | 179 | 5.2 | 202 | 6.2 | 234 | 7.2 | 218 |
| 4.4 | 166 | 221 | 190 | 5.4 | 211 | 6.4 | 243 | 7.4 | 225 |
| 4.6 | 175 | 234 | 202 | 5.6 | 221 | 6.6 | 252 | 7.6 | 232 |
| 4.8 | 185 | 247 | 213 | 5.8 | 230 | 6.8 | 261 | 7.8 | 239 |
| 5.0 | 195 | 260 | 224 | 6.0 | 240 | 7.0 | 270 | 8.0 | 246 |

* Per cubic foot measured at 32° F.

The following figures are given by Poole as a rough approximate guide to the mean effective pressures in lbs. per sq. in. obtained with

different fuels and different compression pressures in a four-cycle engine. In a two-cycle engine the mean effective pressure of the pump diagram should be subtracted. The delivery pressure is usually from 4 to 8 lbs. per sq. in. above the atmosphere, and the corresponding mean effective pressure of the pump about 3.8 to 7.

PROBABLE MEAN EFFECTIVE PRESSURE.

| SUCTION ANTHRACITE PRODUCER GAS. | | | | | | MOND PRODUCER GAS. | | | | | |
|----------------------------------|--|-----|-----|-----|-----|--------------------|-----------------------|-----|-----|-----|-----|
| Engine H.P. | Compression Pressure, abs. lbs. per sq. in. | | | | | Engine H.P. | Compression Pressure. | | | | |
| | 100 | 115 | 130 | 145 | 160 | | 100 | 115 | 130 | 145 | 160 |
| 10 | 55 | 60 | 65 | ... | ... | 10 | ... | 65 | 65 | 65 | ... |
| 25 | 60 | 65 | 70 | 75 | ... | 25 | 60 | 65 | 65 | 70 | 75 |
| 50 | 65 | 70 | 75 | 80 | 80 | 50 | 65 | 70 | 70 | 75 | 80 |
| 100 | 70 | 75 | 80 | 85 | 85 | 100 | 65 | 70 | 75 | 80 | 85 |
| 250 | 75 | 80 | 85 | 90 | 90 | 250 | 70 | 75 | 80 | 85 | 90 |
| 500 | 80 | 85 | 90 | 90 | 90 | 500 | 75 | 80 | 85 | 90 | 90 |

NATURAL AND ILLUMINATING GASES.

| Engine H.P. | Compression Pressure. | | | | | Engine H.P. | Compression Pressures. | | | | |
|----------------|-----------------------|----|----|-----|-----|----------------|------------------------|----|-----|-----|-----|
| | 65 | 75 | 85 | 100 | 115 | | 75 | 85 | 100 | 115 | 130 |
| 10 | 60 | 65 | 70 | 75 | ... | 100 | 80 | 85 | 90 | 95 | 100 |
| 25 | 65 | 70 | 75 | 80 | 85 | 250 | 85 | 90 | 95 | 100 | 105 |
| 50 | 70 | 75 | 80 | 90 | 90 | 500 | ... | 95 | 100 | 105 | 110 |

KEROSENE SPRAY.

| Engine H.P. | Compression Pressures. | | | | | Engine H.P. | Compression Pressures. | | | | |
|----------------|------------------------|----|----|-----|-----|----------------|------------------------|----|----|-----|-----|
| | 65 | 75 | 85 | 100 | 115 | | 65 | 75 | 85 | 100 | ... |
| 5 | 50 | 55 | 60 | 65 | 70 | 5 | 70 | 75 | 80 | 85 | ... |
| 10 | 55 | 60 | 65 | 70 | 75 | 10 | 75 | 80 | 85 | 90 | ... |
| 25 | 60 | 65 | 70 | 75 | 80 | 25 | 80 | 85 | 90 | 90 | ... |
| 50 | 65 | 70 | 75 | 80 | 85 | 50 | 85 | 90 | 95 | 95 | ... |

GASOLINE VAPOR.

Sizes of Large Gas Engines. — From a table of sizes of the Nurnberg gas engine, as built by the Allis-Chalmers Co., the following figures are taken. These figures relate to two-cylinder tandem double-acting engines.

| | | | | | | | | | | |
|--------------------------------|------|------|------|------|------|------|------|------|------|------|
| Diam. cyl., ins..... | 18 | 20 | 21 | 22 | 24 | 24 | 26 | 28 | 30 | 32 |
| Stroke cyl., ins..... | 24 | 24 | 30 | 30 | 30 | 36 | 36 | 36 | 42 | 42 |
| Revs. per min..... | 150 | 150 | 125 | 125 | 125 | 115 | 115 | 115 | 100 | 100 |
| Piston speed, ft. per min..... | 600 | 600 | 625 | 625 | 625 | 690 | 690 | 690 | 700 | 700 |
| Rated B.H.P..... | 260 | 320 | 370 | 405 | 490 | 545 | 630 | 740 | 855 | 985 |
| Factor of C..... | 0.8 | 0.8 | 0.84 | 0.84 | 0.85 | 0.95 | 0.93 | 0.94 | 0.95 | 0.96 |
| Diam., ins..... | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 |
| Stroke, ins..... | 42 | 48 | 48 | 48 | 54 | 54 | 54 | 60 | 60 | 62 |
| Revs. per min..... | 100 | 92 | 92 | 92 | 86 | 86 | 86 | 78 | 78 | 78 |
| Piston speed..... | 700 | 736 | 736 | 736 | 774 | 774 | 774 | 780 | 780 | 780 |
| Rated B.H.P..... | 1105 | 1300 | 1460 | 1630 | 1875 | 2080 | 2280 | 2475 | 2720 | 2950 |
| Factor of C..... | 0.96 | 1 | 1.01 | 1.02 | 1.06 | 1.07 | 1.08 | 1.07 | 1.09 | 1.09 |

The figures "factor C " are the values of C in the equation $B.H.P. = C \times D^2$, in which D = diam. of cylinder in ins. For twin-cylinder double-acting engines, multiply the B.H.P. and the value of C by 0.95; for twin-tandem double-acting engines, multiply by 2; for two-cylinder single-acting, or for single-cylinder double-acting engines, divide by 2; for single-acting single-cylinders, divide by 4. The figures for B.H.P. correspond to mean effective pressures of about 66, 68, and 70 lbs. per sq. in. for 20, 40, and 50 in. cylinders respectively if we assume 0.85 as the mechanical efficiency, or the ratio $B.H.P. \div I.H.P.$

Engine Constants for Gas Engines. — The following constants for figuring the brake H.P. of gas engines are given in *Power*, Dec. 7, 1909. They refer to four-stroke cycle single-cylinder engines, single acting; for double-acting engines multiply by 2. Producer gas, 0.000056. Illuminating gas, 0.000065. Natural gas, 0.00007. Constant \times diam.² \times stroke in ins. \times revs. per min. = probable B.H.P. A deduction should be made for the space occupied by the piston rods, about 5% for small engines up to 10% for very large engines.

Rated Capacity of Automobile Engines. — The standard formula for the American Licensed Automobile Manufacturers Association (called the A. L. A. M. formula) for approximate rating of gasoline engines used in automobiles is $Brake\ H.P. = Diam.^2 \times No.\ of\ cylinders \div 2.5$. It is based on an assumed piston speed of 1000 ft. per min. The following ratings are derived from the formula:

| Bore, ins | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 | 5 | 5 1/2 | 6 |
|-------------------|-------|------|-------|------|-------|-----|-------|------|
| Bore, mm | 64 | 76 | 89 | 102 | 114 | 127 | 140 | 154 |
| H.P., 1 cylinder | 2 1/2 | 3.6 | 4.9 | 6.4 | 8.1 | 10 | 12.1 | 14.4 |
| H.P., 2 cylinders | 5 | 7.2 | 9.8 | 12.8 | 16.2 | 20 | 24.2 | 28.8 |
| H.P., 4 " " | 10 | 14.4 | 19.6 | 25.6 | 32.4 | 40 | 48.4 | 57.6 |
| H.P., 6 " " | 15 | 21.6 | 29.4 | 38.4 | 48.6 | 60 | 72.6 | 86.4 |

Approximate Estimate of the Horse-power of a Gas Engine. — From the formula $I.H.P. = PLAN \div 33,000$, in which P = mean effective pressure in lbs. per sq. in., L = length of stroke in ft., A = area of piston in sq. ins., N = No. of explosion strokes per min., we have $I.H.P. = Pd^2S \div 42,017$, in which d = diam. of piston, and S = piston speed in ft. per min., for an engine in which there are two explosion strokes in each revolution, as in a 4-cycle double-acting, 2-cylinder engine, or a 2-cycle, 2-cylinder, single-acting engine. If the mechanical efficiency is taken at 0.84, then the brake horse power $B.H.P. = Pd^2S \div 50,000$. Under average conditions the product of P and S is in the neighborhood of 50,000, and in that case $B.H.P. = d^2$.

Generally, $B.H.P. = C \times d^2$, in which C is a coefficient having values as below:

| M.E.P. lbs. per sq. in. | Piston speed, ft. per minute. | | | | | |
|-------------------------------|---|------|------|------|------|------|
| | 500 | 600 | 700 | 800 | 900 | 1000 |
| | Value of C for two explosions per revolution. | | | | | |
| 50 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 |
| 60 | 0.60 | 0.72 | 0.84 | 0.96 | 1.08 | 1.20 |
| 70 | 0.70 | 0.84 | 0.98 | 1.12 | 1.26 | 1.40 |
| 80 | 0.80 | 0.96 | 1.12 | 1.28 | 1.44 | 1.60 |
| 90 | 0.90 | 1.08 | 1.26 | 1.44 | 1.62 | 1.80 |
| 100 | 1.00 | 1.20 | 1.40 | 1.60 | 1.80 | 2.00 |
| 110 | 1.10 | 1.32 | 1.54 | 1.76 | 1.98 | 2.20 |

These values of C apply to 4-cylinders, 4-cycle, single-acting, to 2-cyl., 2-cycle, single-acting, and to 1-cyl., 2-cycle double-acting. For single cylinders, 4-cycle, single-acting, divide by 4; for single cylinders, 4-cycle, double-acting, or 2-cycle, single acting, divide by 2.

Oil and Gasoline Engines. — The lighter distillates of petroleum, such as gasoline, are easily vaporized at moderate temperatures, and a gasoline engine differs from a gas-engine only in having an atomizer attached, for spraying a fine jet of the liquid into the air-admission pipe. With kerosene and other heavier distillates, or crude oils, it is necessary to

provide some method of atomizing and vaporizing the oil at a high temperature, such as injecting it into a hot vaporizing chamber at the end of the cylinder, or into a chamber heated by the exhaust gases. In the Diesel oil engine the oil is ignited by the heat of the highly compressed air in the cylinder.

The Diesel Oil Engine.—The distinguishing features of the Diesel engine are: It compresses air only, to a predetermined temperature above the firing point of the fuel. This fuel is blown as a cloud of vapor (by air from a separate small compressor) into the cylinder when compression has been completed, ignites spontaneously without explosion, solely by reason of the heat of the air generated by the compression, and burns steadily with no essential rise in pressure. The temperature of gases, developed and rejected, is much lower than with engines of the explosive type. The engine uses crude oil and residual petroleum products. Guarantees of fuel consumption are made as low as 8 gallons of oil (not heavier than 19° Baumé) for each 100 brake H.P. hour at any load between half and full rated load.

American Diesel engines are built for stationary purposes, in sizes of 120, 170, and 225 H.P. in three cylinders, and in "double units" (six cylinders) of 240, 340 and 450 H.P. See catalogue of the American Diesel Engine Co., St. Louis, 1909.

Much larger sizes have been built in Europe, where they are also built for marine purposes, including submarines in the French and other navies. For the theory of the Diesel engine see a lecture by Rudolph Diesel, in *Zeit. des Ver. Deutscher Ing.*, 1897, trans. in *Progressive Age*, Dec. 1 and 15, 1897, and paper by E. D. Meier in *Jour. Frank. Inst.*, Oct. 1898.

The De La Vergne Oil Engine is described in *Eng. News*, Jan. 13, 1910. It is a four-cycle engine. After the charge of air is compressed to about 200 lbs. per sq. in., the charge of oil is injected, by a jet of air at about 600 lbs. per sq. in., into a vaporizing bulb at the end of the cylinder. Ignition of the oil is caused by the high temperature in this bulb. Average results of tests of an engine developing 128 H.P. showed an oil consumption per B.H.P. hour of 0.408 lb. with Solar fuel oil, and 0.484 lb. with California crude oil.

Alcohol Engines. — Bulletin No. 392 of the U.S. Geol. Survey (1909,) on Comparisons of Gasolene and Alcohol Tests in Internal Combustion Engines, by R. M. Strong, contains the following conclusions:

The "low" heat value of completely denatured alcohol will average 10,500 B.T.U. per lb., or 71,900 B.T.U. per gallon. The low heat value of 0.71 to 0.73 sp. gr. gasolene will average 19,200 B.T.U. per lb.; or 115,800 B.T.U. per gallon.

A gasolene engine having a compression pressure of 70 lbs. but otherwise as well suited to the economical use of denatured alcohol as gasolene, will, when using alcohol, deliver about 10% greater maximum power than when using gasolene.

When the fuels for which they are designed are used to an equal advantage, the maximum B.H.P. of an alcohol engine having a compression pressure of 180 lbs. is about 30% greater than that of a gasolene engine of the same size and speed having a compression pressure of 70 lbs.

Alcohol diluted with water in any proportion, from denatured alcohol, which contains about 10% water, to mixtures containing about as much water as denatured alcohol, can be used in gasolene and alcohol engines if the engines are properly equipped and adjusted.

When used in an engine having constant compression, the amount of pure alcohol required for any given load increases and the maximum available horse-power of the engine decreases with diminution in the percentage of pure alcohol in the diluted alcohol supplied. The rate of increase and decrease, respectively, however, is such that the use of 80% alcohol instead of 90% has but little effect upon the performance; so that if 80% alcohol can be had for 15% less cost than 90% alcohol and could be sold without tax when denatured, it would be more economical to use the 80% alcohol.

Ignition. — The "hot-tube" method of igniting the compressed mixture of gas and air in the cylinder is practically obsolete, and electric systems are used instead. Of these the "make-and-break" and the "jump-spark" systems are in common use. In the former two insulated contact

pieces are located in the end of the cylinder, and through them an electric current passes while they are in contact. A spark-coil is included in the circuit, and when the circuit is suddenly broken at the proper time for ignition, by mechanism operated from the valve-gear shaft, a spark is made at the contacts, which ignites the gas. In the "jump-spark" system two insulated terminals separated about 0.03 in. apart are located in the cylinder, and the secondary or high-tension current of an induction coil causes a spark to jump across the space between them when the circuit of the primary current is closed by mechanism operated by the engine. In some oil engines the mixture of air and oil vapor is ignited automatically by the temperature generated by compression of the vapor, in a chamber at the end of the cylinder, called the vaporizer, which is not water-jacketed and therefore is kept hot by the repeated ignitions. Before starting the engine the vaporizer is heated by a Bunsen burner or other means.

Timing. — By adjusting the cam or other mechanism operated by the valve-gear shaft for causing ignition, the time at which the ignition takes place, with reference to the end of the compression stroke, can be regulated. The mixture is usually ignited before the end of the stroke, the advance depending upon the inflammability of the mixture and on the speed of the engine. A slow-burning mixture requires to be ignited earlier than a rapid-burning one and a high-speed engine earlier than a slow-speed engine.

Governing. — Two methods of governing the speed of an engine are in common use, the "hit-and-miss" and the throttling methods. In the former the engine receives its usual charge of air and gas only when the engine is running at or below its normal speed; at higher speeds the admission of the charge is suspended until the engine regains its normal speed. One method of accomplishing this is to interpose between the valve-rod and its cam or other operating mechanism, a push-rod, or other piece, the position of which with reference to the end of the valve-rod is controlled by a centrifugal governor so that it hits the valve-rod if the speed is at or below normal and misses it if the speed is above normal. The hit-and-miss method is economical of fuel, but it involves irregularity of speed, making a large and heavy fly-wheel necessary if reasonable uniformity of speed is desired. The throttling method of regulating is similar to that used in throttling steam engines; the quantity of mixture admitted at each charge being varied by varying the position of a butterfly valve in the inlet pipe. Cut-off methods of governing are also used, such as varying the time of closing the admission valve during the suction stroke, or varying the time of admission of the gas alone, or "quality regulation."

Gas and Oil Engine Troubles. — The gas engine is subject to a greater number of troubles than the steam engine on account of its greater mechanical complexity and of the variable quality of its operating fluid. Among the causes of troubles are: the variable composition of the fuel; too much or too little air supply; compression ratio not right for the kind of fuel; ignition timer set too late or too early; pre-ignition; back-firing; electrical and mechanical troubles with the igniting system; carbon deposits in the cylinder and on the igniting contacts. For a very full discussion of these and many other troubles and the remedies for them, see Jones on the Gas-Engine.

Conditions of Maximum Efficiency. — The conditions which appear to give the highest thermal efficiency in gas and oil engines are: 1, high temperature of cooling water in the jackets; 2, high pressure at the end of compression; 3, lean mixture; 4, proper timing of the ignition; 5, maximum load. The higher economy of a lean mixture may be due to the fact that high compressions may be used with such a mixture, while with rich mixtures high compression pressures cannot be used without danger of pre-ignition. The effect of different timing on economy is shown in a test by J. R. Bibbins, reported by Carpenter and Diederichs, of an engine using natural gas of a lower heating value* of 934 B.T.U. per cu. ft., delivering 71 H.P. at 297 revs. per min. The maximum thermal efficiency, 23.3%, was obtained when the timing device was set for igni-

* By "lower heating value" is meant the value computed after subtracting the latent heat of evaporation of 9 lbs. of water per pound of hydrogen contained in the gas. See page 533.

tion 30° in advance of the dead center, while the efficiency with ignition at the center was 19%, and with ignition 55° in advance 17.3%.

Other things being equal, the hotter the walls of the cylinder the less heat is transferred into them from the hot gases, and therefore the higher the efficiency. Cool walls, however, allow of higher compression without pre-ignition, and high compression is a cause of high efficiency. Cool walls also tend to give the engine greater capacity, since with hot walls the fuel mixture expands more on entering the cylinder, reducing the weight of charge admitted in the suction stroke.

Heat Losses in the Gas Engine.—The difference between the thermal efficiency, which is the proportion of heat converted into work in the engine, and 100%, is the loss of heat, which includes the heat carried away in the jacket water, that carried away in the waste gases, and that lost by radiation. The relative amounts of these three losses vary greatly, depending on the size of the engine and on the amount of water used for cooling. Thurston, in *Heat as a Form of Energy*, reports a test in which the heat distribution was as follows: Useful work, 17.3%; jacket water, 52%; exhaust gas, 16%; radiation, 15%. Carpenter and Diederichs quote the following, showing that the distribution of the heat losses varies with the rate of compression and with the speed.

| Ratio of Compression. | R.p.m. | M.E.P. lbs. per sq. in. | Ratio Air to Gas. | Heating Value of Charge, B.T.U. | Work done by 1 B.T.U., Ft.-lbs. | Exhaust Temp. Deg. F. | Heat Distribution, Per Cent. | | |
|-----------------------|--------|-------------------------|-------------------|---------------------------------|---------------------------------|-----------------------|------------------------------|---------------|----------|
| | | | | | | | Work. | Jacket Water. | Exhaust. |
| 2.67 | 187 | 54.3 | 7.11 | 18.5 | 140 | 1022 | 18.0 | 51.2 | 30.8 |
| 2.67 | 247 | 51.5 | 7.35 | 17.4 | 141 | 1137 | 18.1 | 45.6 | 36.3 |
| 4.32 | 187 | 69.3 | 7.43 | 17.0 | 190 | 867 | 24.4 | 53.8 | 21.8 |
| 4.32 | 247 | 65.2 | 7.40 | 16.8 | 184 | 992 | 23.7 | 49.5 | 26.8 |

In the long table of results of tests reported by Carpenter and Diederichs, figures of the distribution of heat show that of the total heat received by the engines the heat lost in the jacket water ranged from 25.0 to 50.4%, and that lost in the exhaust gases from 55 to 23.4%.

In small air-cooled gasoline engines, such as those used in some automobile engines, in which the cylinders are surrounded by thin metal ribs to increase the radiating surface, and air is propelled against them by a fan, the air takes the place of the jacket water, and the total loss of heat is that carried away by the air and by the exhaust gases.

Economical Performance of Gas Engines.—The best performance of a gas engine using producer gas (1909) is about 30% better than the best recorded performance of a triple-expansion steam engine, or about 0.71 lb. coal per I.H.P. hour, as compared with 1.06 lbs. for the steam engine. It is probable that the performance of the combination of a high-pressure reciprocating engine, using superheated steam generated in a well-proportioned boiler supplied with mechanical stokers and an economizer, and a low-pressure steam turbine will ere long reduce the steam engine record to 0.9 lb. per I.H.P. hour. As compared with an ordinary steam engine, however, the gas engine with a good producer is far more economical than the steam engine. Where gas can be obtained cheaply, such as the waste-gas from blast furnaces, or natural gas, the gas-engine can furnish power much more cheaply than it can be obtained from the same gas burned under a boiler to furnish steam to a steam engine.

In tests made for the U. S. Geological Survey at the St. Louis Exhibition, 1904, of a 235-H.P. gas engine with different coals, made into gas in the same producer, the best result obtained was 1.12 lbs. of West Virginia coal per B.H.P. hour, and the poorest result 3.23 lbs. per B.H.P. hour, with North Dakota lignite.

A 170-H.P. Crossley (Otto) engine tested in England in 1892, using producer gas, gave a consumption of 0.85 lb. coal per I.H.P. hour, or a thermal efficiency of engine and producer combined of 21.3%.

Experiments on a Taylor gas producer using anthracite coal and a

100-H.P. Otto gas engine showed a consumption of 0.97 lb. carbon per I.H.P. hour. (*Iron Age*, 1893.)

In a table in Carpenter and Diederichs on Internal Combustion Engines the lowest recorded coal consumption per B.H.P. hour is 0.71 lb., with a Tangye engine and a suction gas producer, using Welsh anthracite coal. Other tests show figures ranging from 0.74 lb. to 1.95, the last with a Westinghouse 500-H.P. engine and a Taylor producer using Colorado bituminous coal.

In the same book are given the following figures of the thermal efficiency on brake H.P. with different gas and liquid fuels. Illuminating gas, 6 tests, 16.1 to 31.0%; natural gas, 4 tests, 16.1 to 29.0%; coke-oven gas, 1 test, 27.5%; Mond gas, 1 test, 23.7%; blast-furnace gas, 3 tests, 20.4 to 28.2%; gasoline, 8 tests, 10.2 to 28%; kerosene, Diesel engine, 3 tests, 25.8 to 31.9%; kerosene, other engines, 8 tests, 9.2 to 19.7%; crude oil, Diesel engine, 1 test, 28.1%; alcohol, 4 tests, 21.8 to 32.7%.

Tests of Diesel engines operating centrifugal pumps in India are reported in *Eng. News*, Nov. 25, 1909. Using Borneo petroleum residue of 0.934 sp. gr., and a fuel value of 18,600 B.T.U. per lb., an average of 151 B.H.P. during a season, for a total of 6003 engine hours, was obtained with a consumption of 0.462 lb. of fuel per B.H.P. hour, or one B.H.P. for about 8600 B.T.U. per hour, equal to a thermal efficiency of 29.5%. The pump efficiency at maximum lift of 14 to 16 ft. was 70%, and the fuel consumption per water H.P. hour at the same lift was 0.7 lb.

Utilization of Waste Heat from Gas Engines.—The exhaust gases from a gas engine may be used to heat air by passing them across a nest of tubes through which air is flowing. A design of this kind, for heating the Ives library building, New Haven, Conn., by Harrison Engineering Co., New York, is illustrated in *Heat. and Vent. Mag.*, Jan., 1910.

The waste heat might also be used in a boiler to generate steam at or below atmospheric pressure, for use in a low pressure steam turbine. On account of the comparatively low temperature of the exhaust gases, however, the boiler would require a much greater extent of heating surface for a given capacity than a boiler with an ordinary coal-fired furnace.

RULES FOR CONDUCTING TESTS OF GAS AND OIL ENGINES*. CODE OF 1902.

(From the report of the committee of the A. S. M. E. on Engine Tests.)

[Only a brief abstract is here given. The items, 1, Objects of the Tests; 2, General Conditions of the Engine; 3, Dimensions; 5, Calibration of Instruments, are practically the same as in the report on Steam Engine Tests.]

IV. Fuel. — Decide upon the gas or oil to be used, and if the trial is to be made for maximum efficiency, the fuel should be the best of its class that can readily be obtained, or one that shows the highest calorific power.

VI. Duration of Test. — The duration of a test should depend largely upon the objects in view, and in any case the test should be continued until the successive readings of the rates at which oil or gas is consumed, taken at say half-hourly intervals, become uniform and thus verify each other. If the object is to determine the working economy, and the period of time during which the engine is usually in motion is some part of twenty-four hours, the duration of the test should be fixed for this number of hours. If the engine is one using coal for generating gas, the test should be of at least twenty-four hours' duration.

VII. Starting a Test. — In a test for determining the maximum economy of an engine, it should first be run a sufficient time to bring all the conditions to a normal and constant state.

If a test is made to determine the performance under working conditions, the test should begin as soon as the regular preparations have been made for starting the engine in practical work, and the measurements should then commence and be continued until the close of the period covered by the day's work.

VIII. Measurement of Fuel. — If the fuel used is coal furnished to a gas

* Hot-air engines are not included in this code, those in the market being of comparatively small size, and seldom tested.

producer, the same methods apply for determining the consumption as are used in steam-boiler tests.

If the fuel used be gas, the only practical method of measurement is the use of a meter through which the gas is passed. The temperature and pressure of the gas should be measured, and the quantity of gas should be determined by reference to the calibration of the meter, taking into account the temperature and pressure of the gas.

If the fuel is oil, this can be drawn from a tank which is filled to the original level at the end of the test, the amount of oil required for so doing being weighed; or, for a small engine, the oil may be drawn from a calibrated vertical pipe.

IX. Measurement of Heat-Units Consumed by the Engine. — The number of heat-units used is found by multiplying the number of pounds of coal or oil or the cubic feet of gas consumed, by the total heat of combustion of the fuel as determined by a calorimeter test. In determining the total heat of combustion no deduction is made for the latent heat of the water vapor in the products of combustion.

It is sometimes desirable, also, to have a complete chemical analysis of the oil or gas. The total heat of combustion may be computed, if desired, from the results of the analysis, and should agree well with the calorimeter values.

X. Measurement of Jacket Water. — The jacket water may be measured by passing it through a water meter or allowing it to flow from a measuring tank before entering the jacket, or by collecting it in tanks on its discharge.

XI. Indicated Horse-power. — The directions given for determining the indicated horse-power for steam engines apply in all respects to internal combustion engines.

XII. Brake Horse-power. — The determination of the brake horse-power is the same for internal combustion as for steam engines.

XIII. Speed. — The same directions apply to internal combustion engines as to steam engines for the determination of speed.

In an engine which is governed by varying the number of explosions or working cycles, a record should be kept of the number of explosions per minute; or if the engine is running at nearly maximum load, by counting the number of times the governor causes a miss in the explosions.

XIV. Recording the Data. — The pressures, temperatures, meter readings, speeds, and other measurements should be observed every 20 or 30 minutes when the conditions are practically uniform, and at more frequent intervals if they are variable. Observations of the gas or oil measurements should be taken with special care at the expiration of each hour, so as to divide the test into hourly periods, and reveal the uniformity, or otherwise, of the conditions and results as the test goes forward.

XV. Uniformity of Conditions. — When the object of the test is to determine the maximum economy, all the conditions relating to the operation of the engine should be maintained as constant as possible during the trial.

XVI. Indicator Diagrams. — Sample diagrams nearest to the mean should be selected from those taken during the trial and appended to the tables of the results. If there are separate compression or feed cylinders, the indicator diagrams from these should be taken and the power deducted from that of the main cylinder.

XVII. Standards of Economy and Efficiency. — The hourly consumption of heat, divided by the indicated or the brake horse-power, is the standard expression of engine economy recommended.

In making comparisons between the standard for internal combustion engines and that for steam engines, it must be borne in mind that the steam engine standard does not cover the losses due to combustion, while the internal combustion engine standard, in cases where a crude fuel such as oil is burned in the cylinder, does cover these losses.

The thermal efficiency ratio per indicated horse-power or per brake horse-power for internal combustion engines is expressed by the fraction

$$2545 \div \text{B.T.U. per H.P. per hour.}$$

XVIII. Heat Balance. — For purposes of scientific research, a heat balance should be drawn which shows the manner in which the total

heat of combustion is expended in the various processes concerned in the working of the engine. It may be divided into three parts: first, the heat which is converted into the indicated or brake work; second, the heat rejected in the cooling water of the jackets; and third, the heat rejected in the exhaust gases, together with that lost through incomplete combustion and radiation.

To determine the first item, the number of foot-pounds of work performed by, say, one pound or one cubic foot of the fuel, divided by 778, gives the number of heat-units desired. The second item is determined by measuring the amount of cooling water passed through the jackets, equivalent to one pound or one cubic foot of fuel consumed, and multiplying this quantity by the difference in the sensible heat of the water leaving the jacket and that entering. The third item is obtained by subtracting the sum of the first two items from the total heat supplied. The third item can be subdivided by computing the heat rejected in the exhaust gases as a separate quantity. The data for this computation are found by analyzing the fuel and the exhaust gases, or by measuring the quantity of air admitted to the cylinder in addition to that of the gas or oil.

XIX. Report of Test. — The data and results of a test should be reported in the manner outlined in one of the following tables, the first of which gives a complete summary when all the data are determined, and the second is a shorter form of report in which some of the minor items are omitted. [The short form is given below.]

DATA AND RESULTS OF STANDARD HEAT TEST OF GAS OR OIL ENGINE.

Arranged according to the Short Form advised by the Engine Test Committee, American Society of Mechanical Engineers. Code of 1902.

1. Made by of.....
on engine located at.....
to determine.....
2. Date of trial.....
3. Type and class of engine.....
4. Kind of fuel used..... deg Fahr.
 - (a) Specific gravity.....
 - (b) Burning point.....
 - (c) Flashing point.....
5. Dimensions of engine:

| | 1st Cyl. | 2d Cyl. |
|--|----------|---------|
| (a) Class of cylinder (working or for compressing the charge)..... | | |
| (b) Single or double acting..... | | |
| (c) Cylinder dimensions: | | |
| Bore.....in. | | |
| Stroke.....ft. | | |
| Diameter piston rod.....in. | | |
| (d) Average compression space, or clearance, in per cent..... | | |
| (e) Horse-power constant for one lb. M.E.P. and one revolution per minute..... | | |

Total Quantities.

6. Duration of test..... hours
7. Gas or oil consumed..... cu. ft. or lbs.
8. Cooling water supplied to jackets.....
9. Calorific value of fuel by calorimeter test, determined by calorimeter..... B.T.U.

Pressures and Temperatures.

10. Pressure at meter (for gas engine) in inches of water... ins.
11. Barometric pressure of atmosphere:
 - (a) Reading of barometer.....
 - (b) Reading corrected to 32 degs. Fahr.....

12. Temperature of cooling water:
 (a) Inlet..... deg. Fahr.
 (b) Outlet..... "
 13. Temperature of gas at meter (for gas engine)..... "
 14. Temperature of atmosphere:
 (a) Dry bulb thermometer..... "
 (b) Wet bulb thermometer..... "
 (c) Degree of humidity..... "
 15. Temperature of exhaust gases..... "

Data Relating to Heat Measurement.

16. Heat units consumed per hour (pounds of oil or cubic feet of gas per hour multiplied by the total heat of combustion)..... B.T.U.
 17. Heat rejected in cooling water per hour..... "

Speed, etc.

18. Revolutions per minute..... rev.
 19. Average number of explosions per minute.....

Indicator Diagrams.

20. Pressure in lbs. per sq. in. above atmosphere:
 1st Cyl. 2d Cyl.
 (a) Maximum pressure.....
 (b) Pressure just before ignition.....
 (c) Pressure at end of expansion.....
 (d) Exhaust pressure.....
 (e) Mean effective pressure.....

Power.

21. Indicated horse-power:
 First cylinder..... H.P.
 Second cylinder..... "
 Total..... "
 22. Brake horse-power..... "
 23. Friction horse-power by friction diagrams..... "
 24. Percentage of indicated horse-power lost in friction.. per cent.

Standard Efficiency, and Other Results.

25. Heat units consumed by the engine per hour:
 (a) Per indicated horse-power..... B.T.U.
 (b) Per brake horse-power..... "
 26. Pounds of oil or cubic feet of gas consumed per hour:
 (a) Per indicated horse-power..... lbs. or cu. ft.
 (b) Per brake horse-power..... "

Additional Data.

Add any additional data bearing on the particular objects of the test or relating to the special class of service for which the engine is to be used. Also give copies of indicator diagrams nearest the mean, and the corresponding scales.

LOCOMOTIVES.

Resistance of Trains. — *Resistance due to Speed.* — Various formulæ and tables for the resistance of trains at different speeds on a straight level track have been given by different writers. Among these are the following:

By D. L. Barnes (*Eng. Mag.*), June, 1894:

| | | | | | | |
|---------------------------------------|----|------|------|----|----|-----|
| Speed, miles per hour..... | 50 | 60 | 70 | 80 | 90 | 100 |
| Resistance, pounds per gross ton..... | 12 | 12.4 | 13.5 | 15 | 17 | 20 |

By *Engineering News*, March 8, 1894:

Resistance in lbs. per ton of 2000 lbs. = $\frac{1}{4}v + 4$.

| | | | | | | | | | | | | | | | |
|----------------------------|-----|----------------|----|----------------|-----|-----------------|----|-----------------|------|----|------|----|------|----|-----|
| Speed..... | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 60 | 70 | 80 | 90 | 100 |
| Resistance. $3\frac{1}{4}$ | 4.5 | $5\frac{3}{4}$ | 7 | $8\frac{1}{4}$ | 9.5 | $10\frac{3}{4}$ | 12 | $13\frac{1}{4}$ | 14.5 | 17 | 19.5 | 22 | 24.5 | 27 | |

This formula seems to be more generally accepted than the others. It gives results too small, however, below 10 miles an hour. At starting, the resistance is about 17 lbs. per ton, dropping to 4 or 5 lbs. at 5 miles an hour.

By Baldwin Locomotive Works:

Resistance in lbs. per ton of 2000 lbs. = $3 + v \div 6$.

| | | | | | | | | | | | | | | | | | |
|------------|-------|-----|-----|-----|-----|-----|----|-----|-----|------|------|------|----|------|------|----|------|
| Speed | | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 70 | 80 | 90 | 100 |
| Resistance | | 3.8 | 4.7 | 5.5 | 6.3 | 7.2 | 8 | 8.8 | 9.7 | 10.5 | 11.3 | 12.2 | 13 | 14.7 | 16.3 | 18 | 19.7 |

The resistance due to speed varies with the condition of the track, the number of cars in a train, and other conditions.

For tables showing that the resistance varies with the area exposed to the resistance and friction of the air per ton of loads, see Dashiell, *Trans. A. S. M. E.*, vol. xiii. p. 371.

P. H. Dudley (Bulletin International Ry. Congress, 1900, p. 1734) shows that the condition of the track is an important factor of train resistance which has not hitherto been taken account of. The resistance of heavy trains on the N. Y. Central R. R. at 20 miles an hour is only about $3\frac{1}{2}$ lbs. per ton on smooth 80-lb. $5\frac{1}{8}$ -in. rails. The resistance of an 80-car freight train, 60,000 lbs. per car, as given by indicator cards, at speeds between 15 and 25 miles per hour, is represented by the formula $R = 1 + \frac{1}{8} V$, in which R = resistance in lbs. per ton and V = miles per hour. These values are much below the average and should not be used in estimating the hauling power needed.

New Formulæ for Resistance. — The Amer. Locomotive Co. (Bulletin No. 1001, Feb., 1910) states that the figures obtained from the old formulæ for train resistance are much too high for modern loaded freight cars of 40 to 50 tons capacity, and in some instances too low for very light or empty cars. The best data available show that the resistance varies from about 2.5 to 3 lbs. per ton (of 2000 lbs.) for 72-ton cars (including weight of empty car) to 6 to 8 lbs. for 20-ton cars. From speeds between 5 to 10 and 30 to 35 miles an hour, the resistance of freight cars is practically constant. The resistance of the engine and tender is figured separately, and is composed of the following factors: (a) Engine friction = 22.2 lbs. per ton, or 1.11% of the weight on drivers. (b) Head air resistance = cross-sectional area (taken at 120 sq. ft.) $\times 0.002 V^2$, V being the speed in miles per hour. (c) Resistance due to weight on engine trucks and trailing wheels, and to the tender, the same per ton as that due to the cars. (d) Grade resistance = 20 lbs. per ton for each per cent of grade. (e) Curve resistance, which varies with the wheel-base of the locomotive, and is taken as $0.4 + cD$ lbs. per ton, in which D is the degree of the curve and c a constant whose value is,

| | | | | | | | | | | |
|---------------------|-------|-------|------|------|------|------|------|------|------|-----------|
| For wheel-base, ft. | 5 | 6 | 7 | 8 | 9 | 12 | 13 | 15 | 16 | 20 |
| Value of c | | 0.380 | .415 | .460 | .485 | .520 | .625 | .660 | .730 | .765 .905 |

The sum of these resistances is to be deducted from the tractive force of the locomotive to obtain the available tractive force for overcoming the resistance of the cars. (See Tractive Force, below.) The maximum tractive force is taken for low speeds at 85% of that due to the boiler pressure; for piston speeds over 250 ft. per min. this is to be multiplied by a speed factor to obtain the actual force. Speed factors and percentages of maximum horse-power corresponding to different piston speeds are given below. S = piston speed, ft. per min., F = speed factor, P = % of maximum H.P.

| | | | | | | | | | | | |
|-----------|------|------|------|------|------|------|------|------|------|------|------|
| S | 250 | 300 | 350 | 400 | 450 | 500 | 550 | 600 | 650 | 700 | 750 |
| F | 1.00 | .954 | .908 | .863 | .817 | .772 | .727 | .680 | .636 | .592 | .550 |
| P | 60.4 | 69.1 | 77.2 | 83.7 | 89.0 | 93.5 | 96.8 | 98.7 | 99.7 | 100 | 100 |

| | | | | | | | | | | | |
|-----------|-------|------|------|------|------|------|------|------|------|------|------|
| S | 800 | 850 | 900 | 950 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 |
| F | 0.517 | .487 | .460 | .435 | .412 | .372 | .337 | .307 | .283 | .261 | .241 |
| P | 100 | 100 | 100 | 100 | 100 | 99 | 97.8 | 96.8 | 95.7 | 94.7 | 93.5 |

The resistance of freight cars, according to experiments on the Penna R.R., varies with the weight in tons per car as follows:

| | | | | | | | | | |
|--------------------------|----|-------|------|------|------|------|------|------|-----------|
| Tons per car..... | 10 | 20 | 25 | 30 | 40 | 50 | 60 | 70 | 72 |
| Resistance, lbs. per ton | | 13.10 | 7.84 | 6.62 | 5.78 | 4.66 | 3.94 | 3.44 | 3.06 3.00 |

From plotted curves of resistances of trains of empty and loaded cars the following figures are derived. R = resistance in lbs. per ton.

| | | | | | | |
|----------------------------|------|------|------|------|------|------|
| Wt. loaded, tons..... | 75 | 70 | 65 | 60 | 55 | 50 |
| Wt. empty, tons..... | 21 | 20.3 | 19.5 | 18.6 | 17.6 | 16.5 |
| Per cent of loaded wt..... | 28 | 29 | 30 | 31 | 32 | 33 |
| R loaded | 2.90 | 3.07 | 3.24 | 3.43 | 3.65 | 3.90 |
| R empty | 5.63 | 5.82 | 6.00 | 6.26 | 6.50 | 6.85 |

| | | | | | | | |
|---------------------------|------|------|------|------|------|------|------|
| Wt. loaded, tons..... | 45 | 40 | 35 | 30 | 25 | 20 | 15 |
| Wt. empty, tons..... | 15.3 | 14.0 | 12.6 | 11.1 | 9.5 | 7.8 | 6.0 |
| Per cent of loaded wt.... | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| R loaded | 4.18 | 4.40 | 4.74 | 5.07 | 5.44 | 5.91 | 6.40 |
| R empty | 7.26 | 7.65 | 8.05 | 8.45 | 9.05 | 9.60 | 10.3 |

The resistance of passenger cars is derived from the formula $R = 5.4 + 0.002(V - 15)^2 + 100 \div (V + 2)^3$. V in miles per hour, R = resistance in lbs. per ton (2000 lbs.) H.P. = horse-power per ton.

| | | | | | | | |
|--------------|-------|------|------|------|------|------|------|
| V = | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| R = | 5.89 | 5.51 | 5.42 | 5.46 | 5.60 | 5.85 | 6.20 |
| H.P. = | 0.079 | .147 | .217 | .291 | .374 | .469 | .578 |

| | | | | | | | |
|--------------|------|------|-------|-------|-------|-------|-------|
| V = | 40 | 45 | 50 | 60 | 70 | 80 | 90 |
| R = | 6.65 | 7.20 | 7.85 | 9.45 | 11.45 | 13.85 | 16.65 |
| H.P. = | .709 | .864 | 1.047 | 1.515 | 2.135 | 2.95 | 4.00 |

Resistance of Electric Railway Cars and Trains. — W. J. Davis, Jr. (*Street Ry. Jour.*, Dec. 3, 1904), gives as a result of numerous experiments the following formulæ:

(A) For light open platform street cars, 8 tons to 20 tons; maximum speed, 30 miles per hour; cross-section, 85 sq. ft.

$$R = 6 + 0.11 V + \frac{0.3 V^2}{T} [1 + 0.1 (n - 1)].$$

(B) For standard interurban electric cars, 25 tons to 40 tons; maximum speed, 60 m.p.h.; cross section, 100 sq. ft.

$$R = 5 + 0.13 V + 0.3 V^2/T [1 + 0.1 (n - 1)].$$

(C) For heavy interurban electric cars, or steam passenger coaches, 40 tons to 50 tons; maximum speed, 75 m.p.h.; cross-section, 110 sq. ft.

$$R = 4 + 0.13 V + 0.33 V^2/T [1 + 0.1 (n - 1)].$$

(D) For heavy freight trains, cars weighing 45 tons loaded; maximum speed, 35 m.p.h.; average cross-section, 110 sq. ft.

$$R = 3.5 + 0.13 V + 0.385 V^2/T [1 + 0.1 (n - 1)].$$

R = resistance in lbs. per ton of 2000 lbs., V = speed in miles per hour
 T = weight of train in tons, n = number of cars in train, including leading motor car. The cross-section includes the space bounded by the wheels between the top of rails and the body.

Resistance due to Grade. — The resistance due to a grade of 1 ft. per mile is, per ton of 2000 lbs., $2000 \times 1/5280 = 0.3788$ lb. per ton, or if R_g = resistance in lbs. per ton due to grade and G = ft. per mile $R_g = 0.3788 G$.

If the grade is expressed as a percentage of the length, the resistance is 20 lbs. per ton for each per cent of grade.

Resistance due to Curves. — Mr. G. R. Henderson in his book entitled "Locomotive Operation" gives the resistance due to curvature at 0.7 lb. per ton of 2000 lbs. per degree of the curve. (For definition of degrees of a railroad curve see p. 55.) For locomotives, this factor is sometimes doubled, making the resistance in lbs. per ton = $0.7 c$ for cars and $1.4 c$ for locomotives, c being the number of degrees.

The Baldwin Locomotive Works take the approximate resistance due to each degree of curvature as that due to a straight grade of $1\frac{1}{2}$ ft. per mile. This corresponds to $R_c = 0.5682 c$.

The Amer. Locomotive Co. takes 0.8 lb. per ton per degree of curvature for the resistance of cars on curves.

For mine cars, with short wheel-bases and wheels loose on the axles, experiments quoted by the Baldwin Locomotive Works, 1904, lead to the formula, Resistance due to curvature, in pounds, $= 0.20 \times \text{wheel-base} \times \text{weight of loaded cars in pounds,} \div \text{radius of curve in feet.}$

Resistance due to Acceleration. — This may be calculated by the ordinary formula (see page 504), or reduced to common railroad units, and including the rotative energy of wheels and axles, which increases the effect of the weight of the cars by an equivalent of about 5%, we have

$$P = 70 \frac{V^2}{S} = 95.6 \frac{V}{t} = 70 \frac{V_2^2 - V_1^2}{S}, \text{ where } P = \text{the accelerating force in}$$

pounds per ton, V = the velocity in miles per hour, S = the distance in feet, and t = the time in seconds in which the acceleration takes place. V_1 and V_2 = the smaller and greater velocities, respectively, in miles per hour, for a change of speed.

Total Resistance. — The total resistance in lbs. per ton of 2000 lbs. due to speed, to grade, to curves, and to acceleration is the sum of the resistances calculated above.

The Baldwin Locomotive Works in their "Locomotive Data" take the total resistance on a straight level track at slow speeds at from 6 to 10 lbs. per ton, and in a communication printed in the fourth edition (1898) of this Pocket-book, p. 1076, say: "We know that in some cases, for instance in mine construction, the frictional resistance has been shown to be as much as 60 lbs. per ton at slow speed. The resistance should be approximated to suit the conditions of each individual case, and the increased resistance due to speed added thereto."

Resistance due to Friction. — In the above formulæ no account has been taken of the resistance due to the friction of the working parts. This is rather an obscure subject. Mr. Henderson estimates the percentage of the indicated power consumed by friction to be $0.15 V + c$, where V = speed in miles per hour and c = a constant, whose value may vary from 2 to 8, the latter figure being the safest to use for heavy work at slow speeds. Ordinarily 8% of the indicated power is consumed by internal resistance under these conditions. Professor Goss gives the following formula, obtained from tests at the Purdue locomotive testing laboratory:

Let d = diameter of cylinder; S = stroke of piston; D = diameter of drivers, all in inches. Then the internal friction $= 3.8 d^2 S / D$, in pounds at the circumference of the drivers.

Concerning the effect of increasing speed on tractive force, Mr. Henderson says (1906):

From a number of tests and information from various roads and authorities it seems as if, for ordinary simple engines, the coefficient 0.8

in the equation Actual tractive force $= \frac{0.8 P d^2 s}{D}$ could be modified in accordance with the speed in order to obtain the actual tractive force at various speeds about as follows:

| | | | | | | | | | |
|------------------|------|------|------|------|------|------|------|------|------|
| Revs. per min. = | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | |
| Coefficient = | 0.80 | 0.80 | 0.80 | 0.70 | 0.61 | 0.53 | 0.46 | 0.40 | |
| Revs. per min. = | 180 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 |
| Coefficient = | 0.35 | 0.31 | 0.28 | 0.26 | 0.24 | 0.23 | 0.21 | 0.20 | 0.19 |

Efficiency of the Mechanism of a Locomotive. — Frank C. Wagner (*Proc. A. A. S.*, 1900, p. 140) gives an account of some dynamometer tests which indicate that in ordinary freight service the power used to drive the locomotive and tender and to overcome the friction of the mechanism is from 10% to 35% of the total power developed in the steam-cylinder. In one test the weight of the locomotive and tender was 16% of the total weight of the train, while the power consumed in the locomotive and tender was from 30% to 33% of the indicated horse-power.

Adhesion. — The limit of the hauling capacity of a locomotive is the adhesion due to the weight on the driving wheels. Holmes gives the adhesion, in English practice, as equal to 0.15 of the load on the driving wheels in ordinary dry weather, but only 0.07 in damp weather or when the rails are greasy. In American practice it is generally taken as from $1/4$ to $1/5$ of the load on the drivers.

Tractive Force of a Locomotive. — Single Expansion.Let F = indicated tractive force in lbs. p = average effective pressure in cylinder in lbs. per sq. in. S = stroke of piston in inches. d = diameter of cylinders in inches. D = diameter of driving-wheels in inches. Then

$$F = \frac{4 \pi d^2 p S}{4 \pi D} = \frac{d^2 p S}{D}.$$

The average effective pressure can be obtained from an indicator-diagram, or by calculation, when the initial pressure and ratio of expansion are known, together with the other properties of the valve-motion. The subjoined table from Auchincloss gives the proportion of mean effective pressure to boiler-pressure above atmosphere for various proportions of cut-off.

| Stroke, Cut-off at — | M.E.P. (Boiler- pres. = 1). | Stroke, Cut-off at — | M.E.P. (Boiler- pres. = 1). | Stroke, Cut-off at — | M.E.P. (Boiler- pres. = 1). |
|-------------------------|-----------------------------------|-------------------------|-----------------------------------|-------------------------|-----------------------------------|
| 0.1 | 0.15 | 0.333 = 1/3 | 0.5 = 1/2 | 0.625 = 5/8 | 0.79 |
| .125 = 1/8 | .2 | .375 = 3/8 | .55 | .666 = 2/3 | .82 |
| .15 | .24 | .4 | .57 | .7 | .85 |
| .175 | .28 | .45 | .62 | .75 = 3/4 | .89 |
| .2 | .32 | .5 = 1/2 | .67 | .8 | .93 |
| .25 = 1/4 | .4 | .55 | .72 | .875 = 7/8 | .98 |
| .3 | .46 | | | | |

These values were deduced from experiments with an English locomotive by Mr. Gooch. As diagrams vary so much from different causes, this table will only fairly represent practical cases. It is evident that the cut-off must be such that the boiler will be capable of supplying sufficient steam at the given speed.

We can, however, allow for wire drawing to the steam chest and drop in pressure due to expansion, and internal friction by writing the formula:

Actual Tractive Force = $\frac{0.8 P d^2 S}{D}$, d , S , and D being as before and P representing boiler pressure in lbs. per sq. in.

Compound Locomotives. — The Baldwin Locomotive Works give the following formulæ for compound engines of the Vaucrain four-cylinder type:

$$T = \frac{C^2 S \times 2/3 P}{D} + \frac{c^2 S \times 1/4 P}{D}.$$

T = tractive force in lbs. C = diam. of high-pressure cylinder in ins. c = diam. of low-pressure cylinder in ins. P = boiler-pressure in lbs. S = stroke of piston in ins. D = diam. of driving-wheels in ins.

For a two-cylinder or cross-compound engine it is only necessary to consider the high-pressure cylinder, allowing a sufficient decrease in boiler pressure to compensate for the necessary back-pressure. The formula is

$$T = \frac{C^2 S \times 2/3 P}{D}.$$

The above formulæ are for speeds of from 5 to 10 miles an hour, or less; above that the capacity of the boiler limits the cut-off which can be used, and the available tractive force is rapidly reduced as the speed increases. For a full discussion of this, see page 375 of Henderson's "Locomotive Operation."

The Size of Locomotive Cylinders is usually taken to be such that the engine will just overcome the adhesion of its wheels to the rails under favorable circumstances.

The adhesion is taken by a committee of the Am. Ry. Master Mechanics' Assn. as 0.25 of the weight on the drivers for passenger engines, 0.24 for freight, and 0.22 for switching engines; and the mean effective pressure in the cylinder, when exerting the maximum tractive force, is taken at 0.85 of the boiler-pressure.

Let W = weight on drivers in lbs.; P = tractive force in lbs., = say $0.25 W$; p_1 = boiler-pressure in lbs. per sq. in.; p = mean effective pressure, = $0.85 p_1$; d = diam. of cylinder, S = length of stroke, and D = diam. of driving-wheels, all in inches. Then

$$W = 4P = \frac{4d^2 p S}{D} = \frac{4d^2 \times 0.85 p_1 S}{D}.$$

Whence
$$d = 0.5 \sqrt{\frac{DW}{pS}} = 0.542 \sqrt{\frac{DW}{p_1 S}}.$$

Von Borries's rule for the diameter of the low-pressure cylinder of a compound locomotive is $d^2 = 2ZD \div ph$, in which d = diameter of l.p. cylinder in inches; D = diameter of driving-wheel in inches; p = mean effective pressure per sq. in., after deducting internal machine friction; h = stroke of piston in inches; Z = tractive force required, usually 0.14 to 0.16 of the adhesion.

The value of p depends on the relative volume of the two cylinders, and from indicator experiments may be taken as follows:

| Class of Engine. | Ratio of Cylinder Volumes. | p in percent of Boiler-pressure. | p for Boiler-pressure of 176 lbs. |
|---------------------|----------------------------|------------------------------------|-------------------------------------|
| Large-tender eng's. | 1 : 2 or 1 : 2.05 | 42 | 74 |
| Tank-engines..... | 1 : 2 or 1 : 2.2 | 40 | 71 |

Horse-power of a Locomotive. — For each cylinder the horse-power is $H.P. = pLaN \div 33,000$, in which p = mean effective pressure, L = stroke in feet, a = area of cylinder = $\frac{1}{4} \pi d^2$, N = number of single strokes per minute, LN = piston speed, ft. per min. Let M = speed of train in miles per hour, S = length of stroke in inches, and D = diameter of driving-wheel in inches. Then $LN = M \times 88 \times 2S \div \pi D$. Whence for the two cylinders the horse-power is

$$\frac{2 \times p \times \frac{1}{4} \pi d^2 \times 176 S \times M}{\pi D \times 33,000} = \frac{pd^2 SM}{375 D}.$$

REVOLUTIONS PER MINUTE FOR VARIOUS DIAMETERS OF WHEELS AND SPEEDS.

| Diameter of Wheel. | Miles per Hour. | | | | | | | |
|--------------------|-----------------|-----|-----|-----|-----|-----|-----|-----|
| | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| 50 in. | 67 | 134 | 201 | 268 | 336 | 403 | 470 | 538 |
| 56 in. | 60 | 120 | 180 | 240 | 300 | 360 | 420 | 480 |
| 60 in. | 56 | 112 | 168 | 224 | 280 | 336 | 392 | 448 |
| 62 in. | 54 | 108 | 162 | 217 | 271 | 325 | 379 | 433 |
| 66 in. | 51 | 102 | 153 | 204 | 255 | 306 | 357 | 408 |
| 68 in. | 49 | 99 | 148 | 198 | 247 | 296 | 346 | 395 |
| 72 in. | 47 | 93 | 140 | 187 | 233 | 279 | 326 | 373 |
| 78 in. | 43 | 86 | 129 | 172 | 215 | 258 | 301 | 344 |
| 80 in. | 42 | 84 | 126 | 168 | 210 | 252 | 294 | 336 |
| 84 in. | 40 | 80 | 120 | 160 | 200 | 240 | 280 | 320 |
| 90 in. | 37 | 75 | 112 | 150 | 186 | 224 | 261 | 299 |

The Size of Locomotive Boilers. (Forney's Catechism of the Locomotive.) — They should be proportioned to the amount of adhesive weight and to the speed at which the locomotive is intended to work. Thus a locomotive with a great deal of weight on the driving-wheels could pull a heavier load, would have a greater cylinder capacity than one with little adhesive weight, would consume more steam, and therefore should have a larger boiler.

The weight and dimensions of locomotive boilers are in nearly all cases determined by the limits of weight and space to which they are necessarily confined. It may be stated generally that *within these limits a locomotive boiler cannot be made too large.* In other words, boilers for

locomotives should always be made as large as is possible under the conditions that determine the weight and dimensions of the locomotives. (See also Holmes on the Steam-engine, pp. 371 to 377 and 383 to 389, and the Report of the Am. Ry. M. M. Ass'n. for 1897, pp. 218 to 232.)

Holmes gives the following from English practice:

Evaporation, 9 to 12 lbs. of water from and at 212°.

Ordinary rate of combustion, 65 lbs. per sq. ft. of grate per hour.

Ratio of grate to heating surface, 1 : 60 to 90.

Heating surface per lb. of coal burnt per hour, 0.9 to 1.5 sq. ft.

Mr. Henderson states the approximate heating surface needed per indicated horse-power as follows:

Compound Locomotives..... 2 square feet.

Simple Locomotives (cut-off $\frac{1}{2}$ stroke or less) $2\frac{1}{3}$ square feet.

Simple Locomotives (cut-off $\frac{1}{2}$ to $\frac{3}{4}$ stroke)..... $2\frac{2}{3}$ square feet.

Simple Locomotives (full stroke)..... 3 square feet.

For the ratio of heating surface to grate area the Master Mechanics Ass'n Committee of 1902 advised as below:

| Fuel. | Passenger. | | Freight. | |
|-------------------------------------|------------|-----------|----------|-----------|
| | Simple. | Compound. | Simple. | Compound. |
| Free burning bituminous..... | 65 to 90 | 75 to 95 | 70 to 85 | 65 to 85 |
| Average bituminous..... | 50 to 65 | 60 to 75 | 45 to 70 | 50 to 65 |
| Slow burning bituminous..... | 40 to 50 | 35 to 60 | 35 to 45 | 45 to 50 |
| Bituminous slack and free burning.. | | | | |
| anthracite..... | 35 to 40 | 30 to 35 | 30 to 35 | 40 to 45 |
| Low grade bituminous, lignite and | | | | |
| slow burning anthracite..... | 28 to 35 | 24 to 30 | 25 to 30 | 30 to 40 |

A. E. Mitchell, (*Eng'g News*, Jan. 24, 1891) says: Square feet of boiler-heating surface for bituminous coal should not be less than 4 times the square of the diameter in inches of a cylinder 1 inch larger than the cylinder to be used. One tenth of this should be in the fire-box. On anthracite locomotives more heating-surface is required in the fire-box, on account of the larger grate-area required, but the heating-surface of the flues should not be materially decreased.

Wootten's Locomotive. (Clark's Steam-engine; see also *Jour. Frank. Inst.* 1891, and *Modern Mechanism*, p. 485.) — J. E. Wootten designed and constructed a locomotive boiler for the combustion of anthracite and lignite, though specially for the utilization as fuel of the waste produced in the mining and preparation of anthracite. The special feature of the engine is the fire-box, which is made of great length and breadth, extending clear over the wheels, giving a grate-area of from 64 to 85 sq. ft. The draught diffused over these large areas is so gentle as not to lift the fine particles of the fuel. A number of express-engines having this type of boiler are engaged on the fast trains between Philadelphia and Jersey City. The fire-box shell is 8 ft. 8 in. wide and 10 ft. 5 in. long; the fire-box is $8 \times 9\frac{1}{2}$ ft., making 76 sq. ft. of grate-area. The grate is composed of bars and water-tubes alternately. The regular types of cast-iron shaking grates are also used. The height of the fire-box is only 2 ft. 5 in. above the grate. The grate is terminated by a bridge of fire-brick, beyond which a combustion-chamber, 27 in. long, leads to the flue-tubes, about 184 in number, $1\frac{3}{4}$ in. diam. The cylinders are 21 in. diam., with a stroke of 22 inches. The driving-wheels, four-coupled, are 5 ft. 8 in. diam. The engine weighs 44 tons, of which 29 tons are on driving wheels. The heating-surface of the fire-box is 135 sq. ft., that of the flue-tubes is 982 sq. ft.; together, 1117 sq. ft., or 14.7 times the grate-area. Hauling 15 passenger-cars, weighing with passengers 360 tons, at an average speed of 42 miles per hour, over ruling gradients of 1 in 89, the engine consumes 62 lbs. of fuel per mile, or $34\frac{1}{4}$ lbs. per sq. ft. of grate per hour.

Grate-surface, Smoke-stacks, and Exhaust-nozzles for Locomotives. — A. E. Mitchell, Supt. of Motive Power of the Erie R. R., says (1895) that some roads use the same size of stack, 13½ in. diam. at throat, for all engines up to 20 in. diam. of cylinder.

The area of the orifices in the exhaust-nozzles depends on the quantity and quality of the coal burnt, size of cylinder, construction of stack, and the condition of the outer atmosphere. It is therefore impossible to give rules for computing the exact diameter of the orifices. All that can be done is to give a rule by which an approximate diameter can be found. The exact diameter can only be found by trial. Our experience leads us to believe that the area of each orifice in a double exhaust-nozzle should be equal to 1/400 part of the grate-surface, and for single nozzles 1/200 of the grate-surface. These ratios have been used in finding the diameters of the nozzles given in the following table. The same sizes are often used for either hard or soft coal-burners. [These sizes are small at the present day (1909) as locomotives have enormously increased in size.]

| Size of Cylinders, in inches. | Grate-area for Anthra- cite Coal, in sq. in. | Grate-area for Bitumin- ous Coal, in sq. in. | Diameter of Stacks, in inches. | Double Nozzles. | Single Nozzles. |
|-------------------------------------|---|---|--------------------------------------|-------------------------------------|-------------------------------------|
| | | | | Diam. of Orifices, in inches. | Diam. of Orifices, in inches. |
| 12×20 | 1591 | 1217 | 91/2 | 2 | 2 13/16 |
| 13×20 | 1873 | 1432 | 10 1/2 | 2 1/8 | 3 |
| 14×20 | 2179 | 1666 | 11 1/4 | 2 5/16 | 3 1/4 |
| 15×22 | 2742 | 2097 | 12 1/2 | 2 9/16 | 3 11/16 |
| 16×24 | 3415 | 2611 | 14 | 2 7/8 | 4 1/16 |
| 17×24 | 3856 | 2948 | 15 | 3 1/16 | 4 5/16 |
| 18×24 | 4321 | 3304 | 15 3/4 | 3 1/4 | 4 3/8 |
| 19×24 | 4810 | 3678 | 16 1/2 | 3 7/16 | 4 13/16 |
| 20×24 | 5337 | 4081 | 17 1/2 | 3 5/8 | 5 1/16 |

Exhaust-nozzles in Locomotive Boilers. — A committee of the Am. Ry. Master Mechanics' Ass'n. in 1890 reported that they had, after two years of experiment and research, come to the conclusion that, owing to the great diversity in the relative proportions of cylinders and boilers, together with the difference in the quality of fuel, any rule which does not recognize each and all of these factors would be worthless.

The committee was unable to devise any plan to determine the size of the exhaust-nozzle in proportion to any other part of the engine or boiler. The conditions desirable are: That it must create draught enough on the fire to make steam, and at the same time impose the least possible amount of work on the pistons in the shape of back pressure. It should be large enough to produce a nearly uniform blast without lifting or tearing the fire, and be economical in its use of fuel. The Annual Report of the Association for 1896 contains interesting data on this subject.

Much important information regarding stacks and exhaust nozzles is embodied in the tests at Purdue University, reported to the Master Mechanics' Ass'n. in 1896 and in the tests reported in the *American Engineer* in 1902 and 1903.

Fire-brick Arches in Locomotive Fire-boxes. — A committee of the Am. Ry. Master Mechanics' Ass'n. in 1890 reported strongly in favor of the use of brick arches in locomotive fire-boxes. They say: It is the unanimous opinion of all who use bituminous coal and brick arch, that it is most efficient in consuming the various gases composing black smoke, and by impeding and delaying their passage through the tubes, and mingling and subjecting them to the heat of the furnace, greatly lessens the volume ejected, and intensifies combustion, and does not in the least check but rather augments draught, with the consequent saving of fuel and increased steaming capacity that might be expected from such results. This in particular when used in connection with extension front.

Arches now (1909) are not quite so much in favor, largely on account of the difficulty and delay caused to workmen when flues must be calked, as occurs frequently in bad water districts, and some of their former advocates are now omitting them altogether.

Economy of High Pressures. — Tests of a Schenectady locomotive with cylinders 16 X 24 ins., at the Purdue University locomotive testing plant, gave results as follows: (*Eng. Digest*, Mar., 1909; Bull. No. 26, Univ. of Ill. Expt. Station).

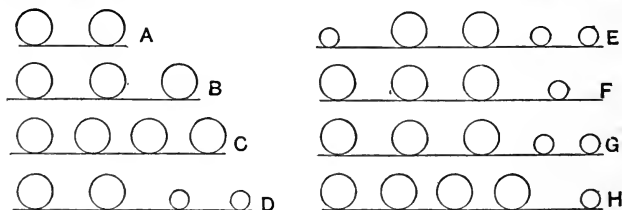
| | | | | | | | |
|-----------------------------------|------|------|------|------|------|------|------|
| Boiler pressure, lbs. per sq. in. | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| Steam per 1 H.P. hour, lbs. | 29.1 | 27.7 | 26.6 | 26. | 25.5 | 25.1 | 24.7 |
| Coal per 1 H.P. hour, lbs. | 4 | 3.77 | 3.59 | 3.50 | 3.43 | 3.37 | 3.31 |

In the same series of tests the economy of the boiler at different rates of driving and different pressures was determined, the results leading to the formula $E = 11.305 - 0.221 H$, in which E = lbs. evaporated from and at 212" per lb. of Youghioghenny coal, and H the equivalent evaporation per sq. ft. of heating surface per hour, with an average error for any pressure which does not exceed 2.1%.

Leading American Types of Locomotive for Freight and Passenger Service.

1. The eight-wheel or "American" passenger type, having four coupled driving-wheels and a four-wheeled truck in front.
2. The "ten-wheel" type, for mixed traffic, having six coupled drivers and a leading four-wheel truck.
3. The "Mogul" freight type, having six coupled driving-wheels and a pony or two-wheel truck in front.
4. The "Consolidation" type, for heavy freight service, having eight coupled driving-wheels and a pony truck in front.

Besides these there is a great variety of types for special conditions of service, as four-wheel and six-wheel switching-engines, without trucks; the Forney type used on elevated railroads, with four coupled wheels under the engine and a four-wheeled rear truck carrying the water-tank and fuel; locomotives for local and suburban service with four coupled driving-wheels, with a two-wheel truck front and rear, or a two-wheel truck front and a four-wheel truck rear, etc. "Decapod" engines for heavy freight service have ten coupled driving-wheels and a two-wheel truck in front.



Classification of Locomotives (Penna. R. R. Co., 1900). — Class A, two pairs of drivers and no truck. Class B, three pairs of drivers and no truck. Class C, four pairs of drivers and no truck. Class D, two pairs of drivers and four-wheel truck. Class E, two pairs of drivers, four-wheel truck, and trailing wheels. Class F, three pairs of driving-wheels and two-wheel truck. Class G, three pairs of drivers and four-wheel truck. Class H, four pairs of drivers and two-wheel truck. Class A is commonly called a "four-wheeler"; B, a "six-wheeler"; D, an "eight-wheeler," or "American" type; E, "Atlantic" type; F, "Mogul"; G, "ten-wheeler"; H, "Consolidation."

Modern Classification. — The classes shown above, lettered A, B, C, etc., are commonly represented respectively by the symbols 0-4-0; 0-6-0; 0-8-0, 4-4-0; 4-4-2, 2-6-0; 4-6-0; 2-8-0; the first figure being the number of wheels in the truck, the second the driving-wheels, and the third the trailers. Other types are the "Pacific," 4-6-2; the "Prairie," 2-6-2;

and the "Santa Fe," 2-10-2. Engines on the Mallet system, with two locomotive engines under one boiler, are classified 0-8-8-0, 2-6-6-2, etc.

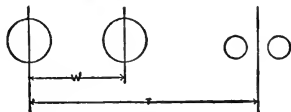
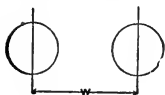
Formulae for Curves. (Baldwin Locomotive Works.)

Approximate Formula for Radius.

$$R = 0.7646 W \div 2 P.$$

Approximate Formula for Swing.

$$WT \div 2 R = S.$$



R = radius of min. curve in feet.

P = play of driving-wheels in decimals of 1 ft.

W = rigid wheel-base in feet.

W = rigid wheel-base.

T = total wheel-base.

R = radius of curve.

S = swing on each side of centre.

Steam-distribution for High-speed Locomotives.

(C. H. Quereau, *Eng'g News*, March 8, 1894.

Balanced Valves. — Mr. Philip Wallis, in 1886, when Engineer of Tests for the C. B. & Q. R. R., reported that while 6 H.P. was required to work unbalanced valves at 40 miles per hour, for the balanced valves 2.2 H.P. only was necessary.

[Later tests were reported by the Master Mechanics' Committee in 1896. Unbalanced valves required from $\frac{3}{4}$ to $2\frac{1}{2}$ per cent of the I.H.P. for their motion, balanced valves from $\frac{1}{3}$ to $\frac{1}{2}$ as much, and piston valves about $\frac{1}{5}$ or $\frac{1}{6}$. Generally in balanced valves, the area of balance = area of exhaust port + area of two bridges + area of one steam port.]

Effect of Speed on Average Cylinder-pressure. — Assume that a locomotive has a train in motion, the reverse lever is placed in the running notch, and the track is level; by what is the maximum speed limited? The resistance of the train and the load increase, and the power of the locomotive decreases with increasing speed till the resistance and power are equal, when the speed becomes uniform. The power of the engine depends on the average pressure in the cylinders. Even though the cut-off and boiler-pressure remain the same, this pressure decreases as the speed increases; because of the higher piston-speed and more rapid valve-travel the steam has a shorter time in which to enter the cylinders at the higher speed. The following table, from indicator-cards taken from a locomotive at varying speeds, shows the decrease of average pressure with increasing speed:

| | | | | | | | | |
|-------------------------------|------|------|------|------|------|------|------|------|
| Miles per hour..... | 46 | 51 | 51 | 53 | 54 | 57 | 60 | 66 |
| Speed, revolutions..... | 224 | 248 | 248 | 258 | 263 | 277 | 292 | 321 |
| Average pressure per sq. in.: | | | | | | | | |
| Actual..... | 51.5 | 44.0 | 47.3 | 43.0 | 41.3 | 42.5 | 37.3 | 36.3 |
| Calculated..... | 46.5 | 46.5 | 44.7 | 43.8 | 41.6 | 39.5 | 35.9 | |

The "average pressure calculated" was figured on the assumption that the mean effective pressure would decrease in the same ratio that the speed increased. The main difference lies in the higher steam-line at the lower speeds, and consequent higher expansion-line, showing that more steam entered the cylinder. The back pressure and compression-lines agree quite closely for all the cards, though they are slightly better for the slower speeds. That the difference is not greater may safely be attributed to the large exhaust-ports, passages, and exhaust tip, which is 5 in. diameter. These are matters of great importance for high speeds.

Boiler-pressure. — Assuming that the train resistance increases as the speed after about 20 miles an hour is reached, that an average of 50 lbs. per sq. in. is the greatest that can be realized in the cylinders of a given engine at 40 miles an hour, and that this pressure furnishes just sufficient power to keep the train at this speed, it follows that, to increase the speed to 50 miles, the mean effective pressure must be increased in the same proportion. To increase the capacity for speed of any locomotive its power must be increased, and at least by as much as the speed is to be increased. One way to accomplish this is to increase the boiler-

pressure. That this is generally realized, is shown by the increase in boiler-pressure in the last ten years. For twenty-three single-expansion locomotives described in the railway journals this year the steam-pressures are as follows: 3, 160 lbs.; 4, 165 lbs.; 2, 170 lbs.; 13 180 lbs.; 1, 190 lbs.

Valve-travel. — An increased average cylinder-pressure may also be obtained by increasing the valve-travel without raising the boiler-pressure, and better results will be obtained by increasing both. The longer travel gives a higher steam-pressure in the cylinders, a later exhaust-opening, later exhaust-closure, and a larger exhaust-opening — all necessary for high speeds and economy. I believe that a 20-in. port and 6½-in. (or even 7-in.) travel could be successfully used for high-speed engines, and that frequently by so doing the cylinders could be economically reduced and the counter-balance lightened. Or, better still, the diameter of the drivers increased, securing lighter counterbalance and better steam-distribution.

Size of Drivers. — Economy will increase with increasing diameter of drivers, provided the work at average speed does not necessitate a cut-off longer than one fourth the stroke. The piston-speed of a locomotive with 62-in. drivers at 55 miles per hour is the same as that of one with 68-in. drivers at 61 miles per hour.

Steam-ports. — The length of steam-ports ranges from 15 in. to 23 in., and has considerable influence on the power, speed, and economy of the locomotive. In cards from similar engines the steam-line of the card from the engine with 23-in. ports is considerably nearer boiler-pressure than that of the card from the engine with 17¼-in. ports. That the higher steam-line is due to the greater length of steam-port there is little room for doubt. The 23-in. port produced 531 H.P. in an 18½-in. cylinder at a cost of 23.5 lbs. of water per I.H.P. per hour. The 17¼ in. port, 424 H.P., at the rate of 22.9 lbs. of water, in a 19-in. cylinder.

Allen Valves. — There is considerable difference of opinion as to the advantage of the Allen ported-valve. (See *Eng. News*, July 6, 1893.)

A Report on the advantage of Allen valves was made by the Master Mechanics' Committee of 1896.

Speed of Railway Trains. — In 1834 the average speed of trains on the Liverpool and Manchester Railway was 20 miles an hour; in 1838 it was 25 miles an hour. But by 1840 there were engines on the Great Western Railway capable of running 50 miles an hour with a train and 80 miles an hour without. (*Trans. A. S. M. E.*, vol. xiii, 363.)

The limitation to the increase of speed of heavy locomotives seems at present to be the difficulty of counterbalancing the reciprocating parts. The unbalanced vertical component of the reciprocating parts causes the pressure of the driver on the rail to vary with every revolution. Whenever the speed is high, it is of considerable magnitude, and its change in direction is so rapid that the resulting effect upon the rail is not inappropriately called a "hammer blow." Heavy rails have been kinked, and bridges have been shaken to their fall under the action of heavily balanced drivers revolving at high speeds. The means by which the evil is to be overcome has not yet been made clear. See paper by W. F. M. Goss, *Trans. A. S. M. E.*, vol. xvi.

Much can be accomplished, however, by carefully designing and proportioning the counter-balance in the wheels and by using light, but strong, reciprocating parts. Pages 41-74 of "Locomotive Operation," gives complete rules and results.

Balanced compound locomotives, with 4 cylinders, the adjacent pistons and crossheads being connected 180° apart have also done much to reduce the disturbance of the moving parts.

Engine No. 999 of the New York Central Railroad ran a mile in 32 seconds equal to 112 miles per hour, May 11, 1893.

$$\left. \begin{array}{l} \text{Speed in} \\ \text{miles per} \\ \text{hour} \end{array} \right\} = \frac{\text{circum. of driving-wheels in in.} \times \text{no. of rev. per min.} \times 60}{63,360}$$

$$= \frac{\text{diam., of driving-wheels in in.} \times \text{no. of rev. per min.} \times .003}{(\text{approximate, giving result } \frac{8}{10} \text{ of 1 per cent too great}).}$$

Performance of a High-speed Locomotive. — The Baldwin compound locomotive No. 1027, on the Phila. & Atlantic City Ry., in 1897 made a record as follows:

For the 52 days the train ran, from July 2d to August 31st, the average time consumed on the run of 55½ miles from Camden to Atlantic City was 48 minutes, equivalent to a uniform rate of speed from start to stop of 69 miles per hour. On July 14th the run from Camden to Atlantic City was made in 46½ min., *an average of 71.6 miles per hour for the total distance.* On 22 days the train consisted of 5 cars and on 30 days it was made up of 6, the weight of cars being as follows: combination car, 57,200 lbs.; coaches, each, 59,200 lbs.; Pullman car, 85,500 lbs.

The general dimensions of the locomotive are as follows: cylinders, 13 and 22 X 26 in.; height of drivers, 84¼ in.; total wheel-base, 26 ft. 7 in.; driving-wheel base, 7 ft. 3 in.; length of tubes, 13 ft.; diameter of boiler, 58¾ in.; diameter of tubes, 1¾ in.; number of tubes, 278; length of fire-box, 1137/8 in.; width of fire-box, 96 in.; heating-surface of fire-box, 136.4 sq. ft.; heating-surface of tubes, 1614.9 sq. ft.; total heating-surface, 1835.1 sq. ft.; tank capacity, 4000 gallons; boiler-pressure, 200 lbs. per sq. in.; total weight of engine and tender, 227,000 lbs.; weight on drivers (about), 78,600 lbs.

Fuel Efficiency of American Locomotives. — Prof. W. M. Goss, as a result of a series of tests run on the Purdue locomotive, finds the disposition of the heat developed by burning coal in a locomotive fire-box to be on the average about as shown in the following table:

Absorbed by steam in the boiler, 52 %; by the superheater, 5 %; total, 57 %. Losses: In vaporizing moisture in the coal, 5 %; discharge of CO., 1 %; high temperature of the products of combustion, 14 %; unconsumed fuel in the form of front-end cinders, 3 %; cinders or sparks passed out of the stack, 9 %; unconsumed fuel in the ash, 4 %; radiation, leakage of steam and water, etc., 7 %. Total losses, 43 %.

It is probable that these losses are considerably less than the losses which are experienced in the average locomotive in regular railway service. — (Bulletin No. 402, U.S. Geol. Survey, 1909.)

Locomotive Link Motion. — Mr. F. A. Halsey, in his work on "Locomotive Link Motion," 1898, shows that the location of the eccentric-rod pins back of the link-arc and the angular vibrations of the eccentric-rods introduce two errors in the motion which are corrected by the angular vibration of the connecting-rod and by locating the saddle-stud back of the link-arc. He holds that it is probable that the opinions of the critics of the locomotive link motion are mistaken ones, and that it comes little short of all that can be desired for a locomotive valve motion. The increase of lead from full to mid gear and the heavy compression at mid gear are both advantages and not defects. The cylinder problem of a locomotive is entirely different from that of a stationary engine. With the latter the problem is to determine the size of the cylinder and the distribution of steam to drive economically a given load at a given speed. With locomotives the cylinder is made of a size which will start the heaviest train which the adhesion of the locomotive will permit, and the problem then is to utilize that cylinder to the best advantage at a greatly increased speed, but under a greatly reduced mean effective pressure.

Negative lead at full gear has been used in the recent practice of some railroads. The advantages claimed are an increase in the power of the engine at full gear, since positive lead offers resistance to the motion of the piston; easier riding; reduced frequency of hot bearings; and a slight gain in fuel economy. Mr. Halsey gives the practice as to lead on several roads as follows, showing great diversity:

| | Full Gear Forward, in. | Full Gear Back, in. | Reversing Gear, in. |
|---|---------------------------|------------------------|------------------------|
| New York, New Haven & Hartford | 1/16 pos. | 1/4 neg. | 1/4 pos. |
| Maine Central | 0 | 1/4 neg. | |
| Illinois Central | 1/32 pos. | | abt 3/16 |
| Lake Shore..... | 1/16 neg. | 9/64 neg. | 5/16 pos. |
| Chicago Great Western..... | 0 | 0 | 3/16 to 9/16 |
| Chicago & Northwestern .. | 3/16 neg. | | 1/4 pos. |

DIMENSIONS OF SOME LARGE AMERICAN LOCOMOTIVES, 1893 AND 1904.

Of the four locomotives described in the table on the next page the first two were exhibited at the Chicago Exposition in 1893. The dimensions are from *Engineering News*, June, 1893. The first, or Decapod engine, has ten-coupled driving-wheels. It is one of the heaviest and most powerful engines built up to that date for freight service. The second is a simple engine, of the standard American 8-wheel type, 4 driving-wheels, and a 4-wheel truck in front. This engine held the world's record for speed in 1893 for short distances, having run a mile in 32 seconds.

The other two engines formed part of the exhibit of the Baldwin Locomotive Works at the St. Louis Exposition in 1904. The Santa Fe type engine has five pairs of driving-wheels, and a two-wheeled truck at the front and at the rear. It is equipped with Vauclain tandem compound cylinders.

Dimensions of Some American Locomotives.
(Baldwin Loco. Wks., 1904-8.)

| Reference Number. | Steam Pressure. | Boilers. | | Tubes. | | | Heating Surface. | | Driving Wheels | Weight, lbs. | |
|-------------------|-----------------|-------------|----------------|--------|-------------|-----------------|-------------------|----------------|----------------|--------------|--------------|
| | | Diam., ins. | Grate, sq. ft. | No. | Diam., ins. | Length, ft. in. | Fire box, sq. ft. | Tubes, sq. ft. | | on Drivers | Total Engine |
| 1 | 150 | 42 | 9 | 97 | 2 | 11 7 | 41 | 586 | 37 | 44,420 | 52,720 |
| 2 | 160 | 50 | 14.6 | 160 | 2 | 10 6 | 75 | 873 | 48 | 72,150 | 84,650 |
| 3 | 200 | 60 | 25.9 | 287 | 2 | 11 7 | 133 | 1733 | 69 | 83,680 | 124,420 |
| 4 | 200 | 62 | 30 | 272 | 2 | 16 1 | 136 | 2279 | 68 | 112,000 | 159,000 |
| 5 | 200 | 76 | 37.2 | 298 | 2 1/4 | 13 10 | 200 | 2414 | 51 | 164,000 | 179,500 |
| 6 | 200 | 68 | 35 | 306 | 2 1/4 | 14 6 | 195 | 2593 | 56 | 166,000 | 186,000 |
| 7 | 200 | 66 | 49.5 | 273 | 2 1/4 | 18 10 | 190 | 3015 | 79 | 101,420 | 193,760 |
| 8 | 200 | 70 | 53.5 | 318 | 2 1/4 | 19 | 195 | 3543 | 79 | 144,600 | 209,210 |
| 9 | 210 | 70 | 55 | 303 | 2 1/4 | 21 | 190 | 3772 | 74 | 151,290 | 230,940 |
| 10 | 225 | 78 | 58.5 | 463 | 2 1/4 | 19 | 210 | 5155 | 57 | 237,800 | 267,800 |
| 11 | 200 | 84 | 68.4 | 401 | 2 1/4 | 21 | 232 | 4941 | 57 | 394,150 | 425,900 |

Type and cylinder size: 1 Mogul, 13×18; 2, Mogul, 16×20; 3, American, 18×24; 4, 10-wheel balanced compound, 16×26 and 26×28; 5, Consolidation, 22×28; 6, Consolidation, 23 and 35×32; 7, Atlantic, 15 and 25×26; 8, Prairie, 17 and 28×28; 9, Pacific, 22×28; 10, Decapod, 19 and 32×32; 11, Mallet, two each 26 and 40×30.

The Mallet Compound Locomotive.—The Mallet articulated locomotive consists principally of two sets of engines flexibly connected under one boiler; the rear, which is a high-pressure engine of two cylinders, fixed rigid with the boiler and receiving the steam direct from the dome. The front or low-pressure engine, also provided with two cylinders, is capable of lateral movement to adjust itself to the curvature of the road on the same general principle as a radial truck. The high-pressure engine exhausts into a receiver flexibly connecting the cylinders of the two sets of engines, from which the low-pressure engine receives its steam supply and is exhausted from the latter through a flexible pipe to the stack. Each cylinder has its independent valve and gear connected to and operated with a common reversing rigging. By this means the tractive power can be doubled over that of the ordinary engine for a given weight of rail with a substantial saving in fuel. (See paper by C. J. Mellin, *Trans. A. S. M. E.*, 1909.)

This type of locomotive is adapted to a wider range of service than perhaps any other design. It was originally intended for narrow-gauge roads of light construction, necessitating sharp curves and steep grades, in combination with light rails. The characteristics of this design are flexibility and uniform distribution of weight combined with the use of two separate engines which would not slip at the same time, and the total weight carried on the drivers, giving great tractive power. The first engine of this class

| | Baldwin. N. Y., L. E. & W. R. R. Decapod Freight. | N. Y. C. & H. R. R. Empire State Express. No. 999. | Baldwin. Santa Fe Type 2-10-2 Freight. | Baldwin. Pacific Type 4-6-2 Passenger. |
|--|---|---|--|---|
| Running-gear: | | | | |
| Driving-wheels, diam. . . | 50 in. | 86 in. | 57 in. | 77 in. |
| Truck " " " " " " | 30 " | 40 " | 29 1/4 & 40" | 33 1/2 & 45" |
| Journals, driving-axles | 9 × 10 in. | 9 × 12 1/2 in. | 11 × 12" | 10 × 12 in. |
| " truck- " " | 5 × 10 " | 6 1/4 × 10 " | 6 1/2 × 10" | 6 × 10 " |
| " tender- " " | 4 1/2 × 9 " | 4 1/8 × 8 " | 7 1/2 × 12" * | 8 × 12 " * |
| Wheel-base: | | | | |
| Driving | 18 ft. 10 in. | 8 ft. 6 in. | 19 ft. 9 in. | 13 ft. 4 in. |
| Total engine | 27 " 3 " | 23 " 11 " | 35 " 11 " | 33 " 4 " |
| " tender | 16 " 8 " | 15 " 2 1/2 " | | |
| " eng. and tender. . . | 53 " 4 " | 47 " 8 1/8 " | 66 ft. 0 in. | 62' 8 3/4 " |
| Wt. in working-order: | | | | |
| On drivers | 170,000 lbs. | 84,000 lbs. | 234,580 lbs. | 141,290 |
| On truck-wheels | 29,500 " | 40,000 " | 52,660 " | 81,230 |
| Engine, total | 192,500 " | 124,000 " | 287,240 " | 222,520 |
| Tender " " " " " " | 117,500 " | 80,000 " | | |
| Eng. and tend., loaded | 310,000 " | 204,000 " | 450,000 " | 357,000 |
| Cylinders: | | | | |
| h.p. (2) | 16 × 28 in. | 19 × 24 in. | 19 × 32 in. | 22 × 28 in. |
| l.p. (2) | 27 × 28 " | | 32 × 32 " | 22 × 28 " |
| Piston-rod, diam. . . . | 4 in. | 3 3/8 in. | | |
| Connecting-rod, l'gth. . | 9' 8 7/16" | 8 ft. 1 1/2 in. | | |
| Steam-ports | 28 1/2 × 2 in. | 1 1/2 × 18 in. | 29 3/4 × 15 8/8" and 13 1/4" | 30 7/8 × 1 1/2" |
| Exhaust-ports | 28 1/2 × 8 " | 23 1/4 × 18 " | 29 3/4 × 6 3/4" | 30 7/8 × 3" |
| Valves, out. lap, h.p. . . | 7/8 in. | 1 in. | 7/8 in. | 1 in. |
| " out. lap, l.p. | 5/8 " | | 3/4 " | |
| " in. lap, h.p. | | 1/10 in. | neg. 1/4 in. | neg. 1/16" |
| " in. lap, l.p. | | | neg. 3/8 " | |
| " max. travel | 6 in. | 5 1/2 in. | 6 in. | 6 in. |
| " lead, h.p. | 1/16 in. | | 0 " | 3/32 in. |
| " lead, l.p. | 5/16 " | | 1/8 " | |
| Boiler.—Type. | Straight | Wagon top | Wagon to | Straight |
| Diam. barrel inside . . . | 6 ft. 2 1/2 in. | 4 ft. 9 in. | 78 3/4 in. | 70 in. |
| Thickness of plates . . . | 3/4 in. | 9/16 in. | 7/8 & 1 1/16" | 1 1/16 in. |
| Height from rail to center line | 8 ft. 0 in. | 7 ft. 11 1/2 in. | | |
| Length of smoke-box . . | 5 " 77/8 " | 4 " 8 " | | |
| Working pressure | 180 lbs. | 190 lbs. | 225 lbs. | |
| Firebox.—type. | Wootten | Buchanan | | |
| Length inside | 10' 11 9/16" | 9 ft. 6 3/8 in. | 108 in. | 108 in. |
| Width " " " " " " | 8 ft. 2 1/8 in. | 3 " 47/8 " | 78 " | 66 " |
| Depth at front | 4 " 6 " | 6 " 11/4 " | 80 1/4 in. | 68 " |
| Thickness side plates . . | 5/16 in. | 5/16 in. | 78 1/4 " | 64 " |
| " back plate | 5/16 " | 5/16 " | 3/8 " | 3/8 " |
| " crown-sheet | 3/8 " | 3/8 " | 3/8 " | 3/8 " |
| " tube sheet | 1/2 " | 1/2 " | 3/8 " | 3/8 " |
| Grate-area | 89.6 sq. ft. | 30.7 sq. ft. | 9/16 " | 1/2 " |
| Stay-bolts, 1 1/8 in. . . . | pitch, 4 1/4 in. | 4 in. | 58.5 sq. ft. | 49.5 sq. ft. |
| Tubes—iron. | 354 | 268 | 391 | 245 |
| Pitch | 23/4 in. | | | |
| Diam., outside | 2 " | 2 in. | 2 1/4 in. | 2 1/4 in. |
| Length | 11 ft. 11 in. | 12 ft. 0 in. | 20 ft. | 20 ft. |
| Heating-surface: | | | | |
| Tubes, exterior | 2,208.8 ft. | 1,697 sq. ft. | 4,586 sq. ft. | 2,874 sq. ft. |
| Fire-box | 234.3 " | 233 " | 210 " | 179 " |
| Miscellaneous: | | | | |
| Exhaust-nozzle, diam. . . | 5 in. | 3 1/2 in. | | |
| Stack, smal'st diam. . . . | 1 ft. 6 in. | 1 ft. 3 1/4 in. | | |
| " height from rail to top | 15 ft. 6 1/2 in. | 14 ft. 10 in. | | |

* Back truck journals.

was built about 1887, and in 1909 there were approximately 500 running in Europe. They are now extensively in use in the United States for the heaviest service. The largest locomotive yet built is described in *Eng. News*, April 29, 1909. It was built by the Baldwin Locomotive Works for use on the heavy grades of the Southern Pacific R.R. The principal dimensions are as follows: Cylinders, 26 and 40 × 30 ins.; valves, balanced piston; boiler (steel): diameter, 84 ins.; thickness, $\frac{13}{16}$ and $\frac{27}{32}$ ins.; working pressure, 200 lbs. per sq. in.; fuel, oil; fire-tubes, 401, $2\frac{1}{4}$ ins. dia. × 21 ft.; firebox: length, 126 ins., width, $78\frac{1}{4}$ ins., depth, front, $75\frac{1}{2}$ ins., depth, back, $70\frac{1}{2}$ ins.; water spaces, 5 ins.; grate area, 68.4 sq. ft.; feed-water heater: length, 63 ins., tubes, 401, $2\frac{1}{4}$ ins. dia.; heating surface: firebox, 232 sq. ft., fire-tubes, 4941 sq. ft., feed-water heater tubes, 1220 sq. ft.; smokebox superheater, 655 sq. ft.; wheels: driving (16), 57 ins. O. dia., main journals, 11×12 ins., other journals, 10×12 ins.; truck (4), $30\frac{1}{2}$ ins. dia., journals, 6×10 ins.; tender (8), $33\frac{1}{2}$ ins. dia., journals, 6×11 ins.; wheelbase: driving, 39 ft. 4 ins., rigid, 15 ft., total engine, 56 ft. 7 ins., total engine and tender, 83 ft. 6 ins.; length over all, 93 ft. $6\frac{1}{2}$ ins.; weight: on drivers, 394,150 lbs., on front truck, 14,500 lbs., on back truck, 17,250 lbs., total engine 425,900 lbs., total engine and tender 596,000 lbs.; tender: water tank capy., 9000 gals., oil tank capy., 2850 gals.

Indicated Water Consumption of Single and Compound Locomotive Engines at Varying Speeds.

C. H. Quereau, *Eng'g News*, March 8, 1894.

| Two-cylinder Compound. | | | Single-expansion. | | |
|------------------------|------------------------|----------------------------|-------------------|-----------------|--------|
| Revolutions. | Speed, miles per hour. | Water per I.H.P. per hour. | Revolutions. | Miles per Hour. | Water. |
| 100 to 150 | 21 to 31 | 18.33 lbs. | 151 | 31 | 21.70 |
| 150 to 200 | 31 to 41 | 18.9 lbs. | 219 | 45 | 20.91 |
| 200 to 250 | 41 to 51 | 19.7 lbs. | 253 | 52 | 20.52 |
| 250 to 275 | 51 to 56 | 21.4 lbs. | 307 | 63 | 20.23 |
| | | | 321 | 66 | 20.01 |

It appears that the compound engine is the more economical at low speeds, the economy decreasing as the speed increases, and that the single engine increases in economy with increase of speed within ordinary limits, becoming more economical than the compound at speeds of more than 50 miles per hour.

The C, B. & Q. two-cylinder compound, which was about 30% less economical than simple engines of the same class when tested in passenger service, has since been shown to be 15% more economical in freight service than the best single-expansion engine, and 29% more economical than the average record of 40 simple engines of the same class on the same division.

The water rate is also affected by the cut-off; the following table gives what we should consider *very good* results in practice, though better (i.e. lower results) have occasionally been obtained. (G. R. Henderson, 1906.)

| | | | | | |
|---|------------------|------------------|------------------|------------------|-----|
| Cut-off per cent of stroke | 10 | 20 | 30 | 40 | 50 |
| Lbs. water per I.H.P. hour — simple . . | 26 | 23 | 22 | 22 | 23 |
| Lbs. water per I.H.P. hour — compound . . | .. | .. | 18 | 18 | 18 |
| Cut-off per cent of stroke | 60 | 70 | 80 | 90 | 100 |
| Lbs. water per I.H.P. hour — simple . . | 24 | 26 | 29 | 33 | 38 |
| Lbs. water per I.H.P. hour — compound | 18 $\frac{1}{2}$ | 19 $\frac{1}{2}$ | 20 $\frac{1}{2}$ | 22 $\frac{1}{2}$ | 25 |

Indicator-tests of a Locomotive at High Speed. (*Locomotive Eng'g*, June, 1893.) — Cards were taken by Mr. Angus Sinclair on the locomotive drawing the Empire State Express.

RESULTS OF INDICATOR-DIAGRAMS.

| Card No. | Revs. | Miles per hour. | I.H.P. | Card No. | Revs. | Miles per hour. | I.H.P. |
|----------|-------|--------------------|--------|----------|-------|--------------------|--------|
| 1 | 160 | 37.1 | 648 | 7 | 304 | 70.5 | 977 |
| 2 | 260 | 60.8 | 728 | 8 | 296 | 68.6 | 972 |
| 3 | 190 | 44 | 551 | 9 | 300 | 69.6 | 1,045 |
| 4 | 250 | 58 | 891 | 10 | 304 | 70.5 | 1,059 |
| 5 | 260 | 60 | 960 | 11 | 340 | 78.9 | 1,120 |
| 6 | 298 | 69 | 983 | 12 | 310 | 71.9 | 1,026 |

The locomotive was of the eight-wheel type, built by the Schenectady Locomotive Works, with 19 × 24 in. cylinders, 78-in. drivers, and a large boiler and fire-box. Details of important dimensions are as follows: Heating-surface of fire-box, 150.8 sq. ft.; of tubes, 1670.7 sq. ft.; of boiler, 1821.5 sq. ft. Grate area, 27.3 sq. ft. Fire-box: length, 8 ft.; width, 3 ft. 47/8 in. Tubes, 268; outside diameter, 2 in. Ports: steam, 18 × 11/4 in.; exhaust, 18 × 23/4 in. Valve-travel, 5 1/2 in. Outside lap, 1 in.; inside lap, 1/64 in. Journals: driving-axle, 8 1/2 × 10 1/2 in.; truck-axle, 6 × 10 in.

The train consisted of four coaches, weighing, with estimated load, 340,000 lbs. The locomotive and tender weighed in working order 200,000 lbs., making the total weight of the train about 270 tons. During the time that the engine was first lifting the train into speed diagram No. 1 was taken. It shows a mean cylinder-pressure of 59 lbs. According to this, the power exerted on the rails to move the train is 6553 lbs., or 24 lbs. per ton. The speed is 37 miles an hour. When a speed of nearly 60 miles an hour was reached the average cylinder-pressure is 40.7 lbs., representing a total traction force of 4520 lbs., without making deductions for internal friction. If we deduct 10% for friction, it leaves 15 lbs. per ton to keep the train going at the speed named. Cards 6, 7, and 8 represent the work of keeping the train running 70 miles an hour. They were taken three miles apart, when the speed was almost uniform. The average cylinder-pressure for the three cards is 47.6 lbs. Deducting 10% again for friction, this leaves 17.6 lbs. per ton as the power exerted in keeping the train up to a velocity of 70 miles. Throughout the trip 7 lbs. of water were evaporated per lb. of coal. The work of pulling the train from New York to Albany was done on a coal consumption of about 3 1/8 lbs. per H.P. per hour. The highest power recorded was at the rate of 1120 H.P.

Locomotive-testing Apparatus at the Laboratory of Purdue University. (W. F. M. Goss, *Trans. A. S. M. E.*, vol. xiv, 826.)—The locomotive is mounted with its drivers upon supporting wheels which are carried by shafts turning in fixed bearings, thus allowing the engine to be run without changing its position as a whole. Load is supplied by four friction-brakes fitted to the supporting shafts and offering resistance to the turning of the supporting wheels. Traction is measured by a dynamometer attached to the draw-bar. The boiler is fired in the usual way, and an exhaust-blower above the engine, but not in pipe connection with it, carries off all that may be given out at the stack.

A Standard Method of Conducting Locomotive-tests is given in a report by a Committee of the A. S. M. E. in vol. xiv of the Transactions, page 1312.

Locomotive Tests of the Penna. R. R. Co.—Eight locomotives were tested in the dynamometer testing plant built by the P. R. R. Co. at the St. Louis Exhibition in 1903. Among the principal results obtained and conclusions derived are the following:

BOILER PERFORMANCE.

| | | | | | | | | |
|---|----------|----------|----------|---------|---------|-----------|---------|--|
| Coal per sq. ft. grate per hour, lbs. | 20 | 40 | 60 | 80 | 100 | 120 | | |
| Equiv. evap. per sq. ft. H. S. per hour | 3-5 | 5-7.5 | 7-10 | 8.2-12 | 10.4-14 | 11.4-15.3 | | |
| Coal per sq. ft. H. S. per hour | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | |
| Equiv. evap. per lb. dry coal | 10-11.5 | 9-10.5 | 8.2-9.7 | 7.7-9.1 | 7.1-8.5 | 6.6-8.1 | 6.2-7.7 | |
| Equiv. evap. per sq. ft. H. S. per hour | 4 | 6 | 8 | 10 | 12 | 14 | | |
| Equiv. evap. per lb. dry coal | 9.7-12.1 | 8.8-11.3 | 7.8-10.5 | 6.8-9.6 | 5.8-8.8 | 5.5-8 | | |

The coal used in these tests was a semi-bituminous, containing 16.25% volatile combustible, 7.00% ash and 0.90% moisture.

The maximum boiler capacity ranged from 8½ to more than 16 lbs. of water evaporated per hour per sq. ft. of heating surface. Little or no advantage was found in the use of *Serve* or ribbed tubes.

The boiler efficiency decreases as the rate of power developed increases.

Furnace losses due to excess air are no greater with large grates properly fired than with smaller ones. The boilers with small grates were inferior in capacity to those with large grates.

No special advantage is derived from large fire-box heating surface; the tube heating surface is effective in absorbing heat not taken up by the fire-box.

ENGINE PERFORMANCE.

Maximum I.H.P., four freight locomotives, 1041, 1050, 1098, 1258

Maximum I.H.P., four passenger locomotives, 816, 945, 1622, 1641

| | Kind of Locomotive. | | |
|---|---------------------|-------------------|---------------------|
| | Simple Freight. | Compound Freight. | Compound Passenger. |
| Minimum water per I.H.P. hour..... | 23.67 | 20.26 | 18.86 |
| Water per I.H.P. hr. at maximum load..... | 23.83 | 22.03 | 21.39 |
| Water per I.H.P. hr. at max. consumption... | 28.95 | 25.31 | 24.41 |

The steam consumption of simple locomotives operating at all speeds and cut-offs commonly employed on the road falls between the limits of 23.4 and 28.3 lbs. per I.H.P. hour; compound locomotives between 18.6 and 27 lbs.; and with superheating the minimum steam consumption was reduced to 16.6 lbs. of superheated steam.

Comparing a simple and a compound locomotive, the simple engine used 40% more steam than the compound at 40 revs. per min., 27% more at 80 revs., and only 7% more at 160 revs. per min.

The frictional resistance of the engines showed an extreme variation ranging from 6 to 38% of the indicated horse-power. The frictional losses increased rapidly at speeds in excess of 160 revs. per min. It appears that the matter of machine friction is closely related to that of lubrication. With oil lubrication a stress at the draw-bar of approximately 500 lbs. is required to overcome the friction of each coupled axle, while with grease the required force is from 800 to 1100 lbs.

The lowest figures for dry coal consumed per dynamometer H.P. hour were approximately as follows:

| | | | | |
|--|------|------|------|------|
| Revs. per min. | 40 | 80 | 160 | 240 |
| Compound freight engine, { lbs. coal | 2.10 | 2.25 | 3.25 | |
| { D.H.P. | 500 | 800 | 800 | |
| Compound passenger engine, { lbs. coal | | 2.8 | 2.3 | 3.0 |
| { D.H.P. | | 600 | 900 | 1000 |

A complete report of the St. Louis locomotive tests is contained in a book of 734 pages and over 800 illustrations, published by the Penna. R.R. Co., Philadelphia, 1906. See also pamphlet on *Locomotive Tests*, published by Amer. Locomotive Co., New York, 1906, and *Trans. A. S. M. E.*, xxvii, 610.

Weights and Prices of Locomotives, 1885 and 1905.

(Baldwin Loco. Wks.)

| 1885 | Type. | W'ght | Price | Price per lb. | 1905 | Type. | W'ght | Price | Price per lb. |
|------|---------------|--------|---------|---------------|------|-----------------|---------|---------|---------------|
| | | | | | | | | | |
| | American.... | 80,857 | \$6,695 | \$.0828 | | American..... | 102,000 | \$9,410 | \$.092 |
| | Mogul..... | 72,800 | 6,662 | .0912 | | Atlantic..... | 187,200 | 15,750 | .083 |
| | Ten wheel... | 85,000 | 7,583 | .0892 | | Pacific..... | 227,000 | 15,830 | .070 |
| | Consolidation | 92,400 | 7,888 | .0854 | | Ten wheel..... | 156,000 | 13,690 | .088 |
| | | | | | | Consolidation . | 192,460 | 14,500 | .075 |

The price per pound is figured from the weight of the engine in working order, without the tender.

Depreciation of Locomotives.—(Baldwin Loco. Wks.)—It is suggested that for the first five years the full second-hand value of the locomotive (75% of first cost) be taken; for the second five years 85% of this value; for the third five years, 70%; after 15 years, 50% of the second-hand value; and after 20 years, and as long as the engine remains in use, 25% of the first cost.

The Average Train Loads of 14 railroads increased from 229 tons of 2000 lbs. in 1895 to 385 tons in 1904. On the Chicago, Milwaukee & St. Paul Ry. the average load increased from 152 tons in 1895 to 281 tons in 1903, and on the Lake Shore & Michigan Southern Ry. from 318 tons in 1895 to 615 tons in 1903. In the same time the average cost of transportation per ton mile on the C., M. & St. P. Ry. decreased from 0.67 to 0.58 cent; and on the L. S. & M. S. Ry. increased from 0.39 to 0.41 cent, the decrease in cost due to heavier train loads being offset by higher cost for labor and material.

Traction Force of Locomotives, 1893 and 1905.

(Baldwin Loco. Wks.)

| Passenger, 1893. | Weight on Driver. | Traction Force. | Passenger, 1905. | Weight on Driver. | Traction Force. |
|------------------------|-------------------|-----------------|---------------------------|-------------------|-----------------|
| American, single-ex. | 75,210 | 17,270 | Atlantic, comp. | 101,420 | 22,180 |
| American, comp. | 83,860 | 12,900 | Atlantic, single-ex. . . | 103,600 | 23,800 |
| American, single-ex. . | 64,560 | 15,550 | Pacific, single-ex. . . . | 141,290 | 29,910 |
| American, comp. | 78,480 | 14,050 | Pacific, single-ex. . . . | 114,890 | 25,610 |
| Ten-wheel type, com. | 93,850 | 16,480 | Atlantic, single-ex. . . | 80,930 | 21,740 |
| Average. | | 15,250 | | | 24,648 |
| Freight, 1893. | | | Freight, 1905. | | |
| Consolidation, comp. | 120,600 | 21,190 | Sante Fe type, comp. | 234,580 | 62,740 |
| Ten-wheel, s'gle-ex. . | 101,000 | 23,310 | Consol., 2-cyl. comp. . | 166,000 | 40,200 |
| Mogul, single-ex. | 91,340 | 21,030 | Consol., single-ex. | 151,490 | 40,150 |
| Decapod, compound | 172,000 | 35,580 | Consol., single-ex. | 171,560 | 44,080 |
| | | | Consol., single-ex. | 165,770 | 45,170 |
| Average. | | 25,277 | | | 46,468 |

Waste of Fuel in Locomotives.—In American practice economy of fuel is necessarily sacrificed to obtain greater economy due to heavy train-loads. D. L. Barnes, in *Eng. Mag.*, June, 1894, gives a diagram showing the reduction of efficiency of boilers due to high rates of combustion, from which the following figures are taken:

| | | | | | | |
|--|----|----|----|-----|-----|-----|
| Lbs. of coal per sq. ft. of grate per hour . . . | 12 | 40 | 80 | 120 | 160 | 200 |
| Per cent efficiency of boiler. | 80 | 75 | 67 | 59 | 51 | 43 |

A rate of 12 lbs. is given as representing stationary-boiler practice, 40 lbs. English locomotive practice, 120 lbs. average American, and 200 lbs. maximum American, locomotive practice.

Pages 473 and 475 of Henderson's "Locomotive Operation" give diagrams of evaporation per lb. of various kinds of coal for different rates of combustion per sq. ft. grate area and heating surface.

Advantages of Compounding.—Report of a Committee of the American Railway Master Mechanics' Association on Compound Locomotives (*Am. Mach.*, July 3, 1890) gives the following summary of the advantages gained by compounding: (a) It has achieved a saving in the fuel burnt averaging 18% at reasonable boiler-pressures, with encouraging possibilities of further improvement in pressure and in fuel and water economy. (b) It has lessened the amount of water (dead weight) to be

hauled, so that (c) the tender and its load are materially reduced in weight. (d) It has increased the possibilities of speed far beyond 60 miles per hour, without unduly straining the motion, frames, axles, or axle-boxes of the engine. (e) It has increased the haulage-power at full speed, or, in other words, has increased the continuous H.P. developed, per given weight of engine and boiler. (f) In some classes has increased the starting-power. (g) It has materially lessened the slide-valve friction per H.P. developed. (h) It has equalized or distributed the turning force on the crank-pin, over a longer portion of its path, which, of course, tends to lengthen the repair life of the engine. (i) In the two-cylinder type it has decreased the oil consumption, and has even done so in the Woolf four-cylinder engine. (j) Its smoother and steadier draught on the fire is favorable to the combustion of all kinds of soft coal; and the sparks thrown being smaller and less in number, it lessens the risk to property from destruction by fire. (k) These advantages and economies are gained without having to improve the man handling the engine, less being left to his discretion (or careless indifference) than in the simple engine. (l) Valve-motion, of every locomotive type, can be used in its best working and most effective position. (m) A wider elasticity in locomotive design is permitted; as, if desired, side-rods can be dispensed with or articulated engines of 100 tons weight, with independent trucks, used for sharp curves on mountain service, as suggested by Mallet and Brunner.

Of 27 compound locomotives in use on the Phila. and Reading Railroad (in 1892), 12 are in use on heavy mountain grades, and are designed to be the equivalent of 22 × 24 in. simple consolidations; 10 are in somewhat lighter service and correspond to 20 × 24 in. consolidations; 5 are in fast passenger service. The monthly coal record shows:

| Class of Engine. | No. | Gain in Fuel Economy. |
|----------------------------|-----|-----------------------|
| Mountain locomotives..... | 12 | 25% to 30% |
| Heavy freight service..... | 10 | 12% to 17% |
| Fast passenger..... | 5 | 9% to 11% |

(Report of Com. A. R. M. M. Assn. 1892.) For a description of the various types of compound locomotive, with discussion of their relative merits, see paper by A. Von Borries, of Germany, the Development of the Compound Locomotive, *Trans. A. S. M. E.*, 1893, vol. xiv, p. 1172.

As a rule compounds cost considerably more for repairs, and require a better class of engineers and machinists to obtain satisfactory results. (Henderson.)

Balanced Compound Locomotives.—There are two high-pressure cylinders placed between the frames and two low-pressure cylinders outside. The inside crank shaft has cranks 90° apart, and each outside crank pin is 180° from the inside crank pin on the same side, so that the engine on each side is perfectly balanced. The balanced piston valve is so made that high-pressure steam may be admitted to the low-pressure cylinder for starting. See circular of the Baldwin Loco. Wks., No. 62, 1907.

Superheating in Locomotives. (*R. R. Age Gazette*, Nov. 20, 1908.)—Superheating steam in locomotives has been found to effect a saving of 10 to 15% in the fuel consumption of a locomotive, and 8 to 12% of the water used, or with the same fuel to increase the horse-power and the tractive force. The Baldwin Locomotive Works builds a superheater in the smoke-box, where it utilizes part of the heat of the waste gases in drying the steam and superheating it 50 to 100° F. The heating surface of the superheater is from 12 to 22% of the heating surface in the tubes and fire-box of the boiler. It is recommended to use a boiler pressure of about 160 lbs. when a superheater is used, and to have cylinders of larger dimensions than when ordinary steam of 200 lbs. pressure is used. For an illustrated and historical description of the use of superheating in locomotives, see paper by H. H. Vaughan, read before the Am. Ry. Mast. Mechs.' Assn., *Eng. News*, June 22, 1905.

Counterbalancing Locomotives.—Rules for counterbalancing, adopted by different locomotive-builders, are quoted in a paper by Prof. Lanza (*Trans. A. S. M. E.*, x, 302.) See also articles on Counterbalancing Locomotives, in *R. R. & Eng. Jour.*, March and April, 1890; *Trans. A. S. M. E.*, vol. xvi, 305; and *Trans. Am. Ry. Master Mechanics' Assn.*,

1897. W. E. Dalby's book on the "Balancing of Engines" (Longmans, Green & Co., 1902) contains a very full discussion of this subject. See also Henderson's "Locomotive Operation" (*The Railway Age*, 1904).

Narrow-gauge Railways in Manufacturing Works. — A tramway of 18 inches gauge, several miles in length, is in the works of the Lancashire and Yorkshire Railway. Curves of 13 feet radius are used. The locomotives used have the following dimensions (*Proc. Inst. M. E.*, July, 1888): The cylinders are 5 in. in diameter with 6 in. stroke, and 2 ft. 3 1/4 in. centre to centre. Wheels 16 1/4 in. diameter, the wheel-base 2 ft. 9 in.; the frame 7 ft. 4 1/4 in. long, and the extreme width of the engine 3 feet. Boiler, of steel, 2 ft. 3 in. outside diam. and 2 ft. long between tube-plates, containing 55 tubes of 1 3/8 in. outside diam.; fire-box, of iron and cylindrical, 2 ft. 3 in. long and 17 in. inside diam. Heating-surface 10.42 sq. ft. in the fire-box and 36.12 in the tubes, total 46.54 sq. ft.; grate-area, 1.78 sq. ft.; capacity of tank, 26 1/2 gallons; working-pressure, 170 lbs. per sq. in. tractive power, say, 1412 lbs., or 9.22 lbs. per lb. of effective pressure per sq. in., on the piston. Weight, empty, 2.80 tons; full and in working order, 3.19 tons.

For description of a system of narrow-gauge railways for manufacturing, see circular of the C. W. Hunt Co., New York.

Light Locomotives. — For dimensions of light locomotives used for mining, etc., and for much valuable information concerning them, see catalogue of H. K. Porter Co., Pittsburgh.

Petroleum-burning Locomotives. (From Clark's Steam-engine.) — The combustion of petroleum refuse in locomotives has been successfully practised by Mr. Thos. Urquhart, on the Grazi and Tsaritsin Railway, Southeast Russia. Since November, 1884, the whole stock of 143 locomotives under his superintendence has been fired with petroleum refuse. The oil is injected from a nozzle through a tubular opening in the back of the fire-box, by means of a jet of steam, with an induced current of air.

A brickwork cavity or "regenerative or accumulative combustion-chamber" is formed in the fire-box, into which the combined current breaks as spray against the rugged brickwork slope. In this arrangement the brickwork is maintained at a white heat, and combustion is complete and smokeless. The form, mass, and dimensions of the brickwork are the most important elements in such a combination.

Compressed air was tried instead of steam for injection, but no appreciable reduction in consumption of fuel was noticed.

The heating-power of petroleum refuse is given as 19,832 heat-units, equivalent to the evaporation of 20.53 lbs. of water from and at 212° F., or to 17.1 lbs. at 8 1/2 atmospheres, or 125 lbs. per sq. in., effective pressure. The highest evaporative duty was 14 lbs. of water under 8 1/2 atmospheres per lb. of the fuel, or nearly 82% efficiency.

There is no probability of any extensive use of petroleum as fuel for locomotives in the United States, on account of the unlimited supply of coal and the comparatively limited supply of petroleum. Texas and California oils are now (1902) used in locomotives of the Southern Pacific Railway and the Santa Fé System.

Self-propelled Railway Cars. — The use of single railway cars containing a steam or gasoline motor has become quite common in Europe. For a description of different systems see a paper on European Railway Motor Cars by B. D. Gray in *Trans. A. S. M. E.*, 1907.

Fireless Locomotive. — The principle of the Francq locomotive is that it depends for the supply of steam on its spontaneous generation from a body of heated water in a reservoir. As steam is generated and drawn off the pressure falls; but by providing a sufficiently large volume of water heated to a high temperature, at a pressure correspondingly high, a margin of surplus pressure may be secured, and means may thus be provided for supplying the required quantity of steam for the trip.

The fireless locomotive designed for the service of the Metropolitan Railway of Paris has a cylindrical reservoir having segmental ends, about 5 ft. 7 in. in diameter, 26 1/4 ft. in length, with a capacity of about 620 cubic feet. Four-fifths of the capacity is occupied by water, which is heated by the aid of a powerful jet of steam supplied from stationary boilers. The water is heated until equilibrium is established between the boilers and the reservoir. The temperature is raised to about 390° F., corresponding to 225 lbs. per sq. in. The steam from the reservoir is

passed through a reducing-valve, by which the steam is reduced to the required pressure. It is then passed through a tubular superheater situated within the receiver at the upper part, and thence through the ordinary regulator to the cylinders. The exhaust-steam is expanded to a low pressure, in order to obviate noise of escape. In certain cases the exhaust-steam is condensed in closed vessels, which are only in part filled with water.

In working off the steam from a pressure of 225 lbs. to 67 lbs., 530 cubic feet of water at 390° F. is sufficient for the traction of the trains, for working the circulating-pump for the condensers, for the brakes, and for electric-lighting of the train. At the stations the locomotive takes from 2200 to 3300 lbs. of steam — nearly the same as the weight of steam consumed during the run between two consecutive charging stations. There is 210 cubic feet of condensing water. Taking the initial temperature at 60° F., the temperature rises to about 180° F. after the longest runs underground.

The locomotive has ten wheels, on a base 24 ft long, of which six are coupled, 4½ ft. in diameter. The extreme wheels are on radial axles. The cylinders are 23½ in. in diameter, with a stroke of 23½ in.

The engine weighs, in working order, 53 tons, of which 35 tons are on the coupled wheels. The speed varies from 15 miles to 25 miles per hour. The trains weigh about 140 tons.

Compressed-air Locomotives. — A compressed-air locomotive consists essentially of a storage tank mounted upon driving wheels, with two engines similar to those of a steam locomotive. One or more reservoirs or storage tanks are located on the line, from which the locomotive tank is charged. These reservoirs are usually riveted steel cylinders, designed for about 1000 lbs. working pressure; but sometimes seamless steel cylinders of small diameter, designed for a working pressure of 2000 lbs. or upwards, are used. The customary maximum pressure in the locomotive tank is 500 lbs. gauge, and the working pressure in the cylinders is from 130 to 140 lbs. The following table is condensed from one in a circular of the Baldwin Locomotive Works, No. 46, 1964.

See account of the Mekarski compressed-air locomotives, page 624 *ante*.

DIMENSIONS AND TRACTIVE POWER OF FOUR COUPLED COMPRESSED-AIR LOCOMOTIVES HAVING TWO STORAGE TANKS.

| Class..... | 4-4-C | 4-6-C | 4-8-C | 4-10-C | 4-12-C | 4-16-C | 4-18-C |
|-------------------------------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Cylinders, inches | 5×10 | 6×10 | 7×12 | 8×14 | 9×14 | 11×14 | 12×16 |
| Diam. of drivers..... | 22" | 24" | 24" | 26" | 28" | 28" | 30" |
| Wheel base..... | 4' 0" | 4' 3" | 4' 5" | 5' 3" | 5' 5" | 5' 6" | 6' 0" |
| Approx. weight, lbs.. | 10,000 | 14,000 | 18,000 | 23,000 | 27,000 | 37,000 | 44,000 |
| Inside dia. of tanks.. | 26" | 28" | 30" | 32" | 34" | 38" | 40" |
| Aggregate tank vol., cu. ft..... | 75 | 100 | 130 | 170 | 200 | 280 | 320 |
| App. height..... | 4' 5" | 4' 10" | 5' 0" | 5' 4" | 5' 8" | 6' 0" | 6' 4" |
| App. width over tanks..... | 4' 10" | 5' 2" | 5' 6" | 5' 10" | 6' 3" | 7' 0" | 7' 4" |
| App. width over cyl- inders..... | Gauge +24" | Gauge +26" | Gauge +27" | Gauge +28" | Gauge +30" | Gauge +32" | Gauge +33" |
| App. length over bumpers..... | 12' 0" | 14' 0" | 15' 0" | 17' 0" | 18' 0" | 20' 0" | 20' 6" |
| Full stroke..... | 1350 | 1785 | 2915 | 4100 | 4820 | 7200 | 9140 |
| ¾ Stroke cut-off | 1290 | 1700 | 2780 | 3900 | 4580 | 6860 | 8705 |
| ½ Stroke cut-off | 940 | 1240 | 2025 | 2840 | 3345 | 4995 | 6340 |
| ¼ Stroke cut-off | 510 | 670 | 1100 | 1540 | 1815 | 2710 | 3440 |

Draw-bar pull on any grade = tractive power - (.0075 + % of grade) × weight of engine.

Working pressure in cylinders 140 lbs.; tank storage pressure, 800 lbs.

Other sizes of engines are 5½ × 10 in., 6 × 12 in., and 8 × 12 in., 24-in. diam. of drivers; 9 × 14 in., 26-in. drivers, and 10 × 14 in., 28-in. drivers.

CUBIC FEET OF AIR, AT DIFFERENT STORAGE PRESSURES, REQUIRED TO HAUL ONE TON ONE MILE AT HALF STROKE CUT-OFF, WITH 20, 30 AND 40 LBS. FRICTIONAL RESISTANCE PER TON. (Baldwin Loco. Wks.)

| Storage pressure | | 600 | 700 | 800 | | 600 | 700 | 800 | | 600 | 700 | 800 |
|--------------------------------|-------|------|------|------|-------|------|------|------|-------|------|------|------|
| Cylinder working pressure..... | | 130 | 135 | 140 | | 130 | 135 | 140 | | 130 | 135 | 140 |
| Grade. | R | V | V | V | R | V | V | V | R | V | V | V |
| Level..... | 20.0 | 1.16 | 0.99 | 0.87 | 30.0 | 1.74 | 1.50 | 1.31 | 40.0 | 2.33 | 1.99 | 1.74 |
| 1/2 %..... | 31.2 | 1.81 | 1.56 | 1.36 | 41.2 | 2.40 | 2.05 | 1.79 | 51.2 | 2.98 | 2.56 | 2.23 |
| 1 %..... | 42.4 | 2.47 | 2.12 | 1.85 | 52.4 | 3.05 | 2.61 | 2.28 | 62.4 | 3.64 | 3.11 | 2.73 |
| 2 %..... | 64.8 | 3.78 | 3.24 | 2.83 | 74.8 | 4.35 | 3.73 | 3.26 | 84.8 | 4.94 | 4.24 | 3.70 |
| 3 %..... | 87.2 | 5.08 | 4.35 | 3.81 | 97.2 | 5.67 | 4.86 | 4.25 | 107.2 | 6.25 | 5.35 | 4.69 |
| 4 %..... | 109.6 | 6.39 | 5.48 | 4.79 | 119.6 | 6.97 | 5.97 | 5.22 | 129.6 | 7.56 | 6.47 | 5.67 |
| 5 %..... | 132.0 | 7.69 | 6.60 | 5.77 | 142.0 | 8.27 | 7.09 | 6.20 | 152.0 | 8.86 | 7.60 | 6.64 |

R=resistance per ton of 2240 lbs. in pounds. V=cubic feet of air.

Air Locomotives with Compound Cylinders and Atmospheric Interheaters are built by H. K. Porter Co. The air enters the high-pressure cylinder at 250 lbs. gauge pressure and is expanded down to 50 lbs., overcoming resistance, while the temperature drops about 140° F. This loss of heat is practically all restored in the atmospheric interheater, which is a cylindrical reservoir filled with brass tubes located in the passage-way from the high- to the low-pressure cylinder. The air enters the low-pressure cylinder at 50 lbs. gauge and a temperature within 10 or 20° of that of the surrounding atmosphere. The exhaust is used to induce a draught of atmospheric air through the tubes of the interheater. This combination permits of expanding the air from 250 lbs. down to atmosphere without unmanageable refrigeration.

The following calculation shows the relative economy of a single-cylinder locomotive using air at 150 lbs. and of a compound using air at 250 lbs. in the high-pressure and 50 lbs. in the low-pressure cylinder, non-expansive working being assumed in both cases.

11.2 cu. ft. of free air at 150 lbs. gauge and atmospheric temperature would fill a cylinder of 1 cu. ft. capacity, and in moving a piston of 1 sq. ft. area one foot would develop $144 \times 150 = 21,600$ ft. lbs. of energy.

11.2 cu. ft. of free air at 250 lbs. gauge if used in a cylinder 0.623 sq. ft. area and 1 ft. stroke would develop $0.623 \times 144 \times 250 = 22,425$ ft. lbs.

If expanded in two cylinders with a ratio of 4 to 1 the energy developed would be $0.623 \times 144 \times 200$ plus $4 \times 0.623 \times 144 \times 50 = 35,880$ ft. lbs., if the heat is restored between the two cylinders. Gain by compounding with interheating, over simple cylinders with 150 lbs. initial pressure, $35,880 \div 21,600 = 1.66$.

These results are about the best that can be obtained with either simple or compound locomotives, as any improvement due to expansive working just about balances the losses due to clearance and initial refrigeration. The work done per cubic foot of free air in the two systems is: with simple cylinders, $21,600 \div 11.2 = 1840$ ft. lbs.; with compound cylinders and atmospheric interheater, $35,880 \div 11.2 = 3205$ ft. lbs.

The above calculations have been practically confirmed by actual tests, which show 1900 ft. lbs. of work per cubic foot of free air with the simple locomotive and 3000 ft. lbs. with the compound, the gain due to expansive working and the losses due to internal friction being somewhat greater in the compound than in the simple machine.

In the operation of compressed-air locomotives the air compressor is generally delivering compressed air at a pressure fluctuating between 800 and 1000 lbs. per sq. in. into the storage reservoir, and it requires an average of about 12,000 ft. lbs. per cubic foot of free air to compress and deliver it at these pressures. The efficiency of the two systems then is: $1900 \div 12000 = 16\%$ for the simple locomotive, and $3000 \div 12000 = 25\%$ for the compound with atmospheric interheater.

SHAFTING.

(See also TORSIONAL STRENGTH; also SHAFTS OF STEAM ENGINES.)

For shafts subjected to torsion only, let d = diam. of the shaft in ins., P = a force in lbs. applied on a lever arm at a distance = a ins. from the axis, S = shearing resistance at the outer fiber, in lbs. per sq. in., then

$$Pa = \frac{\pi d^3 S}{16} = \frac{d^3 S}{5.1} = 0.1936 d^3 S; \quad d = \sqrt[3]{\frac{5.1 Pa}{S}} = \sqrt[3]{\frac{Pa}{K}}$$

If R = revolutions per minute, then the horse-power transmitted =

$$\text{H.P.} = \frac{Pa \ 2 \ \pi R}{33,000 \times 12} = \frac{\pi d^3 S \times 2 \ \pi R}{16 \times 33,000 \times 12} = \frac{RSd^3}{321,000};$$

$$d = \sqrt[3]{\frac{321,000 \text{ H.P.}}{RS}} = \sqrt[3]{\frac{C \times \text{H.P.}}{R}}$$

In practice, empirical values are given to S and to the coefficients $K = 5.1/S$ and $C = 321,000/S$, according to the factor of safety assumed, depending on the material, on whether the shaft is subjected to steady, fluctuating, bending, or reversed strains, on the distance between bearings, etc. Kimball and Barr (Machine Design) state that the following factors of safety are indicated by successful practice: For head shafts, 15; for line shafts carrying pulleys, 10; for small short shafts, countershafts, etc., 7. For steel shafting the allowable stress, S , for the above factors would be about 4000, 6000 and 8500 lbs. respectively, whence

for head shafts $d = \sqrt[3]{\frac{80 \text{ H.P.}}{R}}$; for line shafts $d = \sqrt[3]{\frac{53 \text{ H.P.}}{R}}$; for short

shafts $d = \sqrt[3]{\frac{38 \text{ H.P.}}{R}}$

Jones & Laughlin Steel Co. gives the following for steel shafts:

| | Turned. | Cold-rolled. |
|---|----------------------|----------------------|
| For simply transmitting power and short countershafts, bearings not more than 8 ft. apart | H.P. = $d^3 R + 50$ | H.P. = $d^3 R + 40$ |
| As second movers, or line shafts, bearings 8 ft. apart | H.P. = $d^3 R + 90$ | H.P. = $d^3 R + 70$ |
| As prime movers or head shafts carrying main driving pulley or gear, well supported by bearings | H.P. = $d^3 R + 125$ | H.P. = $d^3 R + 100$ |

Jones & Laughlins give the following notes: Receiving and transmitting pulleys should always be placed as close to bearings as possible; and it is good practice to frame short "headers" between the main tie-beams of a mill so as to support the *main receivers*, carried by the head shafts, with a bearing close to each side as is contemplated in the formulæ. But if it is preferred, or necessary, for the shaft to span the full width of the "bay" without intermediate bearings, or for the pulley to be placed away from the bearings towards or at the middle of the bay, the size of the shaft must be largely increased to secure the *stiffness* necessary to support the load without undue deflection.

Diameter of shaft D to carry load at center of bays from 2 to 12 ft. span, $D = \sqrt[4]{\frac{c}{c_1}} d$, in which d is the diameter derived from the formula for head shafts, c_1 = length of bay in inches, and c = distance in inches between centers of bearings in accordance with the formula for horse-

power of head shafts. (Jones & Laughlin Steel Co.) Values of c_1 for different diameters d are as follows:

| d | c_1 | d | c_1 | d | c_1 | d | c_1 | d | c_1 | d | c_1 |
|------------------|-------|-----------------|-------|----------------|-------|---------------|-------|-------|-------|-------|-------|
| 1 to 1 3/8 | 15 | 2 13/16 | 25 | 3 15/16 & 4 | 37 | 5 1/4 & 5 3/8 | 55 | 6 3/8 | 71 | 7 3/8 | 88 |
| 1 11/16 & 1 3/4 | 16 | 2 7/8 to 3 | 26 | 4 3/16 | 40 | 5 1/2 | 57 | 6 1/2 | 73 | 7 1/2 | 91 |
| 1 13/16 & 1 7/8 | 17 | 3 1/8 to 3 1/4 | 28 | 4 1/4 | 41 | 5 5/8 | 59 | 6 5/8 | 75 | 7 5/8 | 93 |
| 1 15/16 to 2 1/8 | 18 | 3 3/8 | 30 | 4 7/16 & 4 1/2 | 44 | 5 3/4 | 61 | 6 3/4 | 77 | 7 3/4 | 96 |
| 2 3/16 & 2 1/4 | 19 | 3 7/16 & 3 1/2 | 31 | 4 3/4 | 47 | 5 7/8 | 63 | 6 7/8 | 79 | 7 7/8 | 99 |
| 2 5/16 to 2 7/16 | 20 | 3 9/16 & 3 5/8 | 33 | 4 13/16 | 49 | 6 | 65 | 7 | 81 | 8 | 101 |
| 2 1/2 to 2 5/8 | 24 | 3 11/16 & 3 3/4 | 34 | 5 | 51 | 6 1/8 | 67 | 7 1/8 | 84 | 8 1/2 | 112 |
| 2 11/16 & 2 3/4 | 22 | 3 7/8 | 36 | 5 1/8 | 52 | 6 1/4 | 69 | 7 1/4 | 86 | 9 | 123 |

Should the load be applied near one end of the span or bay instead of at the center, multiply the fourth power of the diameter of the shaft required to carry the load at the center of the span or bay by the product of the two parts of the shaft when the load is near one end, and divide this product by the product of the two parts of the shaft when the load is carried at the center. The fourth root of this quotient will be the diameter required.

The shaft in a line which carries a receiving-pulley, or which carries a transmitting-pulley to drive another line, should always be considered a head-shaft, and should be of the size given by the rules for shafts carrying main pulleys or gears.

The greatest admissible distance between bearings of shafts subject to no transverse strain except from their own weight is for cold-rolled shafts, $L = \sqrt[3]{330,608 \times D^2}$, and for turned shafts, $L = \sqrt[3]{319,586 \times D^2}$. D = diam. and L = length of shaft, in inches. These formulæ are based on an allowable deflection at the center of $1/80$ in. per foot of length, weight of steel 490 lbs. per cu. ft., and modulus of elasticity = 29,000,000 for turned and 30,000,000 for cold-rolled shafting. [In deriving these formulæ the weight of the shaft has been taken as a concentrated instead of a distributed load, giving additional safety.]

Kimball and Barr say that the lateral deflection of a shaft should not exceed 0.01 in. per 100 ft. of length, to insure proper contact at the bearings. For ordinary small shafting they give the following as the allowable distance between the hangers: $L = 7 \sqrt[3]{d^2}$, for shaft without pulleys; $L = 5 \sqrt[3]{d^2}$, for shaft carrying pulleys. (L in ft., d in ins.)

Deflection of Shafting. (Pencoyd Iron Works.) — For continuous line-shafting it is considered good practice to limit the deflection to a maximum of $1/100$ of an inch per foot of length. The weight of bare shafting in pounds = $2.6 d^2 L = W$, or when as fully loaded with pulleys as is customary in practice, and allowing 40 lbs. per inch of width for the vertical pull of the belts, experience shows the load in pounds to be about $13 d^2 L = W$. Taking the modulus of transverse elasticity at 26,000,000 lbs., we derive from authoritative formulæ the following:

$$L = \sqrt[3]{873 d^2}, \quad d = \sqrt{L^3/873}, \quad \text{for bare shafting;}$$

$$L = \sqrt[3]{175 d^2}, \quad d = \sqrt{L^3/175}, \quad \text{for shafting carrying pulleys, etc.;$$

L being the maximum distance in feet between bearings for continuous shafting subjected to bending stress alone, d = diam. in inches.

The torsional stress is inversely proportional to the velocity of rotation, while the bending stress will not be reduced in the same ratio. It is therefore impossible to write a formula covering the whole problem and sufficiently simple for practical application, but the following rules are correct within the range of velocities usual in practice.

For continuous shafting so proportioned as to deflect not more than

$\frac{1}{100}$ of an inch per foot of length, allowance being made for the weakening effect of key-seats,

$$d = \sqrt[3]{50 \text{ H.P.} \div R}, L = \sqrt{720 d^2}, \text{ for bare shafts;}$$

$$d = \sqrt[3]{70 \text{ H.P.} \div R}, L = \sqrt[3]{140 d^2}, \text{ for shafts carrying pulleys, etc.}$$

d = diam. in inches, L = length in feet, R = revs. per min.

The following are given by J. B. Francis as the greatest admissible distances between the bearings of continuous steel shafts subject to no transverse strain except from their own weight, as would be the case were the power given off from the shaft equal on all sides, and at an equal distance from the hanger-bearings.

| | | | | | | | | |
|--------------------------|------|------|------|------|------|------|------|------|
| Diam. of shaft, in. ... | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Dist. bet. bearings, ft. | 15.9 | 18.2 | 20.0 | 21.6 | 22.9 | 24.1 | 25.2 | 26.2 |

These conditions, however, do not usually obtain in the transmission of power by belts and pulleys, and the varying circumstances of each case render it impracticable to give any rule which would be of value for universal application.

For example, the theoretical requirements would demand that the bearings be nearer together on those sections of shafting where most power is delivered from the shaft, while considerations as to the location and desired contiguity of the driven machines may render it impracticable to separate the driving-pulleys by the intervention of a hanger at the theoretically required location. (Joshua Rose.)

Horse-Power Transmitted by Cold-rolled Steel Shafting at Different Speeds as Prime Movers or Head Shafts Carrying Main Driving Pulley or Gear, well Supported by Bearings.

$$\text{Formula H.P.} = d^3 R \div 100.$$

| Revolutions per minute. | | | | | | Revolutions per minute. | | | | | |
|-------------------------|------|------|------|------|------|-------------------------|-----|-----|-----|-----|-----|
| Diam. | 100 | 200 | 300 | 400 | 500 | Diam. | 100 | 200 | 300 | 400 | 500 |
| 1 1/2 | 3.4 | 6.7 | 10.1 | 13.5 | 16.9 | 2 7/8 | 24 | 48 | 72 | 95 | 119 |
| 1 9/16 | 3.8 | 7.6 | 11.4 | 15.2 | 19.0 | 2 15/16 | 25 | 51 | 76 | 101 | 127 |
| 1 5/8 | 4.3 | 8.6 | 12.8 | 17.1 | 21 | 3 | 27 | 54 | 81 | 108 | 135 |
| 1 11/16 | 4.8 | 9.6 | 14.4 | 19.2 | 24 | 3 1/8 | 31 | 61 | 91 | 122 | 152 |
| 1 3/4 | 5.4 | 10.7 | 16.1 | 21 | 27 | 3 3/16 | 32 | 65 | 97 | 129 | 162 |
| 1 13/16 | 5.9 | 11.9 | 17.8 | 24 | 30 | 3 1/4 | 34 | 69 | 103 | 137 | 172 |
| 1 7/8 | 6.6 | 13.1 | 19.7 | 26 | 33 | 3 3/8 | 38 | 77 | 115 | 154 | 192 |
| 1 15/16 | 7.3 | 14.5 | 22 | 29 | 36 | 3 7/16 | 41 | 81 | 122 | 162 | 203 |
| 2 | 8.0 | 16.0 | 24 | 32 | 40 | 3 1/2 | 43 | 86 | 128 | 171 | 214 |
| 2 1/16 | 8.8 | 17.6 | 26 | 35 | 44 | 3 9/16 | 45 | 90 | 136 | 180 | 226 |
| 2 1/8 | 9.6 | 19.2 | 29 | 38 | 48 | 3 5/8 | 48 | 95 | 143 | 190 | 238 |
| 2 3/16 | 10.5 | 21 | 31 | 42 | 52 | 3 11/16 | 50 | 100 | 150 | 200 | 251 |
| 2 1/4 | 11.4 | 23 | 34 | 45 | 57 | 3 3/4 | 55 | 105 | 158 | 211 | 264 |
| 2 5/16 | 12.4 | 25 | 37 | 49 | 62 | 3 7/8 | 58 | 116 | 174 | 233 | 291 |
| 2 3/8 | 13.4 | 27 | 40 | 54 | 67 | 3 15/16 | 61 | 122 | 183 | 244 | 305 |
| 2 7/16 | 14.5 | 29 | 43 | 58 | 72 | 4 | 64 | 128 | 192 | 256 | 320 |
| 2 1/2 | 15.6 | 31 | 47 | 62 | 78 | 4 3/16 | 74 | 147 | 221 | 294 | 367 |
| 2 9/16 | 16.8 | 34 | 50 | 67 | 84 | 4 1/4 | 77 | 154 | 230 | 307 | 383 |
| 2 5/8 | 18.1 | 36 | 54 | 72 | 90 | 4 7/16 | 88 | 175 | 263 | 350 | 438 |
| 2 11/16 | 19.4 | 39 | 58 | 77 | 97 | 4 1/2 | 91 | 182 | 273 | 365 | 456 |
| 2 3/4 | 21 | 41 | 62 | 83 | 104 | 4 3/4 | 107 | 214 | 322 | 429 | 537 |
| 2 13/16 | 22 | 44 | 67 | 89 | 111 | 5 | 125 | 250 | 375 | 500 | 625 |

For H.P. transmitted by turned steel shafts, as prime movers, etc., multiply the figures by 0.8.

For shafts, as second movers or line shafts, } Cold-rolled Turned
bearings 8 ft. apart, multiply by } 1.43 1.11

For simply transmitting power, short counter-shafts, etc., bearings not over 8 ft. apart, multiply by

2

2.50

The horse-power is directly proportional to the number of revolutions per minute.

| | |
|---|------------|
| SPEED OF SHAFTING. — Machine shops..... | 120 to 240 |
| Wood-working..... | 250 to 300 |
| Cotton and woollen mills.. | 300 to 400 |

Flange Couplings.—The bolts should be designed so that their combined resistance to a torsional moment around the axis of the shaft is at least as great as the torsional strength of the shaft itself; and the bolts should be accurately fitted so as to distribute the load evenly among them. Let D = diam. of the shaft, d = diam. of the bolts, r = radius of bolt circle, in inches, n = number of bolts, S = allowable shearing stress per sq. in., then $\pi d^3 S \div 16 = \frac{1}{4} \pi d^2 r S$, whence $d = 0.5 \sqrt{D^3/(nr)}$. Kimball and Barr give $n = 3 + D/2$, but this number may be modified for convenience in spacing, etc.

Effect of Cold Rolling.—Experiments by Prof. R. H. Thurston in 1902 on hot-rolled and cold-rolled steel bars (Catalogue of Jones & Laughlin Steel Co.) showed that the cold-rolled steel in tension had its elastic limit increased 15 to 97%; tensile strength increased 20 to 45%; ductility decreased 40 to 69%. In transverse tests the resistance increased 11 to 30% at the elastic limit and 13 to 69% at the yield point. In torsion the resistance at the yield point increased 31 to 64%, and at the point of fracture it decreased 4 to 10%. The angle of torsion at the elastic limit increased 59 to 103%, while the ultimate angle decreased 19 to 28%. Bars turned from 1 3/4 in. diam. to various sizes down to 0.35 in. showed that the change in quality produced by cold rolling extended to the center of the bar. The maximum strength of the cold-rolled bar of full size was 82,200 lbs. per sq. in., and that of the smallest bar 73,600 lbs. In the hot-rolled steel bars the maximum strength of the full-sized bar was 62,900 lbs. and that of the smallest bar 58,600 lbs. per sq. in.

Hollow Shafts.—Let d be the diameter of a solid shaft, and $d_1 d_2$ the external and internal diameters of a hollow shaft of the same material. Then the shafts will be of equal torsional strength when $d^3 = \frac{d_1^4 - d_2^4}{d_1}$.

A 10-inch hollow shaft with internal diameter of 4 inches will weigh 16% less than a solid 10-inch shaft, but its strength will be only 2.56% less. If the hole were increased to 5 inches diameter the weight would be 25% less than that of the solid shaft, and the strength 6.25% less.

Table for Laying Out Shafting.—The table on the opposite page (from the *Stevens Indicator*, April, 1892) is used by Wm. Sellers & Co. to facilitate the laying out of shafting.

The wood-cuts at the head of this table show the position of the hangers and position of couplings, either for the case of extension in both directions from a central head-shaft or extension in one direction from that head-shaft.

Sizes of Collars for Shafting. Wm. Sellers & Co., *Am. Mach.* Jan. 28, 1897. — D , diam. of collar; T , thickness; d , diam. of set screw; l , length. All in inches.

LOOSE COLLARS.

| Shaft | D | T | d | l | Shaft | D | T | d | l | Shaft | D | t | d | l |
|-------|-------|--------|------|------|-------|--------|--------|-----|--------|-------|---------|-------|-----|-----|
| 1 | 1 3/4 | 3/4 | 7/16 | 5/16 | 2 1/4 | 3 3/8 | 1 3/16 | 5/8 | 5/8 | 4 | 5 13/16 | 1 7/8 | 3/4 | 1 |
| 1 1/4 | 1 7/8 | 13/16 | 7/16 | 3/8 | 2 1/2 | 3 3/4 | 1 1/4 | 5/8 | 1 1/16 | 4 1/2 | 6 7/16 | 1 7/8 | 3/4 | 1 |
| 1 1/2 | 2 1/4 | 1 5/16 | 7/16 | 7/16 | 2 3/4 | 4 | 1 5/16 | 5/8 | 1 1/16 | 5 | 6 15/16 | 1 7/8 | 3/4 | 1 |
| 1 5/8 | 2 5/8 | 1 | 7/16 | 7/16 | 3 | 4 1/2 | 1 7/16 | 5/8 | 1 3/16 | 5 1/2 | 7 1/2 | 2 | 3/4 | 1 |
| 1 3/4 | 2 3/4 | 1 1/16 | 1/2 | 9/16 | 3 1/4 | 4 7/8 | 1 5/8 | 3/4 | 1 3/16 | 6 | 8 | 2 | 3/4 | 1 |
| 2 | 3 | 1 1/8 | 5/8 | 9/16 | 3 1/2 | 5 3/16 | 1 3/4 | 3/4 | 1 5/16 | | | | | |

FAST COLLARS.

| Shaft | D | T | Shaft | D | T | Shaft | D | T | Shaft | D | T |
|-------|-------|------|-------|-------|--------|-------|-------|--------|-------|-------|--------|
| 1 1/2 | 2 | 1/2 | 2 1/2 | 3 1/4 | 9/16 | 3 1/2 | 4 5/8 | 7/8 | 5 1/2 | 7 5/8 | 1 3/16 |
| 1 3/4 | 2 1/4 | 1/2 | 2 3/4 | 3 5/8 | 5/8 | 4 | 5 3/8 | 1 5/16 | 6 | 8 1/4 | 1 1/4 |
| 2 | 2 5/8 | 1/2 | 3 | 4 | 1 1/16 | 4 1/2 | 6 | 1 | 6 1/2 | 9 | 1 3/8 |
| 2 1/4 | 3 | 9/16 | 3 1/4 | 4 1/4 | 1 1/16 | 5 | 7 | 1 1/8 | 7 | 9 3/4 | 1 1/2 |

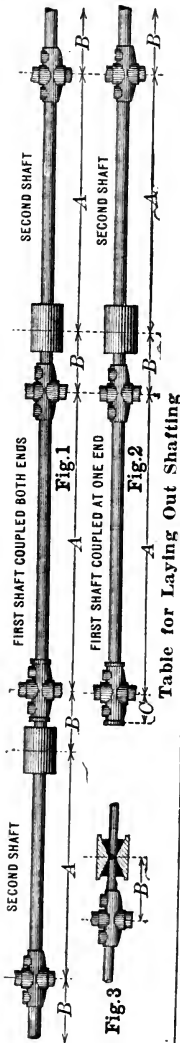


Table for Laying Out Shafting

| Nominal Size of 2d Shaft. | 1 1/2" | 1 3/4" | 2" | 2 1/4" | 2 1/2" | 2 3/4" | 3" | 3 1/4" | 3 1/2" | 4" | 4 1/2" | 5" | 5 1/2" | 6" | 6 1/2" | 7" | 7 1/2" | 8" |
|---------------------------------------|--------|--------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Nominal Size of 1st Shaft, ins. | 1 1/2 | 1 3/4 | 2 | 2 1/4 | 2 1/2 | 2 3/4 | 3 | 3 1/4 | 3 1/2 | 4 | 4 1/2 | 5 | 5 1/2 | 6 | 6 1/2 | 7 | 7 1/2 | 8 |
| Length of Collared and for Fast, ins. | 3 1/2 | 4 | 4 1/2 | 5 | 5 5/8 | 6 1/4 | 6 3/4 | 7 1/4 | 7 7/8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 | 11 1/2 | 12 | 12 1/2 |
| | 3 1/4 | 4 1/4 | 5 1/4 | 6 1/4 | 7 1/4 | 8 1/4 | 9 1/4 | 10 1/4 | 11 1/4 | 12 1/4 | 13 1/4 | 14 1/4 | 15 1/4 | 16 1/4 | 17 1/4 | 18 1/4 | 19 1/4 | 20 1/4 |
| Length of Bearing, ins. | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| | 6 1/2 | 7 1/2 | 8 1/2 | 9 1/2 | 10 1/2 | 11 1/2 | 12 1/2 | 13 1/2 | 14 1/2 | 15 1/2 | 16 1/2 | 17 1/2 | 18 1/2 | 19 1/2 | 20 1/2 | 21 1/2 | 22 1/2 | 23 1/2 |
| Length, inches. | 33/4 | 45/8 | 53/8 | 57/8 | 61/4 | 63/4 | 67/8 | 71/4 | 73/4 | 77/8 | 81/4 | 83/8 | 87/8 | 91/4 | 93/8 | 97/8 | 101/4 | 103/8 |
| | 3 3/4 | 4 5/8 | 5 3/8 | 5 7/8 | 6 1/4 | 6 3/4 | 6 7/8 | 7 1/4 | 7 3/4 | 7 7/8 | 8 1/4 | 8 3/8 | 8 7/8 | 9 1/4 | 9 3/8 | 9 7/8 | 10 1/4 | 10 3/8 |
| Diameter, inches. | 3 1/2 | 4 | 4 1/2 | 5 | 5 5/8 | 6 1/4 | 6 3/4 | 7 1/4 | 7 7/8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 | 11 1/2 | 12 | 12 1/2 |
| | 3 1/2 | 4 | 4 1/2 | 5 | 5 5/8 | 6 1/4 | 6 3/4 | 7 1/4 | 7 7/8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 | 11 1/2 | 12 | 12 1/2 |
| Double Cone-wise Coupling. | 3 1/2 | 4 | 4 1/2 | 5 | 5 5/8 | 6 1/4 | 6 3/4 | 7 1/4 | 7 7/8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 | 11 1/2 | 12 | 12 1/2 |
| | 3 1/2 | 4 | 4 1/2 | 5 | 5 5/8 | 6 1/4 | 6 3/4 | 7 1/4 | 7 7/8 | 8 1/2 | 9 | 9 1/2 | 10 | 10 1/2 | 11 | 11 1/2 | 12 | 12 1/2 |

Distance from Center of Bearing to End of Shaft for Coupling. See B, Figs. 1, 2, and 3.

USE OF TABLE. — Look for size of first shaft in left-hand column, under the head of Size of first shaft, and in the top line of table, marked Size of second shaft, find the size of the shaft to be coupled to it. The intersection gives the length B; this added to the length A, or distance from center to center of bearing, and in cases similar to Fig. 2, to the length C, gives the length of the first shaft, thus; as in Fig. 1, $B + A + B =$ length; Fig. 2, $C + A + B =$ length.

Make bearings at equal distances from each other, when practicable, and always put two bearings on the first, which is the collared shaft. See Figs. 1 and 2.

In coupling shafts of different sizes, either reduce the end of the large shaft in diameter, and use a small coupling, or use a coupling to suit the larger shaft, with one cone bored for the smaller nominal shaft.

PULLEYS.

Proportions of Pulleys. (See also Fly-wheels, page 1031.) — Let n = number of arms, D = diameter of pulley, S = thickness of belt, t = thickness of rim at edge, T = thickness in middle, B = width of rim, β = width of belt, h = breadth of arm at hub, h_1 = breadth of arm at rim, e = thickness of arm at hub, e_1 = thickness of arm at rim, c = amount of crowning; dimensions in inches.

| | | |
|--------------------------------------|----------------------------|---|
| | Unwin. | Reuleaux. |
| B = width of rim..... | $\frac{9}{8}(\beta + 0.4)$ | $\frac{9}{8}\beta$ to $\frac{5}{4}\beta$ |
| t = thickness at edge of rim..... | $0.7S + 0.005D$ | $\left\{ \begin{array}{l} \text{(thick. of rim.)} \\ \frac{1}{5}h \text{ to } \frac{1}{4}h \end{array} \right.$ |
| T = thickness at middle of rim.... | $2t + c$ | |

h = breadth of arm at hub..... $\left\{ \begin{array}{l} \text{For single} \\ \text{belts} = 0.6337 \sqrt{\frac{BD}{n}} \\ \text{For double} \\ \text{belts} = 0.798 \sqrt{\frac{BD}{n}} \end{array} \right. \frac{1}{4} \text{ in.} + \frac{B}{4} + \frac{D}{20n}$

| | | |
|--|----------------------|--|
| h_1 = breadth of arm at rim..... | $\frac{2}{3}h$ | $0.8h$ |
| e = thickness of arm at hub..... | $0.4h$ | $0.5h$ |
| e_1 = thickness of arm at rim..... | $0.4h_1$ | $0.5h_1$ |
| n = number of arms, for a single set | $3 + \frac{BD}{150}$ | $1\frac{1}{2} \left(5 + \frac{D}{2B} \right)$ |

L = length of hub..... $\left\{ \begin{array}{l} \text{not less than } 2.5S, \\ \text{is often } \frac{2}{3}B. \end{array} \right. \left\{ \begin{array}{l} B \text{ for sin-arm} \\ \text{pulleys.} \\ 2B \text{ for double-} \\ \text{arm pulleys.} \end{array} \right.$

M = thickness of metal in hub..... h to $\frac{3}{4}h$
 c = crowning of pulley..... $\frac{1}{24}B$

The number of arms is really arbitrary, and may be altered if necessary, (Unwin.)

Pulleys with two or three sets of arms may be considered as two or three separate pulleys combined in one, except that the proportions of the arms should be 0.8 or 0.7 that of single-arm pulleys. (Reuleaux.)

EXAMPLE. — Dimensions of a pulley 60 in. diam., 16 in. face, for double belt $\frac{1}{2}$ in. thick.

| | | | | | | | | | | |
|-------------|-----|------|-------|------|-------|------|------|------|-----|------|
| Solution by | n | h | h_1 | e | e_1 | t | T | L | M | c |
| Unwin..... | 9 | 3.79 | 2.53 | 1.52 | 1.01 | 0.65 | 1.97 | 10.7 | 3.8 | 0.67 |

| | | | | | | | | | | |
|---------------|---|-----|-----|-----|-----|------|----|---|--|--|
| Reuleaux..... | 4 | 5.0 | 4.0 | 2.5 | 2.0 | 1.25 | 16 | 5 | | |
|---------------|---|-----|-----|-----|-----|------|----|---|--|--|

The following proportions are given in an article in the *Amer. Machinist* authority not stated:

$h = 0.0625D + 0.5$ in., $h_1 = 0.04D + 0.3125$ in., $e = 0.025D + 0.2$ in., $e_1 = 0.016D + 0.125$ in.

These give for the above example: $h = 4.25$ in., $h_1 = 2.71$ in., $e = 1.7$ in., $e_1 = 1.09$ in. The section of the arms in all cases its taken as elliptical.

The following solution for breadth of arm is proposed by the author: Assume a belt pull of 45 lbs. per inch of width of a single belt, that the whole strain is taken in equal proportions on one-half of the arms, and that the arm is a beam loaded at one end and fixed at the other. We have the formula for a beam of elliptical section $fP = 0.0982 Rbd^2 \div l$, in which P = the load, R = the modulus of rupture of the cast iron, b = breadth, d = depth, and l = length of the beam, and f = factor of safety. Assume a modulus of rupture of 36,000 lbs., a factor of safety of 10, and an additional allowance for safety in taking $l = \frac{1}{2}$ the diameter of the pulley instead of $\frac{1}{2}D$ less the radius of the hub.

Take $d = h$, the breadth of the arm at the hub, and $b = e = 0.4h$ the thickness. We then have $fP = 10 \times \frac{45B}{n \div 2} = 900 \frac{B}{n} = \frac{3535 \times 0.4h^2}{\frac{1}{2}D}$,

whence $h = \sqrt[3]{\frac{900BD}{3535n}} = 0.633 \sqrt[3]{\frac{BD}{n}}$, which is practically the same as the value reached by Unwin from a different set of assumptions.

Convexity of Pulleys. — Authorities differ. Morin gives a rise equal to $\frac{1}{10}$ of the face; Molesworth, $\frac{1}{24}$; others from $\frac{1}{8}$ to $\frac{1}{96}$. Scott A. Smith says the crown should not be over $\frac{1}{8}$ inch for a 24-inch face. Pulleys for shifting belts should be "straight," that is, without crowning.

CONE OR STEP PULLEYS.

To find the diameters for the several steps of a pair of cone-pulleys:

1. **Crossed Belts.** — Let D and d be the diameters of two pulleys connected by a crossed belt, L = the distance between their centers, and β = the angle either half of the belt makes with a line joining the centers of the pulleys: then total length of belt $= (D+d) \frac{\pi}{2} + (D+d) \frac{\pi\beta}{180} + 2L \cos \beta$. β = angle whose sine is $\frac{D+d}{2L}$. $L \cos \beta = \sqrt{L^2 - \left(\frac{D+d}{2}\right)^2}$.

The length of the belt is constant when $D+d$ is constant; that is, in a pair of step-pulleys the belt tension will be uniform when the sum of the diameters of each opposite pair of steps is constant. Crossed belts are seldom used for cone-pulleys, on account of the friction between the rubbing parts of the belt.

To design a pair of tapering speed-cones, so that the belt may fit equally tight in all positions: When the belt is crossed, use a pair of equal and similar cones tapering opposite ways.

2. **Open Belts.** — When the belt is uncrossed, use a pair of equal and similar conoids tapering opposite ways, and bulging in the middle, according to the following formula: Let L denote the distance between the axes of the conoids; R the radius of the larger end of each; r the radius of the smaller end; then the radius in the middle, r_0 , is found as follows:

$$r_0 = \frac{R+r}{2} + \frac{(R-r)^2}{6.28 L}. \quad (\text{Rankine.})$$

If D_0 = the diameter of equal steps of a pair of cone-pulleys, D and d = the diameters of unequal opposite steps, and L = distance between the axes, $D_0 = \frac{D+d}{2} + \frac{(D-d)^2}{12.566 L}$.

If a series of differences of radii of the steps, $R-r$, be assumed, then for each pair of steps $\frac{R+r}{2} = r_0 - \frac{(R-r)^2}{6.28 L}$, and the radii of each may be computed from their half sum and half difference, as follows:

$$R = \frac{R+r}{2} + \frac{R-r}{2}; \quad r = \frac{R+r}{2} - \frac{R-r}{2}.$$

A. J. Frith (*Trans. A. S. M. E.*, x, 298) shows the following application of Rankine's method: If we had a set of cones to design, the extreme diameters of which, including thickness of belt, were 40 ins. and 10 ins., and the ratio desired 4, 3, 2, and 1, we would make a table as follows, L being 100 ins.:

| Trial Sum of $D+d$ | Ratio. | Trial Diams. | | Values of $\frac{(D-d)^2}{12.56 L}$ | Amount to be Added. | Corrected Values. | |
|--------------------|--------|--------------|--------|-------------------------------------|---------------------|-------------------|---------|
| | | D | d | | | D | d |
| 50 | 4 | 40 | 10 | 0.7165 | 0.0000 | 40 | 10 |
| 50 | 3 | 37.5 | 12.5 | .4975 | .2190 | 37.7190 | 12.7190 |
| 50 | 2 | 33.333 | 16.666 | .2212 | .4953 | 33.8286 | 17.1619 |
| 50 | 1 | 25 | 25 | .0000 | .7165 | 25.7165 | 25.7165 |

The above formulæ are approximate, and they do not give satisfactory results when the difference of diameters of opposite steps is large and when the axes of the pulleys are near together, giving a large belt-angle. Two more accurate solutions of the problem, one by a graphical method, and another by a trigonometrical method derived from it, are given by C. A. Smith (*Trans. A. S. M. E.*, x, 269). These were copied in earlier editions of this Pocket-book, but are now replaced by the more recent graphical solution by Burmester, given below, and by algebraic formulæ deduced

from it by the author, which give results far more accurate than are required in practice.

In all cases 0.8 of the thickness of the belt should be subtracted from the calculated diameter to obtain the actual diameter of the pulley. This should be done because the belt drawn tight around the pulleys is not the same length as a tape-line measure around them. — (C. A. Smith.)

Burmester's Method, Dr. R. Burmester, in his "Lehrbuch der Kinematik" (*Machinery's Reference Series*, No. 14, 1908), gives a graphical solution of the cone-pulley problem, which while not theoretically exact is much more accurate than practice requires.

From *A* on a horizontal line *AB*, Fig. 170, draw a 45° line, *AC*. Lay off *AS* on *AC* equal, on any convenient scale, the larger the better, to the distance between centers of the shafts, and from *S* draw *ST* perpendicular to *AC*. Make *SK* = $\frac{1}{2}$ *AS*, and with radius *AK* draw an arc of a circle, *XY*. From a convenient point *D* on *AC* draw a vertical line *FDE*, and make *DE* equal the given radius of a step on one cone, and *EF* equal the given radius of the corresponding step on the other cone. Draw *FG* and *EH* parallel to *AC*. From the point *G* on the arc drop a vertical line cutting *EH* in *H*. Through *H* draw a horizontal line *MO*, touching *AC* at *M*. Then if horizontal distances are measured from *M*, as *Ma*, *MH*, *MP*, to equal the radii of the pulleys or steps on one cone, the corresponding vertical distances *ab*, *HG* and *PN* will be the radii of the corresponding steps on the other cone.

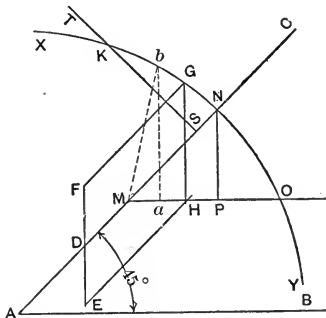


FIG. 170.

If the radii of the two steps of any pair are to bear a certain ratio, as $ab \div Ma$, from *M* draw a line at an angle with *MO* whose tangent equals that ratio, and from the point where it cuts the arc, as *b*, drop a vertical, *ba*. *Ma* and *ba* will be the radii required.

Using Burmester's diagram the author has devised an algebraic solution of the problem (*Industry, Eng.*, June, 1910) which leads to the following equations:

Let *L* = distance between the centers, = *AS* on the diagram.

r_0 = radius of the steps of equal diameter on the two cones, = *MP* = *PN*.

$r_1, r_2 = Ma, ab$, radii of any pair of steps.

a = co-ordinates of *M*, referred to *A*, = $0.79057 L - r_0$.

If r_1 is given, $r_2 = \sqrt{1.25 L^2 - (0.79057 L - r_0 + r_1)^2} - 0.79057 L + r_0$.

If the ratio $r_2 \div r_1$ is given, let $r_2/r_1 = c$: $r_2 = cr_1$.

We then have $a + cr_1 = \sqrt{R^2 - (a + r_1)^2}$, which reduces to

$(1 + c^2) r_1^2 + 2 a (1 + c) r_1 = 1.25 L^2 - 2 a^2$, a quadratic equation, in which $a = 0.79057 L - r_0$. Substituting the value of *a* we have

$$(1 + c^2) r_1^2 + (1.58114 L - 2 r_0) (1 + c) r_1 = 3.16228 L r_0 - 2 r_0^2,$$

in which *L*, r_0 and *c* are given and r_1 is to be found.

Let *L* = 100, *c* = 4, r_0 = 12.858 as in Mr. Frith's example, page 1112.

Then $17 r_1^2 + 10 a r_1 = 12,500 - 8764.62$, from which $r_1 = 5.001$, $r_2 = 20.004$.

If *c* = 3, $r_1 = 6.304$, $r_2 = 18.912$. If *c* = 2, $r_1 = 8.496$, $r_2 = 16.992$.

Checking the results by the approximate formula for length of belt, page 1125, viz, Length = $2 L + \pi (r_1 + r_2) + (r_2 - r_1)^2 \div d$, we have

$$\text{for } c = 1, 200 + 80.79 + 0 = 280.79$$

$$2, 200 + 80.07 + 0.72 = 280.79$$

$$3, 200 + 79.22 + 1.59 = 280.81$$

$$4, 200 + 78.56 + 2.25 = 280.81$$

The maximum difference being only 1 part in 14,000.

J. J. Clark (*Indust. Eng.*, Aug., 1910) gives the following solution:
Using the same notation as above,

$$\frac{(c-1)^2}{L} r_1^2 + \pi(c+1)r_1 = 2\pi r_0 \dots\dots\dots (1)$$

$$\pi(c+1)r_1 + Lx \left(\frac{60-13x}{60-18x} \right) = 2\pi r_0 \dots\dots\dots (2)$$

$$x = (r_2 - r_1)^2 \div L^2 \dots\dots\dots (3)$$

The quadratic equation (1) gives the value of r_1 with an approximation to accuracy sufficient for all practical purposes. If greater accuracy is for any reason desired it may be obtained by (2) and (3), using in (3) the values of r_1 and $r_2 = cr_1$, already found from (1). Taking $\pi = 3.1415927$, the result will be correct to the seventh figure.

Speeds of Shaft with Cone Pulleys. — If S = speed (revs. per min.) of the driving shaft,

s_1, s_2, s_3, s_n = speeds of the driven shaft,

D_1, D_2, D_3, D_n = diameters of the pulleys on the driving cone,

d_1, d_2, d_3, d_n = diams. of corresponding pulleys on the driven cone,

$SD_1 = s_1 d_1$; $SD_2 = s_2 d_2$, etc.

$s_1/S = D_1/d_1 = r_1$; $s_n/S = D_n/d_n = r_n$.

The speed of the driving shaft being constant, the several speeds of the driven shaft are proportional to the ratio of the diameter of the driving pulley to that of the driven, or to D/d .

Speeds in Geometrical Progression. — If it is desired that the speed ratios shall increase by a constant percentage, or in geometrical progression, then $r_2/r_1 = r_3/r_2 = r_n/r_{n-1} = c$, a constant.

$$r_n \div r_1 = c^{n-1}; \quad c = \sqrt[n-1]{r_n - r_1}$$

EXAMPLE. If the speed ratio of the driven shaft at its lowest speed, to the driving shaft be 0.76923, and at its highest speed 2.197, the speeds being in geometrical progression, what is the constant multiplier if $n=5$?

$$\text{Log } 2.197 = 0.341830$$

$$\text{Log } 0.76923 = 1.886056$$

$$\hline 0.455774$$

Divide by $n-1=4$, $0.113943 = \log \text{ of } 1.30$.

If $D_2/d_2 = 1$, then $D_1/d_1 = 1 \div 1.3 = 0.769$; $D_3/d_3 = 1.30$; $D_4/d_4 = 1.69$; $D_5/d_5 = 2.197$.

BELTING.

Theory of Belts and Bands. — A pulley is driven by a belt by means of the friction between the surfaces in contact. Let T_1 be the tension on the driving side of the belt, T_2 the tension on the loose side; then $S = T_1 - T_2$, is the total friction between the band and the pulley, which is equal to the tractive or driving force. Let f = the coefficient of friction, θ the ratio of the length of the arc of contact to the length of the radius, α = the angle of the arc of contact in degrees, e = the base of the Napierian logarithms = 2.71828, m = the modulus of the common logarithms = 0.434295. The following formulæ are derived by calculus (Rankine's *Mach'y and Millwork*, p. 351; *Carpenter's Exper. Eng'g*, p. 173):

$$\frac{T_1}{T_2} = e^{f\theta}; \quad T_2 = \frac{T_1}{e^{f\theta}}; \quad T_1 - T_2 = T_1 - \frac{T_1}{e^{f\theta}} = T_1 (1 - e^{-f\theta}).$$

$$T_1 - T_2 = T_1 (1 - e^{-f\theta}) = T_1 (1 - 10^{-f\theta m}) = T_1 (1 - 10^{-0.00758 f \alpha});$$

$$\frac{T_1}{T_2} = 10^{0.00758 f \alpha}; \quad T_1 = T_2 \times 10^{0.00758 f \alpha}; \quad T_2 = \frac{T_1}{10^{0.00758 f \alpha}}.$$

If the arc of contact between the band and the pulley expressed in turns and fractions of a turn = n , $\theta = 2\pi n$; $e^{f\theta} = 10^{2.7288 f n}$; that is, $e^{f\theta}$ is the natural number corresponding to the common logarithm $2.7288 f n$.

The value of the coefficient of friction f depends on the state and material of the rubbing surfaces. For leather belts on iron pulleys, Morin found $f = 0.56$ when dry, 0.36 when wet, 0.23 when greasy, and 0.15 when oily. In calculating the proper mean tension for a belt, the smallest value, $f = 0.15$, is to be taken if there is a probability of the belt becoming wet with oil. The experiments of Henry R. Towne and Robert Briggs, however (*Jour. Frank. Inst.*, 1868), show that such a state of lubrication is not of ordinary occurrence; and that in designing machinery we may in most cases safely take $f = 0.42$. Reuleaux takes $f = 0.25$. Later writers have shown that the coefficient is not a constant quantity, but is extremely variable, depending on the velocity of slip, the condition of the surfaces, and even on the weather.

The following table shows the values of the coefficient $2.7288 f$, by which n is multiplied in the last equation, corresponding to different values of f ; also the corresponding values of various ratios among the forces, when the arc of contact is half a circumference:

| | | | | |
|---|----------------------------|-------|-------|-------|
| | $f = 0.15$ | 0.25 | 0.42 | 0.56 |
| | $2.7288 f = 0.41$ | 0.68 | 1.15 | 1.53 |
| Let $\theta = \pi$ and $n = 1/2$, then | | | | |
| | $T_1 \div T_2 = 1.603$ | 2.188 | 3.758 | 5.821 |
| | $T_1 \div S = 2.66$ | 1.84 | 1.36 | 1.21 |
| | $T_1 + T_2 \div 2S = 2.16$ | 1.34 | 0.86 | 0.71 |

In ordinary practice it is usual to assume $T_2 = S$; $T_1 = 2S$; $T_1 + T_2 \div 2S = 1.5$. This corresponds to $f = 0.22$ nearly.

For a wire rope on cast iron f may be taken as 0.15 nearly; and if the groove of the pulley is bottomed with gutta-percha, 0.25. (Rankine.)

Centrifugal Tension of Belts. — When a belt or band runs at a high velocity, centrifugal force produces a tension in addition to that existing when the belt is at rest or moving at a low velocity. This centrifugal tension diminishes the effective driving force.

Rankine says: If an endless band, of any figure whatsoever, runs at a given speed, the centrifugal force produces a uniform tension at each cross-section of the band, equal to the weight of a piece of the band whose length is twice the height from which a heavy body must fall in order to acquire the velocity of the band. (See Cooper on Belting, p. 101.)

If T_c = centrifugal tension;

V = velocity in feet per second;

g = acceleration due to gravity = 32.2;

W = weight of a piece of the belt 1 ft. long and 1 sq. in. sectional area, —

Leather weighing 56 lbs. per cubic foot gives $W = 56 \div 144 = 0.388$,

$$T_c = W V^2 \div g = 0.388 V^2 \div 32.2 = 0.012 V^2,$$

Belting Practice. Handy Formulæ for Belting.—Since in the practical application of the above formulæ the value of the coefficient of friction must be assumed, its actual value varying within wide limits (15% to 135%), and since the values of T_1 and T_2 also are fixed arbitrarily, it is customary in practice to substitute for these theoretical formulæ more simple empirical formulæ and rules, some of which are given below.

Let d = diam. of pulley in inches; πd = circumference;

V = velocity of belt in ft. per second; v = vel. in ft. per minute;

α = angle of the arc of contact;

L = length of arc of contact in feet = $\pi da \div (12 \times 360)$;

F = tractive force per square inch of sectional area of belt;

w = width in inches; t = thickness;

S = tractive force per inch of width = $F \div t$;

r.p.m. = revs. per minute; r.p.s. = revs. per second = r.p.m. \div 60.

$$V = \frac{\pi d}{12} \times \text{r.p.s.} = \frac{\pi d}{12} \times \frac{\text{r.p.m.}}{60} = 0.004363 d \times \text{r.p.m.} = \frac{d \times \text{r.p.m.}}{229.2};$$

$$v = \frac{\pi d}{12} \times \text{r.p.m.}; = 0.2618 d \times \text{r.p.m.}$$

$$\text{Horse-power, H.P.} = \frac{Svw}{33000} = \frac{SVw}{550} = \frac{Swd \times \text{r.p.m.}}{126050}.$$

If F = working tension per square inch = 275 lbs., and $t = 7/32$ inch, $S = 60$ lbs. nearly, then

$$\text{H.P.} = \frac{vw}{550} = 0.109 Vw = 0.000476 wd \times \text{r.p.m.} = \frac{wd \times \text{r.p.m.}}{2101}. \quad (1)$$

If $F = 180$ lbs. per square inch, and $t = 1/6$ inch, $S = 30$ lbs., then

$$\text{H.P.} = \frac{vw}{1100} = 0.055 Vw = 0.000238 wd \times \text{r.p.m.} = \frac{wd \times \text{r.p.m.}}{4202}. \quad (2)$$

If the working strain is 60 lbs. per inch of width, a belt 1 inch wide traveling 550 ft. per minute will transmit 1 horse-power. If the working strain is 30 lbs. per inch of width, a belt 1 inch wide traveling 1100 ft. per minute will transmit 1 horse-power. Numerous rules are given by different writers on belting which vary between these extremes. A rule commonly used is: 1 inch wide traveling 1000 ft. per min. = I.H.P.

$$\text{H.P.} = \frac{vw}{1000} = 0.06 Vw = 0.000262 wd \times \text{r.p.m.} = \frac{wd \times \text{r.p.m.}}{3820}. \quad (3)$$

This corresponds to a working strain of 33 lbs. per inch of width.

Many writers give as safe practice for single belts in good condition a working tension of 45 lbs. per inch of width. This gives

$$\text{H.P.} = \frac{vw}{733} = 0.0818 Vw = 0.000357 wd \times \text{r.p.m.} = \frac{wd \times \text{r.p.m.}}{2800}. \quad (4)$$

For double belts of average thickness, some writers say that the transmitting efficiency is to that of single belts as 10 to 7, which would give

$$\text{H.P. of double belts} = \frac{vw}{513} = 0.1169 Vw = 0.00051 wd \times \text{r.p.m.} = \frac{wd \times \text{r.p.m.}}{1960}. \quad (5)$$

Other authorities, however, make the transmitting power of double belts twice that of single belts, on the assumption that the thickness of a double belt is twice that of a single belt.

Rules for horse-power of belts are sometimes based on the number of square feet of surface of the belt which pass over the pulley in a minute. Sq. ft. per min. = $vw \div 12$. The above formulæ translated into this form give:

(1) For $S = 60$ lbs. per inch wide; H.P. = 46 sq. ft. per minute.

(2) " $S = 30$ " " " H.P. = 92 " "

(3) " $S = 33$ " " " H.P. = 83 " "

(4) " $S = 45$ " " " H.P. = 61 " "

(5) " $S = 64.3$ " " " H.P. = 43 " " (double belt).

The above formulæ are all based on the supposition that the arc of contact is 180° . For other arcs, the transmitting power is approximately proportional to the ratio of the degrees of arc to 180° .

Some rules base the horse-power on the length of the arc of contact in feet. Since $L = \frac{\pi da}{12 \times 360}$ and $\text{H.P.} = \frac{Svw}{33000} = \frac{Sw}{33000} \times \frac{\pi d}{12} \times \text{r.p.m.} \times \frac{a}{180}$,

we obtain by substitution $\text{H.P.} = \frac{Sw}{16500} \times L \times \text{r.p.m.}$, and the five formulæ then take the following form for the several values of S :

$$\text{H.P.} = \frac{wL \times \text{r.p.m.}}{275} (1); \frac{wL \times \text{r.p.m.}}{550} (2); \frac{wL \times \text{r.p.m.}}{500} (3); \frac{wL \times \text{r.p.m.}}{367} (4);$$

$$\text{H.P. (double belt)} = \frac{wL \times \text{r.p.m.}}{257} (5).$$

None of the handy formulæ take into consideration the centrifugal tension of belts at high velocities. When the velocity is over 3000 ft. per minute the effect of this tension becomes appreciable, and it should be taken account of, as in Mr. Nagle's formula, which is given below.

Horse-power of a Leather Belt One Inch wide. (Nagle.)

Formula: $\text{H.P.} = CVtw(S - 0.012 V^2) \div 550$.

For $f = 0.40$, $a = 180^\circ$, $C = 0.715$, $w = 1$.

| Laced Belts, $S = 275$. | | | | | | | | Riveted Belts, $S = 400$. | | | | | | | |
|---------------------------|-----------------------------|------|------|------|------|------|------|----------------------------|-----------------------------|------|-------|-------|-------|-------|-------|
| Velocity, ft. per sec. | Thickness in inches = t . | | | | | | | Velocity, ft. per sec. | Thickness in inches = t . | | | | | | |
| | 1/7 | 1/6 | 3/16 | 7/32 | 1/4 | 5/16 | 1/3 | | 7/32 | 1/4 | 5/16 | 1/3 | 3/8 | 7/16 | 1/2 |
| 10 | 0.51 | 0.59 | 0.63 | 0.73 | 0.84 | 1.05 | 1.18 | 15 | 1.69 | 1.94 | 2.42 | 2.58 | 2.91 | 3.39 | 3.87 |
| 15 | 0.75 | 0.88 | 1.00 | 1.16 | 1.32 | 1.66 | 1.77 | 20 | 2.24 | 2.57 | 3.21 | 3.42 | 3.85 | 4.49 | 5.13 |
| 20 | 1.00 | 1.17 | 1.32 | 1.54 | 1.75 | 2.19 | 2.34 | 25 | 2.79 | 3.19 | 3.98 | 4.25 | 4.78 | 5.57 | 6.37 |
| 25 | 1.23 | 1.43 | 1.61 | 1.88 | 2.16 | 2.69 | 2.86 | 30 | 3.31 | 3.79 | 4.74 | 5.05 | 5.67 | 6.62 | 7.58 |
| 30 | 1.47 | 1.72 | 1.93 | 2.25 | 2.58 | 3.22 | 3.44 | 35 | 3.82 | 4.37 | 5.46 | 5.83 | 6.56 | 7.65 | 8.75 |
| 35 | 1.69 | 1.97 | 2.22 | 2.59 | 2.96 | 3.70 | 3.94 | 40 | 4.33 | 4.95 | 6.19 | 6.60 | 7.42 | 8.66 | 9.90 |
| 40 | 1.90 | 2.22 | 2.49 | 2.90 | 3.32 | 4.15 | 4.44 | 45 | 4.85 | 5.49 | 6.86 | 7.32 | 8.43 | 9.70 | 10.98 |
| 45 | 2.09 | 2.45 | 2.75 | 3.21 | 3.67 | 4.58 | 4.89 | 50 | 5.26 | 6.01 | 7.51 | 8.02 | 9.02 | 10.52 | 12.03 |
| 50 | 2.27 | 2.65 | 2.98 | 3.48 | 3.98 | 4.97 | 5.30 | 55 | 5.68 | 6.50 | 8.12 | 8.66 | 9.74 | 11.36 | 13.00 |
| 55 | 2.44 | 2.84 | 3.19 | 3.72 | 4.26 | 5.32 | 5.69 | 60 | 6.09 | 6.96 | 8.70 | 9.28 | 10.43 | 12.17 | 13.91 |
| 60 | 2.58 | 3.01 | 3.38 | 3.95 | 4.51 | 5.64 | 6.02 | 65 | 6.45 | 7.37 | 9.22 | 9.83 | 11.06 | 12.90 | 14.75 |
| 65 | 2.71 | 3.16 | 3.55 | 4.14 | 4.74 | 5.92 | 6.32 | 70 | 6.78 | 7.75 | 9.69 | 10.33 | 11.62 | 13.56 | 15.50 |
| 70 | 2.81 | 3.27 | 3.68 | 4.29 | 4.91 | 6.14 | 6.54 | 75 | 7.09 | 8.11 | 10.13 | 10.84 | 12.16 | 14.18 | 16.21 |
| 75 | 2.89 | 3.37 | 3.79 | 4.42 | 5.05 | 6.31 | 6.73 | 80 | 7.36 | 8.41 | 10.51 | 11.21 | 12.61 | 14.71 | 16.81 |
| 80 | 2.94 | 3.43 | 3.86 | 4.50 | 5.15 | 6.44 | 6.86 | 85 | 7.58 | 8.66 | 10.82 | 11.55 | 13.00 | 15.16 | 17.32 |
| 85 | 2.97 | 3.47 | 3.90 | 4.55 | 5.20 | 6.50 | 6.93 | 90 | 7.74 | 8.85 | 11.06 | 11.80 | 13.27 | 15.48 | 17.69 |
| 90 | 2.97 | 3.47 | 3.90 | 4.55 | 5.20 | 6.50 | 6.93 | 100 | 7.96 | 9.10 | 11.37 | 12.13 | 13.65 | 15.92 | 18.20 |

The H.P. becomes a maximum at 87.41 ft. per sec. = 5245 ft. p. min.

The H.P. becomes a maximum at 105.4 ft. per sec. = 6324 ft. per min.

In the above table the angle of subtension, a , is taken at 180° .
Should it be..... 90° | 100° | 110° | 120° | 130° | 140° | 150° | 160° | 170° | 180° | 200°
Multiply above
values by65 | .70 | .75 | .79 | .83 | .87 | .91 | .94 | .97 | 1 | 1.05

A. F. Nagle's Formula (*Trans. A. S. M. E.*, vol. ii, 1881, p. 91. Tables published in 1882).

$$\text{H.P.} = CVtw \left(\frac{S - 0.012 V^2}{550} \right);$$

$C = 1 - 10^{-0.00758 fa}$;

a = degrees of belt contact;

f = coefficient of friction;

w = width in inches;

t = thickness in inches;

v = velocity in feet per second;

S = stress upon belt per square inch.

Taking S at 275 lbs. per sq. in. for laced belts and 400 lbs. per sq. in. for lapped and riveted belts, the formula becomes

$$\text{H.P.} = CVtw (0.50 - 0.0000218 V^2) \text{ for laced belts;}$$

$$\text{H.P.} = CVtw (0.727 - 0.0000218 V^2) \text{ for riveted belts.}$$

VALUES OF $C = 1 - 10^{-0.00758 fa}$. (NAGLE.)

| f = coefficient of friction. | Degrees of contact = α . | | | | | | | | | | |
|--------------------------------|---------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 90° | 100° | 110° | 120° | 130° | 140° | 150° | 160° | 170° | 180° | 200° |
| 0.15 | 0.210 | 0.230 | 0.250 | 0.270 | 0.288 | 0.307 | 0.325 | 0.342 | 0.359 | 0.376 | 0.408 |
| .20 | .270 | .295 | .319 | .342 | .364 | .386 | .408 | .428 | .448 | .467 | .503 |
| .25 | .325 | .354 | .381 | .407 | .432 | .457 | .480 | .503 | .524 | .544 | .582 |
| .30 | .376 | .408 | .438 | .467 | .494 | .520 | .544 | .567 | .590 | .610 | .649 |
| .35 | .423 | .457 | .489 | .520 | .548 | .575 | .600 | .624 | .646 | .667 | .705 |
| .40 | .467 | .502 | .536 | .567 | .597 | .624 | .649 | .673 | .695 | .715 | .753 |
| .45 | .507 | .544 | .579 | .610 | .640 | .667 | .692 | .715 | .737 | .757 | .792 |
| .55 | .578 | .617 | .652 | .684 | .713 | .739 | .763 | .785 | .805 | .822 | .853 |
| .60 | .610 | .649 | .684 | .715 | .744 | .769 | .792 | .813 | .832 | .848 | .877 |
| 1.00 | .792 | .825 | .853 | .877 | .897 | .913 | .927 | .937 | .947 | .956 | .969 |

The following table gives a comparison of the formulæ already given for the case of a belt one inch wide, with arc of contact 180°.

Horse-power of a Belt One Inch wide, Arc of Contact 180°.

COMPARISON OF DIFFERENT FORMULÆ.

| Velocity in ft. per sec. | Velocity in ft. p. min. | Sq. ft. of Belt p. min. | Form. 1 | Form. 2 | Form. 3 | Form. 4 | Form. 5 | Nagle's Form. | |
|--------------------------|-------------------------|-------------------------|-------------------------|--------------------------|--------------------------|-------------------------|-------------------------------------|-----------------------|---------|
| | | | H.P. = $\frac{wv}{550}$ | H.P. = $\frac{wv}{1100}$ | H.P. = $\frac{wv}{1000}$ | H.P. = $\frac{wv}{733}$ | double belt H.P. = $\frac{wv}{513}$ | 7/32-in. single belt. | |
| | | | | | | | | Laced. | Riv't'd |
| 10 | 600 | 50 | 1.09 | 0.55 | 0.60 | 0.82 | 1.17 | 0.73 | 1.14 |
| 20 | 1200 | 100 | 2.18 | 1.09 | 1.20 | 1.64 | 2.34 | 1.54 | 2.24 |
| 30 | 1800 | 150 | 3.27 | 1.64 | 1.80 | 2.46 | 3.51 | 2.25 | 3.31 |
| 40 | 2400 | 200 | 4.36 | 2.18 | 2.40 | 3.27 | 4.68 | 2.90 | 4.33 |
| 50 | 3000 | 250 | 5.45 | 2.73 | 3.00 | 4.09 | 5.85 | 3.48 | 5.26 |
| 60 | 3600 | 300 | 6.55 | 3.27 | 3.60 | 4.91 | 7.02 | 3.95 | 6.09 |
| 70 | 4200 | 350 | 7.63 | 3.82 | 4.20 | 5.73 | 8.19 | 4.29 | 6.78 |
| 80 | 4800 | 400 | 8.73 | 4.36 | 4.80 | 6.55 | 9.36 | 4.50 | 7.36 |
| 90 | 5400 | 450 | 9.82 | 4.91 | 5.40 | 7.37 | 10.53 | 4.55 | 7.74 |
| 100 | 6000 | 500 | 10.91 | 5.45 | 6.00 | 8.18 | 11.70 | 4.41 | 7.96 |
| 110 | 6600 | 550 | | | | | | 4.05 | 7.97 |
| 120 | 7200 | 600 | | | | | | 3.49 | 7.75 |

Width of Belt for a Given Horse-power. — The width of belt required for any given horse-power may be obtained by transposing the formulæ for horse-power so as to give the value of w . Thus:

$$\text{From formula (1), } w = \frac{550 \text{ H.P.}}{v} = \frac{9.17 \text{ H.P.}}{V} = \frac{2101 \text{ H.P.}}{d \times \text{r.p.m.}} = \frac{275 \text{ H.P.}}{L \times \text{r.p.m.}}$$

$$\text{From formula (2), } w = \frac{1100 \text{ H.P.}}{v} = \frac{18.33 \text{ H.P.}}{V} = \frac{4202 \text{ H.P.}}{d \times \text{r.p.m.}} = \frac{530 \text{ H.P.}}{L \times \text{r.p.m.}}$$

$$\text{From formula (3), } w = \frac{1000 \text{ H.P.}}{v} = \frac{16.67 \text{ H.P.}}{V} = \frac{3820 \text{ H.P.}}{d \times \text{r.p.m.}} = \frac{500 \text{ H.P.}}{L \times \text{r.p.m.}}$$

$$\text{From formula (4), } w = \frac{733 \text{ H.P.}}{v} = \frac{12.22 \text{ H.P.}}{V} = \frac{2800 \text{ H.P.}}{d \times \text{r.p.m.}} = \frac{360 \text{ H.P.}}{L \times \text{r.p.m.}}$$

$$\text{From formula (5), } w = \frac{513 \text{ H.P.}}{v} = \frac{8.56 \text{ H.P.}}{V} = \frac{1960 \text{ H.P.}}{d \times \text{r.p.m.}} = \frac{257 \text{ H.P.}}{L \times \text{r.p.m.}}$$

* For double belts.

Many authorities use formula (1) for double belts and formula (2) or (3) for single belts.

To obtain the width by Nagle's formula, $w = \frac{550 \text{ H.P.}}{C V t (S - 0.012 V^2)}$, or

divide the given horse-power by the figure in the table corresponding to the given thickness of belt and velocity in feet per second.

The formula to be used in any particular case is largely a matter of judgment. A single belt proportioned according to formula (1), if tightly stretched, and if the surface is in good condition, will transmit the horse-power calculated by the formula, but one so proportioned is objectionable, first, because it requires so great an initial tension that it is apt to stretch, slip, and require frequent restretching and relacing; and second, because this tension will cause an undue pressure on the pulley-shaft, and therefore an undue loss of power by friction. To avoid these difficulties, formula (2), (3), or (4), or Mr. Nagle's table, should be used; the latter especially in cases in which the velocity exceeds 4000 ft. per min.

The following are from the notes of the late Samuel Webber. (*Am. Mach.* May 11, 1909.)

Good oak-tanned leather from the back of the hide weighs almost exactly one avoirdupois ounce for each one-hundredth of an inch in thickness, in a piece of leather one foot square, so that

| | Lbs. per Sq. Ft. | Approx. Thick- ness. | Actual Thick- ness. | Vel. per Inch for 1 H.P. | Safe Strain per Inch Width. |
|-------------------|------------------------|----------------------------|---------------------------|--------------------------------|-----------------------------------|
| Single belt..... | 16 oz. | 1/8 in. | 0.16 in. | 625 ft. | 52.8 lbs. |
| Light double..... | 24 " | 1/4 " | 0.24 " | 417 " | 78.1 " |
| Medium..... | 28 " | 5/16 " | 0.28 " | 357 " | 92.5 " |
| Standard..... | 33 " | 1/3 " | 0.33 " | 303 " | 109 " |
| 3-ply..... | 45 " | 9/16 " | 0.45 " | 222 " | 148 " |

The rule for velocity per inch width for 1 H.P. is:

Multiply the denominator of the fraction expressing the thickness of the belt in inches by 100, and divide it by the numerator;

Good, well-calendered rubber belting made with 30-ounce duck and new (i. e., not reclaimed vulcanized) rubber will be as follows:

| Nomenclature. | Approximate Thickness. | Safe Working Strain for 1 Inch Width. | Velocity per Inch for for 1 H.P. |
|---------------|---------------------------|---|-------------------------------------|
| 3-ply | 0.18 in. | 45 pounds | 735 ft. per min. |
| 4 " | 0.24 " | 65 " | 508 " " " |
| 5 " | 0.30 " | 85 " | 388 " " " |
| 6 " | 0.35 " | 105 " | 314 " " " |
| 7 " | 0.40 " | 125 " | 264 " " " |
| 8 " | 0.45 " | 145 " | 218 " " " |

The thickness of rubber belt does not necessarily govern the strength, but the weight of duck does, and with 30-ounce duck, the safe working strains are as above.

Belt Factors. W. W. Bird (*Jour. Worcester Polyt. Inst.*, Jan. 1910.) — The factors given in the table below, for use in the formula $H.P. = vw + F$, in which F is an empirical factor, are based on the following assumptions: A belt of single thickness will stand a stress on the tight side, T_1 , of 60 lbs. per inch of width, a double belt 105 lbs., and a triple belt 150 lbs., and have a fairly long life, requiring only occasional taking up; the ratio of tensions T_1/T_2 should not exceed 2 on small, 2.5 on medium and 3 on large pulleys; the creep (travel of the belt relative to the surface of the pulley due to the elasticity of the belt and not to slip) should not exceed 1% — this requires that the differ-

ence in tensions $T_1 - T_2$ should not be greater than 40 lbs. per inch of width for single, 70 for double and 100 for triple belts.

| Pulley diam. | Under 8 in. | 8 to 36 in. | Over 3 ft. | Under 14 in. | 14 to 60 in. | Over 5 ft. | Under 21 in. | 21 to 84 in. | Over 7 ft. |
|----------------------|----------------|----------------|---------------|-----------------|-----------------|---------------|-----------------|-----------------|---------------|
| Belt thick- ness. | Single. | S'gle. | S'gle. | Dbl. | Dbl. | Dbl. | Triple. | Triple. | Triple. |
| Factor..... | 1100 | 920 | 830 | 630 | 520 | 470 | 440 | 370 | 330 |
| $T_1 - T_2$ | 30 | 36 | 40 | 52.5 | 63 | 70 | 75 | 90 | 100 |
| Creep, %..... | 0.74 | 0.89 | 0.99 | 0.74 | 0.89 | 0.99 | 0.74 | 0.89 | 0.99 |
| $T_1 \div T_2$ | 2 | 2.5 | 3 | 2 | 2.5 | 3 | 2 | 2.5 | 3 |
| T_1 | 60 | 60 | 60 | 105 | 105 | 105 | 150 | 150 | 150 |

These factors are for an arc of contact of 180° . For other arcs they are to be multiplied by the figures given below.

| | | | | | | | | | | |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Arc | 220° | 210° | 200° | 190° | 170° | 160° | 150° | 140° | 130° | 120° |
| Multiply by... | 0.89 | 0.92 | 0.95 | 0.97 | 1.04 | 1.07 | 1.11 | 1.16 | 1.21 | 1.27 |

Taylor's Rules for Belting. — F. W. Taylor (*Trans. A. S. M. E.*, xv, 204) describes a nine years' experiment on belting in a machine shop, giving results of tests of 42 belts running night and day. Some of these belts were run on cone pulleys and others on shifting, or fast-and-loose, pulleys. The average net working load on the shifting belts was only 0.4 of that of the cone belts.

The shifting belts varied in dimensions from 39 ft. 7 in. long, 3.5 in. wide, 0.25 in. thick, to 51 ft. 5 in. long, 6.5 in. wide, 0.37 in. thick. The cone belts varied in dimensions from 24 ft. 7 in. long, 2 in. wide, 0.25 in. thick, to 31 ft. 10 in. long, 4 in. wide, 0.37 in. thick.

Belt-clamps were used having spring-balances between the two pairs of clamps, so that the exact tension to which the belt was subjected was accurately weighed when the belt was first put on, and each time it was tightened.

The tension under which each belt was spliced was carefully figured so as to place it under an initial strain — while the belt was at rest immediately after tightening — of 71 lbs. per inch of width of double belts. This is equivalent, in the case of

| | |
|--------------------------------|----------------------------------|
| Oak tanned and fullled belts, | to 192 lbs. per sq. in. section; |
| Oak tanned, not fullled belts, | to 229 " " " " |
| Semi-raw-hide belts, | to 253 " " " " |
| Raw-hide belts | to 284 " " " " |

From the nine years' experiment Mr. Taylor draws a number of conclusions, some of which are given in an abridged form below.

In using belting so as to obtain the greatest economy and the most satisfactory results, the following rules should be observed:

| | Oak Tanned and Fullled Leather Belts. | Other Types of Leather Belts and 6- to 7-ply Rubber Belts. |
|--|---|--|
| A double belt, having an arc of contact of 180° , will give an effective pull on the face of a pulley per inch of width of belt of..... | 35 lbs. | 30 lbs. |
| Or, a different form of same rule: The number of sq. ft. of double belt passing around a pulley per minute required to transmit one horse-power is..... | 80 sq. ft. | 90 sq. ft. |
| Or: The number of lineal feet of double belting 1 in. wide passing around a pulley per minute required to transmit one horse-power is..... | 950 ft. | 1100 ft. |
| Or: A double belt 6 in. wide, running 4000 to 5000 ft. per min., will transmit..... | 30 H.P. | 25 H.P. |

The terms "initial tension," "effective pull," etc., are thus explained by Mr. Taylor: When pulleys upon which belts are tightened are at rest,

both strands of the belt (the upper and lower) are under the same stress per in. of width. By "tension," "initial tension," or "tension while at rest," we mean the stress per in. of width, or sq. in. of section, to which one of the strands of the belt is tightened, when at rest. After the belts are in motion and transmitting power, the stress on the slack side, or strand, of the belt becomes less, while that on the tight side — or the side which does the pulling — becomes greater than when the belt was at rest. By the term "total load" we mean the total stress per in. of width, or sq. in. of section, on the tight side of belt while in motion.

The difference between the stress on the tight side of the belt and its slack side, while in motion, represents the effective force or pull which is transmitted from one pulley to another. By the terms "working load," "net working load," or "effective pull," we mean the difference in the tension of the tight and slack sides of the belt per in. of width, or sq. in. section, while in motion, or the net effective force that is transmitted from one pulley to another per in. of width or sq. in. of section.

The discovery of Messrs. Lewis and Bancroft (*Trans. A. S. M. E.*, vii, 749) that the "sum of the tension on both sides of the belt does not remain constant," upsets all previous theoretical belting formulæ.

The belt speed for maximum economy should be from 4000 to 4500 ft. per minute.

The best distance from center to center of shafts is from 20 to 25 ft.

Idler pulleys work most satisfactorily when located on the slack side of the belt about one-quarter way from the driving-pulley.

Belts are more durable and work more satisfactorily made narrow and thick, rather than wide and thin.

It is safe and advisable to use: a double belt on a pulley 12 in. diameter or larger; a triple belt on a pulley 20 in. diameter or larger; a quadruple belt on a pulley 30 in. diameter or larger.

As belts increase in width they should also be made thicker.

The ends of the belt should be fastened together by splicing and cementing, instead of lacing, wiring, or using hooks or clamps of any kind.

A V-splice should be used on triple and quadruple belts and when idlers are used. Stepped splice, coated with rubber and vulcanized in place, is best for rubber belts.

For double belting the rule works well of making the splice for all belts up to 10 in. wide, 10 in. long; from 10 in. to 18 in. wide the splice should be the same width as the belt, 18 in. being the greatest length of splice required for double belting.

Belts should be cleaned and greased every five to six months.

Double leather belts will last well when repeatedly tightened under a strain (when at rest) of 71 lbs. per in. of width, or 240 lbs. per sq. in. section. They will not maintain this tension for any length of time, however.

Belt-clamps having spring-balances between the two pairs of clamps should be used for weighing the tension of the belt accurately each time it is tightened.

The stretch, durability, cost of maintenance, etc., of belts proportioned (A) according to the ordinary rules of a total load of 111 lbs. per inch of width, corresponding to an effective pull of 65 lbs. per inch of width, and (B) according to a more economical rule of a total load of 54 lbs., corresponding to an effective pull of 26 lbs. per inch of width, are found to be as follows:

When it is impracticable to accurately weigh the tension of a belt in tightening it, it is safe to shorten a double belt one-half inch for every 10 ft. of length for (A) and one inch for every 10 ft. for (B), if it requires tightening.

Double leather belts, when treated with great care and run night and day at moderate speed, should last for 7 years (A); 18 years (B).

The cost of all labor and materials used in the maintenance and repairs of double belts, added to the cost of renewals as they give out, through a term of years, will amount on an average per year to 37% of the original cost of the belts (A); 14% or less (B).

In figuring the total expense of belting, and the manufacturing cost chargeable to this account, by far the largest item is the time lost on the machines while belts are being relaced and repaired.

The total stretch of leather belting exceeds 6% of the original length.

The stretch during the first six months of the life of belts is 36% of their entire stretch (A); 15% (B).

A double belt will stretch 0.47% of its length before requiring to be tightened (A); 0.81% (B).

The most important consideration in making up tables and rules for the use and care of belting is how to secure the minimum of interruptions to manufacture from this source.

The average double belt (A), when running night and day in a machine-shop, will cause at least 26 interruptions to manufacture during its life, or 5 interruptions per year, but with (B) interruptions to manufacture will not average oftener for each belt than one in sixteen months.

The oak-tanned and fulled belts showed themselves to be superior in all respects except the coefficient of friction to either the oak-tanned not fulled, the semi-raw-hide, or raw-hide with tanned face.

Belts of any width can be successfully shifted backward and forward on tight and loose pulleys. Belts running between 5000 and 6000 ft. per min. and driving 300 H.P. are now being daily shifted on tight and loose pulleys, to throw lines of shafting in and out of use.

The best form of belt-shifter for wide belts is a pair of rollers twice the width of belt, either of which can be pressed onto the flat surface of the belt on its slack side close to the driven pulley, the axis of the roller making an angle of 75° with the center line of the belt.

Remarks on Mr. Taylor's Rules. (W. Kent, *Trans. A. S. M. E.*, xv, 242.)—The most notable feature in Mr. Taylor's paper is the great difference between his rules for proper proportioning of belts and those given by earlier writers. A very commonly used rule is, one horse-power may be transmitted by a single belt 1 in. wide running x ft. per min., substituting for x various values, according to the ideas of different engineers, ranging usually from 550 to 1100.

The practical mechanic of the old school is apt to swear by the figure 600 as being thoroughly reliable, while the modern engineer is more apt to use the figure 1000. Mr. Taylor, however, instead of using a figure from 550 to 1100 for a single belt, uses 950 to 1100 for double belts. If we assume that a double belt is twice as strong, or will carry twice as much power, as a single belt, then he uses a figure at least twice as large as that used in modern practice, and would make the cost of belting for a given shop twice as large as if the belting were proportioned according to the most liberal of the customary rules.

This great difference is to some extent explained by the fact that the problem which Mr. Taylor undertakes to solve is quite a different one from that which is solved by the ordinary rules with their variations. The problem of the latter generally is, "How wide a belt must be used, or how narrow a belt may be used, to transmit a given horse-power?" Mr. Taylor's problem is: "How wide a belt must be used so that a given horse-power may be transmitted with the minimum cost for belt repairs, the longest life to the belt, and the smallest loss and inconvenience from stopping the machine while the belt is being tightened or repaired?"

The difference between the old practical mechanic's rule of a 1-in.-wide single belt, 600 ft. per min., transmits one horse-power, and the rule commonly used by engineers, in which 1000 is substituted for 600, is due to the belief of the engineers, not that a horse-power could not be transmitted by the belt proportioned by the older rule, but that such a proportion involved undue strain from overtightening to prevent slipping, which strain entailed too much journal friction, necessitated frequent tightening, and decreased the length of the life of the belt.

Mr. Taylor's rule substituting 1100 ft. per min. and doubling the belt, is a further step, and a long one, in the same direction. Whether it will be taken in any case by engineers will depend upon whether they appreciate the extent of the losses due to slippage of belts slackened by use under overstrain, and the loss of time in tightening and repairing belts, to such a degree as to induce them to allow the first cost of the belts to be doubled in order to avoid these losses.

It should be noted that Mr. Taylor's experiments were made on rather narrow belts, used for transmitting power from shafting to machinery, and his conclusions may not be applicable to heavy and wide belts, such as engine fly-wheel belts.

Barth's Studies on Belting. (*Trans. A. S. M. E.*, 1909.)—Mr. Carl G. Barth has made an extensive study of the work of earlier writers on the subject of belting, and has derived several new formulæ and diagrams showing the relation of the several variables that enter into the belt problem. He has also devised a slide rule by which calculations of belts may easily be made. He finds that the coefficient of friction depends on the velocity of the belt, and may be expressed by the formula $f = 0.54 A - 140 \div (500 + V)$, in which A is the sum of the tension on the tight side and one-half the tension on the slack side of the belt, and V is the velocity in feet per minute.

Taking Mr. Taylor's data as a starting point, Mr Barth has adopted the rule, as a basis for use of belts on belt-driven machines, that for the driving belt of a machine the *minimum initial tension* must be such that when the belt is doing the maximum amount of work intended, the *sum of the tension in the tight side of the belt and one-half the tension in the slack side will equal 240 lbs. per square inch of cross-section for all belt speeds*; and that for a belt driving a countershaft, or any other belt inconvenient to get at for retightening or more readily made of liberal dimensions, this sum will equal 160 lbs. Further, the maximum initial tension, that is, the initial tension under which a belt is to be put up in the first place, and to which it is to be retightened as often as it drops to the minimum, must be such that the sum defined above is 320 lbs. for a machine belt, and 240 lbs. for a counter-shaft belt or a belt similarly circumstanced.

From a set of curves plotted by Mr. Barth from his formula the following tables are derived. The figures are based upon the conditions named in the above rule, and on an arc of contact = 180° .

Belts on Machines. Tension in tight side + $\frac{1}{2}$ tension in slack side = 240 lbs.

| Velocity, ft. per min... | 500 | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 |
|---|------|------|------|-------|-------|-------|------|
| Initial tension, t_0 | 124 | 120 | 121 | 128 | 136 | 144 | 152 |
| Centrifugal tension t_c . | 0 + | 3 | 13 | 31 | 56 | 86 | 124 |
| Difference, $t_0 - t_c$ | 123 | 117 | 108 | 97 | 80 | 58 | 28 |
| Tension on tight side, t_1 | 210 | 212 | 211 | 207 | 198 | 187 | 173 |
| Tension on slack side, t_2 | 60 | 54 | 57 | 68 | 84 | 107 | 134 |
| Effective pull, $t_1 - t_2$... | 150 | 158 | 154 | 139 | 114 | 80 | 39 |
| Sum of tensions $t_1 + t_2$ | 270 | 268 | 269 | 274 | 282 | 294 | 307 |
| H.P. per sq. in. of section..... | 2.27 | 4.79 | 9.33 | 12.64 | 13.82 | 12.12 | 7.09 |
| H.P. per in. width, $\frac{5}{16}$ in. thick..... | 0.71 | 1.50 | 2.82 | 3.95 | 4.32 | 3.71 | 2.22 |

Belts driving countershafts, $t_1 + \frac{1}{2} t_2 = 160$ lbs.

| Velocity of belt, ft. per min..... | 500 | 1000 | 2000 | 3000 | 4000 | 5000 |
|--|------|------|------|------|------|------|
| Initial tension, t_0 | 82 | 81 | 83 | 89 | 96 | 102 |
| Tension on tight side, t_1 | 140 | 141 | 140 | 134 | 125 | 114 |
| Tension on slack side, t_2 | 40 | 38 | 41 | 53 | 69 | 92 |
| Effective pull, $t_1 - t_2$ | 100 | 103 | 99 | 81 | 56 | 22 |
| Sum of tensions..... | 180 | 179 | 181 | 187 | 194 | 206 |
| H.P. per sq. in. of section..... | 1.51 | 3.12 | 6.04 | 7.36 | 6.79 | 3.33 |
| H.P. per in. width, $\frac{5}{16}$ in. thick | 3.47 | 0.97 | 1.87 | 2.30 | 2.12 | 1.04 |

MISCELLANEOUS NOTES ON BELTING.

Formulæ are useful for proportioning belts and pulleys, but they furnish no means of estimating how much power a particular belt may be transmitting at any given time, any more than the size of the engine is a measure of the load it is actually drawing, or the known strength of a horse is a measure of the load on the wagon. The only reliable means of determining the power actually transmitted is some form of dynamometer. (See *Trans. A. S. M. E.*, vol. xii, p. 707.)

If we increase the thickness, the power transmitted ought to increase in proportion; and for double belts we should have half the width required for a single belt under the same conditions. With large pulleys and moderate velocities of belt it is probable that this holds good. With small pulleys, however, when a double belt is used, there is not such per-

fect contact between the pulley-face and the belt, due to the rigidity of the latter, and more work is necessary to bend the belt-fibers than when a thinner and more pliable belt is used. The centrifugal force tending to throw the belt from the pulley also increases with the thickness, and for these reasons the width of a double belt required to transmit a given horse-power when used with small pulleys is generally assumed not less than seven-tenths the width of a single belt to transmit the same power. (Flather on "Dynamometers and Measurement of Power.")

F. W. Taylor, however, finds that great pliability is objectionable, and favors thick belts even for small pulleys. The power consumed in bending the belt around the pulley he considers inappreciable. According to Rankine's formula for centrifugal tension, this tension is proportional to the sectional area of the belt, and hence it does not increase with increase of thickness when the width is decreased in the same proportion, the sectional area remaining constant.

Scott A. Smith (*Trans. A. S. M. E.*, x, 765) says: The best belts are made from all oak-tanned leather, and curried with the use of cod oil and tallow, all to be of superior quality. Such belts have continued in use thirty to forty years when used as simple driving-belts, driving a proper amount of power, and having had suitable care. The flesh side should not be run to the pulley-face, for the reason that the wear from contact with the pulley should come on the grain side, as that surface of the belt is much weaker in its tensile strength than the flesh side; also as the grain is hard it is more enduring for the wear of attrition; further, if the grain is actually worn off, then the belt may not suffer in its integrity from a ready tendency of the hard grain side to crack.

The most intimate contact of a belt with a pulley comes, first, in the smoothness of a pulley-face, including freedom from ridges and hollows left by turning-tools; second, in the smoothness of the surface and evenness in the texture or body of a belt; third, in having the crown of the driving and receiving pulleys exactly alike, — as nearly so as is practicable in a commercial sense; fourth, in having the crown of pulleys not over $\frac{1}{8}$ in. for a 24-in. face, that is to say, that the pulley is not to be over $\frac{1}{4}$ in. larger in diameter in its center; fifth, in having the crown other than two planes meeting at the center; sixth, the use of any material on or in a belt, in addition to those necessarily used in the currying process, to keep them pliable or increase their tractive quality, should wholly depend upon the exigencies arising in the use of belts; non-use is safer than over-use; seventh, with reference to the lacing of belts, it seems to be a good practice to cut the ends to a convex shape by using a former, so that there may be a nearly uniform stress on the lacing through the center as compared with the edges. For a belt 10 ins. wide, the center of each end should recede $\frac{1}{10}$ in.

Lacing of Belts. — In punching a belt for lacing, use an oval punch, the longer diameter of the punch being parallel with the sides of the belt. Punch two rows of holes in each end, placed zigzag. In a 3-in. belt there should be four holes in each end — two in each row. In a 6-in. belt, seven holes — four in the row nearest the end. A 10-in. belt should have nine holes. The edge of the holes should not come nearer than $\frac{3}{4}$ in. from the sides, nor $\frac{7}{8}$ in. from the ends of the belt. The second row should be at least $1\frac{3}{4}$ ins. from the end. On wide belts these distances should be even a little greater.

Begin to lace in the center of the belt and take care to keep the ends exactly in line, and to lace both sides with equal tightness. The lacing should not be crossed on the side of the belt that runs next the pulley. In taking up belts, observe the same rules as in putting on new ones.

Setting a Belt on Quarter-twist. — A belt must run squarely on to the pulley. To connect with a belt two horizontal shafts at right angles with each other, say an engine-shaft near the floor with a line attached to the ceiling, will require a quarter-turn. First, ascertain the central point on the face of each pulley at the extremity of the horizontal diameter where the belt will leave the pulley, and then set that point on the driven pulley plumb over the corresponding point on the driver. This will cause the belt to run squarely on to each pulley, and it will leave at an angle greater or less, according to the size of the pulleys and their distance from each other.

In quarter-twist belts, in order that the belt may remain on the pulleys,

the central plane on each pulley must pass through the point of delivery of the other pulley. This arrangement does not admit of reversed motion.

To find the Length of Belt required for two given Pulleys. — When the length cannot be measured directly by a tape-line, the following approximate rule may be used: Add the diameter of the two pulleys together, divide the sum by 2, and multiply the quotient by $3\frac{1}{4}$, and add the product to twice the distance between the centers of the shafts. (See accurate formula below.)

To find the Angle of the Arc of Contact of a Belt. — Divide the difference between the radii of the two pulleys in inches by the distance between their centers, also in inches, and in a table of natural sines find the angle most nearly corresponding with the quotient. Multiply this angle by 2, and add the product to 180° for the angle of contact with the larger pulley, or subtract it from 180° for the smaller pulley.

Or, let R = radius of larger pulley, r = radius of smaller;

L = distance between centers of the pulleys;

a = angle whose sine is $(R - r) \div L$.

Arc of contact with smaller pulley = $180^\circ - 2a$;

Arc of contact with larger pulley = $180^\circ + 2a$.

To find the Length of Belt in Contact with the Pulley. — For the larger pulley, multiply the angle a , found as above, by 0.0349, to the product add 3.1416, and multiply the sum by the radius of the pulley. Or length of belt in contact with the pulley

$$= \text{radius} \times (\pi + 0.0349 a) = \text{radius} \times \pi(1 + a/90).$$

For the smaller pulley, length = radius $\times (\pi - 0.0349 a)$

$$= \text{radius} \times \pi(1 - a/90).$$

The above rules refer to **Open Belts**. The accurate formula for length of an open belt is,

$$\text{Length} = \pi R(1 + a/90) + \pi r(1 - a/90) + 2 L \cos a.$$

$$= R(\pi + 0.0349 a) + r(\pi - 0.0349 a) + 2 L \cos a,$$

in which R = radius of larger pulley, r = radius of smaller pulley,

L = distance between centers of pulleys, and a = angle whose sine is

$$(R - r) \div L; \cos a = \sqrt{L^2 - (R - r)^2} \div L.$$

An approximate formula is

$$\text{Length} = 2 L + \pi(R + r) + (R - r)^2/L$$

For $L = 4$, $R = 2$, $r = 1$, this formula gives length = 17.6748, the accurate formula giving 17.6761

For Crossed Belts the formula is

$$\text{Length of belt} = \pi R(1 + \beta/90) + \pi r(1 + \beta/90) + 2 L \cos \beta$$

$$= (R + r) \times (\pi + 0.0349 \beta) + 2 L \cos \beta,$$

in which β = angle whose sine is $(R + r) \div L$; $\cos \beta = \sqrt{L^2 - (R + r)^2} \div L$.

To find the Length of Belt when Closely Rolled. — The sum of the diameter of the roll, and of the eye in inches, \times the number of turns made by the belt and by 1309, = length of the belt in feet.

To find the Approximate Weight of Belts. — Multiply the length of belt, in feet, by the width in inches, and divide the product by 13 for single and 8 for double belt.

Relations of the Size and Speeds of Driving and Driven Pulleys. — The driving pulley is called the driver, D , and the driven pulley the driven, d . If the number of teeth in gears is used instead of diameter, in these calculations, number of teeth must be substituted wherever diameter occurs. R = revs. per min. of driver, r = revs. per min. of driven.

$$D = dr \div R;$$

Diam. of driver = diam. of driven \times revs. of driven \div revs. of driver.

$$d = DR \div r;$$

Diam. of driven = diam. of driver \times revs. of driver \div revs. of driven.

$$R = dr + D;$$

Revs. of driver = revs. of driven \times diam. of driven \div diam. of driver.

$$r = DR \div d;$$

Revs. of driven = revs. of driver \times diam. of driver \div diam. of driven.

Evils of Tight Belts. (Jones and Laughlins.) — Clamps with powerful screws are often used to put on belts with extreme tightness, and with most injurious strain upon the leather. They should be very judiciously used for horizontal belts, which should be allowed sufficient slackness to move with a loose undulating vibration on the returning side, as a test that they have no more strain imposed than is necessary simply to transmit the power.

On this subject a New England cotton-mill engineer of large experience says: I believe that three-quarters of the trouble experienced in broken pulleys, hot boxes, etc., can be traced to the fault of tight belts. The enormous and useless pressure thus put upon pulleys must in time break them, if they are made in any reasonable proportions, besides wearing out the whole outfit, and causing heating and consequent destruction of the bearings. Below are some figures showing the power it takes, in average modern mills with first-class shafting, to drive the shafting alone:

| Mill No. | Whole Load, H.P. | Shafting Alone. | | Mill No. | Whole Load, H.P. | Shafting Alone. | |
|----------|------------------|-----------------|--------------------|----------|------------------|-----------------|--------------------|
| | | Horse-power. | Per cent of whole. | | | Horse-power. | Per cent of whole. |
| 1 | 199 | 51 | 25.6 | 5 | 759 | 172.6 | 22.7 |
| 2 | 472 | 111.5 | 23.6 | 6 | 235 | 84.8 | 36.1 |
| 3 | 486 | 134 | 27.5 | 7 | 670 | 262.9 | 39.2 |
| 4 | 677 | 190 | 28.1 | 8 | 677 | 182 | 26.8 |

These may be taken as a fair showing of the power that is required in many of our best mills to drive shafting. It is unreasonable to think that all that power is consumed by a legitimate amount of friction of bearings and belts. I know of no cause for such a loss of power but tight belts. These, when there are hundreds or thousands in a mill, easily multiply the friction on the bearings, and would account for the figures.

Sag of Belts. Distance between Pulleys. — In the location of shafts that are to be connected with each other by belts, care should be taken to secure a proper distance one from the other. This distance should be such as to allow of a gentle sag to the belt when in motion.

A general rule may be stated thus: Where narrow belts are to be run over small pulleys 15 feet is a good average, the belt having a sag of $1\frac{1}{2}$ to 2 inches.

For larger belts, working on larger pulleys, a distance of 20 to 25 feet does well, with a sag of $2\frac{1}{2}$ to 4 inches.

For main belts working on very large pulleys, the distance should be 25 to 30 feet, the belts working well with a sag of 4 to 5 inches.

If too great a distance is attempted, the belt will have an unsteady flapping motion, which will destroy both the belt and machinery.

Arrangement of Belts and Pulleys. — If possible to avoid it, connected shafts should never be placed one directly over the other, as in such case the belt must be kept very tight to do the work. For this purpose belts should be carefully selected of well-stretched leather.

It is desirable that the angle of the belt with the floor should not exceed 45° . It is also desirable to locate the shafting and machinery so that belts should run off from each shaft in opposite directions, as this arrangement will relieve the bearings from the friction that would result when the belts all pull one way on the shaft.

In arranging the belts leading from the main line of shafting to the counters, those pulling in an opposite direction should be placed as near

each other as practicable, while those pulling in the same direction should be separated. This can often be accomplished by changing the relative positions of the pulleys on the counters. By this procedure much of the friction on the journals may be avoided.

If possible, machinery should be so placed that the direction of the belt motion shall be from the top of the driving to the top of the driven pulley, when the sag will increase the arc of contact.

The pulley should be a little wider than the belt required for the work. The motion of driving should run with and not against the laps of the belts.

Tightening or guide pulleys should be applied to the slack side of belts and near the smaller pulley.

Jones and Laughlins, in their *Useful Information*, say: The diameter of the pulleys should be as large as can be admitted, provided they will not produce a speed of more than 4750 feet of belt motion per minute.

They also say: It is better to gear a mill with small pulleys and run them at a high velocity, than with large pulleys and to run them slower. A mill thus geared costs less and has a much neater appearance than with large heavy pulleys.

M. Arthur Achard (*Proc. Inst. M. E.*, Jan., 1881, p. 62) says: When the belt is wide a partial vacuum is formed between the belt and the pulley at a high velocity. The pressure is then greater than that computed from the tensions in the belt, and the resistance to slipping is greater. This has the advantage of permitting a greater power to be transmitted by a given belt, and of diminishing the strain on the shafting.

On the other hand, some writers claim that the belt entraps air between itself and the pulley, which tends to diminish the friction, and reduce the tractive force. On this theory some manufacturers perforate the belt with numerous holes to let the air escape.

Care of Belts. — Leather belts should be well protected against water, loose steam, and all other moisture, with which they should not come in contact. But where such conditions prevail fairly good results are obtained by using a special dressing prepared for the purpose of water-proofing leather, though a positive water-proofing material has not yet been discovered.

Belts made of coarse, loose-fibered leather will do better service in dry and warm places, but if damp or moist conditions exist then the very finest and firmest leather should be used. (Fayerweather & Ladew.)

Do not allow oil to drip upon the belts. It destroys the life of the leather. Leather belting cannot safely stand above 110° of heat.

Strength of Belting. — The ultimate tensile strength of belting does not generally enter as a factor in calculations of power transmission.

The strength of the solid leather in belts is from 2000 to 5000 lbs. per square inch; at the lacings, even if well put together, only about 1000 to 1500. If riveted, the joint should have half the strength of the solid belt. The working strain on the driving side is generally taken at not over one-third of the strength of the lacing, or from one-eighth to one-sixteenth of the strength of the solid belt. Dr. Hartig found that the tension in practice varied from 30 to 532 lbs. per sq. in., averaging 273 lbs.

Adhesion Independent of Diameter. (Schultz Belting Co.) —

1. The adhesion of the belt to the pulley is the same — the arc or number of degrees of contact, aggregate tension or weight being the same — without reference to width of belt or diameter of pulley.

2. A belt will slip just as readily on a pulley four feet in diameter as it will on a pulley two feet in diameter, provided the conditions of the faces of the pulleys, the arc of contact, the tension, and the number of feet the belt travels per minute are the same in both cases.

3. To obtain a greater amount of power from belts the pulleys may be covered with leather; this will allow the belts to run very slack and give 25% more durability.

Endless Belts. — If the belts are to be endless, they should be put on and drawn together by "belt clamps" made for the purpose. If the belt is made endless at the belt factory, it should never be run on to the pulleys, lest the irregular strain spring the belt. Lift out one shaft, place the belt on the pulleys, and force the shaft back into place.

Belt Data. — A fly-wheel at the Amoskeag Mfg. Co., Manchester, N.H., 30 feet diameter, 110 inches face, running 61 revs. per min., carried two

heavy double-leather belts 40 inches wide each, and one 24 inches wide. The engine indicated 1950 H.P., of which probably 1850 H.P. was transmitted by the belts. The belts were considered to be heavily loaded, but not overtaxed. $(30 \times 3.14 \times 104 \times 61) \div 1850 = 323$ ft. per min. for 1 H.P. per inch of width.

Samuel Webber (*Am. Mach.*, Feb. 22, 1894) reports a case of a belt 30 ins. wide, $\frac{3}{8}$ in. thick, running for six years at a velocity of 3900 ft. per min., on to a pulley 5 ft. diameter, and transmitting 556 H.P. This gives a velocity of 210 ft. per min. for 1 H.P. per in. of width. By Mr. Nagle's table of riveted belts this belt would be designed for 332 H.P. By Mr. Taylor's rule it would be used to transmit only 123 H.P.

The above may be taken as examples of what a belt may be made to do, but they should not be used as precedents in designing. It is not stated how much power was lost by the journal friction due to over-tightening of these belts.

Belt Dressings. — We advise that no belt dressing should be used except when the belt becomes dry and husky, and in such instances we recommend the use of a dressing. Where this is not used beef tallow at blood-warm temperature should be applied and then dried in either by artificial heat or the sun. The addition of beeswax to the tallow will be of some service if the belts are used in wet or damp places. Our experience convinces us that resin should never be used on leather belting. (Fayerweather & Ladew.)

Belts should not be soaked in water before oiling, and penetrating oils should but seldom be used, except occasionally when a belt gets very dry and husky from neglect. It may then be moistened a little, and have neat's-foot oil applied. Frequent applications of such oils to a new belt render the leather soft and flabby, thus causing it to stretch, and making it liable to run out of line. A composition of tallow and oil, with a little resin or beeswax, is better to use. Prepared castor-oil dressing is good, and may be applied with a brush or rag while the belt is running. (Alexander Bros.)

Some forms of belt dressing, the compositions of which have not been published, appear to have the property of increasing the coefficient of friction between the belt and the pulley, enabling a given power to be transmitted with a lower belt tension than with undressed belts. C. W. Evans (*Power*, Dec., 1905), gives a diagram, plotted from tests, which shows that three of these compositions gave increased transmission for a given tension, ranging from about 10% for 90 lbs. tension per inch of width to 100% increase with 20 lbs. tension.

Cement for Cloth or Leather. (Molesworth.) — 16 parts gutta-percha, 4 india-rubber, 2 pitch, 1 shellac, 2 linseed-oil, cut small, melted together and well mixed.

Rubber Belting. — The advantages claimed for rubber belting are perfect uniformity in width and thickness; it will endure a great degree of heat and cold without injury; it is also specially adapted for use in damp or wet places, or where exposed to the action of steam; it is very durable, and has great tensile strength, and when adjusted for service it has the most perfect hold on the pulleys, hence is less liable to slip than leather.

Never use animal oil or grease on rubber belts, as it will greatly injure and soon destroy them.

Rubber belts will be improved, and their durability increased, by putting on with a painter's brush, and letting it dry, a composition made of equal parts of red lead, black lead, French yellow, and litharge, mixed with boiled linseed-oil and japan enough to make it dry quickly. The effect of this will be to produce a finely polished surface. If, from dust or other cause, the belt should slip, it should be lightly moistened on the side next the pulley with boiled linseed-oil. (From circulars of manufacturers.)

The best conditions are large pulleys and high speeds, low tension and reduced width of belt. 4000 ft. per min. is not an excessive speed on proper sized pulleys.

H.P. of a 4-ply rubber belt = (length of arc of contact on smaller pulley in ft. \times width of belt in ins. \times revs. per min.) \div 325. For a 5-ply belt multiply by $1\frac{1}{3}$, for a 6-ply by $1\frac{2}{3}$, for a 7-ply by 2, for an 8-ply by $2\frac{1}{3}$. When the proper weight of duck is used a 3- or 4-ply rubber belt is equal to a single leather belt and a 5- or 6-ply rubber to a double leather belt.

When the arc of contact is 180° , H.P. of a 4-ply belt = width in ins. \times velocity in ft. per min. \div 650. (Boston Belting Co.)

Steel Belts. — The Eloesser-Kraftband-Gesellschaft, of Berlin, has introduced a steel belt for heavy power transmission at high speeds (*Am. Mach.*, Dec. 24, 1908). It is a thin flat band of tempered steel. The ends are soldered and then clamped by a special device consisting of two steel plates, tapered to thin edges, which are curved to the radius of the smallest pulley to be used, and joined together by small screws which pass through holes in the ends of the belt. It is stated that the slip of these belts is less than 0.1%; they are about one-fifth the width of a leather belt for the same power, and they are run at a speed of 10,000 ft. per minute or upwards. The following figures give a comparison of a rope drive with six ropes 1.9 ins. diam., a leather belt 9.6 ins. wide and a steel belt 4 ins. wide, for transmitting 100 H.P. on pulley 3 ft. diam., 30 ft. apart at 200 r.p.m.

| | Rope Drive. | Leather Belt. | Steel Belt. |
|--------------------------------------|-------------|---------------|-------------|
| Weight of pulley, lbs..... | 2200 | 1120 | 460 |
| Weight of rope or belt, lbs..... | 530 | 240 | 30 |
| Total cost of drive..... | \$335 | \$425 | \$250 |
| Power lost, per cent of 100 H.P..... | 13 | 6 | 0.5 |

ROLLER CHAIN AND SPROCKET DRIVES.

The following is abstracted from an article by A. E. Michel, in *Machy*, Feb., 1905.

Steel chain of accurate pitch, high tensile strength, and good wearing qualities, possesses, when used within proper limitations, advantages enjoyed by no other form of transmission. It is compact, affords a positive speed ratio, and at slow speeds is capable of transmitting heavy strains. On short transmissions it is more efficient than belting and will operate more satisfactorily in damp or oily places. There is no loss of power from stretch, and as it allows of a low tension, journal friction is minimized.

Roller chain has been known to stand up at a speed of 2,000 ft. per min., and transmit 25 H.P. at 1,250 ft. per min.; but speeds of 1,000 ft. per min. and under give better satisfaction. Block chain is adapted to slower speeds, say 700 ft. per min. and under, and is extensively used on bicycles, small motor cars and machine tools. Where speed and pull are not fixed quantities, it is advisable to keep the speed high, and chain pull low, yet it should be borne in mind that high speeds are more destructive to chains of large than to those of small pitch.

The following table of tensile strengths, based on tests of "Diamond" chains taken from stock, may be considered a fair standard:

ROLLER CHAIN.

| | | | | | | | | |
|----------------------|---------------|---|---------------|-------|----------------|----------------|----------------|--------|
| Pitch, in..... | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 |
| Tens. strength, lbs. | 1,200 | 1,200 | 4,000 | 6,000 | 9,000 | 12,000 | 19,000 | 25,000 |
| Block chain..... | 1 inch, | 1,200 to 2,500; $1\frac{1}{2}$ inch, 5,000. | | | | | | |

The safe working load of a chain is dependent on the amount of rivet bearing surface, and varies from $\frac{1}{5}$ to $\frac{1}{40}$ of the tensile strength, according to the speed, size of sprockets, and other conditions peculiar to each case. The tendency now is to use the widest possible chain in order to secure maximum rivet bearing surface, thus insuring minimum wear from friction. Manufacturers are making heavier chains than heretofore for the same duty. As short pitch is always desirable, special double and even triple width chains are now made to conform to the requirements when a heavy single width chain of greater pitch is not practical. A double chain has twice the rivet bearing surface and half again as much tensile strength as the similar single one.

The length of chain for a given drive may be found by the following formula:

All dimensions in inches. D = Distance between centers of shafts. A = Distance between limiting points of contact. R = Pitch radius of large sprocket. r = Pitch radius of small sprocket. N = Number of teeth of large sprocket. n = Number of teeth of small sprocket. P = Pitch of chain and sprockets. $(180^\circ + 2\alpha)$ = angle of contact on large sprocket. $(180^\circ - 2\alpha)$ = angle of contact on small sprocket. α = angle whose sine is $(R - r)/D$. $A = D \cos \alpha$.

Length of chain required:

$$L = \frac{180 + 2\alpha}{360} NP + \frac{180 - 2\alpha}{360} nP + 2 D \cos \alpha.$$

For block chain, the total length specified in ordering should be in multiples of the pitch. For roller chain, the length should be in multiples of twice the pitch, as a union of the ends can be effected only with an outside and an inside link.

Wherever possible, the distance between centers of shafts should permit of adjustment in order to regulate the sag of the chain. A chain should be adjusted, in proportion to its length, to show slack when running, care being taken to have it neither too tight nor too loose, as either condition is destructive. If a fixed center distance must be used, and results in too much sag, the looseness should be taken up by an idler, and when there is any considerable tension on the slack side, this idler must be a sprocket. Where an idler is not practical, another combination of sprockets giving approximately the same speed ratio may be tried, and in this manner a combination giving the proper sag may always be obtained.

In automobile drives, too much sag or too great a distance between shafts causes the chain to whip up and down — a condition detrimental to smooth running and very destructive to the chain. In this class of work a center distance of over 4 ft. has been used, but greater efficiency and longer life are secured from the chain on shorter lengths, say 3 ft. and under.

Sprocket Wheels. Properly proportioned and machined sprockets are essential to successful chain gearing. The important dimensions of a sprocket are the pitch diameter and the bottom and outside diameters. For block chain these are obtained as follows:

N = No. of teeth. b = Diameter of round part of chain block. B = Center to center of holes in chain block. A = Center to center of holes in side links. $\alpha = 180^\circ/N$. $\tan \beta = \sin \alpha \div (B/A + \cos \alpha)$.

Pitch diameter = $A/\sin \beta$.

Bottom diam. = pitch diam. - b . Outside diam. = pitch diam. + b .

For roller chain: N = Number of teeth. P = Pitch of chain. D = Diameter of roller. $\alpha = 180^\circ/N$. Pitch diameter = $P/\sin \alpha$.

Bottom diam. = pitch diam. - D .

For sprockets of 17 teeth and over, outside diam = pitch diam. + D .

The outside diameters of small sprockets are cut down so that the teeth will clear the roller perfectly at high speeds.

Outside diam. = pitch diam. + $D - E$.

| Pitch. | Values of E . | |
|-----------------------------|-------------------|--------------------|
| | 8 to 12 Teeth. | 13 to 16 Teeth. |
| $1/2$ in. to $3/4$ in. | 0.062 in. | 0.031 in. |
| 1 in. to 2 ins. | 0.125 in. | 0.062 in. |

Sprocket diameters should be very accurate, particularly the base diameter, which should not vary more than 0.002 in. from the calculated values. Sprockets should be gauged to discover thick teeth and inaccurate diameters. A poor chain may operate on a good sprocket, but a bad sprocket will ruin a good chain. Sprockets of 12 to 60 teeth give best

results. Fewer may be used, but cause undue elongation in the chain, wear the sprockets and consume too much power. Eight-tooth sprockets ruin almost every roller chain applied to them, and ten and eleven teeth are fitted only for medium and slow speeds with other conditions unusually favorable.

Sprocket teeth seldom break from insufficient strength, but the tooth must be properly shaped. A chain will not run well unless the sprockets have sidewise clearance and teeth narrowed at the ends by curves beginning at the pitch line.

Calling W the width of the chain between the links,

$A = \frac{1}{2} W$ = width of tooth at top. B = uniform width below pitch line.

$B = W - \frac{1}{64}$ in. when $W = \frac{1}{4}$ in. or less.

= $W - \frac{1}{32}$ in. when $W = \frac{5}{16}$ to $\frac{5}{8}$ in. inclusive.

= $W - \frac{1}{16}$ in. when $W = \frac{3}{4}$ in. or over.

If the sprocket is flanged the chain must seat itself properly without the side bars coming into contact with the flange.

The principal cause of trouble within the chain is elongation. It is the result of stretch of material or natural wear of rivets and their bearings. To guard against the former, chain makers use special materials of high tensile strength, but a chain subjected to jars and jolts beyond the limit of elasticity of the material may be put in worse condition in an instant than in months of natural wear. If for any reason a link elongates unduly it should be replaced at once, as one elongated link will eventually ruin the entire chain. Such elongation frequently results from all the load being thrown on at once.

To minimize natural wear, chains should be well greased inside and out, protected from mud and heavy grit, cleaned often and replaced to run in the same direction and same side up. A new chain should never be applied to a much-worn sprocket.

Importance of pitch line clearances: In a sprocket with no clearances a new chain fits perfectly, but after natural wear the pitch of chain and sprocket become unlike. The chain is then elongated and climbs the teeth, which act as wedges, producing enormous strain, and it quickly wrecks itself. With the same chain on a driven sprocket, cut with clearances, all rollers seat against their teeth. After long and useful life, the working roller shifts to the top, and the other rollers still seat with the same ease as when new. Theoretically, all the rollers share the load. This never occurs in practice, for infinitesimal wear within the chain causes one, and only one, roller to bear perfectly seated against the working face of the sprocket tooth at any one time. Clearance alone on the driver will not provide for elongation. To operate properly the pitch of the driver must be lengthened, which is done by increasing the pitch diameter by an amount dependent upon the clearance allowed. For theoretical reasoning on this subject see "Roller Chain Gear," a treatise on English practice, by Hans Renold.

When the load reverses, each sprocket becomes alternately driver and driven. This happens in a motor car during positive and negative acceleration, or in ascending or descending a hill. In this event, the above construction is not applicable, for a driven sprocket of longer pitch than the chain will stretch it. No perfect method of equalizing the pitch of a roller chain and its sprockets under reversible load and at all periods of chain elongation has been found. This fault is eliminated in the "silent" type of chain; hence it runs smooth at a very much greater speed than roller chain will stand.

In practice there are comparatively few roller chain drives with chain pull always in the same direction, so manufacturers generally cut the driver sprockets for these with normal pitch diameter, same as the driven. Recent experiments have proven that the difficulties are greatly lessened by cutting both driver and driven with liberal pitch line clearance. Accordingly, chain makers now advise the following pitch line clearance for standard rollers:

| | | | | | | | |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Pitch, in., | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 |
| Clearance, in., | $\frac{1}{32}$ | $\frac{3}{16}$ | $\frac{3}{32}$ | $\frac{1}{16}$ | $\frac{7}{32}$ | $\frac{1}{8}$ | $\frac{5}{32}$ |

Cutters may be obtained from Brown & Sharpe Mfg. Co. with this clearance.

Belting versus Chain Drives. — Chains are suitable for positive transmissions of very heavy powers at slow speed. They are properly used for conveying ashes, sand, chemicals and liquids which would corrode or destroy belting. Chains of this kind are generally made of malleable iron. For conveyers for clean substances, flour, wheat and other grains, belts are preferable, and in the best installations leather is preferred to cotton or rubber, being more durable. Transmission chains have to be carefully made. If the chain is to run smoothly, noiselessly, and without considerable friction, both the links and the sprockets must be mathematically correct. This perfection of design is found only in the highest and best makes of steel chain.

Deterioration of chains starts in with the beginning of service. Even in such light and flexible duty as bicycle transmission, a chain is subjected to sudden severe strains, which either stretch the chain or distort the bearing surfaces. Either mishap is fatal to smooth frictionless running. If the transmission is positive, as from motor or shaft to a machine tool, sudden variations in strain become sledge-hammer blows, and the chain must either break or the parts yield. To avoid the evils arising from the stretching of the chain, self-adjusting forms of teeth have been invented, of which the Renold silent-chain gear is one of the best.

The makers of the Morse rocker chain, also an excellent chain, recommend it for use under the following conditions: (1) Where room is lacking for the proper sized pulleys for belts. (2) Where the centers between shafts are too short for belts. (3) Where a positive speed ratio is desired. (4) Where there is moisture, heat or dust that would prevent a belt working properly. (5) Where a maximum power per inch of width is desired.

The Renold silent chain and the Morse rocker chain find springs necessary in the sprocket wheel. This springiness the belt naturally possesses, and where maximum power is not necessary at a low speed under service conditions of moisture and dirt, as in automobile transmission, the belt will be cheaper to install, cheaper to maintain, cheaper to repair in case of breakdown, and more efficient than any chain. A leather belt will run on very short centers and transmit very high powers, but it should be run at higher speed than a long belt.

For slow service, for positive transmission, for rough service, gears are rivals of chain transmission. For fast service, for springy transmission, for clean, dry work, leather belts are still the best. — Harrington Emerson, *Am. Mach.*, April 6, 1909.

It is to be regretted that there is no standard among chain manufacturers for the correct outline of sprocket cutters and amount of clearance for various sizes of chain. If it is clearly understood that the high quality roller and block chains now on the market require correctly cut sprockets properly proportioned for the particular conditions of service they are to work under, there will be a large increase in their use for power transmission, and the troubles now incident to incorrect installations could be wholly obviated. — C. C. Myers, *Am. Mach.*, Aug. 5, 1909.

A 350-H.P. Silent Chain Drive has been built by the Link Belt Co. The gears are 12 ft. apart, centers. The drive consists of two strands, each 12 ins. wide, of Renold silent chain of 2-in. pitch. The pinion is of forged steel, about 16½ in. diameter, 27-in. face, 26 teeth, bore 29 in. long 10 in. diameter. The main gear is made of two cast-iron wheels, side by side, each 76½-in. diameter, 13½-in. face, 120 teeth. Each wheel is provided with steel flanges and a special hub containing a series of stiff coiled springs in compression through which the driving force is transmitted from the hub to the wheel. The object of this device is to provide an equalizing factor between the power shaft and the teeth of the wheel, so that any unevenness in the rotation and consequent shock will be absorbed by the device. The pinion is mounted on the armature of a motor running 300 r.p.m., and the speed of the driven gear is 65 r.p.m. The speed of the chain belt is 780 ft. per minute. Three of these drives have been constructed to transmit power for wire drawing. — (*Power*, Dec. 28, 1909.)

GEARING.

TOOTHED-WHEEL GEARING.

Pitch, Pitch-circle, etc. — If two cylinders with parallel axes are pressed together and one of them is rotated on its axis, it will drive the other by means of the friction between the surfaces. The cylinders may be considered as a pair of spur-wheels with an infinite number of very small teeth. If actual teeth are formed upon the cylinders, making alternate elevations and depressions in the cylindrical surfaces, the distance between the axes remaining the same, we have a pair of gear-wheels which will drive one another by pressure upon the faces of the teeth, if the teeth are properly shaped. In making the teeth the cylindrical surface may entirely disappear, but the position it occupied may still be considered as a cylindrical surface, which is called the "pitch-surface," and its trace on the end of the wheel, or on a plane cutting the wheel at right angles to its axis, is called the "pitch-circle" or "pitch-line." The diameter of this circle is called the pitch-diameter, and the distance from the face of one tooth to the corresponding face of the next tooth on the same wheel, measured on an arc of the pitch-circle, is called the "pitch of the tooth," or the circular pitch.

If two wheels having teeth of the same pitch are geared together so that their pitch-circles touch, it is a property of the pitch-circles that their diameters are proportional to the number of teeth in the wheels, and vice versa; thus, if one wheel is twice the diameter (measured on the pitch-circle) of the other, it has twice as many teeth. If the teeth are properly shaped the linear velocities of the two wheels are equal, and the angular velocities, or speeds of rotation, are inversely proportional to the number of teeth and to the diameter. Thus the wheel that has twice as many teeth as the other will revolve just half as many times in a minute.

The "pitch," or distance measured on an arc of the pitch-circle from the face of one tooth to the face of the next, consists of two parts — the "thickness" of the tooth and the "space" between it and the next tooth. The space is larger than the thickness by a small amount called the "backlash," which is allowed for imperfections of workmanship. In finely cut gears the backlash may be almost nothing.

The length of a tooth in the direction of the radius of the wheel is called the "depth," and this is divided into two parts: First, the "addendum," the height of the tooth above the pitch line; second, the "dedendum," the depth below the pitch-line, which is an amount equal to the addendum of the mating gear. The depth of the space is usually given a little "clearance" to allow for inaccuracies of workmanship, especially in cast gears.

Referring to Fig. 171, *pl, pl* are the pitch-lines, *al* the addendum-line, *rl* the root-line or dedendum-line, *cl* the clearance-line, and *b* the backlash. The addendum and dedendum are usually made equal to each other.

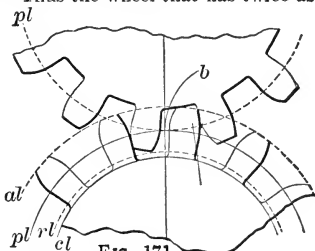


FIG. 171.

$$\text{Diametral pitch} = \frac{\text{No of teeth}}{\text{diam. of pitch-circle in inches}} = \frac{3.1416}{\text{circular pitch}};$$

$$\text{Circular pitch} = \frac{\text{diam.} \times 3.1416}{\text{No. of teeth}} = \frac{3.1416}{\text{diametral pitch}}.$$

Some writers use the term diametral pitch to mean $\frac{\text{diam.}}{\text{No. of teeth}} = \frac{\text{circular pitch}}{3.1416}$, but the first definition is the more common and the more

convenient. A wheel of 12 in. diam. at the pitch-circle, with 48 teeth, is $\frac{48}{12} = 4$ diametral pitch, or simply 4 pitch. The circular pitch of the same wheel is $12 \times 3.1416 \div 48 = 0.7854$, or $3.1416 \div 4 = 0.7854$ in.

Relation of Diametral to Circular Pitch.

| Diametral Pitch. | Circular Pitch. | Diametral Pitch. | Circular Pitch. | Circular Pitch. | Diametral Pitch. | Circular Pitch. | Diametral Pitch. |
|------------------|-----------------|------------------|-----------------|-----------------|------------------|-----------------|------------------|
| 1 | 3.142 in. | 11 | 0.286 in. | 3 | 1.047 | $\frac{15}{16}$ | 3.351 |
| $1\frac{1}{2}$ | 2.094 | 12 | .262 | $2\frac{1}{2}$ | 1.257 | $\frac{7}{8}$ | 3.590 |
| 2 | 1.571 | 14 | .224 | 2 | 1.571 | $\frac{13}{16}$ | 3.867 |
| $2\frac{1}{4}$ | 1.396 | 16 | .196 | $1\frac{7}{8}$ | 1.676 | $\frac{3}{4}$ | 4.189 |
| $2\frac{1}{2}$ | 1.257 | 18 | .175 | $1\frac{3}{4}$ | 1.795 | $\frac{11}{16}$ | 4.570 |
| $2\frac{3}{4}$ | 1.142 | 20 | .157 | $1\frac{5}{8}$ | 1.933 | $\frac{5}{8}$ | 5.027 |
| 3 | 1.047 | 22 | .143 | $1\frac{1}{2}$ | 2.094 | $\frac{9}{16}$ | 5.585 |
| $3\frac{1}{2}$ | .898 | 24 | .131 | $1\frac{7}{16}$ | 2.185 | $\frac{1}{2}$ | 6.283 |
| 4 | .785 | 26 | .121 | $1\frac{3}{8}$ | 2.285 | $\frac{7}{16}$ | 7.181 |
| 5 | .628 | 28 | .112 | $1\frac{5}{16}$ | 2.394 | $\frac{3}{8}$ | 8.378 |
| 6 | .524 | 30 | .105 | $1\frac{1}{4}$ | 2.513 | $\frac{5}{16}$ | 10.053 |
| 7 | .449 | 32 | .098 | $1\frac{3}{16}$ | 2.646 | $\frac{1}{4}$ | 12.566 |
| 8 | .393 | 36 | .087 | $1\frac{1}{8}$ | 2.793 | $\frac{3}{16}$ | 16.755 |
| 9 | .349 | 40 | .079 | $1\frac{1}{16}$ | 2.957 | $\frac{1}{8}$ | 25.133 |
| 10 | .314 | 48 | .055 | 1 | 3.142 | $\frac{1}{16}$ | 50.266 |

$$\text{Since circ. pitch} = \frac{\text{diam.} \times 3.1416}{\text{No. of teeth}}, \text{diam.} = \frac{\text{circ. pitch} \times \text{No. of teeth}}{3.1416}$$

which always brings out the diameter as a number with an inconvenient fraction if the pitch is in even inches or simple fractions of an inch. By the diametral-pitch system this inconvenience is avoided. The diameter may be in even inches or convenient fractions, and the number of teeth is usually an even multiple of the number of inches in the diameter.

Diameter of Pitch-line of Wheels from 10 to 100 Teeth of 1 in. Circular Pitch.

| No. Teeth. | Diam., in. | No. Teeth. | Diam., in. | No. Teeth. | Diam., in. | No. Teeth. | Diam., in. | No. Teeth. | Diam., in. | No. Teeth. | Diam., in. |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 10 | 3.183 | 26 | 8.276 | 41 | 13.051 | 56 | 17.825 | 71 | 22.600 | 86 | 27.375 |
| 11 | 3.501 | 27 | 8.594 | 42 | 13.369 | 57 | 18.144 | 72 | 22.918 | 87 | 27.693 |
| 12 | 3.820 | 28 | 8.913 | 43 | 13.687 | 58 | 18.462 | 73 | 23.236 | 88 | 28.011 |
| 13 | 4.138 | 29 | 9.231 | 44 | 14.006 | 59 | 18.781 | 74 | 23.555 | 89 | 28.329 |
| 14 | 4.456 | 30 | 9.549 | 45 | 14.324 | 60 | 19.099 | 75 | 23.873 | 90 | 28.648 |
| 15 | 4.775 | 31 | 9.868 | 46 | 14.642 | 61 | 19.417 | 76 | 24.192 | 91 | 28.966 |
| 16 | 5.093 | 32 | 10.186 | 47 | 14.961 | 62 | 19.735 | 77 | 24.510 | 92 | 29.285 |
| 17 | 5.411 | 33 | 10.504 | 48 | 15.279 | 63 | 20.054 | 78 | 24.828 | 93 | 29.603 |
| 18 | 5.730 | 34 | 10.823 | 49 | 15.597 | 64 | 20.372 | 79 | 25.146 | 94 | 29.921 |
| 19 | 6.048 | 35 | 11.141 | 50 | 15.915 | 65 | 20.690 | 80 | 25.465 | 95 | 30.239 |
| 20 | 6.366 | 36 | 11.459 | 51 | 16.234 | 66 | 21.008 | 81 | 25.783 | 96 | 30.558 |
| 21 | 6.685 | 37 | 11.777 | 52 | 16.552 | 67 | 21.327 | 82 | 26.101 | 97 | 30.876 |
| 22 | 7.003 | 38 | 12.096 | 53 | 16.870 | 68 | 21.645 | 83 | 26.419 | 98 | 31.194 |
| 23 | 7.321 | 39 | 12.414 | 54 | 17.189 | 69 | 21.963 | 84 | 26.738 | 99 | 31.512 |
| 24 | 7.639 | 40 | 12.732 | 55 | 17.507 | 70 | 22.282 | 85 | 27.056 | 100 | 31.831 |
| 25 | 7.958 | | | | | | | | | | |

For diameter of wheels of any other pitch than 1 in., multiply the figures in the table by the pitch. Given the diameter and the pitch, to find the number of teeth. Divide the diameter by the pitch, look in the table under diameter for the figure nearest to the quotient, and the number of teeth will be found opposite.

Proportions of Teeth. Circular Pitch = 1.

| | 1. | 2. | 3. | 4. | 5. | 6. |
|--------------------------------------|-------|-------|------|-------|-------|-------|
| Depth of tooth above pitch-line..... | 0.35 | 0.30 | 0.37 | 0.33 | 0.30 | 0.30 |
| Depth of tooth below pitch-line..... | .40 | .40 | .43 | | .40 | .35 |
| Working depth of tooth..... | .70 | .60 | .73 | .66 | | |
| Total depth of tooth..... | .75 | .70 | .80 | .75 | .70 | .65 |
| Clearance at root..... | .05 | .10 | .07 | | | |
| Thickness of tooth..... | .45 | .45 | .47 | .45 | .475 | .485 |
| Width of space..... | .54 | .55 | .53 | .55 | .525 | .515 |
| Backlash..... | .09 | .10 | .06 | .10 | .05 | .03 |
| Thickness of rim..... | | | .47 | .45 | .70 | .65 |

| | 7. | 8. | 9. | 10.* |
|--------------------------------------|--------------|------------|------------|-----------------------------------|
| Depth of tooth above pitch-line..... | 0.25 to 0.33 | 0.30 | 0.318 | $1 \div P$ |
| Depth of tooth below pitch-line..... | .35 to .42 | .35 + .08" | .369 | $1.157 \div P$ |
| Working depth of tooth..... | | | .637 | $2 \div P$ |
| Total depth of tooth..... | .6 to .75 | .65 + .08" | .687 | $2.157 \div P$ |
| Clearance at root..... | | | .04 to .05 | $0.157 \div P$ |
| Thickness of tooth..... | .48 to .485 | .48 - .03" | .48 to .5 | $1.51 \div P$ to $1.57 \div P$ |
| Width of space..... | .52 to .515 | .52 + .03" | .52 to .5 | $1.57 \div P$ to $1.63 \div P$ |
| Backlash..... | .04 to .03 | .04 + .06" | .0 to .04 | $0.0 to .06 \div P$ |

* In terms of diametral pitch.

AUTHORITIES. — 1. Sir Wm. Fairbairn. 2, 3. Clark, R. T. D.; "used by engineers in good practice." 4. Molesworth. 5, 6. Coleman Sellers; 5 for cast, 6 for cut wheels. 7, 8. Unwin. 9, 10. Leading American manufacturers of cut gears.

The Chordal Pitch (erroneously called "true pitch" by some authors) is the length of a straight line or chord drawn from center to center of two adjacent teeth. The term is now but little used, except in connection with chain and sprocket gearing.

Chordal pitch = diam. of pitch-circle \times sine of $\frac{180^\circ}{\text{No. of teeth}}$. Chordal pitch of a wheel of 10 in. pitch diameter and 10 teeth, $10 \times \sin 18^\circ = 3.0902$ in. Circular pitch of same wheel = 3.1416. Chordal pitch is used with chain or sprocket wheels, to conform to the pitch of the chain.

Gears with Short Teeth. — There is a tendency in recent years to depart widely from the proportions of teeth given in the above and to use much shorter teeth, especially for heavy machinery. C. W. Hunt gives addendum and dedendum each = 0.25, and the clearance 0.05 of the circular pitch, making the total depth of tooth 0.55 of the circular pitch. The face of the tooth is involute in form, and the angle of action is $14\frac{1}{2}^\circ$, C. H. Logue uses a 20° involute with the following proportions: Addendum $0.25P' = 0.7854 \div P$; dedendum $0.30P' = 0.9424 \div P$; clearance, $0.05P' = 0.157P$; whole depth $0.55P' = 1.7278 \div P$. P' = circular pitch, P = diametral pitch. See papers by R. E. Flanders and Norman Litchfield in *Trans. A. S. M. E.*, 1908.

John Walker (*Am. Mach.*, Mar. 11, 1897) says: For special purposes of slow-running gearing with great tooth stress I should prefer a length of tooth of 0.4 of the pitch, but for general work a length of 0.6 of the pitch. In 1895 Mr. Walker made two pairs of cut steel gears for the Chicago cable railway with 6-in. circular pitch, length = 0.4 pitch. The pinions had 42 teeth and the gears 62, each 20-in. face. The two pairs were set side by side on their shafts, so as to stagger the teeth, making the total face 40 ins. The gears transmitted 1500 H.P. at 60 r.p.m. replacing cast-iron gears of $7\frac{1}{2}$ in. pitch which had broken in service.

Formulae for Determining the Dimensions of Small Gears. (Brown & Sharpe Mfg. Co.)

P = diametral pitch, or the number of teeth to one inch of diameter of pitch-circle;

| | | |
|--------------------------------------|-------------------|----------------------------------|
| D' = diameter of pitch-circle..... | Larger Wheel. | These wheels run together. |
| D = whole diameter..... | | |
| N = number of teeth..... | | |
| V = velocity..... | | |
| d' = diameter of pitch-circle..... | Smaller Wheel. | |
| d = whole diameter..... | | |
| n = number of teeth..... | | |
| u = velocity..... | | |

a = distance between the centers of the two wheels;

b = number of teeth in both wheels;

t = thickness of tooth or cutter on pitch-circle;

s = addendum;

D'' = working depth of tooth;

f = amount added to depth of tooth for rounding the corners and for clearance;

$D'' + f$ = whole depth of tooth;

$\pi = 3.1416$.

P' = circular pitch, or the distance from the center of one tooth to the center of the next measured on the pitch-circle.

Formulae for a single wheel:

$$P = \frac{N+2}{D}; \quad D' = \frac{D \times N}{N+2}; \quad D'' = \frac{2}{P} = 2s; \quad s = \frac{1}{P} = \frac{P'}{\pi} = 0.3183 P';$$

$$P = \frac{N}{D'}; \quad D' = \frac{N}{P}; \quad N = PD - 2; \quad N = PD'; \quad s = \frac{D'}{N} = \frac{D}{N+2};$$

$$P' = \frac{\pi}{P}; \quad D = \frac{N+2}{P}; \quad f = \frac{t}{10}; \quad s + f = \frac{1}{P} \left(1 + \frac{\pi}{20} \right) = 0.3685 P.$$

$$P = \frac{\pi}{P'}; \quad D = D' + \frac{2}{P}; \quad t = \frac{1.57}{P} = 1/2 P'.$$

Formulae for a pair of wheels:

$$b = 2aP; \quad n = \frac{PD'V}{v}; \quad D = \frac{2a(N+2)}{b};$$

$$N = \frac{nv}{V}; \quad v = \frac{PD'V}{n}; \quad d = \frac{2a(n+2)}{b};$$

$$n = \frac{NV}{v}; \quad v = \frac{NV}{n}; \quad a = \frac{b}{2P};$$

$$N = \frac{bv}{v+V}; \quad V = \frac{nv}{N}; \quad a = \frac{D'+d'}{2};$$

$$n = \frac{bV}{v+V}; \quad D' = \frac{2av}{v+V}; \quad d' = \frac{2aV}{v+V}.$$

Width of Teeth. — The width of the faces of teeth is generally made from 2 to 3 times the circular pitch, that is from 6.28 to 9.42 divided by the diametral pitch. There is no standard rule for width.

The following sizes are given in a stock list of cut gears in "Grant's Gears:"

| | | | | | | |
|-------------------|---------|------------|------------|----------|---------------|---------|
| Diametral pitch.. | 3 | 4 | 6 | 8 | 12 | 16 |
| Face, inches..... | 3 and 4 | 2 1/2 13/4 | and 2 11/4 | and 11/2 | 3/4 and 1 1/2 | and 5/8 |

The Walker Company gives:

| | | | | | | | | | | | | |
|----------------------|-------|-------|-------|-----|-------|-------|---|-------|---|----|----|----|
| Circular pitch, in.. | 1/2 | 5/8 | 3/4 | 7/8 | 1 | 1 1/2 | 2 | 2 1/2 | 3 | 4 | 5 | 6 |
| Face, in..... | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/2 | 4 1/2 | 6 | 7 1/2 | 9 | 12 | 16 | 20 |

The following proportions of gear-wheels are recommended by Prof. Coleman Sellers. (*Stevens Indicator*, April, 1892.)

Proportions of Gear-wheels.

| Diametral Pitch. | Circular Pitch. P | Outside of Pitch-line. $P \times 0.3$. | Inside of Pitch-line. | | Width of Space. | |
|------------------|------------------------|--|---|-----------------------------------|---|---|
| | | | For Cast or Cut Bevels or for Cast Spurs. $P \times 0.4$. | For Cut Spurs. $P \times 0.35$ | For Cast Spurs or Bevels. $P \times 0.525$. | For Cut Bevels or Spurs. $P \times 0.51$. |
| | $\frac{1}{4}$ | 0.075 | 0.100 | 0.088 | 0.131 | 0.128 |
| 12 | 0.2618 | .079 | .105 | .092 | .137 | .134 |
| 10 | 0.31416 | .094 | .126 | .11 | .165 | .16 |
| | $\frac{3}{8}$ | .113 | .150 | .131 | .197 | .191 |
| 8 | 0.3927 | .118 | .157 | .137 | .206 | .2 |
| 7 | 0.4477 | .134 | .179 | .157 | .235 | .228 |
| | $\frac{1}{2}$ | .15 | .20 | .175 | .263 | .255 |
| 6 | 0.5236 | .157 | .209 | .183 | .275 | .267 |
| | $\frac{9}{16}$ | .169 | .225 | .197 | .295 | .287 |
| | $\frac{5}{8}$ | .188 | .25 | .219 | .328 | .319 |
| 5 | 0.62832 | .188 | .251 | .22 | .33 | .32 |
| | $\frac{3}{4}$ | .225 | .3 | .263 | .394 | .383 |
| 4 | 0.7854 | .236 | .314 | .275 | .412 | .401 |
| | $\frac{7}{8}$ | .263 | .35 | .307 | .459 | .446 |
| | 1 | .3 | .4 | .35 | .525 | .51 |
| 3 | 1.0472 | .314 | .419 | .364 | .55 | .534 |
| | $\frac{11}{8}$ | .338 | .45 | .394 | .591 | .574 |
| $2\frac{3}{4}$ | 1.1424 | .343 | .457 | .40 | .6 | .583 |
| | $\frac{11}{4}$ | .375 | .5 | .438 | .656 | .638 |
| $2\frac{1}{2}$ | 1.25664 | .377 | .503 | .44 | .66 | .641 |
| | $\frac{13}{8}$ | .413 | .55 | .481 | .722 | .701 |
| | $\frac{11}{2}$ | .45 | .6 | .525 | .788 | .765 |
| 2 | 1.5708 | .471 | .628 | .55 | .825 | .801 |
| | $\frac{13}{4}$ | .525 | .7 | .613 | .919 | .893 |
| | 2 | .6 | .8 | .7 | 1.05 | 1.02 |
| $1\frac{1}{2}$ | 2.0944 | .628 | .838 | .733 | 1.1 | 1.068 |
| | $\frac{21}{4}$ | .675 | .9 | .788 | 1.181 | 1.148 |
| | $\frac{21}{2}$ | .75 | 1.0 | .875 | 1.313 | 1.275 |
| | $\frac{23}{4}$ | .825 | 1.1 | .963 | 1.444 | 1.403 |
| | 3 | .9 | 1.2 | 1.05 | 1.575 | 1.53 |
| 1 | 3.1416 | .942 | 1.257 | 1.1 | 1.649 | 1.602 |
| | $\frac{31}{4}$ | .975 | 1.3 | 1.138 | 1.706 | 1.657 |
| | $\frac{31}{2}$ | 1.05 | 1.4 | 1.225 | 1.838 | 1.785 |

Thickness of rim below root = depth of tooth.

Rules for Calculating the Speed of Gears and Pulleys. — The relations of the size and speed of driving and driven gear-wheels are the same as those of belt pulleys. In calculating for gears, multiply or divide by the diameter of the pitch-circle or by the number of teeth, as may be required. In calculating for pulleys, multiply or divide by their diameter in inches.

If D = diam. of driving wheel, d = diam. of driven, R = revolutions per minute of driver, r = revs. per min. of driven, $RD = rd$;

$$R = rd \div D; r = RD \div d; D = dr \div R; d = DR \div r.$$

If N = No. of teeth of driver and n = No. of teeth of driven, $NR = nr$;

$$N = nr \div R; n = NR \div r; R = rn \div N; r = RN \div n.$$

To find the number of revolutions of the last wheel at the end of a train of spur-wheels, all of which are in a line and mesh into one another, when the revolutions of the first wheel and the number of teeth or the

diameter of the first and last are given: Multiply the revolutions of the first wheel by its number of teeth or its diameter, and divide the product by the number of teeth or the diameter of the last wheel.

To find the number of teeth in each wheel for a train of spur-wheels, each to have a given velocity: Multiply the number of revolutions of the driving-wheel by its number of teeth, and divide the product by the number of revolutions each wheel is to make.

To find the number of revolutions of the last wheel in a train of wheels and pinions, when the revolutions of the first or driver, and the diameter, the teeth, or the circumference of all the drivers and pinions are given: Multiply the diameter, the circumference, or the number of teeth of all the driving-wheels together, and this continued product by the number of revolutions of the first wheel, and divide this product by the continued product of the diameter, the circumference, or the number of teeth of all the driven wheels, and the quotient will be the number of revolutions of the last wheel.

EXAMPLE. — 1. A train of wheels consists of four wheels each 12 in. diameter of pitch-circle, and three pinions 4, 4, and 3 in. diameter. The large wheels are the drivers, and the first makes 36 revs. per min. Required the speed of the last wheel.

$$\frac{36 \times 12 \times 12 \times 12}{4 \times 4 \times 3} = 1296 \text{ r.p.m.}$$

2. What is the speed of the first large wheel if the pinions are the drivers, the 3-in. pinion being the first driver and making 36 revs. per min.?

$$\frac{36 \times 3 \times 4 \times 4}{12 \times 12 \times 12} = 1 \text{ r.p.m. } \textit{Ans.}$$

Milling Cutters for Interchangeable Gears. — The Pratt & Whitney Co. makes a series of cutters for cutting epicycloidal teeth. The number of cutters to cut from a pinion of 12 teeth to a rack is 24 for each pitch coarser than 10. The Brown & Sharpe Mfg. Co. makes a similar series, and also a series for involute teeth, in which eight cutters are made for each pitch, as follows:

| No..... | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. |
|--------------------|-----|----|----|----|----|----|----|----|
| Will cut from..... | 135 | 55 | 35 | 26 | 21 | 17 | 14 | 12 |
| to Rack | 134 | 54 | 34 | 34 | 25 | 20 | 16 | 13 |

FORMS OF THE TEETH.

In order that the teeth of wheels and pinions may run together smoothly and with a constant relative velocity, it is necessary that their working faces shall be formed of certain curves called odontoids. The essential property of these curves is that when two teeth are in contact the common normal to the tooth curves at their point of contact must pass through the pitch-point, or point of contact of the two pitch-circles. Two such curves are in common use — the cycloid and the involute.

The Cycloidal Tooth. — In Fig. 172 let PL and pl be the pitch-circles of two gear-wheels; GC and gc are two equal generating-circles, whose radii should be taken as not greater than one-half of the radius of the smaller pitch-circle. If the circle gc be rolled to the left on the larger pitch-circle PL , the point O will describe an epicycloid, $Oefgh$. If the other generating-circle GC be rolled to the right on PL , the point O will describe a hypocycloid $Oabcd$. These two curves, which are tangent at O , form the two parts of a tooth curve for a gear whose pitch-circle is PL . The upper part Oh is called the face and the lower part Od is called the flank. If the same circles be rolled on the other pitch-circle pl , they will describe the curve for a tooth of the gear pl , which will work properly with the tooth on PL .

The cycloidal curves may be drawn without actually rolling the generating-circle, as follows: On the line PL , from O , step off and mark equal distances, as 1, 2, 3, 4, etc. From 1, 2, 3, etc., draw radial lines toward the center of PL , and from 6, 7, 8, etc., draw radial lines from the same

center, but beyond PL . With the radius of the generating-circle, and with centers successively placed on these radial lines, draw arcs of circles tangent to PL at 1, 2, 3, 6, 7, 8, etc. With the dividers set to one of the equal divisions, as 01, step off on the generating circle gc the points a' , b' , c' , d' , then take successively the chordal distances $0a$, $0b'$, $0c'$, $0d'$, and lay them off on the several arcs $6e$, $7f$, $8g$, $9h$, and 1a, 2b, 3c, 4d; through the points $efgh$ and $abcd$ draw the tooth curves.

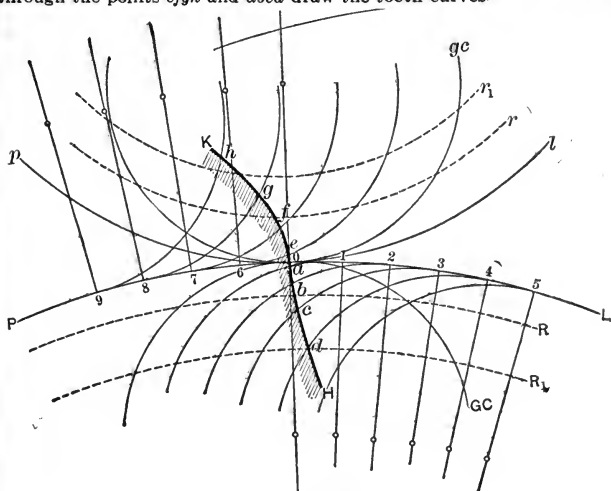


FIG. 172.

The curves for the mating tooth on the other wheel may be found in like manner by drawing arcs of the generating-circle tangent at equidistant points on the pitch-circle pl .

The tooth curve of the face Oh is limited by the addendum-line r or r_1 , and that of the flank OH by the root curve R or R_1 . R and r represent the root and addendum curves for a large number of small teeth, and R_1 and r_1 the like curves for a small number of large teeth. The form or appearance of the tooth therefore varies according to the number of teeth, while the pitch-circle and the generating-circle may remain the same.

In the cycloidal system, in order that a set of wheels of different diameters but equal pitches shall all correctly work together, it is necessary that the generating-circle used for the teeth of all the wheels shall be the same, and it should have a diameter not greater than half the diameter of the pitch-line of the smallest wheel of the set. The customary standard size of the generating-circle of the cycloidal system is one having a diameter equal to the radius of the pitch-circle of a wheel having 12 teeth. (Some gear-makers adopt 15 teeth.) This circle gives a radial flank to the teeth of a wheel having 12 teeth. A pinion of 10 or even a smaller number of teeth can be made, but in that case the flanks will be undercut, and the tooth will not be as strong as a tooth with radial flanks. If in any case the describing circle be half the size of the pitch-circle, the flanks will be radial; if it be less, they will spread out toward the root of the tooth, giving a stronger form; but if greater, the flanks will curve in toward each other, whereby the teeth become weaker and difficult to make.

In some cases cycloidal teeth for a pair of gears are made with the generating-circle of each gear having a radius equal to half the radius of its pitch-circle. In this case each of the gears will have radial flanks.

This method makes a smooth working gear, but a disadvantage is that the wheels are not interchangeable with other wheels of the same pitch but different numbers of teeth.

The rack in the cycloidal system is equivalent to a wheel with an infinite number of teeth. The pitch is equal to the circular pitch of the mating gear. Both faces and flanks are cycloids formed by rolling the generating-circle of the mating gear-wheel on each side of the straight pitch-line of the rack.

Another method of drawing the cycloidal curves is shown in Fig. 173. It is known as the method of tangent arcs. The generating-circles, as before, are drawn with equal radii, the length of the radius being less than half the radius of pl , the smaller pitch-circle. Equal divisions 1, 2,

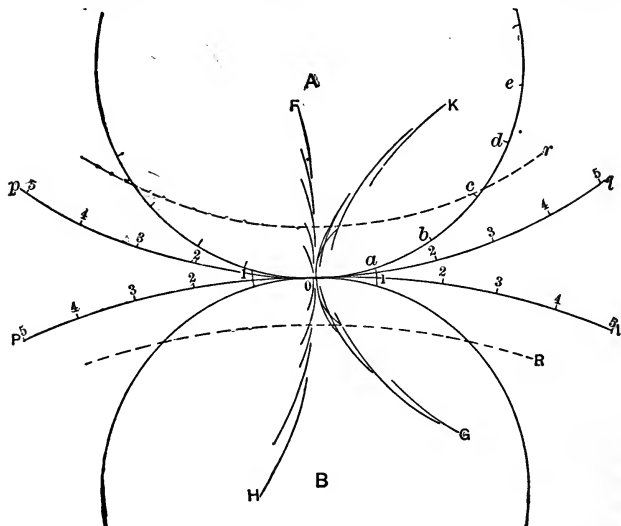


FIG. 173.

3, 4, etc., are marked off on the pitch-circles and divisions of the same length stepped off on one of the generating-circles, as $0, a, b, c$. From the points 1, 2, 3, 4, 5 on the line pO , with radii successively equal to the chord distances $0a, 0b, 0c, 0d, 0e$, draw the five small arcs F . A line drawn through the outer edges of these small arcs, tangent to them all, will be the hypocycloidal curve for the flank of a tooth below the pitch-line pl . From the points 1, 2, 3, etc., on the line Ol , with radii as before, draw the small arcs G . A line tangent to these arcs will be the epicycloid for the face of the same tooth for which the flank curve has already been drawn. In the same way, from centers on the line PO , and OL , with the same radii, the tangent arcs H and K may be drawn, which will give the tooth for the gear whose pitch-circle is PL .

If the generating-circle had a radius just one-half of the radius of pl , the hypocycloid F would be a straight line, and the flank of the tooth would have been radial.

The Involute Tooth. — In drawing the involute-tooth curve, Fig. 174, the angle of obliquity, or the angle which a common tangent to the teeth, when they are in contact at the pitch-point, makes with a line joining the centers of the wheels, is first arbitrarily determined. It is customary to take it at 15° . The pitch-lines pl and PL being drawn in contact at O , the line of obliquity AB is drawn through O normal to a common tangent

to the tooth curves, or at the given angle of obliquity to a common tangent to the pitch-circles. In the cut the angle is 20° . From the centers of the pitch-circles draw circles c and d tangent to the line AB . These circles are called base-lines or base-circles, from which the involutes F and K are drawn. By laying off convenient distances, 0, 1, 2, 3, which should each be less than $\frac{1}{10}$ of the diameter of the base-circle, small arcs can be drawn with successively increasing radii, which will form the involute. The involute extends from the points F and K down to their

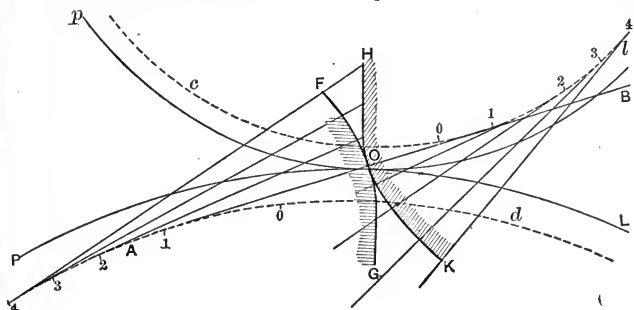


FIG. 174.

respective base-circles, where a tangent to the involute becomes a radius of the circle, and the remainders of the tooth curves, as G and H , are radial straight lines.

In the involute system the customary standard form of tooth is one having an angle of obliquity of 15° (Brown and Sharpe use $14\frac{1}{2}^\circ$), an addendum of about one-third the circular pitch, and a clearance of about one-eighth of the addendum. In this system the smallest gear of a set has 12 teeth, this being the smallest number of teeth that will gear together when made with this angle of obliquity. In gears with less than 30 teeth the points of the teeth must be slightly rounded over to avoid interference (see Grant's Teeth of Gears). All involute teeth of the same pitch and with the same angle of obliquity work smoothly together. The rack to gear with an involute-toothed wheel has straight faces on its teeth, which make an angle with the middle line of the tooth equal to the angle of obliquity, or in the standard form the faces are inclined at an angle of 30° with each other.

To draw the teeth of a rack which is to gear with an involute wheel (Fig. 175). — Let AB be the pitch-line of the rack and $AI = I'I'$ = the pitch. Through the pitch-point I draw EF at the given angle of obliquity.

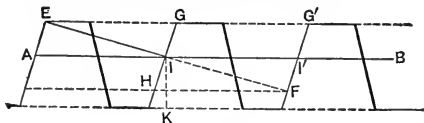


FIG. 175.

Draw AE and $I'F$ perpendicular to EF . Through E and F draw lines EGG' and FH parallel to the pitch-line. EGG' will be the addendum-line and FH the flank-line. From I draw IK perpendicular to AB equal to the greatest addendum in the set of wheels of the given pitch and obliquity plus an allowance for clearance equal to $\frac{1}{8}$ of the addendum. Through K , parallel to AB , draw the clearance-line. The fronts of the teeth are planes perpendicular to EF , and the backs are planes inclined at the same angle to AB in the contrary direction. The outer half of the working face AE may be slightly curved. Mr. Grant makes it a circular

arc drawn from a center on the pitch-line with a radius = 2.1 inches divided by the diametral pitch, or 0.67 in. \times circular pitch.

To Draw an Angle of 15° without using a Protractor. — From C , on the line AC , with radius AC , draw an arc AB , and from A , with the same

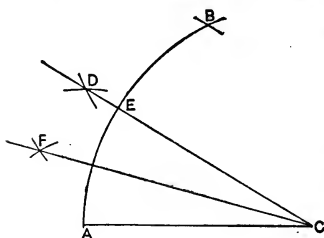


FIG. 176.

radius, cut the arc at B . Bisect the arc BA by drawing small arcs at D from A and B as centers, with the same radius, which must be greater than one-half of AB . Join DC , cutting BA at E . The angle ECA is 30° . Bisect the arc AE in like manner, and the angle FCA will be 15° .

A property of involute-toothed wheels is that the distance between the axes of a pair of gears may be altered to a considerable extent without interfering with their action. The backlash is therefore variable at will, and may be adjusted by moving the wheels farther from or nearer to each other, and

may thus be adjusted so as to be no greater than is necessary to prevent jamming of the teeth.

The relative merits of cycloidal and involute-shaped teeth are a subject of dispute, but there is an increasing tendency to adopt the involute tooth for all purposes.

Clark (R. T. D., p. 734) says: Involute teeth have the disadvantage of being too much inclined to the radial line, by which an undue pressure is exerted on the bearings.

Unwin (Elements of Machine Design, 8th ed., p. 265) says: The obliquity of action is ordinarily alleged as a serious objection to involute wheels. Its importance has perhaps been overrated.

George B. Grant (*Am. Mach.*, Dec. 26, 1885) says:

1. The work done by the friction of an involute tooth is always less than the same work for any possible epicycloidal tooth.

2. With respect to work done by friction, a change of the base from a gear of 12 teeth to one of 15 teeth makes an improvement for the epicycloid of less than one-half of one per cent.

3. For the 12-tooth system the involute has an advantage of $1\frac{1}{3}$ per cent, and for the 15-tooth system an advantage of $\frac{3}{4}$ per cent.

4. That a maximum improvement of about one per cent can be accomplished by the adoption of any possible non-interchangeable radial flank tooth in preference to the 12-tooth interchangeable system.

5. That for gears of very few teeth the involute has a decided advantage.

6. That the common opinion among millwrights and the mechanical public in general in favor of the epicycloid is a prejudice that is founded on long-continued custom, and not on an intimate knowledge of the properties of that curve.

Wilfred Lewis (*Proc. Engrs. Club of Phila.*, vol. x, 1893) says a strong reaction in favor of the involute system is in progress, and he believes that an involute tooth of $22\frac{1}{2}^\circ$ obliquity will finally supplant all other forms.

Approximation by Circular Arcs. — Having found the form of the actual tooth-curve on the drawing-board, circular arcs may be found by trial which will give approximations to the true curves, and these may be used in completing the drawing and the pattern of the gear-wheels. The root of the curve is connected to the clearance by a fillet, which should be as large as possible to give increased strength to the tooth, provided it is not large enough to cause interference.

Molesworth gives the following method of construction by circular arcs:

From the radial line at the edge of the tooth on the pitch-line, lay off the line HK at an angle of 75° with the radial line; on this line will be the centers of the root AB and the point EF . The lines struck from these centers are shown in thick lines. Circles drawn through centers thus

found will give the lines in which the remaining centers will be. The radius DA for striking the root AB is the pitch + the thickness of the tooth. The radius CE for striking the point of the tooth EF = the pitch.

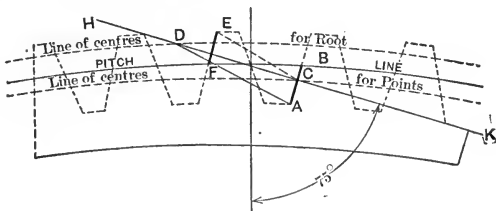


FIG. 177.

George B. Grant says: It is sometimes attempted to construct the curve by some handy method or empirical rule, but such methods are generally worthless.

Stepped Gears. — Two gears of the same pitch and diameter mounted side by side on the same shaft will act as a single gear. If one gear is keyed on the shaft so that the teeth of the two wheels are not in line, but the teeth of one wheel slightly in advance of the other, the two gears form a stepped gear. If mated with a similar stepped gear on a parallel shaft the number of teeth in contact will be twice as great as in an ordinary gear, which will increase the strength of the gear and its smoothness of action.

Twisted Teeth. — If a great number of very thin gears were placed together, one slightly in advance of the other, they would still act as a stepped gear. Continuing the subdivision until the thickness of each separate gear is infinitesimal, the faces of the teeth instead of being in steps take the form of a spiral or twisted surface, and we have a twisted gear. The twist may take any shape, and if it is in one direction for half the width of the gear and in the opposite direction for the other half, we have what is known as the herring-bone or double helical tooth. The obliquity of the twisted tooth if twisted in one direction causes an end thrust on the shaft, but if the herring-bone twist is used, the opposite obliquities neutralize each other. This form of tooth is much used in heavy rolling-mill practice, where great strength and resistance to shocks are necessary. They are frequently made of steel castings (Fig. 178). The angle of the tooth with a line parallel to the axis of the gear is usually 30° .



FIG. 178.

Spiral or Helical Gears. — If a twisted gear has a uniform twist it becomes what is commonly called a spiral gear (properly a helical gear). The line in which the pitch-surface intersects the face of the tooth is part of a helix drawn on the pitch-surface. A spiral wheel may be made with only one helical tooth wrapped around the cylinder several times, in which it becomes a screw or worm. If it has two or three teeth so wrapped, it is a double- or triple-threaded screw or worm. A spiral-gear meshing into a rack is used to drive the table of some forms of planing-machine. For methods of laying out and producing spiral gears see Brown and Sharpe's treatise on Gearing and Halsey's Worm and Spiral Gearing, also *Machy*, May 1906 and *Machy's Reference Series No. 20*.

Worm-gearing. — When the axes of two spiral gears are at right angles, and a wheel of one, two, or three threads works with a larger wheel of many threads, it becomes a worm-gear, or endless screw, the smaller wheel or driver being called the worm, and the larger, or driven wheel, the worm-wheel. With this arrangement a high velocity ratio may be obtained with a single pair of wheels. For a one-threaded wheel the velocity ratio is the number of teeth in the worm-wheel. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm.

To find the diameter of a worm-wheel at the throat, number of teeth and pitch of the worm being given: Add 2 to the number of teeth, multiply the sum by 0.3183, and by the pitch of the worm in inches.

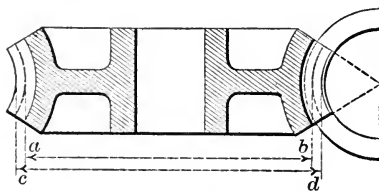


FIG. 179.

To find the number of teeth, diameter at throat and pitch of worm being given: Divide 3.1416 times the diameter by the pitch, and subtract 2 from the quotient.

In Fig. 179 ab is the diam. of the pitch-circle, cd is the diam. at the throat.

EXAMPLE. — Pitch of worm $\frac{1}{4}$ in., number of teeth 70; required the diam. at the throat. $(70 + 2) \times 0.3183 \times 0.25 = 5.73$ in.

For design of worm gearing see Kimball and Barr's Machine Design. For efficiency of worm gears see page .

The Hindley Worm. — In the Hindley worm-gear the worm, instead of being cylindrical in outline, is of an hour-glass shape, the pitch line of the worm being a curved line corresponding to the pitch line of the gear. It is claimed that there is surface contact between the faces of the teeth of the worm and gear, instead of only line contact as in the case of the ordinary worm gear, but this is denied by some writers. For discussion of the Hindley worm see *Am. Mach.*, April 1, 1897 and *Mach.*, Dec. 1908. The Hindley gear is made by the Albro-Clem Elevator Co., Philadelphia.

Teeth of Bevel-wheels. (Rankine's Machinery and Millwork.) — The teeth of a bevel-wheel have acting surfaces of the conical kind, generated by the motion of a line traversing the apex of the conical pitch-surface, while a point in it is carried round the traces of the teeth upon a spherical surface described about that apex.

The operations of drawing the traces of the teeth of bevel-wheels exactly, whether by involutes or by rolling curves, are in every respect analogous to those for drawing the traces of the teeth of spur-wheels; except that in the case of bevel-wheels all those operations are to be performed on the surface of a sphere described about the apex, instead of on a plane, substituting poles for centers and great circles for straight lines.

In consideration of the practical difficulty, especially in the case of large wheels, of obtaining an accurate spherical surface, and of drawing upon it when obtained, the following approximate method, proposed originally by Tredgold, is generally used:

Let O , Fig. 180, be the common apex of the pitch-cones, OBI , $OB'I$, of a pair of bevel-wheels; OC , OC' , the axes of those cones; OI their line of contact. Perpendicular to OI draw AIA' , cutting the axes in A , A' ; make the outer rims of the patterns and of the wheels portions of the cones ABI , $A'B'I$, of which the narrow zones occupied by the teeth will be sufficiently near for practical purposes to a spherical surface described about O . As the cones ABI , $A'B'I$ cut the pitch-cones at right angles in the outer pitch-circles IB , IB' , they may be called the normal cones. To find the traces of the teeth upon the normal cones, draw on a flat surface circular arcs, ID , ID' , with the radii AI , $A'I$; those arcs will be the developments of arcs of the pitch-circles IB , IB' when the conical surfaces ABI , $A'B'I$ are spread out flat. Describe the traces of teeth for the developed arcs as for a pair of spur-wheels, then wrap the

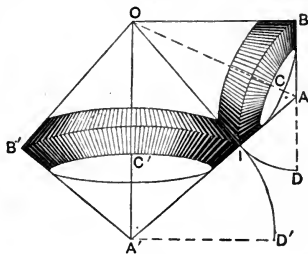


FIG. 180.

developed arcs on the normal cones, so as to make them coincide with the pitch-circles, and trace the teeth on the conical surfaces.

For formulæ and instructions for designing bevel-gears, and for much other valuable information on the subject of gearing, see "Practical Treatise on Gearing," and "Formulas in Gearing," published by Brown & Sharpe Mfg. Co.; and "Teeth of Gears," by George B. Grant, Lexington, Mass. The student may also consult Rankine's Machinery and Millwork, Reuleaux's Constructor, and Unwin's Elements of Machine Design. See also article on Gearing, by C. W. MacCord in App. Cyc. Mech., vol. ii.

Annular and Differential Gearing. (S. W. Balch, *Am. Mach.*, Aug. 24, 1893.) — In internal gears the sum of the diameters of the describing circles for faces and flanks should not exceed the difference in the pitch diameters of the pinion and its internal gear. The sum may be equal to this difference or it may be less; if it is equal, the faces of the teeth of each wheel will drive the faces as well as the flanks of the teeth of the other wheel. The teeth will therefore make contact with each other at two points at the same time.

Cycloidal tooth-curves for interchangeable gears are formed with describing circles of about $\frac{5}{8}$ the pitch diameter of the smallest gear of the series. To admit two such circles between the pitch-circles of the pinion and internal gear the number of teeth in the internal gear should exceed the number in the pinion by 12 or more, if the teeth are of the customary proportions and curvature used in interchangeable gearing.

Very often a less difference is desirable, and the teeth may be modified in several ways to make this possible.

First. The tooth curves resulting from smaller describing circles may be employed. These will give teeth which are more rounding and narrower at their tops, and therefore not as desirable as the regular forms.

Second. The tips of the teeth may be rounded until they clear. This is a cut-and-try method which aims at modifying the teeth to such outlines as smaller describing circles would give.

Third. One of the describing circles may be omitted and one only used, which may be equal to the difference between the pitch-circles. This will permit the meshing of gears differing by six teeth. It will usually prove inexpedient to put wheels in inside gears that differ by much less than 12 teeth.

If a regular diametral pitch and standard tooth forms are determined on, the diameter to which the internal gear-blank is to be bored is calculated by subtracting 2 from the number of teeth, and dividing the remainder by the diametral pitch.

The tooth outlines are the match of a spur-gear of the same number of teeth and diametral pitch, so that the spur-gear will fit the internal gear as a punch fits its die, except that the teeth of each should fail to bottom in the tooth spaces of the other by the customary clearance of one-tenth the thickness of the tooth.

Internal gearing is particularly valuable when employed in differential action. This is a mechanical movement in which one of the wheels is mounted on a crank so that its center can move in a circle about the center of the other wheel. Means are added to the device which restrain the wheel on the crank from turning over and confine it to the revolution of the crank.

The ratio of the number of teeth in the revolving wheel compared with the difference between the two will represent the ratio between the revolving wheel and the crank-shaft by which the other is carried. The advantage in accomplishing the change of speed with such an arrangement, as compared with ordinary spur-gearing, lies in the almost entire absence of friction and consequent wear of the teeth.

But for the limitation that the difference between the wheels must not be too small, the possible ratio of speed might be increased almost indefinitely, and one pair of differential gears made to do the service of a whole train of wheels. If the problem is properly worked out with bevel-gears this limitation may be completely set aside, and external and internal bevel-gears, differing by but a single tooth if need be, made to mesh perfectly with each other.

Differential bevel-gears have been used with advantage in mowing-machines. A description of their construction and operation is given by Mr. Balch in the article from which the above extracts are taken.

EFFICIENCY OF GEARING.

An extensive series of experiments on the efficiency of gearing, chiefly worm and spiral gearing, is described by Wilfred Lewis in *Trans. A. S. M. E.*, vii, 273. The average results are shown in a diagram, from which the following approximate average figures are taken:

EFFICIENCY OF SPUR, SPIRAL, AND WORM GEARING.

| Gearing. | Pitch. | Velocity at pitch-line in feet per min. | | | | |
|---------------------------|--------|---|-------|------|------|-------|
| | | 3 | 10 | 40 | 100 | 200 |
| Spur pinion..... | | 0.90 | 0.935 | 0.97 | 0.98 | 0.985 |
| Spiral pinion..... | 45° | .81 | .87 | .93 | .955 | .965 |
| " "..... | 30 | .75 | .815 | .89 | .93 | .945 |
| " "..... | 20 | .67 | .75 | .845 | .90 | .92 |
| " "..... | 15 | .61 | .70 | .805 | .87 | .90 |
| Spiral pinion or worm.... | 10 | .51 | .615 | .74 | .82 | .86 |
| " "..... | 7 | .43 | .53 | .72 | .765 | .815 |
| " "..... | 5 | .34 | .43 | .60 | .70 | .765 |

The experiments showed the advantage of spur-gearing over all other kinds in both durability and efficiency. The variation from the mean results rarely exceeded 5% in either direction, so long as no cutting occurred, but the variation became much greater and very irregular as soon as cutting began. The loss of power varies with the speed, the pressure, the temperature, and the condition of the surfaces. The excessive friction of worm and spiral gearing is largely due to the end thrust on the collars of the shaft. This may be considerably reduced by roller-bearings for the collars.

When two worms with opposite spirals run in two spiral worm-gears that also work with each other, and the pressure on one gear is opposite that on the other, there is no thrust on the shaft. Even with light loads a worm will begin to heat and cut if run at too high a speed, the limit for safe working being a velocity of the rubbing surfaces of 200 to 300 ft. per minute, the former being preferable where the gearing has to work continuously. The wheel teeth will keep cool, as they form part of a casting having a large radiating surface; but the worm itself is so small that its heat is dissipated slowly. Whenever the heat generated increases faster than it can be conducted and radiated away, the cutting of the worm may be expected to begin. A low efficiency for a worm-gear means more than the loss of power, since the power which is lost reappears as heat and may cause the rapid destruction of the worm.

Unwin (*Elements of Machine Design*, p. 294) says: The efficiency is greater the less the radius of the worm. Generally the radius of the worm = 1.5 to 3 times the pitch of the thread of the worm or the circular pitch of the worm-wheel. For a one-threaded worm the efficiency is only $\frac{2}{5}$ to $\frac{1}{4}$; for a two-threaded worm, $\frac{4}{7}$ to $\frac{2}{5}$; for a three-threaded worm, $\frac{2}{3}$ to $\frac{1}{2}$. Since so much work is wasted in friction it is not surprising that the wear is excessive. The following table gives the calculated efficiencies of worm-wheels of 1, 2, 3, and 4 threads and ratios of radius of worm to pitch of teeth of from 1 to 6, assuming a coefficient of friction of 0.15:

| No. of Threads. | Radius of Worm ÷ Pitch. | | | | | | | | |
|--------------------|-------------------------|-------|-------|-------|------|-------|------|------|------|
| | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/2 | 3 | 4 | 6 |
| 1 | 0.50 | 0.44 | 0.40 | 0.36 | 0.33 | 0.28 | 0.25 | 0.20 | 0.14 |
| 2 | .67 | .62 | .57 | .53 | .50 | .44 | .40 | .33 | .25 |
| 3 | .75 | .70 | .67 | .63 | .60 | .55 | .50 | .43 | .33 |
| 4 | .80 | .76 | .73 | .70 | .67 | .62 | .57 | .50 | .40 |

Efficiency of Worm Gearing. — Worm gearing as a means of transmitting power has generally been looked upon with suspicion, its efficiency being considered necessarily low and its life short. When properly proportioned, however, it is both durable and reasonably efficient. Mr. F. A. Halsey discusses the subject in *Am. Machinist*, Jan. 13 and 20, 1898. He quotes two formulas for the efficiency of worm gearing:

$$E = \frac{\tan \alpha (1 - f \tan \alpha)}{\tan \alpha + f}, \dots (1) \quad E = \frac{\tan \alpha (1 - f \tan \alpha)}{\tan \alpha + 2f} \text{ approx.}, \dots (2)$$

In which E = efficiency; α = angle of thread, being angle between thread and a line perpendicular to the axis of the worm; f = coefficient of friction.

Eq. (1) applies to the worm thread only, while (2) applies to the worm and step combined, on the assumption that the mean friction radius of the two is equal. Eq. (1) gives a maximum for E when $\tan \alpha = \sqrt{1 + f^2} - f$

\dots (3) and eq. (2) a maximum when $\tan \alpha = \sqrt{2 + 4f^2} - 2f \dots$ (4) Using 0.05 for f gives α in (3) = $43^\circ 34'$ and in (4) = $52^\circ 49'$.

On plotting equations (1) and (2) the curves show the striking influence of the pitch-angle upon the efficiency, and since the lost work is expended in friction and wear, it is plain why worms of low angle should be short-lived and those of high angle long-lived. The following table is taken from Mr. Halsey's plotted curves:

RELATION BETWEEN THREAD-ANGLE SPEED AND EFFICIENCY OF WORM GEARS.

| Velocity of Pitch-line, feet per minute. | Angle of Thread. | | | | | |
|--|------------------|----|----|----|----|----|
| | 5 | 10 | 20 | 30 | 40 | 45 |
| | Efficiency. | | | | | |
| 3 | 35 | 52 | 66 | 73 | 76 | 77 |
| 5 | 40 | 56 | 69 | 76 | 79 | 80 |
| 10 | 47 | 62 | 74 | 79 | 82 | 82 |
| 20 | 52 | 67 | 78 | 83 | 85 | 86 |
| 40 | 60 | 74 | 83 | 87 | 88 | 88 |
| 100 | 70 | 82 | 88 | 91 | 91 | 91 |
| 200 | 76 | 85 | 91 | 92 | 92 | 92 |

The experiments of Mr. Wilfred Lewis on worms show a very satisfactory correspondence with the theory. Mr. Halsey gives a collection of data comprising 16 worms doing heavy duty and having pitch-angles ranging between $4^\circ 30'$ and 45° , which show that every worm having an angle above $12^\circ 30'$ was successful in regard to durability, and every worm below 9° was unsuccessful, the overlapping region being occupied by worms some of which were successful and some unsuccessful. In several cases worms of one pitch-angle had been replaced by worms of a different angle, an increase in the angle leading in every case to better results and a decrease to poorer results. He concludes with the following table from experiments by Mr. James Christie, of the Pencoyd Iron Works, and gives data connecting the load upon the teeth with the pitch-line velocity of the worm.

LIMITING SPEEDS AND PRESSURES OF WORM GEARING.

| | Single-thread Worm 1" Pitch, $2\frac{1}{8}$ Pitch Diam. | | | | Double- thread Worm 2" Pitch, $2\frac{1}{8}$ Pitch Diam. | | | Double- thread Worm $2\frac{1}{2}$ " Pitch, $4\frac{1}{2}$ Pitch Diam. | | |
|---|---|------|------|-----|--|------|------|--|-----|-----|
| Revolutions per minute. | 128 | 201 | 272 | 425 | 128 | 201 | 272 | 201 | 272 | 425 |
| Velocity at pitch-line, feet per minute. | 96 | 150 | 205 | 320 | 96 | 150 | 205 | 235 | 319 | 498 |
| Limiting pressure, pounds. | 1700 | 1300 | 1100 | 700 | 1100 | 1100 | 1100 | 1100 | 700 | 400 |

Efficiency of Automobile Gears. (G. E. Quick, *Horseless Age*, Feb. 12, 1908).—A set of slide gears was tested by an electric-driven absorption dynamometer. The following approximate results are taken from a series of plotted curves:

| Horse-power input | | 2 | 4 | 6 | 8 | 10 | 14 | 18 |
|-----------------------------------|--------|-----------------------|------|------|------|------|------|-------|
| | r.p.m. | Efficiency, per cent. | | | | | | |
| Direct driven, third speed..... | 800 | 89 | 95 | 97 | 97.5 | 97.5 | 97.5 | 96 |
| Direct driven, third speed..... | 1,500 | 80 | 89 | 93 | 95 | 96.5 | 97 | 97 |
| Second speed, ratio 1.76 to 1 ... | 800 | 87 | 92.5 | 94 | 95 | 94 | 93 | |
| Second speed, ratio 1.76 to 1 ... | 1,500 | 79 | 88 | 92.5 | 94 | 95 | 95 | 94 |
| First speed, ratio 3.36 to 1..... | 800 | 75 | 87.5 | 93 | 94 | 94 | 93.5 | 92.5 |
| First speed, ratio 3.36 to 1..... | 1,500 | 70 | 84 | 89 | 92 | 93 | 92 | |
| Reverse speed, ratio 4.32 to 1... | 800 | 75 | 84 | 87 | 87 | 86 | 82.5 | |
| Reverse speed, ratio 4.32 to 1... | 1,500 | | 70 | 79 | 83 | 86 | 87 | 85 |
| Worm-gear axle, ratio 6.83 to 1.. | 400 | 85 | 87 | 86.5 | 85.5 | 84 | 80 | 75 |
| Worm-gear axle, ratio 6.83 to 1.. | 800 | 83 | 87 | 88.5 | 89 | 89 | 88 | 87 |
| Worm-gear axle, ratio 6.83 to 1.. | 1,500 | 80 | 85 | 87.5 | 88.5 | 89 | 89 | 89 |

Two bevel-wheel axles were tested, one a floating type, ratio 15 to 32, $14\frac{1}{2}^\circ$ involute; the other a solid wheel and axle type, ratio 13 to 54, 20° involute. Both gave efficiencies of 95 to 96 % at 800 to 1500 r.p.m., and 10 to 26 H.P., with lower efficiencies at lower power and at lower speed. The friction losses include those of the journals and thrust ball bearings.

The worm was 6-threaded, lead, 4.69 in.; pitch diam., 2.08 in.; the gear had 41 teeth; pitch diam., 10.2 in. The worm was of hardened steel and the gear of phosphor-bronze. A test of a steel gear and steel worm gave somewhat lower efficiencies. In both tests the heating was excessive both in the gears and in the thrust bearings, the balls in which were $\frac{7}{16}$ in. diam.

STRENGTH OF GEAR-TEETH.

The strength of gear-teeth and the horse-power that may be transmitted by them depend upon so many variable and uncertain factors that it is not surprising that the formulas and rules given by different writers show a wide variation. In 1879 John H. Cooper (*Jour. Frank. Inst.*, July, 1879) found that there were then in existence about 48 well-established rules for horse-power and working strength, differing from each other in extreme cases about 500%. In 1886 Prof. Wm. Harkness (*Proc. A. A. A. S.*, 1886), from an examination of the bibliography of the subject, beginning in 1796, found that according to the constants and formulæ used by various authors there were differences of 15 to 1 in the power which could be transmitted by a given pair of geared wheels. The various elements which enter into the constitution of a formula to represent the working strength of a toothed wheel are the following: 1. The strength of the metal, usually cast iron, which is an extremely variable quantity. 2. The shape of the tooth, and especially the relation of its thickness at the root or point of least strength to the pitch and to the length. 3. The point at which the load is taken to be applied, assumed by some authors to be at the pitch-line, by others at the extreme end, along the whole face, and by still others at a single outer corner. 4. The consideration of whether the total load is at any time received by a single tooth or whether it is divided between two teeth. 5. The influence of velocity in causing a tendency to break the teeth by shock. 6. The factor of safety assumed to cover all the uncertainties of the other elements of the problem.

Prof. Harkness, as a result of his investigation, found that all the formulæ on the subject might be expressed in one of three forms, viz.:

$$\text{Horse-power} = CVpf, \text{ or } CVp^2, \text{ or } CVp^2f;$$

in which C is a coefficient, V = velocity of pitch-line in feet per second, p = pitch in inches, and f = face of tooth in inches.

From an examination of precedents he proposed the following formula for cast-iron wheels:

$$\text{H.P.} = \frac{0.910 V p f}{\sqrt{1 + 0.65 V}}$$

He found that the teeth of chronometer and watch movements were subject to stresses four times as great as those which any engineer would dare to use in like proportion upon cast-iron wheels of large size.

It appears that all of the earlier rules for the strength of teeth neglected the consideration of the variations in their form; the breaking strength, as said by Mr. Cooper, being based upon the thickness of the teeth at the pitch-line or circle, as if the thickness at the root of the tooth were the same in all cases as it is at the pitch-line.

Wilfred Lewis (*Proc. Eng'rs Club, Phila.*, Jan., 1893; *Am. Mach.*, June 22, 1893) seems to have been the first to use the form of the tooth in the construction of a working formula and table. He assumes that in well-constructed machinery the load can be more properly taken as well distributed across the tooth than as concentrated in one corner, but that it cannot be safely taken as concentrated at a maximum distance from the root less than the extreme end of the tooth. He assumes that the whole load is taken upon one tooth, and considers the tooth as a beam loaded at one end, and from a series of drawings of teeth of the involute, cycloidal, and radial flank systems, determines the point of weakest cross-section of each, and the ratio of the thickness at that section to the pitch. He thereby obtains the general formula,

$$W = spfy;$$

in which W is the load transmitted by the teeth, in pounds; s is the safe working stress of the material, taken at 8000 lbs. for cast iron, when the working speed is 100 ft. or less per minute; p = pitch; f = face, in inches; y = a factor depending on the form of the tooth, whose value for different cases is given in the following table:

| No. of Teeth. | Factor for Strength, y . | | | No. of Teeth. | Factor for Strength, y . | | |
|---------------|----------------------------|----------------------------|----------------|---------------|----------------------------|----------------------------|----------------|
| | Involute 20° Obliquity. | Involute 15° and Cycloidal | Radial Flanks. | | Involute 20° Obliquity. | Involute 15° and Cycloidal | Radial Flanks. |
| 12 | 0.078 | 0.067 | 0.052 | 27 | 0.111 | 0.100 | 0.064 |
| 13 | .083 | .070 | .053 | 30 | .114 | .102 | .065 |
| 14 | .088 | .072 | .054 | 34 | .118 | .104 | .066 |
| 15 | .092 | .075 | .055 | 38 | .122 | .107 | .067 |
| 16 | .094 | .077 | .056 | 43 | .126 | .110 | .068 |
| 17 | .096 | .080 | .057 | 50 | .130 | .112 | .069 |
| 18 | .098 | .083 | .058 | 60 | .134 | .114 | .070 |
| 19 | .100 | .087 | .059 | 75 | .138 | .116 | .071 |
| 20 | .102 | .090 | .060 | 100 | .142 | .118 | .072 |
| 21 | .104 | .092 | .061 | 150 | .146 | .120 | .073 |
| 23 | .106 | .094 | .062 | 300 | .150 | .122 | .074 |
| 25 | .108 | .097 | .063 | Rack. | .154 | .124 | .075 |

SAFE WORKING STRESS, s , FOR DIFFERENT SPEEDS.

| Speed of Teeth in ft. per minute. | 100 or less. | 200 | 300 | 600 | 900 | 1200 | 1800 | 2400 |
|-----------------------------------|--------------|-------|-------|-------|------|------|------|------|
| Cast iron..... | 8000 | 6000 | 4800 | 4000 | 3000 | 2400 | 2000 | 1700 |
| Steel..... | 20000 | 15000 | 12000 | 10000 | 7500 | 6000 | 5000 | 4300 |

The values of s in the above table are given by Mr. Lewis tentatively, in the absence of sufficient data upon which to base more definite values, but they have been found to give satisfactory results in practice.

Mr. Lewis gives the following example to illustrate the use of the tables: Let it be required to find the working strength of a 12-toothed pinion of 1-inch pitch, $2\frac{1}{2}$ -inch face, driving a wheel of 60 teeth at 100 feet or less per minute, and let the teeth be of the 20-degree involute form. In the formula $W = spfy$ we have for a cast-iron pinion $s = 8000$, $pf = 2.5$, and $y = 0.078$; and multiplying these values together, we have $W = 1560$ pounds. For the wheel we have $y = 0.134$ and $W = 2680$ pounds.

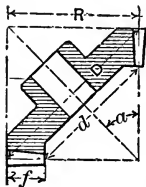


FIG. 181.

The cast-iron pinion is, therefore, the measure of strength; but if a steel pinion be substituted we have $s = 20,000$ and $W = 3900$ pounds, in which combination the wheel is the weaker, and it therefore becomes the measure of strength.

For bevel-wheels Mr. Lewis gives the following, referring to Fig. 181: D = large diameter of bevel; d = small diameter of bevel; p = pitch at large diameter; n = actual number of teeth; f = face of bevel; N = formative number of teeth = $n \times \secant\ \alpha$, or the number corresponding to radius R ; y = factor depending upon shape of teeth and formative number N ; W = working load on teeth.

$$W = spfy \frac{D^3 - d^3}{3 D^2 (D - d)}; \text{ or, more simply, } W = spfy \frac{d}{D},$$

which gives almost identical results when d is not less than $\frac{2}{3} D$, as is the case in good practice.

In *Am. Mach.*, June 22, 1893, Mr. Lewis gives the following formulæ for the working strength of the three systems of gearing, which agree very closely with those obtained by use of the table:

$$\text{For involute, } 20^\circ \text{ obliquity, } W = spf \left(0.154 - \frac{0.912}{n} \right);$$

$$\text{For involute } 15^\circ, \text{ and cycloidal, } W = spf \left(0.124 - \frac{0.684}{n} \right);$$

$$\text{For radial flank system, } W = spf \left(0.075 - \frac{0.276}{n} \right);$$

in which the factor within the parenthesis corresponds to y in the general formula. For the horse-power transmitted, Mr. Lewis's general formula

$W = spfy = \frac{33,000 \text{ H.P.}}{v}$, may take the form $\text{H.P.} = \frac{spf y v}{33,000}$, in which v = velocity in feet per minute; or since $v = d\pi \times \text{r.p.m.} \div 12 = 0.2618 d \times \text{r.p.m.}$, in which d = diameter in inches,

$$\text{H.P.} = \frac{Wv}{33,000} = \frac{spf y \times d \times \text{r.p.m.}}{126,050} = 0.000007933 d spfy \times \text{r.p.m.}$$

It must be borne in mind, however, that in the case of machines which consume power intermittently, such as punching and shearing machines, the gearing should be designed with reference to the maximum load W , which can be brought upon the teeth at any time, and not upon the average horse-power transmitted.

Comparison of the Harkness and Lewis Formulas. — Take an average case in which the safe working strength of the material, $s = 6000$, $v = 200$ ft. per min., and $y = 0.100$, the value in Mr. Lewis's table for an involute tooth of 15° obliquity, or a cycloidal tooth, the number of teeth in the wheel being 27.

$$\text{H.P.} = \frac{spf y v}{33,000} = \frac{6000 pfv \times 0.100}{33,000} = \frac{pfv}{55} = 1.091 pfV,$$

if V is taken in feet per second.

$$\text{Prof. Harkness gives } \text{H.P.} = \frac{0.910 V pf}{\sqrt{1 + 0.65 V}}. \text{ If the } V \text{ in the denominator}$$

be taken at $200 \div 60 = 3\frac{1}{3}$ ft. per sec., $\text{H.P.} = 0.571 pfV$, or about 52% of the result given by Mr. Lewis's formula. This is probably as close an agreement as can be expected, since Prof. Harkness derived his formula from an investigation of ancient precedents and rule-of-thumb practice, largely with common cast gears, while Mr. Lewis's formula was

derived from considerations of modern practice with machine-molded and cut gears.

Mr. Lewis takes into consideration the reduction in working strength of a tooth due to increase in velocity by the figures in his table of the values of the safe working stress s for different speeds. Prof. Harkness gives expression to the same reduction by means of the denominator of his formula, $\sqrt{1 + 0.65 V}$. The decrease in strength as computed by this formula is somewhat less than that given in Mr. Lewis's table, and as the figures given in the table are not based on accurate data, a mean between the values given by the formula and the table is probably as near to the true value as may be obtained from our present knowledge. The following table gives the values for different speeds according to Mr. Lewis's table and Prof. Harkness's formula, taking for a basis a working stress s , for cast-iron 8000, and for steel 20,000 lbs. at speeds of 100 ft. per minute and less:

| v = speed of teeth, ft. per min.. V = speed of teeth, ft. per sec.. | 100 12/3 | 200 31/3 | 300 5 | 600 10 | 900 15 | 1200 20 | 1800 30 | 2400 40 |
|--|-------------|-------------|----------|-----------|-----------|------------|------------|------------|
| Safe stress s , cast iron, Lewis .. | 8000 | 6000 | 4800 | 4000 | 3000 | 2400 | 2000 | 1700 |
| Relative do., $s \div 8000$ | 1 | 0.75 | 0.6 | 0.5 | 0.375 | 0.3 | 0.25 | 0.2125 |
| $1 \div \sqrt{1 + 0.65 V}$ | .6930 | .5621 | .4850 | .3650 | .3050 | .2672 | .2208 | .1924 |
| Relative val. $c \div 0.693$ | 1 | 0.811 | 0.700 | 0.526 | 0.439 | 0.385 | 0.318 | 0.277 |
| $s_1 = 8000 \times (c \div 0.693)$ | 8000 | 6488 | 5600 | 4208 | 3512 | 3080 | 2544 | 2216 |
| Mean of s and s_1 , cast-iron = s_2 .. | 8000 | 6200 | 5200 | 4100 | 3300 | 2700 | 2300 | 2000 |
| Mean of s and s_1 , for steel = s_3 .. | 20000 | 15500 | 13000 | 10300 | 8100 | 6800 | 5700 | 4900 |
| Safe stress for steel, Lewis..... | 20000 | 15000 | 12000 | 10000 | 7500 | 6000 | 5000 | 4300 |

In *Am. Mach.*, Jan. 30, 1902, Mr. Lewis says that 8,000 lbs. was given as safe for cast-iron teeth, either cut or cast, and that 20,000 lbs. was intended for any steel suitable for gearing whether cast or forged. These were the unit stresses for static loads.

The iron should be of good quality capable of sustaining about a ton on a test bar 1 in. square between supports 12 in. apart, and the steel should be solid and of good quality. The value given for steel was intended to include the lower grades, but when the quality is known to be high, correspondingly higher values may be assigned.

Comparing the two formulæ for the case of $s = 8000$, corresponding to a speed of 100 ft. per min., we have

$$\text{Harkness: H.P.} = 1 \div \sqrt{1 + 0.65 V} \times 0.910 V p f = 1.053 p f,$$

$$\text{Lewis: H.P.} = \frac{s p f y v}{33,000} = \frac{s p f y V}{550} = \frac{8000 \times 12/3 p f y}{550} = 24.24 p f y,$$

in which y varies according to the shape and number of the teeth.

For radial-flank gear with 12 teeth $y = 0.052$; $24.24 p f y = 1.260 p f$;
 For 20° inv., 19 teeth, or 15° inv., 27 teeth $y = 0.100$; $24.24 p f y = 2.424 p f$;
 For 20° involute, 300 teeth $y = 0.150$; $24.24 p f y = 3.636 p f$.

Thus the weakest-shaped tooth, according to Mr. Lewis, will transmit 20 per cent more horse-power than is given by Prof. Harkness's formula, in which the shape of the tooth is not considered, and the average-shaped tooth, according to Mr. Lewis, will transmit more than double the horse-power given by Prof. Harkness's formula.

Comparison of Other Formulæ. — Mr. Cooper, in summing up his examination, selected an old English rule, which Mr. Lewis considers as a passably correct expression of good general averages, viz.: $X = 2000 p f$, X = breaking load of tooth in pounds, p = pitch, f = face. If a factor of safety of 10 be taken, this would give for safe working load $W = 200 p f$.

George B. Grant, in his *Teeth of Gears*, page 33, takes the breaking load at $3500 p f$, and, with a factor of safety of 10, gives $W = 350 p f$.

Nystrom's *Pocket-Book*, 20th ed., 1891, says: "The strength and durability of cast-iron teeth require that they shall transmit a force of 80 lbs.

per inch of pitch and per inch breadth of face." This is equivalent to $W = 80$ *pf*, or only 40% of that given by the English rule.

F. A. Halsey (Clark's Pocket-Book) gives a table calculated from the formula $H.P. = pfd \times \text{r.p.m.} \div 850$.

Jones & Laughlins give $H.P. = pfd \times \text{r.p.m.} \div 550$.

These formulæ transformed give $W = 128$ *pf* and $W = 218$ *pf*, respectively.

Unwin, on the assumption that the load acts on the corners of the teeth, derives a formula $p = K \sqrt{W}$, in which K is a coefficient derived from existing wheels, its values being: for slowly moving gearing not subject to much vibration or shock $K = 0.04$; in ordinary mill-gearing, running at greater speed and subject to considerable vibration, $K = 0.05$; and in wheels subjected to excessive vibration and shock, and in mortise gearing, $K = 0.06$. Reduced to the form $W = Cpf$, assuming that $f = 2p$, these values of K give $W = 262$ *pf*, 200 *pf*, and 139 *pf*, respectively.

Unwin also give the following, based on the assumption that the pressure is distributed along the edge of the tooth: $p = K_1 \sqrt{p/f \sqrt{W}}$, where K_1 is about 0.0707 for iron wheels and 0.0848 for mortise wheels when the breadth of face is not less than twice the pitch. For the case of $f = 2p$ and the given values of K_1 this reduces to $W = 200$ *pf* and $W = 139$ *pf*, respectively.

Box, in his Treatise on Mill Gearing, gives $H.P. = 12 p^2 f \sqrt{dn} \div 1000$, in which n = number of revolutions per minute. This formula differs from the more modern formulæ in making the H.P. vary as $p^2 f$, instead of as *pf*, and in this respect it is no doubt incorrect.

Making the H.P. vary as \sqrt{dn} or as \sqrt{v} , instead of directly as v , makes the velocity a factor of the working strength as in the Harkness and Lewis formulæ, the relative strength varying as $1/\sqrt{v}$, which for different velocities is as follows:

| | | | | | | | | |
|---------------------------------------|-----|-------|-------|-------|-------|-------|-------|------|
| Speed of teeth in ft. per min., $v =$ | 100 | 200 | 300 | 600 | 900 | 1200 | 1800 | 2400 |
| Relative strength = | 1 | 0.707 | 0.574 | 0.408 | 0.333 | 0.289 | 0.236 | 0.20 |

showing a somewhat more rapid reduction than is given by Mr. Lewis.

For the purpose of comparing different formulæ they may in general be reduced to either of the following forms:

$$P.P. = Cpfv, \quad H.P. = C_1 pfd \times \text{r.p.m.}, \quad W = cpf,$$

in which p = pitch, f = face, d = diameter, all in inches; v = velocity in feet per minute, r.p.m. revolutions per minute, and C , C_1 and c coefficients. The formulæ for transformation are as follows:

$$H.P. = Wv \div 33,000 = W \times d \times \text{r.p.m.} \div 126,050;$$

$$W = \frac{33,000 H.P.}{v} = \frac{126,050 H.P.}{d \times \text{r.p.m.}} = 33,000 Cpf; pf = \frac{H.P.}{Cv} = \frac{H.P.}{C_1 d \times \text{r.p.m.}} = \frac{W}{c}$$

$$C_1 = 0.2618 C; c = 33,000 C; C = 3.82 C_1, = \frac{c}{33,000}; c = 126,050 C_1.$$

In the Lewis formula C varies with the form of the tooth and with the speed, and is equal to $sy \div 33,000$, in which y and s are the values taken from the table, and $c = sy$.

In the Harkness formula C varies with the speed and is equal to

$$\frac{910}{\sqrt{1 + 0.65 V}} \quad (V \text{ being in feet per second}), = 0.01517 \div \sqrt{1 + 0.011 v}.$$

In the Box formula C varies with the pitch and also with the velocity:

$$\text{and equals } \frac{12 p \sqrt{d \times \text{r.p.m.}}}{1000 v} = 0.02345 \frac{p}{\sqrt{v}}, \quad c = 33,000 C = 774 \frac{p}{\sqrt{v}}.$$

For $v = 100$ ft. per min. $C = 77.4 p$; for $v = 600$ ft. per min., $c = 31.6 p$.

In the other formulæ considered C , C_1 , and c are constants. Reducing the several formulæ to the form $W = cpf$, we have the following:

COMPARISON OF DIFFERENT FORMULÆ FOR STRENGTH OF GEAR-TEETH.

Safe working pressure per inch pitch and per inch of face, or value of c in formula $W = cpf$:

| | | |
|--|------|---------------------|
| $v =$ ft. per min. | 100 | 600 |
| Lewis: Weak form of tooth, radial flank, 12 teeth $c =$ | 416 | 208 |
| Medium tooth, inv. 15° , or cycloid, 27 teeth $c =$ | 800 | 400 |
| Strong form of tooth, inv. 20° , 300 teeth $c =$ | 1200 | 600 |
| Harkness: Average tooth $c =$ | 347 | 184 |
| Box: Tooth of 1 inch pitch $c =$ | 77.4 | 31.6 |
| Box: Tooth of 3 inches pitch $c =$ | 232 | 95 |
| The Gleason Works gives for ft. per min. | 500 | 1000 1500 2000 2500 |
| working stress in pounds = p.f. \times | 480 | 400 340 290 240 |

These are for cut gears, 18 teeth or more, rigidly supported, for average steady loads. Hammering loads, as in rolling mills and saw mills, require heavier gears.

C. W. Hunt, *Trans. A.S.M.E.*, 1908, gives a table of working loads of cut cast gears with a strong shoot form of tooth, which is practically equivalent to $W = 700$ pf.

Various, in which c is independent of form and speed: Old English rule, $c = 200$; Grant, $c = 350$; Nystrom, $c = 80$; Halsey, $c = 128$; Jones & Laughlins, $c = 218$; Unwin, $c = 262$, 200, or 139, according to speed, shock, and vibration.

The value given by Nystrom and those given by Box for teeth of small pitch are so much smaller than those given by the other authorities that they may be rejected as having an entirely unnecessary surplus of strength. The values given by Mr. Lewis seem to rest on the most logical basis, the form of the teeth as well as the velocity being considered; and since they are said to have proven satisfactory in an extended machine practice, they may be considered reliable for gears that are so well made that the pressure bears along the face of the teeth instead of upon the corners. For rough ordinary work the old English rule $W = 200$ pf is probably as good as any, except that the figure 200 may be too high for weak forms of tooth and for high speeds.

The formula $W = 200$ pf is equivalent to $H.P. = pfd \times r.p.m. \div 630 = pfv \div 165$ or, $H.P. = 0.0015873 pfd \times r.p.m. = 0.006063 pfv$.

Raw-hide Pinions. — Pinions of raw-hide are in common use for gearing shafts driven by electric motors to other shafts which carry machine-cut cast-iron or steel gears, in order to reduce vibration, noise and wear. A formula for the maximum horse-power to be transmitted by such gears, given by the New Process Raw-Hide Co., Syracuse, N. Y., is $H.P. = \text{pitch diam.} \times \text{circ. pitch} \times \text{face} \times r.p.m. \div 850$, or $pfd \times r.p.m. \div 850$. This is about $\frac{3}{4}$ of the H.P. for cast-iron teeth by the old English rule. The formula is to be used only when the circular pitch does not exceed 1.65 ins.

Composite gears also are made, consisting of alternate sheets of raw-hide or fibre and steel or bronze, so that a high degree of strength is combined with the smooth-running quality of the fibre.

Maximum Speed of Gearing. — A. Towler, *Eng'g*, April 19, 1889, p. 388, gives the maximum speeds at which it was possible under favorable conditions to run toothed gearing safely as follows, in ft. per min.: Ordinary cast-iron wheels, 1800; Helical, 2400; Mortise, 2400; Ordinary cast-steel wheels, 2600; Helical, 3000; special cast-iron machine-cut wheels, 3000.

Prof. Coleman Sellers (*Stevens Indicator*, April, 1892) recommends that gearing be not run over 1200 ft. per minute, to avoid great noise. The Walker Company, Cleveland, Ohio, say that 2200 ft. per min. for iron gears and 3000 ft. for wood and iron (mortise gears) are excessive, and should be avoided if possible. The Corliss engine at the Philadelphia Exhibition (1876) had a fly-wheel 30 ft. in diameter running 35 r.p.m. geared into a pinion 12 ft. diam. The speed of the pitch-line was 3300 ft. per min.

A Heavy Machine-cut Spur-gear was made in 1891 by the Walker Company, Cleveland, Ohio, for a diamond mine in South Africa, with dimensions as follows: Number of teeth, 192; pitch diameter, 30 ft. 6.66 ins.; face, 30 ins.; pitch, 6 ins.; bore, 27 ins.; diameter of hub, 9 ft. 2 ins.; weight of hub, 15 tons; and total weight of gear, $66\frac{3}{4}$ tons. The

rim was made in 12 segments, the joints of the segments being fastened with two bolts each. The spokes were bolted to the middle of the segments and to the hub with four bolts in each end.

Frictional Gearing. — In frictional gearing the wheels are toothless, and one wheel drives the other by means of the friction between the two surfaces which are pressed together. They may be used where the power to be transmitted is not very great; when the speed is so high that toothed wheels would be noisy; when the shafts require to be frequently put into and out of gear or to have their relative direction of motion reversed; or when it is desired to change the velocity-ratio while the machinery is in motion, as in the case of disk friction-wheels for changing the feed in machine tools.

Let P = the normal pressure in pounds at the line of contact by which two wheels are pressed together, T = tangential resistance of the driven wheel at the line of contact, f = the coefficient of friction, V the velocity of the pitch-surface in feet per second, and H.P. = horse-power; then T may be equal to or less than fP ; H.P. = $TV \div 550$. The value of f for metal on metal may be taken at 0.15 to 0.20; for wood on metal, 0.25 to 0.30; and for wood on compressed paper, 0.20. The tangential driving force T may be as high as 80 lbs. per inch width of face of the driving surface, but this is accompanied by great pressure and friction on the journal-bearings.

In frictional grooved gearing circumferential wedge-shaped grooves are cut in the faces of two wheels in contact. If P = the force pressing the wheels together, and N = the normal pressure on all the grooves, $P = N (\sin a + f \cos a)$, in which $2a$ = the inclination of the sides of the grooves, and the maximum tangential available force $T = fN$. The inclination of the sides of the grooves to a plane at right angles to the axis is usually 30° .

Frictional Grooved Gearing. — A set of friction-gears for transmitting 150 H.P. is on a steam-dredge described in *Proc. Inst. M. E.*, July, 1888. Two grooved pinions of 54 in. diam., with 9 grooves of $13/4$ in. pitch and angle of 40° cut on their face, are geared into two wheels of $127\frac{1}{2}$ in. diam. similarly grooved. The wheels can be thrown in and out of gear by levers operating eccentric bushes on the large wheel-shaft. The circumferential speed of the wheels is about 500 ft. per min. Allowing for engine friction, if half the power is transmitted through each set of gears the tangential force at the rims is about 3960 lbs., requiring, if the angle is 40° and the coefficient of friction 0.18, a pressure of 7524 lbs. between the wheels and pinion to prevent slipping.

The wear of the wheels proving excessive, the gears were replaced by spur-gear wheels and brake-wheels with steel brake-bands, which arrangement has proven more durable than the grooved wheels. Mr. Daniel Adamson states that if the frictional wheels had been run at a higher speed the results would have been better, and says they should run at least 30 ft. per second.

Power Transmitted by Friction Drives. (W. F. M. Goss, *Trans. A. S. M. E.*, 1907.) — A friction drive consists of a fibrous or somewhat yielding driving wheel working in rolling contact with a metallic driven wheel. Such a drive may consist of a pair of plain cylinder wheels mounted upon parallel shafts, or a pair of beveled wheels, or of any other arrangement which will serve in the transmission of motion by rolling contact.

Driving wheels of each of the materials named in the table below were tested in peripheral contact with driving wheels of iron, aluminum and type metal. All the wheels were 16 in. diam.; the face of the driving wheels was $13/4$ in., and that of the driven wheels $1/2$ in. Records were made of the pressure of contact, of the coefficient of friction developed, and of the percentage of slip resulting from the development of the said coefficient of friction. Curves were plotted showing the relation of the coefficient and the slip for pressures of 150 and 400 lbs. per inch width of face in contact. Another series of tests was made in which the slip was maintained constant at 2% and the pressures were varied. In most of the combinations it was found that with constant slip the coefficient of friction diminished very slightly as the pressure of contact was increased, so that it may be considered practically constant for all pressures between 150 and 400 lbs. per sq. in.

The crushing strength of each material under the conditions of the test was determined by running each combination with increasing loads until a load was found under which the wheel failed before 15,000 revolutions had been made. The results showed the failure of the several fiber wheels under loads per inch of width as follows: Straw fiber 750 lbs.; leather fiber, 1,200 lbs.; tarred fiber, 1,200 lbs.; leather, 750 lbs.; sulphite fiber, 700 lbs. One-fifth of these pressures is taken as a safe working load. The coefficient of friction approaches its maximum value when the slip between driver and driven wheel is 2%. The safe working horse-power of the drive is calculated on the basis of 60% the coefficient developed at a pressure of 150 lbs. per inch of width, a reduction of 40% being made to cover possible decrease of the coefficient in actual service and to cover also loss due to friction of the journals. From these data the following table is constructed showing the H.P. that may be transmitted by driving wheels of the several materials named when in frictional contact with iron, aluminum and type metal.

The formula for horse-power is $H.P. = \frac{\pi d}{12} \times \frac{WPN \times 0.6f}{33000} = KdWN$, in which d = diam. in inches, W = width of face in inches, P = safe working pressure in lbs. per in. of width, N = revs. per min., f = coefficient of friction, 0.6 a factor for the decrease of the coefficient in service and for the loss in journal friction, K a coefficient including P , f and the numerical constants.

COEFFICIENTS OF FRICTION AND HORSE-POWER OF FRICTION DRIVES.

| | On iron. | | On aluminum. | | On type metal. | |
|---------------------|----------|---------|--------------|---------|----------------|---------|
| | f | k | f | k | f | k |
| Straw fiber..... | 0.255 | 0.00030 | 0.273 | 0.00033 | 0.186 | 0.00022 |
| Leather fiber..... | 0.309 | 0.00059 | 0.297 | 0.00057 | 0.183 | 0.00035 |
| Tarred fiber..... | 0.150 | 0.00029 | 0.183 | 0.00035 | 0.165 | 0.00031 |
| Sulphite fiber..... | 0.330 | 0.00037 | 0.318 | 0.00035 | 0.309 | 0.00034 |
| Leather..... | 0.135 | 0.00016 | 0.216 | 0.00026 | 0.246 | 0.00029 |

$$\text{Horse-power} = K \times dWN.$$

Friction Clutches. — Much valuable information on different forms of friction clutches is given in a paper by Henry Souther in *Trans. A. S. M. E.*, 1908, and in the discussion on the paper. All friction clutches contain two surfaces that rub on each other when the clutch is thrown into gear, and until the friction between them is increased, by the pressure with which they are forced together, to such an extent that the surfaces bind and enable one surface to drive the other. The surfaces may be metal on metal, metal on wood, cork, leather or other substance, leather on leather or other substance, etc. The surfaces may be disks, at right angles to the shaft, blocks sliding on the outer or inner surface, or both, of a pulley rim, or two cones, internal and external, one fitting in the other, or a band or ribbon around a pulley. The driving force which is just sufficient to cause one part of the clutch to drive the other is the product of the total pressure, exerted at right angles to the direction of sliding, and the coefficient of friction. The latter is an exceedingly variable quantity, depending on the nature and condition of the sliding surfaces and on their lubrication. The surfaces must have sufficient area so that the pressure per square inch on that area will not be sufficient to cause undue heating and wear. The total pressure on the parts of the mechanism that forces the surfaces together also must not cause undue wear of these parts.

For cone clutches, Reuleaux states that the angle of the cone should not be less than 10° , in order that the parts may not become wedged together. He gives the coefficient of cast iron on cast iron, for such clutches, at 0.15.

For clutches with maple blocks on cast iron Mr. Souther gives a coefficient of 0.37, and for a speed of 100 r.p.m. he gives the following table of capacity of such clutches, made by the Dodge Mfg. Co.

| Horse-power. | Block Area. | Diam. at Block, Ins. | Circumferential Pull at Block Center. | Total Pressure. | Total Pressure per sq. in. |
|--------------|-------------|----------------------|---------------------------------------|-----------------|----------------------------|
| | Ins. | | Lbs. | | |
| 25 | 120 | 16 | 1,960 | 5,300 | 44 |
| 32 | 141 | 18 | 2,240 | 6,000 | 44.5 |
| 50 | 208 | 21.5 | 2,900 | 7,800 | 37.5 |
| 98 | 280 | 27.5 | 4,500 | 12,000 | 43.5 |

Prof. I. N. Hollis has found the coefficient of cork on cast iron to be from 0.33 to 0.37, or about double that of cast iron on cast iron or on bronze. A set of cork blocks outlasted a set of maple blocks in the ratio of five to one. Prof. C. M. Allen has found the torque for cork inserts to be nearly double that of a leather-faced clutch for a given dimension.

Disk clutches for automobiles are made with frictional surfaces of leather, bronze, or copper against iron or steel. The Cadillac Motor Car Co. give the following: Mean radius of leather frictional surface $45/16$ ins.; area of do., $361\frac{1}{2}$ sq. ins.; axial pressure, 1000 to 1200 lbs.; H.P. capacity at 400 r.p.m., $5\frac{1}{2}$ H.P.; at 1400 r.p.m., 10 H.P.

C. H. Schlesinger (*Horseless Age*, Oct. 2, 1907) gives the following formula for the ordinary cone clutch:

$$\text{H.P.} = PfrR \div 63,000 \sin. \theta,$$

in which P = assumed pressure of engaging spring in lbs., f = coeff. of friction, which in ordinary practice is about 0.25; r = mean radius of the cone, ins.; R = r.p.m. of the motor; θ = angle of the cone with the axis. Mr. Souther says the value of $f = 0.25$ is probably near enough for a properly lubricated leather-iron clutch.

The Hele-Shaw clutch, with V-shaped rings struck up in the surfaces of disks, is described in *Proc. Inst. M. E.*, 1903. A clutch of this form 18 ins. diam. between the V's transmitted 1000 H.P. at 700 or 800 r.p.m.

Coil Friction Clutches. (H. L. Nachman, *Am. Mach.*, April 1, 1909.) — Friction clutches are now in use which will transmit 1000, and even more, horse-power. A type of clutch which is satisfactory for the transmission of large powers is the coil friction clutch. It consists of a steel coil wound on a chilled cast-iron drum. At each end of the coil a head is formed. The head at one end is attached to the pulley or shaft that is to be set in motion, while that at the other end of the coil serves as a point of application of a force which pulls on the coil to wind it on the drum, thus gripping it firmly.

The friction of the coil on the drum is the same as that of a rope or belt on a pulley. That is, the relation of the tensions at the two ends of the coil may be found from the equation $P/Q = e^{\mu\alpha}$ where P = pull at fixed end of coil; Q = pull at free end of coil; e = base of natural logarithms = 2.718; μ = coefficient of friction between coils and drum; and α = Angle subtended by coil in radian measure, = 6.283 for each turn of coil.

Values of P/Q for different numbers of turns are as follows, assuming $N = 0.05$ for steel on cast iron, lubricated:

| | | | | | | | | |
|--------------|------|------|------|------|------|------|------|-------|
| No. of turns | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| P/Q = | 1.37 | 1.87 | 2.57 | 3.51 | 4.81 | 6.58 | 8.60 | 12.33 |

If D = diam. of drum in ins., N = revs., per min., then $\text{H.P.} = \pi DNP \div (12 \times 33,000) = 0.00000793 DNP$.

HOISTING AND CONVEYING.

Strength of Ropes and Chains.—For the weight and strength of rope for hoisting see notes and tables on pages 386 to 391. For strength of chains see page 251.

Working Strength of Blocks.

(Boston and Lockport Block Co., 1908.)

REGULAR BLOCKS WITH LOOSE HOOKS—LOADS IN POUNDS.

| Size, Inches. | 5 | 6 | 8 | 10 | 12 | 14 |
|----------------------------|----------------|---------------|---------------|------|----------------|----------------|
| Rope diameter, inches..... | $\frac{9}{16}$ | $\frac{3}{4}$ | $\frac{7}{8}$ | 1 | $1\frac{1}{8}$ | $1\frac{1}{4}$ |
| 2 single blocks..... | 150 | 250 | 700 | 2000 | 4000 | 7000 |
| 2 double blocks..... | 250 | 400 | 1200 | 4000 | 8000 | 12000 |
| 2 triple blocks..... | 400 | 650 | 1900 | 6000 | 12000 | 19000 |

LOADS IN TONS.

| | WIDE MORTISE WITH LOOSE HOOKS. | | | | | EXTRA HEAVY WITH SHACKLES. | | | |
|----------------------|-----------------------------------|----------------|----------------|----------------|----------------|-------------------------------|----------------|----------------|-------|
| Size, inches..... | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| Rope, diam., in..... | 1 | $1\frac{1}{4}$ | $1\frac{5}{8}$ | $1\frac{5}{8}$ | $1\frac{3}{4}$ | 2 | $2\frac{1}{4}$ | $2\frac{1}{2}$ | 3 |
| 2 single blocks..... | $\frac{1}{2}$ | 2 | 4 | 6 | 10 | 25 | 30 | 35 | 40 |
| 2 double blocks..... | 1 | 3 | 6 | 8 | 12 | 30 | 35 | 40 | 50 |
| 2 triple blocks..... | 2 | 4 | 8 | 10 | 14 | 40 | 45 | 55 | 70 |
| 2 fourfold blocks .. | | | | | | | | | |

WORKING LOADS FOR A PAIR OF WIRE-ROPE BLOCKS—TONS.

| LOOSE HOOKS. | | | | SHACKLES. | | |
|-------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Sheave Diam., In. | Two Singles. | Two Doubles. | Two Triples. | Two Singles. | Two Doubles. | Two Triples. |
| 8 | 3 | 4 | 5 | 4 | 5 | 6 |
| 10 | 4 | 5 | 6 | 6 | 8 | 10 |
| 12 | 5 | 6 | 7 | 8 | 10 | 12 |
| 14 | 6 | 7 | 8 | 10 | 12 | 15 |
| 16 | 7 | 8 | 10 | 12 | 15 | 20 |
| 18 | 8 | 10 | 12 | 15 | 20 | 25 |

Chain Blocks.—Referring to the table on the next page, the speed of a chain block is governed by the pull required on the hand chain and the distance the hand chain must travel to lift the load the required distance. The speeds are given for short lifts with men accustomed to the work; for continuous easy lifting two-thirds of these speeds are attainable. The triplex block lifts rapidly, and the speed increases for light loads because the length of hand chain to be overhauled is small. This fact also enables the operator to lower the load very quickly with the triplex block. The 12- to 20-ton triplex blocks are provided with two separate hand wheels, thus permitting two men to hoist simultaneously, thereby securing double speed. In the triplex block the power is transmitted to the hoisting-chain wheel by means of a train of spur gearing operated by the hand chain. In the duplex block

Chain Block Hoisting Speeds.

(Yale & Towne Mfg. Co., 1908.)

| Capacity in Tons. | Pull in Pounds re- quired on Hand-Chain to Lift Full Loads. | | | Feet of Hand- Chain to be Pulled by Operator to Lift Load One Foot High. | | | Hoisting Speeds. Feet per Minute Attainable and No. of Men re- quired for Hoisting Full Loads without Pulling over 80 Lb. | | | | | | | |
|----------------------|--|---------|--------------------|--|---------|--------------------|--|---------------|------------------|------------------|--------------------|------------------|------------|------------------|
| | | | | | | | Triplex. | | Duplex. | | Differ- ential. | | | |
| | Triplex. | Duplex. | Differen- tial. | Triplex. | Duplex. | Differen- tial. | Full Load. | Half Load. | Quarter Load. | † No. of Men. | Full Load. | † No. of Men. | Full Load. | † No. of Men. |
| 1/4 | 62 | 68 | 72 | 21 | 40 | 18 | 8 | 16 | 24 | 1 | 4 | 1 | 6 | 1 |
| 1/2 | 82 | 87 | 122 | 31 | 59 | 24 | 4 | 8 | 12 | 1 | 2 | 1 | 6 | 2 |
| 1 | 110 | 94 | 246 | 35 | 80 | 30 | 4 | 8 | 14 | 2 | 2 | 2 | 3.70 | 3 |
| 1 1/2 | 120 | 115 | 308 | 42 | 93 | 36 | 3 | 6 | 10 | 2 | 1 | 2 | 2.50 | 3 |
| 2 | 124 | 132 | 557 | 69 | 126 | 38 | 2 | 3 | 6 | 2 | 1 | 2 | 2.30 | 4 |
| 3 | 114 | 142 | ... | 84 | 155 | ... | 2 | 3 | 5 | 2 | 0 | 2 | 2.30 | 7 |
| 4 | 124 | 142 | ... | 84 | 155 | ... | 1 | 7 | 3 | 2 | 0 | 2 | ... | ... |
| 5 | 110 | 145 | ... | 126 | 195 | ... | 1 | 3 | 2 | 2 | 0 | 2 | ... | ... |
| 6 | 130 | 145 | ... | 126 | 252 | ... | 1 | 1 | 2 | 3 | 0 | 2 | ... | ... |
| 8 | 135 | 160 | ... | 168 | 310 | ... | 0 | 8 | 1 | 2 | 0 | 2 | ... | ... |
| 10 | 140 | 160 | ... | 210 | 390 | ... | 0 | 6 | 1 | 2 | 0 | 2 | ... | ... |
| 12 | 130* | ... | ... | 125* | ... | ... | 1 | 1 | 2 | 3 | 4 | ... | ... | ... |
| 16 | 135* | ... | ... | 168* | ... | ... | 0 | 8 | 1 | 2 | 4 | 4 | ... | ... |
| 20 | 140* | ... | ... | 210* | ... | ... | 0 | 6 | 1 | 2 | 1 | 4 | ... | ... |

* On each of the two hand-chains.

† The number of men is based on each man pulling not over 80 lb. One man pulling 160 lb. or less, as given in the first two columns, can lift the full capacity of any Triplex or Duplex Block.

the power is transmitted through a worm wheel and screw. In the differential block the power is applied by pulling on the slack part of the load chain and the force is multiplied by means of a differential sheave. (See page 513.) The relative efficiency and durability of the three types are as follows:

| | Differential. | Duplex. | Triplex. |
|--------------------------|---------------|---------|----------|
| Relative efficiency..... | 35 | 50 | 100 |
| Relative durability..... | 20 | 80 | 100 |
| Relative cost..... | 40 | 80 | 100 |

Efficiency of Hoisting Tackle. — (S. L. Wonsen, *Eng. News*, June 11, 1903.)

1 1/4 to 2-in. Manila rope.

| Parts of line. | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | ... | ... |
|----------------------------|------|------|------|------|------|------|------|------|-----|-----|
| Ratio of load to pull..... | 1.91 | 2.64 | 3.30 | 3.84 | 4.33 | 4.72 | 5.08 | 5.37 | ... | ... |
| Efficiency, per cent..... | 96 | 88 | 83 | 77 | 72 | 67 | 64 | 60 | ... | ... |

3/4-in. Wire rope.

| Parts of line. | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---------------------------|------|------|------|------|------|------|------|------|------|------|------|
| Ratio load to pull..... | 2.73 | 3.47 | 4.11 | 4.70 | 5.20 | 5.68 | 6.08 | 6.46 | 6.78 | 7.08 | 7.34 |
| Efficiency, per cent..... | 91 | 87 | 82 | 78 | 74 | 71 | 68 | 65 | 62 | 59 | 56 |

Proportions of Hooks. — The following formulæ are given by Henry R. Towne, in his *Treatise on Cranes*, as a result of an extensive experimental and mathematical investigation. They apply to hooks of capacities from 250 lb. to 20,000 lb. Each size of hook is made from some commercial size of round iron. The basis in each case is, therefore, the size of iron of which the hook is to be made, indicated by A in the diagram. The dimension D is arbitrarily assumed. The other dimensions, as given by the formulæ, are those which, while preserving a proper bearing-face on the interior of the hook for the ropes or chains which may be passed through it, give the greatest resistance to spreading and to ultimate rupture, which the amount of material in the original bar admits of. The symbol Δ is used to indicate the nominal capacity of the hook in tons of 2000 lb. The formulæ which determine the lines of the other parts of the hooks of the several sizes are as follows, the measurements being all expressed in inches:

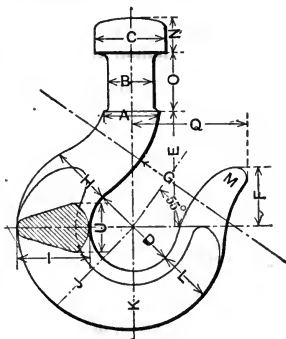


FIG. 182.

$$\begin{aligned} D &= 0.5 \Delta + 1.25; & G &= 0.75 D; & H &= 1.08 A; & L &= 1.05 A; \\ E &= 0.64 \Delta + 1.60; & O &= 0.363 \Delta + 0.66; & I &= 1.33 A; & M &= 0.50 A; \\ F &= 0.33 \Delta + 0.85; & Q &= 0.64 \Delta + 1.60; & J &= 1.20 A; & N &= 0.85 B - 0.16; \\ & & K &= 1.13 A; & U &= 0.866 A. \end{aligned}$$

The dimensions A are necessarily based upon the ordinary merchant sizes of round iron. The sizes which it has been found best to select are the following:

Capacity of hook:

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 3 4 5 6 8 10 tons.

Dimension A :

$\frac{5}{8}$ $\frac{11}{16}$ $\frac{3}{4}$ $\frac{11}{16}$ $\frac{11}{4}$ $\frac{13}{8}$ $\frac{13}{4}$ 2 $2\frac{1}{4}$ $2\frac{1}{2}$ $\frac{27}{8}$ $3\frac{1}{4}$ in.

Experiment has shown that hooks made according to the above formulæ will give way first by opening of the jaw, which, however, will not occur except with a load much in excess of the nominal capacity of the hook. This yielding of the hook when overloaded becomes a source of safety, as it constitutes a signal of danger which cannot easily be overlooked, and which must proceed to a considerable length before rupture will occur and the load be dropped.

Iron versus Steel Hooks. — F. A. Waldron, for over fifteen years connected with the manufacturing of hooks, in the works of the Yale & Towne Mfg. Co., after careful observation of hooks made of different materials and in different forms, says that the only proper material from which hooks can be made and be perfectly reliable is a high-grade puddled iron. While a steel hook, properly made, may stand from 25 to 50% greater load than a wrought-iron hook, it does not follow that the steel hook is better and more reliable than the iron hook.

Iron hooks, made in accordance with the Towne formula, having serious surface defects, have been tested to destruction, and none of them, in spite of these defects, have broken at less than $2\frac{1}{2}$ times the working load, while several steel hooks broke at the working load, without a moment's warning. (*Trans. A. S. M. E.*, 1903.)

Heavy Crane Hooks. — A. E. Holcomb, vice-pres. of the Earth Moving Machinery Co., contributes the following (1908). Seven years ago, while engaged in the design of a 100-ton crane, I made a study of the variations in strength with the different sectional forms for hooks in most common use. As a result certain values which gave the best results were substituted in "Gordon's" formula and a formula was thereby obtained which was good for hooks of any size desired, provided the proper allowable fiber stress per square inch was made use of when designing. From this

formula the enclosed table was made up and was published in the *American Machinist* of Oct. 31, 1901. Since that time hundreds of hooks of cast or hammered steel have been designed and made according to my formula, and not one of them, so far as I know, has ever failed.

The Industrial Works, Bay City, Michigan, manufacturers of heavy cranes, in December, 1904, made the following test under actual working conditions:

A hook was made of hammered steel having an elastic limit or yield point at approximately 36,000 lbs. per sq. in. fiber stress and having the following important dimensions: $d = 7\frac{5}{8}$ in.; $r = 4\frac{1}{2}$ in.; $D = 20\frac{7}{16}$ in.

When the applied load reached 150,000 lbs. the hook straightened out until the opening at the mouth of the hook was $2\frac{1}{2}$ in. larger than formerly, and the distance from center of action line of load to center of gravity of section was found to have decreased $\frac{1}{2}$ in., at which point the hook held the load. Upon increasing the load still further, the hook opened still more. From the dimensions of the hook as originally formed, we find from the formula or table that the fiber stress with a load of 150,000 lbs. was 37,900 lbs. per sq. in., or in excess of the yield point, whereas making use of the dimensions obtained from the hook when it held we find that the fiber stress per square inch was reduced to 35,940 lbs., or under the yield point.

The designer must use his own judgment as to the selection of a proper allowable fiber stress, being governed therein by the nature of the material to be used and the probability of the hook being overloaded at some time. Under average conditions I have made use of the following values for (f):

| | Values of (f) in pounds for a load of — | | | | | |
|---------------------|---|----------------------------|--------------------------------|--------------------------------|---------------------------------|----------------------------|
| | 1,000 to 5,000 lbs. | 5,000 to 15,000 lbs. | 15,000 to 30,000 lbs. | 30,000 to 60,000 lbs. | 60,000 to 100,000 lbs. | 100,000 lbs. and up. |
| Cast iron..... | 2,000 | 2,500 | | | | |
| Steel casting..... | 6,000 | 8,000 | 10,000 | 11,250 | 12,500 | |
| Hammered steel..... | 12,000 | 16,000 | 20,000 | 22,500 | 25,000 | 27,500 |

Mr. Holcomb's formula and his table in condensed form are given below:

DIRECTIONS. — P and f being known, assume r to suit the requirements for which the hook is to be designed. Divide P by f and find the quotient in the column headed by the required r . At the side of the Table, in the same row, will be found the necessary depth of section, d .

Notation. — P = load. S = area of section. R^2 = square of the radius of gyration. f = allowable fiber stress in lbs. per sq. in., 20,000 lbs. for hammered steel. For other letters see Fig. 183.

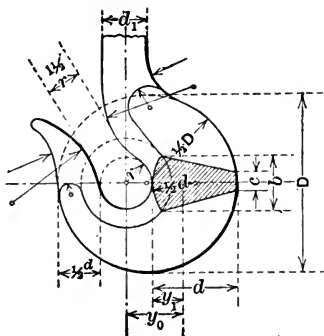


FIG. 183.

$$\frac{P}{f} = \frac{S}{1 + \frac{y_0^2}{R^2}} \quad \text{General formula.}$$

$$S = \frac{b + c}{2} \times d. \quad (1)$$

$$R^2 = \frac{d^2(b^2 + 4bc + c^2)}{18(b^2 + 2bc + c^2)}. \quad (2)$$

$$y_1 = \frac{b + 2c}{b + c} \times \frac{d}{3}. \quad (3)$$

$$y_0 = \left(\frac{b + 2c}{b + c} \times \frac{d}{3} \right) + r. \quad (4)$$

Assuming $b = 0.66d$; $c = 0.22d$, we have:

$$\frac{P}{f} = \frac{d^3}{7.44d + 12.393r} = K. \quad (5)$$

$$d_1 = 0.5d.$$

$$D = 2r + 1.5d.$$

Values of K .

| d . | r . | | | | | | | | | | |
|-------|-------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 | 3.50 | 4.00 | 5.00 | 6.00 |
| 2.00 | 0.379 | 0.331 | 0.292 | 0.240 | 0.203 | 0.176 | 0.155 | | | | |
| 2.25 | .496 | .437 | .391 | .329 | .275 | .239 | .212 | | | | |
| 2.50 | .629 | .559 | .504 | .420 | .360 | .316 | .281 | | | | |
| 2.75 | .778 | .697 | .632 | .532 | .460 | .404 | .360 | | | | |
| 3.00 | .944 | .852 | .776 | .659 | .572 | .506 | .454 | 0.411 | | | |
| 3.25 | 1.143 | 1.039 | .953 | .801 | .700 | .621 | .559 | .508 | | | |
| 3.50 | 1.342 | 1.226 | 1.129 | .957 | .841 | .750 | .677 | .617 | | | |
| 3.75 | 1.558 | 1.429 | 1.321 | 1.148 | .998 | .893 | .808 | .738 | | | |
| 4.00 | 1.790 | 1.649 | 1.530 | 1.336 | 1.187 | 1.067 | .953 | .873 | 0.805 | | |
| 4.25 | 2.038 | 1.886 | 1.754 | 1.544 | 1.373 | 1.239 | 1.129 | 1.038 | 0.943 | | |
| 4.50 | 2.304 | 2.138 | 1.995 | 1.760 | 1.575 | 1.426 | 1.321 | 1.214 | 1.124 | | |
| 4.75 | 2.586 | 2.408 | 2.253 | 1.996 | 1.793 | 1.627 | 1.490 | 1.374 | 1.275 | | |
| 5.00 | 2.884 | 2.694 | 2.527 | 2.248 | 2.072 | 1.843 | 1.691 | 1.563 | 1.453 | | |
| 5.25 | 3.214 | 3.008 | 2.828 | 2.525 | 2.281 | 2.081 | 1.913 | 1.770 | 1.647 | | |
| 5.50 | 3.532 | 3.315 | 3.124 | 2.801 | 2.538 | 2.321 | 2.140 | 1.983 | 1.849 | 1.628 | |
| 5.75 | | 3.651 | 3.447 | 3.101 | 2.818 | 2.583 | 2.385 | 2.215 | 2.067 | 1.825 | |
| 6.00 | | 4.003 | 3.787 | 3.418 | 3.115 | 2.861 | 2.646 | 2.461 | 2.300 | 2.035 | |
| 6.50 | | 4.757 | 4.516 | 4.100 | 3.754 | 3.463 | 3.213 | 2.998 | 2.809 | 2.496 | 2.246 |
| 7.00 | | 5.578 | 5.311 | 4.848 | 4.459 | 4.128 | 3.842 | 3.594 | 3.377 | 3.012 | 2.719 |
| 7.50 | | | 6.173 | 5.661 | 5.227 | 4.855 | 4.533 | 4.252 | 4.003 | 3.584 | 3.244 |
| 8.00 | | | 7.102 | 6.540 | 6.061 | 5.648 | 5.287 | 4.970 | 4.689 | 4.213 | 3.825 |
| 8.50 | | | 8.096 | 7.485 | 6.960 | 6.504 | 6.104 | 5.750 | 5.436 | 4.900 | 4.460 |
| 9.00 | | | 9.158 | 8.496 | 7.924 | 7.424 | 6.984 | 6.593 | 6.243 | 5.645 | 5.152 |
| 9.50 | | | | 9.574 | 8.954 | 8.409 | 7.928 | 7.498 | 7.113 | 6.450 | 5.901 |
| 10.00 | | | | 10.788 | 10.220 | 9.460 | 8.932 | 8.467 | 8.044 | 7.316 | 6.708 |
| 10.50 | | | | 12.098 | 11.381 | 10.746 | 10.008 | 9.499 | 9.039 | 8.241 | 7.573 |
| 11.00 | | | | 13.374 | 12.608 | 11.922 | 11.316 | 10.766 | 10.267 | 9.228 | 8.498 |
| 11.50 | | | | 14.717 | 13.901 | 13.173 | 12.518 | 11.926 | 11.388 | 10.448 | 9.482 |
| 12.00 | | | | 16.126 | 15.261 | 14.485 | 13.785 | 13.150 | 12.572 | 11.558 | 10.697 |
| 12.50 | | | | 17.601 | 16.686 | 15.862 | 15.117 | 14.442 | 13.820 | 12.730 | 11.802 |
| 13.00 | | | | | 18.178 | 17.305 | 16.514 | 15.792 | 15.132 | 13.965 | 12.967 |
| 13.50 | | | | | 19.735 | 18.814 | 17.976 | 17.210 | 16.508 | 15.263 | 14.195 |
| 14.00 | | | | | 21.359 | 20.389 | 19.504 | 18.694 | 17.948 | 16.624 | 15.484 |
| 14.50 | | | | | 23.050 | 22.031 | 21.098 | 20.242 | 19.453 | 18.049 | 16.835 |
| 15.00 | | | | | 24.807 | 23.738 | 22.758 | 21.846 | 21.023 | 19.536 | 18.248 |
| 15.50 | | | | | 26.630 | 25.511 | 24.483 | 23.535 | 22.658 | 21.088 | 19.724 |
| 16.00 | | | | | 28.520 | 27.351 | 26.274 | 25.388 | 24.358 | 22.704 | 21.262 |

For values of K and r intermediate to those given in the table approximate values of d may be found by interpolation. Thus, for $K = 3.700$, $r = 2.75$.

Tabular values, $r = 2.5$ 3.0 Int. for 2.75

$d = 6.50$ $K = 3.462$ 3.213 3.338

$d = 7.00$ $K = 4.128$ 3.842 3.985

Whence: $d = 6.5 + \left\{ \frac{(3.700 - 3.338)}{(3.985 - 3.338)} \right\} \times (7.0 - 6.5) = 6.78$.

In like manner, if d and r are given the value of K and the corresponding safe load may be found.

Strength of Hooks and Shackles. (Boston and Lockport Block Co., 1908.) — Tests made at the Watertown arsenal on the strength of hooks and shackles showed that they failed at the loads given in the table below. In service they should be subjected to only 50% of the figures in the table. Ordinarily the hook of a block gives way first, and where heavy weights are to be handled shackles are superior to hooks and should be used wherever possible.

Strength of Hooks and Shackles.

| HOOKS.* | | SHACKLES. | | HOOKS.* | | SHACKLES. | |
|---------------|---------------------------|---------------------------|--------------------------|---------------|---------------------------|---------------------------|--------------------------|
| Size, Inches. | Tensile Strength, Pounds. | Tensile Strength, Pounds. | Description of Fracture. | Size, Inches. | Tensile Strength, Pounds. | Tensile Strength, Pounds. | Description of Fracture. |
| 1/2 | 1,890 | | | 13/8 | 17,310 | 103,750 | Eye of shackle. |
| 9/16 | 2,560 | | | 11/2 | 20,940 | 119,800 | Eye of shackle. |
| 5/8 | 3,020 | | | 15/8 | 23,670 | 125,900 | Eye of shackle. |
| 3/4 | 4,470 | 20,700 | Eye of shackle. | 13/4 | 27,400 | 146,804 | Sheared shackle pin. |
| 7/8 | 6,280 | 38,100 | Eye of shackle. | 17/8 | 36,120 | 162,700 | Eye of shackle. |
| 1 | 12,600 | 51,900 | Eye of shackle. | 2 | 38,100 | 196,600 | Shackle at neck of eye. |
| 1 1/8 | 13,520 | 62,900 | Sheared shackle pin. | 2 1/2 | 55,380 | 210,400 | Eye of shackle. |
| 1 1/4 | 16,800 | 75,200 | Eye of shackle. | | | | |

* All the hooks failed by straightening the hook.

Horse-power Required to Raise a Load at a Given Speed. — H.P. = $\frac{\text{Gross weight in lb.}}{33,000} \times \text{speed in ft. per min.}$ To this add 25% to 50% for friction, contingencies, etc. The gross weight includes the weight of cage, rope, etc. In a shaft with two cages balancing each other use the net load + weight of one rope, instead of the gross weight.

To find the load which a given pair of engines will start. — Let A = area of cylinder in square inches, or total area of both cylinders, if there are two; P = mean effective pressure in cylinder in lb. per sq. in.; S = stroke of cylinder, inches; C = circumference of hoisting-drum, inches; L = load lifted by hoisting-rope, lb.; F = friction, expressed as a diminution of the load. Then $L = \frac{A \times P \times 2S}{C} - F$.

An example in *Coll'y Engr.*, July, 1891, is a pair of hoisting-engines 24" \times 40", drum 12 ft. diam., average steam-pressure in cylinder = 59.5 lb.; $A = 904.8$; $P = 59.5$; $S = 40$; $C = 452.4$. Theoretical load, not allowing for friction, $A \times P \times 2S \div C = 9589$ lb. The actual load that could just be lifted on trial was 7988 lb., making friction loss $F = 1601$ lb., or 20 + per cent of the actual load lifted, or 162/3% of the theoretical load.

The above rule takes no account of the resistance due to inertia of the load, but for all ordinary cases in which the acceleration of speed of the cage is moderate, it is covered by the allowance for friction, etc. The resistance due to inertia is equal to the force required to give the load the velocity acquired in a given time, or, as shown in *Mechanics*, equal to the product of the mass by the acceleration, or $R = \frac{WV}{gT}$, in which R = resistance in lb. due to inertia; W = weight of load in lb.; V = maximum velocity in ft. per second; T = time in seconds taken to acquire the velocity V ; $g = 32.16$.

Effect of Slack Rope upon Strain in Hoisting. — A series of tests with a dynamometer are published by the Trenton Iron Co., which show that a dangerous extra strain may be caused by a few inches of slack rope. In one case the cage and full tubs weighed 11,300 lb.; the strain when the load was lifted gently was 11,525 lb.; with 3 in. of slack chain it was 19,025 lb.; with 6 in. slack 25,750 lb., and with 9 in. slack 27,950 lb.

Limit of Depth for Hoisting. — Taking the weight of a cast-steel hoisting-rope of 1 1/8 in. diameter at 2 lb. per running foot, and its break-

ing strength at 84,000 lb., it should, theoretically, sustain itself until 42,000 feet long before breaking from its own weight. But taking the usual factor of safety of 7, then the safe working length of such a rope would be only 6000 ft. If a weight of 3 tons is now hung to the rope, which is equivalent to that of a cage of moderate capacity with its loaded cars, the maximum length at which such a rope could be used, with the factor of safety of 7, is 3000 ft., or

$$2x + 6000 = 84,000 \div 7; \therefore x = 3000 \text{ feet.}$$

This limit may be greatly increased by using special steel rope of higher strength, by using a smaller factor of safety, and by using taper ropes. (See paper by H. A. Wheeler, *Trans. A. I. M. E.*, xix, 107.)

Large Hoisting Records. — At a colliery in North Derbyshire during the first week in June, 1890, 6309 tons were raised from a depth of 509 yards, the time of winding being from 7 a.m. to 3.30 p.m.

At two other Derbyshire pits, 170 and 140 yards in depth, the speed of winding and changing has been brought to such perfection that tubs are drawn and changed three times in one minute. (*Proc. Inst. M. E.*, 1890.)

At the Nottingham Colliery near Wilkesbarre, Pa., in Oct., 1891, 70,152 tons were shipped in 24.15 days, the average hoist per day being 1318 mine cars. The depth of hoist was 470 feet, and all coal came from one opening. The engines were fast motion, 22×48 inches, conical drums 4 feet 1 inch long, 7 feet diameter at small end and 9 feet at large end. (*Eng'g News*, Nov., 1891.)

The Most Powerful Hoisting Engines ever built are said to be two 32×72 duplex double-drum units built in 1906 for the Boston and Montana Co., at Butte, Mont. Each is designed to lift a dead load, unbalanced, of 17 tons out of a 3,500-ft. vertical shaft, at the rate of 2,500 ft. per minute. Each hoist has two drums, 12 ft. diameter and 5 ft. 6 ins. face, mounted on the same shaft and driven by 12-ft. diameter flat-disk reversible friction clutches.

Pneumatic Hoisting. (H. A. Wheeler, *Trans. A. I. M. E.*, xix, 107.) — A pneumatic hoist was installed in 1876 at Epinac, France, consisting of two continuous air-tight iron cylinders extending from the bottom to the top of the shaft. Within the cylinder moved a piston from which was hung the cage. It was operated by exhausting the air from above the piston, the lower side being open to the atmosphere. Its use was discontinued on account of the failure of the mine. Mr. Wheeler gives a description of the system, but criticises it as not being equal on the whole to hoisting by steel ropes.

Pneumatic hoisting-cylinders using compressed air have been used at blast-furnaces, the weighted piston counterbalancing the weight of the cage, and the two being connected by a wire rope passing over a pulley-sheave above the top of the cylinder. In the more modern furnaces steam-engine or electric hoists are generally used.

Electric Mine-Hoists. — An important paper on this subject, by D. B. Rushmore and K. A. Paulv, will be found in *Trans. A. I. M. E.*, 1910.

Counterbalancing of Winding-engines. (H. W. Hughes, *Columbia Coll. Qly.*) — Engines running unbalanced are subject to enormous variations in the load; for let W = weight of cage and empty tubs, say 6270 lb.; c = weight of coal, say 4480 lb.; r = weight of hoisting rope, say 6000 lb.; r' = weight of counterbalancing rope hanging down pit, say 6000 lb. The weight to be lifted will be:

If weight of rope is unbalanced. If weight of rope is balanced.

| | | | |
|-----------------------|---|---|---------------|
| At beginning of lift: | $W + c + r - W$ or 10,480 lb. | $W + c + r - (W + r')$, | } 4480 lb. OR |
| At middle of lift: | $W + c + \frac{r}{2} - (W + \frac{r}{2})$ or 4480 lb. | $W + c + \frac{r}{2} + \frac{r'}{2} - (W + \frac{r}{2} + \frac{r'}{2})$, | |
| At end of lift: | $W + c - (W + r)$ or minus 1520 lb. | $W + c + r' - (W + r)$, | |

That counterbalancing materially affects the size of winding-engines is shown by a formula given by Mr. Robert Wilson, which is based on the fact that the greatest work a winding-engine has to do is to get a given mass into a certain velocity uniformly accelerated from rest, and to raise a load the distance passed over during the time this velocity is being obtained.

Let W = the weight to be set in motion: one cage, coal, number of empty tubs on cage, one winding rope from pit head-gear to bottom, and one rope from banking level to bottom.

v = greatest velocity attained, uniformly accelerated from rest;

g = gravity = 32.2;

t = time in seconds during which v is obtained;

L = unbalanced load on engine;

R = ratio of diameter of drum and crank circles;

P = average pressure of steam in cylinders;

N = number of cylinders;

S = space passed over by crank-pin during time t ;

$C = \frac{2}{3}$, constant to reduce angular space passed through by crank to the distance passed through by the piston during the time t ;

A = area of one cylinder, without margin for friction. To this an addition for friction, etc., of engine is to be made, varying from 10 to 30% of A .

1st. Where load is balanced,

$$A = \frac{\left\{ \left(\frac{Wv^2}{2g} \right) + \left(L \frac{vt}{2} \right) \right\} R}{PNSC}.$$

2d. Where load is unbalanced:

The formula is the same, with the addition of another term to allow for the variation in the lengths of the ascending and descending ropes. In this case

h_1 = reduced length of rope in t attached to ascending cage;

h_2 = increased length of rope in t attached to descending cage;

w = weight of rope per foot in pounds. Then

$$A = \frac{\left[\left(\frac{Wv^2}{2g} \right) + \left\{ \left(L \frac{vt}{2} \right) - \frac{h_1w + h_2w}{2} \right\} \right] R}{PNSC}.$$

Applying the above formula when designing new engines, Mr. Wilson found that 30 in. diameter of cylinders would produce equal results, when balanced, to those of the 36-in. cylinder in use, the latter being unbalanced.

Counterbalancing may be employed in the following methods:

(a) *Tapering Rope.* — At the initial stage the tapering rope enables us to wind from greater depths than is possible with ropes of uniform section. The thickness of such a rope at any point should only be such as to safely bear the load on it at that point.

With tapering ropes we obtain a smaller difference between the initial and final load, but the difference is still considerable, and for perfect equalization of the load we must rely on some other resource. The theory of taper ropes is to obtain a rope of uniform strength, thinner at the cage end where the weight is least, and thicker at the drum end where it is greatest.

(b) *The Counterpoise System* consists of a heavy chain working up and down a staple pit, the motion being obtained by means of a special small drum placed on the same axis as the winding drum. It is so arranged that the chain hangs in full length down the staple pit at the commencement of the winding; in the center of the run the whole of the chain rests on the bottom of the pit, and, finally, at the end of the winding the counterpoise has been rewound upon the small drum, and is in the same condition as it was at the commencement.

(c) *Loaded-wagon System.* — A plan, formerly much employed, was to have a loaded wagon running on a short incline in place of this heavy chain; the rope actuating this wagon being connected in the same manner as the above to a subsidiary drum. The incline was constructed steep at the commencement, the inclination gradually decreasing to nothing. At the beginning of a wind the wagon was at the top of the incline, and during a portion of the run gradually passed down it till, at the meet of cages, no pull was exerted on the engine — the wagon by this time being at the bottom. In the latter part of the wind the resistance was all against the engine, owing to its having to pull the wagon up the incline,

and this resistance increased from nothing at the meet of cages to its greatest quantity at the conclusion of the lift.

(d) *The Endless-rope System* is preferable to all others, if there is sufficient sump room and the shaft is free from tubes, cross timbers, and other impediments. It consists in placing beneath the cages a tail rope, similar in diameter to the winding rope, and, after conveying this down the pit, it is attached beneath the other cage.

(e) *Flat Ropes Coiling on Reels.* — This means of winding allows of a certain equalization, for the radius of the coil of ascending rope continues to increase, while that of the descending one continues to diminish. Consequently, as the resistance decreases in the ascending load the leverage increases, and as the power increases in the other, the leverage diminishes. The variation in the leverage is a constant quantity, and is equal to the thickness of the rope where it is wound on the drum.

By the above means a remarkable uniformity in the load may be obtained, the only objection being the use of flat ropes, which weigh heavier and only last about two-thirds the time of round ones.

(f) *Conical Drums.* — Results analogous to the preceding may be obtained by using round ropes coiling on conical drums, which may either be smooth, with the successive coils lying side by side, or they may be provided with a spiral groove. The objection to these forms is, that perfect equalization is not obtained with the conical drums unless the sides are very steep, and consequently there is great risk of the rope slipping; to obviate this, scroll drums were proposed. They are, however, very expensive, and the lateral displacement of the winding rope from the center line of pulley becomes very great, owing to their necessary large width.

(g) *The Koepe System of Winding.* — An iron pulley with a single circular groove takes the place of the ordinary drum. The winding rope passes from one cage, over its head-gear pulley, round the drum, and, after passing over the other head-gear pulley, is connected with the second cage. The winding rope thus encircles about half the periphery of the drum in the same manner as a driving-belt on an ordinary pulley. There is a balance rope beneath the cages, passing round a pulley in the sump; the arrangement may be likened to an endless rope, the two cages being simply points of attachment.

CRANES.

Classification of Cranes. (Henry R. Towne, *Trans. A. S. M. E.*, iv. 288. Revised in *Hoisting*, published by The Yale & Towne Mfg. Co.)

A Hoist is a machine for raising and lowering weights. A Crane is a hoist with the added capacity of moving the load in a horizontal or lateral direction.

Cranes are divided into two classes, as to their motions, viz., *Rotary* and *Rectilinear*, and into four groups, as to their source of motive power, viz.:

Hand. — When operated by manual power.

Power. — When driven by power derived from line shafting.

Steam, Electric, Hydraulic, or Pneumatic. — When driven by an engine or motor attached to the crane, and operated by steam, electricity, water, or air transmitted to the crane from a fixed source of supply.

Locomotive. — When the crane is provided with its own boiler or other generator of power, and is self-propelling; usually being capable of both rotary and rectilinear motions.

Rotary and Rectilinear Cranes are thus subdivided:

ROTARY CRANES.

(1) *Swing-cranes.* — Having rotation, but no trolley motion.

(2) *Jib-cranes.* — Having rotation, and a trolley traveling on the jib.

(3) *Column-cranes.* — Identical with the jib-cranes, but rotating around a fixed column (which usually supports a floor above).

(4) *Pillar-cranes.* — Having rotation only; the pillar or column being supported entirely from the foundation.

(5) *Pillar Jib-cranes.* — Identical with the last, except in having a jib and trolley motion.

(6) *Derrick-cranes.* — Identical with jib-cranes, except that the head of the mast is held in position by guy-rods, instead of by attachment to a roof or ceiling.

(7) *Walking-cranes*. — Consisting of a pillar or jib-crane mounted on wheels and arranged to travel longitudinally upon one or more rails.

(8) *Locomotive-cranes*. — Consisting of a pillar-crane mounted on a truck, and provided with a steam-engine capable of propelling and rotating the crane, and of hoisting and lowering the load.

RECTILINEAR CRANES.

(9) *Bridge-cranes*. — Having a fixed bridge spanning an opening, and a trolley moving across the bridge.

(10) *Tram-cranes*. — Consisting of a truck, or short bridge, traveling longitudinally on overhead rails, and without trolley motion.

(11) *Traveling-cranes*. — Consisting of a bridge moving longitudinally on overhead tracks, and a trolley moving transversely on the bridge.

(12) *Gantries*. — Consisting of an overhead bridge, carried at each end by a trestle traveling on longitudinal tracks on the ground, and having a trolley moving transversely on the bridge.

(13) *Rotary Bridge-cranes*. — Combining rotary and rectilinear movements and consisting of a bridge pivoted at one end to a central pier or post, and supported at the other end on a circular track; provided with a trolley moving transversely on the bridge.

For descriptions of these several forms of cranes see Towne's "Treatise on Cranes."

Stresses in Cranes. — See Stresses in Framed Structures, p. 515, *ante*.

Position of the Inclined Brace in a Jib-crane. — The most economical arrangement is that in which the inclined brace intersects the jib at a distance from the mast equal to four-fifths the effective radius of the crane. (*Hoisting*.)

Electric Overhead Traveling Cranes. (From data supplied by Alliance Machine Co., Alliance, O., and Pawling & Harnischfeger, Milwaukee.) — Electric overhead traveling cranes usually have 3 motors, for hoisting, traversing the hoist trolley on the bridge and for moving the bridge, respectively. The usual range of motor sizes is as follows: Hoist, 15-50 H.P.; trolley, 3-15 H.P.; bridge, 15-50 H.P. The speeds at which the various motions are made range as follows, the figures being feet per minute: Hoist, 8-60; trolley traverse, 75-200; bridge travel, 200-600. These speeds are varied in the same capacity of crane to suit each particular installation. In general, the speed of the bridge in feet per minute should not exceed (length of runway + 100). If the runway is long and covered by more than one crane, the speed may be made equal to the average distance between cranes + 100. Usually 300 ft. per min. is a good speed. For small cranes in special cases, the speeds may be increased, but for cranes of over 50 tons capacity the speed should be below 300 ft. per min. unless the building is made especially strong to stand the strains incident to starting and stopping heavy cranes geared for high speeds. Cranes of over 15 tons capacity usually have an auxiliary hoist of $\frac{1}{5}$ the capacity of the main hoist, and usually operated by the same motor. Wire rope is now almost exclusively used for hoisting with cranes. The diameter of the drums and sheaves should be not less than 30 times the diameter of the hoisting rope, and should have a factor of safety of 5. Cranes are equipped with automatic load brakes to sustain the load when lifted and to regulate the speed when lowering, it being necessary for the hoist to drive the load down.

The voltage now standard for crane service is 220 volts at the crane motor, although 110 volts for small cranes is not objectionable. Voltages of 500-600 are inadvisable, especially in foundries and steel works, where dust and metallic oxides cover many parts of the crane and necessitate frequent cleaning to avoid grounds. On account of the danger from the higher voltages, the operators are apt to neglect this part of their work.

Power Required to Drive Cranes. (Morgan Engineering Co., Alliance, O., 1909.) — The power required to drive the different parts of cranes is determined by allowing a certain friction percentage over the power required to move the dead load. On hoist motions $33\frac{1}{3}\%$ is allowed for friction of the moving parts, thus giving a motor of $\frac{1}{3}$ greater capacity than if friction were neglected. For bridge and trolley motions, a journal friction of the track wheel axles of 10% of the total weight of the crane and load is allowed. There is then added an allowance of $23\frac{1}{3}\%$ of the horse-power required to drive the crane and load plus the track wheel

axle friction, to cover friction of the gearing. In selecting motors, the most important consideration is the maximum starting torque which the motor can exert. With alternating-current motors, this is less than with direct-current motors, requiring a larger motor, particularly on the bridge and trolley motions which require the greatest starting torque.

Walter G. Stephan says (*Iron Trade Rev.*, Jan. 7, 1909) that the bridge girders should be made of two plates latticed, or box girders, their depth varying from $\frac{1}{10}$ to $\frac{1}{20}$ of the span. The important feature of crane girder design is ample strength and stiffness, both vertically and laterally. Especial attention should be given to the transverse strain on the bridge due to sudden stopping or starting of heavy loads. The wheel base on the end trucks should have a ratio to the crane span of 1 to 6, although for long spans this ratio must necessarily be reduced to 1 to 8. Quick-traveling cranes should have as long a wheel base as possible, since the tendency to twist increases with the speed. Where several wheels are necessary at each end to support the crane, equalizing means should be used.

A recent development in cranes is the four- or six-girder crane for handling ladles of molten metal in steel works. The main trolley runs on the outer girders, with the hoist ropes depending between the outer and inner girders. The auxiliary trolley runs on the inner girders, thus being able to pass between the main ropes, and tilt the ladle in either direction.

Dimensions and Wheel Loads of Electric Traveling Cranes.

Based on 60-ft. span and 25-ft. lift; wire rope hoist.

(Alliance Machine Co., 1908.)

| Capacity, Tons (2000 Lb.). | Distance Run- way Rail to Highest Point. | | Distance Center of Rail to Ends of Crane. | Wheel Base of End Truck. | | Maximum Load per Wheel; Trol- ley at End of Bridge. |
|----------------------------------|--|-----|--|-----------------------------|-----|---|
| | Ft. | In. | In. | Ft. | In. | Pounds. |
| 5 | 6 | 0 | 9 | 9 | 0 | 20,000 |
| 10 | 6 | 6 | 10 | 10 | 0 | 27,000 |
| 25 | 7 | 4 | 12 | 11 | 6 | 51,000 |
| 40 | 8 | 0 | 12 | 12 | 3 | 82,000 |
| 50 | 8 | 9 | 12 | 12 | 6 | 48,000* |

* Has 8 track wheels on bridge.

Standard cranes are built in intermediate sizes, varying by 5 tons, up to 40 tons.

Standard Hoisting and Traveling Speeds of Electric Cranes.

(Pawling & Harnischfeger, 1908.)

| Capacity, Tons (2000 Lb.). | Hoisting Speed, Ft. per Min. | Bridge Travel Speed, Ft. per Min. | Capacity Aux. Hoist, Tons. | Speed Aux. Hoist, Ft. per Min. |
|----------------------------------|------------------------------------|---|----------------------------------|--------------------------------------|
| 5 | 25-100 | 300-450 | 3 | 30-75 |
| 10 | 20-75 | 300-450 | 3 | 50-125 } |
| 25 | 10-40 | 250-350 | 10 | 25-60 } |
| 40 | 9-30 | 250-350 | 5 | 40-100 } |
| 50 | 8-30 | 200-300 | 10 | 25-60 } |
| 75 | 6-25 | 200-250 | 5 | 40-100 } |
| 125 | 5-15 | 200-250 | 10 | 25-60 } |
| 150 | 5-15 | 200-250 | 15 | 20-50 } |
| | | | 25 | 20-50 } |
| | | | 25 | 20-50 } |

Trolley travel speed from 100-150 ft. per min. in all cases.

Notable Crane Installations. (1909.)

| Capacity, Tons. | Span. | | No. of Trolleys. | Capacity of Auxiliary Hoist, Tons. | H.P. of Hoist Motor. | | H.P. of Trolley Motor. | H.P. of Bridge Motor. | Hoisting Speed, Ft. per Min. | Bridge Speed, Ft. per Min. | Trolley Traveling Speed, Ft. per Min. | Depth of Main Girders. | Where Installed. | Maker. |
|-----------------|-------|--------|------------------|------------------------------------|----------------------|------------|------------------------|-----------------------|------------------------------|----------------------------|---------------------------------------|------------------------|------------------|--------|
| | | | | | Main. | Auxiliary. | | | | | | | | |
| 150 | 65 | 0 | 1 | 25 | 75† | {35}{10} | 30 | 75 | 8-24 | 150-200 | 100-150 | 7 0 | 4 | 1 |
| 150 | 55 | 0 | 1 | 30 | 120 | 50 | 35 | 50 | 8 | 150-200 | 75-100 | | 5 | 3 |
| 150 | 65 | 0 | 2 | 15 | 75† | 30† | 18† | 75 | 10-25 | 150-200 | 100-150 | 7 6 | 4 | 1 |
| 125* | | | 2 | | 110 | {50}{30} | {30}{15} | 100† | 10 | 200 | {80}{125} | 5 10 | 6 | 2 |
| 120 | 56 | 7 | 2 | 10 | 50† | 18 | 10† | 52† | 10-25 | 150-300 | 100-150 | 5 5 | 7 | 1 |
| 100 | 65 | 0 | 2 | 10 | 50† | 18 | 10† | 50 | 10-25 | 200-250 | 100-150 | 5 5 | 8 | 1 |
| 80 | 74 | 0 | 2 | 10 | 40† | 18† | 10† | 40 | 10-25 | 200-250 | 100-150 | 5 10 1/2 | 9 | 1 |
| 50 | 129 | 11 1/4 | 1 | 15 | 50 | 25 | 7 1/2 | 50 | 10 | 100-150 | 80-100 | 8 6 | 10 | 3 |
| 50 | 125 | 10 | 1 | 15 | 50 | 25 | 7 1/2 | 50 | 10 | 100-150 | 80-100 | 8 6 | 11 | 3 |
| 50 | 121 | 2 | 1 | 5 | 75 | 15 | 15 | 75 | 11 1/2 | 225 | 125 | 8 4 | 12 | 2 |

* Four-girder ladle crane. † On each trolley.

‡ Divided equally between 2 motors for series-parallel control.

1. Pawling & Harnischfeger; 2. Alliance Mach. Co.; 3. Morgan Engineering Co.; 4. Midvale Steel Co., Phila.; 5. Homestead Steel Works, Munhall, Pa.; 6. Indiana Steel Co., Gary, Ind.; 7. Oregon Ry. & Nav. Co., Portland, Ore.; 8. El Paso & S. W. Ry., El Paso, Tex.; 9. C. & E. I. Ry., Danville, Ill.; 10. 3d Ave. Ry., N. Y. City; 11. United Rys. Co., Baltimore; 12. Carnegie Steel Co., Youngstown, Ohio.

A 150-ton Pillar-crane was erected in 1893 on Finnieston Quay, Glasgow. The jib is formed of two steel tubes, each 39 in. diam. and 90 ft. long. The radius of sweep for heavy lifts is 65 ft. The jib and its load are counterbalanced by a balance-box weighted with 100 tons of iron and steel punchings. In a test a 130-ton load was lifted at the rate of 4 ft. per minute, and a complete revolution made with this load in 5 minutes. *Eng'g News*, July 20, 1893.

Compressed-air Traveling-cranes.—Compressed-air overhead traveling-cranes have been built by the Lane & Bodley Co., of Cincinnati. They are of 20 tons nominal capacity, each about 50 ft. span and 400 ft. length of travel, and are of the triple-motor type, a pair of simple reversing-engines being used for each of the necessary operations, the pair of engines for the bridge and the pair for the trolley travel being each 5-inch bore by 7-inch stroke, while the pair for hoisting is 7-inch bore by 9-inch stroke. The air-pressure when required is somewhat over 100 pounds. The air-compressor is allowed to run continuously without a governor, the speed being regulated by the resistance of the air in a receiver. An auxiliary receiver is placed on each traveler, whose object is to provide a supply of air near the engines for immediate demands and independent of the hose connection. Some of the advantages said to be possessed by this type of crane are: simplicity; absence of all moving parts, excepting those required for a particular motion when that motion is in use; no danger from fire, leakage, electric shocks, or freezing; ease of repair; variable speeds and reversal without gearing; almost entire absence of noise; and moderate cost.

Quay-cranes.—An illustrated description of several varieties of stationary and traveling cranes, with results of experiments, is given in a paper on Quay-cranes in the Port of Hamburg by Chas. Nehls, *Trans. A. S. C. E.*, 1893.

Hydraulic Cranes, Accumulators, etc.—See Hydraulic Pressure Transmission, page 779, *ante*.

Electric versus Hydraulic Cranes for Docks.—A paper by V. L. Raven, in *Trans. A. S. M. E.*, 1904, describes some tests of capacity and

efficiency of electric and hydraulic power plants for dock purposes at Middlesbrough, Eng. In loading two cargoes of rails, weighing respectively 1210 and 1225 tons, the first was done with a hydraulic crane, in 7 hours, with 3584 lbs. of coal burned in the power station, and the second with an electric crane in 5¼ hours, with 2912 lbs. of coal. The total cost including labor, per 100 tons, was 327 pence with the hydraulic and 245 pence for the electric crane, a saving by the latter of 25 %.

Loading and Unloading and Storage Machinery for coal, ore, etc., is described by G. E. Titcomb in *Trans. A. S. M. E.*, 1908. The paper illustrates automatic ore unloaders for unloading ore from the hold of a vessel and loading it onto cars, and car-dumping machinery, by which a 50-ton car of coal is lifted, turned over and its contents discharged through a chute into a vessel. Methods of storage of coal and of re-loading it on cars are also described.

Power Required for Traveling-Cranes and Hoists. — Ulrich Peters, in *Machy*, Nov. 1907, develops a series of formulæ for the power required to hoist and to move trolleys on cranes. The following is a brief abstract. Resistance to be overcome in moving a trolley or crane-bridge. P_1 = rolling friction of trolley wheels, P_2 = journal friction of wheels or axles, P_3 = inertia of trolley and load. P = sum of these

resistances = $P_1 + P_2 + P_3 = (T + L) \left(\frac{f_1 + f_2 d}{D} + \frac{v}{1932 t} \right)$ in which T = weight

of trolley, L = load, f_1 = coeff. of rolling friction, about 0.002, (0.001 to 0.003 for cast iron on steel); f_2 = coeff. of journal friction, = 0.1 for starting and 0.01 for running, assuming a load on brasses of 1000 to 3000 lb. per sq. in.; [f_2 is more apt to be 0.05 unless the lubrication is perfect. See Friction and Lubrication, W. K.] d = diam. of journal; D = diam. of wheels; v = trolley speed in ft. per min.; t = time in seconds in which the trolley under full load is required to come to the maximum speed. Horse-power = sum of the resistances \times speed, ft. per min. \div 33,000.

Force required for hoisting and lowering: F_h = actual hoisting force, F_0 = theoretical force or pull, L = load, v = speed in ft. per min. of the rope or chain, c = hoisting speed of the load L , c/v = transmission ratio of the hoist, e = efficiency = F_0/F_h . The actual work to raise the load per minute = $F_h v = Lc = F_0 v \div e$. The efficiency e is the product of the efficiencies of all the several parts of the hoisting mechanism, such as sheaves, windlass, gearing, etc. Methods of calculating these efficiencies, with examples, are given at length in the original paper by Mr. Peters.

Lifting Magnets. — (From data furnished by the Electric Controller and Mfg. Co., Cleveland, and the Cutler-Hammer Clutch Co., Milwaukee). Lifting magnets first came into use about 1898. They have had wide application for handling pig iron, scrap, castings, etc. A lifting magnet comprises essentially a magnet winding, a pole-piece, a shoe and a protecting case, which is ribbed to afford ample radiating surface to dissipate the heat generated in operation. The winding usually consists of coils, each wound with copper ribbon and insulated with asbestos. The insulation must be designed to withstand a higher voltage than the line voltage, due to the inductive kick when the circuit is opened. The wearing plate, which takes the shocks incident to picking up the load, is usually made of manganese steel. The shape of the pole piece or lifting surface of the magnet must be varied, as the same shape is not usually applicable to all classes of materials. For handling pig iron, scrap, etc., a concave pole surface is usually superior to a flat one, which is adapted to handling plates or flat material of similar character, and which bear equally on the piece to be lifted at both the edge and center. A test of a lifting magnet made at the works of the Youngstown Sheet and Tube Co., in 1907, showed the following results:

Total pig iron unloaded, 109,350 pounds; weight of average lift, 785 pounds; time required, 2 hours, 15 minutes; current on magnet, 1 hour 15 minutes; current required, 30 amperes.

The No. 3 and No. 4 magnets are particularly fitted for use on steam-driven locomotive cranes, and when so used are usually supplied with current from a small steam-driven generator set mounted on the crane, steam being drawn from the boiler of the crane. Nos. 5 and 6 are adapted for use with overhead electric traveling cranes in cases where large lifts and high speed of handling are essential.

SIZES AND CAPACITIES OF THE ELECTRIC CONTROLLER & MFG. CO.'S
TYPE S-A LIFTING MAGNETS. (1909).

| Size. | Diam. | Weight. | Average current at 220 volts. | Lifts in machine cast pig iron. | |
|-------|-------|---------|-------------------------------------|------------------------------------|------------------|
| | | | | Maximum lift. | Average lift. |
| | In. | Lb. | Amp. | Lb. | Lb. |
| 3 | 36 | 2,100 | 11 | 1,405 | 750 |
| 4 | 43 | 3,200 | 27 | 2,180 | 1,250 |
| 5 | 52 | 4,800 | 35 | 3,087 | 1,800 |
| 6 | 61 | 6,600 | 45 | 4,589 | 2,600 |

SIZES AND CAPACITIES OF LIFTING MAGNETS (CUTLER-HAMMER), 1908.

| Sizes, in. | Weight lb. | Maximum* Lifting Capacity, lb. | Average Lifting Capacity, lb. | Current Required at 220 volts, amperes. | Head room required, ft. |
|---------------|---------------|---|--|--|-------------------------------|
| 10 | 75 | 800 | 100-300 | 1 | |
| 35 | 1,650 | 5,000 | 500-1,000 | 15-18 | 4 |
| 50 | 5,000 | 20,000 | 1,000-2,000 | 30-35 | 6 |

*This capacity can be obtained only under the most favorable conditions, with complete magnetic contact between the magnet and the piece to be lifted.

The capacity of a lifting magnet in service depends on many other factors than the design of the magnet. Most important is the character of the material handled. Much more can be handled at a single lift with material like billets, ingots, etc., than with scrap, wire, pig iron, etc. The speed of the crane, from which the magnet is suspended, and the distance it must transport the material are also important factors to be considered in calculating the capacity of a given magnet under given conditions. The following results have been selected from a great number of tests of the Electric Controller and Mfg. Co.'s No. 2 Type S magnets in commercial service, and represent what is probably average practice. It should be borne in mind that the average lift is determined from a large number of lifts, including lifts made from a full car of, say, pig iron, where the magnetic conditions are very favorable, and also the "lean" lifts where the car is nearly empty, and magnetic conditions unfavorable; the magnet can reach only a few pigs at one time on the lean lifts, with a consequent heavy decrease in the size of the load. The average lift is therefore less than the maximum lift in handling a given lot of material.

When operated from an ordinary electric overhead traveling crane a magnet of the type used in these trials will handle from 20 to 30 tons per hour of the scrap used by open-hearth furnaces. If operated from a special fast crane, the amount may be somewhat increased. Average lifts in pounds for various materials are as follows:

Skull cracker balls up to 20,000; ingot (or if ground man places magnet, two), each, 6,000; billet slabs, 900-6,000.

The above weights depend on dimensions and whether in pile or stacked evenly.

Machine cast pig iron, 1,250; sand cast pig iron, 1,150.

These are values obtained in unloading railway cars, including lean lifts in cleaning up.

Machine cast pig iron, 1,350; sand cast pig iron, 1,200.

The above are average lifts from stockpile.

Heavy melting stock (billets, crop ends of billets, rails or structural shapes, 1,250; boiler plate scrap, 1,100; farmers' scrap (harvesting machinery parts, plow points, etc.), 900; small risers from steel castings, 1,600; fine wire scrap, scrap tubing not over 3 ft. long, loose even or lamination scrap, 500; bundled scrap, 1,200; miscellaneous junk dealers' scrap, 400-800).

COMMERCIAL RESULTS WITH A 52-INCH, 5,000 POUND MAGNET.
(Electric Controller & Mfg. Co., 1908.)

| Hoist speed, ft. per min. | Crane. | | Distance moved. | | | Total Weight Moved, Tons. | No. of Lifts. | Aver. weight per lift, lbs. | Total time, Minutes. | Conditions of Working. |
|---------------------------|-----------------------------|----------------------------|-----------------|--------------|-------------|---------------------------|---------------|-----------------------------|----------------------|------------------------|
| | Trolley speed, ft. per min. | Bridge speed, ft. per min. | Hoist, Ft. | Trolley, Ft. | Bridge, Ft. | | | | | |
| 60 | 80 | 315 | 5 | 6 | 3 | 60 | 73 | 1,650 | 75 | 1* |
| 60 | 80 | 315 | 3 | 6 | 6 | 35 | 55 | 1,275 | 60 | 2 |
| 60 | 80 | 315 | 10 | 36 | 15 | 39.3 | 60 | 1,328 | 60 | 3 |
| 60 | 80 | 315 | 10 | 20 | 40 | 33.9 | 55 | 1,234 | 55 | 4 |
| 50 | 200 | 550 | 3 | 6 | 3 | 78. | 132 | 1,182 | 135 | 5 |
| 50 | 200 | 550 | 4 | 7 | 8 | 78 | 168 | 929 | 190 | 6 |
| 50 | 200 | 550 | 5 | 8 | 0 | 26 | 30 | 173 | 45 | 7 |
| 50 | 200 | 550 | 4 | 6 | 3 | 80 | 300 | 534 | 300 | 8 |
| 240 | 171 | 160 | 12 | 30 | 12 | 25 | 25 | 2,000 | 80 | 9 |
| 240 | 171 | 160 | 15 | 10 | 150 | 112 | 56 | 4,000 | 120 | 10 |
| 240 | 171 | 160 | 7 | 12 | 5 | 7 | 8 | 1,740 | 15 | 11 |
| 240 | 171 | 160 | 5 | 13 | 0 | 5 | 4 | 2,660 | 10 | 12 |

* 1. Machine cast pig handled from stock pile to charging boxes. 2. Bull heads, ditto. 3. Sand cast pig unloaded from car to stock pile. 4. Baled tin and wire unloaded from car to stock pile. 5. Boiler plate scrap handled from stock pile to charging boxes. 6. Farmers' scrap, comprising knotters and butters from threshing and binding machines, sections of cutter bars from mowers, broken steel teeth from hay rakes, plow points, etc., from stock pile to charging boxes. 7. Small risers from steel castings, handled from stock pile to charging boxes. 8. Laminated plates from armatures and transformers, mixed sizes, from stock pile to charging boxes. 9. Cast iron sewer pipe, 3 feet diameter, weighing 2,000 pounds each, lifted from cars to flat boat. Each pipe had to be blocked and lashed to prevent washing overboard. 10. Pennsylvania Railroad East River tunnel section castings, convex on one side, concave on other, weighing 4,000 pounds each. Handled from local float to barge for shipment. 11. Steel plate $\frac{1}{2}$ -inch \times 10 inches \times 6 feet 0 inches handled from car to float. 12. Steel rails, 40 pounds per yard, 25 feet long. Handled from car to lighter, about 8 rails per lift.

The above results of tests relate to the Electric Controller & Mfg. Co.'s No. 2 Type "S" magnet, 52 in. diameter and weighing 5200 lbs. and are the average of a large number of tests made at various plants between the years 1905 and 1908. This type of magnet is being superseded by the No. 4 Type S-A magnet which is 43 in. diameter, weighs 3200 lbs. and gives substantially the same average lift.

TELPERAGE.

Telpherage is a name given to a system of transporting materials in which the load is suspended from a trolley or small truck running on a cable or overhead rail, and in which the propelling force is obtained from an electric motor carried on the trolley. The trolley, with its motor, is called a "telpher." A historical and illustrated description of the system is given in a paper by C. M. Clark, in *Trans. A. I. E. E.*, 1902. A series of circulars of the United Telpherage Co., New York, show numerous illustrations of the system in operation for handling different classes of materials. Telpherage is especially applicable for moving packages in warehouses, on wharfs, etc. The moving machinery consists of the telpher or the conveying power, with accompanying trailers; the portable electric hoist or the vertical elevating power, and the carriers containing the load. Among the accessories are brakes, switches and controlling devices of many kinds.

An automatic line is controlled by terminal and intermediate switches which are operated by the men who do the loading and unloading, no additional labor being required. A non-automatic line necessitates a boy to accompany the telfer. The advisability of using the non-automatic rather than the automatic line is usually determined by the distance between stations.

COAL-HANDLING MACHINERY.

The following notes and tables are supplied by the Link-Belt Co. •

In large boiler-houses coal is usually delivered from hopper-cars into a track-hopper, about 10 feet wide and 12 to 16 feet long. A feeder set under the track-hopper feeds the coal at a regular rate to a crusher, which reduces it to a size suitable for stokers.

After crushing, the coal is elevated or conveyed to overhead storage-bins. Overhead storage is preferred for several reasons:

1. To avoid expensive wheeling of coal in case of a breakdown of the coal-handling machinery.
2. To avoid running the coal-handling machinery continuously.
3. Coal kept under cover indoors will not freeze in winter and clog the supply-spouts to the boilers.
4. It is often cheaper to store overhead than to use valuable ground-space adjacent to the boiler-house.
5. As distinguished from vault or outside hopper storage, it is cheaper to build steel bins and supports than masonry pits.

Weight of Overhead Bins. — Steel bins of approximately rectangular cross-section, say 10 X 10 feet, will weigh, exclusive of supports, about one-sixth as much as the contained coal. Larger bins, with sloping bottoms, may weigh one-eighth as much as the contained coal. Bag bottom bins of the Berquist type will weigh about one-twelfth as much as the contained coal, not including posts, and about one-ninth as much, including posts.

Supply-pipes from Bins. — The supply-pipes from overhead bins to the boiler-room floor, or to the stoker-hoppers, should not be less than 12 inches in diameter. They should be fitted at the top with a flanged casting and a cut-off gate, to permit removal of the pipe when the boilers are to be cleaned or repaired.

Types of Coal Elevators. — Coal elevators consist of buckets of various shapes attached to one or more strands of link-belt or chain, or to rubber belting. The buckets may either be attached continuously or at intervals. The various types are as follows:

Continuous bucket elevators consist usually of one strand of chain and two sprocket-wheels with buckets attached continuously to the chain. Each bucket after passing the head wheel acts as a chute to direct the flow from the next bucket. This type of elevator will handle the larger sizes of coal. It runs at slow speeds, usually from 90 to 175 feet per minute, and has a maximum capacity of about 120 tons per hour.

Centrifugal discharge elevators consist usually of a single strand of chain, with the buckets attached thereto at intervals. They are used to handle the smaller sizes of coal in small quantities. They run at high speeds, usually 34 to 40 revolutions of the head wheel per minute, and have a capacity up to 40 tons per hour.

Perfect discharge elevators consist of two strands of chain, with buckets at intervals between them. A pair of idlers set under the head wheels cause the buckets to be completely inverted, and to make a clean delivery into the chutes at the elevator head. This type of elevator is useful in handling material which tends to cling to the buckets. It runs at slow speeds, usually less than 150 feet per minute. The capacity depends on the size of the buckets.

Combined Elevators and Conveyors are of the following types:

Gravity discharge elevators, consisting of two strands of chain, with spaced V-shaped buckets fastened between them. After passing the head wheels the buckets act as conveyor-flights and convey the coal in a trough to any desired point. This is the cheapest type of combined elevator and conveyor, and is economical of power. A machine carrying 100 tons of coal per hour, in buckets 20 inches wide, 10 inches deep, and 24 inches long,

spaced 3 feet apart, requires 5 H.P. when loaded and $1\frac{1}{2}$ H.P. when empty for each 100 feet of horizontal run, and $\frac{1}{9}$ H.P. for each foot of vertical lift.

Rigid bucket-carriers consist of two strands of chain with a special bucket rigidly fastened between them. The buckets overlap and are so shaped that they will carry coal around three sides of a rectangle. The coal is carried to any desired point and is discharged by completely inverting the bucket over a turn-wheel.

Pivoted bucket-carriers consist of two strands of long pitch steel chain to which are attached, in a pivotal manner, large malleable iron or steel buckets so arranged that their adjacent lips are close together or overlap. Overlapping buckets require special devices for changing the lap at the corner turns. Carriers in which the buckets do not overlap should be fitted with auxiliary pans or buckets, arranged in such a manner as to catch the spill which falls between the lips at the loading point, and so shaped as to return the spill to the buckets at the corner turns. Pivoted bucket-carriers will carry coal around four sides of a rectangle, the buckets being dumped on the horizontal run by striking a cam suitably placed. Buckets for these carriers are usually of 2 ft. pitch, and range in width from 18 in. to 48 in. They run at low speeds, usually not over 50 ft. per minute, 40 ft. per minute being most usual. At the latter speed, the capacities when handling coal vary from 40 tons per hour for the 18 in. width to 120 tons for the 48 in. width. On account of the superior construction of these carriers and the slow speed at which they run, they are economical of power and durable. The rollers mounted on the chain joints are usually 6 in. diameter, but for severe duty 8-in. rollers are often used. It is usual to make these hollow to carry a quantity of oil for internal lubrication.

Coal Conveyors. — Coal conveyors are of four general types, viz., scraper or flight, bucket, screw, and belt conveyors.

The *flight conveyor* consists of a trough of any desired cross-section and a single or double strand of chain carrying scrapers or flights of approximately the same shape as the trough. The flights push the coal ahead of them in the trough to any desired point, where it is discharged through openings in the bottom of the trough.

For short, low-capacity conveyors, malleable link hook-joint chains are used. For heavier service, malleable pin-joint chains, steel link chains, or monobar, are required. For the heaviest service, two strands of steel link chain, usually with rollers, are used.

Flight conveyors are of three types: plain scraper, suspended flight, and roller flight.

In the plain scraper conveyor, the flight is suspended from the chain and drags along the bottom of the trough. It is of low first cost and is useful where noise of operation is not objectionable. It has a maximum capacity of about 30 tons per hour, and requires more power than either of the other two types of flight conveyors.

Suspended flight conveyors use one or two strands of chain. The flights are attached to cross-bars having wearing-shoes at each end. These wearing-shoes slide on angle-iron tracks on each side of the conveyor trough. The flights do not touch the trough at any point. This type of conveyor is used where quietness of operation is a consideration. It is of higher first cost than the plain scraper conveyor, but requires one-fourth less power for operation. It is economical up to a capacity of about 80 tons per hour.

The *roller flight conveyor* is similar to the suspended flight, except that the wearing-shoes are replaced by rollers. It is highest in first cost of all the flight conveyors, but has the advantages of low power consumption (one-half that of the scraper), low stress in chain, long life of chain, trough, and flights, and noiseless operation. It has an economical maximum capacity of about 120 tons per hour.

The following formula gives approximately the horse-power at the head wheel required to operate flight conveyors:

$$\text{H.P.} = (ATL + BWS) \div 1000.$$

T = tons of coal per hour; L = length of conveyor in feet, center to center; W = weight of chain, flights, and shoes (both runs) in pounds; S = speed in feet per minute; A and B constants depending on angle of incline from horizontal. See example below.

Values of A and B.

| Angle, Deg. | A | B | Angle, Deg. | A | B | Angle, Deg. | A | B |
|----------------|-------|------|----------------|------|-------|----------------|------|-------|
| 0 | 0.343 | 0.01 | 10 | 0.50 | 0.01 | 30 | 0.79 | 0.009 |
| 2 | 0.378 | 0.01 | 14 | 0.57 | 0.01 | 34 | 0.84 | 0.008 |
| 4 | 0.40 | 0.01 | 18 | 0.63 | 0.009 | 38 | 0.88 | 0.008 |
| 6 | 0.44 | 0.01 | 22 | 0.69 | 0.009 | 42 | 0.92 | 0.007 |
| 8 | 0.47 | 0.01 | 26 | 0.74 | 0.009 | 46 | 0.95 | 0.007 |

For suspended flight conveyors take *B* as 0.8, and for roller flights as 0.6, of the values given in the table.

Weight of Chain in Pounds per Foot.

| LINK-BELTING. | | | | | MONOBAR. | | | | | | | |
|---------------|------------------------------|-----|------|-----|---------------|---------------------------|------|------|------|-------|------|-------|
| Chain No. | Pitch of Flights, Inches. | | | | Chain No.* | Pitch of Flights, Inches. | | | | | | |
| | 12 | 18 | 24 | 36 | | 12 | 18 | 24 | 36 | 48 | 54 | 72 |
| 78 | 2.4 | 2.3 | 2.26 | 2.2 | 612 | 3.9 | ... | 3.6 | 3.5 | ... | ... | ... |
| 88 | 2.8 | 2.7 | 2.6 | 2.5 | 618 | ... | 3.0 | ... | 2.8 | ... | 2.7 | ... |
| 85 | 3.1 | 2.8 | 2.7 | 2.6 | 818 | ... | 5.7 | ... | 5.5 | ... | 5.3 | ... |
| 103 | 4.6 | 4.4 | 4.3 | 4.2 | 824 | ... | ... | 4.9 | ... | 4.7 | ... | 4.6 |
| 108 | 4.9 | 4.7 | 4.4 | 4.1 | 1018 | ... | 11.5 | ... | 10.7 | ... | 10.4 | ... |
| 110 | 5.6 | 5.2 | 4.9 | 4.7 | 1024 | ... | ... | 9.6 | ... | 9.07 | ... | 8.8 |
| 114 | 6.3 | 6.0 | 5.9 | 5.7 | 1224 | ... | ... | 14.7 | ... | 14.04 | ... | 13.8 |
| 122 | 8.1 | 7.7 | 7.4 | 7.2 | 1236 | ... | ... | ... | 11.8 | ... | ... | 11.34 |
| 124 | 8.9 | 8.4 | 8.2 | 7.9 | 1424 | ... | ... | 20.5 | ... | 19.7 | ... | 19.4 |

* In monobar the first one or two figures in the number of the chain denote the diameter of the chain in eighths of an inch. The last two figures denote the pitch in inches.

| PIN CHAINS. | | | | | ROLLER CHAINS. | | | | | | | |
|-------------|------------------------------|-----|-----|-----|----------------|---------------------------|-----|-----|-----|--|--|--|
| No. | Pitch of Flights, Inches. | | | | No. | Pitch of Flights, Inches. | | | | | | |
| | 12 | 18 | 24 | 36 | | 12 | 18 | 24 | 36 | | | |
| 720 | 5.9 | 5.6 | 5.4 | 5.3 | 1112 | 7.7 | 6.9 | 6.2 | 5.7 | | | |
| 730 | 6.9 | 6.6 | 6.4 | 6.3 | 1113 | 9.5 | 8.8 | 8.0 | 7.5 | | | |
| 825 | 9.6 | 9.3 | 9.1 | 8.9 | 1130 | 10.5 | 9.5 | 9.0 | 7.8 | | | |

Weight of Flights with Wearing-shoes and Bolts.

| Size, Inches. | Steel. | Malleable Iron. | Suspended Flights. | |
|---------------|--------|-----------------|--------------------|-------------|
| | | | Size. | Weight, Lb. |
| 4×10 | 3.5 | 4.3 | 6×14 | 12.37 |
| 4×12 | 3.9 | 4.7 | 8×19 | 15.55 |
| 5×10 | 4.1 | 5.2 | 10×24 | 25.57 |
| 5×12 | 4.6 | 5.7 | 10×30 | 29.37 |
| 5×15 | 5.8 | 5.9 | 10×36 | 33.17 |
| 6×18 | 8.1 | 9.2 | 10×42 | 34.97 |
| 8×18 | 10.1 | 12.7 | | |
| 8×20 | 11.0 | 13.4 | | |
| 8×24 | 12.6 | 14.4 | | |
| 10×24 | 15.2 | 17.4 | | |

EXAMPLE. — Required the H.P. for a monobar conveyor 200 ft. center to center carrying 100 tons of coal per hour, up a 10° incline at a speed of 100 feet per minute. Conveyor has No. 818 chain and 8×19 suspended flights, spaced 18 inches apart.

$$\text{H.P.} = \frac{0.5 \times 100 \times 200 + 0.008 (400 \times 5.7 + 267 \times 15.55) \times 100}{1000} = 15.15.$$

The following table shows the conveying capacities of various sizes of flights at 100 feet per minute in tons, of 2000 lb., per hour. The values are true for continuous feed only.

| Size of Flight. | Horizontal Conveyors. | | | | Inclined Conveyors. | | |
|-----------------|-----------------------|-------------------|-------------------|-------------------------|-------------------------------|-------------------------------|-------------------------------|
| | Flight Every 16". | Flight Every 18". | Flight Every 24". | Pounds Coal per Flight. | 10° Flights Every 24". | 20° Flights Every 24". | 30° Flights Every 24". |
| | Tons. | Tons. | Tons. | | Tons. | Tons. | Tons. |
| 6×14 | 69.75 | 62 | 46.5 | 31 | 40.5 | 31.5 | 22.5 |
| 8×19 | | 130 | 97.5 | 65 | 78 | 62 | 52 |
| 10×24 | | | 172.5 | 115 | 150 | 120 | 90 |
| 10×30 | | | 220 | 147 | 184 | 146 | 116 |
| 10×36 | | | 268 | 179 | 225 | 177 | 142 |
| 10×42 | | | 315 | 210 | 264 | 210 | 167 |

Bucket Conveyors. — Rigid bucket-carriers are used to convey large quantities of coal over a considerable distance when there is no intermediate point of discharge. These conveyors are made with two strands of steel roller chain. They are built to carry as much as 10 tons of coal per minute.

Screw Conveyors. — Screw conveyors consist of a helical steel flight, either in one piece or in sections, mounted on a pipe or shaft, and running in a steel or wooden trough. These conveyors are made from 4 to 18 inches in diameter, and in sections 8 to 12 feet long. The speed ranges from 20 to 60 revolutions per minute and the capacity from 10 to 30 tons of coal per hour. It is not advisable to use this type of conveyor for coal, as it will only handle the smaller sizes and the flights are very easily damaged by any foreign substance of unusual size or shape.

Belt Conveyors. — Rubber and cotton belt conveyors are used for handling coal, ore, sand, gravel etc., in all sizes. They combine a high carrying capacity with low power consumption.

In some cases the belt is flat, the material being fed to the belt at its center in a narrow stream. In the majority of cases, however, the belt is troughed by means of idler pulleys set at an angle from the horizontal and placed at intervals along the length of the belt. Rubber belts are often made more flexible for deep troughing by removing some of the layers of cotton from the belt and substituting therefor an extra thickness of rubber.

Belt conveyors may be used for elevating materials up to about 23° incline. On greater inclines the material slides back on the belt and spills. With many substances it is important to feed the belt steadily if the conveyor stands at or near the limiting angle. If the flow is interrupted the material may slide back on the belt.

Belt conveyors are run at any speed from 200 to 800 feet per minute, and are made in widths varying from 12 inches to 60 inches.

Capacity of Belt Conveyors in Tons of Coal per Hour.

| Width of Belt, Ins. | Velocity, Feet per Minute. | | | Width of Belt, ins. | Velocity, Feet per Minute. | | | | |
|---------------------|----------------------------|-----|-----|---------------------|----------------------------|-----|-----|-----|-----|
| | 300 | 350 | 400 | | 300 | 350 | 400 | 450 | 500 |
| 12 | 34 | | | 20 | 96 | 112 | 128 | | |
| 14 | 47 | | | 24 | 139 | 162 | 186 | | |
| 16 | 62 | 72 | 82 | 30 | 218 | 254 | 290 | 210 | |
| 18 | 78 | 91 | 104 | 36 | 315 | 368 | 420 | 472 | 520 |

For materials other than coal, the figures in the above table should be multiplied by the coefficients given in the table below:

| Material. | Coefficient. | Material. | Coefficient. |
|-------------------|--------------|----------------------|--------------|
| Ashes (damp)..... | 0.86 | Earth..... | 1.4 |
| Cement..... | 1.76 | Sand..... | 1.8 |
| Clay..... | 1.26 | Stone (crushed)..... | 2.0 |
| Coke..... | 0.60 | | |

Belt Conveyor Construction. (C. K. Baldwin, *Trans. A. S. M. E.*, 1908.) — The troughing idlers should be spaced as follows, depending on the weight of the material carried:

| | | | | |
|--------------|-----------|-----------|-----------|-----------|
| Belt width | 12-16 in. | 18-22 in. | 24-30 in. | 32-36 in. |
| Spacing, ft. | 4 1/2-5 | 4-4 1/2 | 3 1/2-4 | 3-3 1/2 |

The stress in the belt should not exceed 18 to 20 lb. per inch of width per ply with rubber belts. This may be increased about 20% with belts in which 28 oz. duck is used. Where the power required is small the stiffness of the belt fixes the number of plies. The minimum number of plies is as follows:

| | | | | |
|-----------------|-------|-------|-------|-------|
| Belt width, in. | 12-14 | 16-20 | 22-28 | 30-36 |
| Minimum plies | 3 | 4 | 5 | 6 |

Pulleys of small diameter should be avoided on heavy belts, or the constant bending of the belt under heavy stress will cause the friction to lose its hold and destroy the belt. In many cases it is advisable to cover the driving pulley with a rubber lagging to increase the tractive power, particularly in dusty places. The minimum size of driving pulleys to be used is shown in the table below.

Smallest Diameter of Driving Pulleys for Belt Conveyors.

| Width of Belt. | Diameter of Pulley. | Width of Belt. | Diameter of Pulley. | Width of Belt. | Diameter of Pulley. |
|----------------|---------------------|----------------|---------------------|----------------|---------------------|
| In. | In. | In. | In. | In. | In. |
| 12 | 16-18 | 22 | 20-30 | 32 | 30-36 |
| 14 | 16-18 | 24 | 24-30 | 34 | 30-42 |
| 16 | 20-24 | 26 | 24-30 | 36 | 30-48 |
| 18 | 20-24 | 28 | 24-30 | | |
| 20 | 20-24 | 30 | 30-36 | | |

Horse-power to Drive Belt Conveyors. (C. K. Baldwin, *Trans. A. S. M. E.*, 1908.) — The power required to drive a belt conveyor depends on a great variety of conditions, as the spacing of idlers, type of drive, thickness of belt, etc. In figuring the power required, the belt should run no faster than is necessary to carry the desired load. If it should be necessary to increase the speed, the load should be increased in proportion and the power figured accordingly.

For level conveyors

$$\text{H.P.} = C \times T \times L \div 1000.$$

For inclined conveyors

$$\text{H.P.} = (C \times T \times L \div 1000) + (T \times H \div 1000).$$

C = power constant from table below; T = load, tons per hour; L = length of conveyor, center to center, ft.; H = vertical height material is lifted, ft.; S = belt speed, ft. per minute; B = width of belt, in.

For each movable or fixed tripper add horse-power in column 3 of table. Add 20% to horse-power for each conveyor under 50 ft. long. Add 10% to horse-power for each conveyor between 50 ft. and 100 ft. long. The formulae above do not include gear friction, should the conveyor be gear-driven.

Constants for Formulæ Above.

| | 1 | 2 | 3 | 4 | 5 |
|----------------|---|--|--|------------------------|------------------------|
| Width of Belt. | C for Material Weighing from 25 Lb. to 75 Lb. per Cu. Ft. | C for Material Weighing from 75 Lb. to 125 Lb. per Cu. Ft. | H.P. Required for Each Movable or Fixed Tripper. | Minimum Plies of Belt. | Maximum Plies of Belt. |
| In. | | | | | |
| 12 | 0.234 | 0.147 | 1/2 | 3 | 4 |
| 14 | 0.226 | 0.143 | 1/2 | 3 | 4 |
| 16 | 0.220 | 0.140 | 3/4 | 4 | 5 |
| 18 | 0.209 | 0.138 | 1 | 4 | 5 |
| 20 | 0.205 | 0.136 | 1 1/4 | 4 | 6 |
| 22 | 0.199 | 0.133 | 1 1/2 | 5 | 6 |
| 24 | 0.195 | 0.131 | 1 3/4 | 5 | 7 |
| 26 | 0.187 | 0.127 | 2 | 5 | 7 |
| 28 | 0.175 | 0.121 | 2 1/4 | 5 | 8 |
| 30 | 0.167 | 0.117 | 2 1/2 | 6 | 8 |
| 32 | 0.163 | 0.115 | 2 3/4 | 6 | 9 |
| 34 | 0.161 | 0.114 | 3 | 6 | 10 |
| 36 | 0.157 | 0.112 | 3 1/4 | 6 | 10 |

When horse-power and speed are known the stress in the belt in pounds per inch of width is

$$\text{Stress} = \frac{\text{H.P.} \times 33,000}{S \times B}$$

From this the number of plies can be found, using 20 lb. per ply per inch of width as a maximum for rubber belts.

Relative Wearing Power of Conveyor Belts. (T. A. Bennett, *Trans. A. S. M. E.*, 1908.) — Different materials used in the construction of conveyors were subjected to the uniform action of a sand blast for 45 minutes, and the relative abrasive resisting qualities were found to be as follows, taking the volume of rubber belt worn away as 1.0:

| | | | |
|----------------------------------|-----|-------------------------------|-------------|
| Rubber belt..... | 1.0 | Woven cotton belt, high grade | 6.5 |
| Rolled steel bar..... | 1.5 | Stitched duck, high grade | 8.0 |
| Cast iron..... | 3.5 | Woven cotton belt, low grade, | 9.0 to 15.0 |
| Balata belt, including gum cover | 5.0 | | |

A *Symposium on Hoisting and Conveying* was presented at the Detroit meeting of the A. S. M. E., 1908 (*Trans.*, vol. xxx), in papers by G. E. Titcomb, S. B. Peck, C. K. Baldwin, C. J. Tomlinson and E. J. Haddock. Among the subjects discussed are the loading and unloading of cargo steamers; car unloaders; storing of ore and coal; continuous conveying of merchandise; conveying in a Portland cement plant, and suspension cableways.

WIRE-ROPE HAULAGE.

Methods for transporting coal and other products by means of wire rope, though varying from each other in detail, may be grouped in five classes:

- I. The Self-acting or Gravity Inclined Plane.
- II. The Simple Engine-plane.
- III. The Tail-rope System.
- IV. The Endless-rope System.
- V. The Cable Tramway.

The following brief description of these systems is abridged from a pamphlet on Wire-rope Haulage, by Wm. Hildenbrand, C.E., published by John A. Roebling's Sons Co., Trenton, N. J.

I. The Self-acting Inclined Plane. — The motive power for the self-acting inclined plane is gravity; consequently this mode of transporting coal finds application only in places where the coal is conveyed from a higher to a lower point and where the plane has sufficient grade for the loaded descending cars to raise the empty cars to an upper level.

At the head of the plane there is a drum, which is generally constructed of wood, having a diameter of seven to ten feet. It is placed high enough to allow men and cars to pass under it. Loaded cars coming from the pit are either singly or in sets of two or three switched on the track of the plane, and their speed in descending is regulated by a brake on the drum.

Supporting rollers, to prevent the rope dragging on the ground, are generally of wood, 5 to 6 in. in diameter and 18 to 24 in. long, with $\frac{3}{4}$ to $\frac{7}{8}$ in. iron axles. The distance between the rollers varies from 15 to 30 ft., steeper planes requiring less rollers than those with easy grades. Considering only the reduction of friction and what is best for the preservation of rope, a general rule may be given to use rollers of the greatest possible diameter, and to place them as close as economy will permit.

The smallest angle of inclination at which a plane can be made self-acting will be when the motive and resisting forces balance each other. The motive forces are the weights of the loaded car and of the descending rope. The resisting forces consist of the weight of the empty car and ascending rope, of the rolling and axle friction of the cars, and of the axle friction of the supporting rollers. The friction of the drum, stiffness of rope, and resistance of air may be neglected. A general rule cannot be given, because a change in the length of the plane or in the weight of the cars changes the proportion of the forces; also, because the coefficient of friction, depending on the condition of the road, construction of the cars, etc., is a very uncertain factor.

For working a plane with a $\frac{5}{8}$ -in. steel rope and lowering from one to four pit cars weighing empty 1400 lb. and loaded 4000 lb., the rise in 100 ft. necessary to make the plane self-acting will be from about 5 to 10 ft., decreasing as the number of cars increase, and increasing as the length of plane increases.

A gravity inclined plane should be slightly concave, steeper at the top than at the bottom. The maximum deflection of the curve should be at an inclination of 45 degrees, and diminish for smaller as well as for steeper inclinations.

II. The Simple Engine-plane. — The name "Engine-plane" is given to a plane on which a load is raised or lowered by means of a single wire rope and stationary steam-engine. It is a cheap and simple method of conveying coal underground, and therefore is applied wherever circumstances permit it. Under ordinary conditions such as prevail in the Pennsylvania mine region, a train of twenty-five to thirty loaded cars will descend, with reasonable velocity, a straight plane 5000 ft. long on a grade of $1\frac{3}{4}$ ft. in 100, while it would appear that $2\frac{1}{4}$ ft. in 100 is necessary for the same number of empty cars. For roads longer than 5000 ft. or containing sharp curves, the grade should be correspondingly larger.

III. The Tail-rope System. — Of all methods for conveying coal underground by wire rope, the tail-rope system has found the most application. It can be applied under almost any condition. The road may be straight or curved, level or undulating, in one continuous line or with side branches. In general principle a tail-rope plane is the same as an engine-plane worked in both directions with two ropes. One rope, called the "main rope," serves for drawing the set of full cars outward; the other, called the "tail-rope," is necessary to take back the empty set, which on a level or undulating road cannot return by gravity. The two drums may be located at the opposite ends of the road, and driven by separate engines, but more frequently they are on the same shaft at one end of the plane. In the first case each rope would require the length of the plane, but in the second case the tail rope must be twice as long, being led from the drum around a sheave at the other end of the plane and back again to its starting-point. When the main rope draws a set of full cars out, the tail-rope drum runs loose on the shaft, and the rope, being attached to the rear car, unwinds itself steadily. Going in, the reverse takes place. Each drum is provided with a brake to check the speed of the train on a down grade and prevent its overrunning the forward rope. As a rule, the tail rope is strained less than the main rope, but in cases of heavy grades dipping outward it is possible that the strain in the former may become as large, or even larger, than in the latter, and in the selection of the sizes reference should be had to this circumstance.

IV. The Endless-rope System. — The principal features of this system are as follows:

1. The rope, as the name indicates, is endless.
2. Motion is given to

the rope by a single wheel or drum, and friction is obtained either by a grip-wheel or by passing the rope several times around the wheel. 3. The rope must be kept constantly tight, the tension to be produced by artificial means. It is done in placing either the return-wheel or an extra tension wheel on a carriage and connecting it with a weight hanging over a pulley, or attaching it to a fixed post by a screw which occasionally can be shortened. 4. The cars are attached to the rope by a grip or clutch, which can take hold at any place and let go again, starting and stopping the train at will, without stopping the engine or the motion of the rope. 5. On a single-track road the rope works forward and backward, but on a double track it is possible to run it always in the same direction, the full cars going on one track and the empty cars on the other.

This method of conveying coal, as a rule, has not found as general an introduction as the tail-rope system, probably because its efficacy is not so apparent and the opposing difficulties require greater mechanical skill and more complicated appliances. Its advantages are, first, that it requires one-third less rope than the tail-rope system. This advantage, however, is partially counterbalanced by the circumstance that the extra tension in the rope requires a heavier size to move the same load than when a main and tail rope are used. The second and principal advantage is that it is possible to start and stop trains at will without signaling to the engineer. On the other hand, it is more difficult to work curves with the endless system, and still more so to work different branches, and the constant stretch of the rope under tension or its elongation under changes of temperature frequently causes the rope to slip on the wheel, in spite of every attention, causing delay in the transportation and injury to the rope.

Stress in Hoisting-ropes on Inclined Planes.

(Trenton Iron Co., 1906.)

| Rise per 100 Ft. Horizontal. | Angle of Inclination. | Stress in Lb. per Ton of 2000 Lb. | Rise per 100 Ft. Horizontal. | Angle of Inclination. | Stress in Lb. per Ton of 2000 Lb. | Rise per 100 Ft. Horizontal. | Angle of Inclination. | Stress in Lb. per Ton of 2000 Lb. |
|---------------------------------|-----------------------|---|---------------------------------|-----------------------|---|---------------------------------|-----------------------|---|
| Ft. | | | Ft. | | | Ft. | | |
| 5 | 2° 52' | 140 | 55 | 28° 49' | 1003 | 110 | 47° 44' | 1516 |
| 10 | 5° 43' | 240 | 60 | 30° 58' | 1067 | 120 | 50° 12' | 1573 |
| 15 | 8° 32' | 336 | 65 | 33° 02' | 1128 | 130 | 52° 26' | 1620 |
| 20 | 11° 10' | 432 | 70 | 35° 00' | 1185 | 140 | 54° 28' | 1663 |
| 25 | 14° 03' | 527 | 75 | 36° 53' | 1238 | 150 | 56° 19' | 1699 |
| 30 | 16° 42' | 613 | 80 | 38° 40' | 1287 | 160 | 58° 00' | 1730 |
| 35 | 19° 18' | 700 | 85 | 40° 22' | 1332 | 170 | 59° 33' | 1758 |
| 40 | 21° 49' | 782 | 90 | 42° 00' | 1375 | 180 | 60° 57' | 1782 |
| 45 | 24° 14' | 860 | 95 | 43° 32' | 1415 | 190 | 62° 15' | 1804 |
| 50 | 26° 34' | 933 | 100 | 45° 00' | 1450 | 200 | 63° 27' | 1822 |

The above table is based on an allowance of 40 lb. per ton for rolling friction, but an additional allowance must be made for stress due to the weight of the rope proportional to the length of the plane. A factor of safety of 5 to 7 should be taken.

In hoisting the slack-rope should be taken up gently before beginning the lift, otherwise a severe extra strain will be brought on the rope.

V. Wire-rope Tramways. — The methods of conveying products on a suspended rope tramway find especial application in places where a mine is located on one side of a river or deep ravine and the loading station on the other. A wire rope suspended between the two stations forms the track on which material in properly constructed "carriages" or "buggies" is transported. It saves the construction of a bridge or trestlework and is practical for a distance of 2000 feet without an intermediate support.

There are two distinct classes of rope tramways:

1. The rope is stationary, forming the track on which a bucket holding the material moves forward and backward, pulled by a smaller endless wire rope. 2. The rope is movable, forming itself an endless line, which serves at the same time as supporting track and as pulling rope.

Of these two the first method has found more general application, and is especially adapted for long spans, steep inclinations, and heavy loads. The second method is used for long distances, divided into short spans, and is only applicable for light loads which are to be delivered at regular intervals.

For detailed descriptions of the several systems of wire-rope transportation, see circulars of John A. Roebling's Sons Co., The Trenton Iron Co., A. Leschen & Sons Rope Co. See also paper on Two-rope Haulage Systems, by R. Van A. Norris, *Trans. A. S. M. E.*, xii. 626.

In the *Bleichert System* of wire-rope tramways, in which the track rope is stationary, loads up to 2000 lb. are carried at a speed of 3 to 4 miles per hour. While the average spans on a level are from 150 to 200 ft., in crossing rivers, ravines, etc., spans up to 1500 ft. are frequently adopted. In a tramway on this system at Bingham, Utah, the total length of the line is 12,700 ft. with a fall of 1120 ft. The line operates by gravity and carries 35 tons per hour. The cost of conveying on this carrier is $7\frac{3}{4}$ cents per ton of 2000 lb. for labor and repairs, without any apparent deterioration in the condition of track cables and traction rope.

The *Aerial Wire-rope Tramway* of A. Leschen & Sons Co. is of the double-rope type, in which the buckets travel upon stationary track cables and are propelled by an endless traction rope. The buckets are attached to the traction rope by means of clips — spaced according to the desired tonnage. The hold on the rope is positive, but the clip is easily removable. The bucket is held in its normal position in the frame by two malleable iron latches — one on each side. A tripping bar engages these latches at the unloading terminal when the bucket discharges its material. This operation is automatic and takes place while the carriers are moving. At the loading terminal, the bucket is automatically returned to its normal position and latched. Special carriers are provided for the accommodation of any class of material. At each of the terminal stations is a 10-ft. sheave wheel around which the traction rope passes, these wheels being provided with steel grids for the control of the traction rope. When the loaded carriers travel down grade and the difference in elevation is sufficient, this tramway will operate by the force due to gravity, otherwise the power is applied to the sheaves through bevel gearing. Numerous modifications of the system are in use to suit different conditions.

An *Aerial Tramway 21.5 miles long*, with an elevation of the loading end above the discharging end of 11,500 ft., built by A. Bleichert & Co. for the government of the Argentine Republic, connecting the mines of La Mejicana with the town of Chilecito, is described by Wm. Hewitt in *Indust. Eng.*, Aug. 15, 1909. Some of the inclinations are as much as 45 deg., there are some spans nearly 3000 ft. long, and there is a tunnel nearly 500 ft. long. The line is divided into eight sections, each with an independent traction rope. The gravity of the descending loaded carriers is sufficient to make the line self-operating when it is once set in motion, but in order to ensure full control, and to provide for carrying four tons upward while the descending carriers are empty, four steam engines are installed, one for each two sections. The carriers hold 10 cu. ft., or about 1100 lbs. of ore. The speed is 500 ft. per minute, and the interval between carriers 45 seconds. The stress in the traction rope is as high as 11,000 lbs. in some sections.

General Formulæ for Estimating the Deflection of a Wire Cable Corresponding to a Given Tension.

(Trenton Iron Co., 1906.)

Let s = distance between supports or span AB ; m and n = arms into which the span is divided by a vertical through the required point of deflection x , m representing the arm corresponding to the loaded side; y = horizontal distance from load to point of support corresponding with m ; w = wt. of rope per ft.; g = load; t = tension; h = required deflection at any point x ; all measures being in feet and pounds.

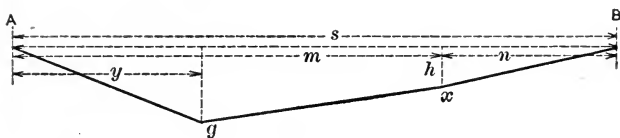


FIG. 184.

For deflection due to rope alone,

$$h = \frac{mnw}{2t} \text{ at } x, \text{ or } \frac{ws^2}{8t} \text{ at center of span.}$$

For deflection due to load alone,

$$h = \frac{gny}{ts} \text{ at } x, \text{ or } \frac{gy}{2t} \text{ at center of span.}$$

$$\text{If } y = \frac{1}{2}s, h = \frac{gn}{2t} \text{ at } x, \text{ or } \frac{gs}{4t} \text{ at center of span.}$$

$$\text{If } y = m, h = \frac{gmn}{ts} \text{ at } x, \text{ or } \frac{gs}{4t} \text{ at center of span.}$$

For total deflection,

$$h = \frac{wmns + 2gny}{2ts} \text{ at } x, \text{ or } \frac{ws^2 + 4gy}{8t} \text{ at center of span.}$$

$$\text{If } y = \frac{1}{2}s, h = \frac{wmn + gn}{2t} \text{ at } x, \text{ or } \frac{ws^2 + 2gs}{8t} \text{ at center of span.}$$

$$\text{If } y = m, h = \frac{wmns + 2gmn}{2ts} \text{ at } x, \text{ or } \frac{ws^2 + 2gs}{8t} \text{ at center of span.}$$

If the tension is required for a given deflection, transpose t and h in above formulæ.

Suspension Cableways or Cable Hoist-conveyors.

(Trenton Iron Co.)

In quarrying, rock-cutting, stripping, piling, dam-building, and many other operations where it is necessary to hoist and convey large individual loads economically, it frequently happens that the application of a system of derricks is impracticable, by reason of the limited area of their efficiency and the room which they occupy. To meet such conditions cable hoist-conveyors are adopted, as they can be operated in clear spans up to 1500 ft., and in lifting individual loads up to 15 tons. Two types are made — one in which the hoisting and conveying are done by separate running ropes, and the other applicable only to inclines in which the carriage descends by gravity, and but one running rope is required. The moving of the carriage in the former is effected by means of an endless rope, and these are commonly known as "endless-rope" hoist-conveyors to distinguish them from the latter, which are termed "inclined" hoist-conveyors.

The general arrangement of the endless-rope hoist-conveyors consists of a main cable passing over towers, A-frames or masts, as may be most convenient, and anchored firmly to the ground at each end, the requisite tension in the cable being maintained by a turnbuckle at one anchorage.

Upon this cable travels the carriage, which is moved back and forth over the line by means of the endless rope. The hoisting is done by a separate rope, both ropes being operated by an engine specially designed for the purpose, which may be located at either end of the line, and is constructed in such a way that the hoisting-rope is coiled up or paid out automatically as the carriage is moved in and out. Loads may be picked up or discharged at any point along the line. Where sufficient inclination can be obtained in the main cable for the carriage to descend by gravity, and the loading and unloading are done at fixed points, the endless rope can be dispensed with. The carriage, which is similar in construction to the carriage used in the endless-rope cableways, is arrested in its descent by a

stop-block, which may be clamped to the main cable at any desired point, the speed of the descending carriage being under control of a brake on the engine-drum.

A *Double-suspension Cableway*, carrying loads of 15 tons, erected near Williamsport, Pa., by the Trenton Iron Co., is described by E. G. Spillsbury in *Trans. A. I. M. E.*, xx. 766. The span is 733 ft., crossing the Susquehanna River. Two steel cables, each 2 in. diam., are used. On these cables runs a carriage supported on four wheels and moved by an endless cable 1 inch in diam. The load consists of a cage carrying a railroad-car loaded with lumber, the latter weighing about 12 tons. The power is furnished by a 50-H.P. engine, and the trip across the river is made in about three minutes.

A hoisting cableway on the endless-rope system, erected by the Lidgerwood Mfg. Co., at the Austin Dam, Texas, had a single span 1350 ft. in length, with main cable $2\frac{1}{2}$ in. diam., and hoisting-rope $1\frac{3}{4}$ in. diam. Loads of 7 to 8 tons were handled at a speed of 600 to 800 ft. per minute.

Another, of still longer span, 1650 ft., was erected by the same company at Holyoke, Mass., for use in the construction of a dam. The main cable is the Elliott or locked-wire cable, having a smooth exterior. In the construction of the Chicago Drainage Canal twenty cableways, of 700 ft. span and 8 tons capacity, were used, the towers traveling on rails.

Tension required to Prevent Slipping of Rope on Drum. (Trenton Iron Co., 1906.)—The amount of artificial tension to be applied in an endless rope to prevent slipping on the driving-drum depends on the character of the drum, the condition of the rope and number of laps which it makes. If T and S represent respectively the tensions in the taut and slack lines of the rope; W , the necessary weight to be applied to the tail-shackle; R , the resistance of the cars and rope, allowing for friction; n , the number of half-laps of the rope on the driving-drum; and f , the coefficient of friction, the following relations must exist to prevent slipping:

$$T = Se^{fn\pi}, \quad W = T + S, \quad \text{and} \quad R = T - S;$$

$$\text{from which we obtain} \quad W = \frac{e^{fn\pi} + 1}{e^{fn\pi} - 1} R,$$

in which $e = 2.71828$, the base of the Napierian system of logarithms.

The following are some of the values of f :

| | Dry. | Wet. | Greasy. |
|---|-------|-------|---------|
| Wire-rope on a grooved iron drum. . . . | 0.120 | 0.085 | 0.070 |
| Wire-rope on wood-filled sheaves. | 0.235 | 0.170 | 0.140 |
| Wire-rope on rubber and leather filling | 0.495 | 0.400 | 0.205 |

The importance of keeping the rope dry is evident from these figures.

The values of the coefficient $\frac{e^{fn\pi} + 1}{e^{fn\pi} - 1}$, corresponding to the above values of f , for one up to six half-laps of the rope on the driving-drum or sheaves, are as follows:

| f | $n = \text{Number of Half-laps on Driving-wheel.}$ | | | | | |
|-------|--|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 0.070 | 9.130 | 4.623 | 3.141 | 2.418 | 1.999 | 1.729 |
| 0.085 | 7.536 | 3.833 | 2.629 | 2.047 | 1.714 | 1.505 |
| 0.120 | 5.345 | 2.777 | 1.953 | 1.570 | 1.358 | 1.232 |
| 0.140 | 4.623 | 2.418 | 1.729 | 1.416 | 1.249 | 1.154 |
| 0.170 | 3.833 | 2.047 | 1.505 | 1.268 | 1.149 | 1.085 |
| 0.205 | 3.212 | 1.762 | 1.338 | 1.165 | 1.083 | 1.043 |
| 0.235 | 2.831 | 1.592 | 1.245 | 1.110 | 1.051 | 1.024 |
| 0.400 | 1.795 | 1.176 | 1.047 | 1.013 | 1.004 | 1.001 |
| 0.495 | 1.538 | 1.093 | 1.019 | 1.004 | 1.001 | |

When the rope is at rest the tension is distributed equally on the two lines of the rope, but when running there will be a difference in the tensions of the taut and slack lines equal to the resistance, and the values of T and S may be readily computed from the foregoing formulæ.

The increase in tension in the endless rope, compared with the main rope of the tail-rope system, where the stress in the rope is equal to the resistance, is about as follows:

| $n =$ | 1 | 2 | 3 | 4 | 5 | 6 |
|--|----|---|----------------|---------------|---------------|----------------|
| Increase in tension in endless rope, compared with direct stress %..... | 40 | 9 | $2\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{5}$ | $\frac{1}{10}$ |

These figures are useful in determining the size of rope. For instance, if the rope makes two half-laps on the driving drum, the strength of the rope should be 9% greater than a main rope in the tail-rope system.

Taper Ropes of Uniform Tensile Strength. — The true form of rope is not a regular taper but follows a logarithmic curve, the girth rapidly increasing toward the upper end. Mr. Chas. D. West gives the following formula, based on a breaking strain of 80,000 lb. per sq. in. of the rope, core included, and a factor of safety of 10: $\log G = F + 3680 + \log g$, in which F = length in fathoms, and G and g the girth in inches at any two sections F fathoms apart. The girth g is first calculated for a safe strain of 8000 lb. per sq. in., and then G is obtained by the formula. For a mathematical investigation see *The Engineer*, April, 1880, p. 267.

TRANSMISSION OF POWER BY WIRE ROPE.

The following notes have been furnished to the author by Mr. Wm. Hewitt, Vice-President of the Trenton Iron Co. (See also circulars of the Trenton Iron Co. and of the John A. Roebling's Sons Co., Trenton, N. J.; "Transmission of Power by Wire Ropes," by A. W. Stahl, Van Nostrand's Science Series, No. 28; and Reuleaux's Constructor.)

The load stress or working tension should not exceed the difference between the safe stress and the bending stress as determined by the table on page 1185.

The approximate strength of iron-wire rope composed of wires having a tensile strength of 75,000 to 90,000 lbs. per sq. in. is half that of cast-steel rope composed of wires of a tensile strength of 150,000 to 190,000 lbs. per sq. in. Extra strong steel wires have a tensile strength of 190,000 to 225,000 and plow-steel wires 225,000 to 275,000 lbs. per sq. in.

The 19-wire rope is more flexible than the 7-wire, and for the same load stress may be run around smaller sheaves, but it is not as well adapted to withstand abrasion or surface wear.

The working tension may be greater, therefore, as the bending stress is less; but since the tension in the slack portion of the rope cannot be less than a certain proportion of the tension in the taut portion, to avoid slipping, a ratio exists between the diameter of sheave and the wires composing the rope corresponding to a maximum safe working tension. This ratio depends upon the number of laps that the rope makes about the sheaves, and the kind of filling in the rims or the character of the material upon which the rope tracks.

For ordinary purposes the maximum safe stress should be about one-third the ultimate, and for shafts and elevators about one-fourth the ultimate. In estimating the stress due to the load for shafts and elevators allowance should be made for the additional stress due to acceleration in starting. For short inclined planes not used for passengers a factor of safety as low as $2\frac{1}{2}$ is sometimes used, and for derricks, in which large sheaves cannot be used, and long life of the rope is not expected, the factor of safety may be as low as 2.

The Seale wire rope is made of six strands of 19 wires, laid 9 around 9 around 1, the intermediate layer being smaller than the others. It is intermediate in flexibility between the 7-wire and the ordinary 19-wire rope.

Approximate Breaking Strength of Steel-Wire Ropes.

| 6 strands of 19 wires each. | | | | | 6 strands of 7 wires each. | | | | |
|-----------------------------|----------------------------|--------------------------------------|---------------------------|----------------|----------------------------|----------------------------|--------------------------------------|---------------------------|----------------|
| Diam. Rope, In. | Wt. per ft., lbs. | Approximate breaking stress, lbs. | | | Diam. Rope, In. | Wt. per ft., lbs. | Approximate breaking stress, lbs. | | |
| | | Cast steel. | Extra strong steel. | Plow steel. | | | Cast steel. | Extra strong steel. | Plow steel. |
| 2 1/4 | 8.00 | 312,000 | 364,000 | 416,000 | 1 1/2 | 3.55 | 136,000 | 158,000 | 182,000 |
| 2 | 6.30 | 248,000 | 288,000 | 330,000 | 1 3/8 | 3.00 | 116,000 | 136,000 | 156,000 |
| 1 3/4 | 4.85 | 192,000 | 224,000 | 256,000 | 1 1/4 | 2.45 | 96,000 | 112,000 | 128,000 |
| 1 1/8 | 4.15 | 168,000 | 194,000 | 222,000 | 1 1/8 | 2.00 | 80,000 | 92,000 | 106,000 |
| 1 1/2 | 3.55 | 144,000 | 168,000 | 192,000 | 1 | 1.58 | 64,000 | 74,000 | 84,000 |
| 1 3/8 | 3.00 | 124,000 | 144,000 | 164,000 | 7/8 | 1.20 | 48,000 | 56,000 | 64,000 |
| 1 1/4 | 2.45 | 100,000 | 116,000 | 134,000 | 3/4 | 0.89 | 37,200 | 42,000 | 48,000 |
| 1 1/8 | 2.00 | 84,000 | 98,000 | 112,000 | 11/16 | 0.75 | 31,600 | 36,800 | 42,000 |
| 1 | 1.58 | 68,000 | 78,000 | 88,000 | 5/8 | 0.62 | 26,400 | 30,200 | 34,000 |
| 7/8 | 1.20 | 52,000 | 60,000 | 68,000 | 9/16 | 0.50 | 21,200 | 24,600 | 28,000 |
| 1/4 | 0.89 | 38,800 | 44,000 | 50,000 | 1/2 | 0.39 | 16,800 | 19,400 | 22,000 |
| 5/8 | 0.62 | 27,200 | 31,600 | 36,000 | 7/16 | 0.30 | 13,200 | 15,000 | 17,100 |
| 9/16 | 0.50 | 22,000 | 25,400 | 29,000 | 3/8 | 0.22 | 9,600 | 11,160 | 12,700 |
| 1/2 | 0.39 | 17,600 | 20,200 | 22,800 | 5/16 | 0.15 | 6,800 | 7,760 | |
| 7/16 | 0.30 | 13,600 | 15,600 | 17,700 | 9/32 | 0.125 | 5,600 | 6,440 | |
| 3/8 | 0.22 | 10,000 | 11,500 | 13,100 | | | | | |
| 5/16 | 0.15 | 6,800 | 8,100 | | | | | | |
| 1/4 | 0.10 | 4,800 | 5,400 | | | | | | |

The sheaves (Fig. 185) are usually of cast iron, and are made as light as possible consistent with the requisite strength. Various materials

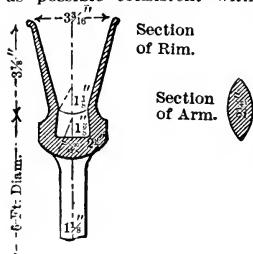


FIG. 185.

have been used for filling the bottom of the groove, such as tarred oakum, jute yarn, hard wood, India-rubber, and leather. The filling which gives the best satisfaction, however, in ordinary transmissions consists of segments of leather and blocks of India-rubber soaked in tar and packed alternately in the groove. Where the working tension is very great, however, the wood filling is to be preferred, as in the case of long-distance transmissions where the rope makes several laps about the sheaves, and is run at a comparatively slow speed.

The Bending Stress is determined by the formula

$$k = \frac{Ea}{2.06 (R + d) + C}$$

k = bending stress in lbs.; E = modulus of elasticity = 28,500,000; a = aggregate area of wires, sq. ins.; R = radius of bend; d = diam. of wires, ins.

For 7-wire rope $d = 1/9$ diam. of rope; $C = 9.27$.

" 19-wire " $d = 1/15$ " " " ; $C = 15.45$.

" the Seale cable $d = 1/12$ " " " ; $C = 12.36$.

From this formula the tables below have been calculated.

Bending Stresses, 7-wire Rope.

| Diam. bend. | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 |
|-------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Diam. Rope. | | | | | | | | | | |
| 1/4 | 826 | 553 | 412 | 333 | 277 | 238 | 208 | 185 | 166 | 151 |
| 9/32 | 1,120 | 750 | 563 | 451 | 376 | 323 | 282 | 251 | 226 | 206 |
| 5/16 | 1,609 | 1,078 | 810 | 649 | 541 | 464 | 406 | 361 | 325 | 296 |
| 3/8 | 2,774 | 1,859 | 1,398 | 1,120 | 934 | 801 | 702 | 624 | 562 | 511 |
| 7/16 | 4,385 | 2,982 | 2,217 | 1,777 | 1,482 | 1,272 | 1,113 | 990 | 892 | 811 |
| 1/2 | 6,200 | 4,161 | 3,131 | 2,510 | 2,095 | 1,797 | 1,574 | 1,400 | 1,260 | 1,146 |
| 9/16 | 9,072 | 6,095 | 4,589 | 3,679 | 3,071 | 2,635 | 2,308 | 2,053 | 1,848 | 1,681 |
| 5/8 | | 8,547 | 6,438 | 5,164 | 4,310 | 3,699 | 3,240 | 2,882 | 2,595 | 2,360 |
| 11/16 | | 10,922 | 8,230 | 6,603 | 5,513 | 4,731 | 4,144 | 3,687 | 3,320 | 3,020 |
| 3/4 | | 14,202 | 10,706 | 8,591 | 7,174 | 6,158 | 5,394 | 4,799 | 4,322 | 3,931 |
| 7/8 | | 22,592 | 17,045 | 13,685 | 11,431 | 9,815 | 8,599 | 7,651 | 6,892 | 6,269 |
| 1 | | | 25,476 | 20,464 | 17,100 | 14,686 | 12,869 | 11,452 | 10,317 | 9,386 |
| 1 1/8 | | | 36,289 | 29,165 | 24,416 | 20,942 | 18,355 | 16,336 | 14,718 | 13,391 |
| 1 1/4 | | | | 40,020 | 33,464 | 28,754 | 25,206 | 22,437 | 20,216 | 18,396 |
| 1 3/8 | | | | | 44,551 | 38,290 | 33,571 | 29,888 | 26,933 | 24,510 |
| 1 1/2 | | | | | 57,835 | 49,718 | 43,599 | 38,821 | 34,987 | 31,842 |

Bending Stresses, 19-wire Rope.

| Diam. Bend. | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
|-------------|-------|--------|--------|--------|---------|---------|---------|---------|---------|--------|
| Diam. Rope. | | | | | | | | | | |
| 1/4 | 993 | 502 | 336 | 252 | 202 | 168 | 144 | 126 | 112 | 101 |
| 5/16 | 1,863 | 944 | 632 | 475 | 380 | 317 | 272 | 238 | 212 | 191 |
| 3/8 | 2,771 | 1,406 | 942 | 708 | 567 | 473 | 406 | 355 | 316 | 285 |
| 7/16 | 4,859 | 2,473 | 1,658 | 1,247 | 1,000 | 834 | 716 | 627 | 557 | 502 |
| 1/2 | 7,125 | 3,635 | 2,440 | 1,836 | 1,472 | 1,228 | 1,054 | 923 | 821 | 739 |
| 9/16 | | 5,319 | 3,573 | 2,690 | 2,157 | 1,800 | 1,545 | 1,353 | 1,203 | 1,084 |
| 5/8 | | 7,452 | 5,011 | 3,774 | 3,027 | 2,526 | 2,169 | 1,900 | 1,690 | 1,522 |
| 11/16 | | 9,767 | 6,572 | 4,953 | 3,973 | 3,317 | 2,847 | 2,494 | 2,219 | 1,998 |
| 3/4 | | 12,512 | 8,427 | 6,352 | 5,098 | 4,257 | 3,654 | 3,201 | 2,848 | 2,565 |
| 7/8 | | 19,436 | 13,111 | 9,891 | 7,941 | 6,633 | 5,696 | 4,990 | 4,440 | 3,999 |
| 1 | | 29,799 | 20,136 | 15,205 | 12,214 | 10,206 | 8,766 | 7,681 | 6,836 | 6,158 |
| 1 1/8 | | | 23,153 | 21,276 | 17,099 | 14,293 | 12,278 | 10,761 | 9,578 | 8,689 |
| 1 1/4 | | | 38,034 | 28,766 | 23,130 | 19,340 | 16,618 | 14,567 | 12,967 | 11,683 |
| 1 3/8 | | | 51,609 | 39,057 | 31,430 | 26,290 | 22,594 | 19,811 | 17,637 | 15,893 |
| 1 1/2 | | | 66,065 | 50,049 | 40,284 | 33,707 | 28,976 | 25,410 | 22,625 | 20,390 |
| 5/8 | | | | 62,895 | 50,647 | 42,391 | 36,450 | 31,969 | 28,470 | 25,661 |
| 13/4 | | | | 79,749 | 64,252 | 53,798 | 46,270 | 40,590 | 36,152 | 32,589 |
| 17/8 | | | | 97,018 | 78,202 | 65,500 | 56,347 | 49,438 | 44,039 | 39,701 |
| 2 | | | | | 94,016 | 78,769 | 67,778 | 59,478 | 52,989 | 47,777 |
| 2 1/4 | | | | | 134,319 | 112,611 | 96,943 | 85,103 | 75,840 | 68,396 |
| 2 1/2 | | | | | | 154,870 | 133,386 | 117,137 | 104,417 | 94,189 |

Horse-Power Transmitted. — The general formula for the amount of power capable of being transmitted is as follows:

$$\text{H.P.} = [cd^2 - 0.000006(w + g_1 + g_2)]v;$$

in which d = diameter of the rope in inches, v = velocity of the rope in feet per second, w = weight of the rope, g_1 = weight of the terminal sheaves and shafts, g_2 = weight of the intermediate sheaves and shafts (all in lbs.), and c = a constant depending on the material of the rope, the filling in the grooves of the sheaves, and the number of laps about the sheaves or drums, a single lap meaning a half-lap at each end. The values of c for one up to six laps for steel rope are given in the following table:

1186 TRANSMISSION OF POWER BY WIRE ROPE.

| c = for steel rope on | Number of laps about sheaves or drums. | | | | | |
|-----------------------|--|-------|-------|-------|-------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Iron..... | 5.61 | 8.81 | 10.62 | 11.65 | 12.16 | 12.56 |
| Wood..... | 6.70 | 9.93 | 11.51 | 12.26 | 12.66 | 12.83 |
| Rubber and leather.. | 9.29 | 11.95 | 12.70 | 12.91 | 12.97 | 13.00 |

The values of *c* for iron rope are one half the above.

When more than three laps are made, the character of the surface in contact is immaterial as far as slippage is concerned.

From the above formula we have the general rule, that *the actual horse-power capable of being transmitted by any wire rope approximately equals c times the square of the diameter of the rope in inches, less six millionths the entire weight of all the moving parts, multiplied by the speed of the rope, in feet per second.*

Instead of grooved drums or a number of sheaves, about which the rope makes two or more laps, it is sometimes found more desirable, especially where space is limited, to use grip-pulleys. The rim is fitted with a continuous series of steel jaws, which bite the rope in contact by reason of the pressure of the same against them, but as soon as relieved of this pressure they open readily, offering no resistance to the egress of the rope.

In the ordinary or "flying" transmission of power, where the rope makes a single lap about sheaves lined with rubber and leather or wood, the ratio between the diameter of the sheaves and the wires of the rope, corresponding to a maximum safe working tension, is: For 7-wire rope, steel, 79.6; iron, 160.5. For 12-wire rope, steel, 59.3; iron, 120. For 19-wire rope, steel, 47.2; iron, 95.8.

Diameters of Minimum Sheaves in Inches, Corresponding to a Maximum Safe Working Tension.

| Diameter of Rope, In. | Steel. | | | Iron. | | |
|-----------------------|---------|----------|----------|---------|----------|----------|
| | 7-Wire. | 12-Wire. | 19-Wire. | 7-Wire. | 12-Wire. | 19-Wire. |
| 1/4 | 20 | 15 | 12 | 40 | 30 | 24 |
| 5/16 | 25 | 19 | 15 | 50 | 38 | 30 |
| 3/8 | 30 | 22 | 18 | 60 | 45 | 36 |
| 7/16 | 35 | 26 | 21 | 70 | 53 | 42 |
| 1/2 | 40 | 30 | 24 | 80 | 60 | 48 |
| 9/16 | 45 | 33 | 27 | 90 | 68 | 54 |
| 5/8 | 50 | 37 | 30 | 100 | 75 | 60 |
| 11/16 | 55 | 41 | 32 | 110 | 83 | 66 |
| 3/4 | 60 | 44 | 35 | 120 | 90 | 72 |
| 7/8 | 70 | 52 | 41 | 140 | 105 | 84 |
| 1 | 80 | 59 | 47 | 160 | 120 | 96 |

Assuming the sheaves to be of equal diameter, and of the sizes in the above table, the *horse-power that may be transmitted by a steel rope making a single lap on wood-filled sheaves* is given in the table on the next page.

The transmission of greater horse-powers than 250 is impracticable with filled sheaves, as the tension would be so great that the filling would quickly cut out, and the adhesion on a metallic surface would be insufficient where the rope makes but a single lap. In this case it becomes necessary to use the Reuleaux method, in which the rope is given more than one lap, as referred to below, under the caption "Long-distance Transmissions."

Horse-power Transmitted by a Steel Rope on Wood-filled Sheaves.

| Diameter of Rope, In. | Velocity of Rope in Feet per Second. | | | | | | | | | |
|-----------------------|--------------------------------------|-----|-----|-------|-------|-------|-------|-------|-------|-------|
| | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 1/4 | 4 | 8 | 13 | 17 | 21 | 25 | 28 | 32 | 37 | 40 |
| 5/16 | 7 | 13 | 20 | 26 | 33 | 40 | 44 | 51 | 57 | 62 |
| 3/8 | 10 | 19 | 28 | 38 | 47 | 56 | 64 | 73 | 80 | 89 |
| 7/16 | 13 | 26 | 38 | 51 | 63 | 75 | 88 | 99 | 109 | 121 |
| 1/2 | 17 | 34 | 51 | 67 | 83 | 99 | 115 | 130 | 144 | 159 |
| 9/16 | 22 | 43 | 65 | 86 | 106 | 128 | 147 | 167 | 184 | 203 |
| 5/8 | 27 | 53 | 79 | 104 | 130 | 155 | 179 | 203 | 225 | 247 |
| 11/16 | 32 | 63 | 95 | 126 | 157 | 186 | 217 | 245 | | |
| 3/4 | 38 | 76 | 103 | 150 | 185 | 223 | | | | |
| 7/8 | 52 | 104 | 156 | 206 | | | | | | |
| 1 | 68 | 135 | 202 | | | | | | | |

The horse-power that may be transmitted by iron ropes is one-half of the above.

This table gives the amount of horse-power transmitted by wire ropes under maximum safe working tensions. In using wood-lined sheaves, therefore, it is well to make some allowance for the stretching of the rope, and to advocate somewhat heavier equipments than the above table would give; that is, if it is desired to transmit 20 horse-power, for instance, to put in a plant that would transmit 25 to 30 horse-power, avoiding the necessity of having to take up a comparatively small amount of stretch. On rubber and leather filling, however, the amount of power capable of being transmitted is 40 per cent greater than for wood, so that this filling is generally used, and in this case no allowance need be made for stretch, as such sheaves will likely transmit the power given by the table, under all possible deflections of the rope.

Under ordinary conditions, ropes of seven wires to the strand, laid about a hemp core, are best adapted to the transmission of power, but conditions often occur where 12- or 19-wire rope is to be preferred, as stated below, under "Limits of Span."

Deflections of the Rope. — The tension of the rope is measured by the amount of sag or deflection at the center of the span, and the deflection corresponding to the maximum safe working tension is determined by the following formulæ, in which S represents the span in feet:

| | Steel Rope. | Iron Rope. |
|--|------------------|---------------------|
| Def. of still rope at center, in feet. h | $= .00004 S^2$ | $h = .00008 S^2$ |
| " driving " " " h_1 | $= .000025 S^2$ | $h_1 = .00005 S^2$ |
| " slack " " " h_2 | $= .0000875 S^2$ | $h_2 = .000175 S^2$ |

Limits of Span. — On spans of less than sixty feet, it is impossible to splice the rope to such a degree of nicety as to give exactly the required deflection, and as the rope is further subject to a certain amount of stretch, it becomes necessary in such cases to apply mechanical means for producing the proper tension in order to avoid frequent splicing, which is very objectionable; but care should always be exercised in using such tightening devices that they do not become the means, in unskilled hands, of overstraining the rope. The rope also is more sensitive to every irregularity in the sheaves and the fluctuations in the amount of power transmitted, and is apt to sway to such an extent beyond the narrow limits of the required deflections as to cause a jerking motion, which is very injurious. For this reason on very short spans it is found desirable to use a considerably heavier rope than that actually required to transmit the power; or in other words, instead of a 7-wire rope corresponding to the conditions of maximum tension, it is better to use a 19-wire rope of the same size wires, and to run this under a tension considerably below the maximum. In this way are obtained the advantages of increased weight and less stretch, without having to use larger sheaves, while the wear will be greater in proportion to the increased surface.

In determining the maximum limit of span, the contour of the ground and the available height of the terminal sheaves must be taken into consideration. It is customary to transmit the power through the lower portion of the rope, as in this case the greatest deflection in this portion occurs when the rope is at rest. When running, the lower portion rises and the upper portion sinks, thus enabling obstructions to be avoided which otherwise would have to be removed, or make it necessary to erect very high towers. The maximum limit of span in this case is determined by the maximum deflection that may be given to the upper portion of the rope when running, which for sheaves of 10 ft. diameter is about 600 feet.

Much greater spans than this, however, are practicable where the contour of the ground is such that the upper portion of the rope may be the driver, and there is nothing to interfere with the proper deflection of the under portion. Some very long transmissions of power have been effected in this way without an intervening support; one at Lockport, N.Y., having a clear span of 1700 feet.

Long-distance Transmissions. — When the distance exceeds the limit for a clear span, intermediate supporting sheaves are used, with plain grooves (not filled), the spacing and size of which will be governed by the contour of the ground and the special conditions involved. The size of these sheaves will depend on the angle of the bend, gauged by the tangents to the curves of the rope at the points of inflection. If the curvature due to this angle and the working tension, regardless of the size of the sheaves, as determined by the table on the next page, is less than that of the minimum sheave (see table p. 1186), the intermediate sheaves should not be smaller than such minimum sheave, but if the curvature is greater, smaller intermediate sheaves may be used.

In very long transmissions of power, requiring numerous intermediate supports, it is found impracticable to run the rope at the high speeds maintained in "flying transmissions." The rope therefore is run under a higher working tension, made practicable by wrapping it several times about grooved terminal drums, with a lap about a sheave on a take-up or counter-weighted carriage, which preserves a constant tension in the slack portion.

Inclined Transmissions. — When the terminal sheaves are not on the same elevation, the tension at the upper sheave will be greater than that at the lower, but this difference is so slight, in most cases, that it may be ignored. The span to be considered is the horizontal distance between the sheaves, and the principles governing the limits of span will hold good in this case, so that for very steep inclinations it becomes necessary to resort to tightening devices for maintaining the requisite tension in the rope. The limiting case of inclined transmissions occurs when one wheel is directly above the other. The rope in this case produces no tension whatever on the lower wheel, while the upper is subject only to the weight of the rope, which is usually so insignificant that it may be neglected altogether, and on vertical transmissions, therefore, mechanical tension is an absolute necessity.

Bending Curvature of Wire Ropes. — The curvature due to any bend in a wire rope is dependent on the tension, and is not always the same as the sheave in contact, but may be greater, which explains how it is that large ropes are frequently run around comparatively small sheaves without detriment, since it is possible to place these so close that the bending angle on each will be such that the resulting curvature will not overstrain the wires. This curvature may be ascertained from the formula and table on the next page, which give the theoretical radii of curvature in inches for various sizes of ropes and different angles for one pound tension in the rope. Dividing these figures by the actual tension in pounds, gives the radius of curvature assumed by the rope in cases where this exceeds the curvature of the sheave. The rigidity of the rope or internal friction of the wires and core has not been taken into account in these figures, but the effect of this is insignificant, and it is on the safe side to ignore it. By the "angle of bend" is meant the angle between the tangents to the curves of the rope at the points of inflection. When the rope is straight the angle is 180° . For angles less than 160° the radius of curvature in most cases will be less than that corresponding to the safe working tension, and the proper size of sheave to use in such

cases will be governed by the table headed "Diameters of Minimum Sheaves Corresponding to a Maximum Safe Working Tension" on page 1186.

Radius of Curvature of Wire Ropes in Inches for 1-lb. Tension.

Formula: $R = E\delta n \div 5.25 t \cos \frac{1}{2} \theta$; in which R = radius of curvature; E = modulus of elasticity = 28,500,000; δ = diameter of wires; n = no. of wires; θ = angle of bend; t = working stress (lbs. and ins.).

Divide by stress in pounds to obtain radius in inches.

| Diam. of Wire. | 160° | 165° | 170° | 172° | 174° | 176° | 178° |
|----------------------|---------|---------|-----------|-----------|-----------|-----------|-----------|
| 19-Wire Rope. | | | | | | | |
| 1/2 | 4,226 | 5,623 | 8,421 | 10,949 | 14,593 | 21,884 | 43,762 |
| 5/8 | 11,090 | 14,753 | 22,095 | 26,731 | 35,628 | 53,429 | 106,841 |
| 3/4 | 22,274 | 29,633 | 45,412 | 54,417 | 72,530 | 108,767 | 217,500 |
| 7/8 | 43,184 | 57,451 | 86,040 | 102,688 | 136,869 | 205,251 | 410,440 |
| 1 | 71,816 | 95,541 | 143,085 | 175,182 | 233,492 | 350,150 | 700,193 |
| 1 1/8 | 112,763 | 150,016 | 224,667 | 280,607 | 374,010 | 560,872 | 1,121,574 |
| 1 1/4 | 169,135 | 225,012 | 336,982 | 427,689 | 570,050 | 854,858 | 1,709,459 |
| 7-Wire Rope. | | | | | | | |
| 1/2 | 12,914 | 17,179 | 25,727 | 31,125 | 41,485 | 62,212 | 124,405 |
| 5/8 | 29,762 | 39,594 | 59,297 | 75,988 | 101,282 | 151,884 | 303,723 |
| 3/4 | 62,313 | 82,899 | 124,151 | 157,570 | 210,018 | 314,948 | 629,800 |
| 7/8 | 116,239 | 154,641 | 231,593 | 291,917 | 389,085 | 583,479 | 1,164,099 |
| 1 | 199,323 | 265,173 | 397,129 | 497,998 | 663,767 | 995,390 | 1,990,478 |
| 1 1/8 | 320,556 | 426,459 | 638,674 | 797,697 | 1,063,217 | 1,594,422 | 3,188,359 |
| 1 1/4 | 504,402 | 671,041 | 1,004,965 | 1,215,817 | 1,620,513 | 2,430,151 | 4,859,561 |

ROPE-DRIVING.

The transmission of power by cotton or manila ropes is a competitor with gearing and leather belting when the amount of power is large, or the distance between the power and the work is comparatively great. The following is condensed from a paper by C. W. Hunt, *Trans. A. S. M. E.*, xii, 230:

But few accurate data are available, on account of the long period required in each experiment, a rope lasting from three to six years. Installations which have been successful, as well as those in which the wear of the rope was destructive, indicate that 200 lbs. on a rope one inch in diameter is a safe and economical working strain. When the strain is materially increased, the wear is rapid.

In the following equations

C = circumference of rope, inches; g = gravity;
 D = sag of the rope in inches; H = horse-power;
 F = centrifugal force in pounds; L = distance between pulleys, ft.;
 P = pounds per foot of rope; w = working strain in pounds;

R = force in pounds doing useful work;

S = strain in pounds on the rope at the pulley;

T = tension in pounds of driving side of the rope;

t = tension in pounds on slack side of the rope;

v = velocity of the rope in feet per second;

W = ultimate breaking strain in pounds.

$W = 720 C^2$; $P = 0.032 C^2$; $w = 20 C^2$.

This makes the normal working strain equal to $1/30$ of the breaking strength, and about $1/25$ of the strength at the splice. The actual strains are ordinarily much greater, owing to the vibrations in running, as well as from imperfectly adjusted tension mechanism.

For this investigation we assume that the strain on the driving side of a rope is equal to 200 lbs. on a rope one inch in diameter, and an equivalent strain for other sizes, and that the rope is in motion at various velocities of from 10 to 140 ft. per second.

The centrifugal force of the rope in running over the pulley will reduce

the amount of force available for the transmission of power. The centrifugal force $F = Pr^2 \div g$.

At a speed of about 80 ft. per second, the centrifugal force increases faster than the power from increased velocity of the rope, and at about 140 ft. per second equals the assumed allowable tension of the rope. Computing this force at various speeds and then subtracting it from the assumed maximum tension, we have the force available for the transmission of power. The whole of this force cannot be used, because a certain amount of tension on the slack side of the rope is needed to give adhesion to the pulley. What tension should be given to the rope for this purpose is uncertain, as there are no experiments which give accurate data. It is known from considerable experience that when the rope runs in a groove whose sides are inclined toward each other at an angle of 45° there is sufficient adhesion when the ratio of the tensions $T \div t = 2$.

For the present purpose T can be divided into three parts: 1. Tension doing useful work; 2. Tension from centrifugal force; 3. Tension to balance the strain for adhesion.

The tension t can be divided into two parts: 1. Tension for adhesion; 2. Tension from centrifugal force.

It is evident, however, that the tension required to do a given work should not be materially exceeded during the life of the rope.

There are two methods of putting ropes on the pulleys; one in which the ropes are single and spliced on, being made very taut at first, and less so as the rope lengthens, stretching until it slips, when it is respliced. The other method is to wind a single rope over the pulleys as many turns as needed to obtain the necessary horse-power and put a tension pulley to give the necessary adhesion and also take up the wear. The tension t on one of the ropes required to transmit the normal horse-power for the ordinary speeds and sizes of rope is computed by formula (1), below. The total tension T on the driving side of the rope is assumed to be the same at all speeds. The centrifugal force, as well as an amount equal to the tension for adhesion on the slack side of the rope, must be taken from the total tension T to ascertain the amount of force available for the transmission of power.

It is assumed that the tension on the slack side necessary for giving adhesion is equal to one half the force doing useful work on the driving

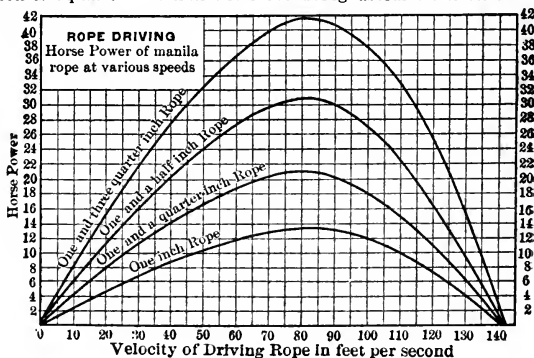


FIG. 186.

side of the rope; hence the force for useful work is $R = \frac{2}{3}(T - F)$; and the tension on the slack side to give the required adhesion is $\frac{1}{3}(T - F)$. Hence

$$t = (T - F)/3 + F \dots \dots \dots (1)$$

The sum of the tensions T and t is not the same at different speeds, as the equation (1) indicates. As F varies as the square of the velocity, there is, with an increasing speed of the rope, a decreasing useful force, and an increasing total tension, t , on the slack side.

With these assumptions of allowable strains the horse-power will be

$$H = 2v(T - F) \div (3 \times 550) \dots \dots \dots (2)$$

Transmission ropes are usually from 1 to 2 inches in diameter. A computation of the horse-power for four sizes at various speeds and under ordinary conditions, based on a maximum strain equivalent to 200 lbs. for a rope one inch in diameter, is given in Fig. 186. The horse-power of other sizes is readily obtained from these. The maximum power is transmitted, under the assumed conditions, at a speed of about 80 feet per second.

The wear of the rope is both internal and external; the internal is caused by the movement of the fibers on each other, under pressure in bending over the sheaves, and the external is caused by the slipping and the wedging in the grooves of the pulley. Both of these causes of wear are, within the limits of ordinary practice, assumed to be directly proportional to the speed.

The rope is supposed to have the strain T constant at all speeds on the driving side, and in direct proportion to the area of the cross-section; hence the catenary of the driving side is not affected by the speed or by the diameter of the rope.

The deflection of the rope between the pulleys on the slack side varies with each change of the load or change of the speed, as the tension equation (1) indicates.

The deflection of the rope is computed for the assumed value of T and t by the parabolic formula $S = \frac{PL^2}{8D} + PD$, S being the assumed strain T on the driving side, and t , calculated by equation (1), on the slack side. The tension t varies with the speed.

Horse-power of Transmission Rope at Various Speeds.

Computed from formula (2) given above.

| Diam. of Ropes. | Speed of the Rope in feet per minute. | | | | | | | | | | | Smallest Diam. of Pulleys in inches. |
|-----------------|---------------------------------------|------|------|------|------|------|------|------|------|------|------|--------------------------------------|
| | 1500 | 2000 | 2500 | 3000 | 3500 | 4000 | 4500 | 5000 | 6000 | 7000 | 8000 | |
| 1/2 | 1.45 | 1.9 | 2.3 | 2.7 | 3 | 3.2 | 3.4 | 3.4 | 3.1 | 2.2 | 0 | 20 |
| 5/8 | 2.3 | 3.2 | 3.6 | 4.2 | 4.6 | 5.0 | 5.3 | 5.3 | 4.9 | 3.4 | 0 | 24 |
| 3/4 | 3.3 | 4.3 | 5.2 | 5.8 | 6.7 | 7.2 | 7.7 | 7.7 | 7.1 | 4.9 | 0 | 30 |
| 7/8 | 4.5 | 5.9 | 7.0 | 8.2 | 9.1 | 9.8 | 10.8 | 10.8 | 9.3 | 6.9 | 0 | 36 |
| 1 | 5.8 | 7.7 | 9.2 | 10.7 | 11.9 | 12.8 | 13.6 | 13.7 | 12.5 | 8.8 | 0 | 42 |
| 1 1/4 | 9.2 | 12.1 | 14.3 | 16.8 | 18.6 | 20.0 | 21.2 | 21.4 | 19.5 | 13.8 | 0 | 54 |
| 1 1/2 | 13.1 | 17.4 | 20.7 | 23.1 | 26.8 | 28.8 | 30.6 | 30.8 | 28.2 | 19.8 | 0 | 60 |
| 1 3/4 | 18 | 23.7 | 28.2 | 32.8 | 36.4 | 39.2 | 41.5 | 41.8 | 37.4 | 27.6 | 0 | 72 |
| 2 | 23.2 | 30.8 | 36.8 | 42.8 | 47.6 | 51.2 | 54.4 | 54.8 | 50 | 35.2 | 0 | 84 |

The following notes are from the circular of the C. W. Hunt Co.:

For a temporary installation, it might be advisable to increase the work to double that given in the table.

For convenience in estimating the necessary clearance on the driving and on the slack sides, we insert a table showing the sag of the rope at different speeds when transmitting the horse-power given in the preceding table. When at rest the sag is not the same as when running, being greater on the driving and less on the slack sides of the rope. The sag of the driving side when transmitting the normal horse-power is the same no matter what size of rope is used or what the speed driven at, because the assumption is that the strain on the rope shall be the same at all speeds when transmitting the assumed horse-power, but on the slack side the strains, and consequently the sag, vary with the speed of the rope and also with the horse-power. The table gives the sag for three speeds. If the actual sag is less than given in the table, the rope is strained more than the work requires.

This table is only approximate, and is exact only when the rope is running at its normal speed, transmitting its full load and strained to the assumed amount. All of these conditions are varying in actual work.

SAG OF THE ROPE BETWEEN PULLEYS.

| Distance between Pulleys in feet. | Driving Side. | | Slack Side of Rope. | | | |
|-----------------------------------|---------------|----------|---------------------|-----------------|------------------|--|
| | All Speeds. | | 80 ft. per sec. | 60 ft. per sec. | 40 ft. per sec. | |
| 40 | 0 feet | 4 inches | 0 feet 7 inches | 0 feet 9 inches | 0 feet 11 inches | |
| 60 | 0 " | 10 " | 1 " 5 " | 1 " 8 " | 1 " 11 " | |
| 80 | 1 " | 5 " | 2 " 4 " | 2 " 10 " | 3 " 3 " | |
| 100 | 2 " | 0 " | 3 " 8 " | 4 " 5 " | 5 " 2 " | |
| 120 | 2 " | 11 " | 5 " 3 " | 6 " 3 " | 7 " 4 " | |
| 140 | 3 " | 10 " | 7 " 2 " | 8 " 9 " | 9 " 9 " | |
| 160 | 5 " | 1 " | 9 " 3 " | 11 " 3 " | 14 " 0 " | |

The size of the pulleys has an important effect on the wear of the rope — the larger the sheaves, the less the fibers of the rope slide on each other, and consequently there is less internal wear of the rope. The pulleys should not be less than forty times the diameter of the rope for economical wear, and as much larger as it is possible to make them. This rule applies also to the idle and tension pulleys as well as to the main driving-pulley.

The angle of the sides of the grooves in which the rope runs varies, with different engineers, from 45° to 60°. It is very important that the sides of these grooves should be carefully polished, as the fibers of the rope rubbing on the metal as it comes from the lathe tools will gradually break fiber by fiber, and so give the rope a short life. It is also necessary to carefully avoid all sand or blow holes, as they will cut the rope out with surprising rapidity.

TENSION ON THE SLACK PART OF THE ROPE.

| Speed of Rope, in feet per second. | Diameter of the Rope and Pounds Tension on the Slack Rope. | | | | | | | | |
|------------------------------------|--|-----|-----|-----|-----|-------|-------|-------|-----|
| | 1/2 | 5/8 | 3/4 | 7/8 | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 |
| 20 | 10 | 27 | 40 | 54 | 71 | 110 | 162 | 216 | 283 |
| 30 | 14 | 29 | 42 | 56 | 74 | 115 | 170 | 226 | 296 |
| 40 | 15 | 31 | 45 | 60 | 79 | 123 | 181 | 240 | 315 |
| 50 | 16 | 33 | 49 | 65 | 85 | 132 | 195 | 259 | 339 |
| 60 | 18 | 36 | 53 | 71 | 93 | 145 | 214 | 285 | 373 |
| 70 | 19 | 39 | 59 | 78 | 101 | 158 | 236 | 310 | 406 |
| 80 | 21 | 43 | 64 | 85 | 111 | 173 | 255 | 340 | 445 |
| 90 | 24 | 48 | 70 | 93 | 122 | 190 | 279 | 372 | 487 |

Much depends also upon the arrangement of the rope on the pulleys, especially where a tension weight is used. Experience shows that the increased wear on the rope from bending the rope first in one direction and then in the other is similar to that of wire rope. At mines where two cages are used, one being hoisted and one lowered by the same engine doing the same work, the wire ropes, cut from the same coil, are usually arranged so that one rope is bent continuously in one direction and the other rope is bent first in one direction and then in the other, in winding on the drum of the engine. The rope having the opposite bends wears much more rapidly than the other, lasting about three quarters as long as its mate. This difference in wear shows in manila rope, both in transmission of power and in coal-hoisting. The pulleys should be arranged, as far as possible, to bend the rope in one direction.

DIAMETER OF PULLEYS AND WEIGHT OF ROPE.

| Diameter of Rope, in inches. | Smallest Diameter of Pulleys, in inches. | Length of Rope to allow for Splicing, in feet. | Approximate Weight, in lbs. per foot of rope. |
|------------------------------|--|--|---|
| 1/2 | 20 | 6 | 0.12 |
| 5/8 | 24 | 6 | 0.18 |
| 3/4 | 30 | 7 | 0.24 |
| 7/8 | 36 | 8 | 0.32 |
| 1 | 42 | 9 | 0.49 |
| 1 1/4 | 54 | 10 | 0.60 |
| 1 1/2 | 60 | 12 | 0.83 |
| 1 3/4 | 72 | 13 | 1.10 |
| 2 | 84 | 14 | 1.40 |

For large amounts of power it is common to use a number of ropes lying side by side in grooves, each spliced separately. For lighter drives some engineers use one rope wrapped as many times around the pulleys as is necessary to get the horse-power required, with a tension pulley to take up the slack as the rope wears when first put in use. The weight put upon this tension pulley should be carefully adjusted, as the overstraining of the rope from this cause is one of the most common errors in rope-driving. We therefore give a table showing the proper strain on the rope for the various sizes, from which the tension weight to transmit the horse-power in the tables is easily deduced. This strain can be still further reduced if the horse-power transmitted is usually less than the nominal work which the rope was proportioned to do, or if the angle of groove in the pulleys is acute.

With a given velocity of the driving-rope, the weight of rope required for transmitting a given horse-power is the same, no matter what size rope is adopted. The smaller rope will require more parts, but the weight will be the same.

Data of Manila Transmission Rope.

From the "Blue Book" of The American Mfg. Co., New York.

| Diam. of Rope. | Square of Diam. | Approximate Weight per ft. | Breaking Strength, lbs. | Maximum Allowable Tension. | Length of Splice, ft. | | | Smallest Diam. of Sheaves, ins. | Maximum No. of Revolutions per Minute. |
|----------------|-----------------|----------------------------|-------------------------|----------------------------|-----------------------|------------|------------|---------------------------------|--|
| | | | | | 3 Strands. | 4 Strands. | 6 Strands. | | |
| 3/4 | 0.5625 | 0.20 | 3,950 | 112 | 6 | 8 | ... | 28 | 760 |
| 7/8 | 0.7656 | 0.26 | 5,400 | 153 | 6 | 8 | ... | 32 | 650 |
| 1 | 1. | 0.34 | 7,000 | 200 | 7 | 10 | 14 | 36 | 570 |
| 1 1/8 | 1.2656 | 0.43 | 8,900 | 253 | 7 | 10 | 16 | 40 | 510 |
| 1 1/4 | 1.5625 | 0.53 | 10,900 | 312 | 7 | 10 | 16 | 46 | 460 |
| 1 3/8 | 1.8906 | 0.65 | 13,200 | 378 | 8 | 12 | 18 | 50 | 415 |
| 1 1/2 | 2.25 | 0.77 | 15,700 | 450 | 8 | 12 | 18 | 54 | 380 |
| 1 5/8 | 2.6406 | 0.90 | 18,500 | 528 | 8 | 12 | 18 | 60 | 344 |
| 1 3/4 | 3.0625 | 1.04 | 21,400 | 612 | 8 | 12 | 18 | 64 | 330 |
| 2 | 4. | 1.36 | 28,000 | 800 | 9 | 14 | 20 | 72 | 290 |
| 2 1/4 | 5.0625 | 1.73 | 35,400 | 1,012 | 9 | 14 | 20 | 82 | 255 |
| 2 1/2 | 6.25 | 2.13 | 43,700 | 1,250 | 10 | 16 | 22 | 90 | 230 |

Weight of transmission rope = $0.34 \times \text{diam.}^2$

Breaking strength = $7,000 \times \text{diam.}^2$

Maximum allowable tension = $200 \times \text{diam.}^2$

Diam. smallest practicable sheave, = $36 \times \text{diam.}$

Velocity of rope (assumed) = 5,400 ft. per min.

Miscellaneous Notes on Rope-Driving. — Reuleaux gives formulæ for calculating sources of loss in hemp-rope transmission due to (1) journal friction, (2) stiffness of ropes, and (3) creep of ropes. The constants in these formulæ are, however, uncertain from lack of experimental data. He calculates an average case giving loss of power due to journal friction = 4%, to stiffness 7.8%, and to creep 5%, or 16.8% in all, and says this is not to be considered higher than the actual loss.

Spencer Miller, in a paper entitled "A Problem in Continuous Rope-driving" (*Trans. A. S. C. E.*, 1897), reviews the difficulties which occur in rope-driving, with a continuous rope from a large to a small pulley. He adopts the angle of 45° as a minimum angle to use on the smaller pulley, and recommends that the larger pulley be grooved with a wider angle to a degree such that the resistance to slipping is equal in both wheels.

Mr. Miller refers to a 250-H.P. drive which has been running ten years, the large pulley being grooved 60° and the smaller 45° . This drive was designed to use a $1\frac{1}{4}$ -in. manila rope, but the grooves were made deep enough so that a $\frac{7}{8}$ -in. rope would not bottom. In order to determine the value of the drive a common $\frac{7}{8}$ -in. rope was put in at first, and lasted six years, working under a factor of safety of only 14. He recommends, however, for continuous rope-driving a factor of safety of not less than 20.

A heavy rope-drive on the separate, or English, rope system is described and illustrated in *Power*, April, 1892. It is in use at the India Mill at Darwen, England, and is driven by a 2000-H.P. engine at 54 revs. per min. The fly-wheel is 30 ft. diameter, weighs 65 tons, and is arranged with 30 grooves for $1\frac{3}{4}$ -in. ropes. These ropes lead off to receiving-pulleys upon the several floors, so that each floor receives its power direct from the fly-wheel. The speed of the ropes is 5089 ft. per min., and five 7-ft. receivers are used. Lambeth cotton ropes are used. (For much other information on this subject see "Rope-Driving," by J. J. Flather, John Wiley & Sons.)

Cotton Ropes are advantageously used as bands or cords on the smaller machine appliances; the fiber, being softer and more flexible than manila hemp, gives good results for small sheaves; but for large drives, where power transmitted is in considerable amounts, cotton rope, as compared with manila, is hardly to be considered, on account of the following disadvantages: It is less durable; it is injuriously affected by the weather, so that for exposed drives, paper-mill work, or use in water-wheel pits, it is absolutely unsatisfactory; it is difficult, if not impossible, to splice uniformly; even the best quality cotton rope is much inferior to manila in strength, the breaking strain of the highest grade being but $4000 \times \text{diam.}^2$ as against $7000 \times \text{diam.}^2$ for manila; while, for the transmission of equal powers, the cost of a cotton rope varies from one-third to one-half more than manila. — ("Blue Book" of the Amer. Mfg. Co.)

A different opinion is found in a paper by E. Kenyon in *Proc. Inst. Engrs. and Shipbuilders of Scotland*, 1904. He says: Evidences of the progress of cotton in the manufacture of driving-ropes are so far-reaching that its superiority may be considered as much an accepted principle in enhanced power-transmitting value, its immunity from frequent atteno-rope transmission as the law of gravitation is in science. As to the longevity of cotton ropes, 24 cotton ropes $1\frac{3}{4}$ -in. diam. are transmitting 820 H.P. at a peripheral speed of 4396 ft. per min., from a driving pulley 28 ft. diam. All the card-room ropes in this drive have been running since 1878, a period of 26 years, without any attention whatever.

FRICTION AND LUBRICATION.

Friction is defined by Rankine as that force which acts between two bodies at their surface of contact so as to resist their sliding on each other, and which depends on the force with which the bodies are pressed together.

Coefficient of Friction. — The ratio of the force required to slide a body along a horizontal plane surface to the weight of the body is called the coefficient of friction. It is equivalent to the tangent of the *angle of repose*, which is the angle of inclination to the horizontal of an inclined plane on which the body will just overcome its tendency to slide. The angle is usually denoted by θ , and the coefficient by f . $f = \tan \theta$.

Friction of Rest and of Motion. — The force required to start a body sliding is called the friction of rest, and the force required to continue its sliding after having started is called the friction of motion.

Rolling Friction is the force required to roll a cylindrical or spherical body on a plane or on a curved surface. It depends on the nature of the surfaces and on the force with which they are pressed together, but is essentially different from ordinary, or sliding, friction.

Friction of Solids. — Rennie's experiments (1829) on friction of solids, usually unlubricated and dry, led to the following conclusions:

1. The laws of sliding friction differ with the character of the bodies rubbing together.

2. The friction of fibrous material is increased by increased extent of surface and by time of contact, and is diminished by pressure and speed.

3. With wood, metal, and stones, within the limit of abrasion, friction varies only with the pressure, and is independent of the extent of surface, time of contact, and velocity.

4. The limit of abrasion is determined by the hardness of the softer of the two rubbing parts.

5. Friction is greatest with soft and least with hard materials.

6. The friction of lubricated surfaces is determined by the nature of the lubricant rather than by that of the solids themselves.

Friction of Rest. (Rennie.)

| Pressure, lbs. per square inch. | Values of f . | | | |
|--|----------------------------------|--------------------------|------------------------|------------------------|
| | Wrought iron on Wrought Iron. | Wrought on Cast Iron. | Steel on Cast Iron. | Brass on Cast Iron. |
| 187 | 0.25 | 0.28 | 0.30 | 0.23 |
| 224 | .27 | .29 | .33 | .22 |
| 336 | .31 | .33 | .35 | .21 |
| 448 | .38 | .37 | .35 | .21 |
| 560 | .41 | .37 | .36 | .23 |
| 672 | Abraded | .38 | .40 | .23 |
| 784 | " | Abraded | Abraded | .23 |

Law of Unlubricated Friction. — A. M. Wellington, *Eng'g News*, April 7, 1888, states that the most important and the best determined of all the laws of unlubricated friction may be thus expressed:

The coefficient of unlubricated friction decreases materially with velocity, is very much greater at minute velocities of 0+, falls very rapidly with minute increases of such velocities, and continues to fall much less rapidly with higher velocities up to a certain varying point, following closely the laws which obtain with lubricated friction.

Friction of Steel Tires Sliding on Steel Rails. (Westinghouse & Galton.)

| | | | | | | |
|-----------------------------------|-------|------|------|------|------|------|
| Speed, miles per hour | 10 | 15 | 25 | 38 | 45 | 50 |
| Coefficient of friction | 0.110 | .087 | .080 | .051 | .047 | .040 |
| Adhesion, lbs. per gross ton | 246 | 195 | 179 | 128 | 114 | 90 |

Rolling Friction is a consequence of the irregularities of form and the roughness of surface of bodies rolling one over the other. Its laws are not yet definitely established in consequence of the uncertainty which exists in experiment as to how much of the resistance is due to roughness of surface, how much to original and permanent irregularity of form, and how much to distortion under the load. (Thurston.)

Coefficients of Rolling Friction. — If R = resistance applied at the circumference of the wheel, W = total weight, r = radius of the wheel, and f = a coefficient, $R = fW \div r$. f is very variable. Coulomb gives 0.06 for wood, 0.005 for metal, where W is in pounds and r in feet. Tredgold made the value of f for iron on iron 0.002.

For wagons on soft soil Morin found $f = 0.065$, and on hard smooth roads 0.02.

A Committee of the Society of Arts (Clark, R. T. D.) reported a loaded omnibus to exhibit a resistance on various loads as below:

| Pavement. | Speed per hour. | Coefficient. | Resistance. |
|-------------------------------|-----------------|--------------|----------------|
| Granite | 2.87 miles. | 0.007 | 17.41 per ton. |
| Asphalt | 3.56 " | 0.0121 | 27.14 " |
| Wood | 3.34 " | 0.0185 | 41.60 " |
| Macadam, graveled | 3.45 " | 0.0199 | 44.48 " |
| Macadam, granite, new | 3.51 " | 0.0451 | 101.09 " |

Thurston gives the value of f for ordinary railroads, 0.003; well-laid railroad track, 0.002; best possible railroad track, 0.001.

The few experiments that have been made upon the coefficients of rolling friction, apart from axle friction, are too incomplete to serve as a basis for practical rules. (Trautwine.)

Laws of Fluid Friction. — For all fluids, whether liquid or gaseous, the resistance is (1) independent of the pressure between the masses in contact; (2) directly proportional to the area of rubbing-surface; (3) proportional to the square of the relative velocity at moderate and high speeds, and to the velocity nearly at low speeds; (4) independent of the nature of the surfaces of the solid against which the stream may flow, but dependent to some extent upon their degree of roughness; (5) proportional to the density of the fluid, and related in some way to its viscosity. (Thurston.)

The Friction of Lubricated Surfaces approximates to that of solid friction as the journal is run dry, and to that of fluid friction as it is flooded with oil.

Angles of Repose and Coefficients of Friction of Building Materials. (From Rankine's Applied Mechanics.)

| | θ . | $f = \tan \theta$. | $\frac{1}{\tan \theta}$. |
|--|---------------------|---------------------|---------------------------|
| Dry masonry and brickwork... | 31° to 35° | 0.6 to 0.7 | 1.67 to 1.4 |
| Masonry and brickwork with damp mortar..... | 36 1/2° | 0.74 | 1.35 |
| Timber on stone..... | 22° | about 0.4 | 2.5 |
| Iron on stone..... | 35° to 162 3/3° | 0.7 to 0.3 | 1.43 to 3.3 |
| Timber on timber..... | 26 1/2° to 111 1/3° | 0.5 to 0.2 | 2 to 5 |
| Timber on metals..... | 31° to 111 1/3° | 0.6 to 0.2 | 1.67 to 5 |
| Metals on metals..... | 14° to 81 1/2° | 0.25 to 0.15 | 4 to 6.67 |
| Masonry on dry clay..... | 27° | 0.51 | 1.96 |
| Masonry on moist clay..... | 181 1/4° | 0.33 | 3. |
| Earth on earth..... | 14° to 45° | 0.25 to 1.0 | 4 to 1 |
| Earth on earth, dry sand, clay, and mixed earth..... | 21° to 37° | 0.38 to 0.75 | 2.63 to 1.33 |
| Earth on earth, damp clay..... | 45° | 1.0 | 1 |
| Earth on earth, wet clay..... | 17° | 0.31 | 3.23 |
| Earth on earth, shingle and gravel..... | 39° to 48° | 0.81 | 1.23 to 0.9 |

Coefficients of Friction of Journals. (Morin.)

| Material. | Unguent. | Lubrication. | |
|-----------------------------|--------------------|--------------|---------------|
| | | Intermittent | Continuous. |
| Cast iron on cast iron..... | Oil, lard, tallow. | 0.07 to 0.08 | 0.03 to 0.054 |
| | Unctuous and wet | 0.14 | |
| Cast iron on bronze..... | Oil, lard, tallow. | 0.07 to 0.08 | 0.03 to 0.054 |
| | Unctuous and wet | 0.16 | |
| Cast iron on lignum vitæ... | Oil, lard. | | 0.09 |
| Wrought iron on cast iron.. | Oil, lard, tallow. | 0.07 to 0.08 | 0.03 to 0.054 |
| Wrought iron on bronze.. | | | |
| Iron on lignum vitæ..... | Oil, lard. | 0.11 | |
| | Unctuous. | 0.19 | |
| Bronze on bronze..... | Olive oil. | 0.10 | |
| | Lard. | 0.09 | |

Prof. Thurston says concerning the above figures that much better results are probably obtained in good practice with ordinary machinery. Those here given are so greatly modified by variations of speed, pressure, and temperature, that they cannot be taken as correct for general purposes.

Friction of Motion. — The following is a table of the angle of repose θ , the coefficient of friction $f = \tan \theta$, and its reciprocal, $1 \div f$, for the materials of mechanism — condensed from the tables of General Morin (1831) and other sources, as given by Rankine:

| No. | Surfaces. | θ . | f . | $1 \div f$. |
|-----|---|----------------|---------------|--------------|
| 1 | Wood on wood, dry..... | 14° to 26 1/2° | 0.25 to 0.5 | 4 to 2 |
| 2 | “ “ “ soaped..... | 11 1/2° to 2° | 0.2 to 0.04 | 5 to 25 |
| 3 | Metals on oak, dry..... | 26 1/2° to 31° | 0.5 to 0.6 | 2 to 1.67 |
| 4 | “ “ “ wet..... | 13 1/2° to 14° | 0.24 to 0.26 | 4.17 to 3.85 |
| 5 | “ “ “ soapy..... | 11 1/2° | 0.2 | 5 |
| 6 | “ “ elm, dry..... | 11 1/2° to 14° | 0.2 to 0.25 | 5 to 4 |
| 7 | Hemp on oak, dry..... | 28° | 0.53 | 1.89 |
| 8 | “ “ “ wet..... | 18 1/2° | 0.33 | 3 |
| 9 | Leather on oak..... | 15° to 19 1/2° | 0.27 to 0.38 | 3.7 to 2.86 |
| 10 | “ “ “ metals, dry..... | 29 1/2° | 0.56 | 1.79 |
| 11 | “ “ “ metals, wet..... | 20° | 0.36 | 2.78 |
| 12 | “ “ “ “ greasy.. | 13° | 0.23 | 4.35 |
| 13 | “ “ “ “ oily..... | 8 1/2° | 0.15 | 6.67 |
| 14 | Metals on metals, dry..... | 8 1/2° to 11° | 0.15 to 0.2 | 6.67 to 5 |
| 15 | “ “ “ “ wet..... | 16 1/2° | 0.3 | 3.33 |
| 16 | Smooth surfaces, occasion- ally greased..... | 4° to 4 1/2° | 0.07 to 0.08 | 14.3 to 12.5 |
| 17 | Smooth surfaces, continu- ously greased..... | 3° | 0.05 | 20 |
| 18 | Smooth surfaces, best results | 13 1/4° to 2° | 0.03 to 0.036 | |
| 19 | Bronze on lignum vitæ, con- stantly wet..... | 3°? | 0.05? | |

Average Coefficients of Friction. — Journal of cast iron in bronze bearing; velocity 720 feet per minute; temperature 70° F.; intermittent feed through an oil-hole. (Thurston on Friction and Lost Work.)

| Oils. | Pressures, pounds per square inch. | | | |
|--------------------------------|------------------------------------|--------------|--------------|--------------|
| | 8 | 16 | 32 | 48 |
| Sperm, lard, neat's-ft., etc.. | .159 to .250 | .138 to .192 | .086 to .141 | .077 to .144 |
| Olive, cotton-seed, rape, etc. | .160 to .283 | .107 to .245 | .101 to .168 | .079 to .131 |
| Cod and menhaden..... | .248 to .278 | .124 to .167 | .097 to .102 | .081 to .122 |
| Mineral lubricating-oils..... | .154 to .261 | .145 to .233 | .086 to .178 | .094 to .222 |

With fine steel journals running in bronze bearings and continuous lubrication, coefficients far below those above given are obtained. Thus with sperm-oil the coefficient with 50 lbs. per square inch pressure was 0.0034; with 200 lbs., 0.0051; with 300 lbs., 0.0057.

For very low pressures, as in spindles, the coefficients are much higher. Thus Mr. Woodbury found, at a temperature of 100° and a velocity of 600 feet per minute,

| Pressures, lbs. per sq. in. | 1 | 2 | 3 | 4 | 5 |
|-----------------------------|------|------|------|------|------|
| Coefficient..... | 0.38 | 0.27 | 0.22 | 0.18 | 0.17 |

These high coefficients, however, and the great decrease in the coefficient at increased pressures are limited as a practical matter only to the smaller pressures which exist especially in spinning machinery, where the pressure is so light and the film of oil so thick that the viscosity of the oil is an important part of the total frictional resistance.

Experiments on Friction of a Journal Lubricated by an Oil-bath (reported by the Committee on Friction, *Proc. Inst. M. E.* Nov., 1883) show that the absolute friction, that is, the absolute tan-

gential force per square inch of bearing, required to resist the tendency of the brass to go round with the journal, is nearly a constant under all loads, within ordinary working limits. Most certainly it does not increase in direct proportion to the load, as it should do according to the ordinary theory of solid friction. The results of these experiments seem to show that the friction of a perfectly lubricated journal follows the laws of liquid friction much more closely than those of solid friction. They show that under these circumstances the friction is nearly independent of the pressure per square inch, and that it increases with the velocity, though at a rate not nearly so rapid as the square of the velocity.

The experiments on friction at different temperatures indicate a great diminution in the friction as the temperature rises. Thus in the case of lard-oil, taking a speed of 450 r.p.m., the coefficient of friction at a temperature of 120° is only one-third of what it was at a temperature of 60°.

The journal was of steel, 4 ins. diameter and 6 ins. long, and a gun-metal brass, embracing somewhat less than half the circumference of the journal, rested on its upper side, on which the load was applied. When the bottom of the journal was immersed in oil, and the oil therefore carried under the brass by rotation of the journal, the greatest load carried with rape-oil was 573 lbs. per sq. in., and with mineral oil 625 lbs.

In experiments with ordinary lubrication, the oil being fed in at the center of the top of the brass, and a distributing groove being cut in the brass parallel to the axis of the journal, the bearing would not run cool with only 100 lbs. per sq. in., the oil being pressed out from the bearing-surface and through the oil-hole, instead of being carried in by it. On introducing the oil at the sides through two parallel grooves, the lubrication appeared to be satisfactory, but the bearing seized with 380 lbs. per sq. in.

When the oil was introduced through two oil-holes, one near each end of the brass, and each connected with a curved groove, the brass refused to take its oil or run cool, and seized with a load of only 200 lbs. per sq. in.

With an oil-pad under the journal feeding rape-oil, the bearing fairly carried 551 lbs. Mr. Tower's conclusion from these experiments is that the friction depends on the quantity and uniformity of distribution of the oil, and may be anything between the oil-bath results and seizing, according to the perfection or imperfection of the lubrication. The lubrication may be very small, giving a coefficient of $\frac{1}{100}$; but it appeared as though it could not be diminished and the friction increased much beyond this point without imminent risk of heating and seizing. The oil-bath probably represents the most perfect lubrication possible, and the limit beyond which friction cannot be reduced by lubrication; and the experiments show that with speeds of from 100 to 200 feet per minute, by properly proportioning the bearing-surface to the load, it is possible to reduce the coefficient of friction to as low as $\frac{1}{1000}$. A coefficient of $\frac{1}{1500}$ is easily attainable, and probably is frequently attained, in ordinary engine-bearings in which the direction of the force is rapidly alternating and the oil given an opportunity to get between the surfaces, while the duration of the force in one direction is not sufficient to allow time for the oil film to be squeezed out.

Observations on the behavior of the apparatus gave reason to believe that with perfect lubrication the speed of minimum friction was from 100 to 150 feet per minute, and that this speed of minimum friction tends to be higher with an increase of load, and also with less perfect lubrication. By the speed of minimum friction is meant that speed in approaching which from rest the friction diminishes, and above which the friction increases.

Coefficients of Friction of Motion and of Rest of a Journal. —

A cast-iron journal in steel boxes, tested by Prof. Thurston at a speed of rubbing of 150 feet per minute, with lard and with sperm oil, gave the following:

| Press. per sq. in., lbs. | 50 | 100 | 250 | 500 | 750 | 1000 |
|---------------------------|-------|--------|--------|--------|--------|-------|
| Coeff., with sperm . . . | 0.013 | 0.008 | 0.005 | 0.004 | 0.0043 | 0.009 |
| Coeff., with lard | 0.02 | 0.0137 | 0.0085 | 0.0053 | 0.0066 | 0.125 |

The coefficients at starting were:

| | | | | | | |
|----------------------|------|-------|------|------|-------|------|
| With sperm | 0.07 | 0.135 | 0.14 | 0.15 | 0.185 | 0.18 |
| With lard | 0.07 | 0.11 | 0.11 | 0.10 | 0.12 | 0.12 |

The coefficient at a speed of 150 feet per minute decreases with increase of pressure until 500 lbs. per sq. in. is reached; above this it increases. The coefficient at rest or at starting increases with the pressure throughout the range of the tests.

Coefficients of Friction of Journal with Oil-bath. — Abstract of results of Tower's experiments on friction (*Proc. Inst. M. E.*, Nov., 1883). Journal, 4 in. diam., 6 in. long; temperature, 90° F.

| Lubricant in Bath. | Nominal Load, in lbs. per sq. in. | | | | | | |
|--|-----------------------------------|--------|-------|-------|-------|-------|-------|
| | 625 | 520 | 415 | 310 | 205 | 153 | 100 |
| | Coefficient of Friction. | | | | | | |
| Lard oil: 157 ft. per min. | | .0099 | .0012 | .0014 | .0020 | .0027 | .0042 |
| 471 | | .0017 | .0021 | .0029 | .0042 | .0052 | .009 |
| Mineral grease: 157 ft. per min. | .001 | .0014 | .0016 | .0022 | .0034 | .0038 | .0076 |
| 471 | .002 | .0022 | .0027 | .004 | .0066 | .0083 | .0151 |
| Sperm-oil: 157 ft. per min. | | seiz'd | .0015 | .0011 | .0016 | .0019 | .003 |
| 471 | | | .0021 | .0019 | .0027 | .0037 | .0064 |
| Rape-oil: 157 ft. per min. | (573 lb. .001 | .001 | .0009 | .0008 | .0014 | .002 | .004 |
| 471 | | .0015 | .0016 | .0016 | .0024 | .004 | .007 |
| Mineral-oil: 157 ft. per min. | .0013 | .0012 | .0012 | .0014 | .0021 | | .004 |
| 471 | | .0018 | .002 | .0024 | .0035 | | .007 |
| Rape-oil fed by | | | | | | | |
| siphon lubricator: { 157 ft. per min. | | | | .0056 | .0098 | | .0125 |
| 314 | | | | .0068 | .0077 | | .0152 |
| Rape-oil, pad | | | | | | | |
| under journal: { 157 ft. per min. | | | | .0099 | .0105 | | .0099 |
| 314 | | | | .0099 | .0078 | | .0133 |

Comparative friction of different lubricants under same circumstances, temperature 90°, oil-bath: sperm-oil, 100; rape-oil, 106; mineral oil, 129; lard, 135; olive oil, 135; mineral grease, 217.

Value of Anti-friction Metals. (Denton.) — The various white metals available for lining brasses do not afford coefficients of friction lower than can be obtained with bare brass, but they are less liable to "overheating," because of the superiority of such material over bronze in ability to permit of abrasion or crushing, without excessive increase of friction.

Thurston (Friction and Lost Work) says that gun-bronze, Babbitt, and other soft white alloys have substantially the same friction; in other words, the friction is determined by the nature of the unguent and not by that of the rubbing-surfaces, when the latter are in good order. The soft metals run at higher temperatures than the bronze. This, however, does not necessarily indicate a serious defect, but simply deficient conductivity. The value of the white alloys for bearings lies mainly in their ready reduction to a smooth surface after any local or general injury by alteration of either surface or form.

Cast Iron for Bearings. (Joshua Rose.) — Cast iron appears to be an exception to the general rule, that the harder the metal the greater the resistance to wear, because cast iron is softer in its texture and easier to cut with steel tools than steel or wrought iron, but in some situations it is far more durable than hardened steel; thus when surrounded by steam it will wear better than will any other metal. Thus, for instance, experience has demonstrated that piston-rings of cast iron will wear smoother, better, and equally as long as those of steel, and longer than those of either wrought iron or brass, whether the cylinder in which it works be composed of brass, steel, wrought iron, or cast iron: the latter being the more noteworthy, since two surfaces of the same metal do not, as a rule, wear or work well together. So also slide-valves of brass are not found to wear so long or so smoothly as those of cast iron, let the metal of which the seating is composed be whatever it may; while, on the other hand, a

cast-iron slide-valve will wear longer of itself and cause less wear to its seat, if the latter is of cast iron, than if of steel, wrought iron, or brass.

Friction of Metals under Steam-pressure. — The friction of brass upon iron under steam-pressure is double that of iron upon iron. (G. H. Babcock, *Trans. A. S. M. E.*, i, 151.)

Morin's "Laws of Friction." — 1. The friction between two bodies is directly proportioned to the pressure; i.e., the coefficient is constant for all pressures.

2. The coefficient and amount of friction, pressure being the same, are independent of the areas in contact.

3. The coefficient of friction is independent of velocity, although static friction (friction of rest) is greater than the friction of motion.

Eng'g News, April 7, 1888, comments on these "laws" as follows: From 1831 till about 1876 there was no attempt worth speaking of to enlarge our knowledge of the laws of friction, which during all that period was assumed to be complete, although it was really worse than nothing, since it was for the most part wholly false. In the year first mentioned Morin began a series of experiments which extended over two or three years, and which resulted in the enunciation of these three "fundamental laws of friction," no one of which is even approximately true.

For fifty years these laws were accepted as axiomatic, and were quoted as such without question in every scientific work published during that whole period. Now that they are so thoroughly discredited it has been attempted to explain away their defects on the ground that they cover only a very limited range of pressures, areas, velocities, etc., and that Morin himself only announced them as true within the range of his conditions. It is now clearly established that there are no limits or conditions within which any one of them even approximates to exactitude, and that there are many conditions under which they lead to the wildest kind of error, while many of the constants were as inaccurate as the laws. For example, in Morin's "Table of Coefficients of Moving Friction of Smooth Plane Surfaces, perfectly lubricated," which may be found in hundreds of text-books now in use, the coefficient of wrought iron on brass is given as 0.075 to 0.103, which would make the rolling friction of railway trains 15 to 20 lbs. per ton instead of the 3 to 6 lbs. which it actually is.

General Morin, in a letter to the Secretary of the Institution of Mechanical Engineers, dated March 15, 1879, writes as follows concerning his experiments on friction made more than forty years before: "The results furnished by my experiments as to the relations between pressure, surface, and speed on the one hand, and sliding friction on the other, have always been regarded by myself, not as mathematical laws, but as close approximations to the truth, within the limits of the data of the experiments themselves. The same holds, in my opinion, for many other laws of practical mechanics, such as those of rolling resistance, fluid resistance, etc."

Prof. J. E. Denton (*Stevens Indicator*, July, 1890) says: It has been generally assumed that friction between lubricated surfaces follows the simple law that the amount of the friction is some fixed fraction of the pressure between the surfaces, such fraction being independent of the intensity of the pressure per square inch and the velocity of rubbing, between certain limits of practice, and that the fixed fraction referred to is represented by the coefficients of friction given by the experiments of Morin or obtained from experimental data which represent conditions of practical lubrication, such as those given in Webber's *Manual of Power*.

By the experiments of Thurston, Woodbury, Tower, etc., however, it appears that the friction between lubricated metallic surfaces, such as machine bearings, is not directly proportional to the pressure, is not independent of the speed, and that the coefficients of Morin and Webber are about tenfold too great for modern journals.

Prof. Denton offers an explanation of this apparent contradiction of authorities by showing, with laboratory testing-machine data, that Morin's laws hold for bearings lubricated by a restricted feed of lubricant, such as is afforded by the oil-cups common to machinery; whereas the modern experiments have been made with a surplus feed or superabun-

dance of lubricant, such as is provided only in railroad-car journals, and a few special cases of practice.

That the low coefficients of friction obtained under the latter conditions are realized in the case of car-journals, is proved by the fact that the temperature of car-boxes remains at 100° at high velocities; and experiment shows that this temperature is consistent only with a coefficient of friction of a fraction of one per cent. Deductions from experiments on train resistance also indicate the same low degree of friction. But these low coefficients do not account for the internal friction of steam-engines as well as do the coefficients of Morin and Webber.

In *American Machinist*, Oct. 23, 1890, Prof. Denton says: Morin's measurements of friction of lubricated journals did not extend to light pressures. They apply only to the conditions of general shafting and engine work.

He clearly understood that there was a frictional resistance, due solely to the viscosity of the oil, and that therefore, for very light pressures, the laws which he enunciated did not prevail.

He applied his dynamometers to ordinary shaft-journals without special preparation of the rubbing-surfaces, and without resorting to artificial methods of supplying the oil.

Later experimenters have with few exceptions devoted themselves exclusively to the measurement of resistance practically due to viscosity alone. They have eliminated the resistance to which Morin confined his measurements, namely, the friction due to such contacts of the rubbing-surfaces as prevail with a very thin film of lubricant between comparatively rough surfaces.

Prof. Denton also says (*Trans. A. S. M. E.*, x, 518): "I do not believe there is a particle of proof in any investigation of friction ever made, that Morin's laws do not hold for ordinary practical oil-cups or restricted rates of feed."

Laws of Friction of Well-lubricated Journals. — John Goodman (*Trans. Inst. C. E.*, 1886, *Eng'g News*, April 7 and 14, 1888), reviewing the results obtained from the testing-machines of Thurston, Tower, and Stroudley, arrives at the following laws:

LAWS OF FRICTION: WELL-LUBRICATED SURFACES.

(Oil-bath.)

1. The coefficient of friction with the surfaces efficiently lubricated is from $\frac{1}{6}$ to $\frac{1}{10}$ that for dry or scantily lubricated surfaces.

2. The coefficient of friction for moderate pressures and speeds varies approximately inversely as the normal pressure; the frictional resistance varies as the area in contact, the normal pressure remaining constant.

3. At very low journal speeds the coefficient of friction is abnormally high; but as the speed of sliding increases from about 10 to 100 ft. per min., the friction diminishes, and again rises when that speed is exceeded, varying approximately as the square root of the speed.

4. The coefficient of friction varies approximately inversely as the temperature, within certain limits, namely, just before abrasion takes place.

The evidence upon which these laws are based is taken from various modern experiments. That relating to Law 1 is derived from the "First Report on Friction Experiments," by Mr. Beauchamp Tower.

| Method of Lubrication. | Coefficient of Friction. | Comparative Friction. |
|------------------------|--------------------------|-----------------------|
| Oil-bath..... | 0.00139 | 1.00 |
| Siphon lubricator..... | 0.0098 | 7.06 |
| Pad under journal..... | 0.0090 | 6.48 |

With a load of 293 lbs. per sq. in. and a journal speed of 314 ft. per min. Mr. Tower found the coefficient of friction to be 0.0016 with an oil-bath, and 0.0097, or six times as much, with a pad. The very low coefficients obtained by Mr. Tower will be accounted for by Law 2, as he found that the frictional resistance per square inch under varying loads is nearly constant, as below:

| | | | | | | | | | |
|--------------------------------|-------|-------|-------|-------|-------|-------|------|-------|------|
| Load in lbs. per sq. in. | 529 | 468 | 415 | 363 | 310 | 258 | 205 | 153 | 100 |
| Frictional resist. per sq. in. | 0.416 | 0.514 | 0.498 | 0.472 | 0.464 | 0.438 | 0.43 | 0.458 | 0.45 |

The frictional resistance per square inch is the product of the coefficient of friction into the load per square inch on horizontal sections of the brass. Hence, if this product be a constant, the one factor must vary inversely as the other, or a high load will give a low coefficient, and *vice versa*.

For ordinary lubrication, the coefficient is more constant under varying loads; the frictional resistance then varies directly as the load, as shown by Mr. Tower in Table VIII of his report (*Proc. Inst. M. E.*, 1883).

With respect to Law 3, A. M. Wellington (*Trans. A. S. C. E.*, 1884), in experiments on journals revolving at very low velocities, found that the friction was then very great, and nearly constant under varying conditions of the lubrication, load, and temperature. But as the speed increased the friction fell slowly and regularly, and again returned to the original amount when the velocity was reduced to the same rate. This is shown in the following table:

Speed, feet per minute:

0 + 2.16 3.33 4.86 8.82 21.42 35.37 53.01 89.28 106.02

Coefficient of friction:

0.118 0.094 0.070 0.069 0.055 0.047 0.040 0.035 0.030 0.026

It was also found by Prof. Kimball that when the journal velocity was increased from 6 to 110 ft. per minute, the friction was reduced 70%; in another case the friction was reduced 67% when the velocity was increased from 1 to 100 ft. per minute; but after that point was reached the coefficient varied approximately with the square root of the velocity.

The following results were obtained by Mr. Tower:

| Feet per minute. . . | 209 | 262 | 314 | 366 | 419 | 471 | Nominal Load per sq. in. |
|---------------------------|--------|--------|--------|--------|--------|--------|--------------------------|
| Coeff. of friction. . . . | 0.0010 | 0.0012 | 0.0013 | 0.0014 | 0.0015 | 0.0017 | 520 lbs. |
| " " " | .0013 | .0014 | .0015 | .0017 | .0018 | .002 | 468 lbs. |
| " " " | .0014 | .0015 | .0017 | .0019 | .0021 | .0024 | 415 lbs. |

The variation of friction with temperature is approximately in the inverse ratio, Law 4. Take, for example, Mr. Tower's results, at 262 ft. per minute:

| Temp. F. | 110° | 100° | 90° | 80° | 70° | 60° |
|---------------------|---------|---------|---------|---------|---------|---------|
| Observed. | 0.0044 | 0.0051 | 0.006 | 0.0073 | 0.0092 | 0.0119 |
| Calculated. | 0.00451 | 0.00518 | 0.00608 | 0.00733 | 0.00954 | 0.01252 |

This law does not hold good for pad or siphon lubrication, as then the coefficient of friction diminishes more rapidly for given increments of temperature, but on a gradually decreasing scale, until the normal temperature has been reached; this normal temperature increases directly as the load per sq. in. This is shown in the following table taken from Mr. Stroudley's experiments with a pad of rape-oil:

| Temp. F. | 105° | 110° | 115° | 120° | 125° | 130° | 135° | 140° | 145° |
|----------------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Coefficient. | 0.022 | 0.0180 | 0.0160 | 0.0140 | 0.0125 | 0.0115 | 0.0110 | 0.0106 | 0.0102 |
| Decrease of coeff. | | 0.0040 | 0.0020 | 0.0020 | 0.0015 | 0.0010 | 0.0005 | 0.0004 | 0.0002 |

In the Galton-Westinghouse experiments it was found that with velocities below 100 ft. per min., and with low pressures, the frictional resistance varied directly as the normal pressure; but when a velocity of 100 ft. per min. was exceeded, the coefficient of friction greatly diminished:

from the same experiments Prof. Kennedy found that the coefficient of friction for high pressures was sensibly less than for low.

Allowable Pressures on Bearing-surfaces. (*Proc. Inst. M. E.*, May, 1888.)—The Committee on Friction experimented with a steel ring of rectangular section, pressed between two cast-iron disks, the annular bearing-surfaces of which were covered with gun-metal, and were 12 in. inside diameter and 14 in. outside. The two disks were rotated together, and the steel ring was prevented from rotating by means of a lever, the holding force of which was measured. When oiled through grooves cut in each face of the ring and tested at from 50 to 130 revs. per min., it was found that a pressure of 75 lbs. per sq. in. of bearing-surface was as much as it would bear safely at the highest speed without seizing, although it carried 90 lbs. per sq. in. at the lowest speed. The coefficient of friction is also much higher than for a cylindrical bearing, and the friction follows the law of the friction of solids much more nearly than that of liquids. This is doubtless due to the much less perfect lubrication applicable to this form of bearing compared with a cylindrical one. The coefficient of friction appears to be about the same with the same load at all speeds, or, in other words, to be independent of the speed; but it seems to diminish somewhat as the load is increased, and may be stated approximately as $\frac{1}{20}$ at 15 lbs. per sq. in., diminishing to $\frac{1}{30}$ at 75 lbs. per sq. in.

The high coefficients of friction are explained by the difficulty of lubricating a collar-bearing. It is similar to the slide-block of an engine, which can carry only about one-tenth the load per sq. in. that can be carried by the crank-pins.

In experiments on cylindrical journals it has been shown that when a cylindrical journal was lubricated from the side on which the pressure bore, 100 lbs. per sq. in. was the limit of pressure that it would carry; but when it came to be lubricated on the lower side and was allowed to drag the oil in with it, 600 lbs. per sq. in. was reached with impunity; and if the 600 lbs. per sq. in., which was reckoned upon the full diameter of the bearing, came to be reckoned on the sixth part of the circle that was taking the greater proportion of the load, it followed that the pressure upon that part of the circle amounted to about 1200 lbs. per sq. in.

In connection with these experiments Mr. Wicksteed states that in drilling-machines the pressure on the collars is frequently as high as 336 lbs. per sq. in., but the speed of rubbing in this case is lower than it was in any of the experiments of the Research Committee. In machines working very slowly and intermittently, as in testing-machines, very much higher pressures are admissible.

Mr. Adamson mentions the case of a heavy upright shaft carried upon a small footstep-bearing, where a weight of at least 20 tons was carried on a shaft of 5 in. diameter, or, say, 20 sq. in. area, giving a pressure of 1 ton per sq. in. The speed was 190 to 200 revs. per min. It was necessary to force the oil under the bearing by means of a pump. For heavy horizontal shafts, such as a fly-wheel shaft, carrying 100 tons on two journals, his practice for getting oil into the bearings was to flatten the journal along one side throughout its whole length to the extent of about an eighth of an inch in width for each inch in diameter up to 8 in. diameter; above that size rather less flat in proportion to the diameter. At first sight it appeared alarming to get a continuous flat place coming round in every revolution of a heavily loaded shaft; yet it carried the oil effectually into the bearing, which ran much better in consequence than a truly cylindrical journal without a flat side.

In thrust-bearings on torpedo-boats Mr. Thornycroft allows a pressure of never more than 50 lbs. per sq. in.

Prof. Thurston (*Friction and Lost Work*, p. 240) says 7000 to 9000 lbs. pressure per square inch is reached on the slow-working and rarely moved pivots of swing bridges.

Mr. Tower says (*Proc. Inst. M. E.*, Jan., 1884): In eccentric-pins of punching and shearing machines very high pressures are sometimes used without seizing. In addition to the alternation in the direction, the pressure is applied for only a very short space of time in these machines, so that the oil has no time to be squeezed out.

In the discussion on Mr. Tower's paper (*Proc. Inst. M. E.*, 1885) it was stated that it is well known from practical experience that with a con-

stant load on an ordinary journal it is difficult and almost impossible to have more than 200 lbs. per square inch, otherwise the bearing would get hot and the oil go out of it; but when the motion was reciprocating, so that the load was alternately relieved from the journal, as with crank-pins and similar journals, much higher loads might be applied than even 700 or 800 lbs. per square inch.

Mr. Goodman (*Proc. Inst. C. E.*, 1886) found that the total frictional resistance is materially reduced by diminishing the width of the brass.

The lubrication is most efficient in reducing the friction when the brass subtends an angle of from 120° to 60° . The film is probably at its best between the angles 80° and 110° .

In the case of a brass of a railway axle-bearing where an oil-groove is cut along its crown and an oil-hole is drilled through the top of the brass into it, the wear is invariably on the off side, which is probably due to the oil escaping as soon as it reaches the crown of the brass, and so leaving the off side almost dry, where the wear consequently ensues.

In railway axles the brass wears always on the forward side. The same observation has been made in marine-engine journals, which always wear in exactly the reverse way to what might be expected. Mr. Stroudley thinks this peculiarity is due to a film of lubricant being drawn in from the under side of the journal to the aft part of the brass, which effectually lubricates and prevents wear on that side; and that when the lubricant reaches the forward side of the brass it is so attenuated down to a wedge shape that there is insufficient lubrication, and greater wear consequently follows.

C. J. Field (*Power*, Feb., 1893) says: One of the most vital points of an engine for electrical service is that of main bearings. They should have a surface velocity of not exceeding 350 feet per minute, with a mean bearing-pressure per square inch of projected area of journal of not more than 80 lbs. This is considerably within the safe limit of cool performance and easy operation. If the bearings are designed in this way, it would admit the use of grease on all the main wearing-surface, which in a large type of engines for this class of work we think advisable.

Oil-pressure in a Bearing. — Mr. Beauchamp Tower (*Proc. Inst. M. E.*, Jan., 1885) made experiments with a brass bearing 4 ins. diameter by 6 ins. long, to determine the pressure of the oil between the brass and the journal. The bearing was half immersed in oil, and had a total load of 8008 lbs. upon it. The journal rotated 150 r.p.m. The pressure of the oil was determined by drilling small holes in the bearing at different points and connecting them by tubes to a Bourdon gauge. It was found that the pressure varied from 310 to 625 lbs. per sq. in., the greatest pressure being a little to the "off" side of the center line of the top of the bearing, in the direction of motion of the journal. The sum of the upward force exerted by these pressures for the whole lubricated area was nearly equal to the total pressure on the bearing. The speed was reduced from 150 to 20 r.p.m., but the oil-pressure remained the same, showing that the brass was as completely oil-borne at the lower speed as at the higher. The following was the observed friction at the lower speed:

| | | | | |
|--|---------|---------|---------|--------|
| Nominal load, lbs. per sq. in. | 443 | 333 | 211 | 89 |
| Coefficient of friction. | 0.00132 | 0.00168 | 0.00247 | 0.0044 |

The nominal load per square inch is the total load divided by the product of the diameter and length of the journal. At the low speed of 20 r.p.m. it was increased to 676 lbs. per sq. in. without any signs of heating or seizing.

Friction of Car-journal Brasses. (J. E. Denton, *Trans. A. S. M. E.*, xii, 405.) — A new brass dressed with an emery-wheel, loaded with 5000 lbs., may have an actual bearing-surface on the journal, as shown by the polish of a portion of the surface, of only 1 square inch. With this pressure of 5000 lbs. per sq. in., the coefficient of friction may be 6%, and the brass may be overheated, scarred and cut, but, on the contrary, it may wear down evenly to a smooth bearing, giving a highly polished area of contact of 3 sq. ins., or more, inside of two hours of running, gradually decreasing the pressure per square inch of contact, and a coefficient of friction of less than 0.5%. A reciprocating motion in the direction of the axis is of importance in reducing the friction. With such polished surfaces any oil will lubricate, and the coefficient of friction then depends

on the viscosity of the oil. With a pressure of 1000 lbs. per sq. in., revolutions from 170 to 320 per min., and temperatures of 75° to 113° F., with both sperm and paraffine oils, a coefficient of as low as 0.11% has been obtained, the oil being fed continuously by a pad.

Experiments on Overheating of Bearings.—Hot Boxes. (Denton.) — Tests with car brasses loaded from 1100 to 4500 lbs. per sq. in. gave 7 cases of overheating out of 32 trials. The tests show how purely a matter of chance is the overheating, as a brass which ran hot at 5000 lbs. load on one day would run cool on a later date at the same or higher pressure. The explanation of this apparently arbitrary difference of behavior is that the accidental variations of the smoothness of the surfaces, almost infinitesimal in their magnitude, cause variations of friction which are always tending to produce overheating, and it is solely a matter of chance when these tendencies preponderate over the lubricating influence of the oil. There is no appreciable advantage shown by sperm-oil, when there is no tendency to overheat — that is, paraffine can lubricate under the highest pressures which occur, as well as sperm, when the surfaces are within the conditions affording the minimum coefficients of friction.

Sperm and other oils of high heat-resisting qualities, like vegetable oil and petroleum cylinder stocks, differ from the more volatile lubricants, like paraffine, only in their ability to reduce the chances of the continual accidental infinitesimal abrasion producing overheating.

The effect of emery or other gritty substance in reducing overheating of a bearing is thus explained:

The effect of the emery upon the surfaces of the bearings is to cover the latter with a series of parallel grooves, and apparently after such grooves are made the presence of the emery does not practically increase the friction over its amount when pure oil only is between the surfaces. The infinite number of grooves constitute a very perfect means of insuring a uniform oil supply at every point of the bearings. As long as grooves in the journal match with those in the brasses the friction appears to amount to only about 10% to 15% of the pressure. But if a smooth journal is placed between a set of brasses which are grooved, and pressure be applied, the journal crushes the grooves and becomes brazed or coated with brass, and then the coefficient of friction becomes upward of 40%. If then emery is applied, the friction is made very much less by its presence, because the grooves are made to match each other, and a uniform oil supply prevails at every point of the bearings, whereas before the application of the emery many spots of the bearing receive no oil between them.

Moment of Friction and Work of Friction of Sliding-surfaces, etc.

| | Moment of Friction, inch-lbs. | Energy lost by Friction in ft.-lbs. per min. |
|---|---|--|
| Flat surfaces..... | | fWS |
| Shafts and journals..... | $\frac{1}{2}fWd$ | $0.2618fWdn$ |
| Flat pivots..... | $\frac{2}{3}fWr$ | $0.349fWrn$ |
| Collar-bearing..... | $\frac{2}{3}fW \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ | $0.349fWn \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ |
| Conical pivot..... | $\frac{2}{3}fWr \operatorname{cosec} a$ | $0.349fWrn \operatorname{cosec} a$ |
| Conical journal..... | $\frac{2}{3}fWr \sec a$ | $0.349fWrn \sec a$ |
| Truncated-cone pivot..... | $\frac{2}{3}fW \frac{r_2^3 - r_1^3}{r_2 \sin a}$ | $0.349fW \frac{r_2^3 - r_1^3}{r_2 \sin a}$ |
| Hemispherical pivot..... | fWr | $0.5236fWrn$ |
| Tractrix, or Schiele's "anti-friction" pivot..... | fWr | $0.5236fWrn$ |

In the above f = coefficient of friction;

W = weight on journal or pivot in pounds;

r = radius, d = diameter, in inches;

S = space in feet through which sliding takes place;

r_2 = outer radius, r_1 = inner radius;

n = number of revolutions per minute;

a = the half-angle of the cone, i.e., the angle of the slope with the axis.

To obtain the horse-power, divide the quantities in the last column by 33,000. Horse-power absorbed by friction of a shaft = $\frac{fWdn}{126,050}$.

The formula for energy lost by shafts and journals is approximately true for loosely fitted bearings. Prof. Thurston shows that the correct formula varies according to the character of fit of the bearing; thus for loosely fitted journals, if U = the energy lost,

$$U = \frac{2f\pi r}{\sqrt{1+f^2}} Wn \text{ inch-pounds} = \frac{0.2618 fWdn}{\sqrt{1+f^2}} \text{ foot-lbs.}$$

For perfectly fitted journals $U = 2.54 f\pi rWn$ inch-lbs. = $0.3325 fWdn$ ft.-lbs.

For a bearing in which the journal is so grasped as to give a uniform pressure throughout, $U = f\pi^2 rWn$ inch-lbs. = $0.4112 fWdn$ ft.-lbs.

Resistance of railway trains and wagons due to friction of trains:

Pull on draw-bar = $f \times 2240 \div R$ pounds per gross ton,

in which R is the ratio of the radius of the wheel to the radius of journal.

A cylindrical journal, perfectly fitted into a bearing, and carrying a total load, distributes the pressure due to this load unequally on the bearing, the maximum pressure being at the extremity of the vertical radius, while at the extremities of the horizontal diameter the pressure is zero. At any point of the bearing-surface at the extremity of a radius which makes an angle θ with the vertical radius the normal pressure is proportional to $\cos \theta$. If p = normal pressure on a unit of surface, w = total load on a unit of length of the journal, and r = radius of journal,

$$w \cos \theta = 1.57 rp, \quad p = w \cos \theta \div 1.57 r.$$

Tests of Large Shaft Bearings are reported by Albert Kingsbury in *Trans. A. S. M. E.*, 1905. A horizontal shaft was supported in two bearings 9×30 ins., and a third bearing 15×40 ins., midway between the other two, was pressed upwards against the shaft by a weighed lever, so that it was subjected to a pressure of 25 to 50 tons. The journals were flooded with oil from a supply tank. The shaft was driven by an electric motor, and the friction H.P. was determined by measuring the current supplied. Following are the principal results:

| | | | | | | | | | |
|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Load, tons* | | | | | | | | | |
| 25 | 25 | 25 | 25 | 33.6 | 42.3 | 47 | 47 | 50.5 | |
| Load per sq. in.* | | | | | | | | | |
| 83 | 83 | 83 | 83 | 112 | 141 | 157 | 157 | 168 | |
| Speed, r.p.m. | | | | | | | | | |
| 309 | 506 | 179 | 301 | 454 | 480 | 946 | 1243 | 1286 | |
| Speed, ft. per min.* | | | | | | | | | |
| 1215 | 1990 | 708 | 704 | 1180 | 1785 | 1890 | 3720 | 4900 | 5050 |
| Friction H.P.† | | | | | | | | | |
| 12.6 | 21.7 | 6.43 | 5.12 | 10.1 | 16 | 17.9 | 41.9 | 47.8 | 52.3 |
| Coeff. of friction† | | | | | | | | | |
| .0045 | .0048 | .0040 | .0037 | .0037 | .0029 | .0024 | .0025 | .0022 | .0022 |

* On the large bearing.

† Three bearings.

The last three tests were with paraffin oil; the others with heavy machine oil.

Clearance between Journal and Bearing. — John W. Upp, in *Trans. A. S. M. E.*, 1905 gives a table showing the diameter of bore of horizontal and vertical bearings according to the practice of one of the leading builders of electrical machinery. The maximum diameter of the journal is the same as its nominal diameter, with an allowable variation below maximum of 0.0005 in. up to 3 in. diam., 0.001 in. from $3\frac{1}{2}$ to 9 in., and 0.0015 in. from 10 to 24 in. The maximum bore of a horizontal bearing is larger than the diam. of the journal by from 0.002 in. for a $\frac{1}{2}$ -in. journal to 0.009 for 6 in., for journals 7 to 15 in. it is $0.004 + 0.001 \times$ diam., and for 16 to 24 in. it is uniformly 0.02 in. For vertical journals the clearance is less by from 0.001 to 0.004 in. according to the diameter. The allowable variation above the minimum bore is from 0.001 to 0.005.

Allowable Pressures on Bearings. — J. T. Nicholson, in a paper read before the Manchester Assoc. of Engrs. (*Am. Mach.*, Jan. 16, 1908,

Eng. Digest, Feb., 1908), as a result of a theoretical study of the lubrication of bearings and of their emission of heat, obtains the formula $p = P/ld = 40 (dN)^{1/4}$, in which p = allowable pressure per sq. in. of projected area, P = total pressure, l = length and d = diam. of journal, N = revs. per min. It appears from this formula that the greater the speed the greater the allowable pressure per sq. in., so that for a 1-in. journal the allowable pressure per sq. in. is 126 lbs. at 100 r.p.m. and 189 lbs. at 500 r.p.m., and for a 5-in. journal 189 lbs. at 100 and 283 lbs. at 500 r.p.m. W. H. Scott (*Eng. Digest*, Feb., 1908) says this is contrary to the teaching of practical experience, and therefore the formula is inaccurate. Mr. Scott, from a study of the experiments of Tower, Lasche, and Stribeck, derives the following formulæ for the several conditions named:

For main bearings of double-acting vertical engines. $p = 750 D^{1/4}/N^{1/4}$
 " " " " " " " horizontal " $p = 660 D^{1/12}/N^{1/4}$
 " " " " " single-acting four-cycle gas engines $p = 1350 D^{1/12}/N^{1/4}$
 For crank pins of vert. and hor. double-acting engines. $p = 1560 D^{1/4}/N^{1/4}$
 " " " " single-acting four-cycle gas engines. $p = 3000 D^{1/4}/N^{1/4}$
 For dead loads with ordinary lubrication $p = 400 N^{-1/5}$
 " " " " forced " $p = 1600 N^{-1/4}$
 p = allowable pressure in lbs. per sq. in. of projected area; D = diam. in ins.; N = revs. per. min.

F. W. Taylor (*Trans. A. S. M. E.*, 1905), as the result of an investigation of line shaft and mill bearings that were running near the limit of durability and heating yet not dangerously heating, gives the formula $PV = 400$. P = pressure in lbs. per sq. in. of projected area, V = velocity of circumference of bearing in ft. per sec.

The formula is applicable to bearings in ordinary shop or mill use on shafting which is intended to run with the care and attention which such bearings usually receive, and gives the maximum or most severe duty to which it is safe to subject ordinary *chain or oiled* ball and socket bearings which are *babbitted*. It is not safe for ordinary shafting to use *cast-iron boxes*, with either sight feed, wick feed, or grease-cup oiling, under as severe conditions as $P \times V = 200$.

Archbutt and Deeley's "Lubrication and Lubricants" gives the following table of allowable pressures in lbs. per sq. in. of projected area of different bearings:

| | |
|---|---------|
| Crank-pin of shearing and punching machine, hard steel, intermittent load bearing | 3000 |
| Bronze crosshead neck journals | 1200 |
| Crank pins, large slow engine | 800-900 |
| Crank pins, marine engines | 400-500 |
| Main crankshaft bearing, fast marine | 400 |
| Same, slow marine | 600 |
| Railway coach journals | 300-400 |
| Flywheel shaft journals | 150-200 |
| Small engine crank pin | 150-200 |
| Small slide block, marine engine | 100 |
| Stationary engine slide blocks | 25-125 |
| Same, usual case | 30- 60 |
| Propeller thrust bearings | 50- 70 |
| Shafts in cast-iron steps, high speed | 15 |

Bearing Pressures for Heavy Intermittent Loads. (Oberlin Smith, *Trans. A. S. M. E.*, 1905.) — In a punching press of about 84 tons capacity, the pressure upon the front journal of the main shaft is about 2400 lbs. per sq. in. of projected area. Upon the eccentric the pressure against the pitman driving the ram is some 7000 lbs. per sq. in. — both surfaces being of cast iron, and sometimes running at a surface speed of 140 feet per minute. Such machines run year in and year out with but little trouble in the way of heating or "cutting." An instance of excessive pressure may be cited in the case of a Ferracute toggle press, where the whole ram pressure of 400 tons is brought to bear upon hardened steel

toggle-pins, running in cast iron or bronze bearings, 3 in. in diam. by nearly 14 in. long. These run habitually, for maximum work, under a load of 20,000 lbs. per sq. in.

Bearings for Very High Rotative Speeds. (*Proc. Inst. M. E.*, Oct., 1888, p. 482.) — In the Parsons steam-turbine, which has a speed as high as 18,000 rev. per min., as it is impossible to secure absolute accuracy of balance, the bearings are of special construction so as to allow of a certain very small amount of lateral freedom. For this purpose the bearing is surrounded by two sets of steel washers $\frac{1}{16}$ in. thick and of different diameters, the larger fitting close in the casing and about $\frac{1}{32}$ in. clear of the bearing, and the smaller fitting close on the bearing and about $\frac{1}{32}$ in. clear of the casing. These are arranged alternately, and are pressed together by a spiral spring. Consequently any lateral movement of the bearing causes them to slide mutually against one another, and by their friction to check or damp any vibrations that may be set up in the spindle. The tendency of the spindle is then to rotate about its axis of mass, and the bearings are thereby relieved from excessive pressure, and the machine from undue vibration. The allowing of the turbine itself to find its own center of gyration is a well-known device in other branches of mechanics: as in the instance of the centrifugal hydro-extractor, where a mass very much out of balance is allowed to find its own center of gyration: the faster it runs the more steadily does it revolve and the less is the vibration. Another illustration is to be found in the spindles of spinning machinery which run at about 10,000 or 11,000 revs. per min.: although of very small dimensions, the outside diameter of the largest portion or driving whorl being perhaps not more than $1\frac{1}{4}$ in., it is found impracticable to run them at that speed in what might be called a hard-and-fast bearing. They are therefore run with some elastic substance surrounding the bearing, such as steel springs, hemp, or cork. Any elastic substance is sufficient to absorb the vibration, and permit of absolutely steady running.

Thrust Bearings in Marine Practice. (G. W. Dickie, *Trans. A. S. M. E.*, 1905.) — The approximate pressure on a thrust bearing of a propeller shaft assuming two thirds of the indicated horse-power to be effective on the propeller is $P = \text{I.H.P.} \times \frac{2 \times 60 \times 33000}{S \times 3 \times 6080} = \frac{\text{I.H.P.}}{S} \times 217.1$, in which S = speed of ship in knots per hour, P = total thrust in lbs. The following are data of water-cooled bearings which have given satisfactory service:

| | | | | |
|--|---------|------------------|--------|---------|
| Speed in knots..... | 22 | 22 $\frac{1}{2}$ | 28 | 21 |
| Thrust-rins surface, horse-shoe type, sq. ins..... | 1188 | 891 | 581 | 2268 |
| Horse-power, one engine, I.H.P. | 11,500 | 6,800 | 4,200 | 15,000 |
| Indicated pressure on bearing, lbs.,... | 112,700 | 89,000 | 33,600 | 154,000 |
| Pressure per sq. in. of surface, lbs. | 95 | 100 | 58 | 68.1 |
| Mean speed of bearing surfaces, ft. per min..... | 642 | 610 | 827 | 504 |

Bearings for Locomotives. (G. M. Basford, *Trans. A. S. M. E.*, 1905.) — Bearing areas for locomotive journals are determined chiefly by the possibilities of lubrication. On driving journals the following figures of pressure in lbs. per sq. in. of projected area give good service: passenger, 190; freight, 200; switching, 220 lbs. Crank pins may be loaded from 1500 to 1700 lbs.; wrist pins to 4000 lbs. per sq. in. Car and tender bearings are usually loaded from 300 to 325 lbs. per sq. in.

Bearings of Corliss Engines. (P. H. Been, *Trans. A. S. M. E.*, 1905.) — In the practice of one of the largest builders the greatest pressure allowed per sq. in. of projected area for all shafts is 140 lbs. On most engines the pressure per sq. in. multiplied by the velocity of the bearing surface in ft. per sec. lies between 1000 and 1300.

Edwin Reynolds says that a main engine bearing to be safe against undue heating should be of such a size that the product of the square root of the speed of rubbing-surface in feet per second multiplied by the pounds per square inch of projected area, should not exceed 375 for a horizontal engine, or 500 for a vertical engine when the shaft is lifted at every revolution. Locomotive driving boxes in some cases give the product as high

as 585, but this is accounted for by the cooling action of the air. (*Am. Mach.*, Sept. 17, 1903.)

Temperature of Engine Bearings. (A. M. Mattice, *Trans. A. S. M. E.*, 1905.)—An examination of the temperature of bearings of a large number of engines of various makes showed more above 135° F. than below that figure. Many bearings were running with a temperature over 150°, and in one case at 180°, and in all of these cases the bearings were giving no trouble.

PIVOT-BEARINGS.

The Schiele Curve.—W. H. Harrison (*Am. Mach.*, 1891) says the Schiele curve is not as good a form for a bearing as the segment of a sphere. He says: A mill-stone weighing a ton frequently bears its whole weight upon the flat end of a hard-steel pivot 1 1/8 in. diam., or 1 sq. in. area of bearing; but to carry a weight of 3000 lbs. he advises an end bearing about 4 ins. diam., made in the form of a segment of a sphere about 1/2 in. in height. The die or fixed bearing should be dish-shaped to fit the pivot. This form gives a chance for the bearing to adjust itself, which it does not have when made flat, or when made with the Schiele curve. If a side bearing is necessary it can be arranged farther up the shaft. The pivot and die should be of steel, hardened; cross-gutters should be in the die to allow oil to flow, and a central oil-hole should be made in the shaft.

The advantage claimed for the Schiele bearing is that the pressure is uniformly distributed over its surface, and that it therefore wears uniformly. Wilfred Lewis (*Am. Mach.*, April 19, 1894) says that its merits as a thrust-bearing have been vastly overestimated; that the term "anti-friction" applied to it is a misnomer, since its friction is greater than that of a flat step or collar of the same diameter. He advises that flat thrust-bearings should always be annular in form, having an inside diameter one-half of the external diameter.

Friction of a Flat Pivot-bearing.—The Research Committee on Friction (*Proc. Inst. M. E.*, 1891) experimented on a step-bearing, flat-topped, 3 in. diam., the oil being forced into the bearing through a hole in its center and distributed through two radial grooves, insuring thorough lubrication. The step was of steel and the bearing of manganese-bronze.

| | | | | | |
|------------------------------|------------|--------|--------|--------|--------|
| At revolutions per min. | 50 | 128 | 194 | 290 | 353 |
| The coefficient of friction} | 0.0181 | 0.0053 | 0.0051 | 0.0044 | 0.0053 |
| varied between } | and 0.0221 | 0.0113 | 0.0102 | 0.0178 | 0.0167 |

With a white-metal bearing at 128 revs. the coefficient of friction was a little larger than with the manganese-bronze. At the higher speeds the coefficient of friction was less, owing to the more perfect lubrication, as shown by the more rapid circulation of the oil. At 128 revs. the bronze-bearing heated and seized on one occasion with a load of 260 lbs., and on another occasion with 300 lbs. per sq. in. The white-metal bearing under similar conditions heated and seized with a load of 240 lbs. per sq. in. The steel footstep on manganese-bronze was afterwards tried, lubricating with three and with four radial grooves; but the friction was from one and a half times to twice as great as with only the two grooves.

Mercury-bath Pivot.—A nearly frictionless step-bearing may be obtained by floating the bearing with its superincumbent weight upon mercury. Such an apparatus is used in the lighthouses of La Heve, Havre. It is thus described in *Eng'g*, July 14, 1893, p. 41:

The optical apparatus, weighing about 1 ton, rests on a circular cast-iron table, which is supported by a vertical shaft of wrought iron 2.36 in. diameter. This is kept in position at the top by a bronze ring and outer iron support, and at the bottom in the same way, while it rotates on a removable steel pivot resting in a steel socket, which is fitted to the base of the support. To the vertical shaft there is rigidly fixed a floating cast-iron ring 17.1 in. diameter and 11.8 in. in depth, which is plunged into and rotates in a mercury bath contained in a fixed outer drum or tank, the clearance between the vertical surfaces of the drum and ring being only 0.2 in., so as to reduce as much as possible the volume of mercury (about 220 lbs.), while the horizontal clearance at the bottom is 0.4 in.

BALL-BEARINGS, ROLLER-BEARINGS, ETC.

Friction-rollers. — If a journal instead of revolving on ordinary bearings be supported on friction-rollers the force required to make the journal revolve will be reduced in nearly the same proportion that the diameter of the axles of the rollers is less than the diameter of the rollers themselves. In experiments by A. M. Wellington with a journal $3\frac{1}{2}$ in. diam. supported on rollers 8 in. diam., whose axles were $1\frac{3}{4}$ in. diam., the friction in starting from rest was $\frac{1}{4}$ the friction of an ordinary $3\frac{1}{2}$ -in. bearing, but at a car speed of 10 miles per hour it was $\frac{1}{2}$ that of the ordinary bearing. The ratio of the diam. of the axle to diam. of roller was $1\frac{3}{4} : 8$, or as 1 to 4.6.

Coefficients of Friction of Roller Bearings. C. H. Benjamin, *Machy.* Oct., 1905. — Comparative tests of plain babbitted, McKeel plain roller, and Hyatt roller bearings gave the following values of the coefficient of friction at a speed of 560 r.p.m.:

| Diameter of Journal. | Hyatt Bearing. | | | McKeel Bearing. | | | Babbitt Bearing. | | |
|----------------------|----------------|------|------|-----------------|-------|-------|------------------|------|------|
| | Max. | Min. | Ave. | Max. | Min. | Ave. | Max. | Min. | Ave. |
| $1\frac{15}{16}$ | .032 | .012 | .018 | .033 | .017 | .022 | .074 | .029 | .043 |
| $2\frac{3}{16}$ | .019 | .011 | .014 | | | | .088 | .078 | .082 |
| $2\frac{7}{16}$ | .042 | .025 | .032 | .028 | .015 | .021 | .114 | .083 | .096 |
| $2\frac{15}{16}$ | .029 | .022 | .025 | .039 | .019 | .027 | .125 | .089 | .107 |

The friction of the roller bearing is from one-fifth to one-third that of a plain bearing at moderate loads and speeds. It is noticeable that as the load on a roller bearing increases the coefficient of friction decreases.

A slight change in the pressure due to the adjusting nuts was sufficient to increase the friction considerably. In the McKeel bearing the rolls bore on a cast-iron sleeve and in the Hyatt on a soft-steel one. If roller bearings are properly adjusted and not overloaded a saving of from 2-3 to 3-4 of the friction may be reasonably expected.

McKeel bearings contained rolls turned from solid steel and guided by spherical ends fitting recesses in cage rings at each end. The cage rings were joined to each other by steel rods parallel to the rolls.

Lubrication is absolutely necessary with ball and roller bearings, although the contrary claim is often advanced. Under favorable conditions an almost imperceptible film is sufficient; a sufficient quantity to immerse half the lowest ball should always be provided as a rust preventive. Rust and grit must be kept out of ball and roller bearings. Acid or rancid lubricants are as destructive as rust. (Henry Hess.)

Both ball and roller bearings, to give the best satisfaction, should be made of steel, hardened and ground; accurately fitted, and in proper alignment with the shaft and load; cleaned and oiled regularly, and fitted with as large-size balls or rollers as possible, depending upon the revolutions per minute and load to be carried. Oil is absolutely necessary on both ball and roller bearings, to prevent rust. (S. S. Eveland.)

Roller Bearings. — The Mossberg roller bearings for journals are made in the sizes given in the table below. D = diam. of journal; d = diam. of roll; N = number of rolls; P = safe load on journals, in lbs. The rolls are enclosed in a bronze supporting cage. (*Trans. A. S. M. E.*, 1905.)

| D | d | N | P | D | d | N | P | D | d | N | P |
|----------------|----------------|-----|--------|-----|-----------------|-----|---------|-------|----------------|-------|---------|
| 2 | $\frac{1}{4}$ | 20 | 3,500 | 6 | $\frac{11}{16}$ | 24 | 50,000 | 15 | $\frac{13}{8}$ | 28 | 255,000 |
| $2\frac{1}{2}$ | $\frac{5}{16}$ | 22 | 7,000 | 7 | $\frac{13}{16}$ | 22 | 70,000 | 18 | $\frac{13}{8}$ | 32 | 325,000 |
| 3 | $\frac{3}{8}$ | 22 | 13,000 | 8 | $\frac{7}{8}$ | 22 | 90,000 | 20 | $\frac{11}{2}$ | 34 | 400,000 |
| 4 | $\frac{7}{16}$ | 24 | 24,000 | 9 | 1 | 24 | 115,000 | 24 | $\frac{11}{2}$ | 38 | 576,000 |
| 5 | $\frac{9}{16}$ | 24 | 37,000 | 12 | $1\frac{1}{4}$ | 26 | 175,000 | | | | |

Surface speed of journal 0 to 50 ft. per min. Length of journal $1\frac{1}{2}$ diameters. The rolls are made of tool steel not too high in carbon, and of spring temper. The journal or shaft should be made not above a medium spring temper. The box should be made of high carbon steel and tempered as hard as possible.

Conical Roller Thrust Bearings. — The Mossberg thrust bearing is made of conical rollers contained in a cage, and two collars, one being stationary and the other fixed to the shaft and revolving with it. One side of each collar is made conical to correspond with the rollers which bear on it. The apex of the cones is at the center of the shaft. The angle of the cones is 6 to 7 degrees. Larger angles are objectionable, giving excessive end thrust. The following sizes are made:

| Diameter of Shaft. Ins. | Outside Diameter of Ring. Ins. | No. of Rolls. | Safe Pressure on Bearing. | | |
|----------------------------------|-----------------------------------|---------------|-------------------------------------|-----------------------|------------------------|
| | | | Area of Pressure Plate. Sq. ins. | Speed 75 Rev. Lbs. | Speed 150 Rev. Lbs. |
| $2\frac{1}{16}$ – $2\frac{1}{4}$ | $5\frac{9}{16}$ | 30 | 10 | 19,000 | 9,500 |
| $3\frac{1}{16}$ – $3\frac{1}{4}$ | 8 | 30 | 20 | 40,000 | 20,000 |
| $4\frac{1}{16}$ – $4\frac{1}{4}$ | $10\frac{5}{16}$ | 30 | 35 | 70,000 | 35,000 |
| $5\frac{1}{16}$ – $5\frac{1}{4}$ | $12\frac{3}{8}$ | 30 | 54 | 108,000 | 56,000 |
| $6\frac{1}{16}$ – $6\frac{1}{2}$ | $14\frac{7}{8}$ | 30 | 78 | 125,000 | 62,000 |
| $8\frac{1}{16}$ – $8\frac{1}{2}$ | $18\frac{3}{4}$ | 32 | 132 | 200,000 | 100,000 |
| $9\frac{1}{16}$ – $9\frac{1}{2}$ | $20\frac{1}{2}$ | 32 | 162 | 300,000 | 150,000 |

Plain Roller Thrust Bearings. — S. S. Eveland, of the Standard Roller Bearing Co., contributes the following data of plain roller thrust bearings in use in 1903. The bearing consists of a large number of short cylindrical rollers enclosed in openings in a disk placed between two hardened steel plates. He says "our plain roller bearing is theoretically wrong, but in practice it works perfectly, and has replaced many thousand ball-bearings which have proven unsatisfactory."

| Size of Bearing. ins. | Number and Size of Rollers. ins. | R.p.m. | Weight on Bearings, lbs. | Lineal inches. | Weight per lin. in., lbs. | Weight on each roll, lbs. |
|--------------------------------------|---------------------------------------|--------|--------------------------|-----------------|---------------------------|---------------------------|
| $4\frac{3}{4} \times 6\frac{11}{16}$ | 36 $5\frac{5}{8} \times 5\frac{1}{8}$ | 500 | 6,000 | $11\frac{1}{4}$ | 546 | 167 |
| $4\frac{3}{4} \times 7\frac{1}{4}$ | 32 $3\frac{3}{4} \times 5\frac{5}{8}$ | 470 | 10,000 | 12 | 833 | 312 |
| $5\frac{1}{2} \times 8\frac{1}{2}$ | 54 $3\frac{3}{4} \times 5\frac{5}{8}$ | 420 | 15,000 | $20\frac{1}{4}$ | 750 | 279 |
| $7 \times 10\frac{3}{8}$ | 48 $1 \times 1\frac{1}{2}$ | 370 | 20,000 | 24 | 833 | 417 |
| $7\frac{1}{2} \times 11\frac{5}{16}$ | 54 $1 \times 1\frac{1}{2}$ | 325 | 25,000 | 27 | 988 | 463 |
| $8 \times 15\frac{1}{2}$ | 70 $1\frac{1}{4} \times 5\frac{5}{8}$ | 300 | 60,000 | 45 | 1334 | 833 |

The Hyatt Roller Bearing. (A. L. Williston, *Trans. A. S. M. E.*, 1905.) — The distinctive feature of the Hyatt roller bearing is a flexible roller, made of a strip of steel wound into a coil or spring of uniform diameter. A roller of this construction insures a uniform distribution of the load along the line of contact of the roller and the surfaces on which it operates. It also permits any slight irregularities in either journal or box without causing excessive pressure. The roller is hollow and serves as an oil reservoir. For a heavy load, a roller of heavy stock can be made, while for a high-speed bearing under light pressure a roller of light weight, made from thin stock, can be used. Following are the results of some tests of the Hyatt bearing in comparison with other bearings:

A shaft 152 ft. long, $2\frac{15}{16}$ in. diam. supported by 20 bearings, belt-driven from one end, gave a friction load of 2.28 H.P. with babbitted bearings, and 0.80 H.P. with Hyatt bearings. With 88 countershafts running in babbitted bearings, the H.P. required was 8.85 when the main shaft was in babbitted bearings and 6.36 H.P. when it was in Hyatt bearings.

Comparative tests of solid rollers and of Hyatt rollers were made in 1898 at the Franklin Institute by placing two sets of rollers between three flat plates, putting the plates under load in a testing machine and measuring the force required to move the middle plate. All the rollers were $\frac{3}{4}$ in. diam., 10 ins. long. The Hyatt rollers were made of $\frac{1}{2} \times \frac{1}{8}$ in. steel strip. With 2000 lbs. load and plain rollers it took 26 lbs. to move the plate, and with the Hyatt rollers 9 lbs. With 3000 lbs. load and plain rollers the resistance was 34 lbs., with Hyatt rollers 17 lbs.

In tests with a pendulum friction testing machine at the Case Scientific School, with a bearing $1\frac{5}{16}$ in. diam. the coefficient of friction with the Hyatt bearing was from 0.0362 down to 0.0196, the loads increasing from 64 to 264 lbs.; with cast-iron bearings and the same loads the coefficient was from 0.165 to 0.098.

In tests at Purdue University with bearings $4 \times 1\frac{1}{2}$ ins. and loads from 1900 to 8300 lbs., the average coefficients with different bearings and different speeds were as follows:

| | | | | | | |
|-------------------|------------|--------|------------|--------|------------|--------|
| Hyatt bearing | 130 r.p.m. | 0.0114 | 302 r.p.m. | 0.0099 | 585 r.p.m. | 0.0147 |
| Cast-iron bearing | 128 | " | 0.0548 | 302 | " | 0.0592 |
| | | | | 410 | " | 0.0683 |
| Bronze bearing | 130 | " | 0.0576 | 320 | " | 0.0661 |
| | | | | 582 | " | 0.140 |

The cast-iron bearing at 128 r.p.m. seized with 8300 lbs., and at 410 r.p.m. with 5900 lbs. The bronze bearing seized at 130 r.p.m. with 3500 lbs., at 320 r.p.m. with 5100 lbs., and at 582 r.p.m. with 2700 lbs.

The makers have found that the advantages of roller bearings of the type described are especially great with either high speeds or heavy loads. Generally, the best results are obtained for line-shaft work up to speeds of 600 rev. per min., when a load of 30 lbs. per square inch of projected area is allowed. For heavy load at slow speed, such as in crane and truck wheels, a load of 500 lbs. gives the best results.

The Friction Coefficient of a well-made annular ball-bearing is 0.001 and 0.002 of the load referred to the shaft diameter and is independent of the speed and load. The friction coefficient of a good roller bearing is from 0.0035 to 0.014; it rises very much if the load is light. It increases also when the speeds are very low, though not so much as with plain bearings. (Henry Hess.)

Notes on Ball Bearings. — The following notes are contributed by Mr. Henry Hess, 1910. Ball bearings in modern use date from the bicycle. That brought in the adjustable cup and cone and three-point contact type. Under the demands for greater load resistance and reliability the two-point contact type, without adjustability, was evolved; that is now used under loads from a few pounds to many tons. Such a bearing consists of an inner race, an outer race and the series of balls that roll in tracks of curved cross section. Various designs are used, differing chiefly in the devices for separating the balls and in the arrangement for introducing the balls between the races. The most widely used type has races that are of the same cross section throughout, unbroken by any openings for the introduction of balls. To introduce the balls the two races are first excentrically placed; the balls will fill slightly more than a half circumference; elastic separators or solid cages are used to space the balls.

Another type has a filling opening of sufficient depth cut into one race; the race continuity is restored by a small piece that is let in. This type is usually filled with balls, without cages or separators. The filling opening is always placed at the unloaded side of the bearing, where the weakening of the race is not important. This type has been almost entirely discarded in favor of the one above described.

A third type has a filling opening cut into each race not quite deep enough to tangent the bottom of the ball track. As this weakened section necessarily comes under the load during each revolution, the carrying capacity is reduced. After slight wear there develops an interference of the balls with the edges of these openings, which seriously reduces the speeds and load capacity. This interference precludes the use of this type to take end thrust.

The carrying capacity of a ball-bearing is directly proportional to the number of balls and to the square of the ball diameter.

It may be written as:

$L = Knd^2$, in which L = load capacity in pounds; n = number of balls; d = ball diameter in eighths of an inch. K varies with the condition and type of bearing, as also with the material and speed.

For a certain special steel that hardens throughout and is also unusually tough, employed by "DWF" or "HB" (the originators of the modern two-point type), the following values apply. For other steels lesser values must be used.

I. For Radial Bearings:

$K = 9$ for uninterrupted race track, cross-section curvature = 0.52 and $\frac{9}{16}$ in. ball diameter respectively for inner and outer races, separated balls, uniform load, and steady speed up to 3000 revs. per min.

$K = 5$ for full ball type, filling opening in one race at the unloaded side, otherwise as above.

$K = 2.5$ for both ball tracks interrupted by filling openings, inelastic cage separators for balls, or full ball, speeds not above 2000 revs. per min., uniform load.

$K = 0.9$ for thrust on a radial bearing of the first type, as above. The larger the balls the smaller K . The type with filling openings in each race is not suitable for end thrust.

The radial load bearing is, up to high speeds, practically unaffected by speed, as to carrying capacity.

II. Thrust Bearings:

With the thrust type, consisting of one flat plate and one seat plate with grooved ball races, the load capacity decreases with speed or

$$L = \frac{K_1 nd^2}{\sqrt[3]{R}}.$$

K_1 = constant for material and race cross-section, etc., R = revolutions per minute. R ranges from about 3000 revs. per min. down to 1 rev. per min. as for crane hooks and similar elements.

$K_1 = 25$ to 40 for material used by the DWF or HB, and race cross-section radius = approx. 1.66 ball radius.

$K_1 = 0.5$ for unhardened steel, occasionally used for very large races; a steel that is fairly hard without tempering must be used, and then only when there is no hammering or sharp load variation.

Balls must be carefully selected to make sure that all that are used in the same bearing do not vary among one another by more than 0.0001 inch. A ball that is more than that larger than its fellows will sustain more than its proportion of the load, and may therefore be overloaded and will in turn overload the races.

The usual test of ball quality, which consists in compressing a ball between flat plates and noting the load at rupture, gives the quality of the plates, but not of the balls. It is the ability of the ball to resist permanent deformation that is of importance. As the deformations involved are very small the test is a difficult one to carry out. Of even greater importance than a small deformation under load is uniformity of such deformation between the balls employed; a hard ball will deform less than its softer mate and so will carry more than its share of the load, and will therefore be overloaded and in turn overload the races.

Coned bearings for balls are objectionable. The defect in all these forms of bearings is their adjustable feature. A bearing properly proportioned with reference to a certain load may be enormously overloaded by a little extra effort applied to the wrench, or on the other hand the bearing may be adjusted with too little pressure, so that the balls will rattle, and the results consequently be unsatisfactory. The prevalent idea that coned ball-bearings can be adjusted to compensate for wear is erroneous.

Mr. Hess's paper, in *Trans. A. S. M. E.*, 1907, contains a great deal of useful information on the practical design of ball-bearings, including different forms of raceways. He prefers a two-point bearing, in which the ball races have a curved section, with sustaining surfaces at right angles with the direction of the load.

Formulae for Number of Balls in a Bearing. (H. Rolfe, *Am. Mach.*, Dec. 3, 1896.) — Let D = diam. of ball circle (the circle passing through

the centers of the balls); d = diam. of balls; n = number of balls; s = average clearance space between the balls. Then $D = (d + s) \div \sin(180^\circ/n)$; $d = D \sin(180^\circ/n) - s$; $s = D \sin(180^\circ/n) - d$; $n = 180^\circ \div$ angle whose sine is $(d + s) \div D$. The clearance s should be about 0.003 in.

VALUES OF $180^\circ/n$ AND OF $\sin 180^\circ/n$.

| n . | $180^\circ/n$. | $\sin 180^\circ/n$. | n . | $180^\circ/n$. | $\sin 180^\circ/n$. | n . | $180^\circ/n$. | $\sin 180^\circ/n$. | n . | $180^\circ/n$. | $\sin 180^\circ/n$. |
|-------|-----------------|----------------------|-------|-----------------|----------------------|-------|-----------------|----------------------|-------|-----------------|----------------------|
| 3 | 60 | 0.86603 | 15 | 12 | 0.20791 | 27 | 6.667 | 0.11609 | 39 | 4.615 | 0.08047 |
| 4 | 45 | .70711 | 16 | 11.250 | .19509 | 28 | 6.429 | .11197 | 40 | 4.500 | .07846 |
| 5 | 36 | .58799 | 17 | 10.588 | .18375 | 29 | 6.207 | .10812 | 41 | 4.390 | .07655 |
| 6 | 30 | .50000 | 18 | 10 | .17365 | 30 | 6 | .10453 | 42 | 4.286 | .07473 |
| 7 | 25.714 | .43388 | 19 | 9.474 | .16454 | 31 | 5.806 | .10117 | 43 | 4.186 | .07300 |
| 8 | 22.500 | .38268 | 20 | 9 | .15643 | 32 | 5.625 | .09801 | 44 | 4.091 | .07134 |
| 9 | 20 | .34202 | 21 | 8.571 | .14904 | 33 | 5.455 | .09506 | 45 | 4 | .06976 |
| 10 | 18 | .30902 | 22 | 8.182 | .14233 | 34 | 5.294 | .09227 | 46 | 3.913 | .06825 |
| 11 | 16.364 | .28173 | 23 | 7.826 | .13616 | 35 | 5.143 | .08963 | 47 | 3.830 | .06679 |
| 12 | 15 | .25882 | 24 | 7.500 | .13053 | 36 | 5 | .08716 | 48 | 3.750 | .06540 |
| 13 | 13.846 | .23931 | 25 | 7.200 | .12533 | 37 | 4.865 | .08510 | 49 | 3.673 | .06407 |
| 14 | 12.857 | .22252 | 26 | 6.923 | .12055 | 38 | 4.737 | .08258 | 50 | 3.600 | .06279 |

Grades of Balls for Bearings. (S. S. Eveland, *Trans. A. S. M. E.*, 1905.) — "A" grade balls vary about 0.0025 in. in diameter; "B" grade, 0.001 to 0.002 in.; while "high-duty" or special balls are furnished varying not over 0.0001 in. The crushing strength of balls is of little importance as to the load a bearing will carry, the revolutions per minute being quite as important as the load.

Saving of Power by Use of Ball-Bearings. — Henry Hess (*Trans. A. S. M. E.*, 1909) describes a series of tests made by Dodge and Day on a $2\frac{15}{16}$ in. line shaft 72 ft. long, alternately equipped with plain ring-oiling babbitted boxes and with Hess-Bright ball-bearings. Eight countershafts were driven from pulleys on the line shaft. The countershaft pulleys had plain bearings. The conclusions from the tests made under normal belt conditions of 44 and 57 lbs. per inch width of angle of single belt are as follows:

a. Savings due to the substitution of ball-bearings for plain bearings on line shafts may be safely calculated by using 0.0015 as the coefficient of ball-bearing friction, 0.03 as the coefficient of line shaft friction, and 0.08 as the coefficient of countershaft friction.

b. When the belts from line shaft to countershaft pull all in one direction and nearly horizontally the saving due to the substitution of ball-bearings for plain bearings on the line shaft may be safely taken as 35% of the bearing friction.

c. When ball-bearings are used also on the countershafts the savings will be correspondingly greater and may amount to 70% or more of the bearing friction.

d. These percentages of savings are percentages of the friction work lost in the plain bearings; they are not percentages of the total power transmitted. The latter will depend upon the ratio of the total power transmitted to that absorbed in the line and countershafts.

e. The power consumed in the plain line and countershafts varies, as is well known, from 10 to 60% in different industries and shops. The substitution of ball-bearings for plain bearings on the line shaft only, under conditions of paragraph "a," will thus result in saving of total power of $35 \times 0.10 = 3.5\%$ to $35 \times 0.60 = 21\%$. By using ball-bearings on the countershafts also, the saving of total power will be from $70 \times 0.10 = 7\%$ to $70 \times 0.60 = 42\%$.

KNIFE-EDGE BEARINGS.

Allowable loads on knife-edges vary with the manner in which the pivots or knife-edges are held in the lever and the pivot supports or seats secured to the base of weighing machines. The extension of the

pivot beyond the solid support is practically worthless. A high-grade uniform tool steel with carbon 0.90% to 1.00% should be used. The temper of the seats should be drawn to a very light straw color; that of the pivots should be slightly darker. The angle of 90° for the knife-edge has given good results for heavy loads. For ordinary weighing machinery and most testing machinery 5000 lbs. per inch of length is ample. Loads of 10,000 lbs. per inch of length are permissible, but the pivot must be flat at its upper portion, normal to the load and supported its whole length, with a minimum deflection of parts to secure reasonable accuracy. The edge may be made perfectly sharp, for loads up to 1000 lbs. per inch of length. For greater loads the sharp edge is rubbed with an oilstone, so that a smoothness is just visible. A pronounced radius of knife-edge will decrease the sensibility of the apparatus. (Jos. W. Bramwell, *Eng. News*, June 14, 1906.)

FRICTION OF STEAM-ENGINES.

Distribution of the Friction of Engines. — Prof. Thurston, in his "Friction and Lost Work," gives the following:

| | 1. | 2. | 3. |
|-------------------------------|-------|-------|-------|
| Main bearings..... | 47.0 | 35.4 | 35.0 |
| Piston and rod..... | 32.9 | 25.0 | 21.0 |
| Crank-pin..... | 6.8 | 5.1 | 13.0 |
| Cross-head and wrist-pin..... | 5.4 | 4.1 | |
| Valve and rod..... | 2.5 | 26.4 | 22.0 |
| Eccentric strap..... | 5.3 | 4.0 | |
| Link and eccentric..... | | | 9.0 |
| Total..... | 100.0 | 100.0 | 100.0 |

No. 1, Straight-line, 6 × 12 in., balanced valve; No. 2, Straight-line, 6 × 12 in., unbalanced valve; No. 3, 7 × 10 in., Lansing traction, locomotive valve-gear.

Prof. Thurston's tests on a number of different styles of engines indicate that the friction of any engine is practically constant under all loads. (*Trans. A. S. M. E.*, viii, 86; ix, 74.)

In a straight-line engine, 8 × 14 in., I.H.P. from 7.41 to 57.54, the friction H.P. varied irregularly between 1.97 and 4.02, the variation being independent of the load. With 50 H.P. on the brake the I.H.P. was only 52.6, the friction being only 2.6 H.P., or about 5%.

A compound condensing-engine, tested from 0 to 102.6 brake H.P., gave I.H.P. from 14.92 to 117.8 H.P., the friction H.P. varying only from 14.92 to 17.42. At the maximum load the friction was 15.2 H.P., or 12.9%.

The friction increases with increase of the boiler-pressure from 30 to 70 lbs., and then becomes constant. The friction generally increases with increase of speed, but there are exceptions to this rule.

Prof. Denton (*Stevens Indicator*, July, 1890), comparing the calculated friction of a number of engines with the friction as determined by measurement, finds that in one case, a 75-ton ammonia ice-machine, the friction of the compressor, 17½ H.P., is accounted for by a coefficient of friction of 7½% on all the external bearings, allowing 6% of the entire friction of the machine for the friction of pistons, stuffing-boxes, and valves. In the case of the Pawtucket pumping-engine, estimating the friction of the external bearings with a coefficient of friction of 6% and that of the pistons, valves, and stuffing-boxes as in the case of the ice-machine, we have the total friction distributed as follows:

| | Horse-power. | Per cent of whole. |
|--|--------------|--------------------|
| Crank-pins and effect of piston-thrust on main shaft | 0.71 | 11.4 |
| Weight of fly-wheel and main shaft..... | 1.95 | 32.4 |
| Steam-valves..... | 0.23 | 3.7 |
| Eccentric..... | 0.07 | 1.2 |
| Pistons..... | 0.43 | 7.2 |
| Stuffing-boxes, six altogether..... | 0.72 | 11.3 |
| Air-pump..... | 2.10 | 32.8 |
| Total friction of engine with load..... | 6.21 | 100.0 |
| Total friction per cent of indicated power. | 4.27 | |

The friction of this engine, though very low in proportion to the indicated power, is satisfactorily accounted for by Morin's law used with a coefficient of friction of 5%. In both cases the main items of friction are those due to the weight of the fly-wheel and main shaft and to the piston-thrust on crank-pins and main-shaft bearings. In the ice-machine the latter items are the larger owing to the extra crank-pin to work the pumps, while in the Pawtucket engine the former preponderates, as the crank-thrusts are partly absorbed by the pump-pistons, and only the surplus effect acts on the crank-shaft.

Prof. Denton describes in *Trans. A. S. M. E.*, x. 392, an apparatus by which he measured the friction of the piston packing-ring. When the parts of the piston were thoroughly devoid of lubricant, the coefficient of friction was found to be about $7\frac{1}{2}\%$; with an oil-feed of one drop in two minutes the coefficient was about 5%; with one drop per minute it was about 3%. These rates of feed gave unsatisfactory lubrication, the piston groaning at the ends of the stroke when run slowly, and the flow of oil left upon the surfaces was found by analysis to contain about 50% of iron. A feed of two drops per minute reduced the coefficient of friction to about 1%, and gave practically perfect lubrication, the oil retaining its natural color and purity.

FRICTION BRAKES AND FRICTION CLUTCHES.

Friction Brakes are used for slowing down or stopping a moving machine by converting its energy of motion into heat, or for controlling the speed of a descending load. The simplest form is the block brake, commonly used for railway car wheels, which resists the motion of the wheel not only with the force due to ordinary sliding friction, but with that due to cutting or grinding away the surface of the metals in contact. If P = total pressure acting normal to the sliding surface, f = coefficient of friction, and v = velocity in feet per minute, then the energy absorbed, in foot-pounds per minute, is Pfv . If the surface is lubricated and the pressure per square inch not great enough to squeeze out the lubricant, then the value of f for different materials may be taken from Morin's tables for friction of motion, page 1196, but if the pressure is great enough to force out the lubricant, then the coefficient becomes much greater and the surface or surfaces will cut and wear, with a rapid rise of temperature.

Other forms of brakes are disk brakes and cone brakes, in which a disk or cone is carried by the rotating shaft and a mating disk or cone is pressed against it by a lever or other means; and band brakes, also called strap or ribbon brakes, in which a flexible band encircles the cylindrical surface of a rotating drum or wheel, and tension applied to one end of the band brings it in contact with that surface. For band brakes the theory of friction of belts applies. See page 1115. For much information on the theory and practice of friction brakes see articles by C. F. Blake in *Mach'y*, Jan., 1901, Mar., 1905, and Aug., 1906, and by E. R. Douglas, *Am. Mach.*, Dec. 26, 1901, and R. B. Brown, *Mach'y*, April, 1909. For friction brake dynamometers see Dynamometers.

Friction Clutches are used for putting shafts in motion gradually, without shock. If two shafts, in line with each other, one in motion and the other at rest, each having a disk keyed to the end, and the disks almost touching, are moved toward each other so that the disks are brought in contact with some pressure, the shaft at rest will be put in motion gradually, while the disks rub on each other, until it acquires the velocity of the driving shaft, when the friction ceases and the disks may then be locked together. This is an elementary form of friction clutch. A great variety of styles are made in which the sliding surfaces may be disks, cones, and gripping blocks of various forms. The work done by a clutch while the surfaces are in sliding contact, and before they are locked together, is the overcoming of the inertia of the driven shaft and of all the mechanism driven by it, and giving it the velocity of the driving shaft. The principles of friction brakes apply to friction clutches. The sliding surfaces must be of sufficient area to keep the normal pressure below that at which they will overheat, cut and wear, and to dissipate the heat generated by friction. The following values of the coefficient of friction to be used in designing clutches are given by C. W. Hunt: cork on iron, 0.35; leather on iron, 0.3; wood on iron, 0.2; iron on iron,

0.25 to 0.3. Lower values than these should be assumed for velocities exceeding 400 ft. per minute. The pressure per square inch in disk clutches should not exceed 25 or 30 lbs., and wooden surfaces should not be loaded beyond 20 to 25 lbs. per sq. in. See Kimball and Barr on Machine Design, also *Trans. A. S. M. E.*, 1903 and 1908.

Electrically Operated Brakes are discussed by H. A Steen in a paper read before the Engrs. Socy. of W. Penna., reprinted in *Iron Trade Rev.*, Dec. 24, 1908. Formulæ are given for the time required for stopping, for the heat generated and the temperature rise, for different types of brakes.

Magnetic and Electric Brakes.—For braking the load on electric cranes a band brake is used which is held off the drum by the action of a magnet or solenoid, and is put on by the action of a spring or weight. The solenoid usually consists of a coil of wire connected in series with the motor, and a plunger working inside of the coil. It should be so proportioned that its action is not delayed by residual magnetism when the current is cut off. Too rapid action is prevented by making the end of the solenoid an air dash-pot.

For electric-driven machinery an electric motor makes a most efficient brake by reversing the direction of the electric current, causing the motor to become a generator supplying current to a rheostat in which it is converted into heat and dissipated. In some cases the electric current generated, instead of being absorbed in a rheostat, is fed into the main electric circuit. In this case the energy of the rotating mass, instead of being wasted in friction or in electrical heating, is converted into electric energy and thus conserved for further use.

Design of Band Brakes. (R. A. Greene, *Am. Mach.*, Oct. 8, 1908.)—In the practice of the Browning Engineering Co., Cleveland, O., in regard to the design of band brakes the equations are:

$$T = PX, t = T - P, S = \frac{2T}{D \times F}, \vartheta = S \times D \times 0.262 \times \text{revolutions per minute, in which } T = \text{the greater tension on the band, } t = \text{the lesser tension on the band, } P = \text{equivalent load on the brake drum, } X = \text{factor from the accompanying table, } X = \frac{N}{N-1} \text{ in which } \log. N = 10^{2.7288} fc,$$

where f = the coefficient of friction and c the length of arc of contact in degrees divided by 360. D = diam. of brake drum, F = width of face of brake drum, S = a checking factor which has a maximum limit of 65, ϑ = a checking factor which has a limit of 54,000 (Yale & Towne practice) or 60,000 (Brown hoist practice).

EXAMPLE.—A band brake is to be designed having an arc of contact of 260°, coefficient of friction = 0.2, drum diameter 30 ins., face 4 ins., speed 100 r.p.m., and a load of 3000 lbs. acting on a diameter of 20 ins.

Then
 $P = 3000 \times 20 \div 30 = 2000$ pounds, $X = 1.68$ (from table), $T = 2000 \times 1.68 = 3360$ pounds, $t = 3360 - 2000 = 1360$ pounds, $S = 2 \times 3360 \div (30 \times 4) = 56$ (within the limit), $\vartheta = 56 \times 30 \times 0.62 \times 100 = 44,000$ (within the limit).

| Degrees. | Values of X. | | | Degrees. | Values of X. ¹ | | |
|----------|--------------|----------|----------|----------|---------------------------|----------|----------|
| | $f=0.2.$ | $f=0.3.$ | $f=0.4.$ | | $f=0.2.$ | $f=0.3.$ | $f=0.4.$ |
| 180 | 2.14 | 1.64 | 1.40 | 260 | 1.68 | 1.35 | 1.19 |
| 195 | 2.03 | 1.56 | 1.35 | 270 | 1.64 | 1.32 | 1.18 |
| 210 | 1.93 | 1.50 | 1.30 | 280 | 1.60 | 1.30 | 1.17 |
| 240 | 1.76 | 1.40 | 1.23 | 290 | 1.57 | 1.28 | 1.15 |
| 250 | 1.72 | 1.37 | 1.21 | 300 | 1.54 | 1.26 | 1.14 |

FRICITION OF HYDRAULIC PLUNGER PACKING.

The "Taschenbuch der Hutte" (15th edition, vol. 1, p. 202) says: "For stuffing boxes with hemp, cotton or leather packing, with water pressures between 1 and 50 atmospheres, the frictional loss is dependent upon the water pressure, the circumference of the packed surface, and a coefficient

μ , which is constant for this range of pressure. The loss is independent of the depth of stuffing-box or leather ring, and is given by the formula $F = Kpd$, in which F = total frictional loss in pounds, p = pressure in pounds per sq. in., d = diameter of plunger in inches.

K is a coefficient, which depends on the kind and condition of the packing, and is given as follows for various cases.

For cotton or hemp, loose or braided, dipped in hot tallow; plungers smooth, glands not pulled down too tight, packing therefore retaining its elasticity; dimensions such as usually occur, $K = 0.072$.

Same conditions, after packing is some months old, $K = 0.132$.

Materials the same, but with hard packing, unfavorable conditions, etc., K = as much as 0.299.

Leather packing; soft leather, well made, etc., $K = 0.036$ to 0.084.

Hard, stiffly tanned leather, $K = 0.12$ to 0.156.

Unfavorable conditions; rough plungers, gritty water, etc., K = as much as 0.239.

Weisbach-Hermann, "Mechanics of Hoisting Machinery," gives a formula which when translated into the same notation as the one in "Hutte" is

$$F = 0.0312 \, pd \text{ to } 0.0767 \, pd.$$

Since the total pressure on a plunger is $\frac{1}{4}\pi d^2 p$, the ratio of the loss of pressure to the total pressure is $Kpd \div \frac{1}{4}\pi d^2 p$, or, using the extreme values of K , 0.0312 and 0.299, the ratio ranges from $0.04 \div d$ to $0.38 \div d$, or from 4 to 38 per cent divided by the diameter in inches.

Walter Ferris (*Am. Mach.*, Feb. 3, 1898) derives from the formula given above the following formula for the pressure produced by a hemp-packed hydraulic intensifier made with two plungers of different diameters:

$$p_2 = p_1 \frac{A - KD}{a + kd},$$

in which p_2 = pressure per sq. in. produced by the intensifier, p_1 = initial pressure, A = area and D = diam. of the larger plunger, a = area and d = diam. of the smaller plunger, and K an experimental coefficient. He gives the following results of tests of an intensifier with a small plunger 8 ins. diam. and two large plungers, $14\frac{1}{4}$ and $17\frac{3}{4}$ ins., either one of which could be used as desired.

| Diam. of large plunger, in. | 14 $\frac{1}{4}$ | 14 $\frac{1}{4}$ | 17 $\frac{3}{4}$ | 17 $\frac{3}{4}$ |
|--|------------------|------------------|------------------|------------------|
| Initial pressure, lbs. per sq. in. | 285 | 475 | 335 | 350 |
| Intensified pressure, lbs. per sq. in. | 750 | 1450 | 1450 | 1510 |
| Intensified if there were no friction | 905 | 1505 | 1650 | 1725 |
| Intensified calculated by formula* | 806 | 1433 | 1572 | 1643 |
| Efficiency of machine | 0.83 | 0.965 | 0.88 | 0.875 |

LUBRICATION.

Measurement of the Durability of Lubricants. — (J. E. Denton, *Trans. A. S. M. E.*, xi, 1013.) — Practical differences of durability of lubricants depend not on any differences of inherent ability to resist being "worn out" by rubbing, but upon the rate at which they flow through and away from the bearing-surfaces. The conditions which control this flow are so delicate in their influence that all attempts thus far made to measure durability of lubricants may be said to have failed to make distinctions of lubricating value having any practical significance. In some kinds of service the limit to the consumption of oil depends upon the extent to which dust or other refuse becomes mixed with it, as in railroad-car lubrication and in the case of agricultural machinery. The economy of one oil over another, so far as the quality used is concerned — that is, so far as durability is concerned — is simply proportional to the rate at which it can insinuate itself into and flow out of minute orifices or cracks. Oils will differ in their ability to do this, first, in proportion to their viscosity, and, second, in proportion to the capillary properties which they may possess by virtue of the particular ingredients used in their composition. Where the thickness of film between rubbing-surfaces must be so great that large amounts of oil pass through bearings in a given time, and the surroundings are such as to permit oil to be fed at high

*Assuming $K = 0.2$. The efficiency calculated by the formula in each case was 0.953.

temperatures or applied by a method not requiring a perfect fluidity, it is probable that the least amount of oil will be used when the viscosity is as great as in the petroleum cylinder stocks. When, however, the oil must flow freely at ordinary temperatures and the feed of oil is restricted, as in the case of crank-pin bearings, it is not practicable to feed such heavy oils in a satisfactory manner. Oils of less viscosity or of a fluidity approximating to lard-oil must then be used.

Relative Value of Lubricants. (J. E. Denton, *Am. Mach.*, Oct. 30, 1890.) — The three elements which determine the value of a lubricant are the cost due to consumption of lubricants, the cost spent for coal to overcome the frictional resistance caused by use of the lubricant, and the cost due to the metallic wear on the journal and the brasses.

The Qualifications of a Good Lubricant, as laid down by W. H. Bailey, in *Proc. Inst. C. E.*, vol. xlv, p. 372, are: 1. Sufficient body to keep the surfaces free from contact under maximum pressure. 2. The greatest possible fluidity consistent with the foregoing condition. 3. The lowest possible coefficient of friction, which in bath lubrication would be for fluid friction approximately. 4. The greatest capacity for storing and carrying away heat. 5. A high temperature of decomposition. 6. Power to resist oxidation or the action of the atmosphere. 7. Freedom from corrosive action on the metals upon which the lubricant is used.

The Examination of Lubricating Oils. (Prof. Thos. B. Stillman, *Stevens Indicator*, July, 1890.) — The generally accepted conditions of a good lubricant are as follows:

1. "Body" enough to prevent the surfaces to which it is applied from coming in contact with each other. (Viscosity.)
2. Freedom from corrosive acid, of either mineral or animal origin.
3. As fluid as possible consistent with "body."
4. A minimum coefficient of friction.
5. High "flash" and burning points.
6. Freedom from all materials liable to produce oxidation or "gumming."

The examinations to be made to verify the above are both chemical and mechanical, and are usually arranged in the following order:

1. Identification of the oil, whether a simple mineral oil, or animal oil, or a mixture.
2. Density.
3. Viscosity.
4. Flash-point.
5. Burning-point.
6. Acidity.
7. Coefficient of friction.
8. Cold test.

Detailed directions for making all of the above tests are given in Prof. Stillman's article. See also Stillman's *Engineering Chemistry*, p. 366.

Notes on Specifications for Petroleum Lubricants. (C. M. Everest, Vice-Pres. Vacuum Oil Co., *Proc. Engineering Congress*, Chicago World's Fair, 1893.) — The specific gravity was the first standard established for determining quality of lubricating oils, but it has long since been discarded as a conclusive test of lubricating quality. However, as the specific gravity of a particular petroleum oil increases the viscosity also increases.

The object of the fire test of a lubricant, as well as its flash test, is the prevention of danger from fire through the use of an oil that will evolve inflammable vapors. The lowest fire test permissible is 300°, which gives a liberal factor of safety under ordinary conditions.

The cold test of an oil, i.e., the temperature at which the oil will congeal, should be well below the temperature at which it is used; otherwise the coefficient of friction would be correspondingly increased.

Viscosity, or fluidity, of an oil is usually expressed in seconds of time in which a given quantity of oil will flow through a certain orifice at the temperature stated, comparison sometimes being made with water, sometimes with sperm-oil, and again with rape-seed oil. It seems evident that within limits the lower the viscosity of an oil (without a too near approach to metallic contact of the rubbing surfaces) the lower will be the coefficient of friction. But we consider that each bearing in a mill or factory would probably require an oil of different viscosity from any other bearing in the mill, in order to give its lowest coefficient of friction, and that slight variations in the condition of a particular bearing would change the requirements of that bearing; and further, that when nearing the "danger point" the question of viscosity alone probably does not govern.

The requirement of the New England Manufacturers' Association, that an oil shall not lose over 5% of its volume when heated to 140° Fahr. for 12 hours, is to prevent losses by evaporation, with the resultant effects.

The precipitation test gives no indication of the quality of the oil itself, as the free carbon in improperly manufactured oils can be easily removed.

It is doubtful whether oil buyers who require certain given standards of laboratory tests are better served than those who do not. Some of the standards are so faulty that to pass them an oil manufacturer must supply oil he knows to be faulty; and the requirements of the best standards can generally be met by products that will give inferior results in actual service.

Penna. R. R. Specifications for Petroleum Products, 1900. — Five different grades of petroleum products will be used.

The materials desired under this specification are the products of the distillation and refining of petroleum unmixed with any other substances.

150° Fire-test Oil. — This grade of oil will not be accepted if sample (1) is not "water-white" in color; (2) flashes below 130° Fahrenheit; (3) burns below 151° Fahrenheit; (4) is cloudy or shipment has cloudy barrels when received, from the presence of glue or suspended matter; (5) becomes opaque or shows cloud when the sample has been 10 minutes at a temperature of 0° Fahrenheit.

300° Fire-test Oil. — This grade of oil will not be accepted if sample (1) is not "water-white" in color; (2) flashes below 249° Fahrenheit; (3) burns below 298° Fahrenheit; (4) is cloudy or shipment has cloudy barrels when received, from the presence of glue or suspended matter; (5) becomes opaque or shows cloud when the sample has been 10 minutes at a temperature of 32° Fahrenheit; (6) shows precipitation when some of the sample is heated to 450° F. The precipitation test is made by having about two fluid ounces of the oil in a six-ounce beaker, with a thermometer suspended in the oil, and then heating slowly until the thermometer shows the required temperature. The oil changes color, but must show no precipitation.

Paraffine and Neutral Oils. — These grades of oil will not be accepted if the sample from shipment (1) is so dark in color that printing with long-primer type cannot be read with ordinary daylight through a layer of the oil $\frac{1}{2}$ inch thick; (2) flashes below 298° F.; (3) has a gravity at 60° F., below 24° or above 35° Baumé; (4) from October 1st to May 1st has a cold test above 10° F., and from May 1st to October 1st has a cold test above 32° F.

The color test is made by having a layer of the oil of the prescribed thickness in a proper glass vessel, and then putting the printing on one side of the vessel and reading it through the layer of oil with the back of the observer toward the source of light.

Well Oil. — This grade of oil will not be accepted if the sample from shipment (1) flashes, from May 1st to October 1st, below 298° F., or from October 1st to May 1st, below 249° F.; (2) has a gravity at 60° F., below 28° or above 31° Baumé; (3) from October 1st to May 1st has a cold test above 10° F., and from May 1st to October 1st has a cold test above 32° F.; (4) shows any precipitation when 5 cubic centimeters are mixed with 95 c.c. of gasoline. The precipitation test is to exclude tarry and suspended matter. It is made by putting 95 c.c. of 88° B. gasoline, which must not be above 80° F. in temperature, into a 100 c.c. graduate, then adding the prescribed amount of oil and shaking thoroughly. Allow to stand ten minutes. With satisfactory oil no separated or precipitated material can be seen.

500° Fire-test Oil. — This grade of oil will not be accepted if sample from shipment (1) flashes below 494° F.; (2) shows precipitation with gasoline when tested as described for well oil.

Printed directions for determining flashing and burning tests and for making cold tests and taking gravity are furnished by the railroad company.

Penna. R. R. Specifications for Lubricating Oils (1894). (In force in 1902.)

| Constituent Oils. | Parts by volume. | | | | | | | | |
|---------------------------|------------------|-------|----------------|----------------|----------------|----------------|----------------|----------------|-------|
| Extra lard-oil..... | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Extra No. 1 lard-oil..... | | | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| 500° fire-test oil..... | | | 4 | 2 | 1 | | | | |
| Paraffine oil..... | | | | | | 4 | 2 | 1 | |
| Well oil..... | 1 | | | | | | | | |
| Used for..... | A | B | C ₁ | C ₂ | C ₃ | D ₁ | D ₂ | D ₃ | E |

A, freight cars; engine oil on shifting-engines; miscellaneous greasing in foundries, etc. B, cylinder lubricant on marine equipment and on stationary engines. C, engine oil; all engine machinery; engine and tender truck boxes; shafting and machine tools; bolt cutting; general lubrication except cars. D, passenger-car lubrication. E, cylinder lubricant for locomotives. C₁, D₁, for use in Dec., Jan., and Feb.; C₂, D₂, in March, April, May, Sept., Oct., and Nov.; C₃, D₃, in June, July, and August. Weights per gallon, A, 7.4 lbs.; B, C, D, E, 7.5 lbs.

Grease Lubricants.—Tests made on an Olsen lubricant testing machine at Cornell University are reported in *Power*, Nov. 9, 1909. It was found that some of the commercial greases stood much higher pressures than the oils tested, and that the coefficients of friction at moderate loads were often as low as those of the oils. The journal of the testing machine was $3\frac{3}{4}$ in. diam., $3\frac{1}{2}$ in. long, and the babbitt bearing shoe had a projected area of 5.8 sq. in. The speed was 240 r.p.m. and each test lasted one hour, except when the bearing showed overheating. The following are the coefficients of friction obtained in the tests:

| Lbs. per sq. in. | Min- eral Grease. | Animal Grease. | Graph- ite Grease. | Min- eral Grease. | Engine Oil. | Engine Oil. | Grease. | Grease. |
|------------------------|-------------------------|-------------------|--------------------------|-------------------------|----------------|----------------|---------|---------|
| 86.2 | 0.024 | 0.023 | 0.04 | 0.023 | 0.019 | 0.015 | 0.020 | 0.025 |
| 172.4 | 0.021 | 0.023 | 0.05 | 0.018 | 0.04 | 0.022 | 0.015 | 0.022 |
| 258.6 | 0.021 | 0.023 | | 0.018 | 0.06 | 0.037 | 0.014 | 0.020 |
| 344.8 | 0.025 | 0.025 | | 0.019 | | | 0.017 | 0.020 |
| 431.0 | 0.050 | 0.035 | | 0.028 | | | 0.026 | 0.019 |

Testing Oil for Steam Turbines. (Robert Job, *Trans. Am. Soc. for Testing Mtls.*, 1909.)—

In some types of steam-turbines, the bearings are very closely adjusted and, if the oil is not clear and free from waxy substances, clogging and heating quickly results. A number of red engine and turbine oils some of which had given good service and others bad service were tested and it was found that clearness and freedom from turbidity were of importance, but mere color, or lack of color, seemed to have little influence, and good service results were obtained with oils which were of a red color, as well as with those which were filtered to an amber color.

Heating Test.—It was found that on heating the oils to 450° F. all which had given bad service showed a marked darkening of color, while those which had proved satisfactory showed little change. With oils that had been filtered or else had been chemically treated in such manner that the so-called "amorphous waxes" had been completely removed, on applying the heating test only a slight darkening of color resulted. It is of advantage in addition to other requirements to specify that an oil for steam turbines on being heated to 450° F. for five minutes shall show not more than a slight darkening of color. The test is that commonly used in test of 300° oil for burning purposes.

Separating Test.—It is known that elimination of the waxes causes an increase in the ease with which the oil separates from hot water when thoroughly shaken with it. This condition can be taken advantage of by prescribing that when one ounce of the oil is placed in a 4-oz. bottle with two ounces of boiling water, the bottle corked and shaken hard for one minute and let stand, the oil must separate from the water within a specified time, depending upon the nature of the oil, and that there must be no appearance of waxy substances at the line of demarcation between the oil and the water.

Quantity of Oil needed to Run an Engine.—The Vacuum Oil Co. in 1892, in response to an inquiry as to cost of oil to run a 1000-H.P. Corliss engine, wrote: The cost of running two engines of equal size of the same make is not always the same. Therefore, while we could furnish figures showing what it is costing some of our customers having Corliss engines of 1000 H.P., we could only give a general idea, which in itself might be considerably out of the way as to the probable cost of cylinder- and engine-oils per year for a particular engine. Such an engine ought to

run readily on less than 8 drops of 600 W oil per minute. If 3000 drops are figured to the quart, and 8 drops used per minute, it would take about two and one half barrels (52.5 gallons) of 600 W cylinder-oil, at 65 cents per gallon, or about \$85 for cylinder-oil per year, running 6 days a week and 10 hours a day. Engine-oil would be even more difficult to guess at what the cost would be, because it would depend upon the number of cups required on the engine, which varies somewhat according to the style of the engine. It would doubtless be safe, however, to calculate at the outside that not more than twice as much engine-oil would be required as of cylinder-oil.

The Vacuum Oil Co. in 1892 published the following results of practice with "600 W" cylinder-oil:

| | |
|--------------------------|--|
| Corliss compound engine, | { 20 and 33 × 48; 83 revs. per min.; 1 drop of oil per min. to 1 drop in two minutes. |
| " triple exp. " | { 20, 33, and 46 × 48; 1 drop every 2 minutes. |
| Porter-Allen " | { 20 and 36 × 36; 143 revs. per min.; 2 drops of oil per min., reduced afterwards to 1 drop per min. |
| Ball " | { 15 and 25 × 16; 240 revs. per min.; 1 drop every 4 minutes. |

Results of tests on ocean-steamers communicated to the author by Prof. Denton in 1892 gave: for 1200-H.P. marine engine, 5 to 6 English gallons (6 to 7.2 U. S. gals.) of engine-oil per 24 hours for external lubrication; and for a 1500-H.P. marine engine, triple expansion, running 75 revs. per min., 6 to 7 English gals. per 24 hours. The cylinder-oil consumption is exceedingly variable, — from 1 to 4 gals. per day on different engines, including cylinder-oil used to swab the piston-rods.

Cylinder Lubrication. — J. H. Spoor, in *Power*, Jan. 4, 1910, has made a study of a great number of records of the amount of oil used for lubricating cylinders of different engines, and has reduced them to a systematic basis of the equivalent number of pints of oil used in a 10-hour day for different areas of surface lubricated. The surface is determined in square inches by multiplying the circumference of the cylinder by the length of stroke. The results are plotted in a series of curves for different types of engines, and approximate average figures taken from these curves are given below:

Compound Engines.

| | | | | | | | |
|------------------------------|-------|-------|-------|-------|--------|--------|--------|
| Sq. ins. lubricated | 2,000 | 4,000 | 6,000 | 8,000 | 10,000 | 12,000 | 18,000 |
| Pints of oil used in 10 hrs. | 2 | 3.5 | 4.3 | 5 | 5.5 | 6 | 6.5 |

Corliss Engines.

| | | | | |
|-----------------------------------|-------|-------|-------|-------|
| Sq. ins. lubricated..... | 1,000 | 2,000 | 3,000 | 4,000 |
| Pints of oil in 10 hrs. Avge..... | 0.9 | 1.65 | 2.25 | 3.75 |
| Max..... | 1.2 | 2.25 | — | — |
| Min. | — | 1.00 | — | — |

Automatic high-speed engines, about 2 pints per 1,000 sq. in.
Simple slide-valve engines, about 0.5 pints per 1,000 sq. ins.

As shown in the figures under 2,000, Corliss, a certain engine may take $2\frac{1}{4}$ times as much oil as another engine of the same size. The difference may be due to smoothness of cylinder surface, kind and pressure of piston rings, quality of oil, method of introducing the lubricant, etc. Variations in speed of a given type of engine and in steam pressure do not appear to make much difference, but the small automatic high-speed engine takes more oil than any other type. Vertical marine engines are commonly run without any cylinder oil, except that used occasionally to swab the piston rods.

Quantity of Oil used on a Locomotive Crank-pin. — Prof. Denton, *Trans. A. S. M. E.*, xi, 1020, says: A very economical case of practical oil-consumption is when a locomotive main crank-pin consumes about six cubic inches of oil in a thousand miles of service. This is equivalent to a consumption of one milligram to seventy square inches of surface rubbed over.

Soda Mixture for Machine Tools. (Penna. R. R. 1894.) — Dissolve 5 lbs. of common sal-soda in 40 gallons of water and stir thoroughly. When needed for use mix a gallon of this solution with about a pint of engine oil. Used for the cutting parts of machine tools instead of oil.

Water as a Lubricant. (C. W. Naylor, *Trans. A. S. M. E.*, 1905.) — Two steel jack-shafts 18 ft. long with bearings 5×14 ins. each receiving 175 H.P. from engines and driving 5 electric generators, with six belts all pulling horizontally on the same side of the shaft, gave trouble by heating when lubricated with oil or grease. Water was substituted, and the shafts ran for 11 years, 10 hours a day, without serious interruption. Oil was fed to the shaft before closing down for the night, to prevent rusting. The wear of the babbitted bearings in 11 years was about $\frac{1}{4}$ in., and of the shaft nil.

Acheson's "Deflocculated" Graphite. (*Trans. A. I. E. E.*, 1907; *Eng. News*, Aug. 1, 1907.) — In 1906, Mr. E. G. Acheson discovered a process of producing a fine, pure, unctuous graphite in the electric furnace. He calls it deflocculated graphite. By treating this graphite in the disintegrated form with a water solution of tannin, the amount of tannin being from 3% to 6% of the weight of the graphite treated, he found that it would be retained in suspension in water, and that it was in such a fine state of subdivision that a large part of it would run through the finest filter paper, the filtrate being an intensely black liquid in which the graphite would remain suspended for months. The addition of a minute quantity of hydrochloric acid causes the graphite to flocculate and group together so that it will no longer flow through filter paper. The same effect has been obtained with alumina, clay, lampblack and siloxicon, by treatment with tannin. The graphite thus suspended in water, known as "*aquedag*," has been successfully used as a lubricant for journals with sight-feed and with chain-feed oilers. It also prevents rust in iron and steel. The deflocculated graphite has also been suspended in oil, in a dehydrated condition, making an excellent lubricant known as "*oildag*." Tests by Prof. C. H. Benjamin of oil with 0.5% of graphite showed that it had a lower coefficient of friction than the oil alone.

SOLID LUBRICANTS.

Graphite in a condition of powder and used as a *solid lubricant*, so called, to distinguish it from a liquid lubricant, has been found to do well where the latter has failed.

Rennie, in 1829, says: "Graphite lessened friction in all cases where it was used." General Morin, at a later date, concluded from experiments that it could be used with advantage under heavy pressures; and Prof. Thurston found it well adapted for use under both light and heavy pressures when mixed with certain oils. It is especially valuable to prevent abrasion and cutting under heavy loads and at low velocities.

For comparative tests of various oils with and without graphite, see paper on lubrication and lubricants, by C. F. Mabery, *Jour. A. S. M. E.*, Feb., 1910.

Soapstone, also called talc and steatite, in the form of powder and mixed with oil or fat, is sometimes used as a lubricant. Graphite or soapstone, mixed with soap, is used on surfaces of wood working against either iron or wood.

Metaline is a solid compound, usually containing graphite, made in the form of small cylinders which are fitted permanently into holes drilled in the surface of the bearing. The bearing thus fitted runs without any other lubrication.

THE FOUNDRY.

(See also Cast-iron, pp. 414 to 429, and Fans and Blowers, pp. 626 to 643.)

Cupola Practice.

The following table and the notes accompanying it are condensed from an article by Simpson Bolland in *Am. Mach.*, June 30, 1892:

| | | | | | | | |
|--------------------------------|------|--------|--------|--------|--------|--------|--------|
| Diam. of lining, in..... | 36 | 48 | 54 | 60 | 66 | 72 | 84 |
| Height to char'g door, ft. . . | 12 | 13 | 14 | 15 | 15 | 16 | 16 |
| Fuel used in bed, lbs..... | 840 | 1380 | 1650 | 1920 | 2190 | 2460 | 3000 |
| First charge of iron, lbs.... | 2520 | 4140 | 4950 | 5760 | 6570 | 7380 | 9000 |
| Other fuel charges, lbs..... | 302 | 554 | 680 | 806 | 932 | 1058 | 1310 |
| Other iron charges, lbs..... | 2718 | 4986 | 6120 | 7254 | 8388 | 9522 | 11,790 |
| Diam. blast pipe, in..... | 14 | 18 | 20 | 22 | 22 | 24 | 26 |
| No. of 6-in. round tuyeres.. | 3.7 | 6.8 | 10.7 | 13.7 | 15.4 | 19 | 31 |
| Equiv. No. flat tuyeres..... | 4 | 6 | 8 | 8 | 8 | 10 | 16 |
| Width of flat tuyeres, in.... | 2 | 2.5 | 2.5 | 3 | 3 | 3 | 3.5 |
| Height of flat tuyeres, in.... | 13.5 | 13.5 | 15.5 | 16.5 | 18.5 | 18.5 | 16 |
| Blast pressure, oz..... | 8 | 12 | 14 | 14 | 14 | 16 | 16 |
| Size of Root blower, No..... | 2 | 4 | 4 | 5 | 5 | 6 | 7 |
| Revs. per min..... | 241 | 212 | 277 | 192 | 240 | 163 | 160 |
| Engine for blower, H.P..... | 2.5 | 10 | 14 | 18 1/2 | 23 | 33 | 47 |
| Sturtevant blower, No..... | 4 | 6 | 7 | 8 | 8 | 9 | 10 |
| Engine for blower, H.P..... | 3 | 9 3/4 | 16 | 22 | 22 | 35 | 48 |
| Melting cap., lbs. per hr.... | 4820 | 10,760 | 13,850 | 16,940 | 21,200 | 26,070 | 37,530 |

Mr. Bolland says that the melting capacities in the table are not supposed to be all that can be melted in the hour by some of the best cupolas, but are simply the amounts which a common cupola under ordinary circumstances may be expected to melt in the time specified.

By height of cupola is meant the distance from the base to the bottom side of the charging door. The distance from the sand-bed, after it has been formed at the bottom of the cupola, up to the under side of the tuyeres is taken at 10 ins. in all cases.

All the amounts for fuel are based upon a bottom of 10 ins. deep. The quantity of fuel used on the bed is more in proportion as the depth is increased, and less when it is made shallower.

The amount of fuel required on the bed is based on the supposition that the cupola is a straight one all through, and that the bottom is 10 ins. deep. If the bottom be more, as in those of the Colliau type, then additional fuel will be needed.

First Charge of Iron. — The amounts given are safe figures to work upon in every instance, yet it will always be in order, after proving the ability of the bed to carry the load quoted, to make a slow and gradual increase of the load until it is fully demonstrated just how much burden the bed will carry.

Successive Charges of Fuel and Iron. — The highest proportions are not favored, for the simple reason that successful melting with any greater proportion of iron to fuel is not the rule, but, rather, the exception.

Diameter of Main Blast-pipe. — The sizes given are of sufficient area for all lengths up to 100 feet.

Tuyeres. — Any arrangement or disposition of tuyeres may be made, which shall answer in their totality to the areas given in the table. On no consideration must the tuyere area be reduced; thus, an 84-inch cupola must have tuyere area equal to 31 pipes 6 ins. diam., or 16 flat tuyeres 16 × 3 1/2 ins. The tuyeres should be arranged in such a manner as will concentrate the fire at the melting-point into the smallest possible compass, so that the metal in fusion will have less space to traverse while exposed to the oxidizing influence of the blast.

To accomplish this, recourse has been had to the placing of additional rows of tuyeres in some instances — the "Stewart rapid cupola" having three rows, and the "Colliau cupola furnace" having two rows, of tuyeres.

Blast-pressure. — About 30,000 cu. ft. of air are consumed in melting a ton of iron, which would weigh about 2400 pounds, or more than both iron and fuel. When the proper quantity of air is supplied, the combustion of the fuel is perfect, and carbonic-acid gas is the result. When the supply of air is insufficient, the combustion is imperfect, and carbonic-oxide gas is the result. The amount of heat evolved in these two cases is as 15 to 4½, showing a loss of over two-thirds of the heat by imperfect combustion. [Combustion is never perfect in the cupola except near the tuyeres. The CO₂ formed by complete combustion is largely reduced to CO in passing through the hot coke above the fusion zone.]

Slag in Cupolas. — A certain amount of slag is necessary to protect the molten iron which has fallen to the bottom from the action of the blast; if it was not there, the iron would suffer from decarbonization.

With clean iron and fuel, slag seldom forms to any appreciable extent in small heats; but when the cupola is to be taxed to its utmost capacity it is then incumbent on the melter to flux the charges all through the heat, carrying it away in the manner directed.

When fuel is bad, or iron is dirty, or both together, it becomes imperative that the slag be kept running all the time.

Fuel for Cupolas.—The best fuel for melting iron is coke, because it requires less blast, makes hotter iron, and melts faster than coal. When coal must be used, care should be exercised in its selection. All anthracites which are bright, black, hard, and free from slate, will melt iron admirably. For the best results, small cupolas should be charged with the size called "egg," a still larger grade for medium-sized cupolas, and what is called "lump" will answer for all large cupolas, when care is taken to pack it carefully on the charges.

| | | | | | | | | | | | |
|-----------------------------------|------|------|------|------|------|------|------|------|------|------|------|
| Diam. of lining, ins. | 44 | 44 | 47 | 49 | 54 | 54 | 54 | 60 | 60 | 60 | 74 |
| Tons per hour .. | 6.7 | 7.3 | 8.4 | 9.1 | 7.7 | 8.8 | 10.2 | 12.4 | 14.8 | 13.8 | 13.0 |
| Pressure, oz. per sq. in. | 12.9 | 16.4 | 17.5 | 11.8 | 13.6 | 11.0 | 20.8 | 15.5 | 16.8 | 12.6 | 8.7 |

From plotted diagrams of records of 46 tests of different cupolas the following figures are obtained:

| | | | | | | | | |
|---------------------------|-----|----|------|-----|------|------|------|----|
| Diam. of lining, ins..... | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| Max. tons per hour..... | 3 | 5 | 7.3 | 9.5 | 12 | 15 | 18 | 21 |
| Avg. " " "..... | 2.5 | 4 | 5.5 | 7.5 | 9 | 11 | 13 | 16 |
| Max. pressure, oz..... | 11 | 12 | 13.5 | 14 | 14.6 | 15.2 | 15.7 | 16 |

For a given cupola and blower the melting rate increases as the square root of the pressure. A cupola melting 9 tons per hour with 10 ounces pressure will melt about 10 tons with 12.5 ounces, and 11 tons with 15 ounces. The power required varies as the cube of the melting rate, so that it would require $(11/9)^3=1.82$ times as much power for 11 tons as for 9 tons. Hence the advantage of large cupolas and blowers with light pressures.

Charging a Cupola. — Chas. A. Smith (*Am. Mach.*, Feb. 12, 1891) gives the following: A 28-in. cupola should have from 300 to 400 lbs. of coke on bottom bed; a 36-in. cupola, 700 to 800 lbs.; a 48-in. cupola, 1500 lbs.; and a 60-in. cupola should have one ton of fuel on bottom bed.

To every pound of fuel on the bed, three, and sometimes four pounds of metal can be added with safety, if the cupola has proper blast; in after-charges, to every pound of fuel add 8 to 10 pounds of metal; any well-constructed cupola will stand ten.

F. P. Wolcott (*Am. Mach.*, Mar. 5, 1891) gives the following as the practice of the Colwell Iron-works, Carteret, N. J.: "We melt daily from twenty to forty tons of iron, with an average of 11.2 pounds of iron to one of fuel. In a 36-in. cupola seven to nine pounds is good melting, but in a cupola that lines up 48 to 60 inches, anything less than nine pounds shows a defect in arrangement of tuyeres or strength of blast, or in charging up."

"The Molder's Text-book," by Thos. D. West, gives forty-six reports in tabular form of cupola practice in thirty States, reaching from Maine to Oregon.

Improvement of Cupola Practice. — The following records are given by J. R. Fortune and H. S. Wells (*Proc. A. S. M. E.*, Mar., 1908) showing how ordinary cupola practice may be improved by making a few changes. The cupola is 13 ft. 4 in. in height from the top of the sand bottom to the charging door, and of three diameters, 50 in. for the first 3 ft. 6 in., then 54 in. for the next 2 ft. 4 in., then 60 in. to the top. When driven with a No. 8 Sturtevant blower, the maximum melting rate, from iron down to blast off, was 8.5 tons per hour. A No. 11 high-pressure blower was then installed. Test No. 1 in the table below gives the result with cupola charges as follows in pounds: Bed, 590 coke, followed by 826 coke, 2000 iron; 400 coke, 2000 iron; 300 coke, 2000 iron; and thereafter all charges were 200 coke, 2000 iron. The time between starting fire and starting blast was 2 hr. 30 min., and the time from blast on to iron down, 11 min. The melting rate, tons per hour, is figured for the time from iron down to blast off. The tuyeres were eight rectangular openings 11 1/4 in. high and of a total area of 1/9.02 of the area of the 54-in. circle.

| No. of Test. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Total tons... | 22.7 | 24. | 22.15 | 24.25 | 24.25 | 22.65 | 24. | 20.30 | 23.85 | 22.35 |
| Tons per hr.. | 9.45 | 8.88 | 8.86 | 9.15 | 9.66 | 10.24 | 10.43 | 10.91 | 11.35 | 11.17 |
| Lbs. per min* | 19.81 | 18.61 | 18.55 | 19.17 | 20.25 | 21.44 | 21.82 | 22.95 | 23.77 | 23.39 |
| Iron ÷ coke† | 7.54 | 7.40 | 7.28 | 8.58 | 8.94 | 8.71 | 9.02 | 9.02 | 10.02 | 9.49 |
| Blast, oz..... | 11.60 | 10.63 | 10.00 | 9.47 | 9.80 | 9.86 | 10.00 | 10.13 | 10.55 | 10.55 |

* Per sq. ft. cupola area at 54 in. diam. from iron down to blast off.

† Including bed.

The tuyeres were then enlarged, making their area 1/5.98 of the cupola (54 in.) area, and the results are shown in tests No. 2 and 3 of the table. The iron was too hot, and the coke charge was decreased to a ratio of 1/13.33 instead of 1/10, the bed of coke being increased. The result, test No. 4, was an increased rate of melting, a decrease in the amount of coke, and a decrease in the blast pressure. Tests 5, 6, 7, 8 and 9 were then made, the coke being decreased, while the blast pressure was increased, resulting in a decided increase in the melting speed. In tests 5, 6 and 7 the iron layer was 13.33 times the weight of the coke layer; in test 8, 14.28 times; and in test 9, 15.38 times. In test 9 it was noticed that the iron was not at the proper temperature, and in test 10 the coke layer was increased to a ratio of 1 to 14.28 without altering the blast pressure; this resulted in a decreased melt per hour. It has been found that a coke charge of 150 lbs. to 2000 lbs. of iron, with a blast pressure of 10.5 ounces, results in a melt of 11.5 tons per hour, the iron coming down at the proper temperature.

An excess of coke decreases the melting rate. Iron in the cupola is melted in a fixed zone, the first charge of iron above the bed being melted by burning coke in the bed. As this iron is melted, the charge of coke above it descends and restores to the bed the amount which has been burned away. If there is too much coke in the charge, the iron is held above the melting zone, and the excess coke must be burned away before it can be melted, and this of course decreases the economy and the melting speed.

Cupola Charges in Stove-foundries. (*Iron Age*, April 14, 1892.) — No two cupolas are charged exactly the same. The amount of fuel on the bed or between the charges differs, while varying amounts of iron are used in the charges. Below will be found charging-lists from some of the prominent stove-foundries in the country:

| | lbs. | | lbs. |
|----------------------------------|-------|--------------------------------|------|
| A —Bed of fuel, coke..... | 1,500 | Four next charges of coke, | |
| First charge of iron..... | 5,000 | each..... | 150 |
| All other charges of iron.... | 1,000 | Six next charges of coke, each | 120 |
| First and second charges of | | Nineteen next charges of coke, | |
| coke, each..... | 200 | each..... | 100 |

Thus for a melt of 18 tons there would be 5120 lbs. of coke used, giving a ratio of 7 to 1. Increase the amount of iron melted to 24 tons, and a ratio of 8 pounds of iron to 1 of coal is obtained.

| | lbs. | | lbs. |
|----------------------------------|-------|-----------------------------|------|
| B —Bed of fuel, coke..... | 1,600 | Second and third charges of | |
| First charge of iron..... | 1,800 | fuel..... | 130 |
| First charge of fuel..... | 150 | All other charges of fuel, | |
| All other charges of iron, | | each..... | 100 |
| each..... | 1,000 | | |

For an 18-ton melt 5060 lbs. of coke would be necessary, giving a ratio of 7.1 lbs. of iron to 1 pound of coke.

| | lbs. | | lbs. |
|----------------------------------|-------|--------------------------------|-------|
| C —Bed of fuel, coke..... | 1,600 | All other charges of iron..... | 2,000 |
| First charge of iron..... | 4,000 | All other charges of coke..... | 150 |
| First and second charges of | | | |
| coke..... | 200 | | |

In a melt of 18 tons 4100 lbs. of coke would be used, or a ratio of 8.5 to 1.

| | lbs. | | lbs. |
|----------------------------------|-------|--------------------------------|-------|
| D —Bed of fuel, coke..... | 1,800 | All charges of coke, each..... | 200 |
| First charge of iron..... | 5,600 | All other charges of iron..... | 2,900 |

In a melt of 18 tons, 3900 lbs. of fuel would be used, giving a ratio of 9.4 pounds of iron to 1 of coke. Very high, indeed, for stove-plate.

| | lbs. | | lbs. |
|----------------------------------|-------|---------------------------------|-------|
| E —Bed of fuel, coal..... | 1,900 | All other charges of iron, each | 2,000 |
| First charge of iron..... | 5,000 | All other charges of coal, each | 175 |
| First charge of coal..... | 200 | | |

In a melt of 18 tons 4700 lbs. of coal would be used, giving a ratio of 7.7 lbs. of iron to 1 lb. of coal.

These are sufficient to demonstrate the varying practices existing among different stove-foundries. In all these places the iron was proper for stove-plate purposes, and apparently there was little or no difference in the kind of work in the sand at the different foundries.

Foundry Blower Practice. (W. B. Snow, *Trans. A. S. M. E.*, 1907.) — The velocity of air produced by a blower is expressed by the formula $V = \sqrt{2gp/d}$. If p , the pressure, is taken in ounces per sq. in., and d , the density, in pounds per cu. ft. of dry air at 50° and atmospheric pressure of 14.69 lbs. or 235 ounces, = 0.77884 lb., the formula reduces to $V = \sqrt{1,746,700 p/(235 + p)}$, no allowance being made for change of temperature during discharge. From this formula the following figures are obtained. Q = volume discharged per min. through an orifice of 1 sq. ft. effective area, H.P. = horse-power required to move the given volume under the given conditions, p = pressure in ounces per sq. in.

| p | Q | H.P. | p | Q | H.P. | p | Q | H.P. | p | Q | H.P. |
|-----|-------|---------|-----|--------|--------|-----|--------|--------|-----|--------|--------|
| 1 | 35.85 | 0.00978 | 6 | 86.89 | 0.1422 | 11 | 116.45 | 0.3493 | 16 | 139.03 | 0.6067 |
| 2 | 50.59 | 0.02759 | 7 | 93.66 | 0.1788 | 12 | 121.38 | 0.3972 | 17 | 143.03 | 0.6631 |
| 3 | 61.83 | 0.05058 | 8 | 99.92 | 0.2180 | 13 | 126.06 | 0.4470 | 18 | 146.88 | 0.7211 |
| 4 | 71.24 | 0.07771 | 9 | 105.76 | 0.2596 | 14 | 130.57 | 0.4986 | 19 | 150.61 | 0.7804 |
| 5 | 79.48 | 0.1084 | 10 | 111.25 | 0.3034 | 15 | 134.89 | 0.5518 | 20 | 154.22 | 0.8412 |

The greatest effective area over which a fan will maintain the maximum velocity of discharge is known as the "capacity area" or "square inches of blast." As originally established by Sturtevant it is represented by $DW/3$, D = diam. of wheel in ins., W = width of wheel at circumference,

in inches. For the ordinary type of fan at constant speed maximum efficiency and power are secured at or near the capacity area; the power per unit of volume and the pressure decrease as the discharge area and volume increase; with closed outlet the power is approximately one-third of that at capacity area.

The following table is calculated on these bases: Capacity area per inch of width at periphery of wheel = $\frac{1}{3}$ of diam. Air, 50° F. Velocity of discharge = circumferential speed of the wheel. Power = double the theoretical. In rotary positive blowers, as well as in fans, the velocity and the volume vary as the number of revolutions, the pressure varies as the square, and the power as the cube of the number of revolutions. In the fan, however, increase of pressure can be had only by increasing the revolutions, while in the rotary blower a great range of pressure is obtainable with constant speed by merely varying the resistance. With a rotary blower at constant speed, theoretically, and disregarding the effect of changes in temperature and density, the volume is constant; the velocity varies inversely as the effective outlet area; the pressure varies inversely as the square of the outlet area, hence as the square of the velocity; and the power varies directly as the pressure. The maximum power is required when a fan discharges against the least, and when a rotary blower discharges against the greatest resistance.

PERFORMANCE OF CUPOLA FAN BLOWERS AT CAPACITY AREA PER INCH OF PERIPHERAL WIDTH.

| Diam. of Wheel, ins. | Item. | Total Pressure in Ounces per Square Inch. | | | | | | | | | | |
|----------------------|---------|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 18 | r.p.m. | 2660.0 | 2860.0 | 3050.0 | 3230.0 | 3400.0 | 3560.0 | 3710.0 | 3850.0 | 3990.0 | 4120.0 | 4250.0 |
| | cu. ft. | 520.0 | 560.0 | 600.0 | 640.0 | 670.0 | 700.0 | 730.0 | 760.0 | 780.0 | 810.0 | 830.0 |
| | h.p. | 1.7 | 2.1 | 2.6 | 3.1 | 3.6 | 4.2 | 4.8 | 5.4 | 6.0 | 6.6 | 7.3 |
| 24 | r.p.m. | 2000.0 | 2150.0 | 2290.0 | 2420.0 | 2550.0 | 2670.0 | 2780.0 | 2890.0 | 2990.0 | 3090.0 | 3190.0 |
| | cu. ft. | 700.0 | 750.0 | 800.0 | 850.0 | 890.0 | 930.0 | 970.0 | 1010.0 | 1040.0 | 1080.0 | 1110.0 |
| | h.p. | 2.3 | 2.9 | 3.5 | 4.2 | 4.9 | 5.6 | 6.4 | 7.1 | 8.0 | 8.8 | 9.7 |
| 30 | r.p.m. | 1590.0 | 1720.0 | 1830.0 | 1940.0 | 2040.0 | 2140.0 | 2230.0 | 2310.0 | 2390.0 | 2470.0 | 2550.0 |
| | cu. ft. | 870.0 | 940.0 | 1000.0 | 1060.0 | 1110.0 | 1160.0 | 1210.0 | 1260.0 | 1310.0 | 1350.0 | 1390.0 |
| | h.p. | 2.8 | 3.6 | 4.4 | 5.2 | 6.1 | 7.0 | 7.9 | 8.9 | 10.0 | 11.0 | 12.1 |
| 36 | r.p.m. | 1330.0 | 1430.0 | 1530.0 | 1620.0 | 1700.0 | 1780.0 | 1850.0 | 1930.0 | 2000.0 | 2060.0 | 2120.0 |
| | cu. ft. | 1040.0 | 1120.0 | 1200.0 | 1270.0 | 1340.0 | 1400.0 | 1460.0 | 1510.0 | 1570.0 | 1620.0 | 1670.0 |
| | h.p. | 3.4 | 4.3 | 5.2 | 6.2 | 7.3 | 8.4 | 9.5 | 10.7 | 11.9 | 13.2 | 14.5 |
| 42 | r.p.m. | 1140.0 | 1230.0 | 1310.0 | 1380.0 | 1460.0 | 1530.0 | 1590.0 | 1650.0 | 1710.0 | 1770.0 | 1820.0 |
| | cu. ft. | 1220.0 | 1310.0 | 1400.0 | 1480.0 | 1560.0 | 1630.0 | 1700.0 | 1770.0 | 1830.0 | 1890.0 | 1950.0 |
| | h.p. | 3.9 | 5.0 | 6.1 | 7.3 | 8.5 | 9.8 | 11.1 | 12.5 | 13.9 | 15.4 | 17.0 |
| 48 | r.p.m. | 1000.0 | 1070.0 | 1150.0 | 1210.0 | 1270.0 | 1330.0 | 1390.0 | 1450.0 | 1500.0 | 1550.0 | 1590.0 |
| | cu. ft. | 1390.0 | 1500.0 | 1600.0 | 1690.0 | 1780.0 | 1860.0 | 1940.0 | 2020.0 | 2090.0 | 2160.0 | 2230.0 |
| | h.p. | 4.5 | 5.7 | 7.0 | 8.3 | 9.7 | 11.2 | 12.7 | 14.3 | 15.9 | 17.7 | 21.0 |

The air supply required by a cupola varies with the melting ratio, the density of the charges, and the incidental leakage. Average practice is represented by the following:

Lbs. iron per lb. coke 6 7 8 9 10
 Cu. ft. air per ton of iron 33,000 31,000 29,000 27,000 25,000

It is customary to provide blower capacity on a basis of 30,000 cu. ft., which corresponds to 75 to 80% of the chemical requirements for complete combustion with average coke, and a melting ratio of 7.5 to 1.

In comparative tests with a 54-inch lining cupola under identical conditions as to contents, alternately run with a No. 10 Sturtevant fan and a 33 cu. ft. Connorsville rotary, with the fan the pressure varied between 12½ and 14⅛ ounces in the wind box, the net power from 25 to 38.5 H.P., while with the rotary blower the pressure varied between 10½ and 25 ounces, and the power between 19 and 45 H.P. With the fan 28.84 tons

were melted in 3.77 hours, or 7.65 tons per hour, while with the rotary blower 2.82 hours were required to melt 31.5 tons, an hourly rate of 10.6 tons, an increase of nearly 40 per cent in output. This reduces to a net input of 4.09 H.P. per ton melted per hour with the fan, and 2.98 H.P. with the rotary blower; an apparent advantage of 27% in favor of the rotary. Had the rotary been of smaller capacity such excessive pressures would not have been necessary, the power would have been decreased, and the duration of the heat prolonged, with probable decrease in the H.P. hours per ton. Had the fan been run at higher speed the H.P. would have increased, the time decreased and the power per ton per hour would have more closely approached that required by the rotary blower.

Theoretically, for otherwise constant conditions, the following relations hold for cupolas and melting rates within the range of practical operation: For a given cupola: For a given melting rate: For a given volume:

$$M \propto V, \sqrt{P} \text{ or } \sqrt[3]{\text{H.P.}}$$

$$V \propto 1 \div D^2$$

$$M \propto D$$

$$V \propto M$$

$$P \propto d$$

For a given cupola

$$P \propto V^2$$

$$\text{H.P.} \propto P \text{ or } 1 \div D^4$$

$$E \propto M^2, \text{ or } P$$

$$\text{H.P.} \propto M^3 \text{ or } \sqrt{P^3}$$

$$E \propto M, P, \text{ or } 1 \div D^4$$

$$\text{Duration of heat} \propto 1 \div \sqrt{P}$$

M = melting rate; V = volume; P = pressure; H.P. = horse-power; D = diam. of lining; E = operating efficiency = power per ton per hour; d = depth of the charge; \propto , varies as.

These relations might be the source of formulæ for practical use were it possible to establish accurate coefficients. But the variety in cupolas, tuyere proportions, character of fuel and iron, and difference in charging practice are bewildering and discouraging. Maximum efficiency in a given case can only be assured after direct experiment. Something short of the maximum is usually accepted in ignorance of the ultimate possibilities.

The actual melting range of a cupola is ordinarily between 0.6 and 0.75 ton per hour per sq. ft. of cross section. The limits of air supply per minute per sq. ft. are roughly 2500 and 4000 cu. ft. The possible power required varies even more widely, ranging from 1.5 to 3.75 H.P. per sq. ft., corresponding to 2.5 and 5 H.P. per ton per hour for the melting rates specified. The power may be roughly calculated, from the theoretical requirement of 0.27 H.P. to deliver 1000 cu. ft. per minute against 1 oz. pressure. The power increases directly with the pressure, and depends also on the efficiency of the blower. Current practice can only be expressed between limits as in the following table.

RANGE OF PERFORMANCE OF CUPOLA BLOWERS.

| Diameter inside Lining, in. | Capacity per Hour, tons. | Pressure per sq. in., oz. | Volume of Air per min., cu. ft. | Horse-power. |
|-----------------------------|--------------------------|---------------------------|---------------------------------|--------------|
| 18..... | 0.25-0.5 | 5-7 | 150-300 | 0.5-1.5 |
| 24..... | 1.00-1.5 | 7-9 | 600-900 | 2.0-6.0 |
| 30..... | 2.00-3.5 | 8-11 | 1,200-2,000 | 5.0-15.0 |
| 36..... | 4.00-5.0 | 8-12 | 2,200-2,800 | 10.0-23.0 |
| 42..... | 5.00-7.0 | 8-13 | 2,700-3,700 | 12.0-32.0 |
| 48..... | 8.00-10.0 | 8-13 | 4,000-5,000 | 18.0-45.0 |
| 54..... | 9.00-12.0 | 9-14 | 4,500-6,000 | 22.0-60.0 |
| 60..... | 12.00-15.0 | 9-14 | 6,000-7,500 | 30.0-75.0 |
| 66..... | 14.00-18.0 | 9-15 | 7,000-9,000 | 35.0-90.0 |
| 72..... | 17.00-21.0 | 10-15 | 8,500-10,500 | 45.0-110.0 |
| 78..... | 19.00-24.0 | 10-16 | 9,500-12,000 | 52.0-130.0 |
| 84..... | 21.00-27.0 | 10-16 | 10,500-13,500 | 60.0-150.0 |

Results of Increased Driving. (Erie City Iron-works, 1891.)—May-Dec., 1890: 60-in. cupola, 100 tons clean castings a week, melting 8 tons per hour; iron per pound of fuel, 7½ lbs.; per cent weight of good castings to iron charged, 75¾. Jan.-May, 1891: Increased rate of melting to 11½ tons per hour; iron per lb. fuel, 9½; per cent weight of good castings, 75; one week, 13¼ tons per hour, 10.3 lbs. iron per lb. fuel; per cent weight of good castings, 75.3. The increase was made by putting in an additional row of tuyeres and using stronger blast, 14 ounces. Coke was used as fuel. (W. O. Webber, *Trans. A. S. M. E.*, xii, 1045.)

Power Required for a Cupola Fan. (Thos. D. West, *The Foundry*, April, 1904.) — The power required when a fan is connected with a cupola depends on the length and diameter of the piping, the number of bends, valves, etc., and on the resistance to the passage of blast through the cupola. The approximate power required in everyday practice is the difference between the power required to run the fan with the outlet open and with it closed. Another rule is to take 75% of the maximum power or that with the outlet open. A fan driving a cupola 66 ins. diam., 1800 r.p.m., driven by an electric motor required horse-power and gave pressures as follows: Outlet open, 146.6; outlet closed, 37.2, pressure 15 oz.; attached to cupola, with no fuel in it, 120.5, 5 oz.; after kindling and coke had been fired, 101.0, 10 oz.; during the run 70.8 to 76.7, 11 to 13 oz., the variations being due to changes in the resistances to the passage of the blast.

Utilization of Cupola Gases. — Jules De Clercy, in a paper read before the Amer. Foundrymen's Assn., advises the return of a portion of the gases from the upper part of the charge to the tuyeres, and thus utilizing the carbon monoxide they contain. He says that A. Baillot has thereby succeeded in melting 15 lbs. of iron per lb. of coke, and at the same time obtained a greater melting speed and a superior quality of castings.

Loss in Melting Iron in Cupolas. — G. O. Vair, *Am. Mach.*, March 5, 1891, gives a record of a 45-in. Colliau cupola as follows:

| | |
|-----------------------------------|-------------|
| Ratio of fuel to iron, 1 to 7.42. | |
| Good castings..... | 21,314 lbs. |
| New scrap..... | 3,005 " |
| Millings..... | 200 " |
| Loss of metal..... | 1,481 " |

Amount melted..... 26,000 lbs.

Loss of metal, 5.69%. Ratio of loss, 1 to 17.55.

Use of Softeners in Foundry Practice. (W. Graham, *Iron Age*, June 27, 1889.) — In the foundry the problem is to have the right proportions of combined and graphitic carbon in the resulting casting; this is done by getting the proper proportion of silicon. The variations in the proportions of silicon afford a reliable and inexpensive means of producing a cast iron of any required mechanical character which is possible with the material employed. In this way, by mixing suitable irons in the right proportions, a required grade of casting can be made more cheaply than by using irons in which the necessary proportions are already found.

Hard irons, mottled and white irons, and even steel scrap, all containing low percentages of silicon and high percentages of combined carbon, could be employed if an iron having a large amount of silicon were mixed with them in sufficient amount. This would bring the silicon to the proper proportion and would cause the combined carbon to be forced into the graphitic state, and the resulting casting would be soft. High-silicon irons used in this way are called "softeners."

Mr. Keep found that more silicon is lost during the remelting of pig of over 10% silicon than in remelting pig iron of lower percentages of silicon. He also points out the possible disadvantage of using ferro-silicons containing as high a percentage of combined carbon as 0.70% to overcome the bad effects of combined carbon in other irons.

The Scotch irons generally contain much more phosphorus than is desired in irons to be employed in making the strongest castings. It is a mistake to mix with strong low-phosphorus irons an iron that would increase the amount of phosphorus for the sake of adding softening qualities, when softness can be produced by mixing irons of the same low phosphorus.

(For further discussion of the influence of silicon see pages 415 and 422.)

Weakness of Large Castings. (W. A. Bole, *Trans. A. S. M. E.*, 1907.) — Thin castings, by virtue of their more rapid cooling, are almost certain to be stronger per unit section than would be the case if the same metal were poured into larger and heavier shapes. Many large iron castings are of questionable strength, because of internal strains and lack of harmony between their elements, even though the casting is poured out of iron of the best quality. This may be due to lack of experience on the part of

the designer, especially in the cooling and shrinking of the various parts or a large casting after being poured.

Castings are often designed with a useless multiplicity of ribs, walls, gussets, brackets, etc., which, by their asynchronous cooling and their inharmonious shrinkage and contraction, may entirely defeat the intention of the designer.

There are some castings which, by virtue of their shapes, can be specially treated by the foundryman, and artificial cooling of certain critical parts may be effected in order to compel such parts to cool more rapidly than they would naturally do, and the strength of the casting may by such means be beneficially affected. As for instance in the case of a fly-wheel with heavy rim but comparatively light arms and hub; it may be beneficial to remove the flask and expose the rim to the air so as to hasten its natural rate of cooling, while the arms and hub are still kept muffled up in the sand of the mold and their cooling retarded as much as possible.

Large fillets are often highly detrimental to good results. Where two walls meet and intersect, as in the shape of a *T*, if a large fillet is swept at the juncture, there will be a pool of liquid metal at this point which will remain liquid for a longer time than either wall, the result being a void, or "draw," at the juncture point.

Risers and sink heads should often be employed on iron castings. In large iron-foundry work interior cavities may exist without detection, and some of these may be avoided by the use of suitable feeding devices, risers and sink heads.

Specimens from a casting having at one point a tensile strength as high as 30,250 lbs. per sq. in. have shown as low as 20,500 in another and heavier section. Large sections cannot be cast to yield the high strength of specimen test pieces cast in smaller sections.

The paper describes a successful method of artificial cooling, by means of a coil of pipe with flowing water, of portions of molds containing cylinder heads with ports cast in them. Before adopting this method the internal ribs in these castings always cracked by contraction.

Shrinkage of Castings.—The allowance necessary for shrinkage varies for different kinds of metal, and the different conditions under which they are cast. For castings where the thickness runs about one inch, cast under ordinary conditions, the following allowance can be made:

| | |
|---|---|
| For cast iron, $\frac{1}{8}$ inch per foot. | For zinc, $\frac{5}{16}$ inch per foot. |
| " brass, $\frac{3}{16}$ " " " | " tin, $\frac{1}{12}$ " " " |
| " steel, $\frac{1}{4}$ " " " | " aluminum, $\frac{3}{16}$ " " " |
| " mal. iron, $\frac{1}{8}$ " " " | " britannia, $\frac{1}{32}$ " " " |

Thicker castings, under the same conditions, will shrink less, and thinner ones more, than this standard. The quality of the material and the manner of molding and cooling will also make a difference. (See also Shrinkage of Cast Iron, page 423.)

Mr. Keep (*Trans. A. S. M. E.*, vol. xvi) gives the following "approximate key for regulating foundry mixtures" so as to produce a shrinkage of $\frac{1}{8}$ in. per ft. in castings of different sections:

| | | | | | | |
|---|---------------|------|------|------|------|-------------|
| Size of casting | $\frac{1}{2}$ | 1 | 2 | 3 | 4 | in. sq. |
| Silicon required, per cent | 3.25 | 2.75 | 2.25 | 1.75 | 1.25 | per cent. |
| Shrinkage of a $\frac{1}{2}$ -in. test-bar .. | 0.125 | .135 | .145 | .155 | .165 | in per. ft. |

Growth of Cast Iron by Heating. (*Proc. I. and S. Inst.*, 1909.)—Investigations by Profs. Rugan and Carpenter confirm Mr. Outerbridge's experiments. (See page 425.) They found: 1. Heating white iron causes it to become gray, and it expands more than sufficient to overcome the original shrinkage. 2. Iron when heated increases in weight, probably due to absorption of oxygen. 3. The change in size due to heating is not only a molecular change, but also a chemical one. 4. The growth of one bar was shown to be due to penetration of gases. When heated in vacuo it contracted.

Hard Iron due to Excessive Silicon.—W. J. Keep in *Jour. Am. Foundrymen's Assn.*, Feb., 1898, reports a case of hard iron containing graphite, 3.04; combined C, 0.10; Si, 3.97; P, 0.61; S, 0.05; Mn, 0.56. He says: For stove plate and light hardware castings it is an advantage to have Si as high as 3.50. When it is much above that the surface of the castings often become very hard, though the center will be very soft.

The surface of heavier parts of a casting having 3.97 Si will be harder than the surface of thinner parts. Ordinarily if a casting is hard an increase of silicon softens it, but after reaching 3.00 or 3.50 per cent, silicon hardens a casting.

Ferro-Alloys for Foundry Use. E. Houghton (*Iron Tr. Rev.*, Oct. 24, 1907.) — The objects of the use of ferro-alloys in the foundry are: 1, to act as deoxidizers and desulphurizers, the added element remaining only in small quantities in the finished casting; 2, to alter the composition of the casting and so to control its mechanical properties. Some of these alloys are made in the blast furnace, but the purest grades are made in the electric furnace. The following table shows the range of composition of blast furnace alloys made by the Darwen & Mostyn Iron Co. All of these alloys may be made of purer quality in the electric furnace.

| | Ferro-Mn. | Spiegel-eisen. | Silicon Spiegel. | Ferro-sil. | Ferro-phos. | Ferro-Chrome. |
|---------|------------|----------------|------------------|------------|-------------|-----------------|
| Mn..... | 41.5- 87.9 | 9.25-29.75 | 17.50-20.87 | 1.17- 2.20 | 3.00- 5.90 | 1.55- 2.30 |
| Si..... | 0.10- 0.63 | 0.42- 0.95 | 9.45-14.23 | 8.10-17.00 | 0.50- 0.84 | 0.13- 0.36 |
| P..... | 0.09- 0.20 | 0.06- 0.09 | 0.07- 0.10 | 0.06- 0.08 | 15.71-20.50 | 0.04- 0.07 |
| C..... | 5.62- 7.00 | 3.94- 5.20 | 1.05- 1.89 | 0.90- 1.75 | 0.27- 0.30 | 5.34- 7.12 |
| S..... | nil | nil-trace | nil | 0.02- 0.05 | 0.16- 0.33 | Cr, 13.50-41.39 |

The following are typical analyses of other alloys made in the electric furnace:

| | Si | Fe | Mn | Al | Ca | Mg | C | S | P | Ti |
|------------------------------|-------|-------|-------|------|-------|-------|------|------|------|-------|
| Ferro-titanium..... | 1.21 | | | 0.30 | | | 3.28 | 0.03 | 0.02 | 53.0 |
| Ferro-aluminum-silicide..... | 45.65 | 44.15 | tr. | 9.45 | nil | nil | 0.55 | 0.01 | 0.03 | |
| Ferro-calcium-silicide..... | 69.80 | 11.15 | 0.22 | 2.55 | 15.05 | 0.26 | 1.14 | 0.01 | 0.04 | |

Ferro-aluminum, Al, 5, 10 and 20%. Metallic manganese, Mn, 95 to 98; Fe, 2 to 4; C, under 5. Do. refined. Mn, 99; Fe, 1; C, 0.

Dangerous Ferro-silicon. — Phosphoretted and arseniuretted hydrogen, highly poisonous gases, are said to be disengaged in a humid atmosphere from ferro-silicon containing between 30 and 40% and between 47 and 65% of Si, and there is therefore danger in transporting it in passenger steamships. A French commission has recommended the abandonment of the manufacture of FeSi of these critical percentages. (*La Lumiere Electrique*, Dec. 11, 1909. *Elec. Rev.*, Feb. 26, 1910.)

Quality of Foundry Coke. (R. Moldenke, *Trans. A. S. M. E.*, 1907.) — Usually the sulphur, ash and fixed carbon are sufficient to give a fair idea of the value of coke, apart from its physical structure, specific gravity, etc. The advent of by-product coke will necessitate closer attention to moisture. Beehive coke, when shipped in open cars, may, through inattention, cause the purchase of from 6 to 10 per cent of water at coke prices.

Concerning sulphur, very hot running of the cupola results in less sulphur in the iron. In good coke, the amount of S should not exceed 1.2 per cent; but, unfortunately, the percentage often runs as high as 2.00. If the coke has a good structure, an average specific gravity, not over 11 per cent of ash and over 86 per cent of fixed carbon, it does not matter much whether it be of the "72 hour" or "24 hour" variety. Departure from the normal composition of a coke of any particular region should place the foundryman on his guard at once, and sometimes the plentiful use of limestone at the right moment may save many castings.

Castings made in Permanent Cast-Iron Molds. — E. A. Custer, in a paper before the Am. Foundrymen's Assn. (*Eng. News*, May 27, 1909), describes the method of making castings in iron molds, and the quality of these castings. Very heavy molds are used, no provision is made against shrinkage, and the casting is removed from the mold as soon as it has set, giving it no time to chill or to shrink by cooling. Over 6000 pieces have been cast in a single mold without its showing any signs of

failure. The mold should be so heavy that it will not become highly heated in use. Casting a 4-in. pipe weighing 65 lbs. every seven minutes in a mold weighing 6500 lbs. did not raise the temperature above 300° F. In using permanent molds the iron as it comes from the cupola should be very hot. The best results in casting pipe are had with iron containing over 3% carbon and about 2% silicon. Iron when cast in an iron mold and removed as soon as it sets, possesses some unusual properties. It will take a temper, and when tempered will retain magnetism. If the casting be taken from the mold at a bright heat and suddenly quenched in cold water, it has the cutting power of a good high-carbon steel, whether the iron be high or low in silicon, phosphorus, sulphur or manganese. There is no evidence of "chill"; no white crystals are shown.

Chilling molten iron swiftly to the point of setting, and then allowing it to cool gradually, produces a metal that is entirely new to the art. It has the chemical characteristics of cast iron, with the exception of combined carbon, and it also possesses some of the properties of high-carbon steel. A piece of cast iron that has 0.44% combined, and over 2% free carbon, has been tempered repeatedly and will do better service in a lathe than a good non-alloy steel. Once this peculiar property is imparted to the casting, it is impossible to eliminate it except by remelting. A bar of iron so treated can be held in a flame until the metal drips from the end, and yet quenching will restore it to its original hardness.

The character of the iron before being quenched is very fine, close-grained, and yet it is easily machined. If permanent molds can be used with success in the foundry, and a system of continuous pouring be inaugurated which in duplicate work would obviate the necessity of having molders, why is it necessary to melt pig iron in the cupola? What could be more ideal than a series of permanent molds supplied with molten iron practically direct from the blast furnace? The interposition of a reheating ladle such as is used by the steel makers makes possible the treatment of the molten iron.

The molten iron from the blast furnace is much hotter than that obtained from the cupola, so that there is every reason to believe that the castings obtained from a blast furnace would be of a better quality than when the pig is remelted in the cupola.

It is immaterial whether an iron contains 1.75 or 3% silicon, so long as the molten mass is at the proper temperature, so that the high temperatures obtained from the blast furnace would go far toward offsetting the variations in the impurities.

R. H. Probert (*Castings*, July, 1909) gives the following analysis of molds which gave the best results: Si, 2.02; S, 0.07; P, 0.89; Mn, 0.29; C.C., 0.84; G.C., 2.76. Molds made from iron with the following analysis were worthless: Si, 3.30; S, 0.06; P, 0.67; Mn, 0.12; C.C., 0.19; G.C., 2.98.

Weight of Castings determined from Weight of Pattern.

(Rose's Pattern-makers' Assistant.)

| A Pattern weighing One Pound, made of — | Will weigh when cast in | | | | |
|---|-------------------------|-------|---------|---------------|------------|
| | Cast Iron. | Zinc. | Copper. | Yellow Brass. | Gun metal. |
| | lbs. | lbs. | lbs. | lbs. | lbs. |
| Mahogany—Nassau..... | 10.7 | 10.4 | 12.8 | 12.2 | 12.5 |
| " Honduras..... | 12.9 | 12.7 | 15.3 | 14.6 | 15. |
| " Spanish..... | 8.5 | 8.2 | 10.1 | 9.7 | 9.9 |
| Pine, red..... | 12.5 | 12.1 | 14.9 | 14.2 | 14.6 |
| " white..... | 16.7 | 16.1 | 19.8 | 19.0 | 19.5 |
| " yellow..... | 14.1 | 13.6 | 16.7 | 16.0 | 16.5 |

Molding Sand. (Walter Bagshaw, *Proc. Inst. M. E.*, 1891.) — The chemical composition of sand will affect the nature of the casting, no matter what treatment it undergoes. Stated generally, good sand is composed of 94 parts silica, 5 parts alumina, and traces of magnesia and oxide of iron. Sand containing much of the metallic oxides, and especially

lime, is to be avoided. Geographical position is the chief factor governing the selection of sand; and whether weak or strong, its deficiencies are made up for by the skill of the molder. For this reason the same sand is often used for both heavy and light castings, the proportion of coal varying according to the nature of the casting. A common mixture of facing-sand consists of six parts by weight of old sand, four of new sand, and one of coal-dust. Floor-sand requires only half the above proportions of new sand and coal-dust to renew it. German foundries adopt one part by measure of new sand to two of old sand; to which is added coal-dust in the proportion of one-tenth of the bulk for large castings, and one-twentieth for small castings. A few foundries mix street-sweepings with the coal in order to get porosity when the metal in the mold is likely to be a long time in setting. Plumbago is effective in preventing destruction of the sand; but owing to its refractory nature, it must not be dusted on in such quantities as to close the pores and prevent free exit of the gases. Powdered French chalk, soapstone, and other substances are sometimes used for facing the mold; but next to plumbago, oak charcoal takes the best place, notwithstanding its liability to float occasionally and give a rough casting.

For the treatment of sand in the molding-shop the most primitive method is that of hand-riddling and treading. Here the materials are roughly proportioned by volume, and riddled over an iron plate in a flat heap, where the mixture is trodden into a cake by stamping with the feet; it is turned over with the shovel, and the process repeated. Tough sand can be obtained in this manner, its toughness being usually tested by squeezing a handful into a ball and then breaking it; but the process is slow and tedious. Other things being equal, the chief characteristics of a good molding-sand are toughness and porosity, qualities that depend on the manner of mixing as well as on uniform ramming.

Toughness of Sand.—In order to test the relative toughness, sand mixed in various ways was pressed under a uniform load into bars 1 in. sq. and about 12 in. long, and each bar was made to project further and further over the edge of a table until its end broke off by its own weight. Old sand from the shop floor had very irregular cohesion, breaking at all lengths of projections from $\frac{1}{2}$ in. to $1\frac{1}{2}$ in. New sand in its natural state held together until an overhang of $2\frac{3}{4}$ in. was reached. A mixture of old sand, new sand, and coal-dust

| | | |
|---------------------------------|--------------------|------------------------------------|
| Mixed under rollers..... | broke at 2 | to $2\frac{1}{4}$ in. of overhang. |
| " in the centrifugal machine... | " " 2 | " $2\frac{1}{4}$ " " " |
| " through a riddle..... | " " $1\frac{3}{4}$ | " $2\frac{1}{8}$ " " " |

showing as a mean of the tests only slight differences between the last three methods, but in favor of machine-work. In many instances the fractures were so uneven that minute measurements were not taken.

Heinrich Ries (*Castings*, July, 1908) says that chemical analysis gives little or no information regarding the bonding power, texture, permeability or use of sand, the only case in which it is of value being in the selection of a highly silicious sand for certain work such as steel casting.

Dimensions of Foundry Ladles.—The following table gives the dimensions, inside the lining, of ladles from 25 lbs. to 16 tons capacity. All the ladles are supposed to have straight sides. (*Am. Mach.*, Aug. 4, 1892.)

| Cap'y. | Diam. | Depth. | Cap'y. | Diam. | Depth. | Cap'y. | Diam. | Depth. |
|---------|-------|--------|------------------|-----------------|-----------------|----------|-----------------|-----------------|
| | in. | in. | | in. | in. | | in. | in. |
| 16 tons | 54 | 56 | 3 tons | 31 | 32 | 300 lbs. | $11\frac{1}{2}$ | $11\frac{1}{2}$ |
| 14 " | 52 | 53 | 2 " | 27 | 28 | 250 " | $10\frac{3}{4}$ | 11 |
| 12 " | 49 | 50 | $1\frac{1}{2}$ " | $24\frac{1}{2}$ | 25 | 200 " | 10 | $10\frac{1}{2}$ |
| 10 " | 46 | 48 | 1 ton | 22 | 22 | 150 " | 9 | $9\frac{1}{2}$ |
| 8 " | 43 | 44 | $\frac{3}{4}$ " | 20 | 20 | 100 " | 8 | $8\frac{1}{2}$ |
| 6 " | 39 | 40 | $\frac{1}{2}$ " | 17 | 17 | 75 " | 7 | $7\frac{1}{2}$ |
| 4 " | 34 | 35 | $\frac{1}{4}$ " | $13\frac{1}{2}$ | $13\frac{1}{2}$ | 50 " | $6\frac{1}{2}$ | $6\frac{1}{2}$ |

THE MACHINE-SHOP.

SPEED OF CUTTING-TOOLS IN LATHES, MILLING MACHINES, ETC.

Relation of diameter of rotating tool or piece, number of revolutions and cutting-speed:

Let d = diam. of rotating piece in inches, n = No. of revs. per min.;

S = speed of circumference in feet per minute;

$$S = \frac{\pi dn}{12} = 0.2618 dn; n = \frac{S}{0.2618 d} = \frac{3.82 S}{d}; d = \frac{3.82 S}{n}$$

Approximate rule: Number of revolutions per minute = $4 \times$ speed in feet per minute \div diameter in inches.

Table of Cutting-speeds.

| Diameter, inches. | Feet per minute. | | | | | | | | | |
|----------------------|-------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | Revolutions per minute. | | | | | | | | | |
| 1/4 | 76.4 | 152.8 | 229.2 | 305.6 | 382.0 | 458.4 | 534.8 | 611.2 | 687.6 | 764.0 |
| 3/8 | 50.9 | 101.9 | 152.8 | 203.7 | 254.6 | 305.6 | 356.5 | 407.4 | 458.3 | 509.3 |
| 1/2 | 38.2 | 76.4 | 114.6 | 152.8 | 191.0 | 229.2 | 267.4 | 305.6 | 343.8 | 382.0 |
| 5/8 | 30.6 | 61.1 | 91.7 | 122.2 | 152.8 | 183.4 | 213.9 | 244.5 | 275.0 | 305.6 |
| 3/4 | 25.5 | 50.9 | 76.4 | 101.8 | 127.3 | 152.8 | 178.2 | 203.7 | 229.1 | 254.6 |
| 7/8 | 21.8 | 43.7 | 65.5 | 87.3 | 109.1 | 130.9 | 152.8 | 174.6 | 196.4 | 218.3 |
| 1 | 19.1 | 38.2 | 57.3 | 76.4 | 95.5 | 114.6 | 133.7 | 152.8 | 171.9 | 191.0 |
| 1 1/8 | 17.0 | 34.0 | 50.9 | 67.9 | 84.9 | 101.8 | 118.8 | 135.8 | 152.8 | 169.7 |
| 1 1/4 | 15.3 | 30.6 | 45.8 | 61.1 | 76.4 | 91.7 | 106.9 | 122.2 | 137.5 | 152.8 |
| 1 3/8 | 13.9 | 27.8 | 41.7 | 55.6 | 69.5 | 83.3 | 97.2 | 111.1 | 125.0 | 138.9 |
| 1 1/2 | 12.7 | 25.5 | 38.2 | 50.9 | 63.6 | 76.4 | 89.1 | 101.8 | 114.5 | 127.2 |
| 1 3/4 | 10.9 | 21.8 | 32.7 | 43.7 | 54.6 | 65.5 | 76.4 | 87.3 | 98.2 | 109.2 |
| 2 | 9.6 | 19.1 | 28.7 | 38.2 | 47.8 | 57.3 | 66.9 | 76.4 | 86.0 | 95.5 |
| 2 1/4 | 8.5 | 17.0 | 25.5 | 34.0 | 42.5 | 50.9 | 59.4 | 67.9 | 76.4 | 84.9 |
| 2 1/2 | 7.6 | 15.3 | 22.9 | 30.6 | 38.2 | 45.8 | 53.5 | 61.1 | 68.8 | 76.4 |
| 2 3/4 | 6.9 | 13.9 | 20.8 | 27.8 | 34.7 | 41.7 | 48.6 | 55.6 | 62.5 | 69.5 |
| 3 | 6.4 | 12.7 | 19.1 | 25.5 | 31.8 | 38.2 | 44.6 | 50.9 | 57.3 | 63.7 |
| 3 1/2 | 5.5 | 10.9 | 16.4 | 21.8 | 27.3 | 32.7 | 38.2 | 43.7 | 49.1 | 54.6 |
| 4 | 4.8 | 9.6 | 14.3 | 19.1 | 23.9 | 28.7 | 33.4 | 38.2 | 43.0 | 47.8 |
| 4 1/2 | 4.2 | 8.5 | 12.7 | 17.0 | 21.2 | 25.5 | 29.7 | 34.0 | 38.2 | 42.5 |
| 5 | 3.8 | 7.6 | 11.5 | 15.3 | 19.1 | 22.9 | 26.7 | 30.6 | 34.4 | 38.1 |
| 5 1/2 | 3.5 | 6.9 | 10.4 | 13.9 | 17.4 | 20.8 | 24.3 | 27.8 | 31.2 | 34.7 |
| 6 | 3.2 | 6.4 | 9.5 | 12.7 | 15.9 | 19.1 | 22.3 | 25.5 | 28.6 | 31.8 |
| 7 | 2.7 | 5.5 | 8.2 | 10.9 | 13.6 | 16.4 | 19.1 | 21.8 | 24.6 | 27.3 |
| 8 | 2.4 | 4.8 | 7.2 | 9.6 | 11.9 | 14.3 | 16.7 | 19.1 | 21.5 | 23.9 |
| 9 | 2.1 | 4.2 | 6.4 | 8.5 | 10.6 | 12.7 | 14.8 | 17.0 | 19.1 | 21.2 |
| 10 | 1.9 | 3.8 | 5.7 | 7.6 | 9.6 | 11.5 | 13.3 | 15.3 | 17.2 | 19.1 |
| 11 | 1.7 | 3.5 | 5.2 | 6.9 | 8.7 | 10.4 | 12.2 | 13.9 | 15.6 | 17.4 |
| 12 | 1.6 | 3.2 | 4.8 | 6.4 | 8.0 | 9.5 | 11.1 | 12.7 | 14.3 | 15.9 |
| 13 | 1.5 | 2.9 | 4.4 | 5.9 | 7.3 | 8.8 | 10.3 | 11.8 | 13.2 | 14.7 |
| 14 | 1.4 | 2.7 | 4.1 | 5.5 | 6.8 | 8.2 | 9.5 | 10.9 | 12.3 | 13.6 |
| 15 | 1.3 | 2.5 | 3.8 | 5.1 | 6.4 | 7.6 | 8.9 | 10.2 | 11.5 | 12.7 |
| 16 | 1.2 | 2.4 | 3.6 | 4.8 | 6.0 | 7.2 | 8.4 | 9.5 | 10.7 | 11.9 |
| 18 | 1.1 | 2.1 | 3.2 | 4.2 | 5.3 | 6.4 | 7.4 | 8.5 | 9.5 | 10.6 |
| 20 | 1.0 | 1.9 | 2.9 | 3.8 | 4.8 | 5.7 | 6.7 | 7.6 | 8.6 | 9.6 |
| 22 | .9 | 1.7 | 2.6 | 3.5 | 4.3 | 5.2 | 6.1 | 6.9 | 7.8 | 8.7 |
| 24 | .8 | 1.6 | 2.4 | 3.2 | 4.0 | 4.8 | 5.6 | 6.4 | 7.2 | 8.0 |
| 26 | .7 | 1.5 | 2.2 | 2.9 | 3.7 | 4.4 | 5.1 | 5.9 | 6.6 | 7.3 |
| 28 | .7 | 1.4 | 2.0 | 2.7 | 3.4 | 4.1 | 4.8 | 5.5 | 6.1 | 6.8 |
| 30 | .6 | 1.3 | 1.9 | 2.5 | 3.2 | 3.8 | 4.5 | 5.1 | 5.7 | 6.4 |
| 36 | .5 | 1.1 | 1.6 | 2.1 | 2.7 | 3.2 | 3.7 | 4.2 | 4.8 | 5.3 |
| 42 | .5 | .9 | 1.4 | 1.8 | 2.3 | 2.7 | 3.2 | 3.6 | 4.1 | 4.5 |
| 48 | .4 | .8 | 1.2 | 1.6 | 2.0 | 2.4 | 2.8 | 3.2 | 3.6 | 4.0 |
| 54 | .4 | .7 | 1.1 | 1.4 | 1.8 | 2.1 | 2.5 | 2.8 | 3.2 | 3.5 |
| 60 | .3 | .6 | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 | 2.5 | 2.9 | 3.2 |

The Speed of Counter-shaft of the lathe is determined by an assumption of a slow speed with the back gear, say 6 feet per minute, on the largest diameter that the lathe will swing.

EXAMPLE. — A 30-inch lathe will swing 30 inches =, say, 90 inches circumference = 7 feet 6 inches; the lowest triple gear should give a speed of 5 or 6 feet per minute.

Spindle Speeds of Lathes. — The spindle speeds of lathes are usually in geometric progression, being obtained either by a combination of cone-pulley and back gears, or by a single pulley in connection with a gear train. Either of these systems may be used with a variable speed motor, giving a wide range of available speeds.

It is desirable to keep work rotating at a rate that will give the most economical cutting speed, necessitating, sometimes, frequent changes in spindle speed. A variable speed motor arranged for 20 speeds in geometric progression, any one of which can be used with any speed due to the mechanical combination of belts and back gears, gives a fine gradation of cutting speeds. The spindle speeds obtained with the higher speeds of the motor in connection with a certain mechanical arrangement of belt and back gears may overlap those obtained with the lower speeds available in the motor in connection with the next higher speed arrangement of belt and gears, but about 200 useful speeds are possible. E. R. Douglas (*Elec. Rev.*, Feb. 10, 1906) presents an arrangement of variable speed motor and geared head lathe, with 22 speed variations in the motor and 3 in the head. The speed range of the spindle is from 4.1 to 500 r.p.m. By the use of this arrangement, and taking advantage of the speed changes possible for different diameters of the work, a saving of 35.4 per cent was obtained in the time of turning a piece ordinarily requiring 289 minutes.

Rule for Gearing Lathes for Screw-cutting. (Garvin Machine Co.) — Read from the lathe index the number of threads per inch cut by equal gears, and multiply it by any number that will give for a product a gear on the index; put this gear upon the stud, then multiply the number of threads per inch to be cut by the same number, and put the resulting gear upon the screw.

EXAMPLE. — To cut $11\frac{1}{2}$ threads per inch. We find on the index that 48 into 48 cuts 6 threads per inch, then $6 \times 4 = 24$, gear on stud, and $11\frac{1}{2} \times 4 = 46$, gear on screw. Any multiplier may be used so long as the products include gears that belong with the lathe. For instance, instead of 4 as a multiplier we may use 6. Thus, $6 \times 6 = 36$, gear upon stud, and $11\frac{1}{2} \times 6 = 69$, gear upon screw.

Rules for Calculating Simple and Compound Gearing where there is no Index. (*Am. Mach.*) — If the lathe is simple-gear, and the stud runs at the same speed as the spindle, select some gear for the screw, and multiply its number of teeth by the number of threads per inch in the lead-screw, and divide this result by the number of threads per inch to be cut. This will give the number of teeth in the gear for the stud. If this result is a fractional number, or a number which is not among the gears on hand, then try some other gear for the screw. Or, select the gear for the stud first, then multiply its number of teeth by the number of threads per inch to be cut, and divide by the number of threads per inch on the lead-screw. This will give the number of teeth for the gear on the screw. If the lathe is compound, select at random all the driving-gears, multiply the numbers of their teeth together, and this product by the number of threads to be cut. Then select at random all the driven gears except one; multiply the numbers of their teeth together, and this product by the number of threads per inch in the lead-screw. Now divide the first result by the second, to obtain the number of teeth in the remaining driven gear. Or, select at random all the driven gears. Multiply the numbers of their teeth together, and this product by the number of threads per inch in the lead-screw. Then select at random all the driving-gears except one. Multiply the numbers of their teeth together, and this result by the number of threads per inch of the screw to be cut. Divide the first result by the last, to obtain the number of teeth in the remaining driver. When the gears on the compounding stud are fast together, and cannot be changed, then the driven one has usually twice as many teeth as the other, or driver, in which case in the calculations consider the lead-screw to have twice as many threads per inch as it actually has, and then ignore

the compounding entirely. Some lathes are so constructed that the stud on which the first driver is placed revolves only half as fast as the spindle. This can be ignored in the calculations by doubling the number of threads of the lead-screw. If both the last conditions are present ignore them in the calculations by multiplying the number of threads per inch in the lead-screw by four. If the thread to be cut is a fractional one, or if the pitch of the lead-screw is fractional, or if both are fractional, then reduce the fractions to a common denominator, and use the numerators of these fractions as if they equaled the pitch of the screw to be cut, and of the lead-screw, respectively. Then use that part of the rule given above which applies to the lathe in question. For instance, suppose it is desired to cut a thread of $\frac{25}{32}$ -inch pitch, and the lead-screw has 4 threads per inch. Then the pitch of the lead-screw will be $\frac{1}{4}$ inch, which is equal to $\frac{8}{32}$ inch. We now have two fractions, $\frac{25}{32}$ and $\frac{8}{32}$, and the two screws will be in the proportion of 25 to 8, and the gears can be figured by the above rule, assuming the number of threads to be cut to be 8 per inch, and those on the lead-screw to be 25 per inch. But this latter number may be further modified by conditions named above, such as a reduced speed of the stud, or fixed compound gears. In the instance given, if the lead-screw had been $2\frac{1}{2}$ threads per inch, then its pitch being $\frac{4}{10}$ inch, we have the fractions $\frac{4}{10}$ and $\frac{25}{32}$, which, reduced to a common denominator, are $\frac{64}{160}$ and $\frac{125}{160}$, and the gears will be the same as if the lead-screw had 125 threads per inch, and the screw to be cut 64 threads per inch.

On this subject consult also "Formulas in Gearing," published by Brown & Sharpe Mfg. Co., and Jamieson's Applied Mechanics.

Change-gears for Screw-cutting Lathes.—There is a lack of uniformity among lathe-builders as to the change-gears provided for screw-cutting. W. R. Macdonald, in *Am. Mach.*, April 7, 1892, proposed the following series, by which 33 whole threads (not fractional) may be cut by changes of only nine gears:

| Screw. | Spindle. | | | | | | | | | Whole Threads. | | | |
|--------|----------|-----|----|-----------------|-----|-----------------|------------------|-----|------------------|----------------|----|----|----|
| | 20 | 30 | 40 | 50 | 60 | 70 | 110 | 120 | 130 | | | | |
| 20 | ... | 8 | 6 | $4\frac{4}{5}$ | 4 | $3\frac{3}{7}$ | $2\frac{2}{11}$ | 2 | $1\frac{11}{13}$ | 2 | 11 | 22 | 44 |
| 30 | 18 | ... | 9 | $7\frac{1}{5}$ | 6 | $5\frac{1}{7}$ | $3\frac{3}{11}$ | 3 | $2\frac{10}{13}$ | 3 | 12 | 24 | 48 |
| 40 | 24 | 16 | 12 | $9\frac{3}{5}$ | 8 | $6\frac{6}{7}$ | $4\frac{4}{11}$ | 4 | $3\frac{9}{13}$ | 4 | 13 | 26 | 52 |
| 50 | 30 | 20 | 15 | ... | 10 | $8\frac{4}{7}$ | $5\frac{5}{11}$ | 5 | $4\frac{8}{13}$ | 5 | 14 | 28 | 66 |
| 60 | 36 | 24 | 18 | $14\frac{2}{5}$ | ... | $10\frac{2}{7}$ | $6\frac{6}{11}$ | 6 | $5\frac{7}{13}$ | 6 | 15 | 30 | 72 |
| 70 | 42 | 28 | 21 | $16\frac{4}{5}$ | 14 | ... | $7\frac{7}{11}$ | 7 | $6\frac{8}{13}$ | 7 | 16 | 33 | 78 |
| 110 | 66 | 44 | 33 | $26\frac{2}{5}$ | 22 | $18\frac{6}{7}$ | ... | 11 | $10\frac{2}{13}$ | 8 | 18 | 36 | |
| 120 | 72 | 48 | 36 | $28\frac{4}{5}$ | 24 | $20\frac{4}{7}$ | $13\frac{1}{11}$ | ... | $11\frac{1}{13}$ | 9 | 20 | 39 | |
| 130 | 78 | 52 | 39 | $31\frac{1}{5}$ | 26 | $22\frac{3}{7}$ | $14\frac{2}{11}$ | 13 | ... | 10 | 21 | 42 | |

Ten gears are sufficient to cut all the usual threads, with the exception of perhaps $11\frac{1}{2}$, the standard pipe-thread; in ordinary practice any fractional thread between 11 and 12 will be near enough for the customary short pipe-thread; if not, the addition of a single gear will give it.

In this table the pitch of the lead-screw is 12, and it may be objected to as too fine for the purpose. This may be rectified by making the real pitch 6 or any other desirable pitch, and establishing the proper ratio between the lathe spindle and the gear-stud.

"Quick Change Gears."—About 1905, lathe manufacturers began building "quick change" lathes in which gear changing for screw cutting is eliminated. The lead-screw carries a cone of gears, one of which is in mesh with a movable gear in a nest of gears driven from the spindle. By changing the position of this movable gear, in relation to the cone of gears, the proper ratio of speeds between the spindle and lead-screws is obtained for cutting any desired thread usual in the range of the machine. About 16 different numbers of threads per inch can usually be cut by means of the "quick change" gear train. Different threads from those usually available can be cut by means of change gears between the spindle

and "quick change" gear train. The threads per inch usually available range from 2 to 46 in a 12-inch lathe to 1 to 24 in a 30-inch lathe. Catalogs of lathe manufacturers should be consulted for constructional details.

Shapes of Tools. — For illustrations and descriptions of various forms of cutting-tools, see Taylor's Experiments, below; also see articles on Lathe Tools in Appleton's Cyc. Mech., vol. ii. and in Modern Mechanism.

Cold Chisels. — Angle of cutting-faces (Joshua Rose): For cast steel, about 65 degrees; for gun-metal or brass, about 50 degrees; for copper and soft metals, about 30 to 35 degrees.

Metric Screw-threads may be cut on lathes with inch-divided lead-screws, by the use of change-wheels with 50 and 127 teeth; since 127 centimeters = 50 inches ($127 \times 0.3937 = 49.9999$ in.).

Rule for Setting the Taper in a Lathe. (*Am. Mach.*) — No rule can be given which will produce exact results, owing to the fact that the centers enter the work an indefinite distance. If it were not for this circumstance the following would be an exact rule, and it is an approximation as it is. To find the distance to set the center over: Divide the difference in the diameters of the large and small ends of the taper by 2, and multiply this quotient by the ratio which the total length of the shaft bears to the length of the tapered portion. **EXAMPLE:** Suppose a shaft three feet long is to have a taper turned on the end one foot long, the large end of the taper being two inches and the small end one inch diameter,

$$\frac{2 - 1}{2} \times \frac{3}{1} = 1\frac{1}{2} \text{ inches.}$$

TAYLOR'S EXPERIMENTS.

Fred W. Taylor directed a series of experiments, extending over 26 years, on feeds, speeds, shape of tool, composition of tool steel, and heat treatment. His results are given in Trans. A. S. M. E., xxviii, "The Art of Cutting Metals." The notes below apply mainly to tools of high speed steel and to heavy work requiring tools not less than $\frac{1}{2}$ by $\frac{3}{4}$ inch in cross-section.

Proper Shape of Lathe Tool. — Mr. Taylor discovered the best shape for lathe tools to be as shown in Fig. 187 with the angles given in the following table, when used on materials of the class shown. The exact outline of the nose of the tool is shown in Fig. 188. The actual dimensions of a 1-inch roughing tool are shown in Fig. 189. Let R = radius of point of tool, A = width of tool, L = length of shank, and H = height of shank, all in inches. Then $L = 14 A + 4$; $H = 1.5 A$; $R = 0.5 A - 0.3125$ for cutting hard steel and cast iron; $R = 0.5 A - 0.1875$ for soft steel. The meaning of the terms back slope, etc., is shown in Fig. 187.

Angles for Tools.

| * Material cut. | a = clearance. | b = back slope. | c = side slope. |
|---------------------------|------------------|-------------------|-------------------|
| Cast iron; Hard steel. | 6 degrees. | 8 degrees. | 14 degrees. |
| Medium steel; Soft steel. | 6 degrees. | 8 degrees. | 22 degrees. |
| Tire steel. | 6 degrees. | 5 degrees. | 9 degrees. |

* As far as the shape of the tool is concerned, Taylor divides metals to be cut into general classes: (a) cast iron and hard steel, steel of 0.45–0.50 per cent carbon, 100,000 pounds tensile strength, and 18 per cent stretch, being a low limit of hardness; (b) soft steel, softer than above; (c) chilled iron; (d) tire steel; (e) extremely soft steel of carbon, say, 0.10–0.15 per cent.

The table presupposes the use of an automatic tool grinder. If tools are ground by hand the clearance angle should be 9 degrees. The lip angles for tools cutting hard steel and cast iron should be 68 degrees;

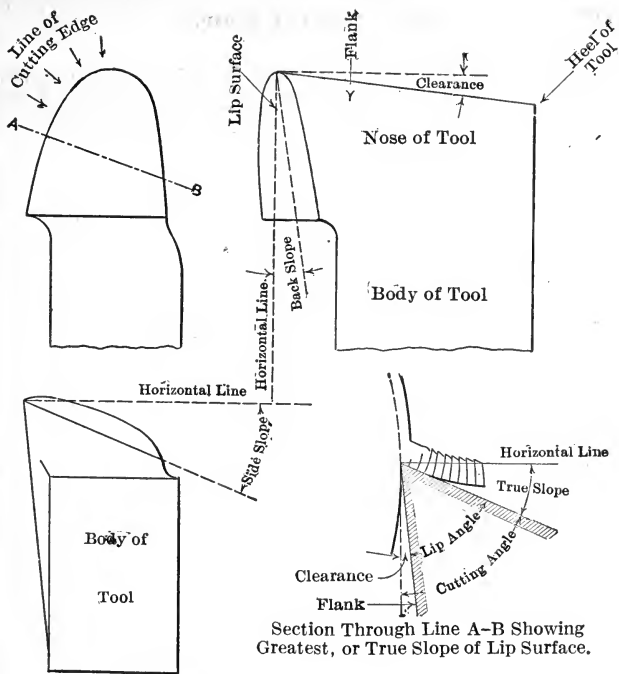


FIG. 187.

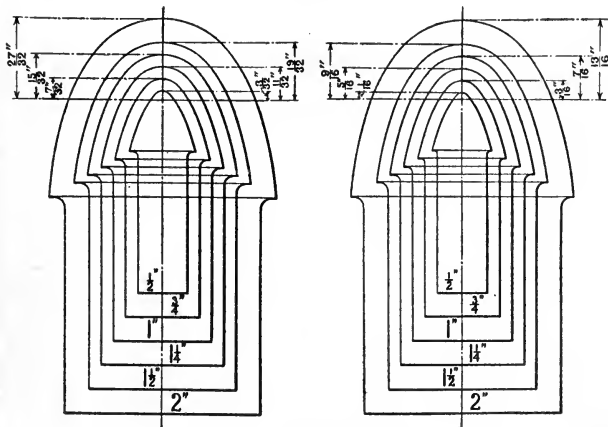


FIG. 188.

for soft steel, 61 degrees; for chilled iron, 86 to 90 degrees; for tire steel, 74 degrees; for extremely soft steel, keener than 61 degrees. A tool should be given more side than back slope; it can then be ground more times without weakening, the chip does not strike the tool post or clamps,

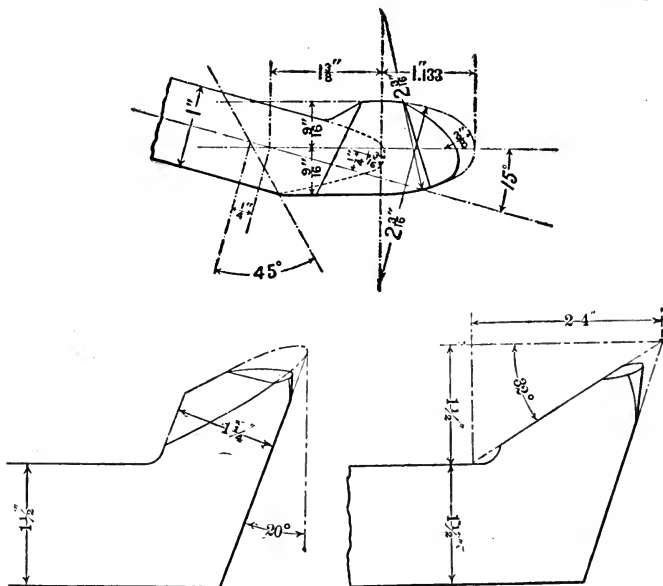


FIG. 189.

and it is also easier to feed. The nose of the tool should be set to one side, as in Fig. 189 above, to avoid any tendency to upset. To use a tool of this shape, lathe tool posts should be set lower below the center of the work than is now (1907) customary.

Forging and Grinding Tools.—The best method of dressing a tool is to turn one end up nearly at right angles to the shank, so that the nose will be high above the top of the body of the tool. Dressing can be thus done in two heats. Tools should leave the smith shop with a clearance angle of 20 degrees. Detailed directions for dressing a tool are given in Mr. Taylor's paper. To avoid overheating the tool in grinding, a stream of water, of at least five gallons a minute, should be thrown at low velocity on the nose of the tool where it is in contact with the emery wheel. In hand grinding, tools should not be held firmly against the wheel, but should be moved over its surface. It is of the utmost importance that high speed steel tools should not be heated above 1200° F. in grinding. Automatic tool grinders are economical, even in a small shop. Grinding machines should have some means for automatically adjusting the pressure of the tool against the grinding wheel. Each size of tool should have adapted to it a pressure, automatically adjusted, and which is just sufficient to grind rapidly without overheating the tool. Standard shapes should be adopted, to which all tools should be ground, there being no economy in automatic grinding without standard shapes.

Best Grinding Wheel.—The best grinding wheel was found to be a corundum wheel, of a mixture of 24 and 30 grit.

Pressure of Tool, etc.—Mr. Taylor found that there is no definite relation between the cutting speed of tools and the pressure with which the chip bears on the lip surface of the tool. He found, however, that the pressure per square inch of sectional area of the chip increases slightly as the thickness of the chip decreases. The feeding pressure of the tool is sometimes equal to the entire driving pressure of the chip against the lip surface of the tool, and the feed gears should be designed to deliver a pressure of this magnitude at the nose of the tool.

Chatter.—Chatter is caused by: too small lathe dogs; imperfect bearing at the points where the face plate drives the dogs; badly made or badly fitted gears; shafts in the machine of too small diameter, or of too great length; loose fits in bearings. A tool which chatters must be run at a cutting speed about 15 per cent slower than can be used if the tool does not chatter, irrespective of the use or non-use of water on the tool. A higher cutting speed can be used with an intermittent cut, as occurs on a planer, or shaper, or in turning, say, the periphery of a gear, than with a steady cut. To avoid chatter, tools should have curved cutting edges, or two or more tools should be used at the same time in the same machine. The body of the tool should be greater in height than width, and should have a true, solid bearing on the tool support, which latter should extend to almost beneath the cutting edge of the tool. Machines should be made massive beyond the metal needed for strength alone, and steady rests should be used on long work. It is advisable to use a steady rest, when turning any cylindrical piece of diameter D , when the length exceeds $12 D$.

Use of Water on Tool.—With the best high speed steel tools, a gain of 16 per cent in cutting speed can be made in cutting cast iron, steel or wrought iron by throwing a heavy stream of water directly on the chip at the point where it is being removed from the forging by the tool. Not less than three gallons a minute should be used for a $2 \times 2\frac{1}{2}$ -inch tool. The gain is practically the same for all qualities of steel, regardless of hardness and whether thick or thin chips are being cut.

Interval between Grindings.—Mr. Taylor derived a table showing how long various sizes of tools should run without regrinding to give the maximum work for the lowest all-around cost. Time a tool should run continuously without regrinding equals $7 \times$ (time to change tool + proper portion of time for redressing + time for grinding + time equivalent to cost of the tool steel ground off).

INTERVAL BETWEEN GRINDINGS, AT MAXIMUM ECONOMICAL CUTTING SPEEDS.

| | | | | | |
|---------------|----------------------------------|------------------------|-----------------------------------|-----------------------------------|-------------------------|
| Size of tool. | | | | | |
| Inches. | $\frac{1}{2} \times \frac{3}{4}$ | $\frac{5}{8} \times 1$ | $\frac{3}{4} \times 1\frac{1}{8}$ | $\frac{7}{8} \times 1\frac{3}{8}$ | $1 \times 1\frac{1}{2}$ |
| Hours. | 1.25 | 1.25 | 1.25 | 1.5 | 1.5 |

| | | | | |
|---------------|------------------------------------|------------------------------------|------------------------------------|--------------|
| Size of tool. | | | | |
| Inches. | $1\frac{1}{4} \times 1\frac{7}{8}$ | $1\frac{1}{2} \times 2\frac{1}{4}$ | $1\frac{3}{4} \times 2\frac{3}{4}$ | 2×3 |
| Hours. | 1.75 | 2.0 | 2.5 | 2.75 |

If the proper cutting speed (A) is known for a cut of given duration, the speed for a cut (B) of different duration can be obtained by multiplying (A) by the factor given in the following table:

Duration of cut in minutes:

| | | | | | | |
|--------------------------------|------|------|------|------|------|------|
| At known speed (A) | 20 | 40 | 20 | 40 | 80 | 80 |
| At derived speed (B) | 40 | 80 | 80 | 20 | 40 | 20 |
| Factor | 0.92 | 0.92 | 0.84 | 1.09 | 1.09 | 1.19 |

For cutting speeds of high-speed lathe tools to last $1\frac{1}{2}$ hours, see tables on pages 1244 and 1245.

Effect of Feed and Depth of Cut on Cutting Speed.—With a given depth of cut, metal can be removed faster with a coarse feed and slow speed, than with fine feed and high speed. With a given depth of cut, a cutting speed of S , and a feed of F , S varies as $1/\sqrt{F}$. With tools of the best high speed steel, varying the feed and depth of cut varies the cutting speed in the same ratio when cutting hard steel as when cutting soft steel.

Best High Speed Tool Steel — Composition — Heat Treatment.

— Mr. Taylor and Maunsel White developed a number of high speed steels, the one showing the best all-around qualities having the following chemical composition: Vanadium, 0.29; tungsten, 18.19; chromium, 5.47; carbon, 0.674; manganese, 0.11; silicon, 0.043. The use of vanadium materially improves high speed steel. The following method of treatment is described as the best for this or any other composition of high speed steel. The tool should be forged at a light yellow heat, and, after forging slowly and uniformly, heated to a bright cherry red, allowing plenty of time for the heat to penetrate to the center of the tool, in order to avoid danger of cracking due to too rapid heating. The tool should then be heated from a bright cherry red to practically its melting-point as rapidly as possible in an intensely hot fire; if the extreme nose of the tool is slightly fused no harm is done. Time should be allowed for the tool to become uniformly hot from the heel to the lip surface.

After the high heat has been given the tools, as above described, they should be cooled rapidly until they are below the "breaking-down point," or, say, down to or below 1550° F. The quality of the tool will be but little affected whether it is cooled rapidly or slowly from this point down to the temperature of the air. Therefore, after all parts of a tool from the outside to the center have reached a uniform temperature below the breaking-down point, it is the practice sometimes to lay it down in any part of the room or shop which is free from moisture, and let it cool in the air, and sometimes to cool it in an air blast to the temperature of the air.

The best method of cooling from the high heat to below the breaking-down point is to plunge the tools into a bath of red-hot molten lead below the temperature of 1550° F. They should then be plunged into a lead bath maintained at a uniform temperature of 1150° F., because the same bath is afterward used for reheating the tools to give them their second treatment. This bath should contain a sufficiently large body of the lead so that its temperature can be maintained uniform; and for this purpose should be used preferably a lead bath containing about 3600 lb. of lead.

Too much stress cannot be laid upon the importance of never allowing the tool to have its temperature even slightly raised for a very short time during the process of cooling down. The temperature must either remain absolutely stationary or continue to fall after the operation of cooling has once started, or the tool will be injured. Any temporary rise of temperature during cooling, however small, will injure the tool. This, however, applies only to cooling the tool to the temperature of about 1240° F. Between the limits of 1240 degrees and the temperature of the air, the tool can be raised or lowered in temperature time after time and for any length of time without injury. And it should also be noted that during the first operation of heating the tool from its cold state to the melting-point, no injury results from allowing it to cool slightly and then reheating. It is from reheating during the operation of cooling from the high heat to 1240° F. that the tool is injured.

The above-described operation is commonly known as the first or high-heat treatment.

To briefly recapitulate, the first or high-heat treatment consists of heating the tool —

- (a) slowly to 1500° F.;
- (b) rapidly from that temperature to just below the melting-point.
- (c) cooling fast to below the breaking-down point, i.e., 1550° F.
- (d) cooling either fast or slowly from 1550° F. to temperature of the air.

Second Treatment, Reheating the Cooled Tool. — After air-temperature has been reached the tool should be reheated to a temperature of from 700 to 1240° F., preferably by plunging it in the before-mentioned lead bath at 1150° F. and kept at that temperature at least five minutes. To avoid danger of fire cracks, the tool should be heated slowly before immersing in the bath. The above tool heated in this fashion possesses a high degree of "red hardness" (ability to cut steel with the nose of the tool at red heat), while it is not extraordinarily hard at ordinary temperatures. It is difficult to injure it by overheating on the grindstone or in the lathe. It will operate at 90 per cent of its maximum cutting speed, even without the second or low-heat treatment. A coke fire is preferable for giving the first heat, and it should be made as deep as possible.

Cooling the tool by plunging it in oil or water, renders it liable to fire cracks and to brittleness in the body. Next to the lead bath an air blast is preferable for cooling.

Best Method of Treating Tools in Small Shops.—For small shops, in treating high-speed tools, Mr. Taylor considers the best method to be as follows for the blacksmith who is equipped only with the apparatus ordinarily found in a smith-shop.

After the tools have been forged and before starting to give them their heat, fuel should be added to the smith's fire so as to give a good deep bed either of coke about the size of a walnut or of first-class blacksmiths' soft coal. A number of tools should then be laid with their noses at a slight distance from the hotter portion of the fire, so that they may all be pre-heating while the fire is being blown up to its proper intensity. After reaching its proper intensity, the tools should be heated one at a time over the hottest part of the fire as rapidly as practicable up to just below their melting-point. During this operation they should be repeatedly turned over and over so as to insure a uniform high heat throughout the whole end of the tool. As soon as each tool reaches its high heat, it should be placed with its nose under a heavy air blast and allowed to cool to the temperature of the air before being removed from the blast.

Unfortunately, however, the blacksmith's fire is so shallow that it is incapable of maintaining its most intense heat for more than a comparatively few minutes, and, therefore, it is only through these few minutes that first-class high-speed tools can be properly heated in the smith's fire. Great numbers of high-speed tools are daily turned out from smiths' fires which are not sufficiently intense in their heat, and they are therefore inferior in red hardness and produce irregular cutting tools.

On the whole, a blacksmith's fire made from coke may be regarded as better for giving the high heat to tools than a soft-coal fire, merely because a coke fire can be more easily made by the smith which will remain capable for a longer period of heating the tools quickly to their melting-points.

Quality of Different Tool Steels.—Mr. Taylor in a letter to the author, Dec. 30, 1907, says:

First. Any of a half dozen makes of high speed tools now on the market are amply good, and but little attention need be paid to the special directions for heating and cooling high speed tools given by the makers of the tool steel. The most important matter is that an intensely hot fire should be used for giving the tools their high heat, and that they should not be allowed to soak a long time in this fire. They should be heated as fast as possible and then cooled in an air blast.

Second. The greatest number of tools are ruined on the emery wheel through overheating, either because a wheel whose surface is glazed is used, or because too small a stream of water is run upon the nose of the tool. The emery wheel should be kept sharp through frequent dressings with a diamond tool.

Third. Uniformity is the most important quality in high speed tools. For this reason, only one make of high speed tool steel should be used in each shop.

Economical Cutting Speeds.—Tools shaped as in Fig. 189, and of the chemical composition and heat treatment given in the preceding paragraphs, should be run at the cutting speeds given in the tables on pages 1244 and 1245 in order to last one hour and 30 minutes without re-grinding.

Cutting Speed of Parting and Thread Tools.—To find the economical cutting speed of a parting tool of the best high speed steel, find the proper value for the size of tool in the tables below and divide by 2.7. The economical speed for a thread tool is similarly found by dividing by 4. The thickness of chip in the latter case is the advance in inches per revolution of the tool toward the center of the work.

Durability of Cutting Tools.—E. G. Herbert (*Am. Mach.*, June 24, 1909) shows that the durability of a tool depends mainly on the temperature to which its extreme edge is raised, and that the rate of evolution of heat and consequently the durability is proportional to the thickness and to the area of the chip and to the cube of the cutting speed. Or if t_1 = thickness or feed, c_1 = depth of cut, a_1 = area of the cut and s_1 = cutting speed, for any given set of working conditions, and $t_2c_2a_2$ and s_2 values for another set of conditions, then the durability of the tool

Cutting Speeds in Feet per Minute of Standard Taylor-White Tools, Tool to Last 1 1/2 Hours between Grindings.

| Tool. | | 1 1/4-Inch. | | | | | | 1-Inch. | | | | | | 7/8-Inch. | | | | | |
|---------------|-----------|-------------|--------------|-------|------------|--------------|-------|---------|--------------|-------|------------|--------------|-------|-----------|--------------|-------|------------|--------------|-------|
| Material Cut. | | Steel. | | | Cast Iron. | | | Steel. | | | Cast Iron. | | | Steel. | | | Cast Iron. | | |
| Cut. in. | Feed. in. | Soft. | Me- dium. | Hard. | Soft. | Me- dium. | Hard. | Soft. | Me- dium. | Hard. | Soft. | Me- dium. | Hard. | Soft. | Me- dium. | Hard. | Soft. | Me- dium. | Hard. |
| 3/32 | 1/64 | 518.0 | 259.0 | 118.0 | 239.0 | 119.6 | 69.8 | 490.0 | 245.0 | 111.0 | 226.0 | 113.0 | 66.0 | 476.0 | 238.0 | 108.0 | 220.0 | 110.0 | 64.2 |
| | 1/32 | 366.0 | 183.0 | 83.2 | 191.0 | 95.3 | 55.6 | 339.0 | 169.0 | 77.0 | 177.0 | 88.4 | 51.6 | 325.0 | 162.0 | 73.8 | 169.0 | 84.6 | 49.4 |
| | 3/16 | 257.0 | 129.0 | 58.4 | 142.0 | 70.8 | 41.3 | 235.0 | 117.0 | 53.4 | 130.0 | 64.8 | 37.8 | 222.0 | 111.0 | 50.4 | 122.0 | 61.2 | 35.7 |
| | 1/8 | 209.0 | 105.0 | 47.5 | 113.0 | 59.1 | 34.4 | 189.0 | 94.5 | 43.0 | 107.0 | 53.5 | 31.2 | 177.0 | 88.4 | 40.2 | 93.8 | 49.9 | 29.1 |
| 1/16 | 3/8 | | | | | | | | | | | | | | | | | | |
| | 1/2 | | | | | | | | | | | | | | | | | | |
| | 5/8 | | | | | | | | | | | | | | | | | | |
| | 3/4 | | | | | | | | | | | | | | | | | | |
| 1/8 | 1/64 | 450.0 | 225.0 | 102.0 | 216.0 | 108.0 | 63.1 | 427.0 | 214.0 | 97.0 | 205.0 | 102.0 | 59.8 | 420.0 | 210.0 | 95.5 | 202.0 | 101.0 | 58.9 |
| | 1/32 | 317.0 | 158.0 | 72.0 | 172.0 | 86.2 | 50.3 | 296.0 | 148.0 | 67.2 | 160.0 | 85.1 | 46.8 | 286.0 | 143.0 | 65.0 | 156.0 | 77.8 | 45.4 |
| | 3/16 | 223.0 | 112.0 | 50.7 | 128.0 | 64.0 | 37.3 | 205.0 | 102.0 | 46.6 | 118.0 | 58.8 | 34.3 | 195.0 | 97.6 | 44.4 | 112.0 | 56.2 | 32.8 |
| | 1/2 | 182.0 | 90.8 | 41.4 | 107.0 | 53.4 | 27.2 | 165.0 | 83.0 | 37.5 | 97.0 | 48.5 | 23.3 | 156.0 | 77.9 | 35.4 | 91.8 | 45.9 | 26.8 |
| 3/16 | 1/4 | 157.0 | 78.5 | 35.7 | 93.4 | 46.7 | 21.3 | 142.0 | 71.0 | 32.3 | 84.2 | 42.1 | 24.6 | 133.0 | 66.4 | 30.2 | 79.3 | 39.7 | 23.2 |
| | 5/8 | | | | | | | | | | | | | | | | | | |
| | 3/4 | | | | | | | | | | | | | | | | | | |
| | 1 | | | | | | | | | | | | | | | | | | |
| 3/16 | 1/64 | 370.0 | 185.0 | 84.1 | 187.0 | 93.5 | 54.6 | 358.0 | 179.0 | 81.3 | 181.0 | 90.6 | 52.9 | 352.0 | 176.0 | 80.0 | 178.0 | 89.0 | 52.0 |
| | 1/32 | 260.0 | 130.0 | 59.1 | 149.0 | 74.6 | 43.6 | 247.0 | 124.0 | 56.1 | 142.0 | 70.8 | 41.3 | 240.0 | 120.0 | 54.5 | 137.0 | 68.6 | 40.1 |
| | 1/16 | 183.0 | 91.7 | 41.6 | 111.0 | 55.5 | 32.7 | 171.0 | 85.5 | 38.8 | 104.0 | 51.9 | 30.3 | 164.0 | 82.0 | 37.3 | 99.4 | 49.7 | 29.0 |
| | 3/8 | 149.0 | 74.6 | 33.8 | 92.5 | 46.3 | 27.0 | 138.0 | 69.0 | 31.3 | 85.8 | 42.9 | 25.0 | 131.0 | 65.5 | 29.8 | 81.0 | 40.5 | 23.7 |
| 1/4 | 1/8 | 129.0 | 64.5 | 29.3 | 73.1 | 36.5 | 21.3 | 118.0 | 59.0 | 26.8 | 74.3 | 37.2 | 21.7 | 112.0 | 56.0 | 25.5 | 70.1 | 35.0 | 20.6 |
| | 3/8 | 105.0 | 52.6 | 23.8 | 66.4 | 33.2 | 19.4 | 95.0 | 47.5 | 21.6 | 60.6 | 30.3 | 17.7 | | | | 56.8 | 28.4 | 16.6 |
| | 1/2 | | | | | | | | | | | | | | | | | | |
| | 3/4 | | | | | | | | | | | | | | | | | | |
| 1/2 | 1/64 | 322.0 | 161.0 | 73.2 | 168.0 | 84.1 | 49.1 | 315.0 | 157.0 | 71.6 | 165.0 | 82.3 | 48.1 | 312.0 | 156.0 | 70.9 | 163.0 | 81.5 | 47.7 |
| | 1/32 | 227.0 | 113.0 | 51.6 | 134.0 | 67.2 | 39.2 | 218.0 | 109.0 | 49.5 | 139.0 | 64.4 | 37.5 | 213.0 | 107.0 | 48.4 | 126.0 | 62.9 | 36.7 |
| | 3/16 | 159.0 | 79.7 | 36.1 | 99.8 | 49.9 | 29.1 | 150.0 | 75.0 | 34.1 | 94.3 | 47.1 | 27.5 | 145.0 | 72.6 | 33.0 | 90.8 | 45.4 | 26.5 |
| | 1/4 | 130.0 | 65.0 | 29.5 | 83.2 | 41.6 | 24.3 | 121.0 | 60.5 | 27.5 | 77.8 | 38.9 | 22.7 | 116.0 | 58.1 | 26.4 | 74.1 | 37.0 | 22.9 |
| 3/8 | 1/8 | 112.0 | 56.1 | 25.5 | 72.6 | 36.3 | 17.2 | 104.0 | 52.0 | 23.6 | 67.5 | 33.7 | 19.7 | | | | 64.1 | 32.0 | 18.7 |
| | 3/8 | 91.4 | 45.7 | 20.8 | 59.7 | 29.8 | 17.4 | | | | | | | | | | 52.0 | 26.0 | 15.2 |
| | 1/2 | | | | | | | | | | | | | | | | | | |
| | 3/4 | | | | | | | | | | | | | | | | | | |
| 5/8 | 1/64 | 264.0 | 132.0 | 60.0 | 144.0 | 71.8 | 41.9 | 263.0 | 132.0 | 59.8 | 143.0 | 71.5 | 41.8 | 264.0 | 132.0 | 60.0 | 144.0 | 71.8 | 41.9 |
| | 1/32 | 186.0 | 93.1 | 42.3 | 115.0 | 57.3 | 33.4 | 182.0 | 91.0 | 41.4 | 121.0 | 56.0 | 32.6 | 180.0 | 90.2 | 41.0 | 111.0 | 55.4 | 32.3 |
| | 1/16 | 131.0 | 65.5 | 31.1 | 85.1 | 42.6 | 24.8 | 126.0 | 62.8 | 28.5 | 81.9 | 41.0 | 23.9 | 122.0 | 61.1 | 27.8 | 80.0 | 40.0 | 23.4 |
| | 3/8 | 107.0 | 53.4 | 24.1 | 70.9 | 35.5 | 20.7 | 101.0 | 50.6 | 23.0 | 67.6 | 33.8 | 19.7 | | | | 65.3 | 32.6 | 19.1 |
| 3/4 | 1/8 | 92.2 | 46.1 | 20.8 | 62.0 | 31.0 | 18.1 | | | | | | | | | | | | |
| | 3/8 | | | | | | | | | | | | | | | | | | |
| | 1/2 | | | | | | | | | | | | | | | | | | |
| | 3/4 | | | | | | | | | | | | | | | | | | |
| 1 | 1/64 | 230.0 | 115.0 | 52.3 | 131.0 | 55.6 | 38.3 | 232.0 | 116.0 | 52.7 | 132.0 | 66.2 | 38.6 | 237.0 | 118.0 | 53.8 | 135.0 | 67.5 | 39.4 |
| | 1/32 | 162.0 | 80.9 | 36.8 | 105.0 | 52.3 | 30.5 | 161.0 | 80.5 | 37.6 | 104.0 | 51.6 | 30.2 | 162.0 | 80.8 | 36.7 | 104.0 | 52.1 | 30.4 |
| | 1/16 | 114.0 | 56.9 | 25.9 | 77.6 | 38.8 | 22.7 | 111.0 | 55.7 | 25.3 | 75.8 | 37.9 | 22.1 | | | | 75.2 | 37.6 | 22.0 |
| | 3/8 | 92.6 | 46.3 | 21.0 | 64.7 | 32.4 | 18.9 | | | | | | | | | | | | |
| 3/8 | 1/8 | | | | | | | | | | | | | | | | | | |
| | 3/16 | | | | | | | | | | | | | | | | | | |
| 3/16 | 1/8 | | | | | | | | | | | | | | | | | | |
| | 3/16 | | | | | | | | | | | | | | | | | | |

will be the same when $t_1 a_1 s_1^3 = t_2 a_2 s_2^3$, or for constant durability $s_2 = s_1 \sqrt[3]{(t_1^2 c_1 \div (t_2^2 c_2))}$.

New High-Speed Steels. — *Am. Mach.*, April 8, May 20 and 27, 1909, describes the operations of some new varieties of high-speed steel made by Sheffield manufacturers, which show results superior to those of the earlier high-speed steels in endurance of tool, ability to cut very hard metals, and higher speeds. The following are the results of some of the tests in lathe-work.

| Tool size. in. | Material Cut. | Diam. in. | Depth. cut in. | Feed in. | Speed ft. per min. | Length of Cut. |
|----------------|-------------------------|-----------|----------------|----------|--------------------|----------------|
| 1 1/4 | Steel, 2.00 C..... | | 3/8 | 1/16 | 36 | 43/4 in.* |
| 1 1/4 | Steel, 0.70 C..... | | 1/4 | 1/16 | 48 | 13 in.† |
| 1 1/4 | Steel, 0.70 C..... | | 3/16 | 1/16 | 65 | 87/8 in. |
| 7/8 | Steel, 0.40 C..... | 4 | 1/8 | 1/16 | 65 | 28 ins., ‡ |
| 7/8 | Steel, 0.40 C..... | 4 | 1/8 | 1/32 | 120 | 28 ins., § |
| 7/8 | Cast iron..... | 5 ft. | 1/8 to 3/16 | 1/10 | 56 | 4 1/2 ins. |
| 1 1/4 | Cast iron..... | 5 ft. | 5/16 | 1/32 | 107 | 6 ins. |
| 1 1/2 | Cast iron..... | 5 ft. | 1/8 | 1/8 | 55 | 8 ins. |
| 1 1/4 | Steel, 0.40 C..... | 53/8 in. | 1/8 | 1/10 | 90 | 54 ins. |
| 1 x 2 | Steel..... | 93/4 in. | 3/8 | 1/8 | 64 | 72 ins. |
| 1 x 2 | Nickel steel..... | 3 1/2 in. | 1/2 | 0.072 | 52 | 124 ins. |
| 1 1/4 | Steel casting, 0.45 C.. | 20 in. | 3/8 | 1/8 | 50 | 15 to 20 min. |
| 1 1/4 | Steel, 0.60 C..... | 7 1/2 in. | 9/64 | 1/26 | 115 | 18 in. |

* Then 13/4 in. at 50 ft. per min. † Then 1 1/8 in. at 65 ft. per min.

‡ Then 28 ins. at 98 ft. § Then 22 ins. at 160 ft. || Required 28 H.P.

Chilled rolls, too hard for ordinary high-speed steel, were cut at a speed of 80 ft. per min., with 5/16 in. depth of cut and 1/8 in. feed.

The following results were obtained in drilling:

| Drill size. | Material. | Rev. per min. | Feed per rev. | Speed per min. | Drilled without Re-grinding. |
|-------------|----------------------|---------------|---------------|----------------|------------------------------|
| 3/4 in. | Close cast iron..... | 466 | 0.018 | 81/2 in. | 70 holes, 3 ins. deep. |
| 3/4 | Steel, 0.25 C..... | 247 | 0.011 | | 60 holes, 23/4 ins. deep. |
| 3/4 | Hard steel..... | 526 | | 6 in. | 12 holes, 2 1/2 ins. deep. |
| 13/16 | Steel..... | 400 | | 3 1/2 | 14 in. at one operation. |

A milling cutter 5 in. diam., with 54 teeth, milling teeth in saw-blanks, at a cutting speed of 56 ft. per min. and a feed of 1 in. per min., cuts 80 blanks (three or more together), each 32 in. diam., 3/8 in. thick, 240 teeth, before re-grinding.

Use of a Magnet to Determine the Hardening Temperature. (Catalogue of Firth-Sterling Steel Co.) — At the proper hardening heat a piece of regular tool steel loses its power to attract a magnet. By touching a magnet against the tool as it heats up in the furnace, the magnet will take hold until the proper heat for quenching is reached, and then it will not take hold at any point. This determines the lowest heat at which it can be hardened.

By heating slowly, trying with a magnet frequently, and dipping the tool when the magnet will not take hold, an extremely hard tool will be secured and one which will do excellent work. The magnet should not be allowed to become heated. In order to guard against the loss of magnetism in a horseshoe magnet an electro-magnet may be made by passing an electric current through a coil of wire wound on an iron rod.

CASE-HARDENING, ETC.

Case-hardening of Iron and Steel. Cementation, Harveyizing. — When iron or soft steel is heated to redness or above in contact with charcoal or other carbonaceous material, the carbon gradually penetrates

the metal, converting it into high carbon steel. The depth of penetration and the percentage of carbon absorbed increase with the temperature and with the length of time allowed for the process. In the old cementation process for converting wrought iron into "blister steel" for re-melting in crucibles flat bars were packed with charcoal in an oven which was kept at a red heat for several days. In the Harvey process of hardening the surface of armor plate, the plate is covered with charcoal and heated in a furnace for a considerable time, and then rapidly cooled by a spray of water.

In case-hardening, a very hard surface is given to articles of iron or soft steel by covering them or packing them in a box or oven with a material containing carbon, heating them to redness while so covered, and then chilling them. Many different substances have been used for the purpose, such as wood or bone charcoal, charred leather, sugar, cyanide of potassium, bichromate of potash, etc. Hydrocarbons, such as illuminating gas, gasoline or naphtha, are also used. *Amer. Machinist*, Feb. 20, 1908, describes a furnace made by the American Gas Furnace Company of Elizabeth, N. J., for case-hardening by gas. The best results are obtained with soft steel, 0.12 to 0.15 carbon, and not over 0.35 manganese, but steel of 0.20 to 0.22 carbon may be used. The carbon begins to penetrate the steel at about 1300° F., and 1700° F. should never be exceeded with ordinary steels. A depth of carbonizing of $\frac{1}{64}$ in. is obtained with gas in one hour, and $\frac{1}{4}$ in. in 12 hours. After carbonizing the steel should be annealed at about 1625° F. and cooled slowly, then re-heated to about 1400° F. and quenched in water. Nickel-chrome steels may be carbonized at 2000° F. and tungsten steels at 2200° F.

Change of Shape due to Hardening and Tempering.—J. E. Storey, *Am. Mach.*, Feb. 20, 1908, describes some experiments on the change of dimensions of steel bars 4 in. long, $\frac{7}{8}$ in. diam. in hardening and tempering. On hardening the length increased in different pieces .0001 to .0014 in., but in two pieces a slight shrinkage, maximum .00017, was found. The diameters increased .0003 to .0036 in. On tempering the length decreased .0017 to .0108 in. as compared with the original 4 ins. length, while the diameter was increased .0003 to .0029; a few samples showing a decrease, max. .0009 in. The general effect of hardening is a slight increase in bulk, which increase is reduced by tempering. The distortion is more important than the increase in bulk.

MILLING CUTTERS.

George Addy (*Proc. Inst. M. E.*, Oct., 1890, p. 537) gives the following: **Analyses of Steel.**—The following are analyses of milling cutter blanks, made from best quality crucible cast steel and from self-hardening "Ivanhoe" steel:

| | C | Si | P | Mn | S | Tungsten | Iron, by difference |
|-----------------|------|-------|-------|------|------|----------|---------------------|
| Crucible Steel, | 1.2 | 0.112 | 0.018 | 0.36 | 0.02 | ... | 98.29 |
| Ivanhoe Steel, | 1.67 | 0.252 | 0.051 | 2.56 | 0.01 | 4.65 | 90.81 |

The first analysis is of a cutter 14 in. diam., 1 in. wide, which gave very good service at a cutting-speed of 60 ft. per min. Large milling cutters are sometimes built up, the cutting-edges only being of tool steel. A cutter 22 in. diam. by $5\frac{1}{2}$ in. wide has been made in this way, the teeth being clamped between two cast-iron flanges. Mr. Addy recommends for this form of tooth one with a cutting-angle of 70°, the face of the tooth being set 10° back of a radial line on the cutter, the clearance-angle being thus 10°. At the Clarence Iron Works, Leeds, the face of the tooth is set 10° back of the radial line for cutting wrought iron and 20° for steel.

Pitch of Teeth.—For obtaining a suitable pitch of teeth for milling-cutters of various diameters there exists no standard rule, the pitch being usually decided in an arbitrary manner according to individual taste. For estimating the pitch of teeth in a cutter of any diameter from 4 in. to 15 in., Mr. Addy has worked out the following rule, which he has found capable of giving good results in practice:

$$\text{Pitch in inches} = \sqrt{(\text{diam. in inches} \times 8) \times 0.0625} = 0.177 \sqrt{\text{diam.}}$$

J. M. Gray gives a rule for pitch as follows: The number of teeth in a milling cutter ought to be 100 times the pitch in inches; that is, if there were 27 teeth, the pitch ought to be 0.27 in. The rules are practically

the same, for if d = diam., n = no. of teeth, p = pitch, c = circumference, $c = pn$; $d = \frac{pn}{\pi} = \frac{100p^2}{\pi} = 31.83p^2$; $p = \sqrt{0.0314d} = 0.177 \sqrt{d}$;

No. of teeth, $n = 3.14d \div p$.

Teeth of Plain or Spiral Milling Cutters. (*Mach'y*, April, 1907.) — Plain milling cutters are usually manufactured in sizes from 2 to 5 in. diameter, and up to 6-in. face. The use of solid plain milling cutters of over 5-in. face is not advised, and cutters over 5-in. face should be made in two or more interlocking sections.

NUMBER OF TEETH AND AMOUNT OF SPIRAL OF PLAIN MILLING CUTTERS.

No. of teeth = $\frac{5 \times \text{diam.} + 24}{2}$; Length of Spiral = $9 \times \text{diam.} + 4$.

Diameter of cutter,

2 2 1/4 2 1/2 2 3/4 3 3 1/2 4 4 1/2 5 5 1/2 6 6 1/2 7 7 1/2 8
Number of teeth,
16 18 18 18 20 20 22 24 24 26 26 28 30 30 32
Length of one turn of spiral, inches,
22 24 1/4 26 1/2 28 3/4 31 35 1/2 40 44 1/2 49 53 1/2 58 62 1/2 67 71 1/2 76.

A cutter with an included angle of 60° (12° on one side and 48° on the other) is recommended for fluting plain milling cutters, although cutters of 52° (12° and 40°) are commonly furnished by manufacturers. The angle of relief of milling cutters should be between 5° and 7° .

Nicked Cutters. — Cutters for milling broad surfaces, whether of the spiral or straight type, usually have nicks cut in the teeth, the nicks being staggered in consecutive teeth. These afford relief from jamming the teeth with chips.

Side Milling Cutters. (*Mach'y*, April, 1907.) — The teeth of side milling cutters should have the same general form as those of plain milling cutters, excepting that the cutter used to form them should have an included angle of about 75° .

NUMBER OF TEETH IN SIDE MILLING CUTTERS.

Number of teeth = $3.1 \text{ diam.} + 11$.

Diam. of cutter,

2 2 1/4 2 1/2 2 3/4 3 3 1/2 4 4 1/2 5 5 1/2 6 6 1/2 7 7 1/2 8 9
Number of teeth,
18 18 18 20 20 22 24 24 26 28 30 32 32 34 36 38

Milling Cutters with Inserted Teeth. — When it is required to use milling cutters of a greater diameter than about 8 in., it is preferable to insert the teeth in a disk or head, so as to avoid the expense of making solid cutters and the difficulty of hardening them, not merely because of the risk of breakage in hardening them, but also on account of the difficulty in obtaining a uniform degree of hardness or temper.

Keyways in Milling Cutters. — A number of manufacturers have adopted the keyways shown below, as standards. The dimensions in inches are given in the tables.

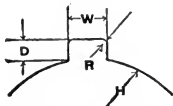


FIG. 190. — Square Keyway.

| Diam. Hole, | 3/8-9/16 | 5/8-7/8 | 15/16-1 1/8 | 1 3/16-1 3/8 | 1 7/16-1 3/4 | 1 13/16-2 | 2 1/16-2 1/2 | 2 9/16-3 |
|-------------|----------|---------|-------------|--------------|--------------|-----------|--------------|----------|
| Width W | 3/32 | 1/8 | 5/32 | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 |
| Depth, D | 3/64 | 1/16 | 5/64 | 3/32 | 1/8 | 5/32 | 3/16 | 3/16 |
| Radius, R | 0.020 | 0.030 | 0.035 | 0.040 | 0.050 | 0.060 | 0.060 | 0.060 |

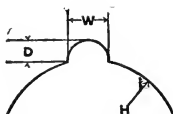


FIG. 191.—Half-round Keyway

| Diam. Hole, H | 3/8-5/8 | 11/16-13/16 | 7/8-1 3/16 | 1 1/4-1 7/16 | 1 1/2-2 | 2 1/16-2 7/16 | 2 1/2-3 |
|---------------|---------|-------------|------------|--------------|---------|---------------|---------|
| Width W | 1/8 | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 |
| Depth, D | 1/16 | 3/32 | 1/8 | 5/32 | 3/16 | 7/32 | 1/4 |

Power Required for Milling. (*Mech. Engr.*, Oct. 26, 1907.) — Mr. S. Strieff made a series of experiments to determine the power required to drive milling cutters of high-speed steel. The results are shown in the table below. A proportionately higher amount of power is required for light than heavy milling, as the power to drive the machine is the same at all loads. The table also shows that the depth of cut does not increase the power required in the same proportion as the width, and that work with a quick feed and a deep but comparatively narrow cut requires less power than a wide cut of moderate depth with slow feed, the amount of metal removed being the same in both cases.

Power Required for Milling.

| Number of Revolutions of Cutter per Minute. | Feed. | | Cutting Speed of Cutter, Feet per Minute. | Depth of Cut, Inches. | Width of Cut, Inches. | Horse-Power Required. | Metal Removed per Hour, Pounds. | Horse-Power Required per Pound-Hour. |
|---|---------------------|-------------------------|---|-----------------------|-----------------------|-----------------------|---------------------------------|--------------------------------------|
| | Per Minute, Inches. | Per Revolution, Inches. | | | | | | |
| 24 | 2.46 | 0.10 | 37 | 0.26 | 23.6 | 25 | 245 | 0.102 |
| 24 | 3.50 | 0.15 | 37 | 0.26 | 10.2 | 17 | 150 | 0.113 |
| 24 | 4.35 | 0.18 | 37 | 0.14 | 9.8 | 17 | 97 | 0.175 |
| 24 | 3.50 | 0.15 | 37 | 0.49 | 9.8 | 27 | 490 | 0.055 |
| 19 | 4.33 | 0.23 | 29.5 | 0.28 | 9.3 | 17 | 331 | 0.051 |
| 23 | 4.17 | 0.18 | 36 | 0.28 | 20.5 | 27 | 386 | 0.070 |
| 23 | 4.17 | 0.18 | 36 | 0.28 | 9.8 | 20 | 183 | 0.109 |
| 40 | 1.89 | 0.05 | 64 | 0.24 | 10.2 | 17 | 74 | 0.230 |
| 40 | 3.94 | 0.10 | 64 | 0.37 | 13.8 | 21 | 331 | 0.063 |
| 40 | 5.79 | 0.14 | 64 | 0.16 | 16.5 | 17 | 123 | 0.138 |

Extreme Results with Milling Machines. — Horace L. Arnold (*Am. Mach.*, Dec. 28, 1893) gives the following results in flat-surface milling, obtained in a Pratt & Whitney milling machine: The mills for the flat cut were 5 in. diam., 12 teeth, 40 to 50 r.p.m. and 47/8 in. feed per min. One single cut was run over this piece at a feed of 9 in. per min., but the mills showed plainly at the end that this rate was greater than they could endure. At 50 r.p.m. for these mills the figures are as follows, with 47/8 in. feed: Surface speed, 64 ft., nearly; feed per tooth, 0.00812 in.; cuts per in., 123. And with 9-in. feed per min.: Surface speed, 64 ft. per min.; feed per tooth, 0.015 in.; cuts per in., 662/3.

At a feed of $47/8$ in. per min., the mills stood up well in this job of cast-iron surfacing, while with a 9-in. feed they required grinding after surfacing one piece; in other words, it did not damage the mill-teeth to do this job with 123 cuts per in. of surface finished, but they would not endure $662/3$ cuts per in. In this cast-iron milling the surface speed of the mills does not seem to be the factor of mill destruction; it is the increase of feed per tooth that prohibits increased production of finished surface. This is precisely the reverse of the action of single-pointed lathe and planer tools in general; with such tools there is a surface-speed limit which cannot be economically exceeded for dry cuts, and so long as this surface-speed limit is not reached, the cut per tooth or feed can be made anything up to the limit of the driving power of the lathe or planer, or to the safe strain on the work itself, which can in many cases be easily broken by a too great feed.

In wrought metal extreme figures were obtained in one experiment made in cutting keyways $5/16$ in. wide by $1/8$ in. deep in a bank of 8 shafts $11/4$ in. diam. at once, on a Pratt & Whitney, No. 3 column milling machine. The 8 mills were successfully operated with 45-ft. surface speed and $191/2$ in. per min. feed; the cutters were 5-in. diam., with 28 teeth, giving the following figures, in steel: Surface speed, 45 ft. per minute; feed per tooth, 0.02024 in.; cuts per inch, 50, nearly. Fed with the revolution of mill. Flooded with oil, that is, a large stream of oil running constantly over each mill. Face of tooth radial. The resulting keyway was described as having a heavy wave or cutter-mark in the bottom, and it was said to have shown no signs of being heavy work on the cutters or on the machine. As a result of the experiment it was decided for economical steady work to run at 17 r.p.m., with a feed of 4 in. per min., flooded cut, work fed with mill revolution, giving the following figures: Surface speed, $221/4$ ft. per min.; feed per tooth, 0.0084 in.; cuts per in., 119.

The Cincinnati Milling Machine Co. (1906) gives the following examples of rapid milling machine work: Gray iron castings $101/4$ in. wide, 14 in. long \times $13/4$ in. thick, finished all over, and a slot $5/8 \times 1$ in. cut from the solid. A gang of five cutters was used, two of 8 in., two of $31/2$ in. and one of $53/4$ in. diameter, respectively. These took a cut $3/16$ in. deep across the top and two edges, and milled the slot in one operation. The table travel was 4.2 in. per minute. The average time, including chucking, was 15.6 minutes.

Gray iron castings 3 in. and $61/2$ in. wide \times $251/4$ in. long, $11/4$ in. thick, were surfaced by a face mill 8 in. diameter at a surface speed of 80 feet per minute. The cut was $3/16$ in., and the table travel 11.4 in. per minute in the 3-in. part and 8 in. per minute in the $61/2$ -in. part. The total time for finishing, including chucking, was seven minutes. The planer required 23 minutes for the same operation. In finishing the opposite side of these castings, two castings are milled at one setting, $5/16$ in. of stock being removed all over and two slots $5/8 \times 5/8$ in. milled from the solid. A gang of seven cutters, 3 of 3 in., 2 of $41/4$ in., and 1 of $81/4$ in. diameter, was used at 38 revolutions per minute and a feed of 0.1 in., giving a table travel of 3.8 in. per minute. These two castings were finished in 18 minutes, including chucking, the actual milling time being eight minutes on each piece. A planer working at 55 ft. cutting speed finished the same job in 36 minutes.

An inserted-tooth face mill 12 in. diameter took a 9-in. cut, $1/8$ in. deep across the entire face of a gray iron casting at a table travel of 5 in. per minute. The length of cut was 18 inches and the time required $61/2$ minutes.

The following table summarizes a number of typical jobs of milling:

Typical Milling Jobs.

(Cincinnati Milling Mach. Co.; Brown & Sharpe Mfg. Co., 1907.)

| Nature of Work. | Material Cut. | Cut, Inches. | | Cutter. | | | Feed per Rev., in. | Table Travel per min., in. | Metal Removed per Minute, cu. in. |
|--------------------|------------------|--------------|-------|------------------|---------------|----------------------------|--------------------|----------------------------|-----------------------------------|
| | | Deep. | Wide. | Diam. in. | Rev. per min. | Surface Speed ft. per min. | | | |
| Spline (R) . . . | Steel | 5/32 | 3/16 | 2 1/2 | 166 | 108 | 0.05 | 8.3 | 0.243 |
| Keyseat (R) . . . | Gray iron . . . | 3/16 | 3/8 | 2 1/2 | 166 | 108 | 0.108 | 17.9 | 1.04 |
| Keyseat (R) . . . | Lumen metal . . | 1/8 | 3/16 | 2 | 211 | 110 | 0.15 | 31.6 | 0.74 |
| Surfacing (F) . . | Brass | 0.01 | 2 1/2 | 31 | 100 | 78 | 0.25 | 25.0 | 0.675 |
| Surfacing (F) . . | Tool steel . . . | 1/16 | 2 1/2 | 31 | 37 | 29 | 0.05 | 1.85 | 0.289 |
| Face Milling (F) | Gray iron . . . | 0.015 | 8 | 10 | 47 | 123 | 0.30 | 14.1 | 1.692 |
| Face Milling (R) | Gray iron . . . | 1/8 | 6 | 8 | 26 | 54 | 0.168 | 4.36 | 3.27 |
| Surfacing (R) . . | Gray iron . . . | 1/8 | 2 1/2 | 3 ² | 100 | 78 | 0.30 | 30.0 | 9.375 |
| Surfacing (F) . . | Bronze casting . | 1/64 | 3 | 3 ² | 166 | 130 | 0.05 | 8.3 | 0.389 |
| T-slotting | Gray iron . . . | See Note 3 | | 1 1/16 | 252 | 75 | 0.05 | 12.6 | 6.693 |
| Surfacing | Gray iron . . . | 0.10 | 12 | 4.5 ⁴ | 45 | 52 | 0.266 | 12. | 14.4 |
| Surfacing | Gray iron . . . | 1/8 | 12 | 3.5 ⁴ | 53 | 55 | 0.226 | 12. | 18.0 |
| Surfacing | Gray iron . . . | 0.20 | 12 | 4 ⁴ | 61 | 65 | 0.148 | 9.02 | 21.6 |

(F) Finishing cut; (R) Roughing cut.

¹ End mill; ² spiral mill with nicked teeth; work done by peripheral teeth. ³ Both sides of cutter engaged, making slot width equal to cutter diameter; slot 1/16 × 1/2 inch. ⁴ Carbon steel nicked spiral cutter.

Tests with a Helical Milling Cutter, 3 in. diam., 6 in. long; 8 teeth; pitch of helix, 18 3/4 in.; notched teeth; on cast iron and on mild steel, are reported by P. V. Vernon in *Am. Mach.*, June 3, 1909. The cutter was run at a constant speed, 84 turns per minute, cutting speed 66 ft. per min. In the tests on cast iron the depth of cut was varied from 0.14 to 1.10 in., and the feed per min. from 109/16 in. to 127/32 in. The material removed per minute ranged from 7.39 to 15.23 cu. in., and the cu. in. per min. per net machine horse-power from 1.06 to 1.52, averaging about 1.30.

In the tests on steel the depth of cut was 0.10 to 1.10 in. and the feed 103/8 to 05/8 in. per min.; the material removed per min. from 2.88 to 6.27 cu. in. per min.; and the cu. in. per min. per net H.P. from 0.47 to 0.71, averaging about 0.57. No regular relation appears between the rate of feed and the metal removed per min., but the maximum output on cast iron was obtained with a cut 5/8 in. deep and a feed of 47/8 in. per min.; and on mild steel with a cut 0.12 in. deep and a feed of 9 1/2 in. per min.

Milling "with" or "against" the Feed.—Tests made with the Brown & Sharpe No. 5 milling-machine (described by H. L. Arnold, in *Am. Mach.*, Oct. 18, 1884) to determine the relative advantage of running the milling cutter with or against the feed—"with the feed" meaning that the teeth of the cutter strike on the top surface or "scale" of cast-iron work in process of being milled, and "against the feed" meaning that the teeth begin to cut in the clean, newly cut surface of the work and cut upwards toward the scale—showed a decided advantage in favor of running the cutter against the feed. The result is directly opposite to that obtained in tests of a Pratt & Whitney machine by experts of the Pratt & Whitney Co.

In the tests with the Brown & Sharpe machine the cutters used were 6 inches face by 4 1/2 and 3 inches diameter, respectively, 15 teeth in each mill, 42 revolutions per minute in each case, or nearly 50 feet per minute surface speed for the 4 1/2-inch and 33 feet per minute for the 3-inch mill. The revolution marks were 6 to the inch, giving a feed of 7 inches per

minute, and a cut per tooth of 0.011 inch. When the machine was forced to the limit of its driving the depth of cut was $11/32$ inch when the cutter ran in the "old" way, or against the feed, and only $1/4$ inch when it ran in the "new" way, or with the feed. The endurance of the milling cutters was much greater when they were run in the "old" way. The Brown & Sharpe Co. says that it is sometimes advisable to mill with the feed, as in surfacing two sides of a piece with straddle mills, the cutters will then tend to hold the work down. In milling deep slots or cutting off stock with thin cutters or saws milling with the feed is less likely to crowd the cutter sidewise and make a crooked slot.

Modern Milling Practice. (Cincinnati Milling Machine Co., 1907.)—The limit of milling operations is determined by the strength and durability of the cutter. A rigid frame on the machine and powerful feed mechanism increase these. The chief causes of low output are: Improperly constructed cutters; insufficient rigidity in the machine; and timidity, due to lack of experience, of both builders and operators. The principal cause of cutter failures is insufficient space for chips between the cutter teeth. Fixed rules cannot be laid down for proper feeds and speeds of milling cutters, these depending on the character and hardness of the metal being cut. On roughing cuts it is desirable to run the cutter at a speed well within its limit, and use as heavy a feed as the machine can pull. The size of chip taken by each tooth of the cutter with the heaviest feeds is comparatively light, and with properly sharpened cutters there is little danger of breaking the cutter by giving too great a feed. It is considered better practice, however, to break an occasional cutter than to run machines at a low rate. It is not considered desirable to run even high speed steel cutters at excessive speeds. The great value of these cutters is their long life and ability to hold a cutting edge as compared with carbon steel cutters. It is important to keep the cutters sharp, as accurate or fast work is impossible with dulled teeth. The clearance angle should be kept low; about 3 degrees for steel, and not more than 5 degrees for gray iron.

The following speeds in feet per minute are a good basis for roughing the materials indicated:

Carbon steel cutters.

| | | | |
|------------|------------------|-------------|-------------------|
| Cast Iron. | Machinery Steel. | Tool Steel. | Brass and Bronze. |
| 40 | 40 | 20 | 60 |

High speed steel cutters.

| | | | |
|----|----|----|-----|
| 80 | 80 | 40 | 120 |
|----|----|----|-----|

On cast-iron work a jet of air delivered to the cutter with sufficient force to blow the chips away as fast as made permits faster feeds and prolongs the cutter's life. A stream of oil fed under heavy pressure to wash the chips away has the same effect when cutting steel. On finishing cuts the rate of feed used determines the grade of the finish. If a spiral mill is used the feed should range from 0.036 in. to 0.05 in. per revolution of a 3-in. diameter cutter. As such cuts are light the speed of cutting can be much higher than used for roughing cuts. The nature of the cut is a factor in determining speeds; a saw can run twice as fast as a surface mill. Keyseating and similar work can be best done with a plain cutter rather than a side mill.

In general small cutters are preferable to large ones, and the hole should be as small as the strength of the arbor will permit. It is advisable in surface milling to have the cutter wider than the work.

Lubricant for Milling Cutters. (Brown & Sharpe Mfg. Co., 1907.)—An excellent lubricant, to use with a pump, for milling cutters is made by mixing together and boiling for one half hour, $1/4$ lb. sal soda, $1/2$ pint lard oil, $1/2$ pint soft soap and water enough to make 10 quarts.

Milling Machine versus Planer.—For comparative data of work done by each see paper by J. J. Grant, Trans. A. S. M. E., ix, 259. He says: The advantages of the milling machine over the planer are many, among which are the following: Exact duplication of work; rapidity of production—the cutting being continuous; lower cost of production, as several machines can be operated by one workman, and he not a skilled mechanic; and lower cost of tools for producing a given amount of work.

DRILLS.

Constant for Finding Speeds of Drills. — For finding the speed in feet when the number of revolutions is given; or the number of revolutions, when the speed in feet is given.

Constant = $12 \div (\text{size of drill} \times 3.1416)$.

Number of revolutions = Constant \times speed in feet.

Speed in feet = Number of revolutions \div constant.

| Size Drill. In. | Con- stant. In. | Size Drill. In. | Con- stant. In. | Size Drill. In. | Con- stant. In. | Size Drill. In. | Con- stant. In. | Size Drill. In. | Con- stant. In. |
|--------------------|-----------------------|--------------------|-----------------------|--------------------|-----------------------|--------------------|-----------------------|--------------------|-----------------------|
| 1/8 | 30.55 | 3/4 | 5.09 | 13/8 | 2.78 | 2 | 1.91 | 25/8 | 1.45 |
| 3/16 | 20.38 | 13/16 | 4.70 | 17/16 | 2.66 | 2 1/16 | 1.85 | 2 11/16 | 1.42 |
| 1/4 | 15.28 | 7/8 | 4.36 | 1 1/2 | 2.55 | 2 1/8 | 1.80 | 2 3/4 | 1.39 |
| 5/16 | 12.22 | 15/16 | 4.07 | 19/16 | 2.44 | 2 3/8 | 1.75 | 2 13/16 | 1.36 |
| 3/8 | 10.19 | 1 | 3.82 | 15/8 | 2.35 | 2 1/4 | 1.70 | 2 7/8 | 1.33 |
| 7/16 | 8.73 | 1 1/16 | 3.59 | 1 11/16 | 2.26 | 2 5/8 | 1.65 | 2 15/16 | 1.30 |
| 1/2 | 7.64 | 1 1/8 | 3.39 | 1 3/4 | 2.18 | 2 3/4 | 1.61 | 3 | 1.27 |
| 9/16 | 6.79 | 1 3/8 | 3.22 | 1 13/16 | 2.11 | 2 7/8 | 1.57 | 3 1/16 | 1.25 |
| 5/8 | 6.11 | 1 1/4 | 3.06 | 1 7/8 | 2.04 | 2 1/2 | 1.53 | 3 1/8 | 1.22 |
| 11/16 | 5.56 | 1 5/16 | 2.91 | 1 15/16 | 1.97 | 2 9/16 | 1.49 | 3 1/4 | 1.18 |

Speed of Drills. — The Cleveland Twist Drill Co. (1907) gives the following speeds in r.p.m. for drilling wrought iron, machinery steel or soft tool steel, with high speed and carbon steel drills.

| Diam., In. | Carbon Steel. | High Speed. | Diam., In. | Carbon Steel. | High Speed. | Diam., In. | Carbon Steel. | High Speed. | Diam., In. | Carbon Steel. | High Speed. |
|---------------|------------------|----------------|---------------|------------------|----------------|---------------|------------------|----------------|---------------|------------------|----------------|
| 1/16 | 1834 | 3057 | 13/16 | 141 | 235 | 19/16 | 73.4 | 122 | 25/16 | 49.5 | 82.7 |
| 1/8 | 917 | 1528 | 7/8 | 131 | 218 | 15/8 | 70.5 | 117 | 23/8 | 48.2 | 80.5 |
| 3/16 | 611 | 1020 | 15/16 | 122 | 204 | 1 11/16 | 67.9 | 113 | 27/16 | 47.0 | 78.5 |
| 1/4 | 458 | 765 | 1 | 115 | 191 | 1 3/4 | 65.5 | 109 | 2 1/2 | 45.8 | 76.5 |
| 5/16 | 367 | 612 | 1 1/16 | 108 | 180 | 1 13/16 | 63.2 | 105.3 | 29/16 | 44.7 | 74.6 |
| 3/8 | 306 | 510 | 1 1/8 | 102 | 170 | 1 7/8 | 61.1 | 102 | 25/8 | 43.7 | 72.8 |
| 7/16 | 262 | 437 | 1 3/16 | 96.5 | 160 | 1 15/16 | 59.2 | 98.7 | 2 11/16 | 42.6 | 71.1 |
| 1/2 | 229 | 382 | 1 1/4 | 91.8 | 153 | 2 | 57.3 | 95.6 | 23/4 | 41.7 | 69.5 |
| 9/16 | 204 | 340 | 1 5/16 | 87.3 | 145 | 2 1/16 | 55.6 | 92.7 | 2 13/16 | 40.7 | 68.0 |
| 5/8 | 184 | 306 | 1 3/8 | 83.3 | 139 | 2 1/8 | 54.0 | 90.0 | 2 7/8 | 39.8 | 66.5 |
| 11/16 | 167 | 277 | 1 7/16 | 79.8 | 133 | 2 3/16 | 52.4 | 87.4 | 2 15/16 | 39.0 | 65.1 |
| 3/4 | 153 | 255 | 1 1/2 | 76.3 | 127 | 2 1/4 | 51.0 | 85.0 | 3 | 38.2 | 63.6 |

The feed per revolution recommended for drills smaller than 1/2-in. is from 0.004 to 0.007 in.; and from 0.005 to 0.01 in. for drills larger than 1/2-in.

High Speed Steel Drills. — The Cleveland Twist Drill Co. says that a high speed steel drill should be started with a peripheral speed between 50 and 60 ft. per minute, and a feed of 0.005 to 0.010 in. per revolution for drills over 1/2-in. A drill with a tendency to wear away on the outside is running too fast; if it breaks or chips on the cutting edges it has too much feed. When used in steel or wrought iron, the drill should be flooded with a good lubricant. For brass, paraffine oil is recommended, and for cast iron, an air blast.

Power Required to Drive High Speed Steel Drills. — The American Tool Works Co. (1907) obtained some remarkable results with drills of high-speed steel as shown in the tables below. The machine used was a triple-gear radial, and the drill was of the "Celfors" type, a flat bar of steel, twisted, affording ease of lubrication, and a free escape for the chips.

Power Required to Drill Steel with High Speed Steel Drills.

| Size of Drill. Inches. | R.P.M. | Cutting Speed, Ft. per Min. | Feeds. | | H.P. Re- quired. |
|---------------------------|--------|--------------------------------|--------------|--------------|---------------------|
| | | | In. per Rev. | In. per Min. | |
| 9/16 | 356 | 52.3 | .012 | 4.27 | 4.2 |
| 3/4 | 313 | 61.5 | .012 | 3.75 | 10.8 |
| 1 1/32 | 188 | 50.9 | .024 | 4.51 | 9.0 |
| 1 5/32 | 188 | 56.9 | .024 | 4.51 | 9.3 |
| 1 23/32 | 128 | 57.6 | .024 | 3.07 | 8.4 |
| 1 31/32 | 167 | 86.2 | .012 | 2.00 | 7.8 |

Power Required to Drill Cast Iron 2 in. thick with High Speed Steel Drill.

| Size of Drill, Inches. | R.P.M. | Cutting Speed, Ft. per Min. | Feeds. | | H.P. |
|---------------------------|--------|--------------------------------|--------------|--------------|------|
| | | | In. per Rev. | In. per Min. | |
| 1 1/32 | 313 | 84.5 | .046 | 14.4 | 13.2 |
| 1 7/32 | 313 | 99.8 | .046 | 14.4 | 15.3 |
| 1 15/32 | 216 | 83.1 | .033 | 7.1 | 12.6 |
| 1 23/32 | 216 | 97.0 | .033 | 7.1 | 16.8 |
| 1 31/32 | 128 | 66.0 | .033 | 4.22 | 15.6 |
| 3 1/2 | 60 | 55.0 | .024 | 1.44 | 10.2 |

Extreme Results with Radial Drills. (F. E. Bocorselski, *Am. Mach.*, Mar. 17, 1910.) — Three different radial drilling machines, designed to drive high-speed steel drills of the twisted type to the limit of their endurance, were tested by drilling steel billets of about 0.70 carbon at speeds and feeds which caused the drills to break after drilling holes from 2 to 11 ins. deep. The following are a few of the results obtained with different sizes of drill.

| Drill size, ins. | Revs. per min. | Cutting speed, ft. per min. | Feed. | | Metal removed. | | Max. H.P. | H.P. per lb. per min. |
|---------------------|----------------------|--------------------------------------|-----------------|------------------|----------------------|------------------|--------------|--------------------------------|
| | | | Per rev. in. | Ins. per min. | Cu. ins. per min. | Lbs. per min. | | |
| 1 1/2 | 290 | 113 | 0.0207 | 6 | 10.56 | 2.95 | 25 | 8.48 |
| 1 1/2 | 312 | 123 | 0.0323 | 10.08 | 17.23 | 4.97 | 56.6 | 11.4 |
| 1 1/4 | 330 | 107 | 0.0207 | 6.83 | 8.33 | 2.33 | 24.8 | 10.6 |
| 1 1/8 | 208 | 61.3 | 0.022 | 4.58 | 4.54 | 1.27 | 22.6 | 17.8 |
| 1 1/16 | 330 | 91 | 0.0207 | 6.83 | 6. | 1.68 | 24.8 | 14.8 |

The H.P. of one of the machines running light at full speed was 4.4; running light at slow speed 2 H.P.

It was concluded from these tests, which were destructive to the drills, that for maximum production and considering the life of the drills, it is best to run a 1-in. drill at about 300 r.p.m. with a feed of 0.015 in. per rev., and a 1 1/2-in. drill 225 r.p.m. with a feed of 0.02 in. per revolution.

Some Data on High-Speed Drilling are given by G. E. Hallenbeck in *Iron Tr. Rev.*, April 29, 1909. A Baker high-speed drilling machine was used. Holes 1 1/8 in. diam. were drilled through 4 1/4-in. blocks of cast iron in 82/3 seconds per hole, or at the rate of 29 in. per min. Holes 15/16 in. diam. were drilled through 3/4 in. steel plate in 3 1/2 seconds.

Experiments on Twist Drills. — An extensive series of experiments on the forces acting on twist drills of high-speed steel when operating

on cast-iron and steel is reported by Dempster Smith and A. Poliakoff, in *Proc. Inst. M. E.*, 1909. Abstracted in *Am. Mach.*, May, 1909, and *Indust. Eng.*, May, 1909. Approximate equations derived from the first set of experiments are as follows:

Torque in pounds-feet, $T = (1800t + 9)d^2$, for medium cast-iron; $T = (3200t + 20)d^2$, for medium steel. End thrust, lbs., $P = 115,000t - 200$, for medium cast-iron; $P = 160,000(d - 0.5)t + 1000$, for medium steel; $d = \text{diam.}$, $t = \text{feed per revolution of drill, both in inches.}$ The steel was of medium hardness, 0.29 C, 0.625 Mn.

The end thrust in enlarging holes in medium steel from one size to a larger was as follows: $3/4$ in. to 1 in., $P = 15,200t - 60$; 1 in. to $1\frac{1}{2}$ in., $P = 25,500t + 3/4$ in. to $1\frac{1}{2}$ in., $P = 30,000t + 200$.

A second series of experiments, with soft cast-iron of C.C., 0.2; G.C., 2.9; Si, 1.41; Mn, 0.68; S, 0.035; P, 1.48, and medium steel of C, 0.31; Si, 0.07; Mn, 0.50; S, 0.018; P, 0.033; tensile strength, 72,600 lbs. per sq. in., gave results from which were derived the following approximate equations:

Torque, lbs.-ft., $T = 740 d^{1.8} t^{0.7}$, or $10 d^2 + 100 t(14 d^2 + 3)$ for cast-iron, $T = 1640 d^{1.8} t^{0.7}$, or $28 d^2(1 + 100 t)$ for medium steel, End thrust, lbs. $P = 35,500 d^{0.7} t^{0.75}$, or $200 d + 10,000 t$ for cast iron, $P = 35,500 d^{0.7} t^{0.6}$, or $750 d + 1000 t(75 d + 50)$ for medium steel,

and for different sizes of drill the following equations:

| Drill. | $3/4$ | 1 | $1\frac{1}{2}$ |
|---------------------------------|-------------------|-------------------|---------------------|
| Cast iron $T = \dots\dots\dots$ | $5 + 1,100 t$ | $10 + 1,750 t$ | $25 + 3,700 t$ |
| Cast iron $P = \dots\dots\dots$ | $125 + 82,000 t$ | $200 + 89,000 t$ | $350 + 103,000 t$ |
| Steel $T = \dots\dots\dots$ | $7.5 + 3,350 t$ | $17.5 + 4,400 t$ | $40 + 9,000 t$ |
| Steel $P = \dots\dots\dots$ | $550 + 109,000 t$ | $750 + 131,000 t$ | $1,250 + 162,000 t$ |

| Drill. | 2 | $2\frac{1}{2}$ | 3 |
|---------------------------------|---------------------|---------------------|---------------------|
| Cast iron $T = \dots\dots\dots$ | $40 + 580 t$ | $60 + 8,800 t$ | $90 + 12,900 t$ |
| Cast iron $P = \dots\dots\dots$ | $500 + 110,000 t$ | $600 + 126,000 t$ | $850 + 140,000 t$ |
| Steel $T = \dots\dots\dots$ | $75 + 12,500 t$ | $112.5 + 19,050 t$ | $175 + 26,250 t$ |
| Steel $P = \dots\dots\dots$ | $1,500 + 181,250 t$ | $1,725 + 224,375 t$ | $2,350 + 280,000 t$ |

The tests above referred to were made without lubricants. When lubricants were used in drilling steel the average torque varied from 72% with $1/400$ in. feed to 92% with $1/35$ in. feed of that obtained when operating dry. The thrust for soft, medium and hard steel is 26%, 37% and 12% respectively less than when operating dry, no marked difference being found, as in the torque, with different feed. The horse-power varies as $t^{0.7}$ and as $d^{0.8}$ for a given drill and speed. The torque and horse-power when drilling medium steel is about 2.1 times that required for cast iron with the same drill speed and feed. The horse-power per cu. in. of metal removed is inversely proportional to $d^{0.2} t^{0.3}$, and is independent of the revolutions.

While the chisel point of the drill scarcely affects the torque it is accountable for about 20% of the thrust. Tests made with a preliminary hole drilled before the main drill was used to enlarge the hole showed that the work required to drill a hole where only one drill is used is greater than that required to drill the hole in two operations, with drills of different diameter.

For economy of power a drill with a larger point angle than 120° is to be preferred, but the increased end thrust strains the machine in proportion, and there is more danger of breaking the drill.

Taking the average recommended speed of 48 ft. per minute for cast iron and 60 ft. for mild steel, and the results obtained in these tests, the figures given in the following table are derived.

REVOLUTIONS PER MINUTE, FEED PER REVOLUTION, CUBIC INCHES REMOVED PER MINUTE, AND HORSE-POWER WHEN DRILLING SOFT CAST-IRON AND MEDIUM HARD STEEL.

| Soft Cast Iron. | | | | | | Medium Hard Steel. | | | | | |
|-------------------------|---|--|---|--------------------|--|-------------------------|---|--|---|--------------------|--|
| Diam. of drill, inches. | R.P.M. at cutting speed of 48 ft. per min. = $12 \times 48/\pi d$. | Feed in ins. per revolution of drill, $t = d^{3/84}$. | Cubic inches removed per min. $1.715 d^3$. | Total horse-power. | H.P. per cu. in. of metal removed per min. | Diam. of drill, inches. | R.P.M. at cutting speed of 60 ft. per min. = $12 \times 60/\pi d$. | Feed in ins. per revolution, $t = d^{3/100}$. | Cubic inches removed per min. $1.8 d^3$. | Total horse-power. | H.P. per cu. in. of metal removed per min. |
| 1/4 | 735 | 0.0075 | 0.27 | 0.295 | 1.092 | 1/4 | 920 | 0.0063 | 0.284 | 0.721 | 2.54 |
| 3/8 | 490 | 0.0086 | 0.462 | 0.4405 | 0.954 | 3/8 | 614 | 0.0072 | 0.485 | 1.078 | 2.22 |
| 1/2 | 368 | 0.0094 | 0.682 | 0.586 | 0.862 | 1/2 | 460 | 0.00795 | 0.716 | 1.426 | 1.99 |
| 3/4 | 245 | 0.0109 | 1.17 | 0.8766 | 0.748 | 3/4 | 306 | 0.0091 | 1.23 | 2.152 | 1.75 |
| 1 | 184 | 0.0119 | 1.715 | 1.167 | 0.681 | 1 | 230 | 0.01 | 1.8 | 2.863 | 1.59 |
| 1 1/4 | 147 | 0.0129 | 2.32 | 1.457 | 0.628 | 1 1/4 | 184 | 0.0108 | 2.44 | 3.574 | 1.47 |
| 1 1/2 | 122 | 0.0136 | 2.92 | 1.748 | 0.598 | 1 1/2 | 153 | 0.0114 | 3.08 | 4.285 | 1.39 |
| 1 3/4 | 105 | 0.0144 | 3.63 | 2.038 | 0.563 | 1 3/4 | 131 | 0.0121 | 3.81 | 5.005 | 1.31 |
| 2 | 92 | 0.015 | 4.32 | 2.328 | 0.539 | 2 | 115 | 0.0126 | 4.54 | 5.715 | 1.26 |
| 2 1/4 | 81.7 | 0.0156 | 5.05 | 2.619 | 0.519 | 2 1/4 | 102 | 0.0131 | 5.3 | 6.436 | 1.21 |
| 2 1/2 | 73.5 | 0.0162 | 5.82 | 2.909 | 0.500 | 2 1/2 | 92 | 0.0136 | 6.12 | 7.136 | 1.165 |
| 2 3/4 | 66.75 | 0.0167 | 6.6 | 3.199 | 0.486 | 2 3/4 | 83.5 | 0.014 | 6.92 | 7.857 | 1.135 |
| 3 | 61.3 | 0.0172 | 7.4 | 3.489 | 0.472 | 3 | 76.5 | 0.0144 | 7.76 | 8.567 | 1.105 |
| 3 1/4 | 56.5 | 0.0176 | 8.22 | 3.78 | 0.46 | 3 1/4 | 70.5 | 0.0148 | 8.66 | 9.267 | 1.07 |
| 3 1/2 | 52.5 | 0.0181 | 9.05 | 4.07 | 0.45 | 3 1/2 | 65.6 | 0.0151 | 9.5 | 9.998 | 1.05 |
| 3 3/4 | 49 | 0.0185 | 10.0 | 4.36 | 0.436 | 3 3/4 | 61.25 | 0.0155 | 10.48 | 10.718 | 1.024 |
| 4 | 46 | 0.019 | 10.8 | 4.65 | 0.431 | 4 | 57.5 | 0.0158 | 11.4 | 11.42 | 1.0 |

POWER REQUIRED FOR MACHINE TOOLS.

Resistance Overcome in Cutting Metal. (Trans. A. S. M. E., viii. 308.) — Some experiments made at the works of William Sellers & Co. showed that the resistance in cutting steel in a lathe would vary from 180,000 to 700,000 pounds per square inch of section removed, while for cast iron the resistance is about one third as much. The power required to remove a given amount of metal depends on the shape of the cut and on the shape and the sharpness of the tool used. If the cut is nearly square in section, the power required is a minimum; if wide and thin, a maximum. The dullness of a tool affects but little the power required for a heavy cut.

Heavy Work on a Planer. — Wm. Sellers & Co. write as follows to the *American Machinist*: The 120-inch planer table is geared to run 18 feet per minute under cut, and 72 feet per minute on the return, which is equivalent, without allowance for time lost in reversing, to continuous cut of 14.4 feet per minute. Assuming the work to be 28 feet long, we may take 14 feet as the continuous cutting speed per minute, the 0.8 of a foot being much more than sufficient to cover time loss in reversing and feeding. The machine carries four tools. At 1/8 inch feed per tool, the surface planed per hour would be 35 square feet. The section of metal cut at 3/4 inch depth would be 0.75 inch \times 0.125 inches \times 4 = 0.375 square inch, which would require approximately 30,000 pounds pressure to remove it. The weight of metal removed per hour would be $14 \times 12 \times 0.375 \times 0.26 \times 60 = 1082.8$ lb. Our earlier form of 36 in. planer has removed with one tool on 3/4 in. cut on work 200 lb. of metal per hour, and the 120 in. machine has more than five times its capacity. The total pulling power of the planer is 45,000 lb.

Horse-power Required to Run Lathes. — The power required to do useful work varies with the depth and breadth of chip, with the

shape of tool, and with the nature and density of metal operated upon; and the power required to run a machine empty is often a variable quantity. For instance, when the machine is new, and the working parts have not become worn or fitted to each other as they will be after running a few months, the power required will be greater than will be the case after the running parts have become better fitted.

Another cause of variation of the power absorbed is the driving-belt; a tight belt will increase the friction.

A third cause is the variation of journal-friction, due to slacking up or tightening the cap-screws, and also the end-thrust bearing screw.

Hartig's investigations show that it requires less total power to turn off a given weight of metal in a given time than it does to plane off the same amount; and also that the power is less for large than for small diameters. (J. J. Flather, *Am. Mach.*, April 23, 1891.)

Horse-power Required to Remove Metal in Lathes.

(Lodge & Shipley Mach. Tool Co., 1906.)

20-INCH CONE-HEAD LATHE.

| Material Cut. | Cutting Speed, ft. per min. | Cut, In. | | Diam. of work, in. | Cu. in. removed per min. | Lb. removed per hour. | H.P. used by Lathe. | | Cu. in. removed per H.P. |
|-------------------------------|-----------------------------|----------|-------|--------------------|--------------------------|-----------------------|---------------------|-----------|--------------------------|
| | | Depth. | Feed. | | | | Idle. | With Cut. | |
| Crucible Steel 0.60 Carbon | 35 | 0.109 | 1/8 | 227/32 | 5.74 | 96 | 0.48 | 3.90 | 1.471 |
| | 65 | 0.055 | 1/8 | 35/8 | 5.33 | 90 | 0.74 | 4.60 | 1.158 |
| | 62.5 | 0.109 | 1/16 | 35/16 | 5.125 | 86 | 0.49 | 4.65 | 1.102 |
| | 32.5 | 0.094 | 1/10 | 35/16 | 3.656 | 62 | 0.49 | 2.64 | 1.384 |
| Cast Iron | 62.5 | 0.273 | 1/12 | 35/32 | 17.09 | 266 | 0.66 | 5.44 | 3.141 |
| | 60 | 0.430 | 1/19 | 221/64 | 16.27 | 253 | 0.59 | 4.77 | 3.410 |
| | 37.5 | 0.334 | 1/16 | 221/32 | 10.76 | 167 | 0.45 | 3.91 | 2.751 |
| | 115 | 0.086 | 1/12 | 155/64 | 9.88 | 153 | 0.21 | 2.54 | 3.889 |
| Open-hearth Steel 0.30 Carbon | 50 | 0.109 | 1/8 | 223/32 | 8.2 | 138 | 0.69 | 5.34 | 1.535 |
| | 45 | 0.117 | 1/8 | 21/2 | 7.91 | 134 | 0.53 | 5.11 | 1.547 |
| | 45 | 0.217 | 1/19 | 217/64 | 6.439 | 109 | 0.69 | 4.10 | 1.570 |
| | 32.5 | 0.109 | 1/8 | 223/64 | 5.33 | 90 | 0.36 | 4.04 | 1.319 |

Average H.P. running idle 0.53; average H.P. with cut 4.25.

20-INCH GEARED-HEAD LATHE,

| Material Cut. | Cutting Speed, ft. per min. | Cut, in. | | Diam. of work in. | Cu. in. removed per min. | Lb. removed per hour. | H.P. used by Lathe. | | Cu. in. removed per H.P. |
|-------------------------------|-----------------------------|----------|-------|-------------------|--------------------------|-----------------------|---------------------|-----------|--------------------------|
| | | Depth. | Feed. | | | | Idle. | With Cut. | |
| 0.50 Carbon Crucible Steel. | 40 | 0.266 | 1/10 | 227/32 | 12.75 | 215 | 2.11 | 11.1 | 1.142 |
| | 50 | 0.281 | 1/15 | 227/32 | 11.25 | 190 | 1.58 | 8.35 | 1.347 |
| | 75 | 0.281 | 1/15 | 227/32 | 16.87 | 285 | 1.58 | 12.69 | 1.329 |
| | 85 | 0.109 | 1/15 | 2 1/4 | 7.43 | 126 | 1.28 | 8.98 | 0.827 |
| Cast Iron | 45 | 0.609 | 1/16 | 721/32 | 20.57 | 320 | 1.34 | 6.94 | 2.963 |
| | 62.5 | 0.609 | 1/16 | 721/32 | 28.56 | 445 | 1.35 | 9.50 | 3.006 |
| | 85 | 0.641 | 1/16 | 721/32 | 40.82 | 636 | 1.64 | 12.69 | 3.216 |
| | 80 | 0.281 | 1/8 | 3 3/32 | 33.75 | 526 | 1.18 | 10.49 | 3.217 |
| Open-hearth Steel 0.15 Carbon | 125 | 0.250 | 1/28 | 421/32 | 13.4 | 226 | 1.62 | 10.60 | 1.265 |
| | 105 | 0.188 | 1/12 | 4 5/32 | 19.68 | 337 | 0.94 | 11.56 | 1.702 |
| | 40 | 0.172 | 1/6 | 327/32 | 13.75 | 232 | 1.75 | 12.49 | 1.100 |
| | 180 | 0.094 | 1/16 | 3 1/16 | 12.65 | 213 | 2.15 | 11.20 | 1.129 |

Average H.P. running idle 1.543; average H.P. with cut 10.55.

Owing to the demand imposed by high speed tool steels stouter machines are more necessary than formerly; these possess more rigid frames and powerful driving gears. The most modern (1907) forms of lathes obtain all speed changes by means of geared head-stocks, power being delivered at a single speed by a belt, or by a motor. If a motor drive is used, a speed variation may be obtained in addition to those available in the head, by using a variable speed motor, whose range usually is about 3:1. The Lodge & Shipley Co. (1906) made an exhaustive series of tests to determine the power required to remove metal, using both the cone-head lathe and the more modern geared-head lathe. The table on page 1257 shows the results obtained with 20-in. lathes of each type.

Power Required to Drive Machine Tools. — The power required to drive a machine tool varies with the material to be cut. There is considerable lack of agreement among authorities on the power required. Prof. C. H. Benjamin (*Mach'y*, Sept., 1902) gives a formula $H.P. = cW$, c being a constant and W the pounds of metal removed per hour. c varies both with the quality of metal and the type of machine.

Values of c .

| | Lathe. | Planer. | Shaper. | Milling Machine. |
|---------------------------|--------|---------|---------|------------------|
| Cast iron | 0.035 | 0.032 | 0.030 | 0.14 |
| Machinery steel | 0.067 | ... | ... | ... |
| Tool steel | ... | ... | ... | 0.30 |
| Bronze | ... | ... | ... | 0.10 |

In each case the power to drive the machine without load should be added. G. M. Campbell (Proc. Engr. Soc. W., Pa., 1906) gives, exclusive of friction losses, $H.P. = Kw$, K being a constant and w the pounds of metal removed per minute. For hard steel $K=2.5$; for soft steel $K=1.8$; for wrought iron, $K=2.0$; for cast iron, $K=1.4$. This formula gives results about 50 % lower than Prof. Benjamin's.

The Westinghouse Elec. and Mfg. Co. (1906) gives a set of formulæ based on the dimensions of the machine.

For Engine Lathes using one cutting tool of water-hardened steel, cutting 20 ft. per minute, $H.P. = 0.15 S - 1$; for heavy engine lathes, as forge lathes, $H.P. = 0.234 S - 2$, S being the swing of the lathe, inches.

For Boring Mills using one cutting tool of water-hardened steel, cutting 20 ft. per min., $H.P. = 0.25 S - 4$. S = swing of mill, inches.

For Milling Machines using water-hardened steel cutters at 20 ft. per minute, $H.P. = 0.3 W$. W = distance between housings, inches.

For Drill Presses using water-hardened steel drills, running at a peripheral cutting speed of 20 feet per minute, $H.P. = 0.06 S$.

For Heavy Radial Drill Presses, $H.P. = 0.1 S$.

S = swing of drill, inches, in both cases.

In general, in all the above Westinghouse formulæ, if high-speed steel tools are used, running at higher cutting speeds than above, the increase in horse-power is proportional to the increase in speed.

Planers. For planers, in which the length of bed in feet is approximately two-tenths of the width between housings in inches, using water-hardened steel tools, cutting at 15 to 20 ft. per minute, $H.P. = 3 W$.

For Heavy Forge Planers, $H.P. = 4.92 W$.

W = width between housings, feet.

These formulæ are for planers having a ratio of return to cutting speeds of about 3:1, and are for planers with two tools in operation. If more than two tools are operated, or if the ratio of cutting and return speeds is increased, or if the length of bed is greater than given above, the horse-power given by the above formulæ should be increased. The horse-power required by motor-driven planers is principally determined by the current inrush at the instant of quick reverse, rather than by that actually required to cut the metal. Motors for operating planers should have greater overload capacity than for any other tool.

Horse-power to Drive Machine Tools.

| Tool. | Material. | Cut, Inches. | | Speed, Ft. per Min. | Wt. Removed, Lb. per Min. | H.P. Re- quired. | | Motor Used. |
|---------------------------|------------|--------------|-------------|------------------------|------------------------------|---------------------|----------|---|
| | | Feed. | Depth. | | | Actual. | Formula. | |
| 72-in. wheel lathe | Hard steel | 1/12 | 3/16 & 1/4 | 13.7 | 1.69 | 4.5 | 4.2 | 25 H.P. shunt wound vari- able speed. |
| | " " | 1/8 | 3/16 & 1/4 | 11.6 | 2.15 | 6.4 | 5.4 | |
| | " " | 3/16 | 5/16 & 3/8 | 13.2 | 5.55 | 8.4 | 13.9 | |
| | " " | 3/16 | 3/8 & 3/8 | 13.2 | 6.3 | 12.0 | 15.7 | |
| 90-in. wheel lathe | Hard steel | 3/16 | 3/16 & 3/16 | 13.0 | 3.1 | 12.0 | 7.7 | 25 H.P. shunt wound vari- able speed. |
| | " " | 3/16 | 5/16 & 5/16 | 8.8 | 3.5 | 8.1 | 8.7 | |
| | " " | 1/5 | 1/4 & 1/4 | 15.5 | 5.3 | 9.0 | 13.2 | |
| 42-in. lathe | Soft steel | 1/16 | 1/4 | 44 | 2.33 | 3.8 | 4.2 | 15 H.P. shunt wound vari- able speed. |
| | " " | 1/16 | 1/8 | 44 | 1.17 | 1.7 | 1.9 | |
| | " " | 1/16 | 1/8 | 44 | 1.17 | 2.6 | 1.9 | |
| | Cast iron | 1/16 | 1/8 | 108 | 2.63 | 5.8 | 3.7 | |
| | " " | 1/16 | 3/16 | 46 | 1.74 | 2.9 | 2.5 | |
| | " " | 1/16 | 3/16 | 58 | 2.12 | 2.2 | 3.0 | |
| 30-in. lathe | Wro't iron | 1/8 | 3/16 | 54 | 4.2 | 6.6 | 8.4 | 10 H.P. shunt wound vari- able speed. |
| | " " | 1/8 | 3/16 | 42 | 3.2 | 4.0 | 6.4 | |
| | Cast iron | 3/32 | 5/32 | 42 | 1.92 | 3.0 | 2.7 | |
| | " " | 3/32 | 1/16 | 61 | 1.12 | 1.5 | 1.6 | |
| | " " | 1/64 | 1/4 | 47 | 2.30 | 2.0 | 3.2 | |
| Axle lathe | Soft steel | 3/16 | 1/4 | 27 | 4.3 | 5.9 | 7.7 | 35 H.P. sh. w'd var. speed. |
| | " " | 1/16 | 1/4 | 51 | 2.7 | 5.0 | 4.9 | |
| 72-in. boring mill . . | Soft steel | 1/8 | 1/16 & 1/32 | 44 | 1.76 | 2.9 | 3.2 | 25 H.P. shunt wound vari- able speed. |
| | " " | 3/16 | 1/32 & 1/16 | 40 | 2.38 | 2.6 | 4.3 | |
| | " " | 1/8 | 1/8 & 1/8 | 51 | 5.41 | 9.6 | 9.7 | |
| | " " | 1/8 | 3/16 | 47 | 3.75 | 7.2 | 6.8 | |
| | Cast iron | 1/16 | 3/8 | 28 | 2.05 | 2.6 | 2.9 | |
| | " " | 1/16 | 1/4 | 39 | 1.90 | 2.7 | 2.7 | |
| 24-in. drill press . | Wro't iron | 1/64 | 1 1/4 to 3* | 25.1 | 0.81 | 2.3 | 1.6 | |
| | " " | 1/64 | 1 1/4 to 3* | 29.7 | 0.96 | 2.7 | 1.9 | |
| | " " | 1/64 | 1 1/4 to 3* | 25.9 | 0.83 | 1.3 | 1.7 | |
| | " " | 1/64 | 1 1/4 drill | 74.5 | 0.52 | 3.5 | 1.0 | |
| | " " | 1/64 | 1 1/4 drill | 20.9 | 0.54 | 1.2 | 1.1 | |
| 60-in. planer | Soft steel | 1/8 | 1/4 | 25.5 | 3.62 | 5.9 | 6.5 | 20 H.P. com- pound wound vari- able speed. |
| | " " | 1/8 | 1/4 | 25.7 | 3.65 | 6.5 | 6.6 | |
| | Wro't iron | 3/16 | 5/16 & 5/16 | 23 | 8.95 | 21.0 | 17.9 | |
| | " " | 1/2 | 1/32 & 1/32 | 17.5 | 1.82 | 2.7 | 3.6 | |
| | Cast iron | 1/8 | 1/8 & 1/16 | 22.2 | 1.72 | 6.5 | 3.4 | |
| | " " | 1/8 & 1/16 | 1/4 & 5/16 | 30 | 4.74 | 9.3 | 6.6 | |
| | " " | 1/7 | 1/4 & 1/4 | 22.6 | 5.03 | 7.6 | 7.1 | |
| | " " | 1/4 | 7/16 & 3/8 | 28.9 | 18.3 | 23.2 | 25.6 | |
| 42-in. planer | Soft steel | 5/32 | 3/8 | 24.3 | 4.73 | 12.1 | 9.5 | 15 H.P. com- pound wound vari- able speed. |
| | " " | 1/8 | 3/8 | 36 | 3.7 | 7.8 | 11.4 | |
| | Cast iron | 3/16 | 3/16 | 37 | 4.06 | 4.7 | 5.7 | |
| | " " | 3/16 | 1/8 | 37 | 2.71 | 4.1 | 3.8 | |
| 19-in. slotter | Hard steel | 1/32 | 1/4 | 30.0 | 0.8 | 2.0 | 2.0 | 13 H.P. comp. w'd var. speed. |
| | Soft steel | 1/32 | 3/8 | 23.3 | 0.93 | 1.3 | 1.7 | |

* Enlarging hole from smaller dimensions to larger.

Actual tests (1906) of a number of machine tools in the shops of the Pittsburgh and Lake Erie R. R. showed the horse-power absorbed in driving under the conditions given in the table on page 1259. The results obtained are compared with those computed by Campbell's formula above.

L. L. Pomeroy (*Gen. Elec. Rev.*, 1908) gives: H.P. required to drive = $12 FDSNC$, in which F = feed and D = depth of cut, in inches, S = speed in ft. per min., N = number of tools cutting, C = a constant, whose values with ordinary carbon steel tools are: for cast iron, 0.35 to 0.5; soft steel or wrought iron, 0.45 to 0.7; locomotive driving-wheel tires, 0.7 to 1.0; very hard steel, 1.0 to 1.1. This formula is based on Prof. Flather's dynamometer tests. An analysis of experiments by Dr. Nicholson of Manchester, which confirm the formula, showed the average H.P. required at the motor per pound of metal removed per minute to be as follows: Medium or soft steel, or wrought iron, 2.4 H.P.; hard steel, 2.65 H.P.; cast-iron, soft or medium, 1.00 H.P.; cast iron, hard, 1.36 H.P.

Size of Motors for Machine Tools. (*Elec. World*, May 27, 1905.)—The average size of motor usually fitted to machine tools is shown by the table below, being compiled by the Electro-Dynamic Co. from published data. In special cases the power required may be several times the value here given.

Boring Mills.

| | H.P. | | H.P. | | H.P. |
|---------------------------|-------|-------------------------|------|----------------|------|
| 34 and 36 in. | 5 | 60 in. | 10 | 8 ft. | 20 |
| 42, 48 and 50 in. | 7 1/2 | 6 ft. and 7 ft. | 15 | 10 ft. | 25 |

Engine Lathes.

| | H.P. | | H.P. | | H.P. |
|-----------------------|-------|-----------------------|------|----------------|-------|
| 12 and 14 in. | 1 | 20 and 30 in. | 3 | 54 in. | 6 |
| 16 in. | 1 1/2 | 36 in. | 4 | 60 in. | 7 1/2 |
| 20 to 25 in. | 2 | 42 and 48 in. | 5 | 72 in. | 10 |

Drill Presses.

| | H.P. | | H.P. | | H.P. |
|----------------------|------|----------------------|------|----------------------|------|
| 21 to 32 in. | 2 | 36 to 48 in. | 3 | 50 to 60 in. | 5 |

Planers.

| | H.P. | | H.P. |
|-----------------------------------|-------|------------------------------------|------|
| 17 × 17 in. × 3 to 6 ft. | 4 | 42 × 42 in. × 10 to 12 ft. | 10 |
| 22 × 24 in. × 4 to 10 ft. | 5 | 48 × 48 in. × 12 to 14 ft. | 15 |
| 26 × 26 in. × 6 to 12 ft. | 6 | 50 × 60 in. × 14 to 18 ft. | 20 |
| 30 × 30 in. × 6 to 14 ft. | 7 1/2 | 60 × 60 in. × 20 to 22 ft. | 25 |
| 36 × 36 in. × 8 to 16 ft. | 7 1/2 | 72 × 72 in. × 20 to 24 ft. | 30 |

Slotters.

| | H.P. | | H.P. | | H.P. |
|----------------------|------|-----------------------|-------|----------------------|------|
| 12 to 14 in. | 5 | 16 and 18 in. | 7 1/2 | 26 to 36 in. | 10 |

Shapers.

| | H.P. | | H.P. | | H.P. |
|----------------------|------|----------------------|------|----------------|------|
| 12 to 16 in. | 2 | 24 to 26 in. | 5 | 36 in. | 8 |
| 18 to 20 in. | 3 | 28 to 30 in. | 6 | | |

The values given above for engine lathes are less than those used by the R. K. LeBlond Mach. Tool Co., which recommends (1907) the following size motors for use with its lathes.

| Swing of lathe. in. | Horse-power of Motor. | | Speed ratio. | Maximum speed range R.P.M. |
|------------------------|-----------------------|-------------|-----------------|-------------------------------|
| | Medium duty. | Heavy duty. | | |
| 12 and 14 | ... | 2 | 3 to 1 | 1500 |
| 16 | 2 | 3 | 3 to 1 | 1500 |
| 18, 20, 22 | 3 | 5 | 3 to 1 | 1500 |
| 24, 27, 30 | 5 | 7 1/2 | 3 to 1 | 1500 |
| 32, 36 | 7 1/2 | 10 | 3 to 1 | 1500 |
| 24* | 15 | 25 | 2 to 1 | 750-1500 |

* High Speed Roughing Lathe.

Horse-power Required to Drive Shafting. — Samuel Webber in his "Manual of Power" gives, among numerous tables of power required to drive textile machinery, a table of results of tests of shafting. A line of 2 1/8-in. shafting, 342 ft. long, weighing 4098 lb., with pulleys weighing 5331 lb., or a total of 9429 lb., supported on 47 bearings, 216 revolutions per minute, required 1.858 H.P. to drive it. This gives a coefficient of friction of 5.52%. In seventeen tests the coefficient ranged from 3.34% to 11.4%, averaging 5.73%.

Horse-power consumed in Machine-shops. — How much power is required to drive ordinary machine tools? and how many men can be employed per horse-power? are questions which it is impossible to answer by any fixed rule. The power varies greatly according to the conditions in each shop. The following table given by J. J. Flather in his work on Dynamometers gives an idea of the variation in several large works. The percentage of the total power required to drive the shafting varies from 15 to 80, and the number of men employed per total H.P. varies from 0.62 to 6.04.

Horse-power; Friction; Men Employed.

| Name of Firm. | Kind of Work. | Horse-power. | | | | Number of Men. | No. of Men per Total H.P. | No. of Men per Effective H.P. |
|---|---------------|--------------|-----------------------------|------------------------------|-----------------------------|----------------|---------------------------|-------------------------------|
| | | Total. | Required to Drive Shafting. | Required to Drive Machinery. | Per cent to Drive Shafting. | | | |
| Lane & Bodley . . . | E. & W. W. | 58 | | | | 132 | 2.27 | |
| J. A. Fay & Co. . . | W. W. | 100 | 15 | 85 | 15 | 300 | 3.00 | 3.53 |
| Union Iron Works . . | E., M. M. | 400 | 95 | 305 | 23 | 1600 | 4.00 | 5.24 |
| Frontier Iron & Brass Works | M. E., etc. | 25 | 8 | 17 | 32 | 150 | 6.00 | 8.82 |
| Taylor Mfg. Co. . . . | E. | 95 | | | | 230 | 2.42 | |
| Baldwin Loco. Works . | L. | 2500 | 2000 | 500 | 80 | 4100 | 1.64 | 8.20 |
| W. Sellers & Co. (one department) | H. M. | 102 | 41 | 61 | 40 | 300 | 2.93 | 4.87 |
| Pond Mach. Tool Co. . | M. T. | 180 | 75 | 105 | 41 | 432 | 2.40 | 4.11 |
| Pratt & Whitney Co. . | " | 120 | | | | 725 | 6.04 | |
| Brown & Sharpe Co. . | " | 230 | | | | 900 | 3.91 | |
| Yale & Towne Co. . . | C. & L. | 135 | 67 | 68 | 49 | 700 | 5.11 | 10.25 |
| Ferracute Mach. Co. . | P. & D. | 35 | 11 | 24 | 31 | 90 | 2.57 | 3.75 |
| T. B. Wood's Sons . . | P. & S. | 12 | | | | 30 | 2.50 | |
| Bridgeport Forge Co. . | H. F. | 150 | 75 | 75 | 50 | 130 | 0.86 | 1.73 |
| Singer Mfg. Co. . . . | S. M. | 1300 | | | | 3500 | 2.69 | |
| Howe Mfg. Co. | " | 350 | | | | 1500 | 4.28 | |
| Worcester Machine Screw Co. | M. S. | 40 | | | | 80 | 2.00 | |
| Hartford Mach. Screw Company | M. S. | 400 | 100 | 300 | 25 | 250 | 0.62 | 0.83 |
| Nicholson File Co. . . | F. | 350 | | | | 400 | 1.14 | |
| Averages | | 346.4 | | | 38.6 | 818.3 | 2.96 | 5.13 |

Abbreviations: E., engine; W. W., wood-working machinery; M. M., mining machinery; M. E., marine engines; L., locomotives; H. M., heavy machinery; M. T., machine tools; C. & L., cranes and locks; P. & D., presses and dies; P. & S., pulleys and shafting; H. F., heavy forgings; S. M., sewing-machines; M. S., machine-screws; F., files.

J. T. Henthorn states (Trans. A. S. M. E., vi. 462) that in print-mills which he examined the friction of the shafting and engine was in 7 cases

below 20% and in 35 cases between 20% and 30%, in 11 cases from 30% to 35% and in 2 cases above 35%, the average being 25.9%. Mr. Barrus in eight cotton-mills found the range to be between 18% and 25.7%, the average being 22%. Mr. Flather believes that for shops using heavy machinery the percentage of power required to drive the shafting will average from 40% to 50% of the total power expended. This presupposes that under the head of shafting are included elevators, fans and blowers.

Power Required to Drive Machines in Groups.—L. P. Alford (*Am. Mach.*, Oct. 31, 1907) gives the results of an investigation to determine the power required to drive machinery in groups. The method employed comprised disconnecting parts of the shafting in a belt-driven plant, and driving the disconnected portion with its machines by an electric motor, readings of the power required being taken every 5 minutes. The average power required for the entire factory was considerably less than the sum of the power required for the individual machines, due to tools being stopped at some portion of the day for adjustment, replacement of work, etc. The conditions of group driving are such that fixed rules cannot be laid down, but a study must be made of each individual case.

ABRASIVE PROCESSES.

Abrasive cutting is performed by means of stones, sand, emery, glass, corundum, carborundum, crocus, rouge, chilled globules of iron, and in some cases by soft, friable iron alone. (See paper by John Richards, read before the Technical Society of the Pacific Coast, *Am. Mach.*, Aug. 20, 1891, and *Eng. & M. Jour.*, July 25 and Aug. 15, 1891.)

The "Cold Saw."—For sawing any section of iron while cold the cold saw is sometimes used. This consists simply of a plain soft steel or iron disk without teeth, about 42 inches diameter and $\frac{3}{16}$ inch thick. The velocity of the circumference is about 15,000 feet per minute. One of these saws will saw through an ordinary steel rail cold in about one minute. In this saw the steel or iron is ground off by the friction of the disk, and is not cut as with the teeth of an ordinary saw. It has generally been found more profitable, however, to saw iron with disks or band-saws fitted with cutting-teeth, which run at moderate speeds and cut the metal as do the teeth of a milling-cutter.

Reese's Fusing-disk.—Reese's fusing-disk is an application of the cold saw to cutting iron or steel in the form of bars, tubes, cylinders, etc., in which the piece to be cut is made to revolve at a slower rate of speed than the saw. By this means only a small surface of the bar to be cut is presented at a time to the circumference of the saw. The saw is about the same size as the cold saw above described, and is rotated at a velocity of about 25,000 feet per minute. The heat generated by the friction of this saw against the small surface of the bar rotated against it is so great that the particles of iron or steel in the bar are actually fused, and the "sawdust" welds as it falls into a solid mass. This disk will cut either cast iron, wrought iron, or steel. It will cut a bar of steel $\frac{13}{8}$ inch diameter in one minute, including the time of setting it in the machine, the bar being rotated about 200 turns per minute.

Cutting Stone with Wire.—A plan of cutting stone by means of a wire cord has been tried in Europe. While retaining sand as the cutting agent, M. Paulin Gay, of Marseilles, has succeeded in applying it by mechanical means, and as continuously as formerly the sand-blast and band-saw, with both of which appliances his system—that of the "helicoidal wire cord"—has considerable analogy. An engine puts in motion a continuous wire cord (varying from five to seven thirty-seconds of an inch in diameter, according to the work), composed of three mild-steel wires twisted at a certain pitch, that is found to give the best results in practice, at a speed of from 15 to 17 feet per second.

The Sand-blast.—In the sand-blast, invented by B. F. Tilghman, of Philadelphia, and first exhibited at the American Institute Fair, New York, in 1871, common sand, powdered quartz, emery, or any sharp cutting material is blown by a jet of air or steam on glass, metal, or other comparatively brittle substance, by which means the latter is cut, drilled, or engraved. To protect those portions of the surface which it is desired shall not be abraded it is only necessary to cover them with a soft or tough material, such as lead, rubber, leather, paper, wax, or rubber-

paint. (See description in App. Cyc. Mech.; also U. S. report of Vienna Exhibition, 1873, vol. iii. 316.)

A "jet of sand" impelled by steam of moderate pressure, or even by the blast of an ordinary fan, depolishes glass in a few seconds; wood is cut quite rapidly; and metals are given the so-called "frosted" surface with great rapidity. With a jet issuing from under 300 pounds pressure, a hole was cut through a piece of corundum $1\frac{1}{2}$ inches thick in 25 minutes.

The sand-blast has been applied to the cleaning of metal castings and sheet metal, the graining of zinc plates for lithographic purposes, the frosting of silverware, the cutting of figures on stone and glass, and the cutting of devices on monuments or tombstones, the recutting of files, etc. The time required to sharpen a worn-out 14-inch bastard file is about four minutes. About one pint of sand, passed through a No. 120 sieve, and 4 H.P. of 60-lb. steam are required for the operation. For cleaning castings, compressed air at from 8 to 10 pounds pressure per square inch is employed. Chilled-iron globules instead of quartz or flint-sand are used with good results, both as to speed of working and cost of material, when the operation can be carried on under proper conditions. With the expenditure of 2 H.P. in compressing air, 2 square feet of ordinary scale on the surface of steel and iron plates can be removed per minute. The surface thus prepared is ready for tinning, galvanizing, plating, bronzing, painting, etc. By continuing the operation the hard skin on the surface of castings, which is so destructive to the cutting edges of milling and other tools, can be removed. Small castings are placed in a sort of slowly rotating barrel, open at one or both ends, through which the blast is directed downward against them as they tumble over and over. No portion of the surface escapes the action of the sand. Plain cored work, such as valve-bodies, can be cleaned perfectly both inside and out. One hundred lbs. of castings can be cleaned in from 10 to 15 minutes with a blast created by 2 H.P. The same weight of small forgings can be scaled in from 20 to 30 minutes. — *Iron Age*, March 8, 1894.

EMERY WHEELS AND GRINDSTONES.

References: "Precision Grinding," by Darbyshire; "Emery Wheels, their Selection and Use," published by Brown & Sharpe Mfg. Co.; "Points on Grinding," C. H. Norton; "Versuche ueber die Leistung von Schmirgel und Karborundum Scheiben bei Wassierzufuehrung," G. Schlesinger; "Die Festigkeit der kuenstlichen Schmirgel und Karborundum Scheiben, ihre Arbeitsleistung und ihre Wirthschaftlichkeit im Werkstattbetriebe," G. Schlesinger.

Selection of Grinding Wheels. (Contributed by Norton Co., 1908.)—The essential features of a modern grinding wheel which should be thoroughly understood by the user are: the definition of grain and grade, and the particular conditions of grinding which cause them to vary.

Grain.—Abrasive grains are numbered according to the meshes per lineal inch of the screen through which they have been graded. The numbers used in wheels are 8, 10, 12, 14, 16, 20, 24, 30, 36, 46, 54, 60, 70, 80, 90, 120, 150, 180, and 200; when finer than 200, the grits are termed flours, being designated as F, FF, FFF and SF; F being the coarsest and SF the finest. Grits from 12 to 30 are generally used on all heavy work, such as snagging; 36 to 80 cover nearly all tool-grinding, saw-gumming, and nearly all operations where precision in measurement is sought; 90 and finer are used for special work, such as grinding steel balls and fine edge work; over 200 is used mostly for oil and hand rubbing stones.

Grade.—When the retentive properties of the bond are great, the wheel is called hard; when the grains are easily broken out, it is called soft. A wheel is of the proper grade when its cutting grains are automatically replaced when dulled. Wheels that are too hard glaze. Dressing re-sharpens them, the points of the dresser breaking out and breaking off the cutting grains by percussion.

Soft wheels are used on hard materials, like hardened steel. Here the cutting particles are quickly dulled and must be renewed. On softer materials, like mild steel and wrought iron, harder grades can be used, the grains not dulling so quickly.

The area of surface to be ground in contact with the wheel is of the utmost importance in determining the grade. If it is a point contact like grinding a ball or if an extremely narrow fin is to be removed, we

must use a very strongly bonded wheel, on account of the leverage exerted on its grain, which tends to tear out the cutting particles before they have done their work. If the contact is a broad one, as in like grinding a hole, or where the work brings a large part of the surface of the wheel into operation, softer grades must be used, because the depth of cut is so infinitely small that the cutting points in work become dulled quickly and must be renewed, or the wheel glazes and loses its efficiency.

Vibrations in grinding machines cause percussion on the cutting grains, necessitating harder wheels. Wheels mounted on rigid machines can be softer in grade and are much more efficient.

Speeds of Grinding Wheels. — The factor of safety in vitrified wheels is proportional to the grade of hardness. Bursting limits are from 12,000 to 25,000 feet per minute, surface speed. Wheels are tested by standard makers at 10,000 feet, corresponding to a stress of 250 lbs. per square inch. Running speeds in practice are from 4000 to 6000 feet, depending on work, condition of machine, and mounting.

Generally speaking, grinding of tools, reamers, cutters, and surface grinding is done at about 4000 feet, snagging and rough forms of hand grinding at 5000 to 5500 feet, cylindrical grinding, or where the work is rigidly held and where the wheel feed is under control, from 5500 to 6500 feet, and in some instances as high as 7500 feet.

These speeds are all for vitrified wheels. The same speeds will apply to wheels made by the elastic and silicate processes.

Grades of Emery. — The numbers representing the grades of emery run from 8 to 120, and the degree of smoothness of surface they leave may be compared to that left by files as follows:

| | |
|--------------|-----------------------------------|
| 8 and 10 | represent the cut of a wood rasp. |
| 16 " 20 | " " " " a coarse-rough file. |
| 24 " 30 | " " " " an ordinary rough file. |
| 36 " 40 | " " " " a bastard file. |
| 46 " 60 | " " " " a second-cut file. |
| 70 " 80 | " " " " a smooth " " |
| 90 " 100 | " " " " a superfine " " |
| 120 F and FF | " " " " a dead-smooth file. |

Speed of Polishing-wheels.

Wood covered with leather, about 7000 ft. per minute.
 Wood covered with a hair brush, about 2500 revs. for largest.
 Wood covered 1 1/2" to 8" diam., hair 1" to 1 1/4" long.
 ab. 4500 revs. for smallest.
 Walrus-hide wheels, about 8000 ft. per minute.
 Rag-wheels, 4 to 8 in. diameter about 7000 ft. per minute.

Safe Speeds for Grindstones and Emery-wheels. — G. D. Hiscox (*Iron Age*, April 7, 1892), by an application of the formula for centrifugal force in fly-wheels (see Fly-wheels), obtains the figures for strains in grindstones and emery-wheels which are given in the tables below. His formulæ are:

Stress per sq. in. of section of a grindstone = $(0.7071D \times N)^2 \times 0.0000795$
 Stress per sq. in. of section of an emery-wheel = $(0.7071D \times N)^2 \times 0.00010226$

D = diameter in feet, N = revolutions per minute.

He takes the weight of sandstone at 0.078 lb. per cubic inch, and that of an emery-wheel at 0.1 lb. per cubic inch; Ohio stone weighs about 0.081 lb. and Huron stone about 0.089 lb. per cubic inch. The Ohio stone will bear a speed at the periphery of 2500 to 3000 ft. per min., which latter should never be exceeded. The Huron stone can be trusted up to 4000 ft., when properly clamped between flanges and not excessively wedged in setting. Apart from the speed of grindstones as a cause of bursting, probably the majority of accidents have really been caused by wedging them on the shaft and over-wedging to true them. The holes being square, the excessive driving of wedges to true the stones starts cracks in the corners that eventually run out until the centrifugal strain becomes greater than the tenacity of the remaining solid stone. Hence the necessity of great caution in the use of wedges, as well as the holding of large quick-running stones between large flanges and leather washers.

The *Iron Age* says the strength of grindstones when wet is reduced 40 to 50%. A section of a stone soaked all night in water broke at a stress

**Revolutions per Minute Required for Specified Rates of
Periphery Speed. Also Stress per Square Inch on
Norton Wheels at the Specified Rates.**

| Diameter, In. | Surface Speeds, Feet per Minute. | | | | | | | | | |
|-------------------------|----------------------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 7000 | 8000 | 9000 | 10000 |
| | Stress per Square Inch, Pounds. | | | | | | | | | |
| | 3 | 12 | 27 | 48 | 75 | 108 | 147 | 192 | 243 | 300 |
| Revolutions per Minute. | | | | | | | | | | |
| 1 | 3820 | 7639 | 11459 | 15279 | 19099 | 22918 | 26738 | 30558 | 34377 | 38197 |
| 2 | 1910 | 3820 | 5730 | 7639 | 9549 | 11459 | 13369 | 15279 | 17189 | 19098 |
| 3 | 1273 | 2546 | 3820 | 5093 | 6366 | 7639 | 8913 | 10186 | 11459 | 12732 |
| 4 | 955 | 1910 | 2865 | 3820 | 4775 | 5729 | 6684 | 7639 | 8594 | 9549 |
| 5 | 764 | 1528 | 2292 | 3056 | 3820 | 4584 | 5347 | 6111 | 6875 | 7639 |
| 6 | 637 | 1273 | 1910 | 2546 | 3183 | 3820 | 4456 | 5093 | 5729 | 6366 |
| 7 | 546 | 1091 | 1637 | 2183 | 2728 | 3274 | 3820 | 4365 | 4911 | 5457 |
| 8 | 477 | 955 | 1432 | 1910 | 2387 | 2865 | 3342 | 3820 | 4297 | 4775 |
| 10 | 382 | 764 | 1146 | 1528 | 1910 | 2292 | 2674 | 3056 | 3438 | 3820 |
| 12 | 318 | 637 | 955 | 1273 | 1591 | 1910 | 2228 | 2546 | 2865 | 3183 |
| 14 | 273 | 546 | 818 | 1091 | 1364 | 1637 | 1910 | 2183 | 2455 | 2728 |
| 16 | 239 | 477 | 716 | 955 | 1194 | 1432 | 1671 | 1910 | 2148 | 2387 |
| 18 | 212 | 424 | 637 | 849 | 1061 | 1273 | 1485 | 1698 | 1910 | 2122 |
| 20 | 191 | 382 | 573 | 764 | 955 | 1146 | 1337 | 1528 | 1719 | 1910 |
| 22 | 174 | 347 | 521 | 694 | 868 | 1042 | 1215 | 1389 | 1563 | 1736 |
| 24 | 159 | 318 | 477 | 637 | 796 | 955 | 1114 | 1273 | 1432 | 1591 |
| 30 | 127 | 255 | 382 | 509 | 637 | 764 | 891 | 1018 | 1146 | 1273 |
| 36 | 106 | 212 | 318 | 424 | 530 | 637 | 743 | 849 | 955 | 1061 |

Table to Figure Surface Speeds of Wheels.
(Circumferences in Feet, Diameters in Inches.)

| Diam. In. | Circumf. Ft. | Diam. In. | Circumf. Ft. | Diam. In. | Circumf. Ft. | Diam. In. | Circumf. Ft. | Diam. In. | Circumf. Ft. | Diam. In. | Circumf. Ft. |
|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|-----------|--------------|
| 1 | .262 | 13 | 3.403 | 25 | 6.546 | 37 | 9.687 | 49 | 12.828 | 61 | 15.970 |
| 2 | .524 | 14 | 3.665 | 26 | 6.807 | 38 | 9.948 | 50 | 13.090 | 62 | 16.232 |
| 3 | .785 | 15 | 3.927 | 27 | 7.069 | 39 | 10.210 | 51 | 13.352 | 63 | 16.493 |
| 4 | 1.047 | 16 | 4.189 | 28 | 7.330 | 40 | 10.472 | 52 | 13.613 | 64 | 16.755 |
| 5 | 1.309 | 17 | 4.451 | 29 | 7.592 | 41 | 10.734 | 53 | 13.875 | 65 | 17.017 |
| 6 | 1.571 | 18 | 4.712 | 30 | 7.854 | 42 | 10.996 | 54 | 14.137 | 66 | 17.279 |
| 7 | 1.833 | 19 | 4.974 | 31 | 8.116 | 43 | 11.257 | 55 | 14.499 | 67 | 17.541 |
| 8 | 2.094 | 20 | 5.236 | 32 | 8.377 | 44 | 11.519 | 56 | 14.661 | 68 | 17.802 |
| 9 | 2.356 | 21 | 5.498 | 33 | 8.639 | 45 | 11.781 | 57 | 14.923 | 69 | 18.064 |
| 10 | 2.618 | 22 | 5.760 | 34 | 8.901 | 46 | 12.043 | 58 | 15.184 | 70 | 18.326 |
| 11 | 2.880 | 23 | 6.021 | 35 | 9.163 | 47 | 12.305 | 59 | 15.446 | 71 | 18.588 |
| 12 | 3.142 | 24 | 6.283 | 36 | 9.425 | 48 | 12.566 | 60 | 15.708 | 72 | 18.850 |

To find surface speed, in feet, per minute, of a wheel.

RULE. — Multiply the circumference (see above table) by its revolutions per minute.

Surface speed and diam. of wheel being given, to find number of revolutions of wheel spindle.

RULE. — Multiply surface speed, in feet, per min., by 12 and divide the product by 3.14 times the diam. of the wheel in inches.

of 80 lb. per sq. in. A section of the same stone dry, broke at 146 lb. per sq. in. A better quality stone broke at stresses of 186 and 116 lb. per sq. in. when dry and wet respectively.

Selection of Emery Wheels. — The Norton Co. (1907) publishes the following table showing the proper grain and grade of wheel for different services. The column headed grain indicates the coarseness of the material composing the wheel, being designated by the number of meshes per inch of a sieve through which the grains pass. A No. 20 grain will pass through a 20-mesh sieve, but not through a 30-mesh, etc.

Table for Selection of Grades.

| Class of Work. | No. of Grain or Degree of Coarseness usually Furnished. | Grade Letters or Degrees of Hardness usually Furnished. | Grade Letters or Degrees of Hardness. Furnished in Exceptional Cases. | |
|---|---|---|---|-------------------|
| | | | Sometimes Soft as | Sometimes Hard as |
| Large cast iron and steel castings | 12 to 20 | Q to R | P | U |
| Small cast iron and steel castings | 20 " 30 | P " Q | O | R |
| Large malleable iron castings | 16 " 20 | Q " R | P | W |
| Small malleable iron castings | 20 " 30 | P " Q | O | U |
| Chilled iron castings | 16 " 20 | Q " R | P | U |
| Wrought iron | 16 " 30 | P " Q | O | R |
| Brass castings | 16 " 30 | O " P | N | Q |
| Bronze castings | 16 " 30 | P " Q | N | R |
| Rough work in general | 16 " 30 | P " Q | O | R |
| General machine-shop use | 30 " 46 | O " P | | |
| Lathe and planer tools | 30 " 46 | N " O | M | P |
| Small tools | 36 " 100 | N " P | | |
| Wood-working tools | 36 " 60 | M " N | L | |
| Twist drills (hand grinding) | 36 " 60 | M " N | | |
| Twist drills (special machines) | 46 " 60 | K " M | H | O |
| Reamers, taps, milling cutters, etc. (hand grinding) | 46 " 100 | N " P | | |
| Reamers, taps, milling cutters, etc. (special machines) | 46 " 60 | H " K | | |
| Edging and joining agricultural implements | 16 " 30 | Q " R | | W |
| Grinding plow points | 16 " 30 | P " Q | | U |
| Surfacing plow bodies | 16 " 30 | N " O | M | Q |
| Stove mounting | 20 " 36 | P " Q | | |
| Finishing edges of stoves | 30 " 46 | O " P | | |
| Drop forgings | 20 " 30 | P " Q | | |
| Gumming and sharpening saws | 36 " 60 | M " N | L | O |
| Planing-mill and paper-cutting knives | 30 " 46 | J " K | I | M |
| Car-wheel grinding | 20 " 30 | O " P | N | R |

EXPLANATION OF GRADE LETTERS.

| Extremely Soft | Soft. | Medium Soft. | Medium. | Medium Hard. | Hard. | Extremely Hard. |
|----------------|-------|--------------|---------|--------------|-------|-----------------|
| A | E | I | M | Q | U | Y |
| B | F | J | N | R | V | Z |
| C | G | K | O | S | W | |
| D | H | L | P | T | X | |

The intermediate letters between those designated as soft, medium soft, etc., indicate so many degrees harder or softer; e.g., L is one grade or degree softer than medium; O, 2 degrees harder than medium but not quite medium hard.

For Grinding High-speed Tool Steel, The American Emery Wheel Co. recommends a wheel one number coarser and one grade softer than a wheel for grinding carbon steel for the same service.

Special Wheels. — Rim wheels and iron-center wheels are specialties that require the maker's guarantee and assignment of speed.

Strains in Grindstones.

LIMIT OF VELOCITY AND APPROXIMATE ACTUAL STRAIN PER SQUARE INCH OF SECTIONAL AREA FOR GRINDSTONES OF MEDIUM TENSILE STRENGTH.

| Diameter. | Revolutions per Minute. | | | | | | |
|-----------|-------------------------|-------|-------|--|-------|-------|-------|
| | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
| feet. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. | lbs. |
| 2 | 1.58 | 3.57 | 6.35 | 9.93 | 14.30 | 18.36 | 25.42 |
| 2½ | 2.47 | 5.57 | 9.88 | 15.49 | 22.29 | 28.64 | 39.75 |
| 3 | 3.57 | 8.04 | 14.28 | 22.34 | 32.16 | | |
| 3½ | 4.86 | 10.93 | 19.44 | 30.38 | | | |
| 4 | 6.35 | 14.30 | 27.37 | | | | |
| 4½ | 8.04 | 18.08 | 32.16 | | | | |
| 5 | 9.93 | 22.34 | | Approximate breaking strain ten times the strain for size opposite the bottom figure in each column. | | | |
| 6 | 14.30 | 32.17 | | | | | |
| 7 | 19.44 | | | | | | |

The figures at the bottom of columns designate the limit of velocity (in revolutions per minute at the head of the columns) for stones of the diameter in the first column opposite the designating figure.

A general rule of safety for any size grindstone that has a compact and strong grain is to limit the peripheral velocity to 47 feet per second.

Joshua Rose (Modern Machine-shop Practice) says: The average circumferential speed of grindstones in workshops may be given as follows:

For grinding machinists' tools, about 900 feet per minute.
 " " carpenters' " " 600 " " " "

The speeds of stones for file-grinding and other similar rapid grinding is thus given in the "Grinders' List."

| | | | | | | | | | | | |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Diam ft..... | 8 | 7½ | 7 | 6½ | 6 | 5½ | 5 | 4½ | 4 | 3½ | 3 |
| Revs. per min. | 135 | 144 | 154 | 166 | 180 | 196 | 216 | 240 | 270 | 308 | 360 |

The following table, from the *Mechanical World*, is for the diameter of stones and the number of revolutions they should run per minute (not to be exceeded), with the diameter of change of shift-pulleys required, varying each shift or change 2½ inches, 2¼ inches, or 2 inches in diameter for each reduction of 6 inches in the diameter of the stone.

| Diameter of Stone. | | Revolutions per Minute. | Shift of Pulleys, in inches. | | |
|--------------------|-----|-------------------------|------------------------------|-----|----|
| | | | 2½ | 2¼ | 2 |
| ft. | in. | | | | |
| 8 | 0 | 135 | 40 | 36 | 32 |
| 7 | 6 | 144 | 37½ | 33¾ | 30 |
| 7 | 0 | 154 | 35 | 31½ | 28 |
| 6 | 6 | 166 | 32½ | 29¼ | 26 |
| 6 | 0 | 180 | 30 | 27 | 24 |
| 5 | 6 | 196 | 27½ | 24¾ | 22 |
| 5 | 0 | 216 | 25 | 22½ | 20 |
| 4 | 6 | 240 | 22½ | 20¼ | 18 |
| 4 | 0 | 270 | 20 | 18 | 16 |
| 3 | 6 | 308 | 17½ | 15¾ | 14 |
| 3 | 0 | 360 | 15 | 13½ | 12 |
| 1 | | 2 | 3 | 4 | 5 |

Columns 3, 4, and 5 are given to show that if we start an 8-foot stone with, say, a countershaft pulley driving a 40-inch pulley on the grindstone spindle, and the stone makes the right number (135) of revolutions per minute, the reduction in the diameter of the pulley on the grinding-stone spindle, when the stone has been reduced 6 inches in diameter, will require to be also reduced $2\frac{1}{2}$ inches in diameter, or to shift from 40 inches to $37\frac{1}{2}$ inches, and so on similarly for columns 4 and 5. Any other suitable dimensions of pulley may be used for the stone when eight feet in diameter, but the number of inches in each shift named, in order to be correct, will have to be proportional to the numbers of revolutions the stone should run, as given in column 2 of the table.

Varieties of Grindstones.

(Joshua Rose.)

FOR GRINDING MACHINISTS' TOOLS.

| Name of Stone. | Kind of Grit. | Texture of Stone. | Color of Stone. |
|--------------------------------|------------------------------------|------------------------------------|-------------------------|
| Nova Scotia, { | All kinds, from finest to coarsest | All kinds, from hardest to softest | Blue or yellowish gray. |
| Bay Chaleur (New Brunswick), { | Medium to finest | Soft and sharp | Uniformly light blue |
| Liverpool or Melling | Medium to fine | Soft, with sharp grit | Reddish |

FOR WOODWORKING TOOLS.

| | | | |
|--------------------------------|------------------|-----------------------|--------------------|
| Wickersley. . . . | Medium to fine | Very soft | Grayish yellow |
| Liverpool or Melling | Medium to fine { | Soft, with sharp grit | Reddish |
| Bay Chaleur (New Brunswick), { | Medium to finest | Soft and sharp | Uniform light blue |
| Huron, Michigan . . | Fine | Soft and sharp | Uniform light blue |

FOR GRINDING BROAD SURFACES, AS SAWS OR IRON PLATES.

| | | | |
|--------------------|-----------------|----------------|-----------------|
| Newcastle | Coarse to med'm | The hard ones | Yellow |
| Independence . . . | Coarse | Hard to medium | Grayish white |
| Massillon. | Coarse | Hard to medium | Yellowish white |

SCREWS, SCREW-THREADS, ETC.

Efficiency of a Screw. — Let a = angle of the thread, that is, the angle whose tangent is the pitch of the screw divided by the circumference of a circle whose diameter is the mean of the diameters at the top and bottom of the thread. Then for a square thread

$$\text{Efficiency} = \frac{1 - f \tan a}{1 + f \cotan a},$$

in which f is the coefficient of friction. (For demonstration, see Cotterill and Slade, Applied Mechanics.) Since $\cotan a = 1 \div \tan a$, we may substitute for $\cotan a$ the reciprocal of the tangent, or if p = pitch, and c = mean circumference of the screw,

$$\text{Efficiency} = \frac{1 - fp/c}{1 + fc/p}.$$

TAP DRILLS.

(The Morse Twist Drill and Machine Co.)

SCREWS, SCREW-THREADS, ETC.

1269

| Diam. of Tap. | No. Threads to Inch. | Drill for V Thread. | Drill for U. S. S. Thread. | Diam. of Tap. | No. Threads to Inch. | Drill for V Thread. | Drill for U. S. S. Thread. |
|---------------|----------------------|---------------------|----------------------------|---------------|----------------------|---------------------|----------------------------|
| 1/4 | 16 | 5/32 | 11/64 | 15 | 3/16 | 11/8 | 7 |
| 9/32 | 18 | 3/16 | 13/64 | 3 | ... | 15/32 | 8 |
| 5/16 | 20 | 7/32 | 15/64 | ... | ... | 13/16 | 8 |
| 11/32 | 16 | 1/4 | 17/64 | ... | C | 17/32 | 7 |
| 3/8 | 18 | 17/64 | 9/32 | M | ... | 11/4 | 8 |
| 13/32 | 14 | 13/32 | 5/16 | P | N | 19/32 | 7 |
| 7/16 | 16 | Q | 11/32 | ... | S | 15/16 | 7 |
| 15/32 | 14 | 23/64 | 3/8 | ... | ... | 11/32 | 7 |
| 1/2 | 12 | 3/8 | W | 25/64 | ... | 13/8 | 6 |
| 17/32 | 14 | 13/32 | 27/64 | 7/16 | 13/32 | 17/16 | 6 |
| 9/16 | 12 | 7/16 | 29/64 | ... | ... | 15/32 | 6 |
| 19/32 | 12 | 31/64 | 1/2 | ... | ... | 17/32 | 6 |
| 5/8 | 10 | 31/64 | 1/2 | 33/64 | ... | 11/2 | 6 |
| 21/32 | 10 | 33/64 | 17/32 | 35/64 | ... | 17/32 | 6 |
| 11/16 | 11 | 9/16 | 37/64 | ... | ... | 19/16 | 6 |
| 23/32 | 11 | 19/32 | 39/64 | ... | ... | 19/32 | 6 |
| 3/4 | 10 | 39/64 | 41/64 | ... | ... | 15/8 | 5 |
| 25/32 | 10 | 41/64 | 21/32 | 43/64 | 5/8 | 21/32 | 5 |
| 13/16 | 10 | 43/64 | ... | ... | ... | 11/16 | 5 |
| 27/32 | 10 | 45/64 | ... | ... | ... | 23/32 | 5 |
| 7/8 | 9 | 23/32 | 47/64 | ... | ... | 13/4 | 5 |
| 29/32 | 9 | 3/4 | 49/64 | ... | 47/64 | 25/32 | 5 |
| 15/16 | 9 | 25/32 | ... | ... | ... | 13/16 | 5 |
| 31/32 | 8 | 13/16 | ... | ... | ... | 27/32 | 5 |
| 1 | 8 | 53/64 | ... | ... | 27/32 | 17/8 | 4 1/2 |
| 11/32 | 8 | 55/64 | ... | ... | ... | 129/32 | 5 |
| 11/16 | 8 | 57/64 | ... | ... | ... | 15/16 | 5 |
| 13/32 | 8 | 59/64 | ... | ... | ... | 131/32 | 5 |
| | | | | | ... | 2 | 4 1/2 |
| | | | | | ... | ... | ... |
| | | | | | ... | ... | 123/32 |

The Morse Twist Drill and Machine Co. gives the above table showing the different sizes of drills that should be used when a suitable thread is to be tapped in a hole. The sizes given are practically correct. For tap-drill diameters for standard A. S. M. E. screws, see page 227.

EXAMPLE. — Efficiency of square-threaded screws of $\frac{1}{2}$ inch pitch.

| | | | | |
|---|----------------|----------------|----------------|----------------|
| Diameter at bottom of thread, in. | 1 | 2 | 3 | 4 |
| Diameter at top of thread, in. | $1\frac{1}{2}$ | $2\frac{1}{2}$ | $3\frac{1}{2}$ | $4\frac{1}{2}$ |
| Mean circumference of thread, in. | 3.927 | 7.069 | 10.21 | 13.35 |
| Cotangent $a = c \div p$ | =7.854 | 14.14 | 20.42 | 26.70 |
| Tangent $a = p \div c$ | =0.1273 | .0707 | .0490 | .0375 |
| Efficiency if $f = 0.10$ | =55.3% | 41.2% | 32.7% | 27.2% |
| Efficiency if $f = 0.15$ | =45% | 31.7% | 24.4% | 19.9% |

The efficiency thus increases with the steepness of the pitch.

The above formulæ and examples are for square-threaded screws, and consider the friction of the screw-thread only, and not the friction of the collar or step by which end thrust is resisted, and which further reduces the efficiency. The efficiency is also further reduced by giving an inclination to the side of the thread, as in the V-threaded screw. For discussion of this subject, see paper by Wilfred Lewis, Jour. Frank. Inst. 1880; also *Trans. A. S. M. E.*, vol. xii, 784.

Efficiency of Screw-bolts. — Mr. Lewis gives the following approximate formula for ordinary screw-bolts (V-threads, with collars): p = pitch of screw, d = outside diameter of screw, F = force applied at circumference to lift a unit of weight, E = efficiency of screw. For an average case, in which the coefficient of friction may be assumed at .15,

$$F = \frac{p + d}{3d}, \quad E = \frac{p}{p + d}.$$

For bolts of the dimensions given above, $\frac{1}{2}$ -inch pitch, and outside diameters $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, and $4\frac{1}{2}$ inches, the efficiencies according to this formula would be, respectively, 0.25, 0.167, 0.125, and 0.10.

James McBride (*Trans. A. S. M. E.*, xii, 781) describes an experiment with an ordinary 2-inch screw-bolt, with a V-thread, $4\frac{1}{2}$ threads per inch, raising a weight of 7500 pounds, the force being applied by turning the nut. Of the power applied 89.8 per cent was absorbed by friction of the nut on its supporting washer and of the threads of the bolt in the nut. The nut was not faced, and had the flat side to the washer.

Professor Ball in his "Experimental Mechanics" says: "Experiments showed in two cases respectively about $\frac{2}{3}$ and $\frac{3}{4}$ of the power was lost."

Trautwine says: "In practice the friction of the screw (which under heavy loads becomes very great) make the theoretical calculations of but little value."

Weisbach says: "The efficiency is from 19 per cent to 30 per cent."

Efficiency of a Differential Screw. — A correspondent of the *American Machinist* describes an experiment with a differential screw-punch, consisting of an outer screw 2 inch diameter, 3 threads per inch, and an inner screw $1\frac{3}{8}$ inch diameter, $3\frac{1}{2}$ threads per inch. The pitch of the outer screw being $\frac{1}{3}$ inch and that of the inner screw $\frac{2}{7}$ inch the punch would advance in one revolution $\frac{1}{3} - \frac{2}{7} = \frac{1}{21}$ inch. Experiments were made to determine the force required to punch an $\frac{11}{16}$ -inch hole in iron $\frac{1}{4}$ inch thick, the force being applied at the end of a lever-arm of $47\frac{3}{4}$ inch. The leverage would be $47\frac{3}{4} \times 2\pi \times 21 = 6300$. The mean force applied at the end of the lever was 95 pounds, and the force at the punch, if there was no friction, would be $6300 \times 95 = 598,500$ pounds. The force required to punch the iron, assuming a shearing resistance of 50,000 pounds per square inch, would be $50,000 \times \frac{11}{16} \times \pi \times \frac{1}{4} = 27,000$ pounds, and the efficiency of the punch would be $27,000 \div 598,500 =$ only 4.5 per cent. With the larger screw only used as a punch the mean force at the end of the lever was only 82 pounds. The leverage in this case was $47\frac{3}{4} \times 2\pi \times 3 = 900$, the total force referred to the punch, including friction, $900 \times 82 = 73,800$, and the efficiency $27,000 \div 73,800 = 36.7$ per cent. The screws were of tool-steel, well fitted, and lubricated with lard-oil and plumbago.

TAPER BOLTS, PINS, REAMERS, ETC.

Taper Bolts for Locomotives. — Bolt-threads, U. S. Standard, except stay-bolts and boiler-studs, V-threads, 12 per inch; valves, cocks, and plugs, V-threads, 14 per inch, and $\frac{1}{8}$ -inch taper per 1 inch. Standard bolt taper $\frac{1}{16}$ inch per foot.

Taper Reamers. — The Pratt & Whitney Co. makes standard taper reamers for locomotive work taper $\frac{1}{16}$ inch per foot from $\frac{1}{4}$ inch diameter;

4 inch length of flute to 2 inch diameter; 18 inch length of flute, diameters advancing by 16ths and 32ds. P. & W. Co.'s standard taper pin reamers taper $\frac{1}{4}$ inch per foot, are made in 15 sizes of diameters, 0.135 to 1.250 inches; length of flute $1\frac{7}{16}$ inches to 14 inches.

Morse Tapers.

| Number of Taper. | Diam of Plug at Small End. | Diam. at End of Socket. | Standard Plug Depth. | Whole Length of Shank. | Depth of Hole. | End of Socket to Key-way. | Length of Key-way. | Width of Key-way. | Length of Tongue. | Diameter of Tongue. | Thickness of Tongue. | Rad. of Mill for Tongue. | Radius of Tongue. | Shank Depth. | Taper per Foot. | Number of Key. |
|------------------|----------------------------|-------------------------|----------------------|------------------------|-----------------|---------------------------|--------------------|-------------------|-------------------|---------------------|----------------------|--------------------------|-------------------|-----------------|-----------------|----------------|
| | D | A | P | B | H | K | L | W | T | d | t | R | a | S | | |
| 0 | .252 | .356 | 2 | $2\frac{11}{32}$ | $2\frac{1}{32}$ | $1\frac{15}{16}$ | $\frac{9}{16}$ | .160 | $\frac{1}{4}$ | .24 | $\frac{5}{32}$ | $\frac{5}{32}$ | .04 | $2\frac{7}{32}$ | .625 | 0 |
| 1 | .369 | .475 | $2\frac{1}{8}$ | $\frac{29}{16}$ | $2\frac{3}{16}$ | $2\frac{1}{16}$ | $\frac{3}{4}$ | .213 | $\frac{1}{5}$ | .35 | $\frac{13}{64}$ | $\frac{3}{16}$ | .05 | $2\frac{3}{8}$ | .600 | 1 |
| 2 | .572 | .700 | $2\frac{9}{16}$ | $\frac{31}{16}$ | $2\frac{5}{8}$ | $2\frac{1}{2}$ | $\frac{7}{8}$ | .26 | $\frac{3}{8}$ | $\frac{17}{32}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | .06 | $2\frac{7}{8}$ | .602 | 2 |
| 3 | .778 | .938 | $3\frac{3}{16}$ | $3\frac{3}{4}$ | $3\frac{1}{4}$ | $3\frac{1}{16}$ | $1\frac{11}{16}$ | .322 | $\frac{7}{16}$ | $\frac{3}{4}$ | $\frac{5}{16}$ | $\frac{9}{32}$ | .08 | $3\frac{9}{16}$ | .602 | 3 |
| 4 | 1.020 | 1.231 | $4\frac{1}{16}$ | $4\frac{3}{4}$ | $4\frac{1}{8}$ | $3\frac{7}{8}$ | $1\frac{1}{4}$ | .478 | $\frac{1}{2}$ | $\frac{31}{32}$ | $\frac{15}{32}$ | $\frac{5}{16}$ | .10 | $4\frac{1}{2}$ | .623 | 4 |
| 5 | 1.475 | 1.748 | $5\frac{3}{16}$ | 6 | $5\frac{1}{4}$ | $4\frac{15}{16}$ | $1\frac{1}{2}$ | .635 | $\frac{5}{8}$ | $\frac{113}{32}$ | $\frac{5}{8}$ | $\frac{3}{8}$ | .12 | $5\frac{3}{4}$ | .630 | 5 |
| 6 | 2.116 | 2.494 | $7\frac{1}{4}$ | $8\frac{5}{16}$ | $7\frac{3}{8}$ | 7 | $\frac{13}{4}$ | .76 | $\frac{7}{8}$ | 2 | $\frac{3}{4}$ | $\frac{1}{2}$ | .15 | 8 | .626 | 6 |
| 7 | 2.75 | 3.27 | 10 | $11\frac{5}{8}$ | $10\frac{1}{8}$ | $9\frac{1}{2}$ | $2\frac{5}{8}$ | 1.135 | $\frac{13}{8}$ | $2\frac{11}{16}$ | $\frac{11}{8}$ | $\frac{3}{4}$ | .18 | $11\frac{1}{4}$ | .625 | 7 |

Brown & Sharpe Mfg. Co. publishes (*Machy's Data Sheets*) a list of 18 sizes of tapers ranging from 0.20 in. to 3 in. diam. at the small end; taper 0.5 in. to 1 ft., except No. 10, which is 0.5161 in. per ft.

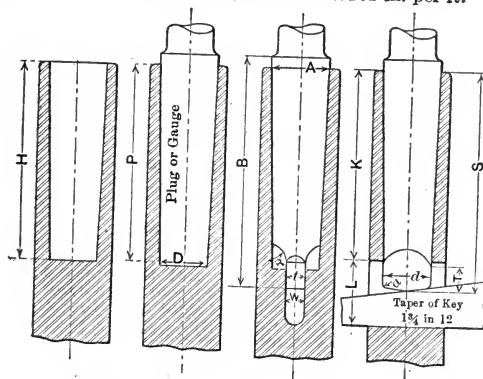


FIG. 192. — Morse Tapers. See table above.

The **Jarno Taper** is 0.05 inch per inch = 0.6 inch per foot. The number of the taper is its diameter in tenths of an inch at the small end, in eighths of an inch at the large end, and the length in halves of an inch.

Thus, No. 3 Jarno taper is $1\frac{1}{2}$ inches long, 0.3 inch diameter at the small end and $\frac{3}{8}$ inch diameter at the large end.

Standard Steel Taper-pins. — The following sizes are made by The Pratt & Whitney Co.: Taper $\frac{1}{4}$ inch to the foot.

Number:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Diameter large end: | | | | | | | | | | |
| 0.156 | 0.172 | 0.193 | 0.219 | 0.250 | 0.289 | 0.341 | 0.409 | 0.492 | 0.591 | 0.706 |

Approximate fractional sizes:

| | | | | | | | | | | |
|----------------|-----------------|----------------|----------------|---------------|-----------------|-----------------|-----------------|----------------|-----------------|-----------------|
| $\frac{5}{32}$ | $\frac{11}{64}$ | $\frac{3}{16}$ | $\frac{7}{32}$ | $\frac{1}{4}$ | $\frac{19}{64}$ | $\frac{11}{32}$ | $\frac{13}{32}$ | $\frac{1}{2}$ | $\frac{19}{32}$ | $\frac{23}{32}$ |
| Lengths from | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{1}{2}$ |

To*

| | | | | | | | | | | |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/4 | 3 1/4 | 3 3/4 | 4 1/2 | 5 1/4 | 6 |
| Diameter small end of standard taper-pin reamer:† | | | | | | | | | | |
| 0.135 | 0.146 | 0.162 | 0.183 | 0.208 | 0.240 | 0.279 | 0.331 | 0.398 | 0.482 | 0.581 |

Standard Steel Mandrels. (The Pratt & Whitney Co.) — These mandrels are made of tool-steel, hardened, and ground true on their centers. Centers are also ground to true 60 degree cones. The ends are of a form best adapted to resist injury likely to be caused by driving. They are slightly taper. Sizes, $\frac{1}{4}$ inch diameter by $3\frac{3}{4}$ inches long to 4 inches diameter by 17 inches long, diameters advancing by 16ths.

PUNCHES AND DIES, PRESSES, ETC.

Clearance between Punch and Die. — For computing the amount of clearance that a die should have, or, in other words, the difference in size between die and punch, the general rule is to make the diameter of die-hole equal to the diameter of the punch, plus $\frac{2}{10}$ the thickness of the plate. Or, $D = d + 0.2t$, in which D = diameter of die-hole, d = diameter of punch, and t = thickness of plate. For very thick plates some mechanics prefer to make the die-hole a little smaller than called for by the above rule. For ordinary boiler-work the die is made from $\frac{1}{10}$ to $\frac{3}{10}$ of the thickness of the plate larger than the diameter of the punch; and some boiler-makers advocate making the punch fit the die accurately. For punching nuts, the punch fits in the die. (*Am. Mach.*)

Kennedy's Spiral Punch. (The Pratt & Whitney Co.) — B. Martell, Chief Surveyor of Lloyd's Register, reported tests of Kennedy's spiral punches in which a $\frac{7}{8}$ -inch spiral punch penetrated a $\frac{5}{8}$ -inch plate at a pressure of 22 to 25 tons, while a flat punch required 33 to 35 tons. Steel boiler-plates punched with a flat punch gave an average tensile strength of 58,579 pounds per square inch, and an elongation in two inches across the hole of 5.2 per cent, while plates punched with a spiral punch gave 63,929 pounds, and 10.6 per cent elongation.

The spiral shear form is not recommended for punches for use in metal of a thickness greater than the diameter of the punch. This form is of greatest benefit when the thickness of metal worked is less than two thirds the diameter of punch.

Size of Blanks used in the Drawing-press. — Oberlin Smith (*Jour. Frank. Inst.*, Nov. 1886) gives three methods of finding the size of blanks. The first is a tentative method, and consists simply in a series of experiments with various blanks, until the proper one is found. This is for use mainly in complicated cases, and when the cutting portions of the die and punch can be finally sized after the other work is done. The second method is by weighing the sample piece, and then, knowing the weight of the sheet metal per square inch, computing the diameter of a piece having the required area to equal the sample in weight. The third method is by computation, and the formula is $x = \sqrt{d^2 + 4dh}$ for a sharp-cornered cup, where x = diameter of blank, d = diameter of cup, h = height of cup. For a round-cornered cup

* Lengths vary by $\frac{1}{4}$ inch each size.

† Taken $\frac{1}{2}$ inch from extreme end. Each size overlaps smaller one about $\frac{1}{2}$ inch.

where the corner is small, say radius of corner less than $\frac{1}{4}$ height of cup, the formula is $x = (\sqrt{d^2 + 4 dh}) - r$, about; r being the radius of the corner. This is based upon the assumption that the thickness of the metal is not to be altered by the drawing operation.

Pressure attainable by the Use of the Drop-press. (R. H. Thurston, *Trans. A. S. M. E.*, v, 53.)—A set of copper cylinders was prepared, of pure Lake Superior copper; they were subjected to the action of presses of different weights and of different heights of fall. Companion specimens of copper were compressed to exactly the same amount, and measures were obtained of the loads producing compression, and of the amount of work done in producing the compression by the drop. Comparing one with the other it was found that the work done with the hammer was 90 per cent of the work which should have been done with perfect efficiency. That is to say, the work done in the testing-machine was equal to 90 per cent of that due the weight of the drop falling the given distance.

$$\text{Formula: Mean pressure in pounds} = \frac{\text{Weight of drop} \times \text{fall} \times \text{efficiency}}{\text{compression}}$$

For pressures per square inch, divide by the mean area opposed to crushing action during the operation.

Similar experiments on Bessemer steel plugs by A. W. Moseley and J. L. Bacon (*Trans. A. S. M. E.*, xxvii, 605) indicated an efficiency for the drop hammer of about 70 per cent.

Flow of Metals. (David Townsend, *Jour. Frank. Inst.*, March, 1878.)—In punching holes $\frac{7}{16}$ -inch diameter through iron blocks $\frac{13}{4}$ inches thick, it was found that the core punched out was only $\frac{11}{16}$ inches thick, and its volume was only about 32 per cent of the volume of the hole. Therefore, 68 per cent of the metal displaced by punching the hole flowed into the block itself, increasing its dimensions.

FORCING, SHRINKING AND RUNNING FITS.

Forcing Fits of Pins and Axles by Hydraulic Pressure.—A 4-inch axle is turned 0.015 inch diameter larger than the hole into which it is to be fitted. They are pressed on by a pressure of 30 to 35 tons. (Lecture by Coleman Sellers, 1872.)

For forcing the crank-pin into a locomotive driving-wheel, when the pinhole is perfectly true and smooth, the pin should be pressed in with a pressure of 6 tons for every inch of diameter of the wheel fit. When the hole is not perfectly true, which may be the result of shrinking the tire on the wheel center after the hole for the crank-pin has been bored, or if the hole is not perfectly smooth, the pressure may have to be increased to 9 tons for every inch of diameter of the wheel-fit. (*Am. Machinist*.)

Shrinkage Fits.—In 1886 the American Railway Master Mechanics' Association recommended the following shrinkage allowances for tires of standard locomotives. The tires are uniformly heated by gas-flames, slipped over the cast-iron centers, and allowed to cool. The centers are turned to the standard sizes given below, and the tires are bored smaller by the amount of the shrinkage designated for each:

| | | | | | | |
|----------------------------------|------|------|------|------|------|------|
| Diameter of center, in. | 38 | 44 | 50 | 56 | 62 | 66 |
| Shrinkage allowance, in. | .040 | .047 | .053 | .060 | .066 | .070 |

This shrinkage allowance is approximately $\frac{1}{80}$ inch per foot, or $\frac{1}{960}$. A common allowance is $\frac{1}{1000}$. Taking the modulus of elasticity of steel at 30,000,000, the strain caused by shrinkage would be 30,000 lb. per sq. in., less an uncertain amount due to compression of the center.

Amer. Machinist published at a later date a table of "M. M. allowances for shrink fits" which correspond to the following: Allowance = 0.001 ($d + 1$) for $d = 20$ to 40 in.; 0.001 ($d + 2$) for $d = 41$ to 60 in.; 0.001 ($d + 3$) for $d = 61$ to 83 in.; 0.088 for $d = 84$ in. d = diam. of wheel center. For running force fits, *Am. Mach.* gives the following allowances: d = diam. of bearing or hole, a = allowance.

| $d =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------------|--------|------|------|-------|-------|------|-------|-------|-------|-------|
| Running, $a =$ | -0.001 | .002 | .003 | .0035 | .0037 | .004 | .0042 | .0042 | .0043 | .0044 |
| Force, $a =$ | +0.001 | .003 | .005 | .006 | .007 | .008 | .0085 | .009 | .01 | .0105 |

| $d =$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|------------------------------|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Running, $a = \dots\dots$ | -0.0045 | .0046 | .0047 | .0048 | .0049 | .005 | .0051 | .0052 | .0053 | .0055 |
| Force, $a = \dots\dots\dots$ | +0.011 | .0115 | .012 | .013 | .014 | .0145 | .015 | .0155 | .016 | .017 |

Allowances for drive fits are one-half these for force fits.

Limits of Diameters for Fits. C. W. Hunt Co. (*Am. Mach.*, July 16, 1903.) — For parallel shafts and bushings (shafts changing): $d =$ diam. in ins.

Shafts: Press fit, $+0.001 d + (0 \text{ to } 0.001 \text{ in.})$. Drive fit, $+0.0005 d + (0. \text{ to } 0.001 \text{ in.})$.

Shafts: Hand fit, $+0.001$ to 0.002 in. for shafts 1 to 3 in.; 0.002 to 0.003 in. for 4 to 6 in.; 0.003 to 0.004 in. for 7 to 10 in.

Holes: all fits 0 to -0.002 in. for 1 to 3 in.; 0 to -0.003 in. for 4 to 6 in.; 0 to -0.004 in. for 7 to 10 in.

Parallel journals and bearings (journals changing):

Close fit $-0.001 d + (0.002 \text{ to } 0.004 \text{ in.})$; Free fit $-0.001 d + (0.007 \text{ to } 0.01 \text{ in.})$; Loose fit, $-0.003 d + (0.02 \text{ to } 0.025)$. Limits of diameters for taper shaft and bushings (holes changing). Shaft turned to standard taper $3/16$ in. per ft., large end to nominal size ± 0.001 in. Holes are reamed until the large end is small by from $0.001 d + 0.004$ to 0.005 in. for press fit, from $0.0005 d + 0.001$ in. for drive fit, and from 0 to 0.001 in. for hand fit. In press fits the shaft is pressed into the hole until the true sizes match, or $1/16$ in. for each $1/1000$ in. that the hole is small. The above formulæ apply to steel shafts and cast-iron wheels or other members.

Shaft Allowances for Electrical Machinery. — In use by General Electric Co. (John Riddell, *Trans. A. S. M. E.*, xxiv, 1174).

| Diam. ins. | 2 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
|---------------|--------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| A, B | 0.0005 | .00075 | .001 | .001 | .0012 | .0012 | .0015 | .0015 | .0017 | .0017 | .002 | .002 | .0023 |
| C | 0.0005 | .00075 | .0015 | .0017 | .0020 | .0023 | .0025 | .0028 | .003 | .0033 | .0035 | .0038 | .004 |
| D | 0.0005 | .00075 | .0017 | .0025 | .0033 | .004 | .0045 | .005 | .0058 | .0063 | .0068 | .0073 | .008 |
| E | 0.0015 | .0027 | .0045 | .0057 | .007 | .008 | .0093 | .0115 | .0125 | .0128 | .0138 | .015 | .016 |

A, minus allowance for sliding fit. B, plus allowance for commutators and split hubs. C, press fit for armature spiders, solid steel. D, do., solid cast iron. E, press fit for couplings, and shrink fit.

Running Fits. — Wm. Sangster (*Am. Mach.*, July 8, 1909) gives the practice of different manufacturers as follows:

An electric manufacturing Co. allows a clearance of 0.003 to 0.004 in. for shafts $1\frac{1}{2}$ to $2\frac{1}{4}$ in. diam.; 0.003 to 0.006 for $2\frac{1}{2}$ ins.; 0.004 to 0.006 for $2\frac{3}{4}$ to $3\frac{1}{2}$ ins.; 0.005 to 0.007 in. for 4 and $4\frac{1}{2}$ ins.; 0.006 to 0.008 in. for 5 ins.; 0.009 to 0.011 in. for 6 ins. Dodge Mfg. Co. allows from $1/64$ for 1-in. ordinary bearings to a little over $1/32$ in. for 6-in. Clutch sleeves, 0.008 to 0.015 in.; loose pulleys as close as 0.003 in. in the smaller sizes, and about $1/64$ in. on a $2\frac{1}{2}$ -in. hole.

Watt Mining Car Wheel Co. allows $1/16$ in. for all sizes of wheels, and $1/16$ in. end play. A large fan-blower concern allows 0.005 to 0.01 in. on fan journals from $9/16$ to $27/16$ ins.

Pressure Required for Press Fits. (*Am. Mach.*, March 7, 1907.) — The following approximate formulæ give the pressures required for press fits of cranks and crank-pins, as used by an engine-building firm. P = total pressure on ram, tons; D = diameter inches.

Crank fits up to $D = 10$.

Crank fits $D = 12$ to 24 .

Straight crank-pins.

Taper crank-pins.

$P = 9.9 D - 14$.

$P = 5 D + 40$.

$P = 13 D$.

$P = 14 D - 7$.

The allowance for cranks and straight pins is 0.0025 inch per inch of diameter. Taper cranks, taper $1/16$ inch per inch, are fitted on the lathe to within $1/8$ inch of shoulder and then forced home.

Stresses due to Force and Shrink Fits. — S. H. Moore, *Trans. A. S. M. E.*, vol. xxiv, gives the following allowances for different fits.

For shrinkage fits, $d = (17/16 D + 0.5) \div 1000$. For forced fits, $d = (2 D + 0.5) \div 1000$. For driven fits, $d = (1/2 D + 0.5) \div 1000$. d = allowance or the amount the diameter of the shaft exceeds the diameter of the hole in the ring and D = nominal diameter of the shaft. A. L. Jenkins, *Eng. News*, Mar. 17, 1910, says the values obtained from the formula for forced fits are about twice as large as those frequently used in practice, and in many cases they lead to excessive stresses in the ring. He calculates from Lamé's formula for hoop stress in a ring subjected to internal pressure the relation between the stress and the allowance for fit, and deduces the following formulæ.

$S_{h_1} = 15,000,000 d \div (k + 0.6)$; $S_{h_2} = 15,000,000 d \div (1 + 0.6/k)$; for a cast-iron ring on a steel shaft.

$S_{h_1} = 30,000,000 d \div (1 + k)$; $S_{h_2} = 30,000,000 d \div (1 + 1/K)$; for a steel ring on a steel shaft.

S_{h_1} = radial unit pressure between the surfaces; S_{h_2} = unit tensile or hoop stress in the ring;

d = allowance per inch of diameter, K a constant whose value depends on t , the thickness, and r , the radius of the ring, as follows.

Values of $t \div r$,

0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.25 1.5 1.75 2.0 3.0

Values of K ,

3.083 2.600 2.282 2.058 1.892 1.766 1.666 1.492 1.380 1.300 1.250 1.133.

The allowances for forced and shrinkage fits should be based on the stresses they produce, as determined by the above formula, and not on the diameter of the shaft.

Force Required to Start Force and Shrink Fits. (*Am. Mach.*, Mar. 7, 1907.) — A series of experiments was made at the Alabama Polytechnic Institute on spindles 1 in. diam. pressed or shrunk into cast-iron disks 6 in. diam., $1\frac{1}{4}$ in. thick. The disks were bored and finished with a reamer to 1 in. diam. with an error believed not to exceed 0.00025 in. The shafts were ground to sizes 0.001 to 0.003 in. over 1 in. Some of the spindles were forced into the disks by a testing machine, the others had the disks shrunk on. Some of each sort were tested by pulling the spindle from the disk in the testing machine, others by twisting the disk on the spindle. The force required to start the spindle in the twisting tests was reduced to equivalent force at the circumference of the spindle, for comparison with the tension tests. The results were as follows: D = diam. of spindle; F = force in lbs.:

| Force Fits, Tension. | | | Force Fits, Torsion. | | | Shrink Fits, Tension. | | | Shrink Fits, Torsion. | | |
|-------------------------|------------|-------------|-------------------------|------------|-------------|--------------------------|------------|-------------|--------------------------|------------|-------------|
| D | F , lbs. | Per sq. in. | D | F , lbs. | Per sq. in. | D | F , lbs. | Per sq. in. | D | F , lbs. | Per sq. in. |
| 1.001 | 1000 | 318 | 1.0015 | 2200 | 700 | 1.001 | 5320 | 1695 | 1.001 | 2200 | 700 |
| 1.0015 | 2150 | 685 | 1.0015 | 2800 | 892 | 1.001 | 5820 | 1853 | 1.0015 | 7200 | 2290 |
| 1.002 | 2570 | 818 | 1.002 | 4200 | 1335 | 1.002 | 7500 | 2385 | 1.0015 | 9800 | 3118 |
| 1.0025 | 4000 | 1272 | 1.0025 | 4600 | 1465 | 1.002 | 8100 | 2580 | 1.0025 | 13800 | 4395 |
| | | | | | | 1.0025 | 9340 | 2974 | 1.003 | 17000 | 5410 |
| | | | | | | 1.0025 | 9710 | 3090 | | | |

PROPORTIONING PARTS OF MACHINES IN A SERIES OF SIZES.

The following method was used by Coleman Sellers (*Stevens Indicator*, April, 1892) to get the proportions of the parts of machines, based upon the size obtained in building a large machine and a small one to any series of machines. This formula is used in getting up the proportion-book and arranging the set of proportions from which any machine can be constructed of intermediate size between the largest and smallest of the series.

Rule to Establish Construction Formulæ. — Take difference between the nominal sizes of the largest and the smallest machines that have been designed of the same construction. Take also the difference between the sizes of similar parts on the largest and smallest machines selected. Divide the latter by the former, and the result obtained will be a "factor," which, multiplied by the nominal capacity of the intermediate machine, and increased or diminished by a constant "increment," will give the size of the part required. To find the "increment:" Multiply the nominal capacity of some known size by the factor obtained, and subtract the result from the size of the part belonging to the machine of nominal capacity selected.

EXAMPLE. — Suppose the size of a part of a 72-inch machine is 3 inches, and the corresponding part of a 42-inch machine is $1\frac{7}{8}$, or 1.875 inches: then $72 \div 42 = 30$, and $3 \text{ inches} - 1\frac{7}{8} \text{ inches} = 1\frac{1}{8} \text{ inches} = 1.125$. $1.125 \div 30 = 0.0375 = \text{the "factor,"}$ and $.0375 \times 42 = 1.575$. Then $1.875 - 1.575 = .3 = \text{the "increment" to be added.}$ Let $D = \text{nominal capacity;}$ then the formula will read: $x = D \times .0375 + .3$.

Proof: $42 \times .0375 + .3 = 1.875$, or $1\frac{7}{8}$, the size of one of the selected parts.

Some prefer the formula: $aD + c = x$, in which $D = \text{nominal capacity}$ in inches or in pounds, c is a constant increment, a is the factor, and $x = \text{the part to be found.}$

KEYS.

Sizes of Keys for Mill-gearing. (*Trans. A. S. M. E.*, xiii, 229.) — E. G. Parkhurst's rule: Width of key = $\frac{1}{8}$ diameter of shaft, depth = $\frac{1}{9}$ diameter of shaft; taper $\frac{1}{8}$ inch to the foot.

Custom in Michigan saw-mills: Keys of square section, side = $\frac{1}{4}$ diameter of shaft, or as nearly as may be in even sixteenths of an inch.

J. T. Hawkins's rule: Width = $\frac{1}{3}$ diameter of hole; depth of side abutment in shaft = $\frac{1}{8}$ diameter of hole.

W. S. Huson's rule: $\frac{1}{4}$ -inch key for 1 to $1\frac{1}{4}$ -in. shafts, $\frac{5}{16}$ -in. key for $1\frac{1}{4}$ to $1\frac{1}{2}$ -inch shafts, $\frac{3}{8}$ -inch key for $1\frac{1}{2}$ to $1\frac{3}{4}$ -inch shafts and so on. Taper $\frac{1}{8}$ inch to the foot. Total thickness at large end of splice, $\frac{4}{5}$ width of key.

Unwin (*Elements of Machine Design*) gives: Width = $\frac{1}{4}d + \frac{1}{8}$ inch. Thickness = $\frac{1}{8}d + \frac{1}{8}$ inch, in which $d = \text{diameter of shaft in inches.}$ When wheels or pulleys transmitting only a small amount of power are keyed on large shafts, he says, these dimensions are excessive. In that case, if H.P. = horse-power transmitted by the wheel or pulley, $N = \text{r.p.m.,}$ $P = \text{force acting at the circumference, in pounds,}$ and $R = \text{radius of pulley in inches,}$ take

$$d = \sqrt[3]{\frac{100 \text{ H.P.}}{N}} \text{ or } \sqrt[3]{\frac{PR}{630}}$$

Prof. Coleman Sellers (*Stevens Indicator*, April, 1892) gives the following: The size of keys, both for shafting and for machine tools, are the proportions adopted by William Sellers & Co., and rigidly adhered to during a period of nearly forty years. Their practice in making keys and fitting them is, that the keys shall always bind tight sidewise, but not top and bottom; that is, not necessarily touch either at the bottom of the key-seat in the shaft or touch the top of the slot cut in the gear-wheel that is

fastened to the shaft; but in practice keys used in this manner depend upon the fit of the wheel upon the shaft being a forcing fit, or a fit that is so tight as to require screw-pressure to put the wheel in place upon the shaft.

Size of Keys for Shafting.

| Diameter of Shaft, in. | | | Size of Key, in. | |
|------------------------|---------|---------|------------------|-------|
| 1 1/4 | 1 7/16 | 1 11/16 | 5/16 | 3/8 |
| 1 15/16 | 2 3/16 | | 7/16 | 1/2 |
| 2 7/16 | | | 9/16 | 5/8 |
| 2 11/16 | 2 15/16 | 3 3/16 | 1 1/16 | 3/4 |
| 3 15/16 | 4 7/16 | 4 15/16 | 1 3/16 | 7/8 |
| 5 7/16 | 5 15/16 | 6 7/16 | 1 5/16 | 1 |
| 6 15/16 | 7 7/16 | 7 15/16 | 8 7/16 | 1 1/8 |

Length of key-seat for coupling = $1\frac{1}{2} \times$ nominal diameter of shaft.

Size of Keys for Machine Tools.

| Diam. of Shaft, in. | Size of Key, in. sq. | Diam. of Shaft, in. | Size of Key, in. sq. |
|---------------------|----------------------|---------------------|----------------------|
| 15/16 and under. | 1/8 | 4 to 5 7/16 | 13/16 |
| 1 to 1 3/16 | 3/16 | 5 1/2 to 6 15/16 | 1 5/16 |
| 1 1/4 to 1 7/16 | 1/4 | 7 to 8 15/16 | 1 11/16 |
| 1 1/2 to 1 11/16 | 5/16 | 9 to 10 15/16 | 1 3/16 |
| 1 3/4 to 2 3/16 | 7/16 | 11 to 12 15/16 | 1 5/16 |
| 2 1/4 to 2 11/16 | 9/16 | 13 to 14 15/16 | 1 7/16 |
| 2 3/4 to 3 15/16 | 1 1/16 | | |

John Richards, in an article in *Cassier's Magazine*, writes as follows: There are two kinds or systems of keys, both proper and necessary, but widely different in nature. 1. The common fastening key, usually made in width one fourth of the shaft's diameter, and the depth five eighths to one third the width. These keys are tapered and fit on all sides, or, as it is commonly described, "bear all over." They perform the double function in most cases of driving or transmitting and fastening the keyed-on member against movement endwise on the shaft. Such keys, when properly made, drive as a strut, diagonally from corner to corner.

2. The other kind or class of keys are not tapered and fit on their sides only, a slight clearance being left on the back to insure against wedge action or radial strain. These keys drive by shearing strain.

For fixed work where there is no sliding movement such keys are commonly made of square section, the sides only being planed, so the depth is more than the width by so much as is cut away in finishing or fitting.

For sliding bearings, as in the case of drilling-machine spindles, the depth should be increased, and in cases where there is heavy strain there should be two keys or feathers instead of one.

The following tables are taken from proportions adopted in practical use.

Flat keys, as in the first table, are employed for fixed work when the parts are to be held not only against torsional strain, but also against movement endwise; and in case of heavy strain the strut principle being the strongest and most secure against movement when there is strain each way, as in the case of engine cranks and first movers generally. The objections to the system for general use are, straining the work out of truth, the care and expense required in fitting, and destroying the evidence of good or bad fitting of the keyed joint. When a wheel or other part is fastened with a tapering key of this kind there is no means of knowing whether the work is well fitted or not. For this reason such keys are not employed by machine-tool-makers, and in the case of accurate work of any kind, indeed, cannot be, because of the wedging strain, and also the difficulty of inspecting completed work.

I. DIMENSIONS OF FLAT KEYS, IN INCHES.

| | | | | | | | | | | | | | |
|-------------------|------|-------|-------|-------|------|-------|------|-------|-----|-------|-------|-------|-------|
| Diam. of shaft .. | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/2 | 3 | 3 1/2 | 4 | 5 | 6 | 7 | 8 |
| Breadth of keys | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 5/8 | 3/4 | 7/8 | 1 | 1 1/8 | 1 3/8 | 1 1/2 | 1 3/4 |
| Depth of keys .. | 5/32 | 3/16 | 1/4 | 9/32 | 5/16 | 3/8 | 7/16 | 1/2 | 5/8 | 11/16 | 13/16 | 7/8 | 1 |

II. DIMENSIONS OF SQUARE KEYS, IN INCHES.

| | | | | | | | | | |
|----------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|
| Diameter of shaft .. | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/2 | 3 | 3 1/2 | 4 |
| Breadth of keys | 5/32 | 7/32 | 9/32 | 11/32 | 13/32 | 15/32 | 17/32 | 9/16 | 11/16 |
| Depth of keys | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 3/4 |

III. DIMENSIONS OF SLIDING FEATHER-KEYS, IN INCHES.

| | | | | | | | | | | |
|-----------------------|-------|-------|-------|------|-------|-------|-----|-------|------|-------|
| Diameter of shaft ... | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/4 | 2 1/2 | 3 | 3 1/2 | 4 | 4 1/2 |
| Breadth of keys | 1/4 | 1/4 | 5/16 | 5/16 | 3/8 | 3/8 | 1/2 | 9/16 | 9/16 | 5/8 |
| Depth of keys | 3/8 | 3/8 | 7/16 | 7/16 | 1/2 | 1/2 | 5/8 | 3/4 | 3/4 | 7/8 |

P. Prybil furnishes the following table of dimensions to the *Am. Machinist*. He says: "On special heavy work and very short hubs we put in two keys in one shaft 90 degrees apart. With special long hubs, where we cannot use keys with noses, the keys should be thicker than the standard.

| Diameter of Shafts, Inches. | Width, Inches. | Thick-ness, In. | Diameter of Shafts, Inches. | Width, Inches. | Thick-ness, In. |
|-----------------------------|----------------|-----------------|-----------------------------|----------------|-----------------|
| 3/4 to 1 1/16 | 3/16 | 3/16 | 3 7/16 to 3 11/16 | 7/8 | 5/8 |
| 1 1/8 to 1 5/16 | 5/16 | 1/4 | 3 15/16 to 4 3/16 | 1 | 11/16 |
| 1 7/16 to 1 11/16 | 3/8 | 5/16 | 4 7/16 to 4 11/16 | 1 1/8 | 3/4 |
| 1 15/16 to 2 3/16 | 1/2 | 3/8 | 4 7/8 to 5 3/8 | 1 1/4 | 15/16 |
| 2 7/16 to 2 11/16 | 5/8 | 1/2 | 5 7/8 to 6 3/8 | 1 1/2 | 1 |
| 2 15/16 to 3 3/16 | 3/4 | 9/16 | 6 7/8 to 7 3/8 | 1 3/4 | 1 1/8 |

Keys longer than 10 inches, say 14 to 16 inches, 1/16 inch thicker; keys longer than 10 inches, say 18 to 20 inches, 1/8 inch thicker; and so on. Special short hubs to have two keys.

For description of the Woodruff system of keying, see circular of the Pratt & Whitney Co.; also *Modern Mechanism*, page 455.

For keyways in milling cutters see page 1248.

HOLDING-POWER OF KEYS AND SET-SCREWS.

Tests of the Holding-power of Set-screws in Pulleys. (G. Lanza, *Trans. A. S. M. E.*, x, 230.) — These tests were made by using a pulley fastened to the shaft by two set-screws with the shaft keyed to the holders; then the load required at the rim of the pulley to cause it to slip was determined, and this being multiplied by the number 6.037 (obtained by adding to the radius of the pulley one-half the diameter of the wire rope, and dividing the sum by twice the radius of the shaft, since there were two set-screws in action at a time) gives the holding-^{and} ^{seat} ^{of} the set-screws. The set-screws used were of wrought iron, 1/2 inch in diameter, and ten threads to the inch; the shaft used was

of steel and rather hard, the set-screws making but little impression upon it. They were set up with a force of 75 pounds at the end of a ten-inch monkey-wrench. The set-screws used were of four kinds, marked respectively A, B, C, and D. The results were as follows:

A, ends perfectly flat, $\frac{9}{16}$ -in. diam. 1412 to 2294 lbs.; average 2064.
 B, radius of rounded ends about $\frac{1}{2}$ -in. 2747 to 3079 lbs.; average 2912.
 C, radius of rounded ends about $\frac{1}{4}$ -in. 1902 to 3079 lbs.; average 2573.
 D, ends cup-shaped and case-hardened 1962 to 2958 lbs.; average 2470.

REMARKS. — A. The set-screws were not entirely normal to the shaft; hence they bore less in the earlier trials, before they had become flattened by wear.

B. The ends of these set-screws, after the first two trials, were found to be flattened, the flattened area having a diameter of about $\frac{1}{4}$ inch.

C. The ends were found, after the first two trials, to be flattened, as in B.

D. The first test held well because the edges were sharp, then the holding-power fell off till they had become flattened in a manner similar to B, when the holding-power increased again.

Tests of the Holding-power of Keys. (Lanza.) — The load was applied as in the tests of set-screws, the shaft being firmly keyed to the holders. The load required at the rim of the pulley to shear the keys was determined, and this, multiplied by a suitable constant, determined in a similar way to that used in the case of set-screws, gives us the shearing strength per square inch of the keys.

The keys tested were of eight kinds, denoted, respectively, by the letters A, B, C, D, E, F, G and H, and the results were as follows: A, B, D, and F, each 4 tests; E, 3 tests; C, G, and H, each 2 tests.

| | | | |
|---|-----------|-----------------------|--------|
| A, Norway iron, $2'' \times \frac{1}{4}'' \times \frac{15}{32}''$, | 40,184 to | 47,760 lbs.; average, | 42,726 |
| B, refined iron, $2'' \times \frac{1}{4}'' \times \frac{15}{32}''$, | 36,482 to | 39,254 lbs.; average, | 38,059 |
| C, tool steel, $1'' \times \frac{1}{4}'' \times \frac{15}{32}''$, | 91,344 & | 100,056 lbs.; | |
| D, mach'y steel, $2'' \times \frac{1}{4}'' \times \frac{15}{32}''$ | 64,630 to | 70,186 lbs.; average, | 66,875 |
| E, Norway iron, $1\frac{1}{3}'' \times \frac{3}{8}'' \times \frac{7}{16}''$ | 36,850 to | 37,222 lbs.; average, | 37,036 |
| F, cast-iron, $2'' \times \frac{1}{4}'' \times \frac{15}{32}''$, | 30,278 to | 36,944 lbs.; average, | 33,034 |
| G, cast-iron, $1\frac{1}{3}'' \times \frac{3}{8}'' \times \frac{7}{16}''$, | 37,222 & | 38,700. | |
| H, cast-iron, $1'' \times \frac{1}{2}'' \times \frac{7}{16}''$, | 29,814 & | 38,978. | |

In A and B some crushing took place before shearing. In E, the keys, being only $\frac{7}{16}$ inch deep, tipped slightly in the key-way. In H, in the first test, there was a defect in the key-way of the pulley.

DYNAMOMETERS.

Dynamometers are instruments used for measuring power. They are of several classes, as: 1. Traction dynamometers, used for determining the power required to pull a car or other vehicle, or a plow or harrow. 2. Brake or absorption dynamometers, in which the power of a rotating shaft or wheel is absorbed or converted into heat by the friction of a brake; and 3. Transmission dynamometers, in which the power in a rotating shaft is measured during its transmission through a belt or other connection to another shaft, without being absorbed.

Traction Dynamometers generally contain two principal parts: (1) A spring or series of springs, through which the pull is exerted, the extension

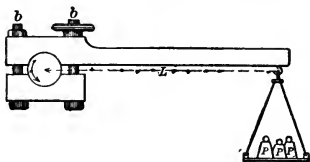


FIG. 193.

of the spring measuring the amount of the pulling force; and (2) a paper-covered drum, rotated either at a uniform speed by clock-work, or at a speed proportional to the speed of the traction, through gearing, on which the extension of the spring is registered by a pencil. From the average height of the diagram drawn by the pencil above the zero-line the average pulling force in pounds is obtained, and this

multiplied by the distance traversed, in feet, gives the work done, in foot-pounds. The product divided by the time in minutes and by 33,000 gives the horse-power.

The Prony brake is the typical form of absorption dynamometer. (See Fig. 193, from Flather on Dynamometers.)

Primarily this consists of a lever connected to a revolving shaft or pulley in such a manner that the friction induced between the surfaces in contact will tend to rotate the arm in the direction in which the shaft revolves. This rotation is counterbalanced by weights P , hung in the scale-pan at the end of the lever. In order to measure the power for a given number of revolutions of pulley, we add weights to the scale-pan and screw up on bolts b, b , until the friction induced balances the weights and the lever is maintained in its horizontal position while the revolutions of the shaft per minute remain constant.

For small powers the beam is generally omitted — the friction being measured by weighting a band or strap thrown over the pulley. Ropes or cords are often used for the same purpose.

Instead of hanging weights in a scale-pan, as in Fig. 107, the friction may be weighed on a platform-scale; in this case, the direction of rotation being the same, the lever-arm will be on the opposite side of the shaft.

In a modification of this brake, the brake-wheel is keyed to the shaft, and its rim is provided with inner flanges which form an annular trough for the retention of water to keep the pulley from heating. A small stream of water constantly discharges into the trough and revolves with the pulley — the centrifugal force of the particles of water overcoming the action of gravity; a waste-pipe with its end flattened is so placed in the trough that it acts as a scoop, and removes all surplus water. The brake consists of a flexible strap to which are fitted blocks of wood forming the rubbing-surface; the ends of the strap are connected by an adjustable bolt-clamp, by means of which any desired tension may be obtained.

The horse-power or work of the shaft is determined from the following:

Let W = work of shaft, equals power absorbed, per minute;

P = unbalanced pressure or weight in pounds, acting on lever-arm at distance L ;

L = length of lever-arm in feet from center of shaft;

V = velocity of a point in feet per minute at distance L , if arm were allowed to rotate at the speed of the shaft;

N = number of revolutions per minute;

H.P. = horse-power.

Then will $W = PV = 2\pi LNP$.

Since H.P. = $PV \div 33,000$, we have H.P. = $2\pi LNP \div 33,000$.

If $L = 33 \div 2\pi$, we obtain H.P. = $NP \div 1000$. $33 \div 2\pi$ is practically 5 ft. 3 in., a value often used in practice for the length of arm.

If the rubbing-surface be too small, the resulting friction will show great irregularity — probably on account of insufficient lubrication — the jaws being allowed to seize the pulley, thus producing shocks and sudden vibrations of the lever-arm.

Soft woods, such as bass, plane-tree, beech, poplar, or maple, are all to be preferred to the harder woods for brake-blocks. The rubbing-surface should be well lubricated with a heavy grease.

The Alden Absorption-dynamometer. (G. I. Alden, *Trans. A. S. M. E.*, vol. xi, 958; also xii, 700 and xiii, 429.) — This dynamometer is a friction-brake, which is capable in quite moderate sizes of absorbing large powers with unusual steadiness and complete regulation. A smooth cast-iron disk is keyed on the rotating shaft. This is inclosed in a cast-iron shell, formed of two disks and a ring at their circumference, which is free to revolve on the shaft. To the interior of each of the sides of the shell is fitted a copper plate, inclosing between itself and the side a water-tight space. Water under pressure from the city pipes is admitted into each of these spaces, forcing the copper plate against the central disk. The chamber inclosing the disk is filled with oil. To the outer shell is fixed a weighted arm, which resists the tendency of the shell to rotate with the shaft, caused by the friction of the plates against the central disk. Four brakes of this type, 56 in. diam., were used in testing the experimental locomotive at Purdue University (*Trans. A. S. M. E.*, xiii, 429). Each was designed for a maximum moment of 10,500 foot-pounds with a water-pressure of 40 lbs. per sq. in. The area in effective contact with the copper plates on either side is represented by an annular surface having its outer radius equal to 28 ins. and its inner radius equal to 10 ins. The apparent coefficient of friction between the plates and the disk was $3\frac{1}{2}\%$.

Capacity of Friction-brakes. — W. W. Beaumont (*Proc. Inst. C. E.*, 1889) has deduced a formula by means of which the relative capacity of brakes can be compared, judging from the amount of horse-power ascertained by their use.

If W = width of rubbing-surface on brake-wheel in inches; V = vel. of point on circum. of wheel in feet per minute; K = coefficient; then

$$K = WV \div \text{H.P.}$$

Prof. Flather obtains the values of K given in the last column of the subjoined table:

| Horse-power. | R.P.M. Brake pulley. | Brake-pulley. | | Length of Arm, inches. | Design of Brake. | Value of K . |
|--------------|----------------------|------------------|--------------------|------------------------|-------------------------------------|----------------|
| | | Face, in inches. | Diameter, in feet. | | | |
| 21 | 150 | 7 | 5 | 33 | Royal Ag. Soc., compensating..... | 785 |
| 19 | 148.5 | 7 | 5 | 33.38 | McLaren, compensating..... | 858 |
| 20 | 146 | 7 | 5 | 32.19 | McLaren, water-cooled and comp..... | 802 |
| 40 | 180 | 10.5 | 5 | 32 | Garrett, water-cooled and comp..... | 741 |
| 33 | 150 | 10.5 | 5 | 32 | Garrett, water-cooled and comp..... | 749 |
| 150 | 150 | 10 | 9 | | Schoenheyder, water-cooled..... | 282 |
| 24 | 142 | 12 | 6 | 38.31 | Balk..... | 1385 |
| 180 | 100 | 24 | 5 | 126.1 | Gately & Kletsch, water-cooled..... | 209 |
| 475 | 76.2 | 24 | 7 | 191 | Webber, water-cooled..... | 84.7 |
| 125 | 290 | 24 | 4 | 63 | Westinghouse, water-cooled..... | 465 |
| 250 | 250 | | | | | |
| 40 | 322 | | | | | |
| 125 | 290 | 13 | 4 | 273/4 | Westinghouse, water-cooled..... | 847 |

The above calculations for eleven brakes give values of K varying from 84.7 to 1385 for actual horse-powers tested, the average being $K = 655$.

Instead of assuming an average coefficient, Prof. Flather proposes the following:

Water-cooled brake, non-compensating, $K = 400$; $W = 400 \text{ H.P.} \div V$.

Water-cooled brake, compensating, $K = 750$; $W = 750 \text{ H.P.} \div V$.

Non-cooling brake, with or without compensating device, $K = 900$; $W = 900 \text{ H.P.} \div V$.

A brake described in *Am. Mach.*, July 27, 1905, had an iron water-cooled drum, 30 in. diam., 20 in. face, with brake blocks of maple attached to an iron strap nearly surrounding the drum. At 250 r.p.m., or a circumferential speed of 1963 ft. per min., the limit of its capacity was about 140 H.P.; above that power the blocks took fire. At 140 H.P. the total surface passing under the brake blocks per minute was 3272 sq. ft., or 23.37 per H.P. This corresponds to a value of $K = 285$.

Several forms of Prony brake, including rope and strap brakes, are described by G. E. Quick in *Am. Mach.*, Nov. 17, 1908. Some other forms are shown in *Am. Electrician*, Feb., 1903.

A 6000 H.P. Hydraulic Absorption Dynamometer, built by the Westinghouse Machine Co., is described by E. H. Longwell in *Eng. News*, Dec. 30, 1909. It was designed for testing the efficiency of the McIlvire and McAlpine turbine reduction gear (see page 1071). This dynamometer consists of a rotor mounted on a shaft coupled to the reduction gear and rotating within a closed casing which is prevented from turning by a $6\frac{1}{2}$ ft. lever arm, the end of which transmits pressure through an I-beam lever to a platform scale. The rotor carries several rows of steam turbine vanes and the casing carries corresponding rows of stationary vanes, so arranged as to baffle and agitate the water passing through the brake, which is heated to boiling temperature by the friction. The dynamometer was run for 40 hours continuously, and proved to be a highly accurate instrument.

Transmission Dynamometers are of various forms, as the Batchelder dynamometer, in which the power is transmitted through a "train-arm" of bevel gearing, with its modifications, as the one described by the author in *Trans. A. S. M. E.*, viii, 177, and the one described by Samuel Webber in *Trans. A. S. M. E.*, x, 514; belt dynamometers, as the Tatham; the Van Winkle dynamometer, in which the power is transmitted from a revolving shaft to another in line with it, the two almost touching, through the medium of coiled springs fastened to arms or disk keyed to the shafts; the Brackett and the Webb cradle dynamometers, used for measuring the power required to run dynamo-electric machines. Descriptions of the four last named are given in Flather on Dynamometers.

The Kenerson transmission dynamometer is described in *Trans. A. S. M. E.*, 1909. It has the form of a shaft coupling, one part of which contains a cavity filled with oil and covered by a flexible copper diaphragm. The other part, by means of bent levers and a thrust ball-bearing, brings an axial pressure on the diaphragm and on the oil, and the pressure of the oil is measured by a gauge.

Much information on various forms of dynamometers will be found in *Trans. A. S. M. E.*, vols. vii to xv, inclusive, indexed under Dynamometers.

ICE-MAKING OR REFRIGERATING MACHINES.

References. — An elaborate discussion of the thermodynamic theory of the action of the various fluids used in the production of cold was published by M. Ledoux in the *Annales des Mines*, and translated in *Van Nostrand's Magazine* in 1879. This work, revised and additions made in the light of recent experience by Professors Denton, Jacobus, and Riesenberger, was reprinted in 1892. (Van Nostrand's Science Series, No. 46.) The work is largely mathematical, but it also contains much information of immediate practical value, from which some of the matter given below is taken. Other references are Wood's *Thermodynamics*, Chap. V. and numerous papers by Professors Wood, Denton, Jacobus, and Linde in *Trans. A. S. M. E.*, vols. x to xiv. Johnson's *Cyclopædia*, article on Refrigerating-machines; and the following books: Siebel's *Compend of Mechanical Refrigeration*; *Modern Refrigerating Machinery*, by Lorenz, translated by Pope; *Refrigerating Machines*, by Gardner T. Voorhees; *Re-*

frigeration, by J. Wemyss Anderson, and Refrigeration, Cold Storage and Ice-making, by A. J. Wallis-Taylor. For properties of Ammonia and Sulphur Dioxide, see papers by Professors Wood and Jacobus, *Trans. A. S. M. E.*, vols. x and xii.

For illustrated descriptions of refrigerating-machines, see catalogues of builders, as Frick & Co., Waynesboro, Pa.; De La Vergne Refrigerating-machine Co., New York; Vilter Mfg. Co., Milwaukee; York Mfg., York, Co., Pa.; Henry Vogt Machine Co., Louisville, Ky.; Carbondale Machine Co., Carbondale, Pa.; and others. See also articles in *Ice and Refrigeration*.

Operations of a Refrigerating-machine. — Apparatus designed for refrigerating is based upon the following series of operations:

Compress a gas or vapor by means of some external force, then relieve it of its heat so as to diminish its volume; next, cause this compressed gas or vapor to expand so as to produce mechanical work, and thus lower its temperature. The absorption of heat at this stage by the gas, in resuming its original condition, constitutes the refrigerating effect of the apparatus.

A refrigerating-machine is a heat-engine reversed.

From this similarity between heat-motors and freezing-machines it results that all the equations deduced from the mechanical theory of heat to determine the performance of the first, apply equally to the second.

The efficiency depends upon the difference between the extremes of temperature.

The useful effect of a refrigerating-machine depends upon the ratio between the heat-units eliminated and the work expended in compressing and expanding.

This result is independent of the nature of the body employed.

Unlike the heat-motors, the freezing-machine possesses the greatest efficiency when the range of temperature is small, and when the final temperature is elevated.

If the temperatures are the same, there is no theoretical advantage in employing a gas rather than a vapor in order to produce cold.

The choice of the intermediate body would be determined by practical considerations based on the physical characteristics of the body, such as the greater or less facility for manipulating it, the extreme pressures required for the best effects, etc.

Air offers the double advantage that it is everywhere obtainable, and that we can vary at will the higher pressures, independent of the temperature of the refrigerant. But to produce a given useful effect the apparatus must be of larger dimensions than that required by liquefiable vapors.

The maximum pressure is determined by the temperature of the condenser and the nature of the volatile liquid; this pressure is often very high.

When a change of volume of a saturated vapor is made under constant pressure, the temperature remains constant. The addition or subtraction of heat, which produces the change of volume, is represented by an increase or a diminution of the quantity of liquid mixed with the vapor.

On the other hand, when vapors, even if saturated, are no longer in contact with their liquids, and receive an addition of heat either through compression by a mechanical force, or from some external source of heat, they comport themselves nearly in the same way as permanent gases, and become superheated.

It results from this property, that refrigerating-machines using a liquefiable gas will afford results differing according to the method of working, and depending upon the state of the gas, whether it remains constantly saturated, or is superheated during a part of the cycle of working.

The temperature of the condenser is determined by local conditions. The interior will exceed by 9° to 18° the temperature of the water furnished to the exterior. This latter will vary from about 52° F., the temperature of water from considerable depth below the surface, to about 95° F., the temperature of surface-water in hot climates. The volatile liquid employed in the machine ought not at this temperature to have a tension above that which can be readily managed by the apparatus.

On the other hand, if the tension of the gas at the minimum temperature is too low, it becomes necessary to give to the compression-cylinder large dimensions, in order that the weight of vapor compressed by a

single stroke of the piston shall be sufficient to produce a notably useful effect.

These two conditions, to which may be added others, such as those depending upon the greater or less facility of obtaining the liquid, upon the dangers incurred in its use, either from its inflammability or unhealthfulness, and finally upon its action upon the metals, limit the choice to a small number of substances.

The gases or vapors generally available are: sulphuric ether, sulphurous oxide, ammonia, methylic ether, and carbonic acid.

The following table, derived from Regnault, shows the tensions of the vapors of these substances at different temperatures between -22° and $+104^{\circ}$.

Pressures and Boiling-points of Liquids available for Use in Refrigerating-machines.

| Temp. of Ebulli- tion. | Tension of Vapor, in lbs. per sq. in., above Zero. | | | | | | |
|---------------------------------|--|---------------------|----------|--------------------|-------------------|------------------|--------------------|
| Deg. Fahr. | Sul- phuric Ether. | Sulphur Dioxide. | Ammonia. | Methylic Ether. | Carbonic Acid. | Pictet Fluid. | Ethyl Chloride. |
| -40 | | | 10.22 | | | | |
| -31 | | | 13.23 | | | | |
| -22 | | 5.56 | 16.95 | 11.15 | | | 2.13 |
| -13 | | 7.23 | 21.51 | 13.85 | 251.6 | | 2.80 |
| -4 | 1.30 | 9.27 | 27.04 | 17.06 | 292.9 | 13.5 | 3.63 |
| 5 | 1.70 | 11.76 | 33.67 | 20.84 | 340.1 | 16.2 | 4.63 |
| 14 | 2.19 | 14.75 | 41.58 | 25.27 | 393.4 | 19.3 | 5.84 |
| 23 | 2.79 | 18.31 | 50.91 | 30.41 | 453.4 | 22.9 | 7.28 |
| 32 | 3.55 | 22.53 | 61.85 | 36.34 | 520.4 | 26.9 | 9.00 |
| 41 | 4.45 | 27.48 | 74.55 | 43.13 | 594.8 | 31.2 | 11.01 |
| 50 | 5.54 | 33.26 | 89.21 | 50.84 | 676.9 | 36.2 | 13.36 |
| 59 | 6.84 | 39.93 | 105.99 | 59.56 | 766.9 | 41.7 | 16.10 |
| 68 | 8.38 | 47.62 | 125.08 | 69.35 | 864.9 | 48.1 | 19.26 |
| 77 | 10.19 | 56.39 | 146.64 | 80.28 | 971.1 | 55.6 | 22.90 |
| 86 | 12.31 | 66.37 | 170.83 | 92.41 | 1085.6 | 64.1 | 27.05 |
| 95 | 14.76 | 77.64 | 197.83 | | 1207.9 | 73.2 | 31.78 |
| 104 | 17.59 | 90.32 | 227.76 | | 1338.2 | 82.9 | 37.12 |

The table shows that the use of ether does not readily lead to the production of low temperatures, because its pressure becomes then very feeble.

Ammonia, on the contrary, is well adapted to the production of low temperatures.

Methylic ether yields low temperatures without attaining too great pressures at the temperature of the condenser. Sulphur dioxide readily affords temperatures of -14 to -5 , while its pressure is only 3 to 4 atmospheres at the ordinary temperature of the condenser. These latter substances then lend themselves conveniently for the production of cold by means of mechanical force.

The "Pictet fluid" is a mixture of 97% sulphur dioxide and 3% carbonic acid. At atmospheric pressure it affords a temperature 14° lower than sulphur dioxide. (It is not now used — 1910.)

Carbonic acid is in use to a limited extent, but the relatively greater compactness of compressor that it requires, and its inoffensive character, are leading to its recommendation for service on shipboard.

Certain ammonia plants are operated with a surplus of liquid present during compression, so that superheating is prevented. This practice is known as the "cold" or "wet" system of compression.

Ethyl chloride, C_2H_5Cl , is a colorless gas which at atmospheric pressure condenses to a liquid at $54.5^{\circ}F$. The latent heat at $23^{\circ}F$. is given at 174 B.T.U. Density of the gas (air=1) = 2.227. Specific heat at constant pressure, 0.274; at constant volume, 0.243.

Nothing definite is known regarding the application of methylic ether or of the petroleum product chymogene in practical refrigerating service. The inflammability of the latter and the cumbrousness of the compressor required are objections to its use.

PROPERTIES OF SULPHUR DIOXIDE AND AMMONIA GAS.

Ledoux's Table for Saturated Sulphur-dioxide Gas.

Heat-units expressed in B.T.U. per pound of sulphur dioxide.

| Temperature of Ebullition in deg. F. | Absolute Pressure in lbs. per sq. in. $P + 144$ | Total Heat reckoned from 32° F. λ | Heat of Liquid reckoned from 32° F. q | Latent Heat of Evaporation. r | Heat Equivalent of External Work. APu | Internal Latent Heat. ρ | Increase of Volume during Evaporation. u | Density of Vapor or Weight of 1 cu. ft. $1 \div v$ |
|--------------------------------------|---|---|---|---------------------------------|---|------------------------------|--|--|
| Deg. F. | Lbs. | B.T.U. | B.T.U. | B.T.U. | B.T.U. | B.T.U. | Cu. ft. | Lbs. |
| -22 | 5.56 | 157.43 | -19.56 | 176.99 | 13.59 | 163.39 | 13.17 | 0.076 |
| -13 | 7.23 | 158.64 | -16.30 | 174.95 | 13.83 | 161.12 | 10.27 | .097 |
| -4 | 9.27 | 159.84 | -13.05 | 172.89 | 14.05 | 158.84 | 8.12 | .123 |
| 5 | 11.76 | 161.03 | -9.79 | 170.82 | 14.26 | 156.56 | 6.50 | .153 |
| 14 | 14.74 | 162.20 | -6.53 | 168.73 | 14.46 | 154.27 | 5.25 | .190 |
| 23 | 18.31 | 163.36 | -3.27 | 166.63 | 14.66 | 151.97 | 4.29 | .232 |
| 32 | 22.53 | 164.51 | 0.00 | 164.51 | 14.84 | 149.68 | 3.54 | .282 |
| 41 | 27.48 | 165.65 | 3.27 | 162.38 | 15.01 | 147.37 | 2.93 | .340 |
| 50 | 33.25 | 166.78 | 6.55 | 160.23 | 15.17 | 145.06 | 2.45 | .407 |
| 59 | 39.93 | 167.90 | 9.83 | 158.07 | 15.32 | 142.75 | 2.07 | .483 |
| 68 | 47.61 | 168.99 | 13.11 | 155.89 | 15.46 | 140.43 | 1.75 | .570 |
| 77 | 56.39 | 170.09 | 16.39 | 153.70 | 15.59 | 138.11 | 1.49 | .669 |
| 86 | 66.36 | 171.17 | 19.69 | 151.49 | 15.71 | 135.78 | 1.27 | .780 |
| 95 | 77.64 | 172.24 | 22.98 | 149.26 | 15.82 | 133.45 | 1.09 | .906 |
| 104 | 90.31 | 173.30 | 26.28 | 147.02 | 15.91 | 131.11 | 0.91 | 1.046 |

E. F. Miller (*Trans. A. S. M. E.*, 1903) reports a series of tests on the pressure of SO_2 at various temperatures, the results agreeing closely with those of Regnault up to the highest figure of the latter, 149° F., 178 lbs. absolute. He gives a table of pressures and temperatures for every degree between -40° and 217°. The results obtained at temperatures between 113° and 212° are as below:

| Temp. °F. | 113 | 122 | 131 | 140 | 149 | 158 | 167 | 176 | 194 | 203 | 212 |
|------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|
| Pres. lbs. per sq. in. | 104.4 | 120.1 | 137.5 | 156.7 | 179.5 | 203.8 | 230.7 | 260.5 | 331.1 | 371.8 | 418. |

104.4 120.1 137.5 156.7 179.5 203.8 230.7 260.5 331.1 371.8 418.

Density of Liquid Ammonia. (D'Andreff, *Trans. A. S. M. E.*, x, 641.)

| | | | | | | | |
|------------------------|--------|-------|-------|-------|-------|-------|-------|
| At temperature C. | -10 | -5 | 0 | 5 | 10 | 15 | 20 |
| At temperature F. | +14 | 23 | 32 | 41 | 50 | 59 | 68 |
| Density | 0.6492 | .6429 | .6364 | .6298 | .6230 | .6160 | .6086 |

These may be expressed very nearly by

$$\delta = 0.6364 - 0.0014^{\circ} \text{ Centigrade};$$

$$\delta = 0.6502 - 0.000777^{\circ} \text{ Fahr.}$$

Latent Heat of Evaporation of Ammonia. (Wood, *Trans. A. S. M. E.*, x, 641.)

$$h_e = 555.5 - 0.613 T - 0.000219 T^2 \text{ (in B.T.U., } ^{\circ} \text{ F)};$$

$$\text{Ledoux found } h_e = 583.33 - 0.5499 T - 0.0001173 T^2.$$

For experimental values at different temperatures determined by Prof. Denton, see *Trans. A. S. M. E.*, xii, 356. For calculated values, see vol. x, 646.

Properties of the Saturated Vapor of Ammonia.
(Wood's Thermodynamics.)

| Temperature. | | Pressure, Absolute. | | Heat of Vaporization, thermal units. | Volume of Vapor per lb., cu. ft. | Volume of Liquid per lb., cu. ft. | Weight of a cu. ft. of Vapor. lbs. |
|--------------|--------------|---------------------|------------------|--------------------------------------|----------------------------------|-----------------------------------|------------------------------------|
| Degs. F. | Absolute, F. | Lbs. per sq. ft. | Lbs. per sq. in. | | | | |
| — 40 | 420.66 | 1540.7 | 10.69 | 579.67 | 24.372 | 0.0234 | 0.0410 |
| — 35 | 425.66 | 1773.6 | 12.31 | 576.69 | 21.319 | .0236 | .0468 |
| — 30 | 430.66 | 2035.8 | 14.13 | 573.69 | 18.697 | .0237 | .0535 |
| — 25 | 435.66 | 2329.5 | 16.17 | 570.68 | 16.445 | .0238 | .0608 |
| — 20 | 440.66 | 2657.5 | 18.45 | 567.67 | 14.507 | .0240 | .0689 |
| — 15 | 445.66 | 3022.5 | 20.99 | 564.64 | 12.834 | .0242 | .0779 |
| — 10 | 450.66 | 3428.0 | 23.80 | 561.61 | 11.384 | .0243 | .0878 |
| — 5 | 455.66 | 3877.2 | 26.93 | 558.56 | 10.125 | .0244 | .0988 |
| 0 | 460.66 | 4373.5 | 30.37 | 555.50 | 9.027 | .0246 | .1108 |
| 5 | 465.66 | 4920.5 | 34.17 | 552.43 | 8.069 | .0247 | .1239 |
| 10 | 470.66 | 5522.2 | 38.34 | 549.35 | 7.229 | .0249 | .1383 |
| 15 | 475.66 | 6182.4 | 42.93 | 546.26 | 6.492 | .0250 | .1544 |
| 20 | 480.66 | 6905.3 | 47.95 | 543.15 | 5.842 | .0252 | .1712 |
| 25 | 485.66 | 7695.2 | 53.43 | 540.03 | 5.269 | .0253 | .1898 |
| 30 | 490.66 | 8556.6 | 59.41 | 536.92 | 4.763 | .0254 | .2100 |
| 35 | 495.66 | 9493.9 | 65.93 | 533.78 | 4.313 | .0256 | .2319 |
| 40 | 500.66 | 10512 | 73.00 | 530.63 | 3.914 | .0257 | .2555 |
| 45 | 505.66 | 11616 | 80.66 | 527.47 | 3.559 | .0259 | .2809 |
| 50 | 510.66 | 12811 | 88.96 | 524.30 | 3.242 | .0261 | .3085 |
| 55 | 515.66 | 14102 | 97.93 | 521.12 | 2.958 | .0263 | .3381 |
| 60 | 520.66 | 15494 | 107.60 | 517.93 | 2.704 | .0265 | .3698 |
| 65 | 525.66 | 16993 | 118.03 | 514.73 | 2.476 | .0266 | .4039 |
| 70 | 530.66 | 18605 | 129.21 | 511.52 | 2.271 | .0268 | .4403 |
| 75 | 535.66 | 20336 | 141.25 | 508.29 | 2.087 | .0270 | .4793 |
| 80 | 540.66 | 22192 | 154.11 | 505.05 | 1.920 | .0272 | .5208 |
| 85 | 545.66 | 24178 | 167.86 | 501.81 | 1.770 | .0273 | .5650 |
| 90 | 550.66 | 26300 | 182.8 | 498.11 | 1.632 | .0274 | .6128 |
| 95 | 555.66 | 28565 | 198.37 | 495.29 | 1.510 | .0277 | .6623 |
| 100 | 560.66 | 30980 | 215.14 | 492.01 | 1.398 | .0279 | .7153 |
| 105 | 565.66 | 33550 | 232.98 | 488.72 | 1.296 | .0281 | .7716 |
| 110 | 570.66 | 36284 | 251.97 | 485.42 | 1.203 | .0283 | .8312 |
| 115 | 575.66 | 39188 | 272.14 | 482.41 | 1.119 | .0285 | .8937 |
| 120 | 580.66 | 42267 | 293.49 | 478.79 | 1.045 | .0287 | .9569 |
| 125 | 585.66 | 45528 | 316.16 | 475.45 | 0.970 | .0289 | 1.0309 |
| 130 | 590.66 | 48978 | 340.42 | 472.11 | 0.905 | .0291 | 1.1049 |
| 135 | 595.66 | 52626 | 365.16 | 468.75 | 0.845 | .0293 | 1.1834 |
| 140 | 600.66 | 56483 | 392.22 | 465.39 | 0.791 | .0295 | 1.2642 |
| 145 | 605.66 | 60550 | 420.49 | 462.01 | 0.741 | .0297 | 1.3495 |
| 150 | 610.66 | 64833 | 450.20 | 458.62 | 0.695 | .0299 | 1.4388 |
| 155 | 615.66 | 69341 | 481.54 | 455.22 | 0.652 | .0302 | 1.5337 |
| 160 | 620.66 | 74086 | 514.40 | 451.81 | 0.613 | .0304 | 1.6343 |
| 165 | 625.66 | 79071 | 549.04 | 448.39 | 0.577 | .0306 | 1.7333 |

Density of Ammonia Gas. — Theoretical, 0.5894; experimental, 0.596. Regnault (*Trans. A. S. M. E.*, x, 633).

Specific Heat of Liquid Ammonia. (Wood, *Trans. A. S. M. E.*, x, 645.) — The specific heat is nearly constant at different temperatures, and about equal to that of water, or unity. From 0° to 100° F., it is

$$c = 1.096 - 0.0012T, \text{ nearly.}$$

In a later paper by Prof. Wood (*Trans. A. S. M. E.*, xii, 136) he gives a higher value, viz., $c = 1.12136 + 0.000438T$.

L. A. Elleau and Wm. D. Ennis (*Jour. Franklin Inst.*, April, 1898) give the results of nine determinations, made between 0° and 20° C., which range from 0.983 to 1.056, averaging 1.0206. Von Strombeck

(*Jour. Franklin Inst.*, Dec., 1890) found the specific heat between 62° and 31° C. to be 1.22876. Ludeking and Starr (*Am. Jour. Science*, iii, 45, 200) obtained 0.886. Prof. Wood deduced from thermodynamic equations $c = 1.093$ at -34° F. or -38° C., and Ledoux in like manner finds $c = 1.0058 + 0.003658 t^{\circ} \text{C.}$ Elleau and Ennis give Ledoux's equation with a new constant derived from their experiments, thus $c = 0.9834 + 0.003658 t^{\circ} \text{C.}$

Strength of Aqua Ammonia at 60° F.

| | | | | | | | | | |
|--------------------------|-------|------|------|------|------|------|------|------|------|
| % NH ₃ by wt. | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| Sp. gr. | 0.986 | .979 | .972 | .966 | .960 | .953 | .945 | .938 | .931 |
| % NH ₃ | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 |
| Sp. gr. | 0.925 | .919 | .913 | .907 | .902 | .897 | .892 | .888 | .884 |

Specific Heat of Ammonia Vapor at the Saturation Point. (Wood, *Trans. A. S. M. E.*, x, 644.) — For the range of temperatures ordinarily used in engineering practice, the specific heat of saturated ammonia is negative, and the saturated vapor will condense with adiabatic expansion. The liquid will evaporate with the compression of the vapor, and when all is vaporized will superheat.

Regnault (*Rel. des. Exp.*, ii, 162) gives for specific heat of ammonia-gas 0.50836. (Wood, *Trans. A. S. M. E.*, xii, 133.)

Weight of Superheated Ammonia Vapor at 15.67 lbs. Gauge Pressure (= 30.67 lbs. abs.) (C. E. Lucke, *Ice and Refrigeration*, Mar., 1908.)
Weight at 0° F. 0.1107 lbs.

| Temp. ° F. | Lb. per cu. ft. | Temp. ° F. | Lb. per cu. ft. | Temp. ° F. | Lb. per cu. ft. | Temp. ° F. | Lb. per cu. ft. |
|---------------|--------------------|---------------|--------------------|---------------|--------------------|---------------|--------------------|
| 5 | 0.1095 | 25 | 0.1050 | 125 | 0.08706 | 225 | 0.07438 |
| 10 | 0.1085 | 50 | 0.09986 | 150 | 0.08351 | 250 | 0.07176 |
| 15 | 0.1072 | 75 | 0.0952 | 175 | 0.08033 | 275 | 0.06932 |
| 20 | 0.1061 | 100 | 0.09096 | 200 | 0.07713 | 300 | 0.06703 |

Specific Heat and Available Latent Heat of Hot Liquid Ammonia at 15.67 lbs. gauge pressure. (Lucke.) Latent heat at 15.67 lbs. and 0° F. = 550.5 B.T.U. Specific heat = $1.096 - 0.0012 T^{\circ}$.

| Temp. of Liquid Supply. | Specific Heat. | Correc- tion for Cooling. | Available Latent Heat for Saturated Vapor. | Temp. of Liquid Supply. | Specific Heat. | Correc- tion for Cooling. | Available Latent Heat for Satur- ated Vapor. |
|----------------------------------|-------------------|------------------------------------|--|----------------------------------|-------------------|------------------------------------|---|
| 5 | 1.090 | 5.45 | 550.05 | 55 | 1.030 | 56.65 | 498.85 |
| 10 | 1.084 | 10.84 | 544.66 | 60 | 1.024 | 61.44 | 494.06 |
| 15 | 1.078 | 16.17 | 539.33 | 65 | 1.018 | 66.17 | 489.33 |
| 20 | 1.072 | 21.44 | 534.06 | 70 | 1.012 | 70.84 | 484.66 |
| 25 | 1.066 | 26.65 | 528.85 | 75 | 1.006 | 75.45 | 480.05 |
| 30 | 1.060 | 31.80 | 523.70 | 80 | 1.000 | 80.00 | 475.50 |
| 35 | 1.054 | 36.89 | 518.61 | 85 | 0.994 | 84.49 | 471.01 |
| 40 | 1.048 | 41.92 | 513.68 | 90 | 0.988 | 88.92 | 466.58 |
| 45 | 1.042 | 46.89 | 508.61 | 95 | 0.982 | 93.29 | 462.21 |
| 50 | 1.036 | 51.80 | 503.70 | 100 | 0.976 | 97.60 | 457.90 |

The latent heat for saturated vapor is subject to three corrections in determining the available latent heat. First, for the temperature of the liquid which must be cooled from its supply temperature to the temperature corresponding to the back pressure, as in the table above; second, for wetness of vapor, a deduction of 5.555 B.T.U. for each 1% of moisture; third, for superheat of vapor in case it leaves the expansion coils or cooler hotter than the temperature corresponding to the pressure, an addition of the number of degrees superheat multiplied by the specific heat, taken as 0.508.

Solubility of Ammonia. (Siebel.) — One pound of water will dissolve the following weights of ammonia at the pressures and temperatures F° stated.

| Abs. Press. per sq. in. | 32° | 68° | 104° | Abs. Press. per sq. in. | 32° | 68° | 104° | Abs. Press. per sq. in. | 32° | 68° | 104° |
|-------------------------|-------|-------|-------|-------------------------|-------|-------|-------|-------------------------|-------|-------|-------|
| lb. | lb. | lb. | lb. | lb. | lb. | lb. | lb. | lb. | lb. | lb. | lb. |
| 14.67 | 0.899 | 0.518 | 0.338 | 21.23 | 1.236 | 0.651 | 0.425 | 27.99 | 1.603 | 0.780 | 0.486 |
| 15.44 | 0.937 | 0.635 | 0.349 | 22.19 | 1.283 | 0.669 | 0.434 | 28.95 | 1.656 | 0.801 | 0.493 |
| 16.41 | 0.980 | 0.556 | 0.363 | 23.16 | 1.330 | 0.685 | 0.445 | 30.88 | 1.758 | 0.842 | 0.511 |
| 17.37 | 1.029 | 0.574 | 0.378 | 24.13 | 1.388 | 0.704 | 0.454 | 32.81 | 1.861 | 0.881 | 0.530 |
| 18.34 | 1.077 | 0.594 | 0.391 | 25.09 | 1.442 | 0.722 | 0.463 | 34.74 | 1.966 | 0.919 | 0.547 |
| 19.30 | 1.126 | 0.613 | 0.404 | 26.06 | 1.496 | 0.741 | 0.472 | 36.67 | 2.070 | 0.955 | 0.565 |
| 20.27 | 1.177 | 0.632 | 0.414 | 27.02 | 1.549 | 0.761 | 0.479 | 38.60 | | 0.992 | 0.579 |

Properties of Saturated Vapors. — The figures in the following table are given by Lorenz, on the authority of Mollier and of Zeuner.

| ° F. | Heat of Vaporization, B.T.U. per lb. | | | Heat of Liquid, B.T.U. per lb. | | | Absolute Pressure, lbs. per sq. in. | | | Volume of 1 lb., cubic feet. | | |
|-------|--------------------------------------|-----------------|-----------------|--------------------------------|-----------------|-----------------|-------------------------------------|-----------------|-----------------|------------------------------|-----------------|-----------------|
| | NH ₃ | CO ₂ | SO ₂ | NH ₃ | CO ₂ | SO ₂ | NH ₃ | CO ₂ | SO ₂ | NH ₃ | CO ₂ | SO ₂ |
| - 4° | 589.0 | 117.6 | 171.0 | -31.21 | -17.19 | -11.16 | 27.1 | 288.7 | 9.27 | 10.33 | 0.312 | 8.06 |
| + 14° | 580.0 | 110.7 | 168.2 | -15.89 | - 9.00 | - 5.69 | 41.5 | 385.4 | 14.75 | 6.92 | 0.229 | 5.27 |
| 32° | 569.0 | 99.8 | 164.2 | 0 | 0 | 0 | 61.9 | 503.5 | 22.53 | 4.77 | 0.167 | 3.59 |
| 50° | 555.5 | 86.0 | 158.9 | 16.51 | 10.28 | 5.90 | 89.1 | 650.1 | 33.26 | 3.38 | 0.120 | 2.44 |
| 68° | 539.9 | 66.5 | 152.5 | 33.58 | 23.08 | 12.03 | 125.0 | 826.4 | 47.61 | 2.47 | 0.083 | 1.71 |
| 86° | 521.4 | 27.1 | 144.8 | 51.28 | 45.45 | 18.34 | 170.8 | 1040. | 66.36 | 1.83 | 0.048 | 1.22 |
| 104° | 500.4 | | 135.9 | 69.58 | | 24.88 | 227.7 | | 90.30 | 1.39 | | 0.88 |

The figures for CO₂ in the above table differ widely from those of Regnault, and are no doubt more reliable.

Heat Generated by Absorption of Ammonia. (Berthelot, from Siebel.) — Heat developed when a solution of 1 lb. NH₃ in n lbs. water is diluted with a great amount of water = $Q = 142/n$ B.T.U. Assuming 925 B.T.U. to be developed when 1 lb. NH₃ is absorbed by a great deal (say 200 lbs.) of water, the heat developed in making solutions of different strengths (1 lb. NH₃ to n lbs. water) = $Q_1 = 925 - 142/n$ B.T.U. Heat developed when b lbs. NH₃ is added to a solution of 1 lb. NH₃ + n lbs. water = $Q_3 = 925 - 142(2b + b^2)/n$ B.T.U.

Let the weak liquor enter the absorber with a strength of 10%, = 1 lb. NH₃ + 9 lbs. water, and the strong liquor leave the absorber with a strength of 25%, = 3 lbs. NH₃ + 9 lbs. water, $b = 2$, $n = 9$; $Q_3 = 925 \times 2 - 142(4 + 4)/9 = 1724$ B.T.U. Hence by dissolving 2 lbs. of ammonia gas or vapor in a solution of 1 lb. ammonia in 9 lbs. water we obtain 12 lbs. of a 25% solution, and the heat generated is 1724 B.T.U.

Cooling Effect, Compressor Volume, and Power Required. — The following table gives the theoretical results computed on the basis of a temperature in the evaporator of 14° F. and in the condenser of 68° F.; in the first three columns of figures the cooling agent is supposed to flow through the regulating valve with this latter temperature; in the last three it is previously cooled to 50° F.

From the stroke-volume per 100,000 B.T.U. the minimum theoretical horse-power is obtained as follows: Adiabatic compression is assumed for the ratio of the absolute condenser pressure to that of the vaporizer, and the mean pressure through the stroke thus found, in lbs. per sq. ft.; multiplying this by the stroke volume per hour and dividing by 1,980,000 gives the net horse-power. The ratio of the mean effective pressure, M.P., to the vaporizer pressure, V.P., for different ratios of condenser pressure, C.P., to vaporizer pressure is given on the next page.

Cooling Effect, Compressor Volume, and Power Required, with Different Cooling Agents. (Lorenz.)

| Cooling Agent. | NH ₃ | CO ₂ | SO ₂ | NH ₃ | CO ₂ | SO ₂ |
|--|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1. Temp. in front of regulating valve..... | 68 | 68 | 68 | 50 | 50 | 50 |
| 2. Vaporizer pressure, lbs. per sq. in..... | 41.5 | 385.4 | 14.75 | 41.5 | 385.4 | 14.75 |
| 3. Condenser pressure, lbs. per sq. in..... | 125.0 | 826.4 | 47.61 | 125.0 | 826.4 | 47.61 |
| 4. Heat of evaporation, B.T.U. per lb..... | 580.2 | 110.7 | 168.2 | 580.2 | 110.7 | 168.2 |
| 5. Heat imparted to the liquid..... | 49.47 | 32.08 | 17.72 | 32.4 | 19.28 | 11.59 |
| 6. Cold produced per lb. B.T.U..... | 530.73 | 78.62 | 150.48 | 547.8 | 91.42 | 156.61 |
| 7. Cooling agent circulated for yield of 100,000 B.T.U. per hour, lbs..... | 188.4 | 1272. | 664.3 | 182.5 | 1094. | 638.5 |
| 8. Stroke volume for 100,000 B.T.U. per hour, cu. ft..... | 1,300 | 292 | 3,507 | 1,264 | 242 | 3,365 |
| 9. Minimum H.P. per 100,000 B.T.U. per hour..... | 4.98 | 4.98 | 4.98 | 4.98 | 4.98 | 4.98 |
| 10. Ratio Heat of evap. + cold produced..... | 1.093 | 1.408 | 1.118 | 1.059 | 1.211 | 1.074 |
| 11. Ratio total work to minimum..... | 1.175 | 1.513 | 1.202 | 1.138 | 1.302 | 1.155 |
| 12. Total I.H.P. per 100,000 B.T.U. per hour..... | 5.85 | 7.53 | 5.99 | 5.67 | 6.48 | 5.75 |
| 13. Cooling effect per I.H.P. hr..... | 17,100 | 13,300 | 16,700 | 17,600 | 15,400 | 17,400 |

RATIOS OF CONDENSER PRESSURE, C. P., AND MEAN EFFECTIVE PRESSURE, M. P., TO VAPORIZER PRESSURE, V. P.

| CP ÷ VP | MP ÷ VP | CP ÷ VP | MP ÷ VP | CP ÷ VP | MP ÷ VP | CP ÷ VP | MP ÷ VP | CP ÷ VP | MP ÷ VP | CP ÷ VP | MP ÷ VP |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.0 | 0. | 2.0 | 0.752 | 3.0 | 1.249 | 4.0 | 1.684 | 5.0 | 1.947 | 6.0 | 2.216 |
| 1.2 | 0.186 | 2.2 | 0.865 | 3.2 | 1.344 | 4.2 | 1.711 | 5.2 | 2.006 | 7.0 | 2.454 |
| 1.4 | 0.350 | 2.4 | 0.970 | 3.4 | 1.414 | 4.4 | 1.766 | 5.4 | 2.062 | 8.0 | 2.666 |
| 1.6 | 0.487 | 2.6 | 1.070 | 3.6 | 1.491 | 4.6 | 1.829 | 5.6 | 2.116 | 9.0 | 2.858 |
| 1.8 | 0.630 | 2.8 | 1.163 | 3.8 | 1.564 | 4.8 | 1.891 | 5.8 | 2.168 | 10.0 | 3.036 |

The minimum theoretical horse-power thus obtained is increased by the ratio of the heat of evaporation to the available cooling action (line 4 ÷ line 6, = line 10 of the table) and by an allowance for the resistance of the valves taken at 7.5% to obtain the total H.P. given in the table.

To the theoretical horse-power given in line 12 Lorenz makes numerous additions, viz.: friction of the compression and driving machine 0.90, 1.10, 0.90, 0.85, 0.95, 0.85 respectively for the six columns in the table; also H.P. for stirring 0.3; for cooling-water pumps, 0.45; for brine pumps, 2.2; for transmission of power, 0.6, making the total H.P. for the six cases 10.30, 12.18, 10.44, 10.07, 10.98, 10.15. He also makes deductions from the theoretical generation of cold of 100,000 B.T.U. per hour, for a brewery cooling installation, for irregularities of valves, etc., for NH₃ and SO₂ machines 10% and for CO₂ machines 5%; for cooling loss through stirring 765 B.T.U., through brine pumps 5610 B.T.U., and through radiation 4500 B.T.U., making the net cooling for NH₃ and SO₂ machines 79,125 B.T.U. and for CO₂ machines 84,125 B.T.U., and the cold generated per effective H.P. in the six cases, 7682, 6908, 7578, 7848, 7662, and 7796 B.T.U.

The figures given in the tables are not to be considered as holding generally or extended to other condenser and evaporator temperatures. Each change of condition requires a separate calculation. The final

results indicate that for the various cooling systems no appreciable difference exists in the work required for the same amount of cold delivered at the place where it is to be applied.

Properties of Brine Used to Absorb Refrigerating Effect of Ammonia. (J. E. Denton, *Trans. A. S. M. E.*, x, 799.) — A solution of Liverpool salt in well-water having a specific gravity of 1.17, or a weight per cubic foot of 73 lbs., will not sensibly thicken or congeal at 0° F.

The mean specific heat between 39° and 16° Fahr. was found by Denton to be 0.805. Brine of the same specific gravity has a specific heat of 0.805 at 65° Fahr., according to Naumann.

Naumann's values are as follows (*Lehr- und Handbuch der Thermochemie*, 1882):

| | | | | | | | |
|---------------------|-------|--------|-------|-------|-------|-------|-------|
| Specific heat | 0.791 | 0.805* | 0.863 | 0.895 | 0.931 | 0.962 | 0.978 |
| Specific gravity... | 1.187 | 1.170 | 1.103 | 1.072 | 1.044 | 1.023 | 1.012 |

Properties of Salt Brine (Carbondale Calcium Co.)

| | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|-------|
| Deg. Baumé 60° F..... | 1 | 5 | 10 | 15 | 19 | 23 |
| Deg. Salinometer 60° F..... | 4 | 20 | 40 | 60 | 80 | 100 |
| Sp. gravity 60° F..... | 1.007 | 1.037 | 1.073 | 1.115 | 1.150 | 1.191 |
| Per cent of salt, by wt.... | 1 | 5 | 10 | 15 | 20 | 25 |
| Wt. of 1 gallon, lbs..... | 8.40 | 8.65 | 8.95 | 9.30 | 9.60 | 9.94 |
| Wt. of 1 cu. ft., lbs..... | 62.8 | 64.7 | 66.95 | 69.57 | 71.76 | 74.26 |
| Freezing point ° F..... | 31.8 | 25.4 | 18.6 | 12.2 | 6.86 | 1.00 |
| Specific heat..... | 0.992 | 0.960 | 0.892 | 0.855 | 0.829 | 0.783 |

Chloride of Calcium solution is commonly used instead of brine. According to Naumann, a solution of 1.0255 sp. gr. has a specific heat of 0.957. A solution of 1.163 sp. gr. in the test reported in *Eng'g*, July 22, 1887, gave a specific heat of 0.827.

H. C. Dickinson (*Science*, April 23, 1909) gives the following values of the specific heat of solutions of chemically pure calcium chloride.

| Density | Specific Heat | Temperature, C. |
|-----------|-------------------|------------------|
| 1.07..... | 0.869 + 0.00057 t | (- 5° to + 15°) |
| 1.14..... | 0.773 + 0.00064 t | (- 10° to + 20°) |
| 1.20..... | 0.710 + 0.00064 t | (- 20° to + 20°) |
| 1.26..... | 0.662 + 0.00064 t | (- 25° to + 20°) |

The advantages of chloride of calcium solution are its lower freezing point and that it has little or no corrosive action on iron and brass. Calcium chloride is sold in the fused or granulated state, in steel drums, containing about 75% anhydrous chloride and 25% water, or in solution containing 40 to 50% anhydrous chloride in tank cars. The following data are taken from the catalogue of the Carbondale Calcium Co.

PROPERTIES OF "SOLVAY" CALCIUM CHLORIDE SOLUTION.

| Deg. Baumé, 60° F. | Spec. Grav., 60° F. | Per cent, CaCl. | Freezes at Deg. F. | Deg. Baumé, 60° F. | Spec. Grav., 60° F. | Per cent, CaCl. | Freezes at Deg. F. | Deg. Baumé, 60° F. | Spec. Grav., 60° F. | Per cent, CaCl. | Freezes at Deg. F. |
|-----------------------|------------------------|--------------------|-----------------------|-----------------------|------------------------|--------------------|-----------------------|-----------------------|------------------------|--------------------|-----------------------|
| 1. | 1.007 | 1 | +31.10 | 21 | 1.169 | 19 | + 1.76 | 32 | 1.283 | 30 | -54.40 |
| 5.5 | 1.041 | 5 | 27.68 | 22 | 1.179 | 20 | - 1.48 | 35 | 1.316 | 33 | -25.24 |
| 11 | 1.085 | 10 | 22.38 | 23 | 1.189 | 21 | - 4.90 | 35.5 | 1.327 | 34 | - 9.76 |
| 17 | 1.131 | 15 | 12.20 | 26 | 1.219 | 24 | -17.14 | 36.5 | 1.338 | 35 | + 2.84 |
| 20 | 1.159 | 18 | 4.64 | 29 | 1.250 | 27 | -32.62 | 37.5 | 1.349 | 36 | 14.36 |

Quantity of 75% calcium chloride required to make solutions of different specific gravities and freezing points.

| | | | | | | | |
|------------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Sp. gravity..... | 1.250 | 1.225 | 1.200 | 1.175 | 1.150 | 1.125 | 1.100 |
| Lbs. per cu ft. solu- tion..... | 28.06 | 25.06 | 22.05 | 19.15 | 16.26 | 13.47 | 10.70 |
| Lbs. per gallon..... | 3.76 | 3.36 | 2.95 | 2.56 | 2.18 | 1.80 | 1.43 |
| Freezing point ° F. . | -32.6 | -19.5 | -8.7 | Zero | +7.5 | +13.3 | +18.5 |

* Interpolated.

Boiling points of calcium chloride solutions:

| | | | | | | |
|--------------------------|-------|-------|-------|-------|-------|-------------------|
| Sp. Gr. at 59° F..... | 1.104 | 1.185 | 1.268 | 1.341 | 1.383 | solid at 59° |
| Boiling point ° F..... | 215.6 | 221.0 | 230.0 | 240.8 | 248.0 | 266.0 282.2 306.5 |
| Sp. gr. at boiling point | 1.085 | 1.119 | 1.209 | 1.308 | 1.365 | 1.452 1.526 1.619 |

“Ice-melting Effect.” — It is agreed that the term “ice-melting effect” means the cold produced in an insulated bath of brine, on the assumption that each 144 B.T.U. represents one pound of ice, this being the latent heat of fusion of ice, or the heat required to melt a pound of ice at 32° to water at the same temperature.

The performance of a machine, expressed in pounds or tons of “ice-melting capacity,” does not mean that the refrigerating-machine would make the same amount of actual ice, but that the cold produced is equivalent to the effect of the melting of ice at 32° to water of the same temperature.

In making artificial ice the water frozen is generally about 70° F. when submitted to the refrigerating effect of a machine; second, the ice is chilled from 12° to 20° below its freezing-point; third, there is a dissipation of cold, from the exposure of the brine tank and the manipulation of the ice-cans: therefore the weight of actual ice made, multiplied by its latent heat of fusion, 144 thermal units, represents only about three-fourths of the cold produced in the brine by the refrigerating fluid per I.H.P. of the engine driving the compressing-pumps. Again, there is considerable fuel consumed to operate the brine-circulating pump, the condensing-water and feed-pumps, and to reboil, or purify, the condensed steam from which the ice is frozen. This fuel, together with that wasted in leakage and drip water, amounts to about one-half that required to drive the main steam-engine. Hence the pounds of actual ice manufactured from distilled water is just about half the equivalent of the refrigerating effect produced in the brine per indicated horse-power of the steam-cylinders.

When ice is made directly from natural water by means of the “plate system,” about half of the fuel, used with distilled water, is saved by avoiding the reboiling, and using steam expansively in a compound engine.

Ether-machines, used in India, are said to have produced about 6 lbs. of actual ice per pound of fuel consumed.

The ether machine is obsolete, because the density of the vapor of ether, at the necessary working-pressure, requires that the compressing-cylinder shall be about 6 times larger than for sulphur dioxide, and 17 times larger than for ammonia.

Air-machines require about 1.2 times greater capacity of compressing cylinder, and are, as a whole, more cumbersome than ether machines, but they remain in use on shipboard. In using air the expansion must take place in a cylinder doing work, instead of through a simple expansion-cock which is used with vapor machines. The work done in the expansion-cylinder is utilized in assisting the compressor.

The *Allen Dense Air Machine* takes for compression air of considerable pressure which is contained in the machine and in a system of pipes. The air at 60 or 70 lbs. pressure is compressed to 210 or 240 lbs. It is then passed through a coil immersed in circulating water and cooled to nearly the temperature of the water. It then passes into an expander, which is, in construction, a common form of steam-engine with a cut-off valve. This engine takes out of the air a quantity of heat equivalent to the work done by the air while expanding, to the original pressure of 60 or 70 lbs., and reduces its temperature to about 90° to 120° F. below the temperature of the cooling water supply. The return stroke of the piston pushes the air out through insulated pipes to the places that are to be refrigerated, from which it is returned to the compressor.

The air pushed out by the expander is commonly about 35 to 55 below zero F. In arrangements where not all the cold is taken out of the air by the refrigerating apparatus, the highly compressed air after cooling in the coil is further cooled by being brought in surface contact with the returning and still cold air, before entering the expander. By this means temperatures of 70 to 90 below zero may be obtained.

The refrigerating effect in B.T.U. per minute is: Lbs. of air handled per

min. $\times 0.2375 \times$ difference of temperature of air passing out of expander and of that returning to the machine.

Carbon-dioxide Machines are in extensive use on shipboard. S. H. Bunnell (*Eng. News*, April 9, 1903) says there are over 1500 CO₂ plants on shipboard. He describes a large duplex CO₂ compressor built by the Brown-Cochrane Co., Lorain, O. Tests of CO₂ machines by a committee of the Danish Agricultural Society were reported in 1899, in "Ice and Cold Storage," of London. Carbon-dioxide machines are built also by Kroeschel Bros., Chicago.

Methyl-Chloride machines are made by Railway and Stationary Refrigerating Co., New York City. The compressor is a rotary pump. When driven by an electric motor the complete apparatus is very compact, and is therefore suitable for refrigerator cars or other places where space is restricted.

Sulphur-Dioxide Machines. — Results of theoretical calculations are given in a table by Ledoux showing an ice-melting capacity per hour per horse-power ranging from 134 to 63 lbs., and per pound of coal ranging from 44.7 to 21.1 lbs., as the temperature corresponding to the pressure of the vapor in the condenser rises from 59° to 104° F. The theoretical results do not represent the actual.

Prof. Denton says concerning Ledoux's theoretical results: The figures given are higher than those obtained in practice, because the effect of superheating of the gas during admission to the cylinder is not considered. This superheating may cause an increase of work of about 25%. There are other losses due to superheating the gas at the brine-tank, and in the pipe leading from the brine-tank to the compressor, so that in actual practice a sulphur-dioxide machine, working under the conditions of an absolute pressure in the condenser of 56 lbs. per sq. in. and the corresponding temperature of 77° F., will give about 22 lbs. of ice-melting capacity per pound of coal, which is about 60% of the theoretical amount neglecting friction, or 70% including friction.

Sulphur-dioxide machines are not now used in the United States (1910).

Refrigerating-Machines using Vapor of Water. (Ledoux.) — In these machines, sometimes called vacuum machines, water, at ordinary temperatures, is injected into, or placed in connection with, a chamber in which a strong vacuum is maintained. A portion of the water vaporizes, the heat to cause the vaporization being supplied from the water not vaporized, so that the latter is chilled or frozen to ice. If brine is used instead of pure water, its temperature may be reduced below the freezing-point of water. The water vapor is compressed from, say, a pressure of 0.1 lb. per sq. in. to 1½ lbs. and discharged into a condenser. It is then condensed and removed by means of an ordinary air-pump. The principle of action of such a machine is the same as that of volatile-vapor machines.

A theoretical calculation for ice-making, assuming a lower temperature of 32° F., a pressure in the condenser of 1½ lbs. per sq. in., and a coal consumption of 3 lbs. per I.H.P. per hour, gives an ice-melting effect of 34.5 lbs. per pound of coal, neglecting friction. Ammonia for ice-making conditions gives 40.9 lbs. The volume of the compressing cylinder is about 150 times the theoretical volume for an ammonia machine for these conditions.

[The Patten Vacuum Ice Co., of Baltimore, has a large plant on this system in operation (1910).]

Ammonia Compression-machines. — "Cold" vs. "Dry" Systems of Compression. — In the "cold" system or "humid" system some of the ammonia entering the compression cylinder is liquid, so that the heat developed in the cylinder is absorbed by the liquid and the temperature of the ammonia thereby confined to the boiling-point due to the condenser-pressure. No jacket is therefore required about the cylinder.

In the "dry" or "hot" system all ammonia entering the compressor is gaseous, and the temperature becomes by compression several hundred degrees greater than the boiling-point due to the condenser-pressure. A water-jacket is therefore necessary to permit the cylinder to be properly lubricated.

Dry, Wet and Flooded Systems. (York Mfg. Co.) — An expansion system, or one where the ammonia leaves the coil slightly superheated, requires about 33½% more pipe surface than a wet compression system,

in which the ammonia leaves the coils containing sufficient entrained liquid to maintain a wet compression condition in the compressor.

The flooded system is one where the ammonia is allowed to flow through the coils and into a trap, where the gas is separated from the liquid, the gas passing on to the compressor, while the liquid goes around through the coils again, together with the fresh liquid, which is fed into the trap. Such a system requires only about one-half the evaporating surface that an expansion system does to do the same work. The relative proportions of the three systems may be expressed as follows:

A Dry Compression plant will need, with an Expansion Evaporating System, a medium size compressor, a large size evaporating system, a small amount of ammonia.

A Dry Compression plant will need, with a Flooded Evaporating System, a small size compressor, a small size evaporating system, a large amount of ammonia.

A Wet Compression plant will need, with a Wet Compression Evaporating System, a large size compressor, a medium size evaporating system, a medium amount of ammonia.

The Ammonia Absorption-machine comprises a generator which contains a concentrated solution of ammonia in water; this generator is heated either directly by a fire, or indirectly by pipes leading from a steam-boiler. The vapor passes first into an "analyzer," a chamber connected with the upper part of the generator which separates some of the water from the vapor, then into a rectifier, where the vapor is partly cooled, precipitating more water, which returns to the generator, and then to the condenser. The upper part of the cooler or brine-tank is in communication with the lower part of the condenser.

An absorption-chamber is filled with a weak solution of ammonia; a tube puts this chamber in communication with the cooling-tank.

The absorption-chamber communicates with the boiler by two tubes: one leads from the bottom of the generator to the top of the chamber, the other leads from the bottom of the chamber to the top of the generator. Upon the latter is mounted a pump, to force the liquid from the absorption-chamber, where the pressure is maintained at about one atmosphere, into the generator, where the pressure is from 8 to 12 atmospheres.

To work the apparatus the ammonia solution in the generator is first heated. This releases the gas from the solution, and the pressure rises. When it reaches the tension of the saturated gas at the temperature of the condenser there is a liquefaction of the gas, and also of a small amount of steam. By means of a cock the flow of the liquefied gas into the refrigerating coils contained in the cooler is regulated. It is here vaporized by absorbing the heat from the substance placed there to be cooled. As fast as it is vaporized it is absorbed by the weak solution in the absorbing-chamber.

Under the influence of the heat in the boiler the solution is unequally saturated, the stronger solution being uppermost. The weaker portion is conveyed by the pipe entering the top of the absorbing-chamber, the flow being regulated by a cock, while the pump sends an equal quantity of strong solution from the chamber back to the boiler.

The working of the apparatus depends upon the adjustment and regulation of the flow of the gas and liquid; by these means the pressure is varied, and consequently the temperature in the cooler may be controlled.

The working is similar to that of compression-machines. The absorption-chamber fills the office of aspirator, and the generator plays the part of compressor. The mechanical force producing exhaustion is here replaced by the affinity of water for ammonia gas, and the mechanical force required for compression is replaced by the heat which severs this affinity and sets the gas at liberty.

Reece's absorption apparatus (1870) is thus described by Wallis-Taylor. The charge of liquid ammonia (26° Baumé) is vaporized by the application of heat, and the mixed vapor passed to the analyzer and rectifier, wherein the bulk of the water is condensed at a comparatively elevated temperature and returned to the generator. The ammoniacal vapor or gas is then passed to the condenser, where it is liquefied under the combined action of the cooling-water and of the pressure maintained in the generator. The liquid ammonia, practically anhydrous, is then used in the refrigerator, and the vapor therefrom, still under considerable pressure, is admitted to

the cylinder of an engine used to drive a pump for returning the strong solution to the generator, after which it is passed to the absorber, where it meets and is absorbed by the weak liquor from the generator, and the strong liquor so formed is forced back into the generator by means of the pump. The temperature exchanger, introduced in 1875, provides for the hot liquor on its way from the generator to the absorber giving up its heat to the cooler liquid from the absorber on its way to the generator.

Wallis-Taylor describes also marine refrigerating, ice-making, cold storage, the application of refrigeration in breweries, dairies, etc.; and the management and testing of apparatus.

For the best results the following conditions are necessary (Voorhees): 1. The generator should have ample liquid evaporating surface to make dry gas. 2. The temperature of the gas to the rectifier should be as low as possible. 3. The drip liquor returned to the generator from the rectifier should be as hot as possible. 4. The gas from the rectifier to the condenser should not be over 10° to 50° hotter than the condensing temperature of the gas. 5. The exchanger should exchange upwards of 90% of the heat of the hot weak liquor to the cold strong liquor. The weight of strong liquor pumped should be from 7 to 8 times that of the anhydrous ammonia circulated in the refrigerator.

To produce one ton of refrigeration at 8.5 lbs. suction and 170 lbs. gauge condenser pressure, about 3.5 times as many heat units are actually used by an absorption machine as by a compression machine (compound condensing engine driven), but, owing to the low efficiency of the steam engine, due to the heat wasted in the exhaust and in cylinder condensation, the actual weight of steam used per hour per ton of refrigeration is the same for both the absorption machine and the compressor.

Relative Performance of Ammonia Compression- and Absorption-machines, assuming no Water to be Entrained with the Ammonia-gas in the Condenser. (Denton and Jacobus, *Trans. A. S. M. E.*, xiii.) — It is assumed in the calculation for both machines that 1 lb. of coal imparts 10,000 B.T.U. to the boiler. The condensed steam from the generator of the absorption-machine is assumed to be returned

| Condenser. | | Refrigerating Coils. | | Temp. of Absorber, degrees F. | Pounds of Ice-melting Effect per lb. of Coal. | | | | | Heat furnished to generator of absorption-machine, B.T.U. per lb. of ammonia circulated. |
|------------------------|-------------------------------------|------------------------|-------------------------------------|-------------------------------|---|--|---|--|------|--|
| Temp. in degrees Fahr. | Absolute pressure, lbs. per sq. in. | Temp. in degrees Fahr. | Absolute pressure, lbs. per sq. in. | | Compress. Machine. | | Absorption-Machine.* | | | |
| | | | | | Using 3 lbs. of coal per hour per I.H.P. | Using 1.6 lbs. of coal per hour per I.H.P. | Absorption-machine in which the ammonia circulating-pump exhausts into the generator. | In which the amm. circ. pump exhausts into the atmosphere through a heater, yielding 212° temp. to the feed-water. | | |
| 61.2 | 110.6 | 5 | 33.7 | 61.2 | 38.1 | 71.4 | 38.1 | 33.5 | 969 | |
| 59.0 | 106.0 | 5 | 33.7 | 59.0 | 39.8 | 74.6 | 38.3 | 33.9 | 967 | |
| 59.0 | 106.0 | 5 | 33.7 | 130.0 | 39.8 | 74.6 | 39.8 | 35.1 | 931 | |
| 59.0 | 106.0 | -22 | 16.9 | 59.0 | 23.4 | 43.9 | 36.3 | 31.5 | 1000 | |
| 86.0 | 170.8 | 5 | 33.7 | 86.0 | 25.0 | 46.9 | 35.4 | 28.6 | 988 | |
| 86.0 | 170.8 | 5 | 33.7 | 130.0 | 25.0 | 46.9 | 36.2 | 29.2 | 966 | |
| 86.0 | 170.8 | -22 | 16.9 | 86.0 | 16.5 | 30.8 | 33.3 | 26.5 | 1025 | |
| 86.0 | 170.8 | -22 | 16.9 | 130.0 | 16.5 | 30.8 | 34.1 | 27.0 | 1002 | |
| 104.0 | 227.7 | 5 | 33.7 | 104.0 | 19.6 | 36.8 | 33.4 | 25.1 | 1002 | |
| 104.0 | 227.7 | -22 | 16.9 | 104.0 | 13.5 | 25.3 | 31.4 | 23.4 | 1041 | |

* 5% of water entrained in the ammonia will lower the economy of the absorption-machine about 15% to 20% below the figures given in the table.

to the boiler at the temperature of the steam entering the generator. The engine of the compression-machine is assumed to exhaust through a feed-water heater that heats the feed-water to 212° F. The engine is assumed to consume $26\frac{1}{4}$ lbs. of water per hour per horse-power. The figures for the compression-machine include the effect of friction, which is taken at 15% of the net work of compression.

(For discussion of the efficiency of the absorption system, see Ledoux's work; paper by Prof. Linde, and discussion on the same by Prof. Jacobus, *Trans. A. S. M. E.*, xiv, 1416, 1436; and papers by Denton and Jacobus, *Trans. A. S. M. E.*, x, 792, xiii, 507.

Relative Efficiency of a Refrigerating-Machine.—The efficiency of a refrigerating-machine is sometimes expressed as the quotient of the quantity of heat received by the ammonia from the brine, that is, the quantity of useful work done, divided by the heat equivalent of the mechanical work done in the compressor. Thus in column 1 of the table of performance of the 75-ton machine (page 1311) the heat given by the brine to the ammonia per minute is 14,776 B.T.U. The horse-power of the ammonia cylinder is 65.7, and its heat equivalent = $65.7 \times 33,000 \div 778 = 2786$ B.T.U. Then $14,776 \div 2786 = 5.304$, efficiency. The apparent paradox that the efficiency is greater than unity, which is impossible in any machine, is thus explained. The working fluid, as ammonia, receives heat from the brine and rejects heat into the condenser. (If the compressor is jacketed, a portion is rejected into the jacket-water.) The heat rejected into the condenser is greater than that received from the brine; the difference (plus or minus a small difference radiated to or from the atmosphere) is heat received by the ammonia from the compressor. The work to be done by the compressor is not the mechanical equivalent of the refrigeration of the brine, but only that necessary to supply the difference between the heat rejected by the ammonia into the condenser and that received from the brine. If cooling water colder than the brine were available, the brine might transfer its heat directly into the cooling water, and there would be no need of ammonia or of a compressor; but since such cold water is not available, the brine rejects its heat into the colder ammonia, and then the compressor is required to heat the ammonia to such a temperature that it may reject heat into the cooling water.

The maximum theoretical efficiency of a refrigerating machine is expressed by the quotient $T_0 \div (T_1 - T_0)$, in which T_1 is the highest and T_0 the lowest temperature of the ammonia or other refrigerating agent.

The efficiency of a refrigerating plant referred to the amount of fuel consumed is

$$\left. \begin{array}{l} \text{Ice-melting capacity} \\ \text{per pound of fuel} \end{array} \right\} = \frac{\left\{ \begin{array}{l} \text{Pounds circulated per hour} \\ \times \text{specific heat} \times \text{range} \\ \text{of temperature} \end{array} \right\} \text{ of brine or other} \\ \text{circulating fluid}}{144 \times \text{pounds of fuel used per hour}}$$

The ice-melting capacity is expressed as follows:

$$\left. \begin{array}{l} \text{Tons (of 2000 lbs.)} \\ \text{ice-melting ca-} \\ \text{pacity per 24 hours} \end{array} \right\} = \frac{\left\{ \begin{array}{l} 24 \times \text{pounds} \\ \times \text{specific heat} \\ \times \text{range of temp.} \end{array} \right\} \text{ of brine circulated per} \\ \text{hour}}{144 \times 2000}$$

The analogy between a heat-engine and a refrigerating-machine is as follows: A steam-engine receives heat from the boiler, converts a part of it into mechanical work in the cylinder, and throws away the difference into the condenser. The ammonia in a compression refrigerating-machine receives heat from the brine-tank or cold-room, receives an additional amount of heat from the mechanical work done in the compression-cylinder, and throws away the sum into the condenser. The efficiency of the steam-engine = work done \div heat received from boiler. The efficiency of the refrigerating-machine = heat received from the brine-tank or cold-room \div heat required to produce the work in the compression-cylinder. In the ammonia absorption-apparatus, the ammonia receives heat from the brine-tank and additional heat from the boiler or generator, and rejects the sum into the condenser and into the cooling water supplied to the absorber. The efficiency = heat received from the brine \div heat received from the boiler.

The Efficiency of Refrigerating Systems depends on the temperature of the condenser water, whether there is sufficient condenser surface for the compressor and whether or not the condenser pipes are free from uncondensable foreign gases. With these things right, condenser pressure for different temperatures of cooling water should be approximately as follows:

| | | | | | | | | | |
|--------------------------------------|-----|-----|-----|-----|-----|-----|-----|--|--|
| 1 gallon per minute per ton per 24 | | | | | | | | | |
| hours—Cooling water, ° F..... | 60 | 65 | 70 | 75 | 80 | 85 | 90 | | |
| Condenser pressure, gage, lb..... | 183 | 200 | 220 | 235 | 255 | 280 | 300 | | |
| Condensed liquid ammonia ° F..... | 95 | 100 | 105 | 110 | 115 | 120 | 125 | | |
| 2 gallons per minute per ton per 24 | | | | | | | | | |
| hours—Condenser pressure, gage, lb., | 130 | 153 | 168 | 183 | 200 | 220 | 235 | | |
| Condensed liquid ammonia, ° F.... | 77 | 85 | 90 | 93 | 100 | 105 | 110 | | |
| 3 gallons per minute per ton per 24 | | | | | | | | | |
| hours—Condenser pressure, gage, lb.. | 125 | 140 | 155 | 170 | 185 | 200 | 215 | | |
| Condensed liquid ammonia, ° F..... | 75 | 85 | 90 | 93 | 95 | 100 | 105 | | |

The evaporating or back pressure within the expansion coils of a refrigerating system depends upon the temperatures on the outside of such coils, i.e., the air or brine to be cooled. For average practice back pressures for the production of required temperatures should be approximately as follows:

| | | | | | | | | | |
|------------------------------------|----|----|----|----|----|----|----|----|----|
| Temperature of room, ° F..... | 10 | 15 | 20 | 28 | 32 | 36 | 40 | 50 | 60 |
| Back pressure, gage, lb..... | 10 | 12 | 15 | 22 | 25 | 27 | 30 | 35 | 40 |
| Temperature of ammonia, ° F... -10 | -5 | 0 | 8 | 12 | 14 | 17 | 22 | 26 | |

The condenser pressure should be kept as low as possible and the back pressure as high as possible, narrow limits between such pressures being as important to the efficiency of a refrigerating system as wide ones are to that of a steam engine in which the economy increases with the range between boiler pressure and condenser pressure. (F. E. Matthews, *Power*, Jan. 26, 1909.)

Cylinder-heating.—In compression-machines employing volatile vapors the principal cause of the difference between the theoretical and the practical result is the heating of the ammonia, by the warm cylinder walls, during its entrance into the compressor, thereby expanding it, so that to compress a pound of ammonia a greater number of revolutions must be made by the compressing-pumps than corresponds to the density of the ammonia-gas as it issues from the brine-tank.

Volumetric Efficiency.—The volumetric efficiency of a compressor is the ratio of the actual weight of ammonia pumped to the amount calculated from the piston displacement. Mr. Voorhees deduces from Denton's experiments the formula: Volumetric efficiency = $E = 1 - (t_1 - t_0)/1330$, in which t_1 = the theoretical temperature of gas after compression and t_0 = temperature of gas delivered to the compressor. The temperature $t_1 = T_1 - 460$, is calculated from the formula for adiabatic compression, $T_1 = T_0 (P_1/P_0)^{0.24}$, in which T_1 and T_0 are absolute temperatures and P_1 and P_0 absolute pressures. In eight tests by Prof. Denton the volumetric efficiency ranged from 73.5% to 84%, and they vary less than 1% from the efficiencies calculated by the formula. The temperature of the gas discharged from the compressor averaged 57° less than the theoretical.

The volumetric efficiency of a dry compressor is greatest when the vapor comes to the compressor with little or no superheat; 30° superheat of the suction gas reduces the capacity of the compressor 4%, and 100° 9%.

The following table (from Voorhees) gives the theoretical discharge temperatures (t_1) and volumetric efficiencies (E) by the formula, and the actual cubic feet of displacement of compressor (F) per ton of refrigeration per minute for the given gauge pressures of suction and condenser.

| Suction pressures. | 0 | | | 15 | | | 30 | | |
|----------------------|-------|------|-------|-------|------|-------|-------|------|-------|
| | t_1 | E | F | t_1 | E | F | t_1 | E | F |
| Cond. press. 140.... | 323° | 0.76 | 10.35 | 358° | 0.73 | 11.02 | 388° | 0.71 | 11.57 |
| Cond. press. 170.... | 221° | 0.83 | 4.57 | 254° | 0.81 | 4.78 | 280° | 0.79 | 5.03 |
| Cond. press. 200.... | 167° | 0.87 | 2.96 | 192° | 0.86 | 3.07 | 216° | 0.84 | 3.21 |

Pounds of Ammonia per Minute to Produce 1 Ton of Refrigeration, and Percentage of Liquid Evaporated at the Expansion Valve.

| Condenser, Pressure and Temperature. | 140 lbs., 80°. | 170 lbs., 90°. | 200 lbs., 100°. |
|--|------------------|------------------|------------------|
| Refrigerator, pressure and temperature 0 lbs., -29°... | 0.431 lb., 19% | 0.441 lb., 20.8% | 0.451 lb., 22.5% |
| Refrigerator pressure and temperature 15 lbs., -0°... | 0.420 lb., 14.4% | 0.430 lb., 16.2% | 0.440 lb., 18.0% |
| Refrigerator pressure and temperature, 30 lbs., -17°. | 0.415 lb., 11.6% | 0.425 lb., 13.4% | 0.434 lb., 15.2% |

Mean Effective Pressure, and Horse-power. — Voorhees deduces the following (*Ice and Refrig.*, 1902): $M.E.P. = 4.333 p_0 [(p_1/p_0)^{0.231} - 1]$, p_0 = suction and p_1 condenser pressure, abs. lbs. per sq. in. The maximum M.E.P. occurs when $p_0 = p_1 \div 3.113$. The percentage of stroke during which the gas is discharged from the compressor is $V_1 = (p_0/p_1)^{0.769}$.

The compressor horse-power, C.H.P., is $0.00437 F \times M.E.P.$

The friction of the compressor and its engine combined is given by Voorhees as $33\frac{1}{3}\%$ of the compressor H.P. or 25% of the engine H.P. Values of the mean effective pressure per ton of refrigeration (M), the compressor horse-power (C) and the engine horse-power (E) are given below for the conditions named.

| Suction pressure. | 0 | | | 15 | | | 30 | | |
|----------------------|------|------|------|------|------|------|------|------|------|
| | (M) | (C) | (E) | (M) | (C) | (E) | (M) | (C) | (E) |
| Cond. press., 140... | 46.5 | 2.10 | 2.80 | 59.5 | 1.19 | 1.59 | 64.5 | 0.83 | 1.11 |
| Cond. press., 170... | 50.5 | 2.42 | 3.23 | 67.0 | 1.40 | 1.87 | 75.0 | 1.00 | 1.33 |
| Cond. press., 200... | 55.0 | 2.78 | 3.71 | 74.5 | 1.64 | 2.19 | 85.0 | 1.19 | 1.59 |

By cooling the liquid between the condenser and the expansion valve the capacity will be increased and the horse-power per ton reduced. With compression from 15 to 170 lbs., if the liquid at the expansion valve is cooled to 76° instead of 90° the H.P. per ton will be reduced 3% .

Prof. Lucke deduces a formula for the I.H.P. per ton of refrigerating capacity, as follows:

p = mean effective pressure, lbs. per sq. in.; L = length of stroke in ft.; a = area of piston in sq. ins.; n = no. of compressions per minute; E_c = apparent volumetric efficiency, the ratio of the volume of ammonia apparently taken in per stroke to the full displacement of the piston; w_c = weight of 1 cu. ft. of ammonia vapor at the back pressure, as it exists in the cylinder when compression begins; L_c = latent heat of vaporization available for refrigeration; $288,000$ = B.T.U. equivalent to 1 ton of refrigeration; T = tons refrigeration per 24 hours.

$$\frac{I.H.P.}{T} = \frac{pLan \div 33,000}{LaE_c n w_c \times L_c \times 60 \times 24} = \frac{0.87}{W_c L_c} \times \frac{p}{E_c}$$

$$144 \times 288,000$$

The Voorhees Multiple Effect Compressor is based upon the fact that both the economy and the capacity of a compression machine vary with the back pressure. In the past it has always been necessary to run a compressor at a gas suction pressure corresponding to the lowest required temperature. The multiple effect compressor takes in gas from two or more refrigerators at two or more different suction pressures and temperatures on the same suction stroke of the compressor. The suction gas of the higher pressure helps to compress the lower suction pressure gas. There are two sets of suction valves in the compressor cylinder; the low temperature and corresponding low back pressure being connected to one suction port, usually in the cylinder head, and the high back pressure connected to the other. At the beginning of the stroke the cylinder is filled with the low pressure gas and as the piston reaches the end of its

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suction stroke, the second or high back pressure port is uncovered, the low pressure suction valve closing automatically, and the cylinder is completely filled with gas at the high pressure. By this means the compressor operates with an economy and capacity corresponding to the higher back pressure, making a gain in capacity of often 50% or more. (*Trans. Am. Soc. Refrig. Engrs.*, 1906.)

Quantity of Ammonia Required per Ton of Refrigeration.—The following table is condensed from one given by F. E. Matthews in *Trans. A. S. M. E.*, 1905. The weight in lbs. per minute is calculated from the formula $P = (144 \times 2000) \div [1440l - (h_1 - h_0)]$ in which l is the latent heat of evaporation at the back pressure in the cooler, and h_1 and h_0 the heat of the liquid at the temperatures of the condenser and the cooler respectively. The specific heat of the liquid has been taken at unity. The ton of refrigeration is 2000 lbs. in 24 hours = 288,000 B.T.U.

B = Pounds of ammonia evaporated per minute.

C = Cubic feet of gas to be handled per minute by the compressor.

| l. w. B.P. | Head or Condenser Gauge Pressure and Corresponding Temperature. | | | | | | | | | | |
|-----------------------|---|-------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--------------------|
| | 100 lb. 63.5° | 110 lb. 68° | 120 lb. 72.6° | 130 lb. 77.4° | 140 lb. 80.3° | 150 lb. 83.8° | 160 lb. 87.4° | 170 lb. 90.8° | 180 lb. 93.8° | 190 lb. 96.9° | 200 lb. 100° |
| 572.78 .0556 0 | B .4159 | .4199 | .4240 | .4284 | .4310 | .4343 | .4376 | .4408 | .4440 | .4470 | .4501 |
| | C 7.482 | 7.551 | 7.626 | 7.703 | 7.761 | 7.812 | 7.870 | 7.929 | 7.986 | 8.041 | 8.095 |
| 566.14 .0133 5 | B .4122 | .4160 | .4202 | .4243 | .4271 | .4308 | .4335 | .4366 | .4397 | .4437 | .4458 |
| | C 5.636 | 5.675 | 5.732 | 5.790 | 5.826 | 5.878 | 5.914 | 5.970 | 5.999 | 6.039 | 6.081 |
| 560.69 .0910 10 | B .4093 | .4130 | .4171 | .4204 | .4237 | .4271 | .4302 | .4332 | .4363 | .4392 | .4423 |
| | C 4.502 | 4.543 | 4.587 | 4.625 | 4.662 | 4.698 | 4.733 | 4.766 | 4.799 | 4.833 | 4.865 |
| 556.11 .1083 15 | B .4068 | .4106 | .4145 | .4186 | .4211 | .4244 | .4276 | .4288 | .4336 | .4365 | .4394 |
| | C 3.756 | 3.791 | 3.827 | 3.866 | 3.889 | 3.918 | 3.948 | 3.975 | 4.003 | 4.030 | 4.058 |
| 552.83 .1258 20 | B .4040 | .4077 | .4116 | .4158 | .4182 | .4214 | .4245 | .4275 | .4304 | .4333 | .4362 |
| | C 3.211 | 3.241 | 3.272 | 3.305 | 3.324 | 3.350 | 3.375 | 3.398 | 3.422 | 3.444 | 3.467 |
| 548.40 .1429 25 | B .4025 | .4062 | .4102 | .4140 | .4167 | .4198 | .4229 | .4258 | .4287 | .4316 | .4345 |
| | C 2.819 | 2.843 | 2.870 | 2.898 | 2.916 | 2.938 | 2.959 | 2.980 | 3.000 | 3.020 | 3.040 |
| 545.13 .1600 30 | B .4013 | .4049 | .4088 | .4128 | .4152 | .4184 | .4213 | .4243 | .4273 | .4300 | .4329 |
| | C 2.507 | 2.530 | 2.555 | 2.580 | 2.600 | 2.615 | 2.633 | 2.653 | 2.671 | 2.687 | 2.706 |
| 542.80 .1766 35 | B .3991 | .4028 | .4066 | .4105 | .4130 | .4161 | .4188 | .4220 | .4249 | .4277 | .4305 |
| | C 2.260 | 2.280 | 2.302 | 2.925 | 2.338 | 2.356 | 2.373 | 2.390 | 2.406 | 2.422 | 2.443 |
| 539.35 .1941 40 | B .3984 | .4020 | .4058 | .4098 | .4122 | .4153 | .4183 | .4211 | .4240 | .4269 | .4296 |
| | C 2.052 | 2.071 | 2.090 | 2.111 | 2.123 | 2.139 | 2.155 | 2.175 | 2.185 | 2.200 | 2.214 |

l , Latent heat of volatilization. w , weight of vapor per cubic foot. B.P. back pressure or suction gauge pressure.

| | | | | | | | | | |
|----------------|--------|--------|-------|-----|-------|-------|-------|-------|------|
| Back Pressures | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| Temperatures | -28.5° | -17.5° | -8.5° | -1° | 5.66° | 11.5° | 16.8° | 21.7° | 26.1 |

Mr. Matthews defines a standard ton of refrigeration as the equivalent of 27 lbs. of anhydrous ammonia evaporated per hour from liquid at 90° F. into saturated vapor at 15.67 lbs. gauge pressure (0° F.), which requires 12,000 B.T.U.; or 20,950 units of evaporation, each of which is equal to 572.78 B.T.U., the heat required to evaporate 1 lb. of ammonia from a temperature of -28.5° F. into saturated vapor at atmospheric pressure.

Size and Capacities of Ammonia Refrigerating Machines.—York Mfg. Co. Based on 15.67 lbs. back-pressure, 185 lbs. condensing pressure, and condensing water at 60° F.

| SINGLE-ACTING COMPRESSORS. | | | | | DOUBLE-ACTING COMPRESSORS. | | | | |
|----------------------------|---------|---------|---------|------------------------------------|----------------------------|---------|---------|---------|------------------------------------|
| Compressors. | | Engine. | | Capacity Tons Refrigeration. | Compressors. | | Engine. | | Capacity Tons Refrigeration. |
| Bore. | Stroke. | Bore. | Stroke. | | Bore. | Stroke. | Bore. | Stroke. | |
| 7 1/2 | 10 | 11 1/2 | 10 | 10 | 9 | 15 | 13 1/2 | 12 | 20 |
| 9 | 12 | 13 1/2 | 12 | 20 | 11 | 18 | 16 | 15 | 30 |
| 11 | 15 | 16 | 15 | 30 | 12 1/2 | 21 | 18 | 18 | 40 |
| 12 1/2 | 18 | 18 | 18 | 40 | 14 | 24 | 20 | 21 | 65 |
| 14 | 21 | 20 | 21 | 65 | 16 | 28 | 24 | 24 | 90 |
| 16 | 24 | 24 | 24 | 90 | 18 | 32 | 26 | 28 | 125 |
| 18 | 28 | 26 | 28 | 125 | 21 | 36 | 28 1/2 | 32 | 175 |
| 21 | 32 | 28 1/2 | 32 | 175 | 24 | 40 | 34 | 36 | 250 |
| 24 | 36 | 34 | 36 | 250 | 26 | 60 | 38 | 54 | 350 |
| 27 | 42 | 36 | 42 | 350 | | | | | |
| 30 | 48 | 44 | 48 | 500 | | | | | |

For larger capacities the machines are built with duplex compressors, driven by simple, tandem or cross compound engines.

DISPLACEMENT AND HORSE-POWER PER TON OF REFRIGERATION.

Dry Compression. S. A., Single-acting; D. A., double-acting.

| Condenser Gauge Pressure and Corresp. Temp. of Liquid at Expansion valve. | Suction Gauge Pressure and Corresponding Temp. | | | | | | | | | |
|--|--|-----------------|-----------------------|-----------------|----------------------|-----------------|---------------------|-----------------|----------------------|-----------------|
| | 5 lb. = - 17.5° F. | | 10 lb. = - 8.5° F. | | 15.67 lb. = 0° F. | | 20 lb. = 5.7° F. | | 25 lb. = 11.5° F. | |
| | Cu. in. Disp.* | I.H.P. per Ton. | Cu. in. Disp. | I.H.P. per Ton. | Cu. in. Disp. | I.H.P. per Ton. | Cu. in. Disp. | I.H.P. per Ton. | Cu. in. Disp. | I.H.P. per Ton. |
| | | | | | | | | | | |
| 145 lb. 82° F., S.A..... | 12,608 | 1.654 | 9,811 | 1.4 | 7829 | 1.195 | 6765 | 1.065 | 5836 | 0.943 |
| 145 lb. 82° F., D.A..... | 14,645 | 1.921 | 11,300 | 1.612 | 8901 | 1.358 | 7625 | 1.2 | 6522 | 1.054 |
| 165 lb. 89° F., S.A..... | 13,045 | 1.834 | 10,148 | 1.56 | 8092 | 1.341 | 6990 | 1.201 | 6027 | 1.071 |
| 165 lb. 89° F., D.A..... | 15,203 | 2.137 | 11,720 | 1.802 | 9224 | 1.529 | 7898 | 1.357 | 6751 | 1.2 |
| 185 lb. 95.5° F., S.A.... | 13,491 | 2.013 | 10,487 | 1.72 | 8362 | 1.4865 | 7219 | 1.336 | 6223 | 1.197 |
| 185 lb. 95.5° F., D.A.... | 15,774 | 2.354 | 12,150 | 1.993 | 9555 | 1.7 | 8176 | 1.513 | 6985 | 1.344 |
| 205 lb. 101.4° F., S.A.... | 13,947 | 2.192 | 10,834 | 1.879 | 8630 | 1.631 | 7450 | 1.47 | 6420 | 1.323 |
| 205 lb. 101.4° F., D.A.... | 16,362 | 2.571 | 12,590 | 2.184 | 9890 | 1.87 | 8459 | 1.67 | 7222 | 1.488 |

* Cu. in. Displacement per Min. per Ton of Refrigeration.

The volumetric efficiency ranges from 63.5 to 76.5% for double-acting, and from 74.5 to 85.5% for single-acting compressors, increasing with the decrease of condenser pressure and with the increase of suction pressure.

Where the liquid is cooled lower than the temperature corresponding to the condensing pressure, there will be a reduction in horse-power and displacement proportional to the increase of work done by each pound of liquid handled. The I.H.P. is that of the compressor. For Engine Horse-Power add 17% up to 20 tons capacity and 15% for larger machines.

1300 ICE-MAKING OR REFRIGERATING MACHINES.

SMALL SIZES OF REFRIGERATING MACHINES.

| Capacity, tons | Single-acting, Vertical. | | | Double-acting, Horizontal. | | |
|-----------------------------|-----------------------------|---|-----|-------------------------------|-------|----|
| | 1 1/4 | 3 | 6 | 2 1/2 | 6 | 10 |
| Compressor, diam., in..... | 4 1/2 | 6 | 2-6 | 4 | 5 1/2 | 7 |
| Compressor, stroke, in..... | 5 | 6 | 6 | 6 | 8 | 10 |
| Engine, diam., in..... | 5 | 6 | 8 | 6 | 8 | 10 |
| Engine, stroke, in..... | 5 | 6 | 6 | 8 | 8 | 10 |

Rated Capacity of Refrigerating Machines. — It is customary to rate refrigerating machines in tons of refrigerating capacity in 24 hours, on the basis of a suction pressure of 15.67 lbs. gauge, corresponding to 0° F. temperature of saturated ammonia vapor, and a condensing pressure of 185 lbs. gauge, corresponding to 95.5° F. The actual capacity increases with the increase of the suction pressure, and decreases with the increase of the condensing pressure. The following table shows the calculated capacities and horse-power of a machine rated at 40 H.P., when run at different pressures. (York Mfg. Co.) The horse-power required increases with the increase of both the suction and the condensing pressure.

| Condenser Press. Temp. | Suction Gauge Pressure and Corresponding Temp. | | | | | | | | | | | |
|---------------------------|--|------|---------------------|------|----------------------|------|---------------------|------|----------------------|------|----------------------|------|
| | 5 lb = -17.5°. | | 10 lb = -8.5° F. | | 15.67 lb. = 0° F. | | 20 lb. = 5.7° F. | | 25 lb. = 11.5° F. | | 30 lb. = 16.8° F. | |
| | Tons. | H.P. | Tons. | H.P. | Tons. | H.P. | Tons. | H.P. | Tons. | H.P. | Tons. | H.P. |
| 145 lb. = 82° F..... | 26.6 | 50.6 | 34.2 | 55.1 | 42.8 | 58.8 | 49.6 | 60.7 | 57.5 | 62.3 | 65.3 | 63.4 |
| 165 lb. = 89° F..... | 25.7 | 54.2 | 33.1 | 59.4 | 41.4 | 63.8 | 48 | 66.3 | 55.7 | 68.6 | 63.2 | 70.1 |
| 185 lb. = 95.5° F..... | 24.8 | 57.4 | 32 | 63.3 | 40 | 68.6 | 46.5 | 71.4 | 53.9 | 74.2 | 61.3 | 76.5 |
| 205 lb. = 101.4° F..... | 24 | 60.5 | 31 | 67 | 38.9 | 72.9 | 45 | 76.1 | 52.3 | 79.6 | 59.4 | 86.2 |

Piston Speeds and Revolutions per Minute. — There is a great diversity in the practice of different builders as to the size of compressor, the piston speed and the number of revolutions per minute for a given rated capacity. F. E. Matthews, *Trans. A. S. M. E.*, 1905, has plotted a diagram of the various speeds and revolutions adopted by four prominent builders, and from average curves the following figures are obtained:

| | | | | | | | | | | | | | | | | |
|------------------|-----|-----|-----|-----|-----|-----|--------|-----|-----|-----|-----|-----|-----|-----|--------|-----|
| Tons..... | 20 | 30 | 40 | 50 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 240 | 300 | 400 | 500 |
| R.P.M..... | 90 | 78 | 73 | 68 | 64 | 60 | 58 1/2 | 57 | 56 | 55 | 54 | 53 | 52 | 51 | 48 1/2 | 46 |
| Piston speeds... | 200 | 215 | 228 | 240 | 250 | 270 | 280 | 286 | 290 | 293 | 296 | 300 | 315 | 340 | 378 | 425 |

Mr. Matthews recommends a standard rating of machines based on these revolutions and speeds and on an apparent compressor displacement of 4.4 cu. ft. per minute per ton rating.

Condensers for Refrigerating Machines are of two kinds: submerged, and open-air evaporative. The submerged condenser requires a large volume of cooling water for maximum efficiency. According to Siebel the amount of condensing surface, the water entering at 70° and leaving at 80°, is 40 sq ft. for each ton of refrigerating capacity, or 64 lineal feet of 2-in. pipe. Frequently only 20 sq. ft., or 90 ft. of 1 1/4-in. pipe, is used, but this necessitates higher condenser pressures. If F = sq. ft. of cooling surface, h = heat of evaporation of 1 lb. ammonia at the condenser temperature, K = lbs. of ammonia circulated per minute, m = B.T.U. transferred per minute per sq. ft. of condenser surface, t = temperature of the ammonia in the coils and t_1 the temperature of the water outside, $F = hK \div m(t - t_1)$. For $t = 80$ and $t_1 = 70$, η

may be taken at 0.5. Practically the amount of water required will vary from 3 to 7 gallons per minute per ton of refrigeration. When cooling water is scarce, cooling towers are commonly used.

E. T. Shinkle gives the average surface of several submerged condensers as equal to 167 lineal feet of 1-in. pipe per ton of refrigeration.

Open air or evaporation surface condensers are usually made of a stack of parallel tubes with return bends, and means for distributing the water so that it will flow uniformly over the pipe surface. Shinkle gives as the average surface of open-air coolers 142 ft. of 1-in. pipe, or 99 ft. of 1 1/4 in. pipe per ton of refrigerating capacity.

CAPACITY OF CONDENSERS. (York Mfg. Co.) — The following table shows the capacities and horse-power per ton refrigeration of one section counter-current double-pipe condenser, 1 1/4-in. and 2-in. pipe, 12 pipes high, 19 feet in length outside of water bends, for water velocities 100 ft. to 400 ft. per minute; initial temperature of condensing water 70°.

High Pressure Constant.

| Condensing Water. | | | | Cap'y Tons Refrig. per 24 hours. | Con- densing Pressure Lbs. per sq. in. | Horse-power per Ton Refrigeration. | | |
|--|---|--|--|--|--|---------------------------------------|--|--|
| Veloc- ity thr'gh 1 1/4-in. pipe. Ft. per min. | Total gallons used per min. | Gallons per min per ton Refrig. | Fric- tion thr'gh Coil. Lbs. per sq. in. | | | Engine driving Com- pressor | Circu- lating Water thr'gh Con- denser. | Total Engine and Water Circu- lation. |
| 100 | 7.77 | 1.16 | 2.28 | 6.7 | 185 | 1.71 | 0.0016 | 1.7116 |
| 150 | 11.65 | 1.165 | 5.75 | 10. | 185 | 1.71 | 0.004 | 1.714 |
| 200 | 15.54 | 1.165 | 9.98 | 13.4 | 185 | 1.71 | 0.007 | 1.717 |
| 250 | 19.42 | 1.18 | 15. | 16.4 | 185 | 1.71 | 0.011 | 1.721 |
| 300 | 23.31 | 1.24 | 21.6 | 18.8 | 185 | 1.71 | 0.016 | 1.726 |
| 400 | 31.08 | 1.30 | 37.8 | 24. | 185 | 1.71 | 0.030 | 1.74 |

Capacity Constant.

| | | | | | | | | |
|-----|-------|-------|------|-----|-----|------|-------|-------|
| 100 | 7.77 | 0.777 | 2.28 | 10. | 225 | 2.04 | 0.001 | 2.041 |
| 150 | 11.65 | 1.165 | 5.75 | 10. | 185 | 1.71 | 0.004 | 1.714 |
| 200 | 15.54 | 1.554 | 9.98 | 10. | 165 | 1.54 | 0.009 | 1.549 |
| 250 | 19.42 | 1.942 | 15. | 10. | 155 | 1.46 | 0.018 | 1.478 |
| 300 | 23.31 | 2.331 | 21.6 | 10. | 148 | 1.40 | 0.030 | 1.43 |
| 400 | 31.08 | 3.108 | 37.8 | 10. | 140 | 1.33 | 0.071 | 1.401 |

The horse-power per ton is for single-acting compressor with 15.67 lbs. suction pressure.

The friction in water pump and connections should be added to water horse-power and to total horse-power.

Cooling-Tower Practice in Refrigerating Plants. (B. F. Hart, Jr., *Southern Engr.*, Mar., 1909.) — The efficiency of a cooling tower depends on exposing the greatest quantity of water surface to the cooling air-currents. In a tower designed to handle 100 gallons per minute the ranges of temperature found when handling different quantities of water were as follows:

| | | | |
|------------------------------------|-------|-------|-----|
| Gallons of water per minute..... | 148 | 109 | 58 |
| Temperature of the atmosphere..... | 78° | 78.5° | 78° |
| Relative humidity, %..... | 47 | 49 | 97 |
| Initial temperature..... | 85.5° | 85° | 86° |
| Final temperature..... | 78° | 76° | 75° |
| Range..... | 7.5° | 9° | 11° |

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The final temperatures which may be obtained when the initial temperature does not exceed 100° are as follows:

| Atmosphere temp. | | 95° | 90° | 85° | 80° | 75° | 70° |
|------------------|----|---|-----|-----|-----|-----|-----|
| | | Final temperature of water leaving tower. | | | | | |
| Humidity, % | 90 | 100 | 95 | 90 | 85 | 80 | 75 |
| | 80 | 98 | 92 | 88 | 83 | 78 | 73 |
| | 70 | 95 | 90 | 86 | 80 | 76 | 71 |
| | 60 | 92 | 88 | 83 | 78 | 74 | 69 |
| | 50 | 89 | 84 | 79 | 75 | 70 | 66 |
| | 40 | 85 | 80 | 76 | 71 | 67 | 63 |

For ammonia condensers we figure on supplying 3 gallons per minute of circulating water per ton of refrigeration, or 6 gallons per minute per ton of ice made per 24 hours, and guarantee a reduction range from 150° to 160° down to about 100° when the temperature of the atmosphere does not exceed 80° nor the relative humidity 60%. When the temperature of the atmosphere and the humidity are both above 90° the speed of the pumps and the ammonia pressure must be increased.

The Refrigerating-Coils of a Pictet ice-machine described by Ledoux had 79 sq. ft. of surface for each 100,000 theoretic negative heat-units produced per hour. The temperature corresponding to the pressure of the dioxide in the coils is 10.4° F., and that of the bath (calcium chloride solution) in which they were immersed is 19.4°.

Comparison of Actual and Theoretical Ice-melting Capacity. — The following is a comparison of the theoretical ice-melting capacity of an ammonia compression machine with that obtained in some of Prof. Schröter's tests on a Linde machine having a compression-cylinder 9.9-in. bore and 16.5-in. stroke, and also in tests by Prof. Denton on a machine having two single-acting compression-cylinders 12 in. × 30 in.:

| No. of Test. | Temp. in Degrees F. Corresponding to Pressure of Vapor. | | Ice-melting Capacity per lb. of Coal, assuming 3 lbs. per hour per Horse-power. | | | |
|--------------|---|----------|---|---------|--|------|
| | Condenser. | Suction. | Theoretical Friction* included. | Actual. | Per cent of Loss Due to Cylinder Superheating. | |
| Schröter { | 1 | 72.3 | 26.6 | 50.4 | 40.6 | 19.4 |
| | 2 | 70.5 | 14.3 | 37.6 | 30.0 | 20.2 |
| | 3 | 69.2 | 0.5 | 29.4 | 22.0 | 25.2 |
| | 4 | 68.5 | - 11.8 | 22.8 | 16.1 | 29.4 |
| Denton { | 24 | 84.2 | 15.0 | 27.4 | 24.2 | 11.7 |
| | 26 | 82.7 | - 3.2 | 21.6 | 17.5 | 19.0 |
| | 25 | 84.6 | - 10.8 | 18.8 | 14.5 | 22.9 |

* Friction taken at figures observed in the tests, which range from 14% to 20% of the work of the steam-cylinder.

TEST-TRIALS OF REFRIGERATING MACHINES.

(G. Linde, *Trans. A. S. M. E.*, xiv, 1414.)

The purpose of the test is to determine the ratio of consumption and production, so that there will have to be measured both the refrigerative effect and the heat (or mechanical work) consumed, also the cooling water. The refrigerative effect is the product of the number of heat-units (Q) abstracted from the body to be cooled, and the quotient ($T_c - T$) ÷ T : in which T_c = absolute temperature at which heat is transmitted to the cooling water, and T = absolute temperature at which heat is taken from the body to be cooled. (*Continued on page 1305.*)

Ammonia Compression-machines.—Ammonia gas possesses the advantage of affording about three times the useful effect of sulphur dioxide for the same volume described by the piston.
The perfection of ammonia apparatus now renders it so convenient and reliable that no practical advantage results from the lower pressures afforded by sulphur dioxide.
The results of the calculations for ammonia are given in the table below:

PERFORMANCE OF AMMONIA COMPRESSION-MACHINES.

Gas superheated during compression as in ordinary practice. Temperature of condenser, 64.4° Fahr. Pressure in condenser, 117.44 lbs per sq. in. (Leducx.)

| Temperature Corresponding to Pressure of Vapor in Refrigerating-coils. | t_2 | Absolute Pressure in Refrigerating-coils. | $P_2 + 144$ | Temperature of Gas at End of Compression. | t_1 | Per Cubic Foot of Piston Displacement. | | | | Performance in British Thermal Units. | | Ice-melting Capacity per place-ment. | Ice-melting Capacity per lb. of Coal, assuming 3 lbs. of Coal per hour per H.P. of Steam-cylinder. With Friction. | Condensing-water. Per Ton of Ice-melting Capacity, as-suming 30° F. Range of Tem-perature. |
|--|---------|---|-------------|---|--------|--|-------------------------------|--|-----------------------|---------------------------------------|--|--------------------------------------|---|--|
| | | | | | | Weight of Gas Com-pressed. | Heat Abstracted at Condenser. | Number of Negative Thermal Units De-veloped. | Work of Com-pression. | Without Friction. | With Friction, or Indicated Steam-power. | | | |
| Lbs. per sq. in. | Deg. F. | Lbs. | B.T.U. | B.T.U. | B.T.U. | Ft.-lbs. | Ft.-lbs. | 1.15W _r | Q | 1.15W _r | Tons. | Lbs. | Gals. | |
| 37.76 | 9.66 | 0.1329 | 78.56 | 69.41 | 7070 | 8130 | 16,900 | 0.00854 | 0.00854 | 0.000244 | 39.6 | 1290 | | |
| 33.67 | 5.00 | 0.1206 | 71.98 | 62.77 | 7120 | 8190 | 15,170 | 0.00766 | 0.00766 | 0.000221 | 35.6 | 1310 | | |
| 16.95 | -22.00 | 0.0639 | 40.45 | 32.58 | 6080 | 6990 | 9,230 | 0.00466 | 0.00466 | 0.000115 | 21.6 | 1410 | | |

The theoretical results for ammonia are higher than the actual, for the same reasons that have been stated for sulphur dioxide. In the case of ammonia the action of the cylinder-walls in superheating the entering vapor has been determined experimentally by Prof. Denton, and the amount found to agree with that indicated by theory. In these experiments the ammonia circulated in a 75-ton refrigerating machine was measured directly by means of a special meter, so that, in addition to determining the effect of superheating, the latent heats can be calculated at the suction and condenser pressure.

Economy of Ammonia Compression-machines at Various Condenser Temperatures. (LEDoux.)

REFRIGERATING EFFECT OF 1 CU. FT. OF AMMONIA EXPANDED THROUGH A SIMPLE COCK TO 33.67 LBS. ABSOLUTE PRESSURE PER SQ. IN., AND TAKEN INTO THE COMPRESSOR AT THIS PRESSURE AND THE CORRESPONDING TEMPERATURE OF 5° F.

| Temp. Due to Press. of Vapor in Condenser. | Absolute Pressure in Condenser. | Deg. F. | Heat Carried away from Condenser. | Refrigerating Effect in Heat Units. | Ratio of Refrigerating Effect to Heat Expended. | Work of Compression, without Friction. | Work of Compression, with Friction, or Indicated Steam-power. | Refrigerating Effect in Heat Units. | | | | Ice-melting Capacity. | | | | | | Condensing-water | | | |
|--|---------------------------------|---------|-----------------------------------|-------------------------------------|---|--|---|---|---|------------------------------------|--------|-----------------------|-------------------|----------------|-------------------------------------|---|----------------------------------|------------------|------|------|-------|
| | | | | | | | | Per ft.-lb. of Work Expended, without Friction. | Per ft.-lb. of Work Expended, including Friction. | Per H.P. Hour, including Friction. | B.T.U. | Per Hour per H.P. | Per Lb. of Coal. | | Per cu. ft. of Piston Displacement. | Per cu. ft. of Piston Displacement, assuming 30° Range of Temp. | Per Ton of Ice-melting Capacity. | Gals. | | | |
| | | | | | | | | | | | | | Without Friction. | With Friction. | | | | | Lbs. | Lbs. | Tons. |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) | (20) | | |
| 59 | 106.0 | 154.6 | 71.81 | 63.47 | 7.61 | 6,450 | 7,410 | .00984 | .00857 | 16,960 | 137.0 | 119.3 | 45.8 | 39.8 | .000223 | 0.2872 | 1290 | 0.89 | | | |
| 68 | 125.1 | 179.9 | 72.05 | 62.31 | 6.40 | 7,530 | 8,660 | .00827 | .00719 | 14,250 | 115.2 | 100.2 | 38.4 | 33.4 | .000219 | .2882 | 1320 | .92 | | | |
| 77 | 146.6 | 205.1 | 72.26 | 61.13 | 5.49 | 8,600 | 9,890 | .00711 | .00618 | 12,240 | 99.0 | 86.1 | 33.0 | 28.7 | .000215 | .2890 | 1350 | .94 | | | |
| 86 | 170.8 | 230.3 | 72.46 | 59.93 | 4.78 | 9,680 | 11,130 | .00619 | .00538 | 10,660 | 86.2 | 75.0 | 28.7 | 25.0 | .000211 | .2898 | 1380 | .96 | | | |
| 95 | 197.8 | 255.4 | 72.61 | 58.70 | 4.22 | 10,750 | 12,360 | .00546 | .00475 | 9,400 | 76.0 | 66.1 | 25.3 | 22.0 | .000206 | .2904 | 1410 | .98 | | | |
| 104 | 227.8 | 280.3 | 72.74 | 57.45 | 3.76 | 11,820 | 13,590 | .00486 | .00423 | 8,380 | 67.7 | 58.9 | 22.6 | 19.6 | .000202 | .2910 | 1440 | 1.00 | | | |

REFRIGERATING EFFECT OF 1 CU. FT. OF AMMONIA EXPANDED THROUGH A SIMPLE COCK TO 16.95 LBS. ABSOLUTE PRESSURE PER SQ. IN., AND TAKEN INTO THE COMPRESSOR AT THIS PRESSURE AND THE CORRESPONDING TEMPERATURE OF -22° F.

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) | (17) | (18) | (19) |
|-----|-------|-------|-------|-------|------|-------|--------|--------|--------|-------|------|------|------|------|---------|--------|------|------|
| 59 | 106.0 | 224.1 | 40.28 | 32.93 | 4.48 | 5,680 | 6,530 | .00580 | .00504 | 9,980 | 80.7 | 70.2 | 26.9 | 23.4 | .000116 | 0.1611 | 1390 | 0.97 |
| 68 | 125.1 | 252.2 | 40.50 | 32.31 | 3.95 | 6,330 | 7,280 | .00510 | .00444 | 8,790 | 71.0 | 61.8 | 23.7 | 20.6 | .000114 | .1620 | 1420 | 0.99 |
| 77 | 146.6 | 280.2 | 40.70 | 31.69 | 3.52 | 6,960 | 8,000 | .00455 | .00396 | 7,840 | 63.3 | 55.1 | 21.1 | 18.4 | .000111 | .1628 | 1470 | 1.02 |
| 86 | 170.8 | 308.3 | 40.90 | 31.05 | 3.15 | 7,610 | 8,750 | .00408 | .00355 | 7,030 | 56.8 | 49.4 | 18.9 | 16.5 | .000109 | .1636 | 1500 | 1.04 |
| 95 | 197.8 | 336.2 | 41.07 | 30.41 | 2.85 | 8,240 | 9,480 | .00369 | .00321 | 6,360 | 51.4 | 44.7 | 17.1 | 14.9 | .000107 | .1643 | 1540 | 1.07 |
| 104 | | 364.0 | 41.23 | 29.75 | 2.59 | 8,870 | 10,200 | .00335 | .00292 | 5,780 | 46.6 | 40.6 | 15.5 | 13.5 | .000105 | .1649 | 1570 | 1.09 |

The determination of the quantity of cold will be possible with the proper exactness only when the machine is employed during the test to refrigerate a liquid; and if the cold be found from the quantity of liquid circulated per unit of time, from its range of refrigeration, and from its specific heat. Sufficient exactness cannot be obtained by the refrigeration of a current of circulating air, nor from the manufacture of a certain quantity of ice, nor from a calculation of the fluid circulating within the machine (for instance, the quantity of ammonia circulated by the compressor). Thus the refrigeration of brine will generally form the basis for tests making any pretension to accuracy. The degree of refrigeration should not be greater than necessary for allowing the range of temperature to be measured with the necessary exactness; a range of temperature of from 5° to 6° Fahr. will suffice.

The condenser measurements for cooling water and its temperatures will be possible with sufficient accuracy only with submerged condensers.

The measurement of the quantity of brine circulated, and of the cooling water, is usually effected by water-meters inserted into the conduits. If the necessary precautions are observed, this method is admissible. For quite precise tests, however, the use of two accurately gauged tanks which are alternately filled and emptied must be advised.

To measure the temperatures of brine and cooling water at the entrance and exit of refrigerator and condenser respectively, the employment of specially constructed and frequently standardized thermometers is indispensable; no less important is the precaution of using at each spot simultaneously two thermometers, and of changing the position of one such thermometer series from inlet to outlet (and *vice versa*) after the expiration of one-half of the test, in order that possible errors may be compensated.

It is important to determine the specific heat of the brine used in each instance for its corresponding temperature range, as small differences in the composition and the concentration may cause considerable variations.

As regards the measurement of consumption, the programme will not have any special rules in cases where only the measurement of steam and cooling water is undertaken, as will be mainly the case for trials of absorption-machines. For compression-machines the steam consumption depends both on the quality of the steam-engine and on that of the refrigerating-machine, while it is evidently desirable to know the consumption of the former separately from that of the latter. As a rule steam-engine and compressor are coupled directly together, thus rendering a direct measurement of the power absorbed by the refrigerating-machine impossible, and it will have to suffice to ascertain the indicated work both of steam-engine and compressor. By further measuring the work for the engine running empty, and by comparing the differences in power between steam-engine and compressor resulting for wide variations of condenser-pressures, the effective consumption of work L_e for the refrigerating-machine can be found very closely. In general, it will suffice to use the indicated work found in the steam-cylinder, especially as from this observation the expenditure of heat can be directly determined. Ordinarily the use of the indicated work in the compressor-cylinder, for purposes of comparison, should be avoided; firstly, because there are usually certain accessory apparatus to be driven (agitators, etc.), belonging to the refrigerating-machine proper; and secondly, because the external friction would be excluded.

Heat Balance. — We possess an important aid for checking the correctness of the results found in each trial by forming the balance in each case for the heat received and rejected. Only those tests should be regarded as correct beyond doubt which show a sufficient conformity in the heat balance. It is true that in certain instances it may not be easy to account fully for the transmission of heat between the several parts of the machine and its environment by radiation and convection, but generally (particularly for compression-machines) it will be possible to obtain for the heat received and rejected a balance exhibiting small discrepancies only.

Report of Test. — Reports intended to be used for comparison with the figures found for other machines will therefore have to embrace at least the following observations:

Refrigerator:

| | |
|---|-------|
| Quantity of brine circulated per hour | |
| Brine temperature at inlet to refrigerator | |
| Brine temperature at outlet of refrigerator | T |
| Specific gravity of brine (at 64° Fahr.) | |
| Specific heat of brine | |
| Heat abstracted (cold produced) | Q_e |
| Absolute pressure in the refrigerator | |

Condenser:

| | |
|---|-------|
| Quantity of cooling water per hour | |
| Temperature at inlet to condenser | |
| Temperature at outlet of condenser | T_c |
| Heat abstracted | Q_1 |
| Absolute pressure in the condenser | |
| Temperature of gases entering the condenser | |

ABSORPTION-MACHINE.

Still:

| | |
|--|--------|
| Steam consumed per hour | |
| Abs. pressure of heating steam. | |
| Temperature of condensed steam at outlet | |
| Heat imparted to still | Q'_e |

Absorber:

| | |
|---------------------------------------|-------|
| Quantity of cooling water per hour .. | |
| Temperature at inlet | |
| Temperature at outlet | |
| Heat removed | Q_2 |

Pump for Ammonia Liquor:

| | |
|---|-----------|
| Indicated work of steam-engine | |
| Steam-consumption for pump | |
| Thermal equivalent for work of pump | ALp |
| Total sum of losses by radiation and convection | $\pm Q_3$ |

Heat Balance:

$$Q_e + Q'_e = Q_1 + Q_2 \pm Q_3.$$

COMPRESSION-MACHINE.

Compressor:

| | |
|-------------------------------|-------|
| Indicated work | L_t |
| Temperature of gases at inlet | |
| Temperature of gases at exit | |

Steam-engine:

| | |
|--|-----------|
| Feed-water per hour | |
| Temperature of feed-water .. | |
| Absolute steam-pressure before steam-engine | |
| Indicated work of steam-engine | L_e |
| Condensing water per hour... | |
| Temperature of do. | |
| Total sum of losses by radiation and convection .. | $\pm Q_3$ |

Heat Balance:

$$Q_e + AL_c = Q_1 \pm Q_3.$$

For the calculation of efficiency and for comparison of various tests, the actual efficiencies must be compared with the theoretical maximum of efficiency $Q \div (AL)$ max. = $T \div (T_c - T)$ corresponding to the temperature range.

Temperature Range. — For the temperatures (T and T_c) at which the heat is abstracted in the refrigerator and imparted to the condenser, it is correct to select the temperature of the brine leaving the refrigerator and that of the cooling water leaving the condenser, because it is in principle impossible to keep the refrigerator pressure higher than would correspond to the lowest brine temperature, or to reduce the condenser pressure below that corresponding to the outlet temperature of the cooling water.

Prof. Linde shows that the *maximum theoretical* efficiency of a compression-machine may be expressed by the formula

$$Q \div (AL) = T \div (T_c - T),$$

in which Q = quantity of heat abstracted (cold produced);

AL = thermal equivalent of the mechanical work expended;

L = the mechanical work, and $A = 1 \div 778$;

T = absolute temperature of heat abstraction (refrigerator);

T_c = absolute temperature of heat rejection (condenser).

If u = ratio between the heat equivalent of the mechanical work AL and the quantity of heat Q' which must be imparted to the motor to produce the work L , then

$$AL \div Q' = u, \text{ and } Q'/Q = (T_c - T) \div (uT).$$

It follows that the expenditure of heat Q' necessary for the production of the quantity of cold Q in a compression-machine will be the smaller, the smaller the difference of temperature $T_c - T$.

Metering the Ammonia. — For a complete test of an ammonia refrigerating-machine it is advisable to measure the quantity of ammonia circulated, as was done in the test of the 75-ton machine described by Prof. Denton. (*Trans. A. S. M. E.*, xii, 326.)

ACTUAL PERFORMANCES OF ICE-MAKING MACHINES.

The table given on page 1308 is abridged from Denton, Jacobus, and Riesenberger's translation of Ledoux on Ice-making Machines. The following shows the class and size of the machines tested, referred to by letters in the table, with the names of the authorities:

| Class of Machines. | Authority. | Dimensions of Compression-cylinder in inches. | |
|---------------------------------------|-------------------------|---|---------|
| | | Bore. | Stroke. |
| A. Ammonia cold-compression..... | Schröter. | 9.9 | 16.5 |
| B. Pictet fluid dry-compression | " | 11.3 | 24.4 |
| C. Bell-Coleman air..... | " | 28.0 | 23.8 |
| D. Closed cycle air..... | { Renwick & Jacobus. | 10.0 | 18.0 |
| E. Ammonia dry-compression..... | Denton. | 12.0 | 30.0 |
| F. Ammonia absorption | " | | |

In class A, a German double-acting machine with compression cylinder 9.9 in. bore, 16 in. stroke, tested by Prof. Schröter, the ice-melting capacity ranges from 46.29 to 16.14 lbs. of ice per pound of coal, according as the suction pressure varies from about 45 to 8 lbs. above the atmosphere, this pressure being the condition which mainly controls the economy of compression machines. These results are equivalent to realizing from 72% to 57% of theoretically perfect performances. The higher per cents appear to occur with the higher suction-pressures, indicating a greater loss from cylinder-heating (a phenomenon the reverse of cylinder condensation in steam-engines), as the range of the temperature of the gas in the compression-cylinder is greater.

In E, an American single-acting compression-machine, two compression cylinders each 12×30 in., operating on the "dry system," tested by Prof. Denton, the percentage of theoretical effect realized ranges from 69.5% to 62.6%. The friction losses are higher for the American machine. The latter's higher efficiency may be attributed, therefore, to more perfect displacement.

The largest "ice-melting capacity" in the American machine is 24.16 lbs. This corresponds to the highest suction-pressures used in American practice for such refrigeration as is required in beer-storage cellars using the direct-expansion system. The conditions most nearly corresponding to American brewery practice in the German tests are those in line 5, which give an "ice-melting capacity" of 19.07 lbs.

For the manufacture of artificial ice, the conditions of practice are those of lines 3 and 4, and lines 25 and 26. In the former the condensing pressure used requires more expense for cooling water than is common in American practice. The ice-melting capacity is therefore greater in the German machine, being 22.03 and 16.14 lbs. against 17.55 and 14.52 for the American apparatus.

CLASS B. Sulphur Dioxide or Pictet Machines. — No records are available for determination of the "ice-melting capacity" of machines using pure sulphur dioxide. In the "Pictet fluid," a mixture of about 97% of sulphur dioxide and 3% of carbonic acid, the presence of the carbonic acid affords a temperature about 14 Fahr. degrees lower than is obtained with pure sulphur dioxide at atmospheric pressure. The latent heat of this mixture has never been determined, but is assumed to be equal to that of pure sulphur dioxide.

For brewery refrigerating conditions, line 17, we have 26.24 lbs. "ice-melting capacity," and for ice-making conditions, line 13, the "ice-melting capacity" is 17.47 lbs. These figures are practically as economical as those for ammonia, the per cent of theoretical effect realized ranging from 65.4 to 57.8. At extremely low temperatures, -15° Fahr., lines 14 and 18, the per cent realized is as low as 42.5.

Actual Performance of Ice-making Machines.

| Machine. | Number of Test. | | Condenser. | | Suction. | Absolute Pressure, in lbs. per square inch. | | Temperature corresponding to Pressure, in degrees Fahr. | | Temperature of Brine, in degrees Fahr. | | Revolutions per minute. | Horse-power of Steam-cylinder. | Per cent of Indicated Power of Steam-cylinder lost in Friction. | Ice-melting Capacity, in tons per 24 hours. | Ice-melting Capacity, in pounds per pound of Coal. Actual.† | Diff. between theoretical Ice-melting Capacity, no Cylinder Heating or Friction, and actual, %‡ | Heat losses. Per cent of Theoretical Amount with Friction.§ | Mean Effective Pressure, in lbs. per square inch |
|----------|-----------------|-----|------------|-----|----------|---|------|---|------|--|------|-------------------------|--------------------------------|---|---|---|---|---|--|
| | | | | | | | | | | | | | | | | | | | |
| A | 1 | 135 | 55 | 72 | 27 | 43 | 37 | 44.9 | 17.9 | 14.4 | 26.2 | 40.63 | 30.8 | 19.1 | 54.8 | | | | |
| " | 2 | 131 | 42 | 70 | 14 | 28 | 23 | 45.1 | 18.0 | 16.7 | 19.5 | 30.01 | 33.5 | 20.2 | 53.4 | | | | |
| " | 3 | 128 | 30 | 69 | 1 | 14 | 9 | 45.1 | 16.8 | 16.0 | 13.3 | 22.03 | 37.1 | 25.2 | 50.3 | | | | |
| " | 4 | 126 | 22 | 68 | -12 | 30 | -5 | 44.8 | 15.5 | 19.5 | 9.0 | 16.14 | 42.9 | 29.1 | 44.7 | | | | |
| " | 5 | 200 | 42 | 95 | 14 | 28 | 23 | 45.0 | 24.1 | 10.5 | 16.5 | 19.07 | 36.0 | 28.5 | 77.0 | | | | |
| " | 6 | 136 | 60 | 72 | 30 | 44 | 37 | 45.2 | 17.9 | 10.7 | 29.8 | 46.29 | 28.5 | 19.9 | 56.8 | | | | |
| " | 7 | 131 | 45 | 71 | 18 | 28 | 23 | 45.1 | 18.0 | 12.1 | 21.6 | 33.23 | 31.3 | 21.9 | 56.4 | | | | |
| " | 8 | 126 | 24 | 68 | -9 | 0 | -6 | 44.7 | 15.6 | 18.0 | 9.9 | 17.55 | 41.1 | 28.3 | 46.1 | | | | |
| " | 9 | 117 | 41 | 64 | 13 | 28 | 23 | 45.0 | 16.4 | 13.5 | 20.0 | 33.77 | 33.1 | 22.9 | 50.6 | | | | |
| " | 10 | 130 | 60 | 70 | 31 | 43 | 37 | 31.7 | 12.0 | 14.8 | 19.5 | 45.01 | 35.2 | 23.8 | 52.0 | | | | |
| B | 11 | 57 | 21 | 77 | 28 | 43 | 37 | 57.0 | 21.5 | 22.9 | 25.6 | 33.07 | 39.9 | 22.2 | 24.1 | | | | |
| " | 12 | 56 | 15 | 76 | 14 | 28 | 23 | 56.8 | 20.6 | 22.9 | 17.9 | 24.11 | 41.3 | 24.0 | 23.1 | | | | |
| " | 13 | 55 | 10 | 75 | -2 | 14 | 9 | 57.1 | 18.5 | 24.0 | 11.6 | 17.47 | 42.2 | 25.2 | 20.4 | | | | |
| " | 14 | 60 | 7 | 81 | -16 | 0 | -6 | 57.6 | 15.7 | 25.7 | 5.7 | 10.14 | 54.5 | 38.5 | 16.8 | | | | |
| " | 15 | 91 | 15 | 104 | 14 | 28 | 23 | 59.3 | 27.2 | 16.9 | 15.7 | 16.05 | 36.2 | 23.1 | 31.5 | | | | |
| " | 16 | 61 | 22 | 81 | 31 | 44 | 37 | 57.3 | 21.6 | 14.0 | 28.1 | 16.19 | 33.4 | 22.5 | 26.8 | | | | |
| " | 17 | 59 | 16 | 80 | 16 | 28 | 23 | 57.5 | 20.5 | 12.8 | 19.3 | 26.24 | 34.6 | 25.0 | 25.6 | | | | |
| " | 18 | 59 | 7 | 79 | -16 | 0 | -6 | 57.8 | 15.9 | 21.1 | 6.8 | 11.93 | 47.5 | 33.4 | 18.0 | | | | |
| " | 19 | 54 | 22 | 75 | 31 | 43 | 37 | 35.3 | 12.4 | 22.3 | 17.0 | 38.04 | 39.5 | 22.6 | 22.6 | | | | |
| " | 20 | 89 | 16 | 103 | 16 | 28 | 23 | 42.9 | 19.9 | 14.7 | 11.9 | 16.68 | 37.7 | 27.0 | 32.7 | | | | |
| " | 21 | 62 | 6 | 82 | -17 | 0 | -5 | 34.8 | 9.9 | 24.3 | 3.5 | 9.86 | 54.2 | 39.5 | 17.7 | | | | |
| C | 22 | 59 | 15 | 65* | -53* | | | 63.2 | 83.2 | 21.9 | 10.3 | 3.42 | 71.7 | 56.9 | 26.6 | | | | |
| D | 23 | 175 | 54 | 81* | -40* | | | 93.4 | 38.1 | 32.1 | 4.9 | 3.0 | 80.0 | 63.0 | 89.2 | | | | |
| E | 24 | 166 | 43 | 84 | 15 | 37 | 28 | 58.1 | 85.0 | 22.7 | 73.9 | 24.16 | 32.8 | 11.7 | 65.9 | | | | |
| " | 25 | 167 | 23 | 85 | -11 | 6 | 2 | 57.7 | 72.6 | 18.6 | 37.9 | 14.52 | 37.4 | 22.7 | 57.6 | | | | |
| " | 26 | 162 | 28 | 83 | -3 | 14 | 2 | 57.9 | 73.6 | 19.3 | 46.5 | 17.55 | 34.9 | 18.6 | 59.9 | | | | |
| " | 27 | 176 | 42 | 88 | 14 | 36 | 28 | 58.9 | 88.6 | 19.7 | 74.4 | 23.31 | 30.5 | 13.5 | 70.5 | | | | |
| F | 28 | 152 | 40 | 79 | 13 | 21 | 16 | | | | 42.2 | 20.1 | 47.8 | | | | | | |

* Temperature of air at entrance and exit of expansion-cylinder.

† On a basis of 3 lbs. of coal per hour per H.P. of steam-cylinder of compression-machine and an evaporation of 11.1 lbs. of water per pound of combustible from and at 212° F. in the absorption-machine.

‡ Per cent of theoretical with no friction.

§ Loss due to heating during aspiration of gas in the compression-cylinder and to radiation and superheating at brine-tank.

|| Actual, including resistance due to inlet and exit valves.

Performance of a 75-ton Ammonia Compression-machine. (J. E. Denton, *Trans. A. S. M. E.*, xii, 326.) — The machine had two single-acting compression cylinders 12 × 30 in., and one Corliss steam-cylinder, double-acting, 18 × 36 in. It was rated by the manufacturers as a 50-ton machine, but it showed 75 tons of ice-refrigerating effect per 24 hours during the test.

The most probable figures of performance in eight trials are as follows:

| No. of Trials. | Ammonia Pressures, lbs. above Atmosphere. | | Brine Temperatures, Degrees F. | | Capacity Tons Refrigerating Effect per 24 hours. | Efficiency lbs. of Ice per lb. of Coal at 3 lbs. per hour per H.P. | Water-consumption, gals. of Water per min. per ton of Capacity. | Ratio of Actual Weights of Ammonia circulated. | Ratio of Capacities. |
|----------------|---|---------------|--------------------------------|---------|--|--|---|--|----------------------|
| | Con- densing | Suc- tion. | Inlet. | Outlet. | | | | | |
| 1 | 151 | 28 | 36.76 | 28.86 | 70.3 | 22.60 | 0.80 | 1.0 | 1.0 |
| 8 | 161 | 27.5 | 36.36 | 28.45 | 70.1 | 22.27 | 1.09 | 1.0 | 1.0 |
| 7 | 147 | 13.0 | 14.29 | 2.29 | 42.0 | 16.27 | 0.83 | 1.70 | 1.60 |
| 4 | 152 | 8.2 | 6.27 | 2.03 | 36.43 | 14.10 | 1.1 | 1.93 | 1.92 |
| 6 | 105 | 7.6 | 6.40 | -2.22 | 37.20 | 17.00 | 2.00 | 1.91 | 1.88 |
| 2 | 135 | 15.7 | 4.62 | 3.22 | 27.2 | 13.20 | 1.25 | 2.59 | 2.57 |

The principal results in four tests are given in the table on page 1311. The fuel economy under different conditions of operation is shown in the following table:

| Condensing Pressure, lbs. | Suction-pressure, lbs. | Pounds of Ice-melting Effect with Engines — | | | | | | B.T.U. per lb. of Steam with Engines — | | |
|---------------------------|------------------------|---|-------------------|------------------------------------|-------------------|------------------------------|-------------------|--|-------------|-------------------------|
| | | Non-con- densing. | | Non-com- pound Con- densing. | | Compound Con- densing. | | Non-condens- ing. | Condensing. | Compound Condensing. |
| | | Per lb. Coal. | Per lb. Steam. | Per lb. Coal. | Per lb. Steam. | Per lb. Coal. | Per lb. Steam. | | | |
| 150 | 28 | 24 | 2.90 | 30 | 3.61 | 37.5 | 4.51 | 393 | 513 | 640 |
| 150 | 7 | 14 | 1.69 | 17.5 | 2.11 | 21.5 | 2.58 | 240 | 300 | 366 |
| 105 | 28 | 34.5 | 4.16 | 43 | 5.18 | 54 | 6.50 | 591 | 725 | 923 |
| 105 | 7 | 22 | 2.65 | 27.5 | 3.31 | 34.5 | 4.16 | 376 | 470 | 591 |

The non-condensing engine is assumed to require 25 lbs. of steam per I.H.P. per hour, the non-compound condensing 20 lbs., and the compound condensing 16 lbs., and the boiler efficiency is assumed at 8.3 lbs. of water per lb. coal under working conditions. The following conclusions were derived from the investigation:

1. The capacity of the machine is proportional, almost entirely, to the weight of ammonia circulated. This weight depends on the suction-pressure and the displacement of the compressor-pumps. The practical suction-pressures range from 7 lbs. above the atmosphere, with which a temperature of 0° F. can be produced, to 28 lbs. above the atmosphere, with which the temperatures of refrigeration are confined to about 28° F. At the lower pressure only about one-half as much weight of ammonia can be circulated as at the upper pressure, the proportion being about in accordance with the ratios of the absolute pressures, 22 and 42 lbs.

respectively. For each cubic foot of piston-displacement per minute a capacity of about one-sixth of a ton of refrigerating effect per 24 hours can be produced at the lower pressure, and of about one-third of a ton at the upper pressure. No other elements practically affect the capacity of a machine, provided the cooling-surface in the brine-tank or other space to be cooled is equal to about 36 sq. ft. per ton of capacity at 28 lbs. back pressure. For example, a difference of 100% in the rate of circulation of brine, while producing a proportional difference in the range of temperature of the latter, made no practical difference in capacity.

The brine-tank was $10\frac{1}{2} \times 13 \times 10\frac{2}{3}$ ft., and contained 8000 lineal feet of 1-in. pipe as cooling-surface. The condensing-tank was $12 \times 10 \times 10$ ft., and contained 5000 lineal feet of 1-in. pipe as cooling-surface.

2. The economy in coal-consumption depends mainly upon both the suction-pressures and condensing-pressures. Maximum economy with a given type of engine, where water must be bought at average city prices, is obtained at 28 lbs. suction-pressure and about 150 lbs. condensing-pressure. Under these conditions, for a non-condensing steam-engine consuming coal at the rate of 3 lbs. per hour per I.H.P. of steam-cylinders, 24 lbs. of ice-refrigerating effect are obtained per lb. of coal consumed. For the same condensing-pressure, and with 7 lbs. suction-pressure, which affords temperatures of 0° F., the possible economy falls to about 14 lbs. of refrigerating effect per lb. of coal consumed. The condensing-pressure is determined by the amount of condensing-water supplied to liquefy the ammonia in the condenser. If the latter is about 1 gallon per minute per ton of refrigerating effect per 24 hours, a condensing-pressure of 150 lbs. results, if the initial temperature of the water is about 56° F. Twenty-five per cent less water causes the condensing-pressure to increase to 190 lbs. The work of compression is thereby increased about 20%, and the resulting "economy" is reduced to about 18 lbs. of "ice effect" per lb. of coal at 28 lbs. suction-pressure and 11.5 at 7 lbs. If, on the other hand, the supply of water is made 3 gallons per minute, the condensing-pressure may be confined to about 105 lbs. The work of compression is thereby reduced about 25%, and a proportional increase of economy results. Minor alterations of economy depend on the initial temperature of the condensing-water and variations of latent heat, but these are confined within about 5% of the gross result, the main element of control being the work of compression, as affected by the back pressure and condensing-pressure, or both. If the steam-engine supplying the motive power may use a condenser to secure a vacuum, an increase of economy of 25% is available over the above figures, making the lbs. of "ice effect" per lb. of coal for 150 lbs. condensing-pressure and 28 lbs. suction-pressure 30.0, and for 7 lbs. suction-pressure, 17.5. It is, however, impracticable to use a condenser in cities where water is bought. The latter must be practically free of cost to be available for this purpose. In this case it may be assumed that water will also be available for condensing the ammonia to obtain as low a condensing-pressure as about 100 lbs., and the economy of the refrigerating-machine becomes, for 28 lbs. back pressure, 43.0 lbs. of "ice-effect" per lb. of coal, or for 7 lbs. back pressure 27.5 lbs. of ice effect per lb. of coal. If a compound condensing-engine can be used with a steam-consumption per hour per horse-power of 16 lbs. of water, the economy of the refrigerating-machine may be 25% higher than the figures last named, making for 28 lbs. back pressure a refrigerating-effect of 54.0 lbs. per lb. of coal, and for 7 lbs. back pressure a refrigerating effect of 34.0 lbs. per lb. of coal.

Performance of a 75-ton Refrigerating-machine. (Denton.)

| | Maximum Capacity and Economy at 28 lbs. Back Pressure. | Maximum Capacity and Economy at Zero, Brine, and 8 lbs. Back Pressure. | Maximum Capacity and Economy for Zero, Brine, 13 lbs. Back Pressure. | Maximum Capacity and Economy at 27.5 lbs. Back Pressure. |
|---|--|--|--|--|
| Av. high ammonia press. above atmos. | 151 lbs. | 152 lbs. | 147 lbs. | 161 lbs. |
| Av. back ammonia press. above atmos. | 28 " | 8.2 " | 13 " | 27.5 " |
| Av. temperature brine inlet. | 36.76° | 6.27° | 14.29° | |
| Av. temperature brine outlet. | 28.86° | 2.03° | 2.29° | 28.45° |
| Av. range of temperature. | 7.9° | 4.24° | 12.00° | 7.91° |
| Lbs. of brine circulated per minute. | 2281 | 2173 | 943 | 2374 |
| Av. temp. condensing-water at inlet. | 44.65° | 56.65° | 46.9° | 54.00° |
| Av. temp. condensing-water at outlet. | 83.66° | 85.4° | 85.46° | 82.86° |
| Av. range of temperature. | 39.01° | 28.75° | 38.56° | 28.80° |
| Lbs. water circulated p. min. thro' cond'ser | 442 | 315 | 257 | 601.5 |
| Lbs. water per min. through jackets. | 25 | 44 | 40 | 14 |
| Range of temperature in jackets. | 24.0° | 16.2° | 16.4° | 29.1° |
| Lbs. ammonia circulated per min. | *28.17 | 14.68 | 16.67 | 28.32 |
| Probable temperature of liquid ammonia, entrance to brine-tank. | *71.3° | *68° | *63.7° | 76.7° |
| Temp. of amm. corresp. to av. back press. | +14° | -8° | -5° | 14° |
| Av. temperature of gas leaving brine-tanks | 34.2° | 14.7° | 3.0° | 29.2° |
| Temperature of gas entering compressor. | *39° | 25° | 10.13° | 34° |
| Av. temperature of gas leaving compressor | 213° | 263° | 239° | 221° |
| Av. temp. of gas entering condenser. | 200° | 218° | 209° | 168° |
| Temperature due to condensing pressure. .. | 84.5° | 84.0° | 82.5° | 88.0° |
| Heat given ammonia: | | | | |
| By brine, B.T.U. per minute. | 14776 | 7186 | 8824 | 14647 |
| By compressor, B.T.U. per minute. | 2786 | 2320 | 2518 | 3020 |
| By atmosphere, B.T.U. per minute. | 140 | 147 | 167 | 141 |
| Total heat rec. by amm., B.T.U. per min. | 17702 | 9653 | 11409 | 17708 |
| Heat taken from ammonia: | | | | |
| By condenser, B.T.U. per min. | 17242 | 9056 | 9910 | 17359 |
| By jackets, B.T.U. per min. | 608 | 712 | 656 | 406 |
| By atmosphere, B.T.U. per min. | 182 | 338 | 250 | 252 |
| Total heat rej. by amm., B.T.U. per min. | 18032 | 10106 | 10816 | 18017 |
| Dif. of heat rec'd and rej., B.T.U. per min. | 330 | 453 | 407 | 309 |
| % work of compression removed by jackets | 22% | 31% | 26% | 13% |
| Av. revolutions per min. | 58.09 | 57.7 | 57.88 | 58.89 |
| Mean eff. press. steam-cyl., lbs. per sq. in. | 32.5 | 27.17 | 27.83 | 32.97 |
| Mean eff. press. amm.-cyl., lbs. per sq. in. | 65.9 | 53.3 | 59.86 | 70.54 |
| Av. H.P. steam-cylinder. | 85.0 | 71.7 | 73.6 | 88.63 |
| Av. H.P. ammonia-cylinder. | 65.7 | 54.7 | 59.37 | 71.20 |
| Friction in per cent of steam H.P. | 23.0 | 24.0 | 20.0 | 19.67 |
| Total cooling water, gallons per min. per ton per 24 hours. | 0.75 | 1.185 | 0.797 | 0.990 |
| Tons ice-melting capacity per 24 hours. | 74.8 | 36.43 | 44.64 | 74.56 |
| Lbs. ice-refrigerating eff. per lb. coal at 3 lbs. per H.P. per hour. | 24.1 | 14.1 | 17.27 | 23.37 |
| Cost coal per ton of ice-refrigerating effect at \$4 per ton. | \$0.166 | \$0.283 | \$0.231 | \$0.170 |
| Cost water per ton of ice-refrigerating effect at \$1 per 1000 cu. ft. | \$0.128 | \$0.200 | \$0.136 | \$0.169 |
| Total cost of 1 ton of ice-refrigerating eff. | \$0.294 | \$0.483 | \$0.467 | \$0.339 |

Figures marked thus (*) are obtained by calculation; all other figures obtained from experimental data; temperatures in Fahrenheit are degrees.

1312 ICE-MAKING OR REFRIGERATING MACHINES.

Ammonia Compression-machine.

ACTUAL RESULTS OBTAINED AT THE MUNICH TESTS.

(Prof. Linde, *Trans. A. S. M. E.*, xiv, 1419.)

| No. of Test..... | 1 | 2 | 3 | 4 | 5 |
|---|----------|---------|---------|---------|---------|
| Temp. of refrigerated brine } Inlet, deg. F.... | 43.194 | 28.344 | 13.952 | -0.279 | 28.251 |
| } Outlet, deg. F.... | 37.054 | 22.885 | 8.771 | -5.879 | 23.072 |
| Specific heat of brine..... | 0.861 | 0.851 | 0.843 | 0.837 | 0.851 |
| Brine circ. per hour, cu. ft..... | 1,039.38 | 908.84 | 633.89 | 414.98 | 800.93 |
| Cold produced, B.T.U. per hour... | 342,909 | 263,950 | 172,776 | 121,474 | 220,284 |
| Cooling water per hour, cu. ft..... | 338.76 | 260.83 | 187.506 | 139.99 | 97.76 |
| I.H.P. in steam-engine cylinder... | 15.80 | 16.47 | 15.28 | 14.24 | 21.61 |
| Cold produced per } Per I.H.P. in comp.-cyl | 24,813 | 18,471 | 12,770 | 10,140 | 11,151 |
| } Per I.H.P. in steam-cyl | 21,703 | 16,026 | 11,307 | 8,530 | 10,194 |
| b., B.T.U. } Per lb. of steam..... | 1,100.8 | 785.6 | 564.9 | 435.82 | 512.12 |

A test of a 35-ton absorption-machine in New Haven, Conn., by Prof. Denton (*Trans. A. S. M. E.*, x, 792), gave an ice-melting effect of 20.1 lbs. per lb. of coal on a basis of boiler economy equivalent to 3 lbs. of steam per I.H.P. in a good non-condensing steam-engine. The ammonia was worked between 138 and 23 lbs. pressure above the atmosphere.

Performance of a Single-acting Ammonia Compressor. — Tests were made at the works of the Eastman Kodak Co., Rochester, N.Y., of a machine fitted with two York Mfg. Co.'s single-acting compressors, 15 in. diam., 22 in. stroke, to determine the horse-power per ton of refrigeration. Following are the principal average results (Bulletin of York Mfg. Co.):

| Date of test, 1908..... | Mar. 6. | Mar. 7 | Mar. 8 | Mar. 9 | Mar. 10. | Mar. 11. | Mar. 14. |
|---------------------------------------|---------|--------|--------|--------|----------|----------|----------|
| Temp. dischg. gas, av. | 216.2 | 217.8 | 250.6 | 245.8 | 253.0 | 242.9 | 255.5 |
| Temp. suction gas, av. | 15.2 | 14.3 | 16.8 | 14.8 | 13.5 | 18.2 | 17.9 |
| Temp. suction at cooler.. | 9.33 | 9.36 | 10.37 | 9.29 | 9.90 | 13.20 | 9.13 |
| Temp. liquid at exp. valve | 74.85 | 74.16 | 71.98 | 77.91 | 76.61 | 82.88 | 76.98 |
| Temp. brine, inlet..... | 22.89 | 23.19 | 25.26 | 22.73 | 27.35 | 28.41 | 23.43 |
| Temp. brine, outlet..... | 13.58 | 13.96 | 14.44 | 13.02 | 15.53 | 16.06 | 12.87 |
| Revs. per min..... | 45.1 | 45.0 | 45.1 | 34.3 | 56.0 | 67.8 | 44.8 |
| Lbs. liquid NH ₃ per min.. | 20.76 | 20.43 | 21.04 | 15.59 | 25.99 | | 20.40 |
| Suc. press. at mach. lb. ... | 20.11 | 19.90 | 19.97 | 20.04 | 20.18 | 18.13 | 20.38 |
| Condenser pressure..... | 183.96 | 184.41 | 186.99 | 187.27 | 187.90 | 186.8 | 183.81 |
| Indicated H.P..... | 69.35 | 69.80 | 70.05 | 52.57 | 89.48 | 105.11 | 68.61 |
| Tons Refrig. Capy, 24 hrs. | 49.08 | 48.79 | 50.38 | 37.01 | 61.39 | 66.65 | 49.31 |
| I.H.P. per ton capacity .. | 1.418 | 1.427 | 1.389 | 1.422 | 1.425 | 1.439 | 1.375 |

Full details of these tests were reported to the Am. Socy. of Refrig. Engrs. and published in *Ice and Refrigeration*, 1908.

Performance of Absorption Machines. — From an elaborate review by Mr. Voorhees of the action of an absorption machine under certain stated conditions, showing the quantity of ammonia circulated per hour per ton of refrigeration, its temperature, etc., at the several stages of the operation, and its course through the several parts of the apparatus, the following condensed statement is obtained:

Generator. — 30.9 lbs. dry steam, 38 lbs. gauge pressure condensed, evaporates 32.2% strong liquor to 22.3% weak liquor.

Exchanger. — 3.01 lbs. weak liquor at 264° cools to 111°.

Absorber. — Adds 0.43 lbs. vapor from the brine cooler, making 3.44 lbs. strong liquor at 111° to go to the pump.

Exchanger. — 3.44 lbs. heated to 224°, some of it is now gas, and the rest liquor of a little less than 32% NH₃.

Analyzer. — (A series of shelves in a tank above the generator) delivers strong liquor to the generator, while the vapor, 91% NH₃, 0.4982 lb., goes to the rectifier.

Rectifier. — Cools the gas to 110° separating water vapor as 0.0682 lb. drip liquor which returns through a trap to the generator.

Condenser. — 0.43 lb. NH₃ gas at 110° cooled and condensed to liquid at 90° by 2 gals. of water per min. heated from 73° to 86°.

Expansion Valve and Cooler. — Reduces liquid to 0° and boils it at 0°, cooling 3 gals. of brine per min. from 12° to 3°. Gas passes to absorber and the cycle is repeated.

Of the 2 gals. per min. of cooling water flowing from the condenser, 0.2 gal. goes to the rectifier, where it is heated to 142°, and 1.8 gal. through the absorber, where it is heated to 110°.

Heat Balance. — Absorbed in the generator 496; in the brine cooler, 200, Total 696 B.T.U. Rejected; condenser, 220; absorber, 383; rectifier, 93; Total 696 B.T.U.

The following table shows the strength of the liquors and the quantity of steam required per hour per ton of refrigeration under the conditions stated:

| Condenser Pressures. | | | | | | | | | |
|----------------------|--------------------|-------|-------|-------|------|-------|------|------|------|
| | 140 | | | 170 | | | 200 | | |
| | Suction Pressures. | | | | | | | | |
| | 0 | 15 | 30 | 0 | 15 | 30 | 0 | 15 | 30 |
| Sl per cent..... | 24 | 35 | 42 | 22 | 32 | 38 | 18 | 28 | 36 |
| Wl per cent..... | 13.13 | 25.75 | 33.70 | 10.85 | 22.3 | 29.15 | 6.28 | 17.7 | 26.9 |
| SG, pounds..... | 30.1 | 27.9 | 22.9 | 41.3 | 30.9 | 26.2 | 48.7 | 34.1 | 27.9 |
| SL, pounds..... | 1.7 | 1.6 | 1.4 | 2.1 | 1.9 | 1.8 | 2.4 | 2.3 | 2.2 |

Sl, strong liquor; Wl, weak liquor; SG, lbs. of steam per hour per ton of refrigeration for the generator, SL, do. for the liquor pump. Pressures are in lbs. per sq. in., gauge.

The following table gives the steam consumption in lbs. per hour per ton of refrigeration, for engine-driven compressors and for absorption machines with liquor pump not exhausting into the generator at the suction and condenser pressures (gauge) given: SC, simple non-condensing engine, CC, compound condensing engine, A, absorption machine.

| Condenser Pressures. | | | | | | | | | |
|----------------------|--------------------|------|------|------|------|------|-------|------|------|
| | 140 | | | 170 | | | 200 | | |
| | Suction Pressures. | | | | | | | | |
| | 0 | 15 | 30 | 0 | 15 | 30 | 0 | 15 | 30 |
| SC..... | 78.3 | 44.5 | 31.1 | 90.5 | 52.5 | 37.2 | 104.0 | 61.4 | 44.5 |
| CC..... | 42.0 | 23.8 | 16.6 | 48.4 | 28.0 | 19.0 | 55.6 | 32.7 | 23.9 |
| A..... | 31.8 | 29.5 | 24.3 | 43.4 | 32.8 | 28.0 | 51.1 | 36.4 | 30.1 |

The economy of the absorption machine is much better for all conditions than that of a simple non-condensing engine-driven compressor. At suction gauge pressures above 8 to 10 lbs. the economy of the compound condensing engine-driven compressor exceeds that of the absorption machine, the absorption machine giving the superior economy at suction pressures below 8 to 10 lbs.

Means for Applying the Cold. (M. C. Bannister, Liverpool Eng'g Soc'y, 1890.) — The most useful means for applying the cold to various uses is a saturated solution of brine or chloride of magnesium, which remains liquid at 5° Fahr. The brine is first cooled by being circulated in contact with the refrigerator-tubes, and then distributed through coils of pipes, arranged either in the substances requiring a reduction of temperature, or in the cold stores or rooms prepared for them; the air coming in contact with the cold tubes is immediately chilled, and the moisture in the air deposited on the pipes. It then falls, making room for warmer air, and so circulates until the whole room is at the temperature of the brine in the pipes.

The Direct Expansion Method consists in conveying the compressed cooled ammonia (or other refrigerating agent) directly to the room to be cooled, and then expanding it through an expansion cock into pipes in the room. Advantages of this system are its simplicity and its rapidity of action in cooling a room; disadvantages are the danger of leakage of the gas and the fact that the machine cannot be stopped without a rapid rise in the temperature of the room. With the brine system, with a large amount of cold brine in the tank, the machine may be stopped for a considerable time without serious cooling of the room.

Air has also been used as the circulating medium. The ammonia-pipes refrigerate the air in a cooling-chamber, and large conduits are used to convey it to and return it from the rooms to be cooled. An advantage of this system is that by it a room may be refrigerated more quickly than by brine-coils. The returning air deposits its moisture on the ammonia-pipes, in the form of snow, which is removed by mechanical brushes.

ARTIFICIAL-ICE MANUFACTURE.

Under summer conditions, with condensing water at 70°, artificial-ice machines use ammonia at a condenser pressure, about 190 lbs. above the atmosphere and 15 lbs. suction-pressure.

In a compression type of machine the useful circulation of ammonia, allowing for the effect of cylinder-heating, is about 13 lbs. per hour per indicated horse-power of the steam-cylinder. This weight of ammonia produces about 32 lbs. of ice at 15° from water at 70°. If the ice is made from distilled water, as in the "can system," the amount of the latter supplied by the boilers is about 33% greater than the weight of ice obtained. This excess represents steam escaping to the atmosphere from the re-boiler and steam-condenser, to purify the distilled water, or free it from air; also, the loss through leaks and drips, and loss by melting of the ice in extracting it from the cans. The total steam consumed per horse-power is, therefore, about $32 \times 1.33 = 43.0$ lbs. About 7.0 lbs. of this covers the steam-consumption of the steam-engines driving the brine circulating-pumps, the several cold-water pumps, and leakage, drips, etc. Consequently, the main steam-engine must consume 36 lbs. of steam per hour per I.H.P., or else live steam must be condensed to supply the required amount of distilled water. There is, therefore, nothing to be gained by using steam at high rates of expansion in the steam-engines, in making artificial ice from distilled water. If the cooling water for the ammonia-coils and steam-condenser is not too hard for use in the boilers, it may enter the latter at about 175° F., by restricting the quantity to $1\frac{1}{2}$ gallons per minute per ton of ice. With good coal $8\frac{1}{2}$ lbs. of feed-water may then be evaporated, on the average, per lb. of coal.

The ice made per pound of coal will then be $32 \div (43.0 \div 8.5) = 6.0$ lbs. This corresponds with the results of average practice.

If ice is manufactured by the "plate system," no distilled water is used for freezing. Hence the water evaporated by the boiler may be reduced to the amount which will drive the steam-motors, and the latter may use steam expansively to any extent consistent with the power required to compress the ammonia, operate the feed and filter pumps, and the hoisting machinery. The latter may require about 15% of the power needed for compressing the ammonia.

If a compound condensing steam-engine is used for driving the compressors, the steam per indicated steam horse-power, or per 32 lbs. of net ice, may be 14 lbs. per hour. The other motors at 50 lbs. of steam per horse-power will use 7.5 lbs. per hour, making the total consumption per steam horse-power of the compressor 21.5 lbs. Taking the evapora-

tion at 8 lbs., the feed-water temperature being limited to about 110°, the coal per horse-power is 2.7 lbs. per hour. The net ice per lb. of coal is then about $32 \div 2.7 = 11.8$ lbs. The best results with "plate-system" plants, using a compound steam-engine, have thus far afforded about 10½ lbs. of ice per lb. of coal.

In the "plate system" the ice gradually forms, in from 8 to 10 days, to a thickness of about 14 inches, on the hollow plates, 10 × 14 feet in area, in which the cooling fluid circulates.

In the "can system" the water is frozen in blocks weighing about 300 lbs. each, and the freezing is completed in from 40 to 48 hours. The freezing-tank area occupied by the "plate system" is, therefore, about twelve times, and the cubic contents about four times, as much as required in the "can system."

The investment for the "plate" is about one-third greater than for the "can" system. In the latter system ice is being drawn throughout the 24 hours, and the hoisting is done by hand tackle. Some "can" plants are equipped with pneumatic hoists and on large hoists electric cranes are used to advantage. In the "plate system" the entire daily product is drawn, cut, and stored in a few hours, the hoisting being performed by power. The distribution of cost is as follows for the two systems, taking the cost for the "can" or distilled-water system as 100, which represents an actual cost of about \$1.25 per net ton:

| | Can System. | Plate System. |
|--|--------------|---------------|
| Hoisting and storing ice | 14.2 | 2.8 |
| Engineers, firemen, and coal-passer | 15.0 | 13.9 |
| Coal at \$3.50 per gross ton | 42.2 | 20.0 |
| Water pumped directly from a natural source at 5 cts. per 1000 cubic feet | 1.3 | 2.6 |
| Interest and depreciation at 10% | 24.6 | 32.7 |
| Repairs | 2.7 | 3.4 |
| | <hr/> 100.00 | <hr/> 75.4 |

A compound condensing engine is assumed to be used by the "plate system."

Test of the New York Hygeia Ice-making Plant. — (By Messrs. Hupfel, Griswold, and Mackenzie; *Stevens Indicator*, Jan., 1894.)

The final results of the tests were as follows:

| | |
|---|---------|
| Net ice made per pound of coal, in pounds | 7.12 |
| Pounds of net ice per hour per horse-power | 37.8 |
| Net ice manufactured per day (12 hours) in tons | 97 |
| Av. pressure of ammonia-gas at condenser, lbs. per sq. in. above atmos. | 135.2 |
| Average back pressure of amm.-gas, lbs. per sq. in. above atmos. | 15.8 |
| Average temperature of brine in freezing-tanks, degrees F. | 19.7 |
| Total number of cans filled per week | 4389 |
| Ratio of cooling-surface of coils in brine-tank to can-surface. | 7 to 10 |

An Absorption Evaporator Ice-making System, built by the Carbon-dale Machine Co. is in operation at the ice plant of the Richmond Ice Co., Clifton, Staten Island, N. Y., which produces the extra distilled water by an evaporator at practically no fuel cost, and thus about 10 tons of distilled water ice per ton of coal is obtained. Steam from the boiler at 100 lbs. pressure enters an evaporator, distilling off steam at 70 lbs., which operates the pumps and auxiliary machinery. These exhaust into the ice machine generator under 10 lbs. pressure, where the exhaust is condensed. In a 100-ton plant the evaporator will condense 43 tons of live steam, distilling off 40 tons of steam to operate the auxiliaries, which exhaust into the generator; 20 tons of live steam has to be added to this exhaust, making 60 tons in all, which is the amount required to operate the generator. The 60 tons of condensation from the generator and 43 tons from the evaporator go to the re-boiler, making 103 tons of distilled water to be frozen into ice. The total steam consumption is the 60 tons condensed in the generator plus 3 tons for radiation, or 63 tons in all. Hence if the boiler evaporates 6.6 lbs. water per pound of coal the economy of the plant will be 10½ lbs. ice per pound of coal, a result which cannot be obtained even with compound condensing engines and compression machines.

Heat-exchanging coils, on the order of a closed feed-water heater, are used to heat the feed-water going to the boiler. The condensation leaving the generator and evaporator at a high temperature is utilized for this purpose; by this means securing a feed-water temperature considerably in excess of 212° .

Ice-Making with Exhaust Steam. — The exhaust steam from electric light plants is being utilized to manufacture ice on the absorption system. A 10-ton plant at the Holdredge Lighting Co., Holdredge, Neb., built by the Carbondale Machine Co., is described in *Elec. World*, April 7, 1910. Here 11 tons of ice were made per day with exhaust steam from the electric engines at $2\frac{1}{2}$ lbs. pressure, using $6\frac{1}{3}$ K.W., or $8\frac{1}{2}$ H.P., for driving the circulating pumps.

Tons of Ice per Ton of Coal. — From a long table by Mr. Voorhees, showing the net tons of plate ice that may be made in well-designed plants under a variety of conditions as to type of engine, the following figures are taken:

| | |
|--|----------|
| Compression, Simple Corliss engine, non-condensing..... | 6.1 tons |
| Absorption liquor pump and auxiliaries not exhausting into generator, simple, non-condensing engine..... | 10.0 |
| Compression, compound condensing engine..... | 11.2 |
| Compression triple-expansion condensing engine..... | 12.8 |
| Absorption, pump and auxiliaries exhausting into generator, Corliss non-condensing engine..... | 13.3 |
| Compression and absorption, compound engine, non-condensing | 16.0 |
| Compression, triple-expansion condensing engine, multiple effect | 16.5 |
| Compression and absorption, triple-expansion non-condensing engine, multiple effect | 19.5 |

Standard Ice Cans or Moulds.

(Buffalo Refrigerating Machine Co.)

| Weight of Block. | Size of Can. | Time of Freezing. | Weight of Block. | Size of Can. | Time of Freezing. |
|------------------|--------------|-------------------|------------------|--------------|-------------------|
| pounds | | hours | pounds | | hours |
| 25 | 4×10×24 | 12 | 100 | 11×11×32 | 48 |
| 50 | 6×12×26 | 20 | 200 | 11×22×32 | 54 |
| 100 | 8×15×32 | 36 | 300 | 11×22×44 | 54 |
| 150 | 8×15×44 | 36 | 400 | 11×22×56 | 54 |
| 150 | 10×15×36 | 48 | 200 | 14×14×40 | 66 |
| 200 | 10×20×36 | 48 | | | |

The above given time of freezing is with a brine temperature of 15° F.

MARINE ENGINEERING.

Rules for Measuring Dimensions and Obtaining Tonnage of Vessels. (Record of American and Foreign Shipping. American Bureau of Shipping, N. Y., 1890.) — The dimensions to be measured as follows;

I. Length, *L*. — From the fore-side of stem to the after-side of stern-post measured at middle line on the upper deck of all vessels, except those having a continuous hurricane-deck extending right fore and aft, in which the length is to be measured on the range of deck immediately below the hurricane-deck.

Vessels having clipper heads, raking forward, or receding stems, or raking stern-posts, the length to be the distance of the fore-side of stem from aft-side of stern-post at the deep-load water-line measured at middle line. (The inner or propeller-post to be taken as stern-post in screw-steamers.)

II. Breadth, *B*. — To be measured over the widest frame at its widest part; in other words, the molded breadth.

III. Depth, *D*. — To be measured at the dead-flat frame and at middle line of vessel. It shall be the distance from the top of floor-plate to the upper side of upper deck-beam in all vessels except those having a continuous hurricane-deck, extending right fore and aft, and not intended for the American coasting trade, in which the depth is to be the distance

from top of floor-plate to midway between top of hurricane deck-beam and the top of deck-beam of the deck immediately below hurricane-deck.

In vessels fitted with a continuous hurricane-deck, extending right fore and aft, and intended for the American coasting trade, the depth is to be the distance from top of floor-plate to top of deck-beam of deck immediately below hurricane-deck.

Rule for Obtaining Tonnage. — Multiply together the length, breadth, and depth, and their product by 0.75; divide the last product by 100; the quotient will be the tonnage. $L \times B \times D \times 0.75 \div 100 = \text{tonnage}$.

The U. S. Custom-house Tonnage Law, May 6, 1864, provides that "the register tonnage of a vessel shall be her entire internal cubic capacity in tons of 100 cubic feet each." This measurement includes all the space between upper decks, however many there may be. Explicit directions for making the measurements are given in the law.

The Displacement of a Vessel (measured in tons of 2240 lbs.) is the weight of the volume of water which it displaces. For sea-water it is equal to the volume of the vessel beneath the water-line, in cubic feet, divided by 35, which figure is the number of cubic feet of sea-water at 60° F. in a ton of 2240 lbs. For fresh water the divisor is 35.93. The U. S. register tonnage will equal the displacement when the entire internal cubic capacity bears to the displacement the ratio of 100 to 35.

The displacement or gross tonnage is sometimes approximately estimated as follows: Let L denote the length in feet of the boat, B its extreme breadth in feet, and D the mean draught in feet; the product of these three dimensions will give the volume of a parallelopipedon in cubic feet. Putting V for this volume, we have $V = L \times B \times D$.

The volume of displacement may then be expressed as a *percentage* of the volume V , known as the "*block coefficient*." This percentage varies for different classes of ships. In racing yachts with very deep keels it varies from 22 to 33; in modern merchantmen from 55 to 90; for ordinary small boats probably 50 will give a fair estimate. The volume of displacement in cubic feet divided by 35 gives the displacement in tons.

Coefficient of Fineness. — A term used to express the relation between the displacement of a ship and the volume of a rectangular prism or box whose lineal dimensions are the length, breadth, and draught.

Coefficient of fineness = $D \times 35 \div (L \times B \times W)$; D being the displacement in tons of 35 cubic feet of sea-water to the ton, L the length between perpendiculars, B the extreme breadth and W the mean draught, all in feet.

Coefficient of Water-lines. — An expression of the relation of the displacement to the volume of the prism whose section equals the midship section of the ship, and length equal to the length of the ship.

Coefficient of water-lines = $D \times 35 \div (\text{area of immersed water section} \times L)$. Seaton gives the following values:

| | Coefficient of Fineness. | Coefficient of Water-lines |
|--|-----------------------------|-------------------------------|
| Finely-shaped ships | 0.55 | 0.63 |
| Fairly-shaped ships | 0.61 | 0.67 |
| Ordinary merchant steamers 10 to 11 knots... | 0.65 | 0.72 |
| Cargo steamers, 9 to 10 knots | 0.70 | 0.76 |
| Modern cargo steamers of large size | 0.78 | 0.83 |

Resistance of Ships. — The resistance of a ship passing through water may vary from a number of causes, as speed, form of body, displacement, midship dimensions, character of wetted surface, fineness of lines, etc. The resistance of the water is twofold; 1st. That due to the displacement of the water at the bow and its replacement at the stern, with the consequent formation of waves. 2d. The friction between the wetted surface of the ship and the water, known as skin resistance. A common approximate formula for resistance of vessels is

Resistance = $\text{speed}^2 \times \sqrt[3]{\text{displacement}^2} \times \text{a constant}$, or $R = S^2 D^{\frac{2}{3}} \times C$.

If D = displacement in pounds, S = speed in feet per minute, R resistance in foot-pounds per minute, $R = CS^2 D^{\frac{2}{3}}$. The work done in overcoming the resistance through a distance equal to S is $R \times S = CS^3 D^{\frac{2}{3}}$; and if E is the efficiency of the propeller and machinery combined, the indicated horse-power I. H.P. = $CS^3 D^{\frac{2}{3}} \div (E \times 33,000)$.

If S = speed in knots, D = displacement in tons, and C a constant

which includes all the constants for form of vessel, efficiency of mechanism, etc., $I.H.P. = S^3 D^{\frac{2}{3}} + C$.

The wetted surface varies as the cube root of the square of the displacement; thus, let L be the length of edge of a cube just immersed, whose displacement is D and wetted surface W . Then $D = L^3$ or $L = \sqrt[3]{D}$, and $W = 5 \times L^2 = 5 \times (\sqrt[3]{D})^2$. That is, W varies as $D^{\frac{2}{3}}$.

Another approximate formula is

$$I.H.P. = \text{area of immersed midship section} \times S^3 + K.$$

The usefulness of these two formulæ depends upon the accuracy of the so-called "constants" C and K , which vary with the size and form of the ship, and probably also with the speed. Seaton gives the following, which may be taken roughly as the values of C and K under the conditions expressed:

| General Description of Ship. | | Speed, knots. | Value of C . | Value of K . |
|--|-----|------------------|-------------------|-------------------|
| Ships over 400 feet long, finely shaped | 400 | 15 to 17 | 240 | 620 |
| | 300 | 15 " 17 | 190 | 500 |
| | " | 13 " 15 | 240 | 650 |
| | " | 11 " 13 | 260 | 700 |
| Ships over 300 feet long, fairly shaped | | 11 " 13 | 240 | 650 |
| | | 9 " 11 | 260 | 700 |
| Ships over 250 feet long, finely shaped | | 13 " 15 | 200 | 580 |
| | | 11 " 13 | 240 | 660 |
| Ships over 250 feet long, fairly shaped | | 9 " 11 | 260 | 700 |
| | | 11 " 13 | 220 | 620 |
| Ships over 200 feet long, finely shaped | | 9 " 11 | 250 | 680 |
| | | 11 " 12 | 220 | 600 |
| Ships over 200 feet long, fairly shaped | | 9 " 11 | 240 | 640 |
| | | 9 " 11 | 220 | 620 |
| Ships under 200 feet long, finely shaped | | 11 " 12 | 200 | 550 |
| | | 10 " 11 | 210 | 580 |
| Ships under 200 feet long, fairly shaped | | 9 " 10 | 230 | 620 |
| | | 9 " 10 | 200 | 600 |

Coefficient of Performance of Vessels. — The quotient

$$\sqrt[3]{(\text{displacement})^2 \times (\text{speed in knots})^3 + \text{tons of coal in 24 hours}}$$

gives a coefficient of performance which represents the comparative cost of propulsion in coal expended. Sixteen vessels with three-stage expansion-engines in 1890 gave an average coefficient of 14,810, the range being from 12,150 to 16,700.

In 1881 seventeen vessels with two-stage expansion-engines gave an average coefficient of 11,710. In 1881 the length of the vessels tested ranged from 260 to 320, and in 1890 from 295 to 400. The speed in knots divided by the square root of the length in feet in 1881 averaged 0.539; and in 1890, 0.579; ranging from 0.520 to 0.641. (*Proc. Inst. M. E.*, July, 1891, p. 329.)

Defects of the Common Formula for Resistance. — Modern experiments throw doubt upon the truth of the statement that the resistance varies as the square of the speed. (See Robt. Mansel's letters in *Engineering*, 1891; also his paper on The Mechanical Theory of Steamship Propulsion, read before Section G of the Engineering Congress, Chicago, 1893.)

Seaton says: In small steamers the chief resistance is the skin resistance. In very fine steamers at high speeds the amount of power required seems excessive when compared with that of ordinary steamers at ordinary speeds.

In torpedo-launches at certain high speeds the resistance increases at a lower rate than the square of the speed.

In ordinary sea-going and river steamers the reverse seems to be the case.

Rankine's Formula for total resistance of vessels of the "wave-line" type is:

$$R = ALBV^2 (1 + 4 \sin^2 \theta + \sin^4 \theta),$$

in which equation θ is the mean angle of greatest obliquity of the stream-lines, A is a constant multiplier, B the mean wetted girth of the surface exposed to friction, L the length in feet, and V the speed in knots. The power demanded to impel a ship is thus the product of a constant to be determined by experiment, the area of the wetted surface, the cube of the speed, and the quantity in the parenthesis, which is known as the "coefficient of augmentation." In calculating the resistance of ships the last term of the coefficient may be neglected as too small to be practically important. In applying the formula, the mean of the squares of the sines of the angles of maximum obliquity of the water-lines is to be taken for $\sin^2 \theta$, and the rule will then read thus:

To obtain the resistance of a ship of good form, in pounds, multiply the length in feet by the mean immersed girth and by the coefficient of augmentation, and then take the product of this "augmented surface," as Rankine termed it, by the square of the speed in knots, and by the proper constant coefficient selected from the following:

| | |
|---|------------------------|
| For clean painted vessels, iron hulls. | $A = 0.01$ |
| For clean coppered vessels. | $A = 0.009$ to 0.008 |
| For moderately rough iron vessels. | $A = 0.011$ + |

The net, or effective, horse-power demanded will be quite closely obtained by multiplying the resistance calculated, as above, by the speed in knots and dividing by 326. The gross, or indicated, power is obtained by multiplying the last quantity by the reciprocal of the efficiency of the machinery and propeller, which usually should be about 0.6. Rankine uses as a divisor in this case 200 to 260.

The form of the vessel, even when designed by skillful and experienced naval architects, will often vary to such an extent as to cause the above constant coefficients to vary somewhat; and the range of variation with good forms is found to be from 0.8 to 1.5 the figures given.

For well-shaped iron vessels, an approximate formula for the horse-power required is $H.P. = SV^3 \div 20,000$, in which S is the "augmented surface." The expression $SV^3 \div H.P.$ has been called by Rankine the *coefficient of propulsion*. In the Hudson River steamer "Mary Powell," according to Thurston, this coefficient was as high as 23,500.

The expression $D^3 V^3 \div H.P.$ has been called the *locomotive performance*. (See Rankine's Treatise on Shipbuilding, 1864; Thurston's Manual of the Steam-engine, part ii, p. 16; also paper by F. T. Bowles, U. S. N., Proc. U. S. Naval Institute, 1883.)

Rankine's method for calculating the resistance is said by Seaton to give more accurate and reliable results than those obtained by the older rules, but it is criticised as being difficult and inconvenient of application.

E. R. Mumford's Method of Calculating Wetted Surfaces is given in a paper by Archibald Denny, *Eng'g*, Sept. 21, 1894. The following is his formula, which gives closely accurate results for medium draughts, beams, and finenesses;

$$S = (L \times D \times 1.7) + (L \times B \times C),$$

in which S = wetted surface in square feet; L = length between perpendiculars in feet; D = middle draught in feet; B = beam in feet; C = block coefficient.

The formula may also be expressed in the form $S = L(1.7 D + BC)$.

In the case of twin-screw ships having projecting shaft-casings, or in the case of a ship having a deep keel or bilge keels, an addition must be made for such projections. The formula gives results which are in general much more accurate than those obtained by Kirk's method. It underestimates the surface when the beam, draught, or block coefficients are excessive; but the error is small except in the case of abnormal forms, such as stern-wheel steamers having very excessive beams (nearly one-fourth the length), and also very full block coefficients. The formula gives a surface about 6% too small for such forms.

The wetted surface of the block is nearly equal to that of the ship of the same length, beam and draught; usually 2% to 5% greater. In exceedingly fine hollow-line ships it may be 8% greater.

Area of bottom of block = $(F + M) \times B$;

Area of sides = $2 M \times H$.

Area of sides of ends = $4 \times \sqrt{F^2 + \left(\frac{B}{2}\right)^2} \times H$;

Tangent of half angle of entrance = $\frac{1}{2} B/F = B/(2 F)$.

From this, by a table of natural tangents, the angle of entrance may be obtained:

| | Angle of Entrance of the Block Model. | Fore-body in parts of length. |
|--|--|----------------------------------|
| Ocean-going steamers, 14 knots and upw'd | 18° to 15° | 0.3 to 0.36 |
| " " 12 to 14 knots | 21° to 18° | 0.26 to 0.3 |
| " cargo steamers, 10 to 12 knots.. | 30° to 22° | 0.22 to 0.26 |

Dr. Kirk's Method. — This method is generally used on the Clyde.

The general idea proposed by Dr. Kirk is to reduce all ships to so definite and simple a form that they may be easily compared; and the magnitude of certain features of this form shall determine the suitability of the ship for speed, etc.

The form consists of a middle body, which is a rectangular parallelepiped, and fore-body and after-body, prisms having isosceles triangles for bases, as shown in Fig. 194.

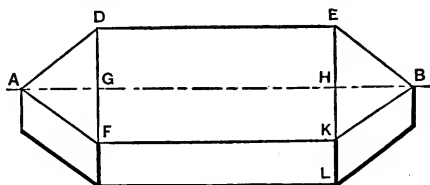


FIG. 194.

This is called a block model, and is such that its length is equal to that of the ship, the depth is equal to the mean draught, the capacity equal to the displacement volume, and its area of section equal to the area of immersed midship section. The dimensions of the block model may be obtained as follows: Let $AG = HB$ = length of fore- or after-body = F ; GH = length of middle body = M ; KL = mean draught = H ; EK = area of immersed midship section $\div KL = B$. Volume of block = $(F + M) \times B \times H$; midship section = $B \times H$; displacement in tons = volume in cubic ft. $\div 35$.

$AH = AG + GH = F + M$ = displacement $\times 35 \div (B \times H)$.

To find the Indicated Horse-power from the Wetted Surface. (Seaton.) — In ordinary cases the horse-power per 100 feet of wetted surface may be found by assuming that the rate for a speed of 10 knots is 5, and that the quantity varies as the cube of the speed. For example: To find the number of I.H.P. necessary to drive a ship at a speed of 15 knots, having a wetted skin of block model of 16,200 square feet:

The rate per 100 feet = $(15/10)^3 \times 5 = 16.875$.

Then I.H.P. required = $16.875 \times 162 = 2734$.

When the ship is exceptionally well-proportioned, the bottom quite clean, and the efficiency of the machinery high, as low a rate as 4 I.H.P. per 100 feet of wetted skin of block model may be allowed.

The gross indicated horse-power includes the power necessary to overcome the friction and other resistance of the engine itself and the shafting, and also the power lost in the propeller. In other words, I.H.P. is no measure of the resistance of the ship, and can only be relied on as a means of deciding the size of engines for speed, so long as the efficiency of the engine and propeller is known definitely, or so long as similar engines and

propellers are employed in ships to be compared. The former is difficult to obtain, and it is nearly impossible in practice to know how much of the power shown in the cylinders is employed usefully in overcoming the resistance of the ship. The following example is given to show the variation in the efficiency of propellers:

| | Knots. | I.H.P. |
|--|--------|-----------|
| H.M.S. "Amazon," with a 4-bladed screw, gave | 12.064 | with 1940 |
| H.M.S. "Amazon," with a 2-bladed screw, increased pitch, and fewer revolutions per minute..... | 12.396 | " 1663 |
| H.M.S. "Iris," with a 4-bladed screw | 16.577 | " 7503 |
| H.M.S. "Iris," with 2-bladed screw, increased pitch, fewer revolutions per knot | 18.587 | " 7556 |

Relative Horse-power Required for Different Speeds of Vessels. (Horse-power for 10 knots = 1.) — The horse-power is taken usually to vary as the cube of the speed, but in different vessels and at different speeds it may vary from the 2.8 power to the 3.5 power, depending upon the lines of the vessel and upon the efficiency of the engines, the propeller, etc. (The power may vary at a much higher rate than the 3.5 power of the speed when the speed is much less than normal, and the machinery is therefore working at less than its normal efficiency.)

| Speed knots. | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
|----------------|-------|------|------|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $H.P. \propto$ | | | | | | | | | | | | | | |
| $S^{2.8}$ | .0769 | .239 | .535 | 1. | 1.666 | 2.565 | 3.729 | 5.185 | 6.964 | 9.095 | 11.60 | 14.52 | 17.87 | 21.67 |
| $S^{2.9}$ | .0701 | .227 | .524 | 1. | 1.697 | 2.653 | 3.908 | 5.499 | 7.464 | 9.841 | 12.67 | 15.97 | 19.80 | 24.19 |
| S^3 | .0640 | .216 | .512 | 1. | 1.728 | 2.744 | 4.096 | 5.832 | 8. | 10.65 | 13.82 | 17.58 | 21.95 | 27. |
| $S^{3.1}$ | .0584 | .205 | .501 | 1. | 1.760 | 2.838 | 4.293 | 6.185 | 8.574 | 11.52 | 15.09 | 19.34 | 24.33 | 30.14 |
| $S^{3.2}$ | .0533 | .195 | .490 | 1. | 1.792 | 2.935 | 4.500 | 6.559 | 9.189 | 12.47 | 16.47 | 21.28 | 26.97 | 33.63 |
| $S^{3.3}$ | .0486 | .185 | .479 | 1. | 1.825 | 3.036 | 4.716 | 6.957 | 9.849 | 13.49 | 17.98 | 23.41 | 29.90 | 37.54 |
| $S^{3.4}$ | .0444 | .176 | .468 | 1. | 1.859 | 3.139 | 4.943 | 7.378 | 10.56 | 14.60 | 19.62 | 25.76 | 33.14 | 41.90 |
| $S^{3.5}$ | .0405 | .167 | .458 | 1. | 1.893 | 3.247 | 5.181 | 7.824 | 11.31 | 15.79 | 21.42 | 28.34 | 36.73 | 46.77 |

EXAMPLE IN USE OF THE TABLE. — A certain vessel makes 14 knots speed with 587 I.H.P. and 16 knots with 900 I.H.P. What I.H.P. will be required at 18 knots, the rate of increase of horse-power with increase of speed remaining constant? The first step is to find the rate of increase, thus: $14^x : 16^x :: 587 : 900$.

$$x \log 16 - x \log 14 = \log 900 - \log 587;$$

$$x (0.204120 - 0.146128) = 2.954243 - 2.768638,$$

whence x (the exponent of S in formula $H.P. \propto S^x$) = 3.2.

From the table, for $S^{3.2}$ and 16 knots, the I.H.P. is 4.5 times the I.H.P. at 10 knots; \therefore H.P. at 10 knots = $900 \div 4.5 = 200$.

From the table for $S^{3.2}$ and 18 knots, the I.H.P. is 6.559 times the I.H.P. at 10 knots; \therefore H.P. at 18 knots = $200 \times 6.559 = 1312$ H.P.

Resistance per Horse-power for Different Speeds. (One horse-power = 33,000 lbs. resistance overcome through 1 ft. in 1 min.) — The resistances per horse-power for various speeds are as follows: For a speed of 1 knot, or 6080 feet per hour = $101\frac{1}{3}$ ft. per min., $33,000 \div 101\frac{1}{3} = 325.658$ lbs. per horse-power; and for any other speed 325.658 lbs. divided by the speed in knots; or for

| | | | | | |
|---------|-------------|---------|------------|----------|------------|
| 1 knot | 325.66 lbs. | 8 knots | 40.71 lbs. | 15 knots | 21.71 lbs. |
| 2 knots | 162.83 " | 9 " | 36.18 " | 16 " | 20.35 " |
| 3 " | 108.55 " | 10 " | 32.57 " | 17 " | 19.16 " |
| 4 " | 81.41 " | 11 " | 29.61 " | 18 " | 18.09 " |
| 5 " | 65.13 " | 12 " | 27.14 " | 19 " | 17.14 " |
| 6 " | 54.28 " | 13 " | 25.05 " | 20 " | 16.28 " |
| 7 " | 48.52 " | 14 " | 23.26 " | | |

More accurate methods than those above given for estimating the horse-power required for any proposed ship are: 1. Estimations calculated from the results of trials of "similar" vessels driven at "corresponding"

speeds; "similar" vessels being those that have the same ratio of length to breadth and to draught, and the same coefficient of fineness, and "corresponding" speeds those which are proportional to the square roots of the lengths of the respective vessels. Froude found that the resistances of such vessels varied almost exactly as wetted surface \times (speed)².

2. The method employed by the British Admiralty and by some Clyde shipbuilders, viz., ascertaining the resistance of a model of the vessel, 12 to 20 ft. long, in a tank, and calculating the power from the results obtained.

Estimated Displacement, Horse-power, etc. — The table on the next page, calculated by the author, will be found convenient for making approximate estimates.

The figures in 7th column are calculated by the formula $H.P. = S^3 D^{\frac{2}{3}} \div c$ in which $c = 200$ for vessels under 200 ft. long when $C = 0.65$, and 210 when $C = 0.55$; $c = 200$ for vessels 200 to 400 ft. long when $C = 0.75$, 220 when $C = 0.65$, 240 when $C = 0.55$; $c = 230$ for vessels over 400 ft. long when $C = 0.75$, 250 when $C = 0.65$, 260 when $C = 0.55$.

The figures in the 8th column are based on 5 H.P. per 100 sq. ft. of wetted surface.

The diameters of screw in the 9th column are from formula $D = 3.31 \sqrt[5]{I.H.P.}$, and in the 10th column from formula $D = 2.71 \sqrt[5]{I.H.P.}$.

To find the diameter of screw for any other speed than 10 knots, revolutions being 100 per minute, multiply the diameter given in the table by the 5th root of the cube of the given speed $\div 10$. For any other revolutions per minute than 100, divide by the revolutions and multiply by 100.

To find the approximate horse-power for any other speed than 10 knots, multiply the horse-power given in the table by the cube of the ratio of the given speed to 10, or by the relative figure from table on p. 1321.

F. E. Cardullo, *Mach'y*, April, 1907, gives the following formula as closely approximating the speed of modern types of hulls: $S = 6.35$

$\sqrt[3]{\frac{I.H.P.}{D^{\frac{2}{3}}}}$, in which S = speed in knots, D = displacement in tons. If we take $S = 10$ knots, then $I.H.P. \div D^{\frac{2}{3}} = 3.906$. Let $D = 10,000$, and $S = 10$, then $H.P. = 1813$. The table on page 1323 gives for a displacement of 10,400 tons and a coefficient of fineness 0.65, 1966 and 1760 H.P., averaging 1863 H.P.

Internal Combustion Marine Engines. — Linton Hope (*Eng'g*, April 8, 1910), in a paper on the application of internal combustion engines to fishing boats and fine-lined commercial vessels, gives a table showing the brake H.P. required to propel such vessels at various speeds. The following table is an abridgment. L = load water line; D = displacement in tons.

| Block Coefficient. | | | | | | | | Speed in Knots. | | | | | | | |
|--------------------|-------|-----|-------|------|--------|-----|-------|--------------------|-------|-------|----|-------|-------|-------|--|
| 0.25 | | 0.3 | | 0.35 | | 0.4 | | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| L | D | L | D | L | D | L | D | Brake Horse-power. | | | | | | | |
| 78 | 105 | 75 | 100 | 72 | 95 | 69 | 90 | 20 | 30 | 43 | 60 | 81 | 110 | 150 | |
| 71 | 81 | 69 | 77 | 66 | 73 | 63 | 70 | 17 | 25 | 37 | 51 | 69 | 93 | | |
| 65 | 62 | 63 | 60 | 60 | 58 | 57 | 55 | 15 | 22 | 32 | 44 | 60 | 82 | | |
| 59 | 47 | 57 | 45 | 54 | 44 | 52 | 42 | 13 | 19 | 27 | 39 | 53 | 76 | | |
| 54 | 36 | 52 | 35 | 50 | 34 | 48 | 32 | 11 | 16 | 24 | 34 | 48 | 71 | | |
| 50 | 28 | 48 | 27 | 46 | 26 | 44 | 25 | 9 | 13 | 20 | 29 | 44 | | | |
| 46 | 22 | 44 | 21 | 42 | 20 | 40 | 19 | 8 | 12 | 17 | 25 | 40 | | | |
| 41 | 17 | 40 | 16 | 38 | 15 | 37 | 14 | 7 | 11 | 15 | 22 | 37 | | | |
| 38 | 13 | 37 | 12 | 35 | 11 1/2 | 34 | 11 | 6 | 9 | 13 | 19 | 34 | | | |
| 35 | 9 | 34 | 8 1/2 | 32 | 8 | 31 | 7 1/2 | 5 | 7 | 11 | 16 | | | | |
| 32 | 6 1/2 | 31 | 6 | 30 | 5 1/2 | 29 | 5 | 4 | 5 1/2 | 9 | 14 | | | | |
| 30 | 4 1/2 | 29 | 4 1/4 | 28 | 3 3/4 | 27 | 3 1/2 | 3 | 5 | 7 | 12 | | | | |
| 28 | 3 1/4 | 27 | 3 | 26 | 2 3/4 | 25 | 2 1/2 | 2 1/2 | 4 1/2 | 6 1/2 | 11 | | | | |

Estimated Displacement, Horse-power, etc., of Steam-vessels of Various Sizes.

| Length, feet, L | Breadth, feet, B | Draught, feet, D | Coefficient of Fine- ness, C | Displacement, $LBD \times C$ | Wetted Surface $L(1.7D + BC)$ sq. ft. | Estimated Horse- power at 10 knots. | | Diam. of Screw for 10, knots speed and 100 revs. per minute. | |
|-----------------------|---------------------|---------------------|------------------------------------|---------------------------------|---|--|----------------------------------|--|-------------------------|
| | | | | 85 tons. | | Calc. from Dis- placement. | Calc. from Wetted Surface. | If Pitch = Diam. | If Pitch = 1.4 Diam. |
| 12 | 3 | 1.5 | 0.55 | 0.85 | 48 | 4.3 | 2.4 | 4.4 | 3.6 |
| 16 | 3 | 1.5 | .55 | 1.13 | 64 | 5.2 | 3.2 | 4.6 | 3.8 |
| | 4 | 2 | .65 | 2.38 | 96 | 8.9 | 4.8 | 5.1 | 4.2 |
| 20 | 3 | 1.5 | .55 | 1.41 | 80 | 6.0 | 4.0 | 4.7 | 3.9 |
| | 4 | 2 | .65 | 2.97 | 120 | 10.3 | 6.0 | 5.3 | 4.3 |
| 24 | 3.5 | 1.5 | .55 | 1.98 | 104 | 7.5 | 5.2 | 5 | 4.1 |
| | 4.5 | 2 | .65 | 4.01 | 152 | 12.6 | 7.6 | 5.5 | 4.5 |
| 30 | 4 | 2 | .55 | 3.77 | 168 | 11.5 | 8.4 | 5.4 | 4.4 |
| | 5 | 2.5 | .65 | 6.96 | 224 | 18.2 | 11.2 | 5.9 | 4.8 |
| 40 | 4.5 | 2 | .55 | 5.66 | 235 | 15.1 | 11.8 | 5.7 | 4.7 |
| | 6 | 2.5 | .65 | 11.1 | 326 | 24.9 | 16.3 | 6.3 | 5.2 |
| 50 | 6 | 3 | .55 | 14.1 | 420 | 27.8 | 21.0 | 6.4 | 5.4 |
| | 8 | 3.5 | .65 | 26 | 558 | 43.9 | 27.9 | 7.1 | 5.8 |
| 60 | 8 | 3.5 | .55 | 26.4 | 621 | 42.2 | 31.1 | 7.0 | 5.7 |
| | 10 | 4 | .65 | 44.6 | 798 | 62.9 | 39.9 | 7.6 | 6.2 |
| 70 | 10 | 4 | .55 | 44 | 861 | 59.4 | 43.1 | 7.5 | 6.1 |
| | 12 | 4.5 | .65 | 70.2 | 1082 | 85.1 | 54.1 | 8.1 | 6.6 |
| 80 | 12 | 4.5 | .55 | 67.9 | 1140 | 79.2 | 57.0 | 7.9 | 6.5 |
| | 14 | 5 | .65 | 104.0 | 1408 | 111 | 70.4 | 8.5 | 7.0 |
| 90 | 13 | 5 | .55 | 91.9 | 1408 | 97 | 70.4 | 8.3 | 6.8 |
| | 16 | 6 | .65 | 160 | 1854 | 147 | 92.7 | 9 | 7.3 |
| 100 | 13 | 5 | .55 | 102 | 1565 | 104 | 78.3 | 8.4 | 6.9 |
| | 15 | 5.5 | .65 | 153 | 1910 | 143 | 95.5 | 8.9 | 7.3 |
| | 17 | 6 | .75 | 219 | 2295 | 202 | 115 | 9.6 | 7.8 |
| 120 | 14 | 5.5 | .55 | 145 | 2046 | 131 | 102 | 8.8 | 7.2 |
| | 16 | 6 | .65 | 214 | 2472 | 179 | 124 | 9.4 | 7.6 |
| | 18 | 6.5 | .75 | 301 | 2946 | 250 | 147 | 10 | 8.2 |
| 140 | 16 | 6 | .55 | 211 | 2660 | 169 | 133 | 9.2 | 7.4 |
| | 18 | 6.5 | .65 | 306 | 3185 | 227 | 159 | 9.8 | 8.0 |
| | 20 | 7 | .75 | 420 | 3766 | 312 | 188 | 10.5 | 8.5 |
| 160 | 17 | 6.5 | .55 | 278 | 3264 | 203 | 163 | 9.6 | 7.8 |
| | 19 | 7 | .65 | 395 | 3880 | 269 | 194 | 10.1 | 8.3 |
| | 21 | 7.5 | .75 | 540 | 4560 | 368 | 228 | 10.8 | 8.8 |
| 180 | 20 | 7 | .55 | 396 | 4122 | 257 | 206 | 10.1 | 8.2 |
| | 22 | 7.5 | .65 | 552 | 4869 | 337 | 243 | 10.6 | 8.7 |
| | 24 | 8 | .75 | 741 | 5688 | 455 | 284 | 11.3 | 9.2 |
| 200 | 22 | 7 | .55 | 484 | 4800 | 257 | 240 | 10.1 | 8.2 |
| | 25 | 8 | .65 | 743 | 5970 | 373 | 299 | 10.8 | 8.8 |
| | 28 | 9 | .75 | 1080 | 7260 | 526 | 363 | 11.6 | 9.5 |
| 250 | 28 | 8 | .55 | 880 | 7250 | 383 | 363 | 10.9 | 8.9 |
| | 32 | 10 | .65 | 1486 | 9450 | 592 | 473 | 11.9 | 9.7 |
| | 36 | 12 | .75 | 2314 | 11850 | 875 | 593 | 12.8 | 10.5 |
| 300 | 32 | 10 | .55 | 1509 | 10380 | 548 | 519 | 11.7 | 9.6 |
| | 36 | 12 | .65 | 2407 | 13140 | 806 | 657 | 12.6 | 10.4 |
| | 40 | 14 | .75 | 3600 | 17140 | 1175 | 857 | 13.6 | 11.1 |
| 350 | 38 | 12 | .55 | 2508 | 14455 | 769 | 723 | 12.5 | 10.2 |
| | 42 | 14 | .65 | 3822 | 17885 | 1111 | 894 | 13.5 | 11.0 |
| | 46 | 16 | .75 | 5520 | 21595 | 1562 | 1080 | 14.4 | 11.8 |
| 400 | 44 | 14 | .55 | 3872 | 19200 | 1028 | 960 | 13.3 | 10.8 |
| | 48 | 16 | .65 | 5705 | 23360 | 1451 | 1168 | 14.2 | 11.6 |
| | 52 | 18 | .75 | 8023 | 27840 | 2006 | 1392 | 15.2 | 12.4 |
| 450 | 50 | 16 | .55 | 5657 | 24515 | 1221 | 1226 | 13.7 | 11.2 |
| | 54 | 18 | .65 | 8123 | 29565 | 1616 | 1478 | 14.5 | 11.9 |
| | 58 | 20 | .75 | 11157 | 34875 | 2171 | 1744 | 15.4 | 12.6 |
| 500 | 52 | 18 | .55 | 7354 | 29600 | 1454 | 1480 | 14.2 | 11.6 |
| | 56 | 20 | .65 | 10400 | 35200 | 1966 | 1760 | 15.1 | 12.4 |
| | 60 | 22 | .75 | 14143 | 41200 | 2543 | 2060 | 15.9 | 13.0 |
| 550 | 56 | 20 | .55 | 9680 | 36245 | 1747 | 1812 | 14.7 | 12.0 |
| | 60 | 22 | .65 | 13483 | 42735 | 2266 | 2137 | 15.5 | 12.7 |
| | 64 | 24 | .75 | 18103 | 49665 | 2998 | 2483 | 16.4 | 13.4 |
| 600 | 60 | 22 | .55 | 12446 | 42900 | 2065 | 2145 | 15.2 | 12.5 |
| | 64 | 24 | .65 | 17115 | 50220 | 2656 | 2511 | 15.4 | 13.1 |
| | 68 | 26 | .75 | 22731 | 58020 | 3489 | 2901 | 16.9 | 13.8 |

The "pitch" of a propeller is the distance which any point in a blade describing a helix will travel in the direction of the axis during one revolution, the point being assumed to move around the axis. The pitch of a propeller with a uniform pitch is equal to the distance a propeller will advance during one revolution, provided there is no slip. In a case of this kind, the term "pitch" is analogous to the term "pitch of the thread" of an ordinary single-threaded screw.

Let P = pitch of screw in feet, R = number of revolutions per second, V = velocity of stream from the propeller = $P \times R$, v = velocity of the ship in feet per second, $V - v$ = slip, A = area in square feet of section of stream from the screw, approximately the area of a circle of the same diameter, $A \times V$ = volume of water projected astern from the ship in cubic feet per second. Taking the weight of a cubic foot of sea-water at 64 lbs., and the force of gravity at 32, we have from the common formula for force of acceleration, viz.: $F = M \frac{v_1}{t} = \frac{W}{g} \frac{v_1}{t}$, or $F = \frac{W}{g} v_1$, when

$$\text{Thrust of screw in pounds} = \frac{64 AV}{32} (V - v) = 2 AV (V - v).$$

Rankine (Rules, Tables, and Data, p. 275) gives the following: To calculate the thrust of a propelling instrument (jet, paddle, or screw) in pounds, multiply together the transverse sectional area, in square feet, of the stream driven astern by the propeller; the speed of the stream relatively to the ship in knots; the real slip, or part of that speed which is impressed on the stream by the propeller, also in knots; and the constant 5.66 for sea-water, or 5.5 for fresh water. If S = speed of the screw in knots, s = speed of ship in knots, A = area of the stream in square feet (of sea-water).

The *real slip* is the velocity (relative to water at rest) of the water projected sternward; the *apparent slip* is the difference between the speed of the ship and the speed of the screw; i.e., the product of the pitch of the screw by the number of revolutions.

This apparent slip is sometimes negative, due to the working of the screw in disturbed water which has a forward velocity, following the ship. Negative apparent slip is an indication that the propeller is not suited to the ship. The apparent slip should generally be about 8% to 10% at full speed in well-formed vessels with moderately fine lines; in bluff cargo boats it rarely exceeds 5%.

The effective area of a screw is the sectional area of the stream of water laid hold of by the propeller, and is generally, if not always, greater than the actual area, in a ratio which in good ordinary examples is 1.2 or thereabouts, and is sometimes as high as 1.4: a fact probably due to the stiffness of the water, which communicates motion laterally amongst its particles. (Rankine's Shipbuilding, p. 89.)

Prof. D. S. Jacobus, *Trans. A. S. M. E.*, xi, 1028, found the ratio of the effective to the actual disk area of the screws of different vessels to be as follows:

| | |
|---|------|
| Tug-boat, with ordinary true-pitch screw | 1.42 |
| Tug-boat, with screw having blades projecting backward | 0.57 |
| Ferryboat "Bergen," with or- { at speed of 12.09 stat. miles per hr.. | 1.53 |
| dinary true-pitch screw { at speed of 13.4 stat. miles per hr.. | 1.48 |
| Steamer "Homer Ramsdell," with ordinary true-pitch screw | 1.20 |

Size of Screw. — Seaton says: The size of a screw depends on so many things that it is very difficult to lay down any rule for guidance, and much must always be left to the experience of the designer, to allow for all the circumstances of each particular case. The following rules are given for ordinary cases (Seaton and Rounthwaite's Pocket-book):

$$P = \text{pitch of propeller in feet} = \frac{10133 S}{R(100 - x)}, \text{ in which } S = \text{speed in}$$

knots, R = revolutions per minute, and x = percentage of apparent slip. For a slip of 10%, pitch = $112.6 S \div R$.

$$D = \text{diameter of propeller} = K \sqrt{\frac{\text{I.H.P.}}{\left(\frac{P \times R}{100}\right)^3}}, K \text{ being a coefficient given}$$

in the table below. If $K = 20$, $D = 20,000 \sqrt{\frac{\text{I.H.P.}}{(P \times R)^3}}$.

Total developed area of blades = $C \sqrt{\text{I.H.P.} \div R}$, in which C is a coefficient to be taken from the table.

Another formula for pitch, given in Seaton's Marine Engineering, is $P = \frac{C}{R} \sqrt[3]{\frac{\text{I.H.P.}}{D^2}}$, in which $C = 737$ for ordinary vessels, and 660 for slow-speed cargo vessels with full lines.

Thickness of blade at root = $\sqrt{\frac{d^3}{nb}} \times k$, in which d = diameter of tail shaft in inches, n = number of blades, b = breadth of blade in inches where it joins the boss, measured parallel to the shaft axis; $k = 4$ for cast iron, 1.5 for cast steel, 2 for gun-metal, 1.5 for high-class bronze.

Thickness of blade at tip: Cast iron $0.04 D + 0.4$ in.; cast steel $0.03 D + 0.4$ in.; gun-metal $0.03 D + 0.2$ in.; high-class bronze $0.02 D + 0.3$ in., where D = diameter of propeller in feet.

Propeller Coefficients.

| Description of Vessel. | Approximate Speed in knots. | Number of Screws. | Number of Blades per Screw. | Values of K . | Values of C . | Usual Material of Blades. |
|------------------------------|-----------------------------|-------------------|-----------------------------|-----------------|-----------------|---------------------------|
| Bluff cargo boats..... | 8-10 | One | 4 | 17 -17.5 | 19 -17.5 | Cast iron |
| Cargo, moderate lines..... | 10-13 | " | 4 | 18 -19 | 17 -15.5 | " " |
| Pass. and mail, fine lines.. | 13-17 | " | 4 | 19.5-20.5 | 15 -13 | C.I. or S. |
| " " " " " | 13-17 | Twin | 4 | 20.5-21.5 | 14.5-12.5 | " " |
| " " " " very fine.. | 17-22 | One | 4 | 21 -22 | 12.5-11 | G.M. or B |
| " " " " " | 17-22 | Twin | 3 | 22 -23 | 10.5- 9 | " " |
| Naval vessels, " " " | 16-22 | " | 4 | 21 -22.5 | 11.5-10.5 | " " |
| " " " " " | 16-22 | " | 3 | 22 -23.5 | 8.5-7 | " " |
| Torpedo-boats, " " " | 20-26 | One | 3 | 25 | 7-6 | B. or F.S. |

C. I., cast iron; G. M., gun-metal; B., bronze; S., steel; F.S., forged steel.

From the formulæ $D = 20,000 \sqrt{\frac{\text{I.H.P.}}{(P \times R)^3}}$ and $P = \sqrt{\frac{737 \text{ I.H.P.}}{R D^2}}$, if $P = D$ and $R = 100$, we obtain $D = \sqrt[5]{400 \times \text{I.H.P.}} = 3.31 \sqrt[5]{\text{I.H.P.}}$.

If $P = 1.4 D$ and $R = 100$, then $D = \sqrt[5]{145.8 \times \text{I.H.P.}} = 2.71 \sqrt[5]{\text{I.H.P.}}$.

From these two formulæ the figures for diameter of screw in the table on page 1323 have been calculated. They may be used as rough approximations to the correct diameter of screw for any given horse-power, for a speed of 10 knots and 100 revolutions per minute.

For any other number of revolutions per minute multiply the figures in the table by 100 and divide by the given number of revolutions. For any other speed than 10 knots, since the I.H.P. varies approximately as the cube of the speed, and the diameter of the screw as the 5th root of the I.H.P., multiply the diameter given for 10 knots by the 5th root of the cube of one-tenth of the given speed. Or, multiply by the following factors:

For speed of knots:

| | | | | | | | | | | | |
|---------------|---------------|---|---|---|---|----|----|----|----|----|----|
| $\frac{4}{5}$ | $\frac{5}{6}$ | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 16 |
|---------------|---------------|---|---|---|---|----|----|----|----|----|----|

$$\sqrt[5]{(S \div 10)^3}$$

= 0.577 0.660 0.736 0.807 0.875 0.939 1.059 1.116 1.170 1.224 1.275 1.327

Speed:

| | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|----|----|----|----|----|----|----|----|----|----|----|----|

$$\sqrt[5]{(S \div 10)^3}$$

| | | | | | | | | | | | |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| = 1.375 | 1.423 | 1.470 | 1.515 | 1.561 | 1.605 | 1.648 | 1.691 | 1.733 | 1.774 | 1.815 | 1.855 |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|

For more accurate determinations of diameter and pitch of screw, the formulae and coefficients given by Seaton, quoted above, should be used.

Efficiency of the Propeller. — According to Rankine, if the slip of the water be s , its weight W , the resistance R , and the speed of the ship v ,

$$R = Ws \div g; Rv = Wsv \div g.$$

This impelling action must, to secure maximum efficiency of propeller, be effected by an instrument which takes hold of the fluid without shock or disturbance of the surrounding mass, and, by a steady acceleration, gives it the required final velocity of discharge. The velocity of the propeller overcoming the resistance R would then be

$$[v + (v + s)] \div 2 = v + s/2;$$

and the work performed would be

$$R(v + s/2) = Wvs \div g + Ws^2 \div 2g,$$

the first of the last two terms being useful, the second the minimum lost work; the latter being the wasted energy of the water thrown backward. The efficiency is $E = v \div (v + s/2)$; and this is the limit attainable with a perfect propelling instrument, which limit is approached the more nearly as the conditions above prescribed are the more nearly fulfilled. The efficiency of the propelling instrument is probably rarely much above 0.60, and never above 0.80.

In designing the screw-propeller, as was shown by Dr. Froude, the best angle for the surface is that of 45° with the plane of the disk; but as all parts of the blade cannot be given the same angle, it should, where practicable, be so proportioned that the "pitch-angle at the center of effort" should be made 45° . The maximum possible efficiency is then, according to Froude, 77%.

In order that the water should be taken on without shock and discharged with maximum backward velocity, the screw must have an axially increasing pitch.

The true screw is by far the more usual form of propeller, in all steamers, both merchant and naval. (Thurston, Manual of the Steam-engine, part ii, p. 176.)

The combined efficiency of screw, shaft, engine, etc., is generally taken at 50%. In some cases it may reach 60% or 65%. Rankine takes the effective H.P. to equal the I.H.P. $\div 1.63$.

Results of Researches on the efficiency of screw-propellers are summarized by S. W. Barnaby, in a paper read before section G of the Engineering Congress, Chicago, 1893. He states that the following general principles have been established:

(a) There is a definite amount of real slip at which, and at which only, maximum efficiency can be obtained with a screw of any given type, and this amount varies with the pitch-ratio. The slip-ratio proper to a given ratio of pitch to diameter has been discovered and tabulated for a screw of a standard type, as below:

Pitch-ratio and Slip for Screws of Standard Form.

| Pitch-ratio | Real Slip of Screw. | Pitch-ratio. | Real Slip of Screw. | Pitch-ratio. | Real Slip of Screw. |
|-------------|---------------------|--------------|---------------------|--------------|---------------------|
| 0.8 | 15.55 | 1.4 | 19.5 | 2.0 | 22.9 |
| 0.9 | 16.22 | 1.5 | 20.1 | 2.1 | 23.5 |
| 1.0 | 16.88 | 1.6 | 20.7 | 2.2 | 24.0 |
| 1.1 | 17.55 | 1.7 | 21.3 | 2.3 | 24.5 |
| 1.2 | 18.2 | 1.8 | 21.8 | 2.4 | 25.0 |
| 1.3 | 18.8 | 1.9 | 22.4 | 2.5 | 25.4 |

(b) Screws of large pitch-ratio, besides being less efficient in themselves, add to the resistance of the hull by an amount bearing some proportion to their distance from it, and to the amount of rotation left in the race.

(c) The best pitch-ratio lies probably between 1.1 and 1.5.

(d) The fuller the lines of the vessel, the less the pitch-ratio should be.

(e) Coarse-pitched screws should be placed further from the stern than fine-pitched ones.

(f) Apparent negative slip is a natural result of abnormal proportions of propellers.

(g) Three blades are to be preferred for high-speed vessels, but when the diameter is unduly restricted, four or even more may be advantageously employed.

(h) An efficient form of blade is an ellipse having a minor axis equal to four-tenths the major axis.

(i) The pitch of wide-bladed screws should increase from forward to aft, but a uniform pitch gives satisfactory results when the blades are narrow, and the amount of the pitch variation should be a function of the width of the blade.

(j) A considerable inclination of screw-shaft produces vibration, and with right-handed twin-screws turning outwards, if the shafts are inclined at all, it should be upwards and outwards from the propellers.

For results of experiments with screw-propellers, see F. C. Marshall, *Proc. Inst. M. E.*, 1881; R. E. Froude, *Trans. Inst. Nav. Archs.*, 1886; G. A. Calvert, *Trans. Inst. Nav. Archs.*, 1887; S. W. Barnaby, *Proc. Inst. C. E.*, 1890, vol. cii, and D. W. Taylor's "Resistance of Ships and Screw Propulsion." Also Mr. Taylor's paper in *Proc. Soc. Nav. Arch. & Marine Engrs.*, 1904. Mr. Taylor found the highest efficiencies, exceeding 70%, in propellers with pitch ratios from 1.0 to 1.5 ratio of width of blade to diameter of $\frac{1}{8}$ to $\frac{1}{5}$, and ratio of developed area of blade to disk area of 0.201 to 0.322.

One of the most important results deduced from experiments on model screws is that they appear to have practically equal efficiencies throughout a wide range both in pitch-ratio and in surface-ratio: so that great latitude is left to the designer in regard to the form of the propeller. Another important feature is that, although these experiments are not a direct guide to the selection of the most efficient propeller for a particular ship, they supply the means of analyzing the performances of screws fitted to vessels, and of thus indirectly determining what are likely to be the best dimensions of screw for a vessel of a class whose results are known. Thus a great advance has been made on the old method of trial upon the ship itself, which was the origin of almost every conceivable erroneous view respecting the screw-propeller. (*Proc. Inst. M. E.*, July, 1891.)

Mr. Barnaby in *Proc. Inst. C. E.*, 1890, gives a table to be used in calculations for determining the best dimensions of screws for any given speed and H.P. from which the following table is abridged. It is deduced from Froude's experiments at Torquay. (*Trans. Inst. Nav. Archs.*, 1886.)

C_A = disk area in sq. ft. $\times V^3/\text{H.P.}$ C_R = revs. per min. $\times D/V$. V = speed in knots, D = diam. of screw in ft. H.P. = effective H.P. on the screw shaft. Disk area = $0.7854 D^2 = C_A \times \text{I.H.P.}/V^3$. Revs. per min. = $C_R \times V/D$. The constants C_A and C_R assume a standard value of the speed of the wake, equal to 10% of the speed of the ship. In a very full ship it may amount to 30%, therefore V should be reduced when using the constants by amounts varying from 20% to 0 as the form varies from "very full" to "fairly fine."

| Effy. of Screw, %. | 63 | | 67 | | 68 | | 69 | | 68 | | 66 | | 63 | |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Pitch ratio. | C_A | C_R | C_A | C_R | C_A | C_R | C_A | C_R | C_A | C_R | C_A | C_R | C_A | C_R |
| 0.80 | 468 | 122 | 304 | 128 | 215 | 134 | 157 | 142 | 115 | 150 | 86 | 160 | 65 | 171 |
| 1.00 | 546 | 99 | 355 | 104 | 251 | 109 | 184 | 115 | 135 | 123 | 100 | 131 | 76 | 140 |
| 1.20 | 625 | 83 | 405 | 87 | 288 | 92 | 210 | 97 | 154 | 104 | 115 | 111 | 87 | 119 |
| 1.40 | 704 | 72 | 456 | 76 | 325 | 80 | 236 | 85 | 173 | 90 | 129 | 97 | 98 | 104 |
| 1.60 | 780 | 63 | 507 | 67 | 360 | 71 | 263 | 75 | 193 | 80 | 144 | 87 | 109 | 93 |
| 1.80 | | | 558 | 60 | 396 | 64 | 290 | 68 | 212 | 73 | 159 | 78 | 120 | 84 |
| 2.00 | | | 609 | 55 | 432 | 58 | 315 | 62 | 231 | 67 | 173 | 72 | 131 | 77 |
| 2.20 | | | 660 | 50 | 469 | 54 | 342 | 57 | 250 | 62 | 187 | 67 | 142 | 72 |
| 2.40 | | | 710 | 47 | 505 | 50 | 369 | 53 | 270 | 57 | 202 | 62 | 153 | 67 |

Dimensions and Performances of Notable Atlantic Steamers (*Eng'g*, Aug. 2, 1907).

| Name. | Date. | Length, ft.* | Breadth, ft. | Depth, ft. | Draft, ft. | Displacement,† | Gross Tonnage,† | Steam Pressure. | Indicated H.P. | Speed on Trial, knots. | Cylinders, diam., ins. | Stroke, ins. | Heating Surface, sq. ft. | Grate Surface, sq. ft. |
|-------------------------|-------|--------------|--------------|------------|------------|----------------|-----------------|-----------------|----------------|-------------------------------------|------------------------|--------------|--------------------------|------------------------|
| Great Eastern..... | 1858 | 680 80 | 83 | 57 5 | 27 | 24 4 | 30 | 7 650 | 14 5 | Screw 4, 84; paddle, 4, 74 | 2, 48; 2, 83 | 60 | | |
| Britannic..... | 1874 | 455 45 | 36 | 23 5 | 8 5 | 5 | 70 | 5 500 | 16 | 1, 62; 2, 90 | 66 | 66 | | |
| Arizona..... | 1879 | 450 45 | 2 37 5 | 22 | 9 | 5 | 90 | 6 300 | 17 | 1, 72; 2, 100 | 72 | 72 | 27 483 | 1014 |
| Servia..... | 1881 | 515 52 | 40 5 | 23 2 | 9 9 | 7 4 | 90 | 10 300 | 17 | 1, 68; 2, 100 | 72 | 72 | 29 286 | 1398 |
| Alaska..... | 1881 | 500 50 | 39 7 | 22 | 11 2 | 8 1 | 90 | 11 900 | 18 | 3, 46; 3, 86 | 72 | 72 | 38 047 | 1428 |
| City of Rome..... | 1881 | 543 52 | 38 8 | 22 | 10 5 | 7 4 | 110 | 13 300 | 18 3 | 1, 70; 2, 104 | 72 | 72 | 38 817 | 1606 |
| Oregon..... | 1883 | 500 54 | 40 | 23 | 10 5 | 7 7 | 110 | 14 320 | 20 18 | 1, 71; 2, 105 | 72 | 72 | 50 265 | 1293 |
| Umbria..... | 1884 | 500 57 | 40 | 22 5 | 10 5 | 7 7 | 110 | 20 000 | 21 8 | 2, 45; 2, 71; 2, 113 | 60 | 60 | 40 072 | 1154 |
| Paris..... | 1888 | 528 63 | 41 1 | 23 | 13 | 10 5 | 150 | 30 000 | 22 01 | 4, 37; 2, 79; 4, 98 | 69 | 69 | 84 285 | 2618 |
| Teutonic..... | 1890 | 565 57 5 | 42 2 | 22 | 12 | 9 7 | 180 | 27 000 | 20 72 | 2, 47 1/2; 2, 79; 4, 93 | 72 | 72 | 74 686 | 1962 |
| Campania..... | 1893 | 600 65 | 41 5 | 23 | 18 | 12 5 | 165 | 36 000 | 23 5 | 4, 37 4; 4, 49 2; 4, 74 8; 4, 112 2 | 73 | 73 | 85 468 | 2188 |
| Kais. Wilhelm d. Grosse | 1897 | 625 66 | 43 | 28 | 20 9 | 14 3 | 178 | 38 000 | 23 5 | Turbines | 71 | 71 | 107 643 | 3121 |
| Oceanic..... | 1899 | 685 68 5 | 49 0 | 32 5 | 28 5 | 17 3 | 192 | 68 000 | 25 | | | | 158 350 | 4048 |
| Deutschland..... | 1901 | 663 67 | 44 | 29 | 23 6 | 16 5 | 220 | | | | | | | |
| Kaiser Wilhelm II..... | 1903 | 678 72 | 52 5 | 29 | 26 | 20 | 225 | | | | | | | |
| Lusitania..... | 1907 | 760 88 | 60 4 | 33 5 | 38 | 32 5 | 195 | | | | | | | |

* Between perpendiculars. † Thousands of tons.

Relative Economy of Turbines and Reciprocating Engine. (C. A. Parsons, *Trans. Inst. Nav. Archts.*, 1910.) — The "Vespasian," a cargo vessel 275 ft. long, 38 ft. 9 in. breadth, 19 ft. 8 in. mean loaded draught, 4350 tons displacement, was at first fitted with a triple-expansion engine, cylinders 22 1/4, 35 and 59 ins., 42-in. stroke; and afterwards with two Parsons turbines, high and low pressure, each connected by a flexible coupling to a 20-tooth pinion, the two pinions gearing into a wheel 8 ft. 3 in. pitch diam., with 398 double helical teeth, 20° angle, 24 in. face, the gear ratio being 19.9 to 1. The boilers, propeller, shafting and thrust block remained the same as with the reciprocating engine. Tests were made before and after the installation of the turbines with the following results: At a speed of 8.87 knots the reciprocating engine used 11,750 lbs. of water per hour, as against 10,750 lbs. taken by the turbines — a saving of 8.5%; at 9.55 knots the figures were 14,500 and 12,600, respectively — a saving of 13.0%; at 10.2 knots, 17,500 and 14,750 lbs., respectively — a saving of 16.0%.

Marine Practice, 1901. — The following tables and "summary of results" are taken from a paper on "Review of Marine Engineering in the Last Ten Years," by Jas. McKechnie, *Proc. Inst. M. E.*, 1901: *Eng. News*, Aug. 29, 1901.

Particulars of Cargo Steamers for North Atlantic Trade, to illustrate Fuel Economy of Large-Capacity Ships. (All are three-decked vessels, with shelter deck, to Class 100 A1 at Lloyd's. Speed of all at sea, 13 knots.)

| Dimensions. | Draft, ft. ins. | Dis- place- ment, tons. | Co-eff. of Dis- placement. | Dead- weight, tons. | I.H.P. | D ² / ₃ constant. | Immersed. | | Coal: 100 ton- miles, * lbs. |
|---------------------------|--------------------|----------------------------------|-------------------------------|---------------------------|--------|--|------------------|--------------|---------------------------------|
| | | | | | | | Area, sq. ft. | Girth ft. | |
| 390' x 45' 9" x 29' 6"... | 24 6 1/2 | 8,640 | 0.69 | 5,000 | 3,475 | 266 | 1,092 | 87.8 | 8.0 |
| 415' x 48' 9" x 31' 0"... | 25 6 | 10,240 | 0.696 | 6,000 | 3,725 | 277 | 1,209 | 92.46 | 7.1 |
| 438' x 51' 5" x 32' 8"... | 26 3 1/2 | 11,870 | 0.702 | 7,000 | 3,970 | 287 | 1,314 | 96.46 | 6.5 |
| 458' x 53' 9" x 34' 0"... | 27 0 1/2 | 13,500 | 0.71 | 8,000 | 4,225 | 295 | 1,412 | 100.0 | 6.05 |
| 475' x 55' 9" x 35' 5"... | 27 11 | 15,100 | 0.715 | 9,000 | 4,475 | 300 | 1,513 | 103.64 | 5.7 |
| 493' x 58' 0" x 36' 7"... | 28 7 | 16,750 | 0.72 | 10,000 | 4,725 | 305 | 1,610 | 107.0 | 5.42 |
| 521' x 61' 2" x 38' 9"... | 30 0 | 19,850 | 0.728 | 12,000 | 5,200 | 311 | 1,780 | 112.8 | 4.97 |
| 535' x 62' 9" x 39' 9"... | 30 7 | 21,470 | 0.732 | 13,000 | 5,430 | 313 | 1,862 | 115.4 | 4.8 |
| 548' x 64' 1" x 40' 9"... | 31 3 | 23,070 | 0.736 | 14,000 | 5,675 | 314 | 1,946 | 118.0 | 4.66 |
| 570' x 66' 9" x 42' 4"... | 32 4 1/2 | 26,150 | 0.742 | 16,000 | 6,130 | 316 | 2,097 | 122.5 | 4.4 |

* The rate of coal consumption is assumed in all cases at 1.5 lbs. per I.H.P. per hour.

Comparison of Marine Engines for the Years 1872, 1881, 1891, 1901.

| Boilers, Engines and Coal. | Average Results. | | | |
|--|------------------|-------|-------|----------|
| | 1872. | 1881. | 1891. | 1901. |
| Boiler press., lbs. per sq. in..... | 52.4 | 77.4 | 158.5 | 197 |
| Heating surface, per sq. ft. grate | | 30.4 | 31.0 | 38 & 43* |
| Heat'g surf., per I.H.P., sq. ft..... | 4.41 | 3.917 | 3.275 | 3.0 |
| Coal, per sq. ft. of grate, lbs. per hr..... | | 13.8 | 15.0 | 18 & 28* |
| Revolutions per minute..... | 55.67 | 59.76 | 63.75 | 87 |
| Piston speed, ft. per min..... | 376 | 467 | 529 | 654 |
| Coal per I.H.P. per hr., lbs..... | 2.11 | 1.83 | 1.52 | 1.48 |
| Av. consumption, long voyage..... | | 2.0 | 1.75 | 1.55 |

* Natural and forced draft respectively.

Summary of Results. — Steam pressures have been increased in the merchant marine from 158 lbs. to 197 lbs. per sq. in., the maximum attained being 267 lbs. per sq. in., and 300 lbs. in the naval service. The piston speed of mercantile machinery has gone up from 529 to 654 ft. per minute, the maximum in merchant practice being about 900 ft., and in naval practice 960 ft. for large engines, and 1300 ft. in torpedo-boat destroyers. Boilers also yield a greater power for a given surface, and thus the average power per ton of machinery has gone up from an average of 6 to about 7 I.H.P. per ton of machinery. The net result in respect of speed is that while ten years ago the highest sustained ocean speed was 20.7 knots, it is now 23.38 knots; the highest speed for large warships was 22 knots and is now 23 knots on a trial of double the duration of those of ten years ago; the maximum speed attained by any craft was 25 knots, as compared with 36.581 knots now, while the number of ships of over 20 knots was 8 in 1891, and is 58 now [1901].

Turbines and Boilers of the "Lusitania." (Thomas Bell, *Proc. Inst. Nav. Archts.*, 1908.)—Some of the principal dimensions of the turbines and boilers of the "Lusitania" are as follows:

| Turbines. | Diameter of Rotor, ins. | Length of Blades, ins. | |
|--------------|-------------------------|------------------------|--------------------|
| | | In First Expansion. | In Last Expansion. |
| H.P. | 96 | 23/4 | 123/8 |
| L.P. | 140 | 81/4 | 22 |
| Astern. | 104 | 21/4 | 8 |

Total cooling surface, main condensers, 82,800 sq. ft.; area of exhaust inlet, 158 sq. ft.; bore of circulating discharge pipes, 32 ins.

BOILERS.—Working pressure, 195 lbs. per sq. in.; 23 double-ended boilers, 17 ft. 6 in. mean diameter by 22 ft. long; 2 single-ended boilers, 17 ft. 6 in. mean diameter by 11 ft. 4 in. long; total number of furnaces, 192; total grate surface, 4048 sq. ft.; total heating surface, 158,352 sq. ft.; total length of boiler-rooms, 336 ft.; total length of main and auxiliary engine rooms, 149 ft. 8 in.

The following are the weights of the various revolving parts, together with the size of bearings and the pressure:

Weight of one H.P. turbine rotor complete, 86 tons; one L.P. rotor, 120 tons; one astern rotor, 62 tons.

| | Main Bearing Journals. | | Pressure Per Sq. In. of Bearing Surface. | At 190 Revs. Surface Speed of Journal. |
|--------------------|------------------------|-------------------|--|--|
| | Diameter. | Effective Length. | | |
| H.P. rotor. | 27 1/8 in. | 44 3/4 in. | 80 lbs. | 1350 ft. per min. |
| L.P. rotor. | 33 1/8 in. | 56 1/2 in. | 72 lbs. | 1650 ft. per min. |
| Astern rotor. | 24 1/8 in. | 34 3/4 in. | 83 lbs. | 1200 ft. per min. |

Performance of the "Lusitania." (Thos. Bell, *Proc. Inst. Nav. Archts.*, 1908: *Power*, May 12, 1908.)—The following records were obtained in the official trials:

| | | | | | |
|--|--------|--------|--------|--------|--------|
| Speed in knots. | 15.77 | 18 | 21 | 23 | 25.4 |
| Shaft horse-power. | 13,400 | 20,500 | 33,000 | 48,000 | 68,850 |
| Steam cons. per shaft, H.P. hr. of turbines, lbs. | 21.23 | 17.24 | 14.91 | 13.92 | 12.77 |
| of auxiliaries, lbs. | 5.3 | 3.72 | 2.6 | 2.01 | 1.69 |
| total lbs. | 26.53 | 20.96 | 17.51 | 15.93 | 14.46 |
| Temperature of feed water, ° F. | 200 | 200 | 199 | 179 | 165 |
| Coal cons. lbs. per shaft H.P. hr. | 2.52 | 2.01 | 1.68 | 1.56 | 1.43 |
| Estimated steam and coal consumption under service conditions, at same speeds: | | | | | |
| Steam cons. of auxiliaries, per shaft H.P. hr., lbs. .. | 6.97 | 4.92 | 3.41 | 2.65 | 2.17 |
| Steam cons. of total per shaft H.P. hr., lbs. | 28.20 | 22.16 | 18.32 | 16.57 | 14.94 |
| Coal cons., lbs. per shaft H.P. hr., lbs. | 2.76 | 2.17 | 1.8 | 1.62 | 1.46 |
| Est. coal cons., on a voyage of 3100 nautical miles, gross tons. | 3,270 | 3,440 | 3,930 | 4,700 | 5,490 |

The following figures are taken from the records of a voyage from Queenstown to Sandy Hook, 2781 nautical miles, Nov. 3-8, 1908, 4 days, 18 hrs. 40 m.; Averages: Steam pressure at boilers, 168 lbs.; temperature hot-well, 74.5°; feed water, 197°; vacuum, 28.1 in.; speed, 24.25 knots;

speed, best day, 24.8 knots; revolutions, 181.1; slip, 15.9%. Total coal, 4976 tons. Steam consumption: main turbines, 851,500 lbs., = 13.1 lbs. per shaft H.P. hr. (on a basis of 65,000 shaft H.P.); auxiliary machinery, 114,000 lbs., = 1.75 per H.P. hr.; evaporating plant and heating, 32,500 lbs., = 0.5 lb. per H.P. hr. Total, 998,000 lbs., = 15.35 lbs. per shaft H.P. hour. Average coal burned, $43\frac{1}{2}$ tons per hour. Water evaporated per lb., coal 10.2 lbs. from feed at 196° , = 10.9 lbs. from and at 212° . Coal for all purposes per shaft H.P. hour, 1.5 lbs. Coal per sq. ft. of grate per hour, 24.1 lbs. The coal was half Yorkshire and half South Wales.

In September, 1909, the "Lusitania" made the westward passage, 2784 miles from Daunt's Rock near Queenstown to Ambrose Channel Lightship, off Sandy Hook, in 4 days 11 h. 42 m., averaging 25.85 knots for the entire passage. Four successive days' runs, from noon to noon, were 650, 652, 651 and 674 miles.

Relation of Horse-Power to Speed. — If S_1 and S_2 are two successive speeds and P_1P_2 the corresponding horse-powers, then to find the value of the exponent x in the equation $H.P. \propto S^x$, we have

$$x = (\log P_2 - \log P_1) \div (\log S_2 - \log S_1).$$

Applying this formula to the horse-powers and speeds of the "Lusitania" we find that between 15.77 and 18 knots $x = 3.21$; between 18 and 21 knots $x = 3.09$; between 21 and 23 knots $x = 4.12$; between 23 and 25.4 knots $x = 3.63$.

Reciprocating Engines with a Low-Pressure Turbine. — The "Laurentic," built for the Canadian trade of the White Star Line, 14,000 tons gross register, is a triple-screw steamer, with the two outer screws driven by four-cylinder triple-expansion engines, and the central screw by a Parsons turbine. The steam, of 200 lbs. boiler pressure, first passes to the reciprocating engines, where it expands to from 14 to 17 lbs. absolute, and then passes to the turbine. For manœuvring the ship into and out of port the turbine is not used, and the steam passes directly from the engines to the condensers. During the trial trip the combined engine-turbine outfit developed 12,000 H.P., with a speed of $17\frac{1}{2}$ knots, and showed a coal consumption of 1.1 lbs. and a water consumption of 11 lbs. per indicated horse-power hour. (*Power*, May 18, 1909.)

The "Kronprinzessin Cecilie" of the North German Lloyd Co., is probably the last high-speed transatlantic steamer of very great power that will be built with reciprocating engines. Its dimensions are: length, 706 ft.; beam, 72 ft.; depth, 44 ft. 2 in.; displacement, 26,000 tons. Four 12,000 H.P. engines, two on each shaft, in tandem. Cylinders, $37\frac{3}{8}$, $49\frac{1}{4}$, $74\frac{7}{8}$ and $112\frac{1}{4}$ ins., by 6 ft. stroke. Steam, 230 lbs., delivered from 19 cylindrical boilers, through four 17-in. steampipes. Coal used in 24 hours, 764 tons, in 124 furnaces; 1.4 lbs. per H.P. hour, including auxiliaries. Speed on trial trip on a 60-mile course, 24.02 knots. (*Sci. Am.*, Aug. 24, 1907.)

THE PADDLE-WHEEL.

Paddle-wheels with Radial Floats. (Seaton's Marine Engineering.) — The effective diameter of a radial wheel is usually taken from the centers of opposite floats; but it is difficult to say what is absolutely that diameter, as much depends on the form of float, the amount of dip, and the waves set in motion by the wheel. The slip of a radial wheel is from 15 to 30 per cent, depending on the size of float.

Area of one float = $C \times I.H.P. \div D$.

D is the effective diameter in feet, and C is a multiplier, varying from 0.25 in tugs to 0.175 in fast-running light steamers.

The breadth of the float is usually about $\frac{1}{4}$ its length, and its thickness about $\frac{1}{8}$ its breadth. The number of floats varies directly with the diameter, and there should be one float for every foot of diameter.

(For a discussion of the action of the radial wheel, see Thurston, *Manual of the Steam-engine*, part ii, p. 182.)

Feathering Paddle-wheels. (Seaton.) — The diameter of a feathering-wheel is found as follows: The amount of slip varies from 12 to 20 per cent, although when the floats are small or the resistance great it is as high as 25 per cent; a well-designed wheel on a well-formed ship should not exceed 15 per cent under ordinary circumstances.

If K is the speed of the ship in knots, S the percentage of slip, and R the revolutions per minute,

Diameter of wheel at centers = $K(100 + S) \div (3.14 \times R)$.

The diameter, however, must be such as will suit the structure of the ship, so that a modification may be necessary on this account, and the revolutions altered to suit it.

The diameter will also depend on the amount of "dip" or immersion of float.

When a ship is working always in smooth water the immersion of the top edge should not exceed $1/8$ the breadth of the float; and for general service at sea an immersion of $1/2$ the breadth of the float is sufficient. If the ship is intended to carry cargo, the immersion when light need not be more than 2 or 3 inches, and should not be more than the breadth of float when at the deepest draught; indeed, the efficiency of the wheel falls off rapidly with the immersion of the wheel.

Area of one float = $C \times \text{I.H.P.} \div D$.

C is a multiplier, varying from 0.3 to 0.35; D is the diameter of the wheel to the float centers, in feet.

The number of floats = $1/2(D + 2)$.

The breadth of the float = $0.35 \times$ the length.

The thickness of floats = $1/12$ the breadth.

Diameter of gudgeons = thickness of float.

Seaton and Rounthwaite's Pocket-book gives:

Number of floats = $60 \div \sqrt{R}$,

where R is number of revolutions per minute.

Area of one float (in square feet) = $\frac{\text{I.H.P.} \times 33,000 \times K}{N \times (D \times R)^3}$,

where N = number of floats in one wheel.

For vessels plying always in smooth water $K = 1200$. For sea-going steamers $K = 1400$. For tugs and such craft as require to stop and start frequently in a tide-way $K = 1600$.

It will be quite accurate enough if the last four figures of the cube $(D \times R)^3$ be taken as ciphers.

For illustrated description of the feathering paddle-wheel see Seaton's Marine Engineering, or Seaton and Rounthwaite's Pocket-book. The diameter of a feathering-wheel is about one-half that of a radial wheel for equal efficiency. (Thurston.)

Efficiency of Paddle-wheels. — Computations by Prof. Thurston of the efficiency of propulsion by paddle-wheels give for light river steamers with ratio of velocity of the vessel, v , to velocity of the paddle-wheel at center of pressure, V , or v/V , = $3/4$, with a dip = $3/20$ radius of the wheel and a slip of 25 per cent, an efficiency of 0.714; and for ocean steamers with the same slip and ratio of v/V , and a dip = $1/3$ radius, an efficiency of 0.685.

JET-PROPULSION.

Numerous experiments have been made in driving a vessel by the reaction of a jet of water pumped through an orifice in the stern, but they have all resulted in commercial failure. Two-jet propulsion steamers, the "Waterwitch," 1100 tons, and the "Squirt," a small torpedo-boat, were built by the British Government. The former was tried in 1867, and gave an efficiency of apparatus of only 18 per cent. The latter gave a speed of 12 knots, as against 17 knots attained by a sister-ship having a screw and equal steam-power. The mathematical theory of the efficiency of the jet was discussed by Rankine in *The Engineer*, Jan. 11, 1867, and he showed that the greater the quantity of water operated on by a jet-propeller, the greater is the efficiency. In defiance both of the theory and of the results of earlier experiments, and also of the opinions of many naval engineers, more than \$200,000 were spent in 1888-90 in New York upon two experimental boats, the "Prima Vista" and the "Evolution," in which the jet was made of very small size, in the latter case only $5/8$ -inch diameter, and with a pressure of 2500 lbs. per square inch. As had been predicted, the vessel was a total failure. (See article by the author in *Mechanics*, March, 1891.)

The theory of the jet-propeller is similar to that of the screw-propeller. If A = the area of the jet in square feet, V its velocity with reference to the orifice, in feet per second, v = the velocity of the ship in reference to

the earth, then the thrust of the jet (see Screw-propeller, *ante*) is $2AV(V-v)$. The work done on the vessel is $2AV(V-v)v$, and the work wasted on the rearward projection of the jet is $\frac{1}{2} \times 2AV(V-v)^2$.

The efficiency is $\frac{2AV(V-v)v}{2AV(V-v)v + AV(V-v)^2} = \frac{2v}{V+v}$. This expression equals unity when $V = v$, that is, when the velocity of the jet with reference to the earth, or $V - v = 0$; but then the thrust of the propeller is also 0. The greater the value of V as compared with v , the less the efficiency. For $V = 20v$, as was proposed in the "Evolution," the efficiency of the jet would be less than 10 per cent, and this would be further reduced by the friction of the pumping mechanism and of the water in pipes.

The whole theory of propulsion may be summed up in Rankine's words: "That propeller is the best, other things being equal, which drives astern the largest body of water at the lowest velocity."

It is practically impossible to devise any system of hydraulic or jet propulsion which can compare favorably, under these conditions, with the screw or the paddle-wheel.

Reaction of a Jet. — If a jet of water issues horizontally from a vessel, the reaction on the side of the vessel opposite the orifice is equal to the weight of a column of water the section of which is the area of the orifice, and the height is twice the head.

The propelling force in jet-propulsion is the reaction of the stream issuing from the orifice, and it is the same whether the jet is discharged under water, in the open air, or against a solid wall. For proof, see account of trials by C. J. Everett, Jr., given by Prof. J. Burkitt Webb, *Trans. A. S. M. E.*, xii, 904.

CONSTRUCTION OF BUILDINGS.*

FOUNDATIONS.

Bearing Power of Soils. — Ira O. Baker, "Treatise on Masonry Construction."

| Kind of Material. | Bearing Power in Tons per Square Foot. | |
|--|---|----------|
| | Minimum. | Maximum. |
| Rock — the hardest — in thick layers, in native bed. | 200 | |
| Rock equal to best ashlar masonry..... | 25 | 30 |
| Rock equal to best brick masonry..... | 15 | 20 |
| Rock equal to poor brick masonry..... | 5 | 10 |
| Clay on thick beds, always dry..... | 4 | 6 |
| Clay on thick beds, moderately dry..... | 2 | 4 |
| Clay, soft..... | 1 | 2 |
| Gravel and coarse sand, well cemented..... | 8 | 10 |
| Sand, compact, and well cemented..... | 4 | 6 |
| Sand, clean, dry..... | 2 | 4 |
| Quicksand, alluvial soils, etc..... | 0.5 | 1 |

* The limitations of space forbid any extended treatment of this subject. Much valuable information upon it will be found in Trautwine's "Civil Engineers' Pocket-book," and in Kidder's "Architects' and Builders' Pocket-book." The latter in its preface mentions the following works of reference: "Notes on Building Construction," 3 vols., Rivingtons, publishers, London; "Building Superintendence," by T. M. Clark (J. R. Osgood & Co., Boston); "The American House Carpenter," and "The Theory of Transverse Strains," both by R. G. Hatfield; "Graphical Analysis of Roof-trusses," by Prof. C. E. Greene; "The Fire Protection of Mills," by C. J. H. Woodbury; "House Drainage and Water Service," by James C. Bayles; "The Builder's Guide and Estimator's Price-book," and "Plastering Mortars and Cements," by Fred. T. Hodgson; "Foundations and Concrete Works," and "Art of Building," by E. Dobson, Weale's Series, London.

The building code of Greater New York specifies the following as the maximum permissible loads for different soils:

- "Soft clay, one ton per square foot;
- "Ordinary clay and sand together, in layers, wet and springy, two tons per square foot;
- "Loam, clay or fine sand, firm and dry, three tons per square foot;
- "Very firm coarse sand, stiff gravel or hard clay, four tons per square foot, or as otherwise determined by the Commissioner of Buildings having jurisdiction."

Bearing Power of Piles. — *Engineering News Formula:* Safe load in tons = $2 Wh \div (S + 1)$. W = weight of hammer in tons, h = height of fall of hammer in feet, S = penetration under last blow, or the average under last five blows.

Safe Strength of Brick Piers, exceeding six diameters in height. (Kidder.)

Piers laid with rich lime mortar, tons per sq. in., $110 - 5 H/D$.

Piers laid with 1 to 2 natural cement mortar, $140 - 5\frac{1}{2} H/D$.

Piers laid with 1 to 3 Portland cement mortar, $200 - 6 H/D$.

H = height; D = least horizontal dimension, in feet.

Thickness of Foundation Walls. (Kidder.)

| Height of Building. | Dwellings, Hotels, etc. | | Warehouses. | |
|---------------------|----------------------------|---------|-------------|---------|
| | Brick. | Stone. | Brick. | Stone. |
| | Inches. | Inches. | Inches. | Inches. |
| Two stories..... | 12 or 16 | 20 | 16 | 20 |
| Three stories..... | 16 | 20 | 20 | 24 |
| Four stories..... | 20 | 24 | 24 | 28 |
| Five stories..... | 24 | 28 | 24 | 28 |
| Six stories..... | 28 | 32 | 28 | 32 |

MASONRY.

Allowable Pressures on Masonry in Tons per Square Foot. (Kidder.)

| Different Cities.* | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|------|------|------------------|------------------|------------------|------|------|
| Granite, cut..... | 60 | | 72-172 | | | | 40 |
| Marble and limestone, cut..... | 40 | | 50-165 | | | | |
| Sandstone, hard, cut..... | 30 | | 28-115 | | | | 12 |
| Hard-burned brick in Portland cement..... | | | 18 | 12 $\frac{1}{2}$ | 15 | | |
| Hard-burned brick in natural cement..... | 15 | 9 | 15 | 9 | | 15 | 9 |
| Hard-burned brick in cement and lime... | 12 | | 11 $\frac{1}{2}$ | | 11 $\frac{1}{2}$ | 12 | |
| Hard-burned brick in lime mortar..... | 8 | 6 | 8 | 6 $\frac{1}{2}$ | 11 | 8 | 8 |
| Pressed brick in Portland cement..... | | 12 | | | | | |
| Pressed brick in natural cement..... | | 9 | | | | | 12 |
| Rubble stone in natural cement..... | | 5 | 8 | | | 10 | 12 |
| In foundations: | | | | | | | |
| Dimension stone..... | | 6 | | 5-7 | | | 30 |
| Portland cement concrete..... | | 4 | 15 | | | 15 | 10 |
| Natural cement concrete..... | | | 8 | 4 | | | 4 |

* From building laws, (1) Boston, 1897; (2) Buffalo, 1897; (3) New York, 1899; (4) Chicago, 1893; (5) St. Louis, 1897; (6) Philadelphia, 1899; (7) Denver, 1898.

Crushing Strength of 12-in. Cubes of Concrete. (Kidder.) — Pounds per square foot. The concrete was made of 1 part Portland cement, 2 parts sand, with average concrete stone and gravel, as below.

| | 10 days. | 45 days. | 3 mos. | 6 mos. | 1 year. |
|---|----------|----------|---------|---------|---------|
| 6 parts stone..... | 130,750 | 172,325 | 324,875 | 361,600 | 440,040 |
| 3 parts stone, 3 gravel..... | 136,750 | 266,962 | | 298,037 | 396,200 |
| 4 parts stone, 2 gravel..... | | | | | 408,300 |
| 6 parts (3/4 stone, 1/4 granolithic)..... | | | | | 388,700 |
| 6 parts average gravel..... | 99,900 | 234,475 | 385,612 | 265,550 | 406,700 |
| 6 parts coarse stone, no fine.. | | | 234,475 | 220,350 | 266,300 |

Reinforced Concrete. — The building laws of New York, St. Louis, Cleveland and Buffalo, and the National Board of Fire Underwriters agree in prescribing the following as the maximum allowable working stresses:

| | |
|--|----------------------|
| Extreme fiber stress in compression in concrete..... | 500 lbs. per sq. in. |
| Shearing stress in concrete..... | 50 " " |
| Direct compression in concrete..... | 350 " " |
| Adhesion of steel to concrete..... | 50 " " |
| Tensile stress in steel..... | 16,000 " " |
| Shearing stress in steel..... | 10,000 " " |

BEAMS AND GIRDERS.

Safe Loads on Beams. — Uniformly distributed load:

$$\text{Safe load in lbs.} = \frac{2 \times \text{breadth} \times \text{square of depth} \times A}{\text{span in feet}}$$

$$\text{Breadth in inches} = \frac{\text{span in feet} \times \text{load}}{2 \times \text{square of depth} \times A}$$

The depth is taken in inches. The coefficient A , is $\frac{1}{18}$ the maximum fiber stress for safe loads, and is the safe load for a beam 1 in. square, 1 ft. span. The following values of A are given by Kidder as one-third of the breaking weight of timber of the quality used in first-class buildings. The values for stones are based on a factor of safety of six.

VALUES FOR A . — COEFFICIENT FOR BEAMS.

| | | | |
|---------------------------|-----|--|---------|
| Cast iron..... | 308 | Pine, Texas yellow..... | 90 |
| Wrought iron..... | 666 | Spruce..... | 70 |
| Steel..... | 888 | Whitewood (poplar)..... | 65 |
| American Woods: | | Redwood (California)..... | 60 |
| Chestnut..... | 60 | Bluestone flagging (Hudson River)..... | 25 |
| Hemlock..... | 55 | Granite, average..... | 17 |
| Oak, white..... | 75 | Limestone..... | 14 |
| Pine, Georgia yellow..... | 100 | Marble..... | 17 |
| Pine, Oregon..... | 90 | Sandstone..... | 8 to 11 |
| Pine, red or Norway..... | 70 | Slate..... | 50 |
| Pine, white, Eastern..... | 60 | | |
| Pine, white, Western..... | 65 | | |

Maximum Permissible Stresses in Structural Materials used in Buildings. (Building Ordinances of the City of Chicago, 1893.) — Cast iron, crushing stress: For plates, 15,000 lbs. per square inch; for lintels, brackets, or corbels, compression 13,500 lbs. per square inch, and tension 3000 lbs. per square inch. For girders, beams, corbels, brackets, and trusses, 16,000 lbs. per square inch for steel and 12,000 lbs. for iron.

For plate girders:

$$\text{Flange area} = \text{maximum bending moment in ft.-lbs.} \div (CD).$$

D = distance between center of gravity of flanges in feet.

C = 13,500 for steel, 10,000 for iron.

Web area = maximum shear $\div C$.

C = 10,000 for steel; 6,000 for iron.

For rivets in single shear per square inch of rivet area:

If shop-driven, steel, 9000 lbs., iron, 7500 lbs.; if field-driven, steel, 7500 lbs., iron, 6000 lbs.

For timber girders: $S = cbd^2 \div l$.

b = breadth of beam in inches, d = depth of beam in inches, l = length of beam in feet, c = 160 for long-leaf yellow pine, 120 for oak, 100 for white or Norway pine.

Safe Loads in Tons, Uniformly Distributed, for White-oak Beams.

(In accordance with the Building Laws of Boston.)

Formula: $W = \frac{4PBD^2}{3L}$. W = safe load in pounds; P , extreme fiber-stress = 1000 lbs. per square inch, for white oak; B , breadth in inches; D , depth in inches; L , distance between supports in inches.

| Size of Timber. | Distance between Supports in Feet. | | | | | | | | | | | | | | |
|--------------------|------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|-------|-------|
| | 6 | 8 | 10 | 11 | 12 | 14 | 15 | 16 | 17 | 18 | 19 | 21 | 23 | 25 | 26 |
| | Safe Load in Tons of 2000 Pounds. | | | | | | | | | | | | | | |
| 2×6 | 0.67 | 0.50 | 0.40 | 0.36 | 0.33 | 0.29 | 0.27 | 0.25 | 0.24 | 0.22 | ... | ... | ... | ... | ... |
| 2×8 | 1.19 | 0.89 | 0.71 | 0.65 | 0.59 | 0.51 | 0.47 | 0.44 | 0.42 | 0.40 | 0.37 | 0.34 | 0.31 | 0.28 | |
| 2×10 | 1.85 | 1.39 | 1.11 | 1.01 | 0.93 | 0.79 | 0.74 | 0.69 | 0.65 | 0.62 | 0.58 | 0.53 | 0.48 | 0.43 | 0.43 |
| 2×12 | 2.67 | 2.00 | 1.60 | 1.45 | 1.33 | 1.14 | 1.07 | 1.00 | 0.94 | 0.89 | 0.84 | 0.76 | 0.70 | 0.64 | 0.62 |
| 3×6 | 1.00 | 0.75 | 0.60 | 0.55 | 0.50 | 0.43 | 0.40 | 0.37 | 0.35 | 0.33 | 0.32 | 0.29 | 0.2 | | |
| 3×8 | 1.78 | 1.33 | 1.07 | 0.97 | 0.89 | 0.76 | 0.71 | 0.67 | 0.63 | 0.59 | 0.56 | 0.51 | 0.46 | 0.43 | 0.41 |
| 3×10 | 2.78 | 2.08 | 1.67 | 1.52 | 1.39 | 1.19 | 1.11 | 1.04 | 0.98 | 0.93 | 0.88 | 0.79 | 0.72 | 0.67 | 0.64 |
| 3×12 | 4.00 | 3.00 | 2.40 | 2.18 | 2.00 | 1.71 | 1.60 | 1.50 | 1.41 | 1.33 | 1.26 | 1.14 | 1.04 | 0.96 | 0.92 |
| 3×14 | 5.45 | 4.08 | 3.27 | 2.97 | 2.72 | 2.37 | 2.18 | 2.04 | 1.92 | 1.82 | 1.72 | 1.56 | 1.42 | 1.31 | 1.25 |
| 3×16 | 7.11 | 5.33 | 4.27 | 3.88 | 3.56 | 3.05 | 2.84 | 2.67 | 2.51 | 2.37 | 2.25 | 2.03 | 1.86 | 1.71 | 1.64 |
| 4×10 | 3.70 | 2.78 | 2.22 | 2.02 | 1.85 | 1.59 | 1.48 | 1.39 | 1.31 | 1.23 | 1.17 | 1.06 | 0.97 | 0.89 | 0.85 |
| 4×12 | 5.33 | 4.00 | 3.20 | 2.91 | 2.67 | 2.29 | 2.13 | 2.00 | 1.88 | 1.78 | 1.68 | 1.52 | 1.39 | 1.28 | 1.23 |
| 4×14 | 7.26 | 5.44 | 4.36 | 3.96 | 3.63 | 3.11 | 2.90 | 2.72 | 2.56 | 2.42 | 2.29 | 2.07 | 1.90 | 1.74 | 1.68 |
| 4×16 | 9.48 | 7.11 | 5.69 | 5.17 | 4.74 | 4.06 | 3.79 | 3.56 | 3.35 | 3.16 | 3.00 | 2.71 | 2.47 | 2.28 | 2.19 |
| 4×18 | 12.00 | 9.00 | 7.20 | 6.55 | 6.00 | 5.14 | 4.80 | 4.50 | 4.24 | 4.00 | 3.79 | 3.43 | 3.13 | 2.88 | 2.77 |

For other kinds of wood than white oak multiply the figures in the table by a figure selected from those given below (which represent the safe stress per square inch on beams of different kinds of wood according to the building laws of the cities named) and divide by 1000.

| | Hem-lock. | Spruce. | White Pine. | Oak. | Yellow Pine. |
|---------------|-----------|---------|-------------|-------|--------------|
| New York..... | 800 | 900 | 900 | 1100 | 1100* |
| Boston..... | | 750 | 750 | 1000† | 1250 |
| Chicago..... | | | 900 | 1080 | 1440 |

* Georgia pine.

† White oak.

WALLS.

Thickness of Walls of Buildings. — Kidder gives the following general rule for mercantile buildings over four stories in height:

For brick equal to those used in Boston or Chicago, make the thickness of the three upper stories 16 ins., of the next three below 20 ins., the next three 24 ins., and the next three 28 ins. For a poorer quality of material make only the two upper stories 16 ins. thick, the next three 20 ins., and so on down.

In buildings less than five stories in height the top story may be 12 ins. in thickness.

THICKNESS OF WALLS IN INCHES, FOR MERCANTILE BUILDINGS AND FOR ALL BUILDINGS OVER FIVE STORIES IN HEIGHT. (The figures show the range of the thicknesses required by the ordinances of eight different cities. — Condensed from Kidder.)

| Stories High. | Stories. | | | | | | | | | | | |
|------------------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 1st. | 2d. | 3d. | 4th. | 5th. | 6th. | 7th. | 8th. | 9th. | 10th | 11th | 12th |
| 2 | 12-18 | 12-13 | | | | | | | | | | |
| 3 | 13-20 | 12-18 | 12-16 | | | | | | | | | |
| 4 | 16-22 | 16-18 | 12-18 | 12-16 | | | | | | | | |
| 5 | 18-22 | 16-22 | 16-20 | 12-20 | 12-16 | | | | | | | |
| 6 | 20-26 | 18-22 | 16-22 | 16-20 | 13-20 | 12-16 | | | | | | |
| 7 | 20-28 | 20-26 | 18-24 | 16-22 | 16-20 | 13-20 | 12-17 | | | | | |
| 8 | 22-32 | 20-28 | 20-26 | 18-24 | 16-22 | 16-20 | 13-20 | 12-17 | | | | |
| 9 | 24-32 | 24-32 | 20-28 | 20-26 | 20-24 | 16-22 | 16-20 | 16-20 | 12-17 | | | |
| 10 | 24-36 | 24-32 | 24-32 | 20-28 | 20-26 | 20-24 | 16-22 | 16-20 | 16-20 | 12-17 | | |
| 11 | 28-36 | 28-36 | 24-32 | 24-30 | 24-28 | 20-26 | 20-24 | 20-22 | 16-20 | 16-20 | 13-17 | |
| 12 | 28-40 | 28-36 | 28-36 | 24-32 | 24-32 | 24-28 | 20-26 | 20-24 | 20-22 | 16-20 | 16-20 | 13-17 |

(Extract from the Building Laws of the City of New York, 1893.)

Walls of Warehouses, Stores, Factories, and Stables. — 25 feet or less in width between walls, not less than 12 in. to height of 40 ft.; If 40 to 60 ft. in height, not less than 16 in. to 40 ft., and 12 in. thence to top;

60 to 80 ft. in height, not less than 20 in. to 25 ft., and 16 in. thence to top;

75 to 85 ft. in height, not less than 24 in. to 20 ft.; 20 in. to 60 ft., and 16 in. to top;

85 to 100 ft. in height, not less than 28 in. to 25 ft.; 24 in. to 50 ft.; 20 in. to 75 ft., and 16 in. to top;

Over 100 ft. in height, each additional 25 ft. in height, or part thereof, next above the curb, shall be increased 4 inches in thickness, the upper 100 feet remaining the same as specified for a wall of that height.

If walls are over 25 feet apart, the bearing-walls shall be 4 inches thicker than above specified for every 12½ feet or fraction thereof that said walls are more than 25 feet apart.

Strength of Floors, Roofs, and Supports.

Floors calculated to bear safely per sq. ft., in addition to their own wt.

| | |
|--|---------|
| Floors of dwelling, tenement, apartment-house or hotel, not less than..... | 70 lbs. |
| Floors of office-building, not less than..... | 100 " |
| Floors of public-assembly building, not less than | 120 " |
| Floors of store, factory, warehouse, etc., not less than..... | 150 " |
| Roofs of all buildings, not less than..... | 50 " |

Every floor shall be of sufficient strength to bear safely the weight to be imposed thereon, in addition to the weight of the materials of which the floor is composed.

Columns and Posts. — The strength of all columns and posts shall be computed according to Gordon's formulæ, and the crushing weights in pounds, to the square inch of section, for the following-named materials, shall be taken as the coefficients in said formulæ, namely: Cast iron, 80,000; wrought or rolled iron, 40,000; rolled steel, 48,000; white pine and spruce, 3500; pitch or Georgia pine, 5000; American oak, 6000. The breaking strength of wooden beams and girders shall be computed according to the formulæ in which the constants for transverse strains for central load shall be as follows, namely: Hemlock, 400; white pine, 450; spruce, 450; pitch or Georgia pine, 550; American oak, 550; and for wooden beams and girders carrying a uniformly distributed load the constants will be doubled.

The factors of safety shall be as one to four for all beams, girders, and other pieces subject to a transverse strain; as one to four for all posts, columns, and other vertical supports when of wrought iron or rolled steel; as one to five for other materials, subject to a compressive strain; as one to six for tie-rods, tie-beams, and other pieces subject to a tensile strain. Good, solid, natural earth shall be deemed to sustain safely a load of four tons to the superficial foot, or as otherwise determined by the superintendent of buildings, and the width of footing-courses shall be at least sufficient to meet this requirement. In computing the width of walls, a cubic foot of brickwork shall be deemed to weigh 115 lbs. Sandstone, white marble, granite, and other kinds of building-stone shall be deemed to weigh 160 lbs. per cubic foot. The safe-bearing load to apply to good brickwork shall be taken at 8 tons per superficial foot when good lime mortar is used, 11½ tons per superficial foot when good lime and cement mortar mixed is used, and 15 tons per superficial foot when good cement mortar is used.

Fire-proof Buildings — Iron and Steel Columns. — All cast-iron, wrought-iron, or rolled-steel columns shall be made true and smooth at both ends, and shall rest on iron or steel bed-plates, and have iron or steel cap-plates, which shall also be made true. All iron or steel trimmer-beams, headers, and tail-beams shall be suitably framed and connected together, and the iron girders, columns, beams, trusses, and all other iron-work of all floors and roofs shall be strapped, bolted, anchored, and connected together, and to the walls, in a strong and substantial manner. Where beams are framed into headers, the angle-irons, which are bolted to the tail-beams, shall have at least two bolts for all beams over 7 inches in depth, and three bolts for all beams 12 inches and over in depth, and these bolts shall not be less than ¾ inch in diameter. Each one of such angles or knees, when bolted to girders, shall have the same number of bolts as stated for the other leg. The angle-iron in no case shall be less in thickness than the header or trimmer to which it is bolted, and the width of angle in no case shall be less than one third the depth of beam, excepting that no angle-knee shall be less than 2½ inches wide, nor required to be more than 6 inches wide. All wrought-iron or rolled-steel beams 8 inches deep and under shall have bearings equal to their depth, if resting on a wall; 9 to 12 inch beams shall have a bearing of 10 inches, and all beams more than 12 inches in depth shall have bearings of not less than 12 inches if resting on a wall. Where beams rest on iron supports, and are properly tied to the same, no greater bearings shall be required than one third of the depth of the beams. Iron or steel floor-beams shall be so arranged as to spacing and length of beams that the load to be supported by them, together with the weights of the materials used in the construction of the said floors, shall not cause a deflection of the said beams of more than 1/30 of an inch per linear foot of span; and they shall be tied together at intervals of not more than eight times the depth of the beam.

Under the ends of all iron or steel beams, where they rest on the walls, a stone or cast-iron template shall be built into the walls. Said template shall be 8 inches wide in 12-inch walls, and in all walls of greater thickness said template shall be 12 inches wide; and such templates, if of stone, shall not be in any case less than 2½ inches in thickness, and no template shall be less than 12 inches long.

No cast-iron post or columns shall be used in any building of a less average thickness of shaft than three quarters of an inch, nor shall it have an unsupported length of more than twenty times its least lateral dimensions or diameter. No wrought-iron or rolled-steel column shall have an unsupported length of more than thirty times its least lateral dimensions or diameter, nor shall its metal be less than one fourth of an inch in thickness.

Lintels, Bearings and Supports. — All iron or steel lintels shall have bearings proportionate to the weight to be imposed thereon, but no lintel used to span any opening more than 10 feet in width shall have a bearing less than 12 inches at each end, if resting on a wall; but if resting on an iron post, such lintel shall have a bearing of at least 6 inches at each end, by the thickness of the wall to be supported.

Strains on Girders and Rivets. — Rolled iron or steel beam girders, or riveted iron or steel plate girders used as lintels or as girders, carrying

a wall or floor or both, shall be so proportioned that the loads which may come upon them shall not produce strains in tension or compression upon the flanges of more than 12,000 lbs. for iron, nor more than 15,000 lbs. for steel per square inch of the gross section of each of such flanges, nor a shearing strain upon the web-plate of more than 6000 lbs. per square inch of section of such web-plate, if of iron, nor more than 7000 pounds if of steel; but no web-plate shall be less than $\frac{1}{4}$ inch in thickness. Rivets in plate girders shall not be less than $\frac{5}{8}$ inch in diameter, and shall not be spaced more than 6 inches apart in any case. They shall be so spaced that their shearing strains shall not exceed 9000 lbs. per square inch, on their diameter, multiplied by the thickness of the plates through which they pass. The riveted plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing strains are resisted entirely by the web-plate. No part of the web shall be estimated as flange area, nor more than one half of that portion of the angle-iron which lies against the web. The distance between the centers of gravity of the flange areas will be considered as the effective depth of the girder.

The building laws of the city of New York contain a great amount of detail in addition to the extracts above, and penalties are provided for violation. See An Act creating a Department of Buildings, etc., Chapter 275, Laws of 1892. Pamphlet copy published by Baker, Voorhies & Co., New York.

FLOORS.

Maximum Load on Floors. (*Eng'g*, Nov. 18, 1892, p. 644.) — Maximum load per square foot of floor surface due to the weight of a dense crowd. Considerable variation is apparent in the figures given by many authorities, as the following table shows:

| Authorities. | Weight of Crowd, lbs. per sq. ft. |
|--|--------------------------------------|
| French practice, quoted by Trautwine and Stoney | 41 |
| Hatfield ("Transverse Strains," p. 80)..... | 70 |
| Mr. Page, London, quoted by Trautwine..... | 84 |
| Maximum load on American highway bridges according to Waddell's general specifications..... | 100 |
| Mr. Nash, architect of Buckingham Palace..... | 120 |
| Experiments by Prof. W. N. Kernot, at Melbourne | 126 |
| Experiments by Mr. B. B. Stoney ("On Stresses," p. 617) | 143.1 |
| Experiments by Prof. L. J. Johnson, <i>Eng. News</i> , April 14, 1904..... | 147.4 |
| | 134.2 |
| | to 156.9 |

The highest results were obtained by crowding a number of persons previously weighed into a small room, the men being tightly packed so as to resemble such a crowd as frequently occurs on the stairways and platforms of a theatre or other public building.

Safe Allowances for Floor Loads. (Kidder.) Pounds per square foot.

| | |
|--|---------|
| For dwellings, sleeping and lodging rooms..... | 40 lbs. |
| For schoolrooms..... | 50 " |
| For offices, upper stories..... | 60 " |
| For offices, first story..... | 80 " |
| For stables and carriage houses..... | 65 " |
| For banking rooms, churches and theaters..... | 80 " |
| For assembly halls, dancing halls, and the corridors of all public buildings, including hotels..... | 120 " |
| For drill rooms..... | 150 " |
| For ordinary stores, light storage, and light manufactur- ing..... | 120* " |

* Also to sustain a concentrated load at any point of 4000 lbs.

STRENGTH OF FLOORS.

(From circular of the Boston Manufacturers' Mutual Insurance Co.)

The following tables were prepared by C. J. H. Woodbury, for determining safe loads on floors. Care should be observed to select the figure giving the greatest possible amount and concentration of load as the one

which may be put upon any beam or set of floor-beams; and in no case should beams be subjected to greater loads than those specified, unless a lower factor of safety is warranted under the advice of a competent engineer.

Beams or heavy timbers used in the construction of a factory or warehouse should not be painted, varnished or oiled, filled or encased in impervious concrete, air-proof plastering, or metal for at least three years, lest fermentation should destroy them by what is called "dry rot."

It is, on the whole, safer to make floor-beams in two parts with a small open space between, so that proper ventilation may be secured.

These tables apply to distributed loads, but the first can be used in respect to floors which may carry concentrated loads by using half the figure given in the table, since a beam will bear twice as much load when evenly distributed over its length as it would if the load was concentrated in the center of the span.

The weight of the floor should be deducted from the figure given in the table, in order to ascertain the net load which may be placed upon any floor. The weight of spruce may be taken at 36 lbs. per cubic foot, and that of Southern pine at 48 lbs. per cubic foot.

Table I was computed upon a working modulus of rupture of Southern pine of 2160 lbs., using a factor of safety of six. It can also be applied to ascertaining the strength of spruce beams if the figures given in the table are multiplied by 0.78; or in designing a floor to be sustained by spruce beams, multiply the required load by 1.28, and use the dimensions as given by the table.

These tables are computed for beams one inch in width, because the strength of beams increase directly as the width when the beams are broad enough not to cripple.

EXAMPLE. — Required the safe load per square foot of floor, which may be safely sustained by a floor on Southern pine 10 × 14 in. beams, 8 ft. on centers, and 20 ft. span. In Table I a 1 × 14 in. beam, 20 ft. span, will sustain 118 lbs. per foot of span; and for a beam 10 ins. wide the load would be 1180 lbs. per foot of span, or 147½ lbs. per sq. ft. of floor for Southern-pine beams. From this should be deducted the weight of the floor, 17½ lbs. per sq. ft., leaving 130 lbs. per sq. ft. as a safe load. If the beams are of spruce, multiply 147½ by 0.78, reducing the load to 115 lbs. Deducting the weight of the floor, 16 lbs., leaves the safe net load as 99 lbs. per sq. ft. for spruce beams.

Table II applies to floors whose strength must be in excess of that necessary to sustain the weight, in order to meet the conditions of delicate or rapidly moving machinery, to the end that the vibration or distortion of the floor may be reduced to the least practicable limit.

In the table the limit is that of a load which would cause a bending of the beams to a curve of which the average radius would be 1250 ft.

This table is based upon a modulus of elasticity obtained from observations upon the deflection of loaded storehouse floors, and is taken at 2,000,000 lbs. for Southern pine; the same table can be applied to spruce, whose modulus of elasticity is taken as 1,200,000 lbs., if six tenths of the load for Southern pine is taken as the proper load for spruce; or, in the matter of designing, the load should be increased one and two thirds times, and the dimension of timbers for this increased load as found in the table should be used for spruce.

It can also be applied to beams and floor-timbers supported at each end and in the middle, remembering that the deflection of a beam supported in that manner is only 0.4 that of a beam of equal span which rests at each end; that is to say, the floor-planks are 2½ times as stiff, cut two bays in length, as they would be if cut only one bay in length. When a floor-plank two bays in length is evenly loaded, ⅓ of the load on the plank is sustained by the beam at each end of the plank, and ⅓ by the beam under the middle of the plank; so that for a completed floor ⅔ of the load would be sustained by the beams under the joints of the plank, and ⅓ of the load by the beams under the middle of the plank: this is the reason of the importance of breaking joints in a floor-plank every 3 ft. in order that each beam shall receive an identical load. If

it were not so, $\frac{3}{8}$ of the whole load upon the floor would be sustained by every other beam, and $\frac{5}{8}$ of the load by the corresponding alternate beams.

Repeating the former example for the load on a mill floor on Southern pine-beams 10×14 ins., and 20 ft. span, 8 ft. centers: In Table II a 1×14 in. beam should receive 61 lbs. per foot of span, or 75 lbs. per sq. ft. of floor, for Southern-pine beams. Deducting the weight of the floor, $17\frac{1}{2}$ lbs. per sq. ft., leaves 57 lbs. per sq. ft. as the advisable load.

If the beams are of spruce, the result of 75 lbs. should be multiplied by 0.6, reducing the load to 45 lbs.: The weight of the floor, in this instance amounting to 16 lbs., would leave the net load as 29 lbs. for spruce beams.

If the beams were two spans in length, they could, under these conditions, support two and a half times as much load with an equal amount of deflection, unless such load should exceed the limit of safe load as found by Table I, as would be the case under the conditions of this problem.

Mill Columns. — Timber posts offer more resistance to fire than iron pillars, and have generally displaced them in millwork. Experiments at the U. S. Arsenal at Watertown, Mass., show that sound timber posts of the proportions customarily used in millwork yield by direct crushing, the strength being directly as the area at the smallest part. The columns yielded at about 4500 lbs. per sq. in., confirming the general practice of allowing 600 lbs. per sq. in. as a safe load. Square columns are one fourth stronger than round ones of the same diameter.

I. Safe Distributed Loads upon Southern-pine Beams One Inch in Width.

(C. J. H. Woodbury.)

(If the load is concentrated at the center of the span, the beams will sustain half the amount given in the table.)

| Span, feet. | Depth of Beam in inches. | | | | | | | | | | | | | | | |
|-------------|----------------------------------|------|------|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|--|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
| | Load in pounds per foot of Span. | | | | | | | | | | | | | | | |
| 5 | 38 | 86 | 154 | 240 | 346 | 470 | 614 | 778 | 960 | | | | | | | |
| 6 | 27 | 60 | 107 | 167 | 240 | 327 | 427 | 540 | 667 | 807 | | | | | | |
| 7 | 20 | 44 | 78 | 122 | 176 | 240 | 314 | 397 | 490 | 593 | 705 | 828 | | | | |
| 8 | 15 | 34 | 60 | 94 | 135 | 184 | 240 | 304 | 375 | 454 | 540 | 634 | 735 | | | |
| 9 | | 27 | 47 | 74 | 107 | 145 | 190 | 240 | 296 | 359 | 427 | 501 | 581 | 667 | 759 | |
| 10 | | 22 | 38 | 60 | 86 | 118 | 154 | 194 | 240 | 290 | 346 | 406 | 470 | 540 | 614 | |
| 11 | | | 32 | 50 | 71 | 97 | 127 | 161 | 198 | 240 | 286 | 335 | 389 | 446 | 508 | |
| 12 | | | 27 | 42 | 60 | 82 | 107 | 135 | 167 | 202 | 240 | 282 | 327 | 375 | 474 | |
| 13 | | | | 36 | 51 | 70 | 90 | 115 | 142 | 172 | 205 | 240 | 278 | 320 | 364 | |
| 14 | | | | 31 | 44 | 60 | 78 | 99 | 123 | 148 | 176 | 207 | 240 | 276 | 314 | |
| 15 | | | | 27 | 38 | 52 | 68 | 86 | 107 | 129 | 154 | 180 | 209 | 240 | 273 | |
| 16 | | | | | 34 | 46 | 60 | 76 | 94 | 113 | 135 | 158 | 184 | 211 | 240 | |
| 17 | | | | | 30 | 41 | 53 | 67 | 83 | 101 | 120 | 140 | 163 | 187 | 217 | |
| 18 | | | | | | 36 | 47 | 60 | 74 | 90 | 107 | 125 | 145 | 167 | 190 | |
| 19 | | | | | | | 43 | 54 | 66 | 80 | 96 | 112 | 130 | 150 | 170 | |
| 20 | | | | | | | 38 | 49 | 60 | 73 | 86 | 101 | 118 | 135 | 154 | |
| 21 | | | | | | | | 44 | 54 | 66 | 78 | 92 | 107 | 122 | 139 | |
| 22 | | | | | | | | | 50 | 60 | 71 | 84 | 97 | 112 | 127 | |
| 23 | | | | | | | | | 45 | 55 | 65 | 77 | 89 | 102 | 116 | |
| 24 | | | | | | | | | | 50 | 60 | 70 | 82 | 94 | 107 | |
| 25 | | | | | | | | | | 46 | 55 | 65 | 75 | 86 | 98 | |

II. Distributed Loads upon Southern-pine Beams Sufficient to Produce Standard Limit of Deflection.

| Span, feet. | Depth of Beam in inches. | | | | | | | | | | | | | | | | Deflection, inches. |
|-------------|----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-----|-----|-----|-----|--|---------------------|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | | |
| | Load in pounds per foot of Span. | | | | | | | | | | | | | | | | |
| 5 | 3 | 10 | 23 | 44 | 77 | 122 | 182 | 259 | | | | | | | | | .0300 |
| 6 | 2 | 7 | 16 | 31 | 53 | 85 | 126 | 180 | 247 | | | | | | | | .0432 |
| 7 | | 5 | 12 | 23 | 39 | 62 | 93 | 132 | 181 | 241 | | | | | | | .0588 |
| 8 | | 4 | 9 | 17 | 30 | 48 | 71 | 101 | 139 | 185 | 240 | 305 | | | | | .0768 |
| 9 | | | 7 | 14 | 24 | 38 | 56 | 80 | 110 | 146 | 190 | 241 | 301 | | | | .0972 |
| 10 | | | 6 | 11 | 19 | 30 | 46 | 65 | 89 | 118 | 154 | 195 | 244 | 300 | | | .1200 |
| 11 | | | | 9 | 16 | 25 | 38 | 54 | 73 | 98 | 127 | 161 | 202 | 248 | 301 | | .1452 |
| 12 | | | | | 13 | 21 | 32 | 45 | 62 | 82 | 107 | 136 | 169 | 208 | 253 | | .1728 |
| 13 | | | | | 11 | 18 | 27 | 38 | 53 | 70 | 91 | 116 | 144 | 178 | 215 | | .2028 |
| 14 | | | | | | 16 | 23 | 33 | 45 | 60 | 78 | 100 | 124 | 153 | 186 | | .2352 |
| 15 | | | | | | 14 | 20 | 29 | 40 | 53 | 68 | 87 | 108 | 133 | 162 | | .2700 |
| 16 | | | | | | | 18 | 25 | 35 | 46 | 60 | 76 | 95 | 117 | 147 | | .3072 |
| 17 | | | | | | | 16 | 22 | 31 | 41 | 53 | 68 | 84 | 104 | 126 | | .3468 |
| 18 | | | | | | | | 20 | 27 | 37 | 47 | 60 | 75 | 93 | 112 | | .3888 |
| 19 | | | | | | | | 18 | 25 | 33 | 43 | 54 | 68 | 83 | 101 | | .4332 |
| 20 | | | | | | | | | 22 | 30 | 38 | 49 | 61 | 75 | 91 | | .4800 |
| 21 | | | | | | | | | 20 | 27 | 35 | 44 | 55 | 68 | 83 | | .5292 |
| 22 | | | | | | | | | | 24 | 32 | 40 | 50 | 62 | 75 | | .5808 |
| 23 | | | | | | | | | | 22 | 29 | 37 | 46 | 57 | 69 | | .6348 |
| 24 | | | | | | | | | | | 27 | 34 | 42 | 52 | 63 | | .6912 |
| 25 | | | | | | | | | | | 25 | 31 | 39 | 48 | 58 | | .7500 |

Maximum Spans for 1, 2 and 3 Inch Plank. (*Am. Mach.*, Feb. 11, 1904.) — Let w = load per sq. ft.; l = length in ins.; $W = wl/12$; S = safe fiber stress, using a factor of safety of 10; b = width of plank; d = thickness; p = deflection, E = coefficient of elasticity, I = moment of inertia = $1/12 bd^3$.

Then $Wl/S = Sbd^2/6$; $s = 5 Wl^3 \div 384 EI$. Taking S at 1200 lbs., E at 850,000 and $s = l \div 360$ for long-leaf yellow pine, the following figures for maximum span, in inches, are obtained:

| Uniform load, lbs. per sq. ft.. | 40 | 60 | 80 | 100 | 150 | 200 | 250 | 300 |
|---------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| 1-in. plank { For strength.. | 75 | 61 | 53 | 48 | 39 | 33 | ... | ... |
| { For deflection.. | 37 | 33 | 30 | 28 | 24 | 22 | ... | ... |
| 2-in. plank { For strength.. | 151 | 123 | 107 | 96 | 78 | 67 | 60 | 55 |
| { For deflection.. | 75 | 66 | 60 | 55 | 48 | 44 | 41 | 38 |
| 3-in. plank { For strength.. | 227 | 185 | 161 | 144 | 117 | 101 | 91 | 83 |
| { For deflection.. | 113 | 99 | 90 | 83 | 73 | 66 | 61 | 58 |

For white oak S may be taken at 1000 and E at 550,000; for Canadian spruce, $S = 800$, $E = 600,000$; for hemlock, $S = 600$, $E = 450,000$.

COST OF BUILDINGS.

Approximate Cost of Mill Buildings. — Chas. T. Main (*Eng. News*, Jan. 27, 1910) gives a series of diagrams of the cost in New England Jan., 1910, per sq. ft. of floor space of different sizes of brick mill buildings, one to six stories high, of the type known as "slow-burning," calculated for total floor loads of about 75 lbs. per sq. ft. Figures taken from the diagrams are given in the table below. The costs include ordinary foundations and plumbing, but no heating, sprinklers or lighting.

COST OF BRICK MILL BUILDINGS PER SQ. FT. OF FLOOR AREA.

| Length, feet. | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 500 |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| One Story. | | | | | | | | | |
| Width 25 ft. | \$1.90 | \$1.66 | \$1.58 | \$1.54 | \$1.51 | \$1.49 | \$1.48 | \$1.47 | \$1.46 |
| 50 | 1.52 | 1.29 | 1.21 | 1.18 | 1.16 | 1.15 | 1.14 | 1.13 | 1.13 |
| 75 | 1.41 | 1.21 | 1.12 | 1.08 | 1.06 | 1.04 | 1.03 | 1.02 | 1.02 |
| 125 | 1.32 | 1.09 | 1.02 | 0.98 | 0.96 | 0.94 | 0.94 | 0.93 | 0.92 |
| Two Stories. | | | | | | | | | |
| 25 | 2.00 | 1.62 | 1.52 | 1.47 | 1.44 | 1.41 | 1.39 | 1.38 | 1.36 |
| 50 | 1.50 | 1.21 | 1.13 | 1.09 | 1.06 | 1.05 | 1.04 | 1.03 | 1.02 |
| 75 | 1.34 | 1.08 | 1.01 | 0.97 | 0.94 | 0.92 | 0.92 | 0.91 | 0.90 |
| 125 | 1.22 | 0.97 | 0.90 | 0.86 | 0.84 | 0.82 | 0.81 | 0.80 | 0.86 |
| Three Stories. | | | | | | | | | |
| 25 | 1.98 | 1.57 | 1.47 | 1.42 | 1.39 | 1.38 | 1.36 | 1.35 | 1.34 |
| 50 | 1.47 | 1.17 | 1.07 | 1.03 | 1.01 | 1.00 | 0.98 | 0.98 | 0.98 |
| 75 | 1.30 | 1.05 | 0.98 | 0.94 | 0.91 | 0.89 | 0.88 | 0.87 | 0.86 |
| 125 | 1.18 | 0.93 | 0.86 | 0.82 | 0.80 | 0.78 | 0.77 | 0.76 | 0.76 |
| Four Stories. | | | | | | | | | |
| 25 | 2.00 | 1.61 | 1.50 | 1.45 | 1.42 | 1.40 | 1.38 | 1.37 | 1.36 |
| 50 | 1.38 | 1.17 | 1.10 | 1.05 | 1.02 | 1.00 | 1.00 | 0.99 | 0.98 |
| 75 | 1.32 | 1.08 | 0.97 | 0.93 | 0.90 | 0.88 | 0.88 | 0.87 | 0.87 |
| 125 | 1.20 | 0.93 | 0.85 | 0.81 | 0.78 | 0.77 | 0.76 | 0.75 | 0.74 |
| Six Stories. | | | | | | | | | |
| 25 | 2.10 | 1.72 | 1.57 | 1.51 | 1.48 | 1.46 | 1.44 | 1.43 | 1.42 |
| 50 | 1.53 | 1.21 | 1.12 | 1.08 | 1.05 | 1.04 | 1.03 | 1.02 | 1.02 |
| 75 | 1.35 | 1.08 | 0.98 | 0.94 | 0.92 | 0.90 | 0.89 | 0.88 | 0.86 |
| 125 | 1.22 | 0.96 | 0.86 | 0.82 | 0.79 | 0.78 | 0.77 | 0.76 | 0.76 |

The cost per sq. ft. of a building 100 ft. wide will be about midway between that of one 75 ft. wide and one 125 ft. wide, and the cost of a five-story building about midway between the costs of a four- and a six-story.

The data for estimating the above costs are as follows:

| | | Stories High. | | | | | |
|---|------------------|---------------|--------|--------|--------|--------|--------|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Foundations, including excavations, cost per lin. ft. | Outside walls.. | \$2.00 | \$2.90 | \$3.80 | \$4.70 | \$5.60 | \$6.50 |
| | Inside walls... | 1.75 | 2.25 | 2.80 | 3.40 | 3.90 | 4.50 |
| Brick walls, cost per sq. ft. of surface... | Outside walls.. | 0.40 | 0.44 | 0.47 | 0.50 | 0.53 | 0.57 |
| | Inside walls.... | 0.40 | 0.40 | 0.40 | 0.43 | 0.45 | 0.47 |

Columns, including piers and castings, cost each \$15.

Assumed Height of Stories. — From ground to first floor, 3 ft. Buildings 25 ft. wide, stories 13 ft. high; 50 ft. wide, 14 ft. high; 75 ft. wide, 15 ft. high; 100 ft. and 125 ft. wide, 16 ft. high.

Floors, 32 cts. per sq. ft. of gross floor space not including columns. Columns included, 38 cts.

Roof, 25 cts. per sq. ft., not including columns. Columns included, 30 cts. Roof to project 18 ins. all around buildings.

Stairways, including partitions, \$100 each flight. Two stairways and one elevator tower for buildings up to 150 ft. long; two stairways and two elevator towers for buildings up to 300 ft. long. In buildings over two stories, three stairways and three elevator towers for buildings over 300 ft. long.

In buildings over two stories, plumbing \$75 for each fixture including piping and partitions. Two fixtures on each floor up to 5000 sq. ft. of floor space and one fixture for each additional 5000 sq. ft. of floor or fraction thereof.

Modifications of the above Costs:

1. If the soil is poor or the conditions of the site are such as to require more than ordinary foundations, the cost will be increased.

2. If the building is to be used for ordinary storage purposes with low stories and no top floors, the cost will be decreased from about 10% for large low buildings to 25% for small high ones, about 20% usually being a fair allowance.

3. If the building is to be used for manufacturing and is substantially built of wood, the cost will be decreased from about 6% for large one-story buildings to 33% for high small buildings; 15% would usually be a fair allowance.

4. If the building is to be used for storage with low stories and built substantially of wood, the cost will be decreased from 13% for large one-story buildings to 50% for small high buildings; 30% would usually be a fair allowance.

5. If the total floor loads are more than 75 lbs. per sq. ft. the cost is increased.

6. For office buildings, the cost must be increased to cover architectural features on the outside and interior finish.

Reinforced-concrete buildings designed to carry floor loads of 100 lbs. per sq. ft. or less will cost about 25% more than the slow-burning type of mill construction.

ELECTRICAL ENGINEERING.

STANDARDS OF MEASUREMENT.

C.G.S. (Centimeter, Gramme, Second) or "Absolute" System of Physical Measurements:

| | |
|---|--------------------------------|
| Unit of space or distance | = 1 centimeter, cm.; |
| Unit of mass | = 1 gramme, gm.; |
| Unit of time | = 1 second, s.; |
| Unit of velocity = space ÷ time | = 1 centimeter in 1 second; |
| Unit of acceleration = change of 1 unit of velocity in 1 second; | |
| Acceleration due to gravity, at Paris, | = 981 centimeters in 1 second; |
| Unit of force = 1 dyne = $\frac{1}{981}$ gramme = $\frac{0.0022046}{981}$ lb. | = 0.000002247 lb. |

A dyne is that force which, acting on a mass of one gramme during one second, will give it a velocity of one centimeter per second. The weight of one gramme in latitude 40° to 45° is about 980 dynes, at the equator 973 dynes, and at the poles nearly 984 dynes. Taking the value of g , the acceleration due to gravity, in British measures at 32.185 feet per second at Paris, and the meter = 39.37 inches, we have

$$1 \text{ gramme} = 32.185 \times 12 \div 0.3937 = 981.00 \text{ dynes.}$$

Unit of work = 1 erg = 1 dyne-centimeter = 0.00000007373 ft.-lb.;

Unit of power = 1 watt = 10 million ergs per second,
 = 0.7373 foot-pound per second,
 = $\frac{0.7373}{550} = \frac{1}{746}$ of 1 horse-power = 0.00134 H.P.

C.G.S. unit magnetic pole is one which reacts on an equal pole at a centimeter's distance with the force of 1 dyne.

C.G.S. unit of magnetic field strength, the *gauss*, is the intensity of field which surrounding unit pole acts on it with a force of 1 dyne.

C.G.S. unit of electro-motive force = the force produced by the cutting of a field of 1 gauss intensity at a velocity of 1 centimeter per second (in a direction normal to the field and to the conductor) by 1 centimeter of conductor. The *volt* is 100,000,000 times this unit.

C.G.S. unit of electrical current = the current in a conductor (located in a plane normal to the field) when each centimeter is urged across a magnetic field of 1 gauss intensity with a force of 1 dyne. One-tenth of this is the *ampere*.

The C.G.S. unit of quantity of electricity is that represented by the flow of 1 C.G.S. unit of current for 1 second. One-tenth of this is the *coulomb*.

The Practical Units used in Electrical Calculations are:

Ampere, the unit of current strength, or rate of flow, represented by I .

Volt, the unit of electro-motive force, electrical pressure, or difference of potential, represented by E .

Ohm, the unit of resistance, represented by R .

Coulomb (or ampere-second), the unit of quantity, Q .

Ampere-hour = 3600 coulombs, Q' .

Watt (ampere-volt, or volt-ampere), the unit of power, P .

Joule (volt-coulomb), the unit of energy or work, W .

Farad, the unit of capacity, represented by C .

Henry, the unit of inductance, represented by L .

Using letters to represent the units, the relations between them may be expressed by the following formulæ, in which t represents one second and T one hour:

$$I = \frac{E}{R}, \quad Q = It, \quad Q' = IT, \quad C = \frac{Q}{E}, \quad W = QE, \quad P = IE.$$

As these relations contain no coefficient other than unity, the letters may represent any quantities given in terms of those units. For example, if E represents the number of volts electro-motive force, and R the number of ohms resistance in a circuit, then their ratio $E \div R$ will give the number of amperes current strength in that circuit.

The above six formulæ can be combined by substitution or elimination, so as to give the relations between any of the quantities. The most important of these are the following:

$$Q = \frac{E}{R}t, \quad C = \frac{I}{E}t, \quad W = IEt = \frac{E^2}{R}t = I^2Rt = Pt,$$

$$E = IR, \quad R = \frac{E}{I}, \quad P = \frac{E^2}{R} = I^2R = \frac{W}{t} = \frac{QE}{t}.$$

The definitions of these units as adopted at the International Electrical Congress at Chicago in 1893, and as established by Act of Congress of the United States, July 12, 1894, are as follows:

The *ohm* is substantially equal to 10^9 (or 1,000,000,000) units of resistance of the C.G.S. system, and is represented by the resistance offered to an unvarying electric current by a column of mercury at 32°F. , 14.4521 grammes in mass, of a constant cross-sectional area, and of the length of 106.3 centimeters.

The *ampere* is $\frac{1}{10}$ of the unit of current of the C.G.S. system, and is the practical equivalent of the unvarying current which when passed through a solution of nitrate of silver in water in accordance with standard specifications deposits silver at the rate of 0.001118 gramme per second.

The *volt* is the electro-motive force that, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere, and is practically equivalent to 1000/1434 (or 0.6974) of the electro-motive force between the poles or electrodes of a Clark's cell at a temperature of 15°C. , and prepared in the manner described in the standard specifications.

The *coulomb* is the quantity of electricity transferred by a current of one ampere in one second.

The *farad* is the capacity of a condenser charged to a potential of one volt by one coulomb of electricity.

The *joule* is equal to 10,000,000 units of work in the C.G.S. system, and is practically equivalent to the energy expended in one second by an ampere in an ohm.

The *watt* is equal to 10,000,000 units of power in the C.G.S. system, and is practically equivalent to the work done at the rate of one joule per second.

The *henry* is the induction in a circuit when the electro-motive force induced in this circuit is one volt, while the inducing current varies at the rate of one ampere per second.

The ohm, volt, etc., as above defined, are called the "international" ohm, volt, etc., to distinguish them from the "legal" ohm, B.A. unit, etc.

The value of the ohm, determined by a committee of the British Association in 1863, called the B.A. unit, was the resistance of a certain piece

of copper wire. The so-called "legal" ohm, as adopted at the International Congress of Electricians in Paris in 1884, was a correction of the B.A. unit, and was defined as the resistance of a column of mercury 1 square millimeter in section and 106 centimeters long, at a temperature of 32° F.

1 legal ohm = 1.0112 B.A. units, 1 B.A. unit = 0.9889 legal ohm;
 1 international ohm = 1.0136 B.A. units, 1 B.A. unit = 0.9866 int. ohm;
 1 international ohm = 1.0023 legal ohm, 1 legal ohm = 0.9977 int. ohm.

DERIVED UNITS.

1 megohm = 1 million ohms;
 1 microhm = 1 millionth of an ohm;
 1 milliampere = $\frac{1}{1000}$ of an ampere;
 1 micro-farad = 1 millionth of a farad.

RELATIONS OF VARIOUS UNITS.

| | |
|---------------------------------|---|
| 1 ampere | = 1 coulomb per second; |
| 1 volt-ampere | = 1 watt = 1 volt-coulomb per sec.; |
| 1 watt..... | { = 0.7373 foot-pound per second; |
| | { = 0.0009477 heat-unit per sec. (Fahr.); |
| | { = $\frac{1}{746}$ of one horse-power; |
| 1 joule | { = 0.7373 foot-pound, |
| | { = work done by one watt in one sec., |
| | { = 0.0009477 heat-unit; |
| 1 British thermal unit..... | = 1055.2 joules; |
| 1 kilowatt, or 1000 watts | = 1000/746 or 1.3405 horse-powers; |
| 1 kilowatt-hour, | { = 1.3405 horse-power hours, |
| 1000 volt-ampere hours, | { = 2,654,200 foot-pounds, |
| 1 British Board of Trade unit, | { = 3412 heat-units; |
| 1 horse-power | { = 746 watts = 746 volt-amperes, |
| | { = 33,000 foot-pounds per minute. |

The ohm, ampere, and volt are defined in terms of one another as follows: Ohm, the resistance of a conductor through which a current of one ampere will pass when the electro-motive force is one volt. Ampere, the quantity of current which will flow through a resistance of one ohm when the electro-motive force is one volt. Volt, the electro-motive force required to cause a current of one ampere to flow through a resistance of one ohm.

For Methods of making Electrical Measurements, Testing, etc.; see Munroe & Jamieson's Pocket-Book of Electrical Rules, Tables, and Data; S. P. Thompson's Dynamo-Electric Machinery; Carhart & Patterson's Electrical Measurements; and works on Electrical Engineering.

Equivalent Electrical and Mechanical Units. — H. Ward Leonard published in *The Electrical Engineer*, Feb. 25, 1895, a table of useful equivalents of electrical and mechanical units, from which the table on page 1347 is taken, with some modifications.

Units of the Magnetic Circuit.

Unit magnetic pole is a pole of such strength that when placed at a distance of one centimeter from a similar pole of equal strength it repels it with a force of one dyne.

Magnetic Moment is the product of the strength of either pole into the distance between the two poles.

Intensity of Magnetization is the magnetic moment of a magnet divided by its volume.

Intensity of Magnetic Field is the force exerted by the field upon a unit magnetic pole. The unit is the gauss.

Gauss = unit of field strength, or density, symbol H, is that intensity of field which acts on a unit pole with a force of one dyne, = one line of force per square centimeter. One gauss produces 1 dyne of force per centimeter length of conductor upon a current of 10 amperes, or an electro-motive force of 1/100,000,000 volt in a centimeter length of conductor when its velocity across the field is 1 centimeter per second. A field of

Equivalent Values of Electrical and Mechanical Units.

| Unit. | Equivalent Value in Other Units. | Unit. | Equivalent Value in Other Units. | Unit. | Equivalent Value in Other Units. |
|---------------------|--|-------------------------|--|--|--|
| 1 K.W. Hour = | 1,000 watt hours. | 1 H.P. = | 746 watts. | 1 Heat-unit = | 1,055 watt seconds. |
| | 1.34 horse-power hours. | | 0.746 K.W. | | 778 ft.-lbs. |
| | 2,654,200 ft.-lbs. | | 33,000 ft.-lbs. per minute. | | 107.6 kilogram meters. |
| | 3,600,000 joules. | | 550 ft.-lbs. per second. | | 0.000293 K.W. hour. |
| 1 Kilo-watt = | 3,412 heat-units. | 1 Watt = | 2,545 heat-units per hour. | 1 lb. Water Evap. from 212° F. = | 0.000393 H.P. hour. |
| | 367,000 kilogram meters. | | 42.4 heat-unit per minute. | | 0.000068 lb. carbon oxidized. |
| | 0.235 lb. carbon oxidized with perfect efficiency. | | 0.707 heat-units per second. | | 0.001036 lb. water evap. from and at 212° F. |
| | 3.53 lbs. water evap. from and at 212° F. | | 0.175 lb. carbon oxidized per hour. | | |
| 1 H.P. Hour = | 22.75 lbs. of water raised from 62° to 212° F. | 1 Joule = | 2.64 lbs. water evap. per hour from and at 212° F. | 1 Heat-unit per Sq. Ft. per min. = | 0.122 watt per square in. |
| | 0.746 K.W. hour. | | 1 watt second. | | 0.0176 K.W. per sq. ft. |
| | 1,980,000 ft.-lbs. | | 0.000000278 K.W. hour. | | 0.0236 H.P. per sq. ft. |
| | 2,545 heat-units. | | 0.102 k.g.m. | | |
| 1 H.P. Hour = | 273,740 k.g.m. | 1 Ft.-lb. = | 0.0009477 heat-units. | 1 Kilogram Meter = | 7.233 ft.-lbs. |
| | 0.175 lb. carbon oxidized with perfect efficiency. | | 0.7373 ft.-lb. | | 0.00000365 H.P. hour. |
| | 2.64 lbs. water evaporated from and at 212° F. | | 1.356 joules. | | 0.00000272 K.W. hour. |
| | 17.0 lbs. water raised from 62° F. to 212° F. | | 0.1383 k.g.m. | | 0.0093 heat-unit. |
| 1 Kilo-watt = | 1,000 watts. | 1 Watt = | 0.000000377 K.W. hour. | 1 lb. Carbon Oxidized with perfect Efficiency = | 14,544 heat-units. |
| | 1.34 horse-power. | | 0.001285 heat-unit. | | 1.11 lbs. Anth'cite coal ox. |
| | 2,654,200 ft.-lbs. per hour. | | 0.0000005 H.P. hour. | | 2.5 lbs. dry wood oxidized. |
| | 44,240 ft.-lbs. per minute. | | 1 joule per second. | | 21 cu. ft. illuminating-gas. |
| 1 Kilo-watt = | 737.3 ft.-lbs. per second. | 1 Watt = | 0.0035 lb. water evap. per hr. | 1 lb. Water Evap. from 212° F. = | 4.26 K.W. hours. |
| | 3,412 heat-units per hour. | | 44.24 ft.-lbs. per minute. | | 5.71 H.P. hours. |
| | 56.9 heat-units per minute. | | 8.19 heat-units per sq. ft. per minute. | | 11,315,000 ft.-lbs. |
| | 0.948 heat-unit per second. | | 6371 ft.-lbs. per sq. ft. per minute. | | 15 lbs. of water evap. from and at 212° F. |
| 1 Kilo-watt = | 0.2275 lb. carbon oxidized per hour. | 1 Watt per sq. in. = | 0.193 H.P. per sq. ft. | 1 lb. Water Evap. from 212° F. = | 0.283 K.W. hour. |
| | 3.53 lbs. water evap. per hour from and at 212° F. | | | | 0.379 H.P. hour. |
| | | | | | 970 heat-units. |
| | | | | | 103,900 k.g.m. |
| 1 Kilo-watt = | | 1 Watt per sq. in. = | | 1 lb. Water Evap. from 212° F. = | 1,019,000 joules. |
| | | | | | 751,300 ft.-lbs. |
| | | | | | 0.0664 lb. of carbon oxidized. |
| | | | | | |

H units is one which acts with H dynes on unit pole, or H lines per square centimeter. A unit magnetic pole has 4π lines of force proceeding from it.

Maxwell = unit of magnetic flux, is the amount of magnetism passing through a square centimeter of a field of unit density. Symbol, ϕ .

In non-magnetic materials a unit of intensity of flux is the same as unit intensity of field. The name maxwell is given to a unit quantity of flux, and one maxwell per square centimeter in non-magnetic materials is the same as the gauss. In magnetic materials the flux produced by the molecular magnets is added to the field (Norris).

Magnetic Flux, ϕ , is equal to the average field intensity multiplied by the cross-sectional area. The unit is the maxwell. Maxwells per square inch = gauss $\times 6.45$.

Magnetic Induction, symbol B, is the flux or the number of magnetic lines per unit of area of cross-section of magnetized material, the area being at every point perpendicular to the direction of the flux. It is equal to the product of the field intensity, H, and the permeability, μ .

Gilbert = unit of magnetomotive force, is the amount of M.M.F. that would be produced by a coil of $10 \div 4\pi$ or 0.7958 ampere-turns. Symbol F.

The M.M.F. of a coil is equal to 1.2566 times the ampere-turns.

If a solenoid is wound with 100 turns of insulated wire carrying a current of 5 amperes, the M.M.F. exerted will be 500 ampere-turns $\times 1.2566 = 628.3$ gilberts.

Oersted = unit of magnetic reluctance; it is the reluctance of a cubic centimeter of an air-pump vacuum. Symbol, R.

Reluctance is that quantity in a magnetic circuit which limits the flux under a given M.M.F. It corresponds to the resistance in the electric circuit.

Permeance is the reciprocal of reluctance.

The reluctivity of any medium is its specific reluctance, and in the C.G.S. system is the reluctance offered by a cubic centimeter of the body between opposed parallel faces. The reluctivity of nearly all substances, other than the magnetic metals, is sensibly that of vacuum, is equal to unity, and is independent of the flux density.

Permeability is the reciprocal of magnetic reluctivity. It is a number and the symbol is μ .

Materials differ in regard to the resistance they offer to the passage of lines of force; thus iron is more permeable than air. The permeability of a substance is expressed by a coefficient, μ , which denotes its relation to the permeability of air, which is taken as 1. If H = number of magnetic lines per square centimeter which will pass through an air-space between the poles of a magnet, and B the number of lines which will pass through a certain piece of iron in that space, then $\mu = B \div H$. The permeability varies with the quality of the iron, and the degree of saturation, reaching a practical limit for soft wrought iron when B = about 18,000 and for cast iron when B = about 10,000 C.G.S. lines per square centimeter.

The permeability of a number of materials may be determined by means of the table on page 1384.

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY.

WATER.

ELECTRICITY.

| | |
|--|--|
| Head, difference of level, in feet. | } Volts; electro-motive force; difference of potential; E. or E.M.F. |
| Difference of pressure, lbs. per sq. in. | |
| Resistance of pipes, apertures, etc., increases with length of pipe, with contractions, roughness, etc.; decreases with increase of sectional area. | |
| Rate of flow, as cubic ft. per second, gallons per min., etc., or volume divided by the time. In the mining regions sometimes expressed in "miners' inches." | Amperes; current; current strength; intensity of current; rate of flow; 1 ampere = 1 coulomb per second. |
| | Amperes = $\frac{\text{volts}}{\text{ohms}}$; $I = \frac{E}{R}$; $E = IR$. |

ANALOGIES BETWEEN THE FLOW OF WATER AND ELECTRICITY — Continued.

WATER.

Quantity, usually measured in cubic ft. or gallons, but is also equivalent to rate of flow \times time, as cu. ft. per second for so many hours.

Work, or energy, measured in foot-pounds; product of weight of falling water into height of fall; in pumping, product of quantity in cubic feet into the pressure in lbs. per square foot against which the water is pumped.

Power, rate of work. Horse-power = ft.-lbs. of work in 1 min. \div 33,000. In water flowing in pipes, rate of flow in cu. ft. per second \times resistance to the flow in lbs. per sq. ft. \div 550.

ELECTRICITY.

Coulomb, unit of quantity, Q , = rate of flow \times time, as ampere-seconds. 1 ampere-hour = 3600 coulombs.

Joule, volt-coulomb, W , the unit of work, = product of quantity by the electro-motive force = volt-ampere-second. 1 joule = 0.7373 foot-pound.

If C (amperes) = rate of flow, and E (volts) = difference of pressure between two points in a circuit, energy expended = $I Et$, = $I^2 Rt$.

Watt, unit of power, P , = volts \times amperes, = current or rate of flow \times difference of potential. 1 watt = 0.7373 foot-pound per sec. = 1746 of a horse-power.

ELECTRICAL RESISTANCE.

Laws of Electrical Resistance. — The resistance, R , of any conductor varies directly as its length, l , and inversely as its sectional area, s , or $R \propto l \div s$.

If r = the resistance of a conductor 1 unit in length and 1 square unit in sectional area, $R = rl \div s$. The common unit of length for electrical calculations in English measure is the foot, and the unit of area of wires is the circular mil = the area of a circle 0.001 in. diameter. 1 mil-foot = 1 foot long 1 circ.-mil area.

Resistance of 1 mil-foot of soft copper wire at 51° F. = 10 international ohms.

EXAMPLE. — What is the resistance of a wire 1000 ft. long, 0.1 in. diam.? 0.1 in. diam. = 10,000 circ. mils.

$$R = rl \div s = 10 \times 1000 \div 10,000 = 1 \text{ ohm.}$$

Specific resistance, also called resistivity, is the resistance of a material of unit length and section as compared with the resistance of soft copper.

Conductivity is the reciprocal of specific resistance, or the relative conducting power compared with copper taken at 100.

Relative Conductivities of Different Metals at 0° and 100° C. (Matthiessen.)

| Metals. | Conductivities. | | Metals. | Conductivities. | |
|-------------------|-----------------------|--------------------------|------------------|-----------------------|--------------------------|
| | At 0° C. At 32° F. | At 100° C. At 212° F. | | At 0° C. At 32° F. | At 100° C. At 212° F. |
| Silver, hard..... | 100 | 71.56 | Tin..... | 12.36 | 8.67 |
| Copper, hard..... | 99.95 | 70.27 | Lead..... | 8.32 | 5.86 |
| Gold, hard..... | 77.96 | 55.90 | Arsenic..... | 4.76 | 3.33 |
| Zinc, pressed.... | 29.02 | 20.67 | Antimony..... | 4.62 | 3.26 |
| Cadmium..... | 23.72 | 16.77 | Mercury, pure... | 1.60 | |
| Platinum, soft... | 18.00 | | Bismuth..... | 1.245 | 0.878 |
| Iron, soft..... | 16.80 | | | | |

Resistance of Various Metals and Alloys. — Condensed from a table compiled by H. F. Parshall and H. M. Hobart from different authorities. R = resistance in ohms per mil foot = resistance per centimeter cube \times 6.015. C = percent increase of resistance per degree C.

| | R | C | | R | C |
|------------------------------|------|-------|------------------------------|------|------|
| Aluminum, 99% pure..... | 15.4 | 0.423 | White cast iron..... | 340 | - |
| Aluminum, 94; copper, 6.. | 17.4 | .381 | Gray cast iron..... | 684 | - |
| Al. bronze, Al 10; Cu, 90 .. | 75.5 | .105 | Wrought iron..... | 82.8 | - |
| Antimony, compressed.... | 211 | .389 | Soft steel, C, 0.045 | 63 | - |
| Bismuth, compressed | 780 | .354 | Manganese steel, Mn, 12.. | 401 | .127 |
| Cadmium, pure..... | 60 | .419 | Nickel steel, Ni, 4.35 | 177 | .201 |
| Copper, annealed, (D).... | 9.35 | .428 | Lead, pure..... | 123 | .411 |
| Copper, annealed, (M) ... | 9.54 | .388 | Manganin, | | |
| Copper, 88; silicon, 12.... | 17.7 | | Cu, 84; Mn, 12; Ni, 4 | 287 | .000 |
| Copper, 65.8; zinc, 34.2.... | 37.8 | .158 | Cu, 80.5; Mn, 3; Ni, 16.5 .. | 294 | .000 |
| Copper, 90; lead, 10 | 31.7 | | Cu, 79.5; Mn, 19.7; Fe, 0.8 | 393 | .000 |
| Copper, 97; aluminum, 3.. | 53.0 | .090 | Mercury | 566 | .072 |
| Cu, 87; Ni, 6.5; Al, 6.5.... | 89.5 | .065 | Nickel | 73.7 | .62 |
| Copper, 65; nickel, 25 | 205 | .019 | Palladium, pure | 61.1 | .354 |
| Cu, 70; manganese, 30 | 605 | .004 | Platinum, annealed | 539 | .247 |
| German silver | | | Platinum, 67; silver, 33 ... | 145 | .133 |
| Cu, 60; Zn, 25; Ni, 15 | 180 | .036 | Phosphor bronze | 33.6 | .394 |
| Gold, 99.9% pure..... | 13.2 | .377 | Silver, pure..... | 8.82 | .400 |
| Gold, 67; silver, 33..... | 61.8 | .065 | Tin, pure..... | 78.5 | .440 |
| Iron, very pure..... | 54.5 | .625 | Zinc, pure..... | 34.5 | .406 |

(D) Dewar and Fleming; (M) Matthiessen.

Conductivity of Aluminum. — J. W. Richards (*Jour. Frank. Inst.*, Mar., 1897) gives for hard-drawn aluminum of purity 98.5, 99.0, 99.5, and 99.75% respectively a conductivity of 55, 59, 61, and 63 to 64%, copper being 100%. The Pittsburg Reduction Co. claims that its purest aluminum has a conductivity of over 64.5%. (*Eng'g News*, Dec. 17, 1896.)

German Silver. — The resistance of German silver depends on its composition. Matthiessen gives it as nearly 13 times that of copper, with a temperature coefficient of 0.0004433 per degree C. Weston, however (*Proc. Electrical Congress*, 1893, p. 179), has found copper-nickel-zinc alloys (German silver) which had a resistance of nearly 28 times that of copper, and a temperature coefficient of about one-half that given by Matthiessen.

Conductors and Insulators in Order of their Value.

CONDUCTORS.

All metals
Well-burned charcoal
Plumbago
Acid solutions
Saline solutions
Metallic ores
Animal fluids
Living vegetable substances
Moist earth
Water

INSULATORS (NON-CONDUCTORS).

Dry air
Shellac
Paraffin
Amber
Resins
Sulphur
Wax
Jet
Glass
Mica
Ebonite
Gutta-percha
India-rubber
Silk
Dry paper
Parchment
Dry leather
Porcelain
Oils

According to Culley, the resistance of distilled water is 6754 million times as great as that of copper. Impurities in water decrease its resistance.

Resistance Varies with Temperature. — For every degree Centigrade the resistance of copper increases about 0.4%, or for every degree F. 0.2222%. Thus a piece of copper wire having a resistance of 10 ohms at 32° would have a resistance of 11.11 ohms at 82° F.

The following table shows the amount of resistance of a few substances used for various electrical purposes by which 1 ohm is increased by a rise of temperature of 1° C.

| | | | |
|-----------------------------|---------|-------------------|---------|
| Platinoid | 0.00021 | Gold, silver..... | 0.00065 |
| Platinum silver..... | 0.00031 | Cast iron..... | 0.00080 |
| German silver (see above).. | 0.00044 | Copper..... | 0.00400 |

Annealing. — Resistance is lessened by annealing. Matthiessen gives the following relative conductivities for copper and silver, the comparison being made with pure silver at 100° C.:

| Metal. | Temp. C. | Hard. | Annealed. | Ratio. |
|--------------|----------|-------|-----------|------------|
| Copper | 11° | 95.31 | 97.83 | 1 to 1.027 |
| Silver | 14.6° | 95.36 | 103.33 | 1 to 1.084 |

Dr. Siemens compared the conductivities of copper, silver, and brass with the following results. Ratio of hard to annealed:

Copper....1 to 1.058 Silver....1 to 1.145 Brass....1 to 1.180

Standard of Resistance of Copper Wire. (*Trans. A. I. E. E.*, Sept. and Nov., 1890.) — Matthiessen's standard is: A hard-drawn copper wire 1 meter long, weighing 1 gramme, has a resistance of 0.1469 B.A. unit at 0° C. Relative conducting power (Matthiessen): silver, 100; hard or unannealed copper, 99.95; soft or annealed copper, 102.21. Conductivity of copper at other temperatures than 0° C., $C_t = C_0 (1 - 0.00387 t \pm 0.000009009 t^2)$.

The resistance is the reciprocal of the conductivity, and is

$$R_t = R_0 (1 + 0.00387 t + 0.00000597 t^2).$$

The shorter formula $R_t = R_0 (1 + 0.00406 t)$ is commonly used.

A committee of the Am. Inst. Electrical Engineers recommend the following as the most correct form of the Matthiessen standard, taking 8.89 as the sp. gr. of pure copper:

A soft copper wire 1 meter long and 1 mm. diam. has an electrical resistance of 0.02057 B.A. unit at 0° C. From this the resistance of a soft copper wire 1 foot long and 0.001 in. diam. (mil-foot) is 9.720 B.A. units at 0° C.

| Standard Resistance at 0° C. | B.A. Units. | Legal Ohms | Internat. Ohms. |
|----------------------------------|---------------|-------------|-----------------|
| Meter-millimeter, soft copper | 0.02057 | 0.02034 | 0.02029 |
| Cubic centimeter | 0.000001616 | 0.000001598 | 0.000001593 |
| Mil-foot | 9.720 | 9.612 | 9.590 |
| 1 mil-ft. of soft copper at 10°. | 22 C. or 50°. | 4 F. | 10. |
| " " " " " " 15°. | 5 | 59°. | 9 F. |
| " " " " " " 23°. | 9 | 75° | 9 F. |
| | | 10.53 | 10.505 |

Hard-drawing and annealing are found to produce proportional changes in the conductivity and the temperature coefficient. The range of conductivity of numerous samples representative of the copper now in common use for electrical purposes is from 94.5% to 101.8% (on the basis of 100% corresponding to 1.7213 micro-ohms per centimeter-cube, at 20° C.

Using this result, a measurement of the conductivity of a sample gives also its temperature coefficient. Thus, a_{20} (in the formula, $R_t = R_{20} [1 + a_{20} (t - 20)]$ for a sample of copper is given by multiplying 0.00393 by the percentage conductivity. The value assumed by the Am. Inst. El. En., $a_0 = 0.0042$, or $a_{20} = 0.00387$, is the true temperature coefficient for copper of 98.6% conductivity. (J. H. Dellinger, *Elec. Rev.*, May 7, 1910.)

For tables of the resistance of copper wire, see pages 1357 and 1358, also page 240.

Taking Matthiessen's standard of pure copper as 100%, some refined metal has exhibited an electrical conductivity equivalent to 103%. Matthiessen found that impurities in copper sufficient to decrease its density from 8.94 to 8.90 produced a marked increase of electrical resistance.

DIRECT ELECTRIC CURRENTS.

Ohm's Law. — This law expresses the relation between the three fundamental units of resistance, electrical pressure, and current. It is:

$$\text{Current} = \frac{\text{electrical pressure}}{\text{resistance}}; I = \frac{E}{R}; \text{whence } E = IR, \text{ and } R = \frac{E}{I}.$$

In terms of the units of the three quantities,

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}; \text{volts} = \text{amperes} \times \text{ohms}; \text{ohms} = \frac{\text{volts}}{\text{amperes}}.$$

EXAMPLES: Simple Circuits. — 1. If the source has an effective electrical pressure of 100 volts, and the resistance is two ohms, what is the current?

$$I = \frac{E}{R} = \frac{100}{2} = 50 \text{ amperes.}$$

2. What pressure will give a current of 50 amperes through a resistance of 2 ohms? $E = IR = 50 \times 2 = 100$ volts.

3. What resistance is required to obtain a current of 50 amperes when the pressure is 100 volts? $R = E \div I = 100 \div 50 = 2$ ohms.

Ohm's law applies equally to a complete electrical circuit and to any part thereof.

Series Circuits. — If conductors are arranged one after the other they are said to be in series, and the total resistance of the circuit is the sum of the resistances of its several parts. Let *A*, Fig. 195, be a source of current, such as a battery or generator, producing a difference of potential or E.M.F. of 120 volts, measured across *ab*, and let the circuit contain four conductors whose resistances, r_1, r_2, r_3, r_4 , are 1 ohm each, and three other resistances, R_1, R_2, R_3 , each 2 ohms. The total resistance is 10 ohms, and by Ohm's law the current $I = E \div R = 120 \div 10 = 12$ amperes. This current is constant throughout the circuit, and a series circuit is therefore one of constant current. The drop of potential in the whole circuit from *a* around to *b* is 120 volts, or $E = RI$. The drop in any portion depends on the resistance of that portion; thus from *a* to

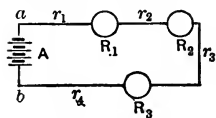


FIG. 195.

R_1 the resistance is 1 ohm, the constant current 12 amperes, and the drop $1 \times 12 = 12$ volts. The drop in passing through each of the resistance R_1, R_2, R_3 is $2 \times 12 = 24$ volts.

Parallel, Divided, or Multiple Circuits. — Let *B*, Fig. 196, be a generator producing an E.M.F. of 220 volts across the terminals *ab*.

The current is divided, so that part flows through the main wires *ac* and part through the "shunt" *s*, having a resistance of 0.5 ohm. Also the current has three paths between *c* and *d*, viz., through the three resistances in parallel R_1, R_2, R_3 , of 2 ohms each. Consider that the resistance of the wires is so small that it may be neglected. Let the conductances of the four paths be represented by C_s, C_1, C_2, C_3 . The total

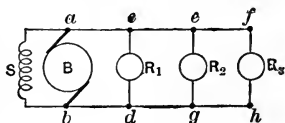


FIG. 196.

conductance is $C_s + C_1 + C_2 + C_3 = C$ and the total resistance $R = 1 \div C$. The conductance of each path is the reciprocal of its resistance, the total conductance is the sum of the separate conductances, and the resistance of the combined or "parallel" paths is the reciprocal of the total conductance.

$$R = 1 + \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) = 1 + 3.5 = 0.286 \text{ ohm.}$$

The current $I = E \div R = 770$ amperes.

Conductors in Series and Parallel. — Let the resistances in parallel be the same as in Fig. 196, with the additional resistance of 0.1 ohm in each of the six sections of the main wires, *ac, bd*, etc., in series. The voltage across *ab* being 220 volts, determine the drop in voltage at the several points, the total current, and the current through each path. The problem is somewhat complicated. It may be solved as follows: Consider first the points *eg*; here there are two paths for the current, *efgh* and *eg*. Find the resistance and the conductance of each and the total resistance (the reciprocal of the joint conductance) of the parallel

paths. Next consider the points cd ; here there are two paths — one through e and the other through cd . Find the total resistance as before. Finally consider the points ab ; here there are two paths — one through c , the other through s . Find the conductances of each and their sum. The product of this sum and the voltage at ab will be the total amperes of current, and the current through any path will be proportional to the conductance of that path. The resistances, R , and conductances, C , of the several paths are as follows:

| | R | C |
|--|----------|--------|
| R_a of $efR_3hg = 0.1 + 2 + 0.1 = 2.2$ | 2.2 | 0.4545 |
| R_b of eR_2g | = 2 | 0.5 |
| Joint $R_c = 1.048$ | | 0.9545 |
| R_d of $ce + dg + R_c$ | = 1.248 | 0.8013 |
| R_e of cR_1d | = 2 | 0.5 |
| Joint $R_f = 0.7687$ | | 1.3013 |
| R_g of $ac + bd + R_f$ | = 0.9687 | 1.0332 |
| R_h of s | = 0.5 | 2 |
| Joint $R_a + R_h = 0.330$ | | 3.0332 |

Total current = $220 \times 3.0332 = 667.3$ amperes.

Current through $s = 220 \times 2 = 440$ amp.; through $c = 227.3$ amp.

" " $cR_1d = 227.3 \times 0.5 \div 1.3013 = 87.34$ amp.

" " $e = 227.3 \times 0.8013 \div 1.3013 = 139.96$ "

" " $eR_2g = 139.96 \times 0.5 \div 0.9545 = 73.31$ "

" " $fR_3 = 139.96 \times 0.4545 \div 0.9545 = 66.65$ "

The drop in voltage in any section of the line is found by the formula $E = RI$, R being the resistance of that section and I the current in it. As the R of each section is 0.1 ohm we find E for ac and bd each = 22.7 volts, for ce and dg each 14.0 volts, and for ef and gh each 6.67 volts. The voltage across cd is $220 - 2 \times 22.7 = 174.6$ volts; across eg , $174.6 - 2 \times 14.0 = 146.6$, and across fh $146.6 - 2 \times 6.67 = 133.3$ volts. Taking these voltages and the resistances R_1, R_2, R_3 , each 2 ohms, we find from $I = E \div R$ the current through each of these resistances 87.3, 73.3, and 66.65 amperes as before.

Internal Resistance. — In a simple circuit we have two resistances, that of the circuit R and that of the internal parts of the source of electro-motive force, called internal resistance, r . The formula of Ohm's law when the internal resistance is considered is $I = E \div (R + r)$.

Power of the Circuit. — The power, or rate of work, in watts = current in amperes \times electro-motive force in volts = $I \times E$. Since $I = E \div R$, watts = $E^2 \div R$ = electro-motive force² \div resistance.

EXAMPLE. — What H.P. is required to supply 100 lamps of 40 ohms resistance each, requiring an electro-motive force of 60 volts?

The number of volt-amperes for each lamp is $\frac{E^2}{R} = \frac{60^2}{40}$, 1 volt-ampere = 0.00134 H.P.; therefore $\frac{60^2}{40} \times 100 \times 0.00134 = 12$ H.P. (electrical) very nearly.

Electrical, Brake, and Indicated Horse-power. — The power given by a dynamo = volts \times amperes \div 1000 = kilowatts, kw. Volts \times out amperes \div 746 = electrical horse-power, E.H.P. The power put into a dynamo shaft by a direct-connected engine or other prime mover is called the shaft or brake horse-power, B.H.P. If e_1 is the efficiency of the dynamo, B.H.P. = E.H.P. $\div e_1$. If e_2 is the mechanical efficiency of the engine, the indicated horse-power, I.H.P. = brake H.P. $\div e_2$ = E.H.P. $\div (e_1 \times e_2)$.

If e_1 and e_2 each = 91.5%, I.H.P. = E.H.P. \times 1.194 = kw. \times 1.60. In direct-connected units of 250 kw. or less the rated H.P. of the engine is commonly taken as $1.6 \times$ the rated kw. of the generator.

Electric motors are rated at the H.P. given out at the pulley or belt. H.P. of motor = E.H.P. supplied \times efficiency of motor.

Heat Generated by a Current. — Joule's law shows that the heat developed in a conductor is directly proportional, 1st, to its resistance; 2d, to the square of the current strength; and 3d, to the time during which the current flows, or $H = I^2 Rt$. Since $I = E \div R$,

$$I^2 Rt = \frac{E}{R} I Rt = E It = E \frac{E}{R} t = \frac{E^2 t}{R}.$$

Or, heat = current² \times resistance \times time
 = electro-motive force \times current \times time.
 = electro-motive force² \times time \div resistance.

Q = quantity of electricity flowing = $It = (Et \div R)$.
 $H = EQ$; or heat = electro-motive force \times quantity.

The electro-motive force here is that causing the flow, or the difference in potential between the ends of the conductor.

The electrical unit of heat, or "joule" = 10^7 ergs = heat generated in one second by a current of 1 ampere flowing through a resistance of one ohm = 0.239 gramme of water raised 1° C. $H = I^2 Rt \times 0.239$ gramme calories = $I^2 Rt \times 0.0009478$ British thermal units.

In electric lighting the energy of the current is converted into heat in the lamps. The resistance of the lamp is made great so that the required quantity of heat may be developed, while in the wire leading to and from the lamp the resistance is made as small as is commercially practicable, so that as little energy as possible may be wasted in heating the wire.

Heating of Conductors. (From Kapp's Electrical Transmission of Energy.) — It becomes a matter of great importance to determine beforehand what rise in temperature is to be expected in each given case, and if that rise should be found to be greater than appears safe, provision must be made to increase the rate at which heat is carried off. This can generally be done by increasing the superficial area of the conductor. Say we have one circular conductor of 1 square inch area, and find that with 1000 amperes flowing it would become too hot. Now by splitting up this conductor into 10 separate wires each one-tenth of a square inch cross-sectional area, we have not altered the total amount of energy transformed into heat, but we have increased the surface exposed to the cooling action of the surrounding air in the ratio of 1: $\sqrt{10}$, and therefore the ten thin wires can dissipate more than three times the heat, as compared with the single thick wire.

Prof. Forbes states that an insulated wire carries a greater current without overheating than a bare wire if the diameter be not too great. Assuming the diameter of the cable to be twice the diam. of the conductor, a greater current can be carried in insulated wires than in bare wires up to 1.9 inch diam. of conductor. If diam. of cable = 4 times diam. of conductor, this is the case up to 1.1 inch diam. of conductor.

Heating of Bare Wires. — The following formulæ are given by Kennelly:

$$T = \frac{I^2}{d^3} \times 90,000 + t; \quad d = 44.8 \sqrt[3]{\frac{I^2}{T - t}}.$$

T = temperature of the wire and t that of the air, in Fahrenheit degrees;
 I = current in amperes, d = diameter of the wire in mils.

If we take $T - t = 90^\circ$ F., $\sqrt[3]{90} = 4.48$, then

$$d = 10 \sqrt[3]{I^2} \quad \text{and} \quad I = \sqrt{d^3 \div 1000}.$$

This latter formula gives for the carrying capacity in amperes of bare wires almost exactly the figures given for weather-proof wires in the Fire Underwriters' table, except in the case of Nos. 18 and 16, B. & S. gauge, for which the formula gives 8 and 11 amperes, respectively, instead of 5 and 8 amperes, given in the table.

Heating of Coils. — The rise of temperature in magnet coils due to the passage of current through the wire is approximately proportional to the watts lost in the coil per unit of effective radiating surface, thus:

$$t \propto \frac{I^2 R}{S} \text{ or } t = \frac{I^2 R}{kS},$$

t being the temperature rise in degrees Fahr.; S , the effective radiating surface; and k a coefficient which varies widely, according to condition. In electromagnet coils of small size and power, k may be as large as 0.015. Ordinarily it ranges from 0.012 down to 0.005; a fair average is 0.007. The more exposed the coil is to air circulation, the larger is the value of k ; the larger the proportion of iron to copper, by weight, in the core and winding, the thinner the winding with relation to its dimension parallel with the magnet core, and the larger the "space factor" of the winding, the larger will be the value of k . The space factor is the ratio of the actual copper cross-section of the whole coil to the gross cross-section of copper, insulation, and interstices.

Fusion of Wires. — W. H. Preece gives a formula for the current required to fuse wires of different metals, viz., $I = ad^{\frac{2}{3}}$, in which d is the diameter in inches and a a coefficient whose value for different metals is as follows: Copper, 10,244; aluminum, 7585; platinum, 5172; German silver, 5230; platinoid, 4750; iron, 3148; tin, 1462; lead, 1379; alloy of 2 lead and 1 tin, 1318.

Allowable Carrying Capacity of Copper Wires.

(For inside wiring, National Board of Fire Underwriters' Rules.)

| B. & S. Gauge. | Circular Mils. | Amperes. | | Circular Mils. | Amperes. | |
|-------------------|-------------------|--------------------|------------------------|-------------------|--------------------|------------------------|
| | | Rubber Covered. | Other In- sulation. | | Rubber Covered. | Other In- sulation. |
| 18 | 1,624 | 3 | 5 | 200,000 | 200 | 300 |
| 16 | 2,583 | 6 | 8 | 300,000 | 270 | 400 |
| 14 | 4,107 | 12 | 16 | 400,000 | 330 | 500 |
| 12 | 6,530 | 17 | 23 | 500,000 | 390 | 590 |
| 10 | 10,380 | 24 | 32 | 600,000 | 450 | 680 |
| 8 | 16,510 | 33 | 46 | 700,000 | 500 | 760 |
| 6 | 26,250 | 46 | 65 | 800,000 | 550 | 840 |
| 5 | 33,100 | 54 | 77 | 900,000 | 600 | 920 |
| 4 | 41,740 | 65 | 92 | 1,000,000 | 650 | 1,000 |
| 3 | 52,630 | 76 | 110 | 1,100,000 | 690 | 1,080 |
| 2 | 66,370 | 90 | 131 | 1,200,000 | 730 | 1,150 |
| 1 | 83,690 | 107 | 156 | 1,300,000 | 770 | 1,220 |
| 0 | 105,500 | 127 | 185 | 1,400,000 | 810 | 1,290 |
| 00 | 133,100 | 150 | 220 | 1,600,000 | 890 | 1,430 |
| 000 | 167,800 | 177 | 262 | 1,800,000 | 970 | 1,550 |
| 0000 | 211,600 | 210 | 312 | 2,000,000 | 1,050 | 1,670 |

Wires smaller than No. 14 B. & S. gauge must not be used except in fixtures and pendant cords.

The lower limit is specified for rubber-covered wires to prevent deterioration of the insulation by the heat of the wires.

For insulated aluminum wire the safe-carrying capacity is 84 per cent of that of copper wire with the same insulation.

See pamphlets published by the National Board of Fire Underwriters, New York, for complete specifications and rules for wiring.

Underwriters' Insulation. — The thickness of insulation required by the rules of the National Board of Fire Underwriters varies with the size of the wire, the character of the insulation, and the voltage. The thickness of insulation on rubber-covered wires carrying voltages up to 600 varies from $\frac{1}{32}$ inch for a No. 18 B. & S. gauge wire to $\frac{1}{8}$ inch for a wire of 1,000,000 circular mils. Weather-proof insulation is required to be slightly thicker. For voltages of over 600 the insulation is required to be at least $\frac{3}{32}$ inch thick for all sizes from No. 14 B. & S. gauge to 500,000 mils and $\frac{1}{8}$ inch thick for larger sizes.

Weights, Lengths, and Resistances of Cool, Warm, and Hot Copper Wires.

| Gauges | | | Diam-eter, inches. | Area, Circular Mils. | Weight. | | Length. | | Resistance in International Ohms. | | | |
|-------------|-------------|---------------|--------------------|----------------------|--------------------------------|-------------|--------------------------------|--------------------------------|-----------------------------------|---------------------------------|---------------------------------|--|
| A W G B & S | B W G Stubs | Lbs per Foot. | | | Lbs per Ohm. at 20° C., 68° F. | Feet per Lb | Ft. per Ohm. at 20° C., 68° F. | Ohms per Lb. at 20° C., 68° F. | Ohms per ft. at 20° C., 68° F. | Ohms per ft. at 50° C., 122° F. | Ohms per ft. at 80° C., 176° F. | |
| 0000 | 0000 | 0.6405 | 0.460 | 211,600 | 13.090 | 1.561 | 20.440 | 0.00007639 | 0.00004893 | 0.00005467 | 0.00006058 | |
| | | 0.6239 | 0.454 | 206,100 | 12.420 | 1.603 | 19.910 | 0.00008051 | 0.00005023 | 0.00005612 | 0.00006220 | |
| | 000 | 0.5468 | 0.425 | 180,600 | 9.538 | 1.829 | 17.450 | 0.0001048 | 0.00005732 | 0.00006404 | 0.00007097 | |
| | 00 | 0.5080 | 0.4096 | 167,800 | 8.232 | 1.969 | 16.210 | 0.0001215 | 0.00006170 | 0.00006893 | 0.00007640 | |
| | 0 | 0.4371 | 0.380 | 144,400 | 6.096 | 2.288 | 13.950 | 0.0001640 | 0.00007170 | 0.00008011 | 0.00008878 | |
| | 0 | 0.4028 | 0.3648 | 133,100 | 5.177 | 2.482 | 12.850 | 0.0001931 | 0.00007780 | 0.00008692 | 0.00009633 | |
| | 0 | 0.3499 | 0.340 | 115,600 | 3.907 | 2.858 | 11.160 | 0.0002560 | 0.00008957 | 0.0001001 | 0.0001109 | |
| | 0 | 0.3195 | 0.3249 | 105,500 | 3.256 | 3.130 | 10.190 | 0.0003071 | 0.00009811 | 0.0001096 | 0.0001215 | |
| | 1 | 0.2724 | 0.3000 | 90,000 | 2.368 | 3.671 | 8.692 | 0.0004223 | 0.0001150 | 0.0001285 | 0.0001424 | |
| | 1 | 0.2533 | 0.2893 | 83,690 | 2.048 | 3.947 | 8.083 | 0.0004883 | 0.0001237 | 0.0001382 | 0.0001532 | |
| | 2 | 0.2441 | 0.2840 | 80,660 | 1.902 | 4.096 | 7.790 | 0.0005258 | 0.0001284 | 0.0001434 | 0.0001589 | |
| | 3 | 0.2031 | 0.2590 | 67,080 | 1.316 | 4.925 | 6.479 | 0.0007601 | 0.0001543 | 0.0001724 | 0.0001911 | |
| | 2 | 0.2009 | 0.2576 | 66,370 | 1.288 | 4.977 | 6.410 | 0.0007765 | 0.0001560 | 0.0001743 | 0.0001932 | |
| | 4 | 0.1715 | 0.2380 | 56,640 | 0.938 | 5.832 | 5.471 | 0.001066 | 0.0001828 | 0.0002042 | 0.0002235 | |
| | 3 | 0.1593 | 0.2294 | 52,630 | 810.0 | 6.276 | 5.084 | 0.001235 | 0.0001967 | 0.0002198 | 0.0002435 | |
| | 5 | 0.1465 | 0.2200 | 48,400 | 684.9 | 6.826 | 4.675 | 0.001460 | 0.0002139 | 0.0002390 | 0.0002649 | |
| | 4 | 0.1264 | 0.2043 | 41,740 | 509.4 | 7.914 | 4.031 | 0.001963 | 0.0002480 | 0.0002771 | 0.0003071 | |
| | 6 | 0.1247 | 0.2030 | 41,210 | 496.5 | 8.017 | 3.980 | 0.002014 | 0.0002513 | 0.0002807 | 0.0003111 | |
| | 5 | 0.1002 | 0.1819 | 33,100 | 320.4 | 9.980 | 3.197 | 0.003128 | 0.0003495 | 0.0003873 | 0.0004249 | |
| | 7 | 0.09808 | 0.1800 | 32,400 | 306.9 | 10.20 | 3.129 | 0.003258 | 0.0003570 | 0.0003957 | 0.0004303 | |
| | 8 | 0.08241 | 0.1650 | 27,230 | 216.7 | 12.13 | 2.629 | 0.004615 | 0.0004249 | 0.0004683 | 0.0005053 | |
| | 6 | 0.07946 | 0.1620 | 26,250 | 201.5 | 12.58 | 2.335 | 0.004963 | 0.0004406 | 0.0004853 | 0.0005281 | |
| | 9 | 0.06302 | 0.1443 | 20,820 | 140.3 | 15.08 | 2.116 | 0.007129 | 0.0005281 | 0.0005756 | 0.0006158 | |
| | 7 | 0.05435 | 0.1340 | 17,960 | 126.7 | 15.87 | 2.011 | 0.007892 | 0.0005493 | 0.0005956 | 0.0006442 | |
| | 8 | 0.04998 | 0.1285 | 16,510 | 94.26 | 18.40 | 1.734 | 0.01061 | 0.0006271 | 0.0006740 | 0.0007140 | |
| | 11 | 0.04359 | 0.1200 | 14,400 | 79.69 | 20.01 | 1.595 | 0.01255 | 0.0007190 | 0.0007707 | 0.0008203 | |
| | 9 | 0.03963 | 0.1144 | 13,090 | 60.62 | 22.94 | 1.391 | 0.01905 | 0.0008335 | 0.0008903 | 0.0009493 | |
| | 12 | 0.03596 | 0.1090 | 10,380 | 50.12 | 25.23 | 1.265 | 0.02423 | 0.0009736 | 0.0010279 | 0.001079 | |
| | 10 | 0.03143 | 0.1019 | 9,025 | 41.27 | 27.81 | 1.147 | 0.03173 | 0.001114 | 0.001167 | 0.0012235 | |
| | 13 | 0.02732 | 0.0950 | 8,234 | 31.52 | 31.82 | 1.033 | 0.04199 | 0.001282 | 0.001340 | 0.001400 | |
| | 11 | 0.02493 | 0.0889 | 7,518 | 23.81 | 36.60 | 0.871 | 0.05045 | 0.001405 | 0.001467 | 0.001527 | |
| | 14 | 0.02085 | 0.08300 | 6,530 | 19.82 | 40.12 | 0.795 | 0.07207 | 0.001503 | 0.001567 | 0.001631 | |
| | 12 | 0.01977 | 0.08081 | 6,364 | 13.87 | 47.95 | 0.665 | 0.08022 | 0.001586 | 0.001653 | 0.001721 | |
| | 15 | 0.01569 | 0.07200 | 5,184 | 12.47 | 50.59 | 0.630 | 0.1275 | 0.001997 | 0.002073 | 0.002143 | |
| | 13 | 0.01568 | 0.07196 | 5,178 | 7.857 | 63.73 | 0.500 | 0.1276 | 0.001999 | 0.002073 | 0.002143 | |
| | 16 | 0.01279 | 0.06500 | 4,225 | 7.840 | 63.79 | 0.500 | 0.1916 | 0.002451 | 0.002534 | 0.002617 | |
| | 14 | 0.01243 | 0.06408 | 4,107 | 5.219 | 78.19 | 0.408 | 0.2028 | 0.002817 | 0.002903 | 0.002989 | |
| | 17 | 0.01018 | 0.05707 | 3,364 | 0.931 | 80.44 | 0.396 | 0.3023 | 0.003439 | 0.003522 | 0.003603 | |
| | 15 | 0.00980 | 0.05582 | 3,257 | 3.308 | 98.23 | 0.324 | 0.3225 | 0.003729 | 0.003811 | 0.003896 | |
| | 16 | 0.007818 | 0.05082 | 2,583 | 0.09858 | 101.4 | 0.314 | 0.3225 | 0.004009 | 0.004094 | 0.004179 | |
| | 18 | 0.007268 | 0.04900 | 2,401 | 1.950 | 127.9 | 0.249 | 0.5128 | 0.004312 | 0.004406 | 0.004491 | |
| | | | | | 1.685 | 137.6 | 0.231 | 0.5933 | 0.004618 | 0.004712 | 0.004807 | |

Weights, Lengths, and Resistances of Cool, Warm, and Hot Copper Wires. — Continued.

| Gauges | | Diam- eter, inches. | Area, Circular Mils. | Weight. | | Length. | | Resistance in International Ohms. | | | |
|--------------------|--------------------|---------------------------|----------------------------|-----------------|-----------------------------------|-----------------|------------------------------------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| A. W. G. B & S. | B. W. G. Stubs. | | | Lbs per Foot | Lbs per Ohm, at 20° C., 68° F. | Feet per Lb. | Feet per Ohm, at 20° C., 68° F. | Ohms per Lb., at 20° C., 68° F. | Ohms per ft., at 20° C., 68° F. | Ohms per ft., at 50° C., 122° F. | Ohms per ft., at 80° C., 176° F. |
| 17 | | 0.04526 | 2.048 | 0.006200 | 1.226 | 161.3 | 197.8 | 0.8153 | 0.005648 | 0.006259 | 0.007267 |
| 18 | 19 | 0.04200 | 1.764 | 0.005340 | 0.9097 | 187.3 | 170.4 | 1.099 | 0.006558 | 0.007267 | 0.008267 |
| 19 | | 0.04030 | 1.624 | 0.004917 | 0.7713 | 203.4 | 156.9 | 1.296 | 0.007122 | 0.007892 | 0.008952 |
| 20 | 21 | 0.03589 | 1.288 | 0.003899 | 0.4851 | 256.5 | 124.4 | 2.061 | 0.008038 | 0.008980 | 0.01047 |
| 21 | | 0.03300 | 1.225 | 0.003708 | 0.4387 | 269.7 | 118.3 | 2.279 | 0.008452 | 0.009443 | 0.01130 |
| 22 | 23 | 0.03196 | 1.022 | 0.003100 | 0.3066 | 322.6 | 98.90 | 3.262 | 0.01011 | 0.01130 | 0.01252 |
| 23 | | 0.02846 | 810.1 | 0.002452 | 0.1919 | 373.4 | 88.66 | 3.278 | 0.01014 | 0.01132 | 0.01255 |
| 24 | 25 | 0.02800 | 784.0 | 0.002373 | 0.1797 | 407.8 | 78.24 | 3.512 | 0.01278 | 0.01428 | 0.01563 |
| 25 | | 0.02535 | 642.4 | 0.001945 | 0.1207 | 421.4 | 75.72 | 5.565 | 0.01321 | 0.01475 | 0.01635 |
| 26 | 27 | 0.02500 | 625.0 | 0.001892 | 0.1142 | 514.2 | 62.05 | 8.287 | 0.01612 | 0.01801 | 0.01996 |
| 27 | | 0.02257 | 509.5 | 0.001542 | 0.07589 | 528.6 | 60.36 | 8.756 | 0.01657 | 0.01851 | 0.02051 |
| 28 | 29 | 0.02200 | 484.0 | 0.001465 | 0.06849 | 648.4 | 49.21 | 13.18 | 0.02032 | 0.02271 | 0.02516 |
| 29 | | 0.02010 | 404.0 | 0.001223 | 0.04773 | 682.6 | 46.75 | 14.60 | 0.02139 | 0.02390 | 0.02649 |
| 30 | 31 | 0.02000 | 400.0 | 0.001211 | 0.04678 | 817.6 | 39.02 | 20.95 | 0.02563 | 0.02863 | 0.03173 |
| 31 | | 0.01800 | 324.0 | 0.0009808 | 0.03069 | 1,025.9 | 38.63 | 21.38 | 0.02588 | 0.02892 | 0.03205 |
| 32 | 33 | 0.01790 | 320.4 | 0.0009699 | 0.03002 | 1,031 | 31.29 | 32.58 | 0.03196 | 0.03570 | 0.03957 |
| 33 | | 0.01600 | 256.0 | 0.0007749 | 0.01916 | 1,290 | 30.95 | 33.32 | 0.03231 | 0.03610 | 0.04001 |
| 34 | 35 | 0.01594 | 254.1 | 0.0007692 | 0.01888 | 1,300 | 24.73 | 52.19 | 0.04045 | 0.04519 | 0.05008 |
| 35 | | 0.0142 | 201.5 | 0.0006100 | 0.01187 | 1,399 | 24.54 | 52.97 | 0.04075 | 0.04552 | 0.05045 |
| 36 | 37 | 0.01400 | 196.0 | 0.0005933 | 0.01123 | 1,639 | 19.46 | 84.23 | 0.05138 | 0.05740 | 0.06362 |
| 37 | | 0.01300 | 169.0 | 0.0005116 | 0.008350 | 1,955 | 18.93 | 89.04 | 0.05283 | 0.05902 | 0.06541 |
| 38 | 39 | 0.01264 | 159.8 | 0.0004837 | 0.007466 | 2,067 | 15.43 | 119.8 | 0.06127 | 0.06845 | 0.07586 |
| 39 | | 0.01200 | 144.0 | 0.0004359 | 0.006062 | 2,294 | 13.91 | 165.0 | 0.06479 | 0.07239 | 0.08022 |
| 40 | 41 | 0.01126 | 126.7 | 0.0003836 | 0.004696 | 2,607 | 12.24 | 213.0 | 0.07190 | 0.08033 | 0.08903 |
| 41 | | 0.01003 | 100.5 | 0.0003042 | 0.002953 | 3,287 | 9.707 | 338.6 | 0.08170 | 0.09128 | 0.1012 |
| 42 | 43 | 0.01000 | 100.0 | 0.0003027 | 0.002924 | 3,304 | 9.658 | 342.0 | 0.1030 | 0.1151 | 0.1276 |
| 43 | | 0.00900 | 81.0 | 0.0002452 | 0.001918 | 4,078 | 7.823 | 421.3 | 0.1035 | 0.1157 | 0.1282 |
| 44 | 45 | 0.008928 | 79.70 | 0.0002413 | 0.001857 | 4,145 | 7.698 | 538.4 | 0.1299 | 0.1451 | 0.1608 |
| 45 | | 0.00800 | 64.0 | 0.0001937 | 0.001197 | 5,162 | 6.181 | 835.1 | 0.1618 | 0.1807 | 0.2003 |
| 46 | 47 | 0.007950 | 63.21 | 0.0001913 | 0.001168 | 5,227 | 6.105 | 856.2 | 0.1638 | 0.1830 | 0.2028 |
| 47 | | 0.007080 | 50.13 | 0.0001517 | 0.0007346 | 6,591 | 4.841 | 1,361 | 0.2066 | 0.2308 | 0.2558 |
| 48 | 49 | 0.00700 | 49.0 | 0.0001483 | 0.0007019 | 6,742 | 4.733 | 1,425 | 0.2113 | 0.2361 | 0.2616 |
| 49 | | 0.006305 | 39.75 | 0.0001203 | 0.0004620 | 8,311 | 3.839 | 2,165 | 0.2605 | 0.2910 | 0.3225 |
| 50 | 51 | 0.006115 | 31.52 | 0.00009543 | 0.0002905 | 10,480 | 3.045 | 3,441 | 0.3284 | 0.3669 | 0.4067 |
| 51 | | 0.00550 | 25.0 | 0.00007568 | 0.0001827 | 13,210 | 2.414 | 4,773 | 0.4142 | 0.4627 | 0.5129 |
| 52 | 53 | 0.004453 | 19.83 | 0.00006001 | 0.0001149 | 16,660 | 1.915 | 6,702 | 0.5222 | 0.5855 | 0.6466 |
| 53 | | 0.00400 | 16.72 | 0.00004843 | 0.00007484 | 20,650 | 1.545 | 13,360 | 0.6471 | 0.7230 | 0.8011 |
| 54 | 55 | 0.003965 | 15.47 | 0.00004759 | 0.00007210 | 21,010 | 1.519 | 13,870 | 0.6585 | 0.7357 | 0.8154 |
| 55 | | 0.003531 | 12.47 | 0.00003774 | 0.00005455 | 26,500 | 1.204 | 22,000 | 0.8304 | 0.9277 | 1.028 |
| 56 | 57 | 0.003145 | 9.888 | 0.00002993 | 0.00002858 | 33,410 | 0.9550 | 34,980 | 1.047 | 1.170 | 1.296 |

ELECTRIC TRANSMISSION, DIRECT CURRENTS.

Cross-section of Wire Required for a Given Current. —

Let R = resistance of a given line of copper wire, in ohms;

r = " " 1 mil-foot of copper;

L = length of wire, in feet;

e = drop in voltage between the two ends;

I = current, in amperes;

A = sectional area of wire, in circular mils;

then $I = \frac{e}{R}$; $R = \frac{e}{I}$; $R = r \frac{L}{A}$; whence $A = \frac{rIL}{e}$.

The value of r for soft copper wire at 75° F. is 10.505 international ohms. For ordinary drawn copper wire the value of 10.8 is commonly taken, corresponding to a conductivity of 97.2 per cent.

For a circuit, going and return, the total length is $2L$, and the formula becomes $A = 21.6 IL \div e$, L here being the distance from the point of supply to the point of delivery.

If E is the voltage at the generator and a the per cent of drop in the line, then $e = Ea \div 100$, and $A = \frac{2160 IL}{aE}$.

If P = the power in watts, $= EI$, then $I = \frac{P}{E}$, and $A = \frac{2160 PL}{aE^2}$.

If P_k = the power in kilowatts, $A = 2,160,000 P_k L \div aE^2$.

If L_m = the distance in miles and A_c the area in circular inches, then $A_c = 6405 P_k L_m \div aE^2$. If A_s = area in square inches, $A_s = 5030 P_k L_m \div aE^2$. When the area in circular mils has been determined by either of these formulæ reference should be made to the table of Allowable Capacity of Wires, to see if the calculated size is sufficient to avoid overheating. For all interior wiring the rules of the National Board of Fire Underwriters should be followed. See Appendix to Vol. II of "Crocker's Electric Lighting."

Weight of Copper for a Given Power. — Taking the weight of a mil-foot of copper at 0.000003027 lb., the weight of copper in a circuit of length $2L$ and cross-section A , in circ. mils, is 0.000006054 LA lbs., $= W$.

Substituting for A its value $2160 PL \div aE^2$ we have

$W = 0.0130766 PL^2 \div aE^2$; P in watts, L in ft.

$W = 13.0766 P_k L^2 \div aE^2$; P_k in kilowatts, L in ft.

$W = 364,556,000 P_k L_m^2 \div aE^2$; P_k in kilowatts, L_m in miles.

The weight of copper required varies directly as the power transmitted; inversely as the percentage of drop or loss; directly as the square of the distance; and inversely as the square of the voltage.

From the last formula the following table has been calculated:

WEIGHT OF COPPER WIRE TO CARRY 1000 KILOWATTS WITH 10% LOSS.

| Distance in miles. | 1 | 5 | 10 | 20 | 50 | 100 |
|--------------------|----------------|-----------|-----------|-----------|-----------|-----------|
| Volts. | Weight in lbs. | | | | | |
| 500 | 145,822 | 3,645,560 | | | | |
| 1,000 | 36,456 | 911,390 | 3,645,560 | | | |
| 2,000 | 9,114 | 227,848 | 911,390 | 3,645,560 | | |
| 5,000 | 1,458 | 36,456 | 145,822 | 593,290 | 3,645,560 | |
| 10,000 | 365 | 9,114 | 36,456 | 145,822 | 911,390 | 3,645,560 |
| 20,000 | 91 | 2,278 | 9,114 | 36,456 | 227,848 | 911,390 |
| 40,000 | | 570 | 2,278 | 9,114 | 56,962 | 227,848 |
| 60,000 | | | 1,013 | 4,051 | 25,316 | 101,266 |

In calculating the distance, an addition of about 5 per cent should be made for sag of the wires.

Short-circuiting. — From the law $I = E/R$ it is seen that with any pressure E , the current I will become very great if R is made very small. In short-circuiting the resistance becomes small and the current therefore great. Hence the dangers of short-circuiting a current.

Economy of Electric Transmission. — Lord Kelvin's rule for the most economical section of conductor for a given voltage is that for which the annual interest on capital outlay is equal to the annual cost of energy wasted.

Tables have been compiled by Professor Forbes and others in accordance with modifications of this rule. For a given entering horse-power the question is merely one as to what current density, or how many amperes per square inch of conductor, should be employed. Kelvin's rule gives about 393 amperes per square inch, and Professor Forbes's tables give a current density of about 380 amperes per square inch as most economical.

Bell ("Electric Transmission of Power") shows that while Kelvin's rule correctly indicates the condition of minimum cost in transmission for a given current and line, it omits many practical considerations and is inapplicable to most power transmission work. Each plant has to be considered on its merits and very various conditions are likely to determine the line loss in different cases. Several cases are cited by Bell to show that neither Kelvin's law nor any modification of it is a safe guide in determining the proper allowance for loss of energy in the line.

Wire Tables. — The tables on this and the following page show the relation between load, distance, and "drop" or loss by voltage in a two-wire direct-current circuit of any standard size of wire. The tables are based on the formula

$$(21.6 IL) \div A = \text{Drop in volts.}$$

I = current in amperes, L = distance in feet from point of supply to point of delivery, A = sectional area of wire in circular mils. The factors I and L are combined in the table, in the compound factor "ampere feet."

WIRE TABLE — RELATION BETWEEN LOAD, DISTANCE, LOSS, AND SIZE OF CONDUCTOR.

NOTE.—The numbers in the body of the tables are Ampere-Feet, i.e., Amperes \times Distance (length of one wire). See examples on next page.

Table I. — 110-volt and 220-volt Two-wire Circuits.

| Wire Sizes; B. & S. Gauge. | | Line Loss in Percentage of the Rated Voltage; and Power Loss in Percentage of the Delivered Power. | | | | | | | | |
|-------------------------------|--------|---|--------|--------|--------|--------|---------|---------|---------|---------|
| 110 V. | 220 V. | 1 | 1 1/2 | 2 | 3 | 4 | 5 | 6 | 8 | 10 |
| | 0000 | 21,550 | 32,325 | 43,100 | 64,650 | 86,200 | 107,750 | 129,300 | 172,400 | 215,500 |
| | 000 | 17,080 | 25,620 | 34,160 | 51,240 | 68,320 | 85,400 | 102,480 | 136,640 | 170,800 |
| | 00 | 13,550 | 20,325 | 27,100 | 40,650 | 54,200 | 67,750 | 81,300 | 108,400 | 135,500 |
| 0000 | 0 | 10,750 | 16,125 | 21,500 | 32,250 | 43,000 | 53,750 | 64,500 | 86,000 | 107,500 |
| 000 | 1 | 8,520 | 12,780 | 17,040 | 25,560 | 34,080 | 42,600 | 51,120 | 68,160 | 85,200 |
| 00 | 2 | 6,750 | 10,140 | 13,520 | 20,280 | 27,040 | 33,800 | 40,560 | 54,080 | 67,600 |
| 0 | 3 | 5,360 | 8,040 | 10,720 | 16,080 | 21,440 | 26,800 | 32,160 | 42,880 | 53,600 |
| 1 | 4 | 4,250 | 6,375 | 8,500 | 12,750 | 17,000 | 21,250 | 25,500 | 34,000 | 42,500 |
| 2 | 5 | 3,370 | 5,055 | 6,740 | 10,110 | 13,480 | 16,850 | 20,220 | 26,960 | 33,700 |
| 3 | 6 | 2,670 | 4,005 | 5,340 | 8,010 | 10,680 | 13,350 | 16,020 | 21,360 | 26,700 |
| 4 | 7 | 2,120 | 3,180 | 4,240 | 6,360 | 8,480 | 10,600 | 12,720 | 16,960 | 21,200 |
| 5 | 8 | 1,680 | 2,520 | 3,360 | 5,040 | 6,720 | 8,400 | 10,800 | 13,440 | 16,800 |
| 6 | 9 | 1,330 | 1,995 | 2,660 | 3,990 | 5,320 | 6,650 | 7,980 | 10,640 | 13,300 |
| 7 | 10 | 1,055 | 1,582 | 2,110 | 3,165 | 4,220 | 5,275 | 6,330 | 8,440 | 10,550 |
| 8 | 11 | 838 | 1,257 | 1,675 | 2,514 | 3,350 | 4,190 | 5,028 | 6,700 | 8,380 |
| 9 | 12 | 665 | 997 | 1,330 | 1,995 | 2,660 | 3,320 | 3,990 | 5,320 | 6,650 |
| 10 | 13 | 527 | 790 | 1,054 | 1,580 | 2,108 | 2,635 | 3,160 | 4,215 | 5,270 |
| 11 | 14 | 418 | 627 | 836 | 1,254 | 1,672 | 2,090 | 2,508 | 3,344 | 4,180 |
| 12 | | 332 | 498 | 665 | 997 | 1,330 | 1,660 | 1,995 | 2,660 | 3,325 |
| 14 | | 209 | 313 | 418 | 627 | 836 | 1,045 | 1,354 | 1,672 | 2,090 |

Table II. — 500, 1000, and 2000 Volt Circuits.

| Wire Sizes; B. & S. Gauge. | | | Line Loss in Percentage of the Rated Voltage; and Power Loss in Percentage of the Delivered Power. | | | | | | |
|-------------------------------|---------|---------|---|---------|---------|---------|---------|---------|---------|
| 500 V. | 1000 V. | 2000 V. | 1 | 1½ | 2 | 2½ | 3 | 4 | 5 |
| | 0000 | 0 | 97,960 | 146,940 | 195,920 | 244,900 | 293,880 | 391,840 | 489,800 |
| | 000 | 1 | 77,690 | 116,535 | 155,380 | 194,225 | 233,970 | 310,760 | 388,450 |
| | 00 | 2 | 61,620 | 92,430 | 123,240 | 154,050 | 184,860 | 246,480 | 308,100 |
| 0000 | 0 | 3 | 48,880 | 73,320 | 97,760 | 122,200 | 146,640 | 195,420 | 244,400 |
| 000 | 1 | 4 | 38,750 | 58,125 | 77,500 | 96,875 | 116,250 | 155,000 | 193,750 |
| 00 | 2 | 5 | 30,760 | 46,140 | 61,520 | 76,900 | 92,280 | 123,040 | 153,800 |
| 0 | 3 | 6 | 24,370 | 36,555 | 48,740 | 60,925 | 73,110 | 97,480 | 121,850 |
| 1 | 4 | 7 | 19,320 | 28,980 | 38,640 | 48,300 | 57,960 | 77,280 | 96,600 |
| 2 | 5 | 8 | 15,320 | 22,980 | 30,640 | 38,300 | 45,960 | 61,280 | 76,600 |
| 3 | 6 | 9 | 12,150 | 18,225 | 24,300 | 30,375 | 36,450 | 48,300 | 60,750 |
| 4 | 7 | 10 | 9,640 | 14,460 | 19,280 | 24,100 | 28,920 | 38,560 | 48,200 |
| 5 | 8 | 11 | 7,640 | 11,460 | 15,280 | 19,100 | 22,920 | 30,560 | 38,200 |
| 6 | 9 | 12 | 6,060 | 9,090 | 12,120 | 15,150 | 18,180 | 24,240 | 30,300 |
| 7 | 10 | 13 | 4,805 | 7,207 | 9,610 | 12,010 | 14,415 | 19,220 | 24,025 |
| 8 | 11 | 14 | 3,810 | 5,715 | 7,620 | 9,525 | 11,430 | 15,220 | 19,050 |
| 9 | 12 | | 3,020 | 4,530 | 6,040 | 7,550 | 9,060 | 12,080 | 15,100 |
| 10 | 13 | | 2,395 | 3,592 | 4,790 | 5,985 | 7,185 | 9,580 | 11,975 |
| 11 | 14 | | 1,900 | 2,850 | 3,800 | 4,750 | 5,700 | 7,600 | 9,500 |
| 12 | | | 1,510 | 2,265 | 3,020 | 3,775 | 4,530 | 6,040 | 7,550 |
| 14 | | | 950 | 1,425 | 1,900 | 2,375 | 2,850 | 3,800 | 4,750 |

EXAMPLES IN THE USE OF THE WIRE TABLES.—1. Required the maximum load in amperes at 220 volts that can be carried 95 feet by No. 6 wire without exceeding 1½% drop.

Find No. 6 in the 220-volt column of Table I; opposite this in the 1½% column is the number 4005, which is the ampere-feet. Dividing this by the required distance (95 feet) gives the load, 42.15 amperes.

Example 2. A 500-volt line is to carry 100 amperes 600 feet with a drop not exceeding 5%; what size of wire will be required?

The ampere-feet will be $100 \times 600 = 60,000$. Referring to the 5% column of Table II, the nearest number of ampere-feet is 60,750, which is opposite No. 3 wire in the 500-volt column.

These tables also show the percentage of the power delivered to a line that is lost in non-inductive alternating-current circuits. Such circuits are obtained when the load consists of incandescent lamps and the circuit wires lie only an inch or two apart, as in conduit wiring.

Efficiency of Electric Systems.—The efficiency of a system is the ratio of the power delivered by the electric motors at the distant end of the line to the power delivered to the dynamo-electric machines at the other end. The efficiency of a dynamo or motor varies with its load and with the size of machine, ranging about as follows for dynamos at full load:

| | | | | | | |
|--------------|----|----|-----|-----|-----|------|
| Kilowatts | 30 | 50 | 100 | 200 | 500 | 1000 |
| Efficiency % | 90 | 91 | 92 | 93 | 94 | 95 |

For motors at full load the efficiencies run about as follows:

| | | | | | | | | |
|---------|----|----|----|------|----|----|------|------|
| H.P. | 1 | 2 | 5 | 10 | 20 | 50 | 75 | 100 |
| Effy. % | 75 | 80 | 85 | 88.5 | 90 | 91 | 91.5 | 91.6 |

The efficiency of both generators and motors decreases, at first very slowly and then more rapidly, as the load decreases. Each machine has its "characteristic" curve of efficiency, showing the ratio of output to input at different loads. The following is a rough approximation for direct-current machines: Decrease of efficiency at half-load, 3%; ¼ load, 10%; ⅛ load, 20%; 1/16 load, 50%. The loss in transmission, due to fall in

Resistances of Pure Aluminum Wire.*

Conductivity 62 in the Matthiessen Standard Scale. Pure aluminum weighs 167.111 pounds per cubic foot.

| Am. Gauge, B. & S. No. | Resistances at 70° F. | | | | Am. Gauge, B. & S. No. | Resistances at 70° F. | | | |
|---------------------------|---------------------------|----------------------|---------------------|--------------------|---------------------------|---------------------------|----------------------|---------------------|--------------------|
| | Ohms per 1000 Feet. | Ohms per Mile. | Feet per Ohm. | Ohms per Pound. | | Ohms per 1000 Feet. | Ohms per Mile. | Feet per Ohm. | Ohms per Pound. |
| 0000 | 0.07904 | 0.41730 | 12652. | 0.00040985 | 19 | 12.985 | 68.564 | 77.05 | 11.070 |
| 000 | .09966 | .52623 | 10034. | .00065102 | 20 | 16.381 | 86.500 | 61.06 | 17.595 |
| 00 | .12569 | .66362 | 7956. | .0010364 | 21 | 20.649 | 109.02 | 48.43 | 27.971 |
| 0 | .15849 | .83684 | 6310. | .0016479 | 22 | 26.025 | 137.42 | 38.44 | 44.450 |
| 1 | .19982 | 1.0552 | 5005. | .0026194 | 23 | 32.830 | 173.35 | 30.45 | 70.700 |
| 2 | .25200 | 1.3305 | 3968. | .0041656 | 24 | 41.400 | 218.60 | 24.16 | 112.43 |
| 3 | .31778 | 1.6779 | 3147. | .0066250 | 25 | 52.200 | 275.61 | 19.16 | 178.78 |
| 4 | .40067 | 2.1156 | 2496. | .010531 | 26 | 65.856 | 347.70 | 15.19 | 284.36 |
| 5 | .50526 | 2.6679 | 1975. | .016749 | 27 | 83.010 | 438.32 | 12.05 | 452.62 |
| 6 | .63720 | 3.3687 | 1569. | .026628 | 28 | 104.67 | 552.64 | 9.55 | 718.95 |
| 7 | .80350 | 4.2425 | 1245. | .042335 | 29 | 132.00 | 697.01 | 7.58 | 1142.9 |
| 8 | 1.0131 | 5.3498 | 987.0 | .057318 | 30 | 166.43 | 878.80 | 6.01 | 1817.2 |
| 9 | 1.2773 | 6.7442 | 783.0 | .071010 | 31 | 209.85 | 1108.0 | 4.77 | 2888.0 |
| 10 | 1.6111 | 8.5065 | 620.8 | .07028 | 32 | 264.68 | 1397.6 | 3.78 | 4595.5 |
| 11 | 2.0312 | 10.723 | 492.4 | .07061 | 33 | 333.68 | 1760.2 | 3.00 | 7302.0 |
| 12 | 2.5615 | 13.525 | 390.5 | .043040 | 34 | 420.87 | 2222.2 | 2.38 | 11627. |
| 13 | 3.2300 | 17.055 | 309.6 | .068437 | 35 | 530.60 | 2801.8 | 1.88 | 18440. |
| 14 | 4.0724 | 21.502 | 245.6 | 1.0877 | 36 | 669.00 | 3532.5 | 1.50 | 29352. |
| 15 | 5.1354 | 27.114 | 194.8 | 1.7308 | 37 | 843.46 | 4453.0 | 1.19 | 46600. |
| 16 | 6.4755 | 34.190 | 154.4 | 2.7505 | 38 | 1064.0 | 5618.0 | 0.95 | 74240. |
| 17 | 8.1670 | 43.124 | 122.5 | 4.3746 | 39 | 1341.2 | 7082.0 | 0.75 | 118070. |
| 18 | 10.300 | 54.388 | 97.10 | 6.9590 | 40 | 1691.1 | 8930.0 | 0.59 | 187700. |

* Calculated on the basis of Dr. Matthiessen's standard, viz.: The resistance of a pure soft copper wire 1 meter long, having a weight of 1 gram = 0.141729 International Ohm at 0° C.

(From Aluminum for Electrical Conductors; Pittsburgh Reduction Co.)

electrical pressure or "drop" in the line, is governed by the size of the wires, the other conditions remaining the same. For a long-distance transmission plant this will vary from 5% upwards.

With generator efficiency and motor efficiency each 90%, and transmission loss 5%, the combined efficiency is $0.90 \times 0.90 \times 0.95 = 76.95\%$.

The methods for long-distance transmission may be divided into three general classes: (1) continuous current; (2) alternating current; and (3) rotary-converter or "motor-dynamo" systems. There are many factors which govern the selection of a system. For each problem considered there will be found certain fixed and certain unfixed conditions. In general the fixed factors are: (1) capacity of source of power; (2) cost of power at source; (3) cost of power by other means at point of delivery; (4) danger considerations at motors; (5) operating conditions; (6) construction conditions (length of line, character of country, etc.). The partly fixed conditions are: (7) power which must be delivered, i.e., the efficiency of the system; (8) size and number of delivery units. The variable conditions are: (9) initial voltage; (10) pounds of copper on line; (11) original cost of all apparatus and construction; (12) expenses, operating (fixed charges, interest, depreciation, taxes, insurance, etc.); (13) liability of trouble and stoppages; (14) danger at station and on line; (15) convenience in operating, making changes, extensions, etc.

Systems of Electrical Distribution in Common Use.

1. DIRECT CURRENT.

A. *Constant Potential.*

110 to 125 and 220 to 250 Volts.—Distances less than, say, 1500 feet.

For incandescent lamps.

For arc-lamps, usually 2 in series.

For motors of moderate sizes.

200 to 250 and 440 Volts, 3-wire.—Distances less than, say, 5000 feet.

For incandescent lamps.

For arc-lamps, usually 2 in series on each branch.

For motors 110 or 220 volts, usually 220 volts.

500 Volts.—Distances less than, say, 20,000 feet.

Incidentally for arc-lamps, usually 10 in series.

For motors, stationary and street-car.

B. *Constant Current.*

Usually 5, 6½, or 9½ amperes, the volts increasing to several thousand, as demanded, for series arc-lamps.

II. ALTERNATING CURRENT.

A. *Constant Potential.*

For incandescent lamps, arc-lamps, and motors.

Polyphase Systems.

For arc and incandescent lamps, motors, and rotary converters for giving direct current.

Polyphase—2- and 3-phase—high tension (25,000 volts and over), for long-distance transmission; transformed by step-up and step-down transformers.

B. *Constant Current.*

Usually 5 to 6.6 amperes. For arc-lamps.

The Relative Advantages of Different Systems vary with each particular transmission problem, but in a general way may be tabulated as below:

| | System. | Advantages. | Disadvantages. |
|-------------|-----------------------|--|--|
| Continuous. | 2-wire { Low voltage. | Safety, simplicity. | Expense for copper. |
| | { High voltage. | Economy, simplicity. | Danger; difficulty of building machines. |
| | 3-wire. | Low voltage on machines and saving in copper. | Not saving enough in copper for long distances. Necessity for "balanced" system. |
| Alternating | Multiple-wire. | Low voltage at machines and saving in copper. | |
| | Single phase. | Economy of copper. | Cannot start under load. Low efficiency. |
| | Multiphase. | Economy of copper, synchronous speed unnecessary; applicable to very long distances. | Requires more than two wires. |
| | Motor-dynamo. | High-voltage A.C. transmission. Low-voltage D.C. delivery. | Expensive. Low efficiency. |

TABLE OF ELECTRICAL HORSE-POWERS.

Formula: $\frac{\text{Volts} \times \text{Amperes}}{746} = \text{H.P.}, \text{ or } 1 \text{ volt ampere} = .0013405 \text{ H.P.}$

Read amperes at top and volts at side or *vice versa*.

| Amperes or Volts. | Volts or Amperes. | | | | | | | | | | | | | |
|----------------------|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| | 1 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | |
| 1 | .00134 | .0134 | .0268 | .0402 | .0536 | .0670 | .0804 | .0938 | .1072 | .1206 | .1341 | .1475 | .1609 | |
| 2 | .00268 | .0268 | .0536 | .0804 | .1072 | .1341 | .1609 | .1877 | .2145 | .2413 | .2681 | .2949 | .3217 | |
| 3 | .00402 | .0402 | .0804 | .1206 | .1609 | .2011 | .2413 | .2815 | .3217 | .3619 | .4022 | .4424 | .4826 | |
| 4 | .00536 | .0536 | .1072 | .1609 | .2145 | .2681 | .3217 | .3753 | .4290 | .4826 | .5362 | .5898 | .6434 | |
| 5 | .00670 | .0670 | .1341 | .2011 | .2681 | .3351 | .4022 | .4692 | .5362 | .6032 | .6703 | .7373 | .8043 | |
| 6 | .00804 | .0804 | .1609 | .2413 | .3217 | .4022 | .4826 | .5630 | .6434 | .7239 | .8043 | .8847 | .9652 | |
| 7 | .00938 | .0938 | .1877 | .2815 | .3753 | .4692 | .5630 | .6568 | .7507 | .8445 | .9384 | 1.032 | 1.126 | |
| 8 | .01072 | .1072 | .2145 | .3217 | .4290 | .5362 | .6434 | .7507 | .8579 | .9652 | 1.072 | 1.180 | 1.287 | |
| 9 | .01206 | .1206 | .2413 | .3619 | .4826 | .6032 | .7239 | .8445 | .9652 | 1.086 | 1.206 | 1.327 | 1.448 | |
| 10 | .01341 | .1341 | .2681 | .4022 | .5362 | .6703 | .8043 | .9383 | 1.072 | 1.206 | 1.341 | 1.475 | 1.609 | |
| 11 | .01475 | .1475 | .2949 | .4424 | .5898 | .7373 | .8847 | 1.032 | 1.180 | 1.327 | 1.475 | 1.622 | 1.769 | |
| 12 | .01609 | .1609 | .3217 | .4826 | .6434 | .8043 | .9652 | 1.126 | 1.287 | 1.448 | 1.609 | 1.769 | 1.930 | |
| 13 | .01743 | .1743 | .3485 | .5228 | .6970 | .8713 | 1.046 | 1.220 | 1.394 | 1.568 | 1.743 | 1.917 | 2.091 | |
| 14 | .01877 | .1877 | .3753 | .5630 | .7507 | .9384 | 1.126 | 1.314 | 1.501 | 1.689 | 1.877 | 2.064 | 2.252 | |
| 15 | .02011 | .2011 | .4022 | .6032 | .8043 | 1.005 | 1.206 | 1.408 | 1.609 | 1.810 | 2.011 | 2.212 | 2.413 | |
| 16 | .02145 | .2145 | .4290 | .6434 | .8579 | 1.072 | 1.287 | 1.501 | 1.716 | 1.930 | 2.145 | 2.359 | 2.574 | |
| 17 | .02279 | .2279 | .4558 | .6837 | .9115 | 1.139 | 1.367 | 1.595 | 1.823 | 2.051 | 2.279 | 2.507 | 2.735 | |
| 18 | .02413 | .2413 | .4826 | .7239 | .9652 | 1.206 | 1.448 | 1.689 | 1.930 | 2.172 | 2.413 | 2.654 | 2.895 | |
| 19 | .02547 | .2547 | .5094 | .7641 | 1.019 | 1.273 | 1.528 | 1.783 | 2.037 | 2.292 | 2.547 | 2.801 | 3.056 | |
| 20 | .02681 | .2681 | .5362 | .8043 | 1.072 | 1.340 | 1.609 | 1.877 | 2.145 | 2.413 | 2.681 | 2.949 | 3.217 | |
| 21 | .02815 | .2815 | .5630 | .8445 | 1.126 | 1.408 | 1.689 | 1.971 | 2.252 | 2.533 | 2.815 | 3.097 | 3.378 | |
| 22 | .02949 | .2949 | .5898 | .8847 | 1.180 | 1.475 | 1.769 | 2.064 | 2.359 | 2.654 | 2.949 | 3.244 | 3.539 | |
| 23 | .03083 | .3083 | .6166 | .9249 | 1.233 | 1.542 | 1.850 | 2.158 | 2.467 | 2.775 | 3.083 | 3.391 | 3.700 | |
| 24 | .03217 | .3217 | .6434 | .9652 | 1.287 | 1.609 | 1.930 | 2.252 | 2.574 | 2.895 | 3.217 | 3.539 | 3.861 | |
| 25 | .03351 | .3351 | .6703 | 1.005 | 1.341 | 1.676 | 2.011 | 2.346 | 2.681 | 3.016 | 3.351 | 3.686 | 4.022 | |
| 26 | .03485 | .3485 | .6971 | 1.046 | 1.394 | 1.743 | 2.091 | 2.440 | 2.788 | 3.137 | 3.485 | 3.834 | 4.182 | |
| 27 | .03619 | .3619 | .7239 | 1.086 | 1.448 | 1.810 | 2.172 | 2.534 | 2.895 | 3.257 | 3.619 | 3.981 | 4.343 | |
| 28 | .03753 | .3753 | .7507 | 1.126 | 1.501 | 1.877 | 2.252 | 2.627 | 3.003 | 3.378 | 3.753 | 4.129 | 4.504 | |
| 29 | .03887 | .3887 | .7775 | 1.166 | 1.555 | 1.944 | 2.332 | 2.721 | 3.110 | 3.499 | 3.887 | 4.276 | 4.665 | |
| 30 | .04022 | .4022 | .8043 | 1.206 | 1.609 | 2.011 | 2.413 | 2.815 | 3.217 | 3.619 | 4.022 | 4.424 | 4.826 | |
| 31 | .04156 | .4156 | .8311 | 1.247 | 1.662 | 2.078 | 2.493 | 2.909 | 3.324 | 3.740 | 4.156 | 4.571 | 4.987 | |
| 32 | .04290 | .4290 | .8579 | 1.287 | 1.716 | 2.145 | 2.574 | 3.003 | 3.432 | 3.861 | 4.290 | 4.719 | 5.148 | |
| 33 | .04424 | .4424 | .8847 | 1.327 | 1.769 | 2.212 | 2.654 | 3.097 | 3.539 | 3.986 | 4.424 | 4.866 | 5.308 | |
| 34 | .04558 | .4558 | .9115 | 1.367 | 1.823 | 2.279 | 2.735 | 3.190 | 3.646 | 4.102 | 4.558 | 5.013 | 5.469 | |
| 35 | .04692 | .4692 | .9384 | 1.408 | 1.877 | 2.346 | 2.815 | 3.284 | 3.753 | 4.223 | 4.692 | 5.161 | 5.630 | |
| 40 | .05362 | .5362 | 1.072 | 1.609 | 2.145 | 2.681 | 3.217 | 3.753 | 4.290 | 4.826 | 5.363 | 5.898 | 6.434 | |
| 45 | .06032 | .6032 | 1.206 | 1.810 | 2.413 | 3.016 | 3.619 | 4.223 | 4.826 | 5.439 | 6.032 | 6.635 | 7.239 | |
| 50 | .06703 | .6703 | 1.341 | 2.011 | 2.681 | 3.351 | 4.022 | 4.692 | 5.362 | 6.032 | 6.703 | 7.373 | 8.043 | |
| 55 | .07373 | .7373 | 1.475 | 2.212 | 2.949 | 3.686 | 4.424 | 5.161 | 5.898 | 6.635 | 7.373 | 8.110 | 8.847 | |
| 60 | .08043 | .8043 | 1.609 | 2.413 | 3.217 | 4.022 | 4.826 | 5.630 | 6.434 | 7.239 | 8.043 | 8.847 | 9.652 | |
| 65 | .08713 | .8713 | 1.743 | 2.614 | 3.485 | 4.357 | 5.228 | 6.099 | 6.970 | 7.842 | 8.713 | 9.584 | 10.46 | |
| 70 | .09384 | .9384 | 1.877 | 2.815 | 3.753 | 4.692 | 5.630 | 6.568 | 7.507 | 8.445 | 9.384 | 10.32 | 11.26 | |
| 75 | .10054 | 1.005 | 2.011 | 3.016 | 4.021 | 5.027 | 6.032 | 7.037 | 8.043 | 9.048 | 10.05 | 11.06 | 12.06 | |
| 80 | .10724 | 1.072 | 2.145 | 3.217 | 4.290 | 5.362 | 6.434 | 7.507 | 8.579 | 9.652 | 10.72 | 11.80 | 12.87 | |
| 85 | .11394 | 1.139 | 2.279 | 3.418 | 4.558 | 5.697 | 6.836 | 7.976 | 9.115 | 10.26 | 11.39 | 12.53 | 13.67 | |
| 90 | .12065 | 1.206 | 2.413 | 3.619 | 4.826 | 6.032 | 7.239 | 8.445 | 9.652 | 10.86 | 12.06 | 13.27 | 14.48 | |
| 95 | .12735 | 1.273 | 2.547 | 3.820 | 5.094 | 6.367 | 7.641 | 8.914 | 10.18 | 11.46 | 12.73 | 14.01 | 15.28 | |
| 100 | .13405 | 1.341 | 2.681 | 4.022 | 5.362 | 6.703 | 8.043 | 9.384 | 10.72 | 12.06 | 13.41 | 14.75 | 16.09 | |
| 200 | .26810 | 2.681 | 5.362 | 8.043 | 10.72 | 13.41 | 16.09 | 18.77 | 21.45 | 24.13 | 26.81 | 29.49 | 32.17 | |
| 300 | .40215 | 4.022 | 8.043 | 12.06 | 16.09 | 20.11 | 24.13 | 28.15 | 32.17 | 36.19 | 40.22 | 44.24 | 48.26 | |
| 400 | .53620 | 5.362 | 10.72 | 16.09 | 21.45 | 26.81 | 32.17 | 37.53 | 42.90 | 48.26 | 53.62 | 58.98 | 64.34 | |
| 500 | .67025 | 6.703 | 13.41 | 20.11 | 26.81 | 33.51 | 40.22 | 46.92 | 53.62 | 60.32 | 67.03 | 73.73 | 80.43 | |
| 600 | .80430 | 8.043 | 16.09 | 24.13 | 32.17 | 40.22 | 48.26 | 56.30 | 64.34 | 72.39 | 80.43 | 88.47 | 96.52 | |
| 700 | .93835 | 9.384 | 18.77 | 28.15 | 37.53 | 46.92 | 56.30 | 65.68 | 75.07 | 84.45 | 93.84 | 103.2 | 112.6 | |
| 800 | 1.07240 | 10.72 | 21.45 | 32.17 | 42.90 | 53.62 | 64.34 | 75.07 | 85.79 | 96.52 | 107.2 | 118.0 | 128.7 | |
| 900 | 1.20650 | 12.06 | 24.13 | 36.19 | 48.26 | 60.32 | 72.39 | 84.45 | 96.52 | 108.6 | 120.6 | 132.7 | 144.8 | |
| 1,000 | 1.34050 | 13.41 | 26.81 | 40.22 | 53.62 | 67.03 | 80.43 | 93.84 | 107.2 | 120.6 | 134.1 | 147.5 | 160.9 | |
| 2,000 | 2.68100 | 26.81 | 53.62 | 80.43 | 107.2 | 134.1 | 160.9 | 187.7 | 214.5 | 241.3 | 268.1 | 294.9 | 321.7 | |
| 3,000 | 4.02150 | 40.22 | 80.43 | 120.6 | 160.9 | 201.1 | 241.3 | 281.5 | 321.7 | 361.9 | 402.2 | 442.4 | 482.6 | |
| 4,000 | 5.36200 | 53.62 | 107.2 | 160.9 | 214.5 | 268.1 | 321.7 | 375.3 | 429.0 | 482.6 | 536.2 | 589.8 | 643.4 | |
| 5,000 | 6.70250 | 67.03 | 134.1 | 201.1 | 268.1 | 335.1 | 402.2 | 469.2 | 536.2 | 603.2 | 670.3 | 737.3 | 804.3 | |
| 6,000 | 8.04300 | 80.43 | 160.9 | 241.3 | 321.7 | 402.2 | 482.6 | 563.0 | 643.4 | 723.9 | 804.3 | 884.7 | 965.2 | |
| 7,000 | 9.38350 | 93.84 | 187.7 | 281.5 | 375.3 | 469.2 | 563.0 | 656.8 | 750.7 | 844.5 | 938.4 | 1032 | 1126 | |
| 8,000 | 10.72400 | 107.2 | 214.5 | 321.7 | 429.0 | 536.2 | 643.4 | 750.7 | 857.9 | 965.2 | 1072 | 1180 | 1287 | |
| 9,000 | 12.06500 | 120.6 | 241.3 | 361.9 | 482.6 | 603.2 | 723.9 | 844.5 | 965.2 | 1086 | 1206 | 1327 | 1448 | |
| 10,000 | 13.40500 | 134.1 | 268.1 | 402.2 | 536.2 | 670.3 | 804.3 | 938.3 | 1072 | 1206 | 1341 | 1475 | 1609 | |

Cost of Copper for Long-distance Transmission.

(Westinghouse El. and Mfg. Co.)

COST OF COPPER REQUIRED FOR THE DELIVERY OF ONE MECHANICAL HORSE-POWER AT MOTOR SHAFT WITH VARIOUS VOLTAGES AT MOTOR TERMINALS, OR AT TERMINALS OF LOWERING TRANSFORMERS.

Loss of energy in conductors (drop) equals 20%. Motor efficiency, 90%. Length of conductor per mile of single distance, 11,000 ft., to allow for sag. Cost of copper taken at 16 cents per pound.

| Miles. | 1000 v. | 2000 v. | 3000 v. | 4000 v. | 5000 v. | 10,000 v. |
|--------|---------|---------|---------|---------|---------|-----------|
| 1 | \$2.08 | \$0.52 | \$ 0.23 | \$0.13 | \$0.08 | \$0.02 |
| 2 | 8.33 | 2.08 | 0.93 | 0.52 | 0.33 | 0.08 |
| 3 | 18.70 | 4.68 | 2.08 | 1.17 | 0.75 | 0.19 |
| 4 | 33.30 | 8.32 | 3.70 | 2.08 | 1.33 | 0.33 |
| 5 | 52.05 | 13.00 | 5.78 | 3.25 | 2.08 | 0.52 |
| 6 | 74.90 | 18.70 | 8.32 | 4.68 | 3.00 | 0.75 |
| 7 | 102.00 | 25.50 | 11.30 | 6.37 | 4.08 | 1.02 |
| 8 | 133.25 | 33.30 | 14.80 | 8.32 | 5.33 | 1.33 |
| 9 | 168.60 | 42.20 | 18.75 | 10.50 | 6.74 | 1.69 |
| 10 | 208.19 | 52.05 | 23.14 | 13.01 | 8.33 | 2.08 |
| 11 | 251.90 | 63.00 | 28.00 | 15.75 | 10.08 | 2.52 |
| 12 | 299.80 | 75.00 | 33.30 | 18.70 | 12.00 | 3.00 |
| 13 | 352.00 | 88.00 | 39.00 | 22.00 | 14.08 | 3.52 |
| 14 | 408.00 | 102.00 | 45.30 | 25.50 | 16.32 | 4.08 |
| 15 | 468.00 | 117.00 | 52.00 | 29.25 | 18.72 | 4.68 |
| 16 | 533.00 | 133.00 | 59.00 | 33.30 | 21.32 | 5.33 |
| 17 | 600.00 | 150.00 | 67.00 | 37.60 | 24.00 | 6.00 |
| 18 | 675.00 | 169.00 | 75.00 | 42.20 | 27.00 | 6.75 |
| 19 | 750.00 | 188.00 | 83.50 | 47.00 | 30.00 | 7.50 |
| 20 | 833.00 | 208.00 | 92.60 | 52.00 | 33.32 | 8.33 |

COST OF COPPER REQUIRED TO DELIVER ONE MECHANICAL HORSE-POWER AT MOTOR-SHAFT WITH VARYING PERCENTAGES OF LOSS IN CONDUCTORS, UPON THE ASSUMPTION THAT THE POTENTIAL AT MOTOR TERMINALS IS IN EACH CASE 3000 VOLTS.

Motor efficiency, 90%. Cost of copper equals 16 cents per pound. Length of conductor per mile of single distance, 11,000 ft.

| Miles. | 10% | 15% | 20% | 25% | 30% |
|--------|--------|--------|--------|--------|--------|
| 1 | \$0.52 | \$0.33 | \$0.23 | \$0.17 | \$0.13 |
| 2 | 2.08 | 1.31 | 0.93 | 0.69 | 0.54 |
| 3 | 4.68 | 2.95 | 2.08 | 1.55 | 1.21 |
| 4 | 8.32 | 5.25 | 3.70 | 2.77 | 2.15 |
| 5 | 13.00 | 8.20 | 5.78 | 4.33 | 3.37 |
| 6 | 18.70 | 11.75 | 8.32 | 6.23 | 4.85 |
| 7 | 25.50 | 16.00 | 11.30 | 8.45 | 6.60 |
| 8 | 33.30 | 21.00 | 14.80 | 11.00 | 8.60 |
| 9 | 42.20 | 26.60 | 18.75 | 14.00 | 10.90 |
| 10 | 52.05 | 32.78 | 23.14 | 17.31 | 13.50 |
| 11 | 63.00 | 39.75 | 28.00 | 21.00 | 16.30 |
| 12 | 75.00 | 47.20 | 33.30 | 24.90 | 19.40 |
| 13 | 88.00 | 55.30 | 39.00 | 29.20 | 22.80 |
| 14 | 102.00 | 64.20 | 45.30 | 33.90 | 26.40 |
| 15 | 117.00 | 73.75 | 52.00 | 38.90 | 30.30 |
| 16 | 133.00 | 83.80 | 59.00 | 44.30 | 34.50 |
| 17 | 150.00 | 94.75 | 67.00 | 50.00 | 39.00 |
| 18 | 169.00 | 106.00 | 75.00 | 56.20 | 43.80 |
| 19 | 188.00 | 118.00 | 83.50 | 62.50 | 48.70 |
| 20 | 208.00 | 131.00 | 92.60 | 69.25 | 54.00 |

ELECTRIC RAILWAYS.

Space will not admit of a proper treatment of this subject in this work. Consult Crosby and Bell, *The Electric Railway in Theory and Practice*; Fairchild, *Street Railways*; Merrill, *Reference Book of Tables and Formulæ for Street Railway Engineers*; Bell, *Electric Transmission of Power*; Dawson, *Engineering and Electric Traction Pocket-book*; *The Standard Handbook for Electrical Engineers*; and Foster's *Electrical Engineers' Pocket-book*. The last named devotes 240 pages to the subject of electric railways.

Electric Railway Cars and Motors. (Foster.) — Small cars weighing 10 to 12 tons may be fitted with two 35-H.P. motors and be geared for a maximum speed of 25 to 30 miles per hour. Larger cars of the single-truck variety weighing close to 15 tons may be equipped with 40-H.P. motors. Suburban cars weighing 18 to 25 tons and measuring 45 ft. over all may be equipped with four 50-H.P. motors and be geared for a maximum speed of 40 m.p.h. Larger types of suburban cars, 50 ft. over all, seating 52 passengers, weigh 28 to 30 tons and are equipped with four 75-H.P. motors geared for a maximum speed of 45 m.p.h. The largest type of suburban car is equipped with four 125-H.P. motors, and is geared for a maximum speed of 60 m.p.h.

Grades upon city lines may run as high as 13 per cent, and to surmount these it is necessary to have every axle on the car equipped with motors; thus single-truck cars require two, and double-truck cars four motors; and even then the cars will be unable to surmount these grades with very bad conditions of track. The motor capacity per car should be liberal, not so much from the danger of overheating the motors as to prevent undue sparking when surmounting the heavy grades.

A 4000-H.P. Electric Locomotive, built by the Westinghouse El. & Mfg. Co., for the New York terminal and tunnel of the Penna. R.R., is described in *Eng'g News*, Nov. 11, 1909.

In wheel arrangement, weight distribution, and general character of the running gear it is the practical equivalent of two American-type steam locomotives coupled back to back. The motors are mounted upon the frame and are side-connected through jack shafts to driving wheels by a system of cranks and parallel connecting rods. The connecting rods are all rotating links between rotating elements, and thus can be perfectly counterbalanced for all speeds. The center of gravity is approximately 72 ins. above the rails.

In these electric locomotives the variable pressure of the unbalanced piston of the steam locomotive is replaced by the more constant torque and more constant rotating effort of the drive wheels, so that the pull upon the drawbar is thereby constant and uniform. The engine will start a train of 550 tons trailing load upon grades of approximately 2%. A tractive effort of 60,000 lbs., and a normal speed of 60 miles per hour, with full train load on a level track, are guaranteed.

The total weight of the locomotive is 332,100 lbs., of which 208,000 lbs. is on the eight drivers. The locomotive is claimed to develop 4000 H.P. for short times, say 20 minutes, without abnormal temperature rise. Each half of the locomotive carries a single motor, four 68-in. drive wheels and one four-wheel, swing-bolster, swivel truck, with 36-in. wheels. Each section has its own steel cab, the two cabs being connected by a vestibule.

The rigid wheel-bases are 7 ft. 2 in. and the total wheel-base of each half is 23 ft. The motive power consists of two motors of a 600-volt, 2000-H.P., commutating-pole type. Each motor weighs complete without its crank, 42,000 lbs. The main-field winding is in two sections, both of which are used together at low-speed operation. At high speeds only one-half is needed, and at intermediate control points one is shunted with resistance. These field positions are available for all series and parallel groupings of the motors, so that eight running positions (or speeds) are possible. Bridging connections are used in passing from series to parallel groupings of the motors, so that the main circuits are not opened in the process.

ELECTRIC LIGHTING. — ILLUMINATION.

Illumination. — Some writers distinguish "lighting" and "illumination." Lighting refers to the character of the lights themselves, as dazzling, brilliant, or soft and pleasing, and illumination to the quantity of light reflected from objects, by which they are rendered visible. If the objects in a room are clearly seen, then the room is well illuminated.

The quantity of light is estimated in candle-power per square foot of area or per cubic foot of space. The amount of illumination given by one candle at a distance of 1 ft. is known as a candle-foot. Since the illumination varies inversely as the square of the distance, one candle-foot is given by a 16-candle-power lamp at a distance of 4 ft., or by a 25-c.-p. lamp at a distance of 5 ft.

Terms, Units, Definitions. — Quantity of light proceeding from a source of light, measured in units of luminous flux, or lumens.

Intensity with which the flux is emitted from a radiant in a single direction, called candle-power.

Illumination, density of the light flux incident upon an area.

Luminosity, brightness of surface; flux emitted per unit area of surface.

Candle-power, the unit of luminous intensity. A spermaceti candle burning at the rate of 120 grains per hour is the old standard used in the gas industry. It is very unsatisfactory as a standard and is being displaced by others.

The hefner lamp, burning amyl acetate, is the legal standard in Germany. The unit of luminous intensity produced by this lamp when constructed and operated as prescribed is called a *hefner*. The standard laboratories of Great Britain, France and America have agreed upon the following relative values of the units used in the several countries: 1 International Candle = 1 Pentane Candle = 1 Bougie Decimale = 1 American Candle = 1.11 Hefners = 0.104 Carcel unit. 1 Hefner = 0.90 International Candle.

Intrinsic Brilliancy of a source of light = candle-power per square inch of surface exposed in a given direction.

Lumen, the unit of luminous flux, is the quantity of light included in a unit solid angle and radiated from a source of unit intensity. A unit solid angle is the angular space subtended at the surface of a sphere by an area equal to the square of the radius, or by $1 \div 4\pi$, or $1/12.5664$ of the surface of the sphere. The light of a source whose average intensity in all directions is 1 candle-power, or one mean spherical candle-power, has a total flux of 12.5664 lumens.

Foot-candle, the unit of illumination, = 1 lumen per square foot; the illumination received by a surface every point of which is distant one foot from a source of one candle-power.

Lux, or meter-candle, 1 lumen per square meter; 1 foot-candle = 10.76 meter-candles.

Law of Inverse Squares. — The illumination of any surface is inversely proportional to the square of its distance from the source of light. This is strictly true when the source of light is a point, and is very nearly true in all cases when the distance is more than ten times the greatest dimension of the light-giving surface.

Law of Cosines. When a surface is illuminated by a beam of light striking it at an angle other than a right angle, the illumination is proportional to the cosine of the angle the beam makes with a normal to the surface.

If E = the illumination at any point in a surface, I the intensity of light coming from a source, θ the angle of deviation of the direction of the beam from a normal to the surface, and l the distance from the source, then $E = I \cos \theta \div l^2$.

Relative Color Values of Various Illuminants. — The light proceeding from any source may be analyzed in terms of the elementary color elements, red, green and blue, by means of the spectroscope or by a colorimeter. The following relative values have been obtained by the Ives colorimeter (*Trans. Ill., Eng. Soc.* iii, 631). In all cases the red rays in the light are taken as 100, and the two figures given are respectively the proportions of green and blue relative to 100 red.

Average daylight, 100, 100. Blue sky, 106, 120. Overcast sky, 92, 85. Afternoon sunlight, 91, 56. Direct-current carbon arc, 64, 39. Mercury

arc (red 100), 130, 190. Moore carbon dioxide tube, 120, 520. Welsbach mantle, $\frac{3}{4}\%$ cerium, 81, 28. Do., $1\frac{1}{4}\%$ cerium, 69, 14.5. Do., $1\frac{3}{4}\%$ cerium, 63, 12.3. Tungsten lamp, 1.25 watts per mean horizontal candle-power, 55, 12.1. Nernst glower, bare, 51.5, 11.3. Tantalum lamp, 2 watts per m. h. c.-p., 49, 8.3. Gem lamp, 2.5 watts per m. h. c.-p., 48, 8.3. Carbon incandescent lamp, 3.1 watts per m. h. c.-p., 45, 7.4. Flaming arc, 36.5, 9. Gas flame, open fish-tail burner, 40, 5.8. Moore nitrogen tube, 28, 6.6. Hefner lamp, 35, 3.8.

Relation of Illumination to Vision. — Wickenden gives the following summary of the principles of effective vision:

1. The eye works with approximately normal efficiency upon surfaces possessing an effective luminosity of one lumen per square foot or more.

2. Excessive illumination and inadequate illumination strain and fatigue the eye in an effort to secure sharp perception.

3. Intrinsic brilliancy of more than 5 c.-p. per sq. in. should be reduced by a diffusing medium when the rays enter the eye at an angle below 60° with the horizontal.

4. Flickering, unsteady, and streaky illumination strains the retina in the effort to maintain uniform vision.

5. True color values are obtained only from light possessing all the elements of diffused daylight in approximately equivalent proportions.

6. An excess of ultra-violet rays is to be avoided for hygienic reasons.

7. Aesthetic considerations commend light of a faint reddish tint as warm and cheerful in comparison with the cold effects of the green tints, although the latter are more effective in revealing fine detail.

Arc Lamps are divided into three classes: 1. Those which produce light by the incandescence of intensely hot refractory electrodes. 2. Those which produce light mainly from the luminescence in the arc of mineral salts vaporized from carbon electrodes. 3. Those which produce light by the luminescence of metallic vapor derived solely from the cathode, the anode being unconsumed.

The Carbon Arc.—In direct-current open arcs the anodes are consumed at the rate of 1 to 2 inches per hour, and the cathodes, or negatives, at half this rate. In alternating-current open arcs the consumption is equal in both carbons, 1 to 1.5 inches per hour. Enclosed arcs have longer life owing to the restricted oxidation of the carbons, but they are of reduced brilliancy and lower efficiency. Carbons of the ordinary sizes burn $\frac{1}{16}$ to $\frac{1}{8}$ in. per hour, giving a life of 100 to 150 hours for direct-current and 80 to 100 hours for alternating-current lamps. The enclosing globes absorb from 8 to 40% of the light.

The Flaming Arc.—The carbons are impregnated with calcium fluoride or other luminescent salts. The current is usually 8 to 12 amperes and the voltage per lamp 35 to 60. The regenerative flame arc is a highly efficient variety of the flame arc.

The Magnetite Arc has for a cathode a thin iron tube packed with a mixture of magnetite, Fe_3O_4 , and titanium and chromium oxides. The anode consists of copper or brass. It is well adapted to series operation with low currents. The 4-ampere lamp, using 80 volts per lamp, is highly successful for street illumination.

Illumination by Arc Lamps at Different Distances. — Several diagrams and curves showing the light distribution in a vertical plane and the illumination at different distances of different types of lamps are given by Wickenden. From the latter are taken the approximate figures in the table below. The carbon and the magnetite lamps were 25 ft. high, the flame arcs 21 ft.

| Horizontal Distance from Lamp, Feet. | | 20 | 30 | 40 | 50 | 100 | 150 | 200 | 250 |
|--------------------------------------|-----------|------------------------------------|------|------|------|------|------|------|------|
| Kind of Lamp. | | Foot-candles, normal illumination. | | | | | | | |
| A. Open carbon arc, D.C., | 6.6 amps. | ... | 0.40 | 0.29 | 0.20 | .032 | .014 | .006 | .002 |
| B. Enclosed carbon arc, A.C. | 6.6 " | 0.30 | .19 | .135 | .10 | .027 | .013 | .006 | .002 |
| C. Flame arc, | 10 " | ... | ... | ... | 1.10 | .31 | .14 | .08 | .05 |
| D. Regenerative arc, | 7 " | ... | ... | .85 | .65 | .15 | .055 | .03 | .02 |
| E. Magnetite arc, | 6.6 " | ... | 1.00 | .69 | .51 | .15 | .075 | .045 | .025 |
| F. Magnetite arc, | 4. " | .47 | .40 | .30 | .21 | .07 | .035 | .022 | .018 |

A. 6.6 amp., D. C., open arc, clear globe.

B. 6.6 amp., A. C., enclosed arc, opal inner and clear outer globe, small reflector.

C. 10 amp., flame arc, vertical electrodes; 50 volts, 1520 M.H.C.-P.;* 0.33 watt per M.L.H.C.-P.;* 10 hours per trim.

D. 7 amp., regenerative flame arc, 70 volts, 2440 M.L.H.C.-P., 0.2 watt per M.L.H.C.-P., 70 hours per trim.

E. 6.6 amp., D.C. series magnetite arc, 79 volts, 510 watts, 1450 M.L.H.C.-P. 75 to 100 hours per trim.

F. 4 amp., D.C. series magnetite arc, 80 volts, 320 watts, 575 M.L.H.C.-P., 150 to 200 hours per trim.

Data of Some Arc Lamps.

| Type of Lamp. | Hours per Trim. | Amperes. | Terminal Volts. | Terminal Watts. | Watts per m.l.h. c.-p. |
|-------------------------------------|-----------------|----------|-----------------|-----------------|------------------------|
| D.C. series carbon, open..... | 9 to 12 | 9.6 | 50 | 480 | 0.6 |
| D.C. series carbon, enclosed. | 100 to 150 | 6.6 | 72 | 475 | 0.9 |
| A.C. series carbon, enclosed. | 70 to 100 | 7.5 | 75 | 480 | 1.25 |
| D.C. multiple carbon, enclosed..... | 100 to 150 | 5.0 | 110 | 550 | 2.25 |
| A.C. multiple carbon, enclosed..... | 70 to 100 | 6.0 | 110 | 430 | 2.40 |
| D.C. flame arcs, open..... | 10 to 16 | 10 | 55 | 440 | 0.45 |
| Regenerative, semi-enclosed | 70 | 5 | 70 | 350 | 0.26 |
| A.C. flame arcs, open..... | 10 to 16 | 10 | 55 | 467 | 0.55 |
| Magnetite, open..... | 70 to 100 | 6.6 | 80 | 528 | 0.45 |

Values of watts per m.l.h. c.-p. approximate for open carbon arcs and magnetite arcs with clear globes, enclosed arcs with opalescent inner and clear outer globes, and for flame and regenerative arcs with opal globes.

Watts per Square Foot Required for Arc Lighting. — W. D'A. Ryan (*Am. Elect'n*, Feb., 1903) gives the following table, deduced from experience, showing the amount of energy required for good illumination by means of enclosed arcs, based on watts at lamp terminals.

| Building. | Watts per sq. ft. |
|---|-------------------|
| Machine-shops; high roofs, electrically driven machinery, no belts..... | 0.5 to 1 |
| Machine-shops; low roofs, belts and other obstructions..... | 0.75 to 1.25 |
| Hardware and shoe stores..... | 0.5 to 1 |
| Department stores; light material, bric-a-brac, etc..... | 0.75 to 1.25 |
| Department stores; colored material..... | 1 to 1.5 |
| Mill lighting; plain white goods..... | 0.9 to 1.3 |
| Mill lighting; colored goods, high looms..... | 1.1 to 1.5 |
| General office; no incandescents..... | 1.25 to 1.75 |
| Drafting rooms..... | 1.5 to 2 |

The space in sq. yds. properly illuminated by 450-watt enclosed arc lamps is given as follows in the *Int. Library of Technology*, vol. 13: Outdoor areas, 2000-2500 sq. yds.; trainsheds, 1400-1600; foundries (general illumination), 600-800; machine-shops, 200-250; thread and cloth mills, 200-230.

The Mercury Vapor Lamp, invented by Peter Cooper Hewitt, is an arc of luminous mercury vapor contained in a glass tube from which the air has been exhausted. A small quantity of mercury is contained in the tube, and platinum wires are inserted in each end. When the tube is placed in a horizontal position, so that a thin thread of mercury lies along it, making electric connection with the wires, and a current is passed through it, part of the mercury is vaporized, and on the tube being inclined so that the liquid mercury remains at one end, an electric arc is

* M.H.C.-P. = mean horizontal candle-power; M.L.H.C.-P. = mean lower hemispherical candle-power.

formed in the vapor throughout the tube. The tubes are made about 1 in. in diameter and of different lengths, as below. The mercury vapor lamp is very efficient, but the color of the light is unsatisfactory, being deficient in red rays. The spectrum consists of three bands, of yellow, green and violet, respectively. The intrinsic brilliancy of the lamp is very moderate, about 17 candle-power per square inch. Commercial lamps are made of the sizes given below. The lamp is essentially a direct-current lamp, but it may be adapted to alternating-current by use of the principle of the mercury arc rectifier. The tubes have a life ordinarily of about 1000 hours.

MERCURY ARC LAMPS.

| Type. | Kind of Circuit. | Length, inches. | Volts. | Am-peres. | Watts. | Hemi-spher. Candle-power. | Watts per Candle |
|-------|------------------|--------------------------------|---------|-----------|---------|---------------------------|------------------|
| H | d.c. | 20 ³ / ₄ | 52-55 | 3.5 | 177-193 | 300 | 0.64 |
| K | d.c. | 45 | 100-120 | 3.5 | 350-420 | 700 | 0.55 |
| U | d.c. | 78 | 206-240 | 2.0 | 412-480 | 900 | 0.48 |
| P | d.c. | 50 | 100-120 | 3.5 | 350-420 | 800 | 0.48 |
| F | a.c. | 50 | 100-120 | | 400-520 | 750-900 | 0.53-0.58 |

Incandescent Lamps. — Candle-power of nominal 16-c.p. 110-volt carbon lamp:

Mean horizontal 15.7 to 16.6, mean spherical 12.7 to 13.8, mean hemispherical 14.0 to 14.6, mean within 30° from tip 7.9 to 10.9.

Ordinary carbon lamps take from 3 to 4 watts per candle-power. A 16-candle-power lamp using 3.5 watts per candle-power or 56 watts at 110 volts takes a current of $56 \div 110 = 0.51$ ampere. For a given efficiency or watts per candle-power the current and the power increase directly as the candle-power. An ordinary lamp taking 56 watts, 13 lamps take 1 H.P. of electrical energy, or 18 lamps 1.008 kilowatts.

Rating of Incandescent Lamps. — Lamps are commonly rated in terms of their mean horizontal candle-power, and their energy consumption in terms of watts per mean horizontal candle-power. The mean spherical intensity differs from the horizontal intensity by a factor which varies with different kinds and styles of lamp. In carbon lamps it is usually about 82%, and in tantalum and tungsten lamps about 76 to 78% of the mean horizontal candle-power.

The new lamp ratings (May, 1910) of the National Electric Lamp Association designate all lamps by wattage instead of by candle-power as formerly.

Lamps are labeled with a three-voltage label and the regular type of 16 c.-p. carbon lamp, in general use, will be made on the basis of 3.1 watts per c.-p. at top voltage.

CARBON LAMPS.

| Nom-inal Watts. | Actual Watts. | Actual Watts per Candle. | Actual Candle-power. | Hours Life. | Nom-inal Watts | Actual Watts. | Actual Watts per Candle. | Actual Candle-power. | Hours Life. |
|-----------------|---------------|--------------------------|----------------------|-------------|----------------|---------------|--------------------------|----------------------|-------------|
| 10 | 10 | 5.00 | 2.0 | 2000 | 60 | T. 60.0 | 2.97 | 20.2 | 700 |
| 20 | 20 | 4.15 | 4.8 | 2000 | | M. 57.9 | 3.18 | 18.3 | 1000 |
| 25 | T. 25.0 | 3.10 | 8.1 | 500 | | B. 55.7 | 3.39 | 16.4 | 1500 |
| | M. 24.1 | 3.31 | 7.3 | 725 | 100 | T. 100.0 | 2.97 | 33.6 | 600 |
| | B. 23.2 | 3.52 | 6.6 | 1050 | | M. 96.4 | 3.18 | 30.5 | 850 |
| 30 | T. 30.0 | 3.23 | 9.3 | 1050 | | B. 92.9 | 3.39 | 27.4 | 1350 |
| | M. 28.9 | 3.46 | 8.4 | 1500 | 120 | T. 120.0 | 2.97 | 40.4 | 600 |
| | B. 27.8 | 3.69 | 7.5 | 2100 | | M. 115.8 | 3.18 | 36.6 | 850 |
| 50 | T. 50.0 | 2.97 | 16.8 | 700 | | B. 111.4 | 3.39 | 32.8 | 1350 |
| | M. 48.2 | 3.18 | 15.2 | 1000 | | | | | |
| | B. 46.4 | 3.39 | 13.7 | 1500 | | | | | |

T, top; M, middle; B, bottom voltage.

The 50- and 60-watt sizes correspond respectively to the old 16-c.-p., 3.1-watt lamp (at top voltage) and the old 16-c.-p., 3.5-watt lamp (at bottom voltage).

The hours life of all of the listed carbon lamps shows the total life and not the useful life or that formerly given as to 80% of initial c.-p.

The Gem Lamp is an improved type of the carbon lamp, having a carbon filament heated to such a degree in an electric oven that it takes on the properties of metal and hence the name, Gem "Metalized Filament."

Variation in Candle-Power, Efficiency, and Life. — The following table shows the variation in candle-power, etc., of standard 100 to 125 volt, 3.1 and 3.5 watt carbon lamps, due to variation in voltage supplied to them. It will be seen that if a 3.1-watt lamp is run at 10% below its normal voltage, it may have over 9 times as long a life, but it will give only 53% of its normal lighting power, and the light will cost 50% more in energy per candle-power. If it is run at 6% above its normal voltage, it will give 37% more light, will take nearly 20% less energy for equal light power, but it will have less than one-third of its normal life.

| Per cent Normal Voltage. | Per cent of Normal Candle-power. | Watts per Candle, 3.1 watt Lamp. | Relative Life, 3.1 watt | Watts per Candle, 3.5 watts. | Relative Life, 3.5 watts. |
|--------------------------|----------------------------------|----------------------------------|-------------------------|------------------------------|---------------------------|
| 90 | 53 | 4.65 | 9.41 | 5.36 | |
| 92 | 61 | 4.24 | 5.55 | 4.85 | |
| 94 | 69.5 | 3.90 | 3.45 | 4.44 | 3.94 |
| 96 | 79 | 3.60 | 2.20 | 4.09 | 2.47 |
| 98 | 89 | 3.34 | 1.46 | 3.78 | 1.53 |
| 99 | 94.5 | 3.22 | 1.21 | 3.64 | 1.26 |
| 100 | 100 | 3.10 | 1.00 | 3.50 | 1.00 |
| 101 | 106 | 2.99 | .818 | 3.38 | .84 |
| 102 | 112 | 2.90 | .681 | 3.27 | .68 |
| 104 | 124 | 2.70 | .452 | 3.05 | .47 |
| 106 | 137 | 2.54 | .310 | 2.85 | .31 |

The candle-power of a lamp falls off with its length of life, so that during the latter half of its life it has only 60 per cent or 70 per cent of its rate candle-power, and the watts per candle-power are increased 60 per cent or 70 per cent. After a lamp has burned for 500 or 600 hours it is more economical to break it and supply a new one if the price of electrical energy is that usually charged by central stations.

Incandescent Lamp Characteristics. — From a series of curves given in Wickenden's "Illumination and Photometry" the following approximate figures have been derived:

LIFE, CANDLE-POWER AND WATTS PER CANDLE-POWER.

| Hours | 0 | 50 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
|----------|-----|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Lamps | . | Per cent of candle-power. | | | | | | | | | |
| Carbon | 100 | 102 | 96 | 95 | 91 | 88 | 86 | 83 | 81 | | |
| Tantalum | 100 | 144 | 119 | 100 | 97 | 95 | 93 | 90 | 88 | 84 | 80 |
| Tungsten | 100 | 104 | 110 | 112 | 110 | 104 | 100 | 98 | 95 | 92 | 90 |

Per cent Watts per candle.

| | | | | | | | | | | | |
|----------|-----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| Carbon | 100 | 99 | 98 | 103 | 107 | 109 | 111 | 112 | 115 | 119 | |
| Tantalum | 100 | 80 | 90 | 101 | 104 | 106 | 107 | 109 | 109 | 110 | 112 |
| Tungsten | 100 | 97 | 96 | 97 | 100 | 102 | 103 | 107 | 108 | 110 | 111 |

RELATION OF CANDLE-POWER TO TERMINAL VOLTS.

| | | | | | | | | |
|-----------------------|-------------------------------|----|----|----|-----|-----|-----|-----|
| Per cent normal volts | 84 | 88 | 92 | 96 | 100 | 104 | 108 | 112 |
| | Per cent normal candle-power. | | | | | | | |

| | | | | | | | | |
|----------|----|----|----|----|-----|-----|-----|-----|
| Carbon | .. | 46 | 60 | 78 | 100 | 123 | 154 | |
| Tantalum | 46 | 56 | 68 | 82 | 100 | 118 | 139 | 161 |
| Tungsten | 54 | 63 | 73 | 86 | 100 | 115 | 134 | 158 |

The above figures show the necessity of close regulation of voltage of lighting circuits. Slight reductions of voltage cause the light to fall far below normal, while excess voltage greatly diminishes the life of the lamps.

RELATION OF ENERGY CONSUMPTION TO TERMINAL VOLTS.

| Per cent normal volts | 92 | 94 | 96 | 98 | 100 | 102 | 104 | 106 | 108 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Per cent normal watts per candle-power. | | | | | | | | | |
| Carbon | | 124 | 116 | 108 | 100 | 94 | 88 | 82 | |
| Tantalum | 126 | 118 | 112 | 106 | 100 | 95 | 90 | 87 | 83 |
| Tungsten | 120 | 115 | 110 | 105 | 100 | 96 | 92 | 88 | 85 |

Average Performance of Tantalum and Tungsten Lamps. — (Winchenden.) 100 to 125 volts.

| | Tantalum. | | | Tungsten. | | | | |
|------------------------------|-----------|------|------|-----------|-------|-------|--------|---------|
| Rated horizontal c-p. | 12.5 | 20 | 40 | 20 | 32 | 48 | 80 | 200 |
| Mean spherical c-p. | 8.9 | 15.8 | 31.6 | 15.6 | 24.0 | 37.6 | 62.9 | 152 |
| Rated watts per c-p. | 2.5 | 2.0 | 2.0 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| Watts per m. spher. c-p. ... | 2.53 | 2.53 | 2.53 | 1.60 | 1.62 | 1.59 | 1.58 | 1.64 |
| Total watts. | 25. | 40. | 80 | 25 | 35-45 | 50-70 | 85-115 | 230-270 |
| Useful hours. | 900* | 900* | 800† | 800 | 800 | 800 | 800 | 800 |

*For direct current; 500 hrs. for 60 cycle alt. current. †500 to 700 hrs. for alt. current.

Specifications for Lamps. (Crocker.) — The initial candle-power of any lamp at the rated voltage should not be more than 9 per cent above or below the value called for. The average candle-power of a lot should be within 6 per cent of the rated value. The standard efficiencies (of the carbon lamp) are 3.1, 3.5, and 4 watts per candle-power. Each lamp at rated voltage should take within 6 per cent of the watts specified, and the average for the lot should be within 4 per cent. The useful life of a lamp is the time it will burn before falling to a certain candle-power, say 80 per cent of its initial candle-power. For 3.1 watt lamps the useful life is about 400 to 450 hours, for 3.5 watt lamps about 800, and 4 watt lamps about 1600 hours.

Special Lamps. — The ordinary 16 c-p. 110-volt is the old standard for interior lighting. Improved forms of incandescent lamp, such as the tungsten, are now, 1910, rapidly coming into use, so that no one style of lamp can be considered the standard. Thousands of varieties of lamps for different voltages and candle-power are made for special purposes, from the primary lamp, supplied by primary batteries using three volts and about 1 ampere and giving $\frac{1}{2}$ c-p., and the $\frac{3}{4}$ c-p. bicycle lamp, 4 volts and 0.5 ampere, lamps of 100 c-p. at 220 volts. Series lamps of 1 c-p. are used in illuminating signs, $\frac{2}{3}$ ampere and 12.5 to 15 volts, eight lamps being used on a 110-volt circuit. Standard sizes for different voltages, 50, 110, or 220, are 8, 16, 24, 32, 50, and 100 c-p.

The Nernst Lamp depends for its operation upon the peculiar property of certain rare earths, such as yttrium, thorium, zirconium, etc., of becoming electrical conductors when heated to a certain temperature; when cold, these oxides are non-conductors. The lamp comprises a "glower" composed of rare earths mixed with a binding material and pressed into a small rod; a heater for bringing the glower up to the conducting temperature; an automatic cut-out for disconnecting the heater when the glower lights up, and a "ballast" consisting of a small resistance coil of wire having a positive temperature-resistance coefficient.

The ballast is connected in series with the glower; its presence is required to compensate the negative temperature-resistance coefficient of

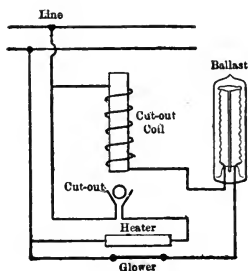


FIG. 197.

the glower; without the ballast, the resistance of the glower would become lower and lower as its temperature rose, until the flow of current through it would destroy it. Fig. 195 shows the elementary circuits of a simple Nernst lamp. The cut-out is an electromagnet connected in series with the glower. When current begins to flow through the glower, the magnet pulls up the armature lying across the contacts of the cut-out, thereby cutting out the heater. The heater is a coil of fine wire either located very near the glower or encircling it. The glower is from $\frac{1}{32}$ to $\frac{1}{16}$ inch in diameter and about 1 inch long.

In the original Nernst lamp the glowers were adapted only for alternating-current, but direct-current glowers are now made.

The lamps are made with one glower, or with two, three, or six glowers connected in parallel, and for operation on 100 to 120 and 200 to 240 volt circuits. A 30-glower lamp for 220 volts, rated at 2000 c.-p., is also made.

Lamps with one glower are rated at 66 watts (110 volt), 88 (220 v.), 110 and 132 watts (110 or 220 v.) with a corresponding mean horizontal candle-power of 50, 77, 96 and 114, respectively. The 2- 3- and 4-glower lamps are multiples of the 132 watt (220 v.) single glower lamps, their m.h.c.-p. being respectively 231, 359 and 504. The Nernst lamp is commonly used where units of intermediate size between incandescent and arc lamps are desired.

Cost of Electric Lighting. A. A. Wohlauer (*El. World*, July, 1908.) — The following table shows the relative cost of 1000 candle-hours of illumination by lamps of different kinds, based on costs of 2, 4 and 10 cents per K.W. hour for electric energy. The life, K , is that of the lamp for incandescent lamps, of the glower for Nernst lamps, of the electrode for arc lamps, and of the vapor tube for vapor lamps.

L_s = mean spherical candle-power.

S_s = watts per mean spherical candle.

P = renewal cost per trim or life, cents.

K = life in hours.

C_r = $1000 P/(KL_s)$.

$C_t = (S_s \times R) + C_t$ = cost per 1000 candle hours.

R = rate in cts. per K.W. hour.

| Illuminant. | Amp. | Volts. | L_s | S_s | P | K | C_r | Rating. | $C_t = (S_s \times R) + C_r$ | | |
|--------------------------------|------|--------|-------|-------|------|-----|-------|----------|------------------------------|------|------|
| Incandescent Lamps. | | | | | | | | | R=2 4 10 | | |
| Carbon..... | 0.31 | 110 | 13.2 | 3.8 | 8 | 450 | 1.35 | 16 c.p. | 10.3 | 17.9 | 40.7 |
| Gem..... | 0.45 | 110 | 16.5 | 3.05 | 10 | 450 | 1.35 | 20 c.p. | 8.8 | 14.8 | 33.2 |
| Meridian..... | 2.3 | 110 | 82 | 3.05 | 35 | 450 | 0.95 | 100 c.p. | 8 | 14.1 | 32.4 |
| Nernst..... | 1.0 | 110 | 42.5 | 2.6 | 32.5 | 500 | 1.5 | 110 Watt | 8.2 | 13.4 | 29 |
| Tantalum..... | 0.36 | 110 | 17 | 2.3 | 25 | 700 | 2.25 | 20 c.p. | 9.1 | 13.7 | 27.5 |
| Tungsten..... | 0.91 | 110 | 72 | 1.4 | 100 | 800 | 1.8 | 80 c.p. | 6.4 | 9.2 | 17.6 |
| Direct-Current Arc Lamps. | | | | | | | | | | | |
| Open arc..... | 10 | 55 | 400 | 1.3 | 4 | 10 | 1 | 10 amp. | 4.6 | 7.2 | 15 |
| Enclosed..... | 5.0 | 110 | 260 | 2.1 | 4.5 | 150 | 0.1 | 5 | 4.4 | 8.6 | 21.2 |
| Carbon..... | 10 | 110 | 550 | 2.0 | 4 | 16 | 0.5 | 10 | 5 | 9 | 21 |
| Miniature..... | 2.5 | 110 | 150 | 1.8 | 3 | 20 | 1. | 2.5 | 5.6 | 9.2 | 20 |
| Magnetite..... | 3.5 | 110 | 225 | 1.7 | 5 | 150 | 0.155 | 3.5 | 3.71 | 7.11 | 17.2 |
| Flaming..... | 10 | 55 | 600 | 0.75 | 8.5 | 10 | 1.2 | 10 | 3.9 | 5.4 | 9.9 |
| Blondel..... | 5 | 55 | 550 | 0.5 | 17.5 | 18 | 1.25 | 5 | 3.5 | 4.5 | 7.5 |
| Inclined flaming..... | 10 | 55 | 1100 | 0.5 | 9 | 10 | 0.8 | 10 | 2.6 | 3.6 | 6.6 |
| Inclined enclosed flaming..... | 5.5 | 100 | 1500 | 0.365 | 15 | 70 | 1.55 | 5.5 | 1.03 | 1.76 | 4 |

| Illuminant. | Amp. | Volts. | L_s | S_s | P | K | C_r | Rating. | $Ct = (S_s \times R) + C_r$ |
|-------------|------|--------|-------|-------|---|---|-------|---------|-----------------------------|
|-------------|------|--------|-------|-------|---|---|-------|---------|-----------------------------|

Alternating-Current Arc Lamps.

| | | | | | | | | | | | |
|--------------------|-----|-------|------|------|------|-----|------|---------|-----|------|------|
| Open arc..... | 15 | | 300 | 1.75 | 5 | 13 | 1.1 | 15 amp. | 5.7 | 9.2 | 19.7 |
| Enclosed..... | 7.5 | | 230 | 2.6 | 4.5 | 100 | 0.2 | 7.5 | 5.6 | 10.8 | 26.4 |
| Flaming | 10 | | 425 | 0.8 | 8.5 | 7 | 2.8 | 10 | 7.2 | 8.8 | 13.6 |
| Inclined flaming.. | 10 | | 1000 | 0.55 | 9 | 10 | 0.65 | 10 | 2.9 | 4 | 7.3 |
| Blondel..... | 10 | | 715 | 0.5 | 12.5 | 15 | 1.15 | 10 | 3.3 | 4.3 | 37. |

Mercury-Vapor Lamps.

| | | | | | | | | | | | |
|-------------------|-----|-----|------|------|-----|------|-------|----------|------|------|------|
| Cooper Hewitt.... | 3.5 | 110 | 770 | 0.5 | 600 | 4000 | 0.2 | 3.5 amp. | 1.4 | 2.4 | 5.4 |
| Quartz..... | 4.0 | 220 | 2740 | 0.33 | 350 | 1000 | 0.125 | 4.2 | 0.85 | 1.45 | 3.25 |

ELECTRIC WELDING.

The apparatus most generally used consists of an alternating-current dynamo, feeding a comparatively high-potential current to the primary coil of an induction-coil or transformer, the secondary of which is made so large in section and so short in length as to supply to the work currents not exceeding two or three volts, and of very large volume or rate of flow. The welding clamps are attached to the secondary terminals. Other forms of apparatus, such as dynamos constructed to yield alternating currents direct from the armature to the welding-clamps, are used.

The conductivity for heat of the metal to be welded has a decided influence on the heating, and in welding iron its comparatively low heat conduction assists the work materially. (See papers by Sir F. Bramwell, Proc. Inst. C. E., part iv., vol. cii. p. 1; and Elihu Thomson, Trans. A. I. M. E., xix. 877.)

Fred. P. Royce, *Iron Age*, Nov. 28, 1892, gives the following figures showing the amount of power required to weld axles and tires:

AXLE-WELDING.

| | Seconds |
|--|---------|
| 1-inch round axle requires 25 H.P. for..... | 45 |
| 1-inch square axle requires 30 H.P. for..... | 48 |
| 1 1/4-inch round axle requires 35 H.P. for..... | 60 |
| 1 1/4-inch square axle requires 40 H.P. for..... | 70 |
| 2-inch round axle requires 75 H.P. for..... | 95 |
| 2-inch square axle requires 90 H.P. for..... | 100 |

The slightly increased time and power required for welding the square axle is not only due to the extra metal in it, but in part to the care which it is best to use to secure a perfect alignment.

TIRE-WELDING.

| | Seconds. |
|--|----------|
| 1 \times 3/16-inch tire requires 11 H.P. for..... | 15 |
| 1 1/4 \times 3/8-inch tire requires 23 H.P. for..... | 25 |
| 1 1/2 \times 3/8-inch tire requires 20 H.P. for..... | 30 |
| 1 1/2 \times 1/2-inch tire requires 23 H.P. for..... | 40 |
| 2 \times 1/2-inch tire requires 29 H.P. for..... | 55 |
| 2 \times 3/4-inch tire requires 42 H.P. for..... | 62 |

The time above given for welding is of course that required for the actual application of the current only, and does not include that consumed by placing the axles or tires in the machine, the removal of the upset and other finishing processes. From the data thus submitted, the cost of welding can be readily figured for any locality where the price of fuel and cost of labor are known.

In almost all cases the cost of the fuel used under the boilers for producing power for electric welding is practically the same as the cost of fuel used in forges for the same amount of work, taking into consideration the difference in price of fuel used in either case.

Prof. A. B. Kennedy found that 2 1/2-inch iron tubes 1/4-inch thick were welded in 61 seconds, the net horse-power required at this speed being 23.4 (say 33 indicated horse-power) per square inch of section. Brass tubing required 21.2 net horse-power. About 60 total indicated horse-power would be required for the welding of angle-irons 3 X 3 X 1/2-inch in from two to three minutes. Copper requires about 80 horse-power per square inch of section, and an inch bar can be welded in 25 seconds. It takes about 90 seconds to weld a steel bar 2 inches in diameter.

ELECTRIC HEATERS.

Wherever a comparatively small amount of heat is desired to be automatically and uniformly maintained, and started or stopped on the instant without waste, there is the province of the electric heater.

The elementary form of heater is some form of resistance, such as coils of thin wire introduced into an electric circuit and surrounded with a substance which will permit the conduction and radiation of heat, and at the same time serve to electrically insulate the resistance.

This resistance should be proportional to the electro-motive force of the current used and to the equation of Joule's law:

$$H = I^2 R t \times 0.24,$$

where I is the current in amperes; R , the resistance in ohms; t , the time in seconds; and H , the heat in gram-centigrade units.

Since the resistance of metals increases as their temperature increases, a thin wire heated by current passing through it will resist more, and grow hotter and hotter until its rate of loss of heat by conduction and radiation equals the rate at which heat is supplied by the current. In a short wire, before heat enough can be dispelled for commercial purposes, fusion will begin; and in electric heaters it is necessary to use either long lengths of thin wire, or carbon, which alone of all conductors resists fusion. In the majority of heaters, coils of thin wire are used, separately embedded in some substance of poor electrical but good thermal conductivity.

The Consolidated Car-heating Co.'s electric heater consists of a galvanized iron wire wound in a spiral groove upon a porcelain insulator. Each heater is 30 5/8 in. long, 8 7/8 in. high, and 6 5/8 in. wide. Upon it is wound 392 ft. of wire. The weight of the whole is 23 1/2 lbs.

Each heater is designed to absorb 1000 watts of a 500-volt current. Six heaters are the complement for an ordinary electric car. For ordinary weather the heaters may be combined by the switch in different ways, so that five different intensities of heating-surface are possible, besides the position in which no heat is generated, the current being turned off.

For heating an ordinary electric car the Consolidated Co. states that from 2 to 12 amperes on a 500-volt circuit is sufficient. With the outside temperature at 20° to 30°, about 6 amperes will suffice. With zero or lower temperature, the full 12 amperes is required to heat a car effectively.

Compare these figures with the experience in steam-heating of railway-cars, as follows:

1 B. T. U. = 0.29084 watt-hours.

6 amperes on a 500-volt circuit = 3000 watts.

A current consumption of 6 amperes will generate $3000 \div 0.29084 = 10,315$ B.T.U. per hour.

In steam-car heating, a passenger coach usually requires from 60 lbs. of steam in freezing weather to 100 lbs. in zero weather per hour. Supposing the steam to enter the pipes at 20 lbs. pressure, and to be discharged at 200° F., each pound of steam will give up 983 B.T.U. to the car. Then

the equivalent of the thermal units delivered by the electrical-heating system in pounds of steam, is $10,315 \div 983 = 10\frac{1}{2}$, nearly.

Thus the Consolidated Co.'s estimates for electric-heating provide the equivalent of $10\frac{1}{2}$ lbs. of steam per car per hour in freezing weather and 21 lbs. in zero weather.

Suppose that by the use of good coal, careful firing, well-designed boilers and triple-expansion engines we are able in daily practice to generate 1 H.P. delivered at the fly-wheel with an expenditure of $2\frac{1}{2}$ lbs. of coal per hour.

We have then to convert this energy into electricity, transmit it by wire to the heater, and convert it into heat by passing it through a resistance-coil. We may set the combined efficiency of the dynamo and line circuit at 85%, and will suppose that all the electricity is converted into heat in the resistance-coils of the radiator. Then 1 brake H.P. at the engine = 0.85 electrical H.P. at the resistance coil = 1,683,000 ft.-lbs. energy per hour = 2180 heat-units. But since it required $2\frac{1}{2}$ lbs. of coal to develop 1 brake H.P., it follows that the heat given out at the radiator per pound of coal burned in the boiler furnace will be $2180 \div 2\frac{1}{2} = 872$ H.U. An ordinary steam-heating system utilizes 9652 H.U. per lb. of coal for heating; hence the efficiency of the electric system is to the efficiency of the steam-heating system as 872 to 9652, or about 1 to 11. (*Eng'g News*, Aug. 9, '90; Mar. 30, '92; May 15, '93.)

Electric Furnaces. (Condensed from an article by J. Wright in *Elec. Age*, May, 1904. The original contains illustrations of many styles of furnace.)—Electric furnaces may be divided into two main classes, (1) those in which the heating effect is produced by the electric arc established between two carbon or other electrodes connected with the source of current, commonly known as arc furnaces; and (2) those in which the heating effect is produced by the passage of the current through a resistance, which either forms part of the furnace proper, or is constituted, by a suitable conducting train, of the material to be treated in the furnace. Such furnaces are known as resistance furnaces.

The Moissan arc furnace consists of two chalk blocks, bored out to receive a carbon crucible which encloses the center or hearth of the furnace proper. Into this cavity pass two massive carbon electrodes, through openings provided for them in the walls of the structure, which is held together by clamps. The arc established between the ends of the carbons when the current is turned on plays over the center of the crucible, heating its contents.

In the Siemens arc furnace a refractory crucible of plumbago, magnesia, lime, or other suitable material is supported at the center of a cylinder or jacket, and packed around with broken charcoal, or other poor conductor of heat. The negative electrode consists of a massive carbon rod passing vertically through the lid of the crucible, and free to move vertically therein. The positive electrode, which may be of iron, platinum or carbon, consists of a rod passing up through the base of the crucible. The furnace was originally designed for the fusion of refractory metals and their ores. Electrical contact is established between the lower electrode and the semi-metallic mass in the crucible, and the arc continues to play between the surface of the mass and the movable carbon rod. As the current through the furnace increases, that through the shunt winding of a solenoid which controls the position of the movable rod diminishes, thereby raising the negative electrode and restoring equilibrium.

The Willson furnace is a modification of the Siemens, the solenoid regulation of the upper movable carbon being replaced by a worm and hand wheel, while the furnace is made continuous in operation by the provision of a tapping hole for drawing off the molten products. This type of furnace was employed by Willson in the manufacture of calcium carbide; many other types of arc furnaces have been developed from these earlier forms. (See *El. Age*, May, 1904, for illustrations.)

The Borchers furnace is typical of that class in which a core, forming part of the furnace itself, is heated by the passage of the current through it, and imparts its heat to the surrounding mass of material contained in the furnace. It consists of a block of refractory material, in the center of which is an opening forming the crucible, into which is fed the material to

be treated. This space is bridged by a thin carbon rod which is attached, at its extremities, to two carbon electrodes, passing through the walls of the furnace. The current heats the smaller rod to a very high temperature, and the rod diffuses its heat throughout the mass, from its center outwards.

H. I. Irvine has brought out a resistance furnace in which the heated column consists of a fused electrolyte, maintained in a state of fusion by the passage of the current, and communicating its heat by radiation and diffusion, to the encircling charge, which is packed around it.

A novel type of resistance furnace, patented independently, with some slight variation of detail, by Colby, Ferranti, and Kjellin, is worked on the inductive principle, and consists of an annular, or helical, channel in a refractory base, filled with a conducting, or semi-conducting, medium, which constitutes the furnace charge, and has a heavy current induced in it by a surrounding coil of many turns, carrying an alternating current. The device, in fact, acts as the closed-circuit secondary of a step-down transformer.

The Acheson furnace for the manufacture of carborundum is a rough firebrick structure, through the end walls of which project the electrodes consisting of composite bundles of carbon rods set in metal clamps. The space between the two electrodes is bridged by a conducting path of coke, which constitutes the core of the furnace. This core is packed round with the raw material, consisting of coke, sand, sawdust and common salt.

A 2½ ton Héroult electric steel furnace has been installed by the Firth-Sterling Steel Co. at Demmler, Pa. In this furnace an arc is formed between the bath of metal and two graphite electrodes which are suspended over it. Single-phase, sixty-cycle alternating current is used and is stepped down to 110 volts by transformers from the 11,000-volt mains. The furnace consumes about 250 kilowatts. It produces steel equal in quality to crucible steel, at a cost little greater than open-hearth steel. (*El. Review*, May 14, 1910.)

The *Iron Trade Review*, 1906, contains a series of illustrated articles on electric furnaces, by J. B. C. Kershaw. See also paper by C. F. Burgess, in *Trans. Western Socy. of Engrs.*, 1905, and papers in *Trans. Am. Electro Chemical Society*, 1902 and later dates.

Silundum, or silicified carbon, is a product obtained when carbon is heated in the vapor of silicon in an electric furnace. It is a form of carborundum, and has similar properties; it is very hard, resists high temperatures and is acid-proof. It is a conductor of electricity, its resistance being about three times that of carbon. It can be heated in the air up to 1600° C. without showing any sign of oxidation. At about 1,700°, however, the silicon leaves the carbon and combines with the oxygen of the air. Silundum cannot be melted. The first use to which the material was applied was for electric cooking and heating. For heating purposes the silundum rods can be used single, in lengths up to 32 ins., depending on the diameter, as solid, round, flat or square rods or tubes, or in the form of a grid mounted in a frame and provided with contact wires. (*El. Review*, London. *Eng. Digest*, Feb., 1909.)

PRIMARY BATTERIES.

Following is a partial list of some of the best known primary cells or batteries.

| Name. | Elements. | | Electrolyte. | Depolarizer. | E.M.F. volts. |
|---------------------|-----------|----|---------------------------------------|---|---------------|
| | — | + | | | |
| Daniell..... | Cu | Zn | Dilute H ₂ SO ₄ | Concent. CuSO ₄ | 1.07 |
| Gravity..... | Cu | Zn | ZnSO ₄ | Concent. CuSO ₄ | 1. |
| Grove..... | Pt | Zn | Dilute H ₂ SO ₄ | HNO ₃ | 1.9 |
| Fuller..... | C | Zn | Dilute H ₂ SO ₄ | K ₂ Cr ₂ O ₇ | 2.1 |
| Edison-Lalande | Cu | Zn | Conc. NaOH | CuO | 0.7-0.9 |
| Leclanche..... | C | Zn | NH ₄ Cl | MnO ₂ | 1.4 |
| Clark..... | Pt | Zn | ZnSO ₄ | Hg ₂ SO ₄ | 1.44 |
| Weston | Pt | Cd | CdSO ₄ | Hg ₂ SO ₄ | 1.02 |
| Dry battery..... | C | Zn | Various electrolyte pastes. | | 1-1.8 |

The gravity cell is used for telegraph work. It is suitable for closed circuits, and should not be used where it is to stand for a long time on open circuit.

The Fuller cell is adapted to telephones or any intermittent work. It can stand on open circuit for months without deterioration.

The Edison-Lalande cell is suitable for either closed or open circuits.

The Leclanché cell is adapted for open circuit intermittent work, such as bells, telephones, etc.

The Clark and Weston cells are used for electrical standards. The Weston cell has largely superseded the Clark.

Dry cells are in common use for house service, igniters for gas engines, etc.

Batteries are coupled in series of two or more to obtain an e.m.f. greater than that of one cell, and in multiple to obtain more amperes without change of e.m.f.

Spark coils, or induction coils with interrupters, are used to obtain ignition sparks for gas engines, etc.

ELECTRICAL ACCUMULATORS OR STORAGE-BATTERIES.

The original, or Planté, storage battery consisted of two plates of metallic lead immersed in a vessel containing sulphuric acid. An electric current being sent through the cell the surface of the positive plate was converted into peroxide of lead, PbO_2 . This was called charging the cell. After being thus charged the cell could be used as a source of electric current, or discharged. Planté and other authorities consider that in charging, PbO_2 is formed on the positive plate and spongy metallic lead on the negative, both being converted into lead oxide, PbO , by the discharge, but others hold that sulphate of lead is made on both plates by discharging, and that during the charging PbO_2 is formed on the positive plate and metallic Pb on the other, sulphuric acid being set free.

The acid being continually abstracted from the electrolyte as the discharge proceeds, the density of the solution becomes less. In the charging operation this action is reversed, the acid being reinstated in the liquid and therefore causing an increase in its density.

The difference of potential developed by lead and lead peroxide immersed in dilute H_2SO_4 is about two volts. A lead-peroxide plate gradually loses its electrical energy by local action, the rate of such loss varying according to the circumstances of its preparation and the condition of the cell.

In the Faure or pasted cells lead plates are coated with minium or litharge made into a paste with acidulated water. When dry these plates are placed in a bath of dilute H_2SO_4 and subjected to the action of the current, by which the oxide on the positive plate is converted into peroxide and that on the negative plate reduced to finely divided or porous lead.

The "Chloride Accumulator" made by The Electric Storage Battery Co., of Philadelphia, consists of modified Planté positives and modified Faure negatives. The positive plate, called the Manchester type, consists of a hard lead grid into which are pressed "buttons" of corrugated pure lead tape, rolled into spirals. When electrolytically "formed," these buttons become coated with lead peroxide. The negative is the so-called "Box" type, in which the grid is made in two halves which are riveted together after "pasting" with lead oxide, the latter upon charging being reduced to spongy lead. The outside faces are covered with perforated lead sheet, which serves to retain the spongy lead or active material.

The following tables give the elements of several sizes of "chloride" accumulators. Type G is furnished in cells containing 11-75 plates, and type H from 21 plates to any greater number desired. The voltage of cells of all sizes is slightly above two volts on open circuit, and during discharge

varies from that point at the begining to 1.75 at the end when working at the normal (eight-hour) rate. At higher rates the final voltage is lower.

Accumulators are largely used in central lighting and power stations, in office buildings and other large isolated plants, for the purpose of absorbing the energy of the generating plant during times of light load, and for giving it out during times of heavy load or when the generating plant is idle. The advantages of their use for such purposes are thus enumerated:

1. Reduction in coal consumption and general operating expenses, due to the generating machinery being run at the point of greatest economy while in service, and being shut down entirely during hours of light load, the battery supplying the whole of the current.

| TYPE, Size of Plates. | "B" 3x3 in. | "C" 4 3/8 x 4 in. | | | | "D" 6x6 in. | | | | | |
|--|-------------------|----------------------|-------|-------|-------|----------------|--------|--------|--------|--------|--------|
| Number of plates..... | 3 | 3 | 5 | 7 | | 3 | 5 | 7 | 9 | 11 | 13 |
| Discharge in } For 8 hours | 5/8 | 1 1/4 | 2 1/2 | 3 3/4 | | 2 1/2 | 5 | 7 1/2 | 10 | 12 1/2 | 15 |
| amperes: } For 5 hours | 7/8 | 1 3/4 | 3 1/2 | 5 1/4 | | 3 1/2 | 7 | 10 1/2 | 14 | 17 1/2 | 21 |
| } For 3 hours | 1 1/4 | 2 1/2 | 5 | 7 1/2 | | 5 | 10 | 15 | 20 | 25 | 30 |
| Normal charge rate..... | 5/8 | 1 1/4 | 2 1/2 | 3 3/4 | | 2 1/2 | 5 | 7 1/2 | 10 | 12 1/2 | 15 |
| Outside dimensions of rubber jar, inches: | Length | 13/4 | 13/4 | 23/4 | 37/8 | 13/4 | 23/4 | 37/8 | 5 | 6 1/8 | 7 1/4 |
| | Width | 35/8 | 4 1/2 | 4 1/2 | 4 1/2 | 6 1/2 | 6 1/2 | 6 1/2 | 6 1/2 | 6 1/2 | 6 1/2 |
| | Height | 5 | 7 | 7 | 7 | 9 | 9 | 9 | 9 | 9 | 9 |
| Outside dimensions of glass jar, inches: | Length | 2 1/2 | 3 1/2 | 4 1/4 | 5 1/4 | 3 1/4 | 4 3/4 | 6 1/2 | 8 3/4 | 8 3/4 | 8 3/4 |
| | Width | 4 | 5 1/4 | 5 1/4 | 5 1/4 | 7 7/8 | 7 7/8 | 7 7/8 | 8 | 8 | 8 1/4 |
| | Height | * | 7 1/4 | 7 1/4 | 7 1/4 | 9 1/2 | 9 1/2 | 9 1/2 | 9 1/2 | 9 1/2 | 9 1/2 |
| Weight of electrolyte, lbs.: | glass jars | † | 3 1/4 | 4 | 5 | 7 1/2 | 10 1/2 | 15 | 17 3/4 | 17 1/4 | 21 |
| | rubber jars... | 1 1/2 | 1 1/2 | 2 1/4 | 2 3/4 | 2 1/4 | 3 3/4 | 5 1/4 | 6 3/4 | 7 3/4 | 10 |
| Weight of cell com- plete, with acid, lbs.: | glass jars | 5 3/4 | 11 | 15 | 19 | 20 | 28 | 38 | 48 | 53 | 63 |
| | rubber jars... | 3 1/2 | 6 1/2 | 10 | 13 | 12 | 18 1/2 | 24 1/2 | 32 1/2 | 39 3/4 | 47 1/4 |
| Height of cell over all, inches: | glass jars | † | 15 | 15 | 15 | 18 | 18 | 18 | 18 | 18 | 18 |
| | rubber jars... | 6 1/2 | 8 1/2 | 8 1/2 | 8 1/2 | 10 1/2 | 10 1/2 | 10 1/2 | 10 1/2 | 10 1/2 | 10 1/2 |

* 4 1/2, 5 1/2, and 6 1/2 ins. † 3/4, 1, and 1 1/4, lbs. ‡ 7 1/2, 9 1/2, and 11 1/2 lbs.

"D" Yacht type, rubber jars, 5, 7, and 9 plates, 2 1/2 in. higher than standard.

| TYPE "E." | | | | | | | TYPE "F." | | | | | | |
|--|-----------------------------------|--------|--------|--------|--------|--------|---------------------------------|----------------|--------|--------|--------|--------|-------|
| Size of Plates, 7 3/4 x 7 3/4 in. | | | | | | | Size of Plates, 11 x 10 1/2 in. | | | | | | |
| Number of plates.... | 5 | 7 | 9 | 11 | 13 | 15 | 9 | 11 | 13 | 15 | 17 | D* | |
| Discharge in am- peres: | For 8 hrs. | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 50 | 60 | 70 | 80 | 5 |
| | For 5 hrs. | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 70 | 84 | 98 | 112 | 7 |
| | For 3 hrs. | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 100 | 120 | 140 | 160 | 10 |
| | For 1 hr. | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 200 | 240 | 280 | 320 | 20 |
| Normal charge rate... | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 50 | 60 | 70 | 80 | 5 | |
| Outside dimensions: | Length, in. } rub- ber jar. | 27/8 | 37/8 | 5 | 6 1/8 | 8 1/8 | 8 1/2 | 9 lead tank | 15 | 16 3/4 | 18 3/8 | 20 | 7/8 |
| | Width, in. } rub- ber jar. | 8 1/2 | 8 1/2 | 8 1/2 | 8 1/2 | 8 1/2 | 8 1/2 | | 15 1/8 | 15 | 15 | 15 | |
| | Height, in. } rub- ber jar. | 11 | 11 | 11 | 11 | 11 | 11 | | 20 1/4 | 20 1/4 | 20 1/4 | 20 1/4 | |
| | Length, in. } glass jar. | 5 1/2 | 6 3/4 | 8 | 8 3/8 | 11 | 11 3/8 | | 9 | 10 3/8 | 10 5/8 | 12 | |
| | Width, in. } glass jar. | 9 1/8 | 9 1/8 | 9 1/8 | 9 1/8 | 9 1/8 | 9 1/8 | | 12 1/2 | 12 1/2 | 12 3/4 | 12 3/4 | |
| | Height, in. } glass jar. | 11 3/8 | 11 3/8 | 11 3/8 | 11 3/8 | 11 3/8 | 11 3/8 | | 17 1 | 17 | 17 | 17 | |
| Weight of electrolyte: | glass jars. | 18 1/2 | 20 | 24 1/2 | 26 | 35 | 34 | 63 | 69 | 67 | 79 | | |
| | rub- ber jars. | 5 1/2 | 8 | 10 1/2 | 12 | 17 | 18 1/2 | | 99 | 111 | 123 | 133 | 6 |
| Weight of cell com- plete, with acid: | glass jar. | 49 | 60 | 74 | 86 1/2 | 104 | 112 | | | | | | |
| | rub- ber jar. | 29 1/2 | 40 1/2 | 52 | 63 | 77 | 87 | tank | | 332 | 372 | 411 | 20 |
| Height of cell over all, in inches: | glass jar. | 20 | 20 | 20 | 20 | 20 | 20 | 27 3/4 | 27 3/4 | 27 3/4 | 27 3/4 | 27 3/4 | |
| | rub- ber jar. | 12 1/2 | 12 1/2 | 12 1/2 | 12 1/2 | 12 1/2 | 12 1/2 | tank | 33 1/4 | 33 1/4 | 33 1/4 | 33 1/4 | |

*D = addition per plate from 25 to 75 plates; approximate as to dimensions and weights.

| TYPE "G." | | | | | | | | | | TYPE "H." | | | | |
|--|------------------|------------------|------------------|------------------|------------------|------------------|-----------------|------------------|------------------|---|------------------|-----------------|--|--|
| Size of Plates, $15\frac{3}{16} \times 15\frac{5}{16}$ in. | | | | | | | | | | Size of Plates, $15\frac{3}{16} \times 30\frac{11}{16}$ in. | | | | |
| Number of plates... | 11 | 13 | 15 | 17 | 25 | 75 | D* | 21 | 23 | 25 | 75 | D* | | |
| Discharge in amperes: { For 8 hrs. | 100 | 120 | 140 | 160 | 240 | 740 | 10 | 400 | 440 | 480 | 480 | 20 | | |
| { For 5 hrs. | 140 | 168 | 196 | 224 | 336 | 1036 | 14 | 560 | 616 | 672 | 2072 | 28 | | |
| { For 3 hrs. | 200 | 240 | 280 | 320 | 480 | 1480 | 20 | 800 | 880 | 960 | 2960 | 40 | | |
| { For 1 hr. | 400 | 480 | 560 | 640 | 960 | 2960 | 40 | 1600 | 1760 | 1920 | 5920 | 20 | | |
| Normal charge rate. | 100 | 120 | 140 | 160 | 240 | 740 | 10 | 400 | 440 | 480 | 480 | 20 | | |
| Outside dimensions of tank, inches: { Length. | 15 $\frac{1}{2}$ | 16 $\frac{1}{4}$ | 18 $\frac{1}{2}$ | 20 | 27 $\frac{5}{8}$ | 69 $\frac{7}{8}$ | 7 $\frac{7}{8}$ | 25 $\frac{1}{2}$ | 26 $\frac{3}{4}$ | 28 $\frac{3}{8}$ | 69 $\frac{7}{8}$ | 7 $\frac{7}{8}$ | | |
| { Width. | 19 $\frac{3}{4}$ | 19 $\frac{3}{4}$ | 19 $\frac{3}{4}$ | 19 $\frac{3}{4}$ | 20 $\frac{3}{4}$ | 21 $\frac{1}{2}$ | ... | 21 $\frac{1}{2}$ | 21 $\frac{1}{2}$ | 21 $\frac{1}{2}$ | 21 $\frac{1}{2}$ | ... | | |
| { Height. | 26 | 26 | 26 | 26 | 26 $\frac{1}{2}$ | 27 $\frac{7}{8}$ | ... | 48 $\frac{1}{8}$ | 48 $\frac{7}{8}$ | 48 $\frac{7}{8}$ | 49 $\frac{7}{8}$ | ... | | |
| Weight of electrolyte in pounds..... | 188 | 210 | 231 | 253 | 338 | 876 | 10.5 | 583 | 625 | 668 | 1741 | 21.5 | | |
| Weight of cell, complete, with electrolyte in lead-lined tank, pounds..... | 568 | 645 | 719 | 798 | 1165 | 3300 | 40 | 1967 | 2121 | 2278 | 6215 | 78 | | |
| Height of cell over all, inches..... | 39 | 39 | 39 | 39 | 40 | 41 $\frac{1}{2}$ | ... | 62 $\frac{1}{4}$ | 62 $\frac{1}{4}$ | 62 $\frac{1}{4}$ | 63 $\frac{1}{4}$ | | | |

*D = addition per plate from 25 to 75 plates; approximate as to dimensions and weights.

2. The possibility of obtaining good regulation in pressure during fluctuations in load, especially when the day load consists largely of elevators and similar disturbing elements.

3. To meet sudden demands which arise unexpectedly, as in the case of darkness caused by storm or thunder-showers; also in case of emergency due to accident or stoppage of generating-plant.

4. Smaller generating-plant required where the battery takes the peak of the load, which usually only lasts for a few hours, and yet where no battery is used necessitates sufficient generators, etc., being installed to provide for the maximum output, which in many cases is about double the normal output.

The Working Current, or Energy Efficiency, of a storage-cell is the ratio between the value of the current or energy expended in the charging operation, and that obtained when the cell is discharged at any specified rate.

In a lead storage-cell, if the surface and quantity of active material be accurately proportioned, and if the discharge be commenced immediately after the termination of the charge, then a current efficiency of as much as 98% may be obtained, provided the rate of discharge is low and well regulated. Since the current efficiency decreases as the discharge rate increases, and since very low discharge rates are seldom used in practice, efficiencies as high as this are never obtained practically, the average being about 90%.

As the normal average discharging electro-motive force of a lead secondary cell never exceeds 2 volts, and as an average electro-motive force during normal charge of about 2.35 volts is required at its poles to overcome both its opposing electro-motive force and its internal resistance, there is an initial loss of at least 15% between the voltage required to charge it and that at which it discharges. Thus with a current efficiency of 90% and a volt efficiency of 85% the energy efficiency under the best conditions cannot be much over 75%, while in practice it is nearer 70%.

Important General Rules.—Storage cells should not be excessively charged, undercharged or allowed to stand when completely discharged.

In setting up new cells the manufacturer should always be consulted as to the proper purity and specific gravity of the electrolyte (solution) to be used in the cells and also as to the duration of the initial charge.

Charging should be done at the normal rate (as given by the manufacturer) or as near to it as possible. At regular periods once each week or two weeks, depending on whether the cells have to be charged daily or not, an overcharge should be given, lasting until the specific gravity of the electrolyte and the cell voltage have risen to a maximum and remained constant for about one hour. The end of charge voltage may vary from 2.40 to 2.70 volts per cell. All other charges termed "regular charges" should cease shortly before the maximum values obtained on the preceding overcharge are reached. If cells are standing idle they should receive an overcharge once every two weeks.

Discharges should be stopped when the cell voltage has fallen to 1.80 volts with current flowing at or about the normal rate. The fall in specific gravity of the electrolyte is also useful as a guide on the discharge and the manufacturer should be consulted as to the proper limits.

The level of the electrolyte should be kept above the top of the plates by adding pure fresh water. Addition of new electrolyte is seldom necessary and should be done only on advice from the manufacturer.

The sediment which collects in the bottom of the cells should always be removed before it touches the plates.

The battery room should be well ventilated, especially when charging, and great care taken not to bring an exposed flame near the cells when charging or shortly after.

Metals or impurities of any kind must not be allowed to get into the cells. If such should happen, the impurity should be removed at once, and if badly contaminated, the electrolyte replaced with new. If in doubt as to the purity of electrolyte or water, the manufacturers should be consulted.

To take cells out of commission, the electrolyte should be drawn off; the cells filled with water and allowed to stand for 12 or 15 hours. The water can then be drawn off and the plates allowed to dry. When putting into service again, the same procedure should be followed as with the initial charge.

ELECTROLYSIS.

The separation of a chemical compound into its constituents by means of an electric current. Faraday gave the nomenclature relating to electrolysis. The compound to be decomposed is the Electrolyte, and the process Electrolysis. The plates or poles of the battery are Electrodes. The plate where the greatest pressure exists is the Anode, and the other pole is the Cathode. The products of decomposition are Ions.

Lord Rayleigh found that a current of one ampere will deposit 0.017253 grain, or 0.001118 gram, of silver per second on one of the plates of a silver voltameter, the liquid employed being a solution of silver nitrate containing from 15% to 20% of the salt. The weight of hydrogen similarly set free by a current of one ampere is 0.00001038 gram per second.

Knowing the amount of hydrogen thus set free, and the chemical equivalents of the constituents of other substances, we can calculate what weight of their elements will be set free or deposited in a given time by a given current. Thus, the current that liberates 1 gram of hydrogen will liberate 8 grams of oxygen, or 107.7 grams of silver, the numbers 8 and 107.7 being the chemical equivalents for oxygen and silver respectively.

To find the weight of metal deposited by a given current in a given time, find the weight of hydrogen liberated by the given current in the given time, and multiply by the chemical equivalent of the metal.

The table on page 1382 (from "Practical Electrical Engineering") is calculated upon Lord Rayleigh's determination of the electro-chemical equivalents and Roscoe's atomic weights,

ELECTRO-CHEMICAL EQUIVALENTS.

| Elements. | Valency.* | Atomic Weight.† | Chemical Equivalent. | Electro-chemical Equivalent (milligrams per coulomb). | Coulombs per gram. | Grams per ampere hour. |
|--------------------------|-----------------|-----------------|----------------------|---|--------------------|------------------------|
| ELECTRO-POSITIVE. | | | | | | |
| Hydrogen..... | H ₁ | 1.00 | 1.00 | 0.010384 | 96293.00 | 0.03738 |
| Potassium..... | K ₁ | 39.04 | 39.04 | 0.40539 | 2467.50 | 1.45950 |
| Sodium..... | Na ₁ | 22.99 | 22.99 | 0.23873 | 4188.90 | 0.85942 |
| Aluminum..... | Al ₃ | 27.3 | 9.1 | 0.09449 | 1058.30 | 0.34018 |
| Magnesium..... | Mg ₂ | 23.94 | 11.97 | 0.12430 | 804.03 | 0.44747 |
| Gold..... | Au ₃ | 196.2 | 65.4 | 0.67911 | 1473.50 | 2.44480 |
| Silver..... | Ag ₁ | 107.66 | 107.66 | 1.11800 | 894.41 | 4.02500 |
| Copper (cupric)..... | Cu ₂ | 63.00 | 31.5 | 0.32709 | 3058.60 | 1.17700 |
| (cuprous)..... | Cu ₁ | 63.00 | 63.00 | 0.65419 | 1525.30 | 2.35500 |
| Mercury (mercuric)..... | Hg ₂ | 199.8 | 99.9 | 1.03740 | 963.99 | 3.73450 |
| (mercurous)..... | Hg ₁ | 199.8 | 199.8 | 2.07470 | 481.99 | 7.46900 |
| Tin (stannic)..... | Sn ₄ | 117.8 | 29.45 | 0.30581 | 3270.00 | 1.10090 |
| (stannous)..... | Sn ₂ | 117.8 | 58.9 | 0.61162 | 1635.00 | 2.20180 |
| Iron (ferric)..... | Fe ₃ | 55.9 | 18.64‡ | 0.19356 | 5166.4 | 0.69681 |
| (ferrous)..... | Fe ₂ | 55.9 | 27.95 | 0.29035 | 3445.50 | 1.04480 |
| Nickel..... | Ni ₂ | 58.6 | 29.3 | 0.30425 | 3286.80 | 1.09530 |
| Zinc..... | Zn ₂ | 64.9 | 32.45 | 0.33696 | 2967.10 | 1.21330 |
| Lead..... | Pb ₂ | 206.4 | 103.2 | 1.07160 | 933.26 | 3.85780 |
| ELECTRO-NEGATIVE. | | | | | | |
| Oxygen..... | O ₂ | 15.96 | 7.98 | 0.08286 | | |
| Chlorine..... | Cl ₁ | 35.37 | 35.37 | 0.36728 | | |
| Iodine..... | I ₁ | 126.53 | 126.53 | 1.31300 | | |
| Bromine..... | Br ₁ | 79.75 | 79.75 | 0.82812 | | |
| Nitrogen..... | N ₃ | 14.01 | 4.67 | 0.04849 | | |

*Valency is the atom-fixing or atom-replacing power of an element compared with hydrogen, whose valency is unity.

†Atomic weight is the weight of one atom of each element compared with hydrogen, whose atomic weight is unity.

‡Becquerel's extension of Faraday's law showed that the electro-chemical equivalent of an element is proportional to its chemical equivalent. The latter is equal to its combining weight, and not to atomic weight ÷ valency, as defined by Thompson, Hospitalier, and others who have copied their tables. For example, the ferric salt is an exception to Thompson's rule, as are sesqui-salts in general.

Thus: Weight of silver deposited in 10 seconds by a current of 10 amperes = weight of hydrogen liberated per second × number of seconds × current strength × 107.7 = 0.00001038 × 10 × 10 × 107.7 = 0.11178 gram.

Weight of copper deposited in 1 hour by a current of 10 amperes =

$$0.00001038 \times 3600 \times 10 \times 31.5 = 11.77 \text{ grams.}$$

Since 1 ampere per second liberates 0.00001038 gram of hydrogen, strength of current in amperes

$$= \text{weight in grams of H liberated per second} \div 0.00001038$$

$$= \frac{\text{weight of element liberated per second}}{0.00001038 \times \text{chemical equivalent of element}}$$

THE MAGNETIC CIRCUIT.

For units of the magnetic circuit, see page 1346.

Lines and Loops of Force.—It is conventionally assumed that the attractions and repulsions shown by the action of a magnet or a conductor upon iron filings are due to "lines of force" surrounding the magnet or conductor. The "number of lines" indicates the magnitude of the forces acting. As the iron filings arrange themselves in concentric circles, we may assume that the forces may be represented by closed curves or "loops of force." The following assumptions are made concerning the loops of force in a conductive circuit:

1. That the lines or loops of force in the conductor are parallel to the axis of the conductor.

2. That the loops of force external to the conductor are proportional in number to the current in the conductor, that is, a definite current generates a definite number of loops of force. These may be stated as the strength of field in proportion to the current.

3. That the radii of the loops of force are at right angles to the axis of the conductor.

The magnetic force proceeding from a point is equal at all points on the surface of an imaginary sphere described by a given radius about that point. A sphere of radius 1 cm. has a surface of 4π square centimeters. If ϕ = total flux, expressed as the number of lines of force emanating from a magnetic pole having a strength M ,

$$\phi = 4\pi M; M = \phi \div 4\pi.$$

Magnetic moment of a magnet = product of strength of pole M and its length, or distance between its poles L . Magnetic moment = $\phi L \div 4\pi$.

If B = number of lines flowing through each square centimeter of cross-section of a bar-magnet, or the "specific induction," and A = cross-section

$$\text{Magnetic Moment} = LAB \div 4\pi.$$

If the bar-magnet be suspended in a magnetic field of density H and so placed that the lines of the field are all horizontal and at right angles to the axis of the bar, the north pole will be pulled forward, that is, in the direction in which the lines flow, and the south pole will be pulled in the opposite direction, the two forces producing a torsional moment or torque,

$$\text{Torque} = MLH = LABH \div 4\pi, \text{ in dyne-centimeters.}$$

Magnetic attraction or repulsion emanating from a point varies inversely as the square of the distance from that point. The law of inverse squares, however, is not true when the magnetism proceeds from a surface of appreciable extent, and the distances are small, as in dynamo-electric machines and ordinary electromagnets.

The Magnetic Circuit.—In the electric circuit

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Resistance}}, \text{ or } I = \frac{E}{R}; \text{ Amperes} = \frac{\text{volts}}{\text{ohms}}.$$

Similarly, in the magnetic circuit

$$\text{Flux} = \frac{\text{Magnetomotive Force}}{\text{Reluctance}}, \text{ or } \phi = \frac{F}{R}. \quad \text{Maxwells} = \frac{\text{Gilberts}}{\text{Oersteds}}.$$

Reluctance is the reciprocal of permeance, and permeance is equal to permeability \times path area \div path length (metric measure); hence

$$\phi = F\mu a \div l.$$

One ampere-turn produces 1.257 gilberts of magnetomotive force and one inch equals 2.54 centimeters; hence, in inch measure,

$$\phi = (1.257 A_t) \mu 6.45 a \div 2.54 l = 3.192 \mu a A_t \div l.$$

The ampere-turns required to produce a given magnetic flux in a given path will be

$$A_t = \phi l \div 3.192 \mu a = 0.3133 \phi l \div \mu a.$$

Since magnetic flux \div area of path = magnetic density, the ampere-turn required to produce a density B , in lines of force per square inch of area of path, will be

$$A_t = 0.3133 B l \div \mu.$$

This formula is used in practical work, as the magnetic density must be predetermined in order to ascertain the permeability of the material under its working conditions. When a magnetic circuit includes several qualities of material, such as wrought iron, cast iron, and air, it is most direct to work in terms of ampere-turns per unit length of path. The

ampere-turns for each material are determined separately, and the winding is designed to produce the sum of all the ampere-turns. The following table gives the average results from a number of tests made by Dr. Samuel Sheldon:

VALUES OF B AND H

| H | Ampere-turns per cent length. | Ampere-turns per inch length. | Cast Iron. | | Cast Steel. | | Wrought Iron. | | Sheet Metal. | |
|-----|-------------------------------------|-------------------------------------|------------------------|----------------------------------|------------------------|----------------------------------|------------------------|----------------------------------|------------------------|----------------------------------|
| | | | B Kilo- gausses. | Kilomax- wells per sq. in. | B Kilo- gausses. | Kilomax- wells per sq. in. | B Kilo- gausses. | Kilomax- wells per sq. in. | B Kilo- gausses. | Kilomax- wells per sq. in. |
| 10 | 7.95 | 20.2 | 4.3 | 27.7 | 11.5 | 74.2 | 13.0 | 83.8 | 14.3 | 92.2 |
| 20 | 15.90 | 40.4 | 5.7 | 36.8 | 13.8 | 89.0 | 14.7 | 94.8 | 15.6 | 100.7 |
| 30 | 23.85 | 60.6 | 6.5 | 41.9 | 14.9 | 96.1 | 15.3 | 98.6 | 16.2 | 104.5 |
| 40 | 31.80 | 80.8 | 7.1 | 45.8 | 15.5 | 100.0 | 15.7 | 101.2 | 16.6 | 107.1 |
| 50 | 39.75 | 101.0 | 7.6 | 49.0 | 16.0 | 103.2 | 16.0 | 103.2 | 16.9 | 109.0 |
| 60 | 47.70 | 121.2 | 8.0 | 51.6 | 16.5 | 106.5 | 16.3 | 105.2 | 17.3 | 111.6 |
| 70 | 55.65 | 141.4 | 8.4 | 59.2 | 16.9 | 109.0 | 16.5 | 106.5 | 17.5 | 112.9 |
| 80 | 63.65 | 161.6 | 8.7 | 56.1 | 17.2 | 111.0 | 16.7 | 107.8 | 17.7 | 114.1 |
| 90 | 71.60 | 181.8 | 9.0 | 58.0 | 17.4 | 112.2 | 16.9 | 109.0 | 18.0 | 116.1 |
| 100 | 79.50 | 202.0 | 9.4 | 60.6 | 17.7 | 114.1 | 17.2 | 110.9 | 18.2 | 117.3 |
| 150 | 119.25 | 303.0 | 10.6 | 68.3 | 18.5 | 119.2 | 18.0 | 116.1 | 19.0 | 122.7 |
| 200 | 159.0 | 404.0 | 11.7 | 75.5 | 19.2 | 123.9 | 18.7 | 120.8 | 1.96 | 126.5 |
| 250 | 198.8 | 505.0 | 12.4 | 80.0 | 19.7 | 127.1 | 19.2 | 123.9 | 20.2 | 130.2 |
| 300 | 238.5 | 606.0 | 13.2 | 85.1 | 20.1 | 129.6 | 19.7 | 127.1 | 20.7 | 133.5 |

H = 1.257 ampere-turns per cm. = 0.495 ampere-turns per inch.

EXAMPLE.— A magnetic circuit consists of 12 ins. of cast steel of 8 sq. ins. cross-section; 4 ins. of cast iron of 22 sq. ins. cross-section; 3 ins. of sheet iron of 8 sq. ins. cross-section; and two air-gaps each $\frac{1}{16}$ in. long and of 12 sq. ins. area. Required, the ampere-turns to produce a flux of 768,000 maxwells, which is to be uniform throughout the magnetic circuit.

The flux density in the steel is $768,000 \div 8 = 96,000$ maxwells; the ampere-turns per inch of length, according to Sheldon's table, are 60.6, so that the 12 in. of steel will require 727.2 ampere-turns.

The density in the cast iron is $768,000 \div 22 = 34,900$; the ampere-turns $= 4 \times 40 = 160$.

The density in the sheet iron $= 768,000 \div 8 = 96,000$; ampere-turns per inch = 30; total ampere-turns for sheet iron = 90.

The air-gap density is $768,000 \div 12 = 64,000$; ampere-turns per in. = 0.3133B; ampere-turns required for air-gap $= 0.3133 \times 64,000 \div 8 = 2506.4$.

The entire circuit will require $727.2 + 160 + 90 + 2506.4 = 3483.6$ ampere-turns, assuming uniform flux throughout.

In practice there is considerable "leakage" of magnetic lines of force; that is, many of the lines stray away from the useful path, there being no material opaque to magnetism and therefore no means of restricting it to a given path. The amount of leakage is proportional to the permeance of the leakage paths available between two points in a magnetic circuit which are at different magnetic potentials, such as opposite ends of a magnet coil. It is seldom practicable to predetermine with any approach to accuracy the magnetic leakage that will occur under given conditions unless one has profuse data obtained experimentally under similar conditions. In dynamo-electric machines the leakage coefficient varies from 1.3 to 2.

Tractive or Lifting Force of a Magnet.— The lifting power or "pull" exerted by an electro-magnet upon an armature in actual contact with its pole-faces is given by the formula

$$\text{Lbs.} = B^2 a \div 72,134,000,$$

a being the area of contact in square inches and B the magnetic density over this area. If the armature is very close to the pole-faces this formula also applies with sufficient accuracy for all practical purposes, but a considerable air-gap renders it inapplicable.

The design of solenoids for the coil-and-plunger type of electro-magnets

is discussed in a series of articles by C. R. Underhill, in *Elec. World*, April 29, May 13, and Oct. 7, 1905.

Various forms of magnetic chucks are illustrated and described by O. S. Walker, in *Am. Mach.*, Feb. 11, 1909.

For magnets used in hoisting, see page 1169.

Determining the Polarity of Electro-magnets.—If a wire is wound around a magnet in a right-handed helix, the end at which the current flows into the helix is the south pole. If a wire is wound around an ordinary wood-screw, and the current flows around the helix in the direction from the head of the screw to the point, the head of the screw is the south pole. If a magnet is held so that the south pole is opposite the eye of the observer, the wire being wound as a right-handed helix around it, the current flows in a right-handed direction, with the hands of a clock.

Determining the Direction of a Current.—Place a wire carrying a current above and parallel to a pivoted magnetic needle. If the current be flowing along the wire from N. to S., it will cause the N.-seeking pole to turn to the eastward; if it be flowing from S. to N., the pole will turn to the westward. If the wire be below the needle, these motions will be reversed.

Maxwell's rule. The direction of the current and that of the resisting magnetic force are related to each other as are the rotation and the forward travel of an ordinary (right-handed) corkscrew.

DYNAMO-ELECTRIC MACHINES.

There are three classes of dynamo-electric machines, viz.:

1. Generators, for the conversion of mechanical into electrical energy.
2. Motors, for the conversion of electrical into mechanical energy.

Generators and motors are both subdivided into direct-current and alternating-current machines.

3. Transformers, for the conversion of one character or voltage of current into another, as direct into alternating or alternating into direct, or from one voltage into a higher or lower voltage.

Kinds of Dynamo-electric Machines as regards Manner of Winding.

1. *Separately-excited Dynamo.*—The field magnet coils have no connection with the armature-coils, but receive their current from a separate machine or source.

2. *Series-wound Dynamo.*—The field winding and the external circuit are connected in series with the armature winding, so that the entire armature current must pass through the field-coils.

Since in a series-wound dynamo the armature-coils, the field, and the external circuit are in series, any increase in the resistance of the external circuit will decrease the electromotive force from the decrease in the magnetizing currents. A decrease in the resistance of the external circuit will, in a like manner, increase the electromotive force from the increase in the magnetizing current. The use of a regulator avoids these changes in the electromotive force.

3. *Shunt-wound Dynamo.*—The field magnet coils are placed in a shunt to the armature circuit, so that only a portion of the current generated passes through the field magnet coils, but all the difference of potential of the armature acts at the terminals of the field-circuit.

In a shunt-wound dynamo an increase in the resistance of the external circuit increases the electromotive force, and a decrease in the resistance of the external circuit decreases the electromotive force. This is just the reverse of the series-wound dynamo.

In a shunt-wound dynamo a continuous balancing of the current occurs, the current dividing at the brushes between the field and the external circuit in the inverse proportion to the resistance of these circuits. If the resistance of the external circuit becomes greater, a proportionately greater current passes through the field magnets, and so causes the electromotive force to become greater. If, on the contrary, the resistance of the external circuit decreases, less current passes through the field, and the electromotive force is proportionately decreased.

4. *Compound-wound Dynamo.*—The field magnets are wound with two separate sets of coils, one of which is in series with the armature and the external circuit, and the other in shunt with the armature or the external circuit.

Motors.—The above classification in regard to winding applies also to motors.

Moving Force of a Dynamo-electric Machine.—A wire through which a current passes has, when placed in a magnetic field, a tendency to move perpendicular to itself and at right angles to the lines of the field. The force producing this tendency is $P = lBI$ dynes, in which l = length of the wire, I = the current in C.G.S. units, and B = the induction, or flux density, in the field in gaussses or lines per square centimeter.

If the current I is taken in amperes, $P = lBI \div 10 = lBI \times 10^{-1}$.

If P_k is taken in kilograms,

$$P_k = lBI \div 9,810,000 = 10.1937 lBI \times 10^{-8} \text{ kilograms.}$$

EXAMPLE.—The mean strength of field, B , of a dynamo is 5000 C.G.S. lines; a current of 100 amperes flows through a wire; the force acts upon 10 centimeters of the wire = $10.1937 \times 10 \times 100 \times 5000 \times 10^{-8} = 0.5097$ kilograms.

Torque of an Armature.—The torque of an armature is the moment tending to turn it. In a generator it is the moment which must be applied to the armature to turn it in order to produce current. In a motor it is the turning moment which the armature gives to the pulley.

Let I = current in the armature in amperes, E = the electromotive force in volts, T = the torque in pound-feet, ϕ = the flux through the armature in maxwells, N = the number of conductors around the armature, and n = the number of revolutions per second. Then

$$\text{Watts} = IE = 2\pi nT \times 1.356.*$$

In any machine if the flux be constant, E is directly proportional to the speed and = $\phi Nn \div 10^8$; whence

$$\begin{aligned} \phi NI \div 10^8 &= 2\pi T \times 1.356; \\ T &= \frac{\phi NI}{10^8 \times 2\pi \times 1.356} = \frac{\phi NI}{8.52 \times 10^8} \text{ pound-feet.} \end{aligned}$$

Let l = length of armature in inches, d = diameter of armature in inches, B = flux density in maxwells per square inch, and let m = the ratio of the conductors under the influence of the pole-pieces to the whole number of conductors on the armature. Then

$$\phi = \frac{1}{2}\pi d \times l \times B \times m.$$

These formulæ apply to both generators and motors. They show that torque is independent of the speed and varies directly with the current and the flux. The total peripheral force is obtained by dividing the torque by the radius (in feet) of the armature, and the drag on each conductor is obtained by dividing the total peripheral force by the number of conductors under the influence of the pole-pieces at one time.

EXAMPLE.—Given an armature of length $l = 20$ inches, diameter $d = 12$ inches, number of conductors $N = 120$, of which 80 are under the influence of the pole-pieces at one time; let the flux density $B = 30,000$ maxwells per sq. in. and the current $I = 400$ amperes.

$$\phi = \frac{12\pi}{2} \times 20 \times 30,000 \times \frac{80}{120} = 7,540,000.$$

$$T = \frac{7,540,000 \times 120 \times 400}{8.52 \times 100,000,000} = 424.8 \text{ pound-feet.}$$

Total peripheral force = $424.8 \div 0.5 = 849.6$ lbs.

Drag per conductor = $849.6 \div 120 = 7.08$ lbs.

The work done in one revolution = torque \times circumference of a circle of 1 foot radius = $424.8 \times 6.28 = 2670$ foot-pounds.

Let the revolutions per minute equal 500, then the horse-power

$$= \frac{2670 \times 500}{33000} = 40.5 \text{ H.P.}$$

Torque, Horse-power and Revolutions.— T = torque in pound-feet, H.P. = $T \times \text{Rpm.} \times 6.2832 \div 33,000 = IE \div 746$. Whence Torque = $7.0403 EI \div \text{Rpm.}$ or 7 times the watts \div the revs. per min. nearly.

Electromotive Force of the Armature Circuit.—From the horse-power, calculated as above, together with the amperes, we can obtain the E.M.F., for $IE = \text{H.P.} \times 746$, whence E.M.F. or $E = \text{H.P.} \times 746 \div I$.

* 1 ft.-lb. per second = 1.356 watts.

If H.P., as above, = 40.5, and $I = 400$, $E = \frac{40.5 \times 746}{400} = 75.5$ volts.

The E.M.F. may also be calculated by the following formulæ:

I = Total current through armature;

e_a = E.M.F. in armature in volts;

N = Number of active conductors counted all around armature;

p = Number of pairs of poles ($p = 1$ in a two-pole machine);

n = Speed in revolutions per minute;

ϕ = Total flux in maxwells.

$$\text{Electromotive force:} \quad \begin{cases} e_a = \phi N \frac{n}{60} 10^{-8} \text{ for two-pole machines.} \\ e_a = \frac{p\phi N}{10^8} \frac{n}{60} \text{ for multipolar machines with series-wound armature} \end{cases}$$

Strength of the Magnetic Field. — Let I = current in amperes, N = number of turns in the coil, A = area of the cross-section of the core in square centimeters, l = length of core in centimeters, μ the permeability of the core, and ϕ = flux in maxwells. Then

$$\phi = \frac{\text{Magnetomotive Force}}{\text{Reluctance}} = \frac{1.257 NI}{(l + A\mu)}$$

In a dynamo-electric machine the reluctance will be made up of three separate quantities, viz.: that of the field magnet cores, that of the air spaces between the field magnet pole-pieces and the armature, and that of the armature. The total reluctance is the sum of the three. Let L_1 , L_2 , L_3 be the length of the path of magnetic lines in the field magnet cores,* in the air-gaps, and in the armature respectively; and let A_1 , A_2 , A_3 be the areas of the cross-sections perpendicular to the path of the magnetic lines in the field magnet cores, the air-gaps, and the armature respectively. Let the permeability of the field magnet cores be μ_1 , and of the armature μ_3 . The permeability of the air-gaps is taken as unity. Then the total reluctance of the machine will be

$$\frac{L_1}{A_1\mu_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3\mu_3}.$$

$$\text{The flux, } \phi = \frac{1.257 NI}{(L_1 + A_1\mu_1) + (L_2 + A_2) + (L_3 + A_3\mu_3)}.$$

The ampere-turns necessary to create a given flux in a machine may be found by the formula,

$$NI = \phi \frac{[(L_1 + A_1\mu_1) + (L_2 + A_2) + (L_3 + A_3\mu_3)]}{1.257}.$$

But the total flux generated by the field coils is not available to produce current in the armature. There is a leakage between the field magnets, and this must be allowed for in calculations. The leakage coefficient varies from 1.3 to 2 in different machines. The meaning of the coefficient is that if a flux of say 100 maxwells per square cm. are desired in the field coils, it will be necessary to provide ampere turns for $1.3 \times 100 = 130$ maxwells, if the leakage coefficient be 1.3.

Another method of calculating the ampere-turns necessary to produce a given flux is to calculate the magnetomotive force required in each portion of the machine, separately, introducing the leakage coefficient in the calculation for the field magnets, and dividing the sum of the magnetomotive forces by 1.257.

In the ordinary type of multipolar machine there are as many magnetic circuits as there are poles. Each winding energizes part of two circuits. The calculation is made in the same manner as for a single magnetic circuit.

*The length of the path in the field magnet cores L_1 includes that portion of the path which lies in the piece joining the cores of the various field magnets.

ALTERNATING CURRENTS.*

The advantages of alternating over direct currents are: 1. Greater simplicity of dynamos and motors, no commutators being required; 2. The feasibility of obtaining high voltages, by means of static transformers, for cheapening the cost of transmission; 3. The facility of transforming from one voltage to another, either higher or lower, for different purposes.

A direct current is uniform in strength and direction, while an alternating current rapidly rises from zero to a maximum, falls to zero, reverses its direction, attains a maximum in the new direction, and again returns to zero. This series of changes can best be represented by a curve the abscissas of which represent time and the ordinates either current or electromotive force (e.m.f.). The curve usually chosen for this purpose is the sine curve, Fig. 172; the best forms of alternators give a curve that is a very close approximation to the sine curve, and all calculations and deductions of formulæ are based on it. The equation of the sine curve is $y = \sin x$, in which y is any ordinate, and x is the angle passed over by a moving radius vector.

After the flow of a direct current has been once established, the only opposition to the flow is the resistance offered by the conductor to the passage of current through it. This resistance of the conductor, in treating of alternating currents, is sometimes spoken of as *ohmic resistance*. The word resistance, used alone, always means the ohmic resistance. In alternating currents, in addition to the resistance, several other quantities, which affect the flow of current, must be taken into consideration. These quantities are inductance, capacity, and skin effect. They are discussed under separate headings.

The current and the e.m.f. may be in phase with each other, that is, they may attain their maximum strength at the same instant, or they may not, depending on the character of the circuit. In a circuit containing only resistance, the current and e.m.f. are in phase; in a current containing inductance the e.m.f. attains its maximum value before the current, or leads the current. In a circuit containing capacity the current leads the e.m.f. If both capacity and inductance are present in a circuit, they will tend to neutralize each other.

Maximum, Average, and Effective Values.—The strength and the e.m.f. of an alternating current being constantly varied, the maximum value of either is attained only for an instant in each period. The maximum values are little used in calculations, except in deducing formulæ and for proportioning insulation, which must stand the maximum pressure.

The average value is obtained by averaging the ordinates of the sine curve representing the current, and is $2 \div \pi$ or 0.637 of the maximum value.

The value of greatest importance is the effective, or "square root of the mean square," value. It is obtained by taking the square root of the mean of the squares of the ordinates of the sine curve. The effective value is the value shown on alternating-current measuring instruments. The product of the square of the effective value of the current and the resistance of the circuit is the heat lost in the circuit.

The comparison of the maximum, average, and effective values is as follows:

$E_{\text{Effec.}} = E_{\text{Max.}} \times 0.707$; $E_{\text{Aver.}} = E_{\text{Max.}} \times 0.637$; $E_{\text{Max.}} = 1.41 \times E_{\text{Effec.}}$

Frequency.—The time required for an alternating current to pass through one complete cycle, as from one maximum point to the next (a and b , Fig. 172), is termed the period. The number of periods in a second is termed the frequency of the current. Since the current changes its direction twice in each period, the number of reversals or alternations is

*Only a very brief treatment of the subject of alternating currents can be given in this book. The following works are recommended as valuable for reference: *Alternating Currents and Alternating Current Machinery*, by D. C. and J. P. Jackson; *Standard Polyphase Apparatus and Systems*, by M. A. Oudin; *Polyphase Electric Currents*, by S. P. Thompson; *Electric Lighting*, by F. B. Crocker, 2 vols.; *Electric Power Transmission*, by Louis Bell; *Alternating Currents*, by Bedell and Crehore; *Alternating-current Phenomena*, by Chas. P. Steinmetz. The two last named are highly mathematical.

double the frequency. A current of 120 alternations per second has a period of $\frac{1}{60}$ and a frequency of 60. The frequency of a current is equal to one-half the number of poles on the generator, multiplied by the number of revolutions per second. Frequency is denoted by the letter f .

The frequencies most generally used in the United States are 25, 40, 60, 125, and 133 cycles per second. The Standardization Report of the A. I. E. E. recommends the adoption of three frequencies, viz. 25, 60 and 120.

With the higher frequencies both transformers and conductors will be less costly in a circuit of a given resistance but the capacity and inductance effects in each will be increased, and these tend to increase the cost. With high frequencies it also becomes difficult to operate alternators in parallel.

A low frequency current cannot be used on lighting circuits, as the lights will flicker when the frequency drops below a certain figure. For arc lights the frequency should not be less than 40. For incandescent lamps it should not be less than 25. If the circuit is to supply both power and light a frequency of 60 is usually desirable. For power transmission to long distances a low frequency, say 25, is considered desirable, in order to lessen the capacity effects. If the alternating current is to be converted into direct current for lighting purposes a low frequency may be used, as the frequency will then have no effect on the lights.

Inductance.—Inductance is that property of an electrical circuit by which it resists a change in the current. A current flowing through a conductor produces a magnetic flux around the conductor. If the current be changed in strength or direction, the flux is also changed, producing in the conductor an e.m.f. whose direction is opposed to that of the current in the conductor. This counter e.m.f. is the *counter e.m.f. of inductance*. It is proportional to the rate of change of current, provided that the permeability of the medium around the conductor remains constant. The unit of inductance is the *henry*, symbol L . A circuit has an inductance of one henry if a uniform variation of current at the rate of one ampere per second produces a counter e.m.f. of one volt.

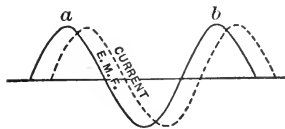


FIG. 198.

The effect of inductance on the circuit is to cause the current to lag behind the e.m.f. as shown in Fig. 198, in which abscissas represents time, and ordinates represent e.m.f. and current strengths respectively.

Capacity.—Any insulated conductor has the power of holding a quantity of static electricity. This power is termed the *capacity* of the body. The capacity of a circuit is measured by the quantity of electricity in it when at unit potential. It may be increased by means of a condenser. A condenser consists of two parallel conductors, insulated from each other by a non-conductor. The conductors are usually in sheet form.

The unit of capacity is a *farad*, symbol C . A condenser has a capacity of one farad when one coulomb of electricity contained in it produces a difference of potential of one volt, or when a rate of change of pressure of one volt per second produces a current of one ampere. The farad is too large a unit to be conveniently used in practice, and the micro-farad or one-millionth of a farad is used instead.

The effect of capacity on a circuit is to cause the e.m.f. to lag behind the current. Both inductance and capacity may be measured with a Wheatstone bridge by substituting for a standard resistance a standard of inductance or a standard of capacity.

Power Factor.—In direct-current work the power, measured in watts, is the product of the volts and amperes in the circuit. In alternating-current work this is only true when the current and e.m.f. are in phase. If the current either lags or leads, the values shown on the volt and ammeters will not be true simultaneous values. Referring to Fig. 172, it will be seen that the product of the ordinates of current and e.m.f. at any particular instant will not be equal to the product of the effective values which are shown on the instruments. The power in the circuit at any instant is the product of the simultaneous values of current and e.m.f., and the volts and amperes shown on the recording instruments must be multiplied together and their product multiplied by a power factor before the true

watts are obtained. This power factor, which is the ratio of the volt-amperes to the watts, is also the cosine of the angle of lag or lead of the current. Thus

$$P = I \times E \times \text{power factor} = I \times E \times \cos \theta,$$
where θ is the angle of lag or lead of the current.

A watt-meter, however, gives the true power in a circuit directly. The method of obtaining the angle of lag is shown below, in the section on Impedance Polygons.

Reactance, Impedance, Admittance.—In addition to the ohmic resistance of a circuit there are also resistances due to inductive, capacity, and skin effect. The virtual resistance due to inductance and capacity is termed the reactance of the circuit. If inductance only be present in circuit, the reactance will vary directly as the inductance. If capacity only be present, the reactance will vary inversely as the capacity.

Inductive reactance = $2 \pi fL$.
Condensive reactance = $\frac{1}{2 \pi fC}$.

The total apparent resistance of the circuit, due to both the ohmic resistance and the total reactance, is termed the impedance, and is equal to the square root of the sum of the squares of the resistance and the reactance.

Impedance = $Z = \sqrt{R^2 + (2 \pi fL)^2}$ when inductance is present in the circuit.

Impedance = $Z = \sqrt{R^2 + \left(\frac{1}{2 \pi fC}\right)^2}$ when capacity is present in the circuit.

Admittance is the reciprocal of impedance, = $1 \div Z$.
If both inductance and capacity are present in the circuit, the reactance of one tends to balance that of the other; the total reactance is the algebraic sum of the two reactances; thus,

Total reactance = $X = 2 \pi fL - \frac{1}{2 \pi fC}$; $Z = \sqrt{R^2 + \left(2 \pi fL - \frac{1}{2 \pi fC}\right)^2}$.
In all cases the tangent of the angle of lag or lead is the reactance divided by the resistance. In the last case

$$\tan \theta = \frac{2 \pi fL - \frac{1}{2 \pi fC}}{R}.$$

Skin Effect.—Alternating currents tend to have a greater density at the surface than at the axis of a conductor. The effect of this is to make the virtual resistance of a wire greater than its true ohmic resistance. With low frequencies and small wires the skin effect is small, but it becomes quite important with high frequencies and large wires.

The skin effect factor, by which the ohmic resistance is to be multiplied to obtain the virtual resistance, for different sizes of wire and frequencies is as follows:

| Wire No. | 0 | 00 | 000 | 0000 | 1/2 in. | 3/4 in. | 1 in. |
|--------------------------|-------|-------|-------|-------|---------|---------|-------|
| 25 cycles, factor | | | | 1.001 | 1.002 | 1.007 | 1.020 |
| 60 cycles, factor | 1.001 | 1.002 | 1.005 | 1.006 | 1.008 | 1.040 | 1.111 |
| 130 cycles, factor | 1.008 | 1.010 | 1.017 | 1.027 | 1.039 | 1.156 | 1.397 |

Ohm's Law applied to Alternating-Current Circuits.—To apply Ohm's law to alternating-current circuits a slight change is necessary in the expression of the law. Impedance is substituted for resistance. The law should read

$$I = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{Z}.$$

Impedance Polygons.—1. *Series Circuits.*—The impedance of a circuit can be determined graphically as follows. Suppose a circuit to contain a resistance R and an inductance L , and to carry a current I of frequency f . In Fig. 199 draw the line ab proportional to R , and representing the direction of current. At b erect bc perpendicular to ab and proportional to $2 \pi fL$. Join a and c . The line ac represents the impedance of the circuit. The angle θ between ab and ac is the angle of lag of the cur-

rent behind the e.m.f., and the power factor of the circuit is cosine θ . The e.m.f. of the circuit is $E = IZ$.

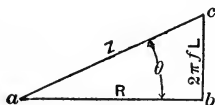


FIG. 199.

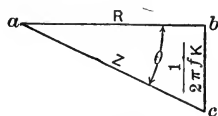


FIG. 200.

If the above circuit contained, instead of the inductance L , a capacity C , then would the polygon be drawn as in Fig. 200. The line bc would be proportional to $\frac{1}{2\pi fC}$ and would be drawn in a direction opposite to that of bc in Fig. 199. The impedance would again be Z , the e.m.f. would be $Z \times I$, but the current would lead the e.m.f. by the angle θ .

Suppose the circuit to contain resistance, inductance, and capacity. The lines of the impedance polygon would then be laid off as in Fig. 201. The impedance of the circuit would be represented by ad , and the angle of lag by θ . If the capacity of the circuit had been such that cd was less than bc , then would the e.m.f. have led the current.

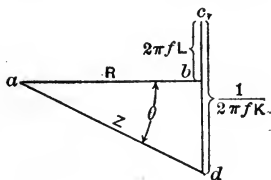


FIG. 201.

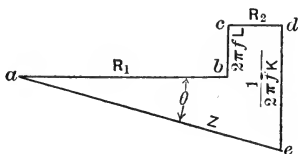


FIG. 202.

If between the inductance and capacity in the circuit in the previous examples there be interposed another resistance, the impedance polygon will take the form of Fig. 202. The lines representing either resistances, inductances, or capacities in the circuit follow each other in all cases as do the resistances, inductances, and capacities in the circuit, each line having its appropriate direction and magnitude.

EXAMPLE.—A circuit (Fig. 203) contains a resistance, R_1 , of 15 ohms, a capacity, C_1 , of 100 microfarads (0.000100 farad), a resistance, R_2 , of 12

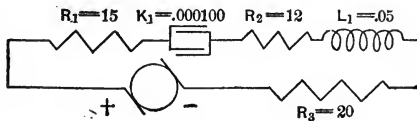


FIG. 203.

ohms, and inductance of L_1 , of 0.05 henry, and a resistance R_3 , of 20 ohms. Find the impedance and electromotive force when a current of 2 amperes is sent through the circuit, and the current when e.m.f. of 120 volts is impressed on the circuit, frequency being taken as 60. Also find the angle of lag, the power factor, and the power in the circuit when 120 volts are impressed.

The resistance is represented in Fig. 204 by the horizontal line ab , 15

units long. The capacity is represented by the line bc , drawn downwards from b and whose length is

$$\frac{1}{2\pi f C_1} = \frac{1}{2 \times 3.1416 \times 60 \times 0.0001} = 26.55.$$

From the point c a horizontal line cd , 12 units long, is drawn to represent R_2 . From the point d the line de is drawn vertically upwards to represent the inductance L_1 . Its length is

$$2\pi f L_1 = 2 \times 3.1416 \times 60 \times 0.05 = 18.85.$$

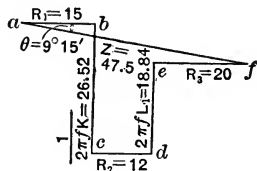


FIG. 204.

From the point e a horizontal line ef , 20 units long, is drawn to represent R_3 . The line adjoining a and f will represent the impedance of the circuit in ohms. The angle θ , between ab and af , is the angle of lag of the e.m.f. behind the current. The impedance in this case is 47.5 ohms, and the angle of lag is $9^\circ 15'$.

The e.m.f. when a current of 2 amperes is sent through is

$$IZ = E = 2 \times 47.5 = 95 \text{ volts.}$$

If an e.m.f. of 120 volts be impressed on the circuit, the current flowing through will be

$$I = \frac{120}{Z} = \frac{120}{47.5} = 2.53 \text{ amperes.}$$

The power factor = $\cos \theta = \cos 9^\circ 15' = 0.987$.

The power in the circuit at 120 volts is

$$I \times E \times \cos \theta = 2.53 \times 120 \times 0.987 = 299.6 \text{ watts.}$$

2. *Parallel Circuits.*—If two circuits be arranged in parallel, the current flowing in each circuit will be inversely proportional to the impedance of that circuit. The e.m.f. of each circuit is the e.m.f. across the terminals at either end of the main circuit, where the various branches separate. Consider a circuit, Fig. 205, consisting of two branches. The first branch contains a resistance R_1 and an inductance L_1 in series with it. The second branch contains a resistance R_2 in series with an inductance L_2 . The impedance of the circuit may be determined by treating each of the two branches as a separate series circuit, and drawing the impedance polygon for each branch on that assumption. Having found the impedance the current flowing in either branch will be the reciprocal of the impedance multiplied by the e.m.f. across the terminals. The current in the entire circuit is the geometrical sum of the current in the two branches.

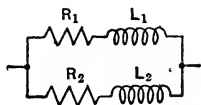


FIG. 205.

The admittance of the equivalent simple circuit may be obtained by drawing a parallelogram, two of whose adjoining sides are made parallel to the impedance lines of each branch and equal to the two admittances respectively.

The diagonal of the parallelogram will represent the admittance of the equivalent simple circuit. The admittance multiplied by the e.m.f. gives the total current in the circuit.

EXAMPLE.—Given the circuit in Fig. 206, consisting of two branches. Branch 1 consists of a resistance $R_1 = 12$ ohms, an inductance $L_1 = 0.05$ henry, a resistance $R_2 = 4$ ohms, and a capacity $C_1 = 120$ microfarads (0.00012 farad). Branch 2 consists of an inductance $L_2 = 0.015$ henry, a resistance $R_3 = 10$ ohms, and an inductance $L_3 = 0.03$ henry. An e.m.f. of 100 volts is impressed on the circuit at a frequency of 60. Find the admittance of the entire circuit, the current, the power factor, and the power in the circuit. Construct the impedance polygons for the two branches separately as shown in Fig. 207, a and b . The impedance in branch 1 is 16.4 ohms, and the current is $(1/16.4) \times 100 = 6.19$ amperes. The angle of lead of the current is $1^\circ 45'$. The impedance in branch 2 is 19.5 ohms and the current is $(1/19.5) \times 100 = 5.13$ amperes. The angle of lag of the current is 61° .

The current in the entire circuit is found by taking the admittances of

the two branches, and drawing them from the point *o*, in Fig. 207 *c*, parallel to the impedance lines in their respective polygons. The diagonal from *o* is the admittance of the entire circuit, and in this case is equal to 0.092.

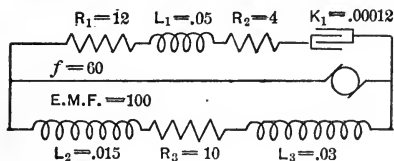


FIG. 206.

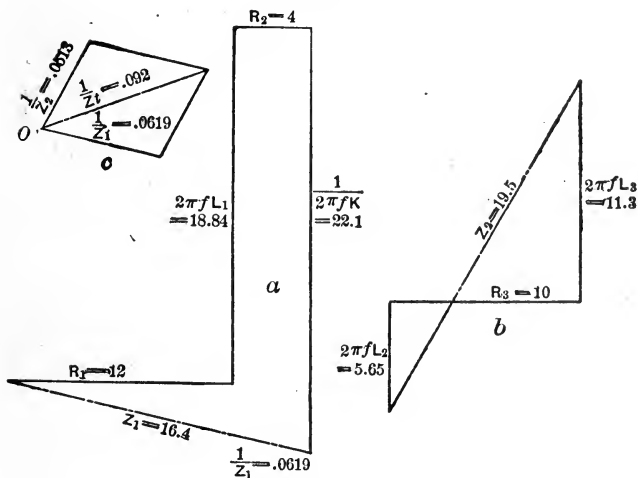


FIG. 207.

The current in the circuit is $0.092 \times 100 = 9.2$ amperes. The power factor is 0.944 and the power in the circuit is $100 \times 0.944 \times 9.2 = 868.48$ watts.

Self-Inductance of Lines and Circuits.—The following formulæ and table, taken from Crocker's "Electric Lighting," give a method of calculating the self-inductance of two parallel aerial wires forming part of the same circuit and composed of copper, or other non-magnetic material:

$$L \text{ per foot} = \left(15.24 + 140.3 \log \frac{2A}{d} \right) 10^{-9}.$$

$$L \text{ per mile} = \left(80.5 + 740 \log \frac{2A}{d}\right) 10^{-6}.$$

in which L is the inductance in henrys of each wire, A is the interaxial distance between the two wires, and d is the diameter of each, both in inches. If the circuit is of iron wire, the formulæ become

$$L \text{ per foot} = \left(2286 + 140.3 \log \frac{2A}{d} \right) 10^{-9}.$$

$$L \text{ per mille} = \left(12070 + 740 \log \frac{2A}{d} \right) 10^{-6},$$

INDUCTANCE, IN MILLIHENRYS PER MILE, FOR EACH OF TWO PARALLEL COPPER WIRES.

| Interaxial Distance, Ins. | American Wire Gauge Number. | | | | | | | | | | | |
|---------------------------|-----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 0000 | 000 | 00 | 0 | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 |
| 6 | 1.130 | 1.168 | 1.205 | 1.242 | 1.280 | 1.317 | 1.354 | 1.392 | 1.466 | 1.540 | 1.615 | 1.690 |
| 12 | 1.353 | 1.391 | 1.428 | 1.465 | 1.502 | 1.540 | 1.577 | 1.614 | 1.689 | 1.764 | 1.838 | 1.913 |
| 24 | 1.576 | 1.614 | 1.651 | 1.688 | 1.725 | 1.764 | 1.800 | 1.838 | 1.912 | 1.986 | 2.061 | 2.135 |
| 36 | 1.707 | 1.745 | 1.784 | 1.818 | 1.856 | 1.893 | 1.931 | 1.968 | 2.043 | 2.117 | 2.192 | 2.266 |
| 60 | 1.871 | 1.909 | 1.946 | 1.982 | 2.023 | 2.058 | 2.095 | 2.132 | 2.208 | 2.282 | 2.356 | 2.432 |
| 96 | 2.023 | 2.059 | 2.097 | 2.134 | 2.172 | 2.210 | 2.246 | 2.283 | 2.358 | 2.433 | 2.507 | 2.582 |

Capacity of Conductors. — All conductors are included in three classes, viz.: 1. Insulated conductors with metallic protection; 2. Single aerial conductor with earth return; 3. Metallic circuit consisting of two parallel aerial wires. The capacity of the lines may be calculated by means of the following formulæ taken from Crocker's "Electric Lighting."

$$\text{Class 1. } C \text{ per foot} = \frac{7361 k 10^{-15}}{\log(D \div d)}, \quad C \text{ per mile} = \frac{38.83 k 10^{-9}}{\log(D \div d)}.$$

$$\text{Class 2. } C \text{ per foot} = \frac{7361 \times 10^{-15}}{\log(4h \div d)}, \quad C \text{ per mile} = \frac{38.83 \times 10^{-9}}{\log(4h \div d)}.$$

$$\text{Class 3. } \begin{cases} C \text{ per foot of each wire} = \frac{3681 \times 10^{-15}}{\log(2A \div d)}, \\ C \text{ per mile of each wire} = \frac{19.42 \times 10^{-9}}{\log(2A \div d)}. \end{cases}$$

In which C is the capacity in farads, D the internal diameter of the metallic covering, d the diameter of the conductor, h the height of the conductor above the ground, and A the interaxial distance between two parallel wires all in inches; k is a dielectric constant which for air is equal to 1 and for pure rubber is equal to 2.5. The formulæ in classes 2 and 3 assume the wires to be bare. If they are insulated, k must be introduced in the numerator and given a value slightly greater than 1.

Single-phase and Polyphase Currents. — A single-phase current is a simple alternating current carried on a single pair of wires, and is generated on a machine having a single armature winding. It is represented by a single sine curve.

Polyphase currents are known as two-phase, three-phase, six-phase, or any other number, and are represented by a corresponding number of sine curves. The most commonly used systems are the two-phase and three-phase.

1. **Two-phase Currents.** — In a two-phase system there are two single-phase alternating currents bearing a definite time relation to each other and represented by two sine curves (Fig. 208).

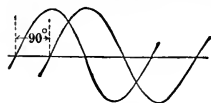


FIG. 208.

The two separate currents may be generated by the same or by separate machines. If by separate machines, the armatures of the two should be positively coupled together. Two-phase currents are usually generated by a machine with two armature windings, each winding terminating in two collector rings. The two windings are so related that the two currents will be 90° apart. For this reason two phase-currents are also called "quarter-phase" currents.

Two-phase currents may be distributed on either three or four wires. The three-wire system of distribution is shown in Fig. 209. One of the return wires is dispensed with, connection being made across to the other as shown. The common return wire should be made 1.41 times the area of either of the other two wires, these two being equal in size.

The four-wire system of distribution is shown in Fig. 210. The two phases are entirely independent, and for lighting purposes may be operated as two single-phase circuits.

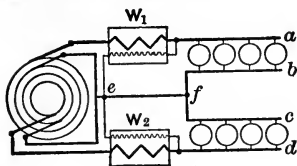


FIG. 209.

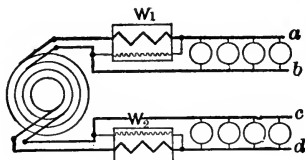


FIG. 210.

2. *Three-phase Currents.*—Three-phase currents consist of three alternating currents, differing in phase by 120° , and represented by three sine curves, as in Fig. 211. They may be distributed by three or six wires. If distributed by the six-wire system, it is analogous to the four-wire, two-phase system, and is equivalent to three single-phase circuits. In the three-wire system of distribution the circuits may be connected in two different ways, known respectively as the Y or star connection, and the Δ (delta) or mesh connection.

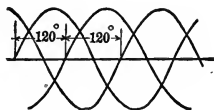


FIG. 211.

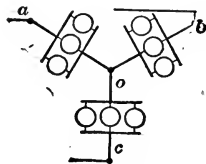


FIG. 212.

The Y connection is shown in Fig. 212. The three circuits are joined at the point o , known as the neutral point, and the three wires carrying the current are connected at the points a , b , and c , respectively. If the three circuits ao , bo , and co are composed of lights, they must be equally loaded or the lights will fluctuate. If the three circuits are perfectly balanced, the lights will remain steady. In this form of connection each wire may be considered as the return wire for the other two. If the three circuits are unbalanced, a return wire may be run from the neutral point o to the neutral point of the armature winding on the generator. The system will then be four-wire, and will work properly with unbalanced circuits.

The Δ connection is shown in Fig. 213. Each of the three circuits ab , ac , bc , receives the current due to a separate coil in the armature winding. This form of connection will work properly even if the circuits are unbalanced; and if the circuit contains lamps, they will not fluctuate when the circuit changes from a balanced to an unbalanced condition, or *vice versa*.

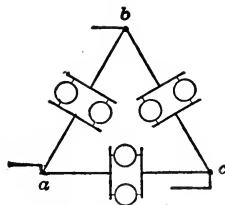


FIG. 213.

Measurement of Power in Polyphase Circuits.—1. *Two-phase Circuits.*—The power of two-phase currents distributed by four wires may be measured by two wattmeters introduced into the circuit as shown in Fig. 210. The sum of the readings of the two instruments is the total power. If but one wattmeter is available, it should be introduced first in one circuit, and then in the other. If the current or e.m.f. does not vary during the operation, the result will be correct. If the circuits are perfectly balanced, twice the reading of one wattmeter will be the total power.

The power of two-phase currents distributed by three wires may be measured by two wattmeters as shown in Fig. 209. The sum of the two readings is the total power. If but one wattmeter is available, the coarse-wire coil should be connected in series with the wire *ef* and one extremity of the pressure-coil should be connected to some point on *ef*. The other end should be connected first to the wire *a* and then to the wire *d*, a reading being taken in each position of the wire. The sum of the readings gives the power in the circuits.

2. *Three-phase Circuits.*—The power in a three-phase circuit may be measured by three wattmeters, connected as in Fig. 214 if the system is Y-connected, and as in Fig. 215 if the system is Δ -connected. The sum of

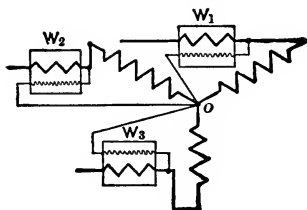


FIG. 214.

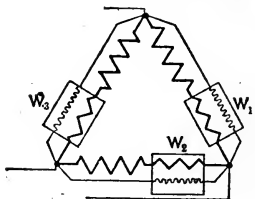


FIG. 215.

the wattmeter readings gives the power in the system. If the circuits are perfectly balanced, three times the reading of one wattmeter is the total power.

The power in a Δ -connected system may be measured by two wattmeters, as shown in Fig. 216. If the power factor of the system is greater than 0.50, the arithmetical sum of the readings is the power in the circuit. If the power factor is less than 0.50, the arithmetical difference of the readings is the power. Whether the power factor is greater or less than 0.50 may be discovered by interchanging the wattmeters without disturbing the relative connection of their coarse- and fine-wire coils. If the deflections of the needles are reversed, the difference of the readings is the power. If the needles are deflected in the same direction as at first, the sum of the readings is the power.

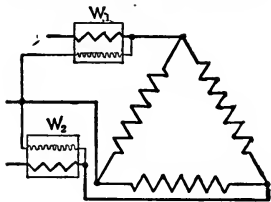


FIG. 216.

Alternating-current Generators.—These differ little from direct current generators in many respects. Any direct-current generator, if provided with collector rings instead of a commutator, could be used as a single-phase alternator. The frequency would in most cases, however, be too low for any practical use. The fields of alternators are always separately excited; the machines are

sometimes compounded by shunting some of their own current around the fields through a rectifying device which changes the current to pulsating direct current. In all large machines the armature is stationary and the field magnets revolve.

ALTERNATING-CURRENT CIRCUITS.

Calculation of Alternating-current Circuits.—The following formulæ and tables are issued by the General Electric Co. They afford a convenient method of calculating the sizes of conductors for, and determining the losses in, alternating-current circuits. They apply only to circuits in which the conductors are spaced 18 inches apart, but a slight increase or decrease in this distance does not alter the figures appreciably. If the conductors are less than 18 inches apart, the loss of voltage is decreased, and vice versa.

Let W = total power delivered in watts;

D = distance of transmission (one way) in feet;

P^* = per cent loss of delivered power (W);

E = voltage between main conductors at consumer's end of circuit;

K = a constant; for continuous current = 2160;

T = a variable depending on the system and nature of the load; for continuous current = 1;

M = a variable, depending on the size of wire and frequency; for continuous current = 1;

A = a factor; for continuous current = 6.04.

$$\text{Area of conductor, circular mils} = \frac{D \times W \times K}{P \times E^2};$$

$$\text{Current in main conductors} = W \times T \div E$$

$$\text{Volts lost in lines} = P \times E \times M \div 100;$$

$$\text{Pounds copper} = \frac{D^2 \times W \times K \times A}{P \times E^2 \times 1,000,000}.$$

The following tables give values for the various constants:

VALUES OF M — WIRES 18 IN. APART.*

| Power Factors— | 25 Cycles | | | | 40 Cycles | | | | 60 Cycles | | | | 125 Cycles | | | |
|----------------------|-----------|------|------|------|--|------|------|------|-----------|------|------|------|------------|------|------|------|
| | .95 | .90 | .85 | .80 | .95 | .90 | .85 | .80 | .95 | .90 | .85 | .80 | .95 | .90 | .85 | .80 |
| Wire Sizes. | | | | | | | | | | | | | | | | |
| 0000 | 1.17 | 1.16 | 1.12 | 1.06 | 1.32 | 1.36 | 1.36 | 1.32 | 1.53 | 1.64 | 1.67 | 1.66 | 2.21 | 2.54 | 2.72 | 2.76 |
| 000 | 1.12 | 1.09 | 1.05 | .99 | 1.24 | 1.26 | 1.24 | 1.19 | 1.41 | 1.49 | 1.50 | 1.47 | 1.97 | 2.22 | 2.34 | 2.37 |
| 00 | 1.08 | 1.04 | .99 | .92 | 1.18 | 1.18 | 1.14 | 1.09 | 1.32 | 1.36 | 1.35 | 1.31 | 1.77 | 1.96 | 2.04 | 2.04 |
| 0 | 1.05 | 1.00 | .94 | .87 | 1.13 | 1.11 | 1.06 | 1.01 | 1.24 | 1.26 | 1.24 | 1.19 | 1.61 | 1.74 | 1.80 | 1.79 |
| 1 | 1.02 | .96 | .90 | .83 | 1.09 | 1.05 | 1.00 | .94 | 1.18 | 1.17 | 1.14 | 1.08 | 1.47 | 1.57 | 1.59 | 1.56 |
| 2 | 1.00 | .93 | .86 | .79 | 1.05 | 1.01 | .95 | .88 | 1.12 | 1.10 | 1.06 | 1.00 | 1.37 | 1.42 | 1.42 | 1.39 |
| 3 | .98 | .91 | .84 | .76 | 1.02 | .97 | .90 | .83 | 1.08 | 1.05 | .99 | .93 | 1.27 | 1.30 | 1.25 | 1.24 |
| 4 | .96 | .89 | .81 | .74 | 1.00 | .94 | .86 | .80 | 1.05 | 1.00 | .94 | .87 | 1.20 | 1.21 | 1.18 | 1.13 |
| 5 | .95 | .88 | .80 | .72 | .98 | .92 | .84 | .77 | 1.02 | .97 | .90 | .83 | 1.15 | 1.13 | 1.09 | 1.03 |
| 6 | .94 | .86 | .78 | .70 | .97 | .90 | .82 | .74 | 1.00 | .94 | .87 | .79 | 1.10 | 1.07 | 1.02 | .96 |
| 7 | .94 | .85 | .77 | .69 | .95 | .88 | .80 | .72 | .98 | .91 | .84 | .76 | 1.06 | 1.02 | .96 | .90 |
| 8 | .93 | .85 | .76 | .68 | .94 | .87 | .79 | .71 | .97 | .89 | .82 | .74 | 1.03 | .98 | .92 | .85 |
| 9 | .92 | .84 | .76 | .67 | .94 | .86 | .77 | .69 | .95 | .88 | .80 | .72 | 1.01 | .95 | .88 | .81 |
| 10 | .92 | .83 | .75 | .67 | .93 | .85 | .76 | .68 | .94 | .86 | .79 | .71 | .99 | .92 | .85 | .78 |
| Wires 36 in. Apart.† | 1.22 | 1.23 | 1.20 | 1.15 | $M = \left(1 + \frac{X}{R} \tan \alpha\right) \cos^2 \alpha.$ X = Reactance. R = Resistance, ohms per 1000 ft. at 60° F. (Wire 100% Matthiessen's standard.) $X = 0.000882 \left[\log_{10} \left(\frac{d}{r}\right) + 0.109 \right]$ in which d = inches between wires, r = radius of wire, inches. f = cycles per sec. | | | | | | | | | | | |
| 0000 | 1.16 | 1.15 | 1.11 | 1.05 | | | | | | | | | | | | |
| 000 | 1.11 | 1.08 | 1.04 | .97 | | | | | | | | | | | | |
| 00 | 1.07 | 1.03 | .98 | .91 | | | | | | | | | | | | |
| 0 | 1.04 | .99 | .93 | .86 | | | | | | | | | | | | |
| 1 | 1.02 | .95 | .89 | .82 | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | | | |

* As corrected by Harold Pender, see *Elect. World*, July 1, 1905. The formula for M is approximate, and gives values correct within 2% for any case likely to arise in practice.

| Per cent of Power Factor. | Value of K . | | | | Value of T . | | | | Value of A . |
|---------------------------|----------------|------|------|------|----------------|------|------|------|----------------|
| | 100 | 95 | 85 | 80 | 100 | 95 | 85 | 80 | |
| System: | | | | | | | | | |
| Single-phase..... | 2160 | 2400 | 3000 | 3380 | 1.00 | 1.05 | 1.17 | 1.25 | 6.04 |
| Two-phase, 4-wire..... | 1080 | 1200 | 1500 | 1690 | 0.50 | 0.53 | 0.59 | 0.62 | 12.08 |
| Three-phase, 3-wire..... | 1080 | 1200 | 1500 | 1690 | 0.58 | 0.61 | 0.68 | 0.72 | 9.06 |

* P should be expressed as a whole number, not as a decimal; thus a 5 per cent loss should be written 5 and not .05.

Relative Weight of Copper Required in Different Systems for Equal Effective Voltages.

| | |
|---|-------|
| Direct current, ordinary two-wire system..... | 1.000 |
| “ “ three-wire system, all wires same size..... | 0.375 |
| “ “ “ neutral one-half size..... | 0.313 |
| Alternating current, single-phase two-wire, and two-phase four-wire..... | 1.000 |
| Two-phase three-wire, voltage between outer and middle wire same as in single-phase two-wire..... | 0.729 |
| “ “ voltage between two outer wires same..... | 1.457 |
| Three-phase three-wire..... | 0.750 |
| “ “ four-wire..... | 0.333 |

The weight of copper is inversely proportional to the squares of the voltages, other things being equal. The maximum value of an alternating e.m.f. is 1.41 times its effective rating. For derivation of the above figures see Crocker's Electric Lighting, vol. ii.

Approximate Rule for Size of Wires for Three-Phase Transmission Lines. (General Electric Co.)

The table given below is for use in making rough estimates for the sizes of wires for three-phase transmission, as in the following example.

Required.—The size of wires to deliver 500 Kw. at 6000 volts, at the end of a three-phase line 12 miles long, allowing an energy loss of 10% and a power factor of 85%. If the example called for the transmission of 100 Kw. (on which the table is based), we should look in the 6000-volt column for the nearest figure to the given distance, and take the size of wire corresponding. But the example calls for the transmission of five times this amount of power, and the size of wire varies directly as the distance, which in this case is 12 miles. Therefore we look for the product $5 \times 12 = 60$ in the 6000-volt column of the table. The nearest value is 60.44 and the size of wire corresponding is No. 00, which is, therefore, the size capable of transmitting 100 Kw. over a line 60.44 miles long, or 500 Kw. over a line 12 miles long, as required by the example.

If it is desired to ascertain the size of wire which will give an energy loss of 5%, or one-half the loss for which the table is computed, it is only necessary to multiply the value obtained by 2, since the area varies inversely as the per cent energy loss.

DISTANCES TO WHICH 100 KW. THREE-PHASE CURRENT CAN BE TRANSMITTED OVER DIFFERENT SIZES OF WIRES AT DIFFERENT POTENTIALS, ASSUMING AN ENERGY LOSS OF 10% AND A POWER FACTOR OF 85%.

| Number B. & S. | Area in Circular Mils. | Distance of Transmission for Various Potentials at Receiving End, in feet | | | | | | | | | | | |
|-------------------|------------------------------|---|-------|--------|--------|--------|--------|--------|--------|---------|--------|---------|--------|
| | | 2,000 | 3,000 | 4,000 | 5,000 | 6,000 | 8,000 | 10,000 | 12,000 | 15,000 | 20,000 | 25,000 | 30,000 |
| 6 | 26,250 | 1 32 | 2 98 | 5 28 | 8 27 | 11 92 | 21 12 | 33 1 | 47 68 | 74 50 | 132 4 | 206 75 | 298 |
| 9 | 33,100 | 1 66 | 3 75 | 6 64 | 10 40 | 15 00 | 26 56 | 41 6 | 60 00 | 93 75 | 166 4 | 260 00 | 375 |
| 4 | 41,740 | 2 10 | 4 74 | 8 40 | 13 15 | 18 96 | 33 60 | 52 6 | 75 84 | 118 50 | 210 4 | 328 75 | 474 |
| 3 | 52,630 | 2 54 | 5 96 | 10 16 | 16 55 | 23 84 | 40 64 | 66 2 | 95 36 | 149 00 | 254 8 | 413 75 | 596 |
| 2 | 66,370 | 3 33 | 7 51 | 13 32 | 20 85 | 30 04 | 53 28 | 83 4 | 120 16 | 187 75 | 333 6 | 521 25 | 751 |
| 1 | 83,690 | 4 21 | 9 48 | 16 84 | 26 32 | 37 92 | 67 36 | 105 3 | 151 68 | 212 00 | 421 2 | 658 00 | 948 |
| 0 | 105,500 | 5 29 | 11 92 | 21 16 | 33 10 | 47 68 | 84 64 | 132 4 | 191 72 | 298 00 | 529 6 | 827 50 | 1192 |
| 00 | 133,100 | 6 71 | 15 11 | 26 84 | 41 97 | 60 44 | 107 36 | 167 9 | 241 76 | 377 75 | 671 6 | 1049 25 | 1511 |
| 000 | 167,800 | 8 45 | 19 04 | 33 80 | 52 85 | 76 16 | 135 20 | 211 4 | 304 64 | 476 00 | 845 6 | 1321 25 | 1904 |
| 0000 | 211,600 | 10 62 | 23 92 | 42 48 | 66 42 | 95 68 | 169 92 | 265 7 | 382 72 | 598 00 | 1062 8 | 1660 50 | 2392 |
| | 250,000 | 12 58 | 28 33 | 50 32 | 78 67 | 113 32 | 201 28 | 314 7 | 453 28 | 708 25 | 1258 8 | 1966 75 | 2833 |
| | 500,000 | 25 17 | 56 66 | 100 68 | 157 35 | 226 64 | 402 72 | 629 4 | 906 56 | 1416 50 | 2517 6 | 3933 75 | 5666 |

Notes on High-tension Transmission. (General Electric Co., 1909.)—The cross-sectional area and, consequently, weight of conductors varies inversely as the square of the voltage for a given power transmission. The cost of conductors is therefore reduced 75% every time the voltage is doubled. The cost of other apparatus and appliances increases with increasing voltage. In the longest lines, from about 190 miles up, the saving in copper with the highest practicable voltages is so great that the

other expenses are rendered practically negligible. In the shorter lines, however, from about one mile to 60 or 75 miles, the most suitable voltage must be determined in each individual case. The voltages in the following table will serve as a guide.

VOLTAGES ADVISABLE FOR VARIOUS LINE LENGTHS.

| Miles. | Volts. | Miles. | Volts. | Miles. | Volts. |
|--------|-----------|--------|---------------|--------|----------------|
| 1 | 500-1000 | 3-10 | 6,600-13,200 | 20-40 | 44,000-66,000 |
| 1-2 | 1000-2300 | 10-15 | 13,200-22,000 | 40-60 | 66,000-88,000 |
| 2-3 | 2300-6600 | 15-20 | 22,000-44,000 | 60-100 | 88,000-110,000 |

Standard machinery is made for 2300, 6600, 13,200, 22,000, 33,000, 44,000, 66,000, 88,000 and 110,000 volts, and standard generators are made for the above voltages up to and including 13,200 volts. When the line voltage is higher than 13,200, step-up transformers must be employed. In a given case the saving in cost of conductor by using the higher voltage may be more than offset by the cost of transformers, and the question of voltage must be determined for each case.

Line Spacing.—Line conductors should be so spaced as to lessen the tendency to leakage and to prevent the wires from swinging together or against the towers. With suspended disk insulators the radius of free movement is increased, and special account should be taken of spacing when these insulators are used. The spacing should be only sufficient for safety, since increased spacing increases the self-induction of the line, and while it lessens the capacity, it does so only in a slight degree. The following spacing is in accordance with average practice.

CONDUCTOR SPACING ADVISABLE FOR VARIOUS VOLTAGES.

| Volts. | Inches. | Volts. | Inches. | Volts. | Inches. |
|--------|---------|--------|---------|---------|---------|
| 5,000 | 28 | 45,000 | 60 | 90,000 | 96 |
| 15,000 | 40 | 60,000 | 72 | 105,000 | 108 |
| 30,000 | 48 | 75,000 | 84 | 120,000 | 120 |

Skin Effects.—For the frequencies and sizes of cables used in transmission lines, skin effect does not appreciably alter the resistance; for example, the resistance of a solid copper wire $\frac{3}{4}$ in. diameter at 60 cycles is increased only $2\frac{1}{2}\%$, the resistance of a stranded cable of the same external diameter being increased a still smaller amount. This refers only to non-magnetic materials; with steel cable skin effect cannot be neglected, and a calculation must be made for it.

Frequency.—So far as the transmission line alone is concerned, the lower frequencies are the more desirable, because they reduce the inductance drop and charging current. Oscillations of dangerous magnitude are less likely with the lower frequencies than with the higher. The A.I.E.E. recognizes two frequencies, viz: 25 and 60, as standard, but frequencies of 15 and in some cases 12.5 are being advocated.

Aluminum Conductors.—The conductivity of aluminum is generally taken at 63.3% that of hard-drawn copper of the same cross-sectional area. The weight of Al is 30.2% that of copper, and therefore an Al conductor of the same length and conductivity as a given copper conductor weighs 47.7% as much. The cost of Al must therefore be 2.097 times that of hard-drawn copper to give equal cost for the same length and conductivity. Owing to the mechanical unreliability of solid Al conductors, stranded conductors are used in all sizes, including even the smallest.

TRANSFORMERS, CONVERTERS, ETC.

Transformers.—A transformer consists essentially of two coils of wire, one coarse and one fine, wound upon an iron core. The function of a transformer is to convert electrical energy from one potential to another. If the transformer causes a change from high to low voltage, it is known as a "step-down" transformer; if from low to high voltage, it is known as a "step-up" transformer.

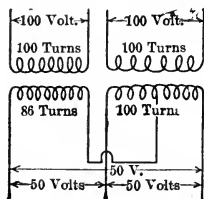


FIG. 217.

The relation of the primary and secondary voltages depends on the number of turns in the two coils. Transformers may also be used to change current of one phase to current of another phase. The windings and the arrangement of the transformers must be adapted to each particular case. In Fig. 217 an arrangement is shown whereby two-phase currents may be converted into three-phase. Two transformers are required, one having its primary and secondary coils in the relation of 100 to 100, and the other having its primary and secondary in the relation of 100 to 86. The secondary of the 100-to-100 transformer is tapped at its middle point and joined to one terminal of the other secondary. Between any pair of the three remaining terminals of the secondaries there will exist a difference of potential of 50.

There are two sources of loss in the transformer, viz., the copper loss and the iron loss. The copper loss is proportional to the square of the current, being the I^2R loss due to heat. If I_1, R_1 , be the current and resistance respectively of the primary, and I_2, R_2 , the current and resistance respectively of the secondary, then the total copper loss is $W_c = I_1^2 R_1 + I_2^2 R_2$ and

the percentage of copper loss is $\frac{I_1^2 R_1 + I_2^2 R_2}{W_p}$, where W_p is the energy

delivered to the primary. The iron loss is constant at all loads, and is due to hysteresis and eddy currents.

Transformers are sometimes cooled by means of forced air or water currents or by immersing them in oil, which tends to equalize the temperature in all parts of the transformer.

Efficiency of Transformers.—The efficiency of a transformer is the ratio of the output in watts at the secondary terminals to the input at the primary terminals. At full load the output is equal to the input less the iron and copper losses. The full-load efficiency of a transformer is usually very high, being from 92 per cent to 98 per cent. As the copper loss varies as the square of the load, the efficiency of a transformer varies considerably at different loads. Transformers on lighting circuits usually operate at full load but a very small part of the day, though they use some current all the time to supply the iron losses. For transformers operated only a part of the time the "all-day" efficiency is more important than the full-load efficiency. It is computed by comparing the watt-hours output to the watt-hours input.

The all-day efficiency of a 10-K.W. transformer, whose copper and iron losses at full load are each 1.5 per cent, and which operates 3 hours at full load, 2 hours at half load, and 19 hours at no load, is computed as follows:

Iron loss, all loads = $10 \times 0.015 = 0.15$ K.W.

Copper loss, full load = $10 \times 0.015 = 0.15$ K.W.

Copper loss, $\frac{1}{2}$ load = $0.15 \times (\frac{1}{2})^2 = 0.0375$ K.W.

Iron loss K. W. hours = $0.15 \times 24 = 3.6$.

Copper loss, full load, K. W. hours = $0.15 \times 3 = 0.45$.

Copper loss, $\frac{1}{2}$ load, K. W. hours = $0.0375 \times 2 = 0.075$.

Output, K. W. hours = $\{ (10 \times 3) + (5 \times 2) \} = 40$.

Input, K. W. hours = $40 + 3.6 + 0.45 + 0.075 = 44.125$.

All-day efficiency = $40 \div 44.125 = 0.907$.

The transformers heretofore discussed are constant-potential transformers and operate at a constant voltage with a variable current. For the operation of lamps in series a constant-current transformer is required. There are a number of types of this transformer. That manufactured by the General Electric Co. operates by causing the primary and secondary coils to approach or to separate on any change in the current.

Converters, etc. — In addition to static transformers, various machines are used for the purpose of changing the voltage of direct currents or the voltage phase or frequency of alternating currents, and also for changing alternating currents to direct or vice versa. These machines are all rotary and are known as rotary converters, motor-dynamos, and dynamotors.

A rotary converter consists of a field excited by the machine itself, and an armature which is provided with both collector rings and a commutator. It receives direct current and changes it to alternating, working as a direct-current motor, or it changes alternating to direct current working as a synchronous motor.

A motor-dynamo consists of a motor and a dynamo mounted on the same base and coupled together by a shaft.

A dynamotor has one field and two armature windings on the same core. One winding performs the functions of a motor armature, and the other those of a dynamo armature.

A booster is a machine inserted in series in a direct-current circuit to change its voltage. It may be driven either by an electric motor or otherwise.

The Mercury Arc Rectifier consists of a mercury vapor arc enclosed in an exhausted glass vessel into which are sealed two terminal anodes connected to the two wires of an alternating-current circuit. A third terminal, at the bottom of the vessel, is a mercury cathode. When an arc is operating, it is a good conductor from either anode to the cathode, but practically an insulator in the other direction. The two anodes connected across the terminals of the alternating-current line become alternately positive and negative. While either anode is positive, there is an arc carrying the current between it and the cathode. When the polarity of the alternating-current reverses, the arc passes from the other anode to the mercury cathode, which is always negative. The current leading out from the mercury cathode is uni-directional. By means of reactances, the pulsations are smoothed out and the current at the cathode becomes a true direct current with pulsations of small amplitude.

ELECTRIC MOTORS.

Classification of Motors. — (From the Standardization Rules of the A. I. E. E.)

a. Constant-speed Motors, in which the speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors.

b. Multi-speed Motors (two-speed, three-speed, etc.), which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as motors with two armature windings.

c. Adjustable-speed Motors, in which the speed can be varied gradually over a considerable range; but when once adjusted remains practically unaffected by the load, such as shunt motors designed for a considerable range of field variation.

d. Varying-speed Motors, or motors in which the speed varies with the load, decreasing when the load increases; such as series motors.

The selection of a motor for a specified service involves,

a. Mechanical ability to develop the requisite torque and speeds, as given by its speed-torque curve.

b. Ability to commutate successfully the current demanded.

c. Ability to operate in service without occasioning a temperature rise in any part which will endanger the life of the insulation.

The nominal rating, or the horse-power output which a motor can give with a rise of temperature not exceeding 90 degrees at the commutator and 75 degrees at any other part after an hour's run on a test stand is a method of designating motors which is in common usage, though it is not a proper measure of service capacity.

Motor Classification of the Am. Assn. of Electric Motor Manufacturers. (*Elec. Jour.*, Aug. 1909.) — Alternating-current motors and direct-current motors can easily be classified under the same speed headings, and this has been done as below.

A. — Constant Speed Motors — in which the speed is either constant or does not vary materially, such as synchronous motors, induction motors with small slip, ordinary direct-current shunt motors, and direct current

compound-wound motors, the no-load speed of which is not more than 20 per cent higher than the full-load speed.

B. — Multi-Speed Motors — (two-speed, three-speed, etc.) — which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as direct-current motors with two armature windings and induction motors with primary windings capable of being grouped so as to form different numbers of poles.

C. — Adjustable Speed Motors. — (1) Shunt-wound motors in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load, such as motors designed for a considerable range of speed by field variation.

(2) Compound-wound motors in which the speed can be varied gradually over a considerable range, as in (1), and, when once adjusted, varies with the load, similar to compound-wound constant-speed motors or varying-speed motors, depending upon the percentage of compounding.

D. — Varying Speed Motors, or motors in which the speed varies with the load, decreasing when the load increases, such as series motors and heavily compounded motors. Examples of heavily compounded motors are those designed for bending roll service and mill service, in which shunt-winding is provided only to limit the light-load operating speed.

Many motor applications can be made more intelligently if, in addition to using the classification given above, the service is described in terms of continuous or intermittent duty, and load constant or varying. In order to make this point clear, the following table has been prepared, giving one example of each of the different classes of service. Practically every motor application can be listed under one or the other of these headings.

CLASSIFICATION OF MOTORS.

| Speed. | Duty. | Load. | Example. |
|--------------|---------------|-------------|-----------------|
| Constant. | Continuous. | { Constant. | Fan. |
| | | { Varying. | Line-shaft. |
| | Intermittent. | { Constant. | Vacuum pump. |
| Adjustable. | | { Varying. | Paper-cutter. |
| | Continuous. | { Constant. | Paper calender. |
| | | { Varying. | Printing press. |
| | Intermittent. | { Constant. | Vacuum pump. |
| Varying. | | { Varying. | Lathe. |
| | Continuous. | { Constant. | Small fan. |
| | | { Varying. | Bending press. |
| | Intermittent. | { Constant. | House pump. |
| Multi-speed. | | { Varying. | Crane. |
| | Continuous. | { Constant. | Fan. |
| | | { Varying. | * |
| | Intermittent. | { Constant. | Fire pump. |
| | | { Varying. | * |

The Auxiliary-pole Type of Motor. (J. M. Hipple, *El. Jour.*, May, 1906.) —

Among the methods of controlling the motor speed, the most satisfactory is the single voltage direct-current system in which the variation of speed is obtained by shunt-field control. The insertion of resistance in the shunt-field circuit varies the strength of the magnetic field, and as the strength of field is decreased the speed of the motor is increased in direct proportion.

An ordinary shunt-wound motor operating under the above conditions over a speed range of four to one will spark excessively at the brushes unless the motor is rated considerably under its normal capacity. This sparking is due principally to the weakened magnetic field and to the distortion or shifting of this field due to reaction on it by the field produced by the ampere turns in the armature.

The use of an auxiliary field by correcting this condition produces

*Multi-speed motors are at present almost exclusively alternating-current motors. The classes of service in which these motors are used are limited, but a considerable field may develop later.

sparkless commutation and a condition of practical stability of field and consequently of speed in the motor. This auxiliary field is produced by a winding in series with the armature and placed on pole-pieces midway between the main pole-pieces. The distortion at the point of commutation which would occur if there was no auxiliary winding is prevented by the field produced by the auxiliary winding. This field being always proportional to the load the commutation is accomplished sparklessly at all loads up to heavy overloads.

Motors of this type are reversible with no change in setting of brushes or other adjustment. The brushes being fixed in the neutral position it is only necessary to reverse the current in both auxiliary field and armature to secure exactly similar operating conditions in the reverse as in the forward direction.

Speed of Electric Motors.—Any direct-current motor, no matter what its type of field winding, if supplied with current of constant potential at its terminals, will run at constant speed if its field strength and the load do not change. The speed of a given motor is directly proportional to the net impressed e.m.f. divided by the effective field strength. The net impressed e.m.f. is that part of the supply e.m.f. which must be exactly opposed by the counter e.m.f. of the armature. Thus, if the supply voltage is 250 volts, the load 50 amperes and the armature circuit resistance 0.2 ohm, the net impressed e.m.f. will be 240 volts, because the armature drop is $0.2 \times 50 = 10$ volts. The "effective" field strength is the actual field flux set up by the field winding after overcoming the armature reaction, which always weakens the field slightly.

In the case of a shunt-wound motor operated on a constant-potential circuit with an adjustable external resistance in series with the armature, no matter at what point the external resistance may be set, so long as it remains at that point, giving unchanging voltage at the motor terminals, the speed will be constant unless the field strength or load be altered. The speed of a series-wound motor increases very rapidly with decreasing load when operated on a constant-potential circuit, becoming so high at no load as to be destructive to the armature. The reason for this is that the armature current passes also through the field winding, so that any decrease in armature current weakens the field and causes the speed to increase far beyond the rate it would attain with a constant field. (C.P. Poole, *Power*, July, 1907.)

The speed of a shunt motor is dependent upon the details of its entire design. The following equation shows the relation of the speed to the main elements of the machine:

$$n = \frac{(E - I_a R_a) c 10^8}{M p N},$$

where E is the impressed electromotive force, R_a the resistance of the armature, I_a the current through it, c the number of parallel circuits for the current through the armature, M the magnetic flux (number of lines of force) per pole, p the number of poles, N the number of armature conductors, and n the speed in revolutions per second. (*El. Review*, July 17, 1909.)

The simplest form of an electric motor is the shunt-wound machine. When connected with an ordinary electric lighting circuit, it runs at a steady speed, drawing hardly any current until it is required to furnish power, and at that moment it consumes power only in proportion to the work done. If connected to a circuit of lower pressure, it will run equally well, but at lower speed. If required to make extra effort, as in starting machinery, it will furnish up to five times its full power without trouble.

When running free, if its speed is increased by the application of external power, as by a belt, it becomes a dynamo and pumps current into the line; this, in turn, throws work upon the machine and tends to slow it down. The machine is, therefore, in itself a factor tending to the preservation of constancy of speed and to the preservation of constancy in the pressure on the circuit, and it is ideal in its simplicity, having absolutely no governing or accessory parts.

The shunt-wound motor runs at practically constant speed under all loads, and if closer uniformity of speed is desired, it can be arranged to run within any desired limits of variation by setting the brushes in a position shifted slightly from their usual place, or by adding to the field

winding a few turns, connected in series with the armature, and reversed in comparison with the main winding. Either of these arrangements causes the motor to speed up under load, and the extent of this action may be adjusted to equal precisely the tendency ordinarily met of slowing down under load. (S. S. Wheeler, *Elec. Age*, Dec., 1904.)

Speed Control of Electric Motors. Rheostats. (The Electric Controller and Mfg. Co.)—A motor of any size, when its armature is at rest, offers a very low resistance to the flow of current and an excessive and perhaps destructive current would flow through it if it were connected across the supply mains while at rest. Take the case of a motor adapted to a normal full-load current of 100 amperes and having a resistance of 0.25 ohm; if this motor were connected across a 250-volt circuit a current of 1,000 amperes would flow through its armature—in other words, it would be overloaded 900% with consequent danger to its windings and also to the driven machine. In the case of the same motor, with a rheostat having a resistance of 2.25 ohms inserted in the motor circuit, at the time of starting the total resistance to the flow of current would be the resistance of the motor (0.25 ohm) plus the resistance of the rheostat (2.25 ohms), or a total of 2.5 ohms. Under these conditions exactly full-load current, or 100 amperes, would flow through the motor, and neither the motor nor the driven machine would be overstrained in starting. This shows the necessity of a rheostat for limiting the flow of current in starting the motor from rest.

An electric motor is simply an inverted generator or dynamo—consequently when its armature begins to revolve a voltage is generated within its windings just as a voltage is generated in the windings of a generator when driven by a prime-mover. This voltage generated within the moving armature of a motor opposes the voltage of the circuit from which the motor is supplied, and hence is known as a "counter-electromotive force." The net voltage tending to force current through the armature of a motor when the motor is running is, therefore, the line voltage minus the counter-electromotive force.

In the case of the motor above cited, when the armature reaches such a speed that a voltage of 125 is generated within its windings, the effective voltage will be 250 minus 125, or 125 volts, and, therefore, the resistance of the rheostat may be reduced to one ohm without exceeding the full-load current of the motor. As the armature further increases its speed the resistance of the rheostat may be further reduced until when the motor has almost reached full speed all of the rheostat may be cut out, and the counter-electromotive force generated by the motor will almost equal the voltage supplied by the line so that an excessive current cannot flow through the armature.

In practice, a rheostat is provided for starting an electric motor, the resistance conductor being divided into sections, such that the entire length or maximum resistance of the rheostat is in circuit with the motor at the instant of starting and the effective length of the conductor, and hence its resistance may be reduced as the motor comes up to speed.

In cutting out the resistance of a starting rheostat care must be used not to cut it out too rapidly. If the resistance is cut out more rapidly than the armature can speed up, a sufficient counter-electromotive force will not be generated to properly oppose the flow of current, and the motor will be overloaded.

If all the resistance of the starting rheostat is not cut out the motor will operate at reduced voltage, and hence at less than normal speed. A rheostat so arranged that all or a portion of its resistance may be left in a motor circuit to secure reduced speeds is called a "rheostatic controller." Such rheostatic controllers are used for controlling series and compound-wound motors driving cranes, and similar machinery requiring variable speed under the control of an operator.

In a series-wound motor the speed varies inversely as the load—the lighter the load the higher the speed. A series-wound motor of any size when supplied with full voltage under no load, or a very light load, will "run away" just as will a steam-engine without a governor when given an open throttle.

For a given load a series-wound motor draws the same current irrespective of the speed and for a given load the speed varies directly as the voltage. The speed at a given load may be varied by varying the resistance

in the motor circuit — in the meantime if the load on the motor be constant the current drawn from the line will be constant regardless of the speed.

The above statements relate to the use of a rheostat in series with a series-wound motor. If a resistance or rheostat be placed in parallel with the field of a series-wound motor the speed will be increased instead of decreased at a given load. This is known as shunting the field of the motor. This shunt would never be applied till the motor has been brought up to normal full speed by cutting out the starting resistance. With a "shunted field" a motor is driving a load at a speed higher than normal and therefore requires a correspondingly increased current.

If a resistance is placed in parallel with the armature of a series motor, the motor will operate at less than normal speed when all of the starting resistance has been cut out. This connection is known as a "shunted armature connection" and is useful where a low speed is desired at light loads and is particularly useful in some cases where the load becomes a negative one, that is, where the load tends to overhaul the motor, as in lowering a heavy weight.

A shunt-wound motor, unlike a series motor, when supplied with full voltage, maintains practically a constant speed regardless of variations in load within the limits of its capacity. It automatically acts like a steam-engine having a very efficient governor.

The speed of a shunt-wound motor may be decreased below normal by a rheostatic controller in series with its armature and may be increased above normal by means of a rheostat in series with its field winding. The latter rheostat is known as a "field rheostat," and, to be effective, must have a high resistance owing to the small current which flows through the shunt-field winding.

A compound-wound motor is a hybrid between a series and shunt-wound motor and its characteristics are likewise of a hybrid nature.

A compound-wound motor will not "run away" under no load as will a series motor, but its speed decreases as the load increases, though not so rapidly as is the case with a series-wound motor.

The characteristics of a compound-wound motor are particularly valuable in cases where the load is subject to wide variation. It will give a strong torque in starting and driving heavy loads and at the same time will not race dangerously when the load is suddenly relieved.

The speed of a compound-wound motor may be reduced below normal by means of a rheostat in the circuit of its armature. The speed may be increased above normal by shunting and even short-circuiting the series field winding, and may be still further increased by means of a field rheostat in series with the shunt-field winding.

Rheostatic controllers are also employed for the control of alternating current induction motors of the so-called "slip-ring type." Such motors have characteristics in many ways similar to those of direct current shunt-wound motors, and speeds lower than normal may be obtained by inserting resistance in series with the windings of the secondary or rotor.

Selection of Motors for Different Kinds of Service. (F. B. Crocker and M. Arendt, *El. World*, Nov., 1907.) — The types of direct-current motor are as follows:

DIRECT-CURRENT MOTORS.

| <i>Type.</i> | <i>Operative Characteristics.</i> |
|------------------------------|--|
| Shunt-wound motors..... | Starting torque usually 50 to 100 per cent greater than rated running torque, and fairly constant speed over wide load ranges. |
| Series-wound motors..... | Powerful starting torque, speed varying greatly (inversely) with load changes. |
| Compound-wound motors.... | Compromise between shunt and series types. |
| Differently-wound motors ... | Starting torque very small, speed can be made almost absolutely constant for load changes within rated capacity. |

The conditions under which machinery operates, in regard to varying speed and power required of the driving motor, may be divided into four

classes, and certain types of motors are usually best suited to these divisions, which are as follows:

(a) Work which requires the motor to operate automatically at a practically constant speed, regardless of load changes or other conditions.

(b) Work requiring frequent starting and stopping and wide variations in speed, including sometimes rapid acceleration.

(c) An approximately steady load or work that varies as some function of the speed should it change.

(d) Work in which the power varies regardless of the speed, or where speed variations with constant torque may be desired.

The first case (a) applies to line-shaft equipments with many machines operated by the same motor and where slight speed variations may be allowed; the direct-current shunt or slightly compounded motor or the alternating-current induction motor would answer, depending upon the character of electric current available. A refinement of this problem is encountered in the driving of textile machinery, especially silk looms, with which even a slight speed variation might affect the appearance of the finished product. In such instances the alternating-current motors, polyphase induction or polyphase synchronous, are generally employed because the speed of direct-current motors varies considerably with voltage changes and the variation in temperature which occurs after several hours of operation, whereas the speed of the alternating-current motors, unless the voltage varies greatly, is primarily dependent upon the frequency of the supplied current.

The second class (b) is divided into two parts, the first being electric traction and crane service, in which the motor is frequently started and stopped and rapidly accelerated at starting; or where the speed is to be adjusted automatically to the load, slowing down when heavily loaded or climbing a steep grade. These conditions are well satisfied by the series motor of either the direct or alternating-current types, depending upon the current supplied. Elevator service is of this character as regards frequent starting and stopping, but after rapid acceleration it calls for a speed independent of the load. Hence, to fulfill both requirements, elevator motors when of direct-current type are heavily over-compounded to give the series characteristic at starting; then, when the motor is up to speed, the series field winding is short-circuited and it operates as a shunt machine. Recently, however, two-speed shunt motors have been employed for this service, the field being of maximum strength for starting and sparking prevented by use of inter-poles. If only alternating current is available the polyphase induction motor should be employed, but for powerful starting torque either slip-ring or compensator control would be necessary. For the second subdivision of this class the motor must be started and stopped frequently and not rapidly accelerated, but on the contrary simply "inched" forward at the start, as in the operation of printing presses, gun turrets, etc. These conditions of service are satisfied by a direct-current compound motor provided with double armature and series-parallel control of the machine.

The third class (c) of work is the operation of pumps, fans or blower equipments and its requirements are satisfied by the series motor, whose speed adjusts itself to the work, and also because it exerts the maximum torque required at starting. It must be, however, either geared or directly connected to the apparatus, because the breaking of the belt or the sudden removal of the load would cause a series motor to race and become injured. The operation of pumps by electric motors is usually effected by gearing, since ordinary plunger pumps do not operate efficiently if driven in excess of fifty strokes per minute, and to accomplish this by direct connection would demand a very low speed and costly motor. Centrifugal pumps operating at high speed may be direct driven.

The fourth class (d) is found in individual machine-tool service, for which the maximum allowable cutting or turning speed requires the number of revolutions of the work or tool to vary inversely as the diameter of the cut. This condition is satisfied best by the direct-current shunt or slightly compounded motors, as they are readily controlled in speed by variation of the applied voltage, shunt field weakening, etc.

It is to be noted that (a) and (c) regulate automatically to maintain a constant speed while (b) and (d) are controlled by hand to give variable speeds. Furthermore, (b) is usually under control of the hand all the time,

whereas (d) is set to operate at a desired speed for some time and regulates automatically when so adjusted.

The Electric Drive in the Machine-Shop. (A. L. De Leeuw, *Trans. A.S.M.E.*, 1909.)—Absence of reliable data is apparent all over the field of this subject, and it will therefore be impossible to say beforehand with any fair degree of certainty how much, if anything, can be gained by the conversion of a shop from a shaft to motor drive.

Nothing but an exhaustive study of the entire plant in all its aspects will clearly show what may be accomplished. The saving of power is by no means the only nor the most important economy resulting from a conversion to electric drive, and such a conversion may even be highly economical, though there be an actual loss in power consumed.

The question whether alternating or direct current should be used is especially difficult of solution, and there is a wide difference of opinion among engineers as to which is best. Given a plant covering a large area and using large amounts of current, of which only a small portion is used for variable-speed machinery, and of sufficient size to permit of the use of a separate unit for lighting current, then alternating current would be the logical solution. On the other hand, given a compact plant, using a large portion of the power for variable-speed machinery, direct-driven by motors, and of which the lighting load is small in the daytime, then it would be natural to select direct current. As a rule, however, conditions are not so simple. Of late the problem has been complicated by the fact that many machine tools may be had with single-pulley drive, to which an alternating-current or a direct-current motor is equally applicable.

The points in favor of the alternating-current motor are:

a High break-down point; that is, the motor goes on with no material change of speed under very heavy overload.

b Freedom from commutator trouble. This is especially valuable where fine chips are made, or where compressed air is used in connection with the machine. The better makes of direct-current motors are now equally free from this kind of trouble.

c Most cities are now lighted by alternating current, so that city current can be used in smaller plants, provided the machine tools are arranged for this kind of motor.

The points in favor of the direct-current motor are:

a Wider air-gap, allowing a greater amount of wear in the bearings before the motor has to be repaired.

b The possibility of power and lighting-loads on the same circuits without the poor regulation due to inductive load.

c The possibility of using variable-speed motors. This is, perhaps, the greatest argument in favor of the direct-current motor. Though it is possible to run a great many machine tools by a motor, yet one of the greatest advantages of such a drive is not available, unless the motor is of the variable-speed variety.

The combination of alternating and direct current has its advantages, especially where it is possible to purchase current from some large power company which delivers its product as alternating current. Transformers reduce the voltage at the entrance to the shop, and the low-voltage alternating current can be used for all purposes except for driving variable-speed motors, and perhaps some auxiliary apparatus such as magnetic clutches, lifting magnets, etc.

See also papers on this subject by Chas. Robbins and John Riddell, *Trans. A.S.M.E.*, 1910.

Choice of Motors for Machine Tools. (Chas. Fair, *Proc. A. I. E. E.*, 1910.)—Shunt-wound direct-current, or squirrel-cage rotor, alternating current: For bolt cutter; boring machine; boring mill; boring bar; centering machine; chucking machine; boring, milling and drilling machines; drill, radial; drill press; grinder-tool, etc.; keyseater, milling-broach; lathe; milling machine; pipe-cutter; saw, small circular; screw machine; tapper.

Compound-wound direct-current, or squirrel-cage rotor: For grinder-castings; reciprocating keyseater; saw, cold bar and I-beam; saw, hot; shaper; slotter; tumbling barrel or mill.

Compound-wound direct-current or squirrel-cage rotor, or squirrel-cage rotor with high starting torque: For bolt and rivet header; bulldozer; bending machine; corrugating roll; punch press; shear.

Other machines may be driven as indicated below. (a) shunt, (b)

compound, (c) series, direct-current motors, (d) squirrel-cage rotor, (e) ditto, high starting torque, (f) slip ring induction motor with external rotor resistance. Raising and lowering cross rails on boring mills and planers, (b), (c), (e). Bending rolls, (b), (c), (f). Gear cutters, (a), (b), (d). Drop hammers, (b), (e). Tire lathes, (f) may be used, as it allows for slowing down when cutting hard spots. Lathe carriages, (c), (e). Heavy slab milling, (a), (b), (d). Planers, (b), (d), (e). Planers, rotary, (a), (b), (d). Swaging, (b), (d), (e). Shunt motors are used in the following cases: when the work is of a fairly steady nature; when considerable range of adjustment of speed is required, as on lathes and boring mills; and on group and lineshaft drives, etc.

Compound-wound motors are used where there are sudden calls for excessive power of short duration, as on planers, punch presses, etc.

Series motors should be used where speed regulation is not essential, and where excessive starting torque and slow starting speeds are required, as for operating cranes.

When in doubt as to the choice of compound or series motors of small horse-power, the choice might be determined by the simplicity of control in favor of the series motor. Series motors, however, should never be used when the motor can run without load, as the speed would accelerate beyond the point of safety.

The alternating current motor of the squirrel-cage rotor type corresponds to the constant-speed, shunt, direct-current motor, but with a high-resistance rotor it approaches more closely the characteristics of a compound direct-current motor. Variable speed machines, driven by squirrel-cage rotors must have the necessary mechanical speed changes.

The slip-ring induction motor with external rotor resistance would be used for variable speed, but this must not be construed to mean that it corresponds to a direct-current, adjustable-speed motor, as it has the characteristics of a direct-current shunt motor with armature control.

The self-contained, rotor resistance type would be used for lineshaft drives, and for groups when of sufficient size.

Multi-speed, alternating-current motors are those giving a number of definite speeds, usually 600 and 1200 or 600, 900, 1200 and 1800 rev. per min., and are made for both constant horse-power and constant torque. These motors would be used where alternating current only was available, or direct current limited; and the speed range of the motor, together with one or two change gears, would give the required speeds.

ALTERNATING-CURRENT MOTORS.

Synchronous Motors. — Any alternator may be used as a motor, provided it be brought into synchronism with the generator supplying the current to it. The operation of the alternating-current motor and generator is similar to the operation of two generators in parallel. It is necessary to supply direct current to the field. The field circuit is left open until the machine is in phase with the generator. If the motor has the same number of poles as the generator, it will run at the same speed; if a different number, the speed will be that of the generator multiplied by the ratio of the number of poles of the motor to that of the generator. Single-phase, synchronous motors are not self-starting. Polyphase motors may be made self-starting, but it is better to bring the machines to speed by independent means before supplying the current. The machines may be started by a small induction motor, the load on the synchronous motor being thrown off, or the field may be excited by a small direct-current generator belted to the motor, and this generator may be used as a motor to start the machine, current to run it being taken from a storage battery. If the field of a synchronous motor be properly regulated to the load, the motor will exercise no inductive effect on the line, and the power factor will be 1. If the load varies, the current in the motor will either lead or lag behind the e.m.f. and will vary the power factor. If the motor be overloaded so that there is a diminution of speed, the motor will fall out of step with the generator and stop.

Synchronous motors are often put on the same circuit with induction motors. The synchronous motor in this case may, by increasing the field excitation, be made to cause the current to lead, while the induction motor will cause it to lag. The two effects will thus tend to balance each other and cause the power factor of the circuit to approach 1.

Synchronous motors are best used for large units of power at high voltages, where the load is constant and the speed invariable. They are unsatisfactory where the required speed is variable and the load changes. Two great disadvantages of the synchronous motor are its inability to start under load and the necessity of direct-current excitation.

Induction Motors. — The distinguishing feature of an induction motor is the rotating magnetic field. It is thus explained: In Fig. 218 let *ab*, *cd* be two pairs of poles of a motor, *a* and *b* being wound from one leg or pair of wires of a two-phase alternating circuit, and *c* and *d* from the other leg, the two-phases being 90° apart. At the instant when *a* and *b* are receiving maximum current so as to make *a* a north pole and *b* a south pole, *c* and *d* are demagnetized, and a needle placed between the poles would stand as shown in the cut. During the progress of the cycle of the current the magnetic flux at *a* decreases and that at *c* increases, causing the point of resultant maximum intensity to shift, and the needle to move clockwise toward *c*. A complete rotation of the resultant point is performed during each cycle of the current. An armature placed within the ring is caused to rotate simply by the shifting of the magnetic field without the use of a collector ring. The words "rotating magnetic field" refer to an area of magnetic intensity and must be distinguished from the words "revolving field," which refer to the portion of the machine constituting the field magnet.

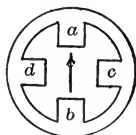


FIG. 218.

The field or "primary" of an induction motor is that portion of the machine to which current is supplied from the outside circuit.

The armature or "secondary" is that portion of the machine in which currents are induced by the rotating magnetic field. Either the primary or the secondary may revolve. In the more modern machines the secondary revolves. The revolving part is called the "rotor," the stationary part the "stator." The rotor may be either of the ring or the drum type, the drum type being more common. A common type of armature is the "squirrel-cage." It consists of a number of copper bars placed on the armature-core and insulated from it. A copper ring at each end connects the bars. The field windings are always so arranged that more than one pair of poles are produced. This is necessary in order to bring the speed down to a practical limit. If but one pair of poles were produced, with a frequency of 60, the revolutions per minute would be 3600.

The revolving part of an induction motor does not rotate as fast as the field, except at no load. When loaded, a slip is necessary, in order that the lines of force may cut the conductors in the rotor and induce currents therein. The current required for starting an induction motor of the squirrel-cage type under full load is 7 or 8 times as great as the current for running at full load. A type of induction motor known as "Form L," built by the General Electric Co., will start with the full-load current, provided the starting torque is not greater than the torque when running at full load.

Induction motors should be run as near their normal primary e.m.f. as possible, as the output and torque are directly proportional to the square of the primary pressure. A machine which will carry an overload of 50 per cent at normal e.m.f. will hardly carry its full load at 80 per cent of the normal e.m.f.

Induction Motor Applications. (A. M. Dudley, *Elec. Jour.*, July, 1908.) *Squirrel-Cage Motors for Constant Speed Service.* —

Motor-Generator Sets. — Small starting torque is required and good speed regulation, which characteristics are preeminently met by a squirrel-cage motor with very low resistance in the secondary rings. A fair specification on a large set is that it shall start on 30 to 40% of full voltage, and draw current not in excess of 1 1/4 times full-load current.

Pumps. — With a centrifugal pump decreasing the head pumped against increases the load on the motor. This type of pump will raise considerably more than four-thirds the amount of water 30 feet that it will 40 feet, with the result that the motor is overloaded if it is designed for 40 ft.

head. In this the centrifugal pump is exactly opposite to the plunger or reciprocating pump, which, being positive in its action, increases its load with increase of head and vice versa. [In some modern types of centrifugal pump the load decreases with decrease of head after reaching the maximum load corresponding to the head for which the pump is designed. See catalogue of the De Laval Steam Turbine Co., 1910. W. K.]

INDUCTION MOTOR APPLICATIONS.

| Squirrel Cage. | | Phase-Wound. | |
|---|--|--|--|
| Constant Speed. | Variable Speed. | Constant Speed. | Variable Speed. |
| 1—Motor-generator sets. | 1—Starting motors. | 1—Flour mills. | 1—Hoists and winches. |
| 2—Pumps. | 2—Crane motors | 2—Paper machinery, pulp grinders, beaters. | 2—Cranes. |
| 3—Blowers. | 3—Fly-wheel service. Punches, Shears, etc. | 3—Belt conveyors. | 3—Elevators. |
| 4—Line-shaft drive. | 4—Sugar centrifugals. | 4—Wood planers. | 4—Fly-wheel motor-generator sets. |
| 5—Cement machinery. | 5—Laundry extractors. | 5—Air compressors. | 5—Steel mill machinery, charging machines, hoists. |
| 6—Wood-working machinery (except planers). | 6—Brake motors | 6—Line shafting. | 6—Coal and ore unloaders. |
| 7—Cotton-mill machinery. | 7—Cross-head motors. | 7—Driving-wheel lathes. | 7—Dredging machinery. |
| 8—Paper machinery, calenders, Jordan engines. | 8—Valve motors. | | 8—Shovels. |
| 9—Concrete mixers. | | | 9—Mine haulage. |

Blowers.—Rotary blowers, except positive blowers, have a characteristic similar to centrifugal pumps, in that the load varies with the amount of air delivered and becomes less as the pressure against which the blower is working increases. That is to say, the maximum load which could be put on a motor driving a blower of this nature would be to take away all delivery pipes and let the blower exhaust into the open air.

Line Shafting.—Squirrel-cage motors are used very successfully for driving line-shafting where the idle belts are run on loose pulleys, in this way keeping down the starting torque.

Cement Mills.—The possibility of entirely covering the bearings and the absence of all moving contacts make the squirrel-cage motor successful where the more complicated construction and moving contact surfaces of the wound secondary motor or the direct-current machine are damaged by accumulation of dust. In starting up a tube mill it must be rotated through nearly 90% before the charge of pebbles and cement begins to roll. This makes the starting condition severe and a motor should have a starting torque of not less than twice full-load torque to do the work.

Wood-working Machinery.—On account of high friction and great inertia, the starting torque is sometimes so high and of so long duration (thirty seconds to one minute) that it is better to apply a wound-secondary motor.

Paper Machinery.—If calenders are driven with a constant speed motor it is necessary to make some provision either by mechanical speed-changing devices or a small auxiliary motor for securing a slow threading speed.

Squirrel-Cage Variable Speed Motors.—These motors in general have high resistance end rings, high slip and high starting torque. The torque increases automatically as the speed decreases. In these general respects they resemble a direct-current series motor and are in fact fitted for the same class of work, with the added advantage that they have a limiting speed and cannot run away under light load.

Fly-Wheel Service.—In driving tools which are used with fly-wheels such as punches, shears, straightening rolls and the like, the usefulness

of high slip comes in, as if the fly-wheel is to give up its energy, it is obliged to slow down in speed when the load comes on. A motor with good regulation and low slip would try to run at constant speed, carrying the fly-wheel and load as well, but the motor in question "lies down" and allows the fly-wheel to carry the peak load, speeding up again when the peak has passed.

Centrifugals. — In sugar centrifugals is an application where the sole purpose of the motor is to accelerate the load to full speed, in say thirty seconds, where it is allowed to run one minute and then shut down to repeat the cycle a minute later. The centrifugal consists of a cylindrical basket with perforated walls and mounted around a vertical shaft as an axis. The same principle is used in laundry extractors where the wet linen is placed in a similarly perforated basket and the water whirled out by centrifugal force.

Constant-Speed Motors with Phase-wound Secondaries. — There are classes of service which require a heavy starting torque combined with close speed regulation after the motor is up to speed. These requirements are exactly met by a motor with a phase-wound secondary. The secondary winding itself has a very low resistance, which means a small "slip," high running efficiency and power-factor and good regulation when the secondary is short-circuited. The insertion of external resistance enables the motor to develop maximum torque at the start with a moderate starting current.

Flour-Mills. — The number of line shafts, belts and gears in flour mills makes a very heavy starting condition and the nature of the product and its quality demand absolute speed within a few revolutions per minute. The best solution is the phase-wound rotor.

Other Examples. — There is another class of machinery which is not so exacting about regulation but which has the same feature of heavy starting and runs continuously after once up to speed. Under this head come most of the applications of this type of motor. They are, paper pulp grinders, which, on account of the inertia of the grindstones, are hard to start; pulp beaters; belt conveyors, which may be required to start when full of coal, rock or cement crushers; air compressors, which have a high starting friction because of the construction and the number of parts; line shafting where the belts run for the most part on the working pulleys and are therefore heavy to start. Under the best possible conditions, if line shafting is employed, the loss of power from this source alone, due to friction, is 25 to 30% and may run up to 40 or 50%. This is a strong argument for individual drive of machines wherever practicable.

Motors with Phase-wound Secondaries for Variable Speed Service. — The application, which is typical of this class, is found in hoist and crane service. Motors for this work are designed for intermittent operation and given a nominal rating based upon the horse-power which they will develop for one-half hour with a temperature rise of 40° C. They never operate for as long a period as thirty minutes continuously and they are called upon at times to develop a torque greatly in excess of their nominal rating. For these reasons motors of this class should never be applied on a horse-power basis, but always on a torque basis. Since torque is the main consideration and the service is intermittent these motors are usually wound for the maximum torque which they will develop and given a nominal rating based upon one-third to one-half of this torque. Double drum hoists, hoisting in balance, and large mine haulage propositions in general require a motor rated on a different basis. For this service the motor should have the necessary maximum torque and be able to develop for about two or three hours, with a safe rise in temperature, a horse-power equivalent to the square root of the mean square requirement of the hoisting cycle. These are only general rules and the most careful consideration should be given in each individual case to secure a motor which will perform the work satisfactorily.

Coal and Ore Unloading Machinery. — Dredges — Power-Shovels. — Owing to the complication of the cycle of operation there is more difficulty in providing a motor for this apparatus than in the case of a plain hoist. Usually the number of cycles per hour given is the maximum which the apparatus can develop and in practice it will not be possible to operate at so high a speed. This in itself is somewhat of a factor of

safety, though not one which can be relied upon, as the test for acceptance is ordinarily made at the contract number of operations per hour.

The most impressive application of motors of this class and perhaps in the operation of any electrical apparatus is the fly-wheel motor-generator set for hoisting or heavy reversing roll service in steel mills. Service of this nature is extremely fluctuating in its requirements, having very great peaks one instant and almost nothing the next. This is a severe strain on the generating plant from which power is being drawn.

Alternating-Current Motors for Variable Speed. (W. I. Slichter, *Trans. A.S.M.E.*, 1903.) —

The speed of an alternating-current motor may be controlled in a number of ways:

(a) By varying the potential applied to the primary of a motor having a suitable resistance in the secondary.

(b) By varying the resistance in the secondary circuit.

(c) By changing the connections of the primary in a manner to change the number of poles.

(d) By varying the frequency of the applied voltage.

The changeable pole and variable frequency methods are the most efficient, but do not permit of a variation through a wide range of speed. The rheostatic control is the simplest and easiest of control, giving a range from standstill to full speed, but is not as efficient as the first two, although more efficient than potential control. The last mentioned has the disadvantages of low efficiency and considerably increased heating in the motor itself, and is also unstable at low speeds, say below one-third speed. That is, a small variation in torque or a smaller variation in voltage will cause a considerable variation in speed.

Mr. Geo. W. Colles, in a discussion of Mr. Schlichter's paper, says that the variable-speed induction-motor problem has not yet been solved.

Of the four possible methods given, the first is the simplest, as here it is merely necessary to insert a compensator in circuit with the motor. This, however, is decidedly unsatisfactory, as, owing to the necessity of having a high-resistance secondary, even the full-speed efficiency of the motor is largely reduced, while at quarter-speed it is about 17%, and even at half-speed only 37%.

All the other solutions given are too complicated, and they cannot be regarded as other than makeshifts. The resistance-in-secondary method is the only one that has been used to any extent. This nullifies the meritorious natural features of the squirrel-cage motor, whose complete freedom from exposed contacts, commutator and slip-rings made it much simpler, and therefore cheaper, than the direct-current motor; and it now becomes more expensive and delicate, and considerably less efficient. The efficiency is now but 65% at $\frac{3}{4}$ load, 43% at $\frac{1}{2}$ load, and only 22% at $\frac{1}{4}$ load.

SIZES OF ELECTRIC GENERATORS AND MOTORS.

(Condensed from Bulletins of the General Electric Co., 1910.)

Direct-connected Engine-driven Railway Generators. Form S.

| | | | | | |
|---|--------------------------------------|-----|-----|-----|-----|
| 6-pole, Kw..... | 100 | 150 | 200 | 200 | 200 |
| Speed, r.p.m..... | 275 | 200 | 200 | 150 | 120 |
| 8-pole, 300 Kw., 120 and 100 r.p.m.; 400 Kw., 150, 120 and 100 r.p.m.; 500 Kw., 120 r.p.m. | | | | | |
| 10-pole, 500 Kw., 100 and 90 r.p.m. | 14-pole, 800 Kw., 100 and 80 r.p.m.; | | | | |
| 14-pole, 1000 Kw., 100 and 80 r.p.m.; 1200-Kw., 80 r.p.m., 16-pole, 1600 Kw., 100 and 75 r.p.m. | | | | | |
| 20-pole, 2000 Kw., 75 r.p.m.; 24-pole, 2500 Kw., 75 r.p.m.; 26-pole, 2700 Kw., 90 r.p.m. | | | | | |

Slow and Moderate Speed Belt-driven Generators. Type CL.

Form B.

| | | | | | | |
|-------------------|----------------------------|------|------|-----|-----|-----|
| Slow Speed | 6 poles, Kw. | 16 | 22 | 22 | 30 | 40 |
| | Speed, 125 and 250 volts | 750 | 900 | 725 | 700 | 650 |
| | Speed, 500 volts | 815 | 850 | 725 | 700 | 650 |
| | 6-poles, Kw. | 55 | 75 | 100 | 150 | |
| | Speed, 125 and 250 volts | 625 | 550 | 550 | 550 | |
| Moderate Speed | Speed, 500 volts | 625 | 550 | 525 | 455 | |
| | 6 poles, Kw..... | 25 | 35 | 45 | 60 | 75 |
| | Speed, 125, 250 and 500 v. | 1100 | 1050 | 975 | 925 | 850 |

Slow and Moderate Speed Belt-driven Motors. Type CL. Form B.

| | | | | | | |
|----------------|------------------------------------|------|-----|-----|-----|-------------|
| Slow Speed | 6 poles, Kw..... | 20 | 25 | 25 | 35 | 50 |
| | 125 and 250 volts, speed. | 690 | 785 | 675 | 650 | 600 |
| | 110 and 220 volts, speed. | 650 | 730 | 635 | 610 | 560 |
| | 500 volts, speed..... | 750 | 800 | 675 | 650 | 600 |
| | 6 poles, Kw..... | 65 | 90 | 125 | 185 | |
| | 125 and 250 volts, speed. | 575 | 500 | 470 | 440 | |
| Moderate Speed | 110 and 220 volts, speed. | 540 | 470 | 440 | 410 | |
| | 500 volts, speed..... | 575 | 500 | 500 | 430 | |
| | 6 poles, Kw..... | 30 | 40 | 55 | 70 | 85 105 150 |
| | 125, 250 and 500 volts, speed..... | 1025 | 975 | 900 | 850 | 800 700 ... |
| | 110 and 220 volts, speed | 965 | 915 | 845 | 800 | 750 655 490 |

After a continuous run of 10 hours, at full-rated load, the rise in temperature above that of the surrounding air, as measured by the thermometer, will not exceed the following: Armature, 35° C.; Commutator, 40° C.; Field, 45° C. The motors will operate for two hours at 25% overload, and withstand a momentary overload of 50% without injurious heating.

Belt-driven Alternators. Form P. Revolving Field.

| | | | | | | |
|-------------------------------------|------|------|------|-----|------|-----|
| Poles..... | 6 | 6 | 8 | 8 | 12 | 12 |
| Kw..... | 30 | 50 | 75 | 100 | 150 | 200 |
| Speed..... | 1200 | 1200 | 900 | 900 | 600 | 600 |
| Amperes at { balanced 3-phase load | 7.5 | 12.5 | 18.8 | 25 | 37.5 | 50 |
| full load { balanced 2-phase load | 6.5 | 11 | 16.5 | 22 | 33 | 44 |
| 2300 volts { single-phase load | 10 | 16.5 | 24.5 | 33 | 49 | 65 |

Built with or without direct-connected exciters. Adapted to 2- or 3-phase windings without change except in the armature coils. Potentials, 3-phase, 240, 480, 600, 1150, 2300; 2-phase, 240, 480, 1150, 2300. When used as synchronous motors these machines have a condenser effect, and in consequence can be used to improve the power factor when used in combination with induction motors.

The full-load single-phase rating at 100% power factor is 80% of the full-load 3-phase rating at both 100% and 80% power factor. The full-load single-phase rating at any power factor from 100 to 80% is the unity power factor single-phase rating multiplied by the power factor. For instance, for the 8-100-900 machine, which is the full-load 3-phase rating unity and 80% power factor, the single-phase rating for 100% at both power factors is 80 Kw., and for 80% power factor it is $80 \times 0.8 = 64$ Kw.

Slow and Moderate Speed Machines with Commutating Poles.**GENERATORS, TYPE DLC, FORM A.**

| Slow Speed. | | | | | | Moderate Speed. | | | |
|-------------|--------|-----|------------------|--------|--------|-----------------|------------------|--------|--------|
| Frame. | Poles. | Kw. | Speed. | | | Kw. | Speed. | | |
| | | | 125 v. 250 v. | 500 v. | 575 v. | | 125 v. 250 v. | 500 v. | 575 v. |
| 1 | 4 | 20 | 950 | 950 | 1050 | 30 | 1300 | 1300 | 1425 |
| 2 | 4 | 25 | 900 | 900 | 1000 | 40 | 1200 | 1200 | 1325 |
| 3 | 4 | 35 | 850 | 850 | 950 | 50 | 1150 | 1150 | 1250 |
| 4 | 6 | 45 | 775 | 775 | 850 | 65 | 1100 | 1100 | 1200 |
| 5 | 6 | 60 | 750 | 750 | 825 | 80 | 1050 | 1050 | 1150 |
| 6 | 6 | 75 | 700 | 700 | 775 | 100 | 1000 | 1000 | 1000 |
| 7 | 6 | 100 | 675 | 675 | 750 | 125 | 950 | 950 | 1050 |
| 8 | 6 | 125 | 650 | 650 | 700 | 150 | 900 | 875 | 900 |
| 9 | 6 | 150 | 600 | 600 | 650 | 200 | *850 | 775 | 850 |
| 10 | 6 | 200 | *500 | 500 | 550 | 300 | *750 | 700 | 750 |

* Not to be made for 125 volts.

MOTORS, TYPE DLC.

| Slow Speed. | | | | | | Moderate Speed. | | | |
|-------------|-------|------|------------------|------------------|--------|-----------------|------------------|------------------|--------|
| Frame | Poles | H.P. | Speed. | | | H.P. | Speed. | | |
| | | | 125 v. 250 v. | 115 v. 230 v. | 550 v. | | 125 v. 250 v. | 115 v. 230 v. | 550 v. |
| 1 | 4 | 20 | 825 | 800 | 925 | 30 | 1150 | 1100 | 1250 |
| 2 | 4 | 25 | 775 | 750 | 875 | 40 | 1100 | 1050 | 1200 |
| 3 | 4 | 35 | 725 | 700 | 825 | 55 | 1050 | 1000 | 1150 |
| 4 | 6 | 50 | 675 | 650 | 750 | 70 | 1000 | 950 | 1100 |
| 5 | 6 | 65 | 650 | 625 | 700 | 90 | 950 | 900 | 1025 |
| 6 | 6 | 80 | 625 | 600 | 675 | 115 | 900 | 850 | 975 |
| 7 | 6 | 100 | 600 | 575 | 650 | 150 | 825 | 800 | 925 |
| 8 | 6 | 125 | 575 | 550 | 625 | 175 | 775 | 750 | 850 |
| 9 | 6 | 175 | 525 | 500 | 575 | 250 | *725 | 700 | 750 |
| 10 | 6 | 250 | *450 | 425 | 500 | 350 | *675 | 650 | 675 |

* Not to be made for 125 or 115 volts.

The first eight sizes are made with enclosed and partly enclosed as well as open casings. For the several types of casings the horse-powers are as below:

| H.P., Slow Speed. | | | | | H.P., Moderate Speed. | | | | |
|-------------------|------|-------------------------|-----------------------------------|--------------------------------|-----------------------|------|-------------------------|-----------------------------------|---------------------------|
| Frame | Open | Semi- En- closed. | En- closed Venti- lated. | Totally En- closed. | Frame | Open | Semi- En- closed. | En- closed Venti- lated. | Totally En- closed. |
| 1 | 20 | 20 | 20 | 10 | 1 | 30 | 30 | 30 | 15 |
| 2 | 25 | 25 | 25 | 12 ¹ / ₂ | 2 | 40 | 40 | 40 | 20 |
| 3 | 35 | 35 | 35 | 17 ¹ / ₂ | 3 | 55 | 55 | 55 | 27 |
| 4 | 50 | 50 | 50 | 25 | 4 | 70 | 70 | 70 | |
| 5 | 65 | 65 | 65 | 30 | 5 | 90 | 90 | 90 | |
| 6 | 80 | 80 | 80 | 40 | 6 | 125 | 125 | 125 | |
| 7 | 100 | 100 | 100 | 50 | 7 | 150 | 150 | 150 | |
| 8 | 125 | 125 | 125 | 60 | 8 | 175 | 175 | 175 | |

Small Moderate Speed Engine-driven Alternators.

| | | | | | |
|--|------|------|------|------|-----|
| Poles..... | 24 | 26 | 28 | 32 | 36 |
| Kw..... | 50 | 75 | 105 | 150 | 240 |
| Speed..... | 300 | 276 | 257 | 225 | 200 |
| Amperes at { balanced 3-phase load. | 12.6 | 17.6 | 26.5 | 37.6 | 60 |
| full load { balanced 2-phase load. | 10.8 | 15.2 | 23 | 33 | 52 |
| 2300 volts { single-phase load..... | 15 | 21 | 32 | 45 | 73 |
| Potentials, 3-phase, 240, 480, 600, 1150, 2300; 2-phase, 240, 480, 1150, 2300. | | | | | |

Box-Frame Type of Railway Motors. Four Field Coils.

H.P., 18, 42, 45, 75, 50, 75, 100, 125, 160, 170, 200, 225.

The first two sizes are for 24-in. gauge, the next two for 36-in., and the others for standard gauge.

Commutating Pole Railway Motors.

Made in six sizes 50 to 200 H.P. Wound for 600 volts. The two smallest have split frames; the others box frames.

The commutating poles, located between the main exciting pole pieces, are connected up with their windings in series with one another and with the armature. The magnetic strength of the commutating poles varies

therefore with the current through the armature, and a magnetic field is produced of such intensity as to properly reverse the current in the armature coils short-circuited during commutation. The pole pieces are so proportioned and wound as to compensate for armature reaction, and practically non-flashing and sparkless commutation is insured up to the severest overloads. As the magnetizing current around the commutating poles is reversed with the armature, the poles perform their functions equally well in whichever direction the motors are running.

Due to the good commutating characteristics of commutating pole railway motors, their overload capacities are considerably increased, and a more rugged form of motor is obtained which is less subject to injury through careless handling by motormen than the present standard railway motor.

Small Polyphase Motors.

60-cycle, 4-pole, 1800 r.p.m., H.P., $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3, 5, $7\frac{1}{2}$, 10, 15.

60-cycle, 4-pole, 1200 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, 3, 5, $7\frac{1}{2}$.

60-cycle, 8-pole, 900 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, 2, 3, 5. 12-pole, 600 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, 2, 3.

40-cycle, 4-pole, 1200 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 5. 6-pole, 800 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3.

25-cycle, 2-pole, 1500 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3, 5, $7\frac{1}{2}$.

25-cycle, 4-pole, 750 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2, 3, 5. 6-pole, 500 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3.

The speeds given are synchronous speeds. Full-load speeds are from 93 to 97% of the synchronous.

Motors below 1 H.P. are adapted for 110 and 220 volts; others for 110, 220, 440 and 550 volts.

Single-phase Motors, 110 and 220 volts.

60-cycle, 4-pole, 1800 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3, 5, $7\frac{1}{2}$, 10, 15.

60-cycle, 6-pole, 1200 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 5, $7\frac{1}{2}$, 10.

25-cycle, 2-pole, 1500 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 2, 3, 5, $7\frac{1}{2}$, 10.

25-cycle, 4-pole, 750 r.p.m., H.P., $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 5.

• Type CQ Motors. Continuous Current.

| Type and Class. | No. of Poles | H.P. | * Speed (Shunt-Wound Motors). | | | | | | | | |
|-----------------|--------------|-------|-------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | | 110 v. | 115 v. | 125 v. | 220 v. | 230 v. | 250 v. | 500 v. | 550 v. | 600 v. |
| CQ 1/6 | 2 | 1/6 | 2200 | 2300 | 2450 | 2200 | 2300 | 2450 | | | |
| CQ 1/4 | 2 | 1/4 | 1800 | 1850 | 1950 | 1800 | 1850 | 1950 | 2100 | 2250 | 2400 |
| CQ 1/2 | 2 | 1/2 | 1600 | 1650 | 1750 | 1600 | 1650 | 1750 | 1850 | 2000 | 2150 |
| CQ 3/4 | 2 | 3/4 | 1425 | 1475 | 1550 | 1425 | 1475 | 1550 | 1675 | 1800 | 1925 |
| | | 1 | 1935 | 2000 | 2100 | 1935 | 2000 | 2100 | 2240 | 2400 | 2575 |
| CQ 1 | 2 | 1 | 1240 | 1275 | 1350 | 1240 | 1275 | 1350 | 1450 | 1575 | 1700 |
| | | 2 | 1825 | 1900 | 2090 | 1825 | 1900 | 2050 | 2200 | 2350 | 2500 |
| CQ 2 | 2 | 2 | 1060 | 1100 | 1175 | 1060 | 1100 | 1175 | 1250 | 1350 | 1450 |
| | | 3 | 1600 | 1650 | 1750 | 1600 | 1650 | 1750 | 1850 | 2000 | 2150 |
| CQ 3 | 2 | 3 | 1060 | 1100 | 1175 | 1060 | 1100 | 1175 | 1250 | 1350 | 1450 |
| | | 5 | 1600 | 1650 | 1750 | 1600 | 1650 | 1750 | 1850 | 2000 | 2150 |
| CQ 5 | 2 | 5 | 1060 | 1100 | 1175 | 1060 | 1100 | 1175 | 1250 | 1350 | 1450 |
| | | 7 1/2 | 1475 | 1525 | 1625 | 1475 | 1525 | 1625 | 1725 | 1850 | 1975 |
| CQ 7 1/2 | 4 | 7 1/2 | 800 | 825 | 875 | 800 | 825 | 875 | 1050 | 1125 | 1200 |
| | | 10 | 1220 | 1250 | 1310 | 1220 | 1250 | 1310 | 1400 | 1500 | 1600 |
| CQ 10 | 4 | 10 | 635 | 650 | 685 | 635 | 650 | 685 | 835 | 900 | 965 |
| | | 15 | 975 | 1000 | 1050 | 975 | 1000 | 1050 | 1250 | 1350 | 1450 |
| CQ 15 | 4 | 15 | 610 | 625 | 660 | 610 | 625 | 660 | 775 | 835 | 890 |
| | | 20 | 900 | 925 | 975 | 900 | 925 | 975 | 1150 | 1250 | 1350 |

* Speed at full load is subject to a maximum variation of 4% above or below standard.

The standard CQ open motor will deliver its rated horse-power output continuously without a temperature rise in any part exceeding 45°C . by the thermometer above the surrounding air. An overload of 25% may be maintained for *one hour* continuously without injurious heating or sparking, or a 40% overload *momentarily*.

Motors developed from the CQ1 frame and smaller will operate semi or totally enclosed within the same load limits as when open. Owing to the fact that the CQ2 and larger frames have less radiating surface per horse-power than the smaller frames, the ratings attainable with them when enclosed are necessarily reduced to keep the heating within established limits.

The voltages for which standard motors are built are 115, 230 and 550. When motors are rated at 115 volts, they may be used on circuits ranging between 110 and 125 volts, and when rated at 230 volts, they may be used on circuits ranging between 220 and 250 volts, and standard heating guarantees will be maintained.

When motors are rated at 550 volts, they may be used on circuits ranging between 500 and 600 volts, inclusive, and standard heating guarantees will be maintained up to 550 volts, and at 600 volts the heating will not be injurious.

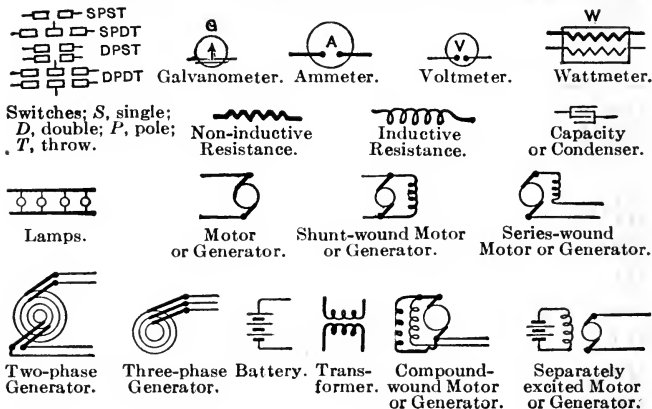
Sewing-Machine Motors.

Ratings, H.P., $\frac{1}{30}$, $\frac{1}{15}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{6}$.
Speed, r.p.m., 1800, 1800, 1500, 1800, 2300, for direct current; alternating current 1800 r.p.m. for all sizes.

Wound for 115 and 230 volts, D.C., and 110 and 220 volts, A.C., 60 cycles.

On special order, machines may be furnished for any commercial voltage between 50 and 250, and for any standard frequency between 25 and 145 cycles.

SYMBOLS USED IN ELECTRICAL DIAGRAMS.



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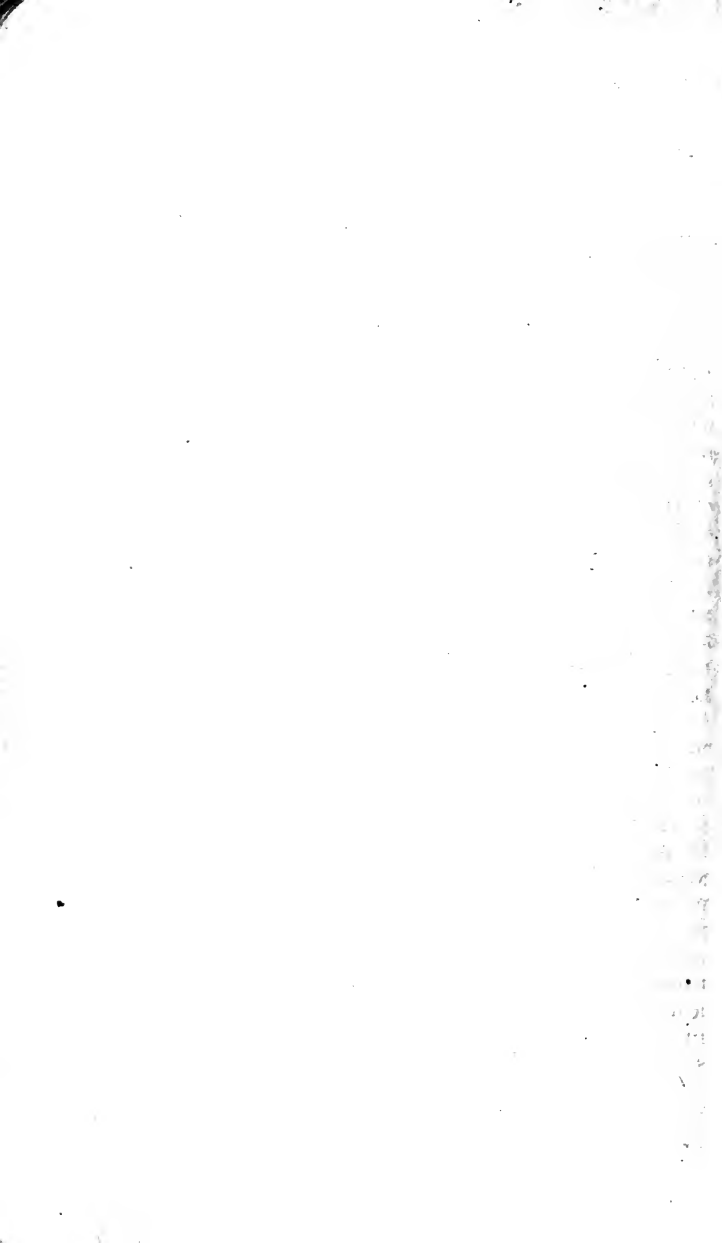
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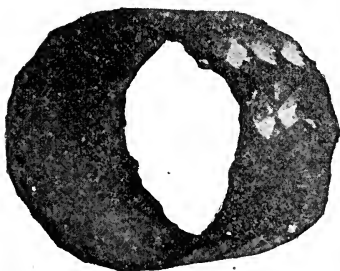
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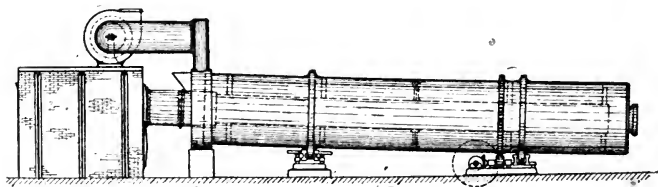
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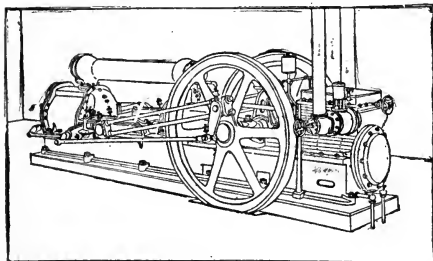
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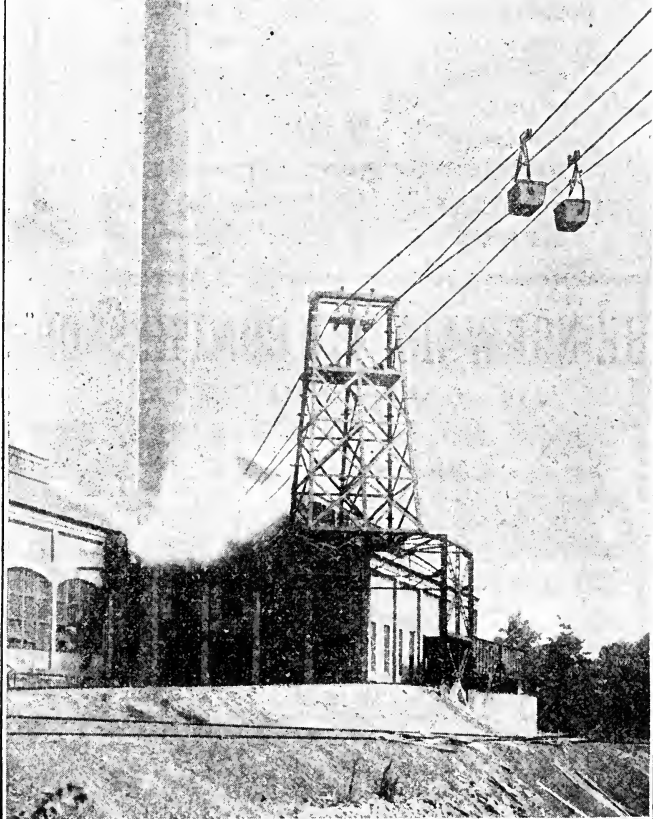
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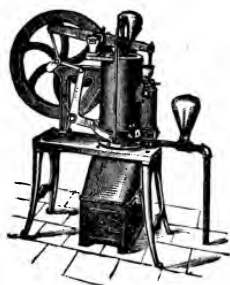
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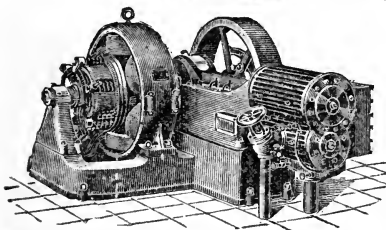
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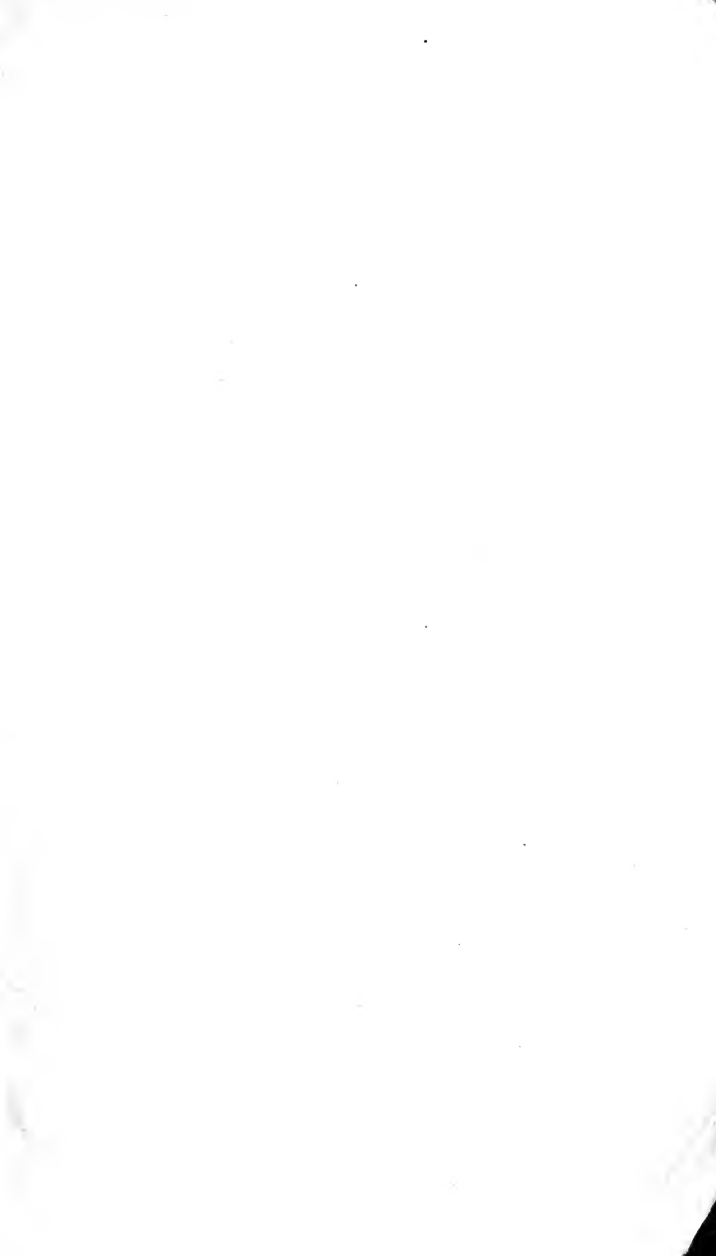
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